Unit 7

Positive and Negative Numbers

Think back to when you first learned about whole numbers and used them to count. Later, you saw there were numbers between them: fractions and decimals. Up until now, many numbers you've encountered have been greater than zero. There is an entire set of numbers (just as many, in fact), lurking on the other side of every number line.

Essential Questions

- What does it mean for a value to be less than zero?
- How do we represent all the numbers that are less than or greater than a value?
- How do we represent points with negative numbers on a coordinate plane?

Summary | Lesson 1

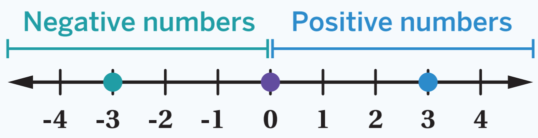
Positive numbers are numbers that are greater than 0. **Negative numbers** are numbers that are less than 0. Zero is neither positive nor negative.

You can extend a number line to the right of 0 to show positive numbers, and you can extend a number line to the left of 0 to show negative numbers.

For example:

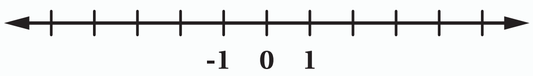
The number 3 is 3 units to the right of 0 on the number line.

The number -3 is 3 units to the left of 0 on the number line.



Try This

a Draw a star at -2 on the number line.

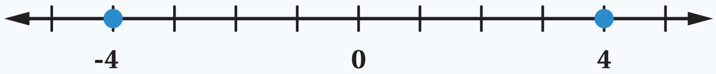


b Label each of the remaining tick marks on the number line.

c Sora says he is thinking of a number that is 3 units away from -2. What number could it be? Explain your thinking.

Summary | Lesson 2

Two numbers are **opposites** if they are the same distance from 0 on different sides of the number line. For example, -4 and 4 are opposites because they are both 4 units away from 0.



Every number has an opposite, including fractions and decimals. 0 is its own opposite. The opposite of the opposite of a number is the number itself. For example, $-(-2) = 2$.

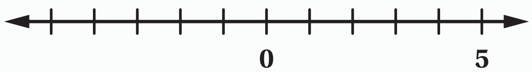
All positive and negative whole numbers and 0 are a group of numbers called *integers*. All positive and negative numbers that can be written as fractions, including whole numbers, are called *rational numbers*.

2 and -2 are both integers and rational numbers.

8.3, -8.3, $\frac{3}{2}$, and $-\frac{3}{2}$ are rational numbers, but *are not* integers.

Try This

Here is a number line.



a Plot point A at 2.5.

b Plot point B at $-\frac{3}{4}$.

c Plot point C at $-(-4)$.

Explain your thinking.

You can use a vertical number line to represent positive and negative numbers.
On a vertical number line, points above 0 are positive and points below 0 are negative.

When talking about elevation, 0 feet represents sea level. This means that a positive elevation is above sea level and a negative elevation is below sea level.

When talking about temperature, 0°C means the temperature is freezing. If the temperature in Mt. Olympus is -10°C, that means it has a temperature of 10°C below 0°C, or below freezing.

If the temperature in Cotopaxi is -3°C, you can write $-10 < -3$, which means that it is colder in Mt. Olympus than it is in Cotopaxi.

Try This

Order these cities in California from *lowest* to *highest* elevation.

City	Coachella	El Centro	Imperial	Niland
Elevation (ft)	-72	-39	-59	-141

Lowest **Highest**

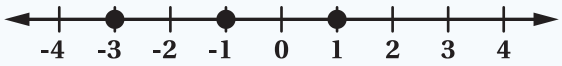
Summary | Lesson 4

You can use a number line to compare numbers with different **signs** (like -4 and 3) or numbers with the same sign (like -4 and -3).

The order of numbers from least to greatest is the same order as they appear on the number line from left to right. This means that negative numbers farther from 0 are less than negative numbers that are closer to 0.

For example, let's say you want to compare -3 and -1. On a number line, -1 is to the right of -3. This means that -1 is greater than -3, or $-3 < -1$. This also makes sense because -1 is closer to 0 than -3 is.

A number line can also help you order numbers from least to greatest. 1 is greater than -1 and -3 because it is the farthest to the right on the number line.



