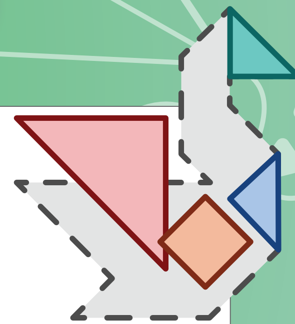
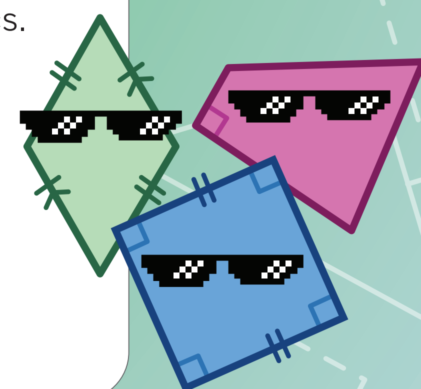


Unit 2

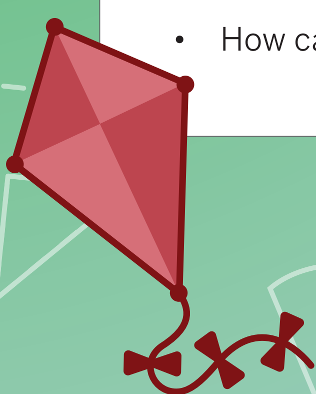
Congruence

In geometry, we often try to prove when two figures are exactly the same. In this unit, you'll use rigid transformations to prove congruent relationships in angles, triangles, and parallelograms. You'll also grow your skills for making a convincing argument in mathematics.



Essential Questions

- How can I use rigid transformations to prove relationships in angles, triangles, and quadrilaterals?
- How can I prove a conjecture?



Summary | Lesson 1

Definitions help us communicate with one another. They are often created to help us understand one another as we live in community. We can use examples and non-examples to help us think about what to include in a definition.

Definitions are often as unique as the people that create them. For example, one person's definition of a vehicle can be very different from another person's, and that is okay. When members of the same community use different definitions, we can begin by asking how each person defines a term so that we can better understand each other.

There are also times when a shared definition can help a community speak clearly and precisely with one another. Creating a shared definition can have consequences, and a community that has a shared definition can make sense of those consequences together.

Try This

a Select *all* the activities that you think are sports.

- ☐ Poker
- ☐ Chess
- ☐ Fishing
- ☐ Dominoes
- ☐ Bowling

b Write a definition of *sport* that is true for you and the choices you made above.

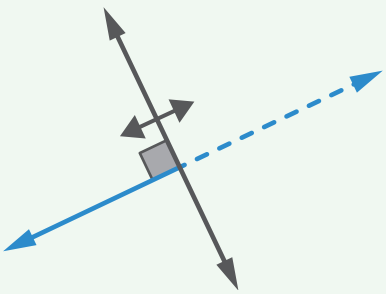
Summary | Lesson 2

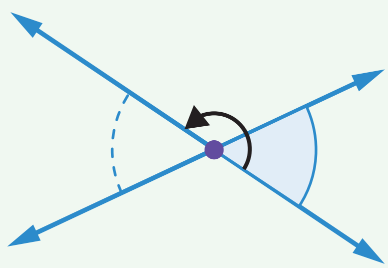
You can use rigid transformations and their definitions to explore geometric relationships.

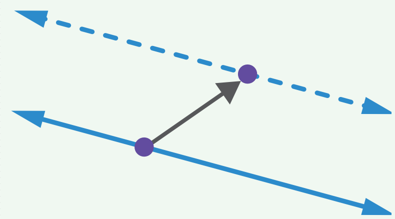
- A translation moves any point a specific distance and direction.
- A reflection moves any point to a point on the opposite side of a line of reflection.
- A rotation moves any point around a center of rotation by a specific angle and direction.

Because rigid transformations keep all angles and distances the same, they can be helpful tools for proving **conjectures** about geometric relationships.

Reflection	Rotation	Translation
You can reflect a ray across a perpendicular line to make another line.	You can rotate an angle 180° around its vertex to prove that *vertical angles* are *congruent*.	You can translate a line to a point off of the line in order to create a parallel new line.





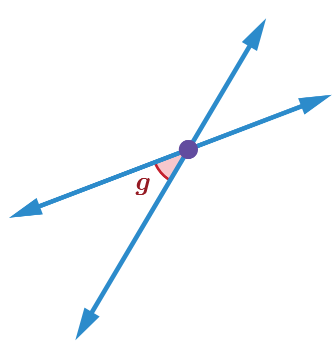


Try This

a If you applied a 180° rotation to a line, what could be the result? Select *all* that apply.

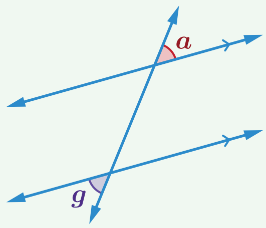
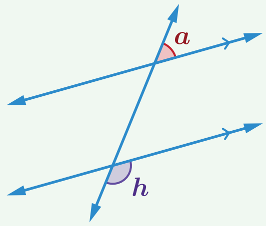
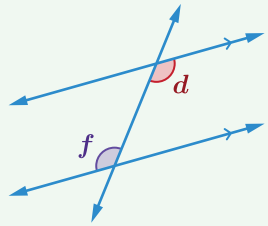
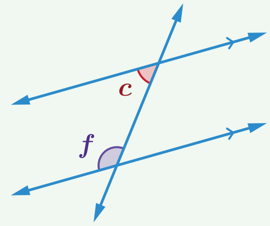
- ☐ **A.** A line that's parallel to the original line
- ☐ **B.** A line that's perpendicular to the original line
- ☐ **C.** Just the original line

b The diagram shows two lines and the angle g. Describe the result of rotating everything by 180° around the intersection point.



