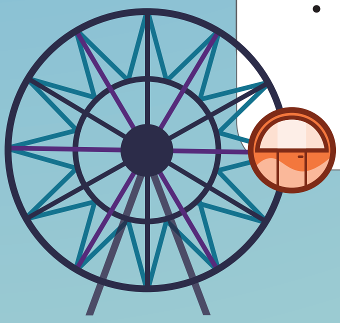
Unit 1

Transformations of Functions

We can shift, reflect, rotate, and dilate figures to change their positions, orientations, and sizes. In this unit, you'll think about how to use equations to transform functions in different ways and use those equations to model swimming, roller coasters, and mental health. You'll also revisit key features of functions, like domain and range.

Essential Questions

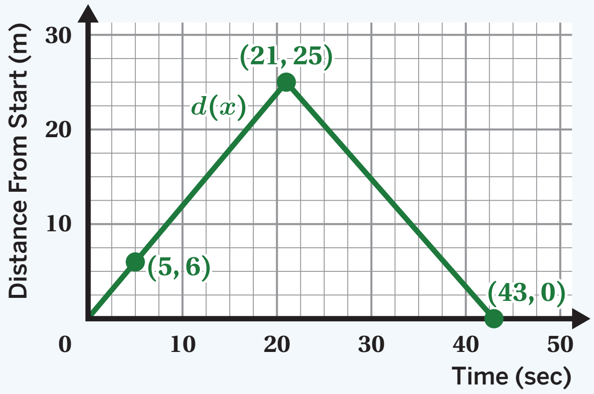
- How can you describe functions and use them to tell stories?
- How do transformations apply to functions?
- How can you use equations to model transformed functions?



Summary | Lesson 1

You can use a graph to help you understand a situation, interpret its key moments, and determine whether it represents a *function*.

This graph represents a swimmer's distance from the start, $d(t)$, after t seconds.



- Every *input* (time) has only one *output* (distance). That means distance is a function of time.
- The function statement $d(5) = 6$ means that at 5 seconds, the swimmer was 6 meters from the start.
- The maximum distance from the start was 25 meters. In context, $d(21) = 25$ represents that after 21 seconds, the swimmer reached the other side of the pool.
- At 43 seconds, the swimmer was back at the start. You can represent this with $d(43) = 0$.

Try This

This graph represents a diver's height, $c(t)$, after t seconds.

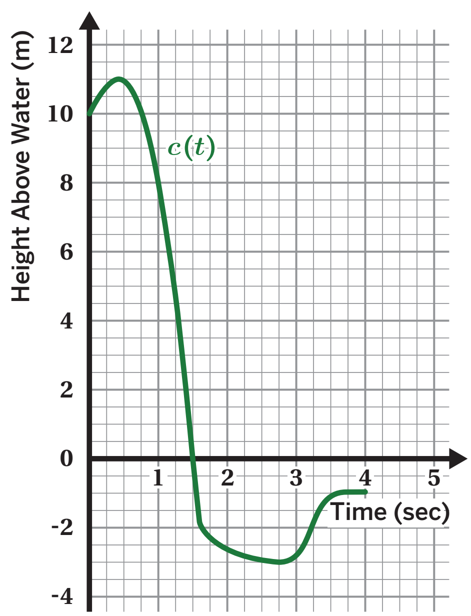
Which statement about this diver's story is false?

A. Their height above the water is a function of time.

B. Their maximum height is 11 meters.

C. $c(0) = 1.5$

D. At 2 seconds, the diver is below the surface.



Summary | Lesson 2

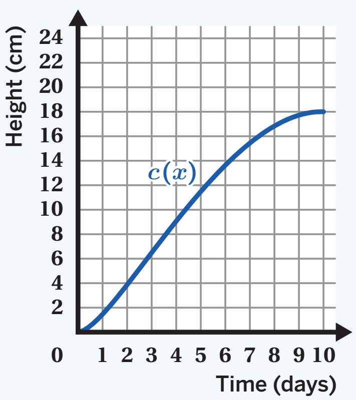
The *domain* of a function is the set of all possible input values. It can be either **discrete** or **continuous**.

The inputs of a function with a discrete domain include distinct or separate values.

The inputs of a function with a continuous domain include *all* the values in an interval.

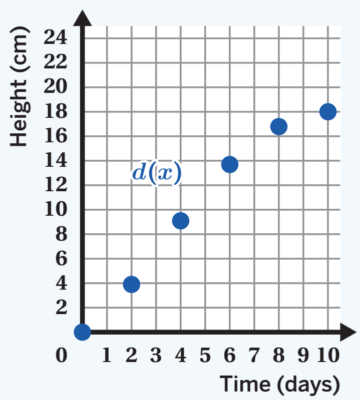
The *range* of a function, which is the set of all possible output values, can also be either discrete or continuous.

You can describe continuous domains and ranges using inequalities. You can describe discrete domains and ranges with lists.



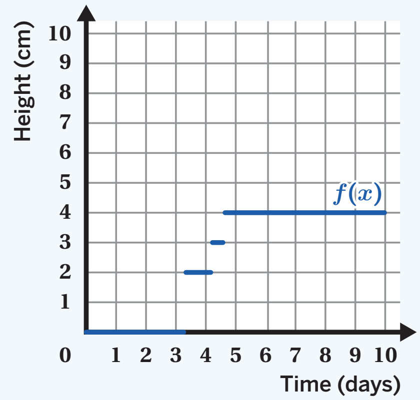
Domain (continuous):
$0 \le x \le 10$

Range (continuous):
$0 \le c(x) \le 18$



Domain (discrete):
$x = \{0, 2, 4, 6, 8, 10\}$

Range (discrete):
$d(x) = \{0, 3.9, 9.1, 13.7, 16.8, 18\}$

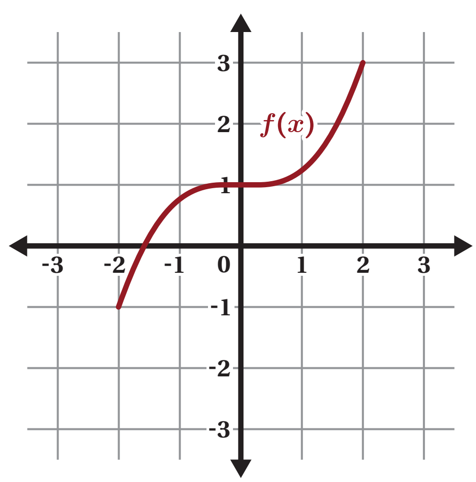


Domain (continuous):
$0 \le x \le 10$

Range (discrete):
$f(x) = \{0, 2, 3, 4\}$

Try This

Here is the graph of the function $f(x)$.



a Write the domain of $f(x)$.

b Select the range of $f(x)$.