Try This

a Complete each number sentence using > or <.

2.5 ________ -0.4

$-2\frac{2}{3}$ ________ $-\frac{5}{4}$

b Order these numbers from *least* to *greatest*.

-0.4	$-\frac{5}{4}$	$-2\frac{2}{3}$	2.5

Least **Greatest**

Summary | Lesson 5

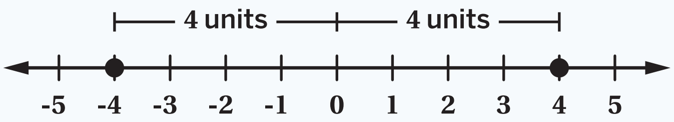
The **absolute value** of a number is a way to describe its distance from 0. For example:

The absolute value of -4 is 4, because it is 4 units away from 0.

$|-4| = 4$

The absolute value of 4 is also 4, because it is 4 units away from 0.

$|4| = 4$



The distance from 0 to itself is 0, so $|0| = 0$.

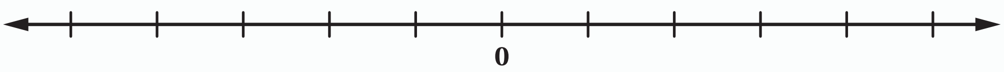
Absolute values are helpful when you are interested in the size of a difference or measurement but its direction is not important.

Try This

Determine the value of each expression.

Use the number line if it helps with your thinking.

Expression	Value
$\|-4\|$	
$\|2.5\|$	
$\left\|-\frac{7}{20}\right\|$	

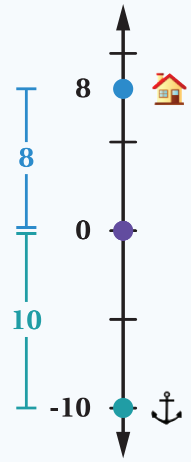


Summary | Lesson 6

You can use absolute value to compare quantity changes or distances.

For example, suppose an anchor has an elevation of -10 meters and a house has an elevation of 8 meters.

- To compare their elevations and describe the anchor as having a lower elevation than the house, you can write $-10 < 8$.
- To compare their distances from sea level and describe the anchor as being farther away from sea level than the house, you can write $|-10| > |8|$.



Try This

The table shows the amount in Oscar's bank account from January through June.

a When was Oscar's account balance the lowest?

b When was Oscar's balance closest to zero?

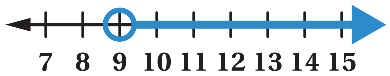
Month	Account Balance ($)
January	55
February	121
March	-23
April	20
May	-17
June	45

Summary | Lesson 7

You can use variables, verbal descriptions, symbols, and number lines to represent inequalities related to real-world situations.

To represent an inequality on a number line, you can shade part of the number line to indicate that every point covered by the shaded region is a solution. Then draw an arrow on one end of the number line to show the possible solutions continue on in that direction.

Here is an example:

Situation	Verbal Description	Inequality	Number Line
A two-year-old sleeps more than 9 hours a day.	Any value greater than 9.	$x > 9$	

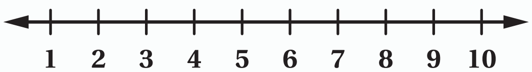
Try This

Nyanna made a game for players over 3 years old.

a List three possible ages game players could be.

b Write an inequality to represent the age of game players, a.

c Graph all the possible ages of game players.



Summary | Lesson 8

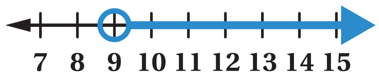
A **solution to an inequality** is any value that makes the inequality true. You can use a number line to represent the solutions to an inequality.

For example, for the inequality $c < 10$, you could say:

- 5 is a solution because $5 < 10$ is a true statement.
- 12 is not a solution because $12 < 10$ is not a true statement.

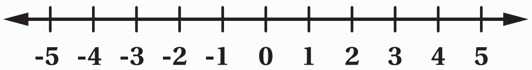
Some inequalities like $c < 10$ have an infinite number of solutions. We use inequality statements with variables and the symbols $<$ or $>$ to represent all the solutions.

