

Unit **3**

Fractions and Decimals

Division is useful for answering questions like “How many groups?” or “How many in 1 group?” You’ve already divided using whole numbers, but how do you divide when the number of groups is a fraction? When you add, subtract, multiply, and divide numbers with decimal places, instead of just estimating, you can get far more precise answers. Doing so can help you make sense of all sorts of real-world situations — like working with money, comparing grocery prices, and even buying a car!

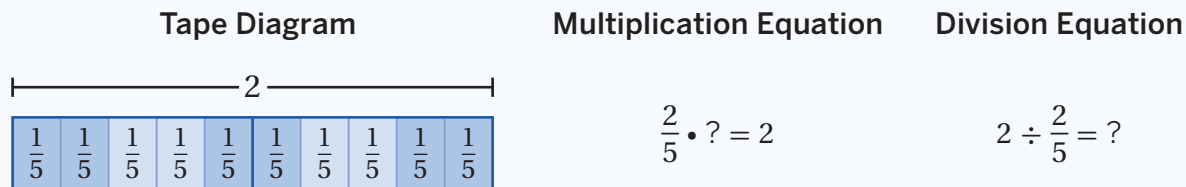
Essential Questions

- What are two ways to think about dividing by a fraction?
- How are division and multiplication related to each other?
- How do the place values in each decimal number in a calculation affect the place value of the result?
- How are strategies for multiplying decimals like strategies for dividing decimals? How are they different?

Summary | Lesson 1

You can answer the question “How many groups?” using different representations that include both whole numbers and fractions.

Here’s the problem “How many $\frac{2}{5}$ s are in 2?” represented using a tape diagram, a multiplication equation, and a division equation.

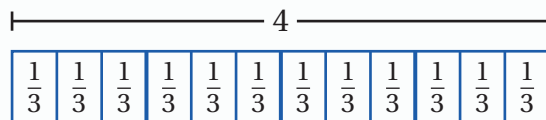


Because there are 5 groups of $\frac{2}{5}$ in 2, the value 5 makes both equations $\frac{2}{5} \cdot 5 = 2$ and $2 \div \frac{2}{5} = 5$ true.

Try This

Maneli needs 4 cups of flour. She has a $\frac{1}{3}$ -cup measuring scoop.

Maneli drew this diagram to determine how many scoops to use.



- Write *at least* one equation to represent Maneli’s diagram.
- How many scoops should Maneli use?

You can answer “How many are in *one* group?” by:

- Evaluating division and multiplication expressions.
- Using tape diagrams that represent division and multiplication expressions.

Situation	Diagram	Expressions	Number of Flowers in 1 Planter
3 flowers fill $\frac{1}{3}$ of a planter.		$3 \div \frac{1}{3} = ?$ or $\frac{1}{3} \cdot ? = 3$	9
18 flowers fill $1\frac{1}{2}$ planters.		$18 \div 1\frac{1}{2} = ?$ or $1\frac{1}{2} \cdot ? = 18$	12

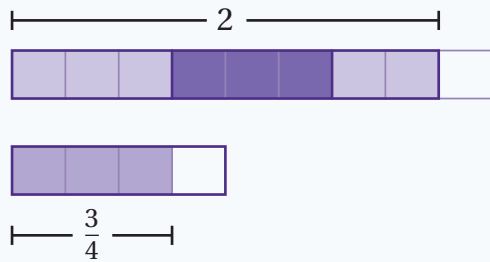
Try This

Caasi picked 12 strawberries, which filled $\frac{3}{4}$ of her basket.

How many strawberries will fill her whole basket? Draw a diagram if it helps with your thinking.

You can use division to determine how many groups fit into a whole. For example, the expression $2 \div \frac{3}{4}$ can represent how many $\frac{3}{4}$ -foot-long bricks fit along a 2-foot garden wall. You can use tape diagrams or reasoning about equal groups to determine how many groups (bricks) fit into the whole (along the garden wall).

Tape Diagram



$$2 \div \frac{3}{4} = 2\frac{2}{3}$$

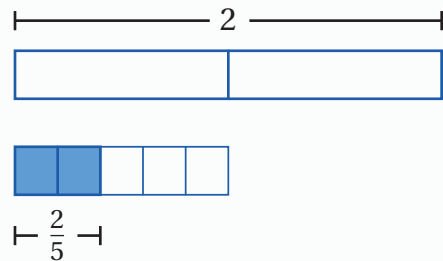
Reasoning About Equal Groups

- To calculate how many lengths of $\frac{3}{4}$ fit into 2, it would help to determine how many $\frac{1}{4}$ s there are in 2 wholes.
- I can rewrite 2 as $\frac{8}{4}$.
- There are two groups of $\frac{3}{4}$ in $\frac{8}{4}$, with $\frac{2}{4}$ left over.
- The leftover $\frac{2}{4}$ has 2 of the 3 parts needed to complete a whole group of $\frac{3}{4}$. That means there are $2\frac{2}{3}$ groups of $\frac{3}{4}$ in 2.