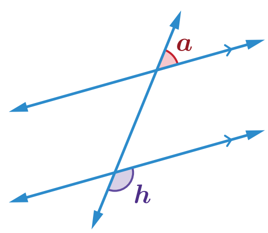
Summary | Lesson 3

There are often several different ways to explain why statements are true. Comparing the different ways can lead to new insights or more flexible understanding. Consider the different angles that form when two parallel lines are cut by a transversal.

Alternate Exterior Angles	Same Side Exterior Angles	Alternate Interior Angles	Same Side Interior Angles
			

You can **prove** that there are relationships between these angles using rigid transformations, such as translations, reflections, and rotations. As an example for alternate exterior angles, by translating angle a to the vertex of angle g, you can prove that angle a is congruent to angle g because they become vertical angles.

Try This



Here is a conjecture.

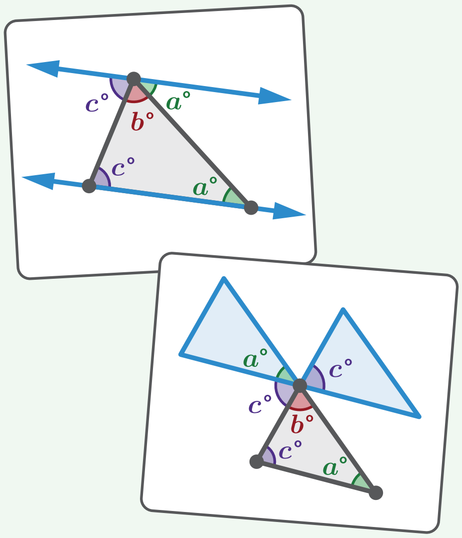
When two parallel lines are crossed by a transversal, same side exterior angles are supplementary.

a Describe a transformation that would show this relationship.

b Explain how this proves the conjecture.

Summary | Lesson 4

Conjectures can be proven in many different ways. As you make a plan to prove a conjecture, it can be helpful to begin with a relationship or property that has already been proven true. For example, to prove that the interior angles of a triangle always sum to $180°$, you might begin with what you know about vertical angles, alternate interior angles, *straight angles*, or other angle relationships.



Proving a conjecture is true for one case is not the same as proving that a conjecture is always true.

There are several strategies for proving a conjecture is always true.

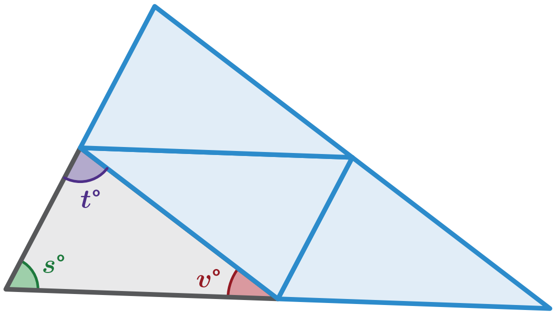
- You can explain why the reasoning that applies to one case can be generalized to all other possible cases.
- You can use draggable points to show that a relationship stays the same for all of the possible cases.
- You can also replace numbers with variables to show that a relationship stays the same for *all* cases.

Try This

A student is trying to prove this conjecture: *The interior angles of a triangle sum to* $180°$.

The student starts by making a triangle, and then uses rigid transformations to make three copies of the triangle.

Label each angle in the copies with the appropriate variable.



Summary | Lesson 5

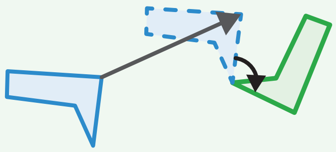
You can use sequences of rigid transformations to decide if two figures are **congruent**. *Translations*, *rotations*, and *reflections* are all *rigid transformations*, which means they keep the size or shape of a figure the same. If two figures stack perfectly using rigid transformations, then they are congruent.

Let's look at two examples.

These figures are congruent because there is a sequence of transformations that will make them stack perfectly.

One way to stack the figures perfectly is to translate and then rotate the figure.

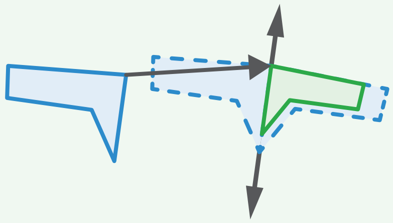
Congruent



These two figures are *not* congruent.

There is no sequence of rigid transformations that will stack the figures perfectly.

Not Congruent

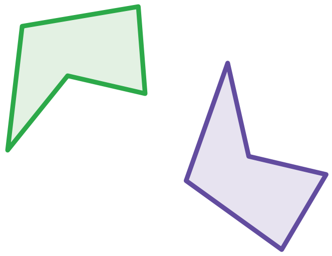


Try This

Are these two figures congruent? Circle one.

