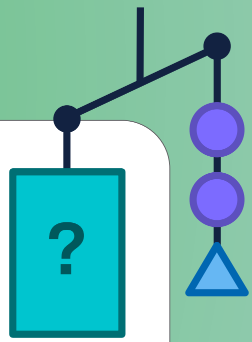
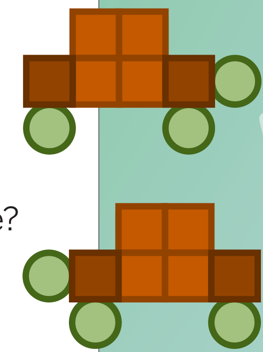


Unit 6

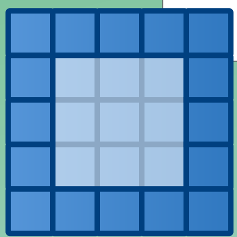
Expressions and Equations

Up until now, an equal sign meant you were being asked to calculate an answer. In this unit, you'll learn about its other meaning: balance. When things are in balance, it becomes possible to know the unknown.



Essential Questions

- What does it mean for an equation to be true? Can an equation be false?
- What does it mean for two expressions to be equivalent?
- If multiplication is repeated addition, how do we represent repeated multiplication?

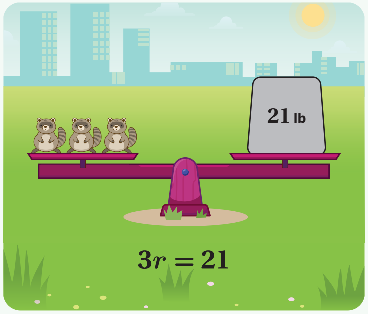


Summary | Lesson 1

You can use seesaws and tape diagrams to represent *equations* and help determine unknown values.

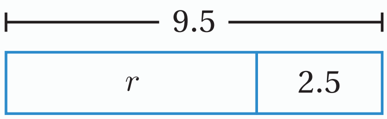
We often use a letter, such as x or a, as a placeholder for an unknown number in tape diagrams and equations. This letter is called a **variable**.

For example, if 3 equal-weight raccoons weigh a total of 21 pounds, you can represent the weight of each raccoon with r and write the equation $3r = 21$.



Try This

A raccoon and a 2.5-pound weight balance with a 9.5-pound weight.



Nekeisha drew this tape diagram to help determine the weight of the raccoon.

a Write an equation to represent this situation.

b How much does the raccoon weigh?

Use the equation or tape diagram if it helps with your thinking.

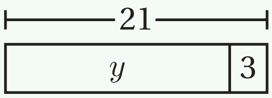
Summary | Lesson 2

A tape diagram can help us visualize an equation and determine its solution. The **solution to an equation** is a value of the variable that makes the equation true.

When we work with an equation that represents a situation, it is important to determine what the variable represents when we determine the solution.

Here is an example.

Emmanuel needed $21 to buy a gift. He had $3 and borrowed the rest from his parents. The variable y represents the amount Emmanuel borrowed from his parents.

Equation	Tape Diagram	Solution to the Equation	Solution's Meaning
$3 + y = 21$		$y = 18$	Emmanuel borrowed $18 from his parents.

Try This

Here is a situation, along with an equation that represents it.

Kiandra sold 4 hats and made $32. The hats cost h dollars each.	$4h = 32$

a Draw a tape diagram to represent this situation.

b Determine the solution to the equation.

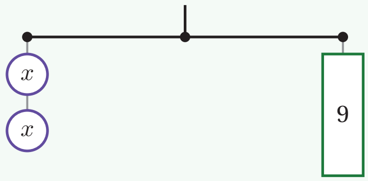
c Explain what the solution means in this situation.

Summary | Lesson 3

Hangers are a helpful way to represent equations. A hanger is balanced when the weight on both sides is equal.

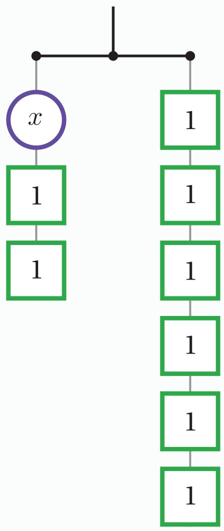
Here is an example.

