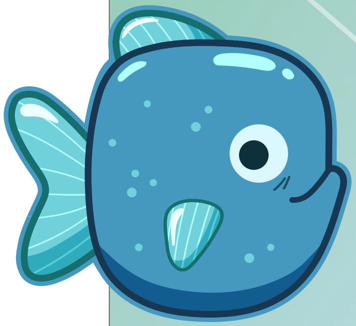
Unit 6

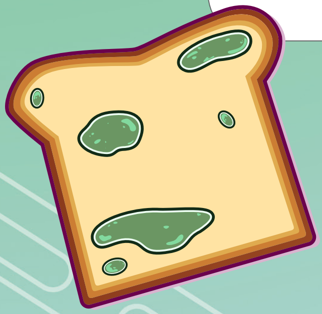
Exponential Functions

Sometimes a small change can make a big difference over time. In this unit, you will learn about relationships that grow quickly over time. You will learn to distinguish between linear and exponential relationships, and create equations to represent them. You will model situations that increase or decrease by a percentage. You will also model data and compound interest with exponential functions.

Essential Questions

- How do exponential and linear functions compare?
- What type of function models repeated percent increase or decrease?
- How can you model situations involving compound interest and population growth?





Summary | Lesson 1

Situations that include repeated multiplication can be modeled using an equation of the form $y = a \cdot b^x$, where a represents the initial value and b represents the base. The base determines whether the quantity in the situation will increase or decrease over time. You can use the equation to solve problems about the situation.

For example, Carlos bought a new mega-growing fish. The fish weighed 4 grams when Carlos bought it, and its mass grows 1.5 times greater every hour.

He wrote the equation $m = 4 \cdot 1.5^t$ to represent the relationship, where:

- m represents the mass of Carlos's fish in grams.
- t represents the time in hours.

You can use the equation to determine the mass of the fish after 4 hours by substituting $t = 4$ and solving for m.

$m = 4 \cdot 1.5^t$

$m = 4 \cdot 1.5^4$

$m = 4 \cdot 5.0625$

$m = 20.25$

After 4 hours, the fish will have a mass of 20.25 grams.

Try This

Here is an equation that models the amount of subscribers to Marc's streaming channel, where s represents the amount of subscribers and m represents the number of months since Marc started the channel.

$$s = 12 \cdot 2^m$$

a Explain what 12 means in this situation.

b Explain what 2 means in this situation.

c How many subscribers will Marc's channel have after 8 months?

Summary | Lesson 2

An **exponential function** increases or decreases by a *constant ratio*. The constant ratio will multiply consecutive y-values. Another name for the constant ratio is the **growth factor**.

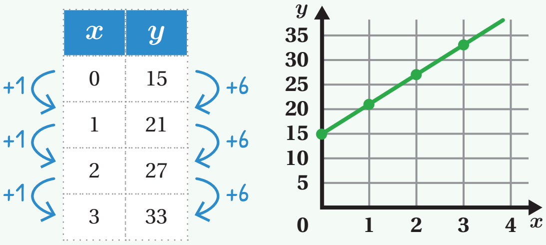
A **linear function** increases or decreases by a *constant difference*. Another name for the constant difference is the *rate of change*.

Here are two examples.

Linear Function

This pattern has a constant difference of 6, so the rate of change is 6.

The graph of this linear function is a straight line.

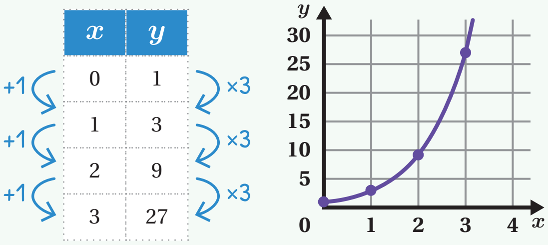


x	y
0	15
1	21
2	27
3	33

Exponential Function

This pattern has a constant ratio of 3, so the growth factor is 3.

The graph of this exponential function is a curve that gets steeper and steeper.



x	y
0	1
1	3
2	9
3	27

Try This

Determine whether each table shows a linear function, exponential function, or neither. Explain your thinking. Draw on the tables if it helps you to show your thinking.

a

x	y
3	5
4	15
5	45

Function type: ....

Explanation:

b

x	y
2	25
3	50
4	75

Function type: ....

Explanation:

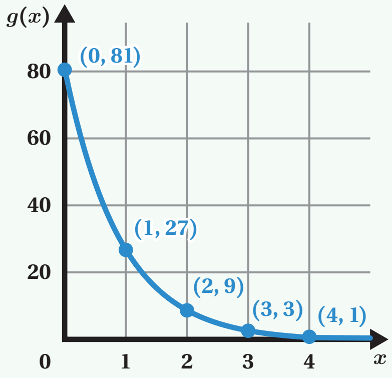
Summary | Lesson 3

In the exponential function $f(x) = a \cdot b^x$, a represents the y-*intercept* and b represents the growth factor. Both parts can be seen on a graph.

Let's look at two examples.

Here is the graph of the exponential function $g(x) = 81 \cdot \left(\frac{1}{3}\right)^x$.