Try This

Determine the value of $2 \div \frac{2}{5}$, then explain your thinking.

Use the tape diagram if it helps with your thinking.



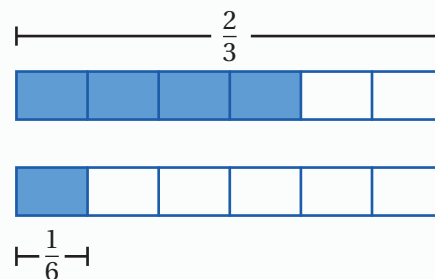
Creating equal-sized pieces, or using a *common denominator*, is a helpful strategy for calculating quotients involving fractions and determining when there is more or less than 1 group.

	Expression and Tape Diagram	Expression and Tape Diagram Using a Common Denominator
More Than 1 Group	$\frac{3}{4} \div \frac{2}{3}$	$\frac{9}{12} \div \frac{8}{12} = \frac{9}{8}$ <p>A common denominator of 4 and 3 is 12.</p>
Less than 1 Group	$\frac{1}{2} \div 1\frac{1}{4}$	$\frac{2}{4} \div \frac{5}{4} = \frac{2}{5}$ <p>A common denominator of 2 and 4 is 4.</p>

Try This

Determine the value of $\frac{2}{3} \div \frac{1}{6}$.

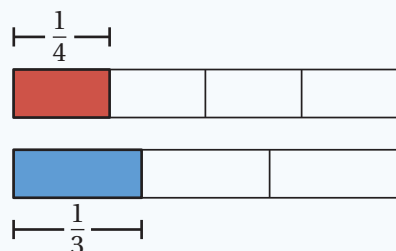
Use the diagram if it helps with your thinking.



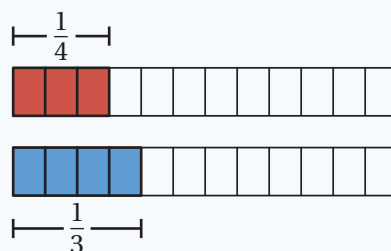
You can use common denominators to determine quotients involving fractions.

For example, in $\frac{1}{4} \div \frac{1}{3}$, you can use 12 as a common denominator of 4 and 3. Then you can rewrite the division expression as $\frac{3}{12} \div \frac{4}{12}$. This helps you determine that there are $\frac{3}{4}$ groups of $\frac{4}{12}$ in $\frac{3}{12}$.

Tape Diagram of Original Problem



Tape Diagram With Common Denominator



Equivalent Fractions With Common Denominator

$$\begin{aligned} \frac{1}{4} \div \frac{1}{3} \\ \frac{3}{12} \div \frac{4}{12} \\ 3 \div 4 \\ \frac{3}{4} \end{aligned}$$

Try This

Determine the value of each expression.

a $\frac{2}{5} \div \frac{3}{5}$

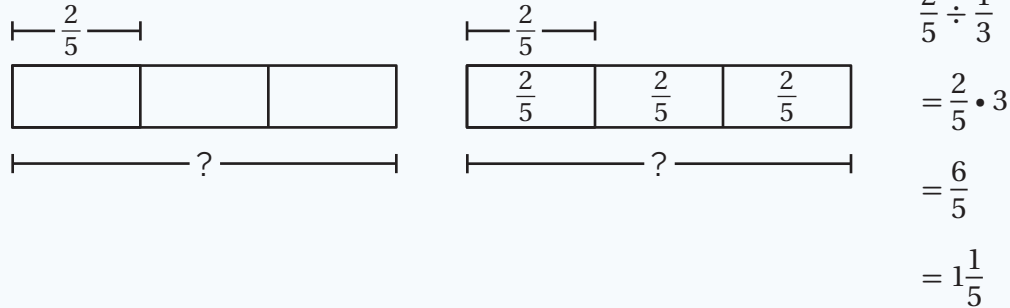
b $1\frac{1}{3} \div \frac{3}{5}$

Summary | Lesson 6

When you divide a number by a unit fraction $\frac{1}{b}$, it's generally the same as multiplying the number by b .

