Amplify Desmos Math CALIFORNIA

Grade 8

Volume 1: Units 1–4

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Dear Student,

Welcome to Amplify Desmos Math California! We are excited to be partnering with you this year. You play an essential role in math class, so we wanted to reach out to introduce ourselves and tell you a bit about who we are.

Amplify Desmos Math California is a team of math educators on a mission to support you and your classmates in learning math. We hope each lesson inspires you to use your creativity, ask questions, and discover connections between math concepts and the world around us.

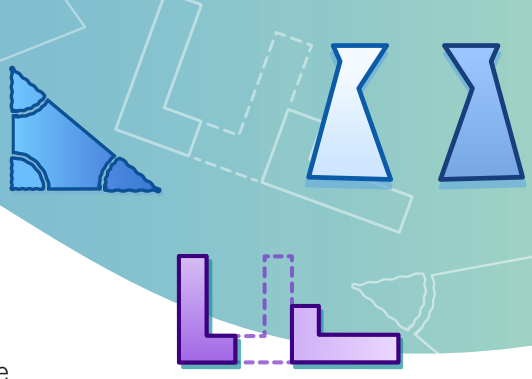
Here is what you can expect this year:

- A blend of learning on both paper and devices.
- Interactive lessons that encourage you to ask questions, explore, settle disputes, create challenges for your classmates, and more!
- Digital activities that show what your ideas mean with plenty of chances for you to revise.
- Opportunities for you to engage with interesting and important big ideas in mathematics.

–The Amplify Desmos Math California Team



Unit 1 Rigid Transformations and Congruence



In this unit, you will investigate translations, reflections, and rotations, and use these transformations to make arguments about congruence. You will also explore angle relationships on parallel lines and the triangle sum theorem.

Sub-Unit 1 Rigid Transformations 4

Explore: Tessellations | How can patterns be created using shapes? 5

1.01 Transformers | Describing Movement 9

1.02 Spinning, Flipping, Sliding | Naming Transformations 16

1.03 Transformation Targets | Sequences of Transformations 23

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1.05 Getting Coordinated, Part 1 | Coordinates of Translations and Reflections 37

1.06 Getting Coordinated, Part 2 | Coordinates of Transformations 44

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1.08 Are They the Same? | Defining Congruence 61

1.09 Are They Congruent? | Rigid Transformations and Congruent Figures 69

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1.10 Transforming Angles | Angle Measures in Parallel Lines 77

1.11 Tearing It Up | Angle Sums in Triangles 85

1.12 Puzzling It Out | Determining Unknown Angle Measures 91

1.13 Tessellate | Using Transformations to Create Art 99

Practice Day 2 107



Unit 2 Dilations, Similarity, and Slope

In this unit, you will study dilations and similar figures. You will use similar triangles to explain, understand, and apply the concept of slope.

Sub-Unit 1 Dilations 114

Explore: Standard Paper Sizes | Are all rectangles scaled copies of each other? 115

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2.02 Dilation Mini Golf | Dilations with No Grid 125

2.03 Transformation Targets With Dilations | Sequences of Transformations 132

2.04 Match My Dilation | Dilations on a Square Grid 139

2.05 Dilations on a Plane | Dilations With Coordinates 146

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2.07 Are Angles Enough? | Similar Triangles 166

2.08 Shadows | Side Length Relationships in Similar Triangles 173

Sub-Unit 3 Slope 180

2.09 Water Slide | Similarity and Slope 181

2.10 Slope Challenges | Developing Fluency with Slope 189

Practice Day 2 196



Unit 3 Proportional and Linear Relationships

In this unit, you will make connections between proportional relationships and non-proportional linear relationships. You will compare representations of linear relationships and interpret them in context. You will also determine solutions to two-variable linear equations of different forms.

Sub-Unit 1 Proportionality Revisited 204

Explore: Visual Patterns | What patterns do you notice in shapes and numbers? 205

3.01 Turtle Time Trials | Understanding Proportional Relationships 209

3.02 Water Tank | Graphs of Proportional Relationships 216

3.03 Proportional Posters | Comparing Proportional Relationships 222

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3.05 Water Cooler | Slopes Don't Have to Be Positive 237

3.06 Ups and Downs | Analyzing Linear Relationships 245

3.07 Stacking Cups | More With Linear Relationships 252

3.08 Translations | Translating $y = mx + b$ 260

Practice Day 1 267

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3.10 Coin Capture | Horizontal and Vertical Lines 278

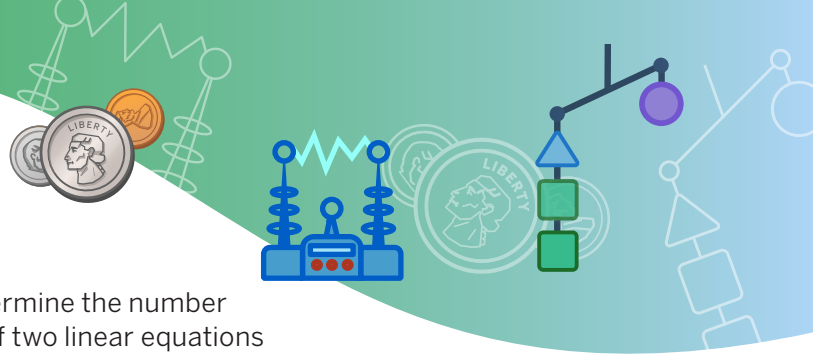
3.11 Why Intercepts? | Equations of All Kinds of Lines 284

3.12 Solutions | Solutions to Linear Equations 292

3.13 Pennies and Quarters | Using Linear Relationships to Solve Problems 299

Practice Day 2 306

Unit 4 Linear Equations and Linear Systems



In this unit, you will solve linear equations and determine the number of possible solutions. You will also solve systems of two linear equations algebraically and graphically.

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Explore: Equation Puzzles | How is an equation like a puzzle? 315

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4.09 On or Off the Line? | Interpreting Points On or Off the Line 383

4.10 On Both Lines | Representing Systems of Linear Equations 390

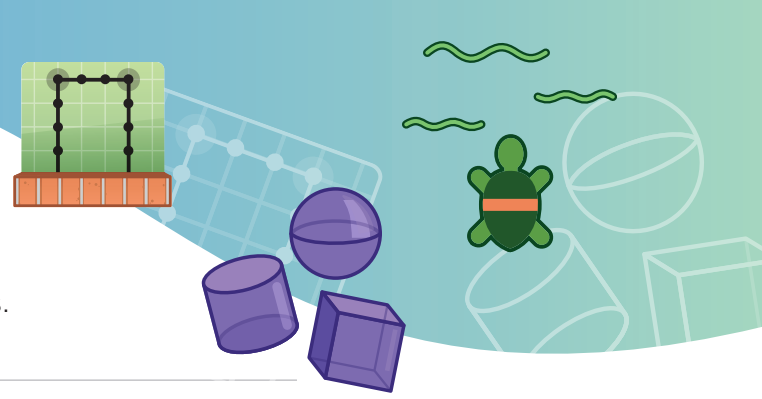
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4.14 Strategic Solving, Part 2 | Solving More Systems of Equations 419

Practice Day 2 426



Unit 5 Functions and Volume

In this unit, you will learn about functions, analyze representations of functions, and examine functions in the context of the volume of cylinders, cones, and spheres.

Sub-Unit 1 Introduction to Functions 434

Explore: Graphs of Sounds | What could a graph of sound look like? 435

5.01 Turtle Crossing | Making Sense of Graphs 439

5.02 Guess My Rule | Introduction to Functions 446

5.03 Function or Not? | Graphs of Functions and Non-Functions 453

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5.09 Piecing It Together | Modeling With Functions 497

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5.13 Cones | Volumes of Cones 531

5.14 Unknown Dimensions | Determining Cylinder and Cone Dimensions 538

5.15 Spheres | Volumes of Spheres 545

Practice Day 2 553



Unit 6 Associations in Data

In this unit, you will analyze data with two variables. You will use scatter plots and fitted lines to analyze numerical data and use two-way tables, bar graphs, and segmented bar graphs to analyze categorical data.

Sub-Unit 1 Organizing Numerical Data 560

Explore: Changes in the Ozone Layer | How is the ozone layer changing? 561

6.01 Click Battle | Organizing Data 565

6.02 Wingspan | Plotting Data 571

6.03 Robots | What a Point on a Scatter Plot Means 577

Sub-Unit 2 Analyzing Numerical Data 584

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6.06 Find the Fit | Fitting a Line to Data 598

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6.09 Animal Brains | Analyzing Bivariate Data 620

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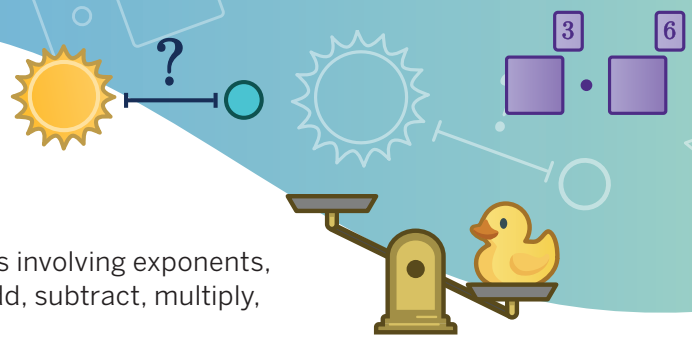
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6.10 Tasty Fruit | Two-Way Tables and Bar Graphs 633

6.11 Finding Associations | Using Data Displays to Find Associations 639

Practice Day 2 646

Unit 7 Exponents and Scientific Notation



In this unit, you will develop fluency with expressions involving exponents, powers of 10, and scientific notation. You will also add, subtract, multiply, and divide numbers written in scientific notation.

Sub-Unit 1 Exponent Properties 652

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7.06 Write a Rule | Generalizing Exponent Properties 692

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7.08 Scales and Weights, Part 2 | Describing Small Numbers Using Powers of 10 708

7.09 Specific and Scientific | Definition of Scientific Notation 717

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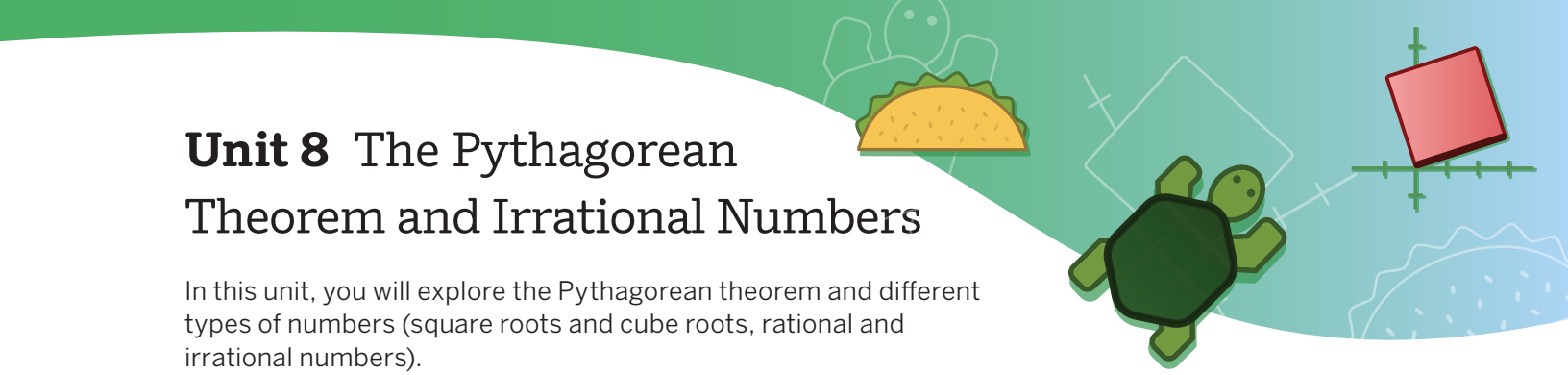
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7.12 Use Your Powers | Applications of Arithmetic With Scientific Notation 737

7.13 City Lights | Adding and Subtracting With Scientific Notation 743

7.14 Star Power | Let's Put It to Work 751

Practice Day 2 757



Unit 8 The Pythagorean Theorem and Irrational Numbers

In this unit, you will explore the Pythagorean theorem and different types of numbers (square roots and cube roots, rational and irrational numbers).

Sub-Unit 1 Square Roots and Cube Roots 764

Explore: The Longest Cut | What's the longest length in a rectangle or rectangular prism? 765

8.01 Tilted Squares | The Areas of Tilted Squares 769

8.02 From Squares to Roots | Side Lengths and Areas 776

8.03 Between Squares | Approximating Square Roots 783

8.04 Root Down | Reasoning About Square Roots 790

8.05 Filling Cubes | Edge Lengths, Volumes, and Cube Roots 797

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8.07 Pictures to Prove It | A Proof of the Pythagorean Theorem 817

8.08 Triangle-Tracing Turtle | Finding Unknown Side Lengths 824

8.09 Make It Right | The Converse of the Pythagorean Theorem 832

8.10 Taco Truck | Applications of the Pythagorean Theorem 840

8.11 Pond Hopper | Finding Distances on the Coordinate Plane 847

Sub-Unit 3 Rational and Irrational Numbers 854

8.12 Fractions to Decimals | Decimal Representations of Rational Numbers 855

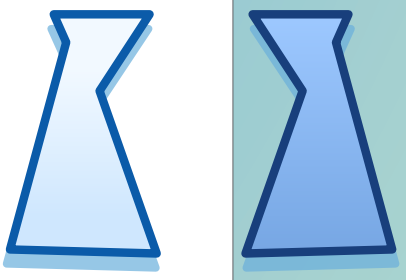
8.13 Decimals to Fractions | Infinite Decimal Expansions 862

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Practice Day 2 876

Unit 1

Rigid Transformations and Congruence

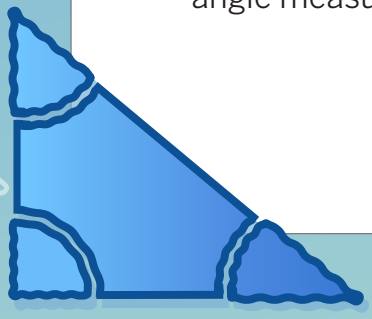


Big Ideas in This Unit

CC4 Transformational Geometry

Questions for Investigation

- What are different ways to transform a figure?
- How can we use rigid transformations to decide whether two figures are congruent?
- How can transformations help make sense of congruent angle measures?



Explore: Tessellations
How can patterns be created using shapes?

Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!

Not yet — Almost — I got it!

I can . . .	Before	After
Describe a translation.		
Describe a reflection.		
Describe a rotation.		
Determine whether a translation, reflection, or rotation was applied on a figure.		
I describe a sequence of transformations that maps one figure onto another.		
Explain how a reflection, rotation, and translation impacts the coordinates of a polygon.		
Explain how a rigid transformation affects the side lengths and angle measures of a figure.		
Justify whether two figures are congruent using a sequence of rigid transformations.		
Justify which angles on two parallel lines cut by a transversal are congruent.		
Use rigid transformations to explain why vertical angles are congruent.		

I can . . .	Before	After
Use rigid transformations to explain why corresponding angles on parallel lines are congruent.		
Explain the relationship between the sum of a triangle's interior angle measures and the angle of a straight line.		
Use transformations and angle relationships to determine unknown angle measurements.		

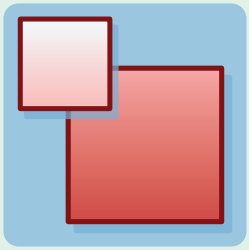
Sub-Unit

1

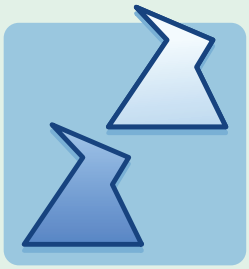
Rigid Transformations



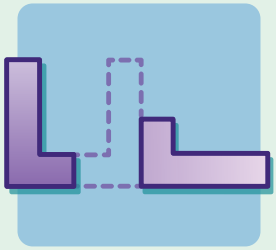
Explore
Tessellations



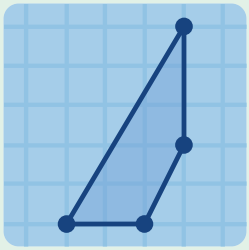
Lesson 1
Transformers



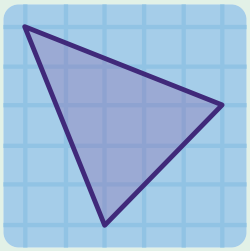
Lesson 2
Spinning, Flipping, Sliding



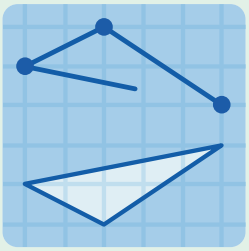
Lesson 3
Transformation Targets



Lesson 4
Moving Day



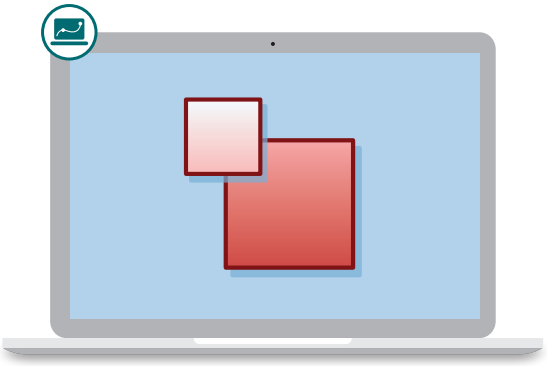
Lesson 5
Getting Coordinated, Part 1



Lesson 6
Getting Coordinated, Part 2

Name: ..... Date: ..... Period: .....

Transformational Geometry | Building Toward 8.G.1, SMP.6, SMP.7



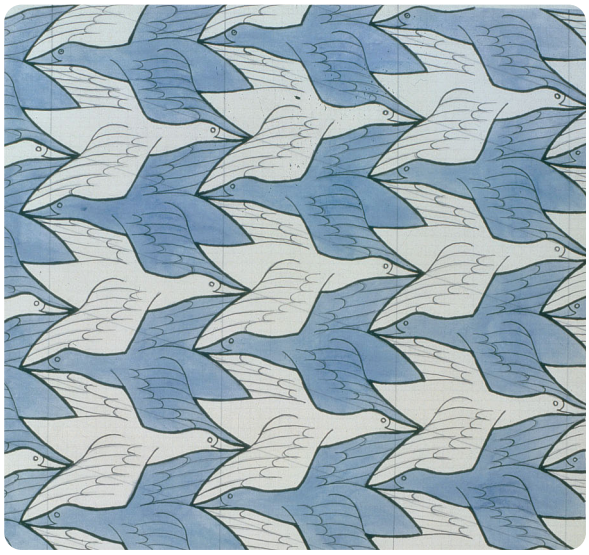
Explore:
Tessellations

How can patterns be created using shapes?

Warm-Up

The artwork shown was created by the Dutch artist Maurits C. (M.C.) Escher (1898–1972).

Discuss: What do you notice? What do you wonder?



M.C. Escher's "Two Birds" © 2020 The M.C. Escher Company-The Netherlands. All rights reserved. ww.mcescher.com

I notice . . .

I wonder . . .

Name: Date: Period:

Triangle Tessellations

1. You will use blank paper, a ruler, and scissors. Draw a triangle and cut it out. Create a tessellation using your triangle. Draw a sketch of your tessellation here.

2. Exchange triangles with your partner and create a tessellation using your partner's triangle. Draw a sketch of your tessellation here.

Name: Date: Period:

Triangle Tessellations (continued)

3. Compare each of your sketches with your partner's. Consider how the sketches that used the same cut-out triangles look different from one another.

4. **Discuss**

- Were the cut-out triangles positioned differently in your tessellations than your partner's tessellations?
- How did the position of the triangles affect the patterns in the tessellations?

5. Can you think of a triangle that does not work for making tessellations?

Name: ..... Date: ..... Period: .....

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet Almost I got it!

I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem.

Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed.

Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

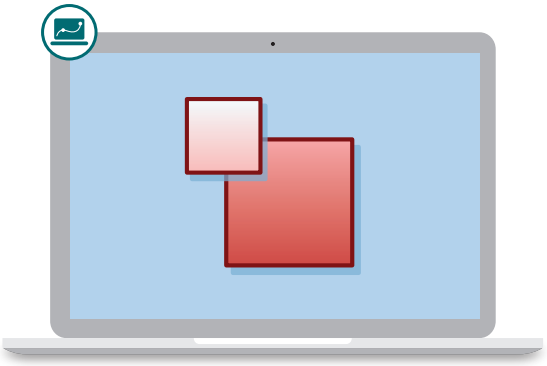
Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

Not yet Almost I got it!

Name: ..... Date: ..... Period: .....

Transformational Geometry | Building Toward 8.G.1, SMP.7



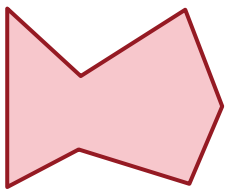
Transformers

Let's describe how figures are transformed.

Warm-Up

1 Titus created this figure by dragging some points. Create your own figure and sketch it below.

Titus's Figure



Our Figure

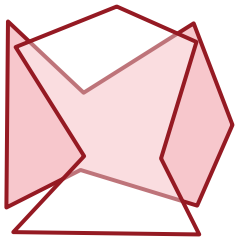
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Transformers

2 Let's watch an animation of Transformer #1. What happened to the figure?

Transformer #1

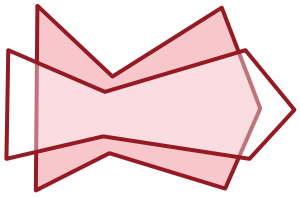
(Titus's figure)



3 Let's watch an animation of Transformer #2. What happened to the figure?

Transformer #2

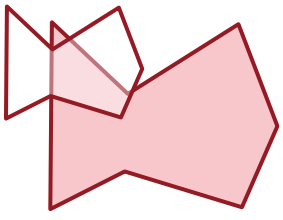
(Titus's figure)



4 Let's watch an animation of Transformer #3. What happened to the figure?

Transformer #3

(Titus's figure)

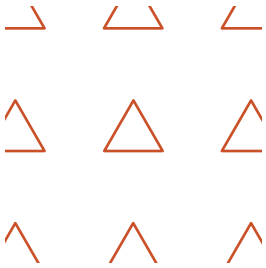
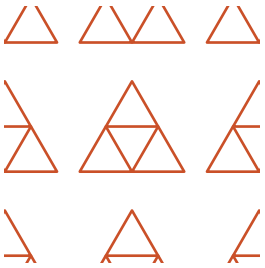


Activity 2

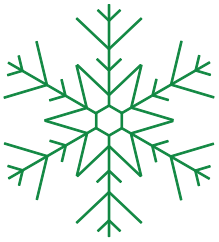
Name: ..... Date: ..... Period: .....