A. $f(x) = \{-1, 0, 1, 2, 3\}$

B. $3 \le f(x) \le -1$

C. $-1 \le f(x) \le 3$

D. $-2 \le f(x) \le 2$

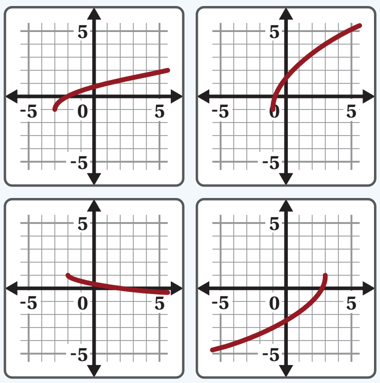
Summary | Lesson 3

A **function family** is a group of functions that is made by transforming the same basic function. Functions in the same family have graphs with similar key features. You can describe these features using terms like *minimum*, *maximum*, *increasing*, *decreasing*, *positive*, *negative*, *domain*, or *range*.

Some function families you may have seen from Algebra 1 include *quadratic*, *absolute value*, *exponential*, and *linear*. Here are some new function families from this lesson.

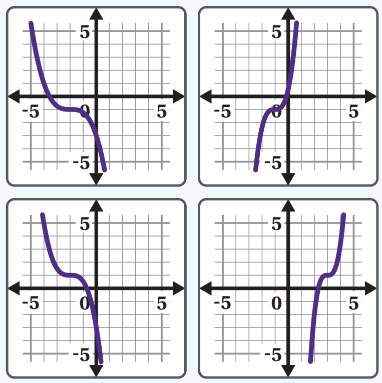
Square Root

$f(x) = \sqrt{x}$



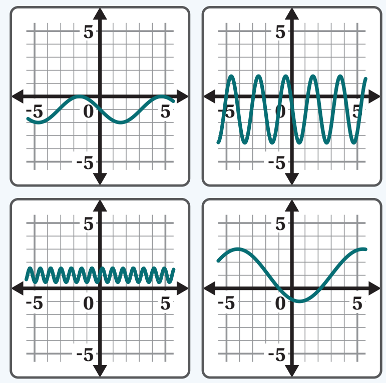
Cubic

$f(x) = x^3$



Sine

$f(x) = \sin(x)$



Try This

For each function family, place a checkmark under any statement that describes its graphs. Refer to the summary if it helps with your thinking.

	Has a maximum and a minimum	Is always increasing or always decreasing	Domain includes all x-values
Square Root			
Cubic			
Sine			

Summary | Lesson 4

Sometimes we write functions in terms of other functions by applying operations to their inputs or outputs.

For instance, $g(x) = f(x + 1) - 2$ is a function defined in terms of the function $f(x)$. If you're given a graph or a table representing $f(x)$, you can determine values for $g(x)$.

Here is a table that represents the function $f(x)$.

x	$f(x)$
-2	3
-1	9
0	15
1	21
2	27

One way you can evaluate $g(1)$ is by using algebraic substitution:

$g(1) = f((1) + 1) - 2$
$g(1) = f(2) - 2$
$g(1) = 27 - 2$
$g(1) = 25$

Another way is to use a multi-column table:

x	$x + 1$	$f(x + 1)$	$f(x + 1) - 2$
1	2	27	25

Try This

This table represents the function $j(x)$.

x	$j(x)$
0	13
1	10
2	6
3	1
4	-4
5	-8

$k(x)$ is a function written in terms of $j(x)$:

$k(x) = 2j(x - 1)$

Determine the following values:

a $k(2) =$ ______

b $k(5) =$ ______

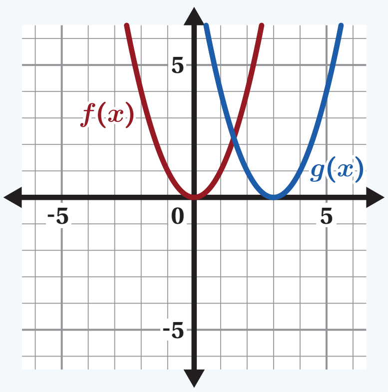
Summary | Lesson 5

You can *transform* graphs using *translations*, *reflections*, and **scales**.

- Translations shift the graph left, right, up, or down by a number of units.
- Reflections flip the graph over the x- or y-axis.
- Scales stretch the graph of a function away from (or compress it toward) an axis.

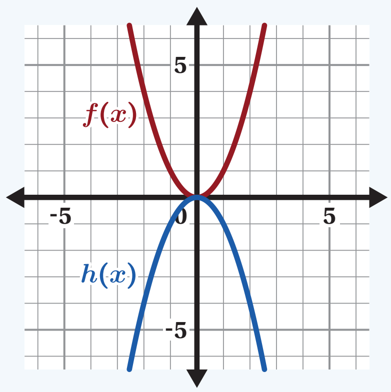
Here are three transformations of the function $f(x) = x^2$.

Translation



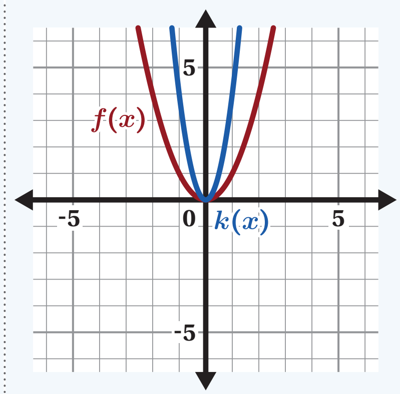
$f(x)$ was translated right 3 units.

Reflection



$f(x)$ was reflected over the x-axis.

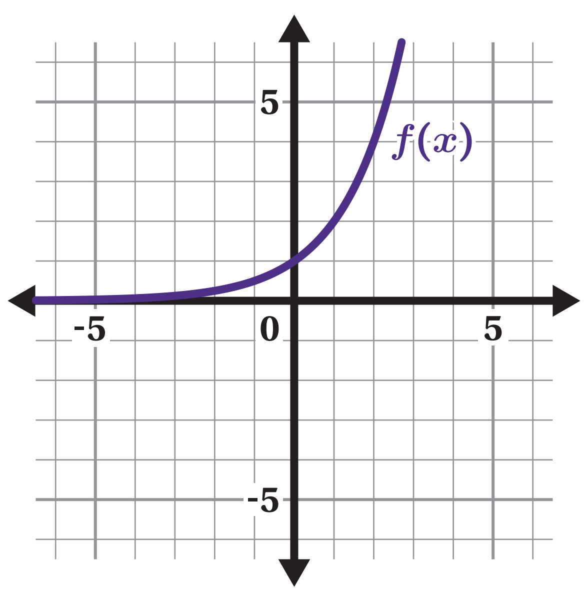
Scale



$f(x)$ was scaled vertically by a factor of 4.

