Unit **2**

Fractions as Quotients and Fraction Multiplication

Essential Questions

- How are division and fractions related?

- How can we determine the area of a rectangle with fractional side lengths?

✦ Unit Story: The More the Merrier

You can read the Unit Story with your student by visiting the Unit Story page on the Caregiver Hub.

🔍 Unit Investigation

Lesson 1 is the Unit Investigation. Students interpret and represent an equal-sharing situation in which each person's share includes a part of a whole to build curiosity and apply their own knowledge in a variety of ways. Use the **Caregiver Connection** to help students continue to explore the math they will see in the unit.
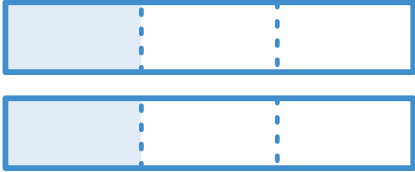
Caregiver Connection

Students may enjoy exploring equal-sharing situations such as sharing food, playing games, filling containers, or wrapping presents. You may ask:

- "How much do you want to share?"
- "How many people do you want to share with?"
- "How will you share the leftover amount?"

Equal-sharing story problems can be represented with a diagram and a division expression. Sometimes, the quotient, or amount each person receives, is a fraction less than or greater than 1.

3 friends equally share 2 sandwiches.
How many sandwiches does each friend receive?

Diagram	Division expression	Amount each friend receives
	$2 \div 3$ what is being shared the number of shares	$\frac{2}{3}$ of a sandwich

Try This

1   4 students equally share 5 pages of stickers for an art project. Determine how many pages of stickers each student receives. Then write a division expression to represent the story problem.

ℹ Show or explain your thinking.

answer: _____   expression: _____

You can represent an equal-sharing situation with a division equation. The dividend and numerator represent the amount being shared, and the divisor and denominator represent the number of shares.

5 liters of water are shared by 6 people. How many liters of water does each person receive?

what is being shared

$$5 \div 6 = \frac{5}{6}$$

how much each person receives (size of 1 group)

number of equal shares

Try This

1 Jada cuts an 11-inch strip of paper into 5 equal parts. Write an equation to represent the length of each part in inches.

_____

2 Han shared 7 liters of water equally among his 6 dogs. Write an equation to represent the amount of water each dog received in liters.

_____

A fraction can be used to write and solve equal-sharing story problems.

amount being shared ⟵ 3
number of equal shares ⟵ 2 } amount each person receives (size of 1 group)

Jada has 3 yards of wrapping paper. If she uses all of the wrapping paper to wrap 2 same-sized presents, how many yards of wrapping paper does she use to wrap each present?

Try This

For Problems 1–4, match the division equation with a situation it could represent.

Equation	Situation

1 $8 \div 11 = \dfrac{8}{11}$ ———   Grandpa made 5 loaves of banana bread and divided 7 cups of bananas equally among the loaves.

2 $11 \div 8 = \dfrac{11}{8}$ ———   The animal shelter had 11 bird cages and 8 cups of bird feed to share equally among the bird cages.

3 $7 \div 5 = \dfrac{7}{5}$ ———   7 cats shared 5 cups of food equally.

4 $5 \div 7 = \dfrac{5}{7}$ ———   8 students shared some water equally. Each student received more than 1 cup of water.

A fraction can be interpreted as division of the numerator by the denominator. Likewise, a division expression can be interpreted as a fraction in which the dividend is the numerator and the divisor is the denominator.

$$\frac{7}{8} = 7 \div 8$$

$$8 \div 7 = \frac{8}{7}$$

Try This

1 Priya divided 3 cups of blueberries equally to make muffins. Each muffin used $\frac{3}{14}$ cups of blueberries. How many muffins did Priya make?
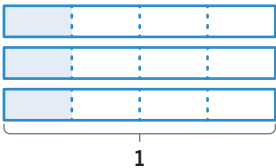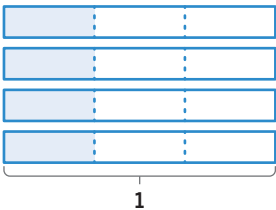
_____

2 Han made 6 cups of chicken potato soup for his family. He poured $\frac{6}{4}$ cups of soup into each bowl. How many bowls of soup did Han prepare?

_____

In this sub-unit . . .

- We represented equal-sharing situations involving 2 whole numbers with division expressions. The quotient represents the size of each share.