This hanger represents the equation $2x = 9$, or $x + x = 9$. The solution to this equation is the value of x that will keep the hanger balanced. The solution for this hanger is 4.5 because $4.5 + 4.5 = 9$ or $2(4.5) = 9$.



Try This

Here is a balanced hanger.



a Write an equation to represent this hanger.

b Determine the value of x that balances this hanger.

Summary | Lesson 4

There are many strategies to solve equations, such as drawing models, using number sense to determine the value that makes an equation true, making a hanger balanced, or using inverse operations to isolate a variable.

Here are two examples that use inverse operations to solve an equation.

Equation	Explanation
$x + 1.5 = 3.25$	Original equation
$x + 1.5 - 1.5 = 3.25 - 1.5$	Subtract 1.5 from both sides.
$x = 1.75$	The solution to this equation is 1.75.

Equation	Explanation
$\frac{1}{2}y = 54$	Original equation
$\frac{1}{2}y \div \frac{1}{2} = 54 \div \frac{1}{2}$	Divide both sides by $\frac{1}{2}$.
$y = 108$	The solution to this equation is 108.

Try This

Determine the solution to each equation.

Draw a hanger or a tape diagram if it helps with your thinking.

a $y + 1.8 = 14.7$

b $1.8 = 3t$

Summary | Lesson 5

Writing an equation to match a situation is a helpful tool when trying to determine an unknown value. You can solve the equation using a variety of strategies such as tape diagrams, hangers, or inverse operations. We can check the solution to an equation by substituting the value of the variable to see if it makes the equation true. Once we have a solution to the equation, it's important to determine the meaning of the solution.

Here is an example.

Situation	Equation	Solution	Solution Check	Solution's Meaning
Adah has \$42 to spend on music downloads. Each download costs \$7. Adah can buy x downloads.	$7x = 42$	$x = 6$	$7 \bullet 6 = 42$	Adah can buy 6 music downloads.

Try This

Riders must be at least 3 feet tall to ride the Calculator 3000, a roller coaster at a math-themed amusement park. Mauricio is visiting the park and is $2\frac{1}{4}$ feet tall.

a Write an equation to determine the number of feet Mauricio must grow, f, to ride the roller coaster.

b Solve your equation.

c Explain what the solution represents in this situation.

Summary | Lesson 6

We can use an expression with a variable to represent a situation. Each part in the expression represents a different value in the situation. Here are two examples.

- The cost of 1 pound of grapes is $\$2.25$. Let p represent pounds of grapes. You can use the expression $2.25p$ to calculate the total cost for any number of pounds of grapes. This expression only has one **term**, $2.25p$.

Grapes (lb)	Total Cost ($)
1	2.25
2	4.50
5	11.25
p	$2.25p$

- A grocery store adds a $\$10$ fee to the cost of groceries for delivery. Let c represent the cost of groceries. You can use the expression $c + 10$ to calculate the total cost for any cost of groceries. This expression has two terms, c and 10.

Cost of Groceries	Total Cost ($)
1	11
2	12
5	15
c	$c + 10$

Try This

Mangoes cost $\$1.80$ per pound.

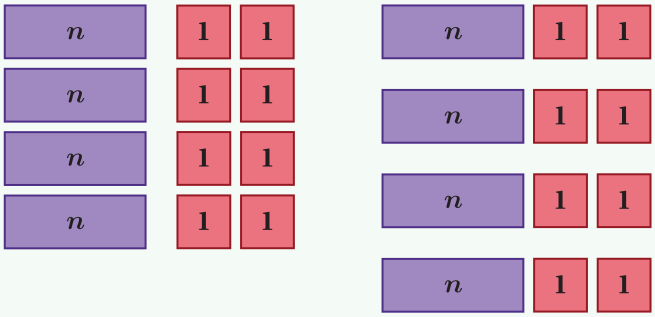
Complete the table to show the cost of other quantities of mangoes.

Mangoes (lb)	Total Cost ($)
1	1.80
2	
5	
10	
p	

Summary | Lesson 7

Equivalent expressions are expressions that are equal for every value of a variable, such as $4n + 8$ and $4(n + 2)$. Diagrams that represent these expressions can help us visually decide if the expressions are equivalent.

$4n + 8$ $\qquad$ $4(n + 2)$



The diagrams for $4n + 8$ and $4(n + 2)$ both show 4 n-tiles and 8 one-tiles. Therefore, $4n + 8$ and $4(n + 2)$ are equivalent expressions because they are equal for every value of n.

