Amplify Desmos Math
FLORIDA

Student Edition

A1

Volume 2

Amplify Desmos Math FLORIDA

Algebra 1

Volume 2: Units 5–7

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

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Dear Student,

Welcome to Amplify Desmos Math Florida! We are excited to be partnering with you this year. You play an essential role in math class, so we wanted to reach out to introduce ourselves and tell you a bit about who we are.

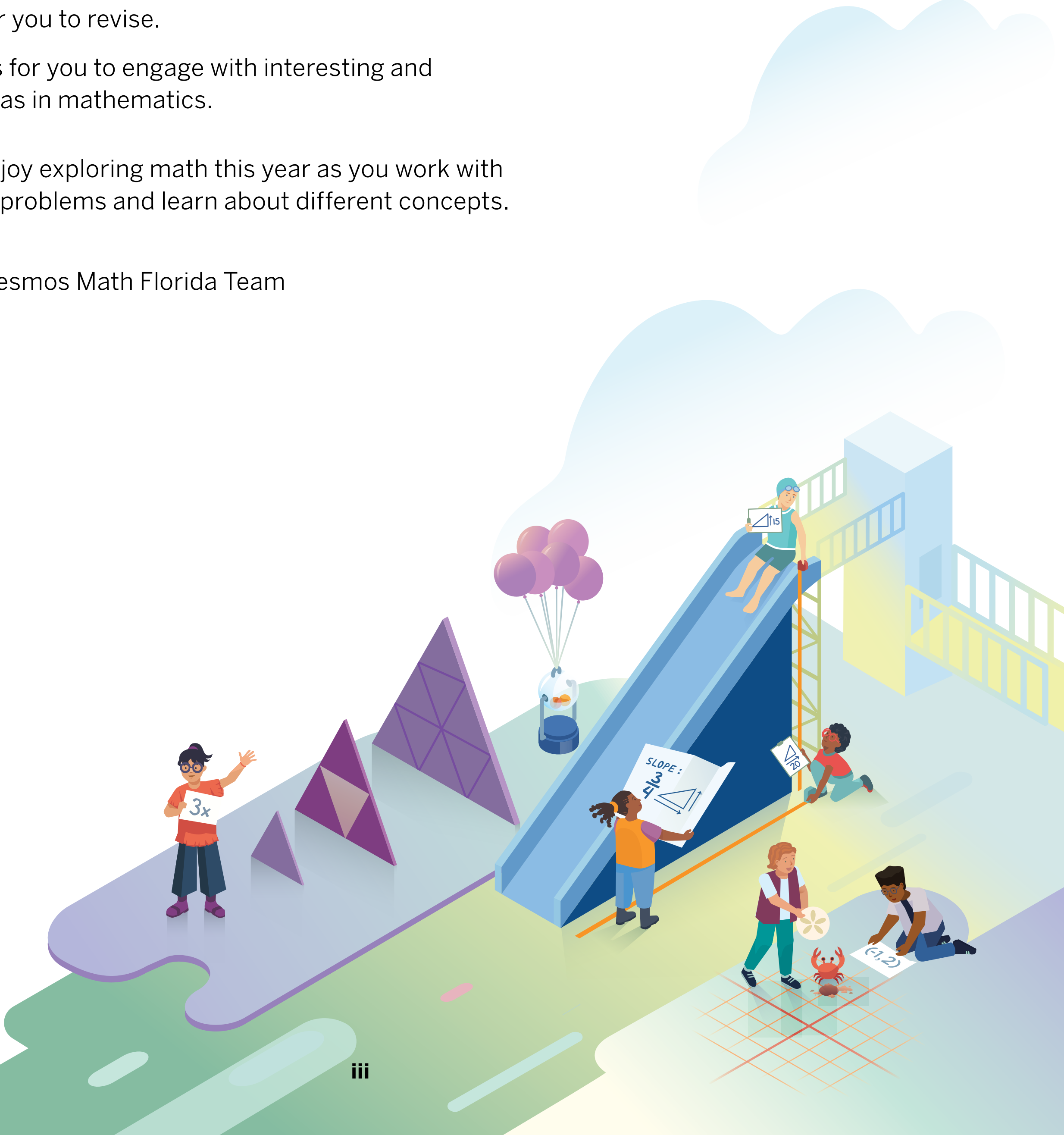
Amplify Desmos Math Florida is a team of math educators on a mission to support you and your classmates in learning math. We hope each lesson inspires you to use your creativity, ask questions, and discover connections between math concepts and the world around us.

Here is what you can expect this year:

- Lessons that encourage you to ask questions, explore, settle disputes, create challenges for your classmates, and more!
- Activities that show what your ideas mean with plenty of chances for you to revise.
- Opportunities for you to engage with interesting and important ideas in mathematics.

We hope you enjoy exploring math this year as you work with friends to solve problems and learn about different concepts.

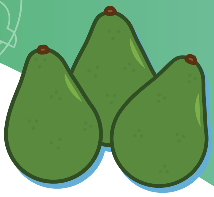
–The Amplify Desmos Math Florida Team

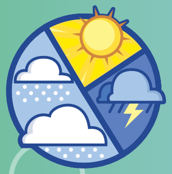


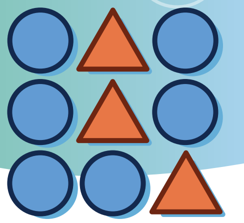
Unit 1 Linear Equations and Inequalities

You will revisit strategies for solving one-variable equations and inequalities and extend your knowledge to make sense of multi-variable equations and two-variable inequalities.

Unit 2 Describing Data



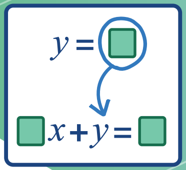


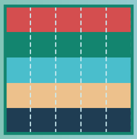


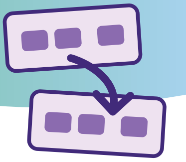
You will analyze, describe, and compare one- and two-variable data sets.

Unit 3 Describing Functions

You will model situations with functions and use function notation to describe key features of functions, compare different functions, and define functions.





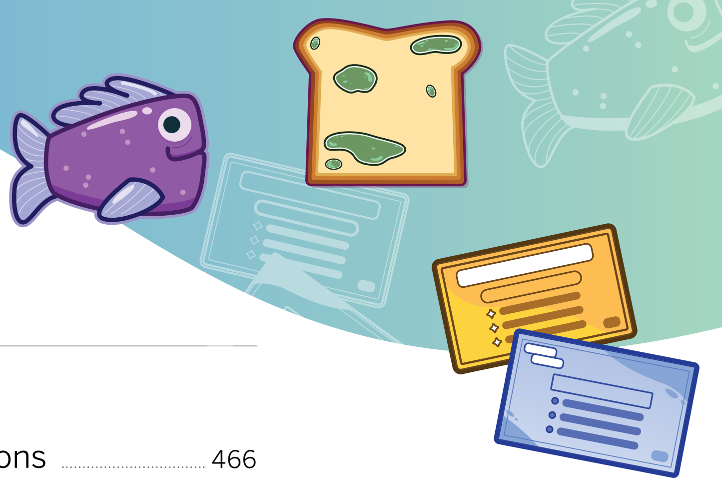


Unit 4 Systems of Linear Equations and Inequalities

You will write and solve systems of linear equations and inequalities, interpreting their solutions in context.

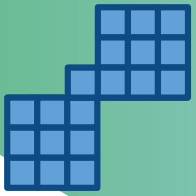
Sub-Unit 1 Systems of Equations 386

Sub-Unit 2 Systems of Inequalities 434



Unit 5 Exponential Functions

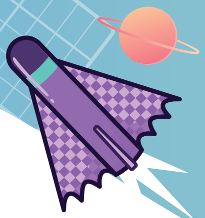
You will create and interpret exponential functions to model situations and data.

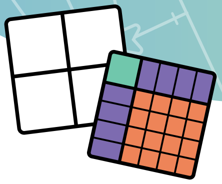


Unit 6 Quadratic Functions

You will analyze graphs, tables, and equations in three forms to identify and interpret key features of quadratic functions.

Unit 7 Quadratic Equations





You will solve quadratic equations and systems of equations using reasoning, factoring, the zero-product property, completing the square, and the quadratic formula.

Florida B.E.S.T. Standards and Benchmarks for Algebra 1

Benchmark	B.E.S.T Mathematics Benchmark
Number Sense and Operations	
MA.912.NSO.1.1	Extend previous understanding of the Laws of Exponents to include rational exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions involving rational exponents.
MA.912.NSO.1.2	Generate equivalent algebraic expressions using the properties of exponents.
MA.912.NSO.1.4	Apply previous understanding of operations with rational numbers to add, subtract, multiply and divide numerical radicals.
Algebraic Reasoning	
MA.912.AR.1.1	Identify and interpret parts of an equation or expression that represent a quantity in terms of a mathematical or real-world context, including viewing one or more of its parts as a single entity.
MA.912.AR.1.2	Rearrange equations or formulas to isolate a quantity of interest.
MA.912.AR.1.3	Add, subtract and multiply polynomial expressions with rational number coefficients.
MA.912.AR.1.4	Divide a polynomial expression by a monomial expression with rational number coefficients.
MA.912.AR.1.7	Rewrite a polynomial expression as a product of polynomials over the real number system.
MA.912.AR.2.1	Given a real-world context, write and solve one-variable multi-step linear equations.
MA.912.AR.2.2	Write a linear two-variable equation to represent the relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.
MA.912.AR.2.3	Write a linear two-variable equation for a line that is parallel or perpendicular to a given line and goes through a given point.
MA.912.AR.2.4	Given a table, equation or written description of a linear function, graph that function, and determine and interpret its key features.
MA.912.AR.2.5	Solve and graph mathematical and real-world problems that are modeled with linear functions. Interpret key features and determine constraints in terms of the context.
MA.912.AR.2.6	Given a mathematical or real-world context, write and solve one-variable linear inequalities, including compound inequalities. Represent solutions algebraically or graphically.
MA.912.AR.2.7	Write two-variable linear inequalities to represent relationships between quantities from a graph or a written description within a mathematical or real-world context.
MA.912.AR.2.8	Given a mathematical or real-world context, graph the solution set to a two-variable linear inequality.
MA.912.AR.3.1	Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real number system.
MA.912.AR.3.4	Write a quadratic function to represent the relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.
MA.912.AR.3.5	Given the x-intercepts and another point on the graph of a quadratic function, write the equation for the function.
MA.912.AR.3.6	Given an expression or equation representing a quadratic function, determine the vertex and zeros and interpret them in terms of a real-world context.

Florida B.E.S.T. Standards and Benchmarks for Algebra 1

MA.912.AR.3.7	Given a table, equation or written description of a quadratic function, graph that function, and determine and interpret its key features.
MA.912.AR.3.8	Solve and graph mathematical and real-world problems that are modeled with quadratic functions. Interpret key features and determine constraints in terms of the context.
MA.912.AR.4.1	Given a mathematical or real-world context, write and solve one-variable absolute value equations.
MA.912.AR.4.3	Given a table, equation or written description of an absolute value function, graph that function and determine its key features.
MA.912.AR.5.3	Given a mathematical or real-world context, classify an exponential function as representing growth or decay.
MA.912.AR.5.4	Write an exponential function to represent a relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.
MA.912.AR.5.6	Given a table, equation or written description of an exponential function, graph that function and determine its key features.
MA.912.AR.9.1	Given a mathematical or real-world context, write and solve a system of two-variable linear equations algebraically or graphically.
MA.912.AR.9.4	Graph the solution set of a system of two-variable linear inequalities.
MA.912.AR.9.6	Given a real-world context, represent constraints as systems of linear equations or inequalities. Interpret solutions to problems as viable or non-viable options.
Functions	
MA.912.F.1.1	Given an equation or graph that defines a function, determine the function type. Given an input-output table, determine a function type that could represent it.
MA.912.F.1.2	Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.
MA.912.F.1.3	Calculate and interpret the average rate of change of a real-world situation represented graphically, algebraically or in a table over a specified interval.
MA.912.F.1.5	Compare key features of linear functions each represented algebraically, graphically, in tables or written descriptions.
MA.912.F.1.6	Compare key features of linear and nonlinear functions each represented algebraically, graphically, in tables or written descriptions.
MA.912.F.1.8	Determine whether a linear, quadratic or exponential function best models a given real-world situation.
MA.912.F.2.1	Identify the effect on the graph or table of a given function after replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ for specific values of k.
Financial Literacy	
MA.912.FL.3.2	Solve real-world problems involving simple, compound and continuously compounded interest.
MA.912.FL.3.4	Explain the relationship between simple interest and linear growth. Explain the relationship between compound interest and exponential growth and the relationship between continuously compounded interest and exponential growth.

Florida B.E.S.T. Standards and Benchmarks for Algebra 1

Data Analysis & Probability	
MA.912.DP.1.1	Given a set of data, select an appropriate method to represent the data, depending on whether it is numerical or categorical data and on whether it is univariate or bivariate.
MA.912.DP.1.2	Interpret data distributions represented in various ways. State whether the data is numerical or categorical, whether it is univariate or bivariate and interpret the different components and quantities in the display.
MA.912.DP.1.3	Explain the difference between correlation and causation in the contexts of both numerical and categorical data.
MA.912.DP.1.4	Estimate a population total, mean or percentage using data from a sample survey; develop a margin of error through the use of simulation.
MA.912.DP.2.4	Fit a linear function to bivariate numerical data that suggests a linear association and interpret the slope and y-intercept of the model. Use the model to solve real-world problems in terms of the context of the data.
MA.912.DP.2.6	Given a scatter plot with a line of fit and residuals, determine the strength and direction of the correlation. Interpret strength and direction within a real-world context.
MA.912.DP.3.1	Construct a two-way frequency table summarizing bivariate categorical data. Interpret joint and marginal frequencies and determine possible associations in terms of a real-world context.

Mathematical Thinking and Reasoning Standards

MA.K12.MTR.1.1	Actively participate in effortful learning both individually and collectively.
MA.K12.MTR.2.1	Demonstrate understanding by representing problems in multiple ways.
MA.K12.MTR.3.1	Complete tasks with mathematical fluency.
MA.K12.MTR.4.1	Engage in discussions that reflect on the mathematical thinking of self and others.
MA.K12.MTR.5.1	Use patterns and structure to help understand and connect mathematical concepts.
MA.K12.MTR.6.1	Assess the reasonableness of solutions.
MA.K12.MTR.7.1	Apply mathematics to real-world contexts.

English Language Arts B.E.S.T. Standards

ELA.K12.EE.1.1	Cite evidence to explain and justify reasoning.
ELA.K12.EE.2.1	Read and comprehend grade-level complex texts proficiently.
ELA.K12.EE.3.1	Make inferences to support comprehension.
ELA.K12.EE.4.1	Use appropriate collaborative techniques and active listening skills when engaging in discussions in a variety of situations.
ELA.K12.EE.5.1	Use the accepted rules governing a specific format to create quality work.
ELA.K12.EE.6.1	Use appropriate voice and tone when speaking or writing.

English Language Development Standards

ELD.K12.ELL.MA.1	English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

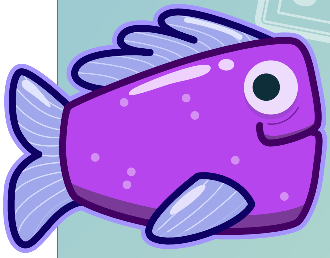
Unit 5

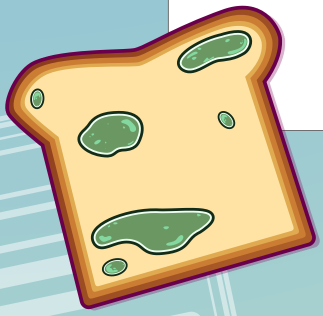
Exponential Functions

Sometimes a small change can make a big difference over time. In this unit, you will learn about relationships that grow quickly over time. You will learn to distinguish between linear and exponential relationships, and create equations to represent them. You will model situations that increase or decrease by a percentage. You will also model compound interest with exponential functions.

Essential Questions

- How do exponential and linear functions compare?
- What type of function models repeated percent increase or decrease?
- How can you model situations involving compound interest?

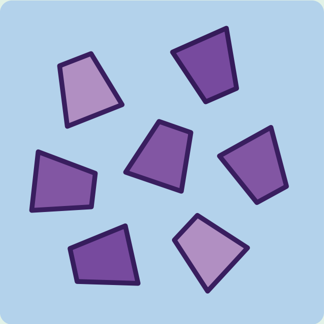




Sub-unit

1

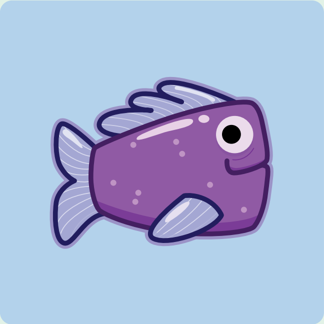
Comparing Linear and Exponential Functions



Lesson 1
Growing Globs

Lesson 2
Going Viral

Lesson 3
Return of the Globs



Lesson 4
Carlos and Corals

Lesson 5
Differences and Factors

Unit 5

Lesson

1

Name: .... Date: .... Period: ....

MA.912.F.1.1, MA.912.F.1.6, MA.912.F.1.8, MTR.4.1, MTR.5.1, MTR.7.1

Growing Globs

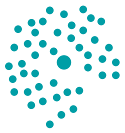
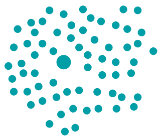
Let's identify and compare two different patterns of growth.

Warm-Up

1. The images to the right show how the number of teal globs grow over the course of 3 days.

a Watch how the number of globs grows.

Write a story about these globs.

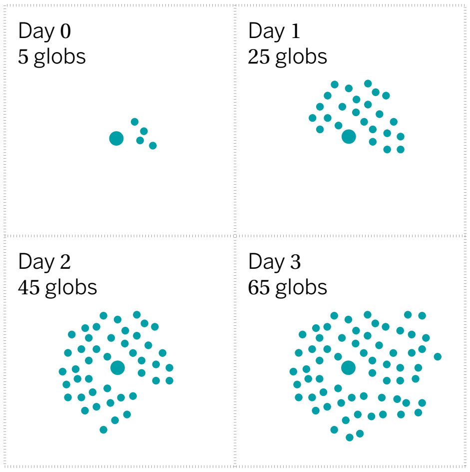
Day 0 5 globs 	Day 1 25 globs 
Day 2 45 globs 	Day 3 65 globs 

Name: .... Date: .... Period: ....

Purple vs. Teal

2. How many teal globs will there on day 4?

Day	Teal Globs
0	5
1	25
2	45
3	65
4	



3. Here is a new group of globs.

How many purple globs will there be on day 4?

Day	Purple Globs
0	2
1	6
2	18
3	54
4	

4. The graph shows the number of each type of glob for the first 3 days.

Which do you think there will be more of on day 10? Circle one.

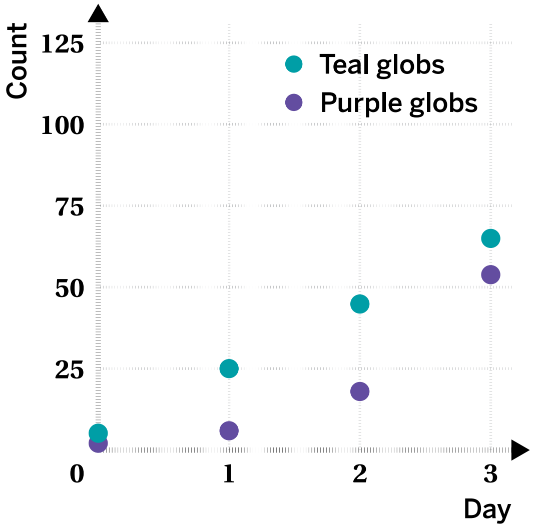
Teal Globs

Purple Globs

There will be the same

Not enough information

Explain your thinking.

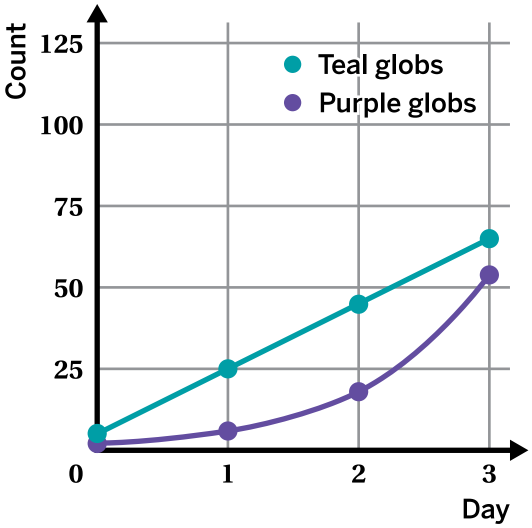


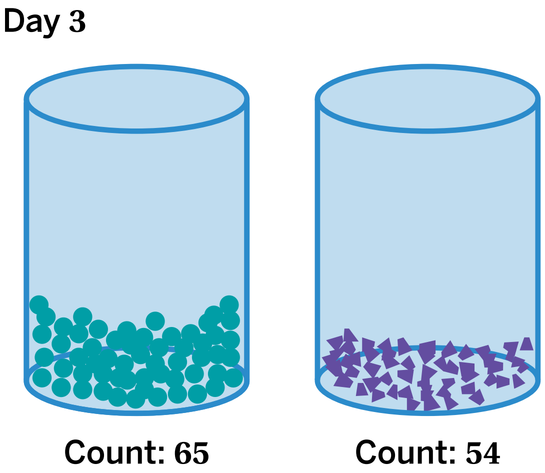
Purple vs. Teal (continued)

5. The graph below shows how the number of teal and purple globs grow over the course of three days.

a What do you notice? What do you wonder?

b Share your observations with a partner.



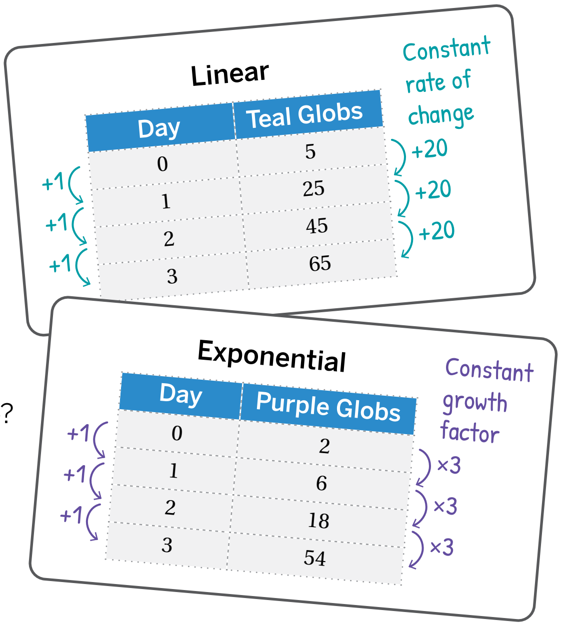


6. Here are tables for the teal and purple growing globs.

- Teal globs grow are modeled by a **linear function** and have a constant *rate of change*.
- Purple globs are modeled by an **exponential function** and have a constant **growth factor**.

Discuss:

- How are *rate of change* and *growth factor* alike?
- How are they different?



Linear

Constant rate of change

Day	Teal Globs
0	5
1	25
2	45
3	65

+1 +1 +1

+20 +20 +20

Exponential

Constant growth factor

Day	Purple Globs
0	2
1	6
2	18
3	54

+1 +1 +1

×3 ×3 ×3

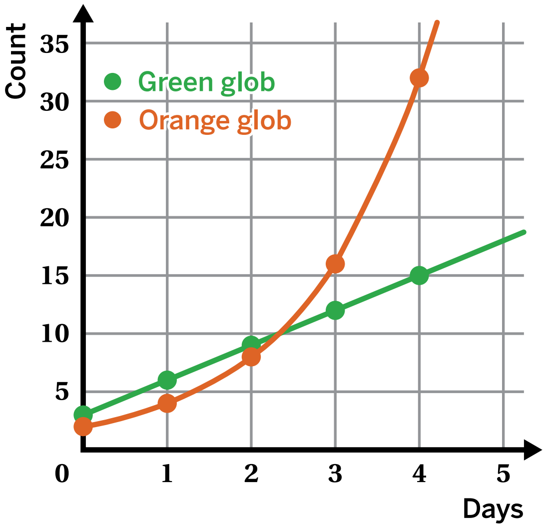
Name: .... Date: .... Period: ....

Comparing Growth

7. Let’s compare globs with different starting amounts and a constant rate of change or a constant growth factor.

Day	Green Globs
0	3
1	6
2	9
3	12
4	

Day	Green Globs
0	2
1	4
2	8
3	16
4	



Fabiana says: *Globs that grow by a constant growth factor will always eventually outnumber globs that grow by a constant rate of change*.

Lukas says: *If the constant rate of change is large enough, then this won't be true*.

Whose idea do you agree with? Circle one.

Fabiana’s Lukas’s Both Neither

Explain your thinking.

Name: .... Date: .... Period: ....

Comparing Growth (continued)

8. Group these cards by their function type.

Card A

x	y
0	2
1	4
2	6
3	8
4	10

Card B

x	y
0	0
1	1
2	4
3	9
4	16

Card C

x	y
0	1
1	2
2	4
3	8
4	16

Card D

x	y
0	0
1	4
2	8
3	12
4	16

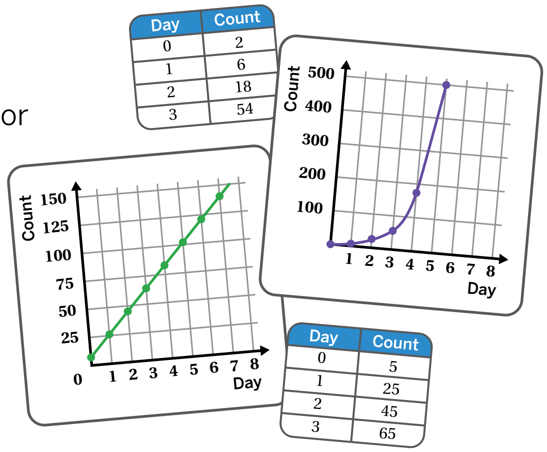
Linear	Exponential	Neither

Synthesis

9. Quantities that grow by a constant rate of change can be modeled by linear functions.

Quantities that grow by a constant growth factor can be modeled by exponential functions.

Describe strategies for determining whether a function is linear or exponential.



Lesson Practice 5.01

Lesson Summary

An **exponential function** increases or decreases by a *constant ratio*. Another name for the constant ratio is the **growth factor**.

A **linear function** increases or decreases by a *constant difference*. Another name for the constant difference is the *rate of change*.

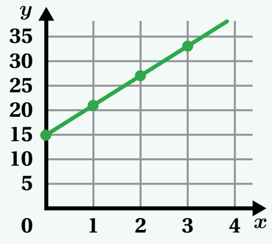
Here are two examples.

Linear Function

This pattern has a constant difference of 6, so the rate of change is 6.

The graph of this linear function is a straight line.

	x	y	
+1	0	15	+6
+1	1	21	+6
+1	2	27	+6
	3	33	

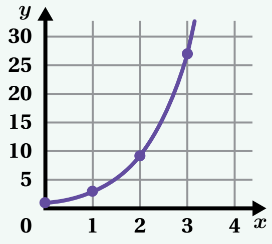


Exponential Function

This pattern has a constant ratio of 3, so the growth factor is 3.

The graph of this exponential function is a curve that gets steeper and steeper.

	x	y	
+1	0	1	×3
+1	1	3	×3
+1	2	9	×3
	3	27	



Lesson Practice
5.01

Name: .... Date: .... Period: ....

Problems 1–3: These tables show the number of red and yellow globs each day.

Day	0	1	2	3	4
Red Globs	50	70	90	110	

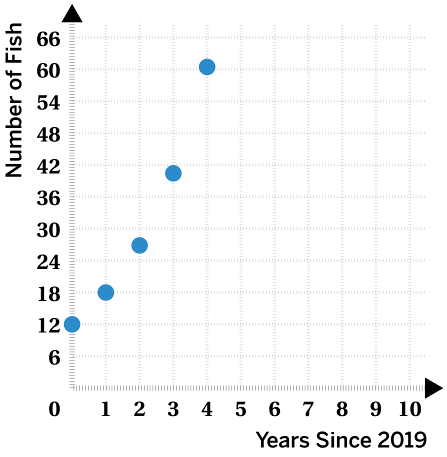
Day	0	1	2	3	4
Yellow Globs	5	10	20	40	

1. How many of each type of glob will there be on day 4?

2. Will there be more red or yellow globs on day 10? Show or explain your thinking.

3. Which group of globs changes by a constant growth factor? Show or explain how you know.

Problems 4–5: This graph shows the number of fish in a pond from 2019–2023.

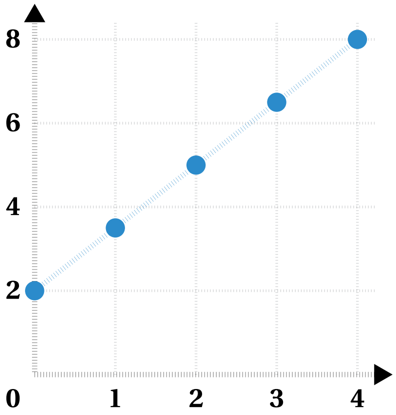


4. How many fish are in the pond in 2019?

5. Does the number of fish grow by a *constant difference?* Show or explain how you know.

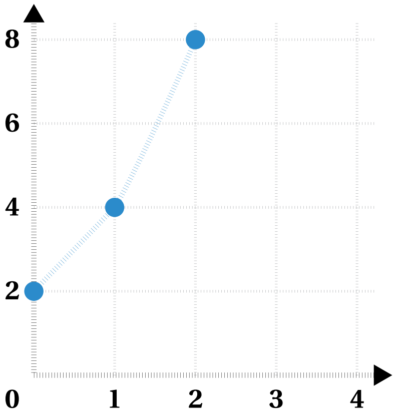
Problems 6–7: Determine whether each graph shows a constant rate of change or a constant growth rate. Circle your choice.

6.



Constant rate of change　　Constant growth rate

7.



Constant rate of change　　Constant growth rate

Lesson Practice
5.01

Name: ______ Date: ______ Period: ______

Test Practice

8. Determine whether each table shows a linear or exponential function.

Table A

x	y
0	4
1	8
2	16
3	32

Table B

x	y
0	4
1	8
2	12
3	16

A. Both represent exponential functions.
B. Both represent linear functions.
C. Table A is exponential and Table B is linear.
D. Table A is linear and Table B is exponential.

Spiral Review

9. Determine the value of each expression when $n = 4$.

a $n^2 - 5$

b $n(n + 6)$

c $3n^2$

10. Approximate the value of each radical expression.

a $\sqrt{10}$

Between ______ & ______

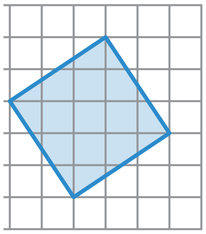
b $\sqrt{54}$

Between ______ & ______

c $\sqrt{41}$

Between ______ & ______

11. Each grid square represents 1 square unit. What is the *exact* side length of the shaded square?



Unit 5

Lesson

2

Name: .... Date: .... Period: ....

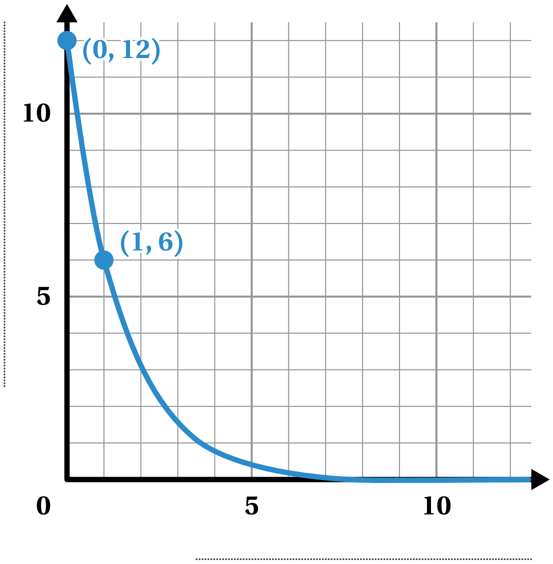
MA.912.AR.1.1, MA.912.AR.5.4, MA.912.F.1.6, MA.912.F.1.8, MTR.4.1, MTR.5.1, MTR.7.1

Going Viral

Let's describe connections between graphs and equations, and use graphs to write equations of exponential functions.

Warm-Up

1. Here is a graph of an exponential relationship.



a Label the axes with any quantities you'd like.

b Write a story about the quantities based on the graph.

Name: .... Date: .... Period: ....

Three Memes

2. The table below represents the number of likes for every hour after a kitty meme goes viral.

Hours since noon: 0:00

❤ 65

Time (hr)	Likes
0	65
1	130
2	260
3	520
4	1,040
5	2,080

What type of relationship is this?

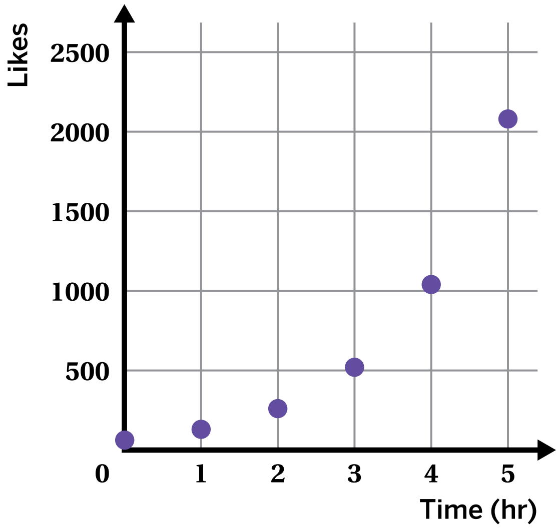
A. Linear

B. Exponential

C. Something else

3. There is an exponential relationship between the number of likes and the hours since noon.

Time (hr), x	Likes, $f(x)$
0	65
1	130
2	260
3	520
4	1,040
5	2,080



Use the formula $f(x) = a(b)^x$, where a represents the initial value and b represents the growth factor, to write an exponential function for this relationship.

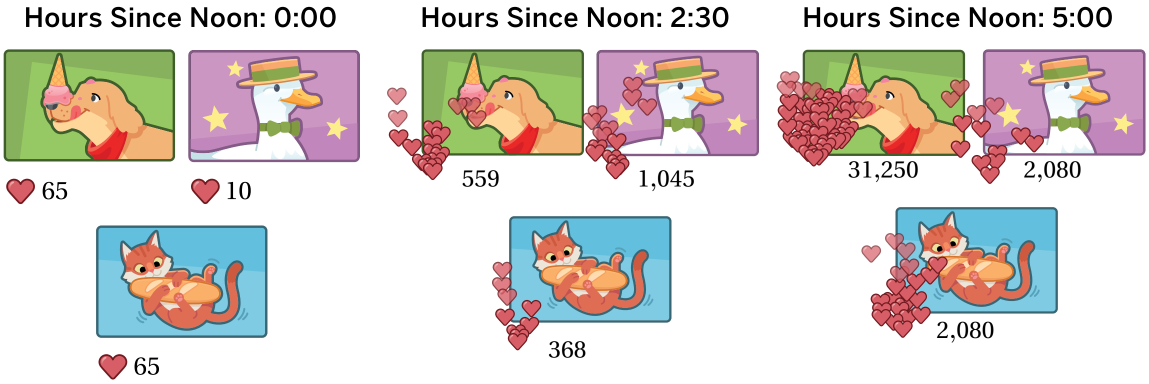
$f(x) =$ ....

Activity
1

Name: .... Date: .... Period: ....

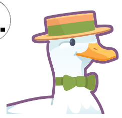
Three Memes (continued)

4. Let’s look at the likes for these memes at different times.

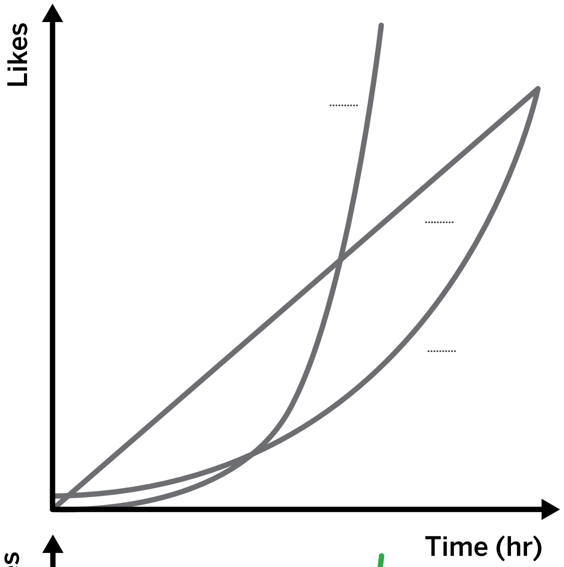


Match each meme to the graph that represents it.

Ⓐ


Ⓑ


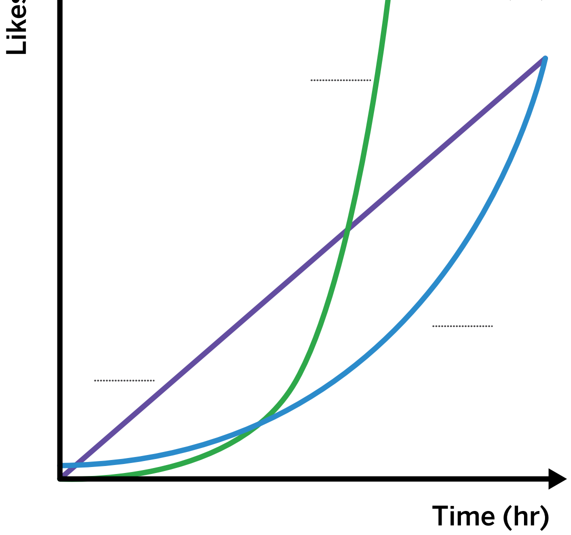
Ⓒ

5. Match each function to its graph.

- $f(x) = 10 + 414x$
- $g(x) = 65 \cdot (2)^x$
- $h(x) = 10 \cdot (5)^x$

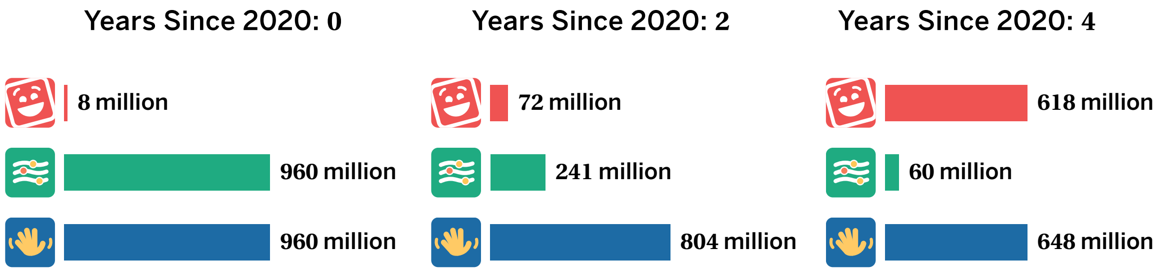
Explain your thinking.



Name: .... Date: .... Period: ....

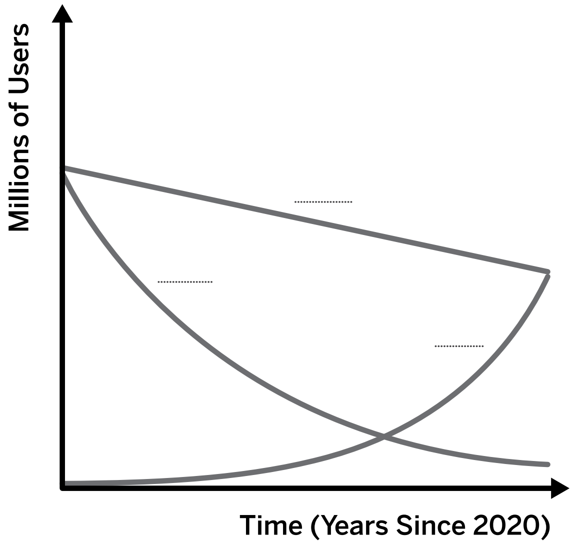
Take It Further

6. Let's look at the number of app users at different times.



Match each app to the graph that represents it.

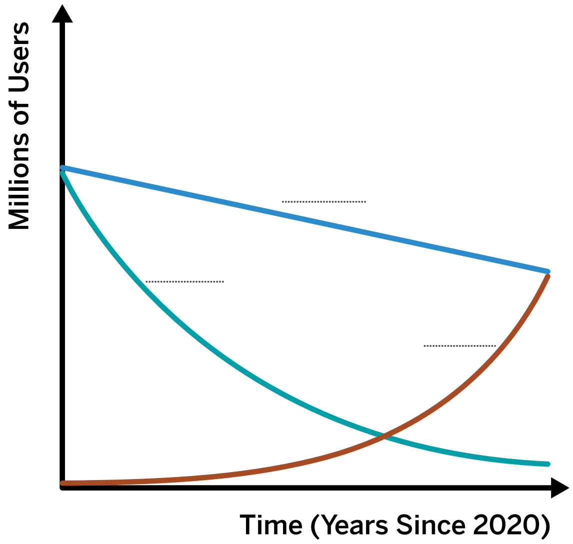
A. B. C.



7. Match each function to its graph.

- $m(x) = 960 - 78x$
- $n(x) = 960 \cdot \left(\frac{1}{2}\right)^x$
- $p(x) = 8 \cdot 3^x$

Explain your thinking.



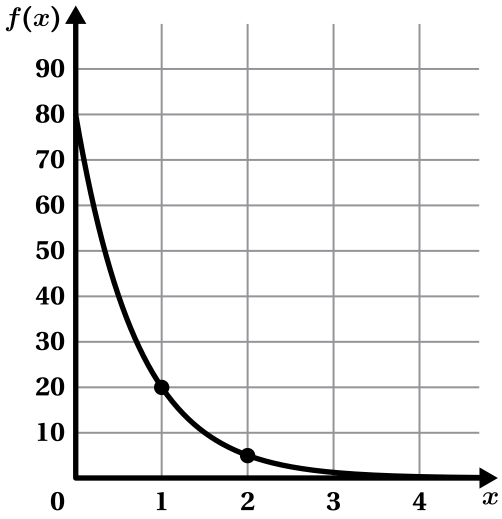
Name: .... Date: .... Period: ....

Take It Further (continued)

8. Here is the graph of $f(x) = 80 \cdot \left(\frac{1}{4}\right)^x$.

What might $g(x) = 80 \cdot \left(\frac{1}{2}\right)^x$ look like?

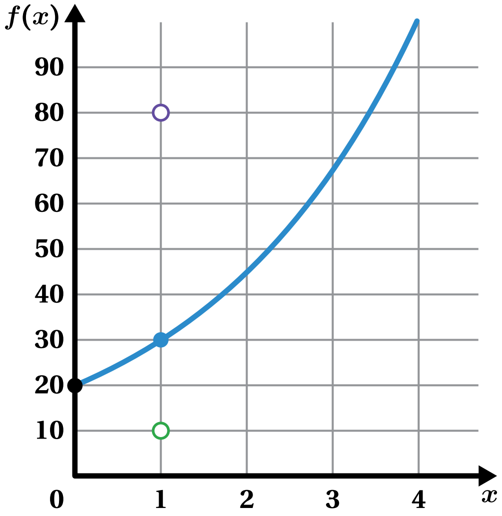
Show or explain your thinking.



9. Here are three different exponential relationships.

Each relationship includes the point $(0, 20)$ and one other point shown on the graph.

One function has been written for you. Write the other two functions.



	Includes the Point	Function
Graph 1	$(1, 80)$	$f(x) =$
Graph 2	$(1, 30)$	$g(x) = 20 \cdot \left(\frac{3}{2}\right)^x$
Graph 3	$(1, 10)$	$h(x) =$

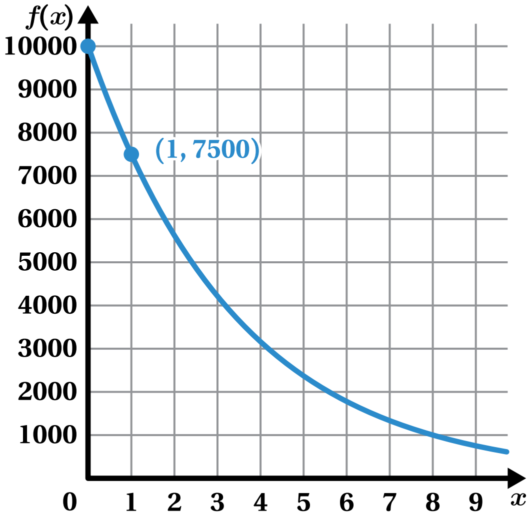
You're invited to explore more.

10. Use the You're Invited to Explore More Sheet to answer a question about a pattern.

Synthesis

11. Here is a graph of $f(x) = 10000 \cdot \left(\frac{3}{4}\right)^x$.

Explain where you can see $10{,}000$ and $\frac{3}{4}$ on the graph.



Lesson Practice 5.02

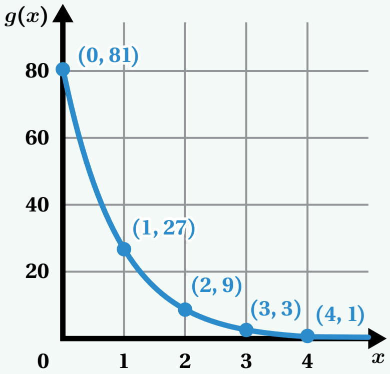
Lesson Summary

In the exponential function $f(x) = a \cdot b^x$, a represents the y-*intercept* and b represents the growth factor. Both parts can be seen on a graph.

Let's look at two examples.

Here is the graph of the exponential function $g(x) = 81 \cdot \left(\frac{1}{3}\right)^x$.

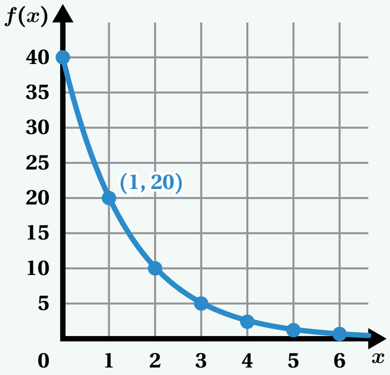
- 81 means that $(0, 81)$ is the y-intercept.
- $\frac{1}{3}$ is the growth factor. As x increases by 1, the y-values are multiplied by a factor of $\frac{1}{3}$. $81 \cdot \left(\frac{1}{3}\right) = 27$.



You can also write an exponential equation to represent a graph.

- The y-intercept is $(0, 40)$, so 40 is the a-value.
- The points $(1, 20)$, $(2, 10)$, and $(3, 5)$ are on the graph of the function. Since 10 is $\frac{1}{2}$ of 20 and 5 is $\frac{1}{2}$ of 10, this function has a growth factor of $\frac{1}{2}$, which is the b-value in the equation.

One way to write the equation of this exponential function is $f(x) = 40 \cdot \left(\frac{1}{2}\right)^x$.



Lesson Practice
5.02

Name: .... Date: .... Period: ....

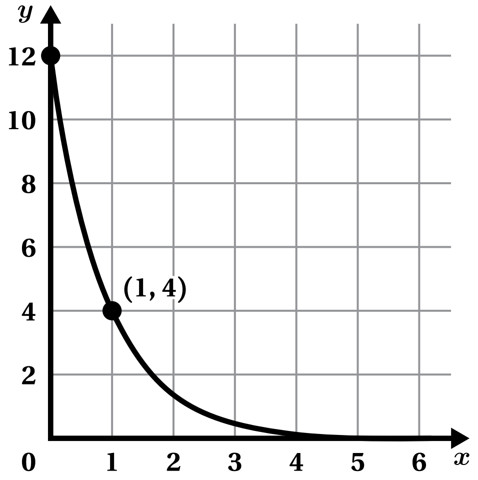
Problems 1–3: Determine the value of each expression when $x = 2$.

1. 4^x

2. $\left(\frac{1}{3}\right)^x$

3. $5(6^x)$

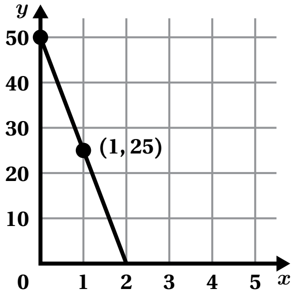
4. Here is a graph of $y = 12 \bullet \left(\frac{1}{3}\right)^x$. Explain where you can see the 12 and the $\frac{1}{3}$ in the graph.



Problems 5–7: Match each equation to the graph that represents it.

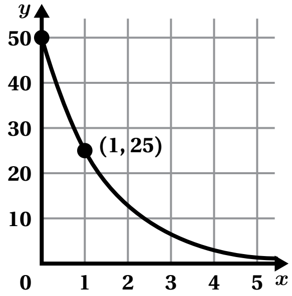
Equation A	Equation B	Equation C
$y = 50 \bullet \left(\frac{1}{2}\right)^x$	$y = 50 \bullet 2^x$	$y = 50 - 25x$

5.



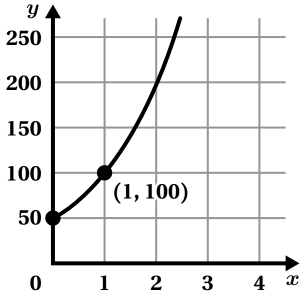
Equation ....

6.



Equation ....

7.



Equation ....

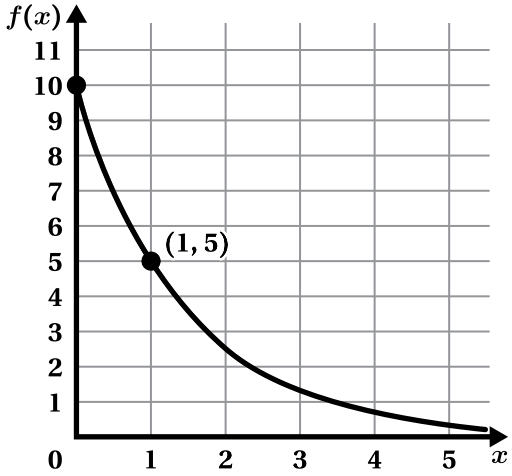
8. Explain how you determined which equation matches with which graph in Problems 5–7.

Lesson Practice
5.02

Name: .... Date: .... Period: ....

Test Practice

9. Which equation represents the exponential graph?



[✓ A.] $f(x) = 10 \bullet \left(\frac{1}{2}\right)^x$

B. $f(x) = 10 \bullet (2)^x$

C. $f(x) = 10 \bullet (x)^{\frac{1}{2}}$

D. $f(x) = 10 \bullet (x)^2$

Spiral Review

Problems 10–13: Rewrite each expression as a single power.

10. $4^4 \bullet 4^8$

11. $\frac{3^5}{3^8}$

12. $(12^3)^5$

13. $\frac{7^3 \bullet 7^4}{7^5}$

Unit 5
Lesson
3

Name: ______________ Date: ________ Period: ________

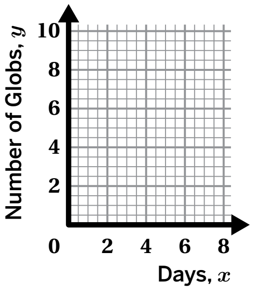
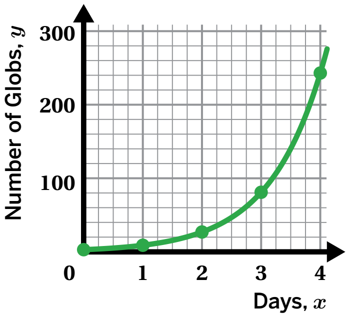
MA.912.AR.1.1, MA.912.F.1.6, MA.912.F.1.8, MA.912.AR.5.3, MA.912.AR.5.4, MA.912.AR.5.6, MTR.3.1, MTR.4.1, MTR.5.1, MTR.7.1

Return of the Globs

Let’s make connections between different representations of linear and exponential functions.

Warm-Up

1. Look at the different representations for how the green and orange globs are growing. Fill in the missing information.

	Green Globs		**Orange Globs**	
Situation	At first, there are 2 globs. 2 more are added each day.			
Table	**Days, x**	**Number of globs, y**	**Days, x**	**Number of globs, y**
	0	2	0	
	1	4	1	
	2	6	2	
	3	8	3	
	4	10	4	
Graph				
Equation			$y = 3 \cdot 3^x$	

2. **Discuss:** How are these globs growing?

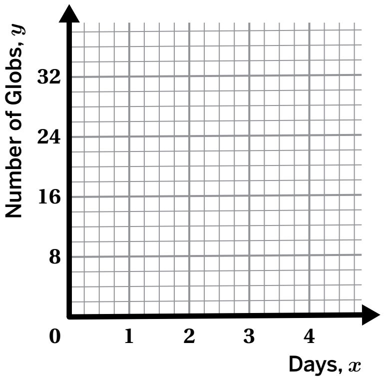
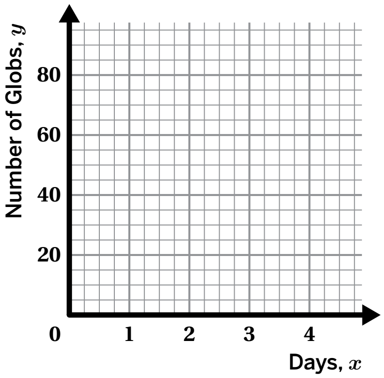
Activity

1

Name: ....   Date: ....   Period: ....

Return of the Globs

3. For each kind of glob, fill in the situation, table, graph, and equation.

<table>
<tr><th></th><th>Green Globs</th><th>Orange Globs</th></tr>
<tr><td>Situation</td><td>At first, there are 4 globs. 6 more globs are added each day.</td><td></td></tr>
<tr><td>Table</td><td><table><tr><th>Days, x</th><th>Number of Globs, y</th></tr><tr><td>0</td><td></td></tr><tr><td>1</td><td></td></tr><tr><td>2</td><td></td></tr><tr><td>3</td><td></td></tr><tr><td>4</td><td></td></tr></table></td><td><table><tr><th>Days, x</th><th>Number of Globs, y</th></tr><tr><td>0</td><td></td></tr><tr><td>1</td><td></td></tr><tr><td>2</td><td></td></tr><tr><td>3</td><td></td></tr><tr><td>4</td><td></td></tr></table></td></tr>
<tr><td>Graph</td><td>
</td><td>
</td></tr>
<tr><td>Equation</td><td>$y =$</td><td>$y = 5 \cdot 2^x$</td></tr>
</table>

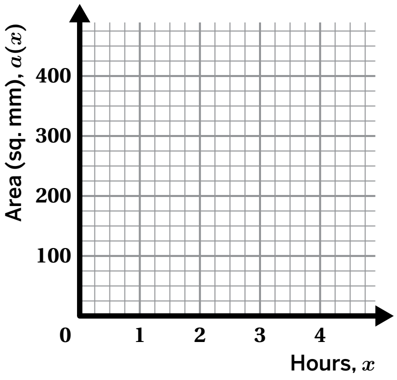
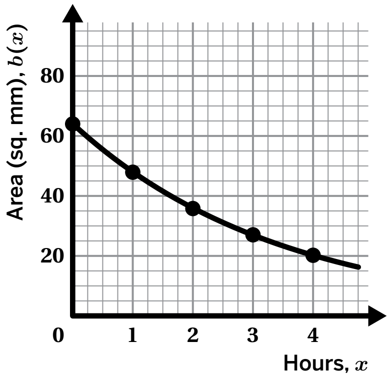
4. Parv claims that the graph of $y = 5 \cdot 2^x$ will have the same graph as $f(x) = 5 \cdot 2^x$. Is Parv's claim correct? Explain your thinking.

Activity
2

Name: ______________________ Date: __________ Period: __________

Bacteria

5. For each Petri dish, fill in the table, graph, and equation written in function notation.

<table>
<tr><th></th><th>Petri Dish A</th><th>Petri Dish B</th></tr>
<tr><td>Situation</td><td>There are 64 sq. mm of bacteria in a Petri dish. One hour later, there are 96 sq. mm of bacteria in the Petri dish. Two hours later, there are 144 sq. mm of bacteria.</td><td>There are 64 sq. mm of bacteria in a Petri dish. Each hour, $\frac{1}{4}$ of the bacteria die.</td></tr>
<tr><td>Table</td><td><table><tr><th>Hours, x</th><th>Area, $a(x)$</th></tr><tr><td>0</td><td>64</td></tr><tr><td>1</td><td>96</td></tr><tr><td>2</td><td>144</td></tr><tr><td>3</td><td></td></tr><tr><td>4</td><td></td></tr></table></td><td><table><tr><th>Hours, x</th><th>Area, $b(x)$</th></tr><tr><td>0</td><td></td></tr><tr><td>1</td><td></td></tr><tr><td>2</td><td></td></tr><tr><td>3</td><td></td></tr><tr><td>4</td><td></td></tr></table></td></tr>
<tr><td>Graph</td><td> </td><td> </td></tr>
<tr><td>Equation</td><td>$a(x) =$</td><td>$b(x) =$</td></tr>
</table>

6. **Discuss:** One of these functions shows **exponential growth** and one shows **exponential decay**. Which do you think is which, and why?

Synthesis

7. Explain where you see the *growth factor* of an exponential relationship in each representation. Use the examples if they help with your thinking.

x	$f(x)$
0	2
1	6
2	18
3	54

$f(x) = 20 \cdot (3)^x$

The area of the bacteria triples each day.

Table:

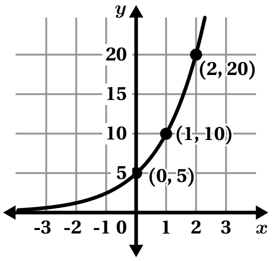
Situation:

Equation:

Lesson Practice 5.03

Lesson Summary

The *growth factor* is the constant ratio that each term is multiplied by to generate an exponential relationship. You can identify the growth factor of an exponential function from a situation, an equation, a table, or a graph. Here is an example.

<table>
<tr><th></th><th>Exponential Function</th><th>The growth factor is 2 because . . .</th></tr>
<tr><td>Situation</td><td>There are 5 sq. mm of bacteria in a Petri dish. Each day, the number of cells doubles.</td><td>The cells are doubling every day.</td></tr>
<tr><td>Table</td><td><table><tr><th>x</th><th>$g(x)$</th></tr><tr><td>0</td><td>5</td></tr><tr><td>1</td><td>10</td></tr><tr><td>2</td><td>20</td></tr></table></td><td>$5 \cdot 2 = 10$ and $10 \cdot 2 = 20$.</td></tr>
<tr><td>Graph</td><td>
</td><td>The distance between the points $(0, 5)$ and $(1, 10)$ doubles in the y direction.</td></tr>
<tr><td>Equation</td><td>$g(x) = 5 \cdot (2)^x$</td><td>2 is the base of the exponential term and it is raised to the power of x.</td></tr>
</table>

Lesson Practice

5.03

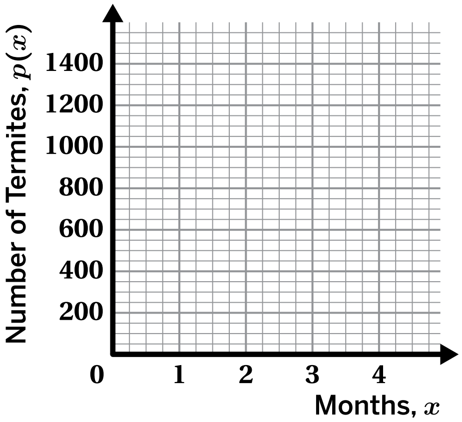
Name: .... Date: .... Period: ....

Problems 1–2: A scientist monitors a population of termites that is growing exponentially. She writes the equation $p(x) = 40 \cdot 2.5^x$ to model the total number of termites after x months.

1. Complete the table.

Months, x	Number of Termites, $p(x)$
0	40
1	100
2	
3	
4	

2. Graph the situation.



Problems 3–4: The table models an exponential function with a growth factor of $\frac{1}{3}$.

3. Fill in the table with values for $f(x)$ that could represent this function.

x	0	1	2	3
$f(x)$				

4. Write an equation to model your function.

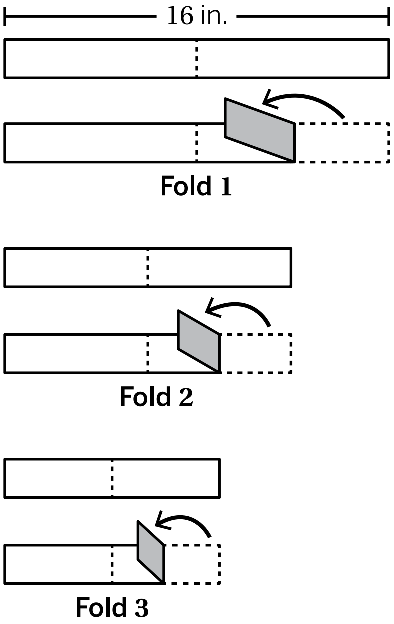
5. There are 50 sq. mm of bacteria in a Petri dish. The bacteria grow exponentially. 1 hour later, there are 70 sq. mm of bacteria in the Petri dish.

What is the growth factor that models the hourly growth of this bacteria?

Name: ______ Date: ______ Period: ______

6. One side of a strip of paper is folded to the middle several times. The new length of the paper is recorded after each fold. Determine the length of the strip of paper after each fold.

Folds	Length (in.)
0	16
1	
2	
3	
4	



Test Practice

7. Neena writes the function $f(x) = 4 \cdot (-0.5)^x$ to model a population that has an initial value of 4 and decreases by half every x days. Is she correct? Explain your thinking.

- **A.** The population has an initial value of 4 and decreases by half every x days.
- **B.** The population has an initial value of 4 and increases by half on every even day, and decreases by half every odd day.
- **C.** The population has an initial value of 4 and increases by .5 every x days.
- **D.** The population has an initial value of 4 and decreases by .5 every x days.

Spiral Review

Problems 8–10: Here is a data set: [1, 2, 3, 3, 3, 5, 5, 6, 7, 8, 22].

8. Complete the table. Use a calculator if it helps with your thinking.

Min.	Q1	Median	Q3	Max.

9. What number is an outlier of the data set?

10. What number could be added to the data set that wouldn't change the 5-number summary?

Name: .... Date: .... Period: ....

MA.912.AR.1.1, MA.912.F.1.6, MA.912.F.1.8, MA.912.AR.5.4, MA.912.AR.5.6, MA.912.F.1.2, MA.912.NSO.1.2, MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1

Carlos and Corals

Let's evaluate exponential functions with positive, negative, and zero inputs.

Warm-Up

1. Select *all* the expressions that are equivalent to $2^{(-3)}$.

- ☐ **A.** $\frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2}$
- ☐ **B.** $\frac{1}{2 \bullet 2 \bullet 2}$
- ☐ **C.** $2 \bullet -3$
- ☐ **D.** $-2 \bullet -2 \bullet -2$
- ☐ **E.** $8^{(-1)}$

Activity
1

Name: .... Date: .... Period: ....

Carlos's Fish

2. Carlos's apartment does not allow pets, so he decided to buy a new toy fish.

The mass of the fish doubles every hour.

What type of function do you think will model the mass of this fish over time?

Linear   Exponential   Neither

Explain your thinking.

3. Carlos's new toy fish has a constant growth factor when placed in water.

What is the mass of the toy fish after 4 hours?

Time (hr)	Mass (g)
1	20
2	40
3	80
4	

4. Carlos writes $m(t)$ to model the fish's mass over time.

$$m(t) = 10 \cdot 2^t$$

What is the value of $m(0)$? Explain your thinking.

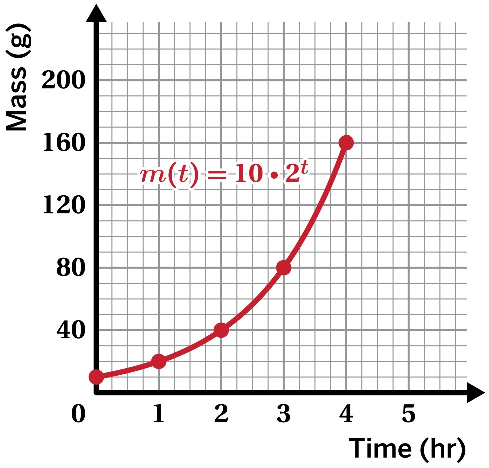
Activity 1

Name: ________ Date: ________ Period: ________

Carlos's Fish (continued)

5. Explain where you see the growth factor in the graph, the table, or the function.

Time, t	Mass, $m(t)$
0	10
1	20
2	40
3	80
4	160



6. **a** What is the value of $m(5)$?

b What is the value of $m(-1)$?

c **Discuss:**

- What does each value say about the fish's mass?
- How would you describe the *domain* of $m(t)$?

Name: .... Date: .... Period: ....

Coral Reefs

Problems 7–9: A marine biologist is studying a coral reef. In 2024, she estimated that its volume was 320 cubic meters.

She wrote the function $v(t)$ to represent the volume of the coral reef t years after 2024:

$$v(t) = 320\left(\frac{4}{5}\right)^t$$

7. Based on $v(t)$, what was the reef's volume in 2025?

- **A.** Less than 320 cubic meters
- **B.** Equal to 320 cubic meters
- **C.** Greater than 320 cubic meters

Explain your thinking.

8. **a** Determine the value of $v(2)$.

b **Discuss:** What domain could make sense for $v(t)$?

9. Determine the missing values.

Years Since 2024	-3	-2	-1	0	1	2
Volume (cubic meters)				320	256	204.8

Name: .... Date: .... Period: ....

Coral Reefs (continued)

10. Here is how Angel and Sora determined the volume of the coral reef in 2021 (3 years before 2024).

Angel

Year Since 2024	Volume (cubic meters)
-3	625
-2	$\cdot \frac{4}{5}$ 500
-1	400 $\cdot \frac{5}{4}$
0	320

Sora

$$v(-3) = 320\left(\frac{4}{5}\right)^{-3}$$

$$v(-3) = 320 \cdot \left(\frac{5}{4}\right)^{3}$$

$$v(-3) = 320 \cdot \frac{125}{64}$$

$$v(-3) = 625$$

Discuss: What is each student's strategy?

11. Here is a new function: $f(x) = 18 \cdot 3^x$.

a **Discuss:** Will $f(-2)$ be less than or greater than 18?

b Determine the value of $f(-2)$.

Synthesis

12. Describe a strategy for evaluating exponential functions for negative inputs.

Use the examples if they help with your thinking.

$a(-4) = 5 \cdot 10^{(-4)}$

$b(-3) = 10 \cdot \left(\frac{1}{2}\right)^{(-3)}$

Lesson Practice 5.04

Lesson Summary

You can evaluate exponential functions for inputs that are positive, negative, or zero.

Let's evaluate the function $f(x) = 9 \cdot 4^x$ for $f(-3)$ using the equation or a table.

You can substitute a value into the *equation*.

Steps	Explanation
$f(-3) = 9 \cdot 4^{(-3)}$	Substitute $x = (-3)$ into the function.
$f(-3) = 9 \cdot \left(\frac{1}{4}\right)^3$	Apply the property of negative *exponents* to rewrite the expression with a positive exponent.
$f(-3) = 9 \cdot \left(\frac{1}{64}\right)$	Rewrite the expression without any exponents.
$f(-3) = \frac{9}{64}$	Multiply to determine the value of $f(-3)$.

You can move backward in a *table*.

x	$f(x)$	
-3	$\frac{9}{64}$	$\times \frac{1}{4}$
-2	$\frac{9}{16}$	$\times \frac{1}{4}$
-1	$\frac{9}{4}$	$\times \frac{1}{4}$
0	9	

Multiply 9 by $\frac{1}{4}$ three times to determine the value of $f(-3)$.

Lesson Practice

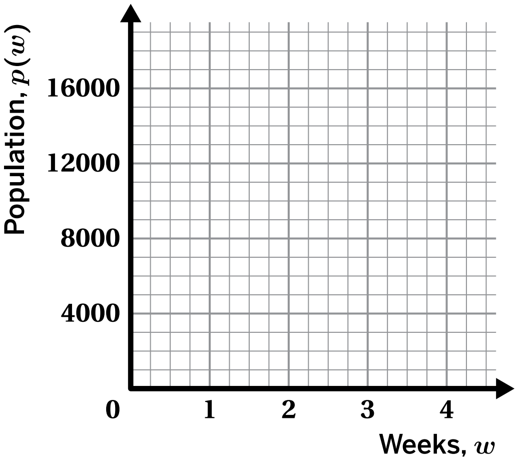
5.04

Name: .... Date: .... Period: ....

Problems 1–3: The equation $p(w) = 1000 \cdot 2^w$ models a population of mosquitos, $p(w)$, where w is the number of weeks after the population was first measured.

1. Complete the table and plot the values on the graph.

Weeks, w	Population, $p(w)$
0	
1	
2	
3	
4	



2. Where on the graph do you see the 1,000 from the equation?

3. Determine the value of $p(-2)$ and explain what it means in this situation.

Problems 4–6: The equation $f(t) = 800 \cdot \left(\frac{1}{2}\right)^t$ models a fish population, $f(t)$, where t is time in years since the beginning of 2015.

4. What is the population of fish at the beginning of 2015?

5. What is the population of fish at the beginning of 2018?

6. What is the population of fish at the beginning of 2012? Do you think this makes sense in this situation?

Lesson Practice
5.04

Name: ________ Date: ________ Period: ________

Test Practice

7. Which equation best models the data in the table?

A. $f(x) = 80(1.25)^x$

B. $f(x) = 64(1.25)^x$

C. $f(x) = 64 + 1.25x$

D. $f(x) = 60 + 20x$

x	$f(x)$
1	80
2	100
3	125
4	156.25

Spiral Review

Problems 8–9: Charlie's gaming club wants to make at least $300 selling boxes of cookies and pies. The gaming club makes $9 for each box of cookies and $15 for each pie.

8. Write an inequality to represent the number of boxes of cookies, c, and pies, p, the club can sell to make at least $300.

9. If the club sells 5 boxes of cookies, what is the minimum number of pies they need to sell in order to meet their goal?

Problems 10–13: Determine the value of each expression.

10. 2^4

11. -2^4

12. $(-2)^4$

13. $\left(\frac{1}{2}\right)^4$

Unit 5

Lesson

5

Name: ________ Date: ________ Period: ________

MA.912.AR.1.1, MA.912.F.1.2, MA.912.F.1.8, MA.912.NSO.1.1, MA.912.NSO.1.2, MTR.2.1, MTR.5.1, MTR.7.1

Differences and Factors

Let's generalize how linear and exponential functions change over equal intervals.

Warm-Up

1. Rewrite each expression using a single exponent.

a $3^2 \cdot 3^5$

b $\frac{4^5}{4^2}$

c $\frac{5^{x+2}}{5^x}$

2. Rewrite this expression using as few terms as possible.

$(4(n+1)+5)-(4n+5)$

Activity

1

Name: ______ Date: ______ Period: ______

Linear Functions

3. Complete the table for the function $f(x) = 2x + 5$.

x	$f(x)$
0	5
1	7
2	
. . .	. . .
7	
8	

4. What happens to $f(x)$ when x increases by 1?

5. Precious says: *Without calculating, I know that* $f(5009) - f(5008)$ *must be* 2.

Is this correct? Explain how you know.

6. What happens to $f(x)$ when x increases to $\boldsymbol{x} + 1$? In other words, what is $f(x + 1) - f(x)$?

7. Precious wrote this expression to help answer the previous problem. Rewrite it using as few terms as possible.

$(2(x + 1) + 5) - (2x + 5)$

x	$f(x)$
x	$2x + 5$
$x + 1$	$2(x + 1) + 5$

8. 💬 **Discuss:** Why does Precious's expression have the same value as your answer to Problem 4?

9. What do you think happens to $f(x)$ when x increases to $x + 3$?

10. Write an expression like Precious's expression in Problem 7. Then rewrite your expression using the fewest terms and check if it matches your prediction.

Activity
2

Name: ____________ Date: ________ Period: ________

Exponential Functions

11. Complete the table for the function $g(x) = 3^x$.

12. What happens to $g(x)$ when x increases by 1?

13. Precious says: *Without calculating, I know that* $\frac{g(5009)}{g(5008)}$ *must be* 3.

Is this correct? Explain how you know.

x	$g(x)$
0	1
1	3
2	
$\ldots$	$\ldots$
6	
7	

14. What happens to $g(x)$ when x increases to $x + 1$? In other words, what is $\frac{g(x+1)}{g(x)}$?

15. Precious wrote this expression to help answer the previous problem. Rewrite it using a single exponent.

$$\frac{3^{x+1}}{3^x}$$

x	$g(x)$
x	3^x
$x + 1$	3^{x+1}

16. 💬 **Discuss:** Why does Precious's expression have the same value as your answer to Problem 12?

17. What do you think happens to $g(x)$ when x increases to $x + 3$?

18. Write an expression like Precious's expression in Problem 15. Then rewrite your expression using a single exponent and check if it matches your prediction.

Synthesis

19. Precious says: *If intervals are equal, then linear functions grow by constant differences and exponential functions grow by constant growth factors.*

Explain what this means in your own words.

Lesson Practice 5.05

Lesson Summary

A linear function always increases (or decreases) by equal differences over equal *intervals*, and an exponential function increases (or decreases) by equal *factors* over equal intervals.

Here are two examples.

	Linear function $g(x) = 2x + 3$	**Exponential function** $h(x) = 3^x$
Describe how the function changes when x grows by 1	When x grows by 1, $g(x)$ will always increase by $2(1) = 2$: $g(x + 1) - g(x)$ $= (2(x + 1) + 3) - (2x + 3)$ $= (2x + 2 + 3) - (2x + 3)$ $= 2x + 5 - 2x - 3$ $= 5 - 3$ $= 2$	When x grows by 1, $h(x)$ will always multiply by a factor of $3^1 = 3$. $\frac{h(x + 1)}{h(x)}$ $= \frac{3^{(x + 1)}}{3^x}$ $= \frac{3^x \cdot 3^1}{3x}$ $= 3$
Describe how the function changes when x grows by 4	When x grows by 4, $g(x)$ will always increase by $2(4) = 8$. $g(x + 4) - g(x)$ $= (2(x + 4) + 3) - (2x + 3)$ $= (2x + 8 + 3) - (2x + 3)$ $= 2x + 11 - 2x - 3$ $= 11 - 3$ $= 8$	When x grows by 4, $h(x)$ will always multiply by a factor of $3^4 = 81$. $\frac{h(x + 4)}{h(x)}$ $= \frac{3^{(x + 4)}}{3^x}$ $= \frac{3^x \cdot 3^4}{3x}$ $= 3^4$ $= 81$

Lesson Practice
5.05

Name: .... Date: .... Period: ....

Problems 1–2: Here are two functions. $f(x)$ is a linear function and $g(x)$ is an exponential function.

	$f(x) = 2x + 1$					
x	0	1	2	3	4	5
$f(x)$	1	3	5	7	9	11

	$g(x) = 2 \cdot 3^x$					
x	0	1	2	3	4	5
$g(x)$	2	6	18	54	162	486

1. How does $f(x)$ change when x grows by 1?

2. How does $g(x)$ change when x grows by 1?

Problems 3–4: Here is a function: $h(x) = 4^x$.

3. Calculate $\frac{h(x+1)}{h(x)}$.

4. Calculate $\frac{h(x+2)}{h(x)}$.

5. Here is a function: $g(x) = 2^x$. Select *all* of the true statements.

☐ **A.** When the input x is increased by 1, the value of $g(x)$ increases by 2.

☐ **B.** When the input x is increased by 1, the value of $g(x)$ multiplies by 2.

☐ **C.** When the input x is increased by 3, the value of $g(x)$ increases by 8.

☐ **D.** When the input x is increased by 3, the value of $g(x)$ multiplies by 8.

☐ **E.** When the input x is increased by 3, the value of $g(x)$ multiplies by 6.

6. Whenever the x-value increases by 1, the value of $j(x)$ increases by 5. Which of these functions could be $j(x)$?

A. $j(x) = 3x + 5$ **B.** $j(x) = 5x + 3$ **C.** $j(x) = 5^x$ **D.** $j(x) = x^5$

7. Whenever the x-value increases by 2, the value of $m(x)$ multiplies by 9. Which of these functions could be $m(x)$?

A. $m(x) = 3x + 9$ **B.** $m(x) = 9x + 3$ **C.** $m(x) = 3^x$ **D.** $m(x) = 9^x$

Lesson Practice
5.05

Name: .... Date: .... Period: ....

Test Practice

8. Select the description for the function $f(x) = 0.19^x$.

A. Decreases by 19% when x increases by 1.

B. Increases by 81% when x increases by 1.

C. Increases by 19% when x increases by 1.

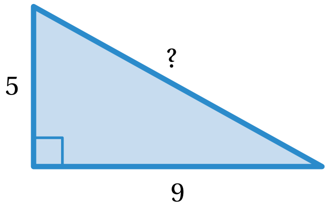
D. Decreases by 81% when x increases by 1.

Spiral Review

9. Determine the value of $f(x) = 15x + 4$ for each function statement.

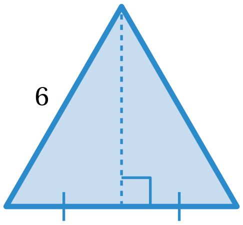
$f(3) =$ .... $f(\text{-}1) =$ .... $f\left(\frac{1}{3}\right) =$ ....

10. Here is a right triangle. What is the *exact* length of the hypotenuse?



11. Here is an equilateral triangle. The length of each side is 6 units.

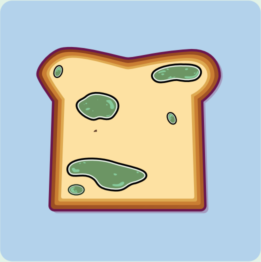
Find the *exact* height of the triangle.



Sub-unit

2

Exponential Growth and Decay



Lesson 6

Growing Mold

Lesson 7

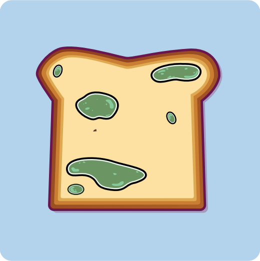
At a Loss

Name: .... Date: .... Period: ....

MA.912.AR.1.1, MA.912.AR.5.3, MA.912.AR.5.4, MA.912.AR.5.6, MA.912.NSO.1.1, MA.912.F.1.2, MTR.2.1, MTR.6.1, MTR.7.1

Growing Mold

Let's explore how to model situations that change by a percent increase with exponential functions.



Warm-Up

Determine the value of each statement.

1. 10% of 30

2. 100% of 30

3. 110% of 30

4. 110% of 50

Activity
1

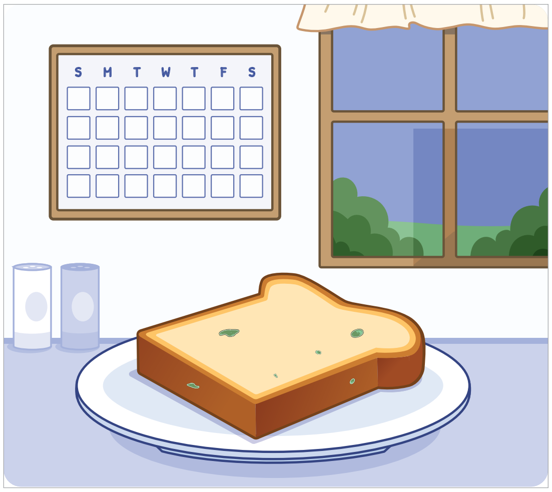
Name: .... Date: .... Period: ....

Growing Mold

5. A piece of bread is left out on a counter.

a Think about how mold might grow over time.

What do you notice? What do you wonder?



b This mold grows by 75% each day. How much mold will there by on day 4?

c How much mold will there be on days 5 and 6?