Try This

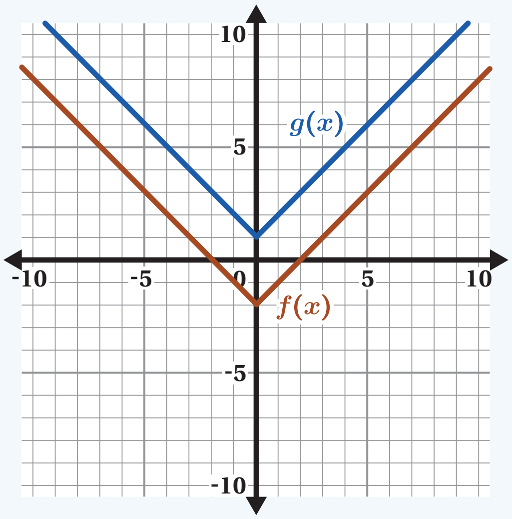
Transform $f(x)$ by reflecting it over the x-axis.



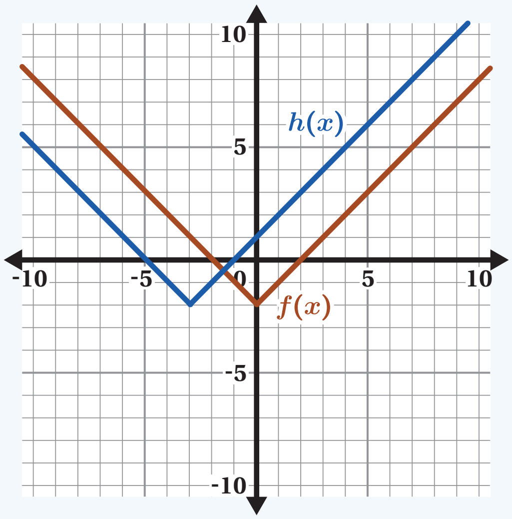
Summary | Lesson 6

You can translate the graph of a function vertically by adding or subtracting from its output. When a positive value is added to $f(x)$, such as $f(x) + k$, the graph will shift up by k units. When a positive value is subtracted from $f(x)$, such as $f(x) - k$, the graph will shift down by k units.

You can translate the graph of a function horizontally by adding or subtracting from the input of a function. When a positive value is added to x, such as $f(x + h)$, the graph will shift left h units. When a positive value is subtracted from x, such as $f(x - h)$, the graph will shift right h units.



The function $g(x) = f(x) + 3$ is the function $f(x)$ translated vertically up 3 units.



The function $h(x) = f(x + 3)$ is the function $f(x)$ translated horizontally left 3 units.

Try This

Here are the graphs of functions $p(x)$ and $q(x)$.

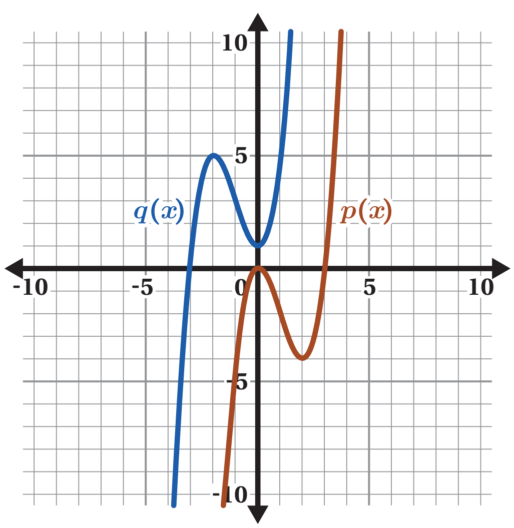
Which of these equations describes $q(x)$ as a transformation of $p(x)$?

A. $q(x) = p(x + 2) + 5$

B. $q(x) = p(x + 2) - 5$

C. $q(x) = p(x - 2) + 5$

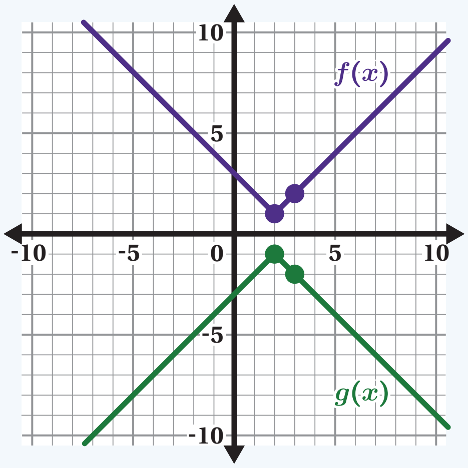
D. $q(x) = p(x - 2) - 5$



Summary | Lesson 7

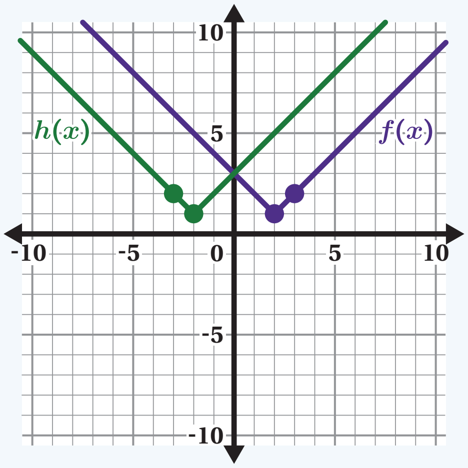
You can reflect a function over the x-axis by multiplying its outputs by -1. You can reflect a function over the y-axis by multiplying its inputs by -1.

In this example, the function $g(x)$ is a reflection of $f(x)$ over the x-axis.



- The outputs of $g(x)$ have the opposite signs of the corresponding outputs of $f(x)$.
- For example, (2, 1) on the graph of $f(x)$ corresponds to (2, -1) on the graph of $g(x)$.

The function $h(x)$ is a reflection of $f(x)$ over the y-axis.