Kaleidoscope Playground

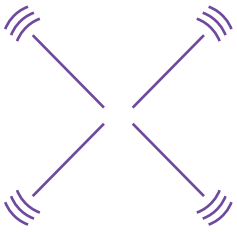
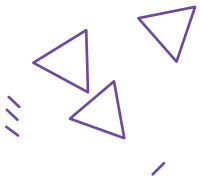
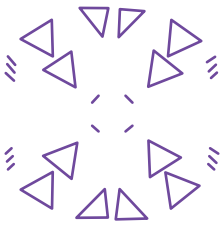
5 Welcome to the kaleidoscope playground. Let's try out some kaleidoscopes, look at examples, and see if you can figure out how each one works.

Kaleidoscope #1			
Sketch A	Kaleidoscope A	Sketch B	Kaleidoscope B
			

a How does Kaleidoscope #1 work?

Kaleidoscope #2			
Sketch C	Kaleidoscope C	Sketch D	Kaleidoscope D
			

b How does Kaleidoscope #2 work?

Kaleidoscope #3			
Sketch E	Kaleidoscope E	Sketch F	Kaleidoscope F
			

c How does Kaleidoscope #3 work?

Name: ..... Date: ..... Period: .....

Kaleidoscope Playground (continued)

Kaleidoscope #4			
Sketch G	Kaleidoscope G	Sketch H	Kaleidoscope H
			

d How does Kaleidoscope #4 work?

e Make your own sketch. Draw what it would look like in at *least one* of the kaleidoscopes.

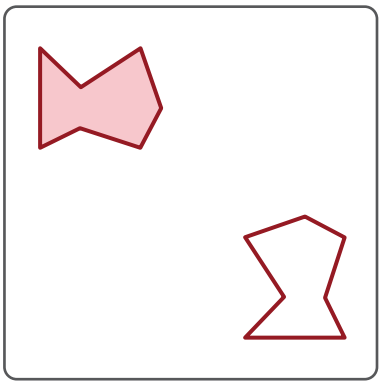
Your Sketch

Kaleidoscope #1	Kaleidoscope #2	Kaleidoscope #3	Kaleidoscope #4

6 Synthesis

Titus created a figure and used a transformer. Here is the figure before (shaded) and after (unshaded).

Discuss: What happened to the figure?



9 Summary 1.01

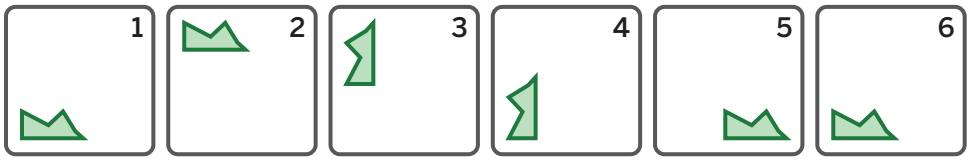
You can describe how a figure moves to get from one position to another in different ways.

- A figure can slide or move up, down, left, or right.
- A figure can spin around the middle or around another point.
- A figure can flip vertically, horizontally, or diagonally.

Practice
1.01

Name: ..... Date: ..... Period: .....

1. These six frames show a figure's different positions.

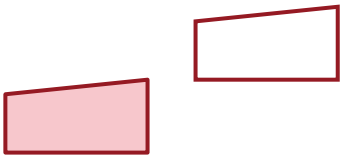


Describe how the figure moves to get from its position in each frame to the next.

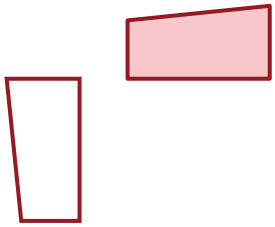
From	To	Description of Moves
Frame 1	Frame 2	
Frame 2	Frame 3	
Frame 3	Frame 4	
Frame 4	Frame 5	
Frame 5	Frame 6	

Problems 2–4: Sai saw a transformer and described the figure before (shaded) and after (unshaded). Rewrite each description to make it stronger and clearer.

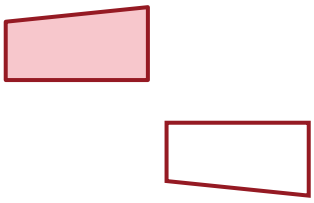
2. The figure moved.



3. The figure spun.



4. The figure flipped.



Name: ..... Date: ..... Period: .....

Problems 5–6: Here is a figure before going through a transformer.

5. Draw what the figure might look like after the transformer based on this description: *The figure got smaller and moved to the left*.



6. Can there be different drawings that would match the description from Problem 5? Explain your thinking.

Spiral Review

Problems 7–9: Determine whether each value of n makes the inequality $2n - 5 < 13$ true. Write *yes* or *no*.

7. $n = 8$

8. $n = 9$

9. $n = 10$

Problems 10–11: Solve each equation. Show your thinking.

10. $-6(x + 2) = 36$

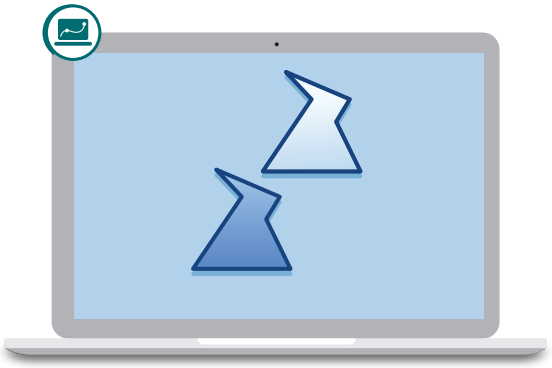
11. $\frac{1}{3}(x - 3) = 10$

Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.1, SMP.3, SMP.6

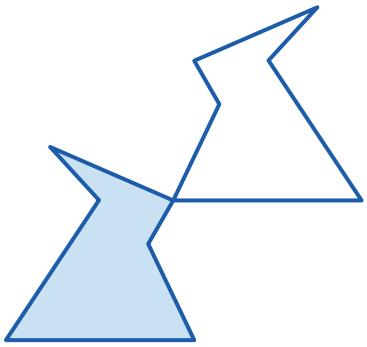
Spinning, Flipping, Sliding

Let's learn some ways to describe how figures move.



Warm-Up

1 Annika created this shaded figure in the previous lesson.



Let's watch a video of a **transformation**.

What happened to the figure?

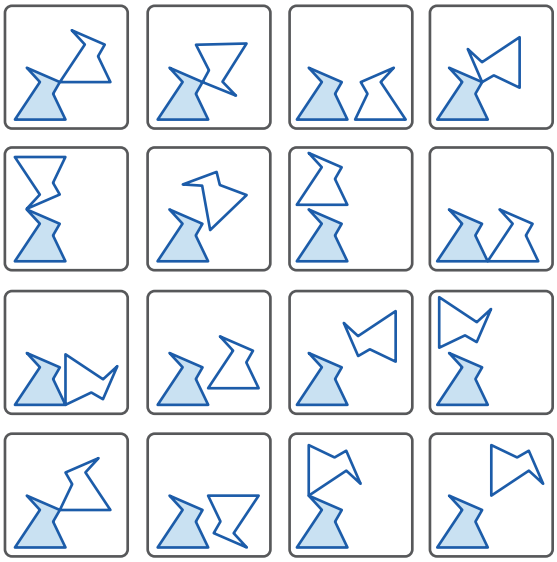
Name: ..... Date: ..... Period: .....

Describing Transformations

2 Play a few rounds of Polygraph with your classmates!

You will use an Activity 1 Sheet with different *transformation* images for four rounds. For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select an image from the Activity 1 Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating images until you're ready to guess which image the Picker chose.



Record helpful questions from each round in this workspace:

Name: ..... Date: ..... Period: .....

Describing Transformations (continued)

In Polygraph, you saw three types of transformations: **rotations**, **reflections**, and **translations**.

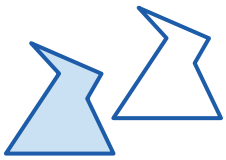
3 Match each word with one of these transformations.

Flip	Mirror	Slide	Spin	Turn

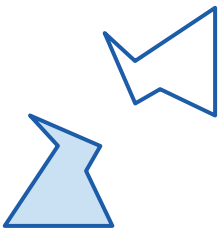
Rotation	Reflection	Translation

4 For each picture, determine whether a rotation, reflection, or translation best describes how one figure can map onto the other in a single transformation.

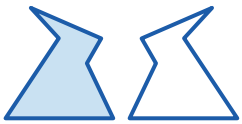
a



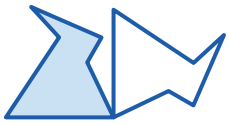
b



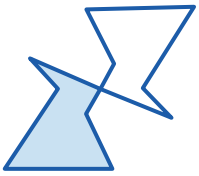
c



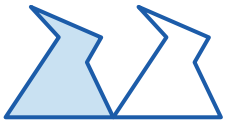
d



e



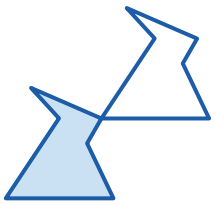
f



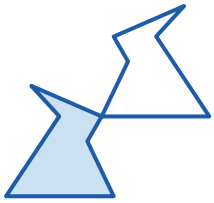
Name: ..... Date: ..... Period: .....

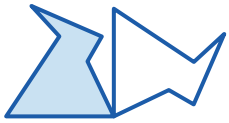
Rotations, Reflections, Translations

5 Circle an image. Then describe how to map the shaded figure onto the unshaded one. Use at least one of these words in your description: *reflection*, *rotation*, *translation*.

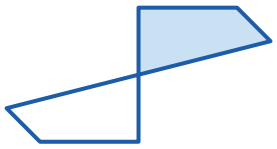
A. 

B. 

C. 

D. 

6 Matias says you can use one *reflection* to map the shaded figure onto the unshaded one.



Dyani says you can use *one rotation*.

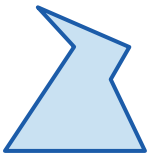
Whose claim is correct? Circle one.

Matias's Dyani's Both Neither

Explain your thinking.

7 Synthesis

In your own words, describe what each *transformation* does to a figure.



Rotation:

Reflection:

Translation:

10 Summary 1.02

Transformations are actions that you can perform to change a figure. They are applied to every point on the figure. Some examples include **rotations**, **reflections**, and **translations**.

reflection A transformation that moves every point on a figure across a line (known as the **line of reflection**) to a point on the opposite side of the line. The new point is the same distance from the line as it was in the original figure.

rotation A movement that moves every point on a figure around a center (known as the **center of rotation**) by a given angle in a specific direction.

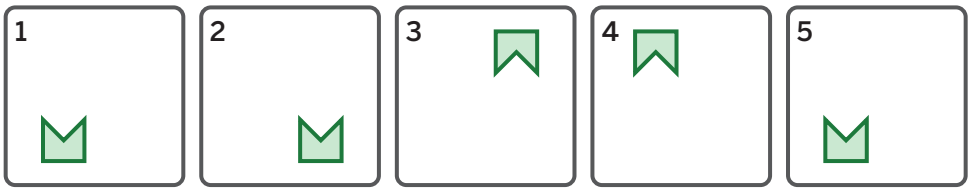
transformation An action or rule for moving or changing figures on a plane. Transformations include translations, reflections, rotations, and dilations.

translation A movement that moves every point in a figure a given distance in a given direction.

Practice
1.02

Name: ..... Date: ..... Period: .....

1. These five frames show a figure's different positions.

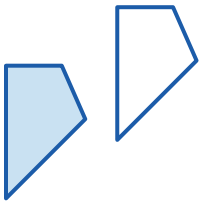


Describe how the figure moves to get from its position in each frame to the next.

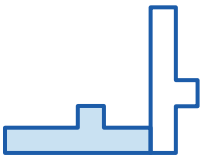
From	To	Description of Moves
Frame 1	Frame 2	
Frame 2	Frame 3	
Frame 3	Frame 4	
Frame 4	Frame 5	

Problems 2–4: Determine whether each transformation shows a translation, reflection, or rotation.

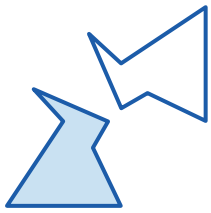
2.



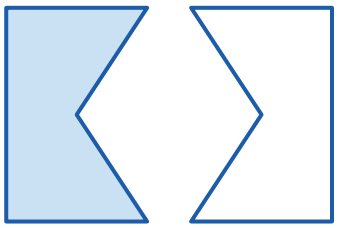
3.



4.



5. Which transformation could map the shaded figure to the unshaded one? Select *all* that apply.



- ☐ **A.** Reflection
- ☐ **B.** Rotation
- ☐ **C.** Translation
- ☐ **D.** None of these

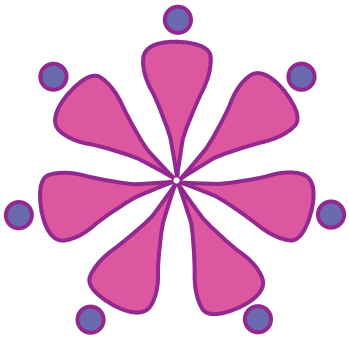
Practice
1.02

Name: ..... Date: ..... Period: .....

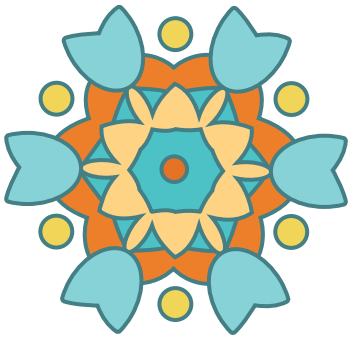
Problems 6–7: A kaleidoscope is a tube with mirrors and colored glass inside. When you point it toward the light and look through it as you turn it, you can see patterns appear.

Analyze the designs and describe how the pattern could be created using reflections, rotations, or translations.

6.



7.



Spiral Review

Problems 8–11: Evaluate each expression.

8. $-5 \cdot (-2.4)$

9. $-7.4 \div 10$

10. $-\frac{4}{7} \div (-2)$

11. $4 \cdot \left(-\frac{3}{8}\right)$

12. Select *all* of the expressions that have a negative value.

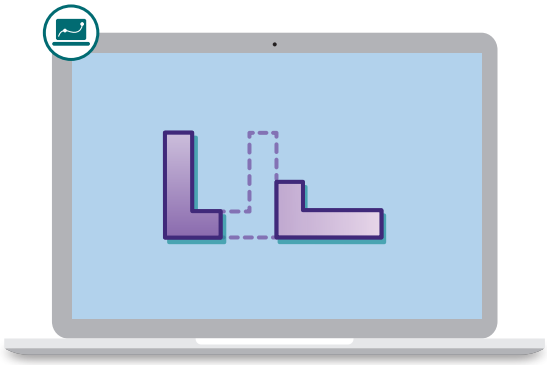
- ☐ A. $-\frac{11}{8}$
- ☐ B. $\frac{-11}{8}$
- ☐ C. $\frac{11}{-8}$
- ☐ D. $\frac{-11}{-8}$
- ☐ E. $\frac{11}{8}$

Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.1, SMP.1, SMP.7

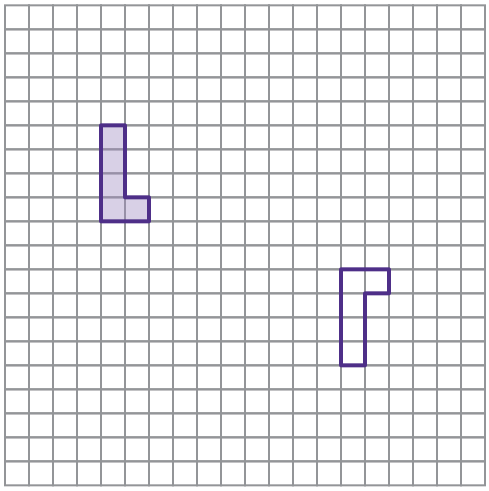
Transformation Targets

Let's explore sequences of transformations.



Warm-Up

1 We have previously seen three types of transformations: translation, rotation, and reflection.



a Let's watch the shaded figure map onto the unshaded figure.

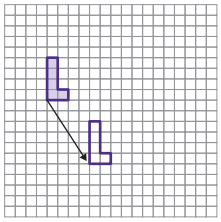
b Describe the **sequence of transformations** you saw.

Name: ..... Date: ..... Period: .....

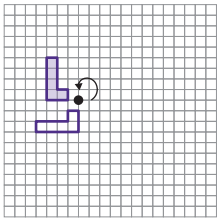
Transformation Targets

Take a look at examples of a translation, rotation, and reflection.

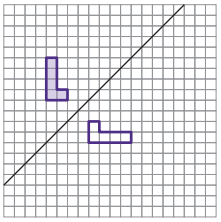
A Translation Down and to the Right



A Rotation 90° Counterclockwise



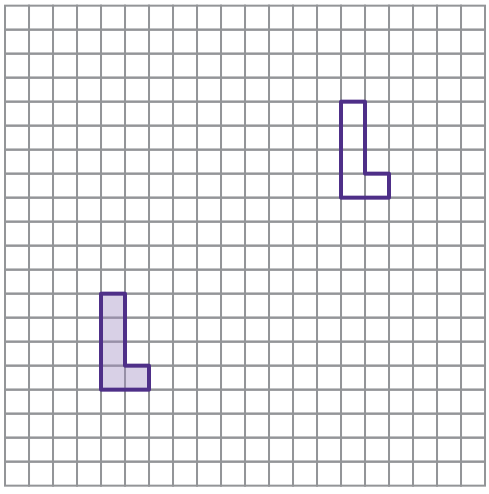
A Reflection Over the Line



You will solve a variety of challenges. For each challenge:

- Show or describe a *sequence of transformations* that maps the shaded figure onto the unshaded figure.
- If it's helpful, draw the new figure after *each* transformation.
- Try to use two or fewer transformations.

2 Challenge #1



3 Let's look at the different translations Ama and Basheera used on the previous challenge. Whose translation is correct? Circle one.

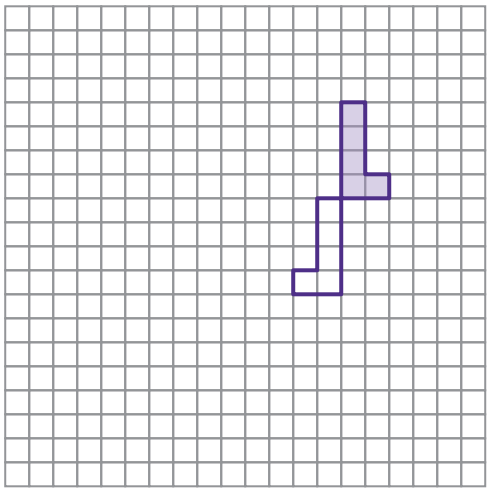
Ama's Basheera's Both Neither

Explain your reasoning.

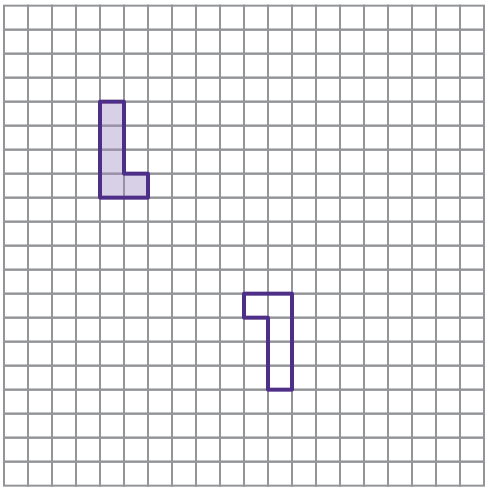
Name: ..... Date: ..... Period: .....

Sequences of Transformations

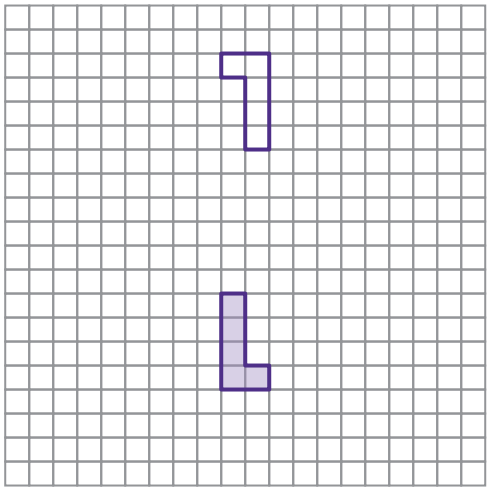
4 Challenge #2



5 Challenge #3



6 Describe a sequence of transformations to complete this challenge.

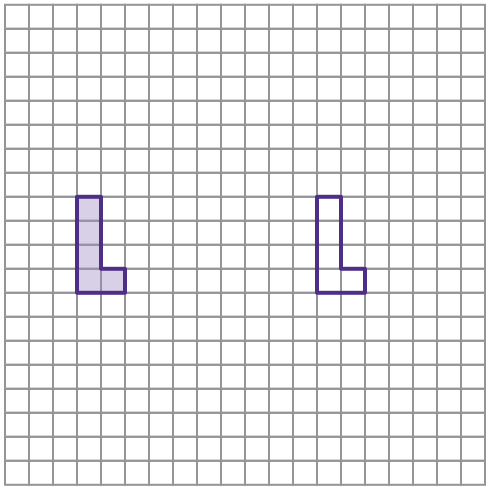


Name: ..... Date: ..... Period: .....