For example, think about the expression $\frac{2}{5} \div \frac{1}{3}$. In our planter and soil situation, this means it takes $\frac{2}{5}$ bags of soil to fill $\frac{1}{3}$ of a planter.

To fill the entire planter, you would need 3 times $\frac{2}{5}$ bags of soil, or $\frac{2}{5} \cdot 3$.



Try This

- a** It takes $\frac{1}{4}$ of a box of dried pasta to make $2\frac{1}{2}$ cups of cooked pasta. How many cups of cooked pasta could the whole box make?
- b** Calculate $\frac{3}{4} \div \frac{1}{5}$.

You don't have to use tape diagrams to determine the quotient of two fractions!

Here are two ways to calculate the quotient of the expression $\frac{9}{10} \div \frac{3}{4}$: by using common denominators and by simplifying numerators.

Common Denominators

- Rewrite the expression using common denominators.

$$\frac{18}{20} \div \frac{15}{20}$$

- Then divide the numerator of the first fraction by the numerator of the second fraction.

$$18 \div 15 = \frac{18}{15} \text{ or } \frac{6}{5}$$

Simplifying Numerators

- Divide the first fraction by the numerator of the divisor to create a unit fraction.

$$\frac{3}{10} \div \frac{1}{4}$$

- To divide by the unit fraction, multiply the dividend by the denominator of the divisor.

$$\frac{3}{10} \cdot 4 = \frac{12}{10} \text{ or } \frac{6}{5}$$

Try This

Determine the value of each expression using any strategy. Show or explain your thinking.

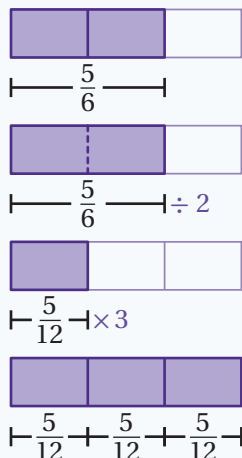
a $4 \div \frac{1}{5}$

b $\frac{3}{2} \div \frac{4}{5}$

In general, when you divide a number by a unit fraction, $\frac{1}{b}$, it's the same as multiplying the number by b (which is the **reciprocal** of $\frac{1}{b}$). This can also help you divide by a fraction $\frac{a}{b}$.

Here are three strategies you can use to solve the problem $\frac{5}{6} \div \frac{2}{3}$.

Tape Diagram



Simplifying Numerators

Divide by the numerator, a , then multiply the result by the denominator, b .

$$\begin{aligned} & \frac{5}{6} \div \frac{2}{3} \\ &= \frac{5}{12} \div \frac{1}{3} \\ &= \frac{5}{12} \cdot 3 \\ &= \frac{15}{12} \\ &= \frac{5}{4} \end{aligned}$$

Multiplying by the Reciprocal

Multiply by the reciprocal of the fraction $\left(\frac{b}{a}\right)$.

$$\begin{aligned} & \frac{5}{6} \div \frac{2}{3} \\ &= \frac{5}{6} \cdot \frac{3}{2} \\ &= \frac{15}{12} \\ &= \frac{5}{4} \end{aligned}$$

Try This

- a** Select *all* the expressions that are equivalent to $\frac{3}{4} \div \frac{2}{3}$.

☐ A. $\frac{3}{4} \cdot \frac{3}{2}$

☐ B. $\frac{3}{8} \cdot 3$

☐ C. $\frac{3}{4} \cdot \frac{2}{3}$

☐ D. $\frac{3}{4} \div \frac{1}{3}$

☐ E. $\frac{9}{8}$

- b** Write a multiplication expression that is equivalent to $\frac{1}{8} \div \frac{4}{5}$.

There are many real-life situations where you can use fraction division.

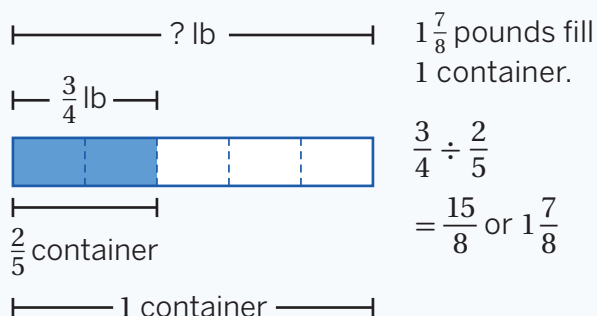
For example, let's say $\frac{3}{4}$ pounds of rice fills $\frac{2}{5}$ of a container.