- The inputs of $h(x)$ have the opposite signs of the corresponding inputs of $f(x)$.
- For example, (2, 1) on the graph of $f(x)$ corresponds to (-2, 1) on the graph of $h(x)$.

Try This

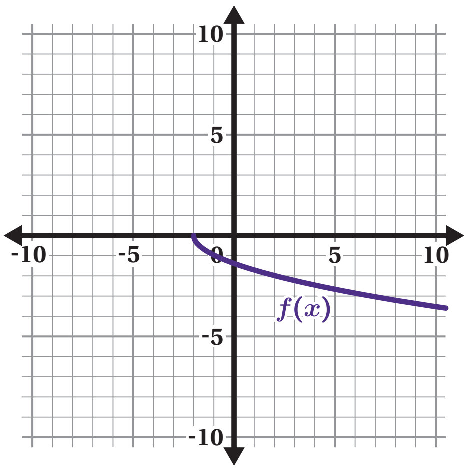
Here is the graph of the function $f(x)$.

$g(x) = -f(x)$

$h(x) = f(-x)$

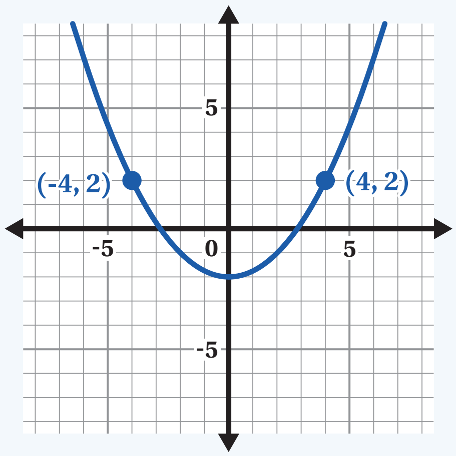
a Sketch the graph of $g(x)$.

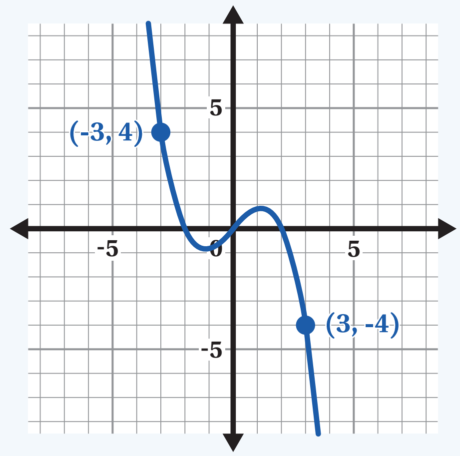
b Sketch the graph of $h(x)$.



Summary | Lesson 8

Functions with symmetry are sometimes classified as even or odd.





The general equation for an **even function** is $f(x) = f(-x)$

The graph of an even function is the same when you reflect it over the y-axis. This means opposite input values have the same output value.

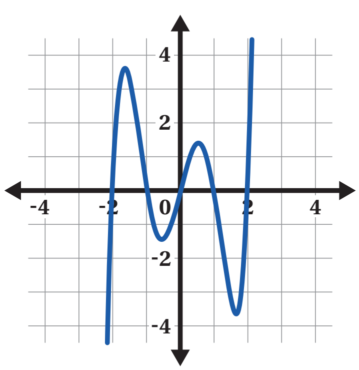
The general equation for an **odd function** is $f(x) = -f(-x)$.

The graph of an odd function is the same when you reflect it over both the x-axis and the y-axis. This means the input and output values are opposites.

Try This

Determine whether each function is even, odd, or neither. Circle one.

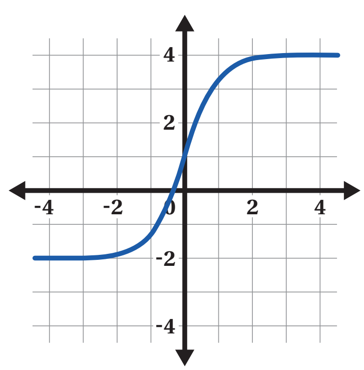
a



Even Odd

Neither

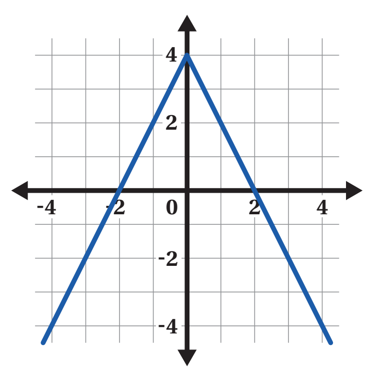
b



Even Odd

Neither

c

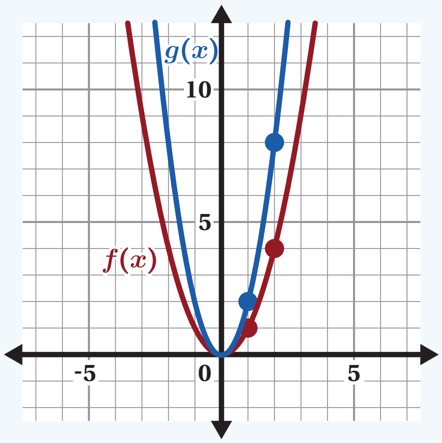


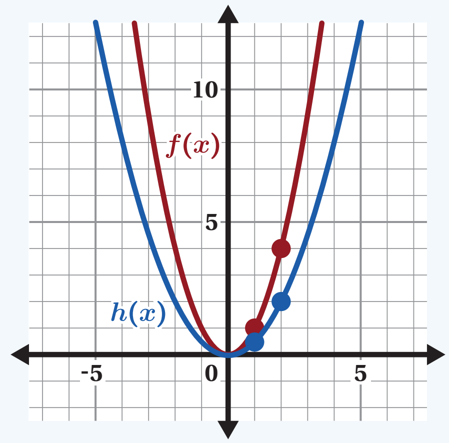
Even Odd

Neither

Summary | Lesson 9

You can scale a function vertically by multiplying its outputs by a factor. When the factor is greater than 1, the graph stretches vertically. When the factor is between 0 and 1, the graph compresses vertically.





The function $g(x)$ scales $f(x)$ vertically by a factor of 2. Its equation is $g(x) = 2f(x)$.

- The outputs of $f(x)$ are scaled by 2.
- For example, $(1, 1)$ on $f(x)$ corresponds to $(1, 2)$ on $g(x)$ and the point $(2, 4)$ on $f(x)$ corresponds to $(2, 8)$ on $g(x)$.

