Unit 3

Multiplying and Dividing Fractions

Essential Questions

- How can we represent situations in which we determine a part of a part?
- How can we represent equal-sharing situations when a fraction of a whole is being shared?
- How can we determine how many fractional-sized parts are in a number of wholes?

Unit Story: Princess Sweetsocks

You can read the Unit Story with your student by visiting the Unit Story page on the Caregiver Hub.

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Unit Investigation

Lesson 1 is the Unit Investigation. Students explore fraction multiplication by predicting the number of pieces a paper has been folded into to build curiosity and apply their own knowledge in a variety of ways. Use the **Caregiver Connection** to help students continue to explore the math they will see in the unit.

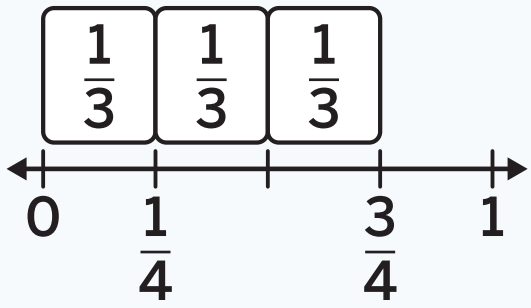
Caregiver Connection

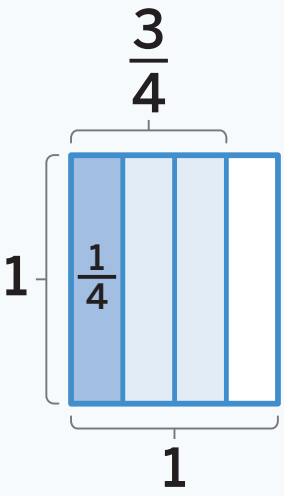
Students may enjoy challenging friends or family members to the same task of predicting the number of parts before unfolding. Encourage students to fold different sizes of paper to see if they notice patterns that hold true, no matter the size of the original shape.

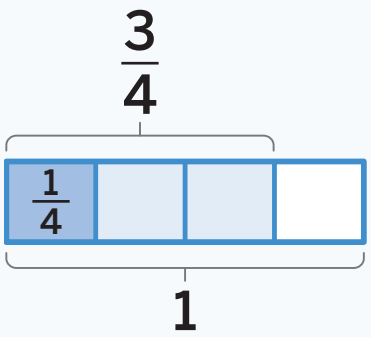
Summary | Lesson 2

You can use different diagrams to represent a fraction of a fraction, or a part of a part of the whole. Often, you can use the same diagram in different ways to represent your thinking.

$\frac{1}{3}$ of $\frac{3}{4}$







Try This

1 Select the true statement.

(A) Priya had $\frac{4}{6}$ of a jar of beads. She used $\frac{1}{2}$ of the beads to make a bracelet. Priya used $\frac{2}{6}$ of the whole jar.

(B) Priya had $\frac{4}{6}$ of a jar of beads. She used $\frac{1}{2}$ of the beads to make a bracelet. Priya used $\frac{4}{8}$ of the whole jar.

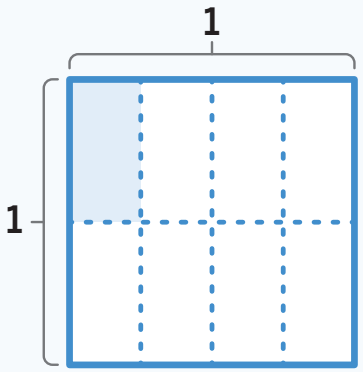
(C) Priya had $\frac{4}{6}$ of a jar of beads. She used $\frac{1}{2}$ of the beads to make a bracelet. Priya used $\frac{5}{8}$ of the whole jar.

(D) Priya had $\frac{4}{6}$ of a jar of beads. She used $\frac{1}{2}$ of the beads to make a bracelet. Priya used $\frac{7}{6}$ of the whole jar.