Yes   No   I'm not sure

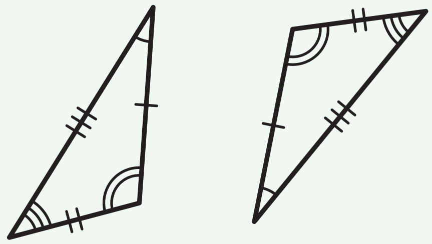
Explain your thinking.



Summary | Lesson 6

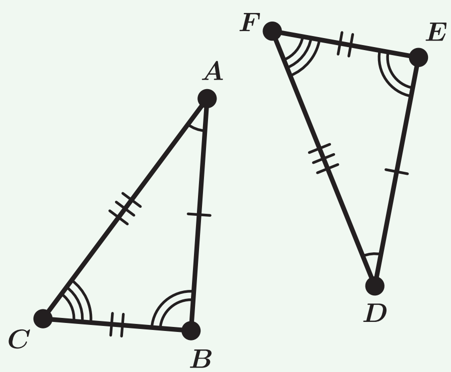
You can determine if two figures are congruent in two different ways:

- Determining a sequence of rigid transformations that stacks the figures perfectly.
- Verifying that all the **corresponding** parts in the figures are congruent.



Corresponding parts are in the same position and orientation in each figure. If *all* the pairs of congruent sides and angles are corresponding in each figure, then the figures are congruent.

To show that congruent segments are corresponding, you can mark them with tick marks. To show that congruent angles are corresponding, you can mark them with arcs.



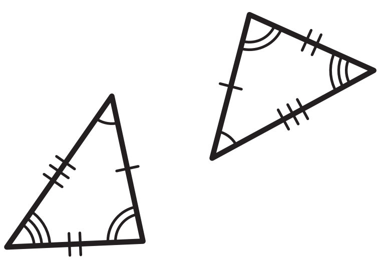
If two triangles are congruent, then they have 6 sets of corresponding congruent parts: 3 pairs of sides and 3 pairs of angles.

$\overline{AB} \cong \overline{DE}$ $\overline{BC} \cong \overline{EF}$ $\overline{AC} \cong \overline{DF}$

$\angle A \cong \angle D$ $\angle B \cong \angle E$ $\angle C \cong \angle F$

Try This

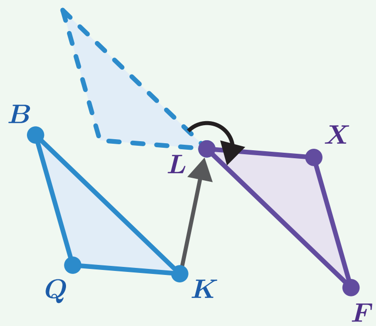
Explain how you know that these two triangles must be congruent.



Summary | Lesson 7

The congruent parts that are in the same position and orientation within congruent figures are called *corresponding parts*. You can identify them in multiple ways:

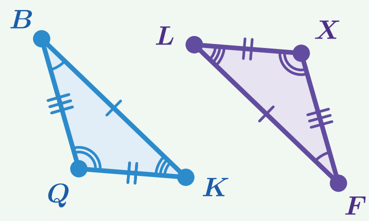
Transformations



The sequence of transformations shows a translation that stacks K on L and a rotation that stacks Q on X and B on F.

Since $\overline{BQ}$ stacks perfectly on $\overline{FX}$, $\overline{BQ} \cong \overline{FX}$.

Diagrams



The markings on the pair of triangles show which parts are congruent and corresponding.

Since $\angle Q$ and $\angle X$ are both marked with a double arc, $\angle Q \cong \angle X$.

Congruence Statements

$\triangle BQK \cong \triangle FXL$

The order in which vertices appear in triangle congruence statements shows which vertices correspond in two figures.

Since K and L are both the third letter in the triangle congruence statement, $\angle K \cong \angle L$.

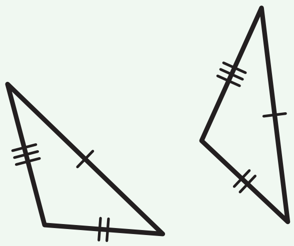
Try This

Draw and label two triangles so that $\triangle YMK \cong \triangle JFK$. Remember to include congruence markings.

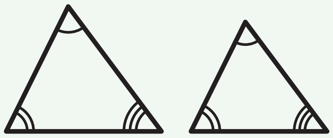
Summary | Lesson 8

If you compare two triangles and every corresponding pair of sides and angles is congruent, then you know the two triangles are congruent. Sometimes, comparing just a few corresponding parts is enough to determine if two triangles are congruent.

Having three pairs of corresponding congruent sides means the triangles will always be congruent.



But there are exceptions to keep in mind! For example, having three pairs of corresponding congruent angles does not mean that the triangles will always be congruent.



Try This

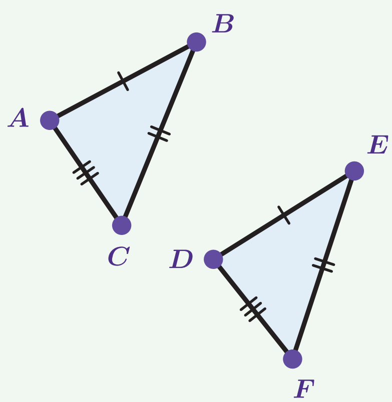
Azul made two triangles with three pairs of corresponding congruent angles.