There are two possible questions you can ask:

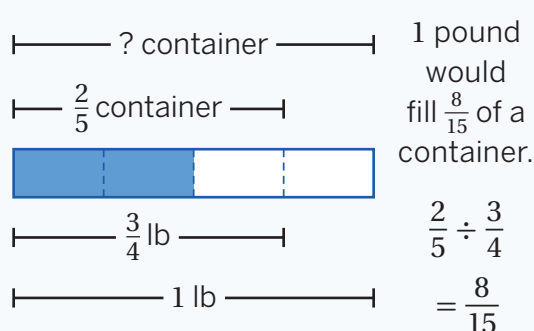
- How many pounds fill 1 container?
- How many containers for 1 pound?

Here's how you can use different division expressions and tape diagrams to answer each question.

How many pounds fill 1 container?



How many containers for 1 pound?



Try This

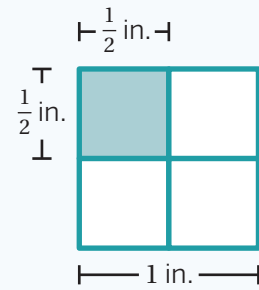
Marquis walked $\frac{3}{5}$ of a mile, which is $\frac{2}{3}$ of the distance between his home and school.

- Write an expression to represent the total distance between Marquis's home and school.
- Calculate the total distance.

Summary | Lesson 10

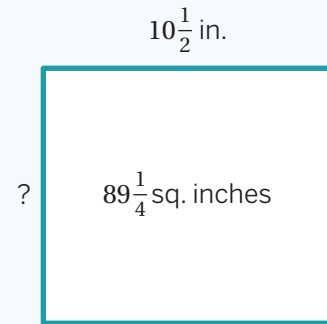
You can determine the area of a polygon that has fractional side lengths just like you would a polygon that has whole-number side lengths.

For example, you can calculate the area of the shaded square using the formula $A = l \cdot w$. The area is equal to $\frac{1}{2} \cdot \frac{1}{2}$, or $\frac{1}{4}$ square inches.



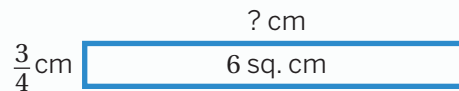
You can also use area formulas to determine an unknown length. If you know the area and one side length of a rectangle, you can divide to determine the other side length.

For example, to determine the missing side length of this rectangle, you can calculate $89\frac{1}{4} \div 10\frac{1}{2} = 8\frac{1}{2}$. The missing side length is $8\frac{1}{2}$ inches.



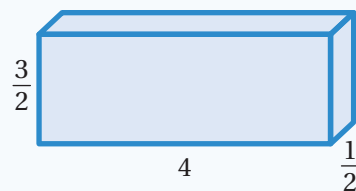
Try This

Use any strategy to determine the unknown value.



You can determine the *volume* of a prism by multiplying its dimensions.

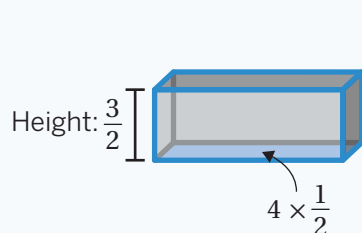
For example, here is a rectangular prism with side lengths measuring 4 units, $\frac{3}{2}$ units, and $\frac{1}{2}$ units.



Volume

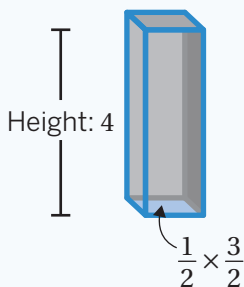
$$4 \cdot \frac{3}{2} \cdot \frac{1}{2} = 3 \text{ cubic units}$$

You can also calculate the volume of a prism as the product of its base area and the height. You can choose any of the rectangles as the base.



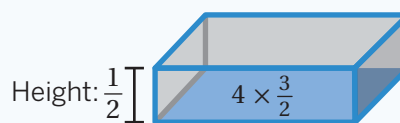
If you choose the 4-by- $\frac{1}{2}$ rectangle as the base, then the base area will be 2 square units.

The volume is $2 \cdot \frac{3}{2} = 3$ cubic units.



If you choose the $\frac{3}{2}$ -by- $\frac{1}{2}$ rectangle as the base, then the base area will be $\frac{3}{4}$ square units.