More Transformation Targets

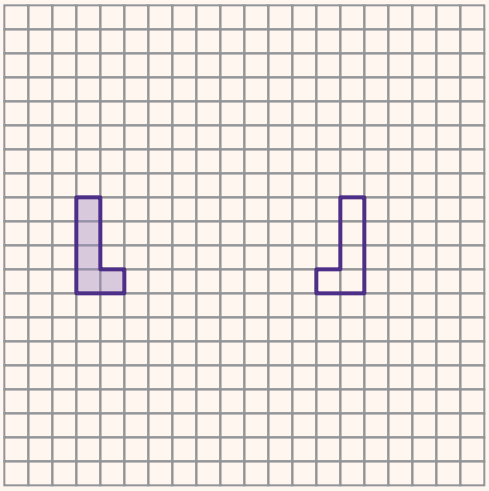
7 Let's try a sequence of transformations for the previous challenge.

8 Challenge #4: Try to solve this challenge using only rotations or only reflections.



You're invited to explore more.

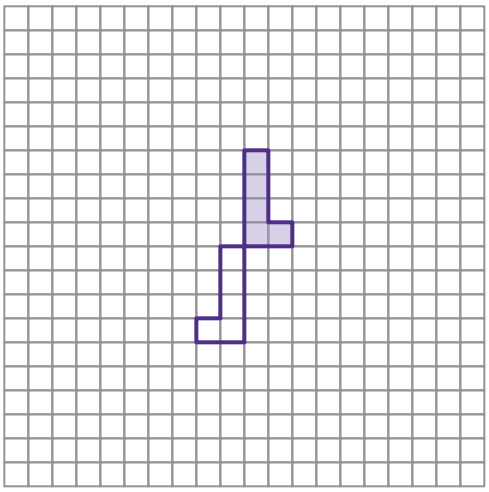
9 **Discuss:** Is it possible to solve this challenge using only translations and rotations? Explain your thinking.



10 Synthesis

Discuss: What are some strategies to determine a sequence of transformations to map a shaded figure onto an unshaded figure?

Use the example if it helps with your thinking.



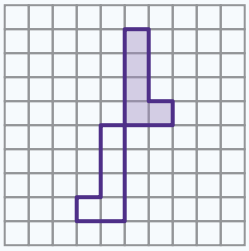
13 Summary 1.03

Here are important details to consider when choosing your **sequence of transformations**:

- For translations, every point on the figure moves the same distance in the same direction.
- For reflections, the new point will be the same distance from the line of reflection as it was in the original figure.
- Rotations are performed around a point by a given angle and in a specific direction. The direction of a rotation can be **clockwise** or **counterclockwise**.
- A reflection changes a figure's orientation, but a rotation and translation do not.

You can use different sequences of transformations on a figure and get the same result.

For example, here are two sequences of transformations that both take the shaded figure to the unshaded figure:



- Sequence #1: Reflect over the longest edge and then translate the figure 4 units down.
- Sequence #2: Rotate 180° clockwise around the lower left corner of the shaded figure, then reflect over the horizontal line that divides the height into two halves.

clockwise In the same direction as the hands of a clock; to the right (of a turn).

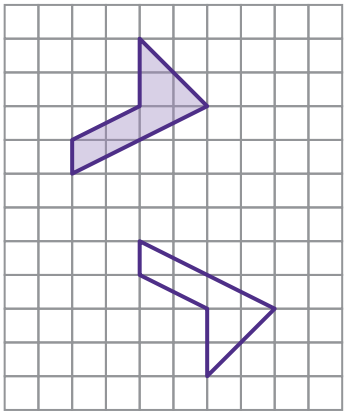
counterclockwise In the opposite direction as the hands of a clock; to the left (of a turn).

sequence of transformations A set of translations, rotations, reflections, and dilations on a figure. The transformations are performed in a given order.

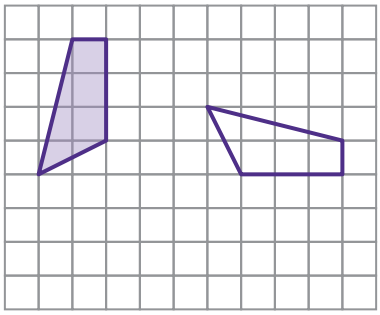
Name: ..... Date: ..... Period: .....

Problems 1–4: Describe a sequence of transformations that maps the shaded figure onto the unshaded figure.

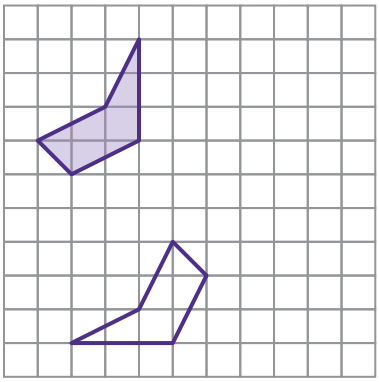
1.



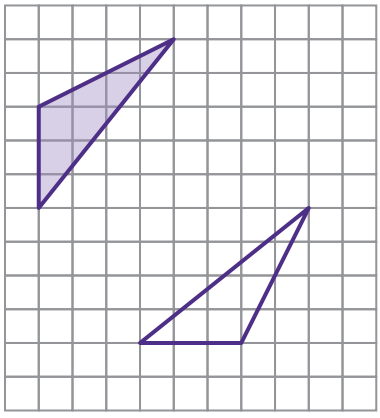
2.



3.



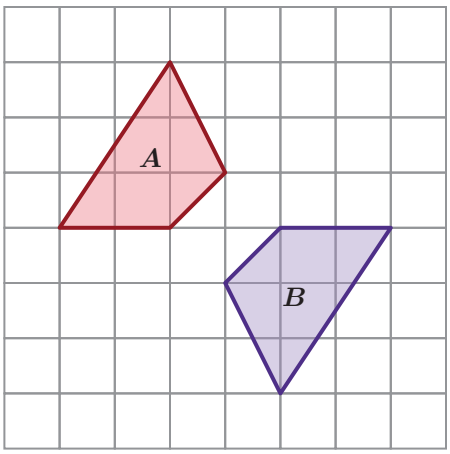
4.



Practice
1.03

Name: ..... Date: ..... Period: .....

Problems 5–6: Here are figures A and B.



5. Jacy says that figure A can map onto figure B using only reflections.

Is Jacy correct? Explain your thinking.

6. Faaria says that she can map figure A onto figure B using only translations.

Is Faaria correct? Explain your thinking.

7. Select *all* the sequences of transformations that could return a figure to its original position.

- ☐ **A.** Reflect the figure over a line and then reflect back over the same line.
- ☐ **B.** Translate the figure 1 unit to the right, then 4 units to the left, and then 3 units to the right.
- ☐ **C.** Reflect the figure over one line and then reflect over a different line.
- ☐ **D.** Rotate the figure 90° counterclockwise around a point and then 270° counterclockwise around the same point.
- ☐ **E.** Rotate the figure 90° counterclockwise around a point and then 270° clockwise around the same point.

Spiral Review

Problems 8–10: Write an operation in the box to make each equation true.

8. $12 \square (-8) = 20$

9. $-17 \square 9 = -26$

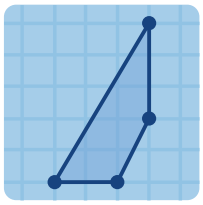
10. $24 \square (-29) = -5$

Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.1, SMP.5, SMP.6, SMP.7

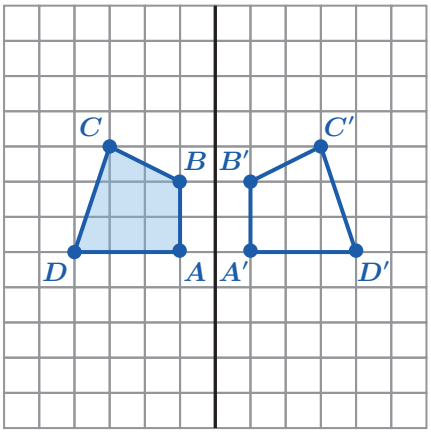
Moving Day

Let's describe transformations.



Warm-Up

1. Here is a transformation. The **pre-image** is shaded and the **image** is unshaded.



What do you notice? What do you wonder?

I notice:

I wonder:

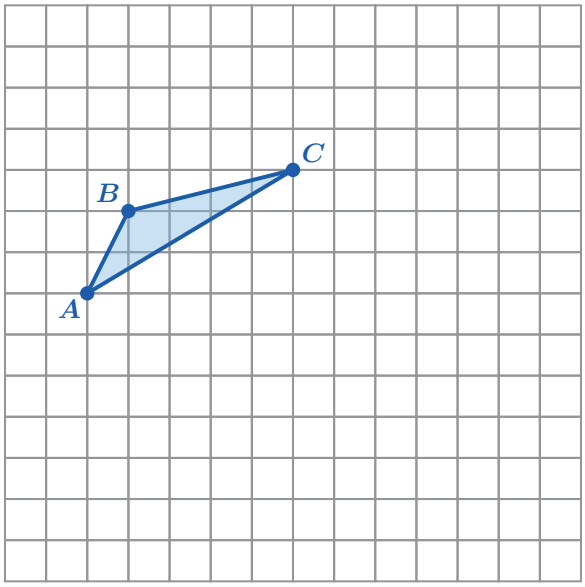
Activity 1

Name: ..... Date: ..... Period: .....

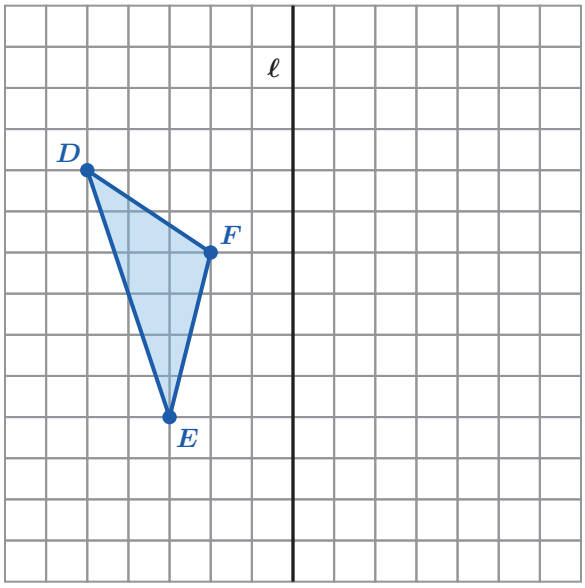
Move It

Perform each transformation. Then label the points in the image using prime notation (′) to correspond with the points in the pre-image.

2. Translate triangle ABC 3 units right and 1 unit down.



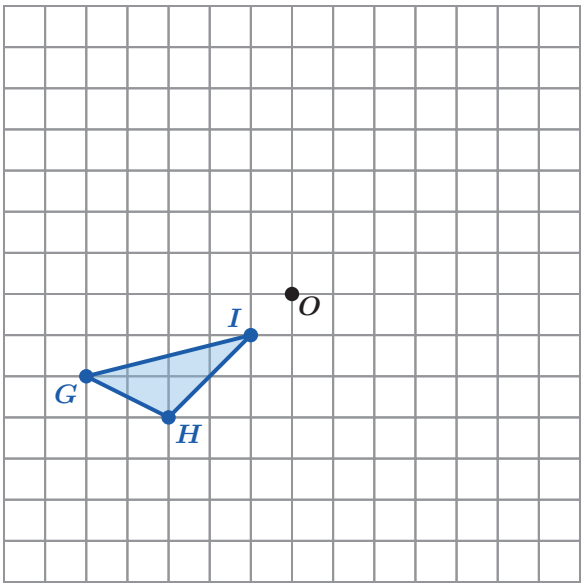
3. Reflect triangle DEF over line ℓ.



Name: ..... Date: ..... Period: .....

Move It (continued)

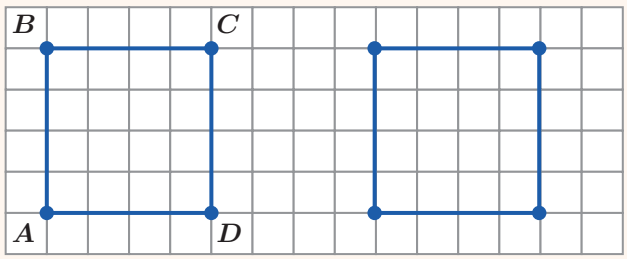
4. Rotate triangle GHI 180° counterclockwise around point O.



You're invited to explore more.

5. Square $ABCD$ is drawn on a grid and a transformation has been applied. Kiran, Clare, and Noah each see different transformations.
For each student:

- Describe a transformation that would map the pre-image to the image.
- Label the image using prime notation (′).



1. Kiran: "I see a translation."
2. Clare: "I see a reflection."
3. Noah: "I see a rotation."

Name: ..... Date: ..... Period: .....

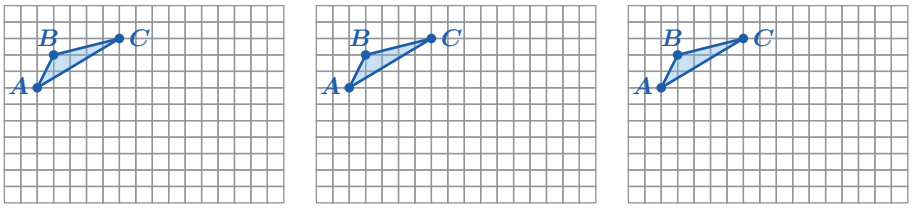
Transformation Information

6. You and your partner will get a set of transformation cards. Place them grid-side down without looking at them.

- Decide who will describe the transformation on a card and who will sketch the image. Start with Card 1.
- **Describer:** Give enough information about the transformation so that the Sketcher can sketch it.
- **Sketcher:** Pause after sketching and share what you think the transformation is.
- **Together:** Compare the card with the sketch and make adjustments as needed. Write a precise description of the transformation.
- Switch roles for Card 2 and repeat. Then do the same for Cards 3 and 4.

a **Sketch 1:** Card 1 or Card 2 (Circle one.)

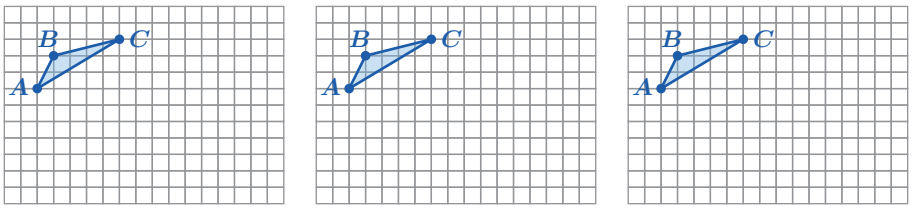
Use as many grids as you need to revise your work.



Description of Transformation:

b **Sketch 2:** Card 3 or Card 4 (Circle one.)

Use as many grids as you need to revise your work.



Description of Transformation:

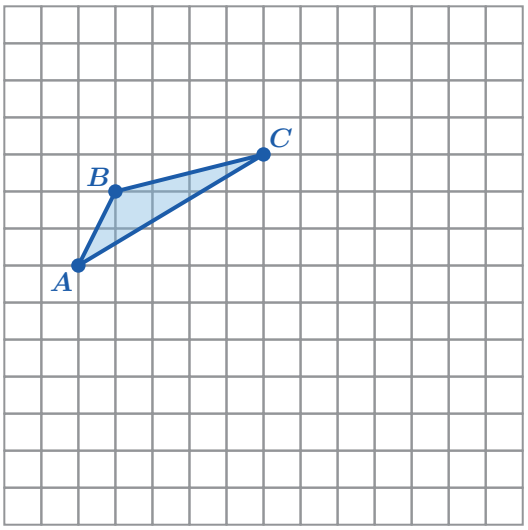
Synthesis

7. What details are helpful to include when precisely describing a transformation?

Translation:

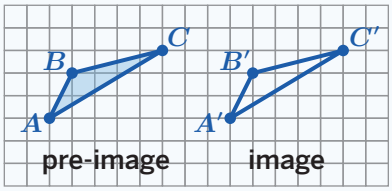
Reflection:

Rotation:



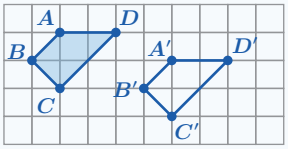
Summary 1.04

All of the points in the **image** correspond to the points in the **pre-image**.
For example, point A' corresponds to point A.



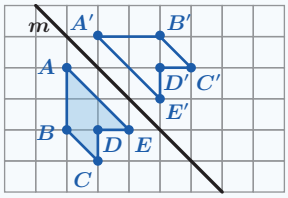
Here are important details to help you describe transformations:

Translations



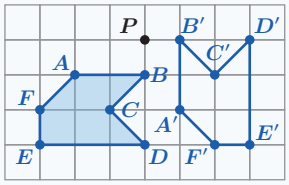
Describe the direction (up or down, left or right) and the number of units.
E.g., Figure $ABCD$ is translated 4 units right and 1 unit down.

Reflections



Describe the line of reflection.
E.g., Figure $ABCDE$ is reflected over line m.

Rotations



Describe the center of rotation, angle of rotation, and direction (clockwise or counterclockwise).
E.g., Figure $ABCDEF$ is rotated $90°$ counterclockwise around point P.

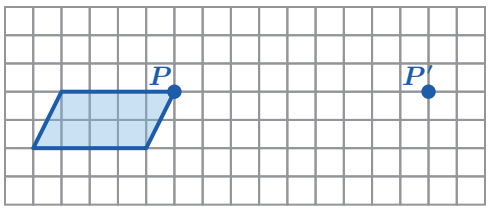
image A new figure that is created after a transformation of an original figure (called the pre-image). Every part of the pre-image moves in the same way to match up with a part of the image.

pre-image The name of a figure before any transformations are performed.

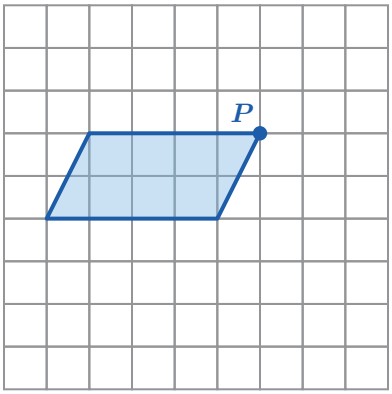
Practice
1.04

Name: ..... Date: ..... Period: .....

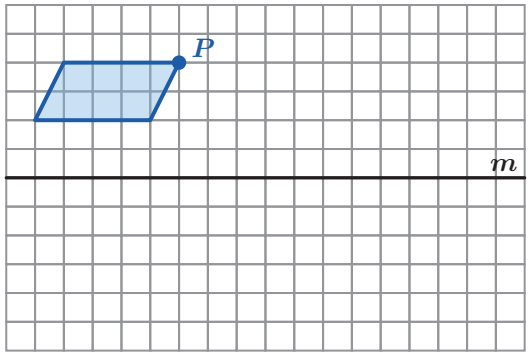
1. Translate this figure to take point P to P'.



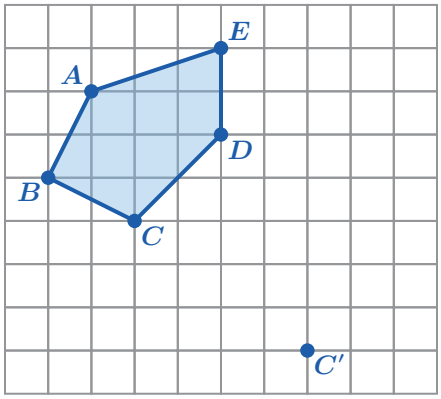
2. Rotate this figure $90°$ counterclockwise around point P.



3. Reflect this figure over line m.

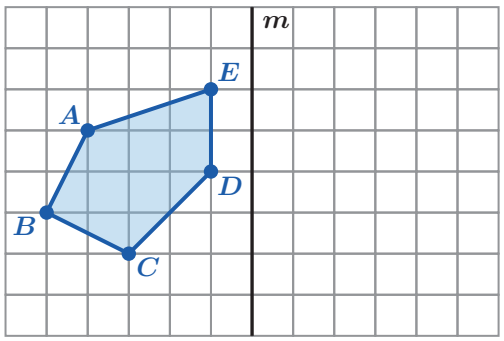


4. Translate figure $ABCDE$ to take point C to C'.



Name: ..... Date: ..... Period: .....

5. Reflect figure $ABCDE$ over line m.



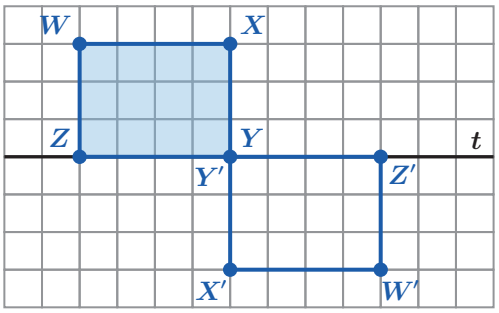
6. Figure $WXYZ$ was transformed to create figure $W'X'Y'Z'$. Which transformation took place?

A. A reflection over line t.

B. A translation 6 units down and 4 units to the right.

C. A rotation 90° clockwise around point Y.

D. A rotation 180° clockwise around point Y.



Spiral Review

7. Match each expression with an equivalent expression.

Expressions	Equivalent Expressions
a $-3x - 7$	..... $-0.9 + 5.7x$
b $-3.4 + 5.7x + 2.5$	..... $-\frac{1}{2}(6x - 14)$
c $1.8x - 5.9 + 3.9x$	..... $5.7x - 5.9$
d $-3x + 7$	..... $\left(-\frac{7}{2} - \frac{3}{2}x\right) \cdot 2$

8. What value of x makes the equation $-5(x - 4) = 40$ true?

A. 4

B. -4

C. 12

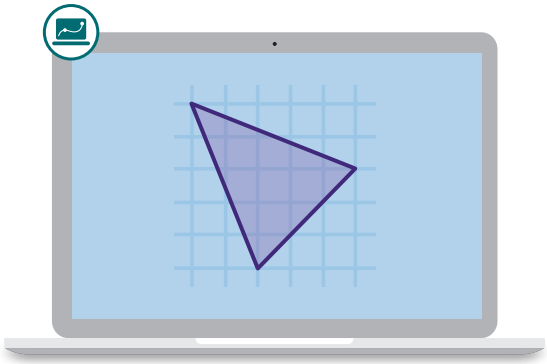
D. -12

Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.1, 8.G.3, SMP.3, SMP.8

Getting Coordinated, Part 1

Let's explore how translations and reflections affect points on the coordinate plane.