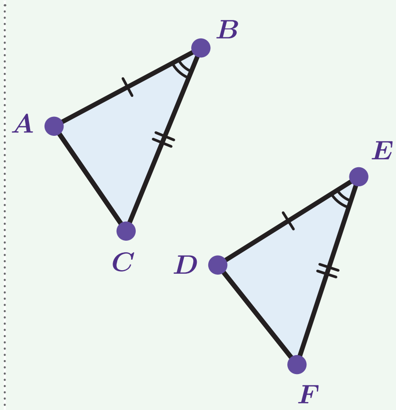
Are these triangles always congruent? Explain your thinking.

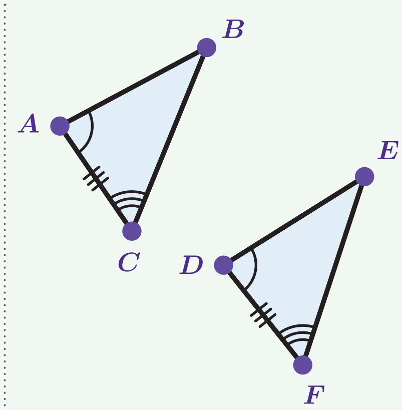
Summary | Lesson 9

You can use specific triangle congruence criteria to create triangles that are always congruent. Two triangles are congruent when they have these pairs of corresponding congruent parts:

SSS (side-side-side)	SAS (side-angle-side)	ASA (angle-side-angle)
Three sides	Two sides and the angle between them	Two angles and the side between them







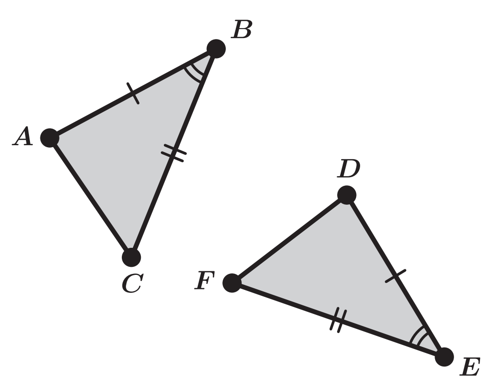
However, two corresponding congruent sides and an adjacent pair of corresponding angles (known as SSA or side-side-angle) do not always create congruent triangles.

Try This

Which criteria helps you know that these triangles are congruent? Circle one.

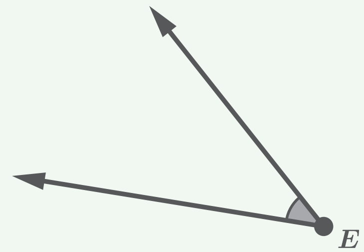
SSS SAS ASA

Explain your thinking.



Summary | Lesson 10

When constructing a copy of an angle, it can be helpful first to construct a copy of a triangle. For example, you can add a point on each ray of $\angle E$. If you connect these points, then you have constructed a triangle.

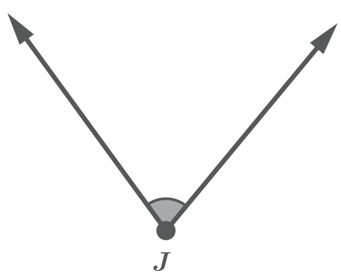


The "circle by length" tool can be used to copy the side lengths of this triangle. The intersection of the new circles represents the vertices of the copied triangle. The constructed triangle is congruent to the original because of the SSS congruence criteria.

By constructing a congruent triangle, you have also constructed a copy of $\angle E$ because corresponding parts of congruent triangles are always congruent.

Try This

Describe how to construct a copy of $\angle J$ using a straightedge and a compass.

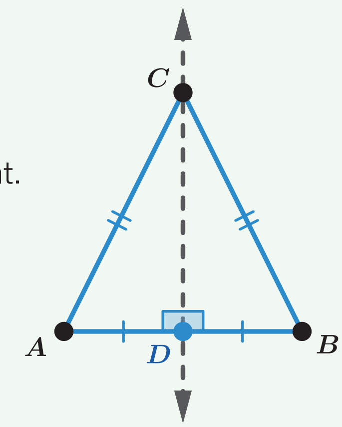


Summary | Lesson 11

The *perpendicular bisector* of a line segment is *exactly* those points that are the same distance from both endpoints of the line segment.

This **theorem** can be split into two conjectures:

1. If a point is on the perpendicular bisector of a segment, then it must be the same distance from both endpoints of the segment.
2. If a point is the same distance from both endpoints of a segment, then it must be on the perpendicular bisector of the segment.



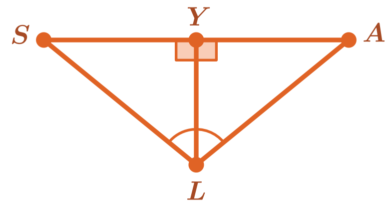
Convincing arguments include a reason explaining how you know that each statement must be true. A table can help you stay organized and make sure there is a reason for each statement.

One first step when writing a convincing argument is to decide if you believe the conjecture is true. This can help you focus on the information that must be true, and you can make a plan that only uses facts to prove the conjecture. Writing a first draft and getting feedback from a partner can help make your proof stronger and clearer.

Try This

Fill in the missing reasons to prove this conjecture:

If $\overline{YL}$ is the perpendicular bisector of $\overline{SA}$, then $\overline{LS} \cong \overline{LA}$.