Try This

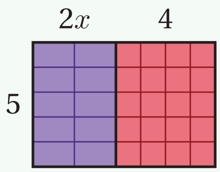
Complete the table by writing equivalent expressions in each row.

	Expression	Equivalent Expression
a	$6(n + 2)$	
b	$5n + 15$	
c	$n + n + n + 1 + 1 + 1$	
d	$(2n + 4) + (2n + 4)$	

Summary | Lesson 8

You can use areas of rectangles to write equivalent expressions. For any rectangle, you can write a *product* expression and a *sum* expression that each represent the area. No matter what value you substitute for the variable, the total area is the same, so the product and sum expressions are equivalent.

Area Model



Product of Two Side Lengths

$5(2x + 4)$

$10x + 20$

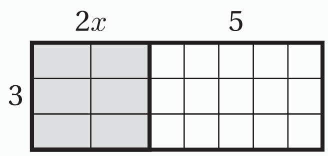
Sum of Two Areas

$5 \bullet 2x + 5 \bullet 4$

$10x + 20$

Try This

Write *two* equivalent expressions that represent the area of this rectangle.



Summary | Lesson 9

The expression $8x + 2$ has two terms, and the term $8x$ has a **coefficient** of 8.
The expression $2(x + 1) + 3(2x)$ also has two terms, $2(x + 1)$ and $3(2x)$, but the terms are more complex.

To decide if two expressions are equivalent, you can draw models, substitute values, or rewrite the expressions. If the expressions are equivalent, you can use the distributive property and other operations to rewrite one expression to look like the other.

Here is an example: Determine whether $2(x + 1) + 3(2x)$ is equivalent to $8x + 2$.

$$\begin{aligned} 2(x + 1) + 3(2x) &= 2x + 2 + 6x \\ &= 2x + 6x + 2 \\ &= 8x + 2 \end{aligned}$$

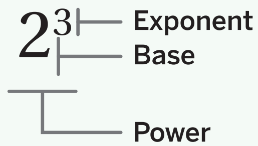
$2(x + 1) + 3(2x)$ and $8x + 2$ are equivalent expressions because after using the distributive property and adding the **like terms**, the expressions are the same.

Try This

Write an expression that is equivalent to $3(2x) + 4(x + 7)$.

Summary | Lesson 10

Exponents are used to represent repeated multiplication. In the expression 2^n, 2 is the **base**, and n is the **exponent**. If n is a positive whole number, it represents how many times 2 should be multiplied to determine the value of the expression.



Here are some examples.

$2^1 = 2$ $\qquad$ $2^3 = 2 \cdot 2 \cdot 2$

There are several different ways to say "2^3."

- "Two to the power of three"
- "Two raised to the power of three"
- "Two to the third power"
- "Two cubed"

Try This

Complete the table.

	Expression With Exponent	Expression Without Exponent	Value
a	3^3		
b		$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$	
c			81

Summary | Lesson 11

There is a specific *order of operations* we use to evaluate expressions with more than one operation, like $5 \cdot 2^4$ or $(5 \cdot 2)^4$.

With Parentheses	Without Parentheses
Evaluate the operations in parentheses first:	Evaluate the term with the exponent first:
$(5 \cdot 2)^4$	$5 \cdot 2^4$
$(10)^4$	$5 \cdot (2 \cdot 2 \cdot 2 \cdot 2)$
$10 \cdot 10 \cdot 10 \cdot 10$	$5 \cdot 16$
$10{,}000$	80

Try This

Calculate the value of each expression.

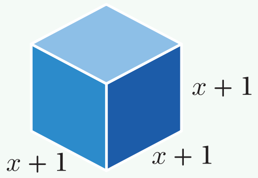
a $7 \cdot 2^3$

b $27 \cdot \left(\frac{1}{3}\right)^2$

c $\frac{(5-3)^2}{4}$

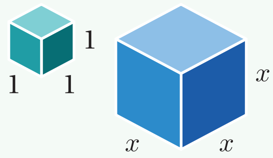
To use the order of operations, *evaluate* the operations in parentheses first. When there are no parentheses, exponents should be evaluated first.

Area is useful for modeling expressions with exponents of 2. Volume is useful for modeling expressions with exponents of 3. When evaluated, these become perfect squares and **perfect cubes**. Here are two examples of expressions evaluated when $x = 2$. Look for the perfect cubes.