Here is an example:

Inequality	Description	Possible Solutions	Number Line
$x > 9$	Any value greater than 9.	$9.75, 10, 11.3, 82$	

Try This

a Make a graph of all the solutions to the inequality $-3.5 < x$.



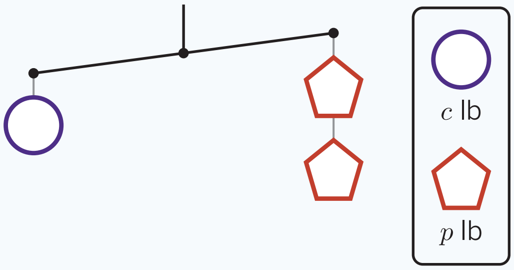
b Is -3.5 a solution to $-3.5 < x$?

Explain your thinking.

Summary | Lesson 9

You can represent inequalities with unbalanced hangers. Inequalities may include a variable as a placeholder for an unknown value.

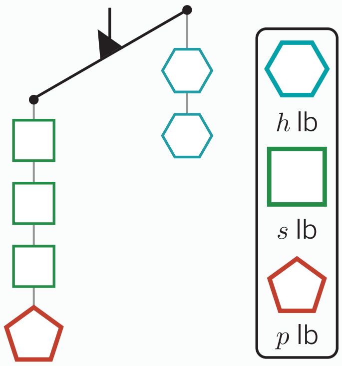
This hanger shows that the weight of the circle, c, is heavier than the weight of two pentagons, $2p$. This relationship can be represented by the inequality $c > 2p$ because these symbols mean that the circle has a greater weight than 2 pentagons.



Try This

Here is an unbalanced hanger.

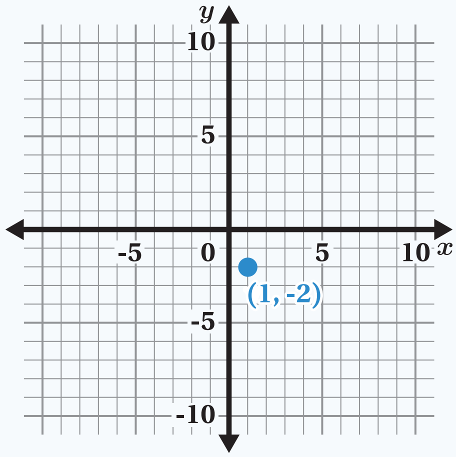
Write an inequality that represents the hanger.



Summary | Lesson 10

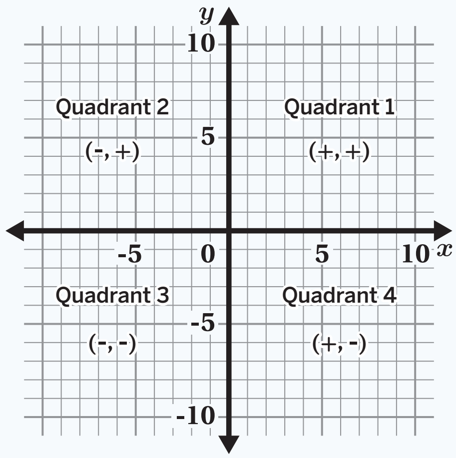
You can include positive and negative numbers along the x- and y-axes, just like on a number line. The x- and y-axes cross at the *origin*, or the point $(0, 0)$.

Ordered pairs are written (x, y), where the x-value is the horizontal location (left and right) and the y-value is the vertical location (up and down). For example, the point $(1, \text{-}2)$ is 1 unit to the right and 2 units down from the origin.



The four regions of the coordinate plane are called **quadrants**. They are numbered 1–4 starting with the top right quadrant and going in a circle counter-clockwise.

The image shows each quadrant, along with the sign of the x- and y-values in that quadrant.