Days	Area of Mold (sq. cm)
3	16
4	
5	
6	

Name: ………… Date: ………… Period: …………

Growing Mold (continued)

6. Arnav made a table to help him write a function to represent the area of mold, $m(x)$, after x days.

Where do you see the 75% increase represented in the function's equation? In the table?

Equation:

Table:

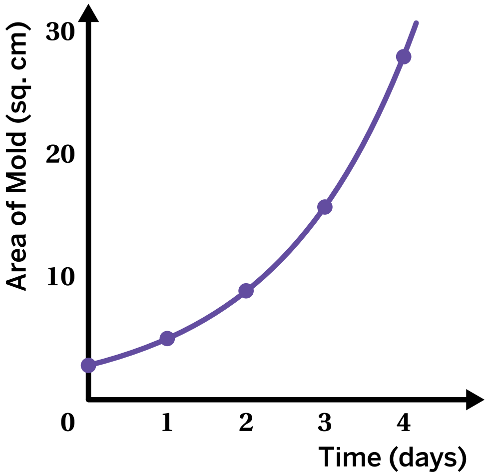
$m(x) = 2.985(1.75)^x$

Days	Area of Mold (sq. cm)
0	2.985
1	5.224
2	9.14
3	16
4	28

7. Here is the graph of $m(x) = 2.985(1.75)^x$.

a Determine how much mold there will be after 10 days.

b **Discuss:** Do you think the mold could grow according to $m(x)$ forever?



Activity

2

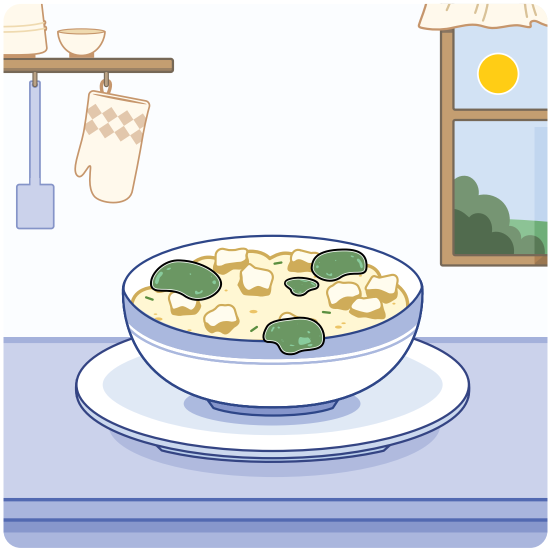
Name: ______________________ Date: __________ Period: __________

Growing with Percents

8. Tyler made potato salad and forgot to put it in the refrigerator.

The amount of bacteria in the potato salad increases by 4% every minute.

Which function, $p(t)$, represents the amount of bacteria in the potato salad after t minutes?



[✓ A.] $p(t) = 5 \bullet 0.4^t$

B. $p(t) = 5 \bullet 1.04^t$

C. $p(t) = 5 \bullet 1.4^t$

Explain your thinking.

9. Let's look at the function Tyler selected.

a What does the 1 represent in this situation?

b What does the 5 represent in the situation?

Name: .... Date: .... Period: ....

Growing with Percents (continued)

10. Match each function with the situation that represents the same relationship. One function will have no match.

Functions

a $a(x) = 20 \cdot 0.85^x$

b $b(x) = 20 \cdot 1.85^x$

c $c(x) = 85 \cdot 1.2^x$

d $d(x) = 85 \cdot 1.02^x$

Situations

.... A population of bacteria starts with 20 cells and grows by 85%.

.... A population of frogs starts with 85 frogs and grows by 20%.

....

Days	Amount of Money ($)
0	85
1	86.7
2	88.4

11. Which function in Problem 10 did not have a match? What do you notice about that function?

Then insert the answer in magenta anno as follows.

You're invited to explore more.

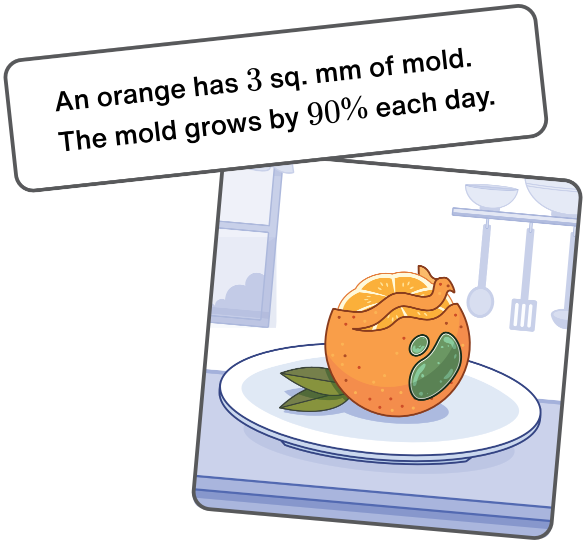
12. Heat and humidity can cause some types of bacteria to grow quickly. Imagine that in a humid room the amount of bacteria in a potato salad *triples* every hour.

By what percent is the bacteria growing per hour? Explain your thinking.

Synthesis

13. How do you write an exponential function that represents growing by a percentage?

Use the example if it helps with your thinking.



An orange has 3 sq. mm of mold. The mold grows by 90% each day.

Lesson Practice 5.06

Lesson Summary

Exponential functions can be written in the form $f(x) = a \cdot b^x$, where a is the *initial value* and b is the growth factor. Exponential functions represent repeated *percent increase* and growth when the growth factor is greater than 1.

In situations that model repeated percent increase, the growth factor b can be written as 1 (representing 100%) plus the percent increase in decimal form.

For example, the value of a baseball card collection increases by 4% every year. In 2020, the collection was valued at $\$500$. Let $f(x)$ represent the value of the collection and x represent the years since 2020.

- At first, the collection was valued at $\$500$. The initial value, or a, is 500.
- The value increases by 4% every year. Because the value of the cards increases by a repeated percent each year, we can represent the growth by changing the percent to a decimal (4% to 0.04) and adding 1. The growth factor, or b, will be 1.04.

We can write the exponential function that represents this situation as $f(x) = 500 \cdot (1.04)^x$. Because 1.04 is greater than 1, this exponential function represents growth.

Lesson Practice

5.06

Name: .... Date: .... Period: ....

Problems 1–3: A group of biologists tracked the number of deer in a forest over several years. There were 600 deer when they first counted. The population has increased by 15% each year.

1. How many deer are in the forest 1 year after the biologists first counted?

2. Write an expression that represents the deer population after 3 years.

3. Write an expression that represents the deer population after t years.

Problems 4–5: Sai gets a $\$500$ loan from the bank with an annual interest rate of 6%.

4. Write a function, $f(t)$, to represent the amount Sai will owe, in dollars, after t years.

5. Complete the table to determine how much money Sai will owe over time if no payments are made.

Time (yr)	Amount Owed ($)
0	500
1	
2	
3	
4	

Problems 6–8: Three cities have the same initial population and different percent increases each year. Match each function $p(t)$, representing the population after t years, with its correct description.

$p(t) = 5000 \cdot (1.20)^t$ $p(t) = 5000 \cdot (1.02)^t$ $p(t) = 5000 \cdot (1.002)^t$

6. City A has a 0.2% annual increase in population.

....

7. City B has a 20% annual increase in population.

....

8. City C has a 2% annual increase in population.

....

Lesson Practice
5.06

Name: .... Date: .... Period: ....

Test Practice

9. Sothy's family paid $1,300 in property tax last year. This year, the county will increase the property tax by 2.1%.

Select *all* the expressions that represent Sothy's family's property taxes this year.

☐ **A.** $1300 + (1.021)$

☐ **B.** $1300(1.21)$

☐ **C.** $1300(1.021)$

☐ **D.** $1300(1.0021)$

☐ **E.** $1300 + 1300(0.021)$

Spiral Review

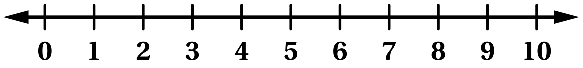
10. Order these values from least to greatest.

75% of 12 25% of 32 50% of 20 10% of 95

Least **Greatest**

11. Create a dot plot that has:

- At least 5 data points.
- A median of 7.
- A mean that is less than the median.



Unit 5

Lesson

7

Name: .... Date: .... Period: ....

MA.912.AR.1.1, MA.912.AR.5.3, MA.912.AR.5.4, MA.912.AR.5.6, MA.912.NSO.1.1, MTR.2.1, MTR.4.1, MTR.5.1, MTR.7.1

At a Loss

Let's make connections between different representations of exponential decay functions.

Warm-Up

1. **Discuss:** How do you think each of these might change over time?

   - The trade-in value of a cell phone
   - The amount of caffeine in the body after drinking coffee
   - The resale value of a laptop

Activity
1

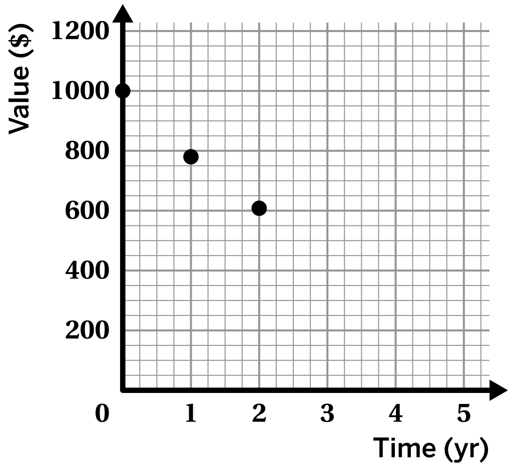
Name: .... Date: .... Period: ....

Des-Phone

Aaliyah bought a Des-Phone for $\$1,000$. Typically, the value of phones decays exponentially. Every year, she checks the trade-in value to see how much money her phone is worth.

2. Complete the table and graph to represent Aaliyah's situation.

Time (yr)	Value ($)
0	1,000
1	780
2	608.40
3	
4	



Aaliyah and Taylor wrote different functions to represent this situation.

Aaliyah	Taylor
$f(x) = 1000(0.78)^x$	$f(x) = 1000(1 - 0.22)^x$

3. 💬 **Discuss:**

- How are the functions alike?
- How are they different?

4. Choose one function. Show or explain what each number represents about the situation.

Name: .... Date: .... Period: ....

Des-Phone (continued)

5. Use any strategy to calculate the trade-in value of the phone after 10 years.

6. Select *all* the statements that are true about this situation.

☐ **A.** The phone retains 78% of its value each year.

☐ **B.** The value of the phone is decreasing by 78% each year.

☐ **C.** The trade-in value of the phone is decreasing by 22% each year.

☐ **D.** The value of the phone will decrease by $220 each year.

Name: .... Date: .... Period: ....

Sorting Percents

7. You will use a set of cards for this activity.

Use the cards to complete each row. Create your own situation for the last row.

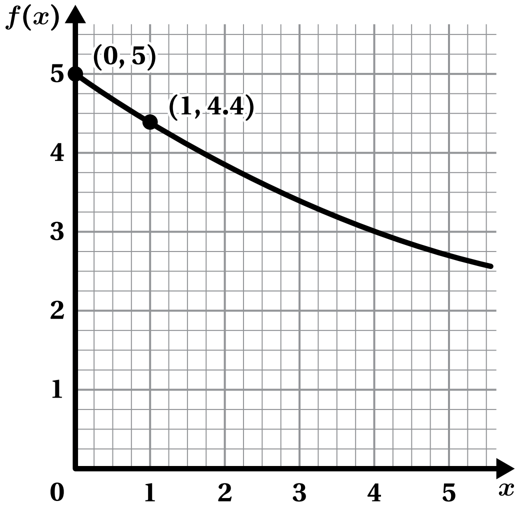
	Situation	Percent Decrease	Functions
Medicine	Victor takes 800 mg of an antiviral medicine. The amount of medicine left in his body is reduced by 25% every hour. Write a function to represent the amount of medicine remaining in Victor's body after x hours.		
Laptops	A laptop that was originally $\$800$ loses 75% of its value each year. Write a function to represent the value of the laptop after x years.		
Coffee	Aditi has 200 mg of caffeine in her body after drinking coffee. Typically, the amount of caffeine in the body decreases by 10% each hour. Write a function to represent the amount of caffeine remaining in her body after x hours.		
Your Situation			$j(x) = 200(0.1)^x$

Activity
3

Name: .... Date: .... Period: ....

Graphs to Functions

This graph represents an exponential function. Diya tried to write its function and made an error.



Diya

(0, 5)

(1, 4.4) $\frac{5}{4.4} = 1.14 \longrightarrow \boxed{f(x)=5(1.14)^x}$

I used the y-values of two points to calculate the growth factor. Then I wrote the function using my growth factor and the y-intercept.

8. What did Diya do correctly?

9. What error did Diya make?

10. Write a correct function.

11. By what percent is the function decreasing when x increases by 1?

12. Show or explain where you see your answer to Problem 11 in your equation from Problem 10.

Synthesis

13. Compare writing functions with percent increase to writing functions with *percent decrease*.

How are they alike? How are they different?

Use the examples if they help with your thinking.

Example 1

A person has 200 mg of caffeine in her body. Typically, the amount of caffeine remaining in her body falls by 10% each hour.
$f(x) = 200(1 - 0.1)^x$

Example 2

An orange has 3 sq. mm of mold. The mold grows by 90% each day.
$g(x) = 3(1 + 0.9)^x$

Lesson Practice 5.07

Lesson Summary

Exponential functions with a growth factor between 0 and 1 represent exponential decay. You can describe a relationship that models exponential decay as a *percent decrease*.

In situations that model exponential decay, the growth factor b can be written as 1 (representing 100%) minus the percent of decay in decimal form.

For example, a potted plant is given 24 milliliters of fertilizer. The amount of fertilizer decreases by 1% every hour. We can write a function to represent this situation in the form $f(x) = a \bullet b^x$. Let $f(x)$ represent the amount of fertilizer left in the potted plant and x represent the time in hours since the potted plant received the fertilizer.

- At first, the amount of fertilizer is 24 milliliters. The *initial value*, or a, is 24.
- The amount of fertilizer decreases by 1% every hour. Because the fertilizer decreases by a constant percent, we can represent the decay by changing the percent to a decimal (1% to 0.01) and subtracting it from 1. The growth factor, or b, will be $1 - 0.01$ or 0.99.

We can write the exponential function that represents this situation as $f(x) = 24 \bullet (0.99)^x$.

Lesson Practice
5.07

Name: .... Date: .... Period: ....

Problems 1–3: Determine the percent decrease for each function.

1. $f(x) = 20(0.9)^x$

2. $g(x) = 45(1 - 0.75)^x$

3. $h(t) = 0.65\left(1 - \frac{25}{100}\right)^t$

4. Select *all* the functions with a 30% decrease.

☐ **A.** $g(t) = 22(0.7)^t$

☐ **B.** $h(x) = 22(0.3)^x$

☐ **C.** $f(x) = 30(1 - 0.7)^x$

☐ **D.** $r(t) = 30(1 - 0.3)^x$

☐ **E.** $v(x) = 70\left(1 - \frac{30}{100}\right)^x$

5. A cell phone is worth \$800 when it is first made, but loses 25% of its value after each year. Write an equation to represent the value, $v(t)$, of the cell phone, t years after it is made.

6. Jamir says that $r(x) = 26(0.3)^x$ and $r(x) = 26\left(1 - \frac{7}{10}\right)^x$ are equivalent equations.

a Is Jamir correct? Explain your thinking.

b Write another equivalent equation that uses fractions.

7. Fill in each blank using the digits 0 to 9 only once each to create the smallest possible value.

$$\square\square(\square.\square\square)^3$$

Name: .... Date: .... Period: ....

Test Practice

8. There are 1,024 players in a tennis tournament. After each round, *half* of the players are eliminated. Which function represents the number of players remaining after x rounds?

Ⓐ $f(x) = 1024(1.50)^x$

Ⓑ $f(x) = 1024(1.05)^x$

Ⓒ $f(x) = 1024(0.50)^x$

Ⓓ $f(x) = 1024(0.05)^x$

Spiral Review

Problems 9–10: A factory makes boxes. Each box's height, $h(x)$, is 2 inches more than 3 times its width, x.

9. Complete the table

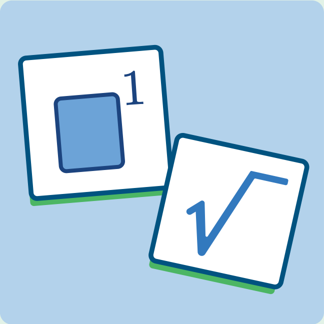
x	1	2	3	4
$h(x)$				

10. Write an equation for the function $h(x)$.

Sub-unit

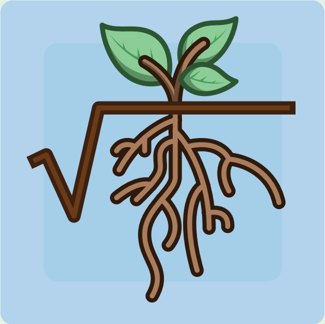
3

Modeling with Exponentials



Lesson 8
Thinking Rationally

Lesson 9
Writing Radicals



Lesson 10
Rule the Roots

Lesson 11
Tame the Terms

Lesson 12
Bank Accounts

Lesson 13
Payday Loan

Lesson 14
Credit Card Compounding

Lesson 15
Exploring Interest

Unit 5

Lesson

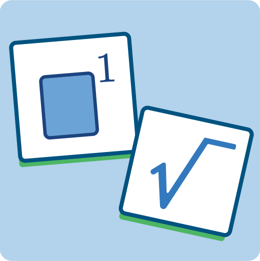
8

Name: .... Date: .... Period: ....

MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.NSO.1.4, MTR.2.1, MTR.4.1, MTR.5.1

Thinking Rationally

Let's write equivalent expressions using radicals and rational exponents.



Warm-Up

Determine the value of the ? that makes each equation true.

1. $9^1 \bullet 9^1 = 9^?$

2. $9^3 \bullet 9^4 = 9^?$

3. $9^? \bullet 9^? = 9^6$

4. $9^? \bullet 9^? = 9^1$

Activity
1

Name: .... Date: .... Period: ....

Express Yourself

5. Here are several equations Lan wrote that are all equal to 9^1.

What is unique about $9^{\frac{1}{2}} \cdot 9^{\frac{1}{2}} = 9^1$?

$9^{\frac{1}{4}} \cdot 9^{\frac{3}{4}} = 9^1$

$9^{\frac{2}{3}} \cdot 9^{\frac{1}{3}} = 9^1$

$9^{\frac{3}{5}} \cdot 9^{\frac{2}{5}} = 9^1$

$9^2 \cdot 9^{-1} = 9^1$

$9^{\frac{1}{2}} \cdot 9^{\frac{1}{2}} = 9^1$

6. Here is an equation Lan wrote. She wondered what the value of $9^{\frac{1}{2}}$ might be. What do you think the value of $9^{\frac{1}{2}}$ is?

$9^{\frac{1}{2}} \cdot 9^{\frac{1}{2}} = 9^1$

$9^{\frac{1}{2}} =$

Name: ______ Date: ______ Period: ______

Express Yourself (continued)

7. Determine the value of each expression.

Exponent Expression	Value
$25^{\frac{1}{2}}$	
$64^{\frac{1}{2}}$	
$1^{\frac{1}{2}}$	

8. Look at your work from the previous problem.

Describe what you know about *any number* raised to the $\frac{1}{2}$ power.

Activity
2

Name: ______________________ Date: ____________ Period: ____________

Thinking about Thirds

9. Here is a new problem.

$$\boxed{64^{?} \cdot 64^{?} \cdot 64^{?} = 64^{1}}$$

$$64^{\frac{1}{3}} \cdot 64^{\frac{1}{3}} \cdot 64^{\frac{1}{3}} = 64^{1}$$

Lan determined the value of the exponents.

Discuss:

- What strategy did she use?
- What is the value of $64^{\frac{1}{3}}$? How do you know?

10. Use Lan's strategy to determine the value of each expression.

Exponent Expression	Value
$64^{\frac{1}{3}}$	
$27^{\frac{1}{3}}$	
$8^{\frac{1}{3}}$	

Name: .... Date: .... Period: ....

Thinking about Thirds (continued)

11. The symbol $\sqrt{\ }$ is called a **radical**.

A **rational exponent** is an exponent that is written as a fraction.

a Let's look at a few examples to explore the relationship between radicals and rational exponents.

$x^{\frac{1}{2}} = \sqrt{x}$

"The square root of x"

$x^{\frac{1}{3}} = \sqrt[3]{x}$

"The cube root of x"

$x^{\frac{1}{4}} = \sqrt[4]{x}$

"The 4th root of x"

$x^{\frac{1}{5}} = \sqrt[5]{x}$

"The 5th root of x"

$x^{\frac{1}{n}} = \sqrt[n]{x}$

"The nth root of x"

b Where do you see each part of the rational exponent expression represented in the radical expression?

Activity
3

Name: .... Date: .... Period: ....

Equivalent Expressions

12. Two students correctly rewrote the equation $27^{\frac{1}{3}} \cdot 27^{\frac{1}{3}} \cdot 27^{\frac{1}{3}} = 27$ without using rational exponents.

Ramon

$3 \cdot 3 \cdot 3 = 27$

Nyanna

$\sqrt[3]{27} \cdot \sqrt[3]{27} \cdot \sqrt[3]{27} = 27$

Whose strategy could you use to rewrite $10^{\frac{1}{2}} \cdot 10^{\frac{1}{2}} = 10$ without using rational exponents? Circle one.

Ramon's Nyanna's Both Neither

Explain your thinking.

13. Write an equivalent radical or exponential expression for each problem.

A. Determine the missing value in each equation.

a $10^{\frac{1}{2}} =$ ....

b $7^{\frac{1}{4}} =$ ....

c $4^{\frac{1}{3}} =$ ....

d $\sqrt{71} =$ ....

e $\sqrt[3]{24} =$ ....

f $\sqrt[4]{16} =$ ....

You're invited to explore more.

14. Create equivalent expressions for 6 using rational exponents or radicals.
Try to create as many expressions as you can (especially ones that you think no one in your class will create)!

Synthesis

15. Show or explain the relationship between expressions written with a rational exponent and radical expressions.

Use the examples if they help with your thinking.

$$4^{\frac{1}{3}} \cdot 4^{\frac{1}{3}} \cdot 4^{\frac{1}{3}} = 4$$
$$\sqrt[3]{4} \cdot \sqrt[3]{4} \cdot \sqrt[3]{4} = 4$$

$$9^{\frac{1}{2}} \cdot 9^{\frac{1}{2}} = 9$$
$$\sqrt{9} \cdot \sqrt{9} = 9$$

Lesson Practice 5.08

Lesson Summary

The symbol $\sqrt{}$ is called a **radical**. You can write equivalent expressions using radicals and **rational exponents**, exponents that are written as a fraction.

Here are some examples of equivalent expressions.

Radical Expression	Rational Expression
$\sqrt{16}$	$16^{\frac{1}{2}}$
$\sqrt[3]{125}$	$125^{\frac{1}{3}}$
$\sqrt[4]{58}$	$58^{\frac{1}{4}}$
$\sqrt[5]{70}$	$70^{\frac{1}{5}}$

Lesson Practice

5.08

Name: .... Date: .... Period: ....

1. Rewrite each exponential expression as a radical expression.

$5^{\frac{1}{2}}$	$7^{\frac{1}{3}}$	$2^{\frac{1}{5}}$

2. Rewrite each radical expression as an exponential expression.

$\sqrt[3]{6}$	$\sqrt[5]{3}$	$\sqrt[8]{4}$

Problems 3–5: Determine a value for b that would make each equation true.

3. $5^{\frac{1}{b}} \cdot 5^{\frac{1}{b}} \cdot 5^{\frac{1}{b}} \cdot 5^{\frac{1}{b}} = 5$

4. $\sqrt[b]{2} \cdot \sqrt[b]{2} \cdot \sqrt[b]{2} \cdot \sqrt[b]{2} \cdot \sqrt[b]{2} = 2$

5. Create three equivalent expressions for 8 using rational exponents or radicals.

6. Priya thinks there's a value for d that will make $\sqrt[d]{d} \cdot \sqrt[d]{d} \cdot \sqrt[d]{d} = d$ true. Is she correct? Show or explain your thinking.

Lesson Practice

5.08

Name: .... Date: .... Period: ....

Test Practice

7. Select all the equations that are true.

☐ **A.** $4^{\frac{1}{2}} \bullet 4^{\frac{1}{2}} = 4$

☐ **B.** $5^{\frac{1}{3}} = \sqrt[3]{5} \bullet \sqrt[3]{5} \bullet \sqrt[3]{5}$

☐ **C.** $\sqrt[4]{10} = 10^{\frac{4}{1}}$

☐ **D.** $\sqrt{7} = 7^{\frac{1}{2}}$

☐ **E.** $6 = 6^{\frac{1}{3}} \bullet 6^{\frac{1}{3}} \bullet 6^{\frac{1}{3}}$

Spiral Review

8. Here is an equation: $2^2 \bullet 3^3 \bullet (3 \bullet 2)^7 = 2^x \bullet 3^y$.

Determine the value of x and y.

$x =$ $y =$

9. Write an equivalent expression to $\frac{3^5}{3^7}$ with a negative exponent.

10. Write an equivalent expression to $5^{-2} \bullet 5^{-3}$ *without* using a negative exponent.

Unit 5
Lesson
9

Name: ______ Date: ______ Period: ______

MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.NSO.1.4, MTR.2.1, MTR.4.1, MTR.5.1

Writing Radicals

Let’s explore rational exponents with numerators other than 1.

Warm-Up

Here are several expressions that are equivalent to $9^{\frac{3}{2}}$.

$9^{\frac{1}{2}} \cdot 9^{\frac{1}{2}} \cdot 9^{\frac{1}{2}}$	$\left(9^{\frac{1}{2}}\right)^3$	$(9^3)^{\frac{1}{2}}$
$\sqrt{9} \cdot \sqrt{9} \cdot \sqrt{9}$	$(\sqrt{9})^3$	$\sqrt{9^3}$

1. What do you notice and wonder about the expressions?

I notice . . .

I wonder . . .

Activity
1

Name: .... Date: .... Period: ....

Rational Exponents

David and Emma both correctly evaluated $4^{\frac{3}{2}}$.

2. Choose one student's work to analyze. Explain what they did at each step.

David	**Emma**
$4^{\frac{3}{2}}$	$4^{\frac{3}{2}}$
$= \left(4^{\frac{1}{2}}\right)^3$	$= (4^3)^{\frac{1}{2}}$
$= (\sqrt{4})^3$	$= \sqrt{4^3}$
$= (2)^3$	$= \sqrt{64}$
$= 8$	$= 8$

3. Find a classmate who chose a different student than you.

Discuss:

- How are David's and Emma's steps similar? How are they different?
- Why did they get the same answer even though their steps were different?

4. Whose steps would you use to evaluate $8^{\frac{4}{3}}$? Circle one.

David's   Emma's   Something else

Explain your thinking.

5. Evaluate $8^{\frac{4}{3}}$.

6. Jacy is trying to decide whether $10^{\frac{6}{7}}$ is equivalent to $(\sqrt[7]{10})^6$ or $(\sqrt[6]{10})^7$. What could you say to help her decide?

Activity
2

Name: .... Date: .... Period: ....

Equivalent Expressions

You will use a set of cards with equivalent expressions.

1. Match Expressions A–D with their equivalent expressions. Each expression has three equivalent matches.

   Record your work in the table below.

Expression A $4^{\frac{2}{5}}$	Expression B $4^{\frac{5}{2}}$	Expression C $27^{\frac{2}{3}}$	Expression D $27^{\frac{3}{2}}$

2. Create as many equivalent expressions of $x^{\frac{5}{4}}$ as you can.

   Try to write expressions no one else in your class will!

3. With a partner, compare all the expressions you created.

   Discuss:

   - Which of your expressions are the same? Which are different?
   - How can you tell if all of the expressions are equivalent?

Synthesis

10. **a** Show or explain why $x^{\frac{3}{5}}$ is equivalent to $\sqrt[5]{x^3}$ using properties of exponents.

b Write $b^{\frac{m}{n}}$ as a radical expression.

Lesson Practice 5.09

Lesson Summary

You can write many equivalent expressions using *radicals* and *rational exponents*.

Here are some examples.

	Equivalent Expressions				
$x^{\frac{6}{5}}$	$(x^6)^{\frac{1}{5}}$	$\left(x^{\frac{1}{5}}\right)^6$	$\sqrt[5]{x^6}$	$(\sqrt[5]{x})^6$	$\sqrt[5]{x \bullet x \bullet x \bullet x \bullet x \bullet x}$
$9^{\frac{5}{2}}$	$(9^5)^{\frac{1}{2}}$	$\left(9^{\frac{1}{2}}\right)^5$	$\sqrt{9^5}$	$(\sqrt{9})^5$	243
$16^{\frac{7}{4}}$	$(16^7)^{\frac{1}{4}}$	$\left(16^{\frac{1}{4}}\right)^7$	$\sqrt[4]{16^7}$	$(\sqrt[4]{16})^7$	128

Lesson Practice
5.09

Name: .... Date: .... Period: ....

Problems 1–3: Write an equivalent expression for each problem.

1. $\sqrt{2^5}$

2. $8^{\frac{5}{3}}$

3. $27^{\frac{1}{4}}$

4. Write two different expressions that are equivalent to $\sqrt[3]{6^8}$.

5. Write or sketch a problem that has $4^{\frac{3}{2}}$ as an answer.

6. Show or explain why $a^{\frac{2}{3}}$ is equivalent to $\sqrt[3]{a^2}$ using properties of exponents.

Lesson Practice
5.09

Name: .... Date: .... Period: ....

Test Practice

7. Match each expression with the equivalent term.

a $9^{\frac{2}{3}}$ .... $\sqrt{3 \bullet 3 \bullet 3 \bullet 3 \bullet 3}$

b $9^{\frac{3}{2}}$ .... $\sqrt[5]{9}$

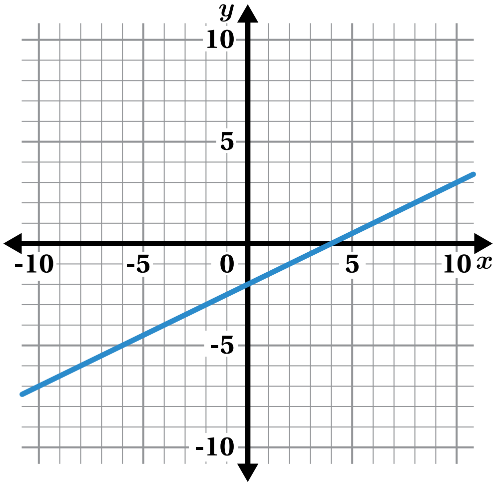
c $3^{\frac{5}{2}}$ 27

d $3^{\frac{2}{5}}$ .... $\sqrt[3]{3^4}$

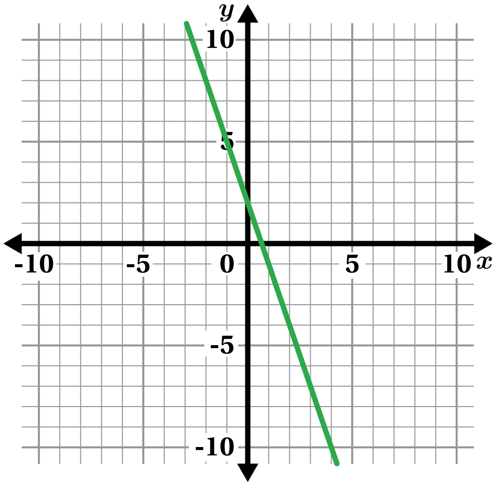
Spiral Review

Problems 8–9: Write an equation for each line.

8. Equation: ....



9. Equation: ....

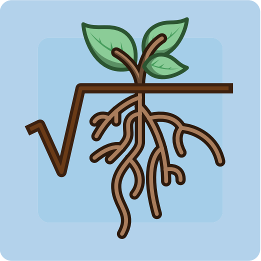


Name: .... Date: .... Period: ....

MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.NSO.1.4, MTR.2.1, MTR.3.1, MTR.4.1, MTR.5.1

Rule the Roots

Let's perform multiplication and division with radicals.



Warm-Up

1. Write a number or expression that is equivalent to 3^4.

2. **Think-Pair-Share:** Compare your expression to your partner's. Did you write the same thing, or do your expressions look different?

Name: .... Date: .... Period: ....

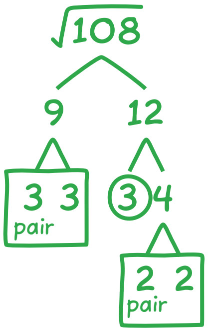
Radicals Two Ways

Amara and Nelly are working on simplifying $\sqrt{108}$. Their work is shown below.

Amara's Work
Rational Exponents

$$\sqrt{108} = \sqrt{9 \cdot 12}$$
$$= \sqrt{3^2 \cdot 2^2 \cdot 3}$$
$$= \sqrt{3^3 \cdot 2^2}$$
$$= (3^3)^{\frac{1}{2}} \cdot (2^2)^{\frac{1}{2}}$$
$$= 3^{\frac{3}{2}} \cdot 2^{\frac{2}{2}}$$
$$= 3^{\frac{1}{2}} \cdot 3^1 \cdot 2^1$$
$$= 6\sqrt{3}$$

Nelly's Work
Factor Trees



$$\sqrt{108} = \sqrt{2^2 \cdot 3^2 \cdot 3}$$
$$= 2 \cdot 3\sqrt{3}$$
$$= 6\sqrt{3}$$

3. **Think-Pair-Share:** Describe how each student approached simplifying $\sqrt{108}$.

4. Simplify $\sqrt{150}$ using both methods.

Name: .... Date: .... Period: ....

Radicals Two Ways (continued)

5. Simplify $\sqrt{72}$ using the method of your choice.

6. What method did you pick? Why did you choose that method?

7. Simplify $\sqrt[3]{72}$ using the method of your choice.

8. 💬 **Think-Pair-Share:** Compare the method you used in Problem 5 and Problem 7. What patterns do you notice when simplifying square roots versus cube roots?

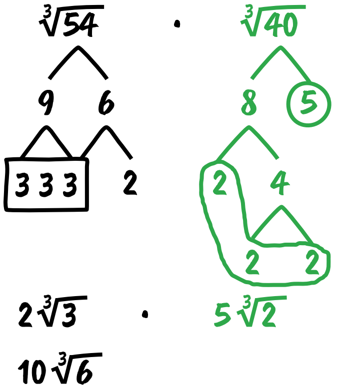
Activity
2

Name: ______________________ Date: ____________ Period: ____________

Multiply Radicals

9. Use Amara's strategy in Activity 1 to try to simplify $\sqrt{24} \cdot \sqrt{8}$. Write your answer as an exact quantity using at most a single radical.

10. Kenji used Nelly's strategy from Activity 1 to simplify $\sqrt[3]{54} \cdot \sqrt[3]{40}$ but made an error. Identify the error and give Kenji advice for next time.



11. Could Kenji have multiplied inside the radicands first and then simplified? Try it!

Name: .... Date: .... Period: ....

Multiply Radicals (continued)

12. Multiply and simplify the radicals. Write your answer as an exact quantity using at most a single radical.

a $-6\sqrt{4} \cdot \sqrt{25}$

b $\sqrt[3]{8} \cdot \sqrt[3]{18}$

c $-2\sqrt{5} \cdot 3\sqrt{10}$

13. What do you notice about how multiplication affects the numbers inside the radical versus outside the radical?

Activity
3

Name: .... Date: .... Period: ....

Divide Radicals

14. Use the information you discovered in Problem 13 to simplify $\frac{\sqrt{100}}{\sqrt{4}}$.

15. Divide and simplify the radicals. Write your answer as an exact quantity using, at most, a single radical.

a $\frac{\sqrt[3]{54}}{\sqrt[3]{6}}$

b $\frac{15\sqrt{24}}{5\sqrt{6}}$

c $\frac{-\sqrt[3]{216}}{\sqrt[3]{8}}$

16. Do you think these properties will work the same way when adding or subtracting radicals? Why or why not?

Synthesis

17. Describe how rational exponents and tree diagrams can help you simplify radical expressions.

Use this radical if it helps you explain your thinking.

$$\sqrt[3]{32}$$

Lesson Practice 5.10

Lesson Summary

- **Rational exponents and tree diagrams** help simplify radicals—rational exponents follow exponent rules, while tree diagrams break numbers into prime factors to find perfect squares or cubes.
- **Multiplying and dividing radicals** involves combining or separating radicands before simplifying.
  - $\sqrt[n]{a} \bullet \sqrt[n]{b} = \sqrt[n]{a \bullet b}$ and $\frac{\sqrt[n]{a}}{\sqrt[b]{b}} = \sqrt[n]{\frac{a}{b}}$

Lesson Practice
5.10

Name: .... Date: .... Period: ....

Problems 1–6: Simplify the radical expressions. Write your answer as an exact quantity using only a single radical.

1. $\sqrt[3]{4} \bullet \sqrt[3]{16}$

2. $2\sqrt{6} \bullet -3\sqrt{2}$

3. $\dfrac{\sqrt[3]{250}}{\sqrt[3]{5}}$

4. $-4\sqrt{27} \bullet 8\sqrt{18}$

5. $\sqrt[3]{9} \times \sqrt[3]{27}$

6. $\dfrac{10\sqrt{50}}{-2\sqrt{2}}$

7. Boris was solving a problem. Review his work, find the mistake, and correct it.

$$\frac{8\sqrt{75}}{-2\sqrt{5}}$$

$$\frac{4\sqrt{75}}{-1\sqrt{5}}$$

$$= -4\sqrt{75-5}$$

$$= -4\sqrt{70}$$

Lesson Practice
5.10

Name: ________ Date: ________ Period: ________

Test Practice

8. Which of the following is equivalent to $\frac{6\sqrt{72}}{-3\sqrt{2}}$?

A. $-6\sqrt{6}$

B. -12

C. $-6\sqrt{2}$

D. $-12\sqrt{3}$

Spiral Review

9. Is $x = 4$ a solution to the inequality $40 + 5x > 10 + 8x$?

Problems 10–12: Solve the equations. Check your work by plugging your answers back in.

10. $4(2x - 3) = 3(x + 5) + x$

11. $5(x - 4) + 2x = 3(2x + 1) + 6$

12. $6(3x + 2) - \frac{1}{2}(4x - 3) = 5x$

Unit 5

Lesson

11

Name: ____________________ Date: __________ Period: __________

MA.912.NSO.1.4, MTR.2.1, MTR.1.1, MTR.4.1, MTR.5.1

Tame the Terms

Let’s perform addition and subtraction with radicals.

Warm-Up

1. Which one doesn’t belong? Explain your thinking.

$$\sqrt{16},\ 8^{\frac{1}{3}},\ 25,\ 2\sqrt{2}$$

Activity
1

Name: .... Date: .... Period: ....

Radical Addition

2. **Think-Pair-Share:** Explain why $5x + 2x = 7x$ but $5x + 2y \neq 7xy$.

3. Using your reasoning from Problem 2, try to simplify the expression $6\sqrt{5} + 9\sqrt{5}$. (Hint: Try rewriting the radicals as expressions with rational exponents)

4. What do you notice about the result from Problem 3?

5. **Think-Pair-Share:** Would $3\sqrt[3]{2} + 3\sqrt[3]{4}$ simplify further? Why or why not?

Name: .... Date: .... Period: ....

Radical Addition (continued)

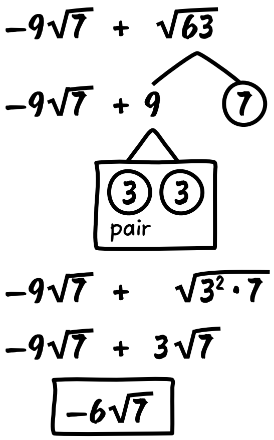
6. Simplify each expression.

a $\sqrt{5} + 4\sqrt{5}$

b $-5\sqrt[3]{19} + 8\sqrt[3]{19}$

c $\frac{4\sqrt{10}}{5} + \frac{2\sqrt{10}}{3}$

7. Mei is simplifying $-9\sqrt{7} + \sqrt{63}$. What do you notice? What do you wonder?



I notice:

I wonder:

8. Simplify each expression. Write your answer as an exact quantity using only a single radical.

a $4\sqrt{50} + 2\sqrt{8}$

b $\sqrt[3]{54} + \sqrt[3]{16}$

c $-2\sqrt[3]{81} + \sqrt[3]{24}$

Activity
2

Name: Date: Period:

Radical Subtraction

9. **Think-Pair-Share:** Kofi was wondering if radical addition works the same as radical subtraction. He was looking at $\sqrt{5} + 4\sqrt{5}$ and decided to change it to $\sqrt{5} - 4\sqrt{5}$. What do you think will happen?

10. Simplify each expression. Write your answer as an exact quantity using only a single radical.

a $\sqrt{75} - (-\sqrt{12})$

b $-4\sqrt[3]{54} - 2\sqrt[3]{16}$

11. Write a radical subtraction problem that simplifies to $\sqrt{2}$.

Name: ______ Date: ______ Period: ______

Find the Matching Pairs

You will use a set of cards for this activity.

12. Here are the instructions for the activity.

- Look over all 12 cards. Cards A–F represent expressions that need to be simplified and Cards 1–6 represent the simplified expressions.
- Use the boxes below to simplify the expressions on Cards A–F and find the correct matching answer within Cards 1–6.
- Compare your solutions and support each other to make adjustments as needed.

Card A simplifies to Card ______.	**Card B simplifies to Card ______.**
Card C simplifies to Card ______.	**Card D simplifies to Card ______.**
Card E simplifies to Card ______.	**Card F simplifies to Card ______.**

Synthesis

13. What must be true for two radical expressions to be added or subtracted?

$6\sqrt{3} + 2\sqrt{3}$

$8\sqrt{32} + 4\sqrt{8}$

Lesson Practice 5.11

Lesson Summary

- When working with radical expressions, you can only add or subtract terms that have the same radicand and index—just like combining like terms in algebraic expressions. If the radicals are different, you should simplify first to check if they can be rewritten in a way that allows you to combine them.

- You can only add or subtract radicals if they have the same index and radicand. Sometimes you may need to simplify each term first to determine whether the radicands are the same.

$$a\sqrt[n]{x} \pm b\sqrt[n]{x} = (a \pm b)\sqrt[n]{x}$$

$$\sqrt{50} + \sqrt{8} = 5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$$

- When simplifying, look for perfect square or perfect cube factors to rewrite each radical in its simplest form. This ensures that you do not overlook possible "like terms." If radicals cannot be simplified so the factors inside the radical and indexes match, they cannot be combined and must remain separate.

Lesson Practice

5.11

Name: .... Date: .... Period: ....

Problems 1–6: Simplify the radical expressions. If possible, write your answer as an exact quantity using only a single radical.

1. $2\sqrt{20} - \sqrt{45}$

2. $-4\sqrt{72} + 3\sqrt{32}$

3. $5\sqrt[3]{24} - 2\sqrt[3]{81}$

4. $-3\sqrt{98} + 4\sqrt{8}$

5. $6\sqrt[3]{16} - 9\sqrt[3]{54}$

6. $-\sqrt[3]{54} + 12\sqrt[3]{25}$

Lesson Practice
5.11

Name: .... Date: .... Period: ....

7. Write a radical addition or subtraction problem that results in $\sqrt[3]{3}$.

Test Practice

8. Which expression is equivalent to $-3\sqrt{72} + 4\sqrt{50}$?

[✓ A.] $-\sqrt{2}$

B. $-\sqrt{3}$

C. $2\sqrt{2}$

D. -5

Spiral Review

Problems 9–11: Solve each equation for the indicated variable.

9. $g = \frac{x}{c}$, for x

10. $z = ma$, for a

11. $u = k - a$, for k

Unit 5

Lesson

12

Name: Date: Period:

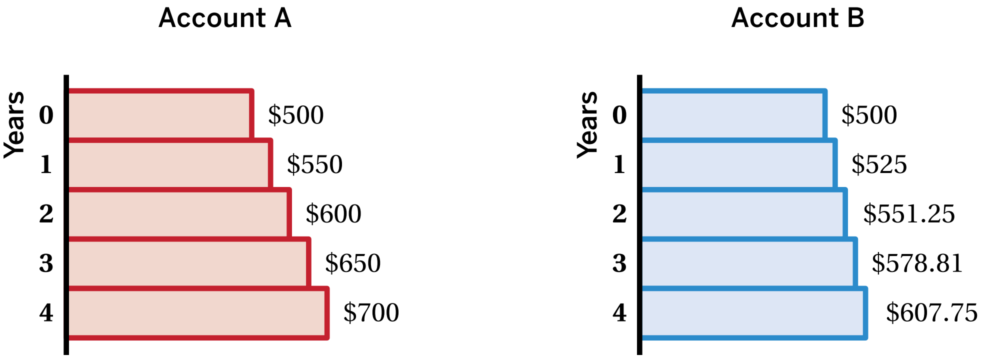
MA.912.FL.3.2, MA.912.FL.3.4, MA.912.F.1.6, MA.912.F.1.8, MTR.2.1, MTR.4.1, MTR.5.1, MTR.7.1

Bank Accounts

Let's learn how to model situations involving simple and compound interest.

Warm-Up

1. Mauricio has $500 to invest. He is researching different kinds of investment accounts. Here are the values of Accounts A and B over time.



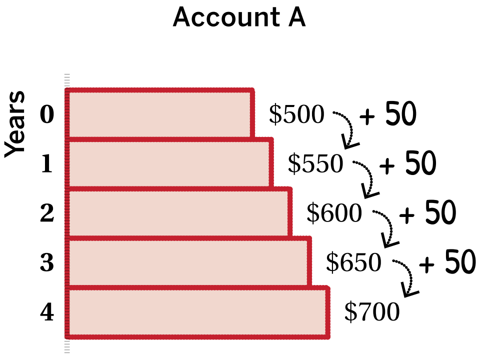
Show or describe how the value of each account grows over time.

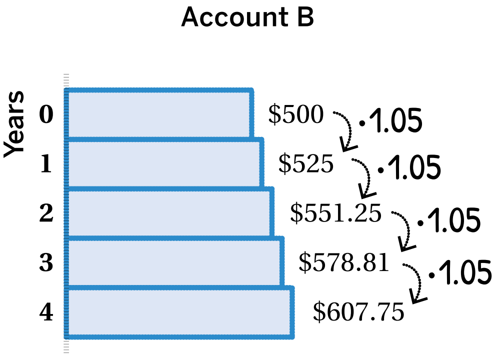
Name: .... Date: .... Period: ....

Earning Interest

3. Here is Mauricio's work to show how each account is growing.

Discuss: Describe his work to a partner. Which account would you recommend he invest in?



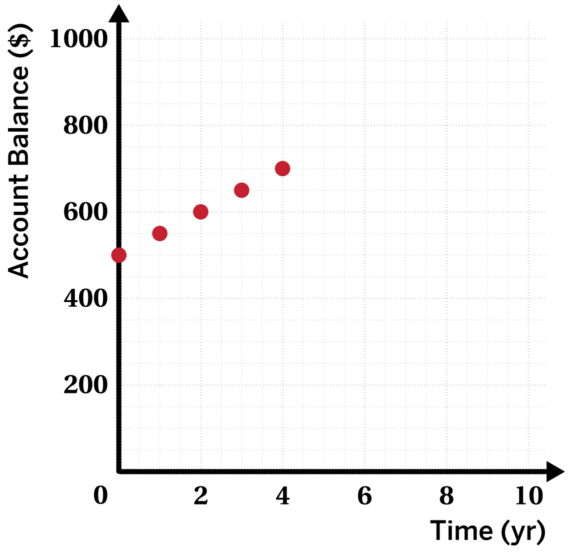


4. Account A earns 10% **simple interest** per year.

a **Discuss:** How do you think *simple interest* works?

b Determine the account balance after 5 years.

Time (yr)	Account Balance ($)
0	500
1	550
2	600
3	650
4	700
5	



Name: .... Date: .... Period: ....

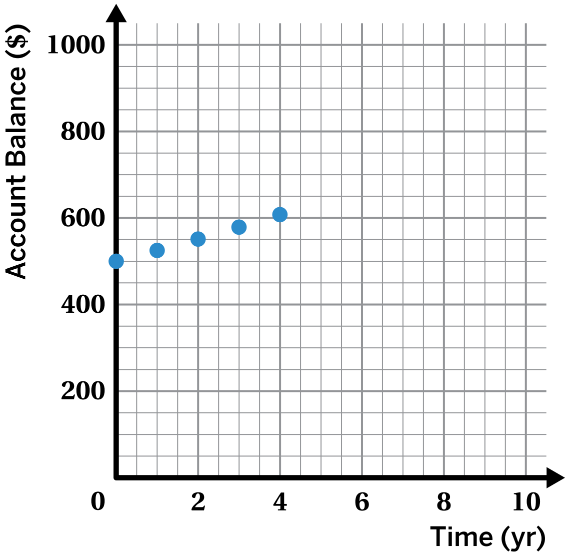
Earning Interest (continued)

5. Account B earns 5% **compound interest** per year.

a **Discuss:** How do you think *compound interest* works?

b Determine the account balance after 5 years.

Time (yr)	Account Balance ($)
0	500
1	525
2	551.25
3	578.81
4	607.75
5	

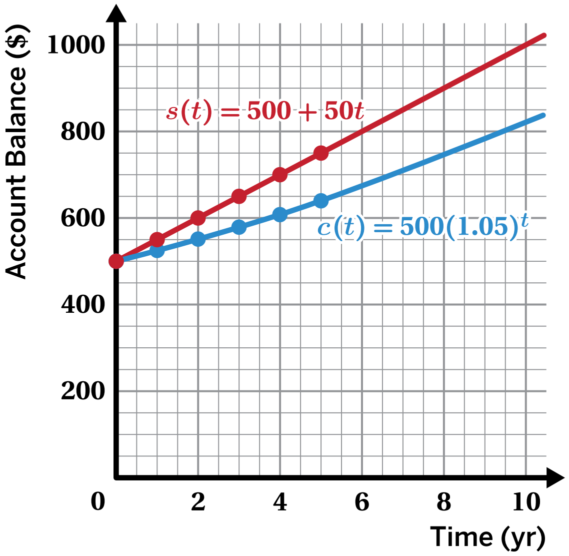


6. We can use functions to describe the account balances after t years.

- Simple interest: $s(t) = 500 + 50t$
- Compound interest: $c(t) = 500(1.05)^t$

How are these functions alike?

How are they different?



7. **Discuss:** Which account would you recommend Mauricio invest in? Why?

Name: .... Date: .... Period: ....

Simple and Compound Interest

8. Mauricio decided to invest in an account that offers 6% compound interest per year.

$a(t) = 500(1.06)^t$ represents its balance after t years.

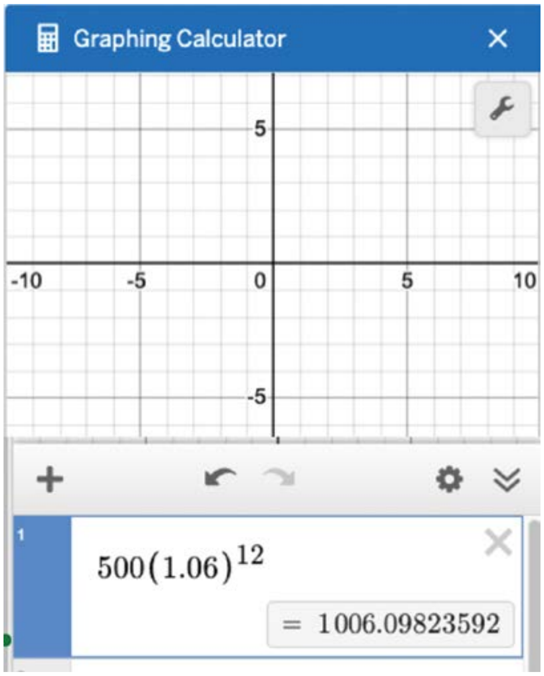
About how many years will it take for the balance to reach $\$1,000$? Explain your thinking.

Use this space or a graphing calculator to help with your thinking.

9. Below are different methods used by two students to determine the time it would take to reach $\$1,000$.

💬 **Discuss:** Which method helps get a more precise answer?

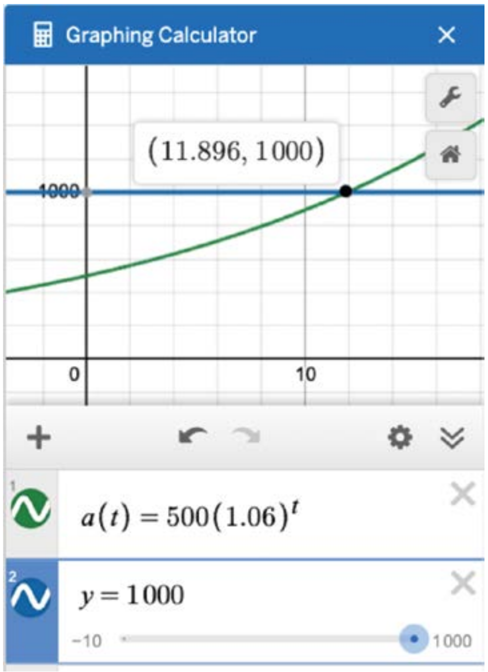
Fabiana



$500(1.06)^{12}$

$= 1006.09823592$

Antwon



$(11.896, 1000)$

$a(t) = 500(1.06)^t$

$y = 1000$

Name: Date: Period:

Simple and Compound Interest (continued)

10. Solve as many challenges as you have time for.

a A $\$1{,}000$ investment earns 4% compound interest.

The function $f(t) = 1000(1.04)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach $\$2{,}500$?

b A $\$200$ investment earns 7% compound interest.

The function $f(t) = 200(1.07)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach $\$450$?

c A $\$1{,}700$ investment earns 3% compound interest.

The function $f(t) = 1700(1.03)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach $\$4{,}150$?

d A $\$1{,}150$ investment earns 2% compound interest.

The function $f(t) = 1150(1.02)^t$ gives the account balance after t years.

About how many years will it take for the balance to reach $\$2{,}700$?

You're invited to explore more.

11. Use the You're Invited to Explore More Sheet to answer questions about an account balance.

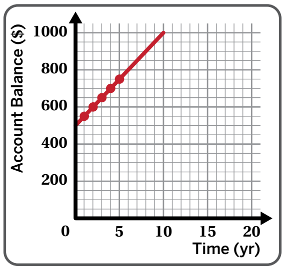
Synthesis

12. Here are some examples of simple and compound interest.

Simple Interest

$s(t) = 500 + 50t$

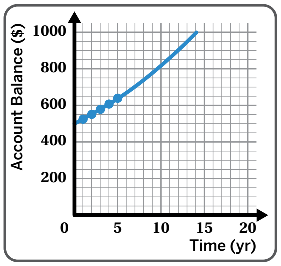
Time (yr)	Account Balance ($)
0	500
1	550
2	600
3	650
4	700



Compound Interest

$c(t) = 500(1.05)^t$

Time (yr)	Account Balance ($)
0	500
1	525
2	551.25
3	578.81
4	607.75



How do investments grow with simple interest?

How do investments grow with simple interest?

Lesson Practice 5.12

Lesson Summary

You can invest money in accounts that earn simple or compound interest. Accounts that earn **simple interest** can be modeled by linear functions, while accounts that earn **compound interest** can be modeled by exponential functions. Which account earns the most interest depends on how much time there is to invest and other variables.

Let's look at an example. Adah has $100 to invest in an account. Which account should she choose if she has 12 years to invest?

Simple Interest	Compound Interest
Adah could invest $100 in an account that earns 10% simple interest annually. The function $a(t) = 100 + 10t$ models the account balance after t years.	Adah could invest $100 in an account that earns 10% compound interest annually. The function $b(t) = 100 \cdot (1.10)^t$ models the account balance after t years.
To determine the balance of the account after 12 years, substitute $t = 12$ into each function and solve for $a(t)$.	
$a(12) = 100 + 10(12)$ $a(12) = 220$ After 12 years, the account balance will be $220.	$b(12) = 100 \cdot (1.10)^{12}$ $b(12) = 313.84$ After 12 years, the account balance will be about $313.84.

Adah may choose to invest in the account that earns compound interest because it earns more money over 12 years.

Lesson Practice
5.12

Name: .... Date: .... Period: ....

Problems 1–4: Determine if each equation or table represents simple or compound interest. Circle your choice.

1. $b(t) = 1000(1.03)^t$

Simple Compound

2. $b(t) = 1000 + 30t$

Simple Compound

3.

Time (yr)	Account Balance ($)
0	300
1	330
2	360

Simple Compound

4.

Time (yr)	Account Balance ($)
0	200
1	230
2	264.50

Simple Compound

Problems 5–7: Jin invests $4,000 in an account that earns 5% compound interest per year.

Time (yr)	Account Balance ($)
0	
1	4,200
2	4,410
3	
4	

5. Complete the table.

6. Which function represents the amount of money in Jin's account after x years?

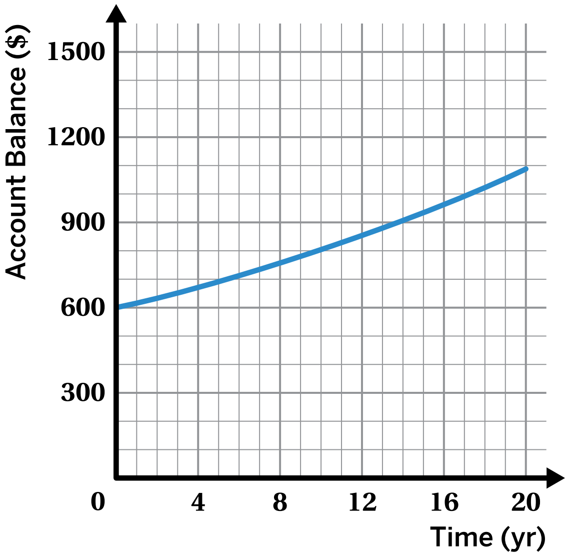
- **A.** $f(x) = 4000 + 1.05x$
- **B.** $f(x) = 4000(1.05)^x$
- **C.** $f(x) = 4000(0.05)^x$
- **D.** $f(x) = 4000 + (1.05)^x$

7. What will the balance of the account be after 10 years?

Lesson Practice
5.12

Name: .... Date: .... Period: ....

Problems 8–9: Keya invests \$600 in an account that earns 3% compound interest per year. The graph shows the function $f(t) = 600(1.03)^t$, which gives Keya's account balance after t years.



8. About how many years will it take for her account balance to reach \$1,000?

9. Use the graph to determine the value of $f(14)$.

What does that tell you about the situation?

10. You just won a contest and have two prize options.

- **Option A:** One payment of \$20 million
- **Option B:** 2 cents on day one, 4 cents on day two, 8 cents on day three, and so on, for 30 days

Which option would you choose? Explain your choice.

Test Practice

11. Which of the following grows the fastest?

A. $y = 400 + 80t$

B. $y = 400(1.08)^t$

C. $y = 400t^2$

D. $y = 400(0.08)^t$

Spiral Review

Problems 12–14: Determine whether each function is linear, exponential, or something else. Circle your choice.

12. $f(x) = x^2 + 5$ Linear Exponential Something else

13. $g(x) = 2x + 5$ Linear Exponential Something else

14. $h(x) = 2x^2 + 5$ Linear Exponential Something else

Unit 5

Lesson

13

Name: Date: Period:

MA.912.FL.3.2, MA.912.FL.3.4, MA.912.NSO.1.1, MA.912.NSO.1.2, MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1

Payday Loan

Let's analyze exponential functions that represent different compound interest scenarios.

Warm-Up

1. Zola says that $x^{12} = (x^4)^3$.

The diagram shows why that is true.

$$\underbrace{(x \cdot x \cdot x \cdot x)}_{x^4}\,(x \cdot x \cdot x \cdot x)\,(x \cdot x \cdot x \cdot x)$$

Write three other expressions equivalent to x^{12}.

Activity
1

Name: ______ Date: ______ Period: ______

Payday Loan

1. A payday loan is a short-term loan designed to be paid back within a month.

Here is an advertisement for a payday loan.

Discuss: What do you notice? What do you wonder?

FASTCASH

PAYDAY LOAN

NO CREDIT CHECK!*

Settle Up by Your Next Payday

Bad credit? No problem! $100

Up to $1000 now! $500

Immediate approval! $1000

APPLY NOW

GET APPROVED

GET CASH FAST

*Proof of regular paychecks required.

2. FastCash offers payday loans that charge 15% compound interest per month.

Marc borrows $\$100$ to help pay his heating bill.

How much will Marc owe after one month?

Explain your thinking.

Name: ________ Date: ________ Period: ________

Payday Loan (continued)

4. The function $f(m)$ represents how much Marc will owe if he doesn't pay back the loan for m months. Write an equation to represent $f(m)$.

$f(m) =$ ________

Months, m	Amount Owed, $f(m)$
0	100
1	115
2	132.25
3	152.09
4	174.90

5. Marc wonders how much money he would owe if he doesn't pay back the loan after 3 years.

He wrote two expressions to represent this situation.

Expression A

$100 \cdot 1.15^{36}$

Expression B

$100 \cdot (1.15^{12})^3$

Discuss: How are the expressions alike? How are they different?

6. Marc wrote a third equivalent expression to represent this situation.

Expression C

$100 \cdot (5.35)^3$

What interest rate does the 5.35 represent?

A. 435% per year

B. 535% per year

C. Neither

Explain your thinking.

Name: .... Date: .... Period: ....

Credit Cards and Other Loans

7. Marc sees an advertisement for a credit card that charges a 2% monthly interest rate.

How much would he owe for a $100 charge on the credit card after 3 years of no payments?

Payday Loan

- $100 loan
- 15% monthly interest

Amount owed after
3 years of no payments:
$15315.19

Credit Card

- $100 charge
- 2% monthly interest

Amount owed after
3 years of no payments:
?

8. Here are three equivalent functions that represent the amount owed on a credit card charge of $100 after t years of 2% monthly interest.

- $g(t) = 100 \bullet 1.02^{12t}$
- $g(t) = 100 \bullet (1.02^{12})^t$
- $g(t) = 100 \bullet 1.2682^t$

Use one or more of the functions to determine the interest rate *per year*.

Explain your thinking.

Activity

3

Name: Date: Period:

Comparing Rates

9. Marc wants to compare interest rates on different types of loans.

a Complete the table.

	Monthly Interest Rate (%)	Monthly Growth Factor	Growth Factor per Year	Interest Rate per Year (%)
Payday Loan	15.00	1.15	5.3503	435.03
Credit Card	2.00	1.02	1.2682	
Private Loan	1.21	1.0121		
30-year Mortgage	0.53			
Federal Student Loan	0.41			

b **Discuss:** In what situations might people take out each of these different types of loans?

Synthesis

10. What can different equivalent expressions tell us about the same situation involving compound interest?

Expression A
$100 \cdot 1.15^{36}$

Expression B
$100 \cdot (1.15^{12})^3$

Expression C
$100 \cdot (5.35)^3$

Lesson Practice 5.13

Lesson Summary

You can write exponential expressions representing compound interest in multiple equivalent ways to reveal different information about the account and situation.

Here is an example.

The amount owed on a $\$400$ loan has a monthly interest rate of 3%. Let t represent the number of years since taking out the loan if no payments are made.

- The expression $400 \cdot 1.03^{12t}$ represents a 3% monthly interest rate 12 times each year, t.
- The expression $400 \cdot (1.03^{12})^t$ uses the powers of powers law to help us think about the interest rate for every t year.
- Since $1.03^{12} = 1.4258$, the expression $400 \cdot (1.4258)^t$ shows the annual interest rate of 42.58%.

Each expression reveals different information about the monthly and annual interest rates applied to the account. While a monthly interest rate of 3% may not seem like it impacts the account balance much, the annual interest rate reveals that the loan amount is increasing by 42.58% each year, and that really adds up!

Lesson Practice
5.13

Name: ______________________ Date: ____________ Period: ____________

Problems 1–2: Alina takes out a $\$1{,}000$ loan with a monthly interest rate of 5%. She makes no additional payments, deposits, or withdrawals.

1. Select *all* the expressions that can be used to calculate her balance after t years.

☐ **A.** $1000 \bullet 1.05^t$

☐ **B.** $1000 \bullet 1.05^{12t}$

☐ **C.** $1000(1.05^{12})^t$

☐ **D.** $1000 \bullet 1.7958^t$

☐ **E.** $1000(1.7958)$

2. What is the interest rate per year for this loan?

Problems 3–7: Alejandro invests money into a college savings account. He writes the expression $750(1.006^{12})^3$ to help him calculate what the account balance will be in 3 years.

3. Explain what 750 represents in the expression.

4. Explain what 1.006 represents in the expression.

5. Explain what 12 represents in the expression.

6. Explain what 3 represents in the expression.

7. Write an equivalent expression that could represent Alejandro's account balance in 3 years.

Lesson Practice
5.13

Name: .... Date: .... Period: ....

Problems 8–9: Rebecca is considering taking out a payday loan that has a 17% monthly interest rate.

Monthly Interest Rate	17%
Monthly Growth Factor	
Growth Factor per Year	
Interest Rate per Year	

8. Complete the table.

9. If Rebecca takes out a \$300 payday loan, how much would she owe after 2 years if she made no additional payments?

Test Practice

10. Wohali takes out an \$8,000 loan with a monthly interest rate of 0.38%. Which function, $g(t)$, will calculate the amount Wohali owes after making no payments for t years?

- **A.** $g(t) = 8000 \cdot (1.0038)^{12t}$
- **B.** $g(t) = 8000 \cdot (0.0038)^{12t}$
- **C.** $g(t) = 8000 \cdot (0.9962)^{12t}$
- **D.** $g(t) = 8000 \cdot (1.38)^{12t}$

Spiral Review

Problems 11–13: Determine the value of each function when $n = 2$.

11. $f(n) = 4 \cdot 2^n$

12. $g(n) = 2 \cdot 4^n$

13. $h(n) = 8 + 2^n$

14. Using the digits 0 to 9, without repeating, fill in each blank to create four equivalent expressions.

$$7^{\square} = 7^{\square} \times 7^{\square} = 7^{\square} \times 7^{\square} \times 7^{\square} = \left(7^{\square}\right)^{\square}$$

Unit 5

Lesson

14

Name: .... Date: .... Period: ....

MA.912.FL.3.2, MA.912.FL.3.4, MA.912.NSO.1.1, MTR.2.1, MTR.4.1, MTR.6.1, MTR.7.1

Credit Card Compounding

Let's explore how to calculate and compare account balances with interest rates that compound at different intervals.

Warm-Up

1. Group each card with the word that it describes.

Card A	**Card B**	**Card C**	**Card D**
2 times per year	4 times per year	$\frac{1}{12}$ of a year	$\frac{1}{4}$ of a year
Card E	**Card F**	**Card G**	
Every 3 months	Every 6 months	Every 12 months	

Monthly	Quarterly	Semi-Annually	Annually

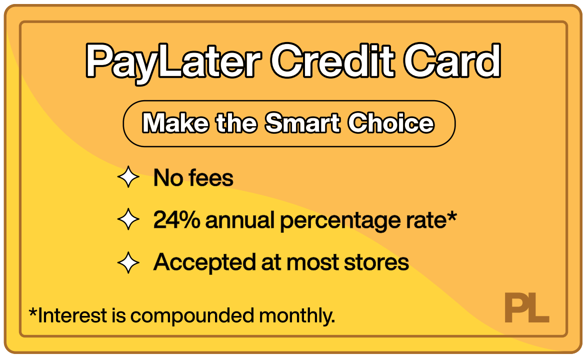
Name: ________ Date: ________ Period: ________

PayLater

2. Alejandro is considering charging $\$1,000$ to this credit card.

He wrote $1000(1 + 0.24)^5$ to determine the balance after 5 years with no payments or additional charges.

Explain what each part of the expression means.



1000:

$1 + 0.24$:

5:

3. The fine print says interest is compounded monthly.

This means the interest is $\frac{24}{12} = 2$, or 2% per month.

Compared to compounding annually, how do you think compounding monthly will affect the total Alejandro owes after 5 years? Circle one.

A. He will owe more

B. He will owe less

C. He will owe the same

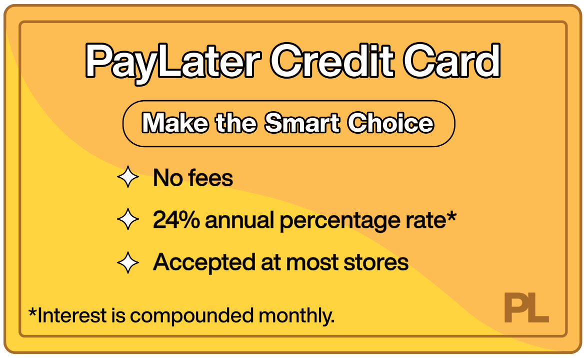
Explain your thinking.

Name: .... Date: .... Period: ....

PayLater (continued)

4. Alejandro is considering charging $\$1{,}000$ to this credit card.

If the interest is compounded at 2% monthly, how much would he owe after 5 years?



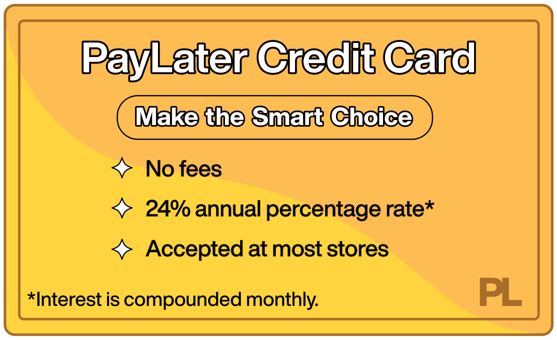
5. Alejandro wrote $1000\left(1+\frac{0.24}{12}\right)^5$ to determine the balance after 5 years, but he made an error.

Find the error and explain why it is incorrect.

Name: .... Date: .... Period: ....

PayLater and Flash Bucks

6. Alejandro is considering charging $\$1,000$ to a different credit card.



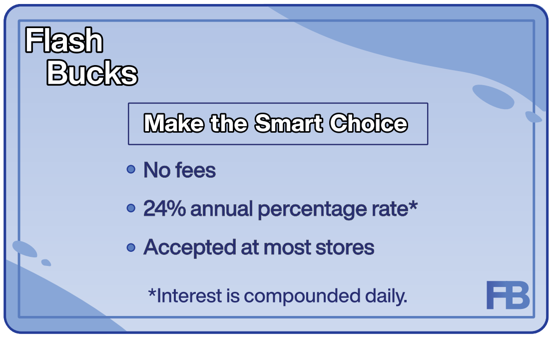
PayLater Credit Card

Make the Smart Choice

- No fees
- 24% annual percentage rate*
- Accepted at most stores

*Interest is compounded monthly.

PL



Flash Bucks

Make the Smart Choice

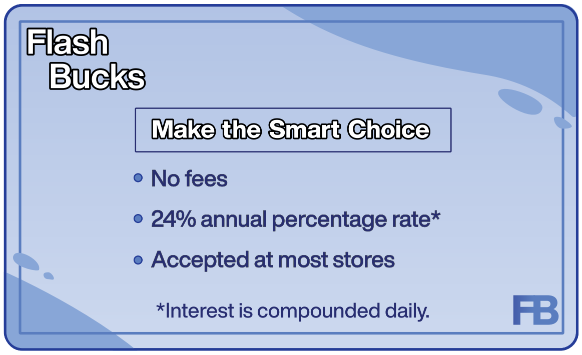
- No fees
- 24% annual percentage rate*
- Accepted at most stores

*Interest is compounded daily.

FB

Discuss: Compared to compounding monthly, how do you think compounding daily will affect the total amount owed?

7. Alejandro is considering charging $\$1,000$ to this credit card.



Flash Bucks

Make the Smart Choice

- No fees
- 24% annual percentage rate*
- Accepted at most stores

*Interest is compounded daily.

FB

a **Discuss:** How would you determine the daily interest rate?

b If interest is compounded daily, how much would Alejandro owe after 5 years with no payments or additional charges?

Activity
3

Name: ______ Date: ______ Period: ______

Compounding Differently

8. Here are some expressions to calculate the total amount for \$800 and a 12% annual interest rate compounded using different *intervals*.

Match each expression with its compounding period and length. One card will have no match.

Card A

$$800\left(1+\frac{0.12}{4}\right)^{(4\cdot 3)}$$

Card B

$$800(1+0.01)^{24}$$

Card C

$$800\left(1+\frac{0.12}{12}\right)^{(12\cdot 2)}$$

Card D

$$800(1+0.04)^{(3\cdot 2)}$$

Card E

$$800(1+0.03)^{12}$$

Compounded Quarterly for 3 Years	Compounded Monthly for 2 Years

9. Compound interest expressions can be represented using this formula:

$$P\left(1+\frac{r}{n}\right)^{nt}$$

Circle one variable and describe what it represents.

P r n t

Name: ______ Date: ______ Period: ______

Compounding Differently (continued)

10. Solve as many challenges as you have time for.

a A person puts $\$500$ into an account with a 10% annual interest rate compounded quarterly.

What is the balance in the account after 4 years?

b A person puts $\$800$ into an account with a 5% annual interest rate compounded daily.

What is the balance in the account after 3 years?

c A person puts $\$3{,}000$ into an account with a 5% annual interest rate compounded daily.

What is the balance in the account after 7 years?

d A person puts $\$1{,}000$ into an account with a 20% annual interest rate compounded yearly.

What is the balance in the account after 8 years?

Synthesis

11. How can you use this formula to calculate the total value of an account or loan with compound interest?

$$P\left(1+\frac{r}{n}\right)^{nt}$$

Lesson Practice 5.14

Lesson Summary

When you take out a loan on a credit card, the annual interest rate may be compounded at different *intervals*, or different lengths of time.

You can use the formula $P\left(1+\frac{r}{n}\right)^{nt}$ to calculate the total amount in an account that accrues compound interest.

- P represents the initial amount of the loan. In finance this is often called the *principal*.
- r represents the interest rate in decimal form.
- n represents the number of compounding intervals in a year.
- t represents the time in years.

Common compounding periods:

Annually	Semi-annually	Quarterly	Monthly	Daily
$n = 1$	$n = 2$	$n = 4$	$n = 12$	$n = 365$

Let's look at the impact of compound interest applied at different intervals on a loan for $\$1,000$.

Interest	Owed in	Compounded Monthly	Compounded Quarterly	Compounded Annually
15% annually	5 years	$1000\left(1+\frac{0.15}{12}\right)^{12\cdot 5}$ $\approx \$2,107.18$	$1000\left(1+\frac{0.15}{4}\right)^{4\cdot 5}$ $\approx \$2,088.15$	$1000(1+0.15)^5$ $\approx \$2,011.36$

Lesson Practice
5.14

Name: .... Date: .... Period: ....

1. Tyrone puts $2,500 into a savings account with a 1.2% annual interest rate, compounded semi-annually. He makes no additional payments, deposits, or withdrawals.

Select *all* the expressions that can be used to calculate his balance after 3 years.

☐ **A.** $2500\left(1+\frac{0.012}{2}\right)^{3 \cdot 2}$

☐ **B.** $2500\left(1+\frac{0.012}{6}\right)^{6}$

☐ **C.** $2500(1+0.012)^{3}$

☐ **D.** $2500(1+0.006)^{6}$

☐ **E.** $2500\left(1+\frac{0.012}{3}\right)^{3 \cdot 2}$

Problems 2–4: Maneli wants to take out a $5,000 loan to help pay for a new washing machine and dryer. The bank offers her the loan with an 18% annual interest rate, compounded quarterly.

Maneli wrote this expression to calculate the balance of the loan in 2 years, but she made an error.

$$5000\left(1+\frac{0.18}{2}\right)^{(4 \cdot 2)}$$

2. Find the error and explain why it is incorrect.

3. Write a correct expression to represent Maneli's balance after 2 years.

4. What will her balance be in 2 years?

Problems 5–6: A payday loan company offers a $1,000 loan with a 25% annual interest rate.

5. If no other charges or payments are made, what will the balance of the loan be after 1 year at each compounding period?

Compounding Period	Balance ($)
Annually	
Monthly	
Daily	

6. Describe how changing the compounding period affects the balance of the loan.

Lesson Practice
5.14

Name: .... Date: .... Period: ....

Problems 7–8: Xavier has $5,000 to invest and has to choose between three investment options.

- Option A: 2.25% interest applied each quarter
- Option B: 3% interest applied every 4 months
- Option C: 4.5% interest applied twice each year

7. Write an expression for each account to represent Xavier's balance after 5 years.

Test Practice

8. How much would Xavier have with Option C after 5 years?

A. $5,450

B. $6,246.02

C. $26,137.66

D. $38,047.92

Spiral Review

Problems 9–10: Irene needs to make at least 25 dinners for a party, including chicken dinners and vegetarian dinners. She has $250 to spend. Chicken dinners cost $8.75 each and vegetarian dinners cost $5.50 each.

- c represents the number of chicken dinners.
- v represents the number of vegetarian dinners.

9. Write a system of inequalities that represents Irene's constraints.

10. Can Irene make 5 chicken dinners and 20 vegetarian dinners? Explain your thinking.

Unit 5
Lesson
15

Name: Date: Period:

MA.912.FL.3.2, MA.912.FL.3.4, MTR.2.1, MTR.4.1, MTR.7.1

Exploring Interest

Let's compare the relationships between different types of interest and different types of functions.

Warm-Up

1. Nadine is offered a birthday gift. She can choose between the following two options.

Option 1: $\$100$ per year for ten years.

Option 2: Invest $\$100$ with 5% annual interest compounded yearly for 10 years.

Decide and Defend: Which option should Nadine choose and why?

2. Calculate how much money each option accrues after 10 years. Does this change your answer to Problem 1?

Activity

1

Name: .... Date: .... Period: ....

Laptop Savings

Amara wants to save money to eventually buy a new laptop. She finds a bank that offers a simple interest savings account with an annual interest rate of 2%. Amara deposits $500 into the account and plans to leave it there for several years without adding or withdrawing money.

3. Amara tracks her account balance over time and notices that the total amount increases by the same amount each year.

a Use the simple interest formula, $A = P(1 + rt)$, to write the growth equation for Amara's money.

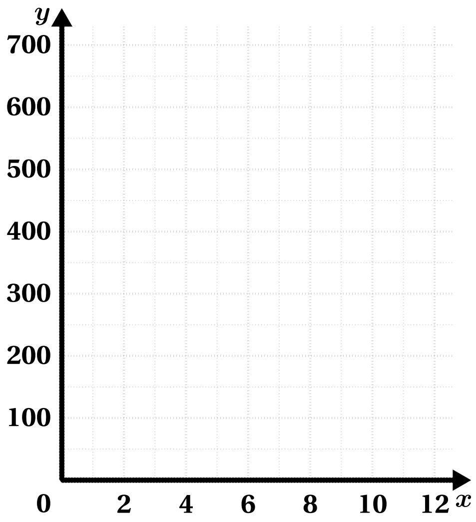
b Compare your formula from part A to a linear equation.

c Explain why the growth in her account is linear and describe what her account balance will look like over time.

4. Amara wants to visualize the progress of her account over the next ten years.

a What will the graph look like?

b Graph the function and label two points.



Activity
2

Name: .... Date: .... Period: ....

Super Savings

Mateo invests $\$1{,}000$ in a savings account with an annual interest rate of 4%, compounded quarterly. His cousin Sofia invests the same amount of $\$1{,}000$, but chooses an account with an annual interest rate of 4%, compounded continuously. Both plan to leave their money in the accounts for 10 years.

In the formula for interest compounded continuously, e is a constant. It is an irrational number approximately equal to 2.71828.

Compound Interest Formula: $A = P\left(1+\frac{r}{n}\right)^{n \cdot t}$

Compound Continuously Formula: $A = P \cdot e^{r \cdot t}$

5. Create an equation for Mateo's and Sofia's accounts and simplify.

6. Is the growth in Mateo's and Sofia's accounts linear or exponential? How do you know?

7. 💬 **Discuss:** How does the growth of both accounts differ over time?