Summary | Lesson 3

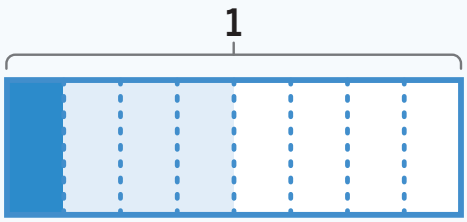
You can represent a unit fraction of a unit fraction with a diagram and a multiplication equation. In the diagram, the whole is equipartitioned twice. In the equation, the second factor represents the starting amount, the first factor represents the fraction of the starting amount, and the product represents the number of shaded parts out of the number of equal parts in the whole.

$\frac{1}{2}$ of $\frac{1}{4}$



$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

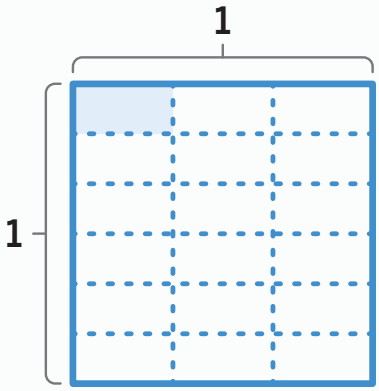
$\frac{1}{4}$ of $\frac{1}{2}$



$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

Try This

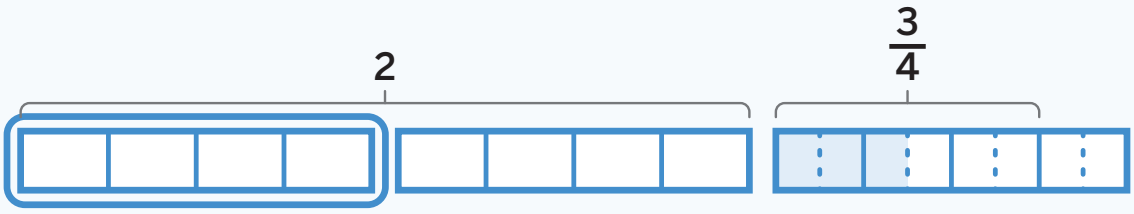
1 Write an equation to represent the area of the shaded rectangular region in the diagram.



Summary | Lesson 4

You can represent the product of a unit fraction and a non-unit fraction or a mixed number with different diagrams and equations. The product is always the number of parts shaded out of the number of equal parts in 1 whole.

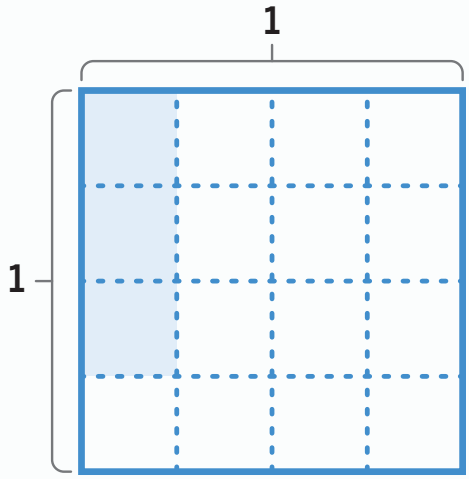
$$\frac{1}{2} \times 2\frac{3}{4}$$



Equation A	$\left(\frac{1}{2} \times 2\right) + \left(\frac{1}{2} \times \frac{3}{4}\right) = 1\frac{3}{8}$
Equation B	$\left(\frac{1}{2} \times \frac{8}{4}\right) + \left(\frac{1}{2} \times \frac{3}{4}\right) = \frac{11}{8}$
Equation C	$\frac{1}{2} \times \frac{11}{4} = \frac{11}{8}$