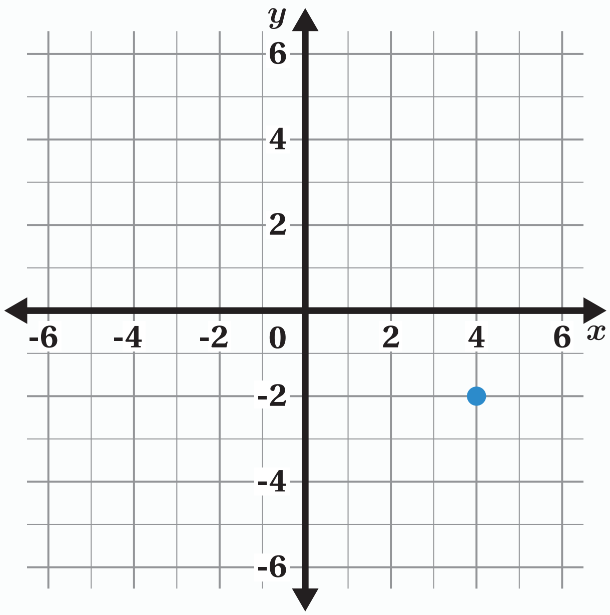
Try This

Renata found a sand dollar on the beach.

She marked the spot where she found it on this map with a dot.

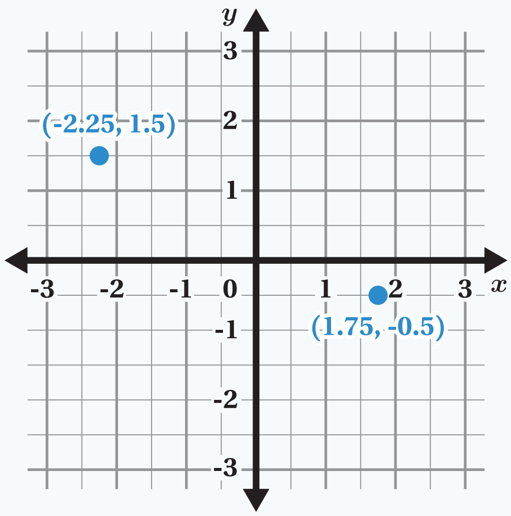
What are the coordinates of the sand dollar?

Explain your thinking.



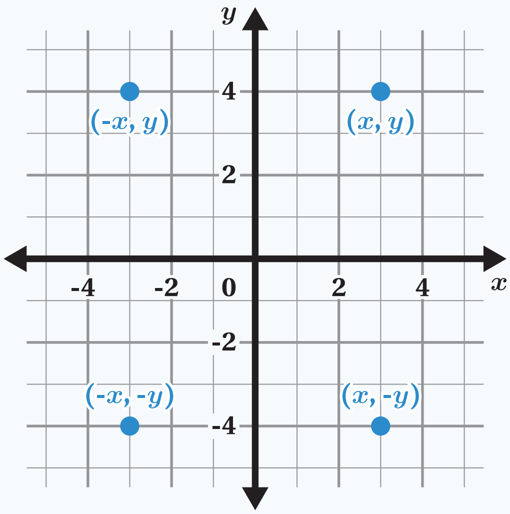
Summary | Lesson 11

You can use different scales to show very big or very small numbers on a coordinate plane. In these cases, the interval is still consistent (e.g., goes by 2s or 0.5s). Sometimes points are plotted in between tick marks. Consider the points (1.75, -0.5) and (-2.25, 1.5) and where they appear on the graph shown.



The points (3, 4) and (3, -4) have the same x-coordinate and the y-coordinates only differ by their sign. We can see on the graph that those points are a reflection, or a mirror, of each other across the x-axis.

The points (-3, -4) and (3, -4) have the same y-coordinate and the x-coordinates only differ by their sign. We can see on the graph that those points are a reflection, or a mirror, of each other across the y-axis.



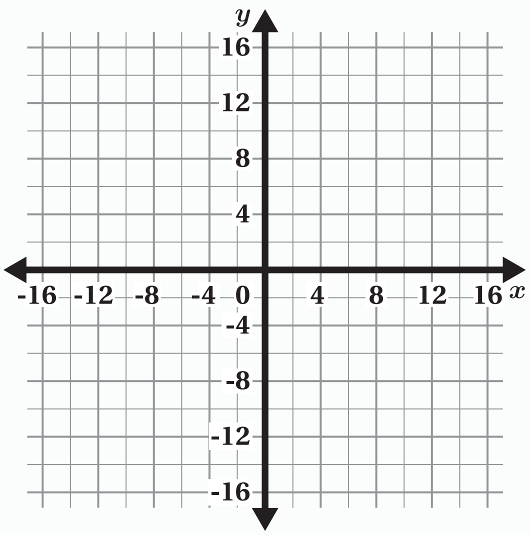
Try This

Plot and label each point on the graph.