The function $h(x)$ scales $f(x)$ vertically by a factor of $\frac{1}{2}$. Its equation is $h(x) = \frac{1}{2}f(x)$.

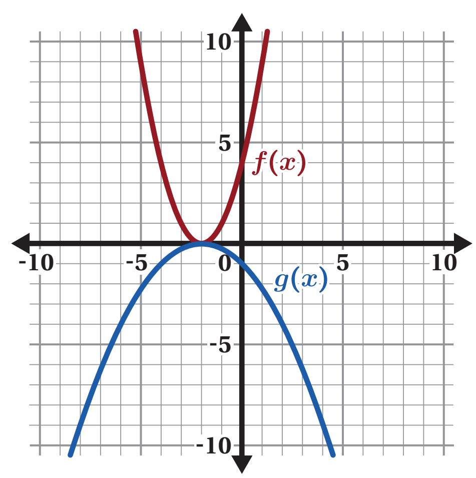
- The outputs of $f(x)$ are scaled by $\frac{1}{2}$.
- For example, $(1, 1)$ on $f(x)$ corresponds to $\left(1, \frac{1}{2}\right)$ on $h(x)$ and the point $(2, 4)$ on $f(x)$ corresponds to $(2, 2)$ on $h(x)$.

Try This

Here are the graphs of the functions $f(x)$ and $g(x)$.

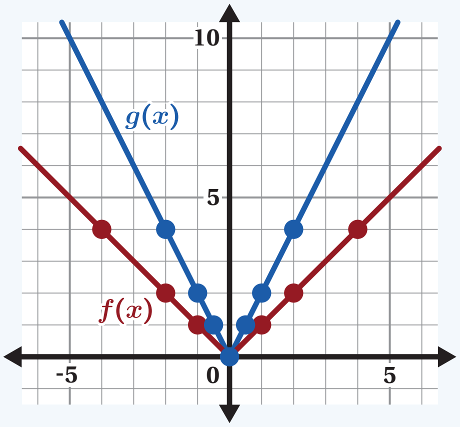
Write an equation for $g(x)$ as a transformation of $f(x)$.

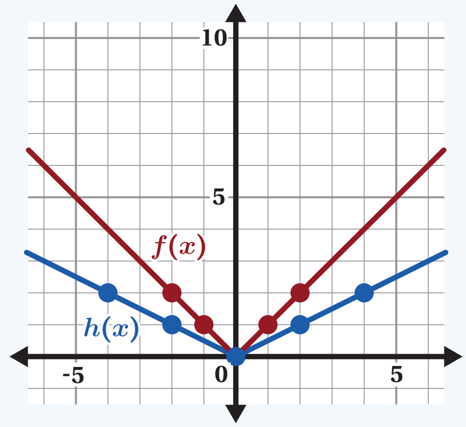
$g(x) =$ ____________



Summary | Lesson 10

You can scale a function horizontally by multiplying its inputs by a factor. The factor you use in the equation is the reciprocal of what you scale the graph by. When the factor is greater than 1, the graph compresses horizontally. When the factor is between 0 and 1, the graph stretches horizontally.





The function $g(x)$ scales $f(x)$ horizontally by a factor of $\frac{1}{2}$.

Its equation is $g(x) = f(2x)$. Its graph is half as close to the y-axis.

The point $(1, 1)$ on $f(x)$ corresponds to $\left(\frac{1}{2}, 1\right)$ on $g(x)$ because $g\left(\frac{1}{2}\right) = f\left(2\left(\frac{1}{2}\right)\right) = f(1)$.

The function $h(x)$ scales $f(x)$ horizontally by a factor of 2.

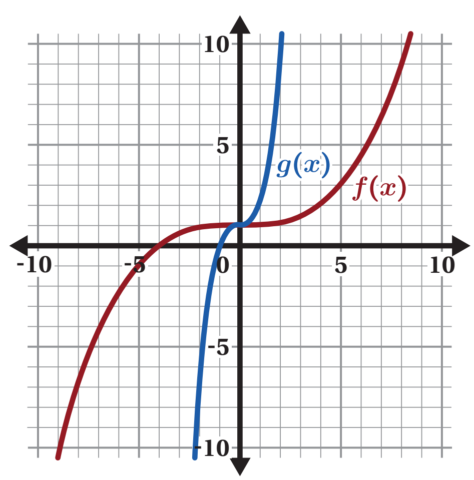
Its equation is $h(x) = f\left(\frac{1}{2}x\right)$. Its graph is twice as far from the y-axis.

The point $(1, 1)$ on $f(x)$ corresponds to $(2, 1)$ on $h(x)$ because $h(2) = f\left(\frac{1}{2}(2)\right) = f(1)$.

Try This

Write an equation for $g(x)$ in terms of $f(x)$.

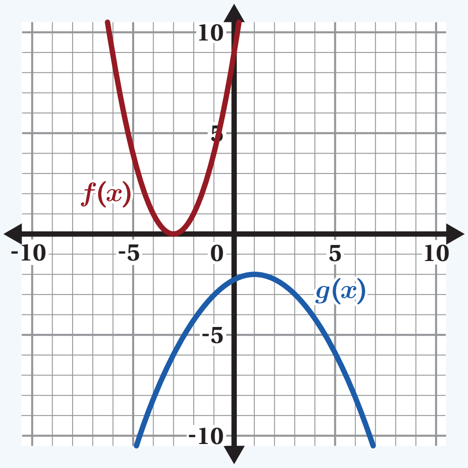
$g(x) =$ ....



Summary | Lesson 11

You can write equations to represent multiple transformations of a function, including translations, reflections, and scales.

Here are the graphs of $f(x)$ and $g(x)$, where $g(x)$ is a transformation of $f(x)$.

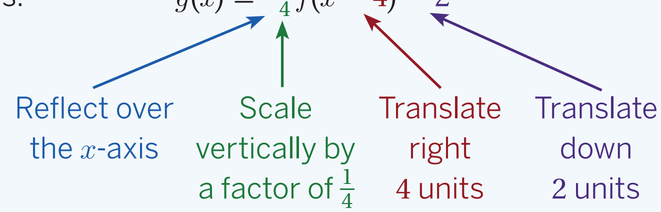


You can describe the function $g(x)$ with a sequence of transformations like this one:

1. Translate $f(x)$ right 4 units.
2. Reflect the result over the x-axis.
3. Scale it vertically by a factor of $\frac{1}{4}$.
4. Translate the result down 2 units.