Try This

1 Which expression represents the area of the shaded rectangular region in the diagram?



(A) $\frac{1}{4} \times \frac{2}{4}$

(B) $\frac{1}{4} \times \frac{3}{4}$

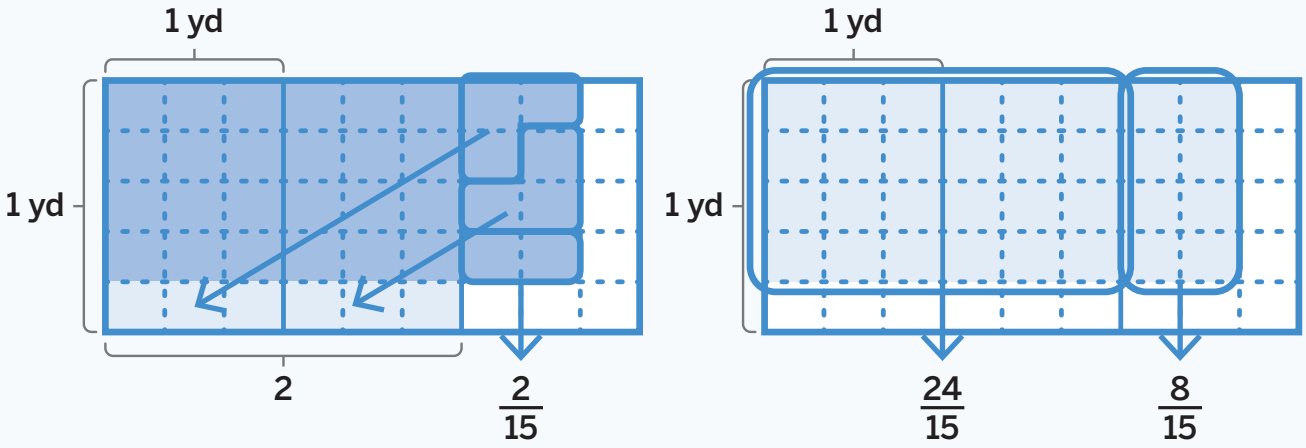
(C) $\frac{1}{4} \times \frac{2}{3}$

(D) $\frac{1}{3} \times \frac{3}{4}$

Summary | Lesson 5

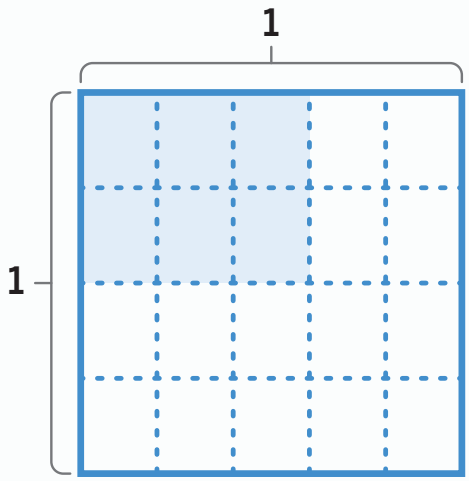
You can draw diagrams with equal-sized parts in each whole to represent the area of a rectangular shaded region with side lengths that are non-unit fractions or mixed numbers. You can interpret the diagrams differently, such as making whole units or grouping equal-sized parts.

$$2\frac{2}{3} \times \frac{4}{5}$$



Try This

1 Which equation represents the area of the shaded rectangular region in the diagram?



(A) $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

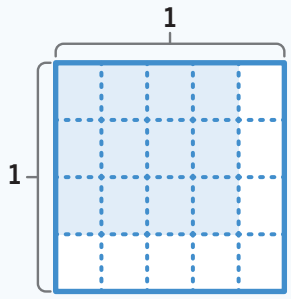
(B) $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15}$

(C) $\frac{2}{5} \times \frac{2}{4} = \frac{4}{20}$

(D) $\frac{3}{5} \times \frac{2}{4} = \frac{6}{20}$

Summary | Lesson 6

The product of the numerators in the factors represents the number of shaded parts, and the product of the denominators in the factors represents the number of equal parts in 1 whole.