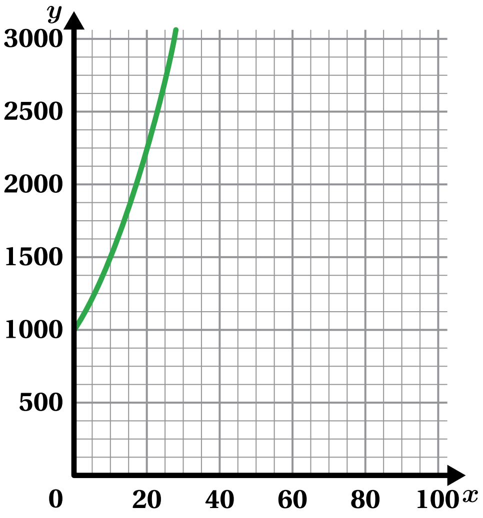
Name: .... Date: .... Period: ....

Super Saving (continued)

8. Mateo wonders what his account would look like if he used simple interest instead of compound interest.

a What is the simple interest formula for Mateo?

b Graph the simple interest function on the same graph as the exponential.



c **Compare** the two functions. Which option should Mateo pick if the money will be in the account for ten years?

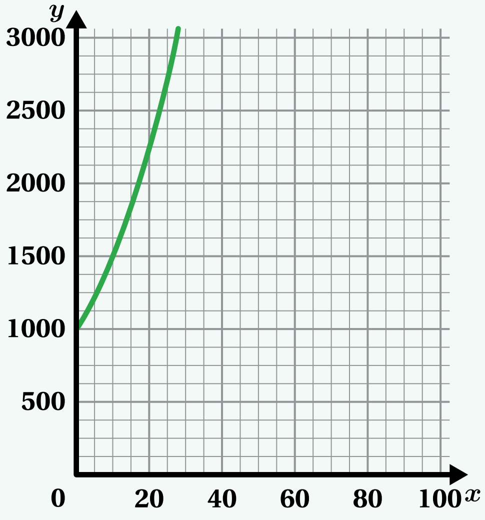
Synthesis

9. Compare and contrast simple and compound interest.

Lesson Practice 5.15

Lesson Summary

- **Simple interest** grows linearly because interest is based only on the original principal.
- **Compound interest** grows exponentially because each interest calculation adds to the balance, creating a feedback loop.
- **Continuous compounding** grows faster than periodic compounding, as interest is added at every moment.



Lesson Practice
5.15

Name: .... Date: .... Period: ....

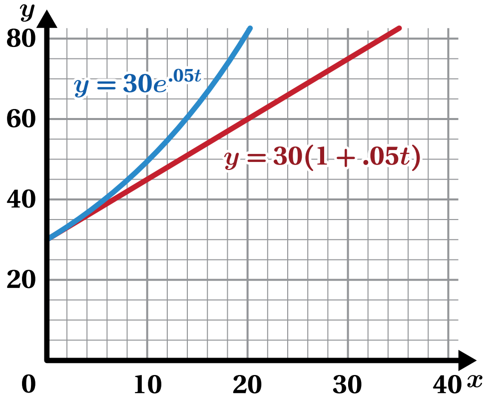
1. A savings account offers simple interest at a rate of 5% annually. If you deposit $800, how much will your total balance be after 3 years?

2. Why does simple interest result in a straight-line graph when plotted over time?

3. Which of the following situations best represents linear growth?

   [✓ A.] A savings account with simple interest
   B. A population doubles every five years
   C. A stock portfolio growing by 6% per year
   D. An allowance that doubles every week

4. Write a story about the two graphs.



5. If $500 is invested in an account with 6% annual interest compounded quarterly, what is the balance after 2 years? Write the equation and then solve it.

Lesson Practice
5.15

Name: ________ Date: ________ Period: ________

6. A student claims that compound interest grows linearly over time. Explain why this is incorrect.

7. Calculate the balance after 5 years if $\$1{,}000$ is invested in an account with 4% annual interest compounded continuously.

Test Practice

8. Which of the following statements correctly describes the relationship between types of interest and their growth patterns? Select **two** correct answers.

☐ **A.** Simple interest represents linear growth because the total interest added is constant over time.

☐ **B.** Continuously compounded interest represents linear growth because the interest rate is constant.

☐ **C.** Compound interest represents exponential growth because the interest is calculated on the increasing principal.

☐ **D.** The formula for simple interest can be written as $y = Pe^{rt}$.

Spiral Review

Problems 9–12. Find the solutions to the systems of equations.

9. $5x + y = -19$

$-4x + 3y = 0$

10. $7x + 2y = 20$

$x - 6y = -16$

11. $6x + 9y = 15$

$4x - 6y = 22$

12. $4x + 6y = -28$

$-10x + 10y = 20$

Career Connection

How is city planning related to solving a puzzle?

KieferPix/Shutterstock.com.

Solving a puzzle can involve thinking about different strategies, trying out ideas, and thinking about constraints. One famous historical puzzle, the "Hundred Fowls Problem," was created by 5th century Chinese mathematician Zhang Quijian.

City planners think about constraints as they design cities to be attractive and functional. Some constraints might be geographic location, access to natural water sources, or available budget to spend. City planners might create systems of equations to help decide where to place buildings, public transit systems, and parks.

B.E.S.T. Mathematics Benchmark Connection

Many city planners rely on algebra concepts in their work. For example, they solve systems of linear equations (MA.912.AR.9.1) when they analyze various costs for materials and labor. They may also represent constraints as systems of equations or inequalities (MA.912.F.1.5) to determine what combination of supplies to use when they want to stay within budget.

Mathematical Thinking and Reasoning Connection

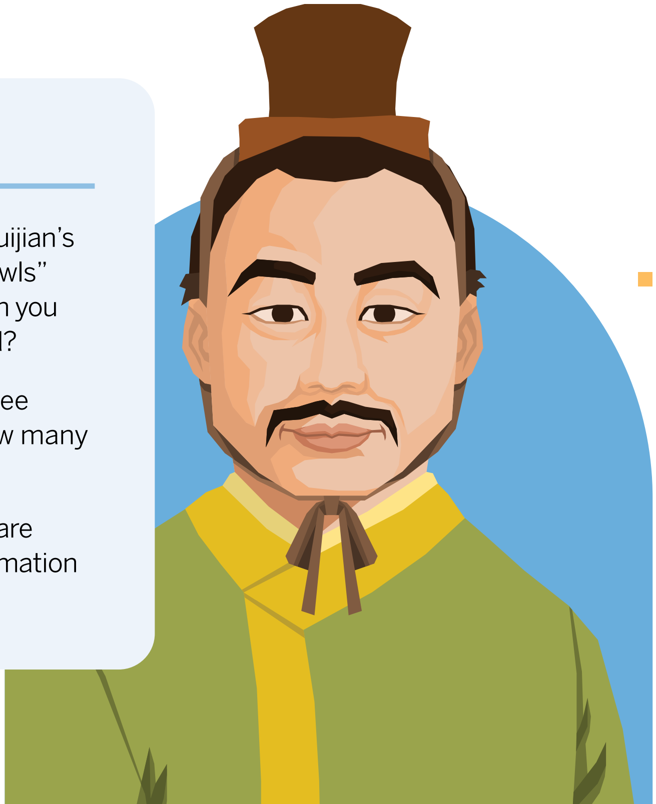
People who work on city planning use thinking and reasoning skills like the ones you use for your math work! For example, teams have different areas of expertise, so when they start a new project, they have to learn about the location, building materials, community requirements, and other key elements as individuals and as a group (MTR.1.1). They use multiple representations to explain budgets, schedules, supplies, and equipment needs so that the other teams involved in the planning understand their work (MTR.2.1).

Meet Zhang Qiujian

While little is known about Chinese mathematician Zhang Quijian's biography, he wrote a famous puzzle called the "Hundred Fowls" problem in his 5th century book, *Zhang Qiujian Suanjing*. Can you solve it? How can a system of equations help you get started?

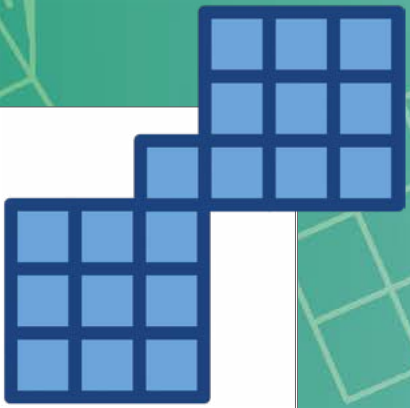
"Roosters cost 5 qian each, hens cost 3 qian each, and three chicks cost 1 qian. If 100 fowls are bought for 100 qian, how many roosters, hens and chicks are there?"

Mathematicians and others often study problems as if they are puzzles. They use equations and graphs to analyze the information they are given, and test values to find succesful solutions.



Unit 6

Quadratic Functions



What does a ball flying through the air have in common with a business estimating how much money they can make selling their products? Both situations can be modeled by quadratic functions. In this unit, you will learn about how quadratic relationships are different from linear and exponential relationships, and graph them. You will model situations with quadratic functions and make predictions and decisions based on your models.

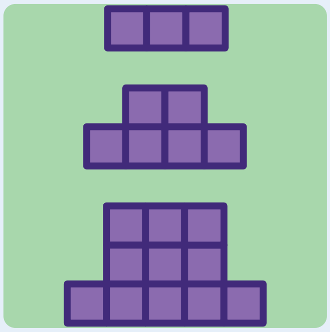
Essential Questions

- What are the important features of quadratic relationships?
- How can we graph quadratic functions?
- What situations can be modeled by quadratic functions?

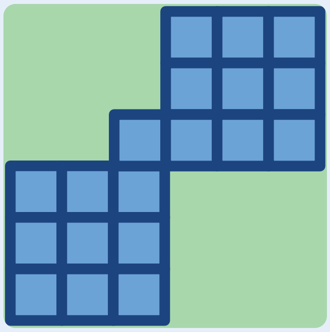
Sub-unit

1

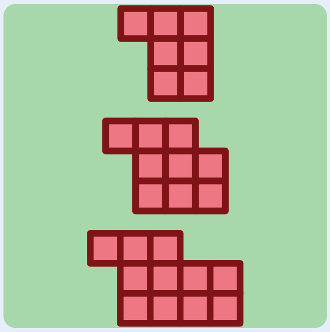
Introduction to Quadratic Functions



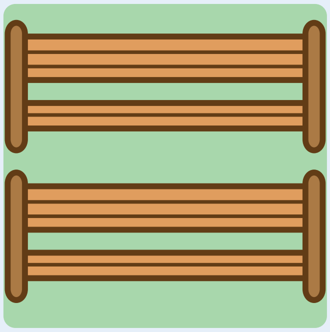
Lesson 1
Revisiting Visual Patterns



Lesson 2
Quadratic Visual Patterns



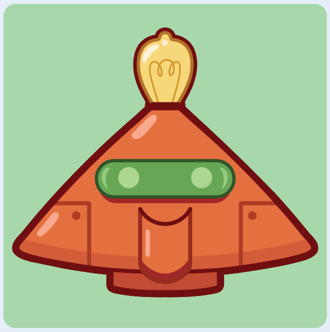
Lesson 3
Sorting Relationships



Lesson 4
On the Fence

Lesson 5
Stomp Rockets

Lesson 6
Plenty of Parabolas



Lesson 7
Robot Launch

Unit 6

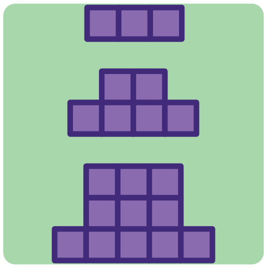
Lesson

1

Name: .... Date: .... Period: ....

MA.912.F.1.8, MA.912.F.1.1, MTR.2.1, MTR.5.1

Revisiting Visual Patterns

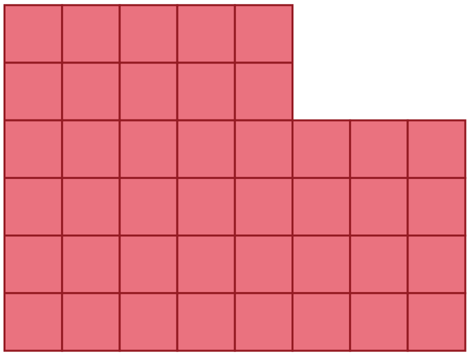


Let's explore a new type of visual pattern.

Warm-Up

1. Here are different ways of counting the tiles.

   Select one expression that you see represented in the diagram.



   A. $5 + 5 + 8 + 8 + 8 + 8$

   B. $4^2 + 4^2 + 10$

   C. $6 \bullet 8 - 2 \bullet 3$

   D. $5 \bullet 6 + 3 \bullet 4$

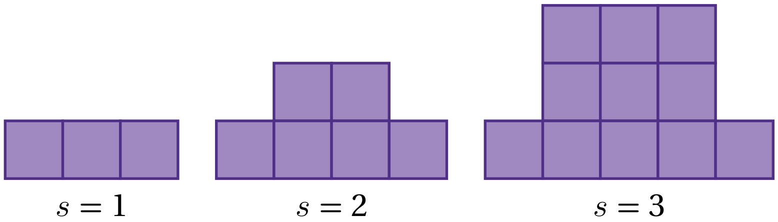
   Show or explain how you see this expression in the diagram.

Activity
1

Name: Date: Period:

A New Type of Pattern

2. Here are the first three steps of a pattern.

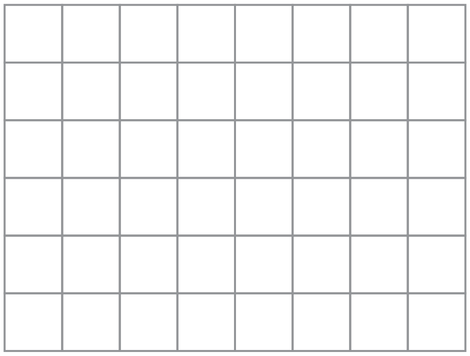


What about the pattern is changing? What is staying the same?

Things that are changing:

Things that are staying the same:

3. Draw the pattern when $s = 4$.



4. How many tiles will there be when $s = 10$?

Name: .... Date: .... Period: ....

A New Type of Pattern (continued)

5. Abdullah used a table to figure out how many tiles there will be when $s = 10$.

What type of relationship is there between s and the number of tiles?

A. Linear

B. Exponential

C. Something else

Explain your thinking.

S	Number of Tiles
1	3
2	6
3	11
4	18
5	27
6	38
7	51
8	66
9	83
10	?

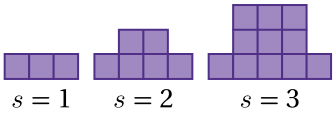
Activity
2

Name: .... Date: .... Period: ....

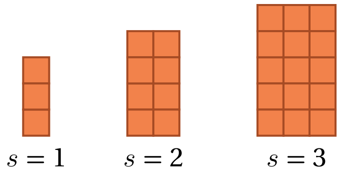
Comparing Patterns

6. Take a look at Pattern A and Pattern B.

Pattern A



Pattern B



How are the two patterns alike? How are they different?

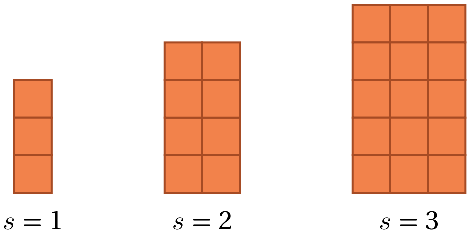
Alike:

Different:

6. Abdullah said: *I see a square plus two rows.*

Deja said: *I see a rectangle where the length is two more than the width.*

a Show how one student saw the pattern.



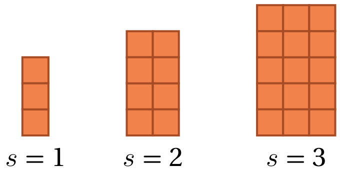
b **Discuss:** How might this student describe how to draw the image when $s = 4$?

Name: .... Date: .... Period: ....

Comparing Patterns (continued)

8. Determine the number of tiles when $s = 4$.

Determine the number of tiles when $s = 10$.



You're invited to explore more.

9. Create the first three steps of your own visual pattern.

$s = 1$ $s = 2$ $s = 3$

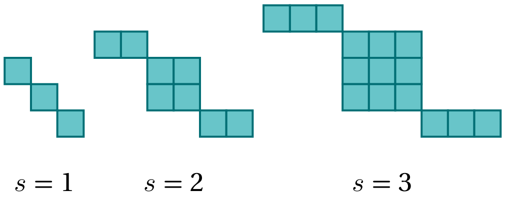
Discuss:

- What about your pattern is changing? What is staying the same?
- How many tiles will there be when $s = 4$? When $s = 10$?

Synthesis

10. What would you say to help a classmate who is trying to describe a pattern?

Use the example if it helps with your thinking.

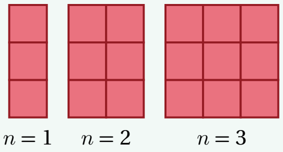


Lesson Practice 6.01

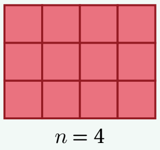
Lesson Summary

You can investigate visual patterns to determine how to build a particular figure or write an expression to represent the pattern.

Here is an example of the first three steps of a visual pattern.



You can describe what changes and what stays the same to help you draw the next figure, where $n = 4$.



Describing how you see a visual pattern changing can help you write a rule or expression to determine the other values in the pattern.

Here are two ways you may determine value of the tenth step of this pattern, or how many tiles are in $n = 10$.

- If you see the pattern increasing by a column of 3 each step, you could add 3 more until you reach $n = 10$.
- If you see each step in the pattern as a rectangle that has the dimensions 3-by-n, you could multiply 3 by 10 to find the total number of tiles.
- In both cases, you would determine that there will be 30 tiles when $n = 10$.

n	Number of Tiles
1	3
2	6
3	9
...	...
10	30

+3
+3
+3
+3

x3

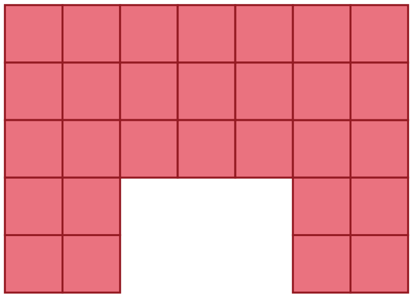
There are many ways to see and describe visual patterns accurately.

Lesson Practice

6.01

Name: .... Date: .... Period: ....

1. *Select all* of the expressions that could represent the number of tiles in this diagram.



☐ **A.** $5 \cdot 7$

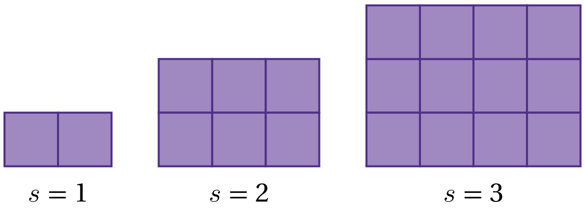
☐ **B.** $5 \cdot 7 - 6$

☐ **C.** $3 \cdot 7 + 2 \cdot 2$

☐ **D.** $5 \cdot 7 - 2 \cdot 3$

☐ **E.** $2 \cdot 5 + 3 \cdot 3 + 2 \cdot 5$

2. Here are the first three steps in a pattern. How many tiles will there be when $s = 10$?



Explain your thinking.

3. What type of relationship does the pattern in the table represent? Circle one.

Linear Exponential Neither

s	Number of Tiles
1	3
2	9
3	27
4	81

Explain your thinking.

Name: .... Date: .... Period: ....

Test Practice

Problems 4–6: A teacher gives her class a table with only the first two rows in the pattern.

s	Number of Tiles
1	5
2	25

4. Rishi says the pattern is an exponential relationship.

Ichiro says there is not enough information to be sure.

Whose thinking is correct? Explain your thinking.

5. How many tiles would be in the next step if the relationship were *linear*?

6. How many tiles would be in the next step if the relationship were *exponential*?

.... tiles

Spiral Review

7. Juana began hiking at 6:00 AM. At noon, she had hiked 12 miles. At 4:00 PM, Juana finished her hike with a total distance of 26 miles.

On average, during which time interval was Juana hiking faster? Circle one.

6:00 AM to noon

Noon to 4:00 PM

Explain your thinking.

Problems 8–10: Use the distributive property to write an equivalent expression for each.

8. $3(n + 5)$

9. $16x + 20$

10. $2(4m - 6)$

Unit 6

Lesson

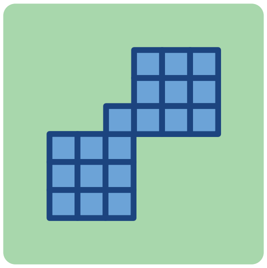
2

Name: .... Date: .... Period: ....

MA.912.AR.3.4, MA.912.F.1.8, MA.912.F.1.1, MTR.2.1, MTR.5.1

Quadratic Visual Patterns

Let's describe a new type of pattern using expressions.



Warm-Up

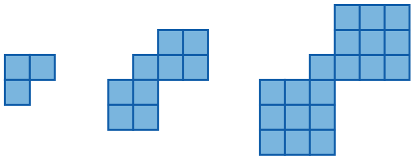
1. Here are Pattern A and Pattern B.

   How are the two patterns alike? How are they different?

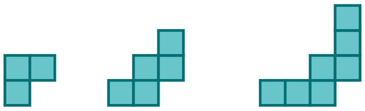
   Alike:

   Different:

Pattern A



Pattern B



Name: .... Date: .... Period: ....

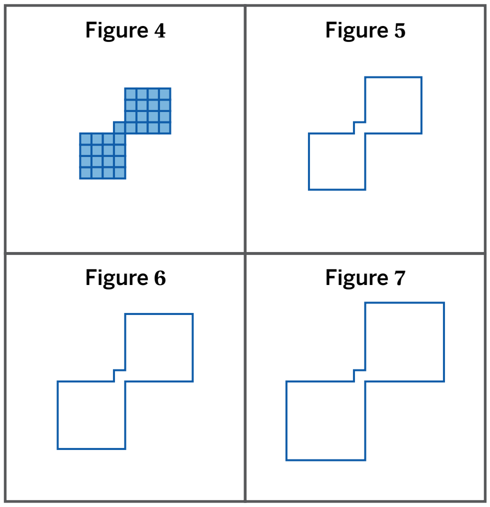
Figure n

2. Let's take a closer look at Pattern A.

Calculate the number of tiles for each figure.

Use an expression if it helps with your thinking.

Figure	Number of Tiles
4	33
5	
6	
7	



3. Here is one student's table from the previous problem.

Write an expression for the number of tiles in Figure n.

Figure	Number of Tiles
4	33
5	$5^2 + 5^2 + 1$
6	$6^2 + 6^2 + 1$
7	$7^2 + 7^2 + 1$
n	?

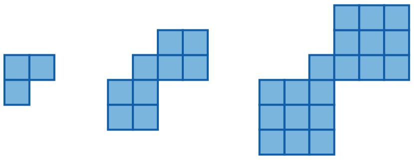
Name: .... Date: .... Period: ....

Writing Expressions

4. Take a look at Pattern A and Pattern C.

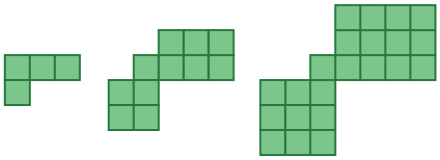
How are the two patterns alike? How are they different?

Pattern A



Alike:

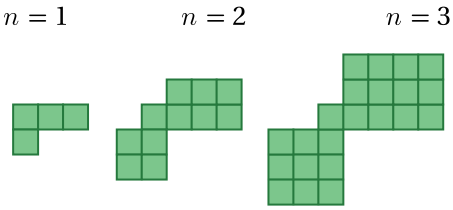
Pattern C



Different:

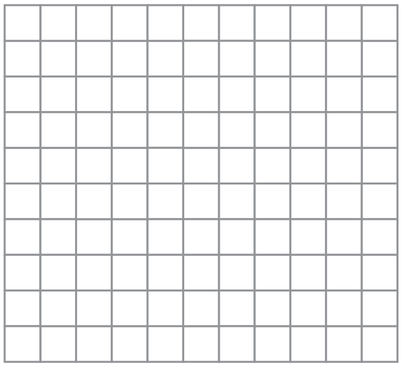
5. Here is Pattern C.

a Sketch the pattern for $n = 4$.



b Calculate the number of tiles when $n = 4$. Use an expression if it helps with your thinking.

$n = 4$



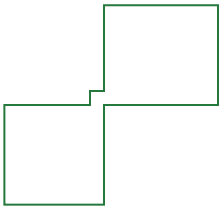
Name: .... Date: .... Period: ....

Writing Expressions (continued)

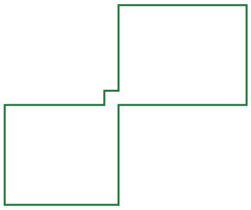
6. Here are three more figures of Pattern C.

Write an expression in terms of n to evaluate them all at once.

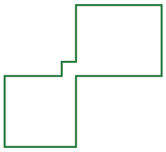
$n = 7$



$n = 8$

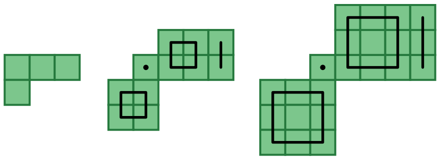


$n = 5$

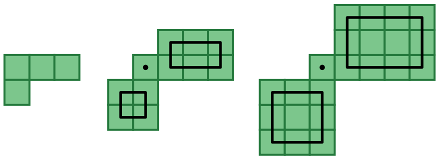


7. Let's look at expressions that two students wrote for Pattern C.

Luis: $n^2 + n^2 + n + 1$



Ishann: $n^2 + n(n + 1) + 1$



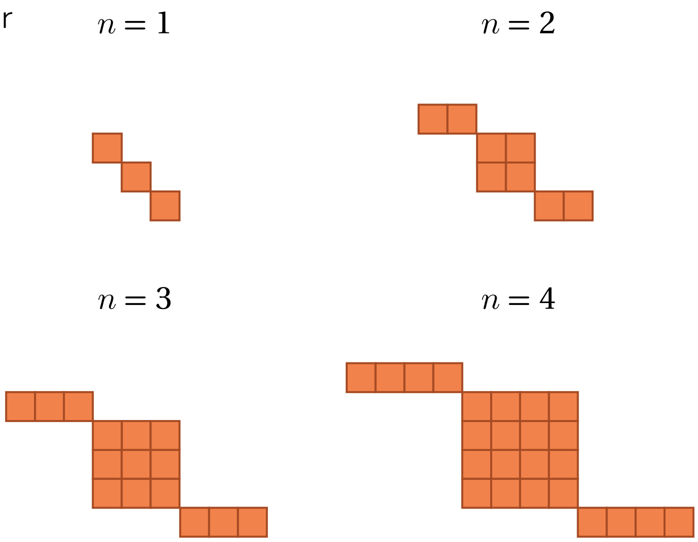
💬 **Discuss:** Where do you see each part of their expressions in their sketches?

Activity
3

Name: ______ Date: ______ Period: ______

Quadratic Relationships

8. Write an expression for the number of tiles in terms of n.



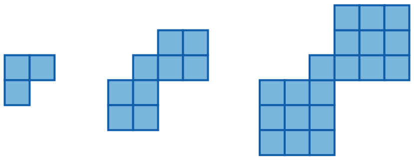
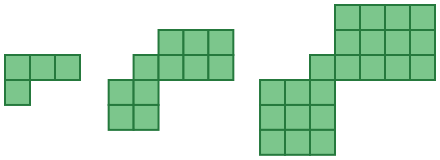
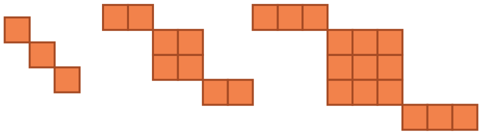
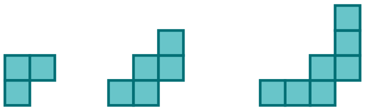
9. Select *all* the expressions that represent the number of tiles in this pattern.

- ☐ **A.** $5n$
- ☐ **B.** $n^2 + 2n$
- ☐ **C.** $(n \cdot n) + (n + n)$
- ☐ **D.** $3n^2$

Name: .... Date: .... Period: ....

Quadratic Relationships (continued)

10. Here are some of the relationships we've explored in this lesson.

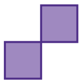
	Expressions	Patterns
Quadratic	$2n^2 + 1$	
	$2n^2 + n + 1$	
	$n^2 + 2n$	
Linear	$2n + 1$	

What do you think **quadratic relationships** all have in common?

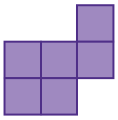
Synthesis

11. What would you say to help a classmate who is trying to write an expression for a quadratic relationship?

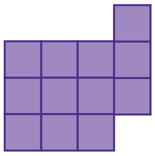
Use this pattern if it helps with your thinking.



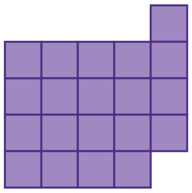
$n = 1$



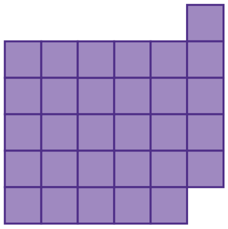
$n = 2$



$n = 3$



$n = 4$



$n = 5$

Lesson Practice 6.02

Lesson Summary

You can determine if a visual pattern represents a linear, exponential, or **quadratic relationship**, or something else.

If you can observe a square that is changing throughout a pattern, the relationship might be quadratic and you can use a squared term to write a quadratic expression.

Here are some strategies you might use to write an expression to represent a visual pattern:

You can look for what is changing and staying the same.

- The 3 outside tiles stay the same.
- The interior square is growing from 1 to 4 to 9.

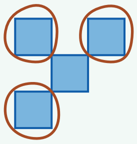


Figure 1

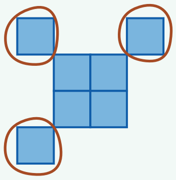


Figure 2

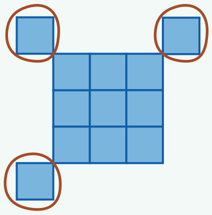


Figure 3

You can look for where you see the figure number, n, in each diagram.

- You can see the figure number as the side length of the growing square.
- You can write each number of tiles as an expression.

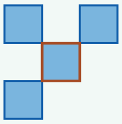


Figure 1

$1^2 + 3$

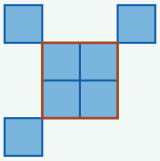


Figure 2

$2^2 + 3$

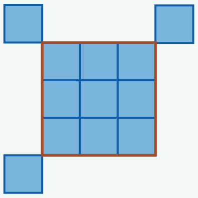


Figure 3

$3^2 + 3$

You can write a quadratic expression to represent the number of tiles in Figure n as $n^2 + 3$.

Lesson Practice

6.02

Name: .... Date: .... Period: ....

Problems 1–4: Circle whether each expression represents a quadratic relationship, linear relationship, or neither.

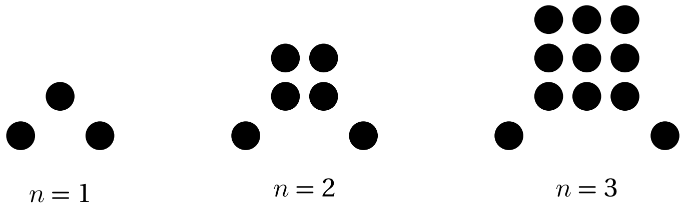
1. $2^n + n$ Quadratic Linear Neither

2. $2n^2 + 2$ Quadratic Linear Neither

3. $2(n - 5)$ Quadratic Linear Neither

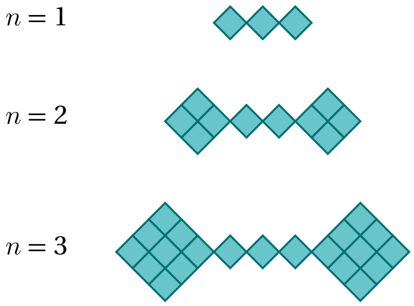
4. $2n(n + 3)$ Quadratic Linear Neither

5. Does this pattern show a quadratic relationship?



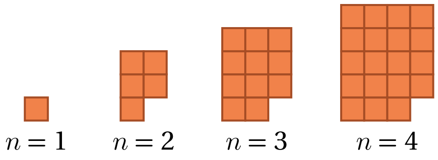
Explain your thinking.

6. Write an expression to represent the relationship between the figure number, n, and the total number of tiles.



7. Three students each wrote different, but correct, expressions to represent this pattern.

Ethan	**Ama**	**Nasir**
$n^2 + (n - 1)$	$n(n + 1) - 1$	$n^2 + n - 1$



Choose a student's expression and describe how it matches the pattern.

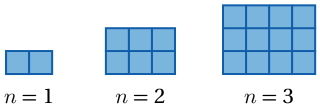
Lesson Practice

6.02

Name: .... Date: .... Period: ....

Test Practice

8. Karima says that she sees a square plus one more row in each pattern.



Circle *all* the expressions Karima could use to represent the number of tiles in this pattern.

$n^2 + 1$ $n^2 + n$ $n(n + 1)$

Spiral Review

9. *Select all* the expressions that are equivalent to $6m + 3q$.

☐ **A.** $4m + 2m + 5q - 2q$

☐ **B.** $3(2m + q)$

☐ **C.** $(6 + 3)(m + q)$

☐ **D.** $3q + 2m + 3q + m$

☐ **E.** $q + 15m + 2q - 9m$

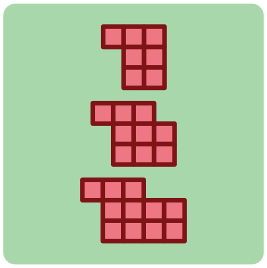
10. Complete the table for the function $h(x) = 5(2)^x$.

x	-2	-1	0	1	2
$h(x)$					

Name: .... Date: .... Period: ....

MA.912.F.1.1, MA.912.F.1.2, MA.912.F.1.8, MTR.2.1, MTR.5.1

Sorting Relationships

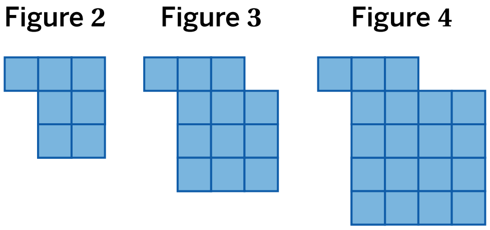


Let’s compare and contrast linear, exponential, and quadratic relationships in tables, patterns, and equations.

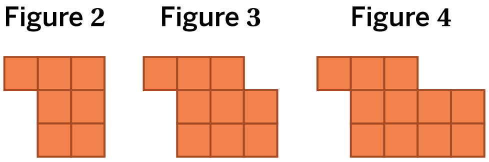
Warm-Up

1. Here are two visual patterns.

Pattern A



Pattern B



How are the patterns alike? How are they different?

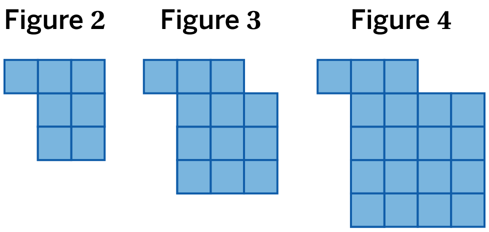
Activity
1

Name: ________________ Date: ________ Period: ________

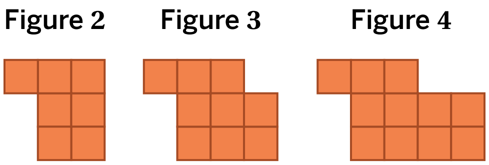
Find the Pattern

Here are the two visual patterns from the Warm-Up.

Pattern A



Pattern B



2. Complete the table for each visual pattern.

Pattern A

Figure	Number of Tiles
1	
2	7
3	12
4	19
5	
6	
7	

Pattern B

Figure	Number of Tiles
1	
2	7
3	9
4	11
5	
6	
7	

3. **Discuss:** What strategies did you use to fill in the tables?

4. Afia said: *Both tables are going up by two*. Do you agree? Explain your thinking.

Find the Pattern (continued)

Here is one way you can use a table to decide if a relationship is linear, exponential, or quadratic.

Linear relationships have a *constant difference*.

Figure	Number of Tiles
1	25
2	45
3	65
4	85

+20
+20
+20

Exponential relationships have a *constant ratio*.

Figure	Number of Tiles
1	9
2	27
3	81
4	243

$\cdot 3$
$\cdot 3$
$\cdot 3$

Quadratic relationships have a constant **second difference**.

Figure	Number of Tiles
0	0
1	5
2	14
3	27

+5
+9
+13

+4
+4

A **quadratic function** is a function that represents a quadratic relationship.

5. What type of relationship is Pattern A? Pattern B?

Pattern A:

Pattern B:

Name: .... Date: .... Period: ....

Sort ’em

You will use a set of cards for this activity.

6. Find Cards A–F. Fill in the missing values for each table.

7. Sort Cards A–F. With your partner, decide what type of relationship each table represents.

Record your thinking in the table.

Linear	Exponential	Quadratic	Something Else

8. How did you determine how to sort Card C?

9. Sort Cards G–L. Record your thinking in the table.

10. Nyanna says that exponential growth functions will eventually exceed all of the other function types. When is Nyanna’s statement true? Circle one.

Always true Sometimes true Never true

Explain your thinking.

You're invited to explore more.

11. Create your own quadratic visual pattern. Show or explain how you know it is quadratic.

Synthesis

12. How can you tell if a table is linear, exponential, or quadratic?

Linear:

Exponential:

Quadratic:

Lesson Practice 6.03

Lesson Summary

You can analyze a table of values, a pattern, or an equation to help you determine whether a relationship is linear, quadratic, exponential, or something else.

- Linear relationships have a constant first difference in the y-values when the x-values change by a constant value.
- Quadratic relationships change by a constant **second difference**, the difference between the first differences. A **quadratic function** is a function that represents a quadratic relationship.
- Exponential relationships have a constant ratio.

Here are some examples.

<table>
<tr><th></th><th>Linear</th><th>Quadratic</th><th>Exponential</th></tr>
<tr><th>Table</th>
<td><table><tr><th>x</th><th>$f(x)$</th><th></th></tr><tr><td>1</td><td>3</td><td>+3</td></tr><tr><td>2</td><td>6</td><td>+3</td></tr><tr><td>3</td><td>9</td><td>+3</td></tr><tr><td>4</td><td>12</td><td></td></tr></table></td>
<td><table><tr><th>x</th><th>$g(x)$</th><th></th><th></th></tr><tr><td>1</td><td>2</td><td>+3</td><td></td></tr><tr><td>2</td><td>5</td><td>+5</td><td>+2</td></tr><tr><td>3</td><td>10</td><td>+7</td><td>+2</td></tr><tr><td>4</td><td>17</td><td></td><td></td></tr></table></td>
<td><table><tr><th>x</th><th>$h(x)$</th><th></th></tr><tr><td>1</td><td>6</td><td>• 2</td></tr><tr><td>2</td><td>12</td><td>• 2</td></tr><tr><td>3</td><td>24</td><td>• 2</td></tr><tr><td>4</td><td>48</td><td></td></tr></table></td>
</tr>
<tr><th>Equation</th><td>$f(x) = 3x$</td><td>$g(x) = x^2 + 1$</td><td>$h(x) = 3 \cdot 2^x$</td></tr>
</table>

Lesson Practice
6.03

Name: .... Date: .... Period: ....

Problems 1–3: Here are two functions: $g(x) = 3x^2$ and $h(x) = 3^x$.
Determine which function is greater for each value of x. Circle your choice.

1. $g(1)$ $h(1)$ They are the same value

2. $g(2)$ $h(2)$ They are the same value

3. $g(5)$ $h(5)$ They are the same value

Problems 4–5: Use the table of $f(x)$.

x	1	2	3	4	5	6	7
$f(x)$	3	12	27	48			

4. What type of relationship does the table represent? Circle one.

Linear Exponential Quadratic Something Else

Explain your thinking.

5. Complete the table.

6. The function $m(x)$ represents a quadratic relationship. Complete the missing values in the table of $m(x)$.

x	0	1	2	3	4	5
$m(x)$	7	12	27			

Problems 7–9: Use the pattern shown.

7. Explain how you see this pattern growing.

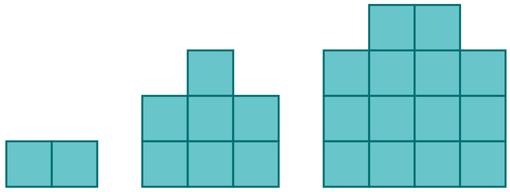


Figure 1 **Figure 2** **Figure 3**

8. How many tiles will there be in Figure 10?

9. Write an expression for the number of tiles in Figure n.

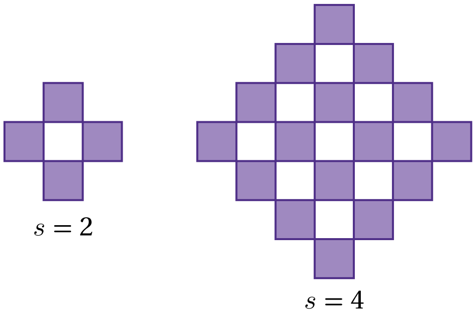
Lesson Practice
6.03

Name: .... Date: .... Period: ....

Test Practice

10. Complete the table for each type of function to show the number of tiles when $s = 3$ and $s = 5$. Choose from the given values to complete the table.

6	8	9	12	15	16	18
20	22	24	25	26	28	32



Choose from the given values to complete the table.

s	Linear	Quadratic	Exponential
2	4	4	4
3			
4	16	16	16
5			

Spiral Review

11. The function $C(x)$ gives the percentage of students at a high school who have cell phones x years after 2004. Explain what $C(10) = 35$ means in this context.

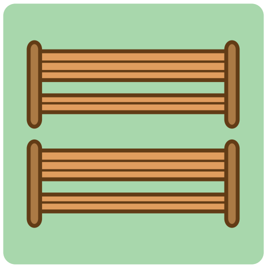
Unit 6
Lesson
4

Name: ________ Date: ________ Period: ________

MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.1.1, MA.912.F.1.8, MTR.2.1, MTR.5.1, MTR.7.1

On the Fence

Let's use the context of building fences to explore symmetry in quadratic relationships.



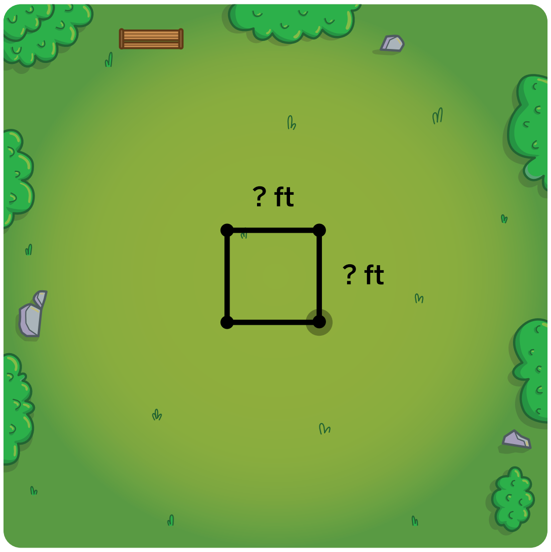
Warm-Up

1. Farmer Farah is building a fence for her sheep.

Each panel of fencing is 5 feet long and she has 100 feet of fencing total.

Build three different fences in the pasture that each use *exactly* 100 feet of fencing.

Width (ft)	Length (ft)



Name: .... Date: .... Period: ....

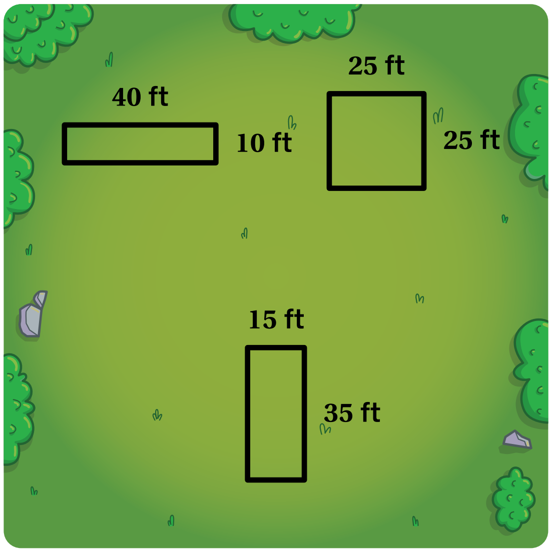
Farmer Farah’s Fencing

2. Let’s look at three fences.

How are these fences alike? How are they different?

Alike:

Different:



3. Farah noticed that for each fence, the perimeter stays the same but the area changes.

Here are Farah’s fences.

Calculate the areas of the three fences.

Width (ft)	Length (ft)	Area (sq. ft)
40	10	
15	35	
5	45	

Name: .... Date: .... Period: ....

Farmer Farah’s Fencing (continued)

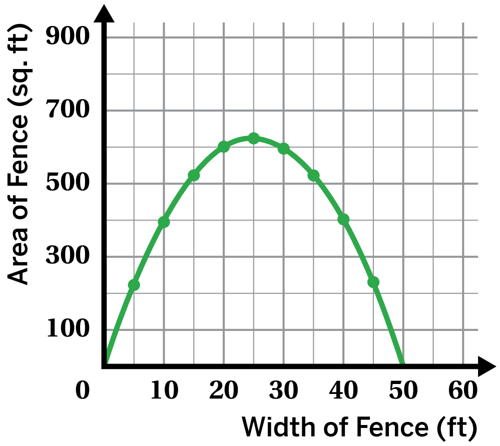
4. The table represents all the possible fences Farmer Farah can build in the pasture.

Width (ft)	Length (ft)	Area (sq. ft)
5	45	225
10	40	400
15	35	525
20	30	
	25	625
		600
35	15	
	10	
	5	

a Complete the missing values in the table.

b **Discuss:** What do you notice? What do you wonder?

5. Here is a graph of a parabola that includes all the possible fences.



What type of relationship is represented?

○ **A.** Linear

○ **B.** Exponential

[✓ **C.** Quadratic]

○ **D.** Something else

Explain your thinking.

6. The graph of a quadratic function is called a **parabola**.

Parabolas have a **line of symmetry**. If you fold a parabola along this line, you get two identical halves.

a Draw the line of symmetry on the graph of possible fences.

b **Discuss:** What does this line mean in the context of the sheep fence?

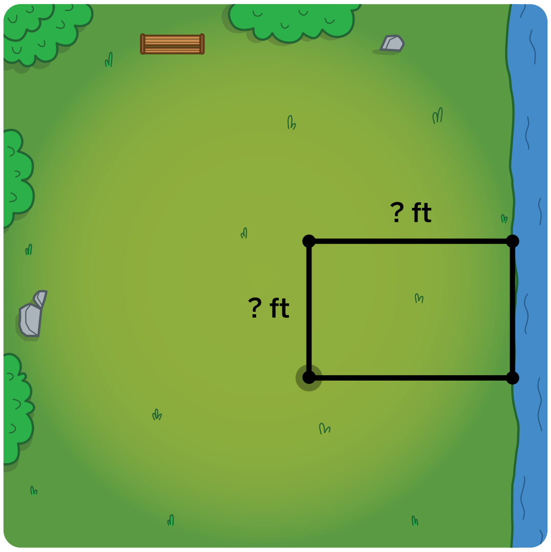
Name: .... Date: .... Period: ....

By the Stream

7. Farmer Farah's sheep don't like to swim. If she builds her fence by the stream, it will only need three sides.

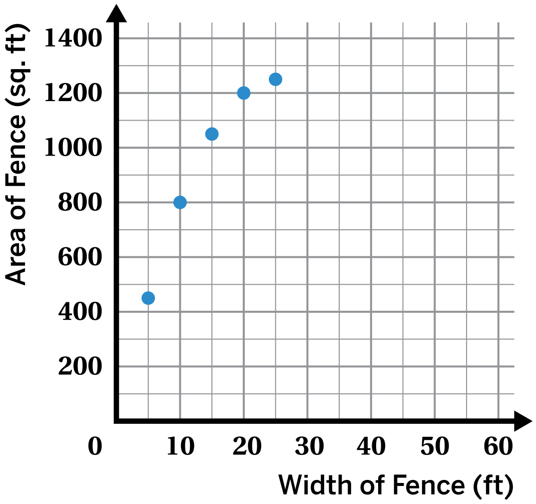
Using 100 feet of fencing, build three possible fences.

Width (ft)	Length (ft)



8. Here are a few of the possible fences Farmer Farah can build by the stream.

Width (ft)	Area (sq. ft)
5	450
10	800
15	1050
20	1200
25	1250



Is this relationship quadratic? Circle one.

Yes No I'm not sure

Explain your thinking.

Name: .... Date: .... Period: ....

By the Stream (continued)

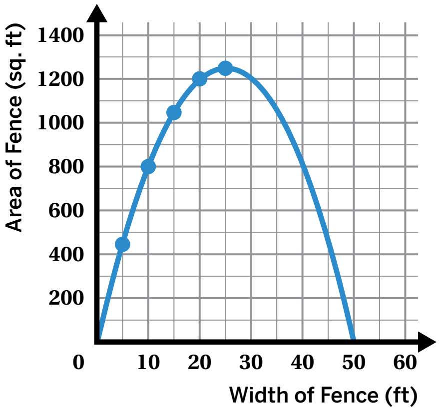
9. What are all the possible fences this parabola could represent?

Complete the table to show all the possible fences Farmer Farah could make by the stream.

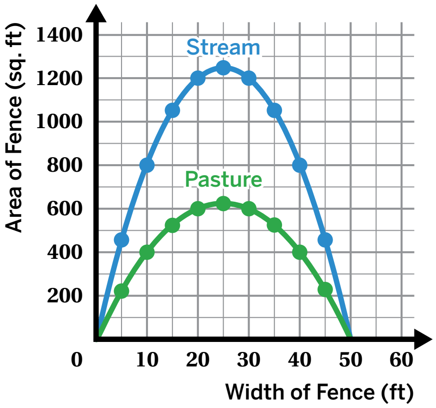
Width (ft)	Area (sq. ft)
5	450
10	800
15	1050
20	1200
25	1250
30	
35	
40	
45	

10. Here is a graph of a parabola that includes all the possible areas of fences along the stream.

Write the equation for the line of symmetry for this parabola.



11. Here are the graphs of parabolas that include all the possible fences Farmer Farah could build in the pasture and by the stream.



How are these relationships alike? How are they different?

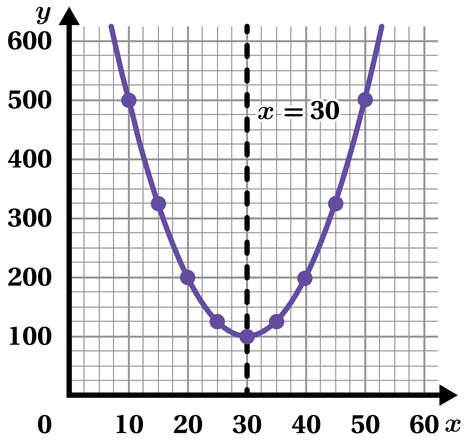
Alike:

Different:

Synthesis

12. The table and the graph represent the same relationship.

x	y
15	325
20	200
25	125
30	100
35	125
40	200
45	325



Describe two ways you know that the relationship in the table and graph is quadratic.

Lesson Practice 6.04

Lesson Summary

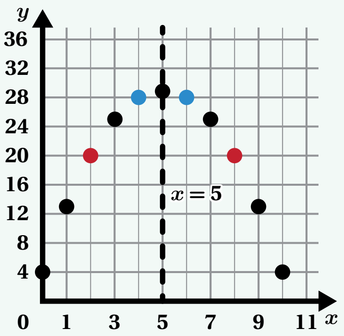
The graph of a quadratic function is called a **parabola**. Parabolas have a **line of symmetry** that goes through the maximum or minimum point. If you fold a parabola along this line, you get two identical halves. Here is a table and graph that represent a quadratic relationship.

Table

x	y
2	20
4	28
5	29
6	28
8	20

You can see the points are symmetrical across the line of symmetry at $x = 5$.

Graph



You can see the points create the shape of a parabola and are symmetrical across the line of symmetry at $x = 5$

Lesson Practice
6.04

Name: .... Date: .... Period: ....

Problems 1–3: For each pair of symmetrical points on a parabola, determine the equation for the line of symmetry.

1. $(0, 0)$ and $(13, 0)$

$x =$ ....

2. $(4, 2)$ and $(20, 2)$

$x =$ ....

3. $(7, 10)$ and $(28, 10)$

$x =$ ....

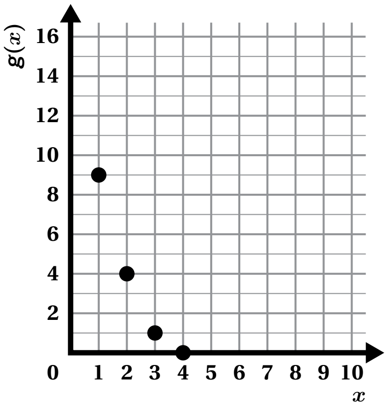
Problems 4–6: Here are a few points that belong to a function $g(x)$.

x	$g(x)$
0	
1	9
2	4
3	1
4	0
5	
6	
7	
8	

4. Does $g(x)$ represent a quadratic relationship? Circle your response and explain your thinking.

Yes   No   Not enough information

5. Complete the table for $g(x)$ and plot the new points on the graph.



6. Write the equation for the line of symmetry.

$x =$ ....

Problems 7–8: Here is an incomplete table that could represent several types of functions.

Figure	Number of Tiles
1	1
2	
3	9

7. Select a function type and determine the number of tiles that would be in Figure 2. Circle one.

Linear   Quadratic   Exponential

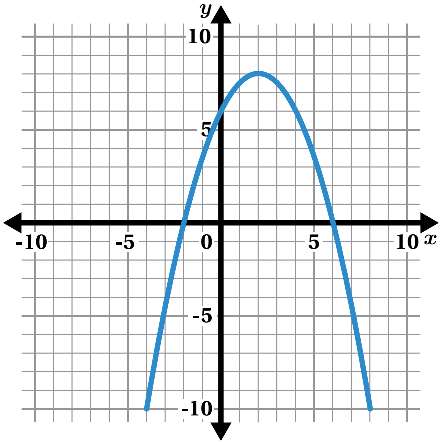
8. Draw three figures to match the pattern in the table.

Lesson Practice

6.04

Name: .... Date: .... Period: ....

Test Practice



Problems 9–10: Here is a graph of a parabola.

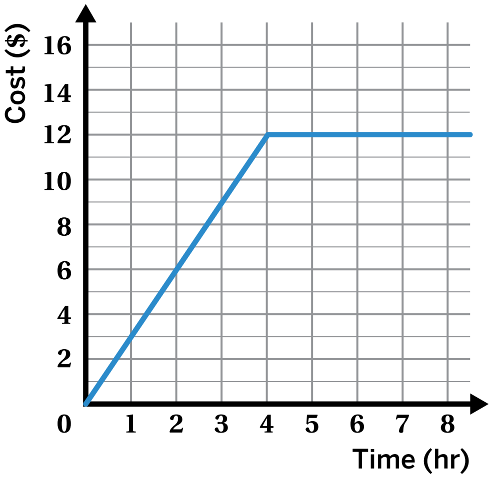
9. Draw the *line of symmetry* where you think it is located on this parabola.

10. Write the equation for the line of symmetry.

$x =$ ....

Spiral Review

Problems 11–12: The graph represents the relationship between the amount of time a car is parked, in hours, and the cost of parking, in dollars.



11. Is the relationship a function?

12. Describe the relationship between the amount of time a car is parked and the cost of parking.

Unit 6
Lesson
5

Name: ______ Date: ______ Period: ______

MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.1.1, MA.912.F.1.8, MTR.1.1, MTR.2.1, MTR.3.1, MTR.5.1

Stomp Rockets

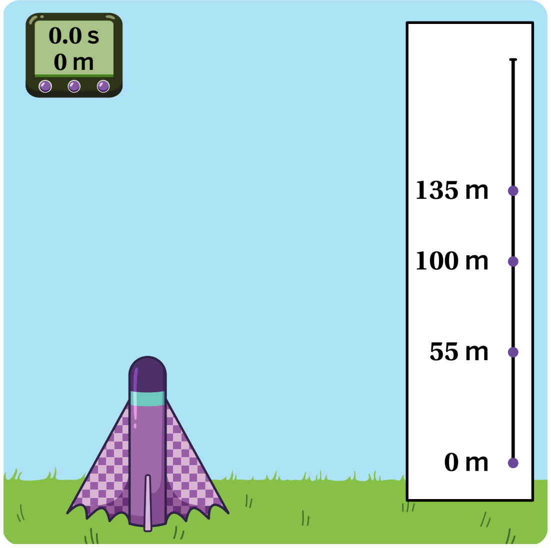
Let's use tables and graphs to make predictions about quadratic relationships in the context of launching stomp rockets.

Warm-Up

1. A stomp rocket is a toy rocket with no engine that is launched by a quick burst of compressed air.

a Look at the data collected for a certain launch of a stomp rocket.

b **Discuss:** What do you notice? What do you wonder?

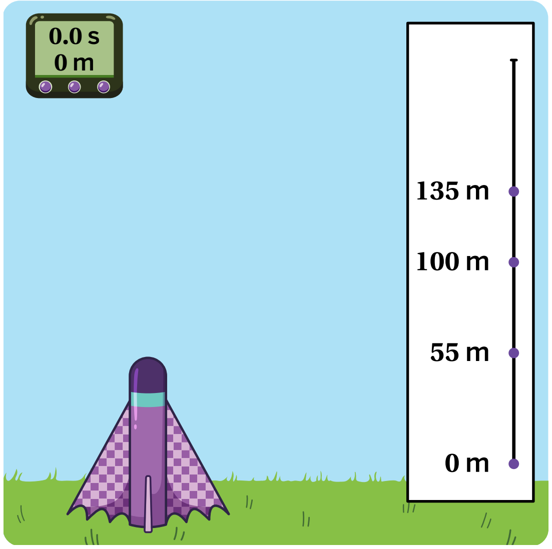


Name: .... Date: .... Period: ....

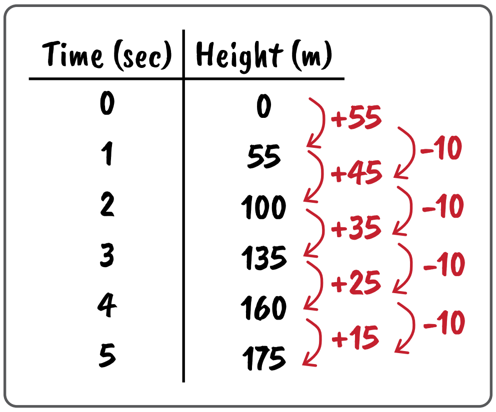
Predicting With Tables

2. Make a prediction: How high do you think the rocket will be after 4 and 5 seconds?

Time (sec)	Height (m)
0	0
1	55
2	100
3	135
4	
5	



3. Let's look at how Maia made her prediction.



Time (sec)	Height (m)		
0	0	+55	-10
1	55	+45	-10
2	100	+35	-10
3	135	+25	-10
4	160	+15	
5	175		

Describe what she did to find the heights at 4 and 5 seconds.

4. What type of function could represent this situation? Explain your thinking.

Name: .... Date: .... Period: ....

Predicting With Tables (continued)

5. Here is a new rocket. The table shows its height at various times.

How high will this rocket go?

Use the table if it helps with your thinking.

Time (sec)	Height (m)
0	0
1	45
2	80

6. How many seconds will it take for the rocket to touch the ground?

Use the table if it helps with your thinking.

Activity

2

Name: ....

Date: ....

Period: ....

Predicting With Tables and Graphs

7. A new stomp rocket is launched from the top of a building.

About how many seconds will it take for the rocket to touch the ground?

Use the table if it helps with your thinking.

A. 6 seconds

B. 8 seconds

C. Between 6 and 7 seconds

D. Between 7 and 8 seconds

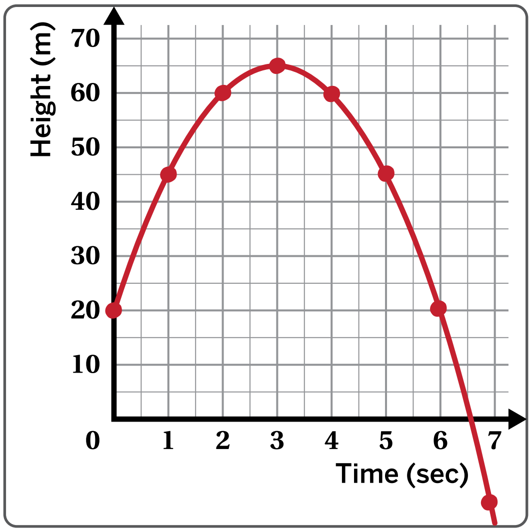
Explain your thinking.

Time (sec)	Height (m)
0	20
1	45
2	60

8. Let's look at Ivan's graph.

a Why do you think Ivan drew a parabola? What type of function could represent this situation?

b How do you think he used his graph to determine when the rocket landed?

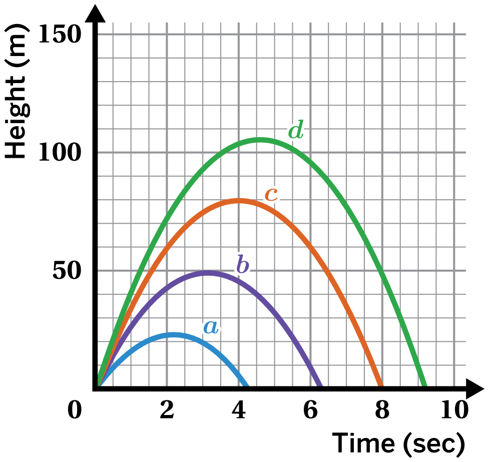


Name: .... Date: .... Period: ....

Predicting With Tables and Graphs (continued)

9. The graph to the right shows the paths of 4 different stomp rockets after they were launched.

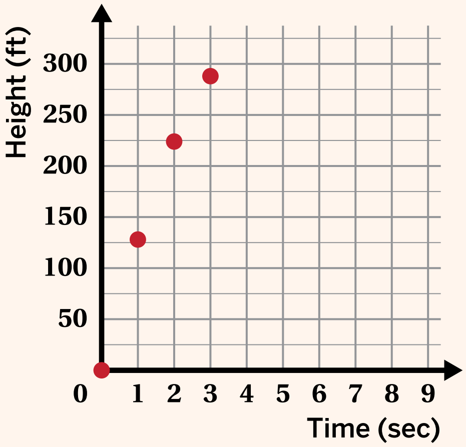
💬 **Discuss:** What is the same and what is different about each rocket launch?



You're invited to explore more.

10. The table and graph show the height of a stomp rocket at various times. How many seconds will it take for this rocket to reach its maximum height?

Time (sec)	Height (m)
0	0
1	128
2	224
3	288



Explain how you know.

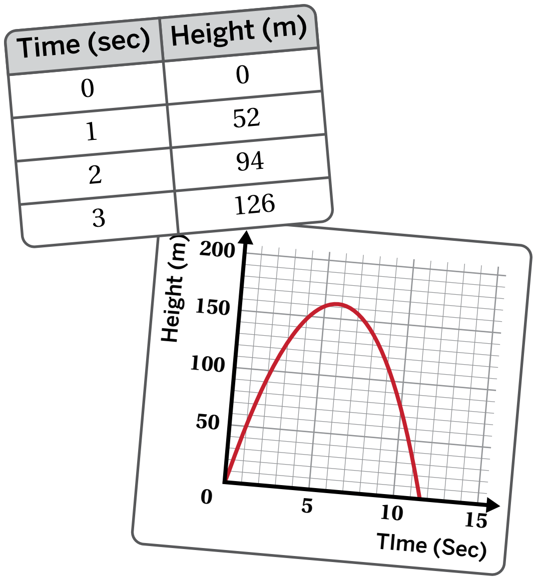
Synthesis

11. The table and graph show the height of a stomp rocket at various times.

Write one question about the rocket that you can answer using the table and one you can answer using the graph.

Table:

Graph:



Time (sec)	Height (m)
0	0
1	52
2	94
3	126

Lesson Practice 6.05

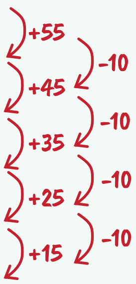
Lesson Summary

You can use tables and graphs to make predictions about quadratic relationships in context.

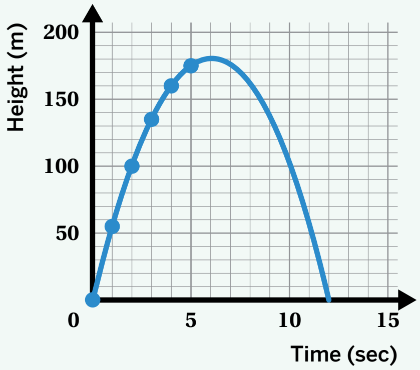
This table and graph show the height of a stomp rocket at various times.

Table

Time (sec)	Height (m)
0	0
1	55
2	100
3	135
4	160
5	175



Graph



You can extend the pattern of the table using the second difference.

From the table, you can see how the rocket is 160 meters high after 4 seconds and 175 meters high after 5 seconds.

You can use the graph to determine the maximum height of the rocket by looking for the highest point on the parabola. The maximum height of the rocket is 180 meters at 6 seconds.

You can also see from the graph that it takes 12 seconds for the rocket to land.

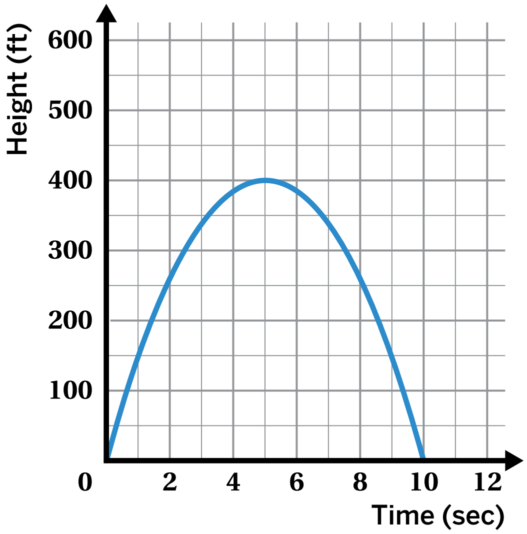
Lesson Practice

6.05

Name: .... Date: .... Period: ....

Problems 1–3: This table and graph show the height of a stomp rocket at various times.

Time (sec)	0	1	2	3
Height (ft)	0	144	256	336



1. What is the maximum height that the stomp rocket reached?

2. How long did it take for the stomp rocket to land?

3. How high was the stomp rocket after 4 seconds?

Problems 4–5: Oliver jumps off a diving board into a swimming pool. The relationship between his height and time can be modeled by a quadratic relationship.

Time (sec)	0	0.2	0.4	0.6	0.8	1	1.2	1.4
Height (m)	3	4.8	6.2	7.2				

4. Complete the table.

5. After how many seconds will Oliver reach his maximum height?

Name: .... Date: .... Period: ....

6. The table shows the height of a stomp rocket from the time it is launched. How many seconds will it take for the rocket to land? Circle one.

Time (sec)	Height (m)
0	0
1	42
2	74
3	96

8 seconds

9 seconds

Between 8 and 9 seconds

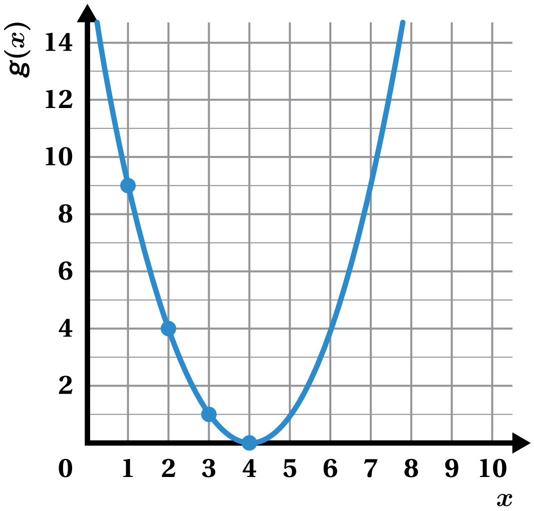
Between 9 and 10 seconds

Explain your thinking.

Test Practice

7. Here is the graph of the function $g(x)$. Write the equation for the line of symmetry of $g(x)$.

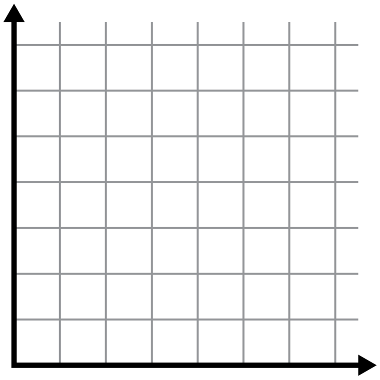
$x =$ ....



Spiral Review

Problems 8–10: Mohamed can run a mile in 9 minutes. The number of miles Mohamed runs is a function of the amount of time he has been running.

8. Graph this function. Be sure to label the axes.



9. Describe the domain of this function.

Domain:

10. Describe the range of this function.

Range:

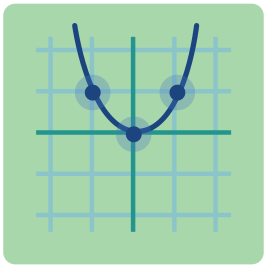
Unit 6
Lesson
6

Name: ____________ Date: ________ Period: ________

MA.912.AR.3.7, MTR.2.1, MTR.4.1, MTR.5.1

Plenty of Parabolas

Let’s describe the key features of a parabola.



Warm-Up

1. Play a few rounds of Polygraph with your classmates!

You will use a Warm-Up Sheet with parabolas for four rounds.

For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a parabola from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating parabolas until you’re ready to guess which parabola the Picker chose.

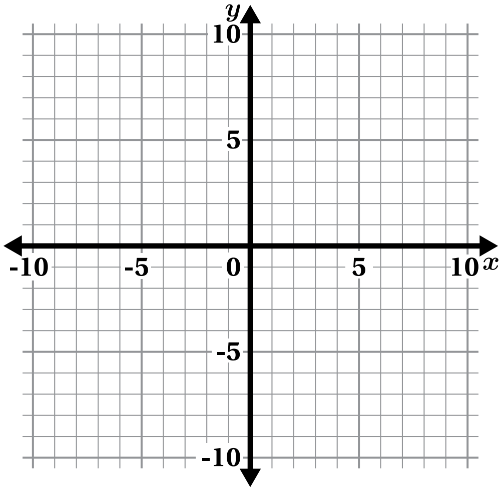
Record helpful questions from each round in the space below.

Activity 1

Name: ________ Date: ________ Period: ________

Describing Parabolas

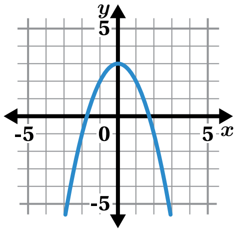
2. Now it’s your turn to graph a parabola.



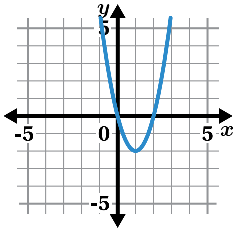
3. Ama says her parabola turns around at 3.

Select a parabola that could be Ama’s.

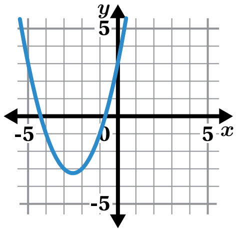
A.



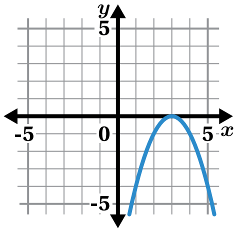
B.



C.



D.

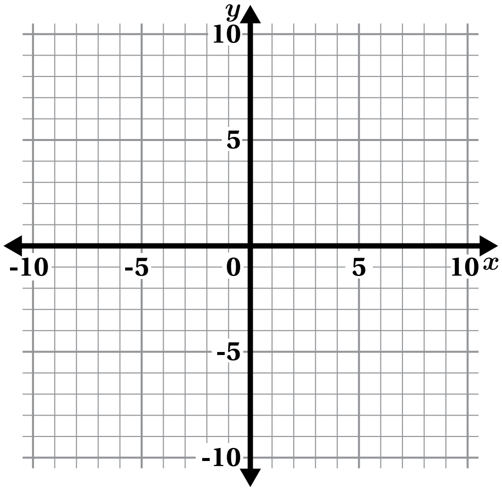


Explain your thinking.

4. The turning point of a parabola is called the **vertex**. This is also its *maximum* or *minimum*.

Draw a parabola with a vertex at (-5, 1).

Try to make a parabola you think none of your classmates will make.

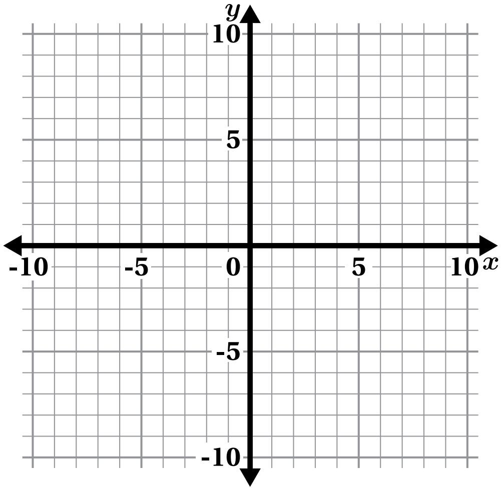


Name: .... Date: .... Period: ....

Describing Parabolas (continued)

5. Katie says her parabola has an x-intercept at -2 and looks like a smile.

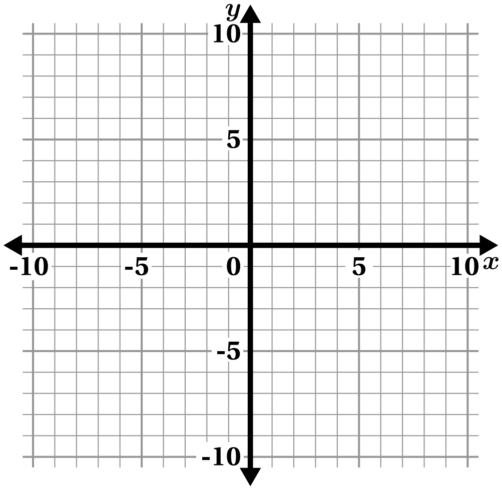
Draw what her parabola could look like.



6. Parabolas that look like a smile are **concave up**.

Parabolas that look like a frown are **concave down**.

Draw a concave down parabola with vertex $(4, -2)$.

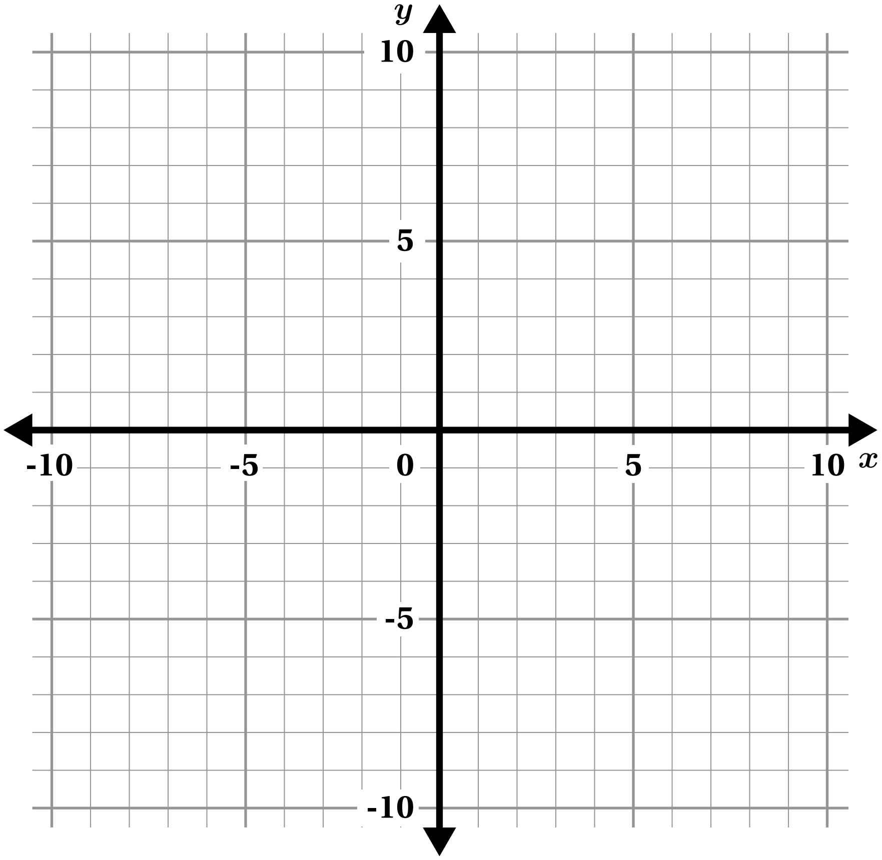


Name: .... Date: .... Period: ....

Parabola Art

7. It's time to make some parabola art!

Create some art by drawing multiple parabolas.



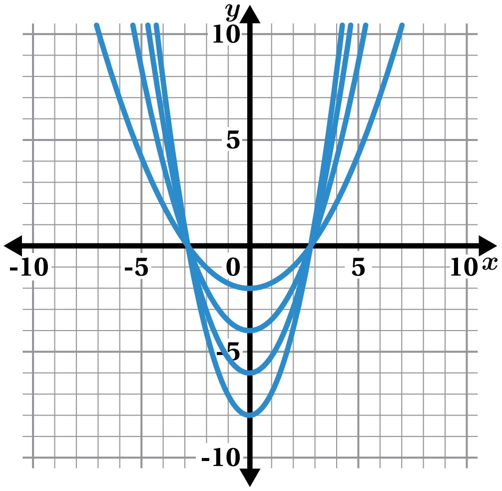
Name: .... Date: .... Period: ....

Parabola Art (continued)

8. Manuel made this design using parabolas.

How are these parabolas alike?

Describe as many similarities as you can.



You're invited to explore more.

9. Try to make concave up and concave down parabolas with different numbers of intercepts.

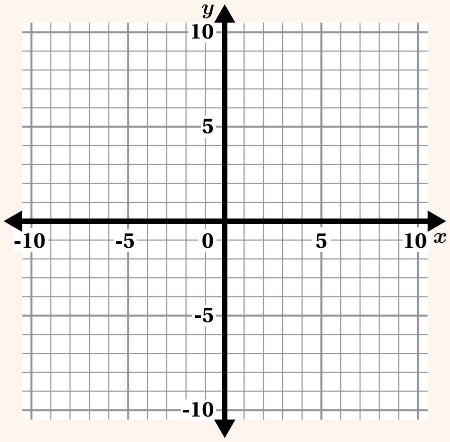
Then select the number of x-intercepts and y-intercepts that are possible.

Number of x-intercepts:

☐ 0 ☐ 1 ☐ 2

Number of y-intercepts:

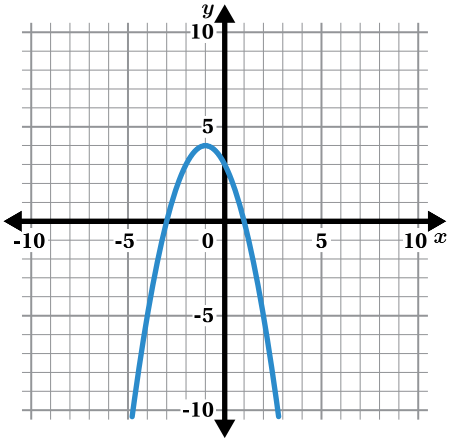
☐ 0 ☐ 1 ☐ 2



Synthesis

10. Describe the graph using vocabulary from this lesson.

Draw on the graph if it helps with your thinking.