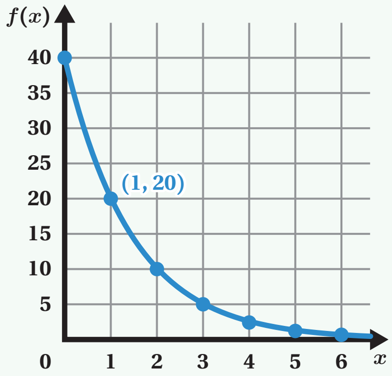
- 81 means that $(0, 81)$ is the y-intercept.
- $\frac{1}{3}$ is the growth factor. As x increases by 1, the y-values are multiplied by a factor of $\frac{1}{3}$. $81 \cdot \left(\frac{1}{3}\right) = 27$.



You can also write an exponential equation to represent a graph.

- The y-intercept is $(0, 40)$, so 40 is the a-value.
- The points $(1, 20)$, $(2, 10)$, and $(3, 5)$ are on the graph of the function. Since 10 is $\frac{1}{2}$ of 20 and 5 is $\frac{1}{2}$ of 10, this function has a growth factor of $\frac{1}{2}$, which is the b-value in the equation.

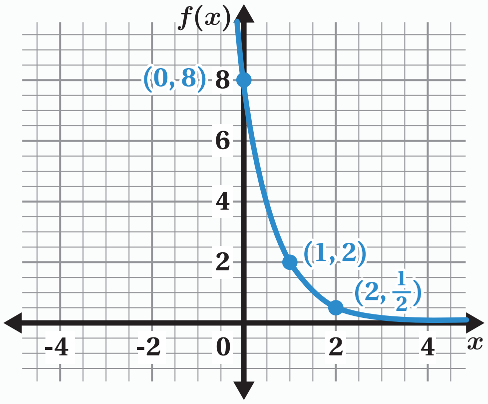
One way to write the equation of this exponential function is $f(x) = 40 \cdot \left(\frac{1}{2}\right)^x$.



Try This

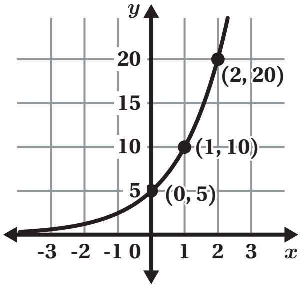
Here is the graph of an exponential function.

Write an equation that could represent this exponential relationship.



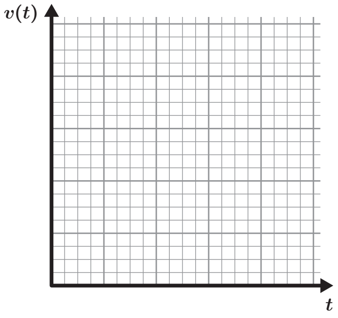
Summary | Lesson 4

The *growth factor* is the constant ratio that each term is multiplied by to generate an exponential relationship. You can identify the growth factor of an exponential function from a situation, an equation, a table, or a graph. Here is an example.

<table>
<tr><th></th><th>Exponential Function</th><th>The growth factor is 2 because . . .</th></tr>
<tr><td>Situation</td><td>There are 5 sq. mm of bacteria in a Petri dish. Each day, the number of cells doubles.</td><td>The cells are doubling every day.</td></tr>
<tr><td>Table</td><td><table><tr><th>x</th><th>$g(x)$</th></tr><tr><td>0</td><td>5</td></tr><tr><td>1</td><td>10</td></tr><tr><td>2</td><td>20</td></tr></table></td><td>$5 \cdot 2 = 10$ and $10 \cdot 2 = 20$.</td></tr>
<tr><td>Graph</td><td>
</td><td>The distance between the points $(0, 5)$ and $(1, 10)$ doubles in the y direction.</td></tr>
<tr><td>Equation</td><td>$g(x) = 5 \cdot 2^x$</td><td>2 is the base of the exponential term and it is raised to the power of x.</td></tr>
</table>

Try This

Fill in the missing parts of each representation.

<table>
<tr><th>Situation</th><th>Equation</th><th>Table</th><th>Graph</th></tr>
<tr><td>In 2020, a coral reef had a volume of 20 cubic meters that changed by a factor of ______.</td><td>$v(t) = \underline{\qquad} \cdot \left(\frac{1}{2}\right)^t$</td><td><table><tr><th>$t$</th><th>$v(t)$</th></tr><tr><td>0</td><td></td></tr><tr><td>1</td><td></td></tr><tr><td>2</td><td></td></tr><tr><td>3</td><td></td></tr></table></td><td>
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Summary | Lesson 5

You can evaluate exponential functions for inputs that are positive, negative, or zero.

Let's evaluate the function $f(x) = 9 \cdot 4^x$ for $f(\text{-}3)$ using the equation or a table.

You can substitute a value into the *equation*.