Situation	Diagram	Fraction	Expression
4 people equally share 3 sandwiches.		$\dfrac{3}{4}$	$3 \div 4$
4 sandwiches are shared equally by 3 people.		$\dfrac{4}{3}$	$4 \div 3$

- We noticed:

  - The dividend in a division expression, which represents an amount being shared, is related to the numerator in its quotient.

  - The divisor in a division expression, which represents a number of equal shares, is related to the denominator in its quotient.

  🔥 *Math tip:* When the number of things being shared is greater than the number of people sharing, each share is greater than 1. When the number of things being shared is less than the number of people sharing, each share is less than 1.

- We saw that a fraction can be interpreted as division of the numerator by the denominator. Likewise, a division expression can be interpreted as a fraction in which the dividend is the numerator and the divisor is the denominator.

Just like with whole numbers, multiplication expressions and division expressions can be used to represent the same equal-sharing or part-of-a-whole story problem.

2 friends share 3 sandwiches. How many sandwiches does each person eat?		
$3 \div 2$	$3 \times \dfrac{1}{2}$	$\dfrac{1}{2} \times 3$

Try This

1 8 cups of dog food is shared equally by 7 dogs. Represent the situation with a multiplication equation and a division equation.

multiplication equation: _____

division equation: _____

2 Diego fed his cats $\frac{1}{5}$ of a 9-pound bag of cat food. Represent the situation with a multiplication equation and a division equation.

multiplication equation: _____

division equation: _____

Dividing 2 whole numbers is equivalent to multiplying a whole number and a unit fraction.

If...	And...	Then...
$6 \div 8 = \dfrac{6}{8}$	$6 \times \dfrac{1}{8} = \dfrac{6}{8}$	$6 \div 8 = 6 \times \dfrac{1}{8}$

Try This

1 Explain how the expressions $2 \div 5$ and $\frac{1}{5} \times 2$ represent the diagram.
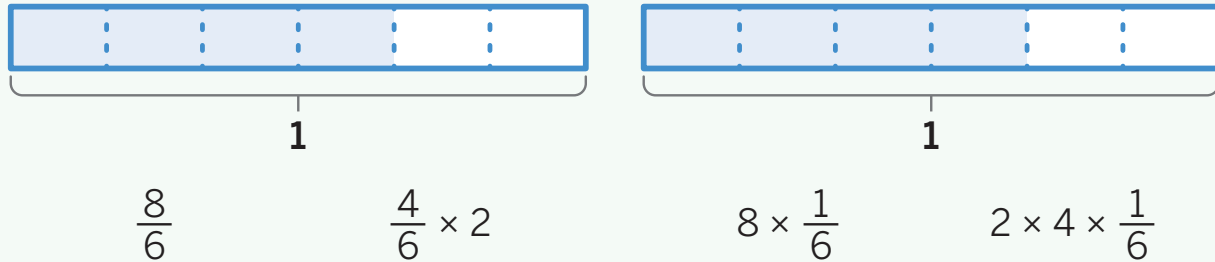

1

_____

_____

_____

_____
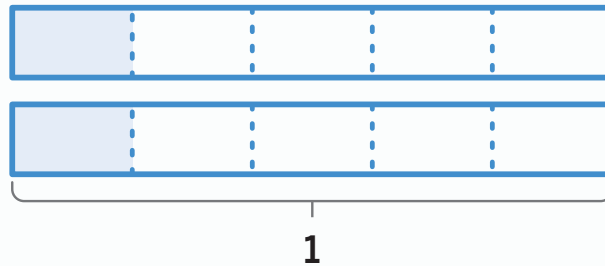
Diagrams and equivalent expressions can be used to represent the product of a non-unit fraction and a whole number.

$$\frac{8}{6} \qquad \frac{4}{6} \times 2 \qquad\qquad 8 \times \frac{1}{6} \qquad 2 \times 4 \times \frac{1}{6}$$

Try This

1   4 people run a relay race. Each person runs $\frac{3}{4}$ kilometers. Write an equation to represent how much they run altogether in kilometers.

_____

2   Shawn cuts a strip of paper into 5 equal parts. Each part is $\frac{7}{5}$ inches long. Write an equation to represent the length of the original strip of paper in inches.

_____

In this sub-unit . . .

- We represented an equal-sharing story problem and a part-of-a-whole story problem with diagrams, division expressions, and multiplication expressions.