$(x + 1)^3$ is

$$(2 + 1)^3 = 3^3$$
$$= 27$$



$x^3 + 1$ is

$$2^3 + 1 = 8 + 1$$
$$= 9$$

If the exponent is larger than 3, substitute the value of the variable and use the order of operations.

For example, when $x = 2$:

$(x + 1)^4$ is

$$(2 + 1)^4 = 3^4$$
$$= 81$$

$x^5 + 1$ is

$$2^5 + 1 = 32 + 1$$
$$= 33$$

Try This

Calculate the value of each expression when $x = 2$.

a $x + 3^3$

b $(x + 1)^4$

c $5x^3$

Summary | Lesson 13

Tables and equations can be used to represent and describe a relationship between two variables or quantities.

- The **dependent variable** is the variable in a relationship that is the effect or result.
- The **independent variable** is the variable in a relationship that is the cause. It is used to calculate the value of the dependent variable.

Let's say a boat can travel 36 miles in 3 hours. How far can the boat travel in 8 hours?

- The dependent variable is the distance traveled, d.
- The independent variable is the amount of time, t.

Table

t (hr)	d (mi)
3	36
1	12
8	96

In 8 hours, the boat can travel 96 miles.

Equation

$$d = 12t$$

In 8 hours, the boat can travel $d = 12 \times 8 = 96$ miles.

In both strategies, the distance *depends* on how many hours the boat travels.

Try This

Adah made a table to represent the number of paper cranes she made during a period of time.

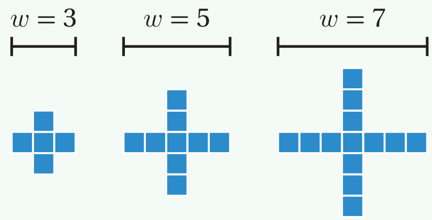
a What is the dependent variable?

b Write an equation to represent this relationship.

Number of Days, d	Number of Paper Cranes, p
1	9
2	18
3	27

Summary | Lesson 14

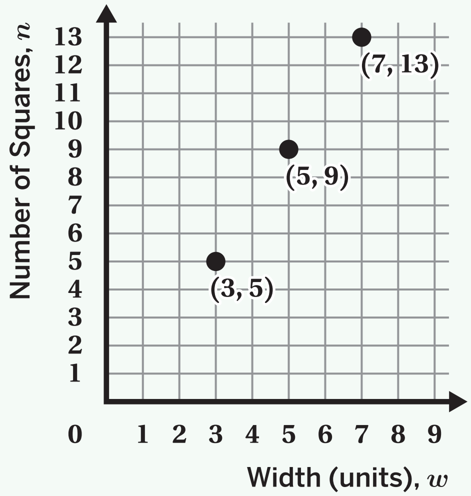
Like tables and equations, graphs are another way of representing the relationship between two quantities. For example, in this pattern, the independent variable is the width of the figure, w, and the dependent variable is the number of squares, n.



Here are the table and graph of this relationship.

Width of the Figure (units), w	Number of Squares, n
3	5
5	9
7	13

The numbers in each row of the table indicate an *ordered pair* on the coordinate plane. In the first row of the table, w is 3 and n is 5, which is represented by the point (3, 5) on the graph.



While representing a relationship using a graph, we usually use the x-axis for the independent variable.

Try This

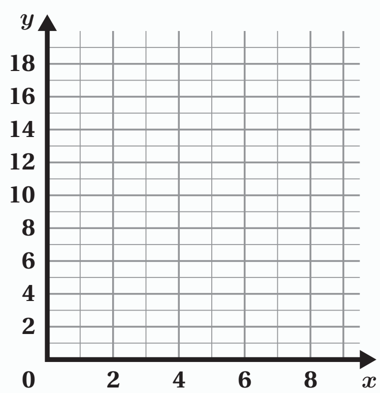
The number of mosquitoes in Kanna's garden keeps increasing!

Kanna made a table to represent the relationship between hours passed, h, and the number of mosquitoes, m, in her garden.

Hours, h	Mosquitoes, m
1	6
2	10
4	18

Use Kanna’s table to create a graph of this relationship.

Label each axis with what it represents.