Steps	Explanation
$f(\text{-}3) = 9 \cdot 4^{(\text{-}3)}$	Substitute $x = (\text{-}3)$ into the function.
$f(\text{-}3) = 9 \cdot \left(\frac{1}{4}\right)^3$	Apply the property of negative *exponents* to rewrite the expression with a positive exponent.
$f(\text{-}3) = 9 \cdot \left(\frac{1}{64}\right)$	Rewrite the expression without any exponents.
$f(\text{-}3) = \frac{9}{64}$	Multiply to determine the value of $f(\text{-}3)$.

You can move backward in a *table*.

x	$f(x)$	
-3	$\frac{9}{64}$	$\times \frac{1}{4}$
-2	$\frac{9}{16}$	$\times \frac{1}{4}$
-1	$\frac{9}{4}$	$\times \frac{1}{4}$
0	9	

Multiply 9 by $\frac{1}{4}$ three times to determine the value of $f(\text{-}3)$.

The domain of an exponential function can include values like zero, all positive, or all negative values. It is important to use the information in the situation to decide which values make sense and which do not. For example, a function that models the amount of caffeine remaining in the body after drinking coffee does not include negatives in the domain. However, an exponential model of the population of ants can include negative numbers in the domain.

Try This

Here are two situations that can be modeled by exponential functions.

Situation A

Haru's social media account had 18 followers in 2020. The function $h(x)$ represents his total subscriber count x years after 2020.

$$h(x) = 18 \cdot \left(\frac{3}{2}\right)^x$$

Situation B

Kiana's cup of green tea has 20 milligrams of caffeine. The function $k(x)$ represents the total amount of caffeine in Kiana's system x hours after she finishes her tea.

$$k(x) = 20\left(\frac{4}{5}\right)^x$$

a Determine the value of $h(\text{-}1)$.

b Determine the value of $k(\text{-}1)$.

c Explain whether $h(\text{-}1)$ or $k(\text{-}1)$ make sense in their corresponding situations.

Summary | Lesson 6

A linear function always increases (or decreases) by equal differences over equal *intervals*, and an exponential function increases (or decreases) by equal *factors* over equal intervals.

Here are two examples.

	Linear function $g(x) = 2x + 3$	**Exponential function** $h(x) = 3^x$
Describe how the function changes when x grows by 1	When x grows by 1, $g(x)$ will always increase by $2(1) = 2$: $g(x + 1) - g(x)$ $= (2(x + 1) + 3) - (2x + 3)$ $= (2x + 2 + 3) - (2x + 3)$ $= 2x + 5 - 2x - 3$ $= 5 - 3$ $= 2$	When x grows by 1, $h(x)$ will always multiply by a factor of $3^1 = 3$. $\dfrac{h(x + 1)}{h(x)}$ $= \dfrac{3^{(x + 1)}}{3^x}$ $= \dfrac{3^x \cdot 3^1}{3x}$ $= 3$
Describe how the function changes when x grows by 4	When x grows by 4, $g(x)$ will always increase by $2(4) = 8$. $g(x + 4) - g(x)$ $= (2(x + 4) + 3) - (2x + 3)$ $= (2x + 8 + 3) - (2x + 3)$ $= 2x + 11 - 2x - 3$ $= 11 - 3$ $= 8$	When x grows by 4, $h(x)$ will always multiply by a factor of $3^4 = 81$. $\dfrac{h(x + 4)}{h(x)}$ $= \dfrac{3^{(x + 4)}}{3^x}$ $= \dfrac{3^x \cdot 3^4}{3x}$ $= 3^4$ $= 81$

Try This

Here is a function: $g(x) = 4^x$.

a Calculate $\dfrac{g(x+1)}{g(x)}$.

b Calculate $\dfrac{g(x+4)}{g(x)}$.

c Consider the function $f(x) = 4x + 10$. Would $f(x + 4)$ grow similarly to the growth in $\dfrac{g(x+4)}{g(x)}$? Explain your thinking.

Summary | Lesson 7

Exponential functions can be written in the form $f(x) = a \bullet b^x$, where a is the *initial value* and b is the growth factor. Exponential functions represent repeated *percent increase* when the growth factor is larger than 1.

In situations that model repeated percent increase, the growth factor b can be written as 1 (representing 100%) plus the percent increase in decimal form.

For example, the value of a baseball card collection increases by 4% every year. In 2020, the collection was valued at $\$500$. Let $f(x)$ represent the value of the collection and x represent the years since 2020.

- At first, the collection was valued at $\$500$. The initial value, or a, is 500.
- The value increases by 4% every year. Because the value of the cards increases by a repeated percent each year, we can represent the growth by changing the percent to a decimal (4% to 0.04) and adding 1. The growth factor, or b, will be 1.04.

We can write the exponential function that represents this situation as $f(x) = 500 \bullet (1.04)^x$.

Try This

Ramon the biologist is studying a sample of mold. At the start of the study, the mold had an area of 12 square centimeters. The area of the mold increases by 60% every day.

a Determine which equation matches the situation where $m(x)$ represents the area of mold after x days:

A. $m(x) = 12(0.06)^x$ **B.** $m(x) = 12(0.6)^x$ **C.** $m(x) = 12(1.06)^x$ **D.** $m(x) = 12(1.6)^x$

b Describe a situation that matches one of the equations you didn't choose.