Situation	Cousin Maxim shared a story about a time when he and his friends, Lin and Andre, ran a 5-mile race as a team. They each ran the same distance. How many miles did each person run?	Cousin Maxim is training for another race. On Monday, he ran $\frac{1}{4}$ of his road, which is 7 miles long. How far did Maxim run in miles?
Diagram	LMA LMA LMA LMA LMA $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$ $\frac{4}{3}$ $\frac{5}{3}$	1 mile
Expressions	$5 \div 3$ $\quad$ $\frac{1}{3} \times 5$ $\quad$ $5 \times \frac{1}{3}$	$7 \div 4$ $\quad$ $\frac{1}{4} \times 7$ $\quad$ $7 \times \frac{1}{4}$

- We saw that the same diagram can be represented with an equation involving equivalent multiplication and division expressions. Dividing 2 whole numbers is equivalent to multiplying a whole number with a unit fraction.

$$3 \div 4 = \frac{3}{4} \qquad 3 \times \frac{1}{4} = \frac{3}{4} \qquad 3 \div 4 = 3 \times \frac{1}{4}$$

- We interpreted diagrams and explained how different expressions involving multiplying a whole number with a non-unit fraction represented the shaded region of a given diagram.

  🔥 *Math tip:* Shaded area diagrams can be used to represent the product of any 2 numbers.

The area of a shaded rectangular region with whole-number and unit-fraction side lengths can be determined by counting the number of shaded parts or by multiplying the number of unit squares by the size of each shaded part in each unit square.

1 unit

$\frac{1}{3}$ **units**
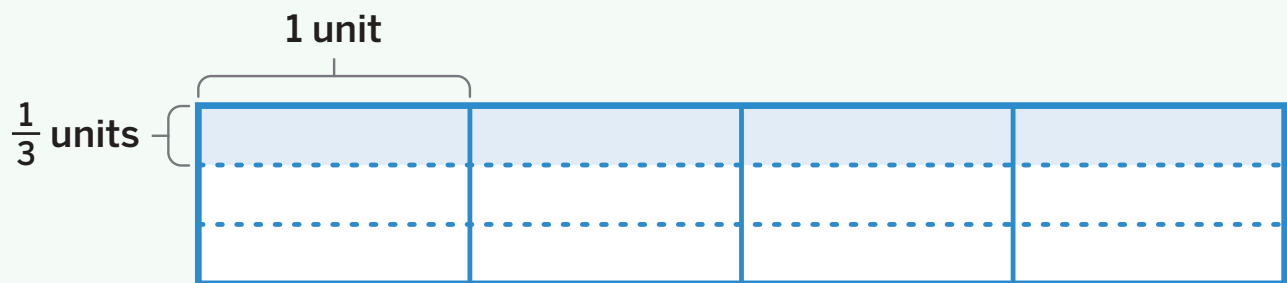
$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{4}{3}$$
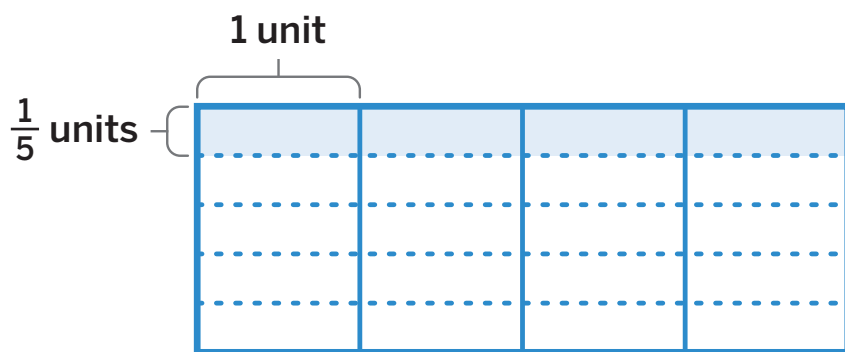
or

$$4 \times \frac{1}{3} = 1\frac{1}{3}$$

Try This

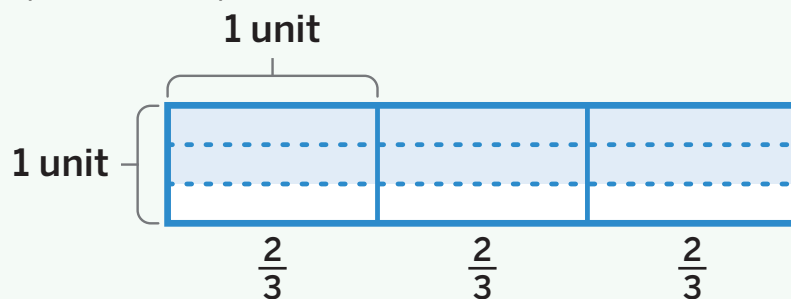**1** Determine the area of the shaded rectangular region in square units.