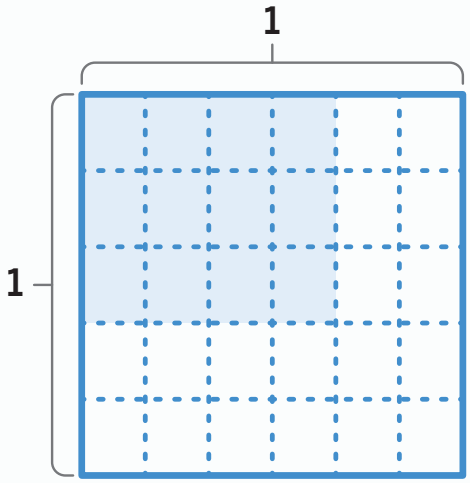
number of rows and columns in the shaded region

$$\frac{4 \times 3}{5 \times 4} = \frac{12}{20} \longrightarrow \frac{\text{number of parts in the shaded region}}{\text{number of equal parts in 1 whole}}$$

number of rows and columns in the whole

Try This

1 Which expressions represent the area of the shaded rectangular region in the diagram? Select *all* that apply.



(A) $\frac{1}{3} \times \frac{3}{5}$

(B) $\frac{2}{3} \times \frac{3}{5}$

(C) $\frac{2}{5} \times \frac{2}{3}$

(D) $\frac{2}{6} \times \frac{2}{5}$

(E) $\frac{3}{5} \times \frac{4}{6}$

(F) $\frac{4}{6} \times \frac{2}{5}$

Summary | Lesson 7

You can determine the product of 2 fractions by multiplying the numerators of the factors and the denominators of the factors. When multiplying with mixed numbers, you can rewrite the mixed number as a fraction greater than 1 or use the Distributive Property.

$\frac{2}{3} \times \frac{4}{5}$	$2\frac{2}{3} \times \frac{4}{5}$
$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5}$	$2\frac{2}{3} \times \frac{4}{5} = \frac{8}{3} \times \frac{4}{5} = \frac{8 \times 4}{3 \times 5}$ **or** $2\frac{2}{3} \times \frac{4}{5} = \left(\frac{6}{3} \times \frac{4}{5}\right) + \left(\frac{2}{3} \times \frac{4}{5}\right)$

Try This

1 Evaluate the expression $\frac{7}{8} \times \frac{1}{7}$.

Show or explain your thinking.

answer: _____

Summary | Lesson 8

You can use the size of factors and the relationship between factors and products to reason about problems involving multiplication with fractions, whole numbers, and mixed numbers.

$$? \times 3 = \frac{24}{7}$$

$$3 = \frac{3}{1}, \text{ so } \frac{? \times 3}{? \times 1} = \frac{24}{7}$$

$$8 \times 3 = 24 \text{ and } 7 \times 1 = 7, \text{ so } \frac{8}{7}.$$

Try This

For Problems 1–3, determine the value that makes the equation true.

1 $\frac{2}{3} \times$ _____ $= \frac{20}{21}$

2 _____ $\times \frac{4}{9} = \frac{28}{45}$

3 $\frac{5}{8} \times$ _____ $= \frac{35}{40}$

Summary | Lesson 9

You can compare the size of a product to 1 factor by reasoning about the size of the other factor.

Expression	The product is . . .
$\frac{4}{4} \times 9{,}276$	Equal to 9,276 because $\frac{4}{4}$ is equal to 1.
$\frac{3}{4} \times 9{,}276$	Less than 9,276 because $\frac{3}{4}$ is less than 1.
$\frac{4}{3} \times 9{,}276$	Greater than 9,276 because $\frac{4}{3}$ is greater than 1.

Try This

For Problems 1–3, complete the comparison statement using <, >, or =.

1. $539 \times \frac{10}{7}$ _____ 539

2. $539 \times \frac{10}{10}$ _____ 539

3. $539 \times \frac{7}{10}$ _____ 539

Summary | Lesson 10

You can compare the size of a product to either of its factors using the size of the other factor. If the other factor is less than, equal to, or greater than 1, the product is also less than, equal to, or greater than the compared factor.

$$\frac{15}{14} \times \frac{23}{30}$$

other factor compared factor $\frac{15}{14} \times \frac{23}{30} > \frac{23}{30}$	compared factor $\frac{15}{14} \times \frac{23}{30} < \frac{15}{14}$ other factor
The product of $\frac{15}{14} \times \frac{23}{30}$ is greater than $\frac{23}{30}$ because $\frac{15}{14}$ is greater than 1.	The product of $\frac{15}{14} \times \frac{23}{30}$ is less than $\frac{15}{14}$ because $\frac{23}{30}$ is less than 1.