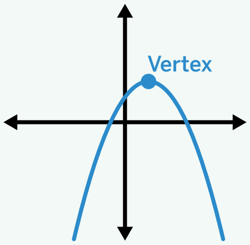


Lesson Practice 6.06

Lesson Summary

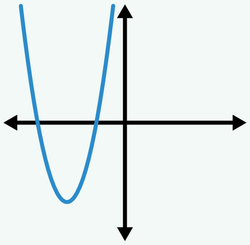
You can describe key features of *parabolas* with terms like: **vertex**, **concave up**, or **concave down**.

Vertex



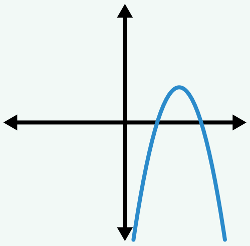
The vertex is the *maximum* or *minimum* point on a parabola (where a parabola changes from increasing to decreasing, or vice versa).

Concave Up



A parabola that opens upward is concave up.

Concave Down

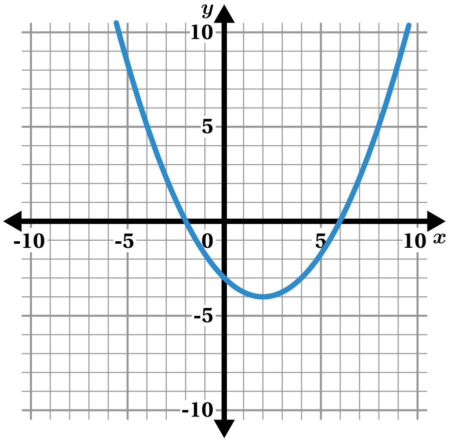


A parabola that opens downward is concave down.

Lesson Practice
6.06

Name: .... Date: .... Period: ....

Problems 1–3: Use the graph to determine the coordinates of each key feature.



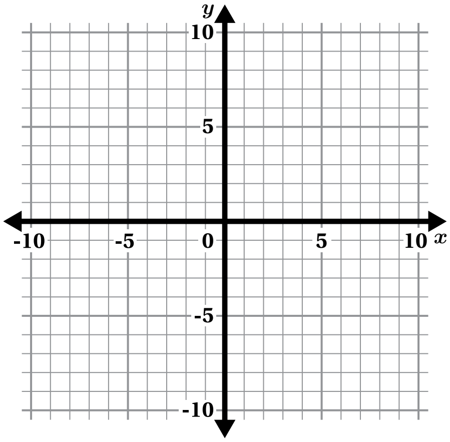
1. vertex:

2. x-intercept(s):

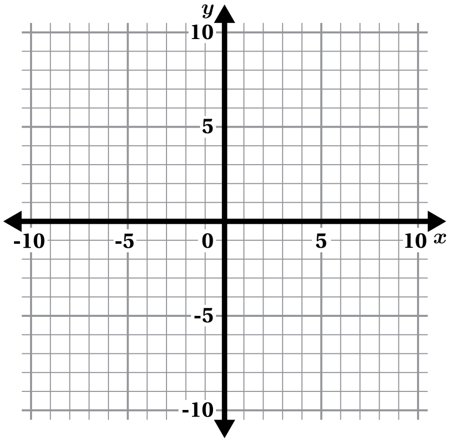
3. y-intercept:

Problems 4–5: Graph a parabola that fits each description.

4. Concave down with a positive y-intercept



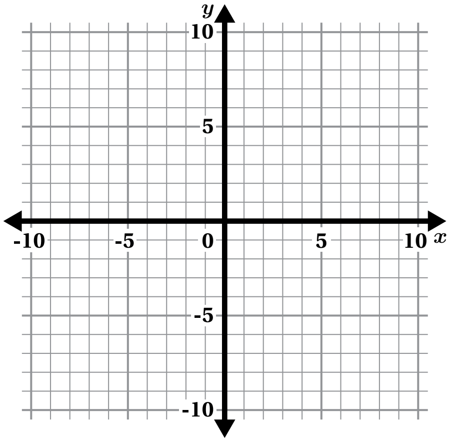
5. Concave up and vertex at $(5, \text{-}5)$



6. A parabola has a vertex at $(4, 2)$. Give two possible coordinates for its x-intercepts.

7. Using only parabolas, make a design that:

- Looks the same when reflected over the x-axis.
- Looks the same when reflected over the y-axis.



Lesson Practice
6.06

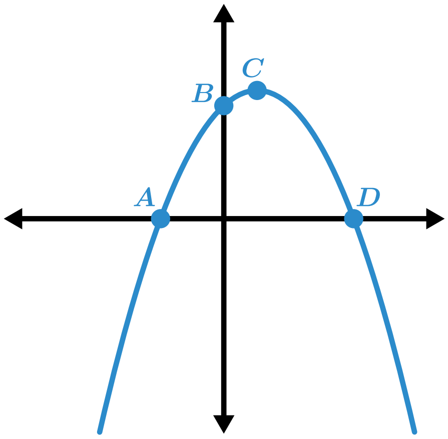
Name: .... Date: .... Period: ....

Test Practice

8. The key features of this parabola are labeled A, B, C, and D.

Match each key feature with a term from the word bank.

vertex	x-intercept	y-intercept



A: ....

B: ....

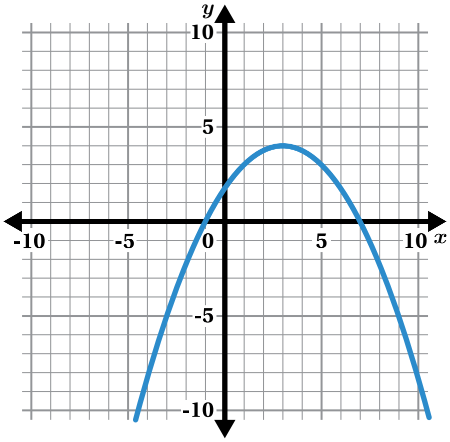
C: ....

D: ....

Spiral Review

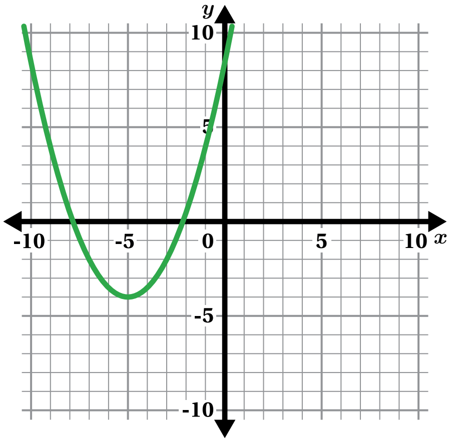
Problems 9–10: Write an equation for the line of symmetry of each parabola.

9.



Equation: ....

10.



Equation: ....

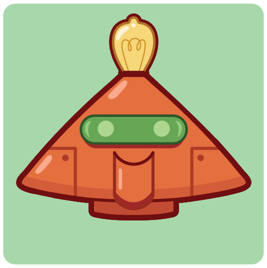
Unit 6
Lesson
7

Name: ______ Date: ______ Period: ______

MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.1.2, MTR.2.1, MTR.4.1, MTR.5.1

Robot Launch

Let's identify features of quadratic functions in graphs, tables, and equations and determine their meanings in context.

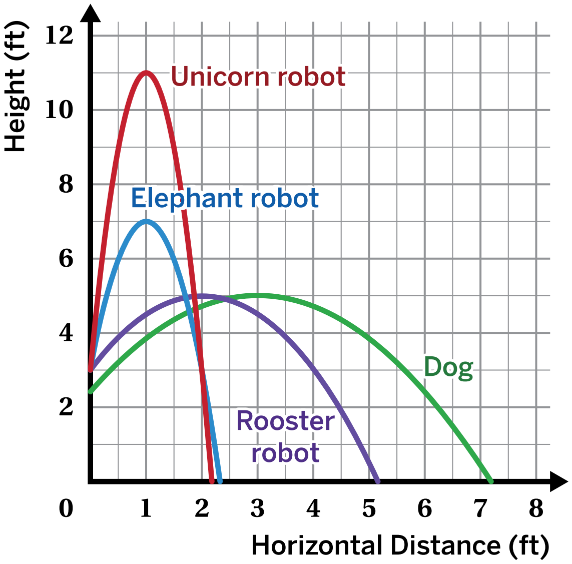


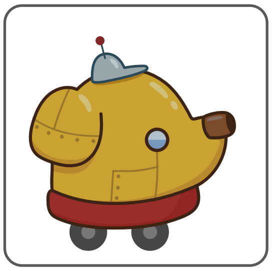
Warm-Up

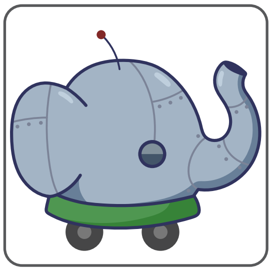
1. Four groups of students enter a robot building contest.

The graph shows the pathway of the ball launched by each robot.

Tell a story about these robots.







Activity
1

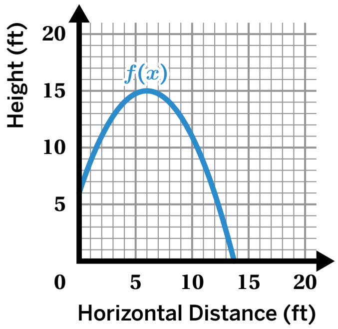
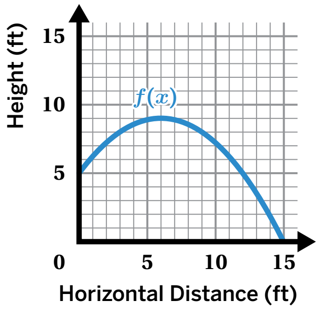
Name: ________ Date: ________ Period: ________

Ball Launch

2. The trajectory of each ball can be modeled using a quadratic function.

a **Discuss:** What do you remember about these words for describing functions?

b You will use a set of cards for this activity. Match the cards to the graphs and table. One card will not match because it has an error.

<table>
<tr><th>Graph or Table</th><th>Card #1</th><th>Card #2</th></tr>
<tr><td>
</td><td></td><td></td></tr>
<tr><td><table>
<tr><th>Horizontal Distance (ft)</th><th>Height (ft)</th></tr>
<tr><td>0</td><td>4</td></tr>
<tr><td>1</td><td>9</td></tr>
<tr><td>2</td><td>12</td></tr>
<tr><td>3</td><td>13</td></tr>
<tr><td>4</td><td>12</td></tr>
</table></td><td></td><td></td></tr>
<tr><td>
</td><td></td><td></td></tr>
</table>

3. Fix the card with the error so that each table or graph has two matching cards.

Activity
2

Name: ____________ Date: ____________ Period: ____________

The Best Robot

Here are robots that four other groups of students built.

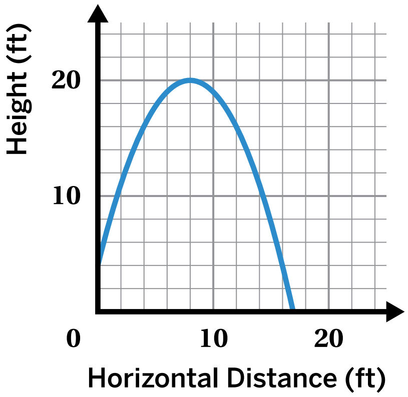
Robot A

Horizontal Distance (ft)	Height (ft)
0	5
1	12
2	17
3	20

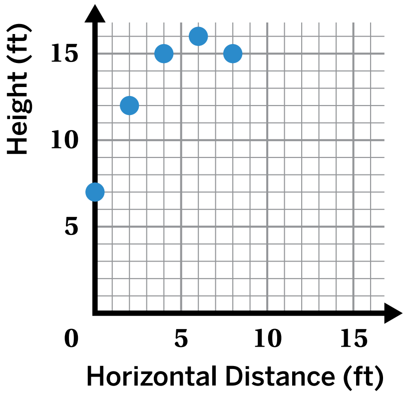
Robot B

The height of the ball can be modeled by $f(x) = 4 - x^2$, where x is the horizontal distance the ball has traveled.

Robot C



Robot D



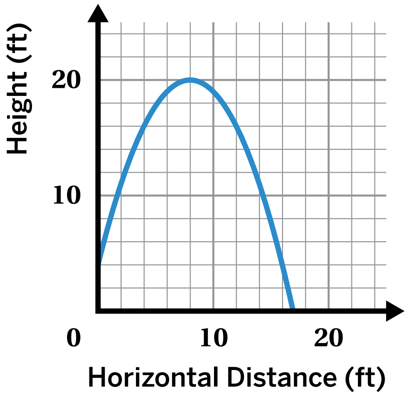
4. Here are the four prizes. Use this workspace to decide which robots get each award.

Highest Launch	Farthest Launch	Cutest Robot	Strongest Robot

Synthesis

5. How do the features of a parabola help you describe the motion of a ball?

Use this parabola if it helps you to explain your thinking.

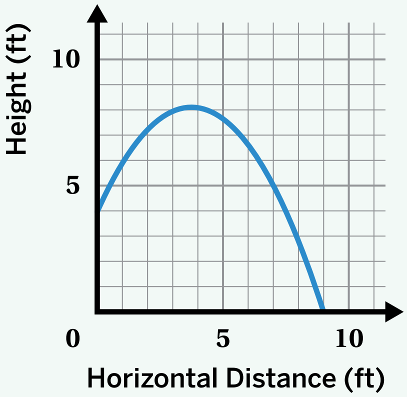


Lesson Practice 6.07

Lesson Summary

The graph of a *quadratic function* is a parabola. You can identify key features of a quadratic function from a graph, table, or equation to help you interpret quadratic functions in a context.

For example, this graph models the path of a stomp rocket.



You can use the y-intercept to describe the starting height of the stomp rocket.

- The y-intercept is $(0, 4)$.
- This means that at the start, or when the horizontal distance was 0 feet, the height was 4 feet.

You can use the *vertex* to describe the maximum height of the stomp rocket.

- The vertex is about $(3.5, 8)$.
- This means that the stomp rocket reached a maximum height of 8 feet.

You can use the x-intercept to describe where the stomp rocket lands.

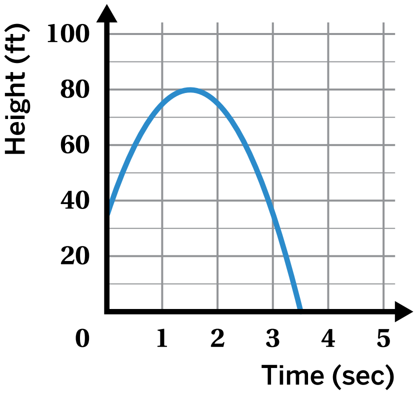
- The x-intercept on the graph is $(9, 0)$.
- This means that the horizontal distance was 9 feet when the rocket landed, or when the height was 0 feet.

Lesson Practice

6.07

Name: ________ Date: ________ Period: ________

Problems 1–4: The graph shows the height of a bird over time.



1. Which key feature tells you the maximum height the bird flew? Circle one:

x-intercept y-intercept vertex

2. When did the bird reach its maximum height?

3. Which key feature tells you when the bird landed? Circle one:

x-intercept y-intercept vertex

4. How long was the bird in flight?

5. Here is a table of values for the function $f(x)$. Circle *two* values of $f(x)$ to change so that the table represents a quadratic function.

x	-3	-2	-1	0	1	2	3
$f(x)$	20	12	6	2	0	2	6

Show or explain what they should change to.

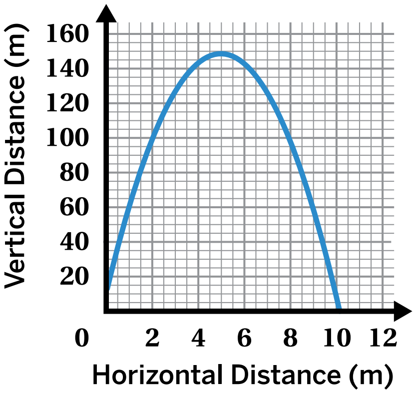
Name: .... Date: .... Period: ....

6. Ava and Mariam launch their stomp rockets at the same time. Ava launches her rocket from the ground. Mariam launches her rocket from a platform.

Ava's Table

Horizontal Distance (m)	Vertical Distance (m)
0	0
1	45
2	80
3	105

Mariam's Graph



Whose rocket goes higher? Describe the key feature that helped you decide.

Test Practice

7. Isaiah throws a ball. The graph shows its height as a function of the horizontal distance from where it was thrown.

Select *all* true statements about the situation.

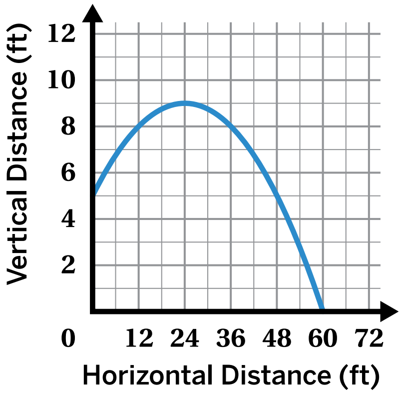
☐ **A.** The maximum height the ball reaches is 9 feet.

☐ **B.** The maximum height the ball reaches is 24 feet.

☐ **C.** The ball lands 24 feet from where it is thrown.

☐ **D.** The ball lands 60 feet from where it is thrown.

☐ **E.** The ball is thrown from a height of 5 feet.



Spiral Review

Problems 8–10: Evaluate each expression for $x = 3$.

8. $(x + 2)(x + 3)$

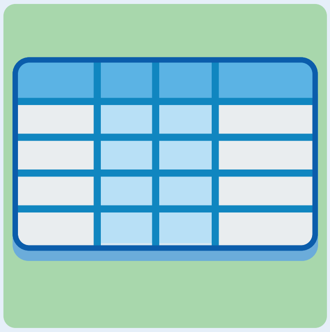
9. $x^2 + 2x + 1$

10. $2(x - 4)^2 - 5$

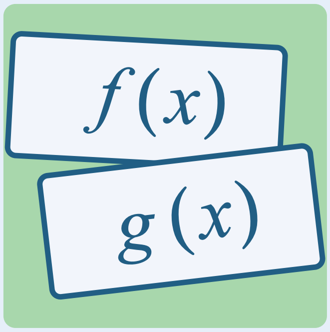
Sub-unit

2

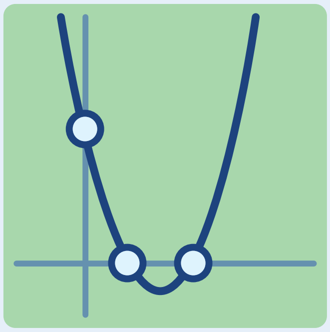
Standard Form and Factored Form



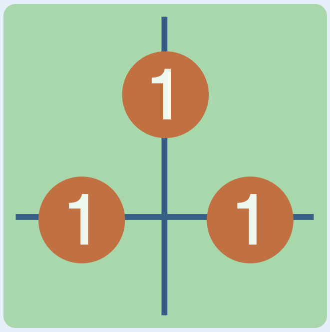
Lesson 8
What's My Graph?



Lesson 9
Two for One



Lesson 10
Interesting Intercepts



Lesson 11
Break Through:
Parabolas

Lesson 12
Sneaker Drop

Unit 6

Lesson

8

Name: ____________ Date: ________ Period: ________

MA.912.AR.3.7, MA.912.F.1.2, MTR.2.1, MTR.4.1

What's My Graph?

Let's develop strategies for graphing quadratic functions by hand.

Warm-Up

1. Here are two functions. Choose two x-values to evaluate for each function.

$a(x) = x^2 + 3x + 1$

x	$a(x)$
-2	
-1	
0	
1	
2	

$b(x) = (x + 3)(x + 1)$

x	$b(x)$
-2	
-1	
0	
1	
2	

2. **Discuss:** What strategy did you use to evaluate each function?

Activity
1

Name: ______________________ Date: __________ Period: __________

Coordinate Co-Op

3. Here is how Ethan evaluated $a(x)$ and $b(x)$ for $x = 2$.

How do each of Ethan's tables work?

$a(x) = x^2 + 3x + 1$

x	x^2	3x	1	$x^2 + 3x + 1$
2	4	6	1	11

$b(x) = (x + 3)(x + 1)$

x	(x + 3)	(x + 1)	(x + 3)(x + 1)
2	5	3	15

You will use a set of cards and graphs to complete Problems 4–6.

4. As a group, complete the table below. Then, using the first column value as the x and last column value as the y, plot each ordered pair (x, y) on Graph #1."

x	x^2	$-2x$	-6	$x^2 - 2x - 6$

5. Repeat the instructions to make $g(x) = (x + 4)(x - 2)$ on Graph #2. Record your thinking.

x	$(x + 4)$	$(x - 2)$	$(x + 4)(x - 2)$

6. Describe a strategy that someone in your group used that you want to celebrate!

Activity
2

Name: .... Date: .... Period: ....

Two Truths and a Lie

Each problem has three statements about the graph of $f(x) = -x^2 - 3x + 2$.

Two are true and one is false. Use the table if it helps with your thinking.

x				

7. Circle the statement that is false.

- **A.** The point $(0, 2)$ is on the graph.
- **B.** The point $(2, -8)$ is on the graph.
- **C.** The point $(2, 0)$ is on the graph.

8. Circle the statement that is false.

- **A.** The point $(-2, 4)$ is on the graph.
- **B.** The point $(-1, 6)$ is on the graph.
- **C.** The point $(1, -2)$ is on the graph.

Each problem has three statements about the graph of $g(x) = (x - 5)(x - 5)$.

Two are true and one is false. Use the table if it helps with your thinking.

x			

9. Circle the statement that is false.

- **A.** The point $(-1, 36)$ is on the graph.
- **B.** The point $(0, -25)$ is on the graph.
- **C.** The point $(1, 16)$ is on the graph.

10. Circle the statement that is false.

- **A.** The point $(4, 1)$ is on the graph.
- **B.** The point $(5, 0)$ is on the graph.
- **C.** The point $(6, 2)$ is on the graph.

You're invited to explore more.

11. Here is another function: $h(x) = 2x^2 - 4$.

- **a** Write two truths and a lie about $h(x)$.
- **b** Share your statements with a classmate. Have them identify the statement that is false.

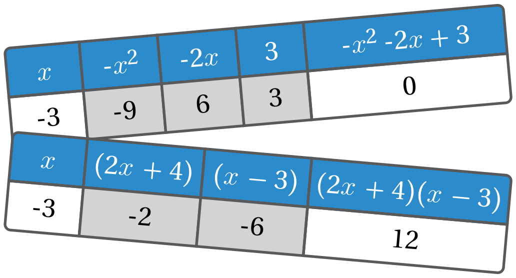
Statement 1:

Statement 2:

Statement 3:

Synthesis

12. How can using a table help you graph quadratic functions?



x	$-x^2$	$-2x$	3	$-x^2 - 2x + 3$
-3	-9	6	3	0

x	$(2x + 4)$	$(x - 3)$	$(2x + 4)(x - 3)$
-3	-2	-6	12

Lesson Practice 6.08

Lesson Summary

You can graph a quadratic function by evaluating its equation at different x-values. A table can be helpful to organize your thinking for quadratics written in standard or factored form.

Here are two examples. You can use the table to determine the value of each function when $x = -3$ and $x = 1$, and determine the corresponding points that are on the graph.

$$f(x) = 2x^2 + 8x - 10$$

x	$2x^2$	$8x$	-10	$2x^2 + 8x - 10$	Point on Graph
-3	$2(-3)^2 = 18$	$8(-3) = -24$	-10	$18 - 24 - 10 = -16$	(-3, -16)
1	$2(1)^2 = 2$	$8(1) = 8$	-10	$2 + 8 - 10 = 0$	(1, 0)

$$g(x) = (3x - 1)(x + 5)$$

x	$(3x - 1)$	$(x + 5)$	$(3x - 1)(x + 5)$	Point on Graph
-3	$3(-3) - 1 = -10$	$-3 + 5 = 2$	$(-10)(2) = -20$	(-3, -20)
1	$3(1) - 1 = 2$	$1 + 5 = 6$	$(2)(6) = 12$	(1, 12)

Lesson Practice

6.08

Name: ______ Date: ______ Period: ______

1. Here is a function: $k(x) = (x + 4)(x - 2)$.

Amoli says that $k(\text{-}2) = \text{-}12$.
Yasmine says that $k(\text{-}2) = \text{-}8$.

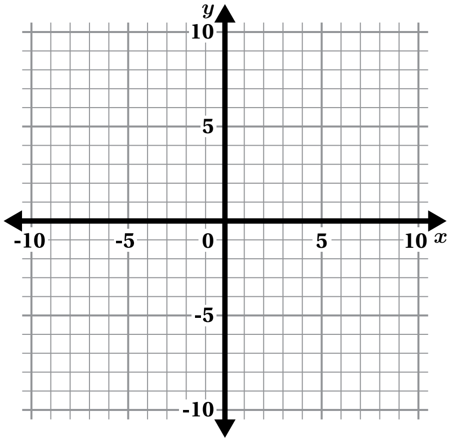
Whose thinking is correct? Explain your thinking.

Problems 2–3: Here is a function: $g(x) = 2x^2 - 2x - 4$.

2. Complete the table for $g(x)$.

x	$2x^2$	$\text{-}2x$	-4	$2x^2 - 2x - 4$
-2			-4	
		2	-4	
0				
1		-2		
	8			

3. Create a graph of $g(x)$.

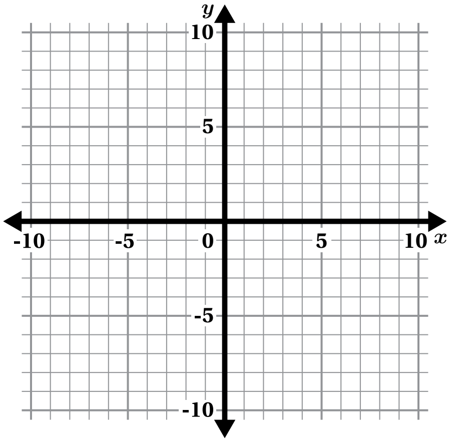


Problems 4–5: Here is a function: $h(x) = 2x(x + 3)$.

4. Complete the table for $h(x)$.

x	$2x$	$x + 3$	$2x(x + 3)$
-3			
-2			
	-2		
0			
1		4	

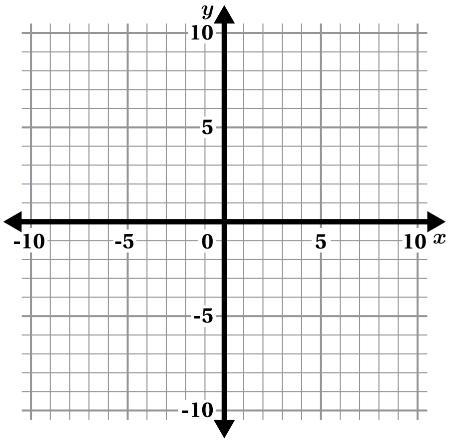
5. Create a graph of $h(x)$.



Lesson Practice
6.08

Name: .... Date: .... Period: ....

6. Plot a point in each quadrant that is on the graph of $g(x) = x^2 - 4$.



Test Practice

7. Here are three statements about the graph of $r(x) = (x - 1)(x + 1)$. Two are true and one is false. Circle the statement that is false.

A. The point $(-1, 0)$ is on the graph.

B. The point $(0, 1)$ is on the graph.

C. The point $(1, 0)$ is on the graph.

Spiral Review

Problems 8–10: Evaluate each expression for $x = 3$.

8. $2(x - 2)$

9. $\frac{1}{9}x + 3$

10. $3(x - 1) + 4$

Unit 6

Lesson

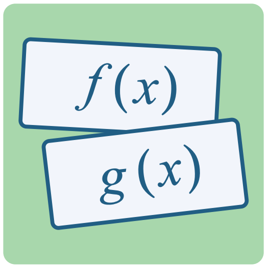
9

Name: ______ Date: ______ Period: ______

MA.912.AR.3.7, MA.912.F.1.2, MTR.2.1, MTR.5.1

Two for One

Let’s explore two forms of quadratic functions and make connections between equations, tables, and graphs.



Warm-Up

Here is a function: $f(x) = (2x + 1)(x - 3)$.

Two students calculated $f(1)$.

Gabriel

x	$2x + 1$	$x - 3$	$(2x + 1)(x - 3)$
1	3	-2	-6

Anya

$f(x) = (2x + 1)(x - 3)$

$f(1) = (2 \cdot 1 + 1)(1 - 3)$

$f(1) = (3)(-2)$

$f(1) = -6$

1. How are the strategies alike? How are they different?

2. Whose strategy do you prefer? Why?

Activity
1

Name: .... Date: .... Period: ....

Card Sort

You will use a set of cards to complete this activity.

3. Work with a partner to sort the equation cards into two or more groups.

Discuss: How did you sort the cards?

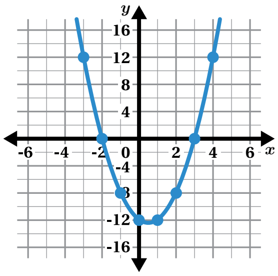
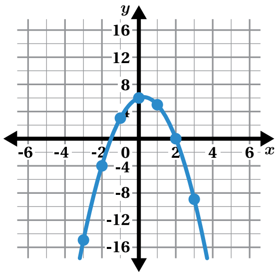
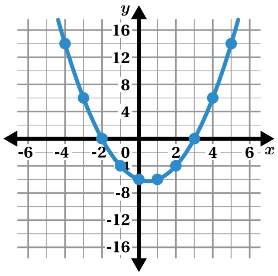
Let's look at examples of quadratic equations written in **factored form** and **standard form**.

4. With your partner, decide whether each card is written in factored form, standard form, or neither. Write the equations from your cards in the table below.

Factored Form	Standard Form	Neither

5. Set $b(x)$, $e(x)$, and $h(x)$ aside. You will not need them for the rest of the lesson.

Match the remaining cards with the graphs below. Two cards will have no match.

Activity
2

Name: .... Date: .... Period: ....

Missing Graphs

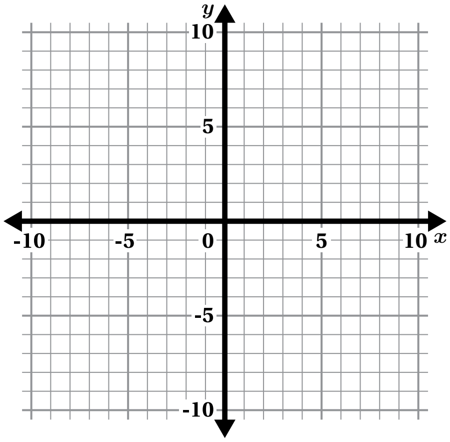
6. Here are the two cards with no match from Activity 1. Complete a table and graph for each equation.

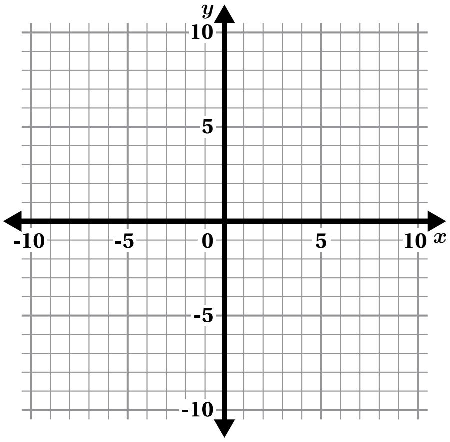
Factored form: $c(x) = (2x - 2)(x + 3)$

x	$c(x)$

Standard form: $j(x) = 2x^2 + 4x - 6$

x	$j(x)$





7. Compare the two functions. How are they alike? How are they different?

Synthesis

8. **a** Which equation do you think would be more challenging to graph? Circle one.

$p(x) = (2x - 2)(x + 5)$

$q(x) = 2x^2 + 8x - 10$

b What makes the equation you chose more challenging to graph?

c What advice would you give someone to help them graph an equation like this?

Lesson Practice 6.09

Lesson Summary

Quadratic equations can be written in many forms. Two of them are **standard form** and **factored form**.

The standard form of a quadratic equation has a squared term, and can have linear and/or constant terms added or subtracted.

The factored form of a quadratic equation has two factors multiplied together.

Here are some examples of functions written in different forms. In the last column, $c(x)$ is a quadratic function written in a different form, and $f(x)$ is an exponential function.

Factored Form	Standard Form	Neither/Not Quadratic
$a(x) = x^2 - 5x + 6$	$b(x) = \frac{1}{2}(x + 1)(x + 5)$	$c(x) = (x + 3)^2 - 1$
$d(x) = 2x^2 + 8x - 10$	$e(x) = (2x - 2)(x + 3)$	$f(x) = 3^x$

Lesson Practice
6.09

Name: .... Date: .... Period: ....

1. Determine whether each equation is written in standard form, factored form, or neither.

$a(x) = 3x^2 + 4$ $b(x) = 3x^2 + 4x + 2$ $c(x) = (x + 3)^2 + 4$

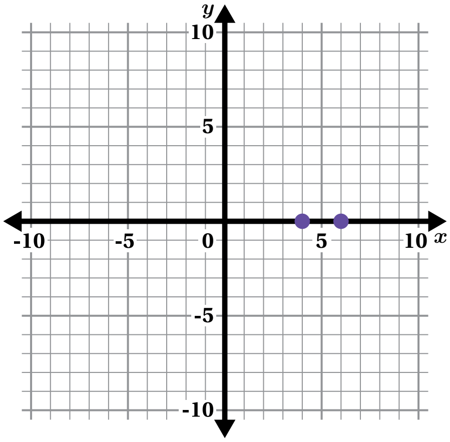
$f(x) = 2x(x + 4)$ $g(x) = 4x^2 - 3x$ $(4x - 4)(x - 3) = h(x)$

Factored Form	Standard Form	Neither

2. Kala is graphing the equation $r(x) = (x - 6)(x - 4)$. Which x-value should she evaluate to best complete her graph? Circle one.

$x = 1$ $x = -1$ $x = 5$

Explain your thinking.

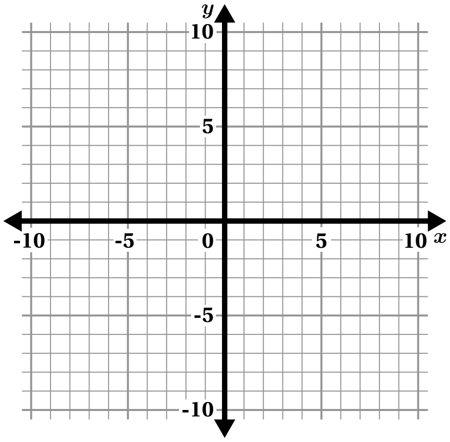


Problems 3–4: Here is a function: $a(x) = x^2 - 2x + 1$.

3. Complete the table for $a(x)$.

x	$a(x)$
-2	
-1	
0	
1	
2	

4. Create a graph of $a(x)$.



Lesson Practice
6.09

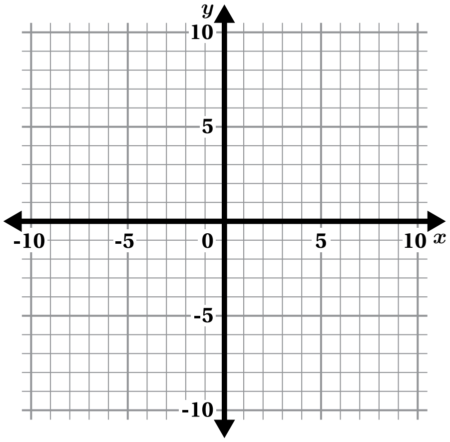
Name: .... Date: .... Period: ....

Problems 5–6: Here is a function: $b(x) = (2x - 2)(x + 3)$.

5. Complete the table for $b(x)$.

x	$b(x)$
-2	
-1	
0	
1	
2	

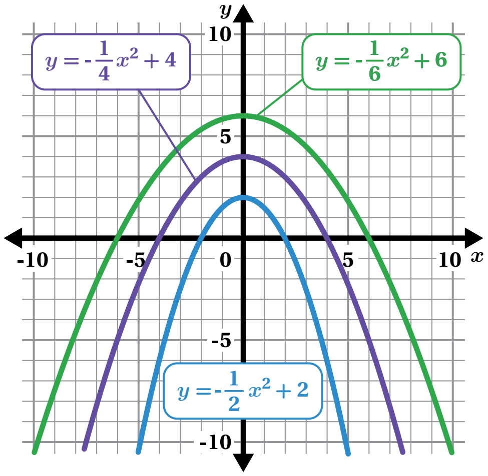
6. Create a graph of $b(x)$.



Test Practice

Problems 7–8: Here are some quadratic functions.

7. Graph the next parabola that follows this pattern.



8. Complete the table for the next parabola that follows this pattern.

x	y
-8	
0	
8	

Spiral Review

9. Write the equation of a line that has a y-intercept of -2.

10. Write the equation of a line that has an x-intercept of 3.

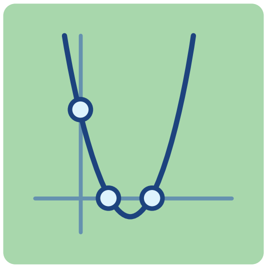
Unit 6
Lesson
10

Name: ________ Date: ________ Period: ________

MA.912.AR.3.4, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.F.1.2, MTR.2.1, MTR.4.1, MTR.5.1

Interesting Intercepts

Let’s make connections between the intercepts of a parabola and the structure of its equation.



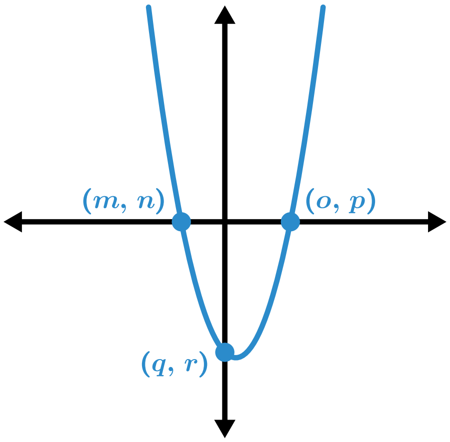
Warm-Up

1. Here is the graph of a function.

Select *all* the values that are equal to 0.

☐ m
☐ n
☐ o
☐ p
☐ q
☐ r

Explain your thinking.



Name: .... Date: .... Period: ....

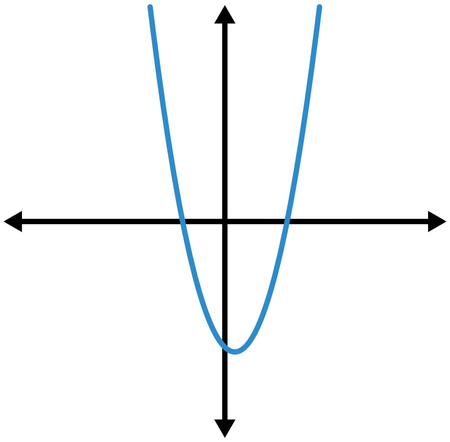
Intercepts in Factored Form

2. Here is the function from the Warm-Up.

Its equation is $f(x) = (x + 2)(x - 3)$.

The x- and y-intercepts of the function are shown. Fill in the circles on the provided graph.

What do you notice about the intercepts?



x-intercepts:

y-intercepts:

Activity
2

Name: ____________________ Date: __________ Period: __________

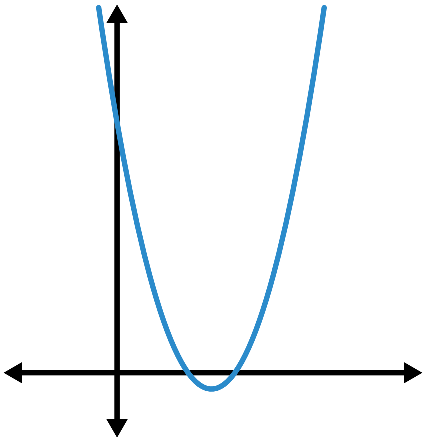
Determine the Intercepts

3. Here is a new function: $g(x) = (2x - 6)(x - 5)$.

The x- and y-intercepts of the function are shown. Fill in the circles on the provided graph

Here is Raven's work from the previous challenge.

x	$(2x - 6)$	$(x - 5)$	$(2x - 6)\ (x - 5)$
5	4	0	0
6	$2(6) - 6$	$6 - 5$	$(2(6) - 6)(6 - 5)$



She found an x-intercept at 5.

She says there will be another x-intercept at 6.

Explain why Raven's thinking is incorrect.

Name: .... Date: .... Period: ....

Determine the Intercepts (continued)

4. Here is a new function:

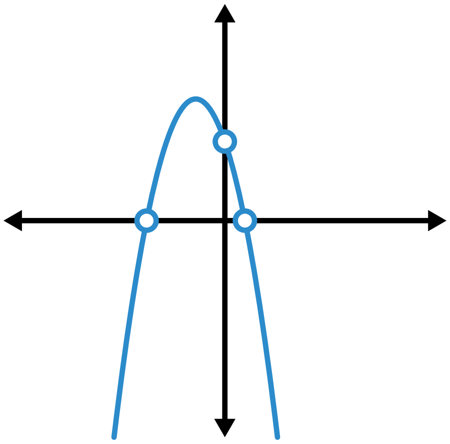
$h(x) = (-x + 0.5)(4x + 8)$

What are the intercepts of h?

x-intercept: ....

x-intercept: ....

y-intercept: ....



5. Two students were asked to determine the x-intercepts of $p(x) = 2x(x + 9)$.

Yolanda says the x-intercepts are at -2 and -9.
Julian says the x-intercepts are at 0 and -9.

Whose thinking is correct? Circle one.

Julian's Yolanda's Both Neither

Explain your thinking.

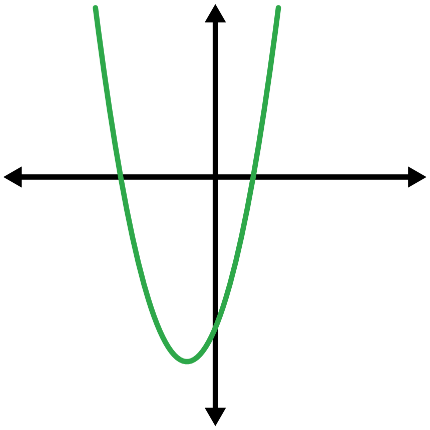
Name: .... Date: .... Period: ....

Intercepts in Standard Form

6. Here is a new function:

$w(x) = x^2 + 3x - 10$

The x- and y-intercepts of the function are shown. Fill in the circles on the provided graph.



Which intercepts were easier for you to determine? Circle one.

x-intercepts

y-intercept

Explain your thinking.

7. Match each equation with its y-intercept. One equation will have no match.

$a(x) = x^2 - 3x + 5$	$b(x) = x^2 + 5x - 3$	$c(x) = x^2 - 5x + 3$
$d(x) = -3x^2 + 5$	$e(x) = 5x^2 - 3$	$f(x) = 3x^2 + 5 + x^2$

(0, 5)	(0, -3)

Synthesis

8. The same function is written in factored and standard form. What does each form tell you about the graph of $f(x)$?

Factored Form

$f(x) = (2x - 1)(x + 3)$

Standard Form

$f(x) = 2x^2 + 5x - 3$

Factored form:

Standard form:

Lesson Practice 6.10

Lesson Summary

Different forms of quadratic equations help us see different key features of a parabola.

In a standard form quadratic equation, the y-*intercept* is the constant term in the equation.

In a factored form quadratic equation, the x-*intercepts* are the values that make each factor equal to 0.

Here is an example of the same function written in standard and factored form. We can use the different forms to determine key features and graph the parabola.

Standard Form

$f(x) = 2x^2 - 4x - 6$

$f(x) = 2x^2 - 4x \boxed{- 6}$

The y-intercept is $(0, \text{-}6)$.

Factored Form

$f(x) = (2x + 2)(x - 3)$

x	$(2x + 2)$	$(x - 3)$	$(2x + 2)\ (x - 3)$
-1	$2(\text{-}1) + 2 = 0$	$(\text{-}1) - 3 = \text{-}4$	$(0)(\text{-}4) = 0$
3	$2(3) + 2 = 8$	$(3) - 3 = 0$	$(8)(0) = 0$

The x-intercepts are $(\text{-}1, 0)$ and $(3, 0)$.

Lesson Practice
6.10

Name: ______ Date: ______ Period: ______

Problems 1–2: Determine an x-intercept(s) from each equation.

1. $a(x) = (x - 2)(x + 1)$

2. $b(x) = (x + 3)^2$

3. Here is the same function written in two forms.

Factored form: $g(x) = (x + 5)(x - 2)$ **Standard form:** $g(x) = x^2 + 3x - 10$

What are the x- and y-intercepts of the function?

x-intercept: ______

x-intercept: ______

y-intercept: ______

4. Evan and Ariel were working on a problem together.

Evan said: *The y-intercept of $y = (x - 3)^2$ is $(0, 3)$.*

Ariel said: *The x-intercept of $y = (x - 3)^2$ is $(3, 0)$.*

Whose thinking is correct? Circle one.

Evan's Ariel's

Explain your thinking

Problems 5–6: Here is a function: $h(x) = x(x + 6)$.

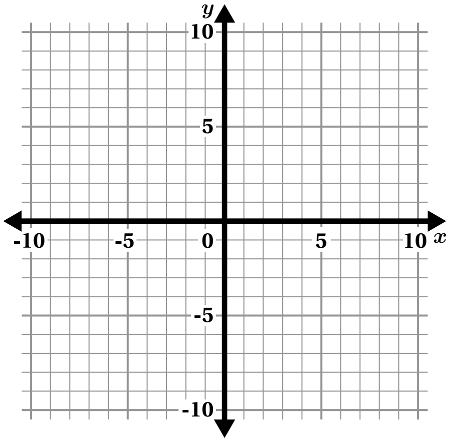
5. Determine the intercepts of $h(x)$.

x-intercept: ______

x-intercept: ______

y-intercept: ______

6. Draw the graph of $h(x)$.



Lesson Practice
6.10

Name: .... Date: .... Period: ....

Problems 7–9: Use a graphing calculator to graph $f(x) = (x + 3)(x + 1)(x - 2)$. Change the numbers in the equation to determine an equation whose graph . . .

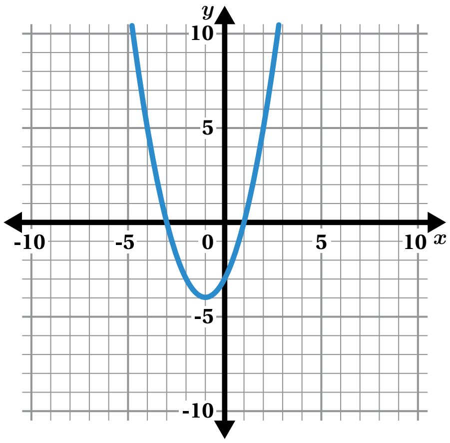
7. Has two x-intercepts. ....

8. Has one x-intercept. ....

9. Has an x-intercept at $(7, 0)$. ....

Test Practice

10. Here is a graph of a quadratic function.

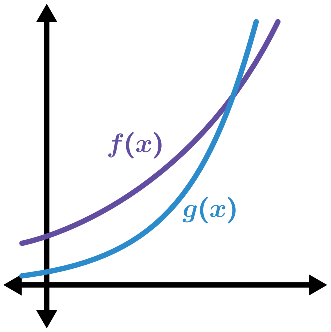


Which function could this be?

- **A.** $y = (x - 3)(x + 1)$
- **B.** $y = (x + 3)(x - 1)$
- **C.** $y = (x - 3)(x - 1)$
- **D.** $y = (x + 3)(x + 1)$

Spiral Review

11. Here are the graphs of two functions, $f(x)$ and $g(x)$.



$f(x) = 100 \cdot 2^x$

Which equation could represent $g(x)$?

- **A.** $g(x) = 25 \cdot 4^x$
- **B.** $g(x) = 50 \cdot 1.5^x$
- **C.** $g(x) = 100 \cdot 4^x$
- **D.** $g(x) = 200 \cdot 1.5^x$

Problems 12–13: Complete each equation with a number that makes the equation true.

12. $2 \cdot$ .... $+ 4 = 0$

13. $3(3 \cdot$ .... $+ 4) = 0$

Unit 6

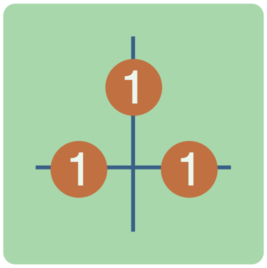
Lesson

11

Name: .... Date: .... Period: ....

MA.912.AR.3.4, MA.912.AR.3.5, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.2.1, MTR.1.1, MTR.3.1, MTR.5.1, MTR.6.1

Break Through: Parabolas

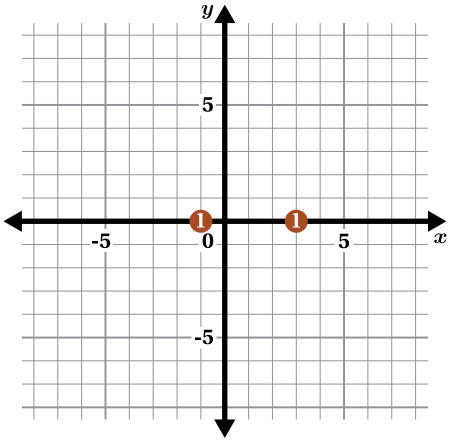


Let's write equations of parabolas in factored form.

Warm-Up

You'll use a graphing calculator or graphing tool for most of this lesson.

1. Your goal is to write equations of parabolas that will break the targets, as shown in the graph to the right.



a Graph $y = (x + 1)(x - 2)$ and see if it passes through the x-intercepts at the location of the targets on the graph.

b Change the equation to break all the targets.

Original equation:

$y = (x + 1)(x - 2)$

Your equation:

c Graph your new quadratic function on the provided graph.

Activity
1

Name: .... Date: .... Period: ....

Building Quadratic Functions

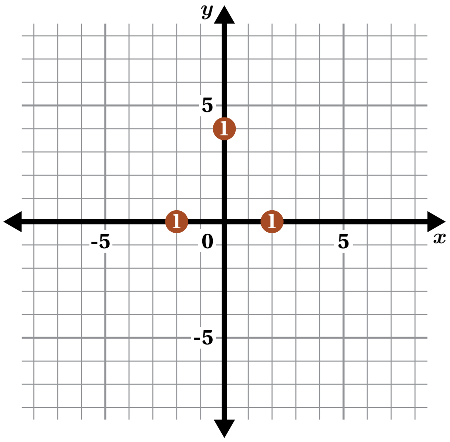
2. Here's another challenge.

Change the equation to break *all* the targets.

Original equation:

$y = (x + 2)(x - 2)$

Your equation:

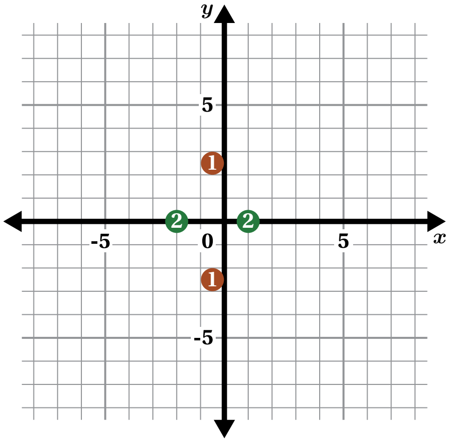


3. Some of the targets will require multiple parabolas.

Write another equation to break the remaining targets.

Equation
$y = (x - 1)(x + 2)$

Graph both quadratic functions on the provided graph.

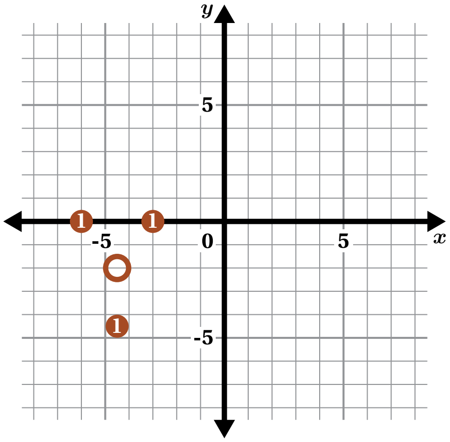


4. Carlos started this challenge.

Write another equation to finish it.

Equation
$y = (x + 6)(x + 3)$

Graph your quadratic function on the provided graph.

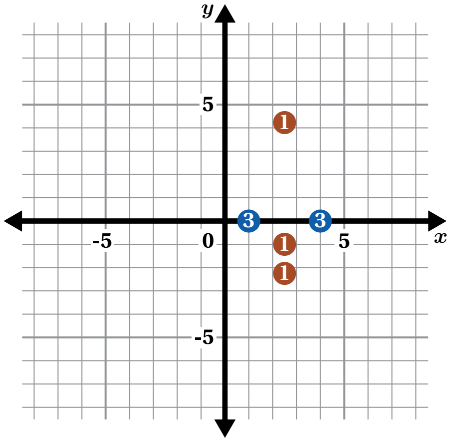


Name: ________ Date: ________ Period: ________

Building Quadratic Functions (continued)

5. Break the targets using three equations.

Write the equations in the table below and graph them on the provided graph to the right. If needed, you may use a graphing calculator or graphing utility to check your thinking.

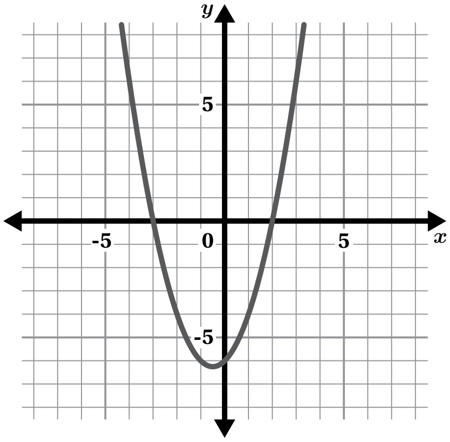


Equation

6. Here is the graph of $y = (x + 3)(x - 2)$.

How might $y = \frac{1}{2}(x + 3)(x - 2)$ look similar? Different?

Show or explain your thinking.

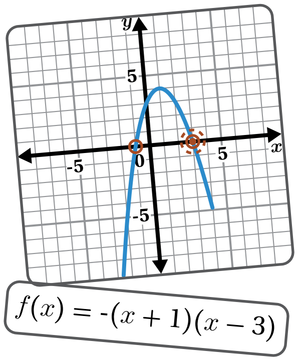


Activity
2

Name: .... Date: .... Period: ....

Lots of Challenges

You'll use a graphing calculator or graphing tool for Problems 7-11.



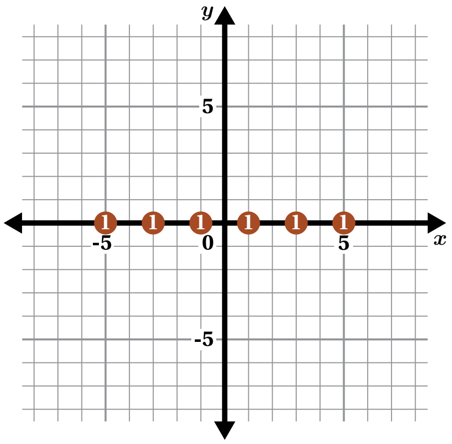
Move on to the final set of challenges.

Use what you know about writing quadratic equations to break all the targets in fun and creative ways!

For each challenge, break the targets by using as few equations as you can. Write the equations in the tables and graph them on the provided graphs to the right.

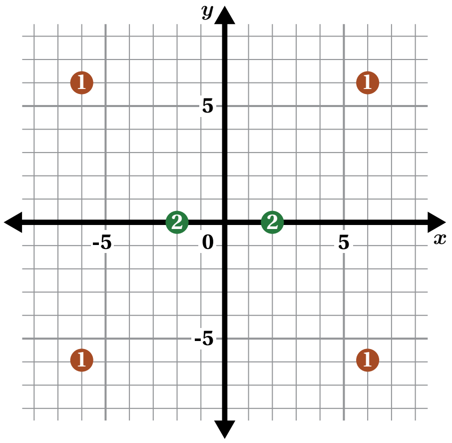
7. Break the targets using as few equations as you can.

Equation



8. Break the targets using as few equations as you can.

Equation

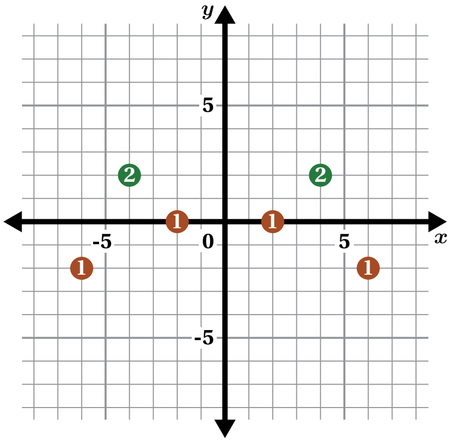


Name: ________________ Date: ________ Period: ________

Lots of Challenges (continued)

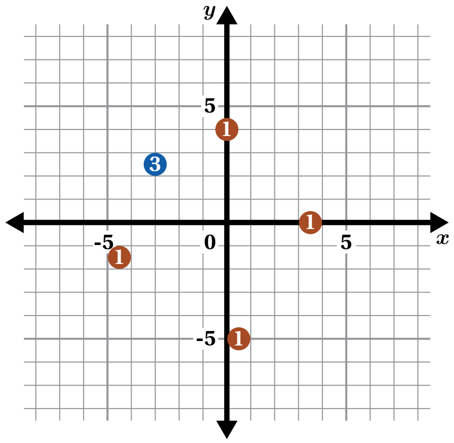
9. Break the targets using as few equations as you can.

Equations



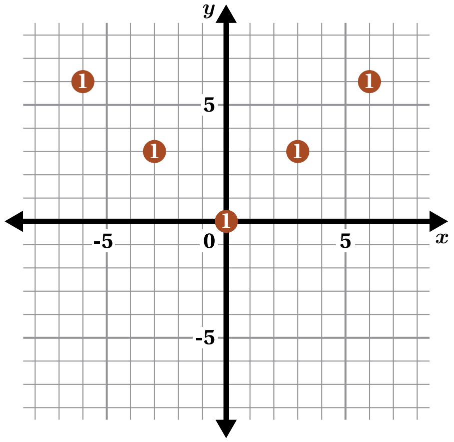
10. Break the targets using as few equations as you can.

Equations



11. Break the targets using as few equations as you can.

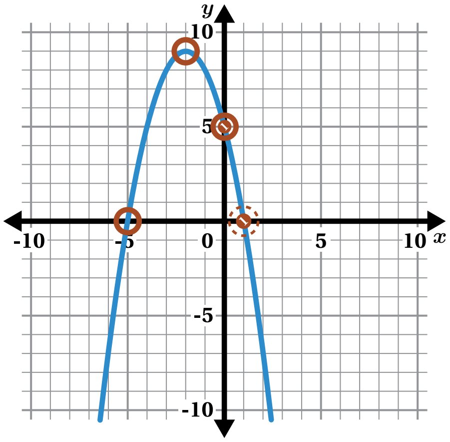
Equations



Synthesis

12. Describe a strategy for writing a quadratic equation in factored form that matches a graph.

Use the example if it helps with your thinking.



Lesson Practice 6.11

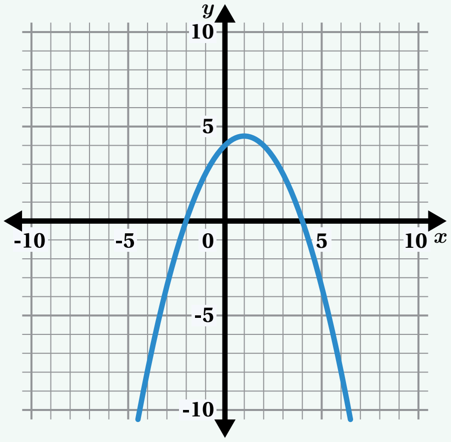
Lesson Summary

We can write quadratic equations to match graphs and key features. One way to do this is in the form $y = a(x - m)(x - n)$.

- Use the x-intercepts to write the *factors* of the equation.
- If the parabola is concave down, make the a-value negative.
- Adjust the a-value to match the vertical position of the vertex and the y-intercept.

Here is an example strategy for writing an equation to match this graph.

- The x-intercepts are $(\text{-}2, 0)$ and $(4, 0)$. You can write the factors as $y = (x + 2)(x - 4)$.
- Since the parabola is concave down, multiply by a negative number, like $y = \text{-}(x + 2)(x - 4)$.
- Adjust the a-value to make the vertical position of the vertex match the graph, like $y = \text{-}\frac{1}{2}(x + 2)(x - 4)$.



Lesson Practice
6.11

Name: .... Date: .... Period: ....

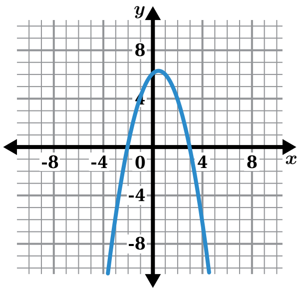
1. Match each equation to the graph it represents. One equation will have no match.

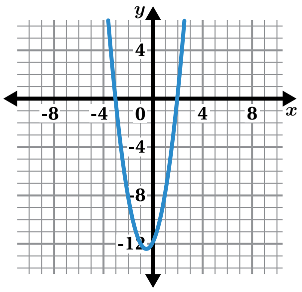
$y = -(x + 2)(x - 3)$

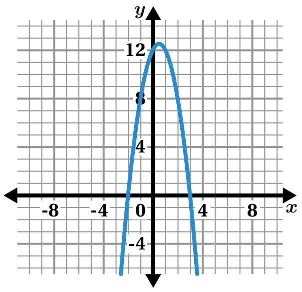
$y = 2(x + 3)(x - 2)$

$y = -2(x + 2)(x - 3)$

$y = (x + 2)(x - 3)$



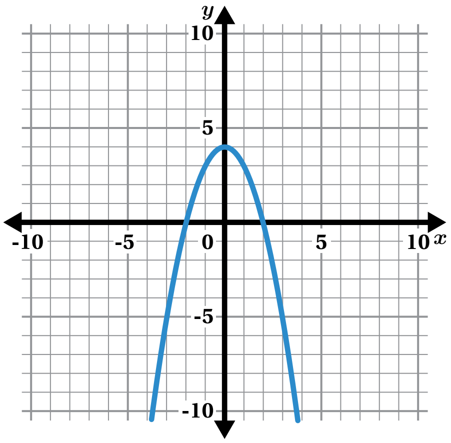




Problems 2–3: Here is the graph of $y = -1(x + 2)(x - 2)$. Change the equation so the vertex goes through:

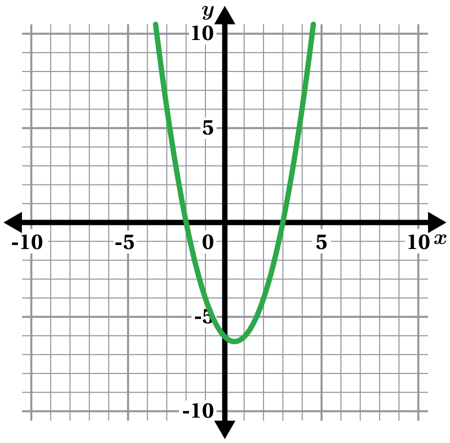
2. (0, 8)

3. (0, -2)

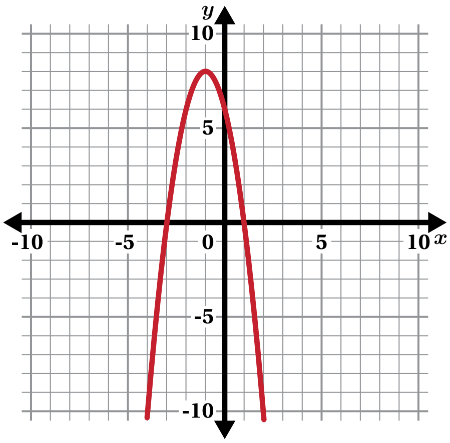


Problems 4–5: Write an equation to match each graph.

4. Equation: ....



5. Equation: ....



Lesson Practice
6.11

Name: .... Date: .... Period: ....

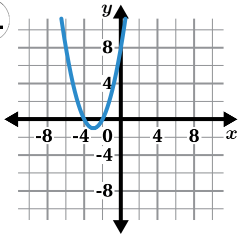
6. Write the equations of three different parabolas that have the same vertex but different x-intercepts.

Equation 1	Equation 2	Equation 3

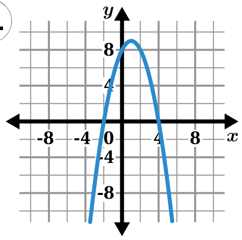
Test Practice

7. Which graph shows the function $y = (x - 4)(x + 2)$?

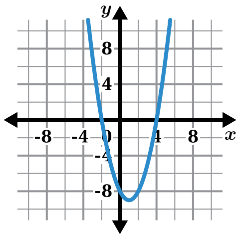
A.



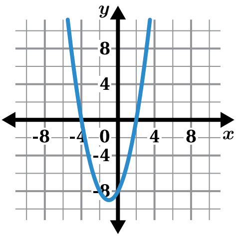
B.



[✓ C.]



D.



Spiral Review

Problems 8–9: Here are two linear functions: $f(x)$ and $h(x)$.

$$f(x) = 3x + 2$$

x	$h(x)$
1	5
2	9
3	13

8. Which function has a greater y-intercept? Circle one.

$f(x)$ $h(x)$

9. Which function has a greater slope? Circle one.

$f(x)$ $h(x)$

Unit 6
Lesson
12

Name: ________ Date: ________ Period: ________

MA.912.AR.3.6, MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.1.2, MTR.2.1, MTR.5.1, MTR.6.1, MTR.7.1

Sneaker Drop

Let's use quadratic equations to make sense of revenue models.

Warm-Up

You've been hired by Des-Kicks to design and sell a new sneaker.

Let's look at some of their most popular designs.

1. Design your own Des-Kicks sneaker.

2. How much would you sell your sneakers for? Why?

Activity
1

Name: ____________ Date: ____________ Period: ____________

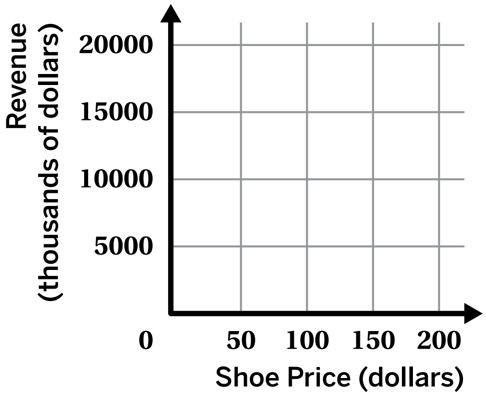
Price Points

The mathematicians at Des-Kicks created a revenue model for your sneakers.

- The model predicts there are 400 thousand people who want to buy your sneakers.
- For every $50 increase in price, Des-Kicks will sell 100 thousand fewer pairs of sneakers.

3. Use the information to create a table and graph of the revenue.

Shoe Price (dollars)	Number of Customers (thousands)	Revenue (thousands of dollars)
0	400	
50	300	
100		
150		
200		



Use the table and graph as evidence for Problems 4–6.

4. Should Des-Kicks sell your sneakers for $50 or $150? Why?

5. According to the model, Des-Kicks will earn $0 in revenue at two prices.

a What are those prices?

b Why is the revenue $0 at each of those prices?

6. What shoe price would maximize revenue for Des-Kicks? Explain your thinking.

Name: .... Date: .... Period: ....

Comparing Companies

Three sneaker companies want you to design their next sneaker.

Use the Company Fact Sheet to answer Problems 7–10.

7. Which company would make the most revenue from selling your sneaker for $50? How do you know?

8. Which company would make the most revenue from selling your sneaker for $150? How do you know?

9. With which company would you earn the highest maximum revenue for selling your shoes? How do you know?

10. Think about all the information you have about the three sneaker companies. Which company would you want to work with? Why?

Name: .... Date: .... Period: ....

Comparing Companies (continued)

You're invited to explore more.

11. A different company shows you their revenue model, which is not a parabola.

Here is a graph of their revenue model. Why do you think this model looks the way it does?

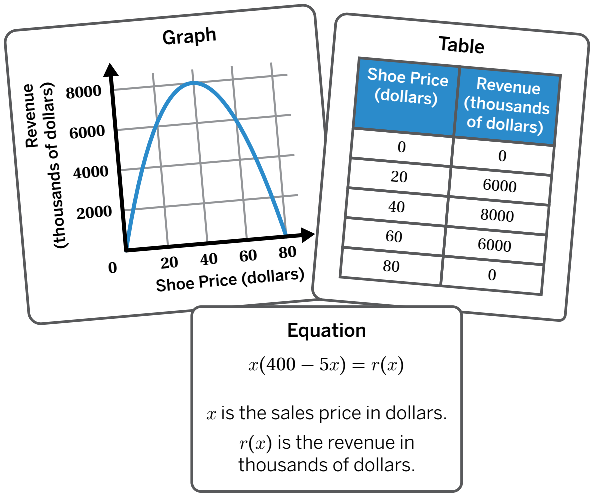
Synthesis

12. The equation, graph, and table all represent the same quadratic function. Explain how to determine the maximum revenue from each representation.

Equation:

Graph:

Table:



Table

Shoe Price (dollars)	Revenue (thousands of dollars)
0	0
20	6000
40	8000
60	6000
80	0

Equation

$x(400 - 5x) = r(x)$

x is the sales price in dollars.

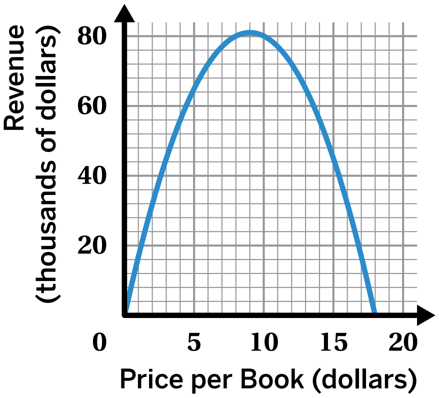
$r(x)$ is the revenue in thousands of dollars.

Lesson Practice 6.12

Lesson Summary

You can use quadratic functions to make sense of the amount of money made from selling an item, also called *revenue*. We can assume that if an item is more expensive, fewer people will buy it.

Here is an example of selling books.

<table>
<tr><th>Equation</th><th>Graph</th><th>Table</th></tr>
<tr>
<td>$r(x) = x(18 - x)$
x is the price per book in dollars.
$r(x)$ is the revenue in thousands of dollars.</td>
<td>
</td>
<td>
<table>
<tr><th>Price per Book (dollars)</th><th>Number of Customers (thousands)</th><th>Revenue (thousands of dollars)</th></tr>
<tr><td>0</td><td>18</td><td>0</td></tr>
<tr><td>1</td><td>17</td><td>17</td></tr>
<tr><td>3</td><td>15</td><td>45</td></tr>
<tr><td>5</td><td>13</td><td>65</td></tr>
<tr><td>10</td><td>8</td><td>80</td></tr>
</table>
</td>
</tr>
</table>

You can see from the vertex on the graph that selling books for $9 will earn a maximum revenue.

You can use the equation to determine the maximum revenue by evaluating $r(x)$ when $x = 9$.

$r(x) = x(18 - x)$

$r(9) = 9(18 - 9)$

$r(9) = 81$ So the maximum revenue will be $81,000.

Lesson Practice
6.12

Name: .... Date: .... Period: ....

1. Match each function with its y-intercept. There will be one function with no match.

$f(x) = 4 + 2.5x - 2x^2$

$h(x) = 4x^2 - 2.5x + 2$

$g(x) = 2.5 + 4x - 2x^2$

$j(x) = 2.5x^2 + 4x - 2$

(0, 2.5)	(0, -2)	(0, 4)

2. Here is a function: $g(x) = (x - 3)^2$. Determine if each statement is true or false, or if there is not enough information.

The y-intercept of g is at $(0, 3)$.	True	False	Not enough information
An x-intercept of g is at $(3, 0)$.	True	False	Not enough information
The vertex of g is at $(3, 0)$.	True	False	Not enough information

Problems 3–6: Store A sells a video game called DesMan. The company asked customers how much they would pay for the video game.

3. Complete the table.

Price ($)	Number of Customers	Revenue ($)
10	15,000	
25	7,500	

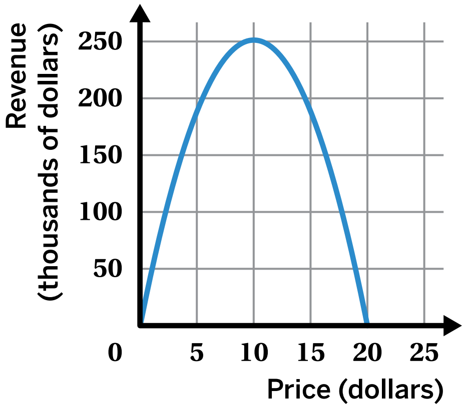
Store A used your table to create a revenue model: $r(p) = p(20000 - 500p)$. p represents the price of the video game.

4. At what prices will Store A make $0 in revenue?

5. At what price will Store A make the most money?

6. Store B also sells DesMan. This graph models their revenue.

Which store has a higher maximum revenue? Explain your thinking.

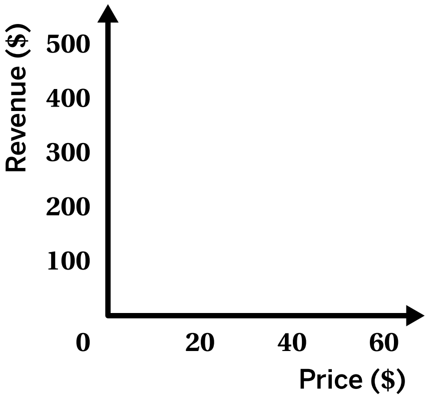


Lesson Practice
6.12

Name: .... Date: .... Period: ....

7. Create a revenue graph for a mystery product that:

- Makes no revenue when priced at \$0 or more than \$60.
- Has a maximum profit that is *not* halfway between the x-intercepts.



Test Practice

8. One possible revenue model for the mysterious product is $r(p) = -60p^2 + 3600p$.

What is the revenue if the price is \$10?

A. \$35,400

B. \$72,000

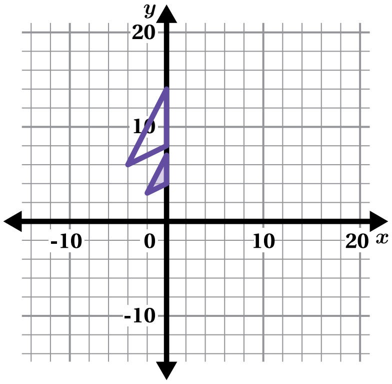
C. \$30,000

D. \$36,000

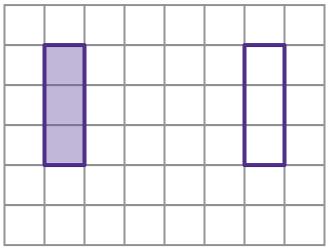
Spiral Review

Problems 9–10: Describe a sequence of transformations that moves the shaded figure onto the unshaded figure.

9.



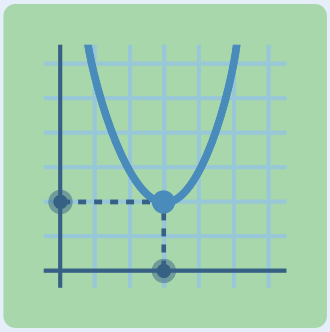
10.



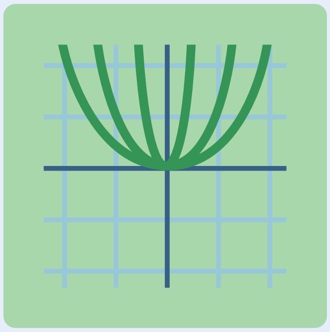
Sub-unit

3

Vertex Form



Lesson 13
Vertex Form



Lesson 14
Stretch It Out

Lesson 15
Predicting Sales

Unit 6

Lesson

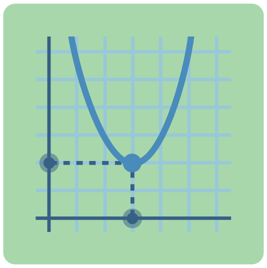
13

Name: .... Date: .... Period: ....

MA.912.AR.3.4, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.2.1, MTR.1.1, MTR.4.1, MTR.5.1

Vertex Form

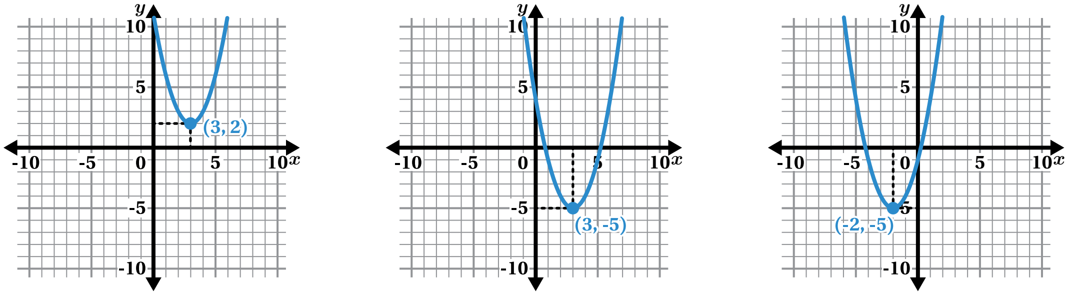
Let’s transform quadratic functions using translations and write their equations in a new form.



Warm-Up

1. Here are a few transformations of a parabola.

$y = (x - 3)^2 + 2$ $y = (x - 3)^2 - 5$ $y = (x + 2)^2 - 5$



What changes? What stays the same?

Changes:

Same:

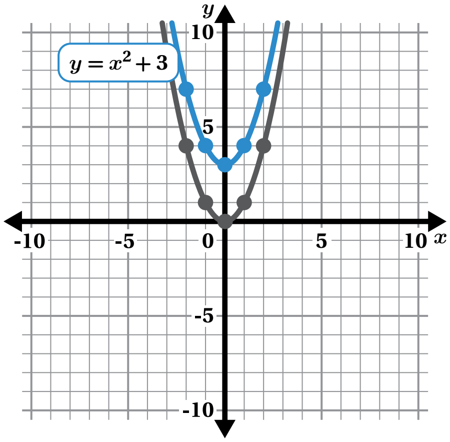
Activity
1

Name: .... Date: .... Period: ....

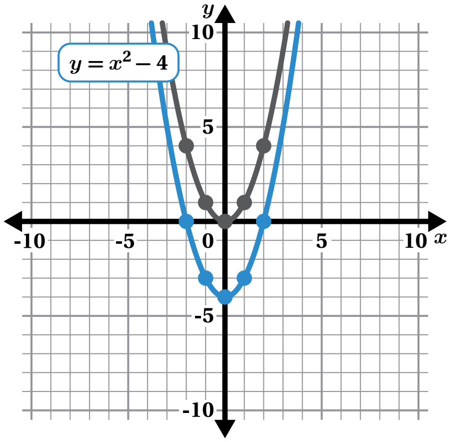
Translating Parabolas

2. Here are two different vertical *translations* of $y = x^2$.

Graph A



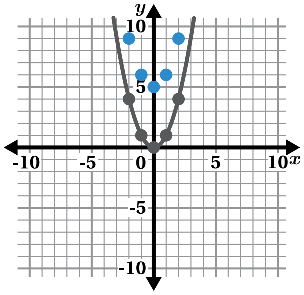
Graph B



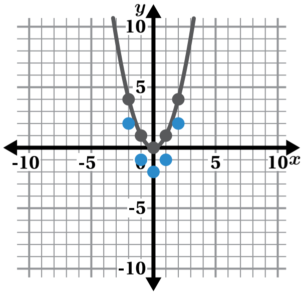
Discuss: What do you notice? What do you wonder?