Summary | Lesson 8

Exponential functions with a growth factor between 0 and 1 represent exponential decay. You can describe a relationship that models exponential decay as a *percent decrease*.

In situations that model exponential decay, the growth factor b can be written as 1 (representing 100%) minus the percent of decay in decimal form.

For example, a potted plant is given 24 milliliters of fertilizer. The amount of fertilizer decreases by 1% every hour. We can write a function to represent this situation in the form $f(x) = a \bullet b^x$. Let $f(x)$ represent the amount of fertilizer left in the potted plant and x represent the time in hours since the potted plant received the fertilizer.

- At first, the amount of fertilizer is 24 milliliters. The *initial value*, or a, is 24.
- The amount of fertilizer decreases by 1% every hour. Because the fertilizer decreases by a constant percent, we can represent the decay by changing the percent to a decimal (1% to 0.01) and subtracting it from 1. The growth factor, or b, will be $1 - 0.01$ or 0.99.

We can write the exponential function that represents this situation as $f(x) = 24 \bullet (0.99)^x$.

Try This

A heated pot of beans has a temperature of $204°$. It cools at a rate of 20% per minute. Use $f(t)$ to represent the temperature of the pot t minutes after taking the pot off the stove.

Select *all* the equations that could model this situation.

☐ **A.** $f(t) = 204(0.2)^t$

☐ **B.** $f(t) = 204(0.8)^t$

☐ **C.** $f(t) = 204(0.98)^t$

☐ **D.** $f(t) = 204(1 - 0.2)^t$

☐ **E.** $f(t) = 204(1 - 0.02)^t$

Summary | Lesson 9

You can interpret exponential functions in the form $f(x) = a \cdot b^x$ and $f(x) = a \cdot b^x + k$ to help understand the situations they represent.

Here are functions that describe two treatment methods for algae blooms. Each function models the size of an algae bloom, in square meters, x weeks after using the treatment in the lake.

$$h(x) = 250 \cdot (0.95)^x$$

- The initial size of algae bloom on the lake is 250 square meters.
- The growth factor is 0.95. The percent decrease each week is 5% because $1 - 0.95 = 0.05$.
- Over a long period, the algae bloom will eventually decrease in size to cover very close to 0 square meters.

$$j(x) = 250 \cdot (0.95)^x + 20$$

- The initial size of the algae bloom on the lake is 270 square meters because $250 \cdot (0.95)^0 + 20 = 250 + 20 = 270$.
- The value of b is 0.95, which means the amount of algae bloom is decreasing since the value is less than 1.
- Over a long period, the algae bloom will eventually decrease in size to cover very close to 20 square meters.

Try This

Write an exponential equation that matches this description:

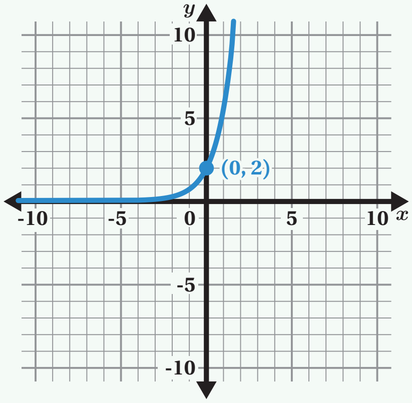
- The initial size of an algae bloom is 150 square meters.
- The size of the algae bloom decreases at a rate of 15% each week.
- Over time, the algae bloom will decrease to a size very close to 7 square meters.

Summary | Lesson 10

Here is the graph of $f(x) = 2 \cdot 3^x$.

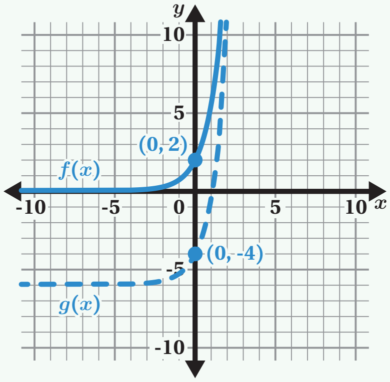
Functions can be translated horizontally and vertically.

Here are two examples of *translations* of $f(x)$.



Vertical Translations

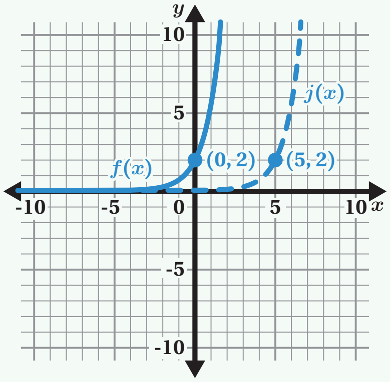
The equation $f(x) = 2 \cdot 3^x + k$ represents a vertical translation by k units.



Here $f(x)$ was translated 6 units down and can be represented with the new equation: $g(x) = 2 \cdot 3^x - 6$.

Horizontal Translations

The equation $f(x) = 2 \cdot 3^{(x-h)}$ represents a horizontal translation by h units.