ⓘ Show or explain your thinking.

1 unit

$\frac{1}{5}$ **units**

answer: _____

The same strategies can be used to determine the area of a shaded rectangular region with a unit-fraction or non-unit fraction side length. When there is a non-unit fraction side length, the area can be represented with different equivalent expressions.
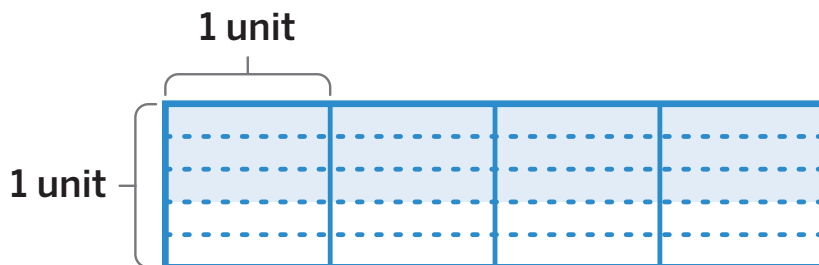
1 unit

1 unit

$\frac{2}{3}$ $\frac{2}{3}$ $\frac{2}{3}$

area:

$\frac{6}{3}$ or 2 square units

expressions:

$3 \times \frac{2}{3}$

$3 \times 2 \times \frac{1}{3}$

Try This

1   Write an expression to represent the area of the shaded rectangular region. Then determine the area of the shaded rectangular region in square units.
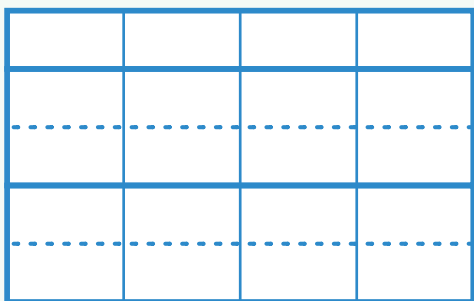
ℹ Show or explain your thinking.

1 unit

1 unit

expression: _____   answer: _____

The same strategies used for determining the areas of rectangles can be used to determine the area of a rectangle with a fractional side length greater than 1. Sometimes it is helpful to think about the side length as a mixed number.
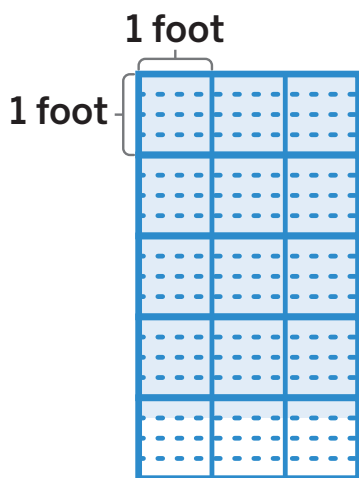
$$4 \times 2\frac{1}{2}$$

$$4 \times \frac{5}{2}$$

Try This

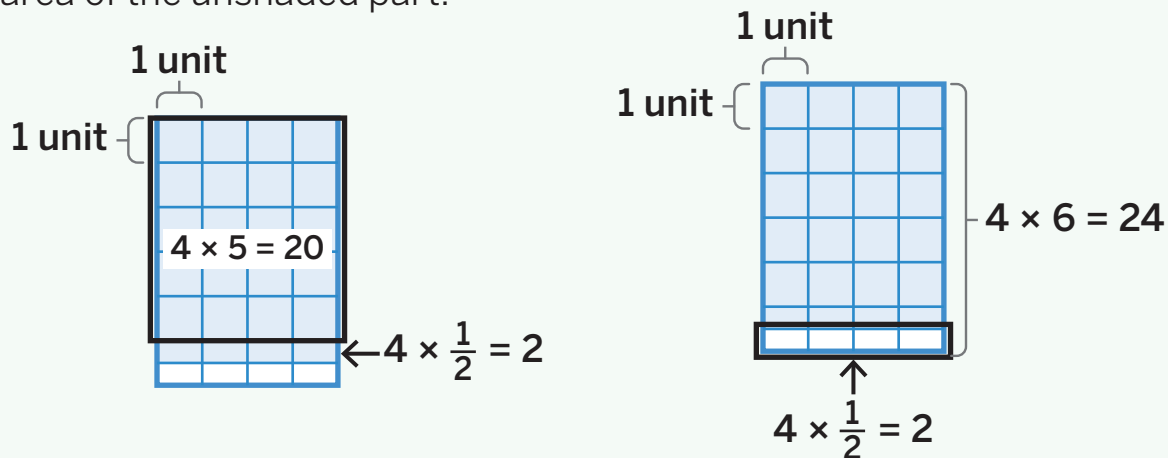**1** Determine the area of the shaded rectangular region in square feet.