One equation for $g(x)$ is:

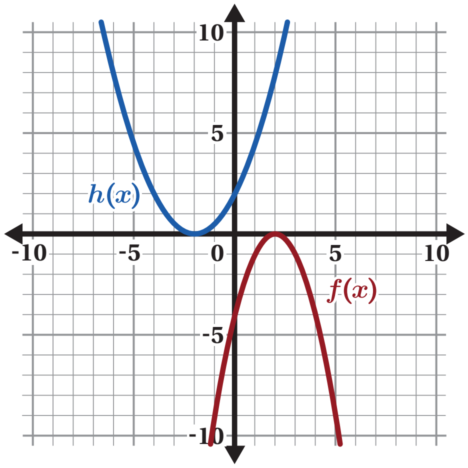
$$g(x) = -\frac{1}{4}f(x-4) - 2$$



Try This

Write an equation for $h(x)$ as a transformation of $f(x)$.

$h(x) =$ ____________



Summary | Lesson 12

You can use transformations of functions to *model* and analyze multiple data sets at once. By selecting a starting function and transforming it to represent other functions, you can interpret the transformations to compare features of the different data sets.

Here are some actions you can use to model and analyze data sets, along with a description of how you completed each action in this lesson, using mental health data.

Modeling Actions	Mental Health Data
Identify variables and make assumptions.	You predicted what a graph about mental illness percentages might include and then selected a function to model the data for one age group.
Create a model, perform operations, then interpret the results.	You transformed your original function to match the data for two other age groups. You compared the vertical scales and vertical translations to learn more about the differences between these age groups.
Support the conclusions.	You zoomed out to see if the functions you created predicted future trends well.
Revise the model.	You considered what you would want to investigate next, including some options for other data that could provide more insights about differences across age groups.
Report results.	You shared your model with a partner and reflected on similarities and differences.

Try This

What are some other real-life situations where you might use transformations of functions to create a mathematical model?

Try This | Answer Key

Lesson 1

C. $c(0) = 1.5$

Lesson 2

a $-2 \leq x \leq 2$

b C. $-1 \leq f(x) \leq 3$

Lesson 3

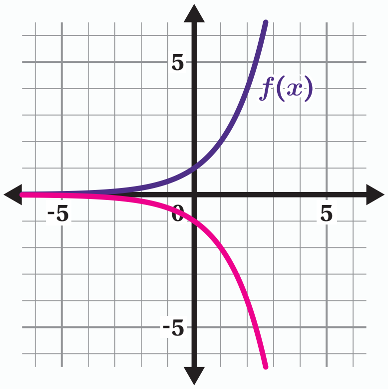
	Has a maximum and a minimum	Is always increasing or always decreasing	Domain includes all x-values
Square Root		✓	
Cubic		✓	✓
Sine	✓		✓

Lesson 4

a $k(2) =$ 20

b $k(5) =$ -8

Lesson 5



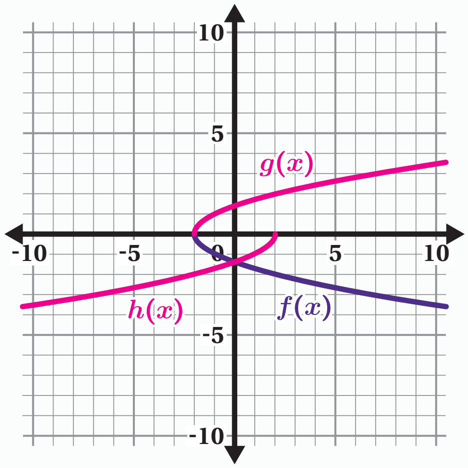
Lesson 6

A. $q(x) = p(x + 2) + 5$

Lesson 7

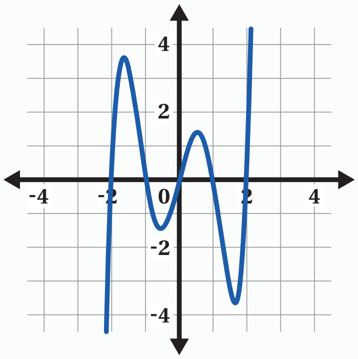
a Sketch the graph of $g(x)$.

b Sketch the graph of $h(x)$.



Lesson 8

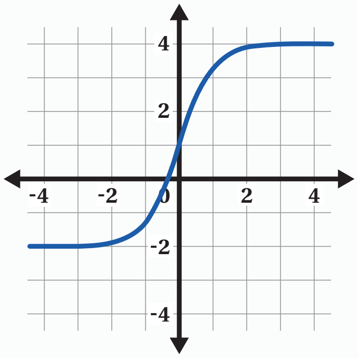
a



Even Odd

Neither

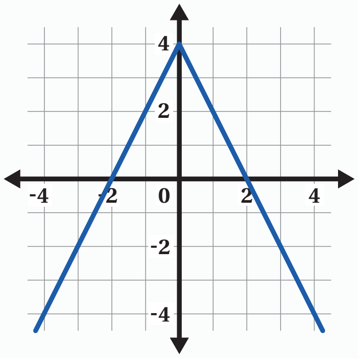
b



Even Odd

Neither

c



Even Odd

Neither

Lesson 9

$g(x) =$ $-\frac{1}{4}f(x)$

Lesson 10

$g(x) =$ $f(4x)$

Lesson 11

Responses vary.

- $-\frac{1}{2}f(x+4)$
- $-\frac{1}{2}f(-x)$

Lesson 12

Responses vary.

- Sunrise and sunset times in different parts of the world
- Birth rates across different cities
- Any data set where the same question was asked across different age groups of people

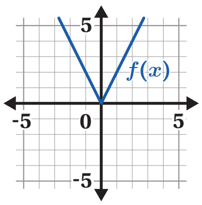
Algebra 2 | Unit 1
Glossary

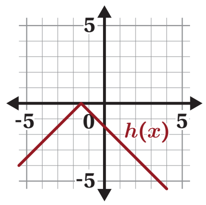
Note: *Spanish glossary coming fall 2026*

A

absolute value function A function that is a transformation of $f(x) = |x|$. The graph of an absolute value is a symmetrical V-shape that has a minimum and opens up or has a maximum and opens down.

For example, $f(x) = 2|x|$, $g(x) = 2|x| + 1$, and $h(x) = -|x + 1.5|$ are absolute value functions.





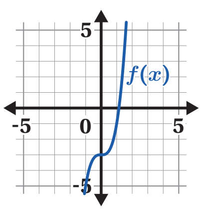
C

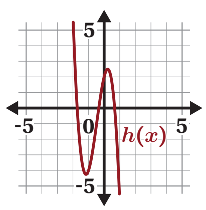
continuous A domain or range of a function is continuous when all the values in an interval are possible inputs (domain) or outputs (range).