3. For each challenge, write the equation for the vertical translation of $y = x^2$.

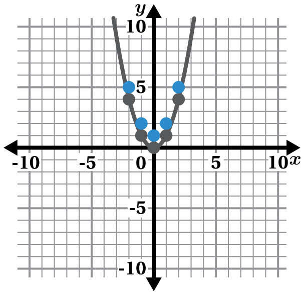
a $y =$ ....



b $y =$ ....



c $y =$ ....



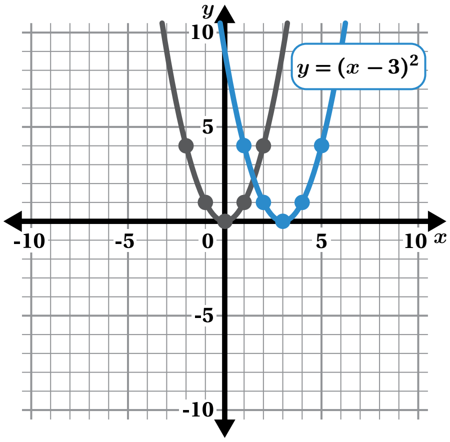
Activity 1

Name: .... Date: .... Period: ....

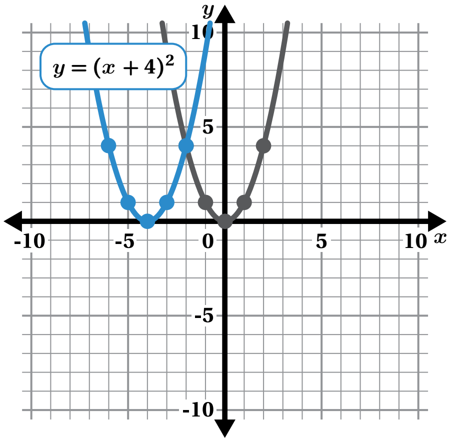
Translating Parabolas (continued)

4. Here are two different horizontal translations of $y = x^2$.

Graph C



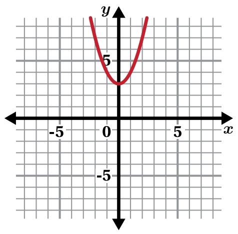
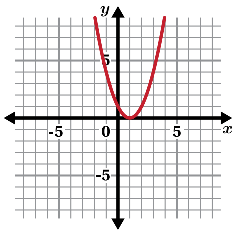
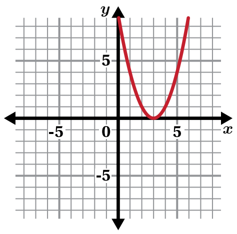
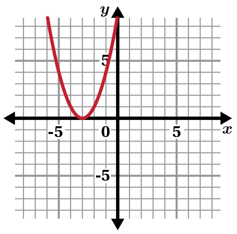
Graph D



How can you see the translation in the equation?

5. Match each equation to a graph. One graph will have no match.

a. $y = (x + 3)^2$ **b.** $y = x^2 + 3$ **c.** $y = (x - 1)^2$

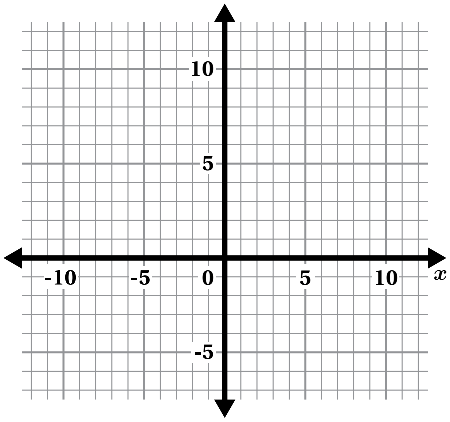
			
Equation:	Equation:	Equation:	Equation: ....

Name: .... Date: .... Period: ....

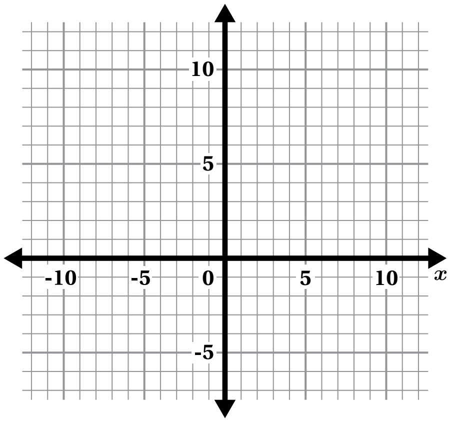
Translating Parabolas (continued)

6. Draw the graph of each parabola.

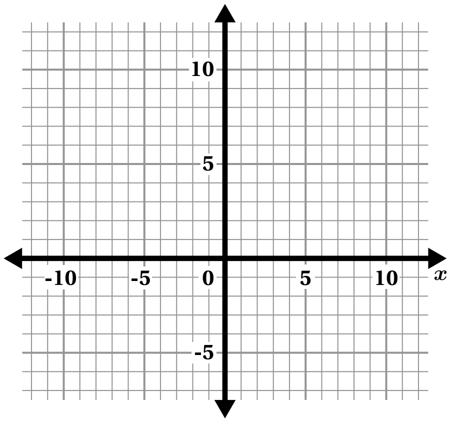
a $y = (x + 2)^2$



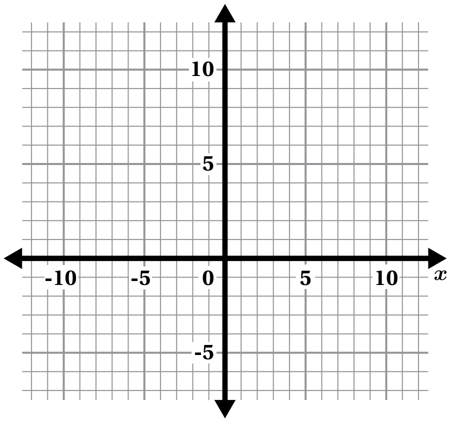
b $y = x^2 - 6$



c $y = (x + 1)^2 + 4$



d $y = (x + 8)^2 - 6$

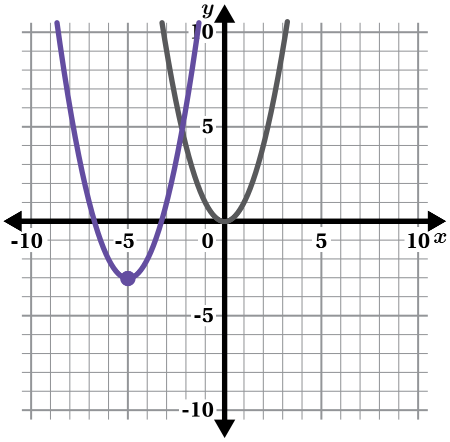


Activity
2

Name: ________ Date: ________ Period: ________

Vertex Form

7. Here are the graphs of two functions, $f(x)$ and $g(x)$. The purple parabola, $g(x)$, is a translation of $f(x) = x^2$ left 5 units and down 3 units to $g(x) = (x + 5)^2 - 3$.



Discuss: Why do you think this type of equation is called **vertex form**?

8. Here is Liam's equation for a parabola with a vertex at (-3, -4). $f(x) = (x - 3)^2 - 4$

a) What did Liam do well?

b) What was Liam's mistake?

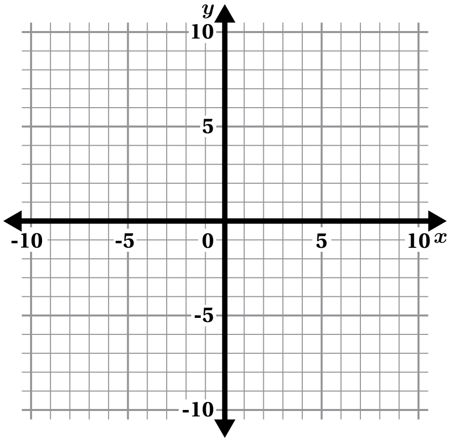
Activity 2

Name: .... Date: .... Period: ....

Vertex Form (continued)

9. Write an equation of a quadratic function with its vertex at at (-6, -2).

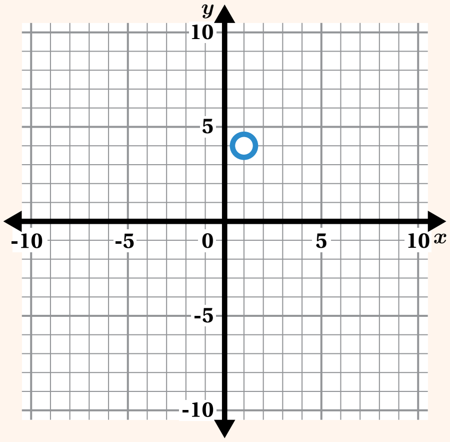
Use the graph if it helps with your thinking.



You're invited to explore more.

10. Write the equations of as many different parabolas as you can that go through the point (1, 4).

Use the graph if it helps with your thinking.

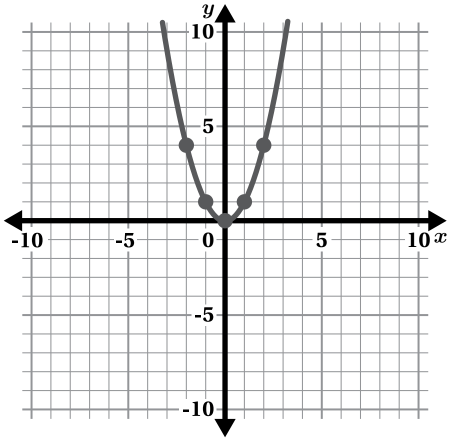


Synthesis

11. Here is a quadratic function written in vertex form: $g(x) = (x - 2)^2 + 3$.

Describe how the graph of $g(x)$ compares to $f(x) = x^2$.

Use the graph if it helps your thinking.



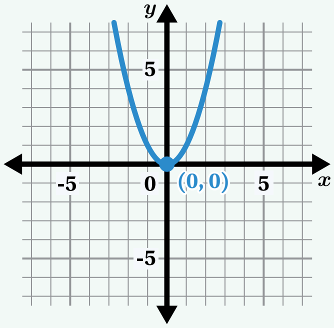
Lesson Practice 6.13

Lesson Summary

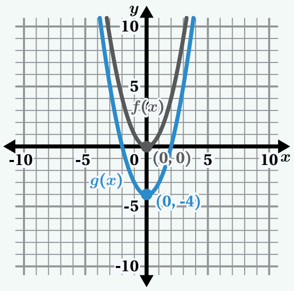
Here is the graph of $f(x) = x^2$.

Functions can be translated horizontally and vertically.

Here are three examples of *translations* of $f(x)$. The equations for these translations are written in **vertex form**, which highlights the coordinates of the vertex in the equation.



Vertical Translations

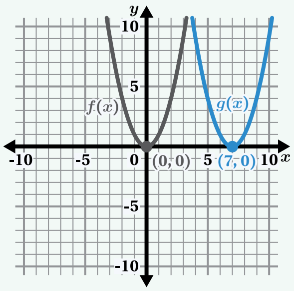


$f(x)$ is translated 4 units down.

Its equation is

$g(x) = x^2 - 4$.

Horizontal Translations

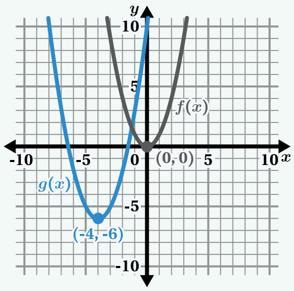


$f(x)$ is translated 7 units right.

Its equation is

$h(x) = (x - 7)^2$.

Vertical and Horizontal Translations



$f(x)$ is translated 4 units left and 6 units down.

Its equation is

$j(x) = (x + 4)^2 - 6$.

Lesson Practice
6.13

Name: ______ Date: ______ Period: ______

Problems 1–2: Circle if the transformation of $y = x^2$ is a *horizontal translation*, *vertical translation*, or *neither*.

1. $y = (x - 3)^2$

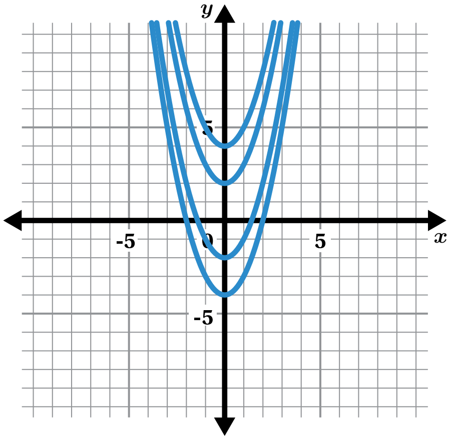
Horizontal Translation   Vertical Translation   Neither

2. $y = x^2 - 5$

Horizontal Translation   Vertical Translation   Neither

3. These parabolas are translations of $y = x^2$.

Select *all* of the equations shown in the graph.

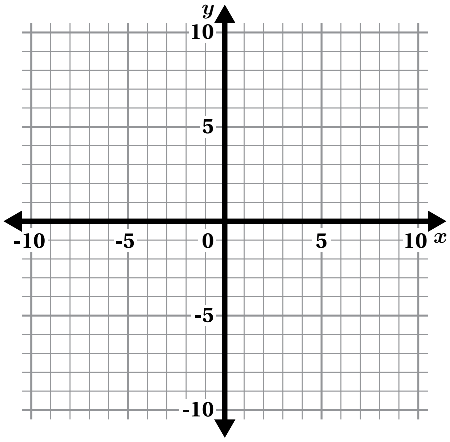


- ☐ **A.** $y = x^2 + 2$
- ☐ **B.** $y = (x - 2)^2$
- ☐ **C.** $y = x^2 - 2$
- ☐ **D.** $y = x^2 - 4$
- ☐ **E.** $y = (x + 4)^2$

4. Match each vertex of a parabola to its equation.

a	$y = (x - 3)^2 + 5$	______ $(0, -4)$
b	$y = (x + 7)^2 + 3$	______ $(-7, 3)$
c	$y = (x - 4)^2$	______ $(-3, 5)$
d	$y =(x + 3)^2 + 5$	______ $(3, 5)$
e	$y = x^2 - 4$	______ $(4, 0)$

5. Draw the graph of $y = (x + 3)^2 + 5$.

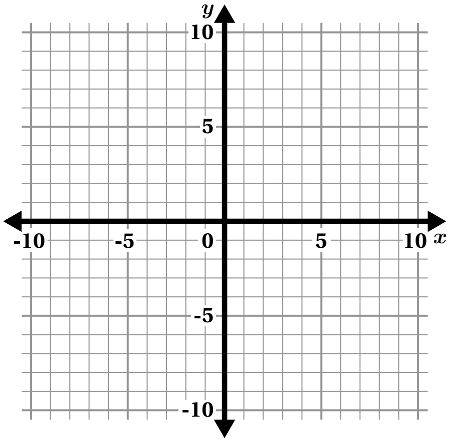


Lesson Practice
6.13

Name: .... Date: .... Period: ....

6. Write and graph the equations of five parabolas so that when you connect their vertices, they form a sixth parabola.

Equations



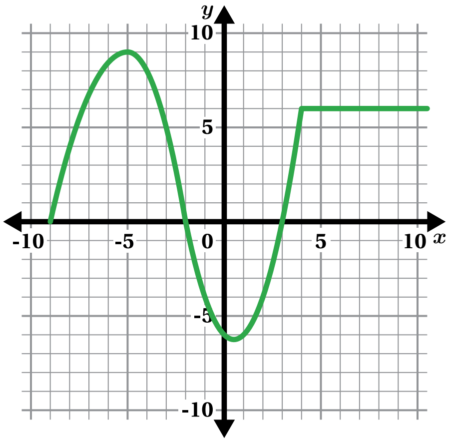
Test Practice

7. Here are four equations in vertex form. Which function has a graph with a vertex at $(1, 3)$?

- **A.** $y = (x + 1)^2 + 3$
- **B.** $y = (x - 1)^2 + 3$
- **C.** $y = (x - 3)^2 + 1$
- **D.** $y = (x + 3)^2 + 1$

Spiral Review

Problems 8–10: Here is a graph of $g(x)$.



8. At what point does the maximum occur?

9. At what points does $g(x) = 0$?

10. What is the y-intercept?

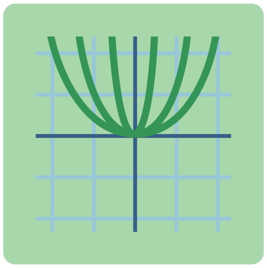
Unit 6
Lesson
14

Name: ______ Date: ______ Period: ______

MA.912.AR.3.4, MA.912.AR.3.7, MA.912.AR.3.8, MA.912.F.2.1, MTR.2.1, MTR.5.1

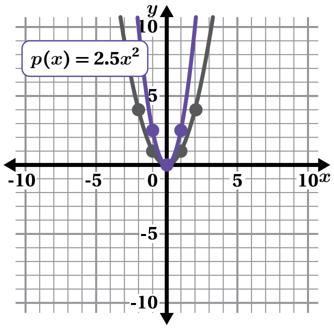
Stretch It Out

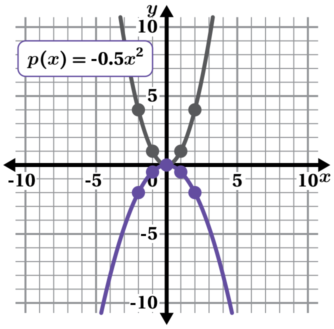
Let's transform quadratic functions using vertical stretches.

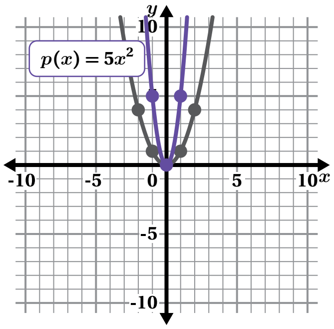


Warm-Up

1. Here is a new kind of transformation.







What changes? What stays the same?

Changes:

Stays the same:

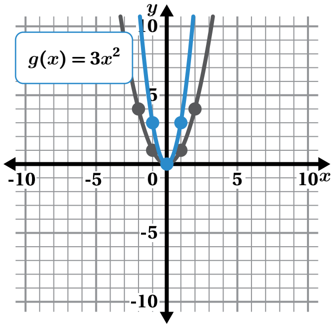
Activity
1

Name: .... Date: .... Period: ....

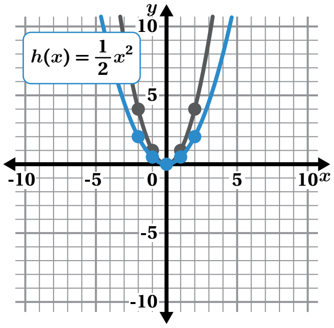
Vertical Stretches

2. Here are three different **vertical stretches** of $f(x) = x^2$.

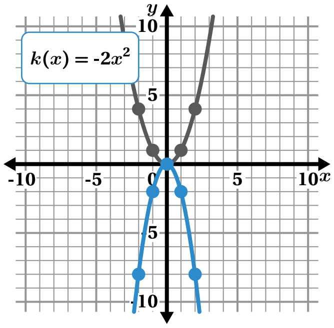
Graph A



Graph B



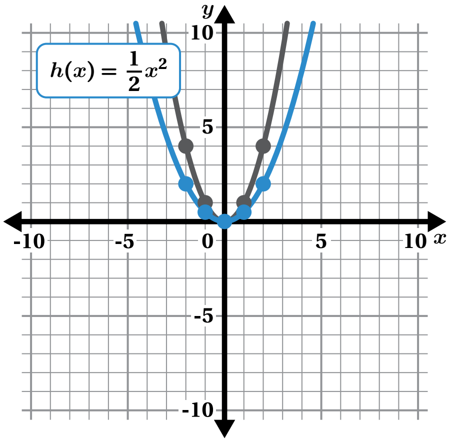
Graph C



Discuss: What do you notice? What do you wonder?

3. Here is Stretch B from the previous problem.

Show or explain where you see the vertical stretch of $\frac{1}{2}$ in the graph of $h(x) = \frac{1}{2}x^2$.



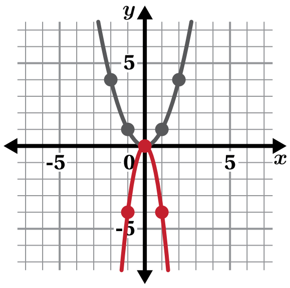
Activity
1

Name: ______ Date: ______ Period: ______

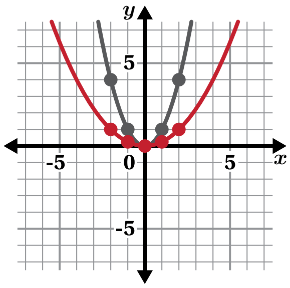
Vertical Stretches (continued)

4. Match the graph of each function with its equation. One equation will have no match.

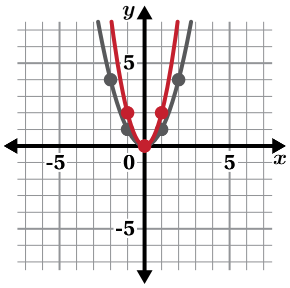
a. $a(x) = \text{-}4x^2$ **b.** $b(x) = \text{-}2x^2$ **c.** $c(x) = \frac{1}{4}x^2$ **d.** $d(x) = 2x^2$



Equation: ______



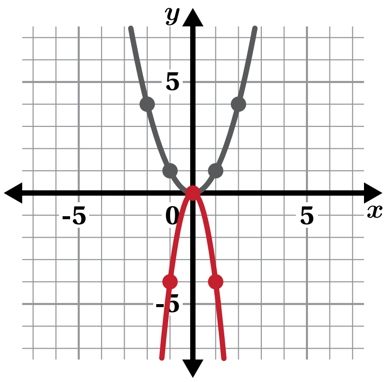
Equation: ______



Equation: ______

5. How did you decide which of these equations matches this graph?

$a(x) = \text{-}4x^2$ $b(x) = \text{-}2x^2$



Activity

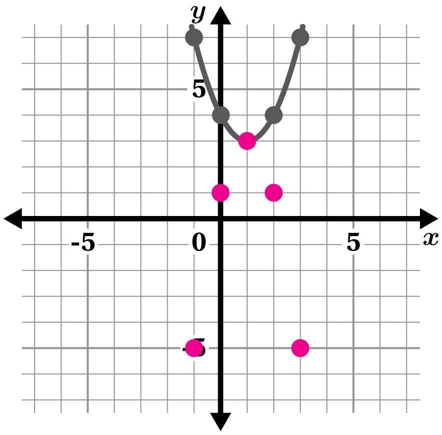
2

Name: .... Date: .... Period: ....

A Bit More Precision

6. The parabola $f(x) = (x - 1)^2 + 3$ has a vertex at $(1, 3)$ with a vertical stretch of 1.

Draw the graph of $g(x) = -2(x - 1)^2 + 3$.



7. Here is Kai's work to graph $g(x) = -2(x - 1)^2 + 3$.

How did Kai's strategy help her graph the parabola?

$y = -2(x - 1)^2 + 3$

$y = -2(2 - 1)^2 + 3$

$y = -2(1)^2 + 3$

$y = -2 + 3$

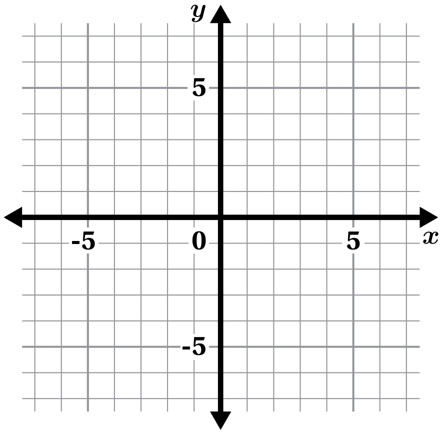
$y = 1$

Name: .... Date: .... Period: ....

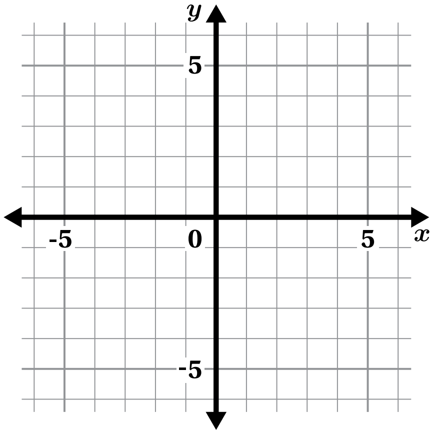
A Bit More Precision (continued)

8. Draw the graph of each quadratic function.

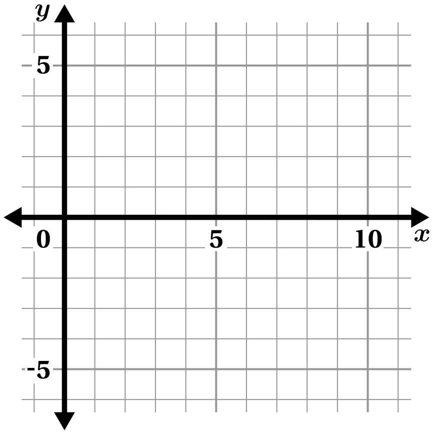
a $a(x) = 4x^2$



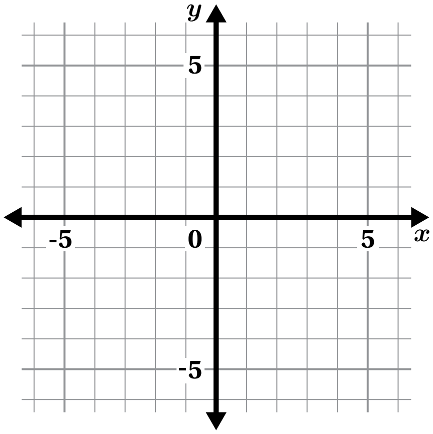
b $b(x) = x^2 - 3$



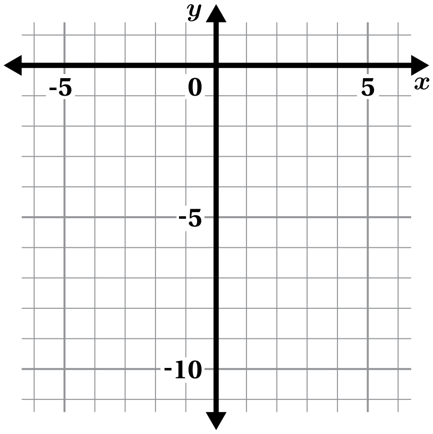
c $c(x) = (x - 5)^2$



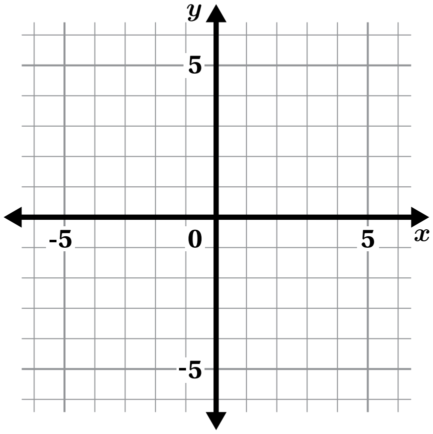
d $d(x) = (x + 1)^2$



e $e(x) = \text{-}0.5x^2$



f $f(x) = \text{-}x^2 - 1$



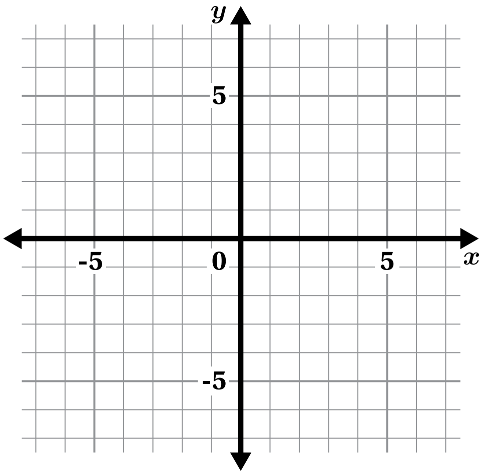
Activity
3

Name: .... Date: .... Period: ....

Parabola Art

9. Create a design by graphing parabolas. Record the functions you use.

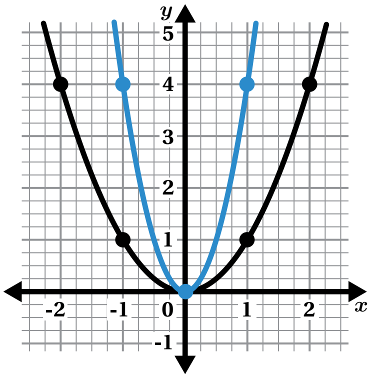
Functions



10. Refer to Problem 8a. How would the graph of the function $g(x) = (4x)^2$ compare to the graph of $a(x) = 4x^2$?

Synthesis

11. Here is the graph of $g(x) = 4x^2$



$g(x)$ is a transformation of $f(x) = x^2$.

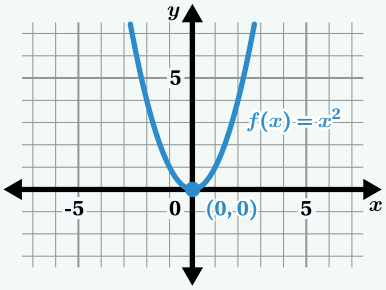
Explain where you see the vertical stretch in the equation and in the graph.

Equation:

Graph:

Lesson Practice 6.14

Lesson Summary



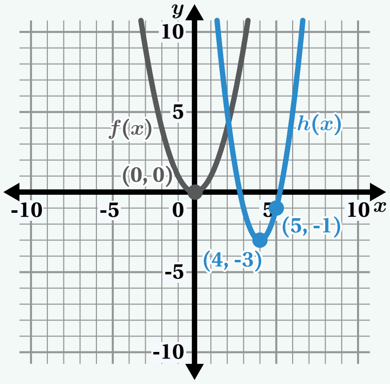
Quadratic functions like $f(x) = a(x - h)^2 + k$ are written in vertex form, where (h, k) is the vertex of the parabola, and a shows the **vertical stretch**.

The a-value:

- Multiplies each output of the function by a constant value.
- Identifies the amount of vertical stretch in the y-direction.
- Tells whether a parabola is concave up or concave down.

Let's look at an example of quadratic functions with a vertical stretch:

Vertical Stretch and Translations



Here $f(x)$ was vertically stretched by a factor of 2 and translated right 4 units and down 3 units to make $h(x) = 2(x - 4)^2 - 3$.

Lesson Practice

6.14

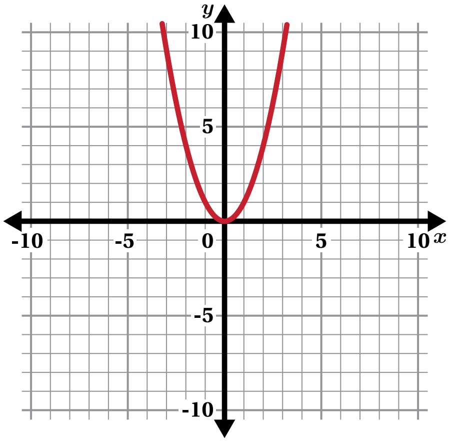
Name: .... Date: .... Period: ....

Problems 1–3: Here's the graph of $y = x^2$. Change one number to make the graph:

1. Wider:

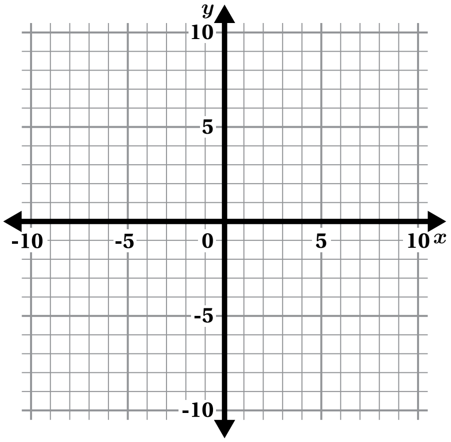
2. Narrower:

3. Open down:



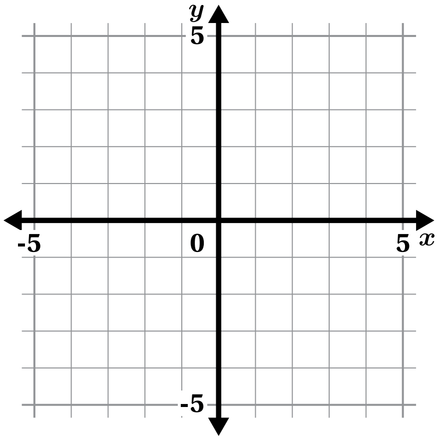
4. Describe how the graph of $y = x^2$ compares to $y = -4x^2$

5. Draw a graph of a parabola that has a vertex at $(3, -1)$ and a vertical stretch of 2.

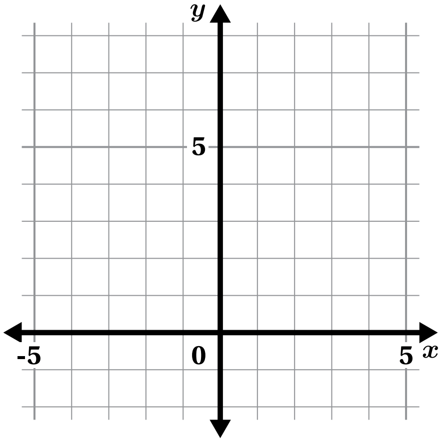


Problems 6–7: Draw a graph for each equation.

6. $a(x) = -2x^2$



7. $c(x) = 3x^2$

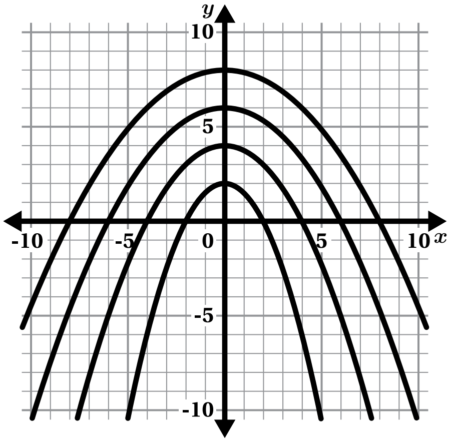


Lesson Practice
6.14

Name: .... Date: .... Period: ....

Test Practice

8. The 4 parabolas remind you of a rainbow. Use what you know about vertex form and choose the function that has the largest vertical stretch.



Ⓐ $y = -\frac{1}{8}x^2 + 8$

Ⓑ $y = -\frac{1}{6}x^2 + 6$

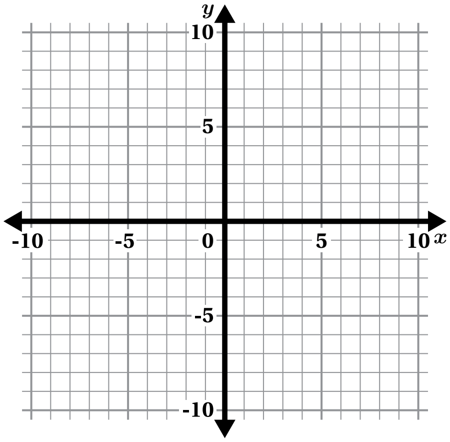
Ⓒ $y = -\frac{1}{4}x^2 + 4$

Ⓓ $y = -\frac{1}{2}x^2 + 2$

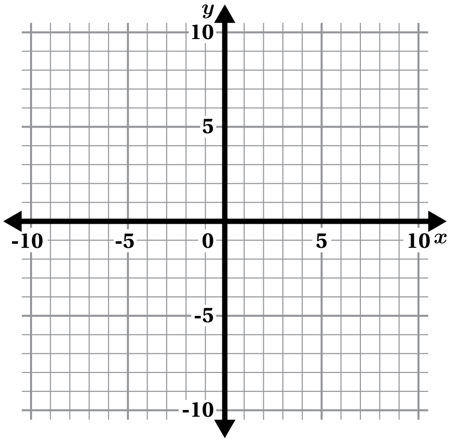
Spiral Review

Problems 9–10: Draw a graph of each equation.

9. $-3x + 3y = 9$



10. $2x + 4y = 8$



Name: .... Date: .... Period: ....

MA.912.F.1.8, MA.912.AR.3.8, MA.912.F.1.2, MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1

Predicting Sales

Let’s model a real-world situation with quadratic functions.

Warm-Up

1. This lesson explores the sale of a new model of cell phone on the market. What do you look for in a cell phone?

Select *all* the factors you consider when buying a cell phone.

☐ **A.** Price

☐ **B.** Color

☐ **C.** Speed

☐ **D.** Size

☐ **E.** Camera

☐ **F.** Weight

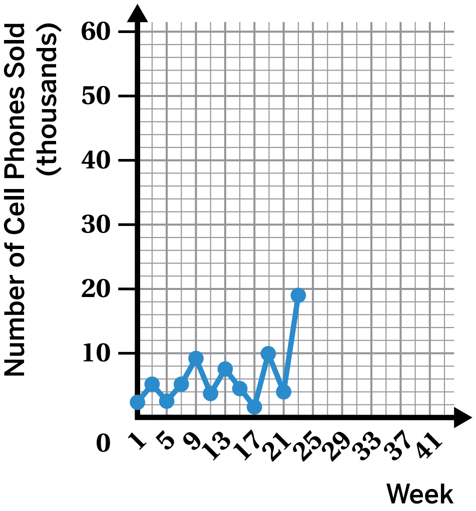
Discuss: What factors would impact the sales of a new cell phone model?

Activity
1

Name: Date: Period:

Modeling Sales

1. Imagine you are the director responsible for marketing a new line of cell phones in Florida. The initial sales data for the state is plotted.



What do you notice? What do you wonder?

I notice:

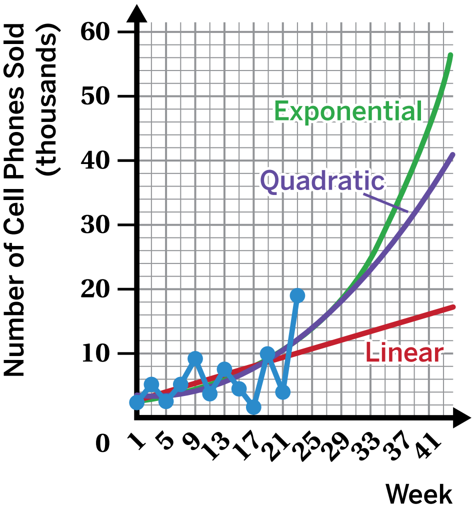
I wonder:

2. You might have wondered: How will sales of the phone continue to perform in the state?

Here are three models of the data.

a Look at each model.

b 💬 **Discuss:** How are the models alike? How are they different?



3. Which model do you think will be most helpful for predicting sales after week 23?

A. Linear **B.** Exponential **C.** Quadratic

How many thousands of phones would be sold in week 39?

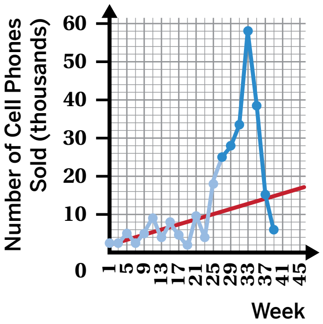
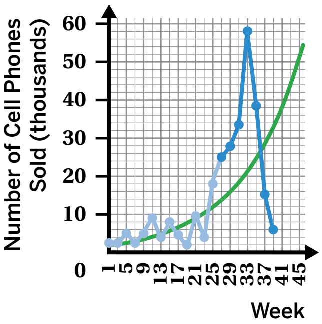
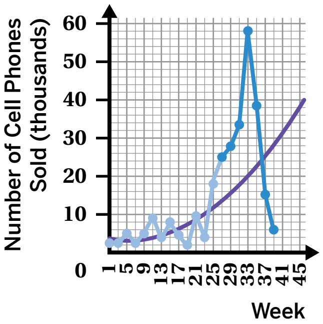
Activity
1

Name: ________ Date: ________ Period: ________

Modeling Sales (continued)

4. Here are three potential models for the number of phones sold between weeks 23 and 39.

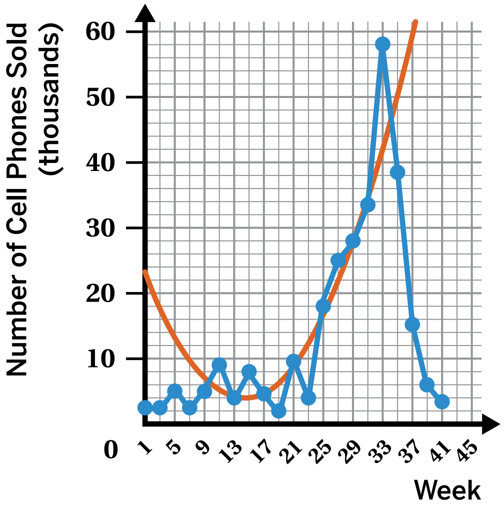
a Circle the model type you picked in Problem 3.

Linear	Exponential	Quadratic
		

b What did your model get right about the number of phones sold? What did it get wrong?

Name: .... Date: .... Period: ....

Making Predictions



5. Ava thinks this quadratic function will be useful for predicting missing values for some of the data:

$f(x) = 0.11(x - 14.3)^2 + 4$

x represents the number of weeks since the phone went on sale.

How many phones were sold in Week 28?

6. Ava says she can use the model to accurately estimate the number of sales in Week 28.

Do you agree? Circle one.

Yes No I'm not sure

Explain your thinking.

7. Ava noticed that her model only fits the data for some of the weeks. In the graph above, highlight a domain where you think this model is a good fit.

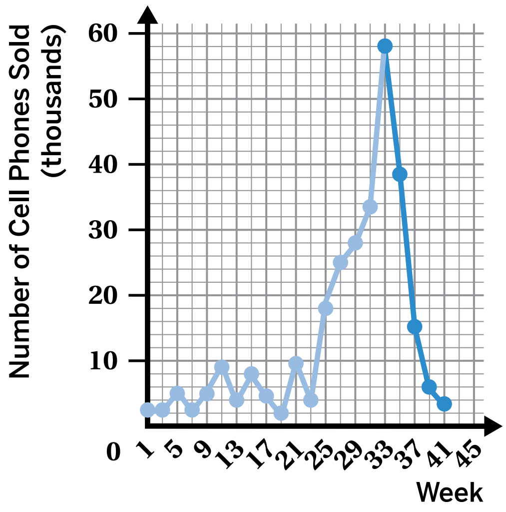
Name: .... Date: .... Period: ....

Making Predictions (continued)

8. Different functions can be useful for modeling different parts of this data.

a Graph a quadratic function that is different from Ava's and fits some of the data well.

b Highlight the interval where your model is most useful.



9. The British statistician George Box once said:

All models are wrong, but some are useful.

💬 **Discuss:** Explain how the models are wrong and useful.

Synthesis

10. Select one of the questions to answer.

- ☐ What is something you learned about making math models?
- ☐ What is a question you have about making math models?

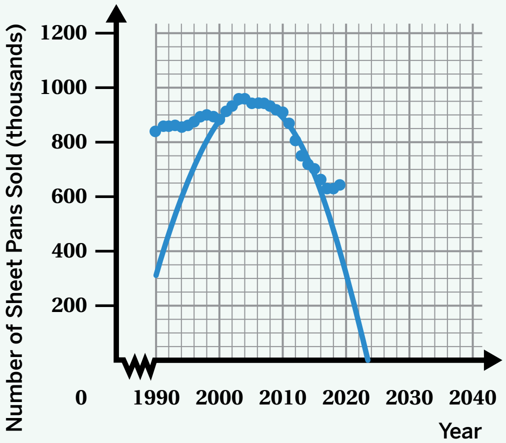
Lesson Practice 6.15

Lesson Summary

Different types of functions can be used to model data and help us predict unknown data values. While models can be useful, they also have limitations. Some models may only be useful for predicting unknown values within a specific domain. Let's look at two quadratic functions that could model the data of a company's sheet pan sales, measured in thousands, since 1990.

Model A

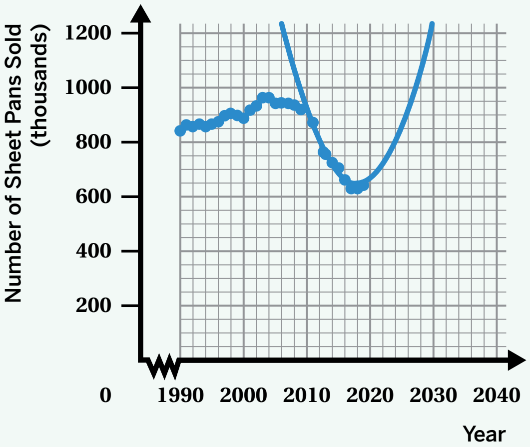
$f(x) = -2.802(x - 28)^2 + 954$



This model is a good fit for the data on the interval from 2000–2016. Beyond 2016, this model may not be useful because it suggests that sales will collapse and eventually go negative, which is impossible.

Model B

$f(x) = 4.253(x - 15)^2 + 954$



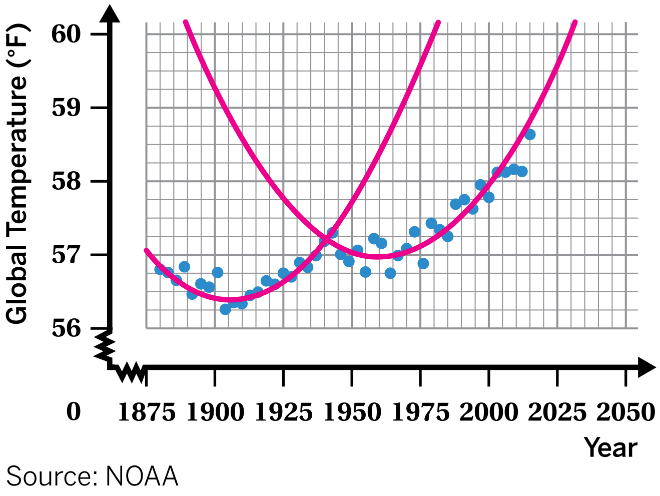
This model is a good fit for the data on the interval from 2010–2020. Beyond 2020, this model may not be useful because it suggests that sales will increase to infinity, which is also impossible.

Lesson Practice

6.15

Name: .... Date: .... Period: ....

Problems 1–3: The graph shows the average global temperature, in degrees Fahrenheit, from 1875 to 2020.



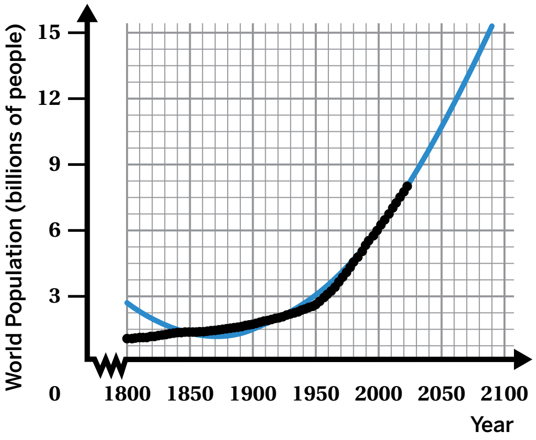
Source: NOAA

1. Graph two different quadratic functions that could be useful for modeling different domains of this data.

2. Use your model to predict the average global temperature, in degrees Fahrenheit, in 2025.

3. Do you think your model is useful for predicting average global temperature, in degrees Fahrenheit, after 2025? Explain your thinking.

Problems 4–6: The graph shows the world population in billions of people. Jacy modeled the data with a quadratic function.



4. Describe an advantage of using Jacy's model.

5. Describe a disadvantage of using Jacy's model.

6. How might Jacy improve this model?

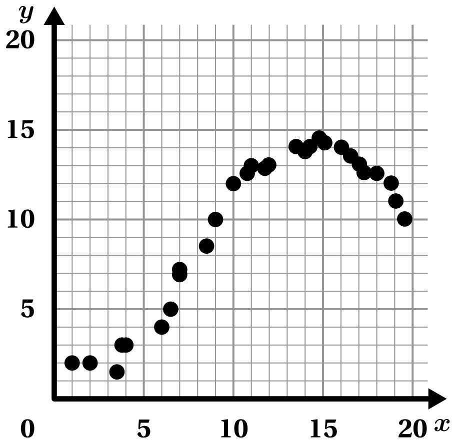
Lesson Practice
6.15

Name: .... Date: .... Period: ....

Test Practice

7. Which type of model could be used to best fit the data?

- **A.** A linear function
- **B.** An exponential function
- **C.** A quadratic function
- **D.** A combination of several different functions



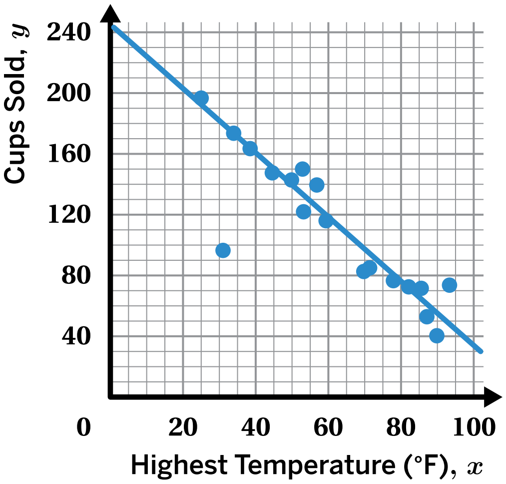
Spiral Review

Problems 8–10: This scatter plot shows the number of cups of hot chocolate sold each day for different highest daily temperatures.

8. Circle the data point(s) that appear to be an outlier(s).

9. Use the linear model to estimate the number of cups sold when the highest temperature is 64°F.

10. The equation $y = -2.11x + 245$ gives the number of cups sold, y, when the highest temperature is x °F. What does the number -2.11 mean in this situation?



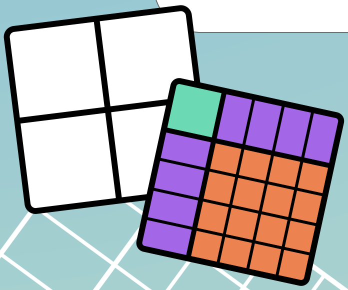
Unit 7

Quadratic Equations

Quadratic equations have fascinated mathematicians for thousands of years. In this unit, you'll learn different methods for solving quadratic equations and develop strategies for choosing your solving method strategically. You'll also connect your solving process to the graphs and key features of quadratic functions.

Essential Questions

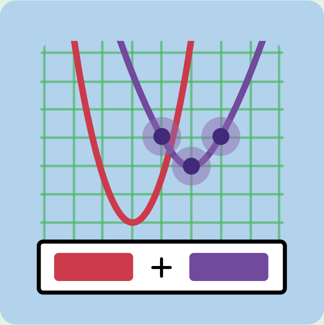
- How can you solve quadratic equations symbolically and graphically?
- How can you use the structure of the equation, available tools, and your personal mathematical preferences to select a solving method strategically?
- How can rewriting expressions in different forms support both solving equations and identifying key features of functions?



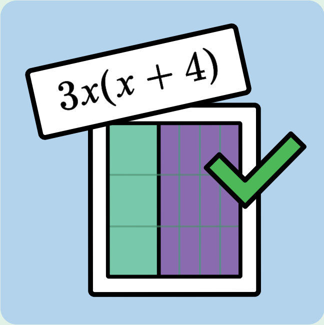
Sub-unit

1

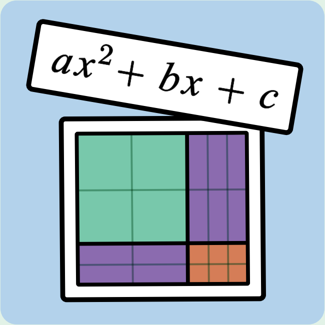
Multiplying and Factoring



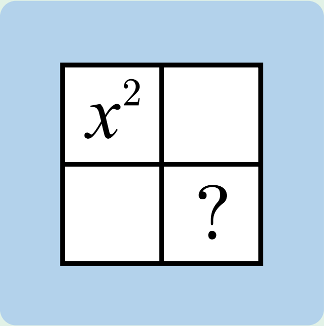
Lesson 1
Sums and Differences



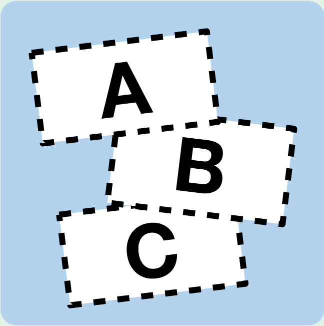
Lesson 2
Two-Factor Multiplication



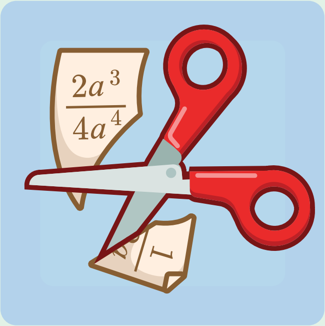
Lesson 3
Standard Feature



Lesson 4
X-Factor



Lesson 5
Form Up



Lesson 6
Divide and Conquer

Lesson 7
Consider the Factors

Lesson 8
Shooting Stars

Lesson 9
Make It Zero

Unit 7

Lesson

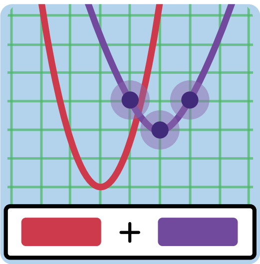
1

Name: ________________ Date: ________ Period: ________

MA.912.AR.1.3, MTR.1.1, MTR.4.1, MTR.5.1

Sums and Differences

Let's explore sums and differences of expressions.



Warm-Up

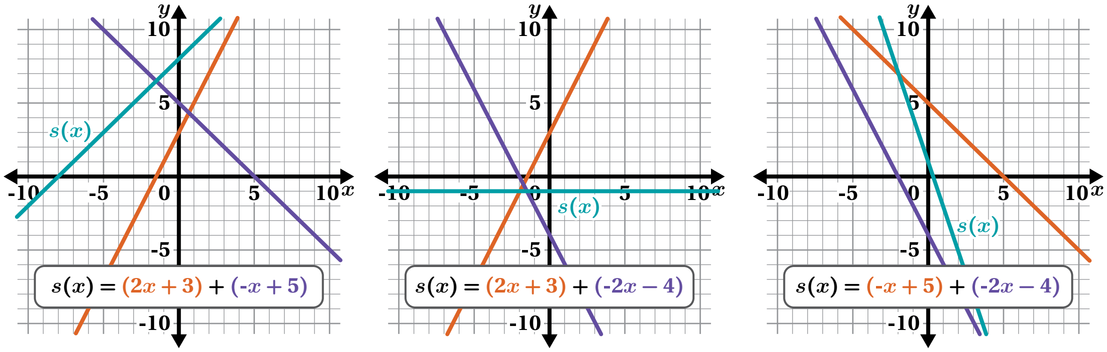
1. $s(x)$ is the *sum* of two linear functions.

a Here are a few examples.

$f(x) = 2x + 3$
$g(x) = -x + 5$

$f(x) = 2x + 3$
$j(x) = -2x - 4$

$g(x) = -x + 5$
$j(x) = -2x - 4$



b **Discuss:** What do you notice? What do you wonder?

Activity
1

Name: ______________________ Date: ____________ Period: ____________

Sum Functions

2. $s(x)$ is the *sum* of two linear functions.

Let’s look at some graphs we can create by combining two linear functions.

Select *all* the graphs you can create by combining two functions.

☐ **A.** Line with a positive slope

☐ **B.** Line with a negative slope

☐ **C.** Line with a slope of 0

3. Here is a function:

$s(x) = (2x + 3) + (\text{-}0.5x - 1)$

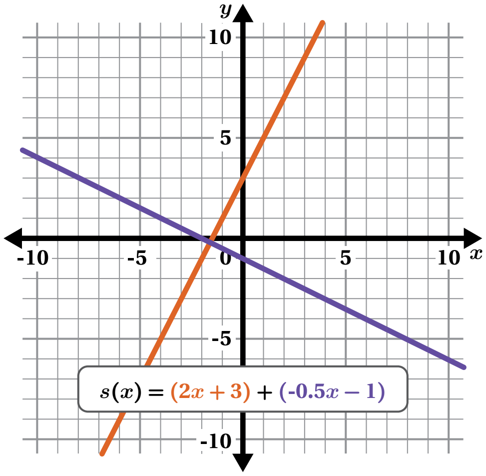
What will the graph of $s(x)$ look like?

○ **A.** Line with a positive slope

○ **B.** Line with a negative slope

○ **C.** Line with a slope of 0

Show or explain your thinking.



4. $s(x)$ is the *sum* of two quadratic functions.

Let’s look at some graphs we can create by combining two quadratic functions.

Select *all* the graphs you can create by combining two functions.

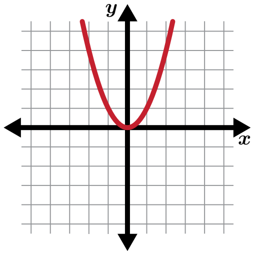
☐ **A.** Parabola that is concave up

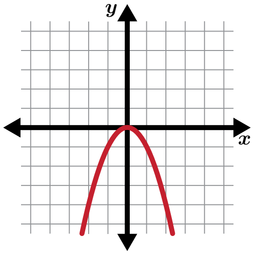
☐ **B.** Parabola that is concave down

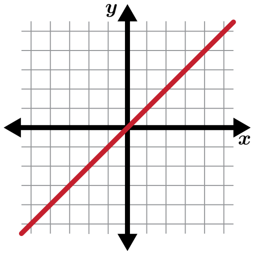
☐ **C.** Line with a positive slope

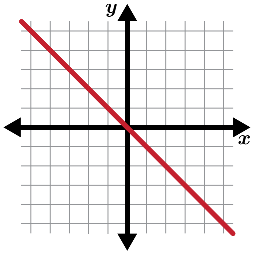
☐ **D.** Line with a negative slope

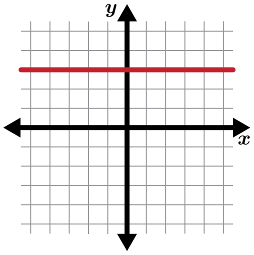
☐ **E.** Line with a slope of 0











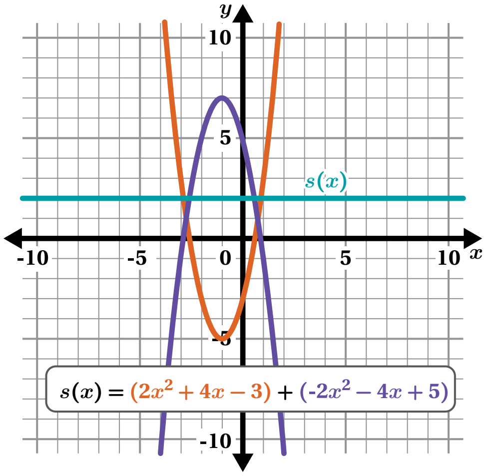
Name: ________________ Date: ________ Period: ________

Sum Functions (continued)

5. Here is a function:

$s(x) = (2x^2 + 4x - 3) + (\text{-}2x^2 - 4x + 5)$.

Why did combining two quadratic functions create a linear function with a slope of 0?



6. Let's look at two strategies for combining the functions on the previous problem.

$$(2x^2 + 4x - 3) + (\text{-}2x^2 - 4x + 5)$$
$$2x^2 - 2x^2 + 4x - 4x - 3 + 5$$
$$0x^2 + 0x + 2$$
$$s(x) = 2$$

$$\begin{array}{rr} & 2x^2 + 4x - 3 \\ + & \text{-}2x^2 - 4x + 5 \\ \hline & 0x^2 + 0x + 2 \end{array}$$
$$s(x) = 2$$

Discuss: How are the two strategies alike? How are they different?

7. Here is a function: $s(x) = (x^2 - 2x + 3) + (\text{-}2x^2 + x - 4)$.

Write the equation of $s(x)$ using the fewest number of terms.

Name: .... Date: .... Period: ....

Difference Functions

8. $d(x)$ is the *difference* of two quadratic functions.

Let's look at some graphs we can create by combining two quadratic functions.

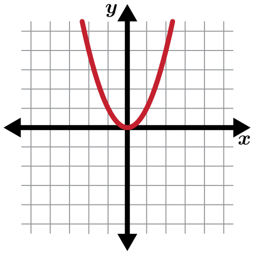
☐ **A.** Parabola that is concave up

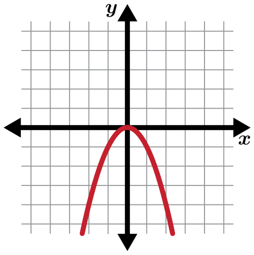
☐ **B.** Parabola that is concave down

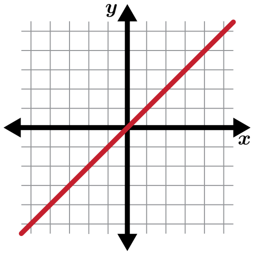
☐ **C.** Line with a positive slope

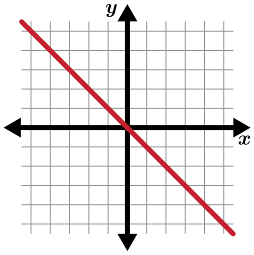
☐ **D.** Line with a negative slope

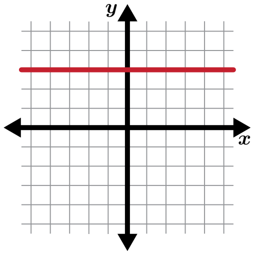
☐ **E.** Line with a slope of 0











9. Here is a function: $d(x) = (2x^2 + 4x - 3) - (2x^2 - 4x + 3)$.

Write the equation of $d(x)$ using the fewest numbers of terms.

10. Diego subtracted the two quadratic functions on the previous problem.

Discuss:

- What did Diego do well? What did Diego not do well?
- How would you correct Diego's work?

Diego

$(2x^2 + 4x - 3) - (2x^2 - 4x + 3)$

$2x^2 - 2x^2 + 4x - 4x - 3 + 3$

$0x^2 + 0x + 0$

Activity
3

Name: .... Date: .... Period: ....

Partner Problems

11.
- Decide who will complete Column A and who will complete Column B.
- Write each expression using the fewest number of terms.
- The expression in each row should be the same. Compare your expressions, then discuss and resolve any differences.

	Column A	Column B
a	$(-3x + 2) + (x + 5)$	$(x + 10) - (3x + 3)$
b	$(9x) - (4x - 3)$	$(4 + 6x) + (-x - 1)$
c	$(-x^2 + x + 2) + (-2x^2 + x - 9)$	$(2x^2 + 4x - 8) - (5x^2 + 2x - 1)$
d	$(10x^2 + 4x + 1) - (10x^2 + 4x - 5)$	$(3x^2 + 7x + 10) + (-3x^2 - 7x - 4)$
e	$(x + 1) + (x^2 + x)$	$(x^2 + 2) + (2x - 1)$
f	$(5x - 3) - (-2x^2 - 5x + 1)$	$(2x^2 + 5x - 3) + (5x - 1)$

Discuss: When adding or subtracting linear and/or quadratic expressions, can you create an expression that isn't linear or quadratic? Why or why not?

Synthesis

12. What's something to remember when adding and subtracting functions?

$s(x) = (2x^2 - 4) + (x^2 - 5x + 6)$

Use these examples if they help with your thinking.

$d(x) = (5x + 8) - (3x - 2)$

Lesson Practice 7.01

Lesson Summary

Quadratic and linear expressions are types of *polynomials*. When adding or subtracting quadratic and linear functions, you can predict what types of functions will be created.

The sum or difference of polynomials will always be a polynomial. For example, when adding or subtracting two quadratic expressions, the result can be either a quadratic, linear, or constant expression. The sum or difference of two quadratic expressions cannot become an exponential expression.

Often the goal of adding and subtracting quadratic and linear expressions is to write the expression with the fewest number of terms possible.

Here are some examples of sums and differences of quadratic and linear expressions written with the fewest number of terms possible.

Sums	Differences
$(2x - 1) + (x + 7) = 3x + 6$	$(4x^2 + 3x) - (3x - 5) = 4x^2 + 5$
$(-5x^2 + 9x - 4) + (5x^2 - 9x + 2) = -2$	$(x^2 + 4x + 1) - (x^2 + 2x - 5) = 2x + 6$

Lesson Practice

7.01

Name: .... Date: .... Period: ....

Problems 1–3: Fill in the blanks to make each equation true.

1. $(6x^2 + 4x - 6) + (\underline{\qquad}x^2 + \underline{\qquad}x + \underline{\qquad}) = 3x^2 + 8x + 4$

2. $(-x^2 + 5x - 4) + (\underline{\qquad}x^2 + \underline{\qquad}x + \underline{\qquad}) = 3x^2 - 2$

3. $(8x^2 - x + 10) - (\underline{\qquad}x^2 + \underline{\qquad}x + \underline{\qquad}) = 5x^2 - 6x$

Problems 4–7: Write an equivalent function using the least number of terms with $f(x) = 10x^2 - 8x - 5$ and $g(x) = -3x^2 - x + 8$.

4. $f(x) + g(x)$

5. $f(x) - g(x)$

6. $g(x) + f(x)$

7. $g(x) - f(x)$

Problems 8–9: Neo made a mistake when subtracting $(-x^2 + 5x - 3) - (7x - 2)$.

Neo

$$\begin{array}{rr} & -x^2 + 5x - 3 \\ + & -7x + 2 \\ \hline & -8x^2 + 7x - 3 \end{array}$$

8. Describe the error that Neo made.

9. Write an expression for the difference using the fewest number of terms.

Lesson Practice

7.01

Name: .... Date: .... Period: ....

Test Practice

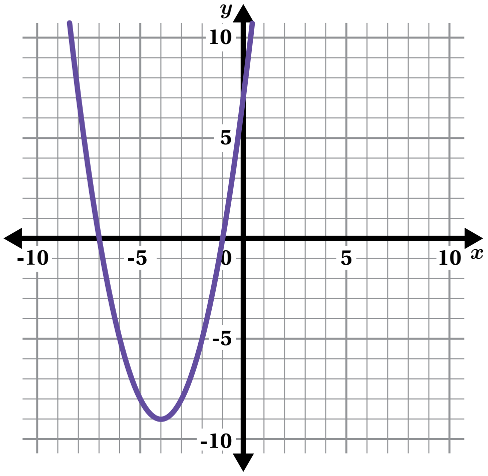
10. Here is a function: $s(x) = (2x^2 + x + 3) + (-2x^2 + 1)$.

What will the graph of $s(x)$ look like?

- **A.** Parabola that is concave up
- **B.** Parabola that is concave down
- **C.** Line with a positive slope
- **D.** Line with a negative slope
- **E.** Line with a slope of 0

Spiral Review

11. Write two quadratic equations that represent this graph.



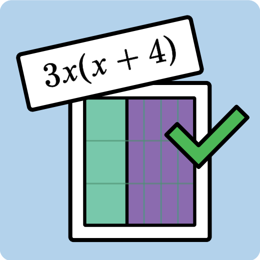
Unit 7
Lesson
2

Name: ______ Date: ______ Period: ______

MA.912.AR.1.3, MA.912.NSO.1.2, MTR.1.1, MTR.3.1, MTR.4.1, MTR.5.1

Two-Factor Multiplication

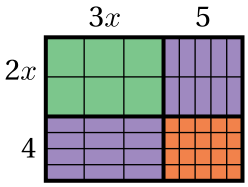
Let's rewrite factored-form quadratic expressions in standard form.



Warm-Up

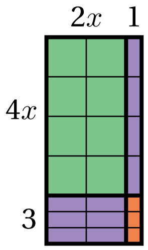
1. An area model shows equivalent quadratic expressions.

a Here are three area models.



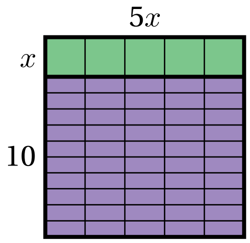
Factored Form
$(2x + 4)(3x + 5)$

Standard Form
$6x^2 + 22x + 20$



Factored Form
$(4x + 3)(2x + 1)$

Standard Form
$8x^2 + 10x + 3$



Factored Form
$(x + 10)(5x)$

Standard Form
$5x^2 + 50x$

b Where on the area model do you see the *factored form*? Where do you see the *standard form*?

Factored Form:

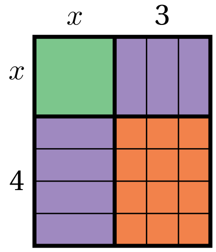
Standard Form:

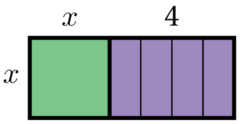
Name: .... Date: .... Period: ....

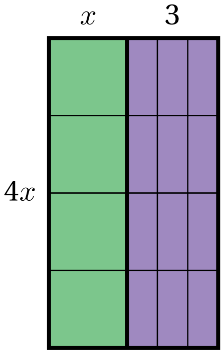
Multiplying With Area Models

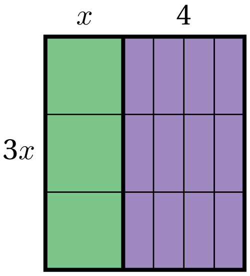
2. Match each expression with an equivalent area model. One area model will have no match.

$x(x + 4)$	$3x(x + 4)$	$(x + 4)(x + 3)$





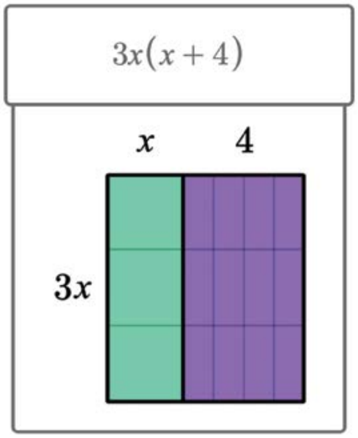




3. Let's look at two cards that Sahana correctly matched.

She wrote this standard-form expression: $3x^2 + 12x$.

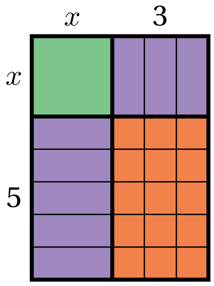
Show or explain where you see $3x^2 + 12x$ in the area model or in the factored-form expression.



Name: .... Date: .... Period: ....

Multiplying With Area Models (continued)

4. Here is a list of equivalent expressions.



Circle *one* expression.

A. $x^2 + 3x + 5x + 15$

B. $x^2 + 8x + 15$

C. $x(x + 3) + 5(x + 3)$

Show or explain how you see it represented in the area model.

5. **a** Draw an area model to represent the expression $(x + 7)(x + 3)$.

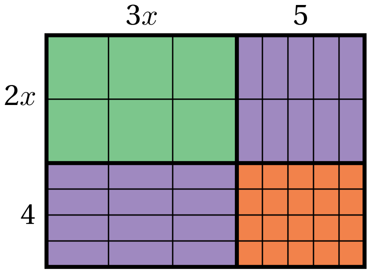
b Rewrite $(x + 7)(x + 3)$ in standard form.

Activity
2

Name: ____________________ Date: __________ Period: __________

Multiplying With Diagrams

6. The diagram on the right shows a different way to represent the area model.



	$3x$	5
$2x$	$6x^2$	$10x$
4	$12x$	20

Discuss: How are the area model and the diagram alike? How are they different?

7. Multiply to rewrite $(2x + 5)(x + 8)$ in standard form.

Use the diagram if it helps with your thinking.

	x	8
$2x$		
5		

Name: ………… Date: ………… Period: …………

Multiplying With Diagrams (continued)

8. Karima tried to rewrite $(5x - 3)(2x + 5)$ in standard form and made an error.

What did Karima do well? What could she improve?

Something Karima did well:

Something Karima could improve:

	5x	-3
2x	$10x^2$	$-6x$
5	$25x$	-15

$10x^2 + 31x - 15$

9. Multiply to rewrite each expression in standard form. Use the diagrams if they help with your thinking.

$(x + 6)(x + 10)$

$(3x + 1)(x + 6)$

$(2x - 6)(3x + 1)$

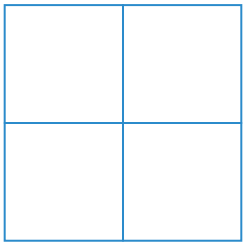
$(4x + 5)(x - 7)$

Synthesis

10. Describe how to write a factored-form expression in standard form.

Use the diagram if it helps with your thinking.

$(2x - 3)(x + 4)$



Lesson Practice 7.02

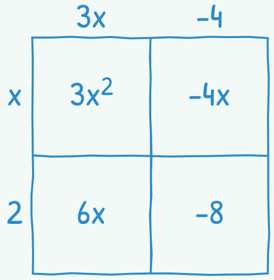
Lesson Summary

Quadratic expressions can be written in *factored form* or *standard form*.

You can use an area model to help you rewrite a factored-form quadratic expression into an equivalent expression in standard form.

Here are two examples.

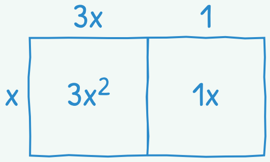
Factored form: $(3x - 4)(x + 2)$



$3x^2 + 6x - 4x - 8$

Standard form: $3x^2 + 2x - 8$

Factored Form: $x(3x + 1)$

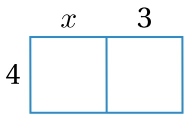


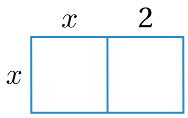
Standard form: $3x^2 + x$

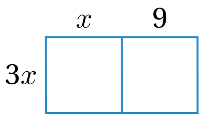
Lesson Practice
7.02

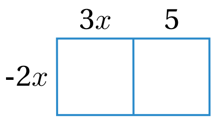
Name: …………………… Date: ………… Period: …………

1. Write an expression that represents each area model.

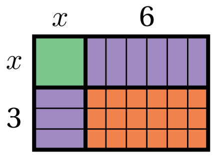








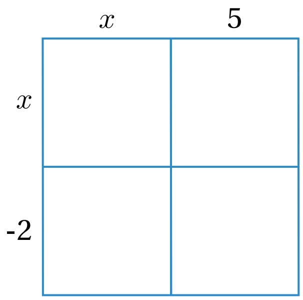
2. Write two expressions that match the area model.



Factored Form: ……………

Standard Form: ……………

3. Multiply to rewrite $(x + 5)\ (x - 2)$ in standard form. Use the diagram if it helps with your thinking.

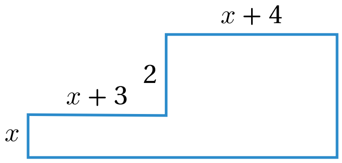


……………

4. Complete the table by writing each expression in standard form.

Factored Form	Standard Form
$(x + 7)(x + 3)$	
$(x - 2)(x - 12)$	

5. Write an expression that represents the area and an expression for the perimeter of this figure:



Area: ……………

Perimeter: ……………

Lesson Practice
7.02

Name: .... Date: .... Period: ....

Test Practice

6. Match each expression to its equivalent expression in standard form.

a $(x+2)(x+6)$ .... $x^2+12x+32$

b $(2x+8)(x+2)$ .... $2x^2+18x+16$

c $(x+8)(x+4)$ .... $2x^2+12x+16$

d $(x+8)(2x+2)$ .... $x^2+8x+12$

Spiral Review

Problems 7–9: Complete each equivalent expression:

7. $14a+21=$ .... (............ a + ...)

8. $3b+2+3b+10=$ .... (............ b + ...)

9. $-3(3c+3)=$ .... c + ...

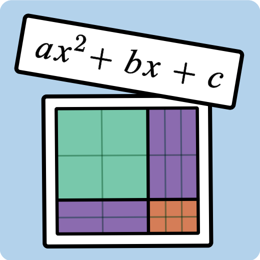
Unit 7
Lesson
3

Name: .... Date: .... Period: ....

MA.912.AR.1.3, MA.912.NSO.1.2, MTR.1.1, MTR.3.1, MTR.5.1

Standard Feature

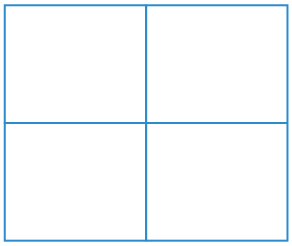
Let's look for patterns that help us rewrite factored-form expressions in standard form.



Warm-Up

1. Which expression is equivalent to $(x + 5)^2$?

Use the diagram if it helps with your thinking.



A. $x^2 + 25$

B. $x^2 + 10x + 25$

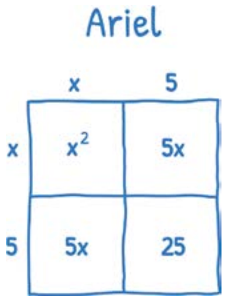
C. Both

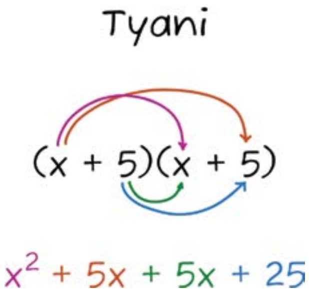
D. Neither

Explain your thinking.

2. Let's look at how two students determined the expression equivalent to $(x + 5)^2$.

Discuss: How are their strategies alike? How are they different?



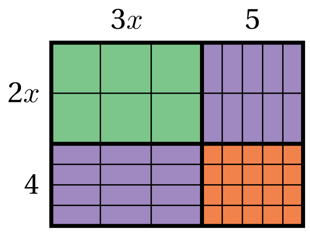


Name: .... Date: .... Period: ....

Standard Form

3. We often use $ax^2 + bx + c$ to represent a quadratic expression in *standard form*.

Where do you see a, b, and c in the area model?



$$\underbrace{6x^2}_{a} + \underbrace{22x}_{b} + \underbrace{20}_{c}$$

4. Group the equivalent expressions. One expression will have no match. Use the diagrams if they help with your thinking.

$(x + 3)(x + 3)$	$3x^2 + 9x$	$x^2 - 9$	$(x + 3)^2$
$(x - 3)(x + 3)$	$3x(x + 3)$	$x^2 + 9$	$x^2 + 6x + 9$

Group 1	Group 2	Group 3

Name: ______________________ Date: ____________ Period: ________

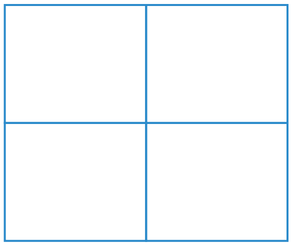
Standard Form (continued)

5. Here are two equivalent expressions:

$(x - 3)(x + 3)$ and $x^2 - 9$

Discuss: Why does $(x - 3)(x + 3)$ have a b-value of 0 when written in standard form?

Use the diagram if it helps with your thinking.

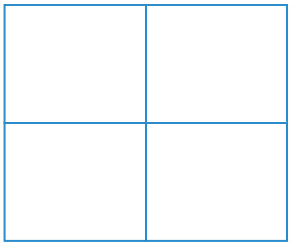


6. Here are two equivalent expressions:

$3x(x + 3)$ and $3x^2 + 9x$

Why is the c-value 0?

Use the diagram if it helps with your thinking.



7. Select *all* the expressions that have a b- or c-value of 0 when written in standard form.

☐ **A.** $(3x - 1)(3x + 1)$

☐ **B.** $(x - 4)(x - 4)$

☐ **C.** $x(x + 4)$

☐ **D.** $(3x + 1)(x - 1)$

☐ **E.** $(x + 10)(x - 10)$

Name: .... Date: .... Period: ....

Now I Know My ABC's

8. Write an expression in *factored form* that has a b*-value of* 0 when written in standard form.

Use the diagram if it helps with your thinking.

9. Write an expression in factored form that has a *positive* b*-value* and a *negative* c*-value* when written in standard form.

Use the diagram if it helps with your thinking.

10. Here are four expressions that have a *positive* b*-value* and a *negative* c*-value*.

Describe any patterns you notice.

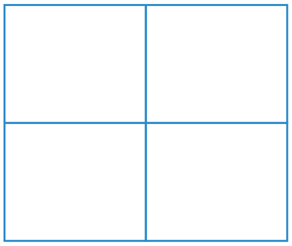
Factored Form	**Standard Form**
$(x-2)(x+9)$	$x^2+7x-18$
$(x+10)(x-8)$	$x^2+2x-80$
$(3x+7)(x-2)$	$3x^2+1x-14$
$(4x-9)(x+3)$	$4x^2+3x-27$

Name: .... Date: .... Period: ....