ℹ **Show or explain your thinking.**

1 foot

1 foot

answer: _____

You can decompose a fractional side length greater than 1 into a whole number and a fraction less than 1 and then add the 2 areas together.
You can also determine the area of the entire diagram and then subtract the area of the unshaded part.

1 unit

1 unit

4 × 5 = 20

←4 × $\frac{1}{2}$ = 2

1 unit
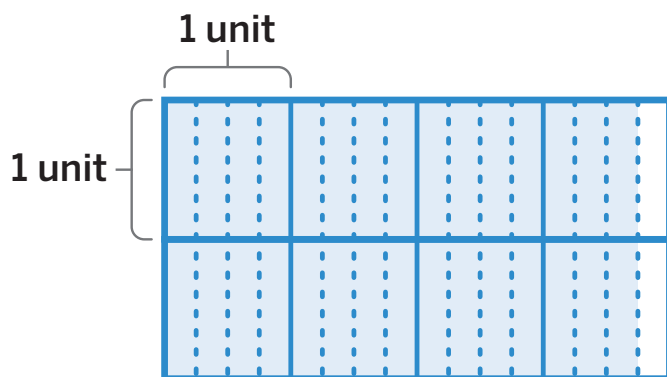
1 unit

4 × 6 = 24

4 × $\frac{1}{2}$ = 2

20 + 2 = 22, so 22 square units.   **24 − 2 = 22**, so 22 square units.

Try This

1   Determine the area of the shaded rectangular region in square units.
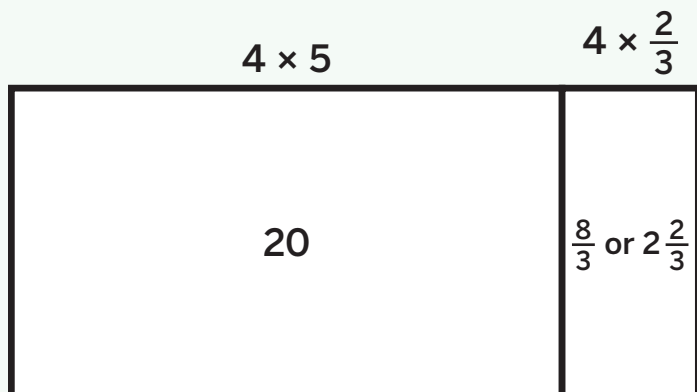
i   Show your thinking.

1 unit

1 unit

answer: _____

The Distributive Property can be used to multiply a mixed number and a whole number to help you determine the product.

$$4 \times 5\frac{2}{3}$$

4×5 $\quad$ $4 \times \frac{2}{3}$

| 20 | $\frac{8}{3}$ or $2\frac{2}{3}$ |

$$4 \times 5\frac{2}{3} = (4 \times 5) + (4 \times \frac{2}{3})$$
$$= 20 + 2\frac{2}{3}$$
$$= 22\frac{2}{3}$$

Try This

1 Select *all* the expressions that represent the area of the shaded rectangular region in square units.

1 unit

1 unit

(A) $20 \times 2 \times \frac{1}{3}$

(B) $3\frac{1}{3} \times \frac{2}{3}$

(C) $(4 \times 3) + (4 \times \frac{2}{3})$

(D) $4 \times 3\frac{1}{3}$

(E) $(4 \times 4) - (4 \times \frac{2}{3})$
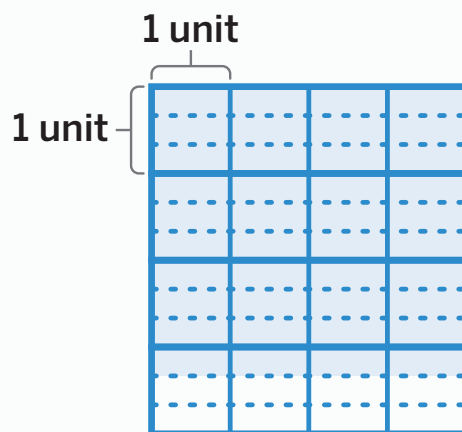
When multiplying whole numbers and fractions greater than 1, you can multiply by a fraction greater than 1 or by a mixed number. Consider the numbers to determine which way would be more helpful.