A(6, 8)

B(-6, 8)

C(-12, -10)

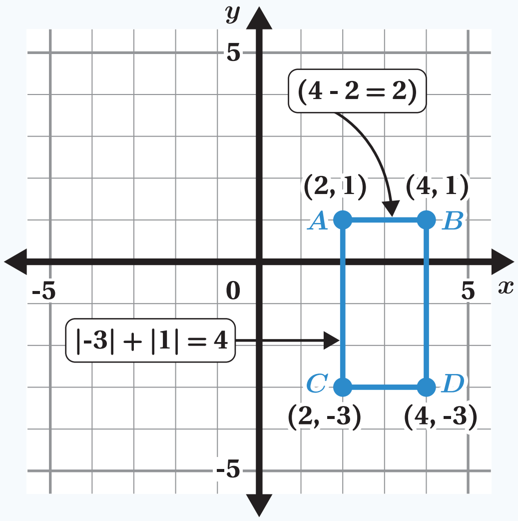


Summary | Lesson 12

You can plot points on the coordinate plane to create *polygons*. When the vertices of a polygon are horizontally or vertically aligned on the graph, you can count the number of units between them to determine the length of that side.

You can also calculate the side lengths using the coordinates of each vertex. Here are two calculation strategies:

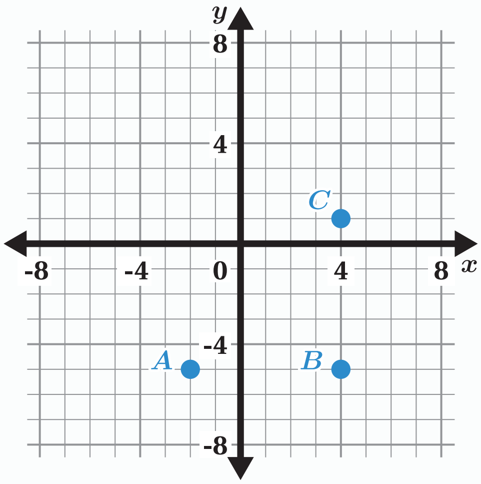
- If the coordinates are in the same quadrant, like points A and B, find the length by subtracting the coordinates that are different.
- If the coordinates are in different quadrants, like points A and C, use the absolute value to determine the distance each point is from the axis between them.



Try This

Here are three of the four coordinates that make a square.

Point	Coordinates
A	(-2, -5)
B	(4, -5)
C	(4, 1)
D	



a Write the coordinates of point D.

b What is the side length of the square?

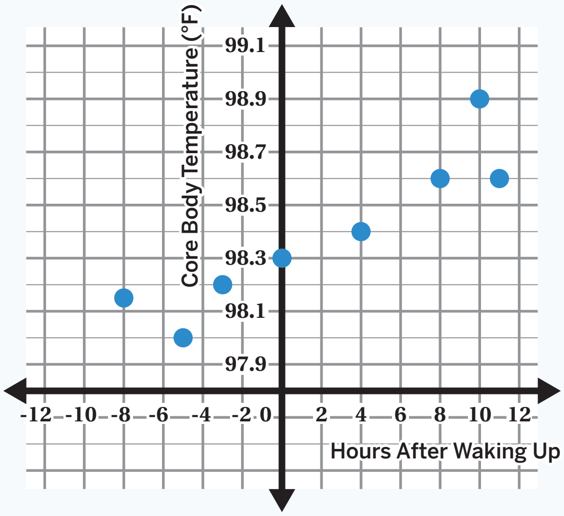
Explain your thinking.

Summary | Lesson 13

You can use points with positive and negative coordinates on a graph to make sense of different situations. Common contexts include elevation, time, temperature, and money.