Try This

For Problems 1–4, complete the comparison statement using <, >, or =.

1. $\frac{103}{104} \times \frac{103}{104}$ ____ $\frac{103}{104}$

2. $\frac{103}{104} \times \frac{104}{103}$ ____ $\frac{103}{104}$

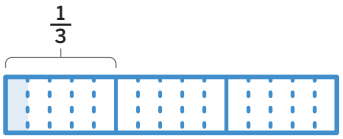
3. $\frac{63}{54} \times \frac{62}{54}$ ____ $\frac{63}{54}$

4. $\frac{63}{54} \times \frac{62}{54}$ ____ $\frac{62}{54}$

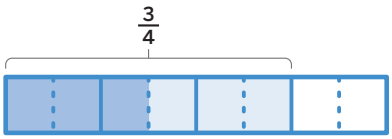
Sub-Unit 1 | Summary

In this sub-unit . . .

- We saw that we can represent fraction multiplication with diagrams and multiplication equations. We can interpret this situation as determining *part of a part*.

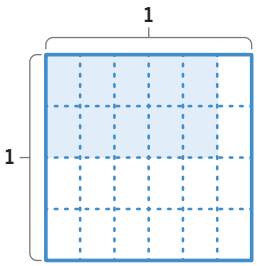


$\frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$

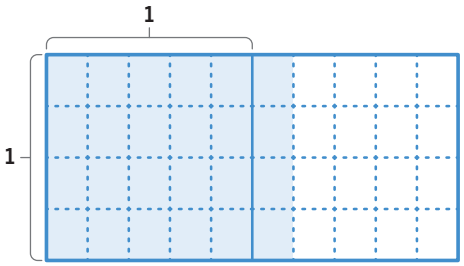


$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

- We determined the area of shaded rectangular regions and represented the areas with equations.



$\frac{2}{4} \times \frac{5}{6} = \frac{10}{24}$



$\frac{6}{5} \times \frac{4}{4} = \frac{24}{20}$

- We saw that the product of 2 fractions can be determined by multiplying the numerators and multiplying the denominators. When multiplying with mixed numbers, you can rewrite the mixed number as a fraction greater than 1 or use the Distributive Property.

$$2\frac{7}{9} \times \frac{5}{8}$$

$$2\frac{7}{9} = \frac{25}{9}$$

$$\frac{25}{9} \times \frac{5}{8} = \frac{125}{72}$$

$$2\frac{7}{9} \times \frac{5}{8}$$

$$\left(\frac{18}{9} \times \frac{5}{8}\right) + \left(\frac{7}{9} \times \frac{5}{8}\right)$$

$$\frac{90}{72} + \frac{35}{72} = \frac{125}{72}$$

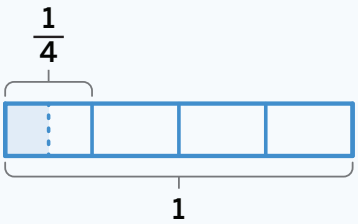
Math tip: Whole numbers are equivalent to fractions with a denominator of 1. Mixed numbers are equivalent to fractions greater than 1 with the same denominator.

Summary | Lesson 11

You can use familiar diagrams and a division equation to represent story problems in which a unit fraction is equally shared. Just like when equal sharing with whole numbers, the dividend is the amount being shared, the divisor is the number of equal shares, and the quotient is the size of each share.

2 cats equally share $\frac{1}{4}$ of a bag of food.

How much of a bag does each cat get?



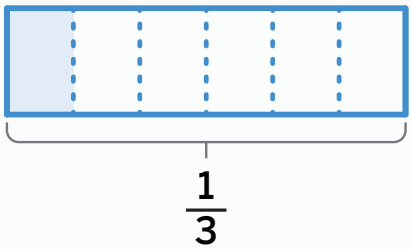
$\frac{1}{4} \div 2 = \frac{1}{8}$

amount being shared | **number of equal shares** | **size of each share out of 1 whole**

Each cat gets $\frac{1}{8}$ of a whole bag of food.