A function is continuous when its domain and range are both continuous. You can draw its graph without picking up your pencil because the graph of a continuous function has no breaks, jumps, or holes.

cubic function A polynomial function of degree 3.

For example, $f(x) = 2x^3 - 3$, $g(x) = \frac{1}{4}(x + 4)(x + 1)(x - 2)$, and $h(x) = -5x^3 - 7x^2 + 4x + 2$ are cubic functions.

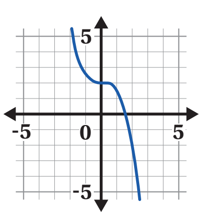


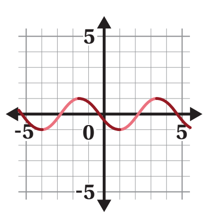


D

decreasing (interval or function) A function or interval of a function is decreasing if the y-values decrease when the x-values increase.

The bold parts of these graphs are intervals where each function is decreasing.



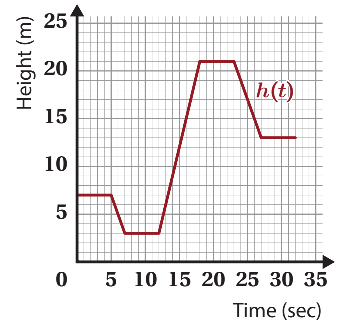


discrete A domain or range of a function is discrete when only specific values in an interval are possible inputs (domain) or outputs (range).

domain The set of all possible input values for a function or relation. The domain can be described in words, set notation, or as an inequality.

A function's domain can be discrete or continuous.

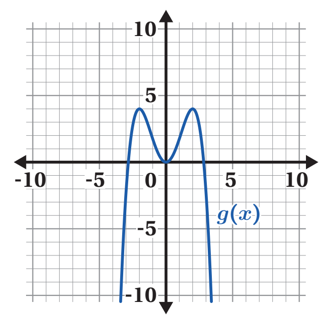
The domain of $h(t)$ can be described as all values from 0 through 32 or $0 \leq t \leq 32$.



E

even function A function whose graph is the same when reflected over the y-axis. A function is even if $f(x) = f(-x)$ for all values of x.

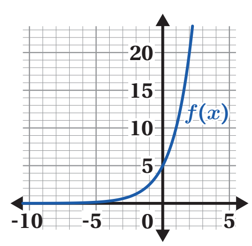
The function $g(x) = -\frac{1}{4}(x + 2)^2(x - 2)^2 + 4$ is even because reflecting it over the y-axis results in the same graph.



exponential function A function that is a transformation of $f(x) = a^x$, where a is a constant. The graph of an exponential function is always increasing or decreasing and approaches a horizontal asymptote.

The table and graph show the exponential function $f(x) = 5 \cdot (2)^x$, which has an initial value of 5 and has a growth factor of 2.

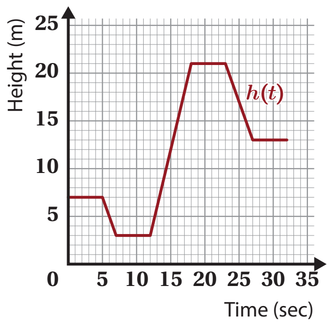
x	$f(x)$
-2	1.25
-1	2.5
0	5
1	10
2	20



F

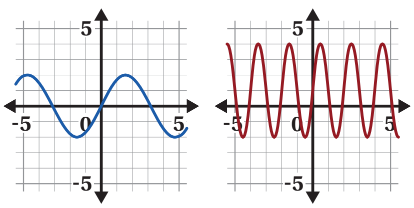
function A rule that assigns exactly one output to each possible input. On a graph of a function, each x-value has no more than one y-value.

For example, $h(t)$ is a function because, for any value of t, there is only one value of $h(t)$.



function family A group of functions made from the same basic equation, with graphs that have consistent key features and shapes.

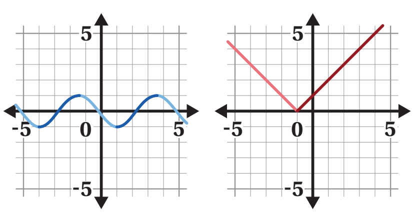
These functions are in the sine family because they are each transformations of the function $f(x) = \sin(x)$.



I

increasing (interval or function) A function or interval of a function is increasing if the y-values increase when the x-values increase.

The bold part of these graphs are intervals where each function is increasing.



input A value that is substituted into a function. The input is sometimes called the independent variable.

The input typically appears on the horizontal axis of a graph and in the left-hand column of a table.

The input of the function $f(x)$ is x.

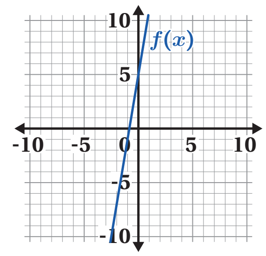
L

linear function A function that increases or decreases by a constant rate of change. The graph of a linear function is a line.

For example, $f(x) = 5 + 6x$ represents a linear function that has an initial value of 5 and increases by a constant difference of 6.

x	f(x)
0	5
1	11
2	17
3	23

+1 +1 +1   +6 +6 +6



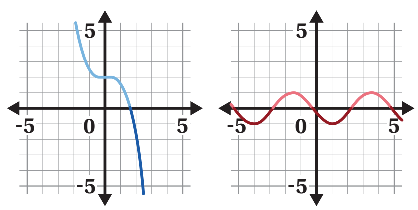
M

model A model is a mathematical representation of real-world data that you can use to solve problems or make predictions and decisions. Modeling is the process of creating a mathematical representation of real-world data.

N

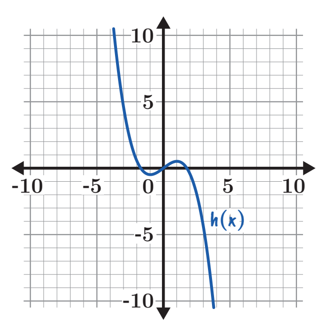
negative (interval or function) A function or internal of a function is negative when its outputs are negative and its graph is below the x-axis.

The bold parts of these graphs are intervals where each function is negative.



O

odd function A function whose graph is the same when reflected over both the x-axis and the y-axis or when rotated 180 degrees about the origin. A function is odd if $f(x) = -f(-x)$ for all values of x.