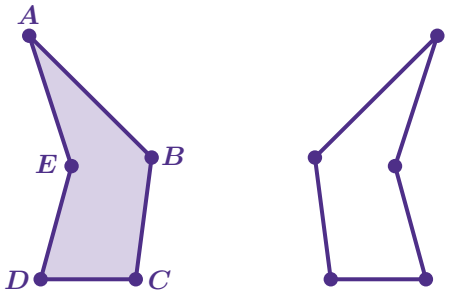


Warm-Up

1 The pre-image (shaded) has been reflected to create the image (unshaded).

Label each corresponding point on the image.

A'	B'	C'	D'	E'



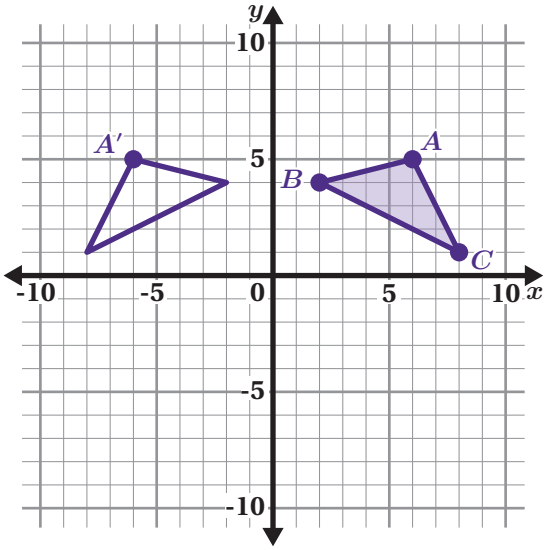
Activity 1

Name: ..... Date: ..... Period: .....

Coordinate Patterns, Part 1

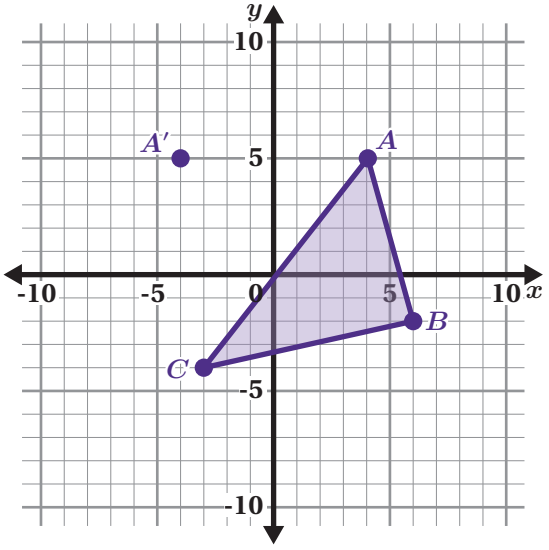
2 Determine the coordinates of each point after a reflection over the y-axis.

Pre-Image Coordinates	Image Coordinates
(6, 5)	(-6, 5)
(2, 4)	
(8, 1)	



3 Determine the coordinates of each point after a reflection over the y-axis.

Pre-Image Coordinates	Image Coordinates
(4, 5)	(-4, 5)
(6, -2)	
(-3, -4)	



4 Take a look at your tables from Problems 2 and 3. Those points show a reflection over the y-axis.

a **Discuss:** What patterns do you see between the pre-image coordinates and the image coordinates?

b Complete the table for the same transformation.

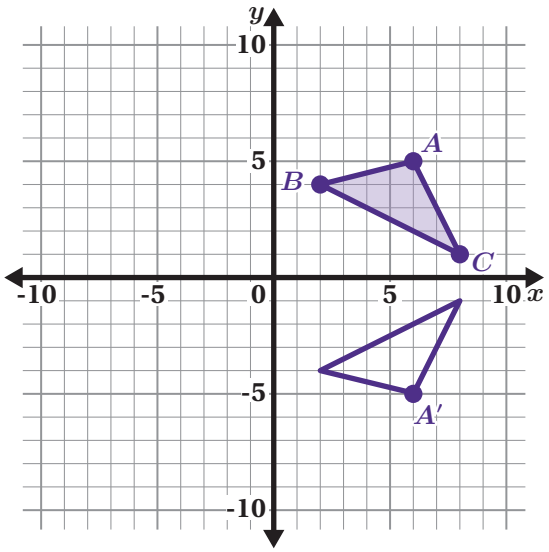
Pre-Image Coordinates	Image Coordinates
(3, 1)	
(x, y)	

Name: Date: Period:

Coordinate Patterns, Part 1 (continued)

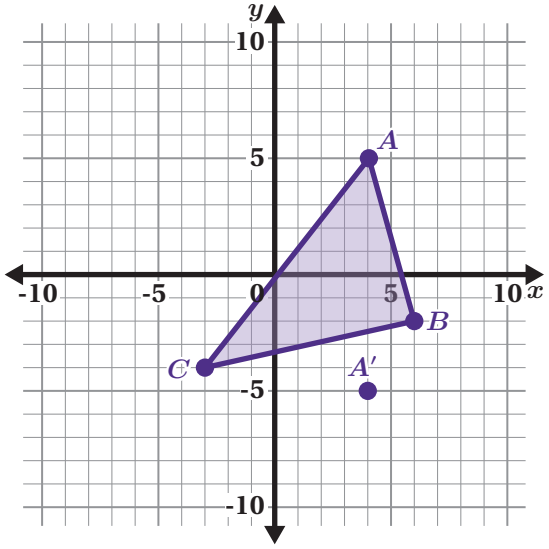
5 Determine the coordinates of each point after a reflection over the x-axis.

Pre-Image Coordinates	Image Coordinates
(6, 5)	(6, -5)
(2, 4)	
(8, 1)	



6 Determine the coordinates of each point after a reflection over the x-axis.

Pre-Image Coordinates	Image Coordinates
(4, 5)	(4, -5)
(6, -2)	
(3, -4)	



7 Take a look at your tables from Problems 5–6. Those points show a reflection over the x-axis.

a **Discuss:** What patterns do you see between the pre-image coordinates and the image coordinates?

b Complete the table for the same transformation.

Pre-Image Coordinates	Image Coordinates
(3, 1)	
(x, y)	

Activity 2

Name: ..... Date: ..... Period: .....

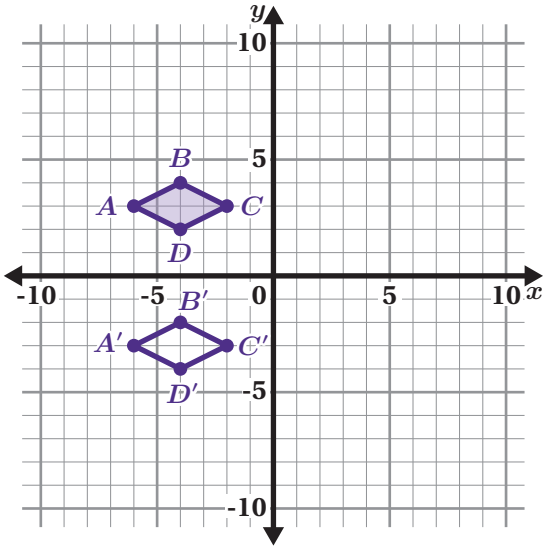
Coordinate Patterns, Part 2

8 Amari says that figure $A'B'C'D'$ is the image of figure $ABCD$ after a reflection over the x-axis.

Is Amari's claim correct? Circle one.

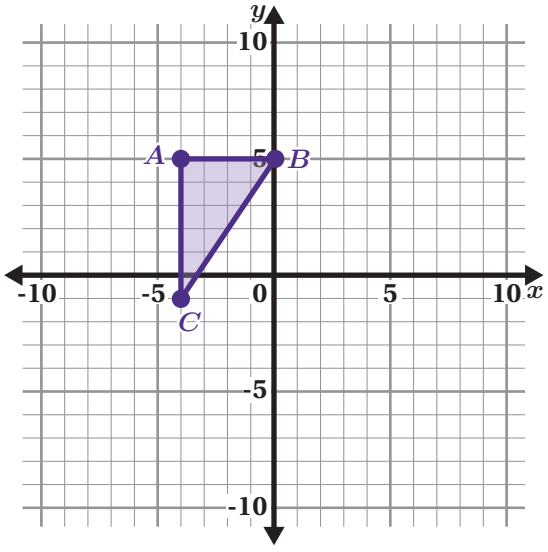
Yes No I'm not sure

Explain your thinking.



9 Determine the coordinates of each point after a translation 4 units right and 3 units down.

Pre-Image Coordinates	Image Coordinates
(-4, 5)	
(0, 5)	
(-4, -1)	



10 Take a look at your table from Problem 9. Those points show a translation 4 units right and 3 units down.

a **Discuss:** What patterns do you see between the pre-image coordinates and the image coordinates?

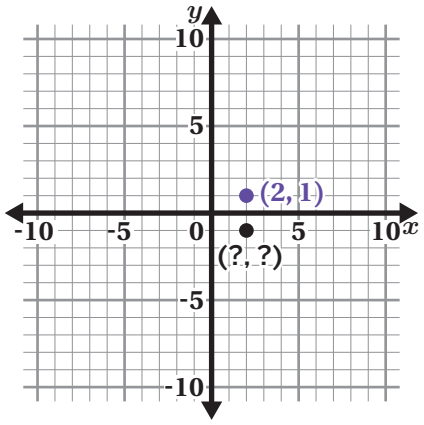
b Complete the table for the same transformation.

Pre-Image Coordinates	Image Coordinates
(3, 1)	
(x, y)	

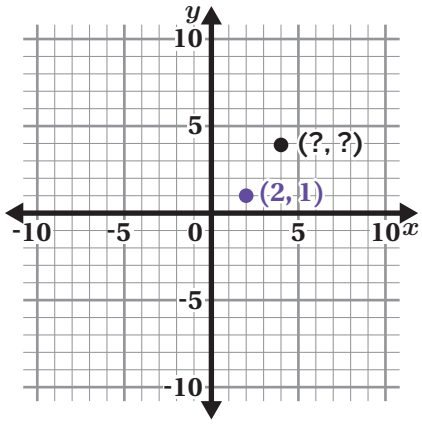
11 Synthesis

If you know the pre-image coordinates, how can you find the image coordinates for any reflection or translation?

Reflection



Translation



Use the examples if they help with your thinking.

Reflection:

Translation:

14 Summary 1.05

When you compare the coordinates of corresponding points in the image and pre-image, you might notice patterns in their values.

Translating a point . . .

- right or left h units changes the x coordinate
- up or down k units changes the y-coordinate.
- $(x, y) \rightarrow (x \pm h, y \pm k)$

Example: If pre-image $(3, 5)$ is translated to the left 2 units and up 1 unit, then the image is $(3 - 2, 5 + 1) \rightarrow (1, 6)$.

Reflecting a point across the . . .

- x-axis changes the sign of the y-coordinate. The x-coordinate remains the same. $(x, y) \rightarrow (x, -y)$
- y-axis changes the sign of the x-coordinate. The y-coordinate remains the same. $(x, y) \rightarrow (-x, y)$

Example:

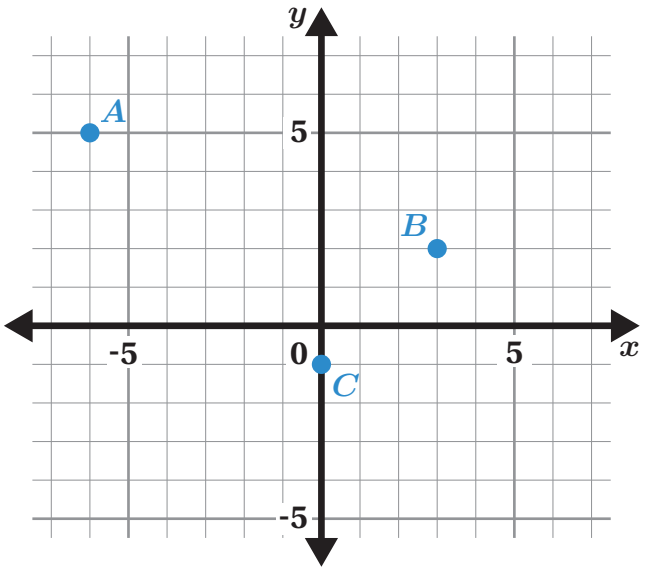
- If pre-image $(3, 5)$ is reflected across the x-axis, then the image is $(3, -5)$.
- If pre-image $(3, 5)$ is reflected across the y-axis, then the image is $(-3, 5)$.

Practice
1.05

Name: ..... Date: ..... Period: .....

1. Plot the location of points A, B, and C after a translation 4 units to the right and 1 unit up. Label the points A', B', and C'. Then write the coordinates in the table.

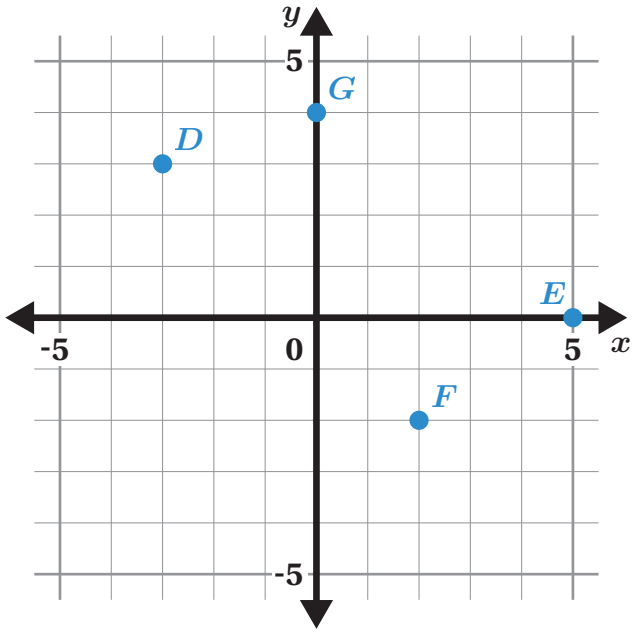
Point	Coordinates
A'	
B'	
C'	



Problems 2–3: Points D, E, F, and G are plotted on the coordinate plane.

2. Plot the coordinates of points D and E after a reflection over the y-axis. Label the images D' and E'. Include the coordinates.

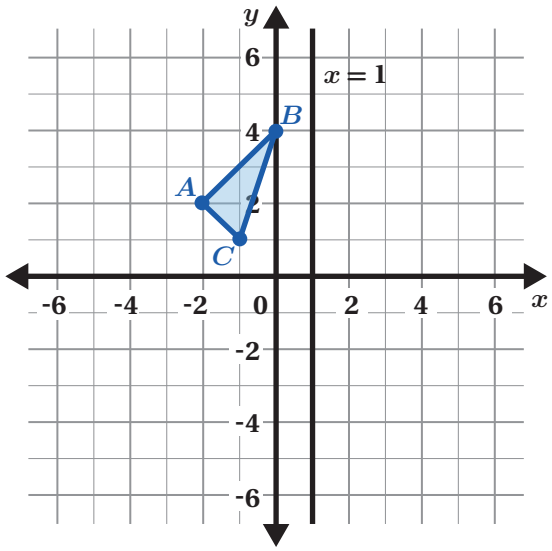
3. Plot the coordinates of points F and G after a reflection over the x-axis. Label the images F' and G'. Include the coordinates.



Name: ..... Date: ..... Period: .....

4. Determine the coordinates of the image after the shaded pre-image is reflected over the line $x = 1$. Use the graph if it helps with your thinking.

Pre-Image Coordinates	Image Coordinates
(-2, 2)	
(0, 4)	
(-1, 1)	



5. Point $H(5, 3)$ is translated 4 units down and 3 units to the left to get point H'. Which of the following are the coordinates of point H'?

A. $(-2, 1)$ **B.** $(1, 0)$ **C.** $(1, 6)$ **D.** $(2, -1)$

6. Point (x, y) is reflected over the y-axis, and then translated 5 units to the right and 2 units down. Write the generalized coordinates of the image.

Spiral Review

Problems 7–12: Compare each of these values using the symbols <, =, or >.

7. -11 ..... -15

8. 8.01 8

9. $\frac{2}{3}$ ..... $-\frac{3}{2}$

10. -8.01 ..... -8

11. -(-6) 6

12. -2.5 ..... $-\frac{10}{4}$

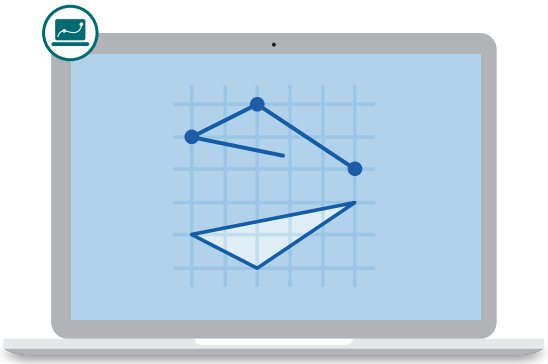
13. What value of x makes the equation $-4x = 2.2$ true?

Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.1, 8.G.3, SMP.3, SMP.8

Getting Coordinated, Part 2

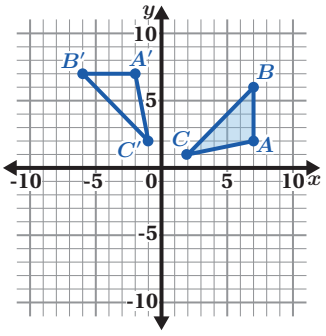
Let's explore how rotations affect coordinates.



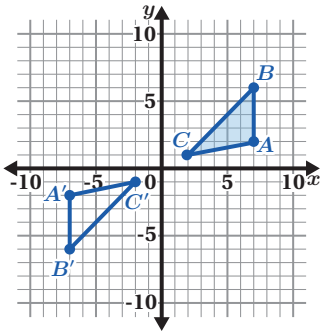
Warm-Up

1 a Take a look at several different rotations.

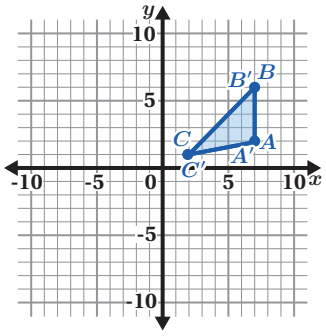
Counterclockwise 90°



Clockwise 180°



Counterclockwise 360°



b **Discuss:** What do you notice? What do you wonder?

Name: Date: Period:

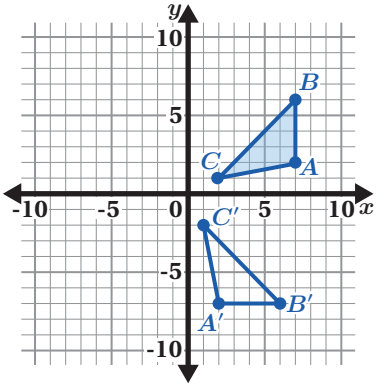
Coordinate Patterns

2 Tyani says this is a rotation of figure ABC 90° clockwise around center (0, 0). Anushka says this is a rotation of figure ABC 270° counterclockwise around the *origin*.

Whose claim is correct? Circle one.

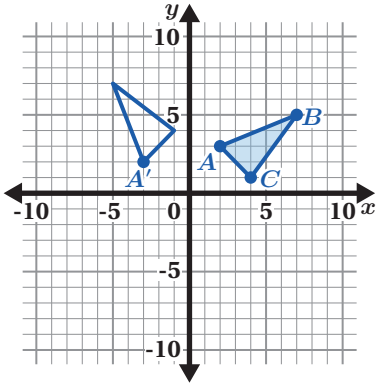
Tyani's Anushka's Both Neither

Explain your thinking.



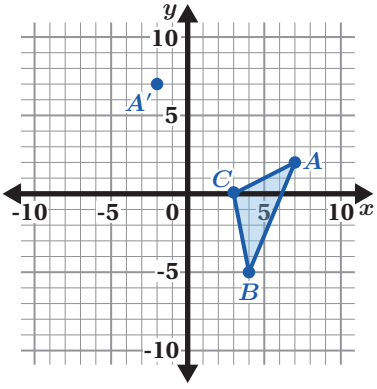
3 Identify the coordinates for the image of figure ABC after a rotation 90° counterclockwise around center (0, 0).

Pre-Image Coordinates	Image Coordinates
(2, 3)	(-3, 2)
(7, 5)	
(4, 1)	



4 Determine the coordinates for the image of figure ABC after a rotation 90° counterclockwise around center (0, 0).

Pre-Image Coordinates	Image Coordinates
(7, 2)	(-2, 7)
(4, -5)	
(3, 0)	

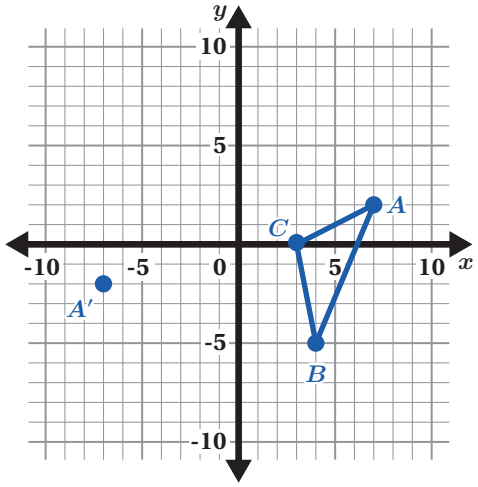


Name: ..... Date: ..... Period: .....