Summary | Lesson 15

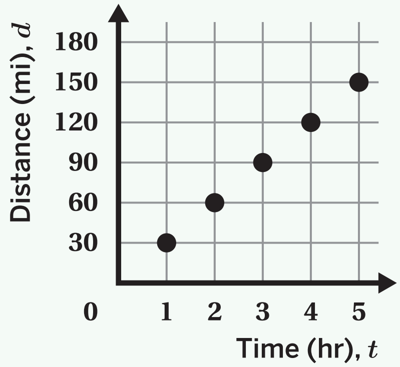
All three representations — tables, equations, and graphs — hold the same mathematical information described in a situation but display it in different ways.

For example, if a car travels 30 miles per hour at a constant speed, you can determine the distance the car traveled in 4 hours using a table, a graph, or an equation.

Table

Time, t (hr)	Distance, d (mi)
1	30
2	60
4	120

Graph



Equation

$d = 30t$

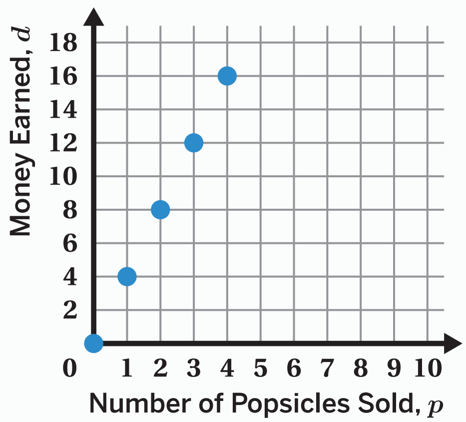
$d = 30(4)$

$d = 120$

In all three representations, we can see that the car traveled 120 miles in 4 hours.

Try This

Here is a graph that shows the money that Jin earned from selling popsicles.



a Create a table that represents this graph.

Number of Popsicles Sold, p	Money Earned, d

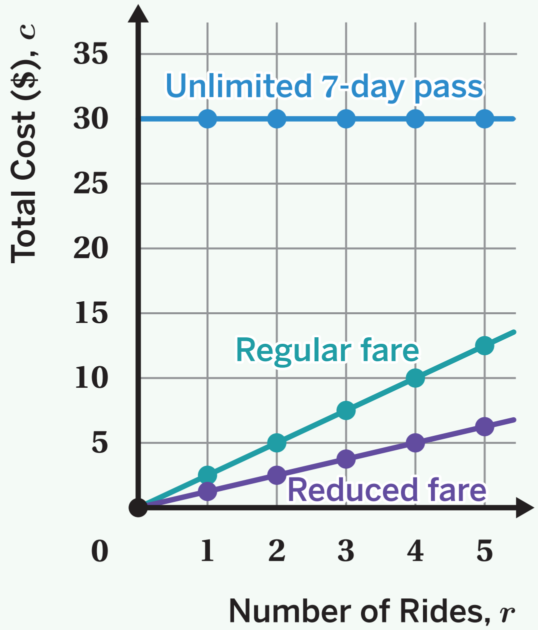
b Write an equation that represents this graph.

Summary | Lesson 16

We can use data in tables, graphs, and equations to help make decisions in real-world situations. When it comes to analyzing subway ticket fares, these representations can help us make informed decisions about what type of transportation ticket to purchase.

Using the graph, we can see that the regular fare ticket is the best choice if we ride 5 times or less and do not qualify for the reduced fare. If we extend each line on the graph, we'll be able to determine when the price of an unlimited 7-day pass will be lower than the regular fare.

We can use these tools to make sure we get the best subway ticket for our needs.



Try This

In 2024, one regular-fare subway ride cost $2.90 in New York City.

a Write an equation to represent the relationship between the total cost, t, and the number of rides, r.

b Use the equation to determine how much 15 rides would cost.

c An unlimited 30-day subway pass costs $132. Explain when it would be a good deal to buy the unlimited monthly pass.

Try This | Answer Key

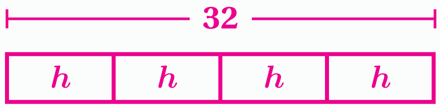
Lesson 1

a $r + 2.5 = 9.5$

b 7 pounds

Lesson 2

a



b $h = 8$

c Each hat costs \$8.

