Unit 1

Constructions and Rigid Transformations

There are many ways to create objects in geometry. You can use drawing tools, such as a compass or straightedge, or use design software to create things digitally. In this unit, you'll explore that creation process by constructing figures using circles and lines and using transformations to move, flip, and rotate those figures.

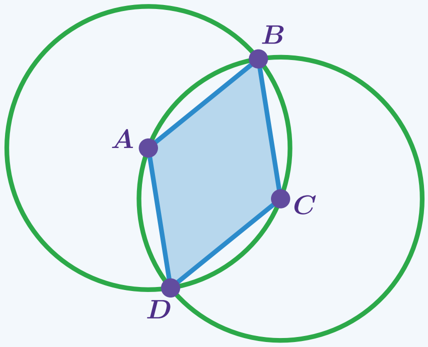
Essential Questions

- How can we construct precise geometric figures using circles and lines?
- How can we describe transforming geometric figures using reflections, rotations, and translations?
- How can we use tools to construct geometric relationships?

Summary | Lesson 1

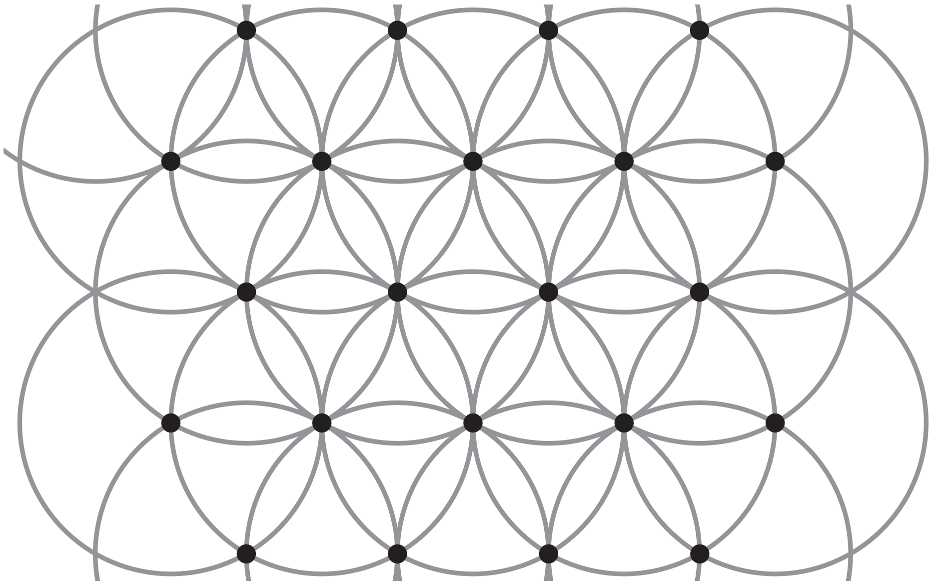
A **construction** is a geometric drawing made using precise tools. Because *circles* have a known radius, you can use them to construct *line segments* of equal length. When circles overlap with their centers on either end of the same radius, you can construct line segments with the same length.

For example, you know that each side length of quadrilateral $ABCD$ is the same because each side is a radius of one of two circles that are the same size.



Try This

a Construct two different rectangles.

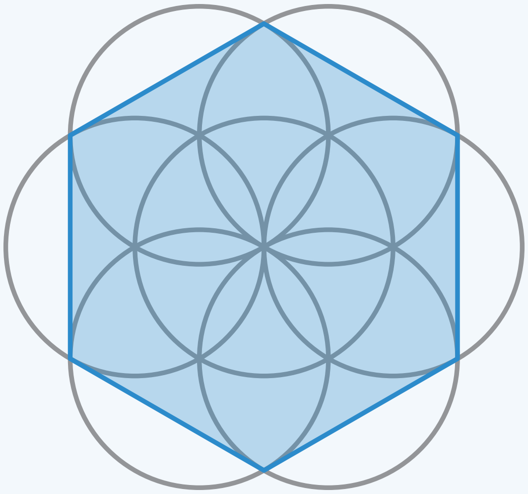


b Explain how you know that opposite sides of your rectangles have the same length.

Summary | Lesson 2

You can use a *compass* to make circles that are all the same size. This is a helpful measurement tool when constructing **regular polygons** where each side is a radius of the circle.

The intersections of lines and circles define points that help you replicate constructions and write instructions. You can use these points to reason about distances, precisely construct designs, and communicate about how to recreate your constructions.



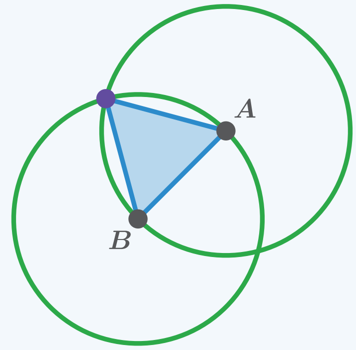
Regular hexagon

Try This

Use a compass and straightedge to construct a polygon where all the sides are the same length.