The volume is $\frac{3}{4} \cdot 4 = 3$ cubic units.

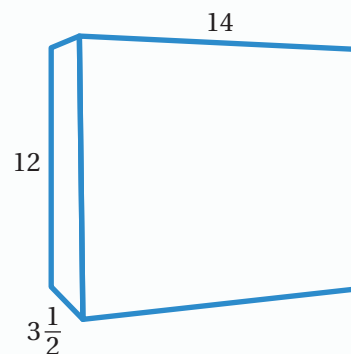


If you choose the 4-by- $\frac{3}{2}$ rectangle as the base, then the base area will be 6 square units.

The volume is $6 \cdot \frac{1}{2} = 3$ cubic units.

Try This

What is the volume of a box that measures 14 inches by 12 inches by $3\frac{1}{2}$ inches?

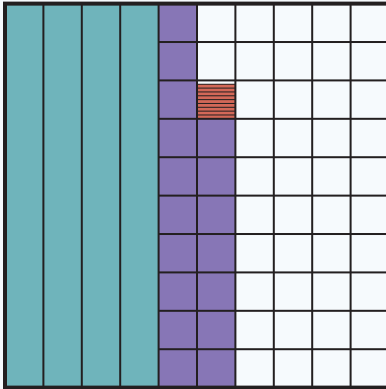


You can represent decimals in more than one way using words, diagrams, and decimal points. For example, six tenths, 0.6, sixty hundredths, and 0.60 all represent the same quantity.

Using multiple representations can help when you're adding or subtracting decimals.

Let's say we're calculating $0.189 + 0.39$.

Hundredths Chart



Vertical Calculation

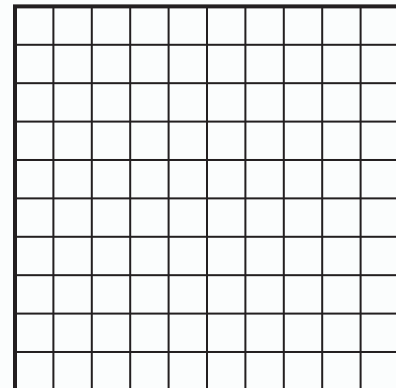
$$\begin{array}{r} 1 \\ 0.189 \\ + 0.390 \\ \hline 0.579 \end{array}$$

Both the hundredths chart and vertical calculation show a total of 4 tenths, 17 hundredths, and 9 thousandths. 10 hundredths equals 1 tenth, so the final answer is 5 tenths, 7 hundredths, and 9 thousandths, or 0.579.

Try This

Calculate the value of $0.472 - 0.081$.

Use the diagram or a vertical calculation if it helps to show your thinking.



When you're adding and subtracting decimals, it's helpful to rewrite them in different ways.

Let's say a lemon and a strawberry together weigh 0.35 pounds, and the lemon on its own weighs 0.235 pounds.

How much does the strawberry weigh?

$$\begin{array}{r} 4 10 \\ 0. 3 5 \\ - 0. 2 3 5 \\ \hline 0. 1 1 5 \end{array}$$

You can use vertical calculations to rewrite the total weight, 0.35, as 3 tenths, 4 hundredths, and 10 thousandths. Now you can subtract the weight of the lemon from the total to determine the weight of the strawberry.

Try This

Calculate the value of $3.725 - 1.14$.

Use a vertical calculation if it helps to show your thinking.

One strategy that can help you make sense of decimal addition and subtraction is vertical calculations.

To use a vertical calculation, you just align numbers by place value so that you're adding or subtracting ones with ones, tenths with tenths, hundredths with hundredths, and thousandths with thousandths.

Here's a vertical calculation. To check if the calculation is correct, you could either estimate or use the opposite operation (addition).

$$\begin{array}{r} 6.2 \\ - 2.5 \\ \hline 3.7 \end{array}$$

You could estimate that $6.2 - 3 = 3.2$, so your difference should be larger than 3.2.

You could also use addition to check your work, adding 2.5 to 3.7 to get 6.2.

Try This

Here is a subtraction problem.

- a** Determine the missing digits.

$$\begin{array}{r} 8.8 \\ - \square.2\square \\ \hline 4.\square4 \end{array}$$

- b** Use addition to check your work.