Lesson 3

a $x + 2 = 6$

b $x = 4$

Lesson 4

a $y = 12.9$

Caregiver Note: One strategy is to use the inverse operation and subtract 1.8 from both sides of the equation.

b $0.6 = t$

Caregiver Note: One strategy is to use the inverse operation and divide both sides of the equation by 3.

Lesson 5

a $2\frac{1}{4} + f = 3$ (or equivalent)

b $f = \frac{3}{4}$

Caregiver Note: One strategy is to use the inverse operation and subtract $2\frac{1}{4}$ from both sides of the equation. Then you can check the solution by substituting $\frac{3}{4}$ for f: $2\frac{1}{4} + \frac{3}{4} = 3$.

c Mauricio must grow $\frac{3}{4}$ feet (or equivalent) in order to ride the roller coaster.

Lesson 6

Mangoes (lb)	Total Cost ($)
1	1.80
2	**3.60**
5	**9.00**
10	**18.00**
p	**$1.80p$**

Lesson 7

Expression	Equivalent Expression
$6(n+2)$	$6n+12$ (or equivalent)
$5n+15$	$5(n+3)$ (or equivalent)
$n+n+n+1+1+1$	$3n+3$ (or equivalent)
$(2n+4)+(2n+4)$	$4n+8$ (or equivalent)

Lesson 8

Responses vary.

a $3(2x+5)$

b $6x+15$

Lesson 9

Responses vary. $10x+28$

Caregiver Note: One strategy is to use the distributive property to expand the expressions. Then you can add the like terms. $3(2x)+4(x+7)=6x+4x+28=10x+28$

Lesson 10

Expression With Exponent	Expression Without Exponent	Value
3^3	$3 \cdot 3 \cdot 3$	27
$\left(\frac{1}{2}\right)^4$ (or equivalent)	$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$	$\frac{1}{16}$ (or equivalent)
9^2 (or equivalent)	$9 \cdot 9$ (or equivalent)	81

Lesson 11

a 56

Caregiver Note: Here is one strategy: $7 \cdot 2^3 = 7 \cdot (2 \cdot 2 \cdot 2) = 7 \cdot 8 = 56$

b 3

Caregiver Note: Here is one strategy: $27 \cdot \left(\frac{1}{3}\right)^2 = 27 \cdot (\frac{1}{3} \cdot \frac{1}{3}) = 27 \cdot \frac{1}{9} = \frac{27}{9} = 3$

c 1

Caregiver Note: Here is one strategy: $\frac{(5-3)^2}{4} = \frac{(2)^2}{4} = \frac{4}{4} = 1$

Lesson 12

a 29

b 81

c 40

Caregiver Note: One strategy is to substitute 2 for x, then use the order of operations.
$5x^3 = 5 \cdot 2^3 = 5 \cdot 8 = 40$

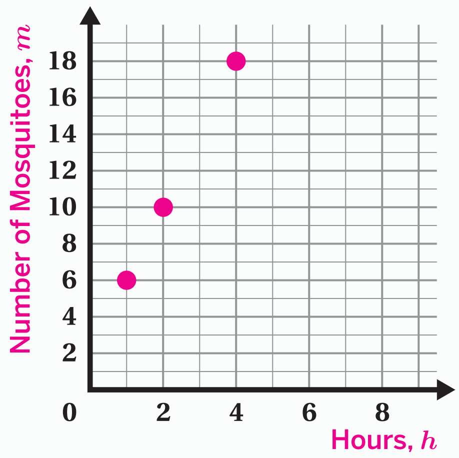
Lesson 13

a Number of paper cranes, p

Caregiver Note: The number of cranes made is dependent upon the number of days.

b $p = 9d$ (or equivalent)

Lesson 14



Lesson 15

a *Responses vary.*

Number of Popsicles Sold, p	Money Earned, d
1	4
2	8
3	12
4	16

b $d = 4p$

Lesson 16

a $t = 2.9r$

b $43.50

Caregiver Note: One strategy is to substitute 15 *for* r, *then solve the equation* for t. $t = 2.9(15) = 43.50$

c *Explanations vary.* The monthly pass would be a good deal if the total cost of all your single-ride tickets is going to be more than $132. You can use the equation to find out how many rides you would need to take. If $132 = 2.9r$, we can divide both sides by 2.9 to solve for r: $\frac{132}{2.9} = \frac{2.9r}{2.9}$ and $45.5 = r$. That means that a rider would start to save money with the monthly pass after 45 rides.