Summary | Lesson 3

Digital tools are helpful in making precise constructions. For circles, the radius lengths will always be exactly what you define (and your compass won't slip like it can on paper).



You can define a center and a point on a circle to construct line segments of exactly the same length.

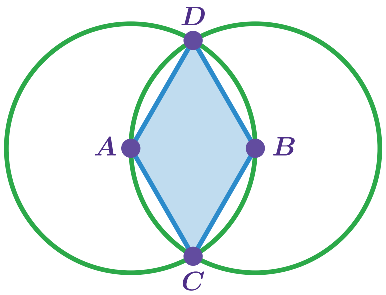
When using digital tools to construct, you can also drag points to see what relationships hold true about the constructions.

Using precise language can help you describe digital constructions. It's helpful to:

- Clearly define the center and radius length of a circle.
- Use points on constructed objects, like circles or line segments.
- Label intersection points to have clear ways to describe constructed points.

Try This

Here are instructions for constructing this polygon.



Step 1: Construct two circles.

Step 2: Label their intersection points C and D.

Step 3: Connect all the points to make a rhombus.

Select a step and revise it to make these instructions more precise.

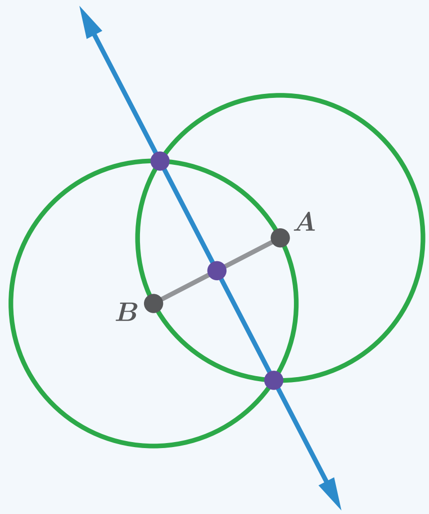
Summary | Lesson 4

A **perpendicular bisector** is a line that passes through the **midpoint** of a line segment. Any points on the perpendicular bisector are **equidistant** from the endpoints of that line segment.

To construct a perpendicular bisector:

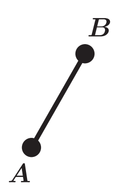
- You can construct two circles centered at each endpoint of a line segment, using the segment as the radius of each circle.
- You can construct two circles with the same radius centered at the endpoints of the segment.

The line that passes through the intersection points of the circles is the perpendicular bisector.



Try This

Construct the perpendicular bisector of segment AB.



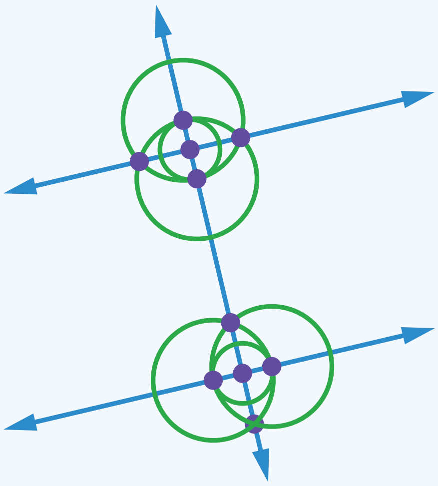
Summary | Lesson 5

You can use perpendicular bisectors to construct **perpendicular lines** and **parallel lines** with precision. Perpendicular lines are pairs of lines that form right angles at their intersection. Lines are parallel when each point on one line is the same distance from a corresponding point on the other line.

To construct a line through a specific point that's perpendicular to a given line:

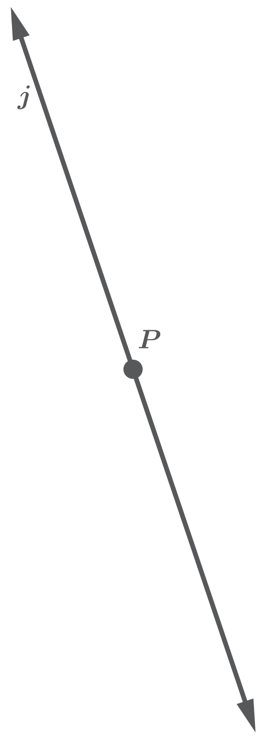
- Construct a circle using the specific point as its center, making sure that it intersects the given line at two points.
- Use the two intersection points as endpoints of a line segment and then construct the perpendicular bisector of that segment.