Coordinate Patterns (continued)

5 Determine the coordinates for the image of figure ABC after a $180°$ counterclockwise rotation around center $(0, 0)$.

Pre-Image Coordinates	Image Coordinates
$(7, 2)$	$(-7, -2)$
$(4, -5)$	
$(3, 0)$	



6 **Discuss**

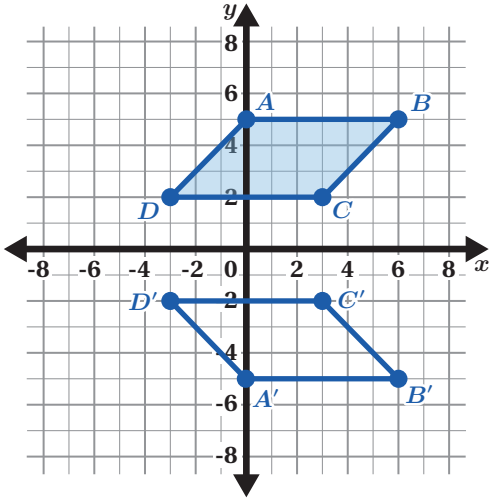
- What patterns do you notice between the pre-image coordinates and image coordinates after a $90°$ counterclockwise around center $(0, 0)$?
- What patterns do you notice between the pre-image coordinates and image coordinates after a $180°$ counterclockwise around center $(0, 0)$?

7 Binta says this is a $180°$ clockwise rotation of figure $ABCD$ around center $(0, 0)$. Chloe says this is a reflection over the x-axis.

Whose claim is correct? Circle one.

Binta's (Rotation) Chloe's (Reflection) Both Neither

Explain your thinking.



Activity 2

Name: ..... Date: ..... Period: .....

Challenge Creator

8 You will use the Activity 2 Sheet to create your own transformation challenge.

a **Make It!** On the Activity 2 Sheet, create a transformation challenge.

b **Solve It!** On this sheet, write the pre-image and image coordinates for your transformation.

Pre-Image Coordinates	Image Coordinates

c **Swap It!** Swap your challenge with one or more partners. Write the pre-image and image coordinates for their transformation.

Partner 1's Challenge

Pre-Image Coordinates	Image Coordinates

Partner 2's Challenge

Pre-Image Coordinates	Image Coordinates

Partner 3's Challenge

Pre-Image Coordinates	Image Coordinates

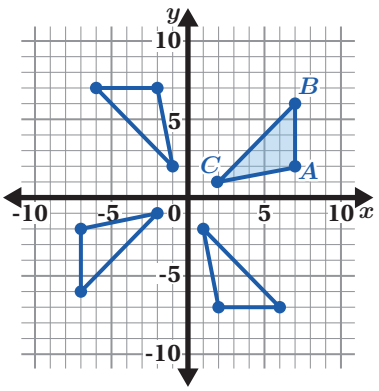
Partner 4's Challenge

Pre-Image Coordinates	Image Coordinates

9 Synthesis

Discuss: What are some patterns in the pre-image and image coordinates for rotations around center $(0, 0)$?

Use the examples if they help with your thinking.



12 Summary 1.06

When rotating a point around center $(0, 0)$, the coordinates of the image are related to the coordinates of the pre-image.

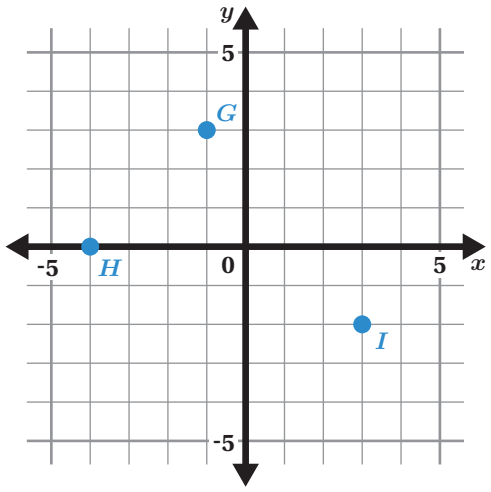
90° counterclockwise or 270° clockwise	90° clockwise or 270° counterclockwise	180° counterclockwise or clockwise
The x- and y- coordinates switch places. The x-coordinate of the image has the opposite sign of the y-coordinate of the pre-image. $(x, y) \rightarrow (-y, x)$ **Example:** Pre-image: (-3, 2) Image: (-2, -3)	The x- and y-coordinates switch places. The y-coordinate of the image has the opposite sign of the x-coordinate of the pre-image. $(x, y) \rightarrow (y, -x)$ **Example:** Pre-image: (-3, 2) Image: (2, 3)	The order of the x- and y-coordinates of the image stay in the same place as the pre-image, but have opposite signs. $(x, y) \rightarrow (-x, -y)$ **Example:** Pre-image: (-3, 2) Image: (3, -2)

Practice
1.06

Name: ..... Date: ..... Period: .....

1. Plot the location of points G, H, and I after a $180°$ rotation around the origin. Label the points G', H', and I'. Then write the coordinates in the table.

Point	Coordinates
G'	
H'	
I'	



2. Point $P(5, 3)$ is rotated $90°$ counterclockwise around the origin and the image is labeled P'. Which of the following are the coordinates of point P'?

A. $(1, -3)$ **B.** $(-3, -5)$ **C.** $(3, -5)$ **D.** $(-3, 5)$

3. Here are the pre-image and image coordinates of points on a graph. Describe the transformation.

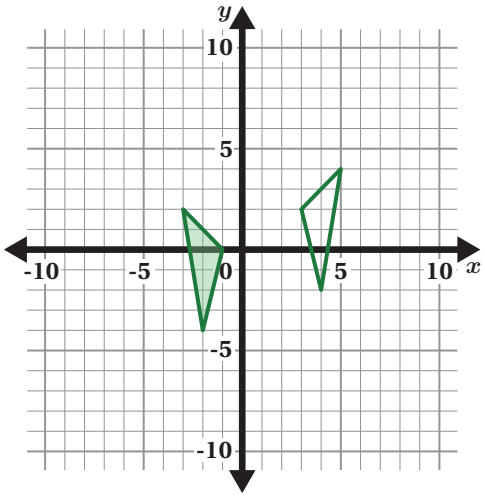
Pre-Image Coordinates	Image Coordinates
$(0, 5)$	$(5, 0)$
$(-2, 1)$	$(1, 2)$
$(4, 3)$	$(3, -4)$
$(6, 0)$	$(0, -6)$
$(-5, -1)$	$(-1, 5)$

4. Point (x, y) is rotated $90°$ counterclockwise around the origin, and then translated 1 unit to the left and 3 units up. Write the generalized coordinates of the image.

Name: ..... Date: ..... Period: .....

5. Write the generalized coordinates of the image after the pre-image is rotated $180°$ around the origin.

6. Describe a sequence of transformations that will map the pre-image (shaded) onto the image (unshaded).



Spiral Review

Problems 7–11: Write an expression that has a value of $\frac{3}{5}$ based on the given rule.

7. An expression that is a sum.

8. An expression that is a difference.

9. An expression that is a product.

10. An expression that is a quotient.

11. An expression that involves at least two operations.

Name: _____ Date: _____ Period: _____

Transformational Geometry 8.G.1, 8.G.3, SMP.6, SMP.7

Practice Day 1

Let's practice what you've learned so far in this unit!



You will use problem cards for this Practice Day. Record all of your responses here.

Card 1

From frame 1 to 2:

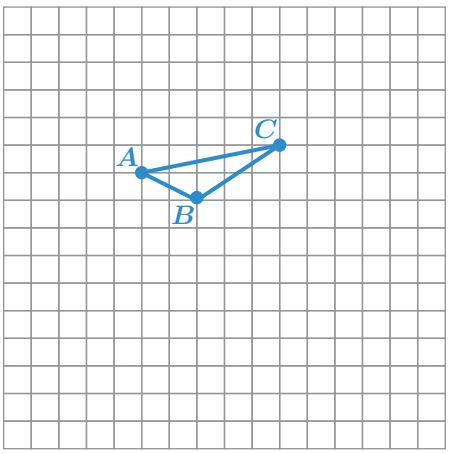
From frame 2 to 3:

Card 2

Description:

Card 3

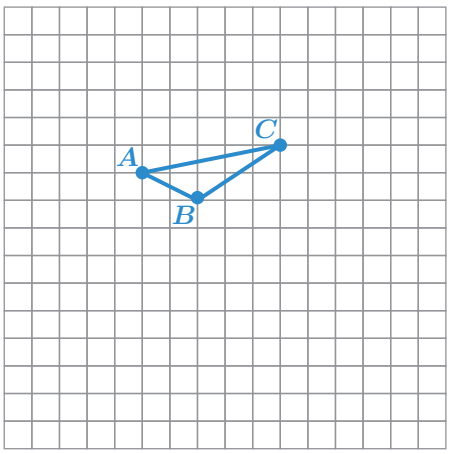
a



b Translation:

Card 4

a



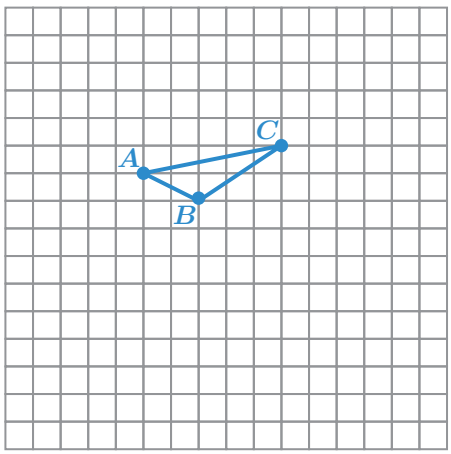
b Rotation:

Name: Date: Period:

Practice Day 1

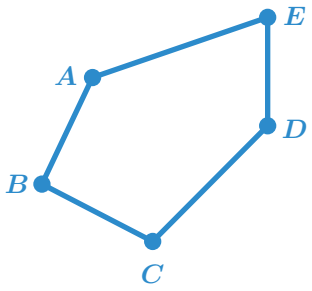
Card 5

a



b Reflection:

Card 6

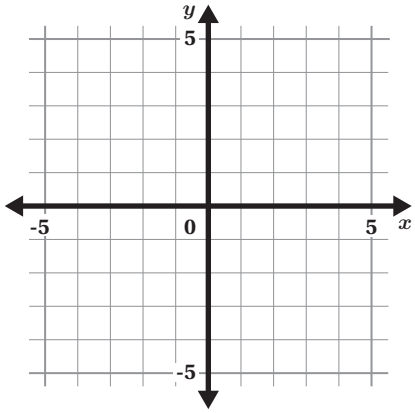


Card 7



Description:

Card 8



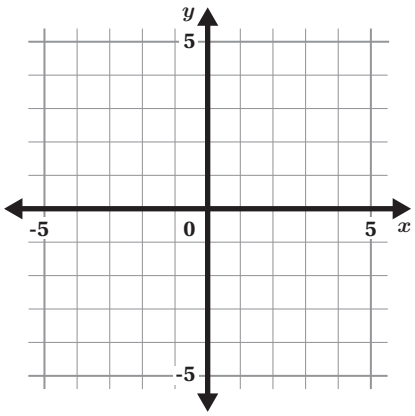
a Coordinates:

b Transformation:

Name: _____ Date: _____ Period: _____

Practice Day 1

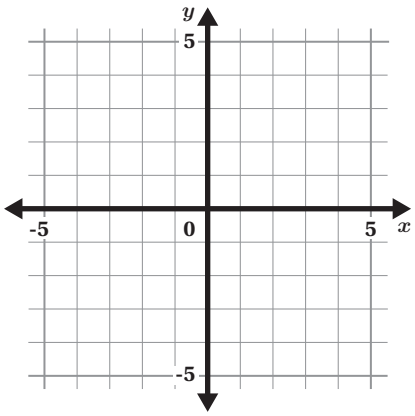
Card 9



a Coordinates:

b Transformation:

Card 10



a Coordinates:

b Transformation:

Card 11

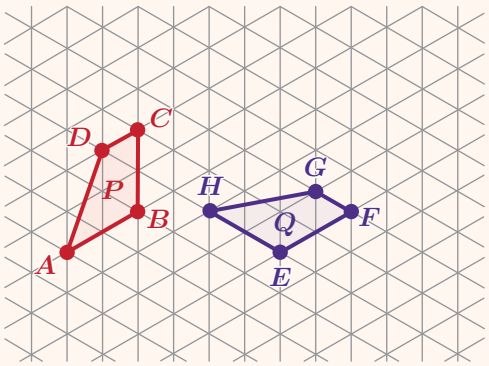
Description:

Card 12

Description:

You're invited to explore more.

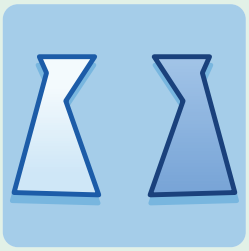
1. Describe a sequence of translations, rotations, and reflections that maps figure P onto figure Q.



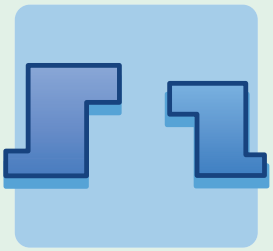
Sub-Unit

2

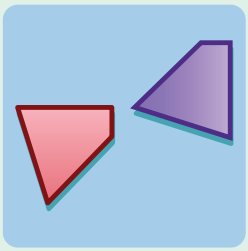
Congruence



Lesson 7
No Bending,
No Stretching



Lesson 8
Are They the Same?

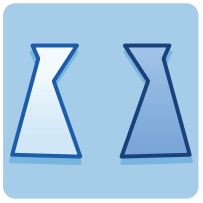


Lesson 9
Are They Congruent?

Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.1, 8.G.1.a, 8.G.1.b, SMP.3, SMP.5, SMP.7

No Bending, No Stretching



Let's compare the measurements of transformed figures.

Warm-Up

1. Describe a sequence of transformations to map the pre-image (shaded) onto the corresponding image (unshaded).

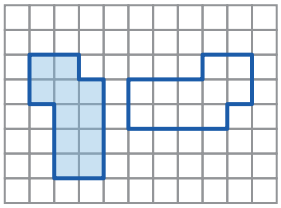
Pair A:

Pair B:

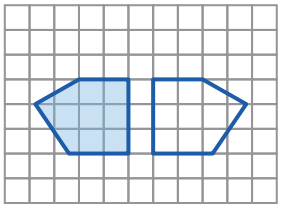
Pair C:

Pair D:

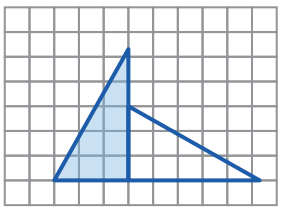
Pair A



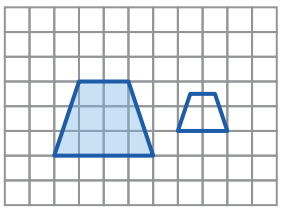
Pair B



Pair C



Pair D



Name: ..... Date: ..... Period: .....

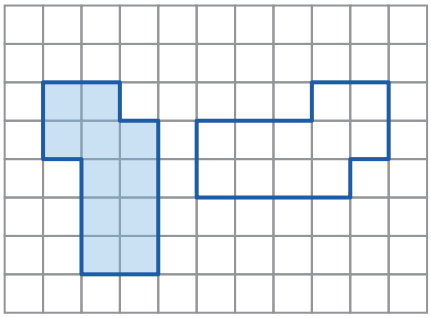
Sides and Angles

2. Here are the pairs from the Warm-Up.

For each pair:

- Use your tools to compare the measurements of the side lengths and interior angles.
- Determine which corresponding measurements are the same and which are different.

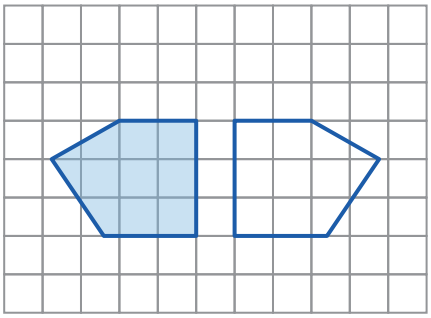
Pair A



Same:

Different:

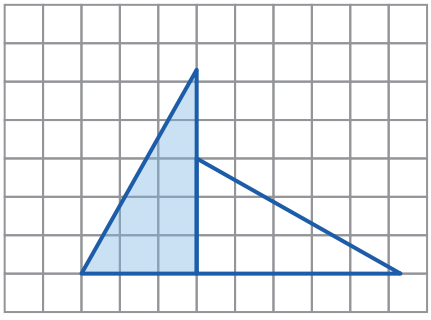
Pair B



Same:

Different:

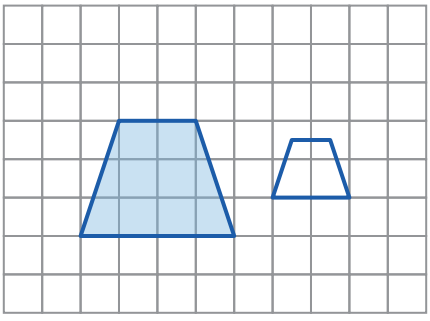
Pair C



Same:

Different:

Pair D



Same:

Different:

Name: ..... Date: ..... Period: .....

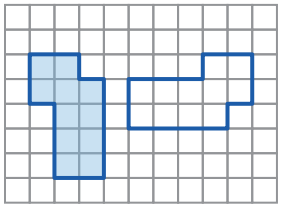
Rigid Transformations

3. A **rigid transformation** is a sequence of transformations that doesn't change any measurements in a figure.

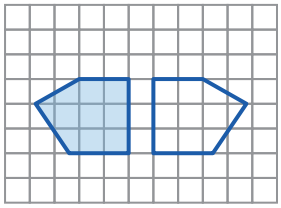
Select *all* the pairs that show a rigid transformation.

- ☐ Pair A
- ☐ Pair B
- ☐ Pair C
- ☐ Pair D

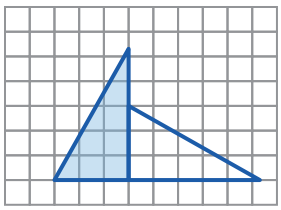
Pair A



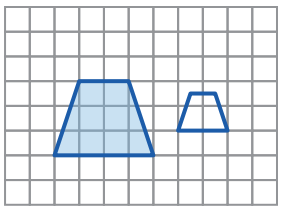
Pair B



Pair C



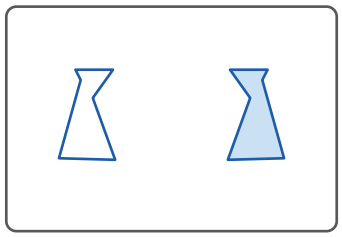
Pair D



4. You will use a set of cards. Sort the cards into two groups according to their type of transformation.

Rigid	Non-Rigid

5. Devon says that this card shows a rigid transformation.



a Is her claim correct?

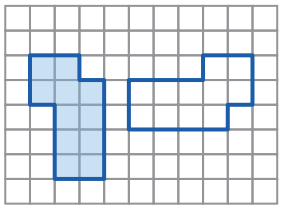
Explain your thinking.

b Describe a transformation that would map the pre-image (shaded) onto the image (unshaded).

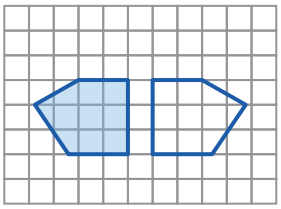
Synthesis

6. How can you determine whether a transformation is rigid? Use the examples if they help with your thinking.

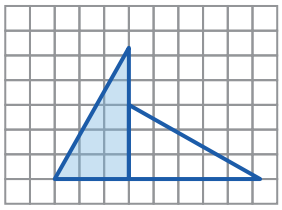
Pair A



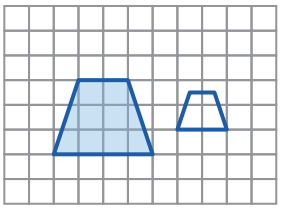
Pair B



Pair C



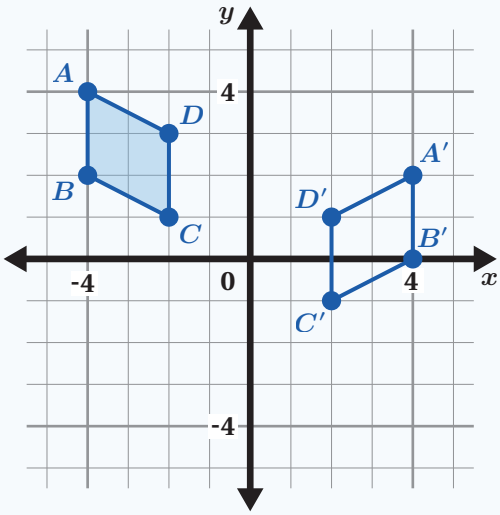
Pair D



Summary 1.07

Translations, rotations, and reflections are all examples of **rigid transformations**.

When a pre-image is transformed using a rigid transformation, corresponding sides will have the same length and corresponding angles will have the same measure. For example, figure $A'B'C'D'$ is the image of figure $ABCD$ after a reflection and a translation. Side AB has the same length as side $A'B'$ and angle C is the same measurement as angle C'.



rigid transformation A move that does not change any measurements of a figure. Translations, rotations, and reflections (or any sequence of these) are rigid transformations.

Practice
1.07

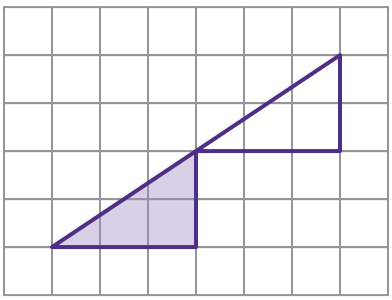
Name: ..... Date: ..... Period: .....

1. Is there a rigid transformation that can map the shaded figure onto the unshaded figure?



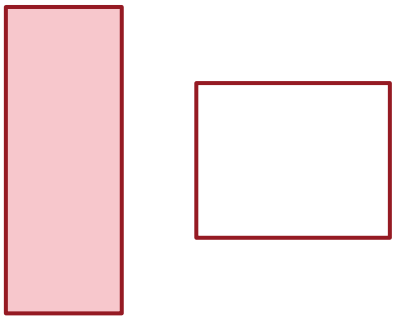
Explain your thinking.