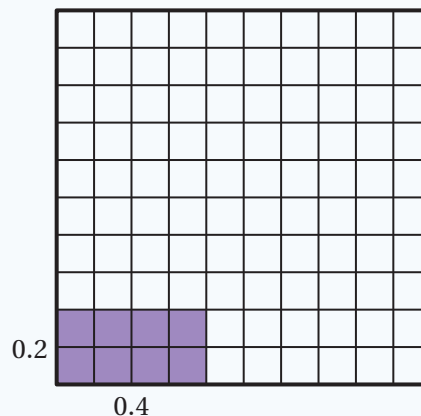
Two strategies for multiplying decimals are:

- Using an area model.
- Writing the decimals as equivalent fractions.

One advantage to using an area model is that you can visualize the product. For example, you can represent $0.4 \cdot 0.2$ as a rectangle with a length of 0.4 and a width of 0.2 . On a hundredths chart, you can count the shaded boxes, each representing $\frac{1}{100}$, to determine the product.

It can, however, be challenging to use an area model to represent decimals smaller than tenths or hundredths.

Your other option is to convert decimals to equivalent fractions. $0.4 \cdot 0.2$ can be written as $\frac{4}{10} \cdot \frac{2}{10}$, which equals $\frac{8}{100}$ or 0.08 .

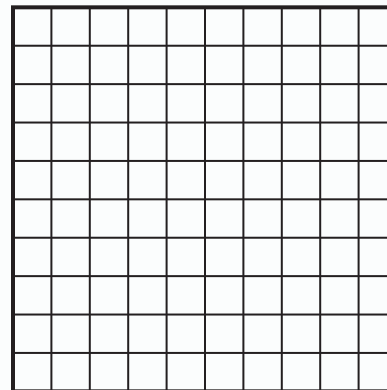


$$\begin{aligned}\text{Area} &= \text{length} \cdot \text{width} \\ &= 0.4 \cdot 0.2 \\ &= 0.08\end{aligned}$$

Try This

Calculate the value of $0.8 \cdot 0.05$.

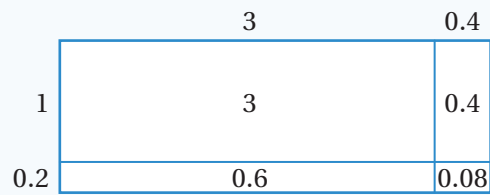
Use the hundredths chart or equivalent fractions if it helps to show your thinking



One way to multiply two decimals is to use an area model.

To use an area model, separate the decimals into parts. This rectangle has side lengths measuring 3.4 and 1.2 units. Each side length has been split apart by place value: 3.4 has been split into $3 + 0.4$ and 1.2 has been split into $1 + 0.2$.

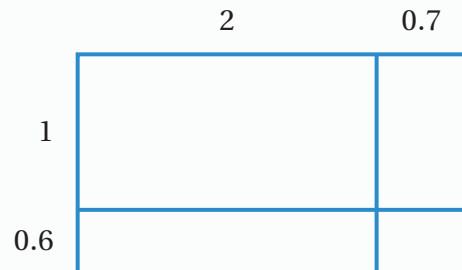
The total area of the rectangle is equal to the sum of the areas of the four smaller rectangles: $3.4 \cdot 1.2 = 3 + 0.4 + 0.6 + 0.08 = 4.08$.



Try This

Here is an area model for $2.7 \cdot 1.6$, split into parts.

- a** Calculate the area of each part.



- b** Use your area model to calculate $2.7 \cdot 1.6$.

There are several different strategies you can use to multiply decimals, such as area models and fractions. You can even convert the decimals to whole numbers and then use place value reasoning. Depending on the problem, one strategy might be more helpful than another.

Let's solve $2.4 \cdot 0.03$ using two strategies: *converting fractions* and *using whole numbers with place value reasoning*.

Strategy 1:

Converting Fractions

Rewrite each value as an equivalent fraction.

$$2.4 \cdot 0.03 = \frac{24}{10} \cdot \frac{3}{100}$$

Multiply the fractions.

$$\frac{24}{10} \cdot \frac{3}{100} = \frac{72}{1000}$$

Use the denominator to determine the place value.

$$\frac{72}{1000} \text{ is 72 thousandths.}$$
$$\frac{72}{1000} = 0.072$$

Strategy 2: Whole Numbers
With Place Value Reasoning

Think of each term as a whole number, then multiply.

$$2.4 \cdot 0.03 \rightarrow 24 \cdot 3$$
$$24 \cdot 3 = 72$$

Think about the place value of each term.

$$2.4 \text{ is 24 tenths.}$$
$$0.03 \text{ is 3 hundredths.}$$

Determine the appropriate place value of the product.

tenths times hundredths = thousandths

$$2.4 \cdot 0.03 = 72 \text{ thousandths}$$
$$2.4 \cdot 0.03 = 0.072$$

Try This

Calculate the value of $1.5 \cdot 0.023$.