$$\frac{5}{3} \times 15$$

$\frac{5}{3} \times 15 = \frac{75}{3}$ $75 \div 3 = 25$	$\frac{5}{3} = 1\frac{2}{3}$ $(15 \times 1) + (15 \times \frac{2}{3})$ $15 + 10 = 25$

Try This

1 Which equations are true? Select *all* that apply.

(A) $\frac{8}{3} \times 4 = \frac{32}{3}$

(B) $3 \times 1\frac{2}{3} = 3\frac{2}{3}$

(C) $10\frac{1}{2} = 2\frac{5}{8} \times 4$

(D) $\frac{14}{8} = \frac{7}{4} \times 2$

(E) $\frac{5}{4} \times 3 = 3\frac{3}{4}$

(F) $1\frac{3}{8} \times 2 = \frac{11}{4}$

2 Determine whether each equation is *true* or *false*. Place a check mark in the correct column.

	True	False
$3 \times 4\frac{4}{5} = (3 \times 4) + (3 \times \frac{4}{5})$		
$6 \times 2\frac{3}{4} = (6 \times 2) - (6 \times \frac{1}{4})$		
$4\frac{1}{2} \times 3 = 4 \times \frac{1}{2} \times 3$		
$4 \times 3\frac{5}{8} = 4 \times \frac{29}{8}$		

You can estimate the product of a whole number and any fraction by reasoning about the relative size of the fraction factor and considering how close the mixed number is to the next benchmark value.

$5 \times 8\frac{6}{7}$	$8 \times \frac{31}{7}$
$8\frac{6}{7}$ is about 9.	$\frac{31}{7}$ is about $4\frac{1}{2}$.
$5 \times 9 = 45$	$8 \times 4\frac{1}{2} = 36$
$5 \times 8\frac{6}{7}$ is about 45.	$8 \times \frac{31}{7}$ is about 36.

Try This

1 Han says that $9\frac{11}{12} \times 5$ is close to but a little less than 50.
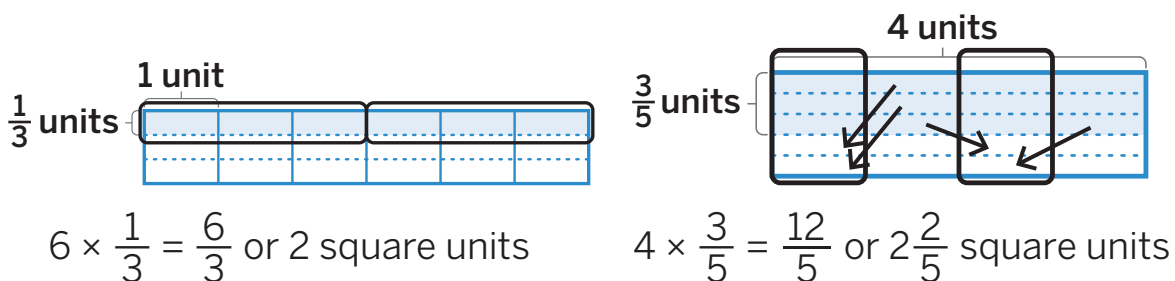
Do you agree with Han? Explain your thinking.

_____

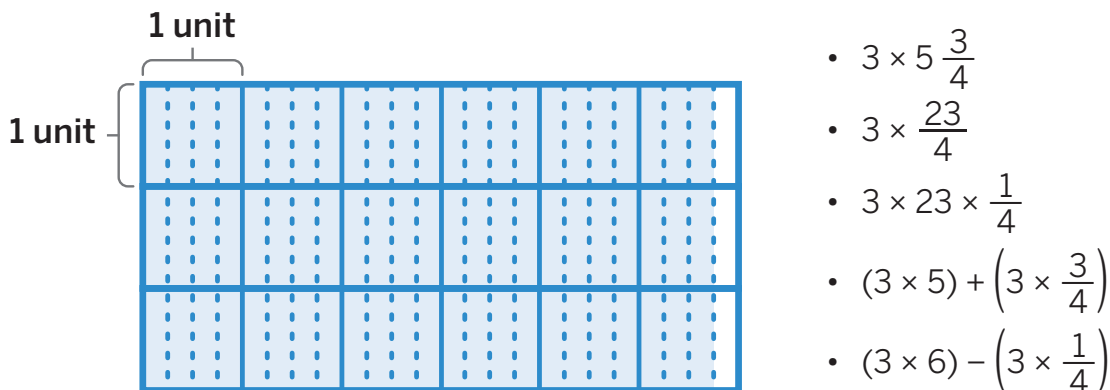_____

_____

In this sub-unit . . .