2. Describe a rigid transformation that maps the shaded figure onto the unshaded figure.

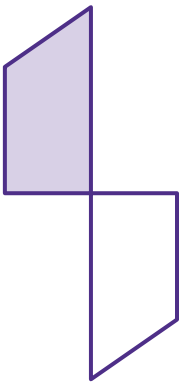


Problems 3–6: Determine whether a rigid transformation can map the shaded figure onto the unshaded figure. If so, describe the rigid transformation.

3.



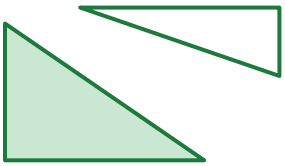
4.



5.



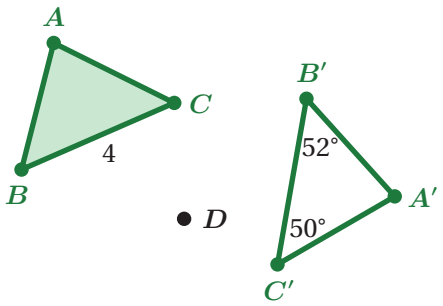
6.



Practice
1.07

Name: ..... Date: ..... Period: .....

Problems 7–9: Triangle $A'B'C'$ is an image of triangle ABC after a rotation. The center of rotation is point D.



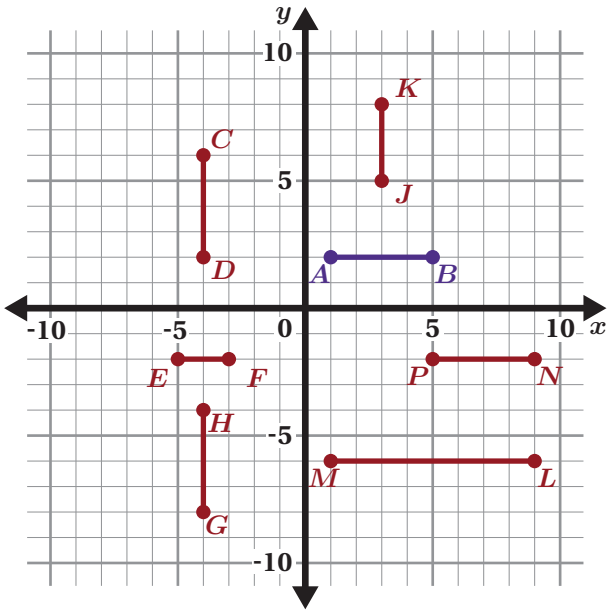
7. What is the length of side $B'C'$?

8. What is the measure of angle B?

9. What is the measure of angle C?

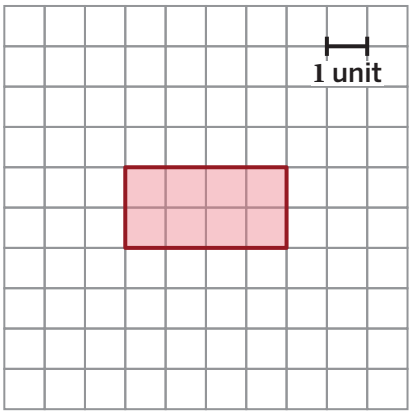
10. Which of the following segments *cannot* be the result of a series of reflections, rotations, and/or translations of segment AB? Select *all* that apply.

- ☐ **A.** Line segment CD
- ☐ **B.** Line segment KJ
- ☐ **C.** Line segment NP
- ☐ **D.** Line segment LM
- ☐ **E.** Line segment GH
- ☐ **F.** Line segment EF



Spiral Review

Problems 11–12: Here is a rectangle.



11. What is the area of the rectangle?

12. What is the perimeter of the rectangle?

13. There are 260 calories in 4 ounces of chocolate milk. How many calories are there in 5 ounces of chocolate milk?

A. 232

B. 236

C. 325

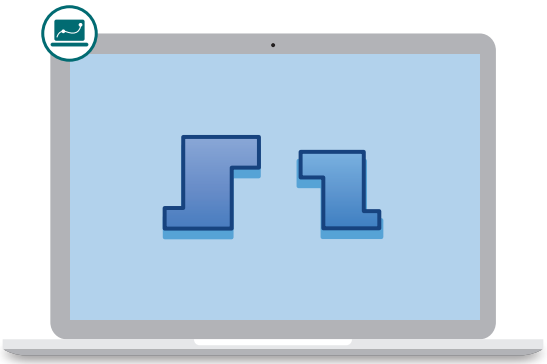
D. 460

Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.1, 8.G.2, SMP.3, SMP.6, SMP.7

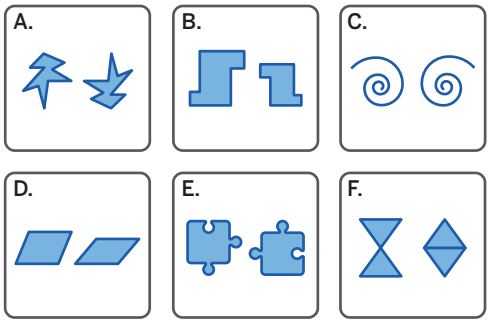
Are They the Same?

Let's explore a type of sameness.



Warm-Up

1 Here are six pairs of figures.



a Circle *all* the pairs with figures that are the same.

b **Discuss:** Which pairs did you choose? Why?

2 Why might someone say Pair B's figures are the same? Are *not* the same?

They are the same because . . .

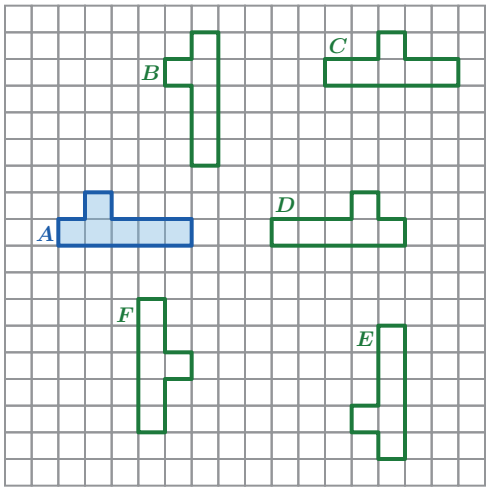
They are not the same because . . .

Name: ..... Date: ..... Period: .....

Defining Congruence

3 One figure is **congruent** to another if there is a sequence of translations, rotations and reflections that map one figure onto the other.

Circle *all* the figures that are congruent to figure A.



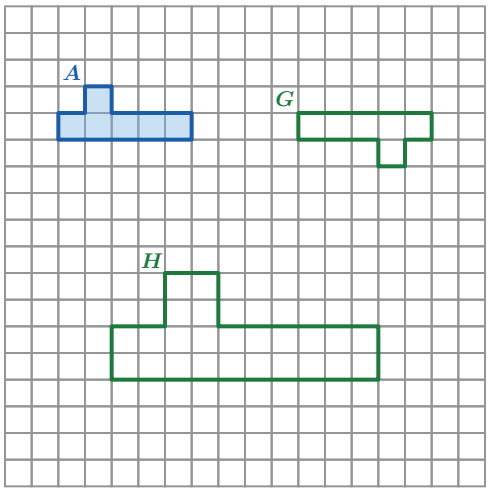
4 Describe a sequence of rigid transformations that would map figure A onto one of the congruent figures you circled in the previous problem.

5 Kweku says figures A and G are congruent.
Lan says figures A and H are congruent.

Whose claim is correct? Circle one.

Kweku's Lan's Both Neither

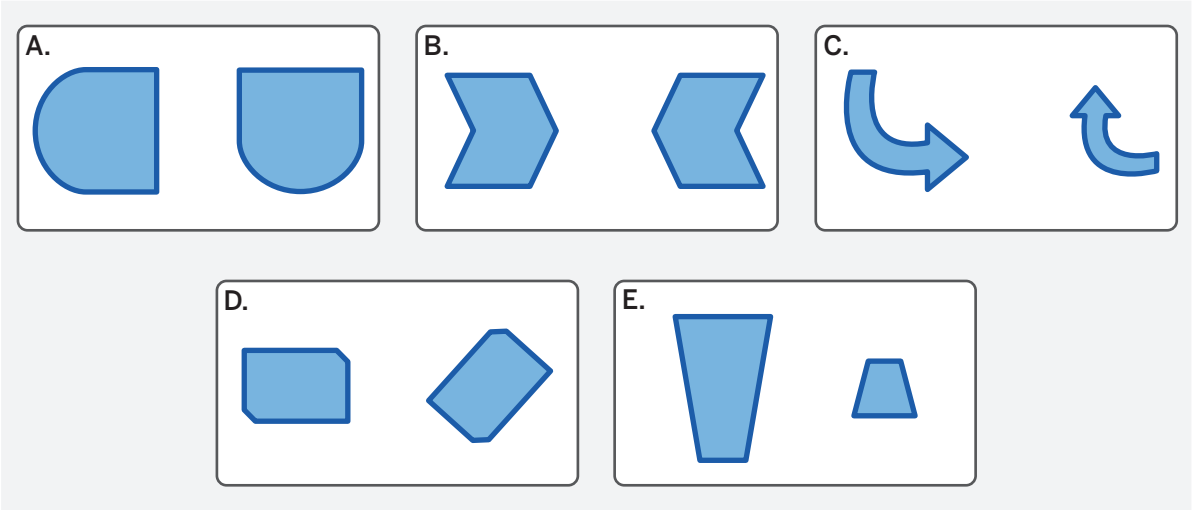
Explain your thinking.



Name: ..... Date: ..... Period: .....

Defining Congruence (continued)

6 Group the pairs of figures based on whether you think they are congruent or not congruent.

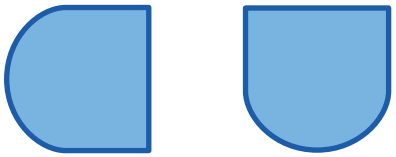


Congruent	Not Congruent

Name: ..... Date: ..... Period: .....

Sequence of Rigid Transformations

7 Which sequence of rigid transformations could be used to show that these two figures are congruent?



A. A translation

B. A rotation and a translation

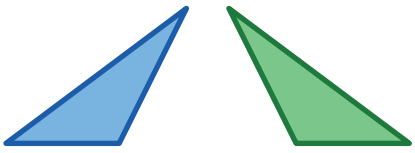
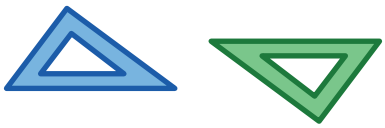
C. A reflection and a translation

D. None. They're not congruent.

Explain your thinking.

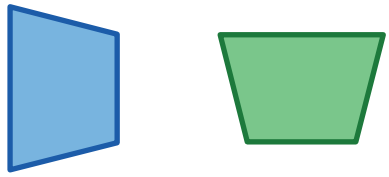
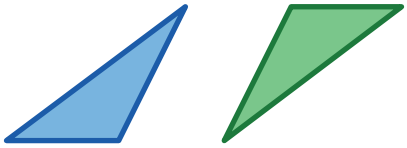
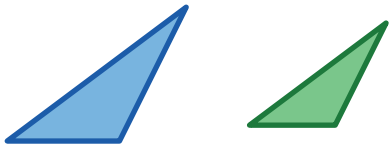
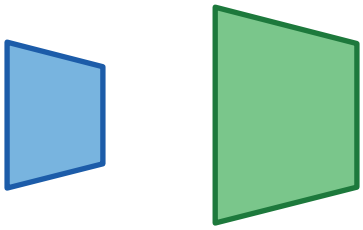
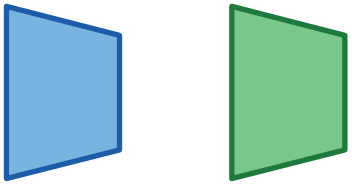
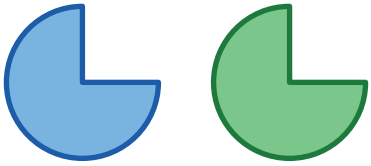
8 For each pair of figures, name a sequence of rigid transformations that could be used to show that they are congruent or write that they aren't congruent. Decide with a partner who will complete Column A and who will complete Column B.

- After each problem, compare your answers. The answers in each row should be the same. Discuss and resolve any differences.
- Complete as many problems as you have time for.

Column A	Column B
	
Sequence:	Sequence:

Name: ..... Date: ..... Period: .....

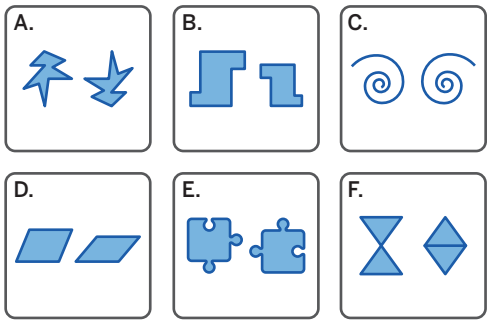
Sequence of Rigid Transformations (continued)

Column A	Column B
 Sequence:	 Sequence:
 Sequence:	 Sequence:
 Sequence:	 Sequence:

9 Synthesis

How can you determine whether two figures are congruent?

Use the examples if they help with your thinking.

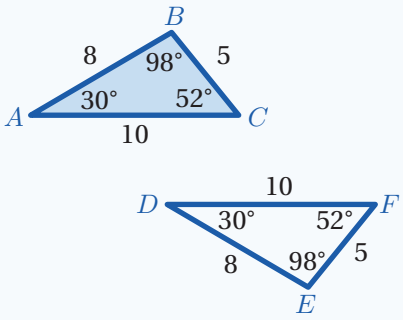


10 Summary 1.08

When two figures are **congruent**, all the corresponding sides have the same length and all the corresponding angles have the same measure.

For example, triangle DEF is congruent to triangle ABC because you can reflect ABC over a horizontal line and translate it to map directly onto triangle DEF.

Angle A and angle D are congruent because they have the same measure. Similarly, the sides DE and AB are congruent because they have the same measurement.

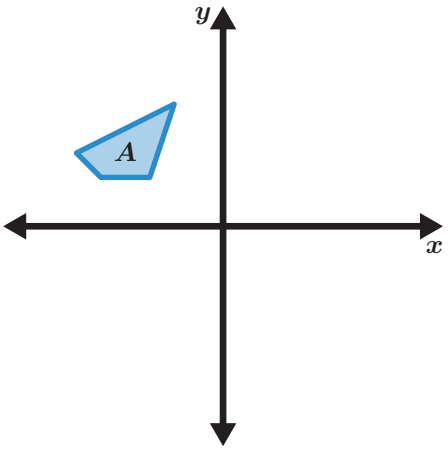


congruent Two figures are congruent to each other if one figure can be mapped onto the other by a sequence of rigid transformations.

Practice
1.08

Name: ..... Date: ..... Period: .....

Problems 1–3: This coordinate plane shows figure A.



1. Reflect figure A over the x-axis. Label the image B.

2. Reflect figure B over the y-axis. Label the image C.

3. Are figures A and C congruent? Explain your thinking.

4. Pentagon $ABCDE$ was graphed on a coordinate plane and then rotated 90° counterclockwise around the origin to form pentagon $A'B'C'D'E'$. Which statement is true?

A. Pentagon $A'B'C'D'E'$ is not congruent to pentagon $ABCDE$.

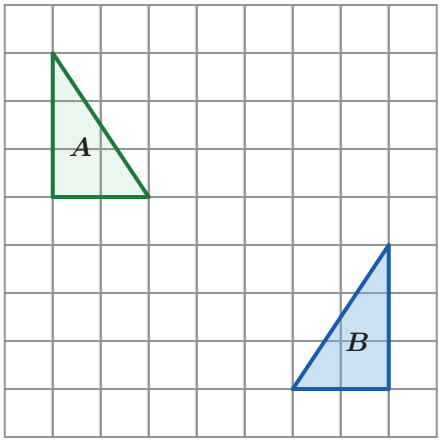
B. The area of pentagon $A'B'C'D'E'$ is not equal to the area of pentagon $ABCDE$.

C. The angle measures of pentagon $A'B'C'D'E'$ are congruent to the corresponding angle measures of pentagon $ABCDE$.

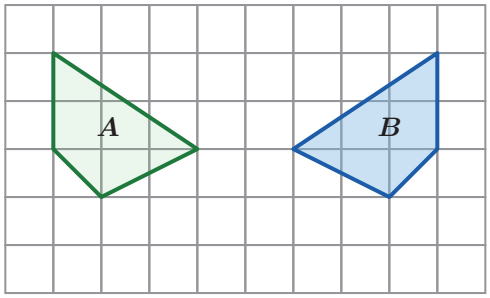
D. The perimeter of pentagon $A'B'C'D'E'$ is greater than the perimeter of pentagon $ABCDE$.

Problems 5–8: Determine whether figure A is congruent to figure B. Explain your thinking.

5.

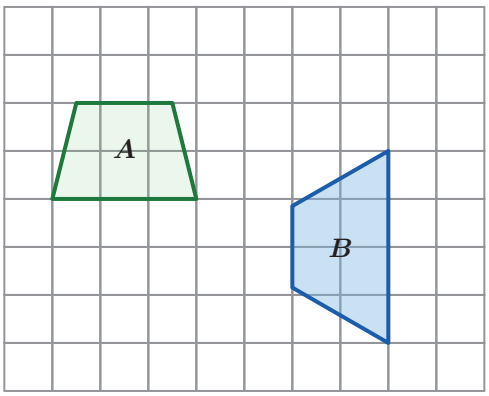


6.

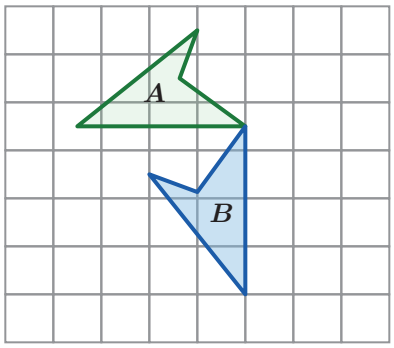


Name: _____ Date: _____ Period: _____

7.



8.



Spiral Review

Problems 9–10: Determine the coordinates of the image of point A (2, -5) after each transformation.

9. A reflection over the x-axis.

10. A reflection over the y-axis.

11. The sum of two numbers is zero. If one of the numbers is 6, what is the other number?

A. 3

B. -6

C. 0

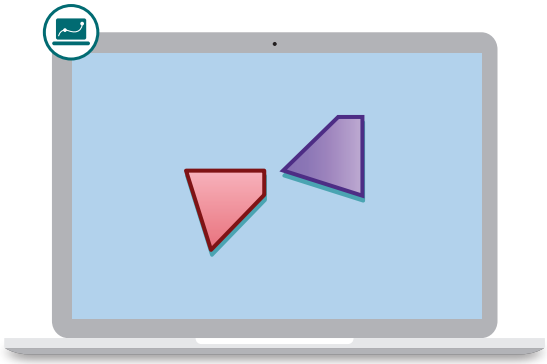
D. -3

Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.1, 8.G.2, SMP.3, SMP.6

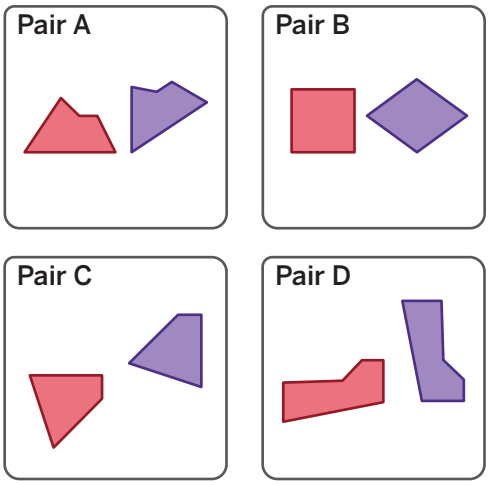
Are They Congruent?

Let's make arguments about whether two figures are congruent.



Warm-Up

1 Here are four pairs of figures.



a Make a prediction! Circle *all* pairs of figures that look congruent.

Pair A Pair B Pair C Pair D

b **Discuss:** How can you be more sure which pairs of figures are congruent?

Activity 1

Name: ..... Date: ..... Period: .....

Congruent or Not?

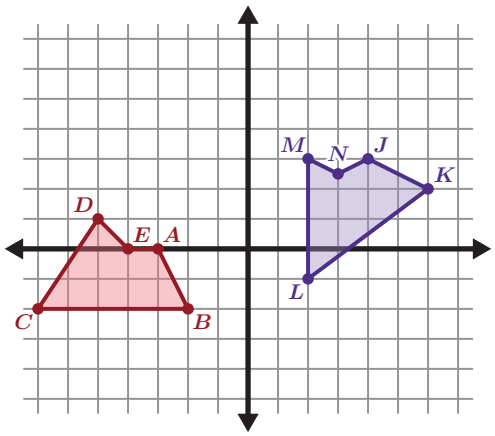
Let's revisit each of the pairs from the Warm-Up.

2 Let's watch a sequence of transformations.

a Precisely describe the sequence of transformations that maps LK onto CB and JK onto AB.

b Explain how you know the figures are *not* congruent.

Pair A



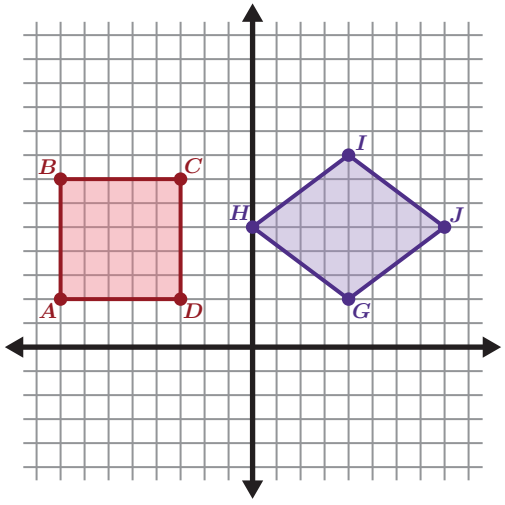
3 Mauricio noticed that each side length in figure $ABCD$ is equal to each side length in figure $GHIJ$. He says this shows that $ABCD \cong GHIJ$ ($ABCD$ is congruent to $GHIJ$).