Use any strategy to show your thinking.

When you get a remainder in a division expression, you can just continue to divide.

Here is how you can calculate $86 \div 4$ using **long division**.

In this strategy, you break down the remaining 2 ones into 20 tenths by writing the dividend of 86 as 86.0.

This allows you to bring a 0 down to the right of the remaining 2 ones.

Then you add a decimal point to the right of the 1 in the quotient, to show that the resulting 5 is in the tenths place.

$$\begin{array}{r} 21.5 \\ 4 \overline{) 86.0} \\ \underline{-8} \\ 6 \\ \underline{-4} \\ 2 \\ \underline{-2} \\ 0 \end{array}$$

So $86 \div 4 = 21.5$.

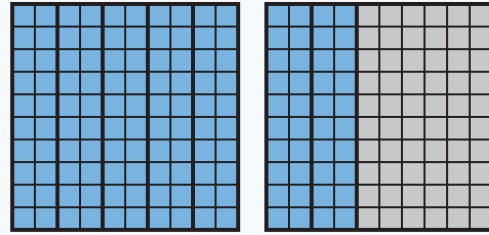
Try This

Use long division to calculate the value of $855 \div 6$.

We can use hundredths charts, thousandths charts, and fractions to help us visualize and divide decimals.

This diagram represents the expression $1.4 \div 0.2$.

- Using the hundredths chart, you can count the number of groups of 2 tenths needed to fill the 1 whole and 4 tenths. It takes 7 groups, so $1.4 \div 0.2 = 7$.
- You can rewrite each decimal as an equivalent fraction, so 1.4 becomes $\frac{14}{10}$ and 0.2 becomes $\frac{2}{10}$. Now you can use your knowledge of fraction division to calculate the quotient.



$$\begin{aligned} 1.4 \div 0.2 &= \frac{14}{10} \div \frac{2}{10} \\ &= 14 \div 2 \\ &= 7 \end{aligned}$$

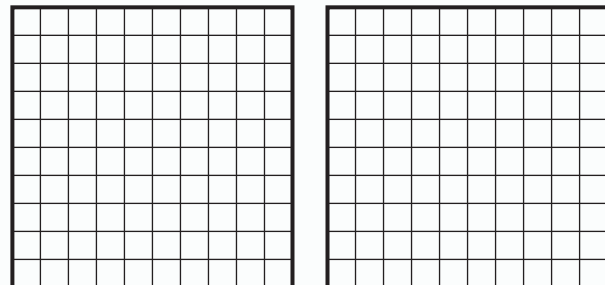
Sometimes you will need to use common denominators to solve expressions with fractions. For example,

$$1.5 \div 0.03 = \frac{15}{10} \div \frac{3}{100} = \frac{150}{100} \div \frac{3}{100} = 150 \div 3 = 50.$$

Try This

Calculate $1.08 \div 0.09$.

Use the diagram if it helps to show your thinking.



Summary | Lesson 20

When dividing by a decimal, it can be helpful to rewrite the expression using whole numbers by multiplying by a power of 10.

For example, you can rewrite $7.65 \div 1.2$ as $765 \div 120$.

$$\begin{aligned} 7.65 \div 1.2 &= \frac{765}{100} \div \frac{12}{10} \\ &= \frac{765}{100} \div \frac{120}{100} \\ &= 765 \div 120 \end{aligned}$$

Once you have an expression with whole numbers, you can use long division to calculate the quotient.

$$\begin{array}{r} \textcolor{blue}{6.375} \\ 120 \overline{) 765.000} \\ \underline{-720} \downarrow \\ 450 \downarrow \\ \underline{-360} \downarrow \\ 900 \downarrow \\ \underline{-840} \downarrow \\ 600 \\ \underline{-600} \\ 0 \end{array}$$

Try This

Calculate $5.62 \div 0.05$.

Use any strategy to show your thinking.

To determine how long it will take to play a movie at a certain speed, you can divide the length of the original movie by the playback speed.

Let's say a movie is 5.6 minutes long.

- When the playing speed is doubled, it will take 2.8 minutes to watch the movie: $5.6 \div 2 = 2.8$.
- When the playing speed is halved, it will take 11.2 minutes to watch the movie: $5.6 \div 0.5 = 11.2$.