Now I Know My ABC’s (continued)

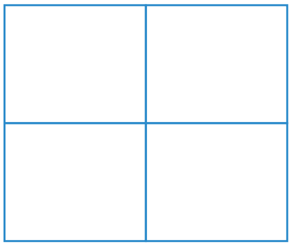
11. Write an expression in factored form that has a *b-value greater than* 5 and a *c-value of* 1 when written in standard form.

Use the diagram if it helps with your thinking.



12. **a** Write an expression in factored form that has a *negative a-value*, a *negative b-value*, and a *negative c-value* when written in standard form.

Use the diagram if it helps with your thinking.



b Compare your expression with another group’s expression.

💬 **Discuss:** What patterns do you notice?

You’re invited to explore more.

13. Do you think it’s possible to write an expression in factored form that has a b-value of 0 *and* a positive c-value when written in standard form? Circle one.

Yes No Not enough information

Explain your thinking.

Synthesis

14. Describe 2–3 patterns you noticed between equivalent expressions in factored form and in standard form.

Use the examples if they help with your thinking.

$(3x - 2)(3x + 2)$

$(x - 6)(x - 3)$

$(x + 5)(x + 5)$

$(5x + 2)(x - 5)$

$2x(x - 4)$

Lesson Practice 7.03

Lesson Summary

You can analyze the structure of a quadratic expression written in *factored form* to make predictions about what the equivalent expression in *standard form*, $ax^2 + bx + c$, will look like.

Here are two strategies for multiplying a factored-form expression to rewrite quadratic expressions in standard form.

Strategy 1

$(2x - 5)(x + 3)$

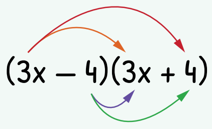
	2x	-5
x	$2x^2$	-5x
3	6x	-15

Standard form: $2x^2 + x - 15$

$a = 2 \quad b = 1 \quad c = -15$

Strategy 2

$(3x - 4)(3x + 4)$



$9x^2 + 12x - 12x - 16$

Standard form: $9x^2 - 16$

$a = 9 \quad b = 0 \quad c = -16$

Here are some patterns demonstrated in these two examples:

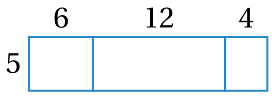
- If the constants in factored form have opposite signs, the c-value in standard form will be negative.
- If the factors have the same *coefficients* but opposite constants, then $b = 0$ in standard form.

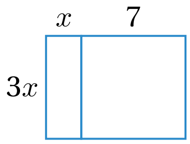
Lesson Practice

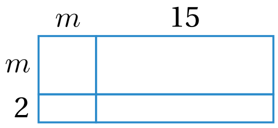
7.03

Name: .... Date: .... Period: ....

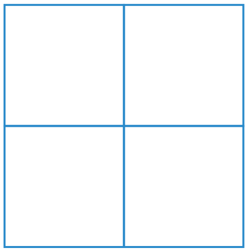
1. Write an expression that represents each area model.







2. Complete the diagram to show that $(x - 10)(x - 3)$ is equivalent to $x^2 - 13x + 30$.



Problems 3–4: Write an expression in factored form that has:

3. A b-*value* less than 4 and a c-*value* greater than 1 when written in standard form.

4. A negative a-*value*, negative b-*value*, and a positive c-*value* when written in standard form.

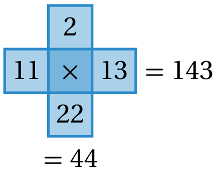
Lesson Practice

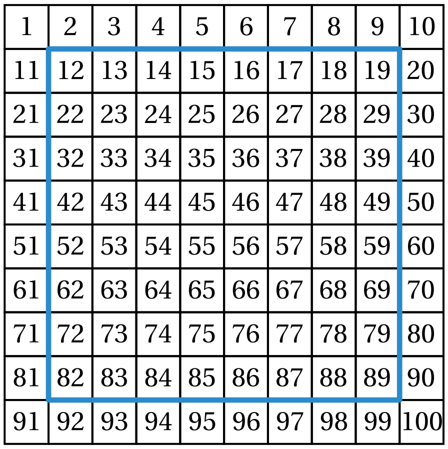
7.03

Name: .... Date: .... Period: ....

5. Select any number from the inner square.

- Multiply the number to the left of your selection by the number to the right.
- Multiply the number above your selection by the number below.
- Here's an example with the number 12 selected:





1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The difference between the numbers will be 99 no matter your selection. Explain why.

Test Practice

Problems 6–8: For each expression in factored form, write an equivalent expression in standard form.

6. $(x - 2)^2$

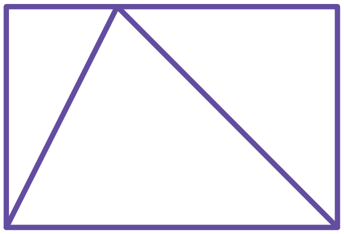
7. $(x + 1)(x - 1)$

8. $(2x + 4)(x - 3)$

Spiral Review

9. Draw a line to show that this rectangle is made only from 2 congruent triangles.

10. Draw a line to show that this rectangle is made only from 2 *pairs* of congruent triangles.



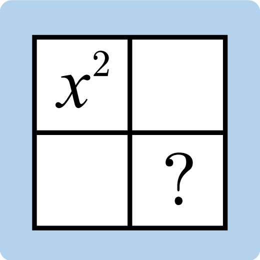
Unit 7
Lesson
4

Name: ________ Date: ________ Period: ________

MA.912.AR.1.7, MA.912.NSO.1.2, MTR.2.1, MTR.3.1, MTR.5.1, MTR.6.1

X-Factor

Let's rewrite standard-form quadratic expressions in factored form.



Warm-Up

1. Match each expression in *factored form* with its equivalent expression in *standard form*.

Factored Form	Standard Form
a $(5x+6)(x-3)$	________ $5x^2+43x-18$
b $(5x-3)(x+6)$	________ $5x^2-9x-18$
c $(5x-2)(x+9)$	________ $5x^2-43x-18$
d $(5x+2)(x-9)$	________ $5x^2+27x-18$

Activity
1

Name: ________ Date: ________ Period: ________

Diagram Puzzles

Complete each diagram puzzle, standard-form expression, and factored-form expression.

<table>
<tr><th>Diagram</th><th>Standard Form</th><th>Factored Form</th></tr>
<tr><td>

2.

<table>
<tr><td></td><td>$3x$</td><td>-5</td></tr>
<tr><td>$4x$</td><td></td><td></td></tr>
<tr><td></td><td>-$9x$</td><td>15</td></tr>
</table>

</td><td>________ $+ 15$</td><td>$(3x - 5)(4x$ ________$)$</td></tr>
<tr><td>

3.

<table>
<tr><td></td><td>$2x$</td><td>3</td></tr>
<tr><td></td><td>$4x^2$</td><td>$6x$</td></tr>
<tr><td></td><td></td><td>-9</td></tr>
</table>

</td><td>$4x^2$ ________</td><td>$(2x + 3)($________$)$</td></tr>
<tr><td>

4.

<table>
<tr><td></td><td></td><td>-3</td></tr>
<tr><td></td><td>$2x^2$</td><td>-$3x$</td></tr>
<tr><td></td><td>$8x$</td><td></td></tr>
</table>

</td><td>$2x^2 + 5x$ ________</td><td></td></tr>
<tr><td>

5.

<table>
<tr><td>$3x^2$</td><td>$4x$</td></tr>
<tr><td>$15x$</td><td>20</td></tr>
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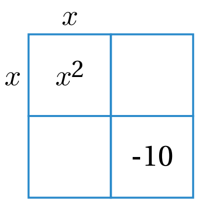
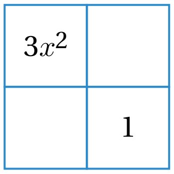
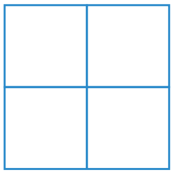
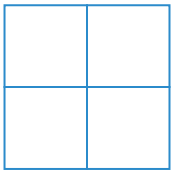
Name: .... Date: .... Period: ....

Diagram Puzzles (continued)

Diagram	Standard Form	Factored Form
6. 	$x^2 - 3x - 10$	
7. 	$3x^2 + 4x + 1$	
8. 	$x^2 + 9x + 20$	
9. 	$6x^2 + 7x + 2$	

Activity
2

Name: .... Date: .... Period: ....

Next Steps

Nicolas is trying to factor $2x^2 + 9x + 7$.

	2x	1
x	$2x^2$	x
7	14x	7

10. **Discuss:**

- What did Nicolas do well?
- Explain what you think is incorrect about Nicolas's work.
- What could he try next?

Sneha is trying to factor $2x^2 + 23x - 12$. She started by creating this diagram.

	2x	
x	$2x^2$	
		-12

11. List pairs of constants Sneha could try in order to complete the outside of the diagram.

Sneha tried the numbers -6 and 2.

	2x	2
x	$2x^2$	2x
-6	-12x	-12

12. **Discuss:**

- How can you tell Sneha's work is incorrect?
- What did Sneha do well?
- What could she try next?

13. Rewrite $2x^2 + 23x - 12$ in factored form.

Activity
2

Name: .... Date: .... Period: ....

Next Steps (continued)

14. Ariana is trying to factor $10x^2 - 7x - 12$. She starts by creating this diagram.

$10x^2$	
	-12

Ariana says: *I have to use factors of* 10. *I also need to use factors of* -12.

What do you think she means?

15. Rewrite $10x^2 - 7x - 12$ in factored form.

16. Here are three other expressions with a c-value of -12. Rewrite each expression in factored form. Use the diagrams if they help with your thinking.

a $x^2 + x - 12$

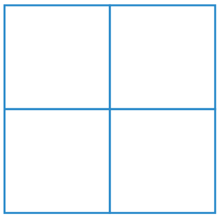
b $3x^2 - 16x - 12$

c $6x^2 - x - 12$

Synthesis

17. Describe how to rewrite a standard-form expression in factored form. Use the example if it helps with your thinking.

$5x^2 - 31x - 28$



Lesson Practice 7.04

Lesson Summary

A diagram can be a helpful tool for rewriting *standard-form* quadratic expressions in *factored form*. Here is an example: Rewrite the expression $3x^2 + 14x + 15$ in factored form.

Work

Explanation

$3x^2$	
	15

Place the ax^2 term in top-left corner of the diagram and the c-term in the bottom right.

	x	15
3x	$3x^2$	45x
1	x	15

Try different *factors* on the outside of the diagram that multiply to get ax^2 and c until the inside of the diagram matches standard form.

This attempt didn't work because $x + 45x \neq 14x$.
The first attempt might not work and that's okay! Try different factors or switching the positions of the current factors.

	3x	5
x	$3x^2$	5x
3	9x	15

This attempt works because the linear terms in the diagram combine to match bx in standard form: $5x + 9x = 14x$.

The factored form is $(3x + 5)(x + 3)$.

Lesson Practice
7.04

Name: …………………… Date: ………… Period: …………

1. Write two possible constants that could complete the outside of the diagram.

	$3x$	☐
x	$3x^2$	
☐		18

Problems 2–3: Complete the diagram puzzles and expressions.

2.

	$2x$	5
x	$2x^2$	
-7		-35

Factored Form: $(2x + 5)(x - 7)$

Standard Form: ……………………

3.

	$3x$	-6
x	$3x^2$	
	$12x$	

Factored Form: ……………………

Standard Form: ……………………

4. Rewrite each expression in factored or standard form.

Factored Form	Standard Form
	$x^2 + 9x + 18$
$(2x - 3)(2x - 7)$	
	$3x^2 + 10x - 8$

5. Determine values that make the equation true.

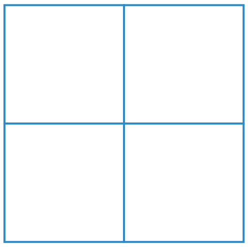
$7x^2 - 10x -$ ………… $= ($…………$x - 2)(7x +$ …………$)$

Lesson Practice

7.04

Name: .... Date: .... Period: ....

6. Create and complete a diagram puzzle that only uses factors of 12 as coefficients and constants.



Test Practice

7. This quadratic expression in standard form has an unknown b-value. If we know the expression can be factored, select the possibilties for the unknown value. Select all that apply.

$$3x^2 + \boxed{?}\,x - 4$$

- ☐ **A.** 12
- ☐ **B.** -11
- ☐ **C.** -8
- ☐ **D.** -1
- ☐ **E.** 4

Spiral Review

Problems 8–10: Fill in the missing values to make each equation true.

8. $3(\text{.........} + \text{.........}) = 18$

9. $\text{.........}(\text{.........} + \text{.........}) = 20$

10. $4(\text{.........} + \text{.........}) = 3(\text{.........}) + 2(\text{.........})$

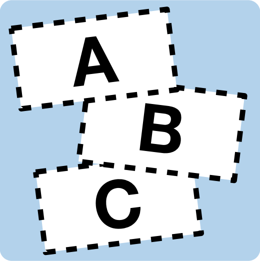
Unit 7
Lesson
5

Name: ______________ Date: ______ Period: ______

MA.912.AR.1.4, MA.912.AR.1.7, MA.912.NSO.1.2, MTR.2.1, MTR.4.1, MTR.5.1

Form Up

Let's factor some special quadratic expressions.



Warm-Up

Eliza is trying to factor $x^2 + x - 56$. She started by listing pairs of numbers that multiply to -56.

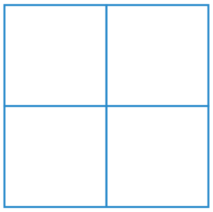
1 and -56 2 and -28 4 and -14 7 and -8

-1 and 56 -2 and 28 -4 and 14 -7 and 8

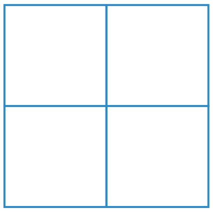
1. **Discuss:** Which pairs might Eliza try first? Why?

2. Factor each expression. Use the diagrams if they help with your thinking.

a $x^2 + x - 56$



b $x^2 + 26x - 56$



Activity

1

Name: ______________________ Date: __________ Period: __________

Spotting Similarities

Here are three groups of expressions.

Group 1	Group 2	Group 3
$4x^2 - 25$	$8x^2 + 32x + 24$	$x^2 - 6x - 27$
$x^2 - 36$	$-4x^2 + 8x + 32$	$x^2 + 2x - 80$
$x^2 - 100$	$-10x^2 - 20x - 10$	$x^2 - 13x + 30$
$25x^2 - 49$	$2x^2 - 22x + 60$	$x^2 + 2x - 63$

3. Explain how the expressions in each group are alike.

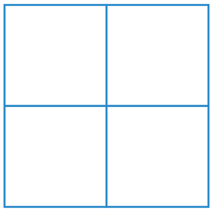
Group 1:

Group 2:

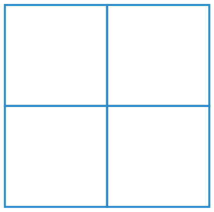
Group 3:

4. Factor one expression from each group. Use the diagrams if they help with your thinking.

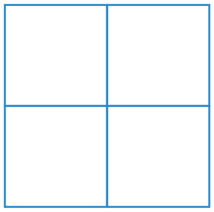
Group 1: ____________



Group 2: ____________



Group 3: ____________



Name: .... Date: .... Period: ....

Spotting Similarities (continued)

Deiondre factored the expression $7x^2 + 28x + 21$.

Deiondre

$7x^2 + 28x + 21$

$7(x^2 + 4x + 3)$

$7(x + 3)(x + 1)$

5. **Discuss:**
   - Are $7x^2 + 28x + 21$ and $7(x^2 + 4x + 3)$ equivalent? How do you know?
   - Why might Deiondre have written $7(x^2 + 4x + 3)$ as a first step?

6. Does Deiondre's expression belong in Group 1, 2, or 3? Explain your thinking.

Yasmine factored the expression $9x^2 - 49$.

Yasmine

$9x^2 - 49$

$9x^2 + 0x - 49$

$(3x - 7)(3x + 7)$

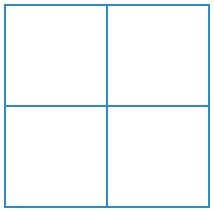
7. **Discuss:** Does Yasmine's expression belong in Group 1, 2, or 3? Explain your thinking.

8. Write a new expression in *standard form* that belongs in the same group as Yasmine's.

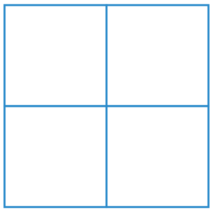
9. Factor the expression you wrote in the previous problem.

10. Factor each expression. Use the diagrams if they help with your thinking.

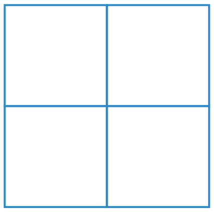
a $3x^2 - 6x - 105$



b $16x^2 - 49$



c $4x^2 + 52x + 120$



Activity
2

Name: .... Date: .... Period: ....

Solve and Swap

You will get a card.

- Factor the expression on your card. Draw a diagram if it helps with your thinking.
- Find a partner and swap cards. Factor your new expression, then check with your partner.
- Find a new partner and repeat this process.

Card ....

Card ....

Card ....

Card ....

Card ....

Card ....

Card ....

Card ....

Synthesis

What do you think is important to remember when factoring an expression in standard form?

Use the expressions if they help with your thinking.

$5x^2 - 18x - 8$

$9x^2 - 16$

$6x^2 - 24x - 30$

Lesson Practice 7.05

Lesson Summary

You can use the structure of a quadratic expression written in *standard form* to predict what the *factored form* will look like. Here are some strategies:

- Try to factor out a common factor first.
- If the standard form expression only has two terms, write the missing term with a coefficient of 0.
- If the c-value is negative, the signs of the constants in factored form will be different.

Here are some examples.

$3x^2 - 9x - 30$

3 is a common factor.
$3(x^2 - 3x - 10)$

Then factor $x^2 - 3x - 10$.

	x	-5
x	x^2	-5x
2	2x	-10

When $a = 1$, the constants in factored form multiply to c and add to b.

Factored form:
$3(x - 5)(x + 2)$

$x^2 - 81$

Expressions with this structure are called a *difference of squares*. Rewriting the expression with three terms might be helpful.

Factor $x^2 + 0x - 81$.

	x	9
x	x^2	9x
-9	-9x	-81

Factored form:
$(x - 9)(x + 9)$

$2x^2 + 3x - 27$

There is not a common factor in this quadratic expression.

Test pairs of expressions that multiply to $2x^2$ and -27.

The c-value is negative so the signs of the constants in factored form will be different.

	2x	9
x	2x^2	9x
-3	-6x	-27

Factored form:
$(2x + 9)(x - 3)$

Lesson Practice

7.05

Name: ____________________ Date: __________ Period: __________

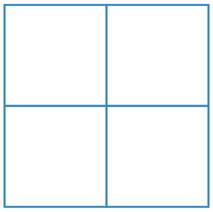
1. Write a + or – sign in each box to make true equations.

$(x \square 18)(x \square 3) = x^2 + 15x - 54$

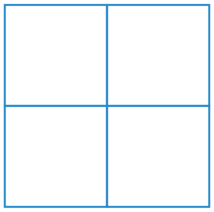
$(x \square 18)(x \square 3) = x^2 - 21x + 54$

Problems 2–5: Fill in the blanks to make each equation true. Use the diagrams if they help your thinking.

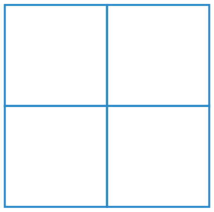
2. x^2 ______ x ______ $= (x - 9)(x - 3)$



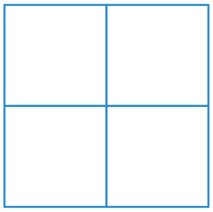
3. $x^2 + 12x$ ______ $= (x + 4)(x$ ______$)$



4. $2x^2 + 11x + 15 = (2x$ ______$)(x$ ______$)$



5. $3x^2 - 11x$ ______ $= (x - 6)(3x$ ______$)$



Problems 6–8: Factor each expression.

6. $x^2 - x - 30$

7. $4x^2 + 20x + 25$

8. $2x^2 + x - 15$

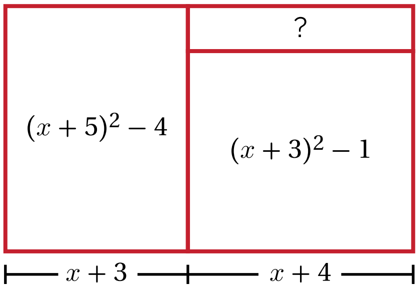
Lesson Practice

7.05

Name: .... Date: .... Period: ....

Test Practice

9. The diagram shows the expressions for areas and lengths. Choose the expression for the unknown area.



[✓ **A.**] $5(x+4)$

B. $4(x+5)$

C. x-2

D. -2$(x+1)$

Spiral Review

Problems 10–12: Solve each equation.

10. $6+2x=0$

11. $2x-5=0$

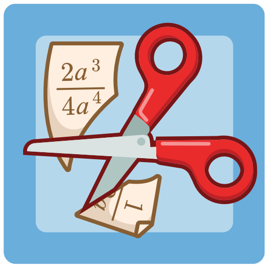
12. $\frac{1}{2}(x-87)=0$

Name: ____________ Date: ________ Period: ________

MA.912.AR.1.4, MA.912.NSO.1.2, MTR.3.1, MTR.4.1, MTR.5.1

Divide and Conquer

Let’s divide polynomials by monomials.



Warm-Up

1. Simplify the following expressions using the Laws of Exponents.

$\frac{2a^3}{4a^4}$ $\frac{5x^2}{15x^6}$

2. Rewrite each expression as one term by factoring out the greatest common factor.

$36x^4 - 6x$ $-96m^3 + 108m$

3. **Think-Pair-Share:** Reflect on how you wrote expressions in Problems 1 and 2. How are the Laws of Exponents and finding the greatest common factor (GCF) similar and different when simplifying expressions?

Name: .... Date: .... Period: ....

Exploring Closure

4. Determine if the following expressions are examples of polynomials or non-examples of polynomials. Write yes or *no* in the table.

Expression	Is it a polynomial?
$3x^2 + 2x - 5$	
$\frac{1}{x} + 2x$	
$4y^4$	
$5\sqrt{x - 3}$	
$7z^3 + 2z - 5$	

5. **Think-Pair-Share:** What are the differences between the examples of polynomials and the non-examples of polynomials?

6. Complete the chart with your own examples of polynomials and non-examples of polynomials.

Polynomial Examples	Non-Polynomial Examples

Activity
2

Name: Date: Period:

Division with No Remainder

Dividing a polynomial by a monomial is the same as dividing each polynomial term individually by the monomial. Here is Kazia's work on dividing polynomials.

$$\frac{6x^3 + 12x^2 - 18x}{3x} = \frac{6x^3}{3x} + \frac{12x^2}{3x} - \frac{18x}{3x}$$

7. Look at Kazia's work.

a Describe what strategy Kazia is using to simplify.

b Finish her work. Is the result a polynomial?

8. Simplify each expression. Then identify if the result is a polynomial and why.

a $\frac{30x^8 + 10x^7 + 2x^6}{10x}$

b $\frac{16x^2y^4 - 8x^2y + 6xy)}{2xy}$

c $\frac{-90x^{11}\,y^{15} + 3x^6y^{10} + 27x^4y^5}{3x^4y^5}$

9. **Help a Friend:** Claude completed the following problem and made a mistake. Define the error and fix it. What advice would you give Claude?

$$\frac{2x^4 + 18x^3 - 6x^2}{6x^2} = 2x^2 + 3x - 1x^2$$

Activity
3

Name: ________ Date: ________ Period: ________

Polynomial No More!

10. Simplify the following expressions and identify if the result is a polynomial.

a $\frac{40r^3 + 8r^2 + 2r}{8r^3}$

b $\frac{20x^6y^4 - 2x^3y}{4x^3y^2}$

11. Determine the quotient of $\frac{1}{3}x^4 - 3x^3 + \frac{1}{2}x^2$ and $3x^3$.

a **Discuss** the strategy you used with a partner. How do your quotients compare to one another?

b Is the quotient a polynomial? Explain.

c Is this allowed to happen? Shouldn't all polynomials turn into polynomials?

Synthesis

12. How do you know if an expression is a polynomial? How do you know if it is not a polynomial?

Lesson Practice 7.06

Lesson Summary

In this lesson, you explored how to divide polynomials by monomials and learned about key concepts to guide your work.

Polynomials consist of terms where variables have whole-number exponents, and there are no variables in denominators or under square roots.

- To divide a polynomial by a monomial, break the numerator into separate terms, divide each by the monomial, and simplify using the Laws of Exponents: $\frac{a^m}{a^n} = a^{(m-n)}$.
- **Remainders** occur when a term in the numerator cannot be fully divided by the monomial, resulting in a fraction.
- **Closure:** Polynomials are closed under addition, subtraction, and multiplication, but not always under division. Dividing by monomials keeps the result as a polynomial, but more complex divisors may result in non-polynomials.

Lesson Practice
7.06

Name: .... Date: .... Period: ....

Problems 1–3: James was finding the quotient of $\frac{12x^4 + 18x^3 - 24x^2}{6x^2}$

$$\frac{12x^4 + 18x^3 - 24x^2}{6x^2}$$

$$\frac{12x^4}{6x^2} + \frac{18x^3}{6x^2} - \frac{24x^2}{6x^2}$$

$$2x^2 + 3 - 4$$

1. Identify his error.

2. What is the correct answer?

3. What advice can you give James to avoid this error in the future?

Problems 4–5: Simplify each expression. State whether the resulting expression is a polynomial or not a polynomial.

4. $\frac{12x^4 + 18x^3 - 24x^2}{6x^2}$

5. $\frac{7x^3 + 9x^2}{2x^3}$

Lesson Practice

7.06

Name: .... Date: .... Period: ....

Test Practice

6. Which two expressions when simplified have the same value?

☐ **A.** $\dfrac{8x^2 + 4x - 10}{2}$

☐ **B.** $\dfrac{8x^2 + 4x + 10x}{2}$

☐ **C.** $\dfrac{12x^3 + 6x - 15x}{3x}$

☐ **D.** $\dfrac{20x^4 + 10x^3 + 25}{5x^2}$

Spiral Review

Problems 7–10: Identify the x- and y-intercepts.

7. $2x + 3y = 12$

8. $5x - 4y = 20$

9. $y = \dfrac{2}{3}x - 4$

10. $y = -5x + 10$

Unit 7

Lesson

7

Name: ____________ Date: ____________ Period: ____________

MA.912.AR.1.4, MA.912.AR.1.7, MA.912.NSO.1.2, MTR.2.1, MTR.5.1

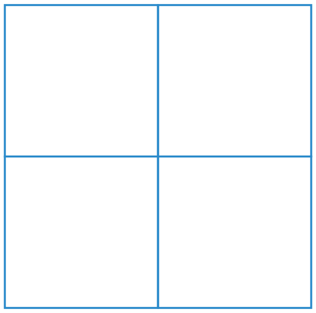
Consider the Factors

Let’s rewrite polynomial expressions by factoring.

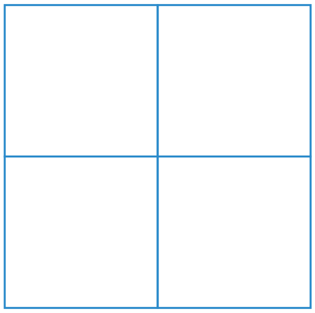
Warm-Up

1. Factor each expression. Use the diagrams if they help with your thinking.

a $x^2 + 6x + 9$



b $2x^2 + 7x + 3$



c $-8x^4 + 2x^2 - 10x$

2. **Discuss:** Is there an equivalent form to your answer in Part a?

Activity 1

Name: Date: Period:

More Variables, More Fun!

Here are two groups of expressions.

Group 1	Group 2
$4x^4 - 40x^2 + 2x$	$4x^4y^5 - 40x^2y^2 + 2xy^2$
$-6y^5 - 42y^3 + y^2$	$-6x^5y^5 - 42xy^3 + x^2y^2$
$-14x^6 - 70x^4$	$-14x^6y^2z^4 - 70x^4yz^2$

3. **Discuss:** What do you notice? What do you wonder?

4. Find the GCF for two expressions from Group 1. Draw a diagram if it helps with your thinking.

Expression 1:

Factored Form:

Expression 2:

Factored Form:

Name: .... Date: .... Period: ....

More Variables, More Fun! (continued)

5. Find the GCF for two expressions from Group 2, and use the expressions that are in the same row as the choices from Group 1. Draw a diagram if it helps with your thinking.

Expression 1: ....

Expression 2: ....

Factored Form: ....

Factored Form: ....

6. 💬 **Discuss:** What is similar about your answers from Problems 4 and 5?

Activity
2

Name: ______ Date: ______ Period: ______

Patterns in Polynomials

Here are six expressions.

Group 1	Group 2
$x^2 + 12x + 36$	$9z^2 + 6z + 1$
$x^2 - 49$	$16y^2 - 25$
$4y^2 - 20y + 25$	$16x^4 - 1$

7. Factor the expressions

8. Organize all six factored expressions into two groups based on how they look. Give each group a title.

Titles vary. Squares	*Titles vary.* No Squares

9. 💬 **Discuss:** Write down a rule or pattern for each group.

10. How can you identify these types of problems before factoring?

Activity
3

Name: ________ Date: ________ Period: ________

Factoring Four Terms

Jamal and Candice are working on factoring polynomials that have four terms. Their work is below.

$$6x^3 + 9x^2 + 4x + 6$$

Jamal's Work (Box Diagram)

	$3x^2$	2
$2x$	$6x^3$	$4x$
$+3$	$9x^2$	6

$(3x^2 + 2)(2x + 3)$

Candice's Work (Factoring by Grouping)

$6x^3 + 9x^2 + 4x + 6$

$(6x^3 + 9x^2) + (4x + 6)$

$3x^2(2x + 3) + 2(2x + 3)$

$(3x^2 + 2)(2x + 3)$

11. **Discuss:** the following questions.

a How are Jamal's and Candice's methods similar?

b What patterns did you notice in both methods?

Name: .... Date: .... Period: ....

Factoring Four Terms (continued)

12. Factor the following polynomials, pick one to factor by grouping and one using the box diagram

a $8x^3 + 16x^2 + 2x + 4$

b $5y^3 - 15y^2 + y - 3$

13. Explain why you choose your strategy for each problem.

Synthesis

14. Explain the different kinds of strategies you use to factor expressions.

Lesson Practice 7.07

Lesson Summary

In this lesson, you explored multiple strategies to factor polynomials, each suited to different types of expressions

- First, you learned how to factor out the greatest common factor (GCF). This involves identifying the largest factor shared by all terms, including variables, and rewriting the polynomial as the product of the GCF and a simpler expression.
- You examined **perfect square trinomials**, which have three terms. These can be identified when the first and last terms are perfect squares, and the middle term equals twice the product of their square roots. These trinomials factor into the form $(a + b)^2$ or $(a - b)^2$.
- You also worked with the **difference of squares**, which consists of two terms that are both perfect squares separated by subtraction. These always factor into $(a - b)(a + b)$.
- Finally, you learned **factoring by grouping**, a method for factoring four-term polynomials. By grouping terms into two pairs, factoring out the GCF from each pair, and identifying a common binomial factor, you can factor an expression into the product of two binomials.

Lesson Practice

7.07

Name: ______ Date: ______ Period: ______

Problems 1–2: Factor the expressions completely.

1. $-36x^4 + 40x + 32$

- **A.** $4x(-9x^3 + 10x + 8)$
- **B.** $4x(-9x^4 + 10x + 8)$
- **C.** $4(-9x^4 + 10x + 8)$
- **D.** $4(-9x^4 + 5x + 8)$

2. $81x^4 - 1$

- **A.** $(9x + 1)(9x - 1)$
- **B.** $(9x^2 - 1)(9x^2 + 1)$
- **C.** $(3x - 1)^2(3x + 1)^2$
- **D.** $(3x - 1)(3x + 1)(9x^2 + 1)$

Problems 3–6: Factor the expressions completely. Draw a diagram if it helps with your thinking.

3. $100z^4 - 25$

4. $20b^3 + 15b^2 - 100b - 75$

5. $4t^2 - 64$

6. $-8n^8m^3 - 6n^7m + 12n^7$

Lesson Practice
7.07

Name: .... Date: .... Period: ....

Test Practice

Problems 7–9: Priya factored the binomial expression $9x^2 - 36$.

$$9x^2 - 36$$
$$(3x + 6)(3x - 6)$$

7. What strategy did Priya use?

8. How do you know that Priya did not completely factor the expression? Explain your reasoning.

9. Which of the following is the completely factored form of $9x^2 - 36$?

A. $(9x^2 - 4)(9x^2 + 4)$

B. $(3x - 2)(3x + 2)(9x^2 + 4)$

C. $(9x - 4)(9x + 4)$

D. $(3x - 2)(3x + 2)(3x^2 + 4)$

Spiral Review

Problems 10–13: Solve the equations.

10. $4x - 7 = 2x + 5$

11. $3y + 2 + 5y = 6y + 10$

12. $-2(3w - 4) = 10w + 12$

13. $5(2c + 1) - 3c = 2c + 8 + c$

Unit 7

Lesson

8

Name: .... Date: .... Period: ....

MA.912.AR.1.1, MA.912.AR.1.4, MA.912.AR.1.7, MA.912.AR.3.6, MTR.2.1, MTR.4.1, MTR.5.1

Shooting Stars

Let's determine the x-intercepts of quadratic functions written in factored form and standard form.

Warm-Up

1. Determine whether each coordinate pair is an x-intercept or a y-intercept.

Ordered Pair	x-intercept	y-intercept	Neither
$(1.7, 0)$			
$(1, 1)$			
$(0, 4)$			
$\left(-\frac{3}{2}, 0\right)$			
$(5,0)$			
$(0, -6)$			

Activity

1

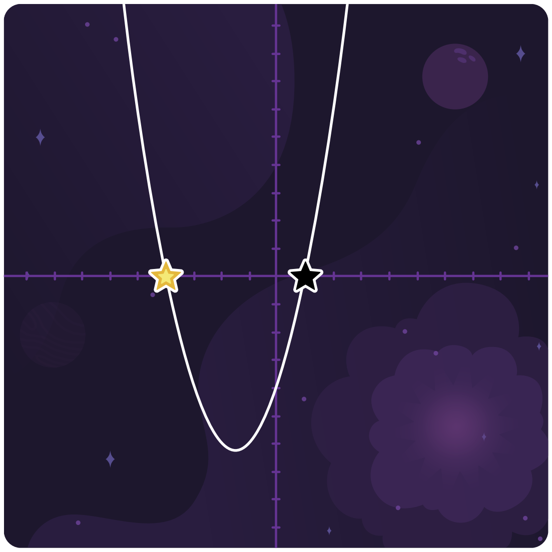
Name: ______________ Date: ______________ Period: ______________

Star Mail

2. Send stars to the x-intercepts of this function:

$$f(x) = (x + 4)(x - 1)$$

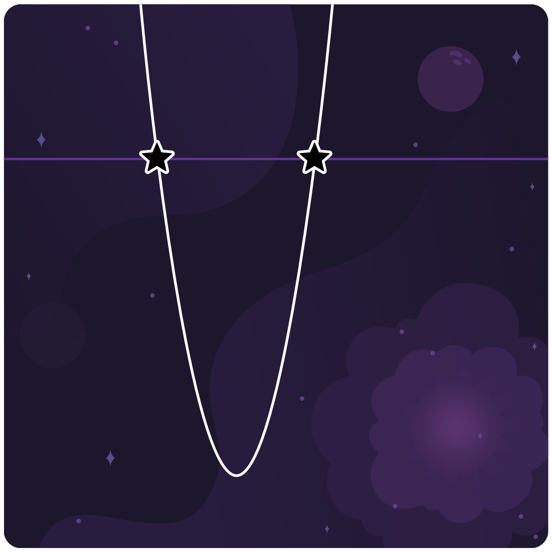
Star	Ordered Pair
Star #1	(-4, 0)
Star #2	



3. Send stars to the x-intercepts of this function:

$$g(x) = (x + 1)(2x - 6)$$

Star	Ordered Pair
Star #1	
Star #2	



4. Let's look at Aba's strategy from the previous problem.

Discuss:

- Why did Aba replace $g(x)$ with 0?
- How did Aba figure out the coordinates of the x-intercepts?

Name: ______ Date: ______ Period: ______

Standard Space Mail

5. Send stars to the x-intercepts of this function:

$$h(x) = x^2 + 3x - 10$$

Star	Ordered Pair
Star #1	
Star #2	

6. Aba, Darius, and Rishi factored the function $a(x) = 4x^2 + 20x + 24$ in three different ways.

Aba	Darius	Rishi
$a(x) = 4(x + 3)(x + 2)$	$a(x) = (4x + 8)(x + 3)$	$a(x) = (2x + 6)(2x + 4)$

a **Discuss:** How can you see that each equation has the same x-intercepts?

b Write the x-intercepts in the table below.

x-intercepts	Ordered Pair
x-intercept #1	
x-intercept #2	

7. Send stars to the x-intercepts of this function:

$$b(x) = 2x^2 - 11x + 12$$

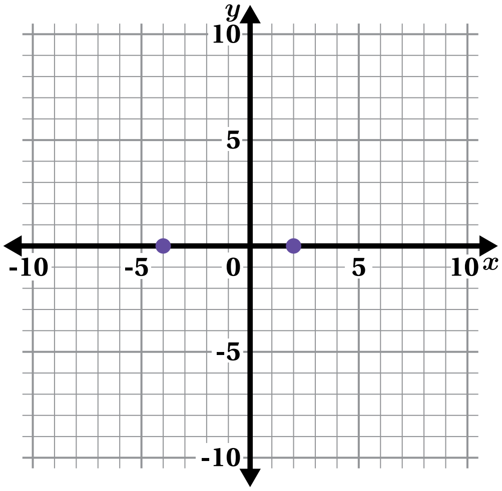
Star	Ordered Pair
Star #1	
Star #2	

Activity
3

Name: .... Date: .... Period: ....

Zero, My Hero

8. A term related to x-intercepts is **zeros**. The zeros of a function are the x-values that make $f(x) = 0$.



a **Discuss:** How are *zeros* related to x-intercepts?

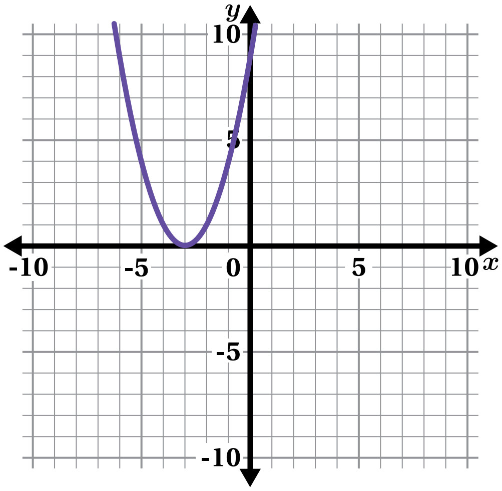
b Write a function whose zeros are $x = -4$ and $x = 2$.

$f(x) =$ ....

9. The function $f(x) = x^2 + 6x + 9$ has exactly one zero.

Write a new quadratic function that has exactly one zero.

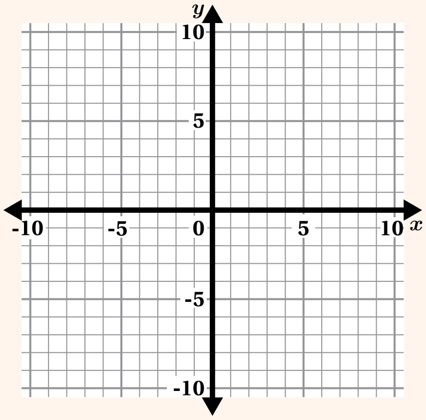
$g(x) =$ ....



You're invited to explore more.

10. Show or describe as much as you can about the graph of this function:

$$f(x) = (x + 1)(x - 2)(x + 3)$$



Synthesis

11. Describe a strategy for determining the x-intercepts or zeros of a quadratic function.

$g(x) = (x + 1)(2x - 4)$

$h(x) = x^2 + 3x - 10$

Lesson Practice 7.08

Lesson Summary

An x-intercept is the coordinate point where the graph crosses the x-axis, and a **zero** is the x-value of the x-intercept.

To determine the zeros or x-intercepts, you determine the x-values that make the function equal to 0. Rewriting the function in factored form is a helpful step for determining those values.

Here are two examples of determining the x-intercepts of functions in different forms.

Factored Form: $h(x) = (x - 1)(x + 6)$

When $x = 1$, the factor $x - 1 = 0$, so $h(1) = 0$.

When $x = -6$, the factor $x + 6 = 0$, so $h(-6) = 0$.

The zeros are $x = 1$ and $x = -6$.

The x-intercepts are $(1, 0)$ and $(-6, 0)$.

Standard Form: $g(x) = x^2 - 6x - 40$

First, I can factor the function.

	x	-10
x	x^2	$-10x$
4	$4x$	-40

$g(x) = (x - 10)(x + 4)$

When $x = 10$, the factor $x - 10 = 0$, so $g(10) = 0$.

When $x = -4$, the factor $x + 4 = 0$, so $g(-4) = 0$.

The zeros are $x = 10$ and $x = -4$.

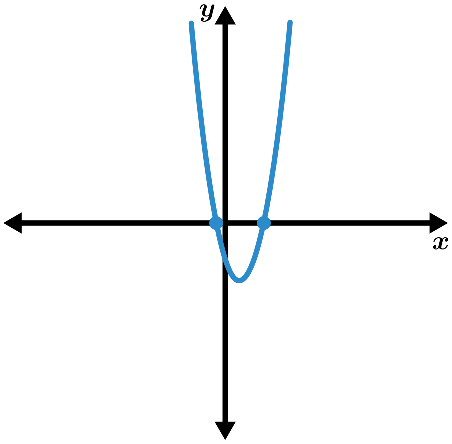
The x-intercepts are $(10, 0)$ and $(-4, 0)$.

Lesson Practice
7.08

Name: .... Date: .... Period: ....

Problems 1–3: What are the x-intercepts of the function?

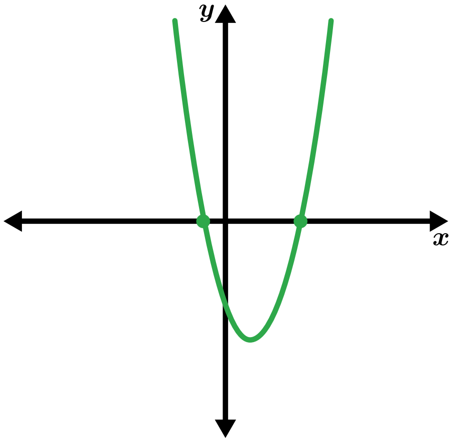
1. $a(x) = (x - 2)(2x + 1)$



x-intercept #1: ....

x-intercept #2: ....

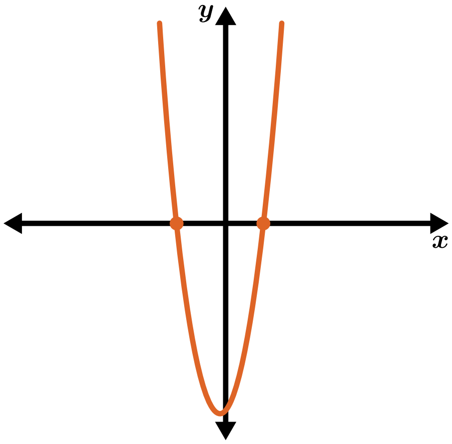
2. $b(x) = x^2 - 3x - 4$



x-intercept #1: ....

x-intercept #2: ....

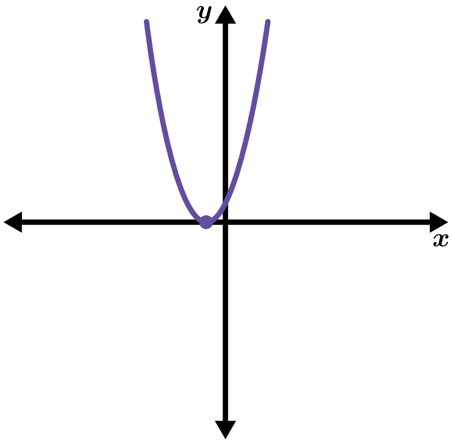
3. $c(x) = 2x^2 + x - 10$



x-intercept #1: ....

x-intercept #2: ....

4. Write an equation that could represent this graph.



$f(m) =$ ....

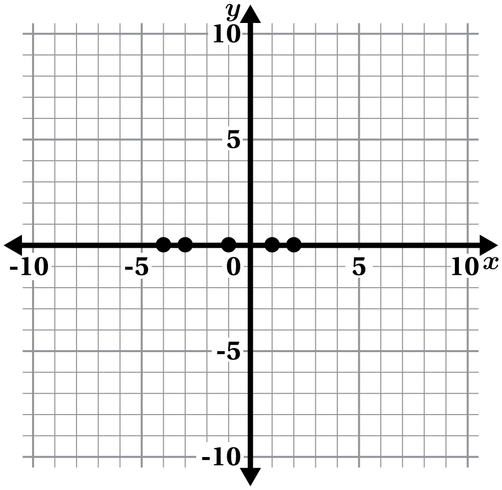
5. Change one number so that the function has x-intercepts at -1 and 3.

$g(x) = x^2 - 1x - 3$

Lesson Practice
7.08

Name: .... Date: .... Period: ....

6. Write the equations of 3 parabolas that go through all of the points.



Test Practice

7. Select *all* the functions that have 5 and -1 as their x-intercepts.

- ☐ **A.** $f(x) = (x + 5)(x - 1)$
- ☐ **B.** $g(x) = (x + 1)(x - 5)$
- ☐ **C.** $h(x) = x^2 + 4x - 5$
- ☐ **D.** $j(x) = 2x^2 - 8x - 10$
- ☐ **E.** $k(x) = (15 - 3x)(4x + 4)$

Spiral Review

Problems 8–9: Solve for x.

8. $2x - 3 \geq 4x + 7$

9. $3(x - 2) < 2x - 12$

Unit 7

Lesson

9

Name: ______ Date: ______ Period: ______

MA.912.AR.1.2, MA.912.AR.1.7, MA.912.AR.3.1, MA.912.AR.3.6, MTR.2.1, MTR.4.1, MTR.5.1

Make It Zero

Let's use the zero-product property to solve quadratic equations.

Warm-Up

1. Determine the *solution* to each equation.

$4a = 0$ $\quad$ $0 = 2\pi b$ $\quad$ $6(c - 5) = 0$ $\quad$ $7 \cdot (d + 8) \cdot 9 = 0$

2. The **zero-product property** states: If the product of two or more *factors* is 0, then at least one of the factors is 0.

We can use this to help solve equations like $4a = 0$ or $6(c - 5) = 0$.

Write a new equation using the variable x that the zero-product property could help solve.

Activity

1

Name: .... Date: .... Period: ....

Solve It

Use the zero-product property to solve the following equations.

3. $(x - 4)(2x + 3) = 0$

$x =$ .... $x =$ ....

4. $x^2 + 5x + 4 = 0$

$x =$ .... $x =$ ....

5. $3x^2 - 18x + 15 = 0$

$x =$ .... $x =$ ....

Name: .... Date: .... Period: ....

Solve It (continued)

6. Here is Hamza's work from the previous problem.

$$3x^2 - 18x + 15 = 0$$
$$(3x - 3)(x - 5) = 0$$
$$x = 3 \text{ or } x = 5$$

a What is something Hamza did well?

b What is something Hamza can improve?

Name: .... Date: .... Period: ....

Zoering In

7. Inola says you can't use the zero-product property to solve the equation $x^2 - 4 = 3x$.

a **Discuss:** Why might Inola think that?

b Describe how you could rewrite the equation so that the zero-product property *can* be used.

8. Solve Inola's equation: $x^2 - 4 = 3x$.

$x =$ .... $x =$ ....

Name: .... Date: .... Period: ....

Zeroing In (continued)

9. The equation $9x^2 = 12x - 4$ has one solution.

What's the solution? Show or explain your thinking.

You're invited to explore more.

10. Write at least one equation with $x = 2$ and $x = 3$ as solutions.

Try to write some equations you think none of your classmates will write.

Synthesis

11. How can you use the zero-product property to solve a quadratic equation?

Consider the examples if they help with your thinking.

A $(2x+4)(x+3)=0$

B $3x^2-18x+15=0$

C $x^2-4=3x$

D $(x-5)(x+1)=7$

Lesson Practice 7.09

Lesson Summary

The **zero-product property** states that if the product of two or more *factors* is 0, then at least one of the factors is 0. You can use this property to determine the x-intercepts of a function or the *solutions* to quadratic equations using the following steps.

- Set the quadratic equation equal to 0.
- Factor the equation.
- Set each factor equal to 0.
- Solve for x.

Here are two examples of solving quadratic equations.

$(5x-3)(2x+3)=0$

Set each factor equal to 0 and solve for x.

$$(5x-3)=0 \qquad (2x+3)=0$$
$$5x=3 \qquad 2x=\text{-}3$$
$$x=\frac{3}{5} \text{ and } x=\text{-}\frac{3}{2}$$

$2x^2-x=21$

First, rewrite the equation so that it is equal to 0.

$2x^2-x-21=0$

Then factor the equation.

$(2x-7)(x+3)=0$

Set each factor equal to 0 and solve for x.

$$(2x-7)=0 \qquad (x+3)=0$$
$$2x=7 \qquad x=\text{-}3$$
$$x=\frac{7}{2} \text{ and } x=\text{-}3$$

Lesson Practice

7.09

Name: .... Date: .... Period: ....

1. Rewrite each standard-form quadratic equation in factored form.

$x^2 + 7x + 6 = 0$ $x^2 - 5x + 6 = 0$ $x^2 + 5x - 6 = 0$

2. Rewrite the equation $x^2 - x = 6$ so that one side is equal to 0. Then solve the equation.

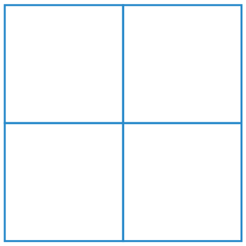
Problems 3–5: Solve each equation.

3. $(4 - 5x)(x + 4) = 0$

4. $x^2 + 14x + 33 = 0$

5. $5x + 12 = x^2 - 5x - 12$

6. Solve the equation $3x^2 - 18x + 24 = 0$.
Use the diagram if it helps with your thinking.

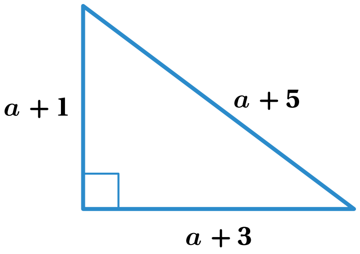


7. Write two equations with $x = \frac{2}{3}$ and $x =$ -9 as solutions.

Lesson Practice
7.09

Name: .... Date: .... Period: ....

8. Determine the value of a in the right triangle. Explain your thinking.



Test Practice

Problems 9–10: Solve each equation.

9. $x^2 - 7x = 0$

10. $(x + 2)(x + 4) = 3$

Spiral Review

11. Decide whether this equation has one solution, no solution, or infinitely many solutions: $-3(x + 2) = -x + 6 - 2x$.

One solution No solution Infinitely many solutions

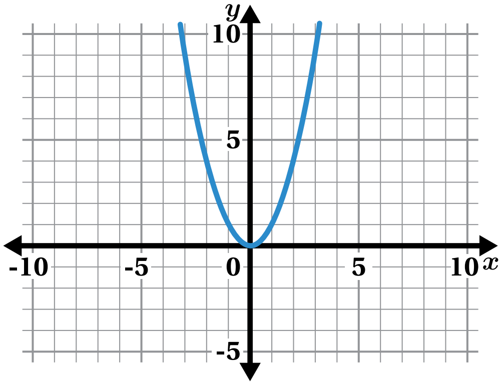
12. Select *all* the true statements about the function $f(x) = x^2$.

☐ **A.** The domain has no negative values.

☐ **B.** The range has no negative values.

☐ **C.** The function has no minimum.

☐ **D.** The function has no maximum.

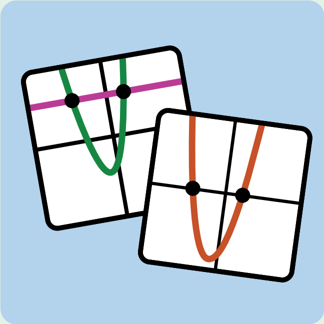


Sub-unit

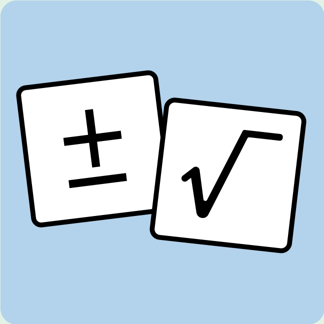
2

Solving Equations and Completing the Square

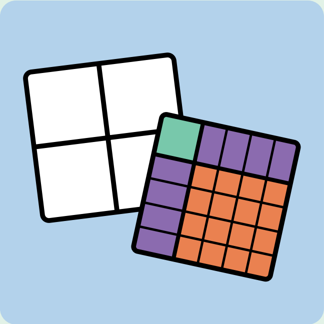
Lesson 10
Zero, One, or Two?



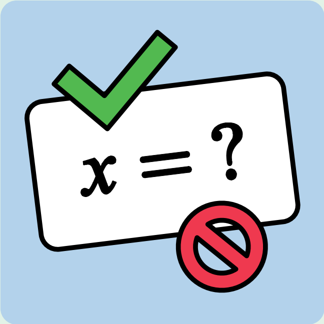
Lesson 11
Graph to Solve



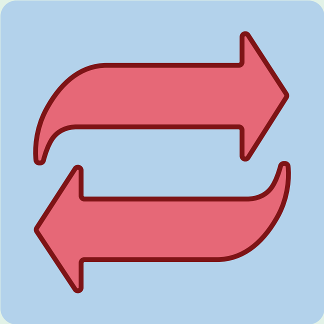
Lesson 12
Couldn’t Square Less



Lesson 13
Square Dance



Lesson 14
Square Tactic



Lesson 15
Back and Forth

Unit 7

Lesson

10

Name: ________ Date: ________ Period: ________

MA.912.AR.3.1, MTR.4.1, MTR.5.1, MTR.6.1

Zero, One, or Two?

Let’s determine whether quadratic equations have zero, one, or two solutions.

Warm-Up

1–2. Determine the value of each expression using mental math.

a 8^2

b -8^2

c $(-8)^2$

d Solve $x^2 = 64$.

Activity
1

Name: .... Date: .... Period: ....

How Many?

3. For each equation, put a check for the number of solutions.

Equation	No Solutions	One Solution	Two Solutions
$(x-3)^2 = 1$			
$(x-3)^2 = 0$			
$(x-3)^2 = -1$			
$(x-3) = 1$			
$(x-3)(x-3) = 1$			

4. Diya says that $x = 4$ is the only solution to $(x-3)^2 = 1$.

a **Discuss:** How do you know that $x = 4$ is a solution to $(x-3)^2 = 1$?

b Write a hint to help Diya determine *another* solution.

5. Here is a new equation: $x^2 - 16 = 9$.

How many solutions does this equation have? Circle one.

No solutions One solution Two solutions

Record any solution(s) here:

$x =$ ....

$x =$ ....

Name: .... Date: .... Period: ....

How Many? (continued)

6. Rewrite the equation $x^2 - 16 = 9$ so that it has no solutions.

Show or explain your thinking.

7. Here is a new equation: $(x - 5)^2 = 36$.

How many solutions does this equation have? Circle one.

No solutions One solution Two solutions

Record any solution(s) here:

$x =$ ....

$x =$ ....

Activity

2

Name: ______ Date: ______ Period: ______

How Many and More

8. For each equation, put a check for the number of solutions.

Equation	No Solutions	One Solution	Two Solutions
$x(x-6)=0$			
$2x^2=50$			
$x^2=-9$			
$x^2+4=0$			
$(x+2)(x+2)=0$			

9. Here are two equations from the previous problem.

$(x+2)(x+2)=0$ $x^2=-9$

Explain how you decided on the number of solutions for $(x+2)(x+2)=0$.

Explain how you decided on the number of solutions for $x^2=-9$.

Name: .... Date: .... Period: ....

How Many and More (continued)

10. Solve as many challenges as you have time for.

- Circle how many solutions each equation has.
- Record any solutions.

	Equation	Number of Solutions			Solution(s)
a	$30 = x^2 - 6$	No solutions	One solution	Two solutions	$x =$ .... $x =$ ....
b	$7x^2 + 1 = 1$	No solutions	One solution	Two solutions	$x =$ .... $x =$ ....
c	$(x - 4)^2 = -12$	No solutions	One solution	Two solutions	$x =$ .... $x =$ ....
d	$x(x + 2) = 15$	No solutions	One solution	Two solutions	$x =$ .... $x =$ ....
e	$(x - 4)(x - 4) = 16$	No solutions	One solution	Two solutions	$x =$ .... $x =$ ....
f	$3x^2 - 3 = -3$	No solutions	One solution	Two solutions	$x =$ .... $x =$ ....

Synthesis

11. **a** Discuss these questions:

- How can you determine the number of solutions to a quadratic equation?
- How can you solve a quadratic equation?

b Select *one question* and record your response.

$(x+2)(x+2)=0$

$x^2=-9$

$(x-5)^2=36$

Lesson Practice 7.10

Lesson Summary

To determine the number of solutions to a quadratic equation, you can use reasoning or use the structure of the equation. Here are some examples:

No Solutions	One Solution	Two Solutions
$(x+\underline{\qquad})^2=$ a negative number No value squared will result in a negative number.	$(x+\underline{\qquad})^2=0$ Only one value squared will equal 0.	$(x+\underline{\qquad})^2=$ a positive number There are two values that when squared will equal a positive number.
$(x+10)^2=-25$ $(x-3)^2+1=0$ $x^2+4=0$	$(x+4)^2=0$ $x^2+9=9$ $(x-3)(x-3)=0$	$(x+4)^2=1$ $x^2-12=-3$ $(x-3)(x-3)=1$

Lesson Practice
7.10

Name: .... Date: .... Period: ....

1. Write a quadratic equation that has . . .

Two solutions One solution No solutions

2. For each equation, determine the number of solutions.

Equation	Number of Solutions
$x(x + 3) = 0$	
$(x + 3)(x + 1) = 0$	
$(x + 1)(x + 1) = 0$	
$x^2 - 10x = -9$	
$(x - 5)(x - 5) = -14$	
$x^2 - 3 = -3$	
$x^2 + 6 = 2$	

Problems 3–4: Determine the solution for each equation.

3. $149 + (x - 2)^2 = 149$

4. $x^2 + 3x = x - 1$

Problems 5–6: Determine the two solutions for each equation.

5. $100 + (x - 2)^2 = 149$

6. $x^2 + 4x = x + 18$

Test Practice

7. Which value for x is a solution to the equation $x^2 + 10 = 9$?

A. $x = 1$ **B.** $x = -1$ **C.** $x = \sqrt{19}$ **D.** There is no solution to the equation.

Lesson Practice

7.10

Name: .... Date: .... Period: ....

Spiral Review

8. Order the expressions by value from *least* to *greatest*: 5^2, $\sqrt{90}$, 8, 3^3, $\sqrt{27}$.

Least **Greatest**

Problems 9–10: Complete each diagram. Then write the corresponding quadratic expression in factored form and standard form.

9.

	$-3x$	
$6x$		$18x$
3		

Factored form:

Standard form:

10.

		-8
$5x$	$30x^2$	
		-16

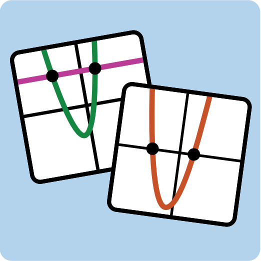
Factored form:

Standard form:

Name: .... Date: .... Period: ....

MA.912.AR.3.1, MA.912.AR.3.6, MTR.2.1, MTR.4.1, MTR.5.1

Graph to Solve



Let's use graphs to solve quadratic equations.

Warm-Up

1. Determine whether each statement is true or false.

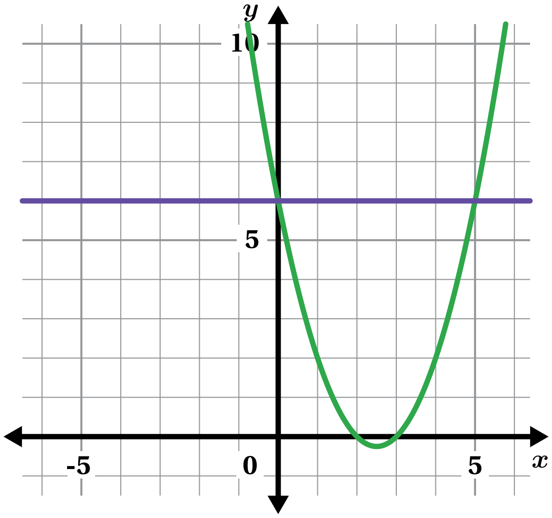
Statement	True	False
$x = 2$ and $x = 3$ are the solutions to $(x - 2)(x - 3) = 6$.		
$x = 3$ is the only solution to $x^2 - 9 = 0$.		
$x(x - 7) = 0$ has two solutions.		
$x =$ -5 is a solution to $x^2 + 25 = 0$.		

Name: ________ Date: ________ Period: ________

When In Doubt, Graph It Out

2. Malik used a graphing tool to determine the solutions to $(x-2)(x-3)=6$.

a Malik's strategy is shown on the graph. He graphed both sides of the equation as different functions.



b **Discuss:** Where in the graph can you see that the solutions are $x=0$ and $x=5$?

3. Use a graphing tool and Malik's strategy to solve $x^2+2x+1=4$.

How many solutions does this equation have? Circle one.

No solutions One solution Two solutions

Record any solution(s) here:

$x=$ ________

$x=$ ________

If you circled *No solutions*, complete the statement:

The equation has no solutions because . . .

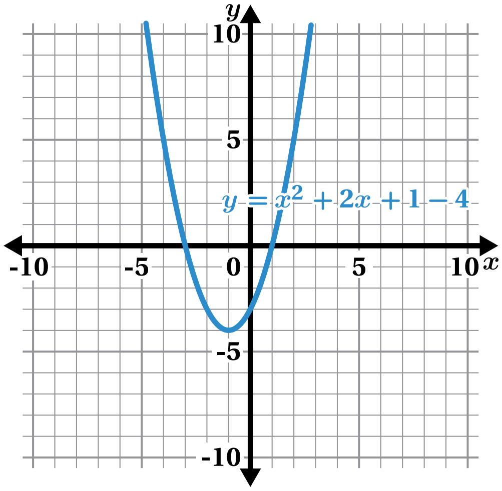
Name: .... Date: .... Period: ....

When In Doubt, Graph It Out (continued)

4. Saanvi also solved $x^2 + 2x + 1 = 4$ by graphing.

She graphed the equation $y = x^2 + 2x + 1 - 4$.

Show or describe where you see the solutions to $x^2 + 2x + 1 = 4$ in Saanvi's graph.



5. Use any strategy to solve $(x - 3)^2 = -1$

How many solutions does this equation have? Circle one.

No solutions One solution Two solutions

Record any solution(s) here:

$x =$ ....

$x =$ ....

If you circled *No solutions*, complete the statement:

The equation has no solutions because . . .

Activity
2

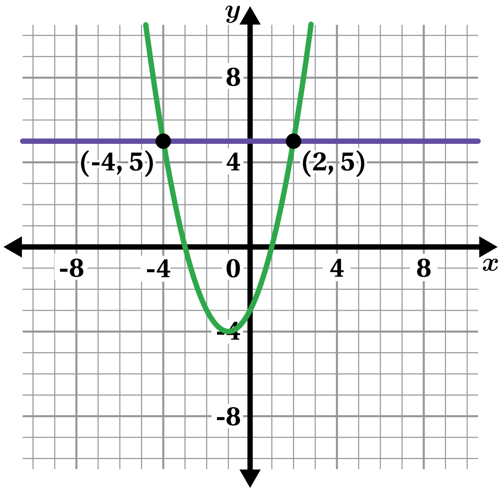
Name: ________ Date: ________ Period: ________

None, One, or Some

6. Malik and Saanvi each used graphing to solve $(x + 3)(x - 1) = 5$.

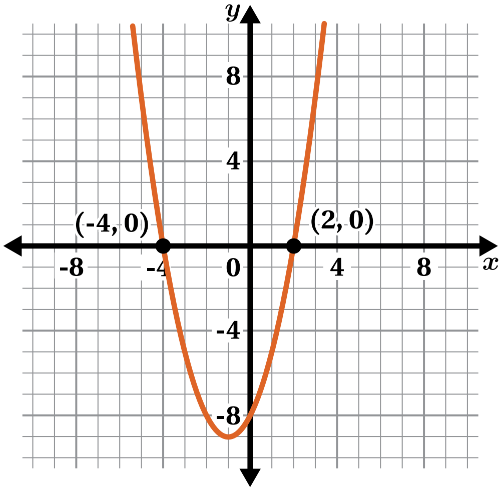
a Take a look at each student's strategy.

Malik



Graph $y = (x + 3)(x - 1)$ and $y = 5$.
Where do the graphs intersect?

Saanvi



Graph $y = (x + 3)(x - 1) - 5$.
What are the x-intercepts?

b **Discuss:**

- How are their strategies alike? How are they different?
- When might you use one strategy or the other?

Name: .... Date: .... Period: ....

None, One, or Some (continued)

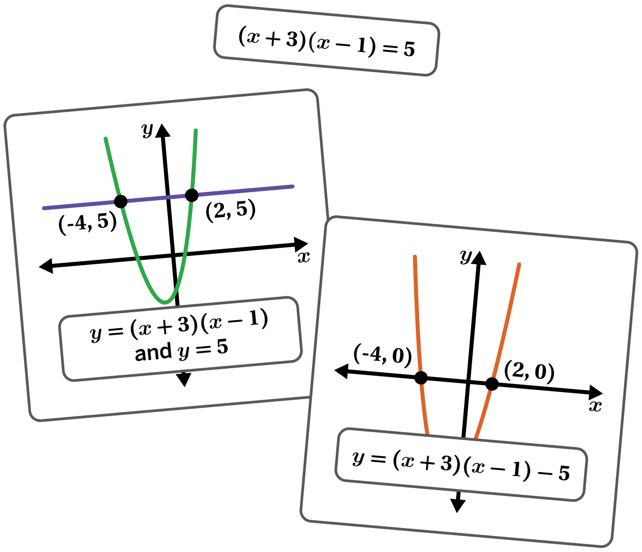
7. Use a graphing tool to solve as many challenges as you have time for.

- Circle how many solutions each equation has.
- Record any solutions.

	Equation	Number of Solutions			Solution(s)
a	$-4x^2 + 5 = 1$	No solutions	One solution	Two solutions	$x =$ ...... $x =$ ......
b	$(x - 4)(x - 2) = -5$	No solutions	One solution	Two solutions	$x =$ ...... $x =$ ......
c	$2x^2 - x - 4 = 2$	No solutions	One solution	Two solutions	$x =$ ...... $x =$ ......
d	$x(x - 2) = -1$	No solutions	One solution	Two solutions	$x =$ ...... $x =$ ......
e	$7 = x(x - 6)$	No solutions	One solution	Two solutions	$x =$ ...... $x =$ ......
f	$(x + 6)(x + 8) = -1$	No solutions	One solution	Two solutions	$x =$ ...... $x =$ ......

Synthesis

8. Describe a strategy for using a graphing tool to solve a quadratic equation.



Lesson Practice 7.11

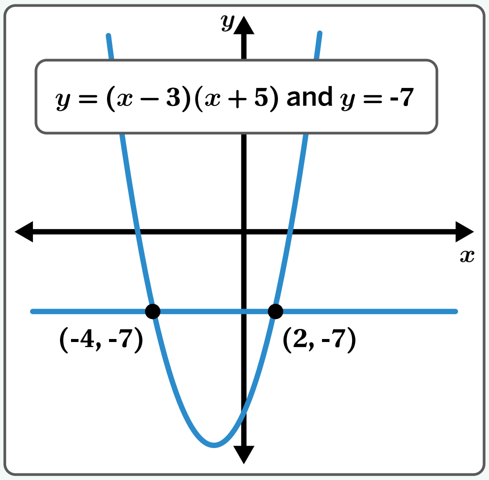
Lesson Summary

Graphs can be used to determine the solutions to a quadratic equation.

Here are two strategies using graphs to solve $(x-3)(x+5)=-7$.

Strategy 1

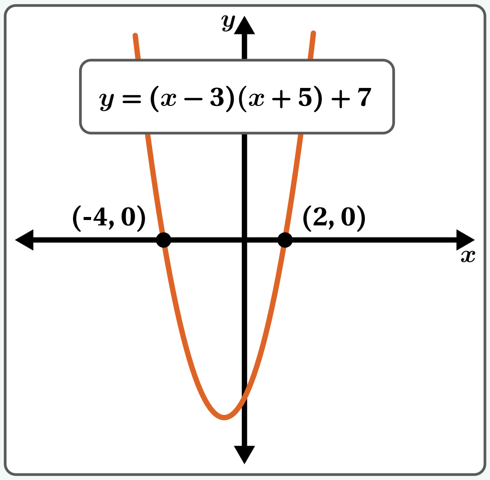
- Graph both sides of the equation as two separate graphs.
- Determine the x-coordinates where the graphs intersect.



Solutions: $x=-4$ and $x=2$

Strategy 2

- Rewrite the equation so that it equals 0.
- Graph the equation. The solutions will be at the x-intercepts.



Solutions: $x=-4$ and $x=2$

Lesson Practice
7.11

Name: .... Date: .... Period: ....

Problems 1–4: Circle how many solutions each equation has. Record any solutions.

1. $-2x^2 + 2 = -6$ No solutions One solution Two solutions $x =$ .... $x =$ ....

2. $x(x + 3) = 4$ No solutions One solution Two solutions $x =$ .... $x =$ ....

3. $0 = (x - 2)(x + 4) + 9$ No solutions One solution Two solutions $x =$ .... $x =$ ....

4. $2x(x + 1) = -1$ No solutions One solution Two solutions $x =$ .... $x =$ ....

Problems 5–6: Fill in the blank so the equation has:

5. One solution

$3x(x + 2) =$ ....

6. No solutions

$(x - 2)(x +$ ....$) = -4$

7. The graphs of $y = -x^2 + 25$, $y = 24$, $y = 21$, and $y = 16$ all intersect at integer values.

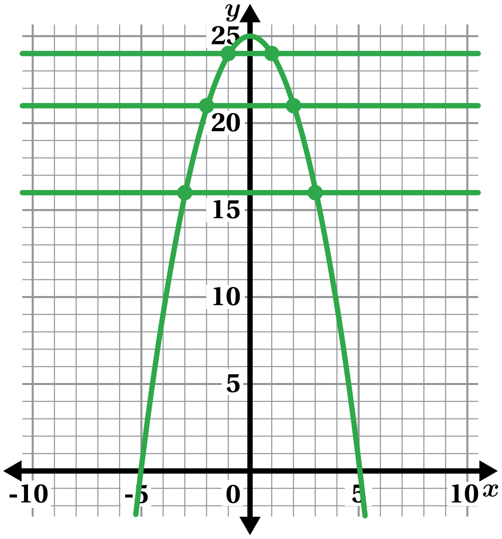
Write the equations of three more lines that follow this property.

$y =$ ....

$y =$ ....

$y =$ ....

What pattern do you notice?

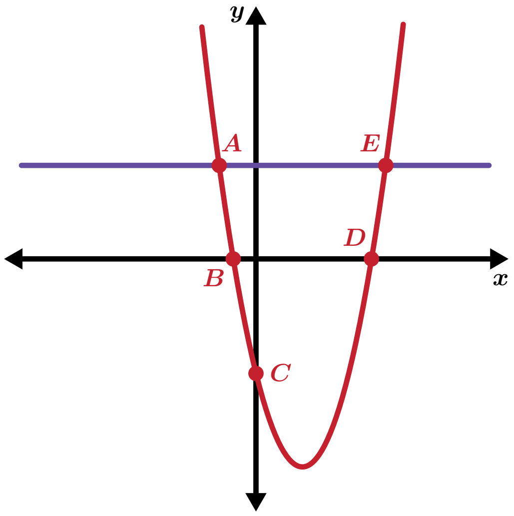


Lesson Practice
7.11

Name: ______ Date: ______ Period: ______

Test Practice

8. Here is a graph of $y = x^2 - 4x - 5$ and $y = 4$. Select *all* the points that are solutions to $x^2 - 4x - 5 = 4$.



☐ **A.** Point A

☐ **B.** Point B

☐ **C.** Point C

☐ **D.** Point D

☐ **E.** Point E

Spiral Review

9. Fill in the blanks to make each equation true.

$6^2 =$ ______ (______)$^2 = 49$ $2^2 =$ ______ (______)$^2 = 11$

10. Match each expression to an equivalent expression. You will have one left over.

A. $1 - x^2$	______ $(y + x)(y - x)$
B. $x^2 - y^2$	______ $(1 + x)(1 - x)$
C. $y^2 - x^2$	______ $(1 + x)(x - 1)$
D. $x^2 - 2xy + y^2$	______ $(x - y)(x - y)$
E. $x^2 - 1$	

Unit 7

Lesson

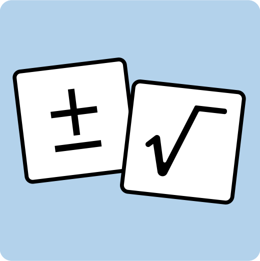
12

Name: .... Date: .... Period: ....

MA.912.AR.1.2, MA.912.AR.3.1, MA.912.NSO.1.4, MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1

Couldn’t Square Less

Let’s solve quadratic equations by taking the square root of each side.



Warm-Up

1. Solve this equation *as many ways* as you can. Show or explain your thinking.

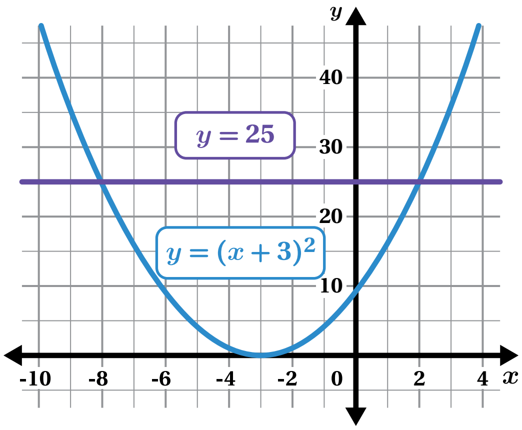
$$(x - 1)^2 = 36$$

Name: Date: Period:

Two Strategies

Binta and Charlie each started solving $(x + 3)^2 = 25$.

Binta



$x =$

Charlie

$(x + 3)^2 = 25$

$x + 3 = \pm\sqrt{25}$

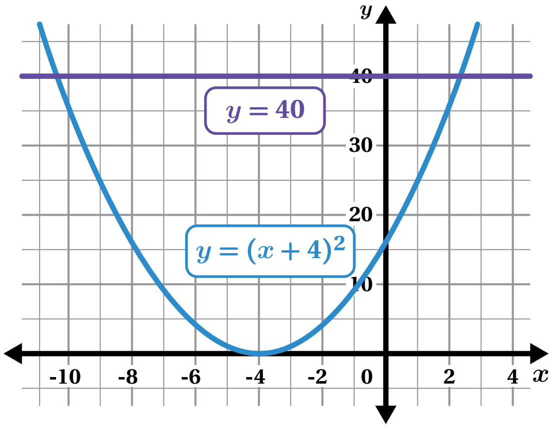
$x =$

2. **Discuss:** Where do you see two solutions in each student's strategy?

3. Finish each student's work.

4. Next, Binta and Charlie each started solving $(x + 4)^2 = 40$. Finish each student's work.

Binta



Charlie

$(x + 4)^2 = 40$

$x + 4 = \pm\sqrt{40}$

5. **Discuss:** What are some advantages to each strategy? What are some disadvantages?

Name: .... Date: .... Period: ....

Pair Solving

Charlie says: *I can solve any equation if I can get it to look like* $(x + \underline{\qquad})^2 = \underline{\qquad}$.

6. Rewrite each equation so that it looks like $(x + \underline{\qquad})^2 = \underline{\qquad}$.

a $2(x + 3)^2 = 20$

b $(x - 4)^2 + 12 = 3$

c $6 - (x - 5)^2 = -2$

Choose one person to be Partner A and the other to be Partner B. Solve each equation in your column, then check in with your partner. Your solutions should be the same.

	Partner A	Partner B
7.	$(x - 6)^2 - 3 = 0$	$4(x - 6)^2 = 12$
8.	$2(x - 1)^2 = 10$	$(x - 1)^2 + 3 = 8$
9.	$(x + 2)^2 + 3 = 3$	$3(x + 2)^2 = 0$

Activity
3

Name: ____ Date: ____ Period: ____

Exact and Approximate

Students in a class worked to solve the equation $(x - 14)^2 = 11$.

Duri says: *The solutions are* $x = 14 \pm \sqrt{11}$.

Yosef says: *The solutions are* $x = 17.317$ *and* $x = 10.683$.

10. How are Duri's and Yosef's solutions alike? How are they different?

Alike: **Different:**

11. Duri and Yosef each checked their first solution by substituting it into the original equation.

Duri	Yosef
$((14 + \sqrt{11}) - 14)^2 = 11$	$(17.317 - 14)^2 = 11$
$(\sqrt{11})^2 = 11$	$(3.317)^2 = 11$
$11 = 11$	$11.002489 = 11$

Discuss:

- What happens in each step of Duri's work?
- What happens in each step of Yosef's work?

12. Abena says: *Duri's solutions are exact. Yosef's solutions are approximate.*

What might Abena mean by that?

Activity
3

Name: .... Date: .... Period: ....

Exact and Approximate (continued)

13. Riku says: *I also got Duri's solutions and kept going*.

What could you say to help Riku realize his mistake?

Riku

$x = 14 + \sqrt{11}$ and $x = 14 - \sqrt{11}$

$x = \sqrt{25}$ and $x = \sqrt{3}$

$x = 5$ and $x = \sqrt{3}$

You're invited to explore more.

14. Prove that these two equations have the same solutions.

a $x^2 + 6x + 9 = 5$

b $4 - (3 + x)^2 = -1$

Synthesis

What are some advantages and disadvantages of using the $\sqrt{\ }$ and ± symbols in solutions instead of writing them as decimals?

Advantages:

Disadvantages:

Lesson Practice 7.12

Lesson Summary

One strategy for solving equations in the form $(x + \_\_\_)^2 = \_\_\_$ is to take the *square root* ($\sqrt{\ }$) of both sides of the equation. When you take the square root of the equation while solving, you can use the ± **(plus/minus symbol)** to represent that there are two solutions that come from this process. Here are two examples:

$$(x - 1)^2 = 36$$

$$\begin{aligned} (x - 1)^2 &= 36 \\ \sqrt{(x - 1)^2} &= \sqrt{36} \\ x - 1 &= \pm\sqrt{36} \\ x - 1 &= \pm 6 \\ x &= 1 \pm 6 \\ x &= 7 \text{ and } x = -5 \end{aligned}$$

$$2(x + 4)^2 = 12$$

$$\begin{aligned} 2(x + 4)^2 &= 12 \\ (x + 4)^2 &= 6 \\ \sqrt{(x + 4)^2} &= \sqrt{6} \\ x + 4 &= \pm\sqrt{6} \end{aligned}$$

Exact solutions: $x = -4 \pm \sqrt{6}$

Approximate solutions: $x \approx -1.55$ and
$x \approx -6.45$

If the solutions to a quadratic equation are *irrational numbers*, you can leave the square root in the expression to represent an exact solution or convert the square root to a decimal to express approximate solutions.