Try This

1 Determine which expressions represent the tape diagram. Select *all* that apply.



(A) $\frac{1}{3} \div 6$

(B) $\frac{1}{3} \div 5$

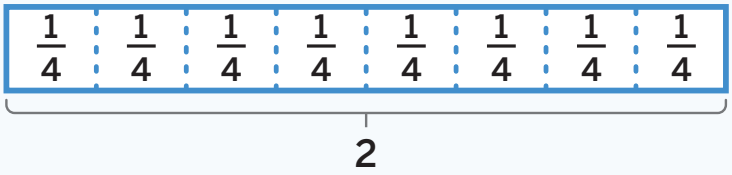
(C) $\frac{1}{6}$

(D) $\frac{1}{18}$

Summary | Lesson 12

You can use division equations and diagrams to represent "how many parts?" problems and equal-sharing situations. In the equation, the dividend is the total amount, the divisor is the size of the equal parts, and the quotient is the number of parts in the total amount.

1 serving of cat food is $\frac{1}{4}$ bags of food.
How many servings are there in 2 bags?



$2 \div \frac{1}{4} = 8$

There are 4 servings in 1 bag, so there are 8 servings in 2 bags.

Try This

1 Priya has a strip of ribbon that is 6 feet long. She cuts it into pieces that are each $\frac{1}{6}$ feet long. Determine how many pieces she has now. Then write a division equation to represent the situation.

Show your thinking.

answer: _____

equation: _____

Summary | Lesson 13

You can use the structure of a division expression to relate expressions to story problems and reason about the size of quotients without drawing diagrams or evaluating.

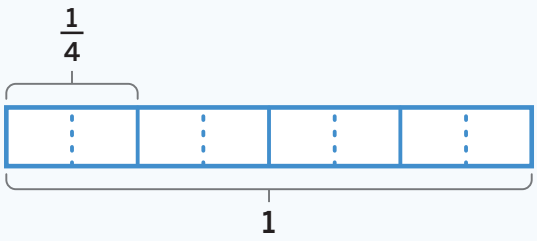
Story problem	1 serving of cat food is $\frac{1}{5}$ of a bag. How many servings are in 10 bags?	10 cats share $\frac{1}{5}$ bags of food. How much food does each cat get?
Problem type	"how many parts?"	equal sharing
Expression	$10 \div \frac{1}{5}$ **starting amount** — **size of each part**	$\frac{1}{5} \div 10$ **starting amount** — **number of shares/groups**
Size of quotient	greater than 10	less than $\frac{1}{5}$

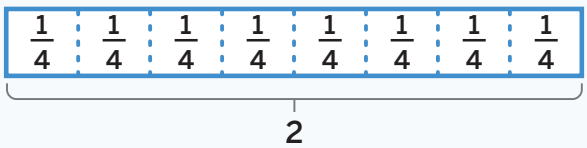
Try This

1 Write a story problem that represents the expression $4 \div \frac{1}{3}$.

Summary | Lesson 14

Just like with whole numbers, a multiplicative situation involving fractions can be represented with multiplication and division equations. You can use the relationship between multiplication and division to reason about story problems.





$\frac{1}{4} \div 2 = \frac{1}{8}$

$2 \div \frac{1}{4} = 8$

$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

$2 \times 4 = 8$

Try This

1 Diego cuts $\frac{1}{5}$ of a large dog treat for his 3 dogs to equally share. Write a multiplication equation and a division equation to represent how much of the dog treat each dog will get.

multiplication equation: _____

division equation: _____

Summary | Lesson 15

Just like with whole numbers, you can create a story problem that involves multiplication or division with fractions by thinking about what the operation means and what the product or quotient will mean. This is particularly helpful before choosing characters and an action for your story problem.

Expression	Meaning	Story problem
$\frac{1}{2} \times \frac{1}{3}$	$\frac{1}{2}$ of $\frac{1}{3}$	A dog had $\frac{1}{3}$ of a bowl of water and drank $\frac{1}{2}$ of the water. How much of a whole bowl of water did the dog drink?
$3 \div \frac{1}{2}$	How many $\frac{1}{2}$s are in 3?	A serving of food is $\frac{1}{2}$ of a bag. How many servings are in 3 bags?