One technique for constructing a parallel line through a specific point is to use the perpendicular bisector construction twice. If two lines are perpendicular to a given line, then those two lines are parallel.



Try This

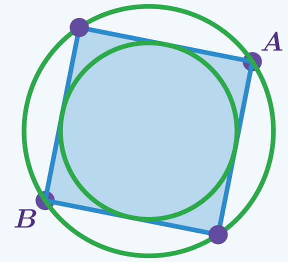
Construct line k so it is perpendicular to line j and passes through point P.



Summary | Lesson 6

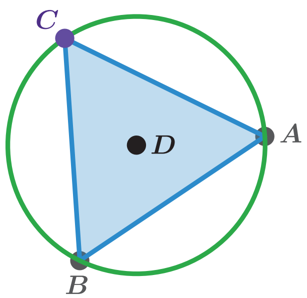
Parallel lines, perpendicular lines, and midpoints are helpful when constructing a square. Midpoints are especially helpful when constructing a square **inscribed** in a circle.

For a square (or any polygon) to be inscribed in a circle, every *vertex* of the square must be on the circle. For a circle to be inscribed in a square (or any polygon), the circle must fit inside the square and a point on the circle must be on each side of the square. When you construct either a square inscribed in a circle or a circle inscribed in a square, it's helpful to determine midpoints and perpendicular lines to identify the center point of the circle.



Try This

Triangle ABC is inscribed in a circle whose center is point D.

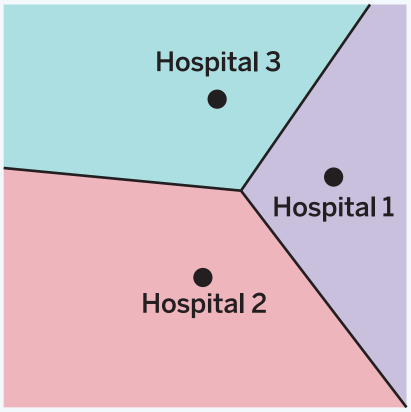


Show or explain how to construct a circle inscribed in $\triangle ABC$.

Summary | Lesson 7

You can use constructions to solve real-world problems. **Voronoi diagrams** are constructed with perpendicular bisectors and divide an area into regions where all the locations inside a region are closest to an identified point.

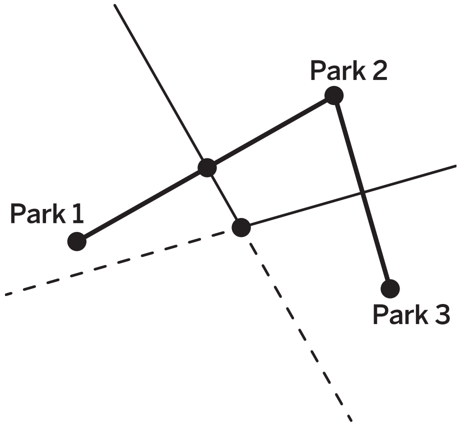
Voronoi diagrams are tools that help us make decisions in community planning. They provide evidence to support where specific services might be located and how these locations might impact a community.



Try This

Here is the start of a Voronoi diagram based on the locations of three parks.

a Complete this Voronoi diagram.



b What factors could be helpful to consider when choosing a location for an additional park?

Summary | Lesson 8

You can use tracing paper to perform *rigid transformations*, which don't change the side lengths or angle measures of a figure. With tracing paper, you can perform:

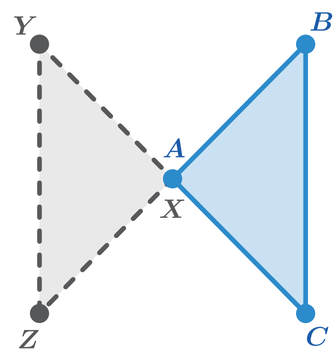
- *Translations*, which slide a figure up, down, left, or right.
- *Reflections*, which flip a figure.
- *Rotations*, which spin a figure.

Using tracing paper can help you make sense of how to use rigid transformations to move one figure on top of another. Sometimes you only need one transformation, but other times you may need to use multiple transformations to perfectly align one figure to another.

Try This

Describe how to transform $\triangle ABC$ onto $\triangle XYZ$.

Use tracing paper if it helps with your thinking.