Is his claim correct? Circle one.

Yes No

Explain your thinking.

Pair B



Name: ..... Date: ..... Period: .....

Congruent or Not? (continued)

4 Students in another class were asked to convince their peers that the figures in Pair C are congruent. Which of their arguments do you think is most convincing?

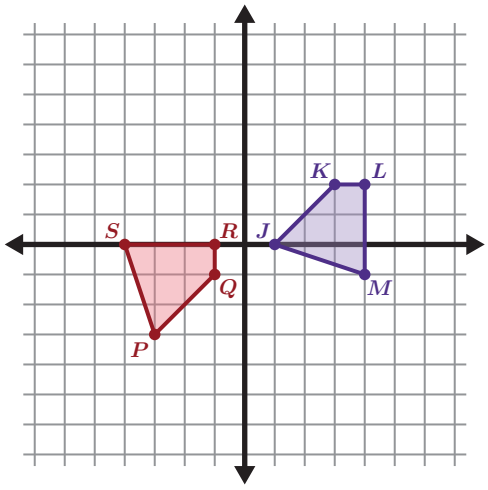
A. Both figures have 4 sides and an area of 5.5 square units.

B. I can map the figures right on top of each other by translating figure $JKLM$ down 3 and left 4, and then reflecting over side QP.

C. When I measure the side lengths of figures $JKLM$ and $PQRS$, I get the same measurements.

Explain your thinking.

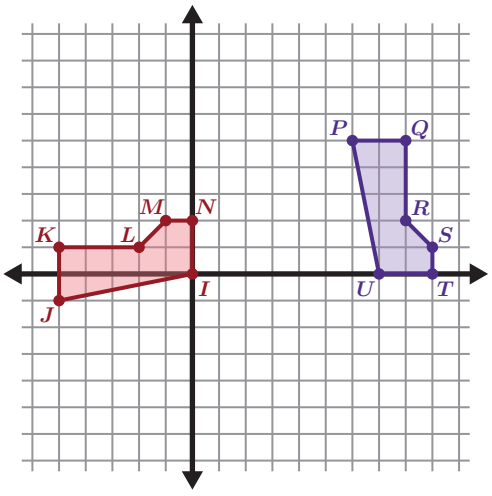
Pair C



5 Describe a sequence of rigid transformations that will convince your classmates that the figures from Pair D are congruent.

Use tracing paper if it helps with your thinking.

Pair D



Name: ..... Date: ..... Period: .....

Prove It!

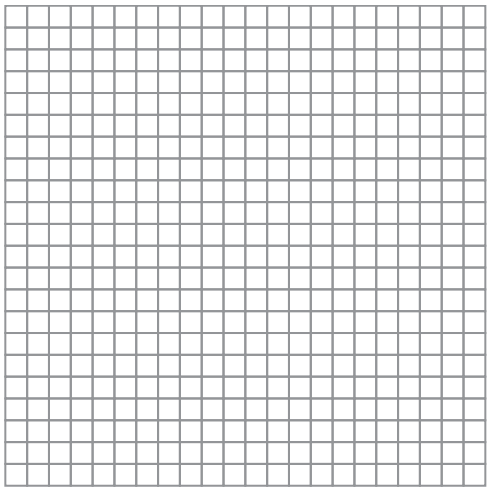
6 Angel says that any two rectangles with the same area are congruent.

Is Angel's claim correct? Circle one.

Draw some rectangles to help you decide.

Yes No I'm not sure

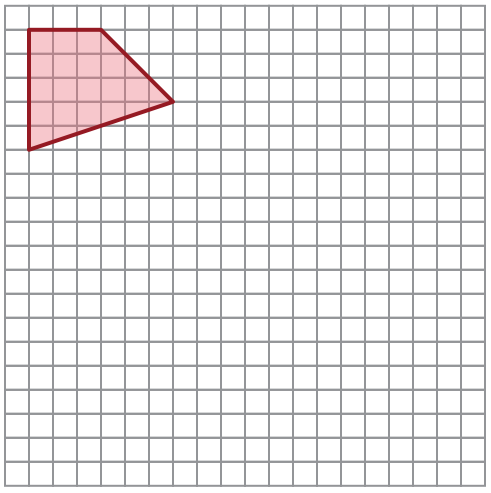
Explain your thinking.



7 Select *all* statements that are true.

- ☐ **A.** If both figures are right triangles, then they are congruent.
- ☐ **B.** If one figure maps directly onto the other figure after a sequence of rigid transformations, then they are congruent.
- ☐ **C.** If both figures have an area of 8 square inches, then they are congruent.
- ☐ **D.** If both figures have the same corresponding measurements, then they are congruent.
- ☐ **E.** If both figures have the same number of sides, then they are congruent.

8 **a** Draw a second figure to create two figures that are *not* congruent but that someone might think are.

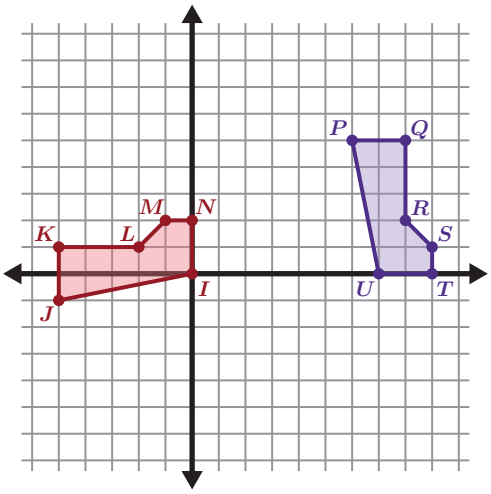


b Explain how you know the two figures are not congruent.

9 Synthesis

How can you show that two figures are congruent?

Use the example if it helps with your thinking.



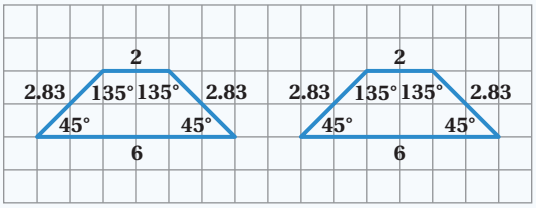
10 Summary 1.09

Here are two ways to determine whether two figures are congruent:

- You can determine a sequence of transformations to map one exactly onto the other.
- You can determine that *all* the corresponding sides have the same length and *all* the corresponding angles have the same measure.

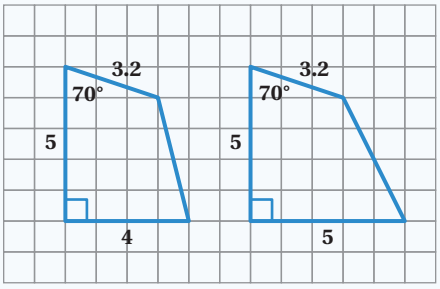
Two figures are not congruent if their corresponding side lengths or angle measures are not the same, or if they have different perimeters or areas.

Congruent



These figures are congruent because all the corresponding side lengths and angle measures are the same.

Not Congruent



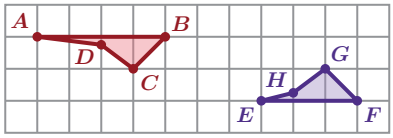
These figures are not congruent because *some* of the side lengths and measures are the same, but some are different.

Practice
1.09

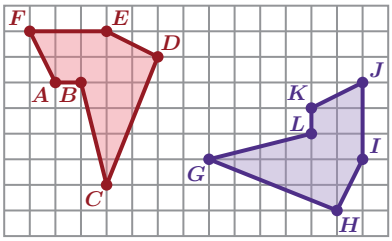
Name: ..... Date: ..... Period: .....

Problems 1–2: For each problem, determine whether the figures are congruent and explain how you know.

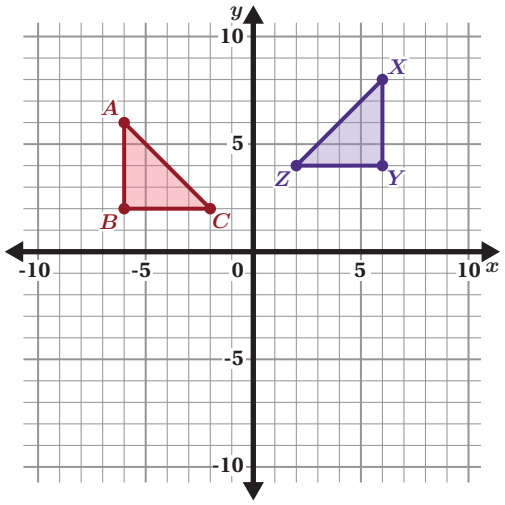
1.



2.



3. Triangle ABC and triangle XYZ are congruent. Which sequence of transformations will map triangle ABC onto triangle XYZ?



A. A reflection over the y-axis, and then a translation 2 units up.

B. A reflection over the x-axis, and then a translation 2 units down.

C. A reflection over the y-axis, and then a translation 2 units right.

D. A reflection over the x-axis, and then a translation 2 units left.

4. If two rectangles have the same perimeter, will they always be congruent? Explain your thinking.

Practice
1.09

Name: ..... Date: ..... Period: .....

Problems 5–7: Here are two congruent figures.

Figure 1

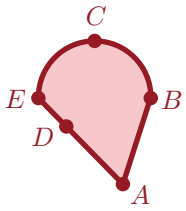
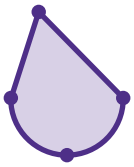


Figure 2



5. Label Figure 2 with points A', B', C', and E' so that they correspond to points A, B, C, and E in Figure 1.

6. If segment AB is 2 centimeters long, how long is segment $A'B'$? Explain your thinking.

7. Plot point D' where you think it is located. Explain your thinking.

Spiral Review

Problems 8–10: A rectangular room has a length of 12 feet and a width of 15 feet. A contractor creates a blueprint of the room using a scale factor of $\frac{1}{6}$.

8. Determine the length and width of the room on the blueprint.

9. How does the perimeter of the actual room compare to the perimeter on the blueprint?

10. How does the area of the actual room compare to the area on the blueprint?

11. A bicyclist travels $6\frac{1}{2}$ miles in $\frac{2}{3}$ hour. What is the average speed, in miles per hour, of the bicyclist?

A. $6\frac{1}{2}$

B. $6\frac{5}{6}$

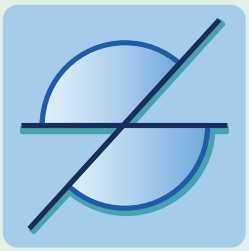
C. $7\frac{1}{6}$

D. $9\frac{3}{4}$

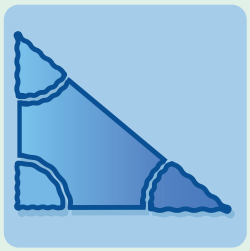
Sub-Unit

3

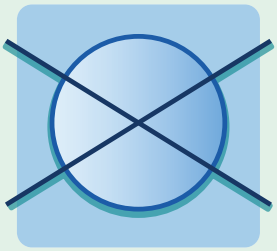
Angle Relationships



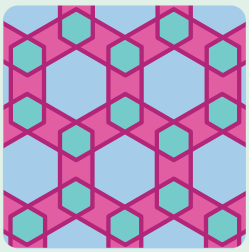
Lesson 10
Transforming Angles



Lesson 11
Tearing It Up



Lesson 12
Puzzling It Out



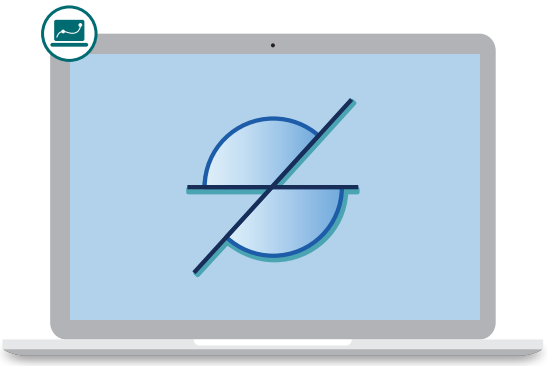
Lesson 13
Tessellate

Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.1, 8.G.1.a, 8.G.1.b, 8.G.1.c, 8.G.5, SMP.3, SMP.7

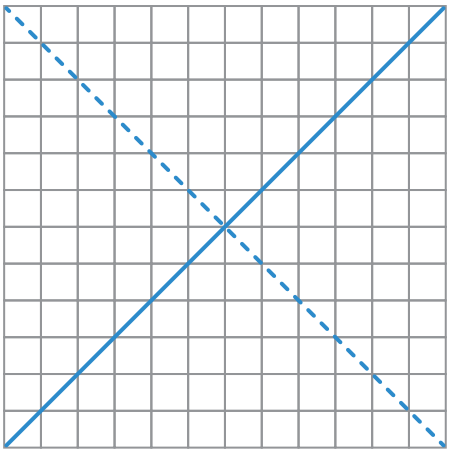
Transforming Angles

Let's explore congruent angles.



Warm-Up

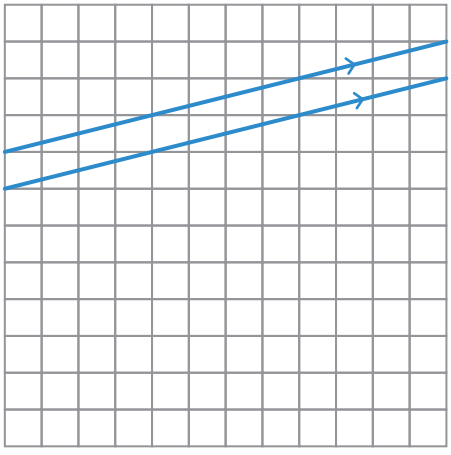
1 Select *all* the transformations that can be used to map the solid line onto the dashed line.



- ☐ **A.** Translations
- ☐ **B.** Rotations
- ☐ **C.** Reflections

2 This pre-image is a pair of lines.

The arrowheads mean that the lines are *parallel*.



a What transformation could map one line onto the other?

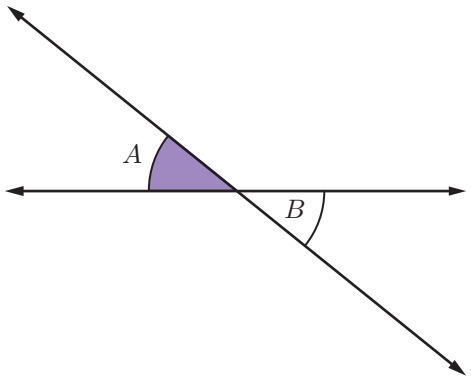
b Draw two lines that *cannot* be rigid transformations of these lines. Explain your thinking.

Name: ..... Date: ..... Period: .....

Vertical Angles

Angles that are opposite each other when two lines cross, like A and B, are called *vertical angles*. We can use transformations to show whether angles are congruent.

3 Describe a single transformation to show that vertical angles A and B are congruent.



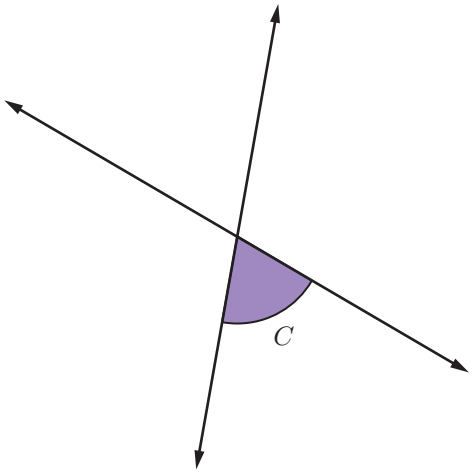
4 Here are another two lines that cross. $\angle C$ has a measure of $70°$.

With this information, Ariel says it's possible to determine the measure of the three remaining angles. Vihaan says it's only possible to determine the measure of one more angle.

Whose claim is correct? Circle one.

Ariel's Vihaan's Neither

Explain your reasoning.

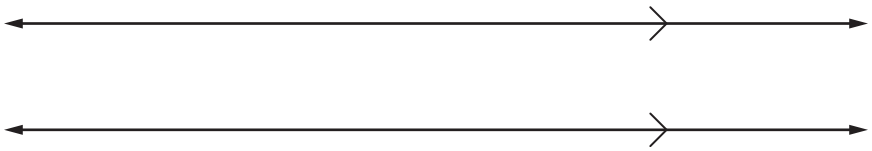


Activity 2

Name: ..... Date: ..... Period: .....

Parallel Lines and a Transversal

5-6 Here are two parallel lines.



a Draw a third line that intersects both the parallel lines. This line is called a **transversal**.

b Choose one angle. Label it with your favorite letter.

c Using tracing paper, trace the angle you selected then compare it to each remaining angle. Mark all of the angles that are congruent to your angle.

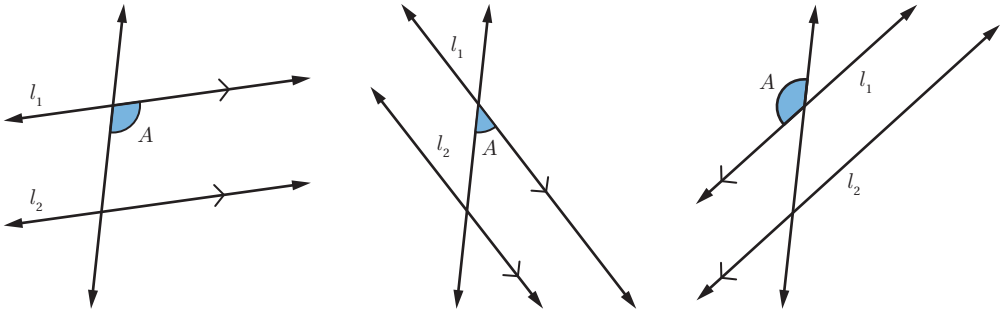
d Compare your drawing with a classmate's.

Activity 3

Name: ..... Date: ..... Period: .....

Congruent Angles?

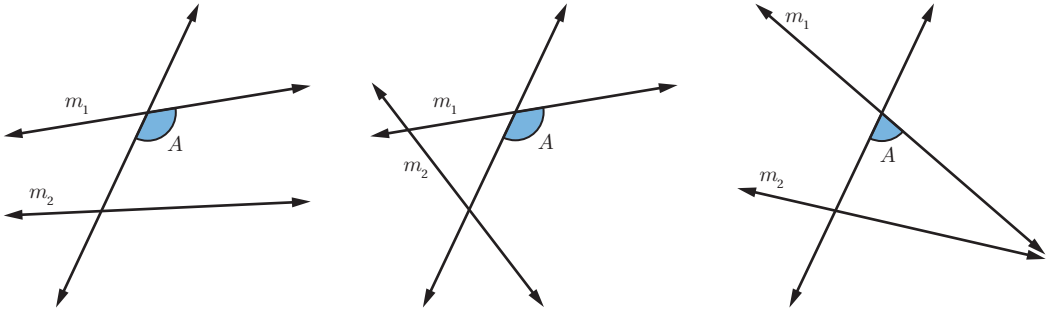
7 Lines l_1 and l_2 are parallel. Take a look at three different positions for l_1 and l_2.



a **Discuss:** How many angles in each diagram are *always* congruent to angle A?

b In each diagram, mark the angles that are congruent to angle A.

8 Lines m_1 and m_2 are *not* parallel. Take a look at three different positions for m_1 and m_2.



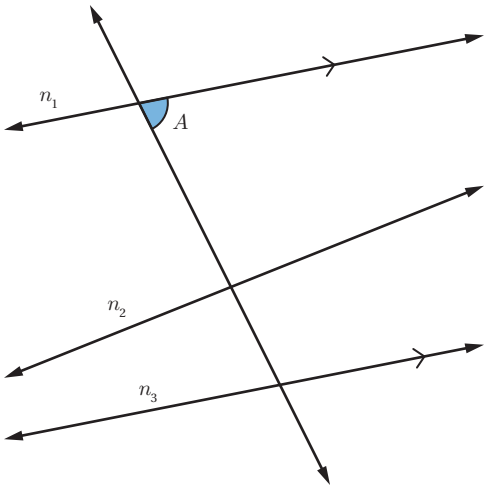
a **Discuss:** How many angles in each diagram are *always* congruent to angle A?

b In each diagram, mark the angles that are congruent to angle A.

Name: ..... Date: ..... Period: .....

Congruent Angles? (continued)

9 Here are three lines: n_1, n_2, and n_3.



a **Discuss:** How many angles in the diagram *must* be congruent to angle A?

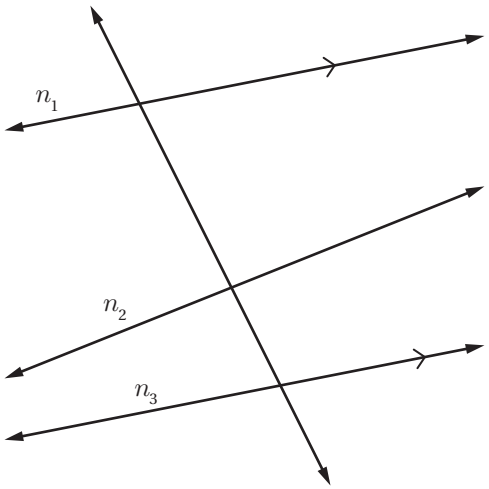
b Mark the angles that are congruent to angle A. Label each one with a different letter.

10 For each angle you marked in the previous problem, describe a single transformation to show that the angle is congruent to angle A.

11 Synthesis

Discuss: How can you use transformations to show that two angles are congruent?