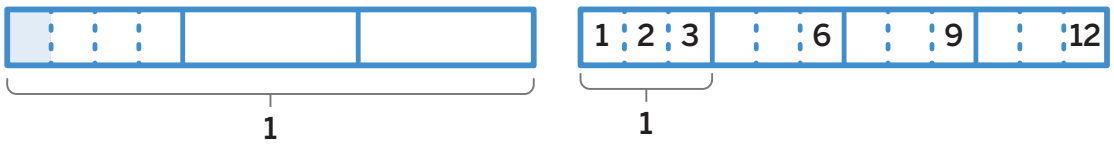
Try This

1 Create a story problem that represents the expression $\frac{1}{10} \times \frac{1}{6}$.

Sub-Unit 2 | Summary

In this sub-unit . . .

- We represented equal-sharing situations in which a unit fraction was divided by a whole number.
- We represented "how many parts?" situations in which a whole number was divided by a unit fraction.



$\frac{1}{3} \div 4 = \frac{1}{12}$

amount being shared | number of shares | size of each share

$4 \div \frac{1}{3} = 12$

amount being shared | size of each share | number of shares

Math tip: When dividing by a number greater than 1, the quotient is less than the dividend. When dividing by a unit fraction, the quotient is greater than the dividend. This makes sense because the size of a unit fraction is much smaller than a number greater than 1.

- We represented the same situation with division equations and multiplication equations.

Situation	Representations
There is $\frac{1}{4}$ of a stack of flyers left. Shay and KT split it equally. How much of the stack of flyers does each person get?	 $\frac{1}{4} \div 2 = \frac{1}{8}$ $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$
A full bottle contains 2 milliliters of medicine. Each kitten needs $\frac{1}{4}$ milliliters of medicine. How many kittens can be given medicine from a full bottle?	 $2 \div \frac{1}{4} = 8$ $2 \times 4 = 8$

Try This | Answer Key

Lesson 2

1 A

Lesson 3

1 $\frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$ or $\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$

Lesson 4

1 B

Lesson 5

1 D

Lesson 6

1 B and E

Lesson 7

1 Sample work:

$\frac{7}{8} \times \frac{1}{7} = \frac{7 \times 1}{8 \times 7} = \frac{7}{56}$

answer: $\frac{7}{56}$ or $\frac{1}{8}$

Lesson 8

1 $\frac{10}{7}$ or $1\frac{3}{7}$ 2 $\frac{7}{5}$ or $1\frac{2}{5}$ 3 $\frac{7}{5}$ or $1\frac{2}{5}$

Lesson 9

1 > 2 = 3 <

Lesson 10

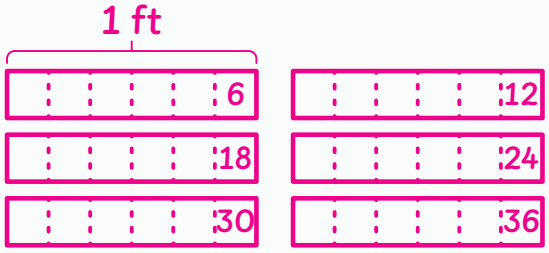
1 < 2 > 3 > 4 >

Lesson 11

1 A and D

Lesson 12

1 Sample work:



answer: 36 pieces

equation: $6 \div \frac{1}{6} = 36$

Lesson 13

1 Sample response: Jada is sharing 4 cups of popcorn with friends. Each friend receives $\frac{1}{3}$ cups of popcorn. How many friends are there?

Lesson 14

1 multiplication equation: $\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$

division equation: $\frac{1}{5} \div 3 = \frac{1}{15}$

Lesson 15

1 Sample response: An artist had $\frac{1}{6}$ cans of blue paint to use on her painting. She used $\frac{1}{10}$ of the blue paint to paint the sky. How much of the can of blue paint did she use for the sky?