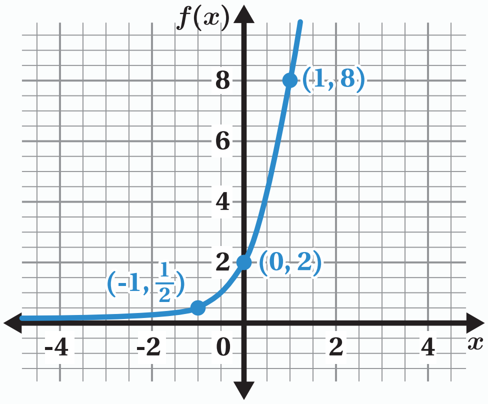
Here $f(x)$ was translated 5 units to the right and can be represented with the new equation: $j(x) = 2 \cdot 3^{(x-5)}$.

Try This

Here is a graph of the exponential function $f(x) = 2 \cdot 4^x$.

a Write and graph an equation for $g(x)$ that represents $f(x)$ translated up 2 units.

b Write and graph an equation for $h(x)$ that represents $f(x)$ translated to the right 3 units.



Summary | Lesson 11

The symbol $\sqrt{}$ is called a **radical**. You can write equivalent expressions using radicals and **rational exponents**, exponents that are written as a fraction.

Here are some examples of equivalent expressions.

Raising a number or expression to the $\frac{1}{n}$ power is equivalent to taking its nth root. Written symbolically that means $x^{\frac{1}{n}} = \sqrt[n]{x}$

Radical Expression	Exponential Expression
$\sqrt{16}$	$16^{\frac{1}{2}}$
$\sqrt[3]{125}$	$125^{\frac{1}{3}}$
$\sqrt[4]{58}$	$58^{\frac{1}{4}}$
$\sqrt[5]{70}$	$70^{\frac{1}{5}}$

Try This

Complete the table.

Radical Expression	Exponential Expression	Value
$\sqrt{81}$		
	$81^{\frac{1}{4}}$	
$\sqrt[3]{343}$		
	$7776^{\frac{1}{5}}$	

You can write many equivalent expressions using *radicals* and *rational exponents*.

Here are some examples.

	Equivalent Expressions				
$x^{\frac{6}{5}}$	$(x^6)^{\frac{1}{5}}$	$\left(x^{\frac{1}{5}}\right)^6$	$\sqrt[5]{x^6}$	$(\sqrt[5]{x})^6$	$\sqrt[5]{x \bullet x \bullet x \bullet x \bullet x \bullet x}$
$9^{\frac{5}{2}}$	$(9^5)^{\frac{1}{2}}$	$\left(9^{\frac{1}{2}}\right)^5$	$\sqrt{9^5}$	$(\sqrt{9})^5$	243
$16^{\frac{7}{4}}$	$(16^7)^{\frac{1}{4}}$	$\left(16^{\frac{1}{4}}\right)^7$	$\sqrt[4]{16^7}$	$(\sqrt[4]{16})^7$	128

Raising a number or expression to the $\frac{m}{n}$ power is the same as raising it to the mth power and taking its nth root, in either order. Written symbolically, this means that $x^{\frac{m}{n}} = \sqrt[n]{x^m}$.

Try This

For each expression, select *all* of the equivalent expressions.

a $x^{\frac{4}{3}}$

- ☐ **A.** $(x^4)^{\frac{1}{3}}$
- ☐ **B.** $\left(x^{\frac{1}{4}}\right)^{\frac{1}{3}}$
- ☐ **C.** $\sqrt[4]{x^3}$
- ☐ **D.** $\sqrt[3]{x^4}$
- ☐ **E.** $(\sqrt[3]{x})^4$

b $k^{\frac{6}{7}}$

- ☐ **A.** $(k^7)^{\frac{1}{6}}$
- ☐ **B.** $\left(k^{\frac{1}{6}}\right)^7$
- ☐ **C.** $\sqrt[7]{k^6}$
- ☐ **D.** $\sqrt[\frac{1}{7}]{k^6}$
- ☐ **E.** $(\sqrt[7]{k})^6$

Summary | Lesson 13

You can invest money in accounts that earn simple or compound interest. Accounts that earn **simple interest** can be modeled by linear functions, while accounts that earn **compound interest** can be modeled by exponential functions. Which account earns the most interest depends on how much time there is to invest and other variables.

Let's look at an example. Adah has $100 to invest in an account. Which account should she choose if she has 12 years to invest?

Simple Interest	Compound Interest
Adah could invest $100 in an account that earns 10% simple interest annually. The function $a(t) = 100 + 10t$ models the account balance after t years.	Adah could invest $100 in an account that earns 10% compound interest annually. The function $b(t) = 100 \bullet (1.10)^t$ models the account balance after t years.
To determine the balance of the account after 12 years, substitute $t = 12$ into each function and solve for $a(t)$.	
$a(12) = 100 + 10(12)$ $a(12) = 220$ After 12 years, the account balance will be $220.	$b(12) = 100 \bullet (1.10)^{12}$ $b(12) = 313.84$ After 12 years, the account balance will be about $313.84.

Adah may choose to invest in the account that earns compound interest because it earns more money over 12 years.