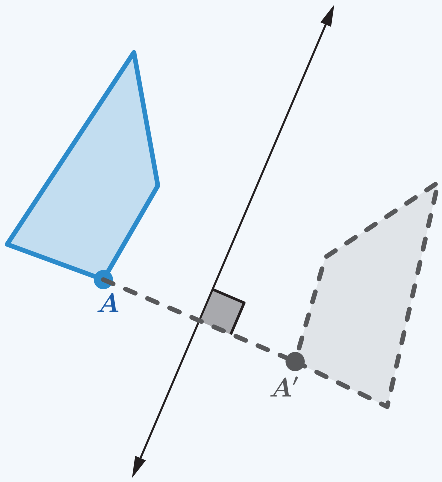


Summary | Lesson 9

You can use tracing paper to explore the relationship between a **line of reflection** and the reflected image. Some properties of reflections are:

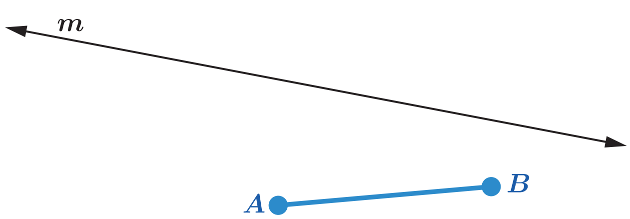
- Reflections are rigid transformations, so the reflected **image** will be the same size and shape as the **pre-image**.
- If you connect a point and its reflected image using a line segment, then the line segment is perpendicular to the line of reflection.
- A line of reflection bisects the line segment between a point and its reflected image.

You can use these properties to construct precise reflections. For example, a perpendicular line construction can help you determine the line that the reflected point must lie on. A circle can help you represent the correct distance from the line of reflection.



Try This

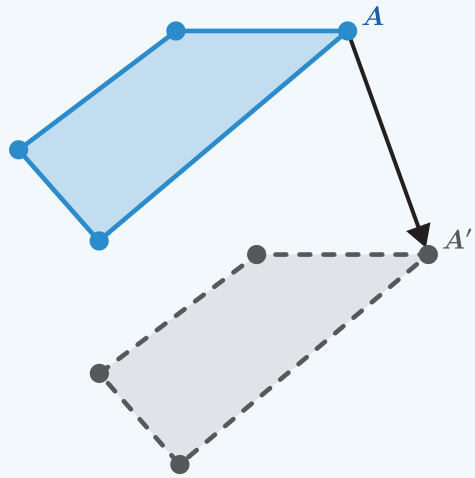
Reflect segment $\overline{AB}$ over line m.



Summary | Lesson 10

A translation is a type of rigid transformation that is defined by two points, a starting point and an endpoint. When you translate a starting point, A, to an endpoint, A', all other points in the translated figure move the same distance and in the same direction.

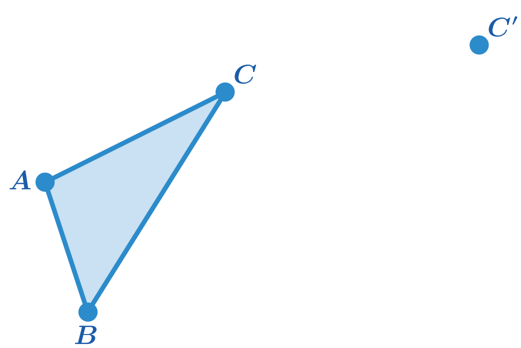
Translations move a figure without spinning or rotating. The translated figure is the same shape and size as the original figure but in a different location. When you translate a line, the image is a line parallel to the original.



When translating a figure, it's helpful to label translated points with **prime notation**, like A' or B', which are read out loud as A *prime* or B *prime*. This notation helps to show how the image relates to the pre-image.

Try This

Translate $\triangle ABC$ so that point C goes to point C'.

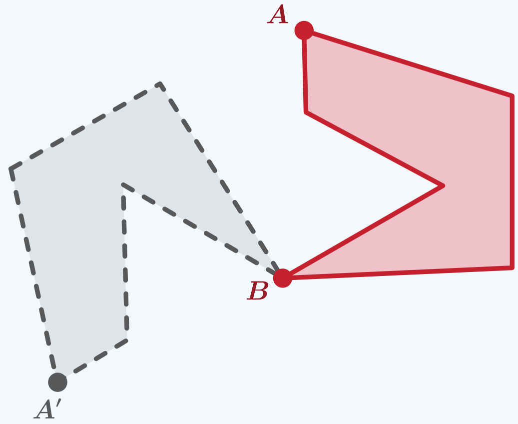


Summary | Lesson 11

A rotation is a rigid transformation that moves all of the points in a figure around a given point called the *center of rotation*. The pre-image points and their images are always the same distance from the center of rotation.

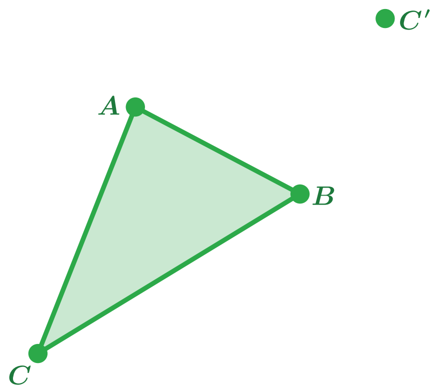
When a figure is rotated, the pre-image and the image have the same size and shape.