Here's an example:

- Santino used a device to track his temperature before and after waking up. He plotted his data on the graph.
- Time is represented on the x-axis. Temperature is represented on the y-axis.
- The points to the left of the y-axis represent temperatures recorded while Santino was sleeping. The points to the right of the y-axis represent temperatures recorded after he woke up at 8:00 AM.



Coordinates on the graph tell different parts of the story. For example, the point $(-5, 98)$ indicates that 5 hours before he woke up, his temperature was 98°F.

Try This

Use the data from Santino's watch, displayed in the graph above, to answer these questions.

a What was Santino's temperature when he woke up?

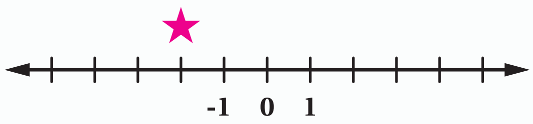
b At what time was Santino's temperature the highest?

c What does the point $(-3, 98.2)$ tell us?

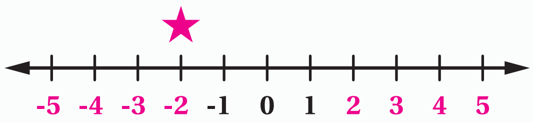
d Tell a story about Santino's temperature throughout the day.

Lesson 1

a

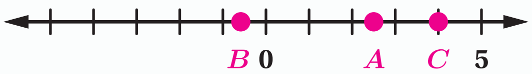


b



c -5 or 1. *Explanations vary*. -5 is three units to the left of the star, and 1 is three units to the right of the star.

Lesson 2



c *Explanations vary*. Another way of saying -(-4) is "the opposite of -4." The opposite of a negative number is a positive number, so -(-4) = 4.

Lesson 3

Niland	Coachella	Imperial	El Centro

Lowest — Highest

Lesson 4

a $2.5 > -0.4$ $-2\frac{2}{3} < -\frac{5}{4}$

b

$-2\frac{2}{3}$	$-\frac{5}{4}$	-0.4	2.5

Least — Greatest

Lesson 5

Expression	Value
$\lvert -4 \rvert$	4
$\lvert 2.5 \rvert$	2.5
$\left\lvert -\frac{7}{20} \right\rvert$	$\frac{7}{20}$

Lesson 6

a March

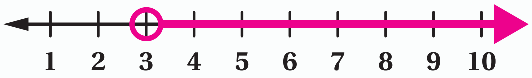
b May

Lesson 7

a *Responses vary.* 4, 7, 13

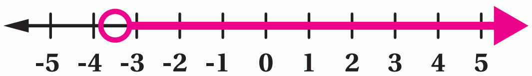
b $a > 3$

c



Lesson 8

a



b No. *Explanations vary*. Solutions to $-3.5 < x$ are numbers that are greater than -3.5. -3.5 is not greater than itself. $-3.5 < -3.5$ is a false statement.

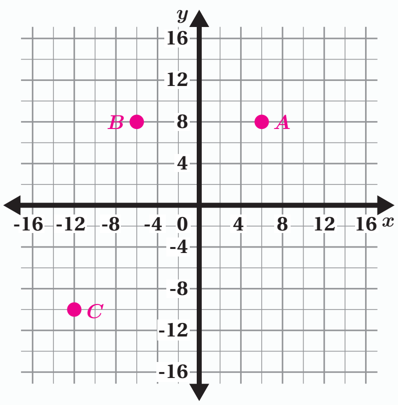
Lesson 9

$3s + p > 2h$

Lesson 10

(4, -2). *Explanations vary*. It is 4 units to the right of the y-axis and 2 units below the x-axis. If this were two number lines, then it would be at 4 on the horizontal one and -2 on the vertical one.

Lesson 11



Lesson 12

a (-2, 1)

b 6 units. *Explanations vary*. For the top side of the square, it is 2 units from -2 to 0 and another 4 units from 0 to 4.

Lesson 13

a 98.3°F

b 6:00 PM (10 hours after he woke up)

c 3 hours before waking up, Santino's body temperature was 98.2°F.

d *Responses vary*. Before waking up, Santino's temperature was at its coldest. He got warmer as he woke up and throughout the day. Then around 6:00 PM, Santino's body temperature started to decrease again.