The function $h(x) = \frac{3}{4}\left(\frac{1}{3}x^3 - x\right)$ is odd because reflecting it over both the x- and y-axes results in the same graph.

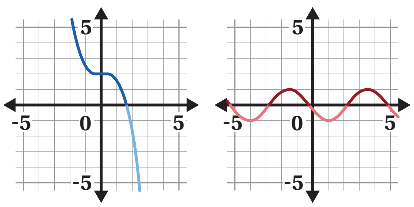
output The value of a function after it has been evaluated. The output is sometimes called the dependent variable.

The output typically appears on the vertical axis of a graph and in the right-hand column of a table.

The output of the function $f(x)$ is $f(x)$.

P

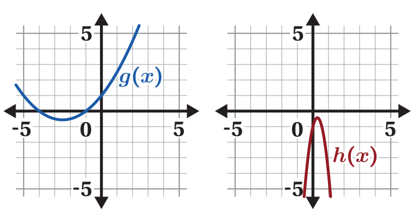
positive (interval or function) A function or interval of a function is positive when its outputs are positive and its graph is above the x-axis.



The bold parts of these graphs are intervals where each function is positive.

Q

quadratic function A function that is a transformation of $y = x^2$. The graph of a quadratic function is a parabola.

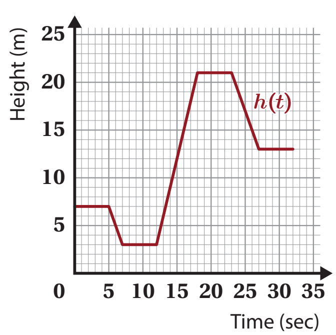


For example, $g(x) = \frac{1}{4}(x + 4)(x + 1)$ and $h(x) = -7x^2 + 4x - 1$ are quadratic functions.

R

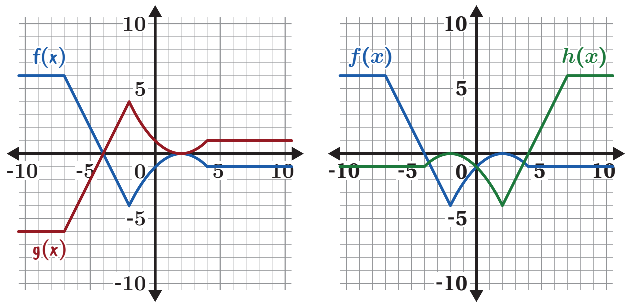
range The set of all possible output values for a function or relation. The range can be described in words, set notation, or as an inequality.

A function's range can be discrete or continuous.



The range of $h(t)$ can be described as all values from 3 through 21 or $3 \le h(t) \le 21$.

reflection (of a function) A transformation that moves every point of a function onto a point on the opposite side of a given line (called the line of reflection). The new points are the same distance from the line of reflection as they were in the original function.

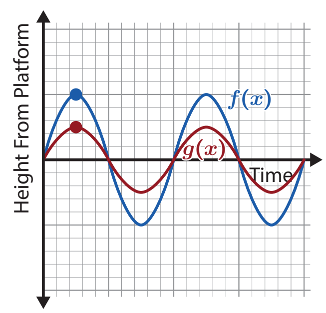


The function $g(x) = -f(x)$ is a reflection of $f(x)$ over the x-axis. The function $h(x) = f(-x)$ is a reflection over the y-axis.

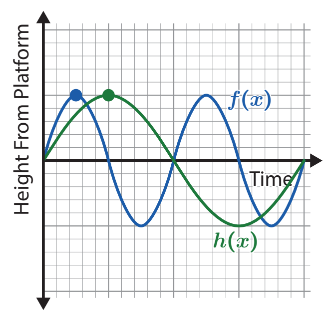
S

scale A transformation that multiplies the input or output of every point of a function by the same factor.

When you scale a function vertically, the y-values change as the points on its graph stretch away from or compress toward the x-axis, but the function's x-values do not change.

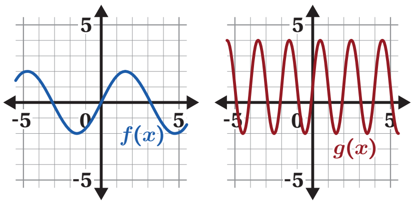


When you scale a function horizontally, the x-values change as the points on its graph stretch away from or compress toward the y-axis, but the function's y-values do not change.



The function $g(x) = \frac{1}{2}f(x)$ is $f(x)$ scaled vertically. The function $h(x) = f\left(\frac{1}{2}x\right)$ is $f(x)$ scaled horizontally.

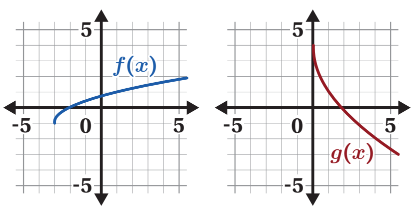
sine function A function that is a transformation of $f(x) = \sin(x)$.



For an angle θ in standard position, $\sin(\theta)$ represents the y-coordinate of its corresponding point on the unit circle.

For example, $f(x) = 2\sin(x)$ and $g(x) = 3\sin(\pi x) + 1$ are sine functions.

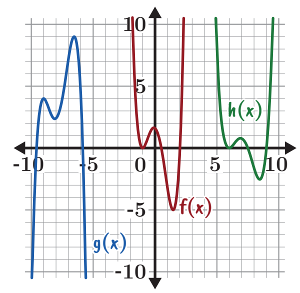
square root function A function that is a transformation of $f(x) = \sqrt{x}$. The graph of a square root function is either always increasing from a minimum value or always decreasing from a maximum value.



For example, $f(x) = \sqrt{x+3} - 1$ and $g(x) = -3\sqrt{x} + 4$ are square root functions.

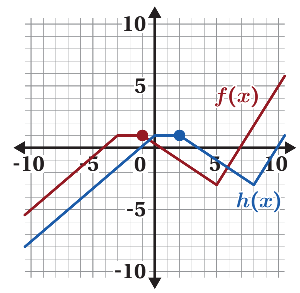
T

transformation (of a function) A rule for moving or changing functions on a graph. Transformations include translations, reflections, rotations, dilations, and scaling.



The functions $g(x)$ and $h(x)$ are transformations of the function $f(x)$.

translation (of a function) A transformation that moves every point on a function the same distance, in the same direction. A translation changes the location of a function, but not its shape or orientation.



The function $h(x) = f(x-3)$ is a translation of $f(x)$ right 3 units.