- We determined the area of rectangular regions with 1 whole-number side length and 1 fractional side length.

$\frac{1}{3}$ units

1 unit

$6 \times \frac{1}{3} = \frac{6}{3}$ or 2 square units

4 units

$\frac{3}{5}$ units

$4 \times \frac{3}{5} = \frac{12}{5}$ or $2\frac{2}{5}$ square units

- We saw that we can represent the area of rectangles with 1 whole-number side length and 1 fractional side length with different, equivalent expressions involving multiplication, addition, and subtraction.

1 unit

1 unit

- $3 \times 5\frac{3}{4}$
- $3 \times \frac{23}{4}$
- $3 \times 23 \times \frac{1}{4}$
- $(3 \times 5) + \left(3 \times \frac{3}{4}\right)$
- $(3 \times 6) - \left(3 \times \frac{1}{4}\right)$

🔥 *Math tip:* To multiply with a mixed number, you can convert the mixed number to a fraction greater than 1 and multiply, or you can use the Distributive Property to multiply with the whole number part and the fraction part separately and then add those products.
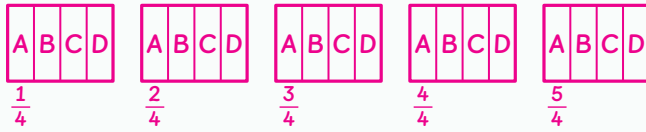
- We used estimation to reason about products of whole numbers and fractions.

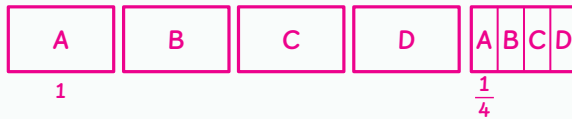$9 \times 4 = 36$. $4\frac{2}{3}$ is greater than 4, so $9 \times 4\frac{2}{3}$ is greater than 36 square feet.

Lesson 2

1   **Sample work:**

| A | B | C | D | | A | B | C | D | | A | B | C | D | | A | B | C | D | | A | B | C | D |
$\frac{1}{4}$ $\qquad$ $\frac{2}{4}$ $\qquad$ $\frac{3}{4}$ $\qquad$ $\frac{4}{4}$ $\qquad$ $\frac{5}{4}$

or

| A | | B | | C | | D | | A | B | C | D |
| 1 | | | | | | | | $\frac{1}{4}$ |

answer: $\frac{5}{4}$ or $1\frac{1}{4}$ pages

expression: **5 ÷ 4**

Lesson 3

1 $11 \div 5 = \frac{11}{5}$ or $2\frac{1}{5}$

2 $7 \div 6 = \frac{7}{6}$ or $1\frac{1}{6}$

Lesson 4

$7 \div 5 = \frac{7}{5}$ Grandpa made 5 loaves of banana bread and divided 7 cups of bananas equally among the loaves.

$8 \div 11 = \frac{8}{11}$ The animal shelter had 11 bird cages and 8 cups of bird feed to share equally among the bird cages.

$5 \div 7 = \frac{5}{7}$ 7 cats shared 5 cups of food equally.

$11 \div 8 = \frac{11}{8}$ 8 students shared some water equally. Each student received more than 1 cup of water.