We can describe this rotation by saying "rotate the figure around point B so that point A goes to point A'."



Try This

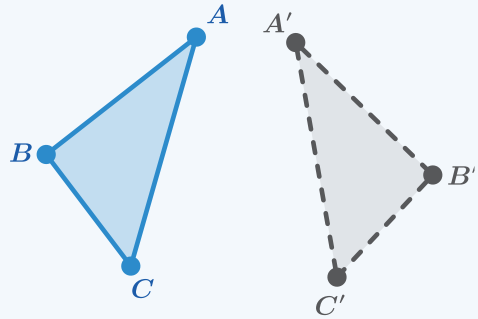
Rotate $\triangle ABC$ around point A until point C goes to point C'.



Summary | Lesson 12

There are many strategies to transform one figure onto another.

Here are three ways you could transform triangle ABC onto triangle $A'B'C'$.

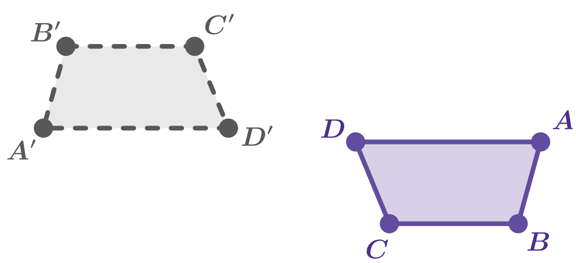


1. Translate triangle ABC so that point B goes to B'. Rotate the image around B' until the point that was originally C moves onto C', and then reflect over segment $B'C'$.
2. Reflect triangle ABC over the perpendicular bisector of segment AA'.
3. Rotate triangle ABC around the midpoint between points A and A' until segment AB aligns with segment $A'B'$. Then reflect the image over segment $A'B'$.

Try This

Describe how to transform figure $ABCD$ onto figure $A'B'C'D'$.

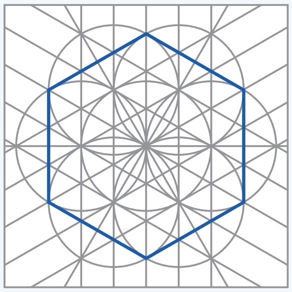
Use tracing paper if it helps with your thinking.



Summary | Lesson 13

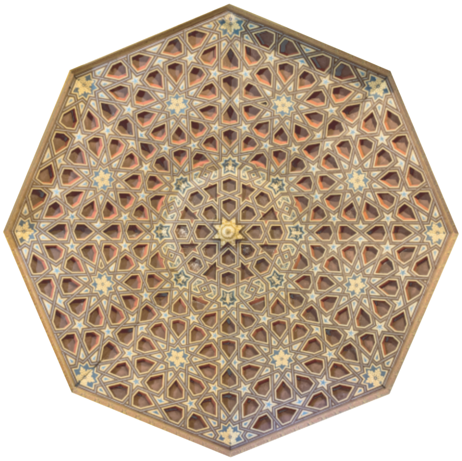
Artists and mathematicians throughout history have developed strategies for making geometric patterns. Here is an example of a geometric pattern from the Tomb of I'timād-ud-Daulah in India.

There are many strategies to construct and recreate designs. You can use circles, intersection points, and line segments to precisely construct figures. You can transform those figures using translations, rotations, and reflections. By combining constructions and transformations, you can create complex designs made up of beautiful shapes and patterns.



Try This

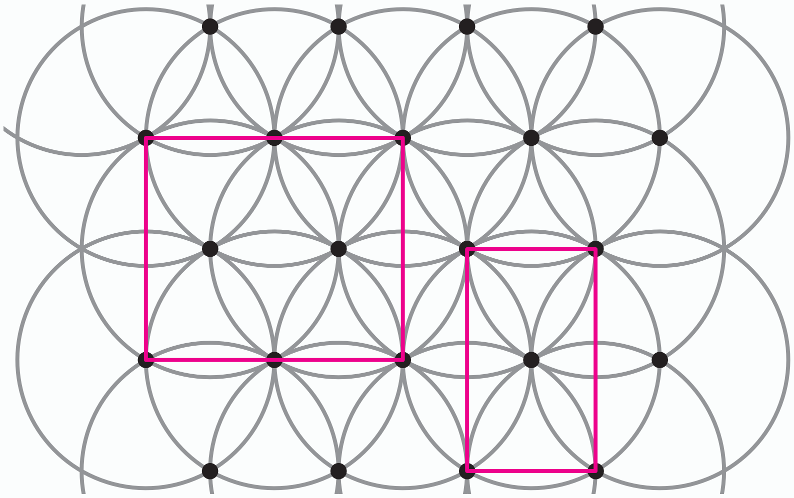
Describe transformations that could be used to recreate this pattern from the Royal Alcazar in Spain. Draw on the image if it helps to show your thinking.