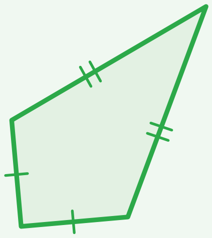
	Statement	Reason
	$\overline{YL}$ is the perpendicular bisector of $\overline{SA}$.	Given in the conjecture.
	$\overline{SY} \cong \overline{AY}$, $m\angle SYL = 90°$, and $m\angle AYL = 90°$.	Definition of a perpendicular bisector.
a	$\angle SYL \cong \angle AYL$	
	$\overline{YL} \cong \overline{YL}$	The same segment in both triangles.
b	$\triangle YSL \cong \triangle YAL$	
c	$\overline{LS} \cong \overline{LA}$	

Summary | Lesson 12

Quadrilaterals can look very different from each other, and some quadrilaterals are defined by specific properties. Here are the names and definitions of five quadrilaterals.

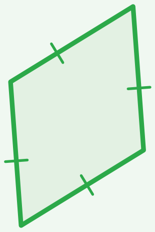
Kite

A quadrilateral with two pairs of adjacent congruent sides.



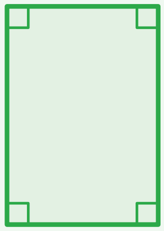
Rhombus

A quadrilateral with four congruent sides.



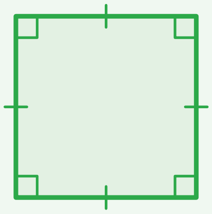
Rectangle

A quadrilateral with four congruent angles.



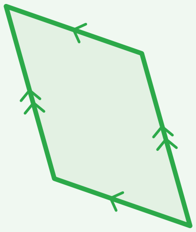
Square

A quadrilateral with four congruent angles and sides.



Parallelogram

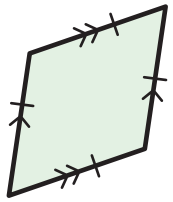
A quadrilateral with opposite sides that are parallel.



These defining properties can sometimes overlap. For example, a rhombus is also a kite because it has two pairs of adjacent sides. A square is both a rhombus and a rectangle because it has four congruent sides and four congruent angles. The definitions of quadrilaterals often imply that there are additional properties. For example, a rectangle has four congruent angles, which means that each of its angles must be a right angle.

Try This

Select *all* the names that apply to this quadrilateral.



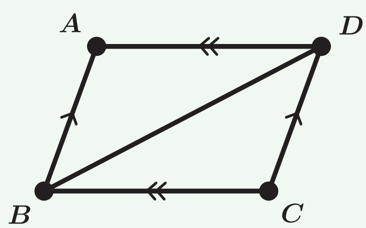
- ☐ **A.** Rectangle
- ☐ **B.** Square
- ☐ **C.** Parallelogram
- ☐ **D.** Kite
- ☐ **E.** Rhombus

Summary | Lesson 13

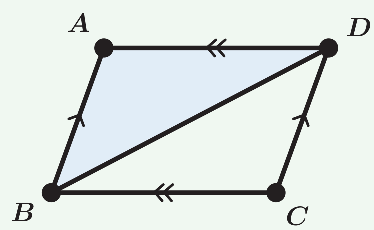
Constructing diagonals can be a helpful strategy for proving the properties of parallelograms.

When you construct a diagonal in a parallelogram, you create congruent triangles. You can use the triangles' corresponding pairs of congruent parts to prove the properties of parallelograms.

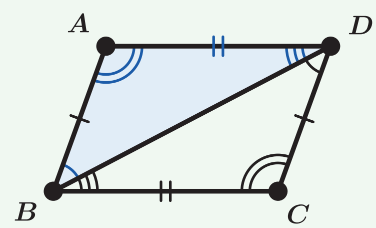
Here's one way you can use diagonals to prove that parallelograms have congruent opposite sides and congruent opposite angles.



Construct the diagonal $\overline{BD}$.



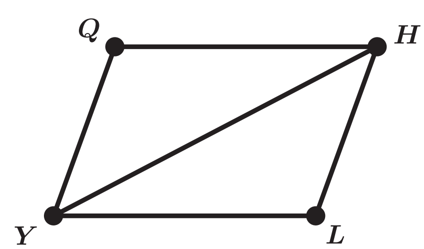
Prove $\triangle ABD \cong \triangle CDB$.



Look for corresponding congruent parts.

Try This

Here is parallelogram $YQHL$ with diagonal $\overline{YH}$.



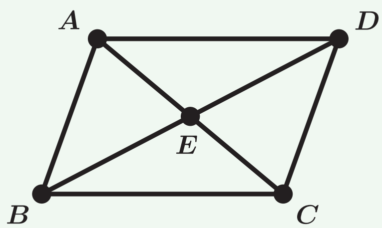
Select *all* the true statements.

- ☐ **A.** $\overline{QY} \cong \overline{LY}$
- ☐ **B.** $\angle QYH \cong \angle LHY$
- ☐ **C.** $\overline{QH} \cong \overline{LY}$
- ☐ **D.** $\angle YHL \cong \angle YHQ$
- ☐ **E.** $\overline{YQ} \cong \overline{HL}$
- ☐ **F.** $\angle YQH \cong \angle HLY$

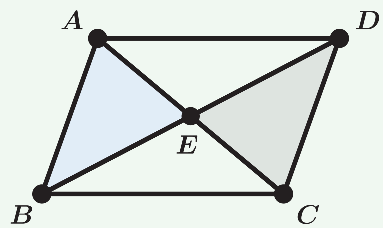
Summary | Lesson 14

When you construct one or two diagonals in a parallelogram, they make congruent triangles. Since congruent triangles have corresponding pairs of congruent parts, this step helps prove the properties of parallelograms, such as opposite sides and angles being congruent.