Lesson Practice

7.12

Name: .... Date: .... Period: ....

1. Is the equation $\sqrt{x^2} = x$ always true? Explain your thinking.

Problems 2–5: Determine the exact solutions to each equation. Write the solutions using ± notation.

2. $x^2 = 144$

3. $x^2 = 5$

4. $6x^2 = 42$

5. $4x^2 - 25 = 0$

Problems 6–8: Determine the exact solutions to each equation. Write the solutions using ± notation.

6. $(x + 4)^2 = 7$

7. $5(x - 4)^2 = 30$

8. $4 + (3 + x)^2 = 9$

9. Adrian says that the two solutions to $(x - 3)^2 = 2$ can be represented by $3 \pm \sqrt{2}$. Substitute each solution into the original equation and show that it makes the equation true.

Lesson Practice
7.12

Name: .... Date: .... Period: ....

Test Practice

10. Match each expression to an equivalent expression.

a -4 ± 1 .... -17 and 5

b $10 \pm \sqrt{4}$ .... $4 + \sqrt{2}$ and $4 - \sqrt{2}$

c -6 ± 11 .... 12 and 8

d $4 \pm \sqrt{10}$ .... -3 and -5

e $\sqrt{16} \pm \sqrt{2}$ .... $4 + \sqrt{10}$ and $4 - \sqrt{10}$

Spiral Review

11. Complete the table by writing each expression in the missing form.

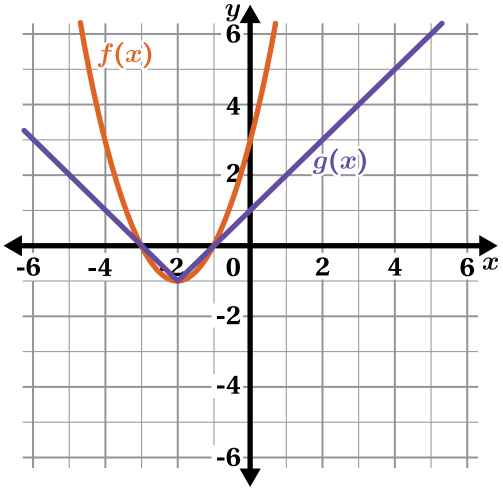
Factored Form	Standard Form
	$x^2 - 14x + 49$
$(4x + 3)(4x + 3)$	
	$4x^2 + 20x + 25$
$\left(x - \frac{1}{8}\right)\left(x - \frac{1}{8}\right)$	

12. Here are the graphs of a quadratic function and an absolute value function.

Write the equation for each function:

$f(x) =$

$g(x) =$



Unit 7

Lesson

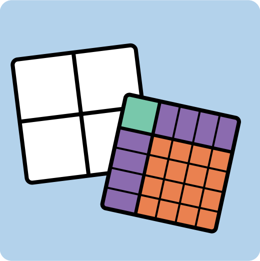
13

Name: .... Date: .... Period: ....

MA.912.AR.1.2, MA.912.AR.1.3, MA.912.AR.1.7, MTR.2.1, MTR.4.1, MTR.5.1

Square Dance

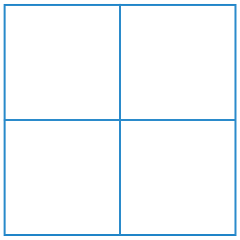
Let's build squares using tiles and algebra.



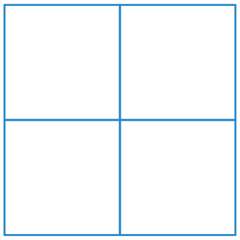
Warm-Up

1. **a** Write each expression in factored form. Use the diagrams if they help with your thinking.

$x^2 + 8x + 16$



$x^2 + 8x + 12$



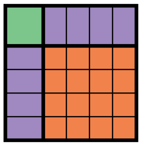
b **Discuss:** How are $x^2 + 8x + 16$ and $x^2 + 8x + 12$ alike? How are they different?

Name: .... Date: .... Period: ....

Perfect Squares

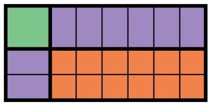
2. $(x + 4)^2$ and $x^2 + 8x + 16$ are **perfect squares**.

	x	4
x	x^2	$4x$
4	$4x$	16



$(x + 6)(x + 2)$ and $x^2 + 8x + 12$ are not perfect squares.

	x	6
x	x^2	$6x$
2	$2x$	12

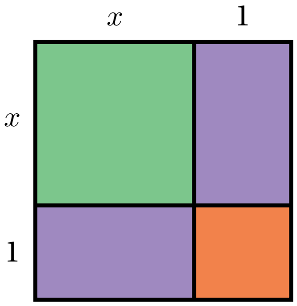


a **Discuss:** What do you think makes an expression a perfect square?

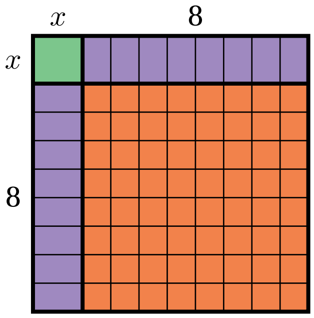
b Write a different expression that is a perfect square.

3. Here are more perfect square expressions written in factored and standard form.

Factored form: $(x + 1)^2$
Standard form: $x^2 + 2x + 1$



Factored form: $(x + 8)^2$
Standard form: $x^2 + 16x + 64$



a **Discuss:** What do you notice? What do you wonder?

b Is $x^2 + 12x + 144$ a perfect square? Circle one. Yes No Not enough information

Explain your thinking.

Name: ______ Date: ______ Period: ______

Perfect Squares (continued)

4. Sort the expressions based on whether they are perfect squares.

$x^2 + 10x + 100$ $x^2 - 24x - 144$ $x^2 + 4$ $x^2 + 5x + 6.25$

$x^2 - 24x + 144$ $x^2 + 10x + 25$ $(x - 4)^2$

Perfect Square	Not a Perfect Square

5. How did you decide whether the expression $x^2 + 5x + 6.25$ was a perfect square?

Name: ______________________ Date: __________ Period: __________

Completing Squares

6. This perfect square is written in factored and standard form. Some numbers are smudged.

Factored Form	**Standard Form**
$(x + \bullet)^2$	$x^2 + 6x + \bullet$

Is there enough information to determine the smudged numbers? Explain your thinking.

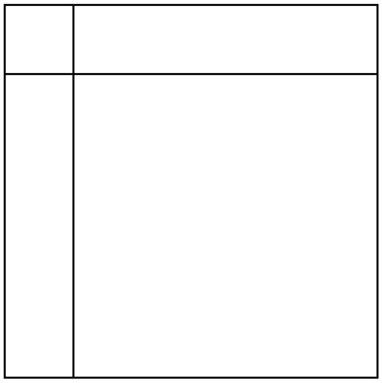
Use algebra tiles if they help with your thinking.

7. Here is a new expression with a smudge.

$x^2 + 22x + \bullet$

If the expression is a perfect square, what number is smudged?

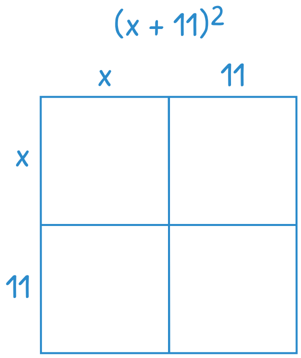
Use the diagram if it helps with your thinking.



8. Sadia wrote the expression $(x + 11)^2$ to help her find the smudged number.

Explain why this makes sense and how it can help her figure out the smudged number.

$x^2 + 22x + \bullet$



Name: .... Date: .... Period: ....

Completing Squares (continued)

9. Solve as many challenges as you have time for.

If each expression is a perfect square, what number is missing?

$x^2 - 10x +$ ....

$x^2 + 20x +$ ....

$x^2 +$ .... $x + 36$

$x^2 +$ .... $x + 100$

$x^2 - 50x +$ ....

$x^2 +$ .... $x + 144$

$x^2 + \frac{1}{5}x +$ ....

$x^2 + 7x +$ ....

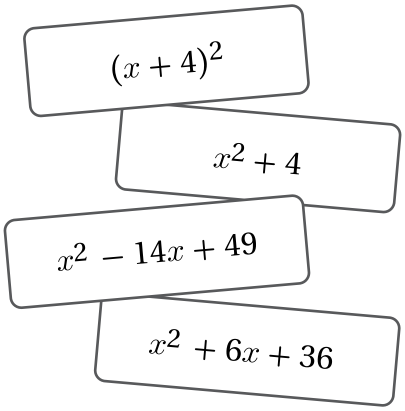
$x^2 +$ .... $x + 4$

You're invited to explore more.

10. Use the You're Invited to Explore More sheet to explore the graph of perfect square equations.

Synthesis

11. How can you determine whether an expression is a perfect square?



Lesson Practice 7.13

Lesson Summary

A quadratic expression is a **perfect square** if it can be represented as something multiplied by itself, like $(x + \underline{\qquad})^2$.

You can determine the missing constant value to add to make a perfect square by dividing the linear coefficient in half and then squaring that number.

Here are two examples of filling in the blanks to make each expression a perfect square.

$x^2 + 14x + \underline{\qquad}$

In perfect square expressions, the b-value, 14, is always double the constant in the factored form $(x + 7)^2$.

So the missing number must be the constant 7 squared, which is 49.

$x^2 + 14x + 49$

	x	7
x	x^2	$7x$
7	$7x$	49

$x^2 - \underline{\qquad} + 81$

The c-value, 81, is a perfect square so the factored form expression must be $(x - 9)^2$.

Rewrite the factored form to standard by using an area model or distributive property. The missing number must be $-18x$.

$x^2 - 18x + 81$

	x	-9
x	x^2	$-9x$
-9	$-9x$	81

Lesson Practice
7.13

Name: .... Date: .... Period: ....

1. Which expression is equivalent to $(x - 6)^2$?

A. $(x + 6)^2$ **B.** $x^2 - 12x - 36$ **C.** $x^2 - 36$ [✓ **D.**] $x^2 - 12x + 36$

2. Write each perfect square expression in factored form.

$x^2 + 6x + 9$ $x^2 - 16x + 64$ $x^2 - 12x + 36$ $x^2 + 5x + \frac{25}{4}$

3. LaShawn claims that $x^2 + bx + \frac{b^2}{4}$ is a perfect square. Do you agree? Explain your thinking.

4. Daniela says that if a perfect square expression is written in the form $x^2 + bx + c$, the value of c cannot be negative. Why is this true?

Problems 5–8: Fill in the blanks to complete each perfect square.

5. $x^2 + 24x$ ....

6. $x^2 - 2x$ ....

7. $x^2 -$ .... $+ 64$

8. $x^2 + \frac{2}{3}x$ ....

Lesson Practice

7.13

Name: ______________________ Date: __________ Period: __________

9. The expressions $(x - 4)^2$ and $(4 - x)^2$ are both perfect squares. Are they equivalent to one another? Explain your thinking.

Test Practice

10. Select *all* the expressions that are perfect squares.

- ☐ **A.** $x^2 + 10x + 25$
- ☐ **B.** $x^2 - 10x + 25$
- ☐ **C.** $x^2 + 4$
- ☐ **D.** $(x + 3.5)(3.5 + x)$
- ☐ **E.** $(x - 10)(10 - x)$
- ☐ **F.** $x^2 + \frac{1}{2}x + \frac{1}{16}$

Spiral Review

Problems 11–12: The equations $y = x^2 + 5x + 6$ and $y = (x + 2)(x + 3)$ are equivalent.

11. Which equation would you use to determine the x-intercepts? Explain your thinking.

12. Which equation would you use to determine the y-intercept? Explain your thinking.

13. Without using a graphing calculator, select *all* the equations with a positive y-intercept.

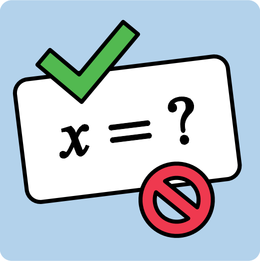
- ☐ **A.** $y = x^2 + 3x - 2$
- ☐ **B.** $y = (x + 1)(x + 5)$
- ☐ **C.** $y = x^2 - 10x$
- ☐ **D.** $y = (x - 3)^2$
- ☐ **E.** $y = -5x^2 + 3x - 12$

Name: ______ Date: ______ Period: ______

MA.912.AR.1.2, MA.912.AR.1.7, MA.912.AR.3.1, MA.912.NSO.1.4, MTR.5.1, MTR.6.1

Square Tactic

Let's develop a new strategy for solving quadratic equations called "completing the square."

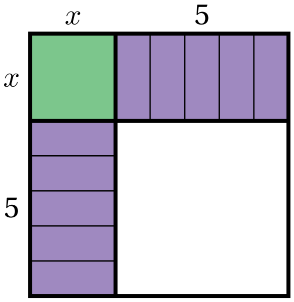


Warm-Up

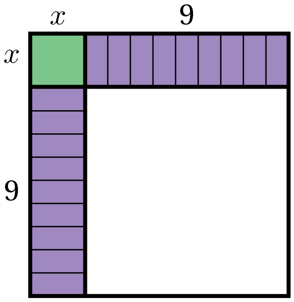
1. Here are seven expressions. The first four are represented with algebra tiles.

For each expression, how many unit tiles do you need to add to make it a perfect square?

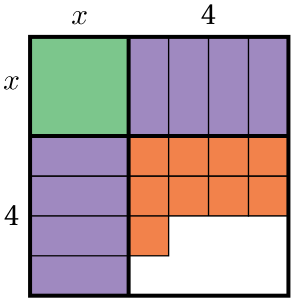
a $x^2 + 10x +$ ______



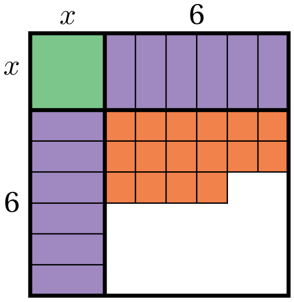
b $x^2 + 18x +$ ______



c $x^2 + 8x + 9 +$ ______



d $x^2 + 12x + 16 +$ ______



e $x^2 + 20x + 70 +$ ______

f $x^2 + 12x + 2 +$ ______

g $x^2 + 16x + 11 +$ ______

Activity
1

Name: .... Date: .... Period: ....

Ancient Equations

2. Here are three equations.

Equation A	**Equation B**	**Equation C**
$(x+3)^2 = 25$	$x^2 + 6x + 9 = 25$	$x^2 + 6x = 16$

Discuss:

- What do you notice about each equation's structure?
- Which equations can be solved by taking the square root? Explain how you know.

3. Solve the equation $x^2 - 8x + 16 = 9$.

Deven	Tay
$x^2 - 8x + 16 = 9$	$x^2 - 8x + 16 = 9$
$x^2 - 8x + 7 = 0$	$(x-4)^2 = 9$
$(x-7)(x-1) = 0$	$x - 4 = \pm\sqrt{9}$
$x = 7 \qquad x = 1$	$x = 4 \pm 3$

4. Let's look at how Deven and Tay each solved the previous equation.

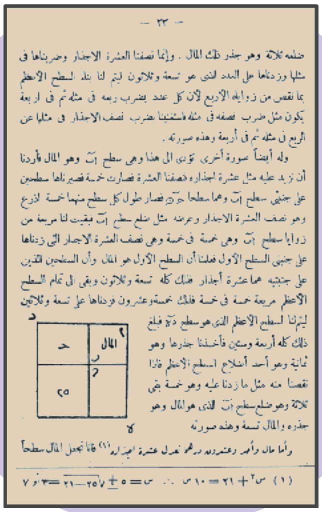
Discuss: What was each student's strategy? When might you use one strategy or the other?

Name: .... Date: .... Period: ....

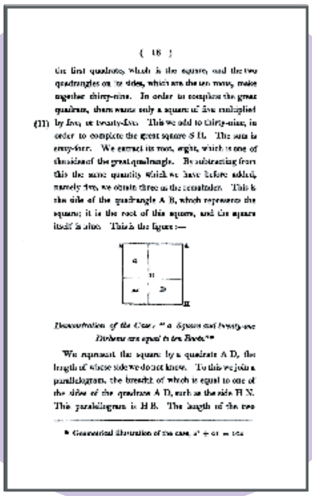
Ancient Equations (continued)

5. The word *algebra* comes from the title of the book *Hisab al-jabr w'al-Muqabala, "The Compendious Book on Calculation by Completion and Balancing."*

Original



English Translation



The book was written in 830 CE by Muhammad ibn Mūsā al-Khwarizmi, the mathematician who many scholars believe began the study of algebra.

The focus of the book is solving equations, including: $x^2 + 10x = 39$.

💬 **Discuss:** What are some different ways you could solve this equation?

6. Here is a translated version of the author's first step in solving the equation.

How does this help solve the equation?

al-Khwarizmi

$x^2 + 10x = 39$

$x^2 + 10x + 25 = 39 + 25$

Activity

2

Name: .... Date: .... Period: ....

Completing the Square

7. al-Khwarizmi's process is called **completing the square**.

Solve $x^2 + 14x = 31$ by completing the square.

8. Solve the equation $x^2 + 6x + 4 = -3$.

Name: .... Date: .... Period: ....

Completing the Square (continued)

9. Roberto made a mistake while solving the equation $x^2 - 12x + 6 = 14$.

What did Roberto do well? What should he fix?

Roberto

$$x^2 - 12x + 6 = 14$$

$$x^2 - 12x + 36 = 14 + 36$$

$$(x - 6)^2 = 50$$

$$x - 6 = \pm\sqrt{50}$$

$$x = 6 \pm \sqrt{50}$$

You're invited to explore more.

10. a **Discuss:** How many solutions does the equation $x2 + 10x$ = -60 have?

b Can you write another equation of the form $x^2 + 10x =$ .... that has . . .

No solutions: $x^2 + 10x =$ ....

One solutions: $x^2 + 10x =$ ....

Two solutions: $x^2 + 10x =$ ....

Synthesis

11. Here are the solving strategies you've seen in this unit: factoring, graphing, and completing the square.

Discuss:

- Which strategy would you use for each equation?
- What are the advantages and disadvantages of completing the square?

$$x^2 + 8x + 2 = 0$$

$$x^2 + 10x = 39$$

$$x^2 + 5x + 6 = 2$$

Lesson Practice 7.14

Lesson Summary

You can solve quadratic equations by graphing, factoring, or **completing the square**, which is the process of rewriting a quadratic expression or equation to include a perfect square. You can analyze the structure of the equation to help you decide which strategy to use.

Here is an example: $x^2 + 10x = 2$. Solving by graphing will not produce exact solutions and factoring is not possible for this equation, so we can solve by completing the square:

Work	**Explanation**
$x^2 + 10x + 25 = 2 + 25$	$x^2 + 10x + 25$ is a perfect square, so add the constant value 25 to both sides of the equation.
$(x + 5)^2 = 27$	Rewrite the perfect square $x^2 + 10x + 25$ in factored form.
$x + 5 = \pm\sqrt{27}$	Take the square root and include both possibilities by writing ±.
$x = -5 \pm 3\sqrt{3}$	Solve for x.

Lesson Practice
7.14

Name: .... Date: .... Period: ....

1. Select *all* the expressions that are perfect squares.

☐ **A.** $(x + 5)(5 + x)$

☐ **B.** $(x - 3)^2$

☐ **C.** $x^2 - 3^2$

☐ **D.** $x^2 + 8x + 64$

☐ **E.** $x^2 + 10x + 25$

2. Add the number that would make the expression a perfect square. Then write an equivalent expression in factored form.

$x^2 - 6x + \boxed{}$ ....

$x^2 + 2x + \boxed{}$ ....

$x^2 - 14x + \boxed{}$ ....

3. Match each equation to an equivalent equation.

a $x^2 - 12x = 6$ .... $(x - 6)^2 = 30$

b $x^2 - 12x + 6 = 0$ .... $(x - 3)^2 = 42$

c $x^2 - 6x = 6$ .... $(x - 6)^2 = 42$

d $x^2 - 6x = 33$ .... $(x - 3)^2 = 15$

4. Alexis solved the equation $x^2 + 12x = 13$ by completing the square, but some parts are blank. Fill in the blanks.

$$x^2 + 12x = 13$$

$$\boxed{}$$

$$(x + 6)^2 = 49$$

$$x + 6 = \pm 7$$

$$x = \boxed{} \text{ and } x = \boxed{}$$

Lesson Practice

7.14

Name: ________________ Date: ________ Period: ________

Problems 5–7: Solve each equation by completing the square.

5. $x^2 - 2x = 8$

6. $7 = x^2 + 4x - 1$

7. $x^2 - 18x + 60 = -11$

Test Practice

8. Write a quadratic equation of the form $x^2 + bx + c = 0$ with solutions that are $x = 5 - \sqrt{2}$ and $x = 5 + \sqrt{2}$.

Spiral Review

9. For each equation, determine the number of solutions.

Equation	Number of Solutions
$x^2 + 144 = 0$	
$x^2 - 144 = 0$	
$(x - 7)^2 = 0$	

10. The graph of $y = (x - 1)^2 + 4$ is the same as the graph of $y = x^2$, but:

- A. It is shifted 1 unit to the right and 4 units up.
- B. It is shifted 1 unit to the left and 4 units up.
- C. It is shifted 1 unit to the right and 4 units down.
- D. It is shifted 1 unit to the left and 4 units down.

Unit 7

Lesson

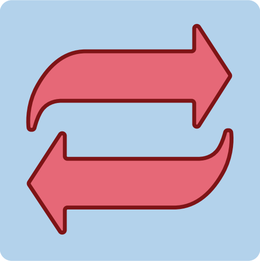
15

Name: ________ Date: ________ Period: ________

MA.912.AR.3.6, MA.912.AR.1.2, MA.912.AR.1.3, MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1

Back and Forth

Let's complete the square in order to reveal the vertex of a quadratic function.



Warm-Up

You will use a set of cards.

1. Try to match each function with its key features.

2. Select a feature you were able to match.

   Discuss: How did you match this card with its function?

3. Select a feature you were *not* able to match.

   Discuss: What would help you match this card?

Name: .... Date: .... Period: ....

Form to Form

Ebony and Jamar want to determine the vertex of the graph of $h(x) = x^2 + 8x + 21$. They use different strategies to rewrite the standard-form expression in *vertex form*.

Ebony	Jamar
$x^2 + 8x + 21$	$x^2 + 8x + 21$
$(x^2 + 8x + 16) - 16 + 21$	$(x^2 + 8x + 16) + 5$
$(x + 4)^2 - 16 + 21$	$(x + 4)^2 + 5$
$(x + 4)^2 + 5$	

4. **Discuss:**

- Why do both students write $+ 16$ in their work?
- Why does Ebony subtract 16? Why doesn't Jamar subtract 16?

5. Use either strategy to rewrite $x^2 + 10x - 3$ in vertex form. Then determine the vertex.

6. Lucia says that $x^2 - 12x + 40$ is equivalent to $(x - 6)^2 + 4$.
Is Lucia correct? Explain or show your thinking.

Name: .... Date: .... Period: ....

Form to Form (continued)

Here is how Omar determined the vertex of the graph of $j(x) = 2x^2 + 20x + 47$.

7. Why did Omar subtract 50?

$$2x^2 + 20x + 47$$

$$2(x^2 + 10x) + 47$$

$$2(x^2 + 10x + 25) - 50 + 47$$

$$2(x + 5)^2 - 3$$

8. Omar began the process of rewriting $3x^2 - 12x + 18$ in vertex form.

$$3x^2 - 12x + 18$$

$$3(x^2 - 4x) + 18$$

$$3(x^2 - 4x + 4) - \underline{\quad\quad} + 18$$

a What number should Omar subtract?

b Determine the vertex of the graph of $k(x) = 3x^2 - 12x + 18$.

c Is the vertex the maximum or minimum? Explain your thinking.

Activity 2

Name: .... Date: .... Period: ....

Card Sort

Look at the set of cards that you used during the Warm-Up.

9. Continue to match each function with *two* key features of its graph. Record your thinking in the table.

$a(x) = 2x^2 + 8x + 6$	y-intercept:	x-intercepts:	vertex:
$b(x) = (2x + 6)(x - 3)$	y-intercept:	x-intercepts:	vertex:
$c(x) = 3(x - 2)^2 + 6$	y-intercept:	x-intercepts:	vertex:
$d(x) = x^2 - 5x - 6$	y-intercept:	x-intercepts:	vertex:
$f(x) = x^2 + 4x + 10$	y-intercept:	x-intercepts:	vertex:
$g(x) = (x + 2)(x + 6)$	y-intercept:	x-intercepts:	vertex:

10. 💬 **Discuss:** What strategies did you use to match the cards this time?

You're invited to explore more.

11. In Activity 2, each function matches with two key features. For each function, determine the missing key feature: its x-intercept, y-intercept, or vertex.

Synthesis

12. Describe a strategy for rewriting a standard-form expression in vertex form. Use this example if it helps with your thinking:

Standard form: $x^2 - 4x + 9$

Vertex form: $(x - 2)^2 + 5$

Lesson Practice 7.15

Lesson Summary

Different forms of a quadratic function reveal different key features of its graph.

- Factored form reveals the x-intercepts or zeros.
- Standard form reveals the y-intercept.
- *Vertex form* reveals the maximum or minimum value of the quadratic function.

You can complete the square to rewrite a standard-form quadratic function into vertex form.

Here are two different strategies to rewrite $f(x) = x^2 - 6x + 17$ in vertex form.

Strategy 1

$$f(x) = x^2 - 6x + 17$$
$$(x^2 - 6x + 9) - 9 + 17$$
$$(x - 3)^2 - 9 + 17$$
$$(x - 3)^2 + 8$$

Strategy 2

$$f(x) = x^2 - 6x + 17$$
$$(x^2 - 6x + 9) + 8$$
$$(x - 3)^2 + 8$$

The vertex of $f(x)$ is $(3, 8)$ and the graph of $f(x)$ is a concave up parabola, so 8 will be the minimum value.

Name: .... Date: .... Period: ....

Problems 1–3: Each of these equations represents the same function in an equivalent form. Complete the table with which feature of the function's graph (vertex, y-intercept, x-intercepts) can easily be determined and the coordinates.

	Function form	Feature of function's graph	Coordinates
1.	$f(x) = x^2 - 20x + 21$		
2.	$f(x) = (x - 7)(x - 3)$		
3.	$f(x) = (x - 5)^2 - 4$		

Problems 4–5: Gabriela made a mistake while rewriting the expression $x^2 - 12x + 10$ in vertex form.

$x^2 - 12x + 10$

$x^2 - 12x + 36 + 10$

$(x - 6)^2 + 10$

4. Describe Gabriela's mistake.

5. Rewrite $x^2 - 12x + 10$ in vertex form. Show your work.

Problems 6–9: Complete the table by writing each expression in the missing form.

	Vertex Form	Standard Form	Factored Form
6.	$(x + 3)^2 - 16$		$(x + 7)(x - 1)$
7.		$x^2 - 6x$	$x(x - 6)$
8.		$x^2 + 14x + 40$	
9.		$2x^2 - 16x - 18$	

Lesson Practice

7.15

Name: .... Date: .... Period: ....

10. Using the digits 0 to 9, without repeating, fill in the boxes to create three equations that produce the exact same parabola.

$$y = (x + \square)^2 - \square$$

$$y = (x + \square)(x + \square)$$

$$y = x^2 + \square x + \square$$

Test Practice

11. Select *all* the expressions that are equivalent to $x^2 - 16x$.

- ☐ **A.** $x(x - 16)$
- ☐ **B.** $(x - 4)(x + 4)$
- ☐ **C.** $x^2 - 16x + 64 - 64$
- ☐ **D.** $(x - 4)^2$
- ☐ **E.** $(x - 8)^2 - 64$

Spiral Review

12. Fill in the blanks to complete each perfect square.

$$x^2 - 18x + \square$$

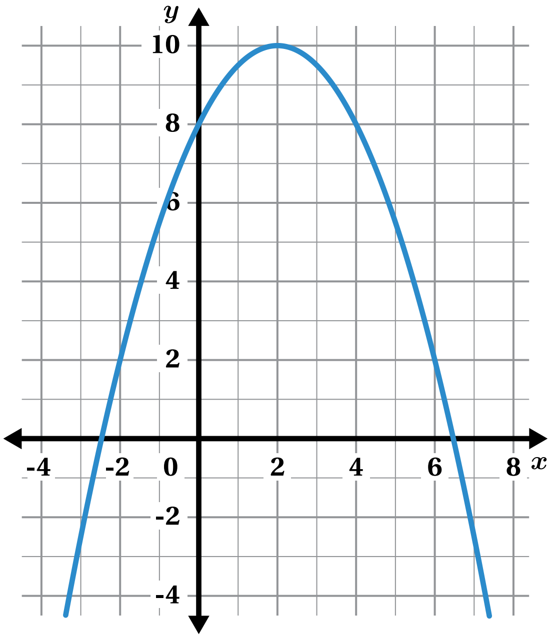
$$x^2 + \frac{4}{3}x + \square$$

13. Determine the following values of the quadratic function $g(x) = -\frac{1}{2}x^2 + 2x + 8$.

$g(4) =$ ....

$g(-2) =$ ....

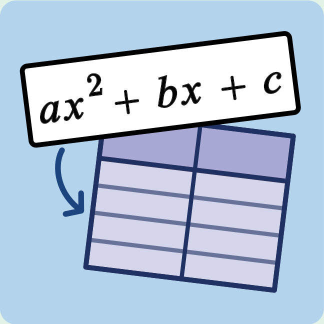
$g(x) = -2.5$; $x =$ ....



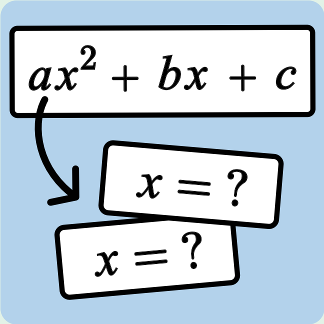
Sub-unit

3

The Quadratic Formula and More



Lesson 16
Formula Foundations



Lesson 17
Formula Fluency

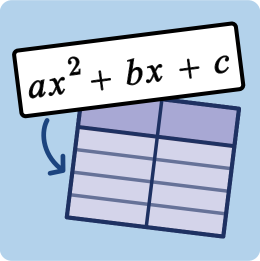
Lesson 18
Stomp Rockets in Space

Unit 7
Lesson
16

Name: Date: Period:

MA.912.AR.1.2, MA.912.NSO.1.4, MA.912.AR.3.1, MTR.3.1, MTR.4.1, MTR.5.1

Formula Foundations



Let's explore how the quadratic formula can be derived from the process of completing the square.

Warm-Up

1. Alma wants to solve $5x^2 + 9x + 3 = 0$ by completing the square.

Here is her first step.

Alma

$$5x^2 + 9x + 3 = 0$$

$$x^2 + \frac{9x}{5} + \frac{3}{5} = 0$$

Discuss:

- What did Alma do?
- Why do you think this is her first step?
- What would you do next?

Name: .... Date: .... Period: ....

Completing Any Square

2. Here are the rest of the steps Alma took to solve $5x^2 + 9x + 3 = 0$. Describe what she does in each step.

Steps	Description
$5x^2 + 9x + 3 = 0$	Original equation
$x^2 + \frac{9}{5}x + \frac{3}{5} = 0$	Divide by 5
$x^2 + \frac{9}{5}x = -\frac{3}{5}$	
$x^2 + \frac{9}{5}x + \left(\frac{9}{2 \cdot 5}\right)^2 = -\frac{3}{5} + \left(\frac{9}{2 \cdot 5}\right)^2$	
$\left(x + \frac{9}{10}\right)^2 = -\frac{3}{5} + \left(\frac{9}{10}\right)^2$	
$x + \frac{9}{10} = \pm\sqrt{-\frac{3}{5} + \left(\frac{9}{10}\right)^2}$	
$x = -\frac{9}{10} \pm \sqrt{-\frac{3}{5} + \left(\frac{9}{10}\right)^2}$	
$x = \frac{-9 \pm \sqrt{9^2 - 4 \cdot 5 \cdot 3}}{2 \cdot 5}$	Rewrite

3. Let's look at how we can use Alma's steps to solve other quadratic equations.

Discuss: What do you notice? What do you wonder?

Name: .... Date: .... Period: ....

Completing Any Square (continued)

4. Felipe notices that you can write the solutions to an equation without completing the square.

How would the solutions change if the original equation were $3x^2 + 8x - 15 = 0$?

Use the table if it helps with your thinking.

Steps	Description
$10x^2 + 7x + 1 = 0$	Original equation
$x^2 + \frac{7}{10}x + \frac{1}{10} = 0$	Divide by 10
$x^2 + \frac{7}{10}x = -\frac{1}{10}$	Subtract $\frac{1}{10}$
$x^2 + \frac{7}{10}x + \left(\frac{7}{2 \cdot 10}\right)^2 = -\frac{1}{10} + \left(\frac{7}{2 \cdot 10}\right)^2$	Complete the square
$\left(x + \frac{7}{20}\right)^2 = -\frac{1}{10} + \left(\frac{7}{20}\right)^2$	Rewrite the left side as a perfect square
$x + \frac{7}{20} = \pm\sqrt{-\frac{1}{10} + \left(\frac{7}{20}\right)^2}$	Take the square root
$x = -\frac{7}{20} \pm \sqrt{-\frac{1}{10} + \left(\frac{7}{20}\right)^2}$	Subtract $\frac{7}{20}$
$x = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 10 \cdot 1}}{2 \cdot 10}$	Rewrite

Activity
2

Name: .... Date: .... Period: ....

Formula-izing

5. We just discovered a way to write the solutions for any quadratic equation without completing the square!

Use the variables a, b, and c to represent the solutions to $ax^2 + bx + c = 0$.

Equation:	Equation:
$7x^2 - 9x + 5 = 0$	$\boxed{a}\, x^2 + \boxed{b}\, x + \boxed{c} = 0$
Solutions:	Solutions:
$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \cdot 7 \cdot 5}}{2 \cdot 7}$	$x = \frac{-\square \pm \sqrt{\square^2 - 4 \cdot \square \cdot \square}}{2 \cdot \square}$

6.

For any quadratic equation: $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This equation is known as the **quadratic formula**.

Discuss:

- What do the a, b, and c in the formula represent?
- Why is there a ± symbol in the formula?
- What new things could this formula help you do?

Step	**Description**
$ax^2 + bx + c = 0$	Original equation
$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$	Divide by a
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	Subtract $\frac{c}{a}$
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Complete the square
$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Rewrite the left side as a perfect square
$\left(x + \frac{b}{2a}\right) = \pm\sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$	Take the square root
$x = -\frac{b}{2a} \pm \sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$	Subtract $\frac{b}{2a}$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Rewrite

Activity
3

Name: .... Date: .... Period: ....

Solution Search

7. You will use a graphing calculator to explore solutions to quadratic equations.

a Here is an equation that has *two integer solutions*. Find two more equations with two integer solutions.

Equation	$1x^2 - 5x + 6 = 0$		
Solutions	$x = \frac{5 \pm \sqrt{1}}{2}$	$x =$	$x =$

b Here is an equation that has *one solution*. Find two more equations with one solution.

Equation	$1x^2 + 4x + 4 = 0$		
Solutions	$x = \frac{-4 \pm \sqrt{0}}{2}$	$x =$	$x =$

c Here is an equation that has *no solution*. Find two more equations with no solution.

Equation	$7x^2 + 2x + 5 = 0$		
Solutions	$x = \frac{-2 \pm \sqrt{-136}}{14}$	$x =$	$x =$

d Examine the equations and solutions you found.

Discuss: What patterns do you notice?

Synthesis

8. A classmate who is absent today asks for your help.

What would you say to help them understand where the quadratic formula came from?

Step	Description
$ax^2 + bx + c = 0$	Original equation
$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$	Divide by a
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	Subtract $\frac{c}{a}$
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Complete the square
$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Rewrite the left side as a perfect square
$\left(x + \frac{b}{2a}\right) = \pm\sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$	Take the square root
$x = -\frac{b}{2a} \pm \sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$	Subtract $\frac{b}{2a}$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Rewrite

Lesson Practice 7.16

Lesson Summary

You can use the **quadratic formula** to solve quadratic equations written in standard form. The quadratic formula states that the solutions to any quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

The quadratic formula comes from completing the square.

Here is an example of completing the square and using the quadratic formula to solve $x^2 + 8x + 13 = 0$.

Completing the Square

Solve $x^2 + 8x + 13 = 0$

$$x^2 + 8x = -13$$
$$x^2 + 8x + 16 = -13 + 16$$
$$x^2 + 8x + 16 = 3$$
$$(x + 4)^2 = 3$$
$$x + 4 = \pm\sqrt{3}$$
$$x = -4 \pm \sqrt{3}$$
$$x = -4 + \sqrt{3} \text{ or } x = -4 - \sqrt{3}$$

Quadratic Formula

Solve $x^2 + 8x + 13 = 0$

$a = 1, b = 8, c = 13$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(13)}}{2(1)}$$
$$x = \frac{-8 \pm \sqrt{12}}{2}$$
$$x = -4 \pm \sqrt{3}$$
$$x = -4 + \sqrt{3} \text{ or } x = -4 - \sqrt{3}$$

Lesson Practice
7.16

Name: .... Date: .... Period: ....

1. The quadratic formula is derived by solving $ax^2 + bx + c = 0$ by . . .

A. Factoring

A. Completing the square

B. Graphing

C. Elimination

Problems 2–4: The quadratic equation $x^2 + 7x + 10 = 0$ is in the form $ax^2 + bx + c = 0$.

2. What are the values of a, b, and c?

$a =$

$b =$

$c =$

3. Substitute the values of a, b, and c into the quadratic formula. (You do not need to perform any operations.)

4. Explain how the expression you wrote is related to solving $x^2 + 7x + 10 = 0$ by completing the square.

5. Part of the quadratic formula is $\pm\sqrt{b^2 - 4ac}$. What value must this equal to have exactly one solution? Explain your thinking.

6. Kiri is using a quadratic equation to figure out the quadratic formula. Here are her first few steps. Why did Kiri add $\left(\frac{b}{2a}\right)^2$ in the bottom row?

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

Lesson Practice
7.16

Name: .... Date: .... Period: ....

Test Practice

7. The quadratic formula can be used to find the solutions to any quadratic equation in the form $ax^2 + bx + c = 0$.

Which equation represents the solutions to

$2x^2 - x + 13 = 0$?

[✓ **A.**] $x = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(13)}}{2(2)}$

B. $x = \frac{-2 \pm \sqrt{(2)^2 - 4(13)(-1)}}{2(13)}$

C. $x = \frac{-1 \pm \sqrt{(-1)^2 - 4(2)(13)}}{2(-1)}$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Spiral Review

Problems 8–10: Rewrite each equation in the form $(x + \_\_\_)^2 = \_\_\_$. An example has been done for you.

Example:

$x^2 + 10x = 4$

$(x + 5)^2 = 29$

8. $x^2 + 6x = -2$

9. $x^2 + 12x + 3 = -7$

10. $x^2 - 32 = -20x$

Problems 11–13: Solve each equation for x.

11. $3x^2 - 5 = 0$

12. $-7x^2 + 2 = 0$

13. $ax^2 + c = 0$

Unit 7

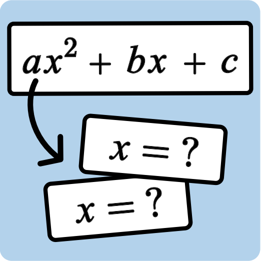
Lesson

17

Name: .... Date: .... Period: ....

MA.912.AR.3.1, MA.912.NSO.1.4, MTR.3.1, MTR.4.1, MTR.5.1

Formula Fluency



Let's use the quadratic formula to solve quadratic equations.

Warm-Up

1. The quadratic formula can be used to find the solutions to any quadratic equation in the form $ax^2 + bx + c = 0$.

Determine the a-, b-, and c-values of the following equations.

a $3x^2 - 8x + 15 = 0$

$a =$ .... $b =$ .... $c =$ ....

b $x^2 + 4 + 3x = 0$

$a =$ .... $b =$ .... $c =$ ....

c $5x^2 - 20 = 0$

$a =$ .... $b =$ .... $c =$ ....

d $-x^2 + 2x = -12$

$a =$ .... $b =$ .... $c =$ ....

Name: .... Date: .... Period: ....

Form Over Function

2. Here are four quadratic equations and their solutions.

Use the quadratic formula to show that the solutions are correct.

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a $x^2 - 8x + 15 = 0$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)}$$

Solutions: $x = 5$ and $x = 3$

b $x^2 + 10x + 18 = 0$

Solutions: $x = -5 \pm \sqrt{7}$

c $9x^2 - 6x = -1$

Solution: $x = \frac{1}{3}$

d $2x^2 + 6x + 5 = 0$

No solutions

3. 💬 **Discuss**: Do you think that the quadratic formula is the best strategy for solving each of these equations? Explain your thinking.

Activity
2

Name: Date: Period:

Error Analysis

You will use a sheet for this activity with the same equations from the previous activity.

There is an error in each attempt to solve the equation.

4. 💬 **Discuss:**

- What is the error in each attempt?
- How would you correct the error?
- Why might someone make this error?

5. Solve the following equation using the quadratic formula, but include an error that you think would be common.

$$3x^2 - 6x - 1 = 0$$

6. Swap equations with a classmate. Identify and describe the error in each other's work.

Name: .... Date: .... Period: ....

Error Analysis (continued)

7. **Reflect:** What kinds of errors do you think you are most likely to make when using the quadratic formula?

8. Write two pieces of advice that will help your future self correctly use the quadratic formula. Include examples if they help with your thinking.

Synthesis

9. What are some advantages of using the quadratic formula to solve quadratic equations?

What are some disadvantages?

Use the examples if they help with your thinking.

$x^2 - 6x + 8 = 0$

$x^2 + 4x - 1 = 0$

$2x^2 + 7x - 10 = 0$

Lesson Practice 7.17

Lesson Summary

You can solve *any* quadratic equation written in standard form, $ax^2 + bx + c = 0$, using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here is an example of using the quadratic formula to solve the equation $2x^2 - 9x + 6 = 0$.

Work	**Explanation**
$a = 2,\ b = -9,\ c = 6$	Identify the a-, b-, and c-values from the standard form quadratic equation.
$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(6)}}{2(2)}$	Substitute the a-, b-, and c-values into the quadratic formula.
$x = \frac{9 \pm \sqrt{33}}{4}$	Use order of operations to simplify the expression.
$x = \frac{9 + \sqrt{33}}{4}$ and $x = \frac{9 - \sqrt{33}}{4}$	Write the solutions.

Lesson Practice

7.17

Name: .... Date: .... Period: ....

Problems 1–2: Each equation is in the form $ax^2 + bx + c = 0$. Determine the values of a, b, and c.

1. $x^2 - 5x + 9 = 0$

$a =$ ..... $b =$ ..... $c =$ .....

2. $-x^2 + 8 = 0$

$a =$ ..... $b =$ ..... $c =$ .....

Problems 3–5: Solve each equation using any method.

3. $2x^2 - 7x = 15$

4. $2x^2 + 5x - 1 = 0$

5. $4x^2 = 5$

6. Santiago determined that the solutions to $3x^2 - 6x - 9 = 0$ are $x = 3$ and $x = -1$. Is he correct? Explain your thinking.

7. Write a quadratic equation that would be simpler to solve *without* using the quadratic equation.

Lesson Practice

7.17

Name: ………………………… Date: ………… Period: …………

Test Practice

8. Choose the a quadratic function with the following x-intercepts.

$$\left(\frac{2 \pm \sqrt{40}}{6}, 0\right)$$

● **A.** $2x^2 + 3x - 2$

○ **B.** $3x^2 - 2x - 3$

○ **C.** $2x^2 + 2x + 3$

○ **D.** $3x^2 - 3x - 2$

Spiral Review

Problems 9–10: Here is the function $f(x) = (x + 1)(x + 5)$.

9. What are the coordinates of the x-intercepts?

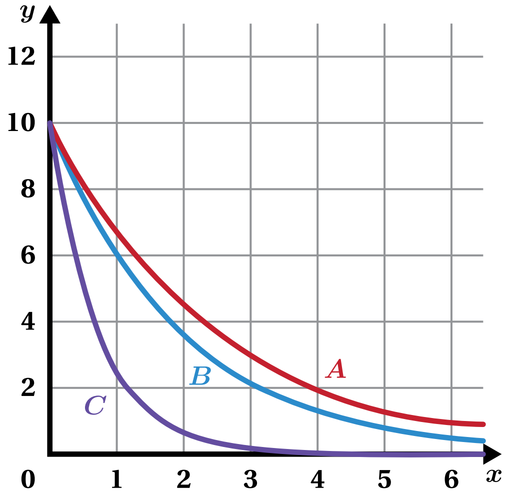
10. What are the coordinates of the vertex? Explain your thinking.

11. Here are the graphs of three equations. Match each graph with its equation.

………… $y = 10\left(\frac{2}{3}\right)^x$

………… $y = 10\left(\frac{1}{4}\right)^x$

………… $y = 10\left(\frac{3}{5}\right)^x$



Unit 7

Lesson

18

Name: ________ Date: ________ Period: ________

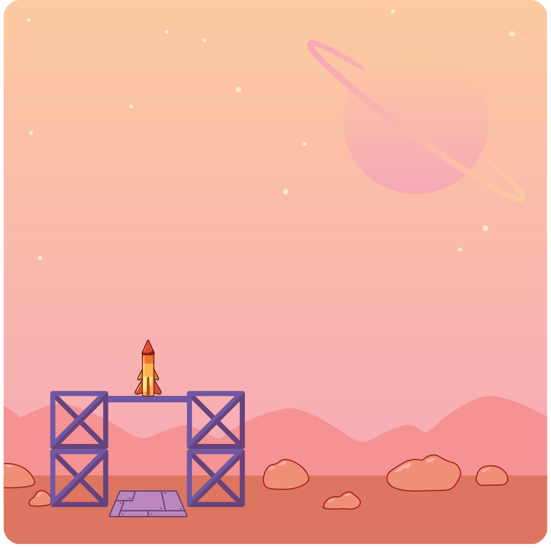
MA.912.AR.3.1, MA.912.AR.3.6, MA.912.AR.3.8, MA.912.NSO.1.4, MTR.3.1, MTR.6.1, MTR.7.1

Stomp Rockets in Space

Let's solve quadratic equations and explain what the solutions mean for a situation.

Warm-Up

1. Here is a stomp rocket on another planet.



The function $h(t) = \text{-}3t^2 + 20t + 4$ represents the height, in meters, of the stomp rocket t seconds after it has been launched.

Write a question about the stomp rocket that $h(t)$ could help you answer.

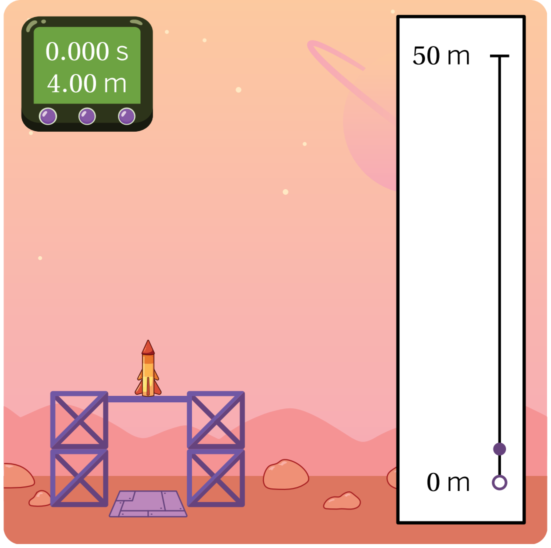
Activity
1

Name: ______________________ Date: __________ Period: __________

Rocket Time

2. The function $h(t) = -3t^2 + 20t + 4$ represents the height, in meters, of the stomp rocket t seconds after it has been launched.

When will the rocket touch the ground? Round to three decimal places if necessary.

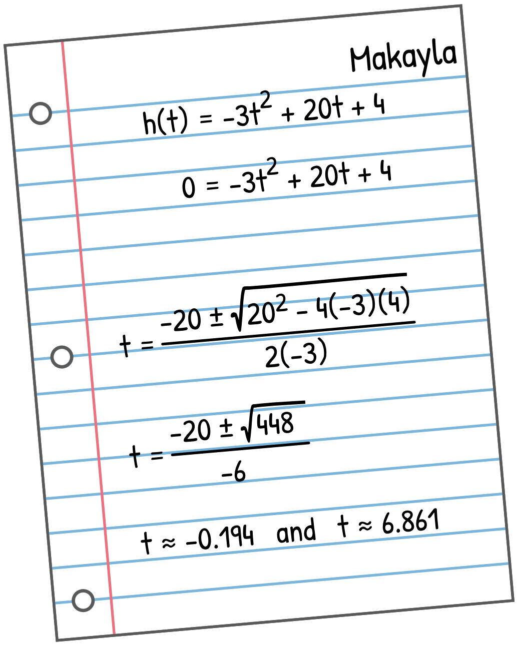


3 Here is Makayla's work from the previous problem.

Makayla says that the rocket will touch the ground at about -0.194 seconds and 6.861 seconds.

Discuss:

- Why did Makayla substitute 0 for $h(t)$?
- What is correct about Makayla's response?
- What is incorrect? Why?



Makayla

$h(t) = -3t^2 + 20t + 4$

$0 = -3t^2 + 20t + 4$

$t = \frac{-20 \pm \sqrt{20^2 - 4(-3)(4)}}{2(-3)}$

$t = \frac{-20 \pm \sqrt{448}}{-6}$

$t \approx -0.194$ and $t \approx 6.861$

Name: ________ Date: ________ Period: ________

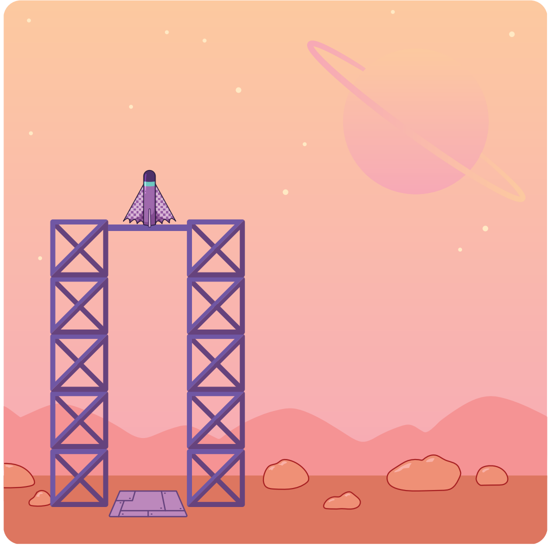
Beam Me Up

4. Here is a new rocket.

The function $h(t) = \text{-}8t^2 + 40t + 10$ represents the height, in meters, of the rocket t seconds after it has been launched.

The rocket reaches a maximum height of 60 meters.

Write an equation that can be solved to determine when the rocket is at its maximum height.

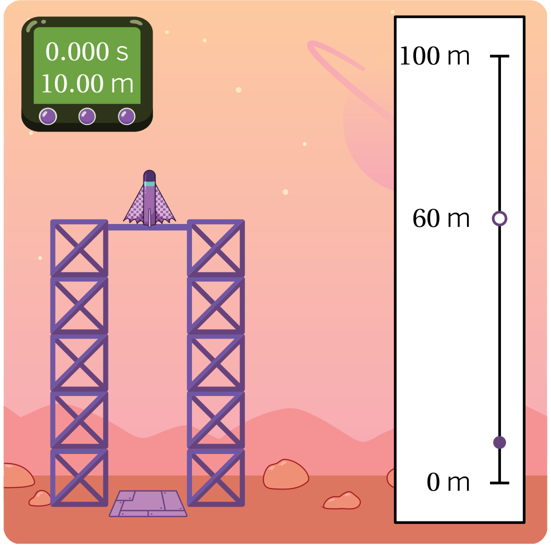


Explain your thinking.

5. Here is an equation someone wrote for the new rocket: $60 = \text{-}8t^2 + 40t + 10$.

How many seconds will it take for the rocket to reach its maximum height?

Round to three decimal places if necessary.



Name: .... Date: .... Period: ....

Beam Me Up (continued)

6. Here is a new rocket.

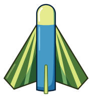
The function $h(t) = -4t^2 + 30t + 10$ represents the height, in meters, of the rocket t seconds after it has been launched.

Does this rocket ever reach a height of 100 meters? Circle one.

Yes No

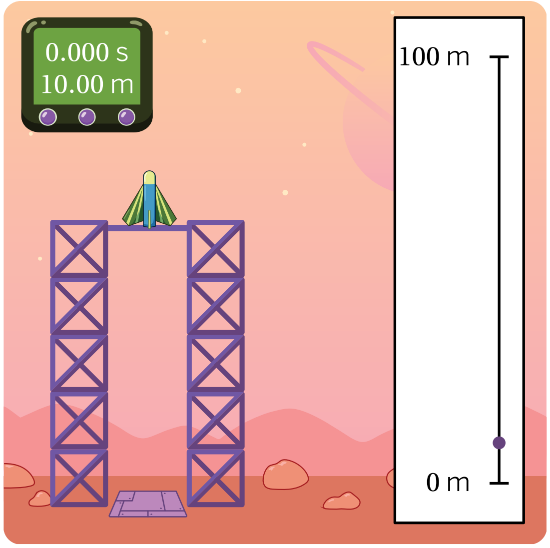
Show or explain your thinking.

$h(t) = -4t^2 + 30t + 10$



7. The function $h(t) = -4t^2 + 30t + 10$ represents the height, in meters, of the stomp rocket t seconds after it has been launched.

Write a question about the height of the rocket that will have two answers.



Activity
3

Name: Date: Period:

Rocket Scientist

8. You will use the Activity 3 Sheet to choose your own stomp rocket.

a Choose It!

- On the activity sheet, choose a stomp rocket.
- In this table, write down the function, the question you selected, and the solution to the question.

Round your solution to three decimal places if necessary.

Function:	$h(t) =$
Question:	
Solution:	

b Swap It!

- Share your stomp rocket with a partner who has a different rocket.
- Solve the question they chose for their stomp rocket.

Round your solution to three decimals places if necessary.

Partner 1's Rocket

Function:	$h(t) =$
Question:	
Solution:	

Partner 2's Rocket

Function:	$h(t) =$
Question:	
Solution:	

Synthesis

9. Describe a strategy that helped you answer the question in today's lesson.

If you learned it from another student, give them a shout-out!

When will the rocket touch the ground?

When will the rocket reach its maximum height?

Will the rocket ever reach a height of 100 meters?

Lesson Practice 7.18

Lesson Summary

You can use quadratic functions to represent the paths of objects that are launched in the air, such as a stomp rocket. You can use functions to answer questions about the launch by:

1. Substituting the given information into the equation.
2. Solving for the missing variable using any strategy (e.g., the quadratic formula or any other available strategy).
3. Interpreting if the solution(s) make sense in the situation.

The function $h(t) = \text{-}2.5t^2 + 6t + 8$ represents the height, in meters, of a stomp rocket t seconds after it has been launched.

When will the rocket touch the ground?	When will the rocket be at a height of 10 meters?	Will the rocket reach a height of 15 meters?
Given information: $h(t) = 0$ $0 = -2.5t^2 + 6t + 8$ Using the quadratic formula, I get the following solutions: $t = -0.954$ and $t = 3.354$ Going back in time does not make sense in this situation, so the only answer is 3.354 seconds.	Given information: $h(t) = 10$ $10 = -2.5t^2 + 6t + 8$ $0 = -2.5t^2 + 6t - 2$ Using the quadratic formula, I get the following solutions: $t = 0.4$ and $t = 2$ The rocket will be at a height of 10 meters at both .4 seconds and 2 seconds.	Given information: $h(t) = 15$ $15 = -2.5t^2 + 6t + 8$ $0 = -2.5t^2 + 6t - 7$ Using the quadratic formula, I get no solution: $t = \frac{-6 \pm \sqrt{-34}}{-5}$ This means the rocket will never reach a height of 15 feet.

Lesson Practice
7.18

Name: .... Date: .... Period: ....

Problems 1–2: The function $f(t) = 4 + 12t - 16t^2$ models the height of a tennis ball, in feet, t seconds after it was hit.

1. Select *all* of the solutions to the equation $0 = 4 + 12t - 16t^2$.

☐ **A.** $-\frac{1}{4}$ ☐ **B.** $\frac{1}{4}$ ☐ **C.** 4 ☐ **D.** 1 ☐ **E.** -1

2. How many seconds until the tennis ball hits the ground? Explain how you know.

Problems 3–5: Katie is planning to go skydiving. She writes the function $h(t) = -16t^2 + 13500$ to represent her height, in feet, t seconds after jumping out of the airplane.

3. According to Katie's function, how high is the airplane when she jumps?

4. It's recommended that skydivers open their parachutes at 5000 feet. Use $h(t)$ to approximate how many seconds after jumping Katie should open her parachute.

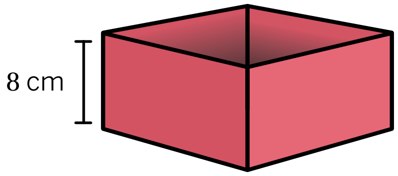
5. When Katie actually jumps, do you think she will reach 5000 feet in less time, more time, or exactly the amount of time you approximated? Explain your thinking.

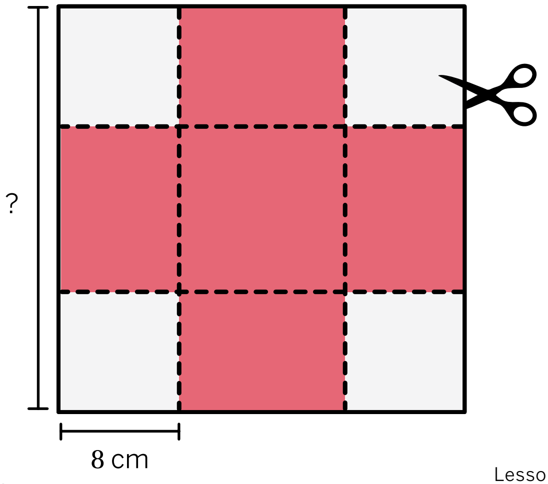
6. A company wants to make a square box with no top. The requirements are:

- It must be 8 cm tall.
- Its volume must be $1{,}000$ cubic cm.

The boxes are made by cutting four corners from a square piece of cardboard and folding the flaps up.

What should the length of the starting square be? Show or explain your thinking.





Lesson Practice
7.18

Name: .... Date: .... Period: ....

Test Practice

Problems 7–9: The function $h(t) = -75t^2 + 60t$ models the height, in inches, of a jumping frog, where t is the number of seconds after it jumped.

7. Solve the equation $-75t^2 + 60t = 0$.

8. What do the solutions tell us about the jumping frog?

9. When does the frog reach its maximum height of 12 inches?

Spiral Review

10. Select all the irrational numbers.

☐ **A.** $\sqrt{13}$

☐ **B.** $-\frac{53}{11}$

☐ **C.** $-\sqrt{17}$

☐ **D.** $-\sqrt{\frac{28}{7}}$

☐ **E.** $\sqrt{81}$

☐ **D.** $(-\sqrt{8})^2$

11. Write an equation of each type that starts with $x^2 + 8x =$ ....

No Solutions	$x^2 + 8x =$ ....
One Solution	$x^2 + 8x =$ ....
Two Solutions	$x^2 + 8x =$ ....

Unit 7 | Math in Action

Career Connection

Did you know that you can make math history today?

Math professor Po-Shen Loh did just that in 2019 when he discovered a new approach for solving quadratic equations. To demonstrate it, consider the quadratic equation $x^2 - 10x + 23 = 0$.

$(5 + u)(5 - u) = 23$	Two numbers with a sum of 10 have an average of $10 \div 2$, or 5. Their product is 23.
$25 - u^2 = 23$	Expand using the Distributive Property.
$2 = u^2$	Subtract 23 from each side.
$\pm\sqrt{2} = u$	Take the square root.

The solutions to the equation are $5 + u$ and $5 - u$. Substituting the values of u into these expressions gives solutions of $5 + \sqrt{2}$ and $5 - \sqrt{2}$.

Mathematicians use their knowledge of math to look for new ways of solving problems.

B.E.S.T. Mathematics Benchmark Connection

Many mathematicians explore algebra concepts in their work. For example, write and solve one-variable quadratic equations (MA.912.AR.3.1) when they analyze an equation, or look for unique ways to solve them by studying the terms of the equation. They may also determine key features of a quadratic function represented by the equation (MA.912.AR.3.6) to interpret the features as they relate to a real-world context.

Mathematical Thinking and Reasoning Connection

Mathematicians who study advanced topics use thinking and reasoning skills like the ones you use for your math work! For example, Po-Shen Loh studied the work of ancient mathematicians to learn enough about quadratic equations to look for a new way to solve them (MTR.4.1). They also use fundamental properties and operations to connect math concepts (MTR.5.1).

Meet Po-Shen Loh

Po-Shen Loh is a professor of mathematics at Carnegie Mellon University, in Pittsburgh, Pennsylvania. The method he discovered in 2019 for solving quadratic equations was based on observations by French mathematician François Viète (hundreds of years ago) and Babylonian and Greek mathematicians (thousands of years ago). Finding new ways to solve equations like Po-Shen Loh did helps people in many career areas do their work more efficiently and with greater accuracy.

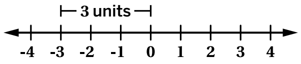
Algebra 1 / Álgebra 1

Glossary/Glosario

English

A

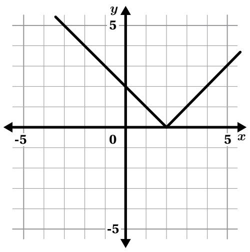
absolute value The absolute value of a number is its distance from 0 on the number line.



For example, the absolute value of -3 is 3 because -3 is 3 units away from 0. This is written as $|-3| = 3$.

$|4| = 4$ and $|-4| = 4$. They are both 4 units away from 0.

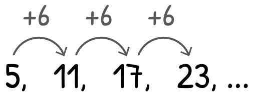
absolute value function A function that is defined using absolute value symbols. When written in the form $f(x) = |x - h|$, its output is the distance of its input from a given value, h.



For example, $f(x) = |x - 2|$ outputs the distance from 2 for every input value.

area The amount of space inside a two-dimensional shape. It is measured in square units, such as square inches or square centimeters.

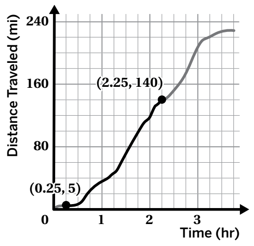
arithmetic sequence A sequence that changes by a constant difference.



In this arithmetic sequence, the first term is 5 and the constant difference is 6.

association If two variables are statistically related to one another, we say that there is an association between the two variables. A *correlation* is a type of association.

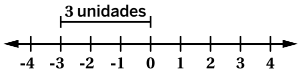
average rate of change A measure of how much a function changes, on average, over an interval. To calculate the average rate of change over an interval, find the slope between the point on the graph of the function where the interval begins and the point where it ends.



To calculate the average rate of change over the interval from 0.25 hours to 2.25 hours, calculate the change in y-values $(140 - 5)$ and divide by the change in x-values $(2.25 - 0.25)$. The average rate of change from 0.25 to 2.25 is 67.5, which means the average speed on that trip was 67.5 miles per hour in that interval.

Español

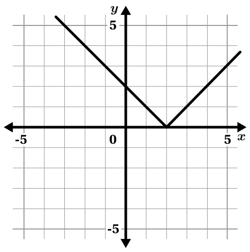
valor absoluto El valor absoluto de un número es su distancia al 0 en la recta numérica.



Por ejemplo, el valor absoluto de -3 es 3 porque -3 está a 3 unidades del 0. Esto se escribe $|-3| = 3$.

$|4| = 4$ y $|-4| = 4$. Ambos están a 4 unidades del 0.

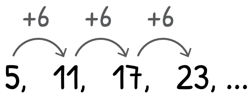
función de valor absoluto Una función que se define utilizando signos de valor absoluto. Cuando se escribe en la forma $f(x) = |x - h|$, su salida es la distancia de su entrada a un valor determinado, h.



Por ejemplo, $f(x) = |x - 2|$ arroja la distancia de cada valor de entrada al 2.

área La cantidad de espacio dentro de una figura bidimensional. Se mide en unidades cuadradas, como pulgadas cuadradas o centímetros cuadrados.

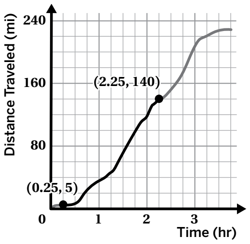
secuencia aritmética Una secuencia que cambia con una diferencia constante.



En esta secuencia aritmética, el primer término es 5 y la diferencia constante es 6.

asociación Si dos variables están relacionadas estadísticamente, decimos que existe una asociación entre ambas variables. Una *correlación* es un tipo de asociación.

tasa de cambio promedio Una medida de cuánto cambia una función, en promedio, en un intervalo. Para calcular la tasa de cambio promedio a lo largo de un intervalo, se halla la pendiente entre el punto donde empieza el intervalo y el punto donde termina en la gráfica de la función.

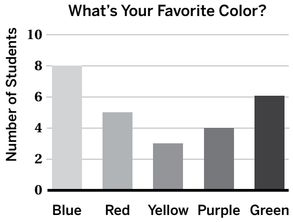


Para calcular la tasa de cambio promedio en el intervalo de 0.25 horas a 2.25 horas, se calcula el cambio en los valores de y $(140 - 5)$ y se divide por el cambio en los valores de x $(2.25 - 0.25)$. La tasa de cambio promedio de 0.25 a 2.25 es 67.5, lo que significa que la velocidad media en ese trayecto fue de 67.5 millas por hora en ese intervalo.

English

B

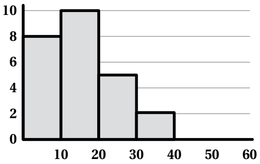
bar chart A visual display of categorical data values that uses bars to show frequencies for each category.



base A number or expression that is raised to an exponent. The exponent describes the number of times to multiply the base by itself.

For example, in the expression 2^3, 2 is the base and 3 is the exponent. In the expression $50(1.6)^{12x}$, 1.6 is the base and $12x$ is the exponent.

bin (of a histogram) The intervals used to group data values in a histogram are called *bins*. The distance of each interval is called the *bin width*.

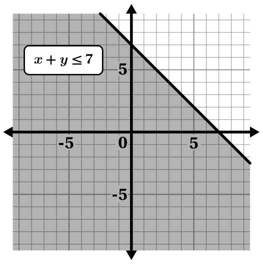


For example, this histogram shows 4 bins, each with a bin width of 10 units.

bivariate data Data that involves two variables. Each data point contains two pieces of information.

For example, a collection of students' heights and shoe sizes would be a bivariate data set.

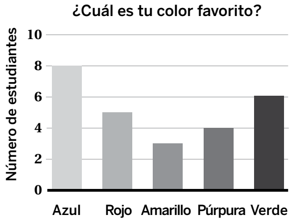
boundary line The line that separates the solution region of a linear inequality from non-solutions. A linear inequality (e.g., $y < 2x + 5$) has a boundary line that is represented symbolically by the corresponding equation (e.g., $y = 2x + 5$). A solid boundary line indicates that these points are included in the solution set (e.g., $y \leq x$). A dashed boundary line indicates that they are not (e.g., $y < x$).



The solution region to $x + y \leq 7$ has a boundary line at $x + y = 7$. The line is solid because the points on the line $x + y = 7$ are included in the solution region.

Español

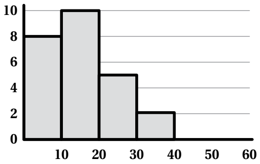
gráfico de barras Una representación visual de valores de datos categóricos que utiliza barras para mostrar frecuencias para cada categoría.



base Un número o una expresión que se eleva a un exponente. El exponente describe el número de veces que se multiplica la base por sí misma.

Por ejemplo, en la expresión 2^3, 2 es la base y 3 es el exponente. En la expresión $50(1.6)^{12x}$, 1.6 es la base y $12x$ es el exponente.

intervalo (de un histograma) Se denominan intervalos a los espacios que se usan para agrupar los valores de datos en un histograma. La distancia de cada intervalo se denomina *ancho del intervalo*.

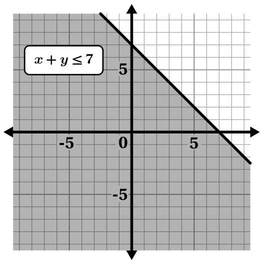


Por ejemplo, este histograma muestra 4 intervalos, cada uno con un ancho de intervalo de 10 unidades.

datos bivariados Datos en los que se incluyen dos variables. Cada punto de datos contiene dos informaciones.

Por ejemplo, una colección de estaturas de estudiantes y tamaños de zapatos sería un conjunto de datos bivariados.

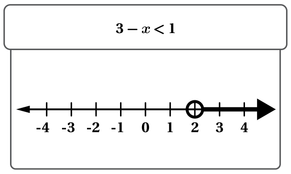
recta límite La línea que separa la región solución de una desigualdad lineal de todos los valores que no son soluciones. Una desigualdad lineal (p. ej., $y < 2x + 5$) tiene una recta límite representada simbólicamente por la ecuación correspondiente (p. ej., $y = 2x + 5$). Una recta límite continua indica que esos puntos están incluidos en el conjunto de soluciones (p. ej., $y \leq x$). Una recta límite discontinua indica que no lo están (p. ej., $y < x$).



La región solución de $x + y \leq 7$ tiene una recta límite en $x + y = 7$. La recta es continua porque los puntos en la recta $x + y = 7$ se encuentran dentro de la región solución.

English

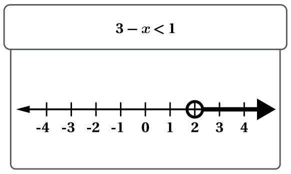
boundary point The value that separates the solution set of an inequality from non-solutions. A solid boundary point indicates that the point is included in the solution set. An empty boundary point indicates the point is not included in the solution set.



The solution set to $3 - x < 1$ has a boundary point at $x = 2$. The point at 2 is empty because 2 is not included in the solution set.

Español

punto límite El valor que separa el conjunto de soluciones de una desigualdad de todos los valores que no son soluciones. Un punto límite sólido (relleno) indica que el punto está incluido en el conjunto de soluciones. Un punto límite vacío indica que el punto no está incluido en el conjunto de soluciones.



El conjunto de soluciones de $3 - x < 1$ tiene un punto límite en $x = 2$. El punto en 2 no está relleno porque 2 no está incluido en el conjunto de soluciones.

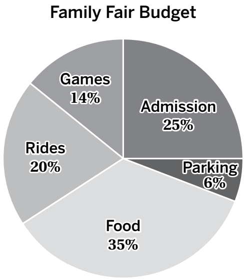
C

categorical data A type of data that is divided into groups or categories. Categorical data describes qualities or characteristics, such as colors, names, or zip codes, rather than numerical values.

For example, the breeds of 10 dogs are categorical data. Another example is the colors of 100 flowers.

causation Describes a relationship between two variables in which a change in one variable causes a change in the other variable. A special type of *correlation*.

circle graph A visual display of categorical data. The whole set of data is represented by a circle and its interior. The categories are represented by fractional parts of the circle. Also called a *pie chart*.



coefficient The number that multiplies a variable in an algebraic expression. If no number is shown, the coefficient is understood to be 1.

For example, in the expression $5x$, the coefficient is 5. In y, the coefficient is 1.

completing the square The process of rewriting a quadratic expression or equation to include a perfect square.

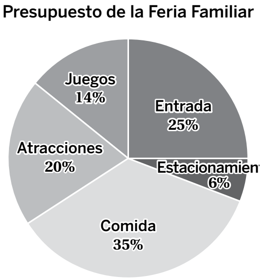
$$x^2 - 6x + 17$$
$$(x^2 - 6x + 9) - 9 + 17$$
$$(x - 3)^2 - 9 + 17$$
$$(x - 3)^2 + 8$$

datos categóricos Un tipo de datos que se dividen en grupos o categorías. Los datos categóricos describen cualidades o características, como colores, nombres o códigos postales, en vez de valores numéricos.

Por ejemplo, las razas de 10 perros diferentes son datos categóricos. Otro ejemplo son los colores de 100 flores diferentes.

causalidad Describe una relación entre dos variables en la que un cambio en una variable provoca un cambio en la otra variable. Un tipo especial de *correlación*.

gráfico circular Una representación visual de datos categóricos. Todo el conjunto de datos está representado por un círculo y su interior. Las categorías están representadas por partes fraccionarias del círculo. También llamado *gráfico de torta*.



coeficiente El número que multiplica una variable en una expresión algebraica. Si no se muestra ningún número, se entiende que el coeficiente es 1.

Por ejemplo, en la expresión $5x$, el coeficiente es 5. En y, el coeficiente es 1.

completación del cuadrado El proceso de reescribir una expresión o ecuación cuadrática para incluir un cuadrado perfecto.

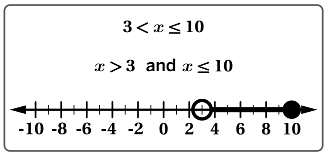
$$x^2 - 6x + 17$$
$$(x^2 - 6x + 9) - 9 + 17$$
$$(x - 3)^2 - 9 + 17$$
$$(x - 3)^2 + 8$$

English

compound To increase in value by applying an operation repeatedly. In mathematics, compounding can involve repeated multiplication or growth over time.

For example, when bacteria multiply by splitting in half every hour, the number of bacteria compounds, doubling each hour.

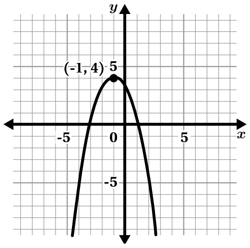
compound inequality Two or more inequalities joined together. A compound inequality can be written using symbols or the words "and" or "or."



The numbers greater than 3 and less than or equal to 10 can be written as: $x > 3$ and $x \leq 10$ or $3 < x \leq 10$.

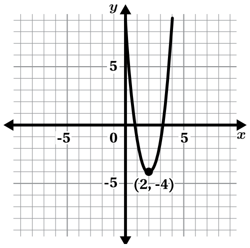
compound interest Interest calculated based on the initial amount and the interest from previous periods. It is calculated at regular intervals (daily, monthly, annually, etc.). The balance in an account that earns compound interest can be modeled by an exponential function.

concave down A parabola that opens downward is described as *concave down*. A function with a negative a-value will produce a concave down parabola.



This parabola is concave down. Two ways to write the equation are $f(x) = -1(x + 1)^2 + 4$ and $f(x) = -x^2 - 2x + 3$.

concave up A parabola that opens upward is described as *concave up*. A function with a positive a-value will produce a concave up parabola.



This parabola is concave up. Two ways to write the equation are $f(x) = 4(x - 2)^2 - 4$ and $f(x) = 4x^2 - 16x + 12$.

confidence interval A range of values used to estimate an unknown population value. It shows how likely it is that the true value is within the range.

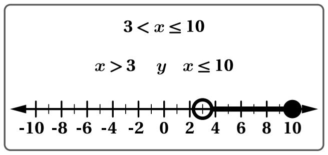
A survey estimates that 60% of students prefer online learning, with a confidence interval of 55% to 65%. This means the true percentage is likely between 55% and 65%.

Español

crecimiento compuesto Aumentar en valor aplicando una operación repetidamente. En matemáticas, el crecimiento compuesto puede implicar la multiplicación repetida o el crecimiento a lo largo del tiempo.

Por ejemplo, cuando las bacterias se multiplican dividiéndose por la mitad cada hora, el número de bacterias crece de forma compuesta y se duplica cada hora.

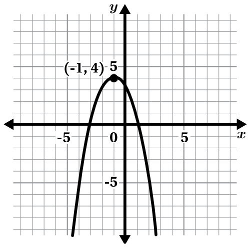
desigualdad compuesta Dos o más desigualdades juntas. Una desigualdad compuesta puede escribirse con signos o las palabras "y" u "o".



Los números mayores que 3 y menores o iguales que 10 pueden escribirse de la siguiente forma: $x > 3$ y $x \leq 10$ o $3 < x \leq 10$.

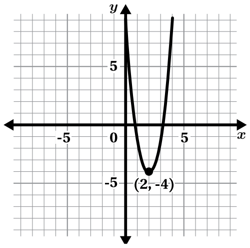
interés compuesto Interés que se calcula según una cantidad inicial y el interés de los períodos anteriores. Se calcula en intervalos regulares (diariamente, mensualmente, anualmente, etc.). El saldo de una cuenta que produce interés compuesto puede modelarse con una función exponencial.

cóncava hacia abajo Una parábola que se abre hacia abajo se describe como cóncava hacia abajo. Una función con un valor a negativo producirá una parábola cóncava hacia abajo.



Esta parábola es cóncava hacia abajo. Dos formas de escribir la ecuación son $f(x) = -1(x + 1)^2 + 4$ y $f(x) = -x^2 - 2x + 3$.

cóncava hacia arriba Una parábola que se abre hacia arriba se describe como cóncava hacia arriba. Una función con un valor a positivo producirá una parábola cóncava hacia arriba.



Esta parábola es cóncava hacia arriba. Dos formas de escribir la ecuación son $f(x) = 4(x - 2)^2 - 4$ y $f(x) = 4x^2 - 16x + 12$.

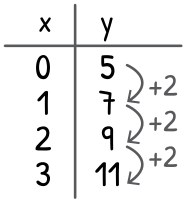
intervalo de confianza Un rango de valores utilizado para estimar un valor poblacional desconocido. Muestra la probabilidad de que el valor real esté dentro del rango.

Una encuesta estima que el 60 % de los estudiantes prefieren el aprendizaje en línea, con un intervalo de confianza del 55 % al 65 %. Esto significa que el porcentaje real probablemente esté entre el 55 % y el 65 %.

English

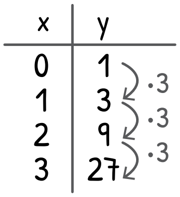
constant difference When the difference between every two consecutive values in a pattern is the same, there is a *constant difference*.

The pattern in the table has a constant difference of 2.



constant ratio When the ratio between every two consecutive values in a pattern is the same, there is a *constant ratio*.

The pattern in the table has a constant ratio of 3.



constraint A limitation on the possible values of variables in a model. Equations and inequalities are often used to represent constraints.

The constraint that "you must be 36 inches or taller to ride the Ferris wheel" can be represented by the inequality $h \geq 36$.

correlation A statistical relationship between two or more variables. Also called an *association*.

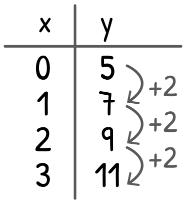
cube root The cube root of a number n (written as $\sqrt[3]{n}$) is the number that can be cubed to get n. The cube root is also the edge length of a cube with a volume of n.

For example, the cube root of 64 ($\sqrt[3]{64}$) is 4 because 4^3 is 64. 4 is also the edge length of a cube that has a volume of 64 cubic units.

Español

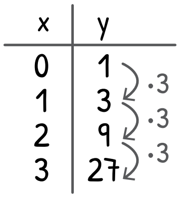
diferencia constante Cuando la diferencia entre dos valores consecutivos en un patrón permanece igual, hay una diferencia constante.

El patrón en la tabla tiene una diferencia constante de 2.



razón constante Cuando la razón entre dos valores consecutivos en un patrón permanece igual, hay una razón constante.

El patrón en la tabla tiene una razón constante de 3.



restricción Una limitación de los posibles valores de las variables en un modelo. Suelen usarse ecuaciones o desigualdades para representar restricciones.

La restricción "debes medir 36 pulgadas o más para subirte a la rueda de la fortuna" puede representarse con la desigualdad $h \geq 36$.

correlación Una relación estadística entre dos o más variables. También se denomina *asociación*.

raíz cúbica La raíz cúbica de un número n (se escribe $\sqrt[3]{n}$) es el número que puede elevarse al cubo para obtener n. La raíz cúbica también es la longitud de la arista de un cubo con un volumen de n.