Try This

Ichiro wants to play a 20-second movie at different speeds.

- Write an expression he could use to calculate how long it would take to play the movie at $4x$.
- How long would it take to play the movie at $0.8x$?

Lesson 1

- a Responses vary. Some possible equations are $4 \div \frac{1}{3} = 12$ and $\frac{1}{3} \cdot 12 = 4$.
- b 12 scoops

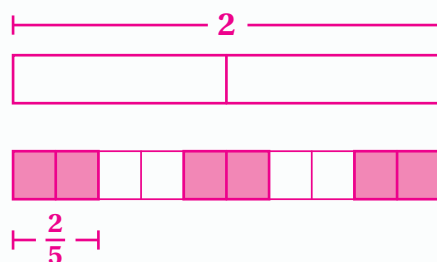
Lesson 2

16 strawberries



Lesson 3

5. Explanations vary. 2 equals $\frac{10}{5}$. There are 5 groups of $\frac{2}{5}$ in $\frac{10}{5}$.



Lesson 4

4. According to the diagram, it would take four $\frac{1}{6}$ pieces to equal $\frac{2}{3}$.

Lesson 5

- a $\frac{2}{3}$.

Caregiver Note: When dividing fractions with the same denominator, you can just divide the numerators and $2 \div 3 = \frac{2}{3}$.

- b $\frac{20}{9}$.

Caregiver Note: One strategy is to find a common denominator. $1\frac{1}{3} \div \frac{3}{5}$ is equivalent to $\frac{20}{15} \div \frac{9}{15}$. Then you can divide the numerators. $20 \div 9 = \frac{20}{9}$.

Lesson 6

- a** 10 cups.

Caregiver Note: Since $\frac{1}{4}$ of the box makes $2\frac{1}{2}$ cups, the whole box would make 4 times as much and $2\frac{1}{2} \cdot 4 = 10$.

- b** $\frac{15}{4}$.

Caregiver Note: Since $\frac{1}{5}$ is a unit fraction, you can multiply by the denominator. $\frac{3}{4} \div \frac{1}{5}$ is equivalent to $\frac{3}{4} \cdot 5 = \frac{15}{4}$.

Lesson 7

- a** 20. *Explanations vary.* Since $\frac{1}{5}$ is a unit fraction, one strategy is to multiply by the denominator. $4 \div \frac{1}{5}$ has the same value as $4 \cdot 5 = 20$.

- b** $\frac{15}{8}$. *Explanations vary.* One strategy is to find a common denominator. $\frac{3}{2} \div \frac{4}{5}$ is equivalent to $\frac{15}{10} \div \frac{8}{10}$. Then you can divide the numerators and $15 \div 8 = \frac{15}{8}$.

Lesson 8

- a** A. $\frac{3}{4} \cdot \frac{3}{2}$, B. $\frac{3}{8} \cdot 3$, and E. $\frac{9}{8}$

- b** *Responses vary.* Two possible expressions are $\frac{1}{8} \cdot \frac{5}{4}$ and $\frac{1}{32} \cdot 5$.

Lesson 9

- a** $\frac{3}{5} \div \frac{2}{3}$

- b** $\frac{9}{10}$ of a mile

Lesson 10

8 centimeters.

Caregiver Note: One strategy is to divide the area by the given dimension and $6 \div \frac{3}{4} = 8$.

Lesson 11

588 cubic inches

Lesson 12

0.391

Lesson 13

2.585

Lesson 14

a

$$\begin{array}{r} 8.8 \\ - 4.26 \\ \hline 4.54 \end{array}$$

b $4.26 + 4.54 = 8.8$

Lesson 15

0.04

Lesson 16

a

	2	0.7
1	2	0.7
0.6	1.2	0.42

b 4.32

Lesson 17

0.0345

Explanations vary. One strategy is to use the converting fractions model to calculate $1.5 \cdot 0.023$.

- Rewrite each value as an equivalent fraction. $1.5 \cdot 0.023 = \frac{15}{10} \cdot \frac{23}{1000}$
- Multiply the fractions. $\frac{15}{10} \cdot \frac{23}{1000} = \frac{345}{10000}$
- Use the denominator to determine the place value. $\frac{345}{10000}$ is 345 ten-thousandths, which is written as 0.0345 in decimal form.

Lesson 21

- a $20 \div 4$ (or equivalent)
- b 25 seconds

Caregiver Note: One strategy is to divide the length of the movie by the playback speed.

$$20 \div 0.8 = 25$$