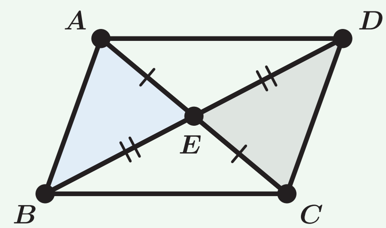
Here is one example of how constructing diagonals can be a helpful strategy for proving this property of parallelograms: *The diagonals of a parallelogram bisect each other.*



Construct the diagonals $\overline{AC}$ and $\overline{BD}$. Label the intersection point E.



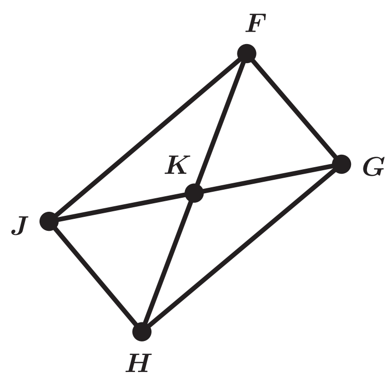
Prove $\triangle AEB \cong \triangle CED$.



Look for corresponding congruent parts. $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$, so point E bisects both $\overline{AC}$ and $\overline{BD}$.

Try This

Figure $FGHJ$ is a rectangle with diagonals $\overline{JG}$ and $\overline{FH}$.



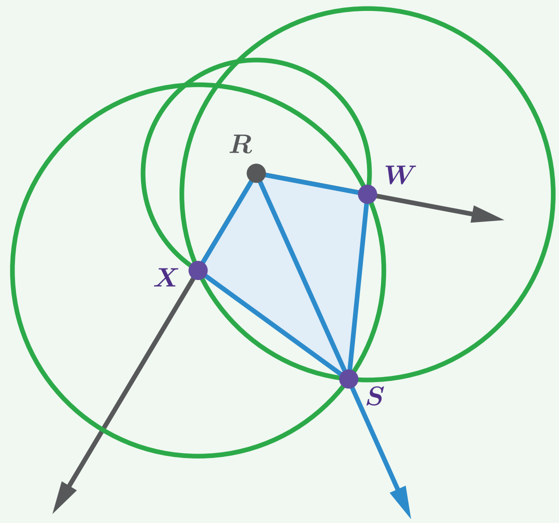
Select *all* the true statements.

- ☐ A. $\overline{JK} \cong \overline{FG}$
- ☐ B. $\overline{JK} \cong \overline{GK}$
- ☐ C. $\overline{JG} \cong \overline{FH}$
- ☐ D. $\overline{FH} \cong \overline{JF}$
- ☐ E. $\overline{JH} \cong \overline{FG}$
- ☐ F. $\overline{GH} \cong \overline{JH}$

Summary | Lesson 15

An **angle bisector** is a line, segment, or ray that divides an angle into two adjacent congruent angles.

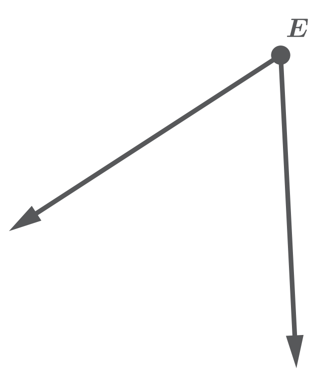
You can make sense of the construction of an angle bisector by using figures that form angle bisectors. For example, the diagonal that connects the vertices between two congruent sides of a kite will also bisect two of the kite's angles. If you can construct a kite, then you can use your understanding of a kite's properties to construct an angle bisector.



Other quadrilaterals, such as squares and rhombuses, also have diagonals that are angle bisectors. You can use figures that you already know how to construct to guide your thinking about figures that you haven't constructed yet.

Try This

Construct the angle bisector of $\angle E$.



Try This | Answer Key

Lesson 1

a *Responses vary.*

b *Responses vary.* A sport is an activity that involves competition and requires physical effort.

Lesson 2

a If you applied a 180° rotation to a line, what could be the result? Select *all* that apply.

- ☑ **A.** A line that's parallel to the original line
- ☐ **B.** A line that's perpendicular to the original line
- ☑ **C.** Just the original line

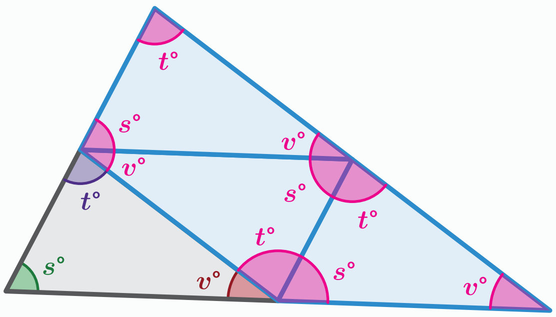
b *Responses vary.* Each line rotates back onto itself. Angle g rotates onto its vertical angle, showing the two angles are congruent.

Lesson 3

a *Responses vary.* I can translate the top line onto the bottom line because they are parallel lines.

b *Responses vary.* Angle a would be adjacent to angle h, and their angles would form a straight angle. This means they are supplementary.