"Royal Alcazar, Seville, Spain" by GaryCampbell-Hall via Flickr

Lesson 1

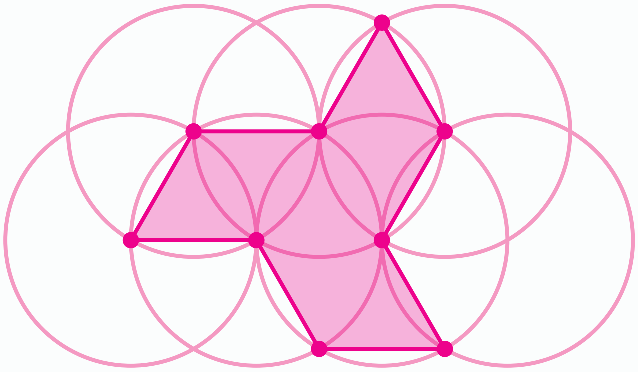
a *Responses vary.*



b *Responses vary.* Since all of the circles are the same size, that means that every radius has the same length and every diameter has the same length. I can also use symmetry to see that the opposite sides of each of my rectangles have the same length.

Lesson 2

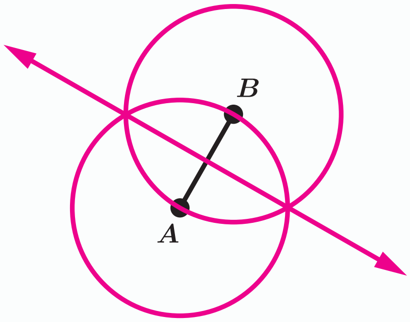
Responses vary.



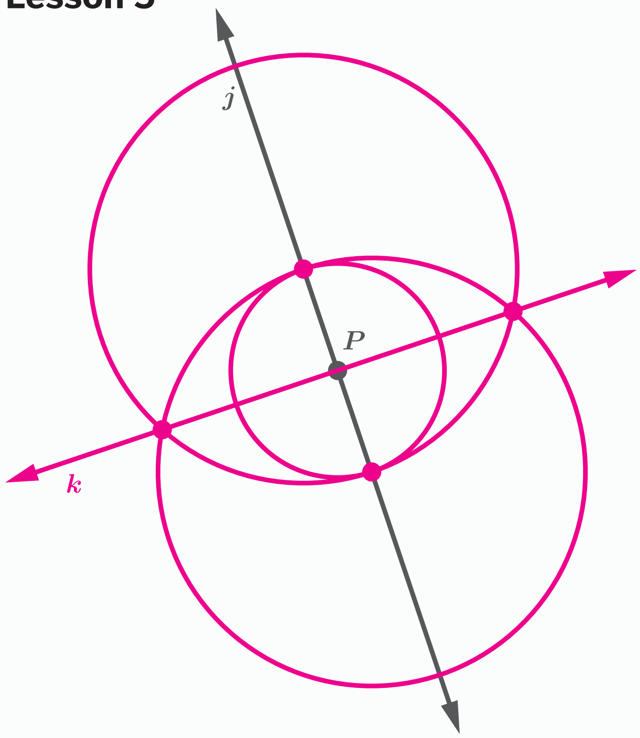
Lesson 3

Responses vary. Step 1: Construct two circles, one centered at point A and one centered at point B, each with radius $\overline{AB}$.

Lesson 4



Lesson 5

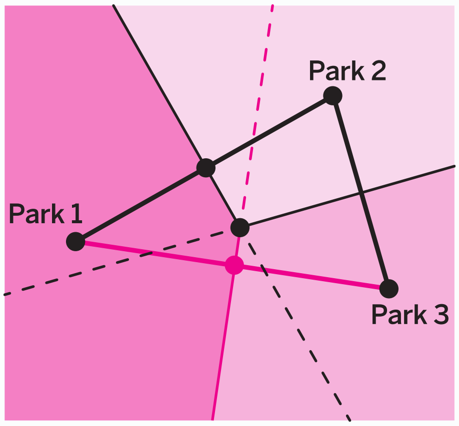


Lesson 6

Responses vary. Construct the midpoint of segment AC and connect it to point D with a line segment. Set that line segment as the radius of a circle that is also centered at point D.