Lesson 5

1   **14 muffins**
2   **4 bowls of soup**

Lesson 6

1 multiplication expression: $8 \times \dfrac{1}{7} = \dfrac{8}{7}$

division expression: $8 \div 7 = \dfrac{8}{7}$

2 multiplication expression: $\dfrac{1}{5} \times 9 = \dfrac{9}{5}$

division expression: $9 \div 5 = \dfrac{9}{5}$

Lesson 7

1 Sample response: $2 \div 5$: There are 2 wholes, each divided into 5 equal parts, and 1 part is shaded in each whole. $\dfrac{1}{5} \times 2$: There are 2 wholes. $\dfrac{1}{5}$ of each whole is shaded.
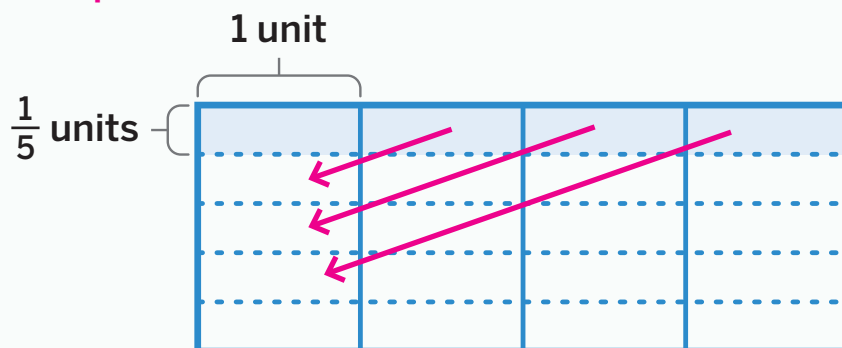
Lesson 8

1 $4 \times \dfrac{3}{4} = \dfrac{12}{4}$ or 3

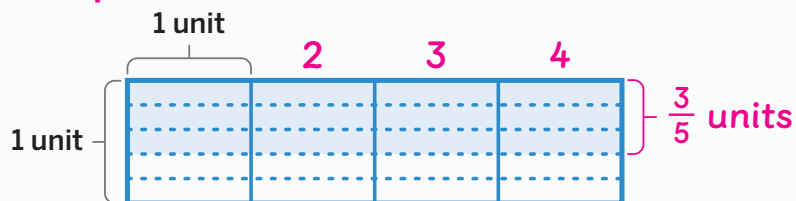**2** $5 \times \dfrac{7}{5} = \dfrac{35}{5}$ or 7

Lesson 9

1 Sample work:

answer: $\dfrac{4}{5}$ square units or equivalent

Lesson 10

1 **Sample work:**
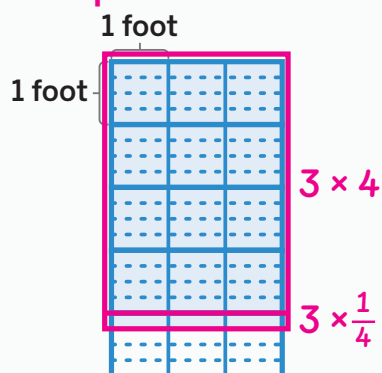
1 unit

1 unit | 1 unit | 2 | 3 | 4 | $\frac{3}{5}$ units

expression: Sample response: $4 \times \frac{3}{5}$

answer: $\frac{12}{5}$ or $2\frac{2}{5}$ square units

Lesson 11

1 **Sample work:**

1 foot

1 foot

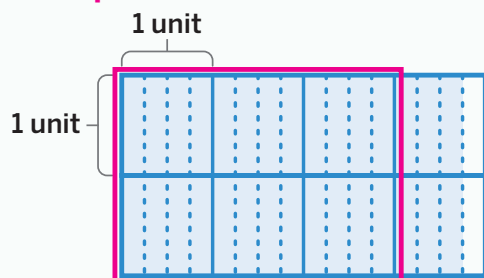3×4

$3 \times \frac{1}{4}$

$(3 \times 4) + \left(3 \times \frac{1}{4}\right)$

$12 + \frac{3}{4}$

answer: $12\frac{3}{4}$ square feet or equivalent

Lesson 12

1 **Sample work:**

1 unit

1 unit

$2 \times 3 = 6$

$2 \times \frac{3}{4} = \frac{6}{4}$

$\frac{6}{4} = 1\frac{1}{2}$

$6 + 1\frac{1}{2} = 7\frac{1}{2}$

answer: $7\frac{1}{2}$ square units or equivalent

Try This | Answer Key

Lesson 13

1 A, D, E

Lesson 14

1 A, C, E, F

2

	True	False
$3 \times 4\frac{4}{5} = (3 \times 4) + (3 \times \frac{4}{5})$	✔	
$6 \times 2\frac{3}{4} = (6 \times 2) - (6 \times \frac{1}{4})$		✔
$4\frac{1}{2} \times 3 = 4 \times \frac{1}{2} \times 3$		✔
$4 \times 3\frac{5}{8} = 4 \times \frac{29}{8}$	✔	

Lesson 15

1 Sample explanation: I agree. $10 \times 5 = 50$, and $9\frac{11}{12}$ is close to but less than 10. So, $9\frac{11}{12} \times 5$ is a little less than 50.