Lesson 4



Lesson 5

(Yes) No I'm not sure

Responses vary. I used tracing paper to translate and rotate until both figures stacked perfectly. This means the figures are congruent.

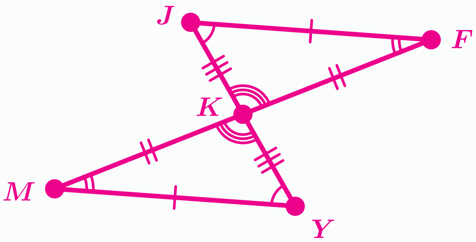
Lesson 6

Responses vary. The triangles are congruent because they have congruent corresponding parts.

Try This | Answer Key

Lesson 7

Responses vary.



Lesson 8

No. *Responses vary.* The triangles can have three pairs of congruent corresponding angles, but one of the triangles can be bigger than the other. This means they're not congruent.

Lesson 9

SSS (SAS) ASA

Explanations vary. For $\triangle ABC$ and $\triangle DEF$, two pairs of sides are congruent: $\overline{BC} \cong \overline{EF}$ and $\overline{AB} \cong \overline{DE}$. The pair of angles between these sides are also congruent: $\angle B \cong \angle E$.

Lesson 10

Responses vary. First, I'll make two more points, K and L, one on each ray. Then I can make a triangle that connects all three points, $\triangle JKL$. I'll use the compass to create a circle by length and copy each side length to make a congruent triangle using SSS congruence criteria. I know the angle is copied because every corresponding part of a congruent triangle is also congruent.

Lesson 11

Responses vary.

	Statement	Reason
	$\overline{YL}$ is the perpendicular bisector of $\overline{SA}$.	Given in the conjecture.
	$\overline{SY} \cong \overline{AY}$, $m\angle SYL = 90°$, and $m\angle AYL = 90°$.	Definition of a perpendicular bisector.
a	$\angle SYL \cong \angle AYL$	All right angles are congruent.
	$\overline{YL} \cong \overline{YL}$	The same segment in both triangles.
b	$\triangle YSL \cong \triangle YAL$	Congruent by SAS.
c	$\overline{LS} \cong \overline{LA}$	Corresponding sides in congruent triangles.

Lesson 12

Select *all* the names that apply to this quadrilateral.

- ☐ **A.** Rectangle
- ☐ **B.** Square
- ☑ **C.** Parallelogram
- ☑ **D.** Kite
- ☑ **E.** Rhombus

Lesson 13

Here is parallelogram $YQHL$ with diagonal $\overline{YH}$.

Select *all* the true statements.

- ☐ **A.** $\overline{QY} \cong \overline{LY}$
- ☑ **B.** $\angle QYH \cong \angle LHY$
- ☑ **C.** $\overline{QH} \cong \overline{LY}$
- ☐ **D.** $\angle YHL \cong \angle YHQ$
- ☑ **E.** $\overline{YQ} \cong \overline{HL}$
- ☑ **F.** $\angle YQH \cong \angle HLY$

Lesson 14

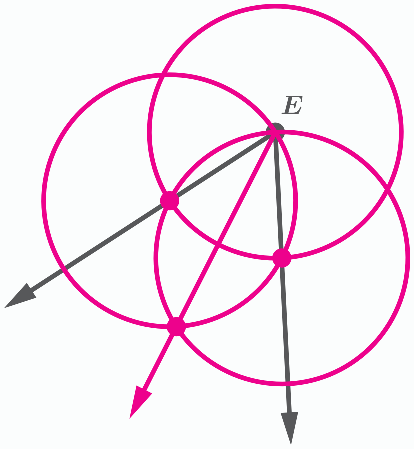
Figure $FGHJ$ is a rectangle with diagonals $\overline{JG}$ and $\overline{FH}$.

Select *all* the true statements.

- ☐ **A.** $\overline{JK} \cong \overline{FG}$
- ☑ **B.** $\overline{JK} \cong \overline{GK}$
- ☑ **C.** $\overline{JG} \cong \overline{FH}$
- ☐ **D.** $\overline{FH} \cong \overline{JF}$
- ☑ **E.** $\overline{JH} \cong \overline{FG}$
- ☐ **F.** $\overline{GH} \cong \overline{JH}$

Lesson 15

Responses vary.



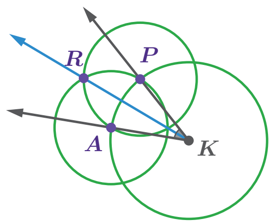
Glossary

Note: Spanish glossary coming fall 2026

A

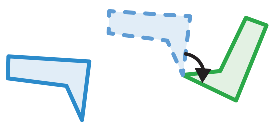
angle bisector A line, segment, or ray that divides an angle into two congruent angles.

Ray KR is the angle bisector of angle AKP.



C

congruent Figures are congruent when they stack perfectly using a sequence of transformations, such as translations, rotations, and reflections.



These figures are congruent because they can be stacked perfectly using a translation followed by a rotation.

conjecture A claim that we are trying to either prove or disprove.

converse A statement that reverses the order of a conjecture. Even if a conjecture is true, its converse might not be true.

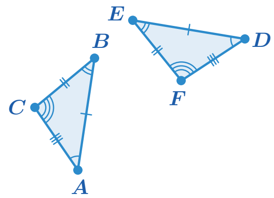
Here is a conjecture:

If $ABCD$ is a parallelogram, then its opposite sides are parallel.

Here is its converse:

If the opposite sides of $ABCD$ are parallel, then it is a parallelogram.