Try This

You save $250 to invest in an account. Here are descriptions of two account options:

Account A	**Account B**
This account earns 12% interest compounded annually.	This account earns 15% simple interest annually.

a Write equations to model each account option where x represents the years after opening the account.

b If you plan on investing for 3 years, which account would you choose? Explain your thinking.

c If you plan on investing for 7 years, which account would you choose? Explain your thinking.

Summary | Lesson 14

You can write exponential expressions representing compound interest in multiple equivalent ways to reveal different information about the account and situation.

Here is an example.

The amount owed on a $\$400$ loan has a monthly interest rate of 3%. Let t represent the number of years since taking out the loan if no payments are made.

- The expression $400 \bullet 1.03^{12t}$ represents a 3% monthly interest rate 12 times each year, t.
- The expression $400 \bullet (1.03^{12})^t$ uses the powers of powers property to help us think about the interest rate for every t year.
- Since $1.03^{12} = 1.4258$, the expression $400 \bullet (1.4258)^t$ shows the annual interest rate of 42.58%.

Each expression reveals different information about the monthly and annual interest rates applied to the account. While a monthly interest rate of 3% may not seem like it impacts the account balance much, the annual interest rate reveals that the loan amount is increasing by 42.58% each year, and that really adds up!

Try This

Select *all* the expressions that will calculate the total value of a $\$1{,}000$ loan after 2 years. The loan has a 7% monthly interest rate and no additional payments.

☐ **A.** $1000 \cdot 1.07^{24}$

☐ **B.** $2000 \cdot 1.07^{12}$

☐ **C.** $1000 \cdot (1.07^{12})^2$

☐ **D.** $1000 \cdot 2.252^2$

☐ **E.** $1000 \cdot 1.1449^{12}$

Summary | Lesson 15

When you take out a loan on a credit card, the annual interest rate may be compounded at different *intervals*, or different lengths of time.

You can use the formula $P\left(1+\frac{r}{n}\right)^{nt}$ to calculate the total amount in an account that accrues compound interest.

- P represents the initial amount of the loan. In finance this is often called the *principal*.
- r represents the interest rate in decimal form.
- n represents the number of compounding intervals in a year.
- t represents the time in years.

Common compounding periods:

Annually	Semi-annually	Quarterly	Monthly	Daily
$n=1$	$n=2$	$n=4$	$n=12$	$n=365$

Let's look at the impact of compound interest applied at different intervals on a loan for $\$1{,}000$.

Interest	Owed in	Compounded Monthly	Compounded Quarterly	Compounded Annually
15% annually	5 years	$1000\left(1+\frac{0.15}{12}\right)^{12\cdot 5}$ $\approx \$2{,}107.18$	$1000\left(1+\frac{0.15}{4}\right)^{4\cdot 5}$ $\approx \$2{,}088.15$	$1000(1+0.15)^5$ $\approx \$2{,}011.36$

Try This

Ivan wants to take out a $\$2{,}500$ loan to open a hot dog stand.

Complete the table to see how much Ivan would owe at different compounding intervals.

Interest	Owed In	Compounded Monthly	Compounded Quarterly	Compounded Annually
20% annually	7 years	$2500\left(1+\frac{0.2}{12}\right)^{12\cdot 7}$ $\approx \$10021.69$		
9% annually	6 months		$2500\left(1+\frac{0.09}{4}\right)^{4\cdot 0.5}$ $\approx \$2613.77$	

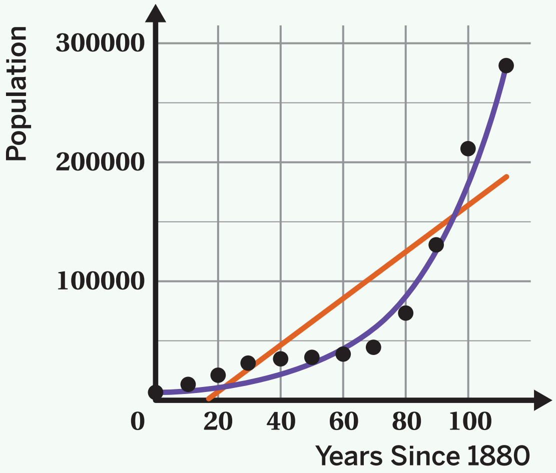
Summary | Lesson 16

Exponential and linear functions are often used to model the population growth of a city or country. *Models* can help us predict unknown data values, including future values. While some models can be useful, they also have limitations.

- Exponential functions increase toward infinity, but populations are limited by space, time, and available land.
- Linear models may generate a negative y-intercept, but populations do not have less than 0 people.
- Models may only be useful for predicting unknown values within a specific *domain*.

Let's look at an example. Here is data about the population of a city since 1880.

An exponential model is a better fit than a linear model. The data fits the shape of a curve more closely than it follows a line.



You can use the exponential function $p(t) = 3992(1.0397)^x$ to model the number of people in 1945.

Since 1945 is 65 years after 1880, substitute 65 into $p(t)$ to estimate the unknown value.