Lesson 7

a

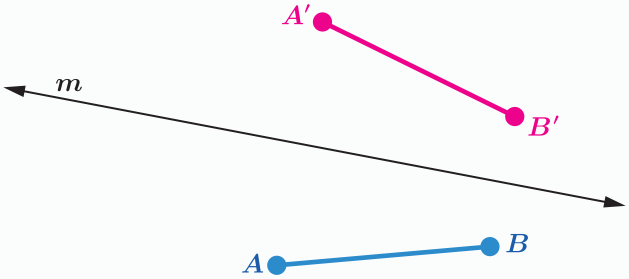


b *Responses vary.* It might be helpful to consider the locations of the other three parks, public transportation options to get to the new locations, the population density, and which populations the park could serve.

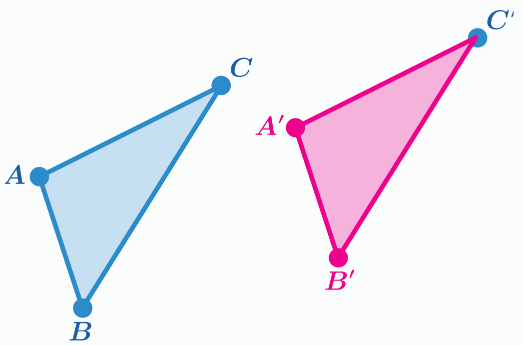
Lesson 8

Responses vary. Reflect $\triangle ABC$ over a vertical line through point A.

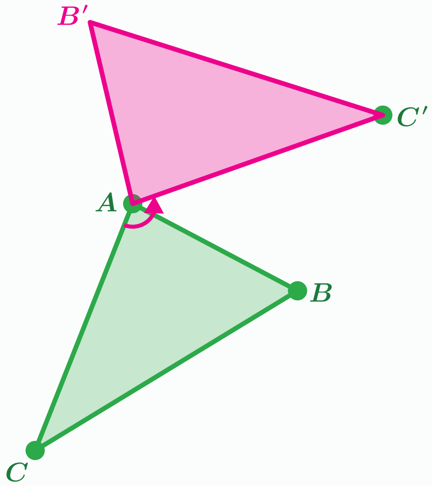
Lesson 9



Lesson 10



Lesson 11

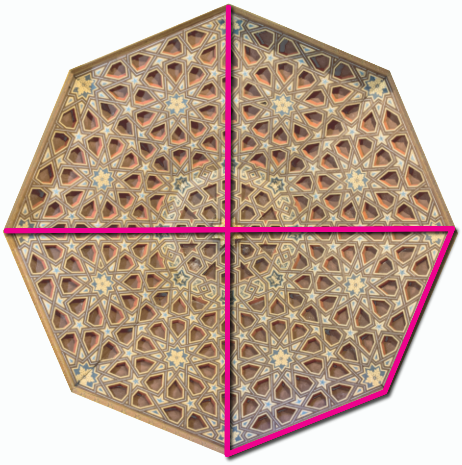


Lesson 12

Responses vary. First, translate figure $ABCD$ so that point D goes to point D'. Then rotate the new figure around point D' so that the point that began as A goes to A'.

Lesson 13

Responses vary. If you construct one quarter of this pattern, you could then reflect that quarter over a vertical line to create half of the pattern, and then reflect that over a horizontal line to create the full pattern.



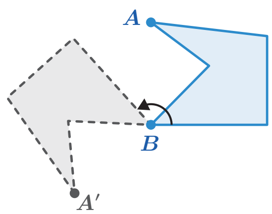
Glossary

Note: Spanish glossary coming fall 2026

C

center of rotation The point around which a figure or point is rotated.

Point B is the center of rotation.

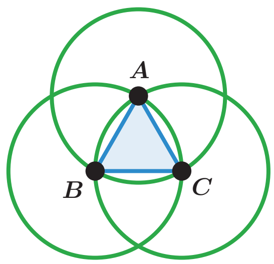


circle A figure made of all the points that are the same distance from a center point.

compass A tool used to draw precise circles or arcs.

construction A geometric drawing made by using precise tools.

Here is a construction of triangle ABC.

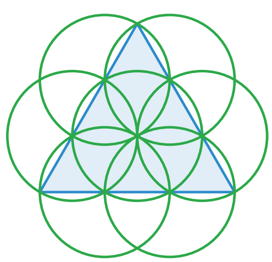


E

equidistant At equal distances from a given reference point.

All points on a circle are equidistant from the center of that circle.

equilateral triangle A triangle with three equal side lengths and three equal angle measures.



I

inscribe To draw a figure within another figure such that each of the inner polygon's vertices are on the outer figure.

A circle is inscribed in a polygon when the circle touches, but does not cross, each of the polygon's sides.

Here is triangle ABC inscribed in a circle.