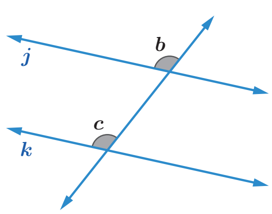
corresponding To correspond is to match. When the parts of two figures stack perfectly, the parts are corresponding.



Triangle ABC is congruent to triangle DEF. This means:

- $\overline{AB}$ corresponds to $\overline{DE}$
- $\overline{BC}$ corresponds to $\overline{EF}$
- $\overline{AC}$ corresponds to $\overline{DF}$
- $\angle A$ corresponds to $\angle D$
- $\angle B$ corresponds to $\angle E$
- $\angle C$ corresponds to $\angle F$

corresponding angles Two angles in the same position and on the same side of a transversal crossed by two lines. When the lines are parallel, corresponding angles are congruent.



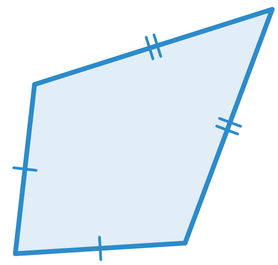
E

equidistant At equal distances from a given reference point.

All points on a circle are equidistant from the center of that circle.

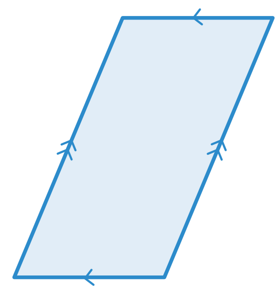
K

kite A quadrilateral with two pairs of adjacent congruent sides.



P

parallelogram
A quadrilateral with opposite sides that are parallel.

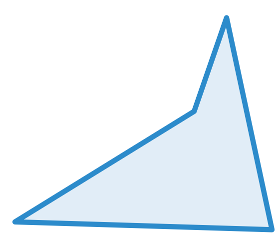


perpendicular bisector A line that is made of a set of points which are equidistant from the endpoints of a line segment. A perpendicular bisector passes through the midpoint of a line segment.

prove To use mathematical reasoning to convince a community that a conjecture is always true.

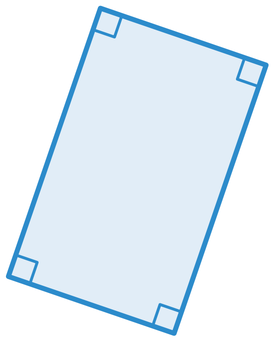
Q

quadrilateral A polygon with exactly four sides.

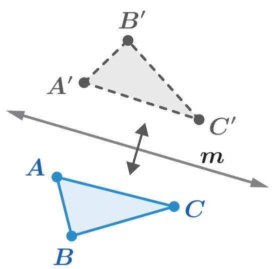


R

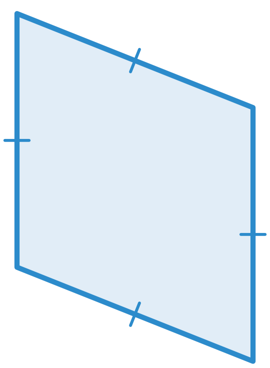
rectangle A quadrilateral with four congruent angles.



reflection A type of rigid transformation that moves every point on a figure to a point directly on the opposite side of the line. The new point is the same distance from the line as it was in the original figure.

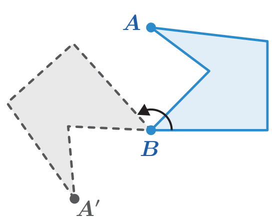


rhombus A quadrilateral with four congruent sides.



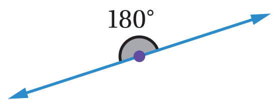
rigid transformation A move that does not change any measurements of a figure. Translations, rotations, and reflections (or any sequence of these) are rigid transformations.

rotation A type of rigid transformation that moves every point on a figure around a center by a given angle in a given direction.

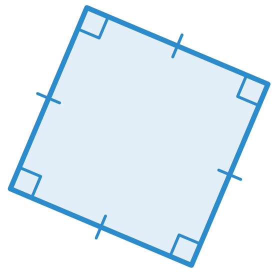


S

straight angle An angle that measures 180°. This angle can be formed by rotating a ray around its endpoint, creating a straight line.



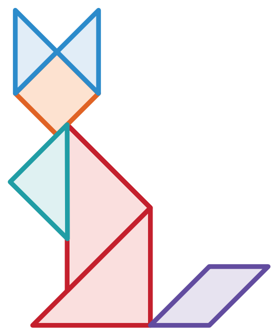
square A quadrilateral with four congruent angles and sides.



T

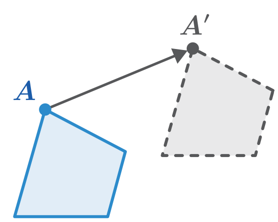
tangram A puzzle that is typically made of seven specific geometric figures, called tans, that can be arranged to form many different shapes.

Here is an example of a tangram puzzle that makes a fox.



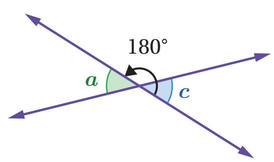
theorem An argument that has already been proven true.

translation A type of rigid transformation that slides every point on a figure by a given distance in a given direction.



V

vertical angles Pairs of angles that are constructed by rotating one angle 180° around its vertex.



Angles a and c are vertical angles.