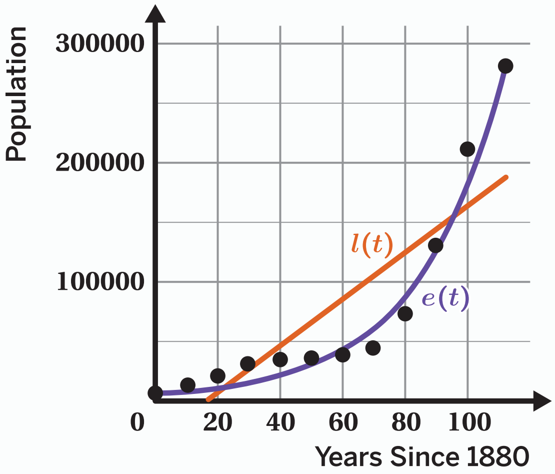
$p(65) = 3992(1.0397)^{65}$

$p(65) = 50143$

The model estimates that there were $50{,}143$ people in 1945.

Try This

Here is a graph of the population in Colorado from 1880 to 1990.



a Which of the two models do you think better fits the data for the population of Colorado from 1880–1990? Circle one.

Linear

Exponential

b What are the limitations of using the model you chose?

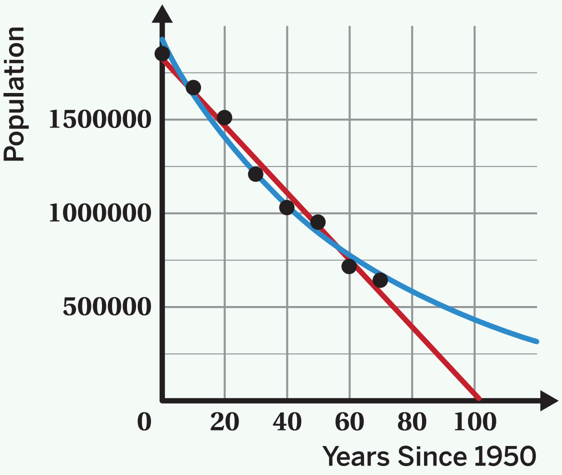
Summary | Lesson 17

You can use a graphing calculator to generate a line of best fit or an exponential curve of best fit to model data. Here is an example of data about the population of Detroit from 1950–2020 that was used to generate a line and exponential curve of best fit.

The slope of the linear *model* means for every 1 year, the population of Detroit is predicted to decrease by about 17,989 people.

The growth factor of the exponential model means for every 1 year, the population of Detroit decreases by about 1.50%.

Which model is used depends on how well the function fits the data. In this case, both models fit the data well and can be used to predict unknown values accurately.



$y_1 \sim mx_1 + b$

Parameters

$m = -17989.3$

$b = 1825460$

$y_1 \sim a \cdot b^{x_1}$

Parameters

$a = 1912612$

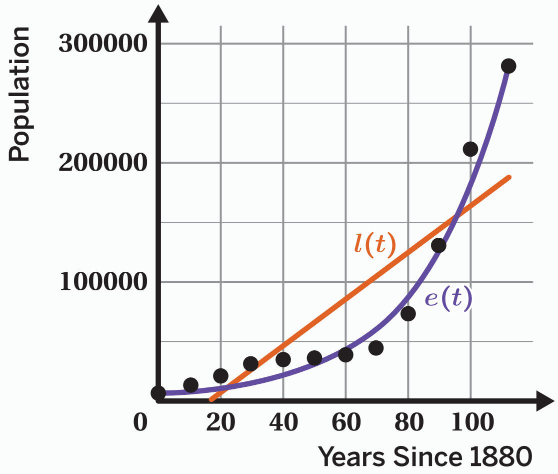
$b = 0.985051$

Try This

Here is a graph of the population in Colorado from 1880 to 1990.

a Describe what the slope of the linear model represents in this situation.

b Describe what the growth factor represents in the exponential model.



Linear model: $l(t) = 2075.25t - 38082.2$

Exponential model: $e(t) = 3991.89 \cdot 1.03967^t$

Try This | Answer Key

Lesson 1

a *Responses vary*. 12 means the number of subscribers Marc’s channel had when he started the channel (at 0 months).

b *Responses vary*. 2 means the amount of subscribers doubles every month.

c 3072 subscribers

Lesson 2

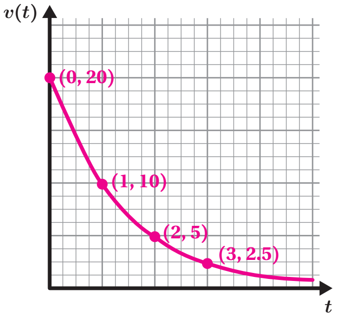
a Exponential. *Explanations vary*. The y-values have a constant growth factor of 3.

b Linear. *Explanations vary*. The y-values have a constant rate of change of 25.