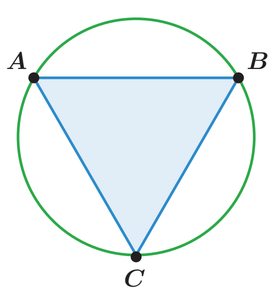
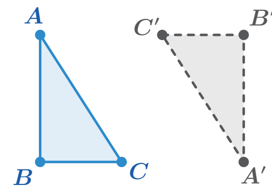


image A new figure that is created after a transformation of an original figure (called the *pre-image*).

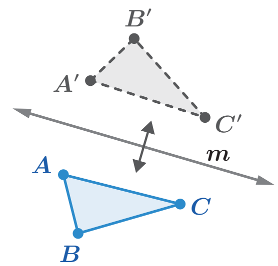
Here is an image and a pre-image. Triangle $A'B'C'$ is the image of triangle ABC.



L

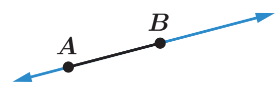
line of reflection The line which reflections are performed over. When points are reflected, the new point (*image*) is the same distance from the line as the original point (*pre-image*).

Line m is the line of reflection.



line segment A part of a line between two endpoints.

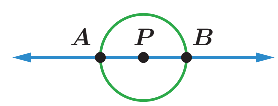
Here is line segment AB, also called $\overline{AB}$.



M

midpoint (of a line segment) The point that divides a line segment into two equal parts.

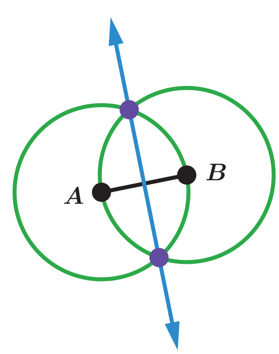
Point P is the midpoint of line segment AB.



P

parallel lines A pair of lines where each point on one line is the same distance from a corresponding point on the other line.

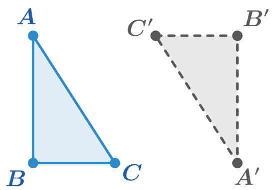
perpendicular bisector A line that is made of a set of points equidistant from two endpoints of a line segment. A perpendicular bisector passes through the midpoint of a line segment.



perpendicular lines A pair of lines that forms right angles at its intersection. Perpendicular lines are formed when a line is exactly one quarter turn from the other.

pre-image The name of a figure before any transformations are performed.

Here is an image and a pre-image. Triangle ABC is the pre-image.



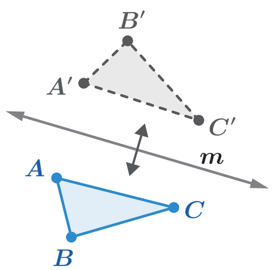
prime notation A way to name corresponding points in an image and pre-image. Points in an image include the prime symbol (′).

Points A and B on a given pre-image would be named A' and B' on the image.

R

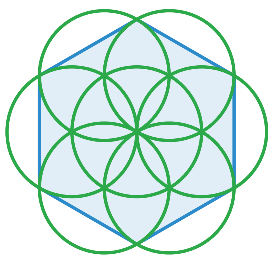
reflection A type of rigid transformation that moves every point on a figure to a point directly on the opposite side of the line. The reflected point is the same distance from the line as it was in the original figure.

Triangle $A'B'C'$ is a reflection of triangle ABC.



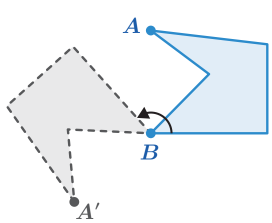
regular polygon A polygon where all sides are the same length and all angles are the same measure.

Here is a regular hexagon.



rigid transformation A move that does not change any measurements of a figure. Translations, rotations, and reflections (or any sequence of these) are rigid transformations.

rotation A type of rigid transformation that moves every point on a figure around a center by a given angle in a given direction.

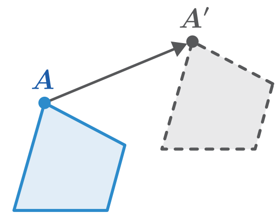


S

straightedge A tool used to draw straight lines.

T

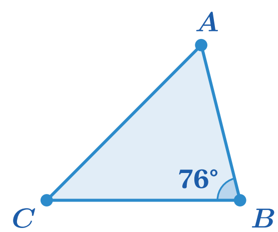
translation A type of rigid transformation that slides every point on a figure a given distance in a given direction.



V

vertex A corner or a point where two or more line segments or rays meet. An angle is named based on its vertex, and polygons are named based on their vertices.

Angle ABC is 76° at vertex B.



Voronoi diagram A diagram that uses perpendicular bisectors to create different regions based on the closest distance to a given point or object.