Por ejemplo, la raíz cúbica de 64 ($\sqrt[3]{64}$) es 4 porque 4^3 es 64. 4 también es la longitud de arista de un cubo que tiene un volumen de 64 unidades cúbicas.

D

data A collection of information, usually in the form of numbers, words, or measurements, that can be analyzed.

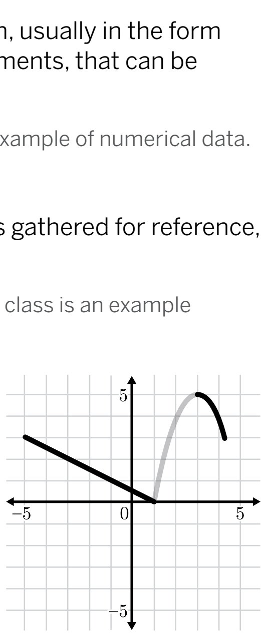
A list of students' test scores is an example of numerical data.

data sets A collection of values gathered for reference, analysis, or interpretation.

A list of test scores collected from a class is an example of data.

decreasing A function, or interval of a function, is decreasing if the y-values go down when the x-values go up.

The bolded parts of this function are decreasing.



datos Una recopilación de información, generalmente en forma de números, palabras o medidas, que se puede analizar.

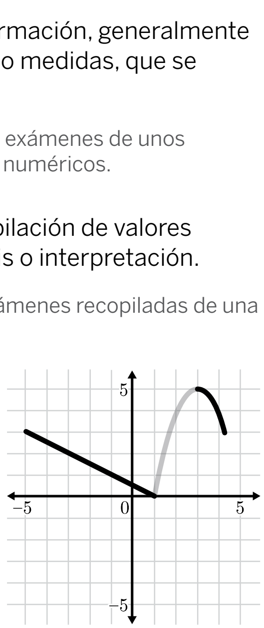
Una lista de las calificaciones de los exámenes de unos estudiantes es un ejemplo de datos numéricos.

conjuntos de datos Una recopilación de valores reunidos para referencia, análisis o interpretación.

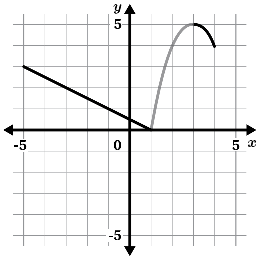
Una lista de las calificaciones de exámenes recopiladas de una clase es un ejemplo de datos.

decreciente Una función, o un intervalo de una función, es decreciente si los valores de y disminuyen cuando los valores de x aumentan.

Las partes resaltadas de esta función son decrecientes.



English



decreasing (interval or function) A function is decreasing when its outputs decrease as its inputs increase. A function can be decreasing for its entire domain or over an interval.

For example, the function $f(x)$ is decreasing when $-5 < x < 1$ and when $3 < x < 4$.

dependent variable The value of a dependent variable is based on the value of another variable or set of variables. In a function, the value of the dependent variable represents the output. The dependent variable is typically on the vertical axis of a graph and in the right-hand column of a table.

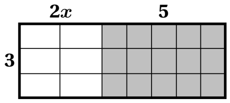
difference of squares An expression that can be written as one perfect square subtracted from another perfect square. It has the structure $r^2 - s^2$ in standard form and $(r - s)(r + s)$ in factored form.

These expressions are differences of squares: $x^2 - 25, 9x^2 - 100,$ and $16x^2 - 1$.

discrete A set of values is discrete if there is separation between the values. A graph can be described as discrete when it consists of unconnected points or intervals.

The set of numbers 1, 2, 3 is discrete, while the set of all values between 1 and 3 is not discrete.

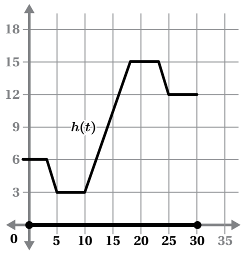
distributive property Multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding them together.



Area as a Product $3(2x + 5)$

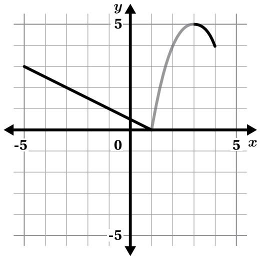
For example, $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

domain The set of all possible input values for a function or relation. The domain can be described in words or as an inequality.



The domain of this graph can be described as: All numbers from 0 to 30 or $0 \leq t \leq 30$.

Español



decreciente (intervalo o función) Una función es decreciente cuando sus valores de salida decrecen a medida que crecen sus valores de entrada. Una función puede ser decreciente en todo su dominio o en un intervalo.

Por ejemplo, la función $f(x)$ es decreciente cuando $-5 < x < 1$ y cuando $3 < x < 4$.

variable dependiente El valor de una variable dependiente se basa en el valor de otra variable o un conjunto de variables. En una función, el valor de la variable dependiente representa la salida.
La variable dependiente suele estar en el eje vertical de una gráfica y en la columna derecha de una tabla.

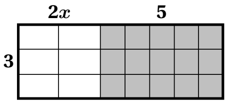
diferencia de cuadrados Una expresión que puede escribirse como un cuadrado perfecto que se resta a otro cuadrado perfecto. Tiene la estructura $r2 - s2$ en forma estándar y $(r - s)(r + s)$ en forma factorizada.

Estas expresiones son diferencias de cuadrados: $x^2 - 25, 9x^2 - 100$ y $16x^2 - 1$.

discreto Un conjunto de valores es discreto si hay separación entre los valores. Una gráfica puede describirse como discreta cuando consta de puntos o intervalos que no se conectan.

El conjunto de números 1, 2, 3 es discreto, mientras que el conjunto de todos los valores entre 1 y 3 no es discreto.

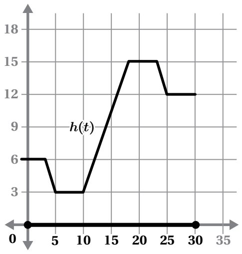
propiedad distributiva Multiplicar un número por la suma de dos o más términos equivale a multiplicar el número por cada término individualmente antes de sumarlos.



El área como un producto $3(2x + 5)$

Ejemplo: $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

dominio El conjunto de todos los valores de entrada posibles de una función o relación. El dominio puede describirse con palabras o como una desigualdad.



El dominio de esta gráfica puede describirse de la siguiente manera: Todos los números del 0 al 30 o $0 \leq t \leq 30$.

English

E

elimination A method of solving systems of equations where you add or subtract the equations to produce a new equation with fewer variables.

$$\begin{array}{r} 9x + y = 2 \\ -(3x + y = 10) \\ \hline 6x + 0 = -8 \end{array}$$

In the example, subtraction is used to eliminate y and create an equation that can be solved for x.

equation Two expressions with an equal sign between them.

$-3(2x + 5) = 5 + x$ is an example of an equation.

equivalent equations Equations that have the same solution(s).

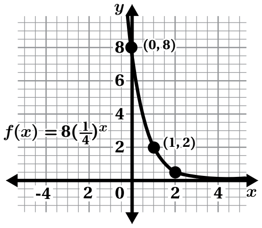
$3x + 4 = 10$ and $9x + 12 = 30$ are equivalent equations because if you multiply the first equation by 3, you create the second. The solution to each equation is $x = 2$.

equivalent systems Two or more systems of equations that have the same set of solutions.

exponent Exponents describe repeated multiplication. The exponent describes the number of times to multiply the base by itself.

For example, in the expression 2^3, 2 is the base and 3 is the exponent. In the expression $50(1.6)^{12x}$, 1.6 is the base and $12x$ is the exponent.

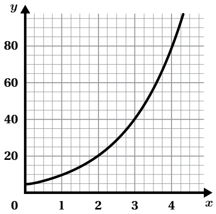
exponential decay An exponential relationship that decreases. An exponential decay relationship has a growth factor between 0 and 1.



For example, $f(x) = 8\left(\frac{1}{4}\right)^x$ is an example of exponential decay because the growth factor, $\frac{1}{4}$, is between 0 and 1.

exponential function A function that increases or decreases by a constant percent rate of change. A function that changes by equal factors over equal intervals.

x	$f(x)$
0	5
1	10
2	20
3	40



For example, the table and graph show the exponential function $f(x) = 5 \cdot (2)^x$, which has an initial value of 5 and has a growth factor of 2.

Español

eliminación Un método para resolver sistemas de ecuaciones en el que se suman o restan las ecuaciones para producir una nueva ecuación con menos variables.

$$\begin{array}{r} 9x + y = 2 \\ -(3x + y = 10) \\ \hline 6x + 0 = -8 \end{array}$$

En el ejemplo, se usa la resta para eliminar y y producir una ecuación que puede resolverse para determinar el valor de x.

ecuación Dos expresiones con un signo igual entre ambas.

$-3(2x + 5) = 5 + x$ es un ejemplo de una ecuación.

ecuaciones equivalentes Ecuaciones que tienen exactamente la misma o las mismas soluciones.

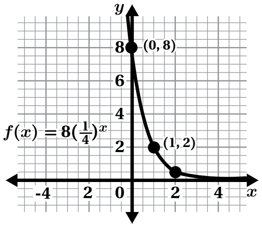
$3x + 4 = 10$ y $9x + 12 = 30$ son ecuaciones equivalentes porque si se multiplica la primera por 3, se forma la segunda. La solución de cada ecuación es $x = 2$.

sistemas equivalentes Dos o más sistemas de ecuaciones que tienen el mismo conjunto de soluciones.

exponente Los exponentes describen una multiplicación repetida. El exponente describe el número de veces que se multiplica la base por sí misma.

Por ejemplo, en la expresión 2^3, 2 es la base y 3 es el exponente. En la expresión $50(1.6)^{12x}$, 1.6 es la base y $12x$ es el exponente.

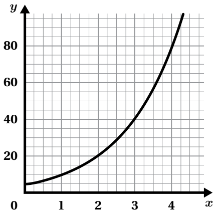
decaimiento exponencial Una relación exponencial que decrece. Una relación de decaimiento exponencial tiene un factor de crecimiento entre 0 y 1.



Por ejemplo, $f(x) = 8\left(\frac{1}{4}\right)^x$ es un ejemplo de decaimiento exponencial porque el factor de crecimiento, $\frac{1}{4}$, está entre 0 y 1.

función exponencial Una función que crece o decrece con una tasa porcentual de cambio constante. Una función que cambia por factores iguales en intervalos iguales.

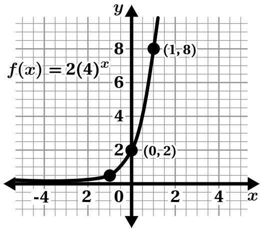
x	$f(x)$
0	5
1	10
2	20
3	40



Por ejemplo, la tabla y la gráfica muestran la función exponencial $f(x) = 5 \cdot (2)^x$, que tiene un valor inicial de 5 y un factor de crecimiento de 2.

English

exponential growth An exponential relationship that increases. An exponential growth relationship has a growth factor greater than 1.



For example, $f(x) = 2(4)^x$ is an example of exponential growth because the growth rate, 4, is greater than 1.

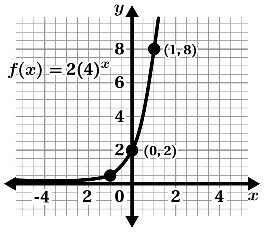
exponential relationship See *exponential function*.

expression A set of numbers, variables, operations, and grouping symbols that represent a quantity.

5, $2x$, $7 - 3^p$, and $4(2 - 5x^2) + 8$ are all examples of expressions.

Español

crecimiento exponencial Una relación exponencial que crece. Una relación con crecimiento exponencial tiene un factor de crecimiento mayor que 1.



Por ejemplo, $f(x) = 2(4)^x$ es un ejemplo de crecimiento exponencial porque la tasa de crecimiento, 4, es mayor que 1.

relación exponencial Ver función exponencial.

expresión Un conjunto de números, variables, operaciones y signos de agrupación que representan una cantidad.

5, $2x$, $7 - 3^p$ y $4(2 - 5x^2) + 8$ son ejemplos de expresiones.

F

factor (of a number or expression) A number or expression multiplied with other numbers or expressions to make a product.

For example, $1, 2, 4$, and 8 are all factors of the number 8 because $1 \cdot 8 = 8$ and $2 \cdot 4 = 8$. Also, $(x + 3)$ and $(x - 5)$ are factors of $x^2 - 2x - 15$ because $(x + 3)(x - 5) = x^2 - 2x - 15$.

factored form One of three common forms of a quadratic equation. A quadratic equation in factored form looks like $f(x) = a(x - m)(x - n)$.

These equations are in factored form:
$g(x) = x(x + 10)$
$2(x - 1)(x + 3) = y$
$y = (5x + 2)(3x - 1)$

frequency table A table that shows the number of times each value or category occurs in a data set.

Favorite Color	Frequency
Red	5
Blue	3
Pink	4

For example, this frequency table shows that 5 students selected red as their favorite color.

function A rule, or relation, that assigns exactly one output to each possible input. Every function is a relation, but not every relation is a function. In a function, the value of the output variable depends on the value of the input variable.

factor (de un número o una expresión) Un número o una expresión que se multiplica por otros números o expresiones para dar como resultado un producto.

Por ejemplo, $1, 2, 4$ y 8 son factores del número 8 porque $1 \cdot 8 = 8$ y $2 \cdot 4 = 8$. Además, $(x + 3)$ y $(x - 5)$ son factores de $x^2 - 2x - 15$ porque $(x + 3)(x - 5) = x^2 - 2x - 15$.

forma factorizada Una de las tres formas comunes de una ecuación cuadrática. Una ecuación cuadrática en forma factorizada tiene el siguiente orden: $f(x) = a(x - m)(x - n)$.

Estas ecuaciones están en forma factorizada:
$g(x) = x(x + 10)$
$2(x - 1)(x + 3) = y$
$y = (5x + 2)(3x - 1)$

tabla de frecuencia Una tabla que muestra el número de veces que aparece cada valor o categoría en un conjunto de datos.

Color favorito	Frecuencia
Rojo	5
Azul	3
Rosado	4

Por ejemplo, esta tabla de frecuencia muestra que 5 estudiantes seleccionaron el rojo como su color favorito.

función Una regla, o relación, que asigna exactamente una salida a cada entrada posible. Toda función es una relación, pero no toda relación es una función. En una función, el valor de la variable de salida depende del valor de la variable de entrada.

English

Español

function notation A notation used to represent the inputs and outputs of a function. For a function f, when x is an input (or domain value), the symbol $f(x)$ shows the corresponding output (or range value).

$$f(x) = 2x + 1$$
$$f(4) = 9$$

For example, $f(4) = 9$ is a statement written in function notation. It says that when the input of the function f is 4, the output is 9. In other words, when the value of the independent variable is 4, the value of the dependent variable is 9.

notación de función Una notación utilizada para representar las entradas y las salidas de una función. Para una función (x), cuando x es una entrada (o valor de dominio), el símbolo $f(x)$ muestra la salida correspondiente (o valor de rango).

$$f(x) = 2x + 1$$
$$f(4) = 9$$

Por ejemplo, $f(4) = 9$ es una expresión escrita en notación de funciones. Indica que cuando la entrada de la función f es 4, la salida es 9. En otras palabras, cuando el valor de la variable independiente es 4, el valor de la variable dependiente es 9.

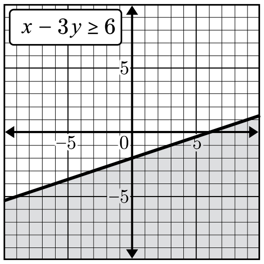
G

growth factor The constant ratio (or common factor) that each term is multiplied by to generate an exponential pattern.

factor de crecimiento La razón constante (o el factor común) que multiplica a cada término para generar un patrón exponencial.

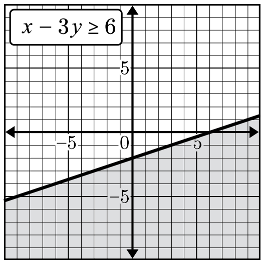
H

half-plane A region that defines the solution set of a single two-variable linear inequality. The boundary line of the inequality splits the coordinate plane into two equal half-planes.



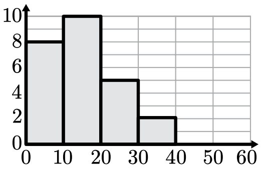
The graph shows the solution region to $x - 3y \geq 6$. The boundary line $x - 3y = 6$ divides the coordinate plane into two equal half-planes.

semiplano Una región que define el conjunto de soluciones de una desigualdad lineal con dos variables. La recta límite de la desigualdad divide el plano de coordenadas en dos semiplanos iguales.



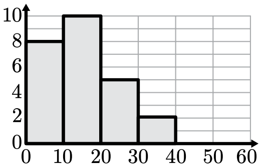
La gráfica muestra la región solución de $x - 3y \geq 6$. La recta límite $x - 3y = 6$ divide el plano de coordenadas en dos semiplanos iguales.

histogram A visual display of quantitative data that groups values into intervals, called *bins*. Each bin is represented by a rectangle, where the height shows the frequency or relative frequency of the data in that bin.



For example, this histogram shows that there are 8 values between 0 and 10.

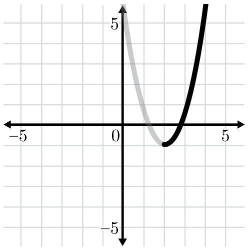
histograma Una representación visual de datos cuantitativos que agrupa los valores en intervalos, llamados *contenedores*. Cada contenedor está representado por un rectángulo, donde la altura muestra la frecuencia o la frecuencia relativa de los datos en ese contenedor.



Por ejemplo, este histograma muestra que hay 8 valores entre 0 y 10.

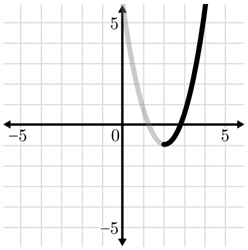
I

increasing (interval or function) A function is increasing when its outputs increase as its inputs increase. A function can be increasing for its entire domain or over an interval.



For example, the function $f(x)$, whose graph is shown, is increasing when $x > 2$.

creciente (intervalo o función) Una función es creciente cuando sus valores de salida crecen a medida que crecen sus valores de entrada. Una función puede ser creciente en todo su dominio o en un intervalo.



Por ejemplo, la función $f(x)$, cuya gráfica se muestra, es creciente cuando $x > 2$.

English

independent variable The value of an independent variable is not based on the value of any other variable. In a function, the value of the independent variable represents the input. The independent variable is typically on the horizontal axis of a graph and in the left-hand column of a table.

index In a radical $\sqrt[n]{x}$ the quantity n is called the *index*. (See *radical*.)

inequality A comparison statement that uses the symbols $<$, $>$, $\leq$, or $\geq$. Inequalities are used to represent the relationship between numbers, variables, or expressions that are not always equal.

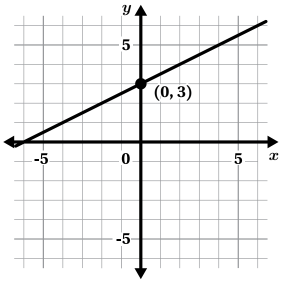
For example, the inequality $y + 2 \geq 30$ means that the value of the expression $y + 2$ will be greater than or equal to 30.

infinitely many solutions An equation has infinitely many solutions if it is true for any value of the variable. A system of equations has infinitely many solutions if the equations in the system are equivalent. In a system of equations with infinitely many solutions, every point on the graph is a solution to the system.

For example, the equation $3x + 6 = 3(x + 2)$ has infinitely many solutions because the equation is true for any value of x.

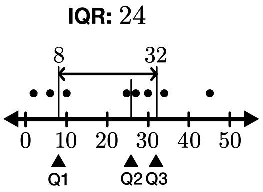
initial value A point where the graph of an equation or function crosses the y-axis or when $x = 0$.

The initial value of the graph $-2x + 4y = 12$ is $(0, 3)$, or just 3.



input In a function, any value that you substitute for x is called an input. The input is sometimes called the *independent variable*. The input typically appears on the horizontal axis of a graph and in the left-hand column of a table.

interquartile range (IQR) A measure of variation in a set of numerical data. The interquartile range is the distance between the first quartile (Q1) and the third quartile (Q3) of the data set.



For example, the IQR of this data set is $32 - 8 = 24$.

Español

variable independiente El valor de una variable independiente no depende del valor de ninguna otra variable. En una función, el valor de la variable independiente representa la entrada. La variable independiente suele estar en el eje horizontal de una gráfica y en la columna izquierda de una tabla.

índice En un radical $\sqrt[n]{x}$, la cantidad n se llama índice. (Ver *radical*).

desigualdad Un enunciado de comparación que utiliza los signos $<$, $>$, $\leq$ o $\geq$. Las desigualdades se usan para representar la relación entre números, variables o expresiones que no siempre son iguales.

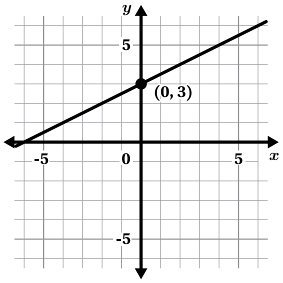
Por ejemplo, la desigualdad $y + 2 \geq 30$ significa que el valor de la expresión $y + 2$ será mayor o igual que 30.

infinitas soluciones Una ecuación tiene infinitas soluciones si es verdadera sea cual sea el valor de la variable. Un sistema de ecuaciones tiene infinitas soluciones si las ecuaciones del sistema son equivalentes. En un sistema de ecuaciones con infinitas soluciones, cada punto de la gráfica es una solución del sistema.

Por ejemplo, la ecuación $3x + 6 = 3(x + 2)$ tiene infinitas soluciones porque la ecuación es verdadera con cualquier valor de x.

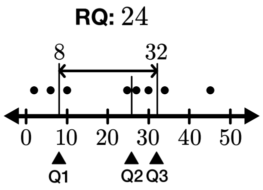
valor inicial Un punto donde la gráfica de una ecuación o función cruza el eje y, o cuando $x = 0$.

El valor inicial de la gráfica de $-2x + 4y = 12$ es $(0, 3)$, o simplemente 3.



entrada En una función, todo valor que sustituya a x se denomina entrada. La entrada a veces se denomina variable independiente.La entrada suele estar en el eje horizontal de una gráfica y en la columna izquierda de una tabla.

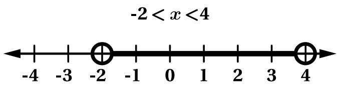
rango intercuartil (RIC) Una medida de variación en un conjunto de datos numéricos. El rango intercuartil es la distancia entre el primer cuartil (Q1) y el tercer cuartil (Q3) del conjunto de datos.



Por ejemplo, el RQ de este conjunto de datos es $32 - 8 = 24$.

English

interval A set of values between two points.



In words: All numbers between -2 and 4.
Inequality: $-2 < x < 4$.

interval notation A way to represent a range of values on a number line using brackets and parentheses.

(2, 6] represents all values between 2 and 6, not including 2 but including 6. [-3, 5) represents all values between -3 and 5, including -3 but not including 5.

inverse operations Two operations that undo each other are called *inverse operations*.

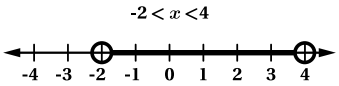
Adding and subtracting are inverse operations. Multiplying and dividing are inverse operations.

irrational number A number that cannot be written as a fraction of two integers, where the denominator is not zero.

2 is a rational number because it can be written as $\frac{2}{1}$, whereas $\sqrt{3}$ is irrational because it cannot be written as a fraction made up of two integers.

Español

intervalo Un conjunto de valores entre dos puntos.



Con palabras: Todos los números entre el -2 y el 4. Desigualdad: $-2 < x < 4$.

notación de intervalo Una forma de representar un rango de valores en una recta numérica usando corchetes y paréntesis.

(2, 6] representa todos los valores entre 2 y 6, excluyendo 2 pero incluyendo 6. [-3, 5) representa todos los valores entre -3 y 5, incluyendo -3 pero excluyendo 5.

operaciones inversas Dos operaciones que se anulan entre sí se llaman operaciones inversas.

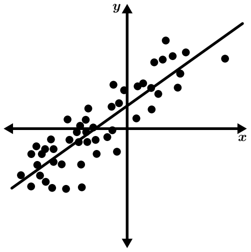
La suma y la resta son operaciones inversas. La multiplicación y la división son operaciones inversas.

número irracional Un número que no se puede escribir como una fracción de dos números enteros, donde el denominador es diferente de cero.

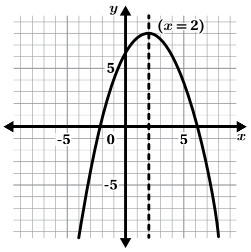
2 es un número racional porque se puede escribir como $\frac{2}{1}$, mientras que $\sqrt{3}$ es irracional porque no se puede escribir como una fracción de dos números enteros.

L

line of fit The line on a scatter plot that best represents the trend created by the points in a data set. There are many lines of fit for a single data set, but there is only one line of best fit.

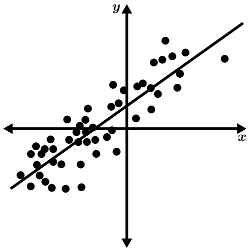


line of symmetry A line that divides a figure or the graph of a function into two halves. For every point (except the vertex), there is a corresponding point on the other side of the line that is the same distance from the line.

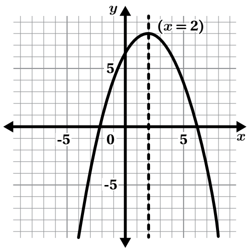


The equation of this line of symmetry is $x = 2$.

línea de (mejor) ajuste La línea de un diagrama de dispersión que mejor representa la tendencia que definen los puntos de un conjunto de datos. Existen muchas líneas de ajuste para un mismo conjunto de datos, pero solo hay una línea de mejor ajuste.



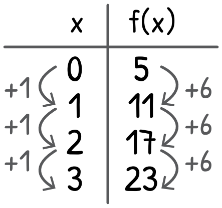
eje de simetría Una línea que divide una figura o la gráfica de una función en dos mitades. Cada punto (excepto el vértice) tiene un punto correspondiente en el otro lado de la línea, el cual está a la misma distancia de la línea.



La ecuación de este eje de simetría es $x = 2$.

English

linear function A function that increases or decreases by a constant rate of change, or shows a linear relationship. A function that changes by equal differences over equal intervals. The graph of a linear function is a line.



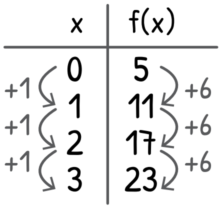
For example, $f(x) = 5 + 6x$ represents a linear function that has an initial value of 5 and increases by a constant difference of 6.

linear relationship See *linear function*.

lurking variable An additional variable that has an effect on the other variables being analyzed. Lurking variables can lead us to make conclusions about data and relationships that are inaccurate.

Español

función lineal Función que crece o decrece con una tasa de cambio constante o muestra una relación lineal. Una función que cambia con diferencias iguales en intervalos iguales. La gráfica de una función lineal es una recta.



Por ejemplo, $f(x) = 5 + 6x$ representa una función lineal que tiene un valor inicial de 5 y crece con una diferencia constante de 6.

relación lineal Ver *función lineal*.

variable de confusión Una variable adicional que tiene un efecto sobre las demás variables que se analizan. Las variables de confusión pueden producir conclusiones imprecisas en torno a los datos y las relaciones.

M

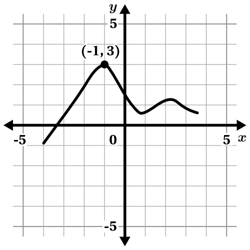
margin of error A measure that shows how much the results of a survey or study might differ from the true population value.

If a survey shows that 60% of people like a product with a margin of error of ±3%, the true percentage is likely between 57% and 63%.

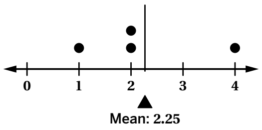
maximum The greatest value in a data set.

maximum (of a function) The highest point on the graph.

The maximum of this function is $(-1, 3)$.



mean A statistic used to describe the typical value of a data set. To calculate the mean, you can add up all the data values and divide by the number of data points. The mean is a measure of center and is also called the *average*.



In this data set, the mean is 2.25.

$1 + 2 + 2 + 4 = 9$

$\frac{9}{4} = 2.25$

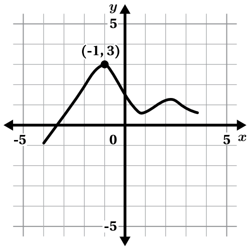
margen de error Una medida que muestra cuánto pueden diferir los resultados de una encuesta o un estudio del valor real de la población.

Si una encuesta muestra que al 60 % de las personas les gusta un producto con un margen de error de ±3 %, el porcentaje real probablemente esté entre el 57 % y el 63 %.

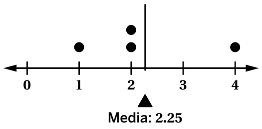
máximo El mayor valor en un conjunto de datos.

máximo (de una función) El punto más alto en una gráfica.

El máximo de esta función está en $(-1, 3)$.



media Una estadística que se usa para describir el valor típico de un conjunto de datos. Para calcular la media, se suman todos los valores y el resultado se divide por el número de puntos de datos. La media es una medida de tendencia central y también se denomina promedio.



En este conjunto de datos, la media es 2.25.

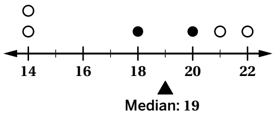
$1 + 2 + 2 + 4 = 9$

$\frac{9}{4} = 2.25$

English

measure of center A numerical value that describes the overall center or clustering of data in a set. The three most common measures of central tendency are the mean, median, and mode.

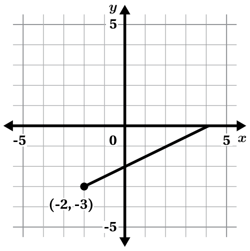
median A statistic used to describe the typical value of a data set. It is the middle value of a data set when the values are in numerical order. If there are two values in the middle of the data set, then the median is the mean of those two values. The median is a measure of center.



maximum The greatest value in a data set.

minimum (of a function) The lowest point on the graph.

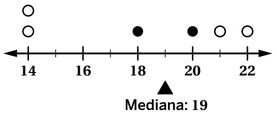
The minimum of this function is (-2, -3).



Español

medida de centro Un valor numérico que describe el centro general o la agrupación de datos en un conjunto. Las tres medidas de tendencia central más comunes son la media, la mediana y la moda.

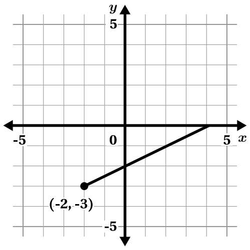
mediana Una estadística que se usa para describir el valor típico de un conjunto de datos. Es el valor del medio de un conjunto de datos cuando los valores se clasifican en orden numérico. Cuando hay dos valores en el medio del conjunto de datos, la mediana es la media de esos dos valores. La mediana es una medida de tendencia central.



máximo El mayor valor en un conjunto de datos.

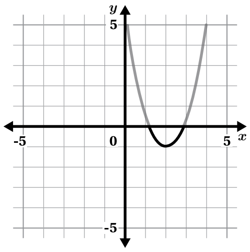
mínimo (de una función) El punto más bajo en una gráfica.

El mínimo de esta función está en (-2, -3).



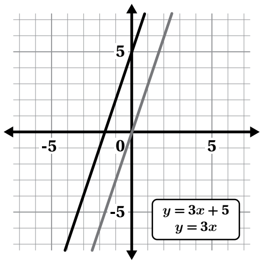
N

negative (interval or function) A function is negative when its outputs are negative and its graph is below the x-axis. A function can be negative for its entire domain or over an interval.



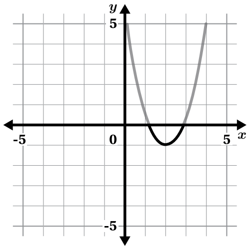
This function is negative when $1 < x < 3$.

no solution An equation has no solution if there is no value of the variable that will make the equation true. A system of equations has no solution if there is no set of values that makes all the equations in the system true. In a system of equations with no solution, there is no point that is on the graph of every equation in the system.



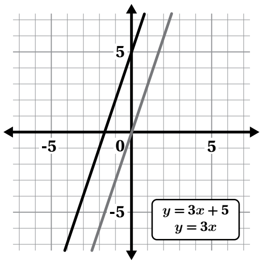
For example, the system of equations containing $y = 3x + 5$ and $y = 3x$ has no solution because the graphs are parallel and never intersect.

negativo (intervalo o función) Una función es negativa cuando sus salidas son negativas y su gráfica está por debajo del eje x. Una función puede ser negativa en todo su dominio o en un intervalo.



Esta función es negativa cuando $1 < x < 3$.

sin solución Una ecuación no tiene solución si no hay ningún valor de la variable que haga que la ecuación sea verdadera. Un sistema de ecuaciones no tiene solución si no hay ningún conjunto de valores que haga que todas las ecuaciones de ese sistema sean verdaderas. En un sistema de ecuaciones sin solución, no hay ningún punto que esté en la gráfica de cada una de las ecuaciones del sistema.



Por ejemplo, el sistema de ecuaciones que contiene $y = 3x + 5$ y $y = 3x$ no tiene solución porque las gráficas son paralelas y nunca se intersecan.

English

non-linear relationship A pattern, table, scenario, graph, or equation that does not have a constant rate of change. A non-linear relationship is called non-linear because its graph is not a line.

Day	Teal Globs
0	1
1	2
2	5
3	10

The equation $y = x^2 + 1$ represents a non-linear relationship because its rate of change is not constant and its graph is not a line.

numerical data See *quantitative data*.

Español

relación no lineal Patrón, tabla, contexto, gráfica o ecuación que no tiene una tasa de cambio constante. Una relación no lineal se denomina no lineal porque su gráfica no es una recta.

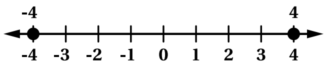
Día	Manchitas turquesa
0	1
1	2
2	5
3	10

La ecuación $y = x^2 + 1$ representa una relación no lineal porque su tasa de cambio no es constante y su gráfica no es una recta.

datos numéricos Ver *datos cuantitativos*.

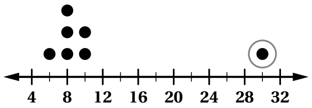
O

opposite Two numbers that are the same distance from 0 and on different sides of the number line. Two terms that are opposites are also referred to as zero pairs and additive inverses.



For example, 4 and -4 are opposites.

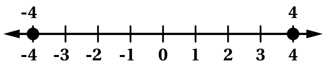
outlier A data value that is far from the other values in the data set. Values that are 1.5 times the IQR from either quartile 1 or quartile 3 are outliers.



The circled point is an outlier.

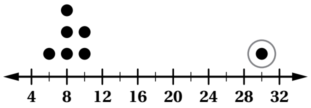
output The value of a function after it has been evaluated for an input value of x is called the *output*. The output is sometimes called the *dependent variable*. The output typically appears on the vertical axis of a graph and in the right-hand column of a table.

opuestos Dos números que están a la misma distancia del 0 y en diferentes lados de la recta numérica. Dos términos que son opuestos también se conocen como inversos aditivos.



Por ejemplo, 4 y -4 son opuestos.

valor atípico Un valor de datos que está lejos de los demás valores del conjunto de datos. Son valores atípicos aquellos que están a 1.5 veces el RQ del cuartil 1 o del cuartil 3.

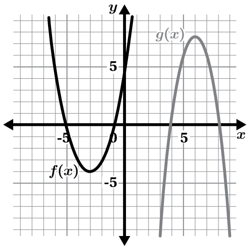


El punto encerrado en un círculo es un valor atípico.

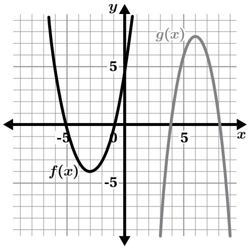
salida El valor de una función tras haber sido evaluada con un valor de entrada x se denomina salida. La salida a veces se denomina variable dependiente. La salida suele estar en el eje vertical de una gráfica y en la columna derecha de una tabla.

P

parabola The graph of a quadratic function, which is a U-shaped curve.

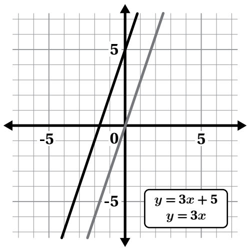


parábola La gráfica de una función cuadrática, que es una curva en forma de U.



English

parallel lines Lines that never cross or intersect. On a graph, two lines with the same slope and different y-intercepts are parallel.

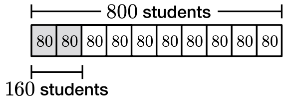


For example, the lines $y = 3x + 5$ and $y = 3x$ are parallel and never intersect.

percent decrease Describes how much a quantity goes down, expressed as a percent of the starting amount.

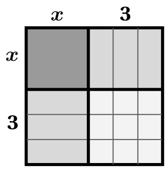
For example, the function $f(x) = 1000(.78)^x$ represents the value of a phone with an initial value of $1,000 with a percent decrease in value of 22% each year, x.

percent (percentage) Percent means for every 100. It is represented by the percent symbol: %. We use percentages to represent ratios and fractions.



For example, 20% means 20 : 100. 20% of a number means $\frac{20}{100}$ or $\frac{1}{5}$ of that number.
Let's say there are 800 students in a school. If 20% of them are on a field trip, that means 160 students because 20 students are on the trip for every 100 students total.

perfect square An expression that can be written as something multiplied by itself.



These expressions are perfect squares: 3^2, 9, $(x + 3)^2$, and $x^2 + 6x + 9$.

perimeter The sum of the lengths of all the sides of a polygon.

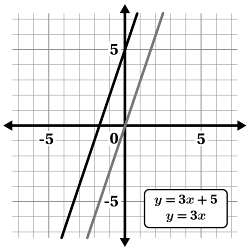
perpendicular Describes a line that crosses or meets another line at a 90° angle.

± (plus/minus symbol) A symbol used to represent both the positive and negative values of a number. It also can be used to represent two expressions.

±9 represents -9 and +9.
2 ± 3 represents 2 − 3 and 2 + 3.

Español

líneas (o rectas) paralelas Líneas que nunca se cruzan o intersecan. En una gráfica, dos rectas con la misma pendiente e intersecciones diferentes con el eje y son paralelas.

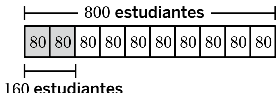


Por ejemplo, las rectas $y = 3x + 5$ y $y = 3x$ son paralelas y nunca se intersecan.

disminución porcentual Describe cuánto disminuye una cantidad y se expresa como un porcentaje de la cantidad inicial.

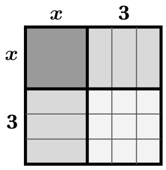
Por ejemplo, la función $f(x) = 1000(.78)^x$ representa el valor de un teléfono con un valor inicial de $1,000 y una disminución porcentual del 22% de su valor cada año, x.

por ciento (porcentaje) Por ciento significa por cada 100. Se representa con el símbolo de porcentaje: %. Usamos porcentajes para representar razones y fracciones.



Por ejemplo, 20% significa 20 : 100. 20% de un número significa $\frac{20}{100}$ o $\frac{1}{5}$ de dicho número.
Supongamos que hay 800 estudiantes en una escuela. Si el 20% de ellos están en una excursión, entonces eso es 160 estudiantes porque 20 están de viaje por cada 100 estudiantes en total.

cuadrado perfecto Una expresión que puede escribirse como algo multiplicado por sí mismo.



Estas expresiones son cuadrados perfectos: 3^2, 9, $(x + 3)^2$ y $x^2 + 6x + 9$.

perímetro La suma de las longitudes de todos los lados de un polígono.

perpendicular Describe una línea que cruza o se une con otra línea formando un ángulo de 90°.

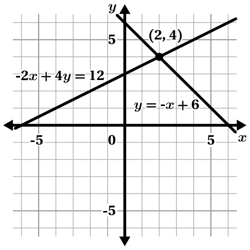
± (signo más menos) Un símbolo que se usa para representar los valores positivos y negativos de un número. También puede usarse para representar dos expresiones.

±9 representa -9 and +9.
2 ± 3 representa 2 − 3 y 2 + 3.

English

point of intersection A point where two lines or curves meet.

For example, $(2, 4)$ is the point of intersection for the lines $-2x + 4y = 12$ and $y = -x + 6$.



polynomial The sum or difference of terms that include variables raised to non-negative integer powers, with coefficients that can be real or complex numbers.

$2x - 7$ and $3x^2 - x + 4$ are polynomial expressions.
$f(x) = 2x - 7$ and $g(x) = 3x^2 - x + 4$ are polynomial functions.

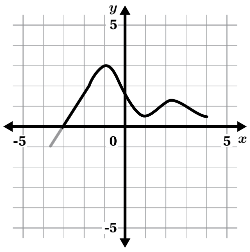
population A set of people or things that are being studied.

For example, if we want to study the heights of people on different sports teams, the population would be all the people on the teams.

population mean The average of all the values in a population. It is calculated by adding all the values and dividing by the total number of values.

positive (interval or function) A function is positive when its outputs are positive and its graph is above the x-axis. A function can be positive for its entire domain or over an interval.

This function is positive when $-3 < x < 4$.



principal The total amount (initial value) of money borrowed or invested, not including any interest.

proportional relationship A set of equivalent ratios. The values for one quantity are each multiplied by the same number to get the values for the other quantity.

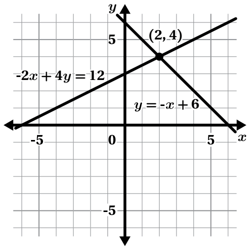
Carpet (sq. ft.)	Cost (dollars)
10 $\xrightarrow{\text{x 1.5}}$	15.00
20 $\xrightarrow{\text{x 1.5}}$	30.00
50 $\xrightarrow{\text{x 1.5}}$	75.00

For example, every cost in the table is equal to 1.5 times the number of square feet of carpet.

Español

punto de intersección Un punto donde se cruzan dos rectas o curvas.

Por ejemplo, $(2, 4)$ es el punto de intersección de las rectas $y = -x + 6$ y $-2x + 4y = 12$.



polinomio La suma o la diferencia de términos que incluyen variables elevadas a potencias enteras no negativas, con coeficientes que pueden ser números reales o complejos.

$2x - 7$ y $3x^2 - x + 4$ son expresiones polinómicas.
$f(x) = 2x - 7$ y $g(x) = 3x^2 - x + 4$ son funciones polinómicas.

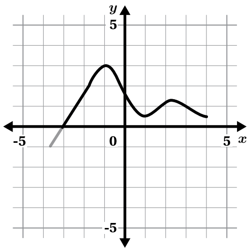
población Conjunto de personas o cosas que se estudian.

Por ejemplo, si queremos estudiar las alturas de las personas de diferentes equipos deportivos, la población sería todas las personas de los equipos.

media poblacional El promedio de todos los valores en una población. Se calcula sumando todos los valores y dividiendo por el número total de valores.

positivo (intervalo o función) Una función es positiva cuando sus salidas son positivas y su gráfica está por encima del eje x. Una función puede ser positiva en todo su dominio o en un intervalo.

Esta función es positiva cuando $-3 < x < 4$.



principal La cantidad total (valor inicial) de un préstamo o una inversión de dinero, sin incluir los intereses.

relación proporcional Un conjunto de razones equivalentes. Cada uno de los valores de una cantidad se multiplica por el mismo número para obtener los valores de la otra cantidad.

Alfombra (pies cuadrados)	Costo (dólares)
10 $\xrightarrow{\text{x 1.5}}$	15.00
20 $\xrightarrow{\text{x 1.5}}$	30.00
50 $\xrightarrow{\text{x 1.5}}$	75.00

Por ejemplo, cada costo en la tabla es igual a 1.5 veces el número de pies cuadrados de alfombra.

Glossary/Glosario

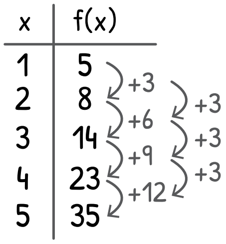
English

Q

quadratic formula A formula that can be used to determine the solutions of a quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

quadratic function A function with output values that change by a constant second difference. Equations of quadratic functions have a squared term when written in standard form. The graph of a quadratic function is a parabola.

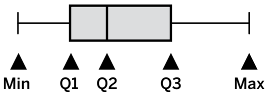


quadratic relationship See *quadratic function*.

quantitative data Data represented as numbers, quantities, or measurements that can be meaningfully compared is called *quantitative data*, or *numerical data*.

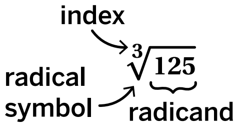
How many pets do you have? is a question that produces quantitative data. This is different from categorical data.

quartile Quartiles divide a data set into four sections. Quartile 1 is the median of the lower half of the data.
Q2 is the median.
Q3 is the median of the upper half of the data.
Q4 is the maximum.



R

radical A square root, cube root, fourth root, etc. The radical symbol is $\sqrt{\ }$.



$$\underbrace{\sqrt[3]{125}}$$

radicand The expression under a radical symbol. In the radical $\sqrt[n]{x}$ the quantity x is called the *radicand*. (See *radical*).

random Happening without a predictable pattern or order. In mathematics, a random event has outcomes that occur by chance.

Rolling a die is a random event because any number from 1 to 6 can appear with equal probability.

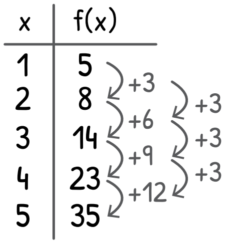
Español

Q

fórmula cuadrática Una fórmula que puede usarse para determinar las soluciones de una ecuación cuadrática $ax2 + bx + c = 0$, donde $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

función cuadrática Una función con valores de salida que cambian de acuerdo con una segunda diferencia constante. Las ecuaciones de las funciones cuadráticas tienen un término elevado al cuadrado cuando se escriben en forma estándar. La gráfica de una función cuadrática es una parábola.

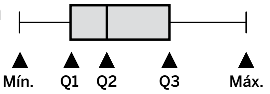


relación cuadrática Ver *función cuadrática*.

datos cuantitativos Los datos que se representan como números, cantidades o medidas y que pueden compararse de forma significativa se denominan datos cuantitativos o datos numéricos.

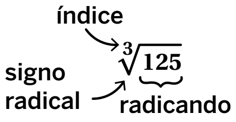
¿Cuántas mascotas tienes? es una pregunta que produce datos cuantitativos, que son diferentes de los datos categóricos.

cuartil Los cuartiles dividen un conjunto de datos en cuatro secciones.
El cuartil 1 es la mediana de la mitad inferior de los datos.
Q2 es la mediana.
Q3 es la mediana de la mitad superior de los datos.
Q4 es el máximo.



R

radical Una raíz cuadrada, raíz cúbica, raíz cuarta, etc. El signo radical es $\sqrt{\ }$.



$$\underbrace{\sqrt[3]{125}}$$

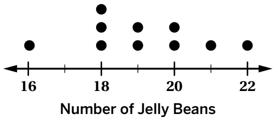
radicando La expresión bajo un signo radical. En un radical $\sqrt[n]{x}$, la cantidad x se llama radicando. (Ver *radical*).

aleatorio Algo que ocurre sin un patrón o un orden predecibles. En matemáticas, un suceso aleatorio tiene resultados que ocurren por casualidad.

Lanzar un dado es un suceso aleatorio porque cualquier número del 1 al 6 puede aparecer con la misma probabilidad.

English

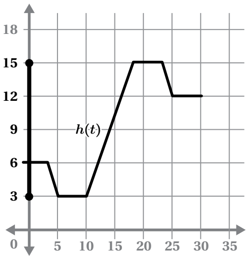
range The difference between the maximum and minimum values in a data set. Range is a measure of spread.



For example, the range of this data set is 6 jelly beans because $22 - 16 = 6$.

range (of a function) The set of all possible output values for a function or relation. The range can be described in words or as an inequality.

The range of this graph can be described as: All numbers from 3 to 15 or $3 \leq h(t) \leq 15$.



rate of change The ratio of the change in one quantity to the corresponding change in another quantity. In a linear relationship, the rate of change is the change in y divided by the change in x, which is also the slope of the graph.

rational exponent A real number that can be expressed as the ratio of two integers, where the denominator is not zero. Rational numbers can also appear as fractional exponents in mathematical expressions.

In the expression $5^{\frac{1}{2}}$, $\frac{1}{2}$ is a rational exponent.

rational number A number that can be written as a fraction of two integers, where the denominator is not zero.

$\frac{1}{3}$, -$\frac{7}{4}$, 0, 0.2, -5, and $\sqrt{9}$ are rational numbers.

reciprocal The reciprocal of a fraction $\frac{a}{b}$ is $\frac{b}{a}$. The product of two fractions that are reciprocals of one another is 1.

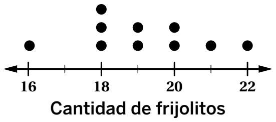
For example, $\frac{3}{2}$ and $\frac{2}{3}$ are reciprocals because $\left(\frac{3}{2}\right)\left(\frac{2}{3}\right) = 1$.

relation A way of creating input-output pairs. When a relation assigns exactly one output to every input, it is called a *function*.

relative frequency table A type of two-way table used to compare data across two categorical variables. Relative frequency tables present the fraction or percent of the data that is in that category, instead of the actual number of data points. You can use this representation to see if the data presents evidence that there is an association between the two variables.

Español

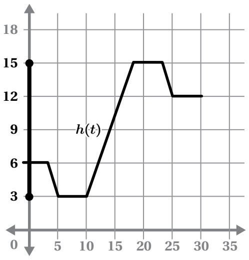
rango La diferencia entre los valores máximo y mínimo de un conjunto de datos. El rango es una medida de dispersión.



Por ejemplo, el rango de este conjunto de datos es 6 frijolitos de jalea porque $22 - 16 = 6$.

fluctuación (de una función) El conjunto de todos los posibles valores de salida de una función o relación. La fluctuación puede describirse con palabras o como una desigualdad.

La fluctuación de esta gráfica puede describirse de la siguiente manera: Todos los números del 3 al 15 o $3 \leq h(t) \leq 15$.



tasa de cambio La razón entre el cambio en una cantidad y el cambio correspondiente en otra cantidad. En una relación lineal, la tasa de cambio es el cambio en (x) dividido por el cambio en x, que también es la pendiente del gráfico.

exponente racional Un número real que puede expresarse como la razón de dos números enteros, donde el denominador no es cero. Los números racionales también pueden aparecer como exponentes fraccionarios en expresiones matemáticas.

En la expresión $5^{\frac{1}{2}}$, $\frac{1}{2}$ es un exponente racional.

número racional Un número que se puede escribir como una fracción de dos números enteros, donde el denominador es diferente de cero.

$\frac{1}{3}$, -$\frac{7}{4}$, 0, 0.2, -5 y $\sqrt{9}$ son números racionales.

recíproco El recíproco de una fracción $\frac{a}{b}$ es $\frac{b}{a}$. El producto de dos fracciones que son recíprocas entre sí es 1.

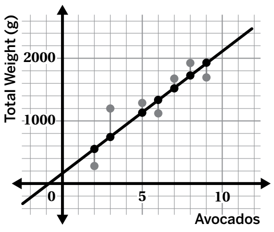
Por ejemplo, $\frac{3}{2}$ y $\frac{2}{3}$ son recíprocos porque $\left(\frac{3}{2}\right)\left(\frac{2}{3}\right) = 1$.

relación Una forma de establecer pares de entrada y salida. Cuando una relación asigna exactamente una salida a cada entrada, se denomina función.

tabla de frecuencia relativa Un tipo de tabla de doble entrada que se usa para comparar datos entre dos variables categóricas. Las tablas de frecuencia relativa presentan la fracción o el porcentaje de los datos que están en esa categoría, en lugar de la cantidad de puntos de datos. Esta representación puede emplearse para saber si los datos muestran que existe una asociación entre las dos variables.

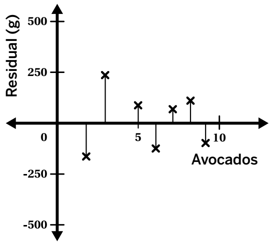
English

residual The difference between the y-value for a point in a scatter plot and the value predicted by the line of best fit.



The short lines connecting each point to the line of fit show the residual value for that point.

residual plot A scatter plot of residual values for a data set. The x-axis represents the value predicted by the line of best fit, and the y-value of each point represents the value of the residual.

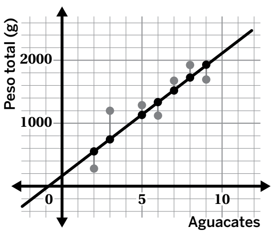


The residual plot shows how the total weight of different numbers of avocados vary from their predicted values.

revenue The amount of money generated by selling a product or service.

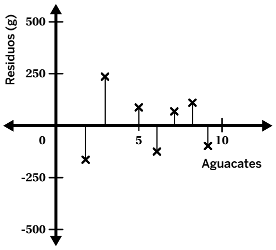
Español

residuo La diferencia entre el valor y de un punto en un diagrama de dispersión y el valor que predice la línea de mejor ajuste.



Las líneas cortas que conectan cada punto con la línea de ajuste muestran el valor del residuo del punto en cuestión.

diagrama de residuos Un diagrama de dispersión de los valores de los residuos de un conjunto de datos. El eje x representa el valor que predice la línea de mejor ajuste, y el valor y de cada punto representa el valor del residuo.



El diagrama de residuos muestra que el peso total de diferentes cantidades de aguacates varía con respecto a sus valores esperados.

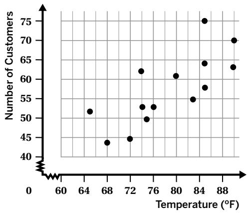
ingresos La cantidad de dinero que genera la venta de un producto o servicio.

S

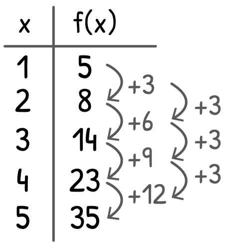
sample A part of a population.

For example, a population could be all the seventh-grade students at one school. One sample of that population is all the seventh-grade students who are in band.

scatter plot A graph in the coordinate plane that shows the relationship between two variables by plotting individual data points. Scatter plots are used to observe patterns in bivariate numerical data.



second difference The differences between consecutive output values in the table of a function are called *first differences*. The differences between those values are called *second differences*. Quadratic functions have constant second differences.

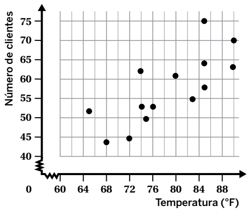


In this example, the first differences are 3, 6, 9, and 12. The second differences are constant at 3, so $f(x)$ is a quadratic function.

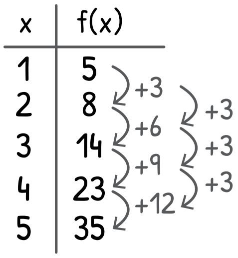
muestra Una parte de una población.

Por ejemplo, una población podría ser todos los estudiantes de séptimo grado de una escuela. Una muestra de esa población son todos los estudiantes de séptimo grado que están en la banda.

diagrama de dispersión Un gráfico en el plano de coordenadas que muestra la relación entre dos variables mediante el trazado de puntos de datos individuales. Los diagramas de dispersión se utilizan para observar patrones en datos numéricos bivariados.



segunda diferencia Las diferencias entre valores de salida consecutivos en la tabla de una función se llaman primeras diferencias. Las diferencias entre dichos valores se llaman segundas diferencias. Las funciones cuadráticas tienen segundas diferencias constantes.



En este ejemplo, las primeras diferencias son 3, 6, 9 y 12. Las segundas diferencias son constantes, de 3, por lo que $f(x)$ es una función cuadrática.

English

segmented bar graph A bar graph where each bar is divided into segments that represent different categories, showing how the total is split into parts.

set-builder notation A shorthand used to describe sets, often those with an infinite number of elements. It is written in the form $\{x : x > 5\}$, which is read as "the set of all x such that x is greater than 5." The colon (:) can also be replaced by a vertical line (|), as in $\{x \mid x > 5\}$, and is read the same way.

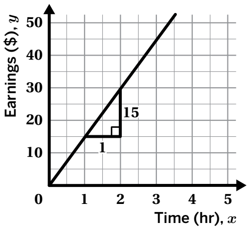
The set of all numbers greater than 8 can be written in set-builder notation as: $\{x \mid x>8\}$
This is read as "the set of all x such that x is greater than 8."

simple interest Interest that is calculated solely based on the initial amount. The balance in an account with simple interest is modeled by a linear function.

simulation An approximate imitation of a statistical experiment, often performed using a computer program to analyze the results of a large number of trials.

A simulation can be used to predict the likelihood of getting heads or tails by flipping a virtual coin many times.

slope (rate of change) The ratio of the change in the vertical direction (y-axis) to the change in the horizontal direction (x-axis).



In this graph, y increases by 15 dollars when x increases by 1 hour. The slope of the line is 15, and the rate of change is 15 dollars per hour.

slope-intercept form A way to write a linear equation that highlights the slope and the y-intercept of the line it represents. Slope-intercept form equations are written as $y = mx + b$, where m represents the slope, b represents the y-intercept of the line, and x and y are variables.

The equations $y = 2x + 4$ and $y = -5x - 10$ are in slope-intercept form. The equation $2x + 5y = 20$ is not in slope-intercept form.

Español

gráfico de barras segmentadas Un gráfico de barras donde cada barra está dividida en segmentos que representan diferentes categorías y muestran cómo el total se divide en partes.

notación de construcción de conjuntos Una abreviatura utilizada para describir conjuntos, a menudo aquellos con un número infinito de elementos. Se escribe en la forma $\{x : x > 5\}$, que se lee como "el conjunto de todas las x, de modo que x es mayor que 5". Los dos puntos (:) también se pueden reemplazar por una línea vertical (|), como en $\{x \mid x > 5\}$, y se leen de la misma manera.

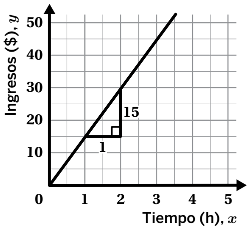
El conjunto de todos los números mayores que 8 se puede escribir en notación de construcción de conjuntos como: $\{x \mid x > 8\}$
Esto se lee como "el conjunto de todas las x, de modo que x es mayor que 8".

interés simple Interés que se calcula únicamente en función de la cantidad inicial. El saldo de una cuenta con interés simple se modela con una función lineal.

simulación Una imitación aproximada de un experimento estadístico, a menudo realizada mediante un programa informático para analizar los resultados de una gran cantidad de ensayos.

Se puede utilizar una simulación para predecir la probabilidad de obtener cara o cruz lanzando una moneda virtual muchas veces.

pendiente (tasa de cambio) La razón entre el cambio en la dirección vertical (eje y) y el cambio en la dirección horizontal (eje x).



En esta gráfica, y incrementa en 15 dólares cuando x incrementa en 1 hora. La pendiente de la recta es 15 y la tasa de cambio es de 15 dólares por hora.

forma pendiente-intersección, forma pendiente-ordenada al origen Una forma de escribir una ecuación lineal que destaca la pendiente y la intersección con el eje y (o la ordena al origen) de la recta que representa. Las ecuaciones en forma pendiente-intersección se escriben $y = mx + b$, donde m representa la pendiente, b representa la intersección con el eje y de la recta, y tanto x como y son variables.

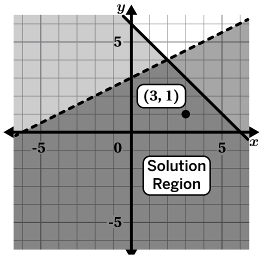
Las ecuaciones $y = 2x + 4$ y $y = -5x - 10$ están en la forma pendiente-intersección. La ecuación $2x + 5y = 20$ no está en la forma pendiente-intersección.

English

solution A value or set of values that makes an equation or inequality true.

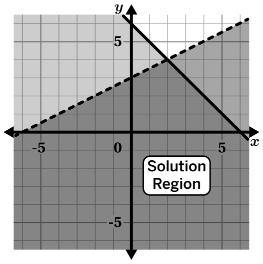
For example, $x = 2$ is a solution to the equation $3x + 4 = 10$.
$x > 2$ are the solutions to the inequality $3x + 4 > 10$.
The ordered pair $(1, 2)$ is a solution to the equation $3x + 4y = 11$.

solution (to a system of inequalities) An ordered pair that makes each inequality in a system true. Every ordered pair that is a solution to a system is located in the solution region where the graphs overlap.

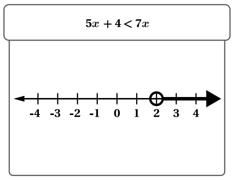


For system $y \leq -x + 6$ and $-2x + 4y < 12$, $(3,1)$ is a solution to this system of inequalities because it makes both inequalities true and falls in the region where the inequalities overlap.

solution region The set of all ordered pairs that make an inequality or each inequality in a system true. For a two-variable linear inequality, the solution region is a half-plane. For a system of inequalities, the solution region is located where the graphs overlap.



solution set The set of all values that makes an inequality true. To describe a solution set symbolically, we often use inequalities. To describe a solution set graphically, we often shade a portion of a number line or a region of the coordinate plane.



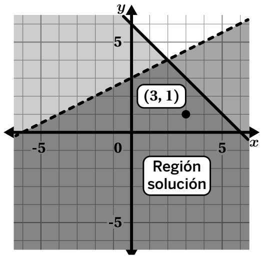
The solution set for the inequality $5x + 4 < 7x$ includes all of the values that are larger than 2.
2 is not included in the solution set.

Español

solución Un valor o conjunto de valores que hacen que una ecuación o una desigualdad sean verdaderas.

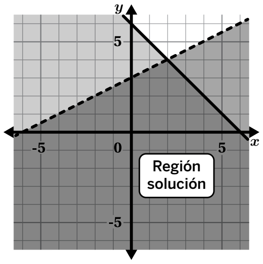
Por ejemplo, $x = 2$ es una solución de la ecuación $3x + 4 = 10$.
$x > 2$ son las soluciónes de la desigualdad $3x + 4 > 10$.
El par ordenado $(1, 2)$ es una solución de la ecuación $3x + 4y = 11$.

solución (de un sistema de desigualdades) Un par ordenado que hace que cada desigualdad de un sistema sea verdadera. Cada par ordenado que sea una solución de un sistema se encuentra en la región solución donde se superponen las gráficas.

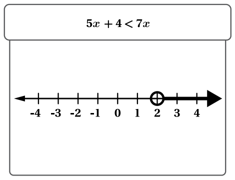


Por ejemplo, en el sistema de desigualdades $y \leq -x + 6$ y $-2x + 4y < 12$, la solución es $(3,1)$ porque hace que ambas desigualdades sean verdaderas y se ubica en la región donde las desigualdades se suporponen.

región solución El conjunto de todos los pares ordenados que hacen que una desigualdad, o cada desigualdad de un sistema, sean verdaderas. En una desigualdad lineal de dos variables, la región solución es un semiplano. En un sistema de desigualdades, la región solución se encuentra donde se superponen las gráficas.



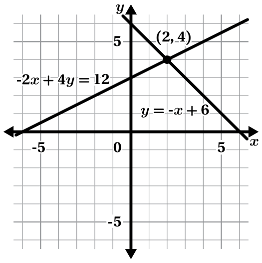
conjunto de soluciones El conjunto de todos los valores que hacen verdadera una desigualdad. Para describir con signos un conjunto de soluciones suelen emplearse desigualdades. Para describir con gráficas un conjunto de soluciones suele colorearse o sombrearse una parte de una recta numérica o una región del plano de coordenadas.



El conjunto de soluciones de la desigualdad $5x + 4 \geq 7x$ incluye todos los valores mayores que 2. El 2 no está incluido en el conjunto de soluciones.

English

solution to a system of equations
A solution to a system of equations is a set of values that makes all equations in that system true. When the equations are graphed, the solution to the system is the point of intersection.



For the system $y = -x + 6$ and $-2x + 4y = 12$, $(2, 4)$ is the solution to this system of equations and the point of intersection on the graph.

square root The square root of a number n (written as $\sqrt{n}$) is the positive number that can be squared to get n. The square root is also the side length of a square with an area of n.

The square root of 16 ($\sqrt{16}$) is 4 because 4^2 is 16. The $\sqrt{16}$ is also the side length of a square that has an area of 16.

standard form (of a linear equation) Linear equations that are written in the form $ax + by = c$, where a, b, and c are constants and x and y are variables.

The equations $2x + 5y = 20$ and $3x - 4y = -10$ are in standard form.
The equation $y = 2x + 4$ is not in standard form.

standard form (of a quadratic equation) One of three common forms of a quadratic equation. A quadratic equation in standard form looks like $f(x) = ax^2 + bx + c$.

These equations are in standard form:
$y = 2x^2 + 5x + 3$
$h(x) = x^2 + 3x$
$4x^2 - 7 = f(x)$

substitute Replace a variable or expression with a value or another expression.

$$4x = 4(5)$$

In this example, 5 is substituted for x in the expression $4x$.

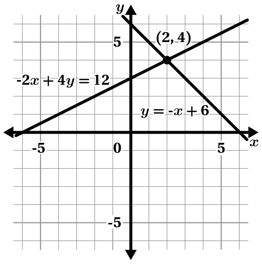
substitution A method of solving systems of equations where a variable is replaced with an equivalent expression in order to produce a new equation with fewer variables.

$y = \boxed{-4x + 6}$ $\quad y = 3x - 15$
$-4x + 6 = 3x - 15$
$-7x = -21$
$\boxed{x = 3}$
$y = 3(3) - 15$
$\boxed{y = -6}$

For example, we can substitute $-4x + 6$ in for y in $y = 3x - 15$ because they are equivalent.

Español

solución de un sistema de ecuaciones Una solución de un sistema de ecuaciones es un conjunto de valores que hace que todas las ecuaciones de ese sistema sean verdaderas. Al graficar las ecuaciones, la solución del sistema es el punto de intersección.



Por ejemplo, en el sistema de ecuaciones $y = -x + 6$ y $-2x + 4y = 12$, la solución y el punto de intersección de la gráfica es $(2, 4)$.

raíz cuadrada La raíz cuadrada de un número n (se escribe $\sqrt{n}$) es el número positivo que puede elevarse al cuadrado para obtener n. La raíz cuadrada también es la longitud de lado de un cuadrado con un área de n.

La raíz cuadrada de 16 ($\sqrt{16}$) es 4 porque 4^2 es 16. La $\sqrt{16}$ también es la longitud de lado de un cuadrado que tiene un área de 16.

forma estándar (de una ecuación lineal) Las ecuaciones lineales que se escriben en la forma $ax + by = c$, donde a, b y c son constantes, y tanto x como y son variables, se conocen como ecuaciones en forma estándar.

Las ecuaciones $2x + 5y = 20$ y $3x - 4y = -10$ están en forma estándar.
La ecuación $y = 2x + 4$ no está en forma estándar.

forma estándar (de una ecuación cuadrática) Una de las tres formas comunes de una ecuación cuadrática. Una ecuación cuadrática en forma estándar tiene el siguiente orden: $f(x) = ax^2 + bx + c$.

Estas ecuaciones están en forma estándar:
$y = 2x^2 + 5x + 3$
$h(x) = x^2 + 3x$
$4x^2 - 7 = f(x)$

sustituir Reemplazar una variable o expresión por un valor u otra expresión.

$$4x = 4(5)$$

En este ejemplo, el 5 sustituye a la x en la expresión $4x$.

sustitución Un método para resolver sistemas de ecuaciones donde una variable se reemplaza con una expresión equivalente para producir una nueva ecuación con menos variables.

$y = \boxed{-4x + 6}$ $\quad y = 3x - 15$
$-4x + 6 = 3x - 15$
$-7x = -21$
$\boxed{x = 3}$
$y = 3(3) - 15$
$\boxed{y = -6}$

Por ejemplo, podemos introducir $-4x + 6$ en lugar de y en $y = 3x - 15$ porque son equivalentes.

English

survey A method of collecting data by asking people to answer a given set of questions.

system of equations Two or more equations that represent the constraints on a shared set of variables form a system of equations.

These equations make a system:
$3b + c = -2$
$b - 5c = 12$

system of inequalities Two or more inequalities that represent the constraints on a shared set of variables form a system of inequalities.

These inequalities make a system:
$10m + 5n > -2$
$m - 5n \leq 12$

Español

encuesta Un método de recopilación de datos que consiste en pedir a personas que respondan una serie de preguntas.

sistema de ecuaciones Dos o más ecuaciones que representan las restricciones de un conjunto compartido de variables forman un sistema de ecuaciones.

Estas ecuaciones forman un sistema:
$3b + c = -2$
$b - 5c = 12$

sistema de desigualdades Dos o más desigualdades que representan las restricciones de un conjunto compartido de variables forman un sistema de desigualdades.

Estas desigualdades forman un sistema:
$10m + 5n > -2$
$m - 5n \leq 12$

T

total relative frequency table A type of two-way table used to compare data across two categorical variables. Total relative frequency tables present the fraction or percentage of the data that is in that category, instead of the number of data points. You can use this representation to see if the data presents evidence that there is an association between the two variables.

	Meditated	Did Not Meditate	Total
Calm	46%	8%	54%
Agitated	24%	22%	46%
Total	70%	30%	100%

translation A transformation that moves every point in a function a given distance in a given direction. A translation changes the location of a function, but does not change its shape.

two-way table A way to compare two categorical variables. It shows one of the variables across the top and the other down one side. Each entry in the table is the frequency or relative frequency of the category shown by the column and row headings.

	Meditated	Did Not Meditate	Total
Calm	45	8	53
Agitated	23	21	44
Total	68	29	97

tabla de frecuencia relativa total Un tipo de tabla de doble entrada que se usa para comparar datos entre dos variables categóricas. Las tablas de frecuencia relativa total presentan la fracción o el porcentaje de los datos que se encuentran en esa categoría, en lugar de la cantidad de puntos de datos. Esta representación puede emplearse para saber si los datos muestran que existe una asociación entre las dos variables.

	Meditaron	No meditaron	Total
Calmados	46%	8%	54%
Agitados	24%	22%	46%
Total	70%	30%	100%

traslación Una transformación que mueve cada punto de una función una determinada distancia en una determinada dirección. Una traslación cambia la ubicación de una función, pero no su forma.

tabla de doble entrada Una manera de comparar dos variables categóricas. Muestra una de las variables en la fila superior y la otra variable en la columna de un costado. Cada entrada de la tabla es la frecuencia o frecuencia relativa de la categoría que describen los títulos de la columna y la fila.

	Meditaron	No meditaron	Total
Calmados	45	8	53
Agitados	23	21	44
Total	68	29	97

English

Español

U

univariate data A data set that involves one variable. Each data point contains one piece of information.

A collection of students' heights is a univariate data set. Tables, dot plots, and bar graphs are useful for displaying univariate data.

datos univariados Un conjunto de datos que incluye una variable. Cada punto de datos contiene una información.

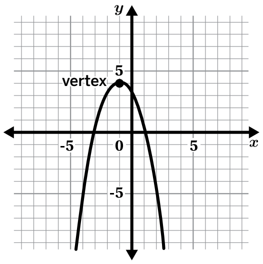
Una colección de las estaturas de estudiantes es un conjunto de datos univariado.
Las tablas, los diagramas de puntos y los diagramas de barras son útiles para mostrar datos univariados.

V

variable A letter or symbol that represents a value or set of values.

In the expression $10 - x$, the variable is x.

vertex On the graph of a quadratic or absolute value function, the vertex is the maximum or minimum point. The vertex is also where the function changes from increasing to decreasing, or vice versa.



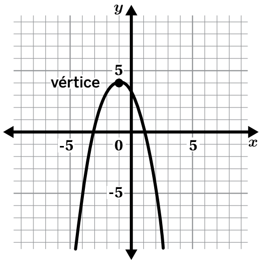
vertex form One of three common forms of a quadratic equation. A quadratic equation in vertex form looks like $f(x) = a(x - h)^2 + k$.

These equations are in vertex form:
$(x - 3)^2 + 10 = g(x)$
$y = 2(x + 8)^2 - 1$
$f(x) = -(x - 6)^2 + 15$

variable Una letra o un símbolo que representa un valor o un conjunto de valores.

En la expresión $10 - x$, la variable es x.

vértice En la gráfica de una función cuadrática o una función de valor absoluto, el vértice es el punto máximo o mínimo. El vértice también es donde la función cambia de creciente a decreciente, o viceversa.



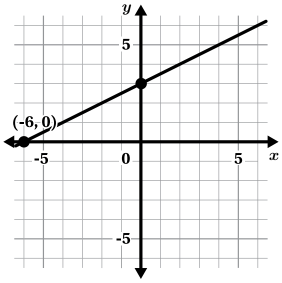
forma de vértice Una de las tres formas comunes de una ecuación cuadrática. Una ecuación cuadrática en forma de vértice tiene el siguiente orden: $f(x) = a(x - h)2 + k$.

Estas ecuaciones están en forma de vértice:
$(x - 3)^2 + 10 = g(x)$
$y = 2(x + 8)^2 - 1$
$f(x) = -(x - 6)^2 + 15$

X

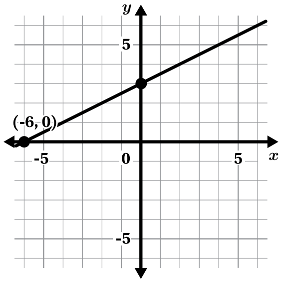
x-intercept A point where the graph of an equation or function crosses the x-axis or when $y = 0$.

The x-intercept of the graph of $-2x + 4y = 12$ is $(-6, 0)$, or just -6.



intersección con el eje x, abscisa al origen Un punto donde la gráfica de una ecuación o función cruza el eje x, o cuando $y = 0$.

La intersección con el eje x de la gráfica de $-2x + 4y = 12$ es $(-6, 0)$, o simplemente -6.



Algebra 1 / Álgebra 1

Glossary/Glosario

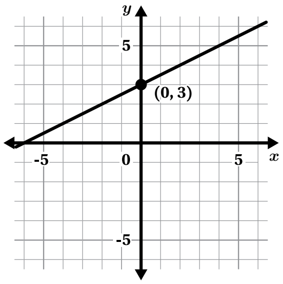
English

Español

Y

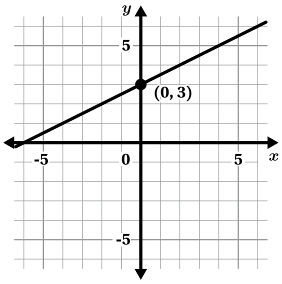
y-intercept A point where the graph of an equation or function crosses the y-axis or when $x = 0$.

The y-intercept of the graph -$2x + 4y = 12$ is $(0, 3)$, or just 3.



intersección con el eje y, ordenada al origen Un punto donde la gráfica de una ecuación o función cruza el eje y, o cuando $x = 0$.

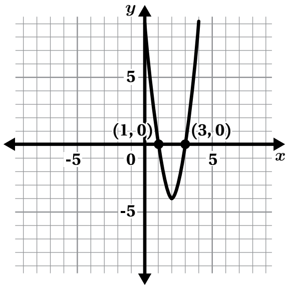
La intersección con el eje y de la gráfica de -$2x + 4y = 12$ es $(0, 3)$, o simplemente 3.



Z

zero-product property A property which states that if the product of two or more factors is 0, then at least one of the factors is 0. This property can be used to help solve equations.

If $(2x - 3)(x + 1) = 0$, then either $2x - 3 = 0$ or $x + 1 = 0$.

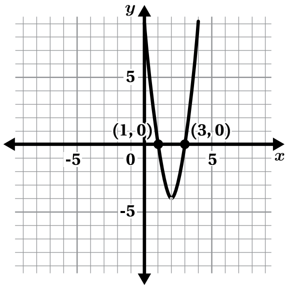


zeros The x-values that make a function equal zero, or $f(x) = 0$.

The zeros of $f(x) = 4(x - 1)(x - 3)$ are 1 and 3.

propiedad del producto cero Una propiedad que establece que si el producto de dos o más factores es 0, entonces al menos uno de los factores es 0. Esta propiedad puede usarse como ayuda para resolver ecuaciones.

Si $(2x - 3)(x + 1) = 0$, entonces $2x - 3 = 0$ o $x + 1 = 0$.



ceros Los valores x que hacen que una función sea igual a cero, o $f(x) = 0$.

Los ceros de $f(x) = 4(x - 1)(x - 3)$ son 1 y 3.

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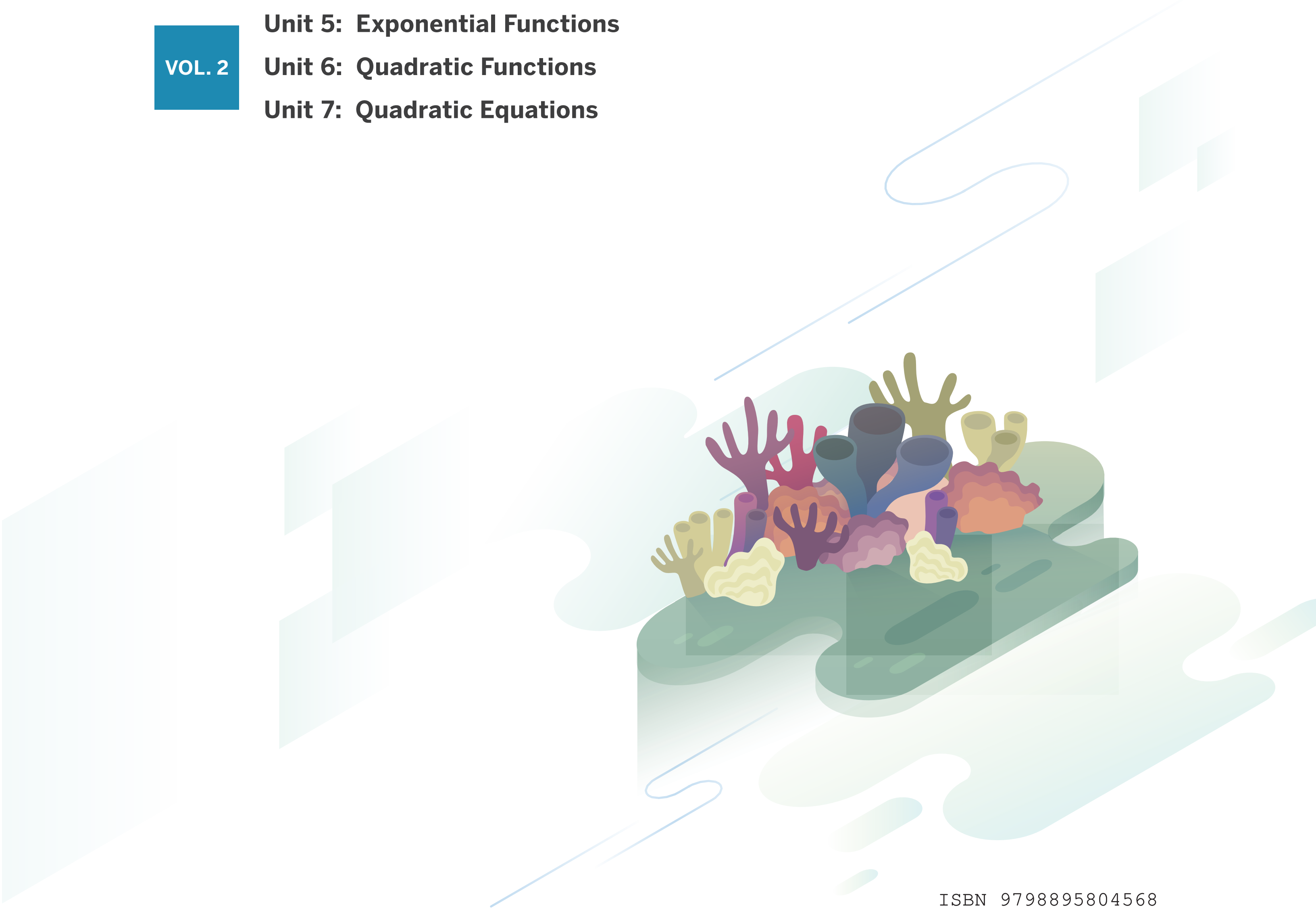

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