Lesson 3

$f(x) = 8 \cdot \left(\frac{1}{4}\right)^x$ (or equivalent)

Lesson 4

<table>
<tr><th>Situation</th><th>Equation</th><th>Table</th><th>Graph</th></tr>
<tr>
<td>In 2020, a coral reef had a volume of 20 cubic meters that changed by a factor of $\frac{1}{2}$.</td>
<td>$v(t) = 20 \cdot \left(\frac{1}{2}\right)^t$</td>
<td>
<table>
<tr><th>t</th><th>$v(t)$</th></tr>
<tr><td>0</td><td>20</td></tr>
<tr><td>1</td><td>10</td></tr>
<tr><td>2</td><td>5</td></tr>
<tr><td>3</td><td>2.5</td></tr>
</table>
</td>
<td> </td>
</tr>
</table>

Lesson 5

a $h(-1) = 12$

b $k(-1) = 25$

c *Responses vary*. In Situation A, $h(-1)$ makes sense because x represents years since 2020. This means $h(-1)$ represents the year 2019. In Situation B, $k(-1)$ does not make sense because x represents the number of hours since Kiana finished her tea.

Lesson 6

a 4

b 256

c No. *Explanations vary*. $f(x+4)$ does not grow similarly to $\frac{g(x+4)}{g(x)}$ because $f(x)$ is a linear function and $g(x)$ is an exponential function. $f(x+4)$ increases $f(x)$ by 16. $\frac{g(x+4)}{g(x)}$ multiplies $g(x)$ by a factor of 256.

Lesson 7

a D. $m(x) = 12(1.6)^x$

b *Responses vary*.
A: Ramon is also studying how well a disinfectant can eliminate mold. The original area of mold was 12 square centimeters. Each hour, the disinfectant decreases the area of mold by 94%.
B: The amount of carbon in a fossil decays at a rate of 40% every 10 years. The fossil originally had 12 moles of carbon.
C: Fintastic Finny the magic fish weighs 12 grams and grows at a rate of 6% per minute.

Lesson 8

B. $f(t) = 204(0.8)^t$

D. $f(t) = 204(1 - 0.2)^t$

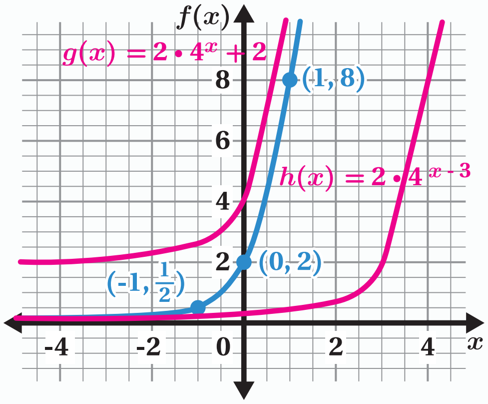
Lesson 9

$f(x) = 143(1 - 0.15)^x + 7$ or $f(x) = 143(0.85)^x + 7$

Lesson 10

a $g(x) = 2 \cdot 4^x + 2$

b $h(x) = 2 \cdot 4^{(x-3)}$



Lesson 11

Radical Expression	Exponential Expression	Value
$\sqrt{81}$	$81^{\frac{1}{2}}$	9
$\sqrt[4]{81}$	$81^{\frac{1}{4}}$	3
$\sqrt[3]{343}$	$343^{\frac{1}{3}}$	7
$\sqrt[5]{7776}$	$7776^{\frac{1}{5}}$	6

Lesson 12

a A. $(x^4)^{\frac{1}{3}}$

D. $\sqrt[3]{x^4}$

E. $(\sqrt[3]{x})^4$

b C. $\sqrt[7]{k^6}$

E. $(\sqrt[7]{k})^6$

Lesson 13

a Account A: $a(x) = 250 \cdot 1.12^x$
Account B: $b(x) = 250 + 37.5x$

b Account B. *Explanations vary*. After 3 years, Account B will have $362.50 while Account A will only have approximately $351.23.

c Account A. *Explanations vary*. After 7 years, Account A will have approximately $552.67 while Account B will only have $512.50.

Lesson 14

A. $1000 \cdot 1.07^{24}$

C. $1000 \cdot (1.07^{12})^2$

D. $1000 \cdot 2.252^2$

E. $1000 \cdot 1.1449^{12}$

Lesson 15

Interest	Owed In	Compounded Monthly	Compounded Quarterly	Compounded Annually
20% annually	7 years	$2500\left(1+\frac{0.2}{12}\right)^{12\cdot7}$ $\approx \$10021.69$	$2500\left(1+\frac{0.2}{4}\right)^{4\cdot7}$ $\approx \$9800.32$	$2500\left(1+\frac{0.2}{1}\right)^{1\cdot7}$ $\approx \$8957.95$
9% annually	6 months	$2500\left(1+\frac{0.09}{12}\right)^{12\cdot0.5}$ $\approx \$2614.63$	$2500\left(1+\frac{0.09}{4}\right)^{4\cdot0.5}$ $\approx \$2613.77$	$2500\left(1+\frac{0.09}{1}\right)^{1\cdot0.5}$ $\approx \$2610.08$

Lesson 16

a Exponential

b *Responses vary*. As time passes, the population and the model both increase toward infinity. This model does not account for restrictions on housing and land. Additionally, this model does not account for any weather or historic events that could drastically affect the population at any given time.

Lesson 17

a The slope of the linear model represents a population increase by about 2075 people every year.

b Every 1 year the population increases by about 4%.