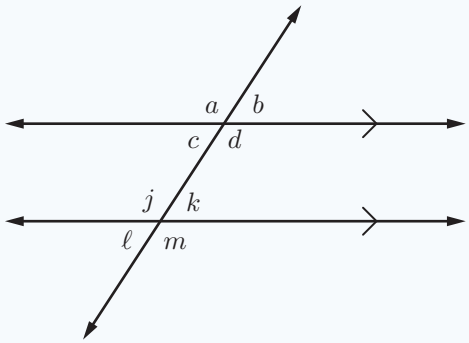
Use the example if it helps with your thinking.



14 Summary 1.10

Transformations of lines can help show which angles in a diagram are congruent. Rigid transformations map lines to lines and *parallel lines* to parallel lines.

We can use transformations to show that *vertical angles* (angles opposite each other when two lines cross) are congruent. For example, vertical angles a and d are congruent because you can rotate angle a 180 degrees around where the lines intersect and it will map onto angle d.

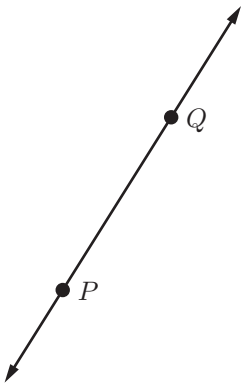


Transformations can also help show which angles are congruent when a **transversal** intersects parallel lines. Angle b is congruent to angle k because you can translate angle b along the transversal until it maps exactly onto angle k.

transversal A line that cuts across parallel lines.

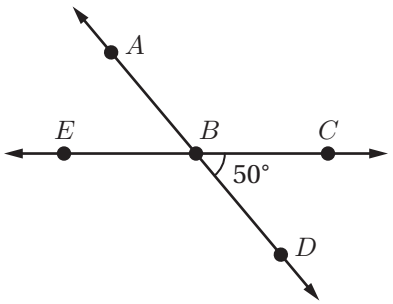
Name: ..... Date: ..... Period: .....

Problems 1–2: Points P and Q are plotted on a line.



1. Plot point R so that a $180°$ rotation with center R maps point P onto point Q and point Q onto point P.
2. Is there more than one point R that works? Explain your thinking.

Problems 3–5: Determine the measure of each angle in the diagram. Explain your thinking.

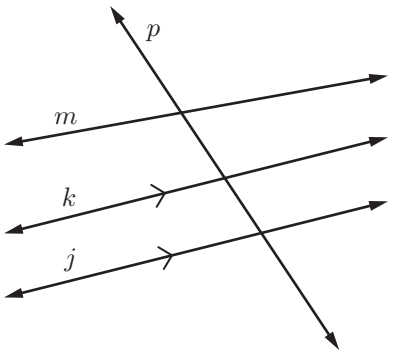


3. $\angle ABC$

4. $\angle EBD$

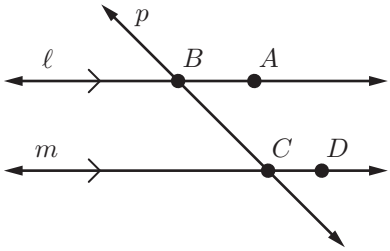
5. $\angle ABE$

6. Lines j and k are parallel. Mark any four congruent angles.



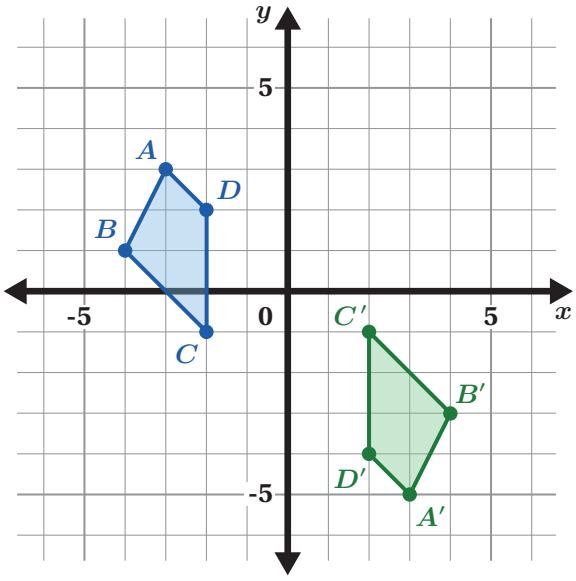
Name: ..... Date: ..... Period: .....

7. Transversal p intersects parallel lines ℓ and m. If the measure of $\angle BCD$ is three times as much as the measure of $\angle ABC$, what is the measure of $\angle ABC$? Show or explain your thinking.



Spiral Review

Problems 8–9: This graph shows two figures.



8. Describe a sequence of transformations that maps figure $ABCD$ onto figure $A'B'C'D'$.

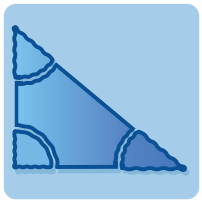
9. Is figure $ABCD$ congruent to figure $A'B'C'D'$? Explain your thinking.

10. Point (x, y) is rotated $180°$ clockwise around the origin, and then translated 3 units to the left and 2 units down. Write the generalized coordinates of the image.

Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.2, 8.G.5, SMP.7, SMP.8

Tearing It Up

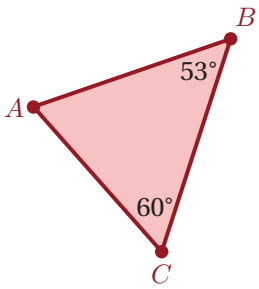


Let's explore the interior angles of triangles.

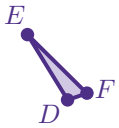
Warm-Up

Here are three triangles.

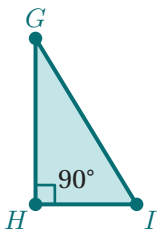
Triangle 1



Triangle 2



Triangle 3



If you add up the three interior angles of each triangle, which triangle do you think has the greatest sum? Explain your thinking.

Activity 1

Name: ..... Date: ..... Period: .....

Find All Three

You will get a card with a picture of a triangle.

1. The measurement of one of the angles is labeled. Estimate the measures of the other two angles.

Labeled angle measure: .....

Estimated angle measure: .....

Estimated angle measure: .....

2. Find two other students with triangles that look congruent to yours, but with a different labeled angle.

Name: ..... Card number: .....

Name: ..... Card number: .....

3. Confirm that all three triangles are congruent and that each card has a different labeled angle. How did you know that the triangles were congruent?

4. Record the three angle measures for your triangle in the table and calculate the sum.

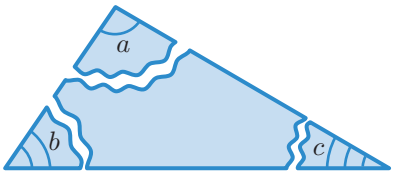
Card Numbers	Angle 1	Angle 2	Angle 3	Angle Sum

Name: ..... Date: ..... Period: .....

Tear It Up

5. You will use a blank sheet of paper to complete this activity. Use a straightedge to draw a triangle that you think will be different from the triangles your classmates will draw.

6. Label the angles of your triangle with the letters a, b, and c. Cut out the triangle, then tear the three angles off of the triangle like in this picture.



7. Rearrange the angles so that the three vertices meet with no overlap.

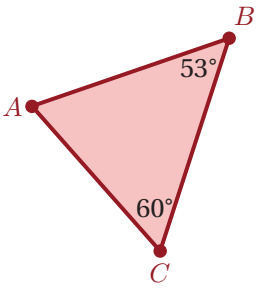
8. Compare your results with your classmates' results.

Discuss:

- What do you notice about your angles?
- What does this mean about the sum of the angles in a triangle?

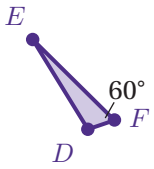
9. Here are the triangles from the Warm-Up, with some additional angle measurements labeled. For each triangle, determine a possible value for the angle listed.

Triangle 1



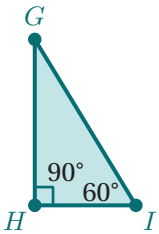
$m\angle A =$ .....

Triangle 2



$m\angle D =$ .....

Triangle 3

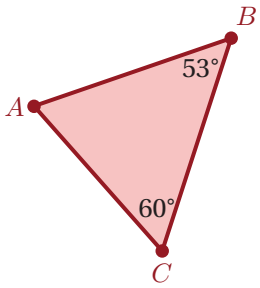


$m\angle G =$ .....

Synthesis

10. **Discuss** What is true about the sum of the three angle measures in a triangle?

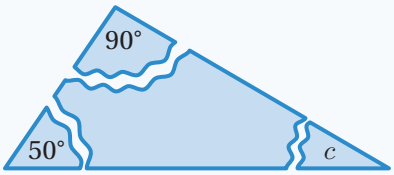
Use the example if it helps with your thinking.

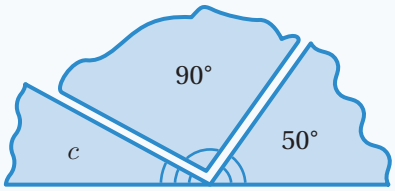


Summary 1.11

The interior angle measures of any triangle always sum to 180 degrees. We can show this by rearranging the angles of any triangle to form a straight line, which has a measure of 180°.

If you know the measures of two angles in a triangle, you can determine the third angle by subtracting the sum of the two known angle measures from 180°. Here is an example.





$c = 180 - 90 - 50$

$c = 40$

Practice
1.11

Name: ..... Date: ..... Period: .....

1. If triangle ABC is a right triangle and $m\angle A = 40°$, what are possible measures for angles B and C?

2. Triangle ABC is an isosceles triangle and the measure of one of the angles is $40°$. List a set of possible angle measures in this triangle.

3. Select *all* of the sets of angles that could make a triangle.

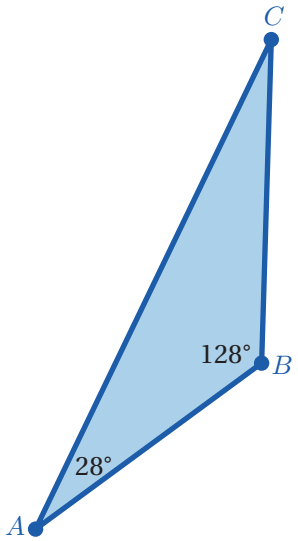
- ☐ **A.** $60°, 60°, 60°$
- ☐ **B.** $90°, 90°, 45°$
- ☐ **C.** $30°, 40°, 50°$
- ☐ **D.** $90°, 45°, 45°$
- ☐ **E.** $120°, 30°, 30°$

4. Angle A in triangle ABC is obtuse. Can angle B or angle C be obtuse?

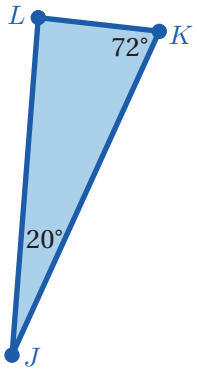
Explain your thinking.

Problems 5–6: Determine the measure of each missing angle.

5.



6.



Name: ..... Date: ..... Period: .....

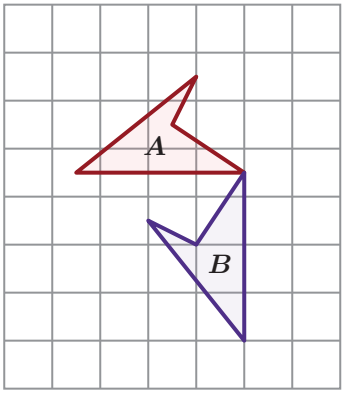
7. Can there be a triangle with two right angles?

Explain your thinking.

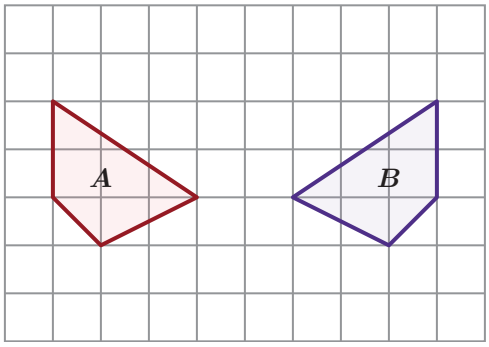
Spiral Review

Problems 8–11: For each pair of polygons, describe a transformation or sequence of transformations that shows polygon A is congruent to polygon B.

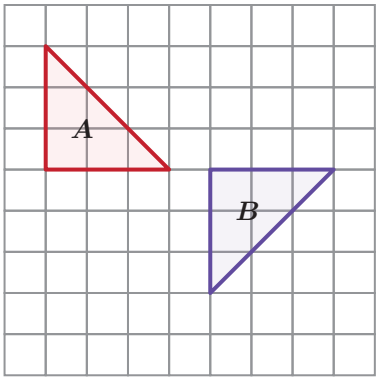
8.



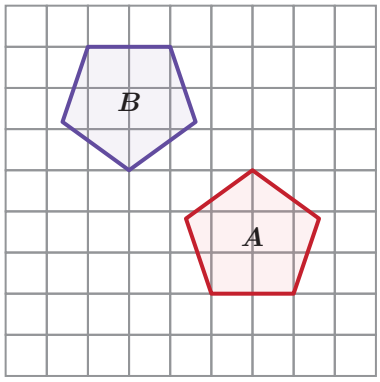
9.



10.

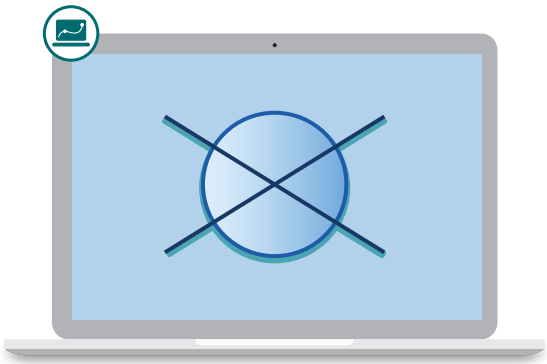


11.



Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.5, SMP.1, SMP.3, SMP.7



Puzzling It Out

Let's solve some puzzles using angle relationships.

Warm-Up

1 Point D is on line AC.

What are possible measures for angles ADB and CDB?

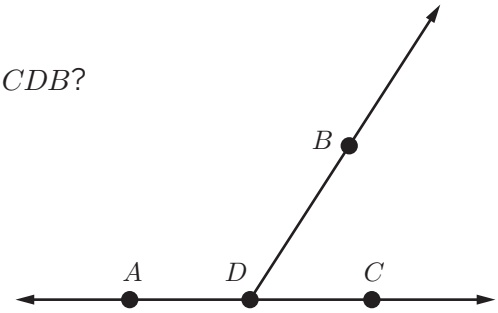


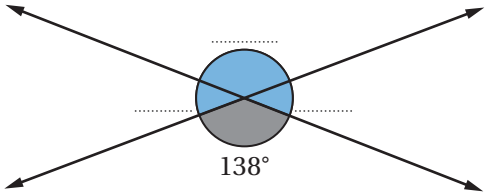
Figure may not be drawn to scale.

Name: ..... Date: ..... Period: .....

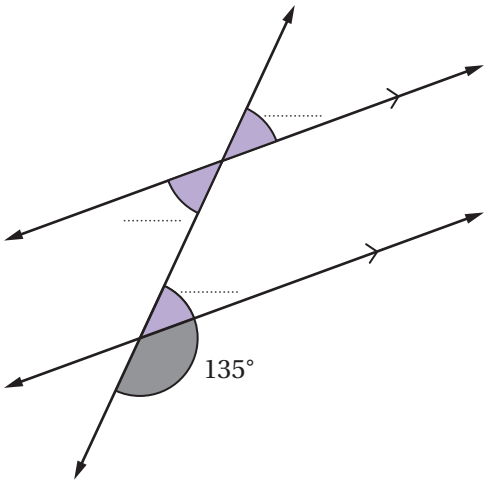
Angle Puzzles

Here are some angle puzzles. For each puzzle, one angle is revealed. You can ask for the measure of one or two more angles, if needed. Determine all the angle measures using as few hints as you can.

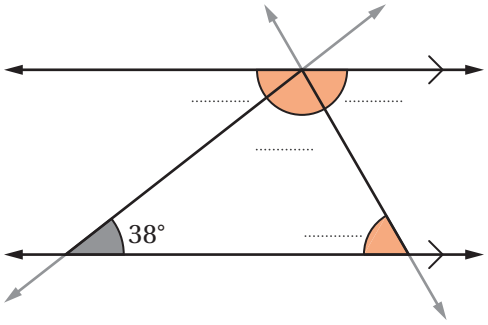
2 Angle Puzzle #1



3 Angle Puzzle #2



4 Angle Puzzle #3

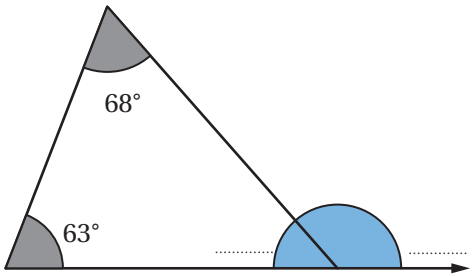


Name: ..... Date: ..... Period: .....

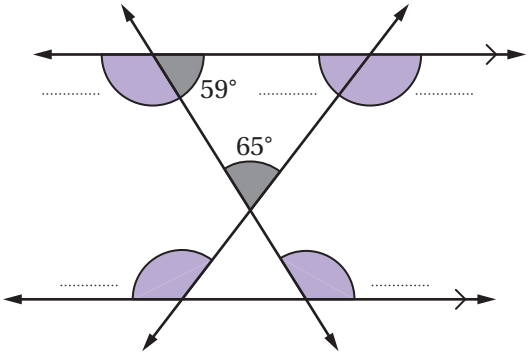
Angle Puzzles (continued)

An exterior angle of a triangle is created when one side of the triangle is extended to form an angle with the adjacent side.

5 Angle Puzzle #4



6 Angle Puzzle #5



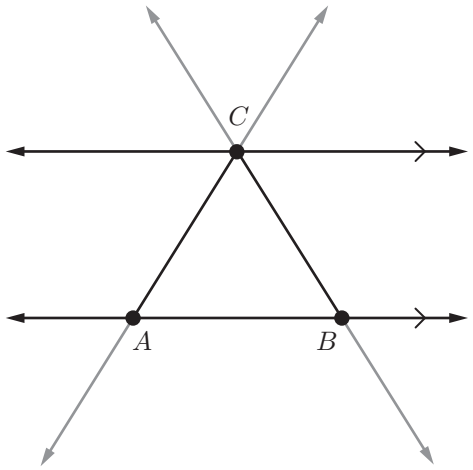
7 **Discuss:** What do you notice about the exterior angle of a triangle and the non-adjacent interior angles?

Activity 2

Name: ..... Date: ..... Period: .....

Triangles and Parallel Lines

8 Imagine that the two lines stay parallel, but you can drag points A, B, and C to make any triangle.



a Mark one set of two or more angles inside the parallel lines that you think will *always* be congruent no matter where you drag the points.

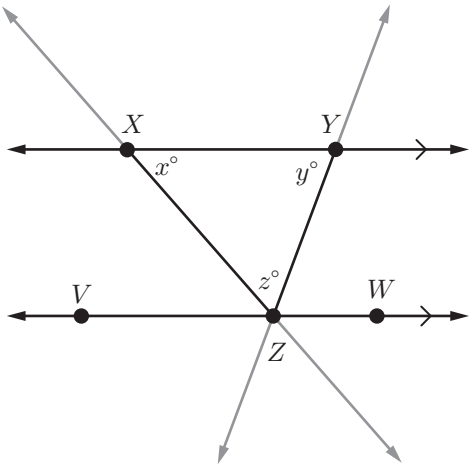
b Explain your thinking.

9 Let's see what happens when you drag points A, B, and C.

Discuss: Are the angles that you thought will always be congruent actually congruent?

10 Fabiana claims that if you tell her the value of x and y, she can use *transformations* to determine the value of z.

What might Fabiana be thinking? Do you believe her claim?



Activity 3

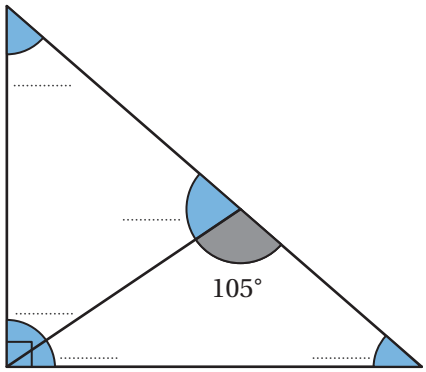
Name: ..... Date: ..... Period: .....

More Angle Puzzles

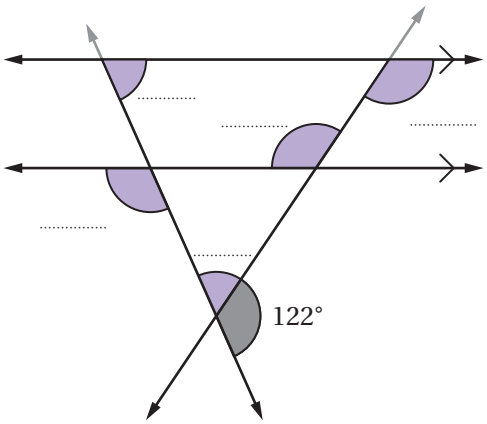
Here are some more angle puzzles. For each puzzle, one angle is revealed. You can ask for the measure of up to two more angles, if needed. Determine all the angle measures using as few hints as you can.

11 Angle Puzzle #6

Note: The square indicates a right angle (90°).



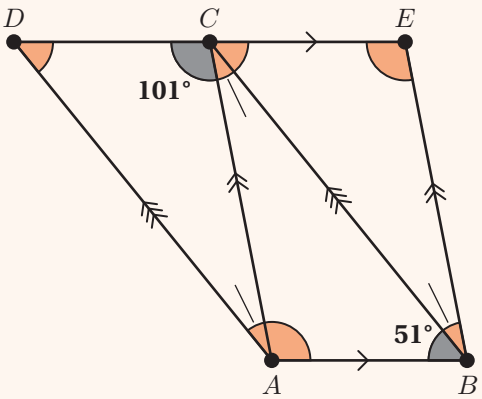
12 Angle Puzzle #7



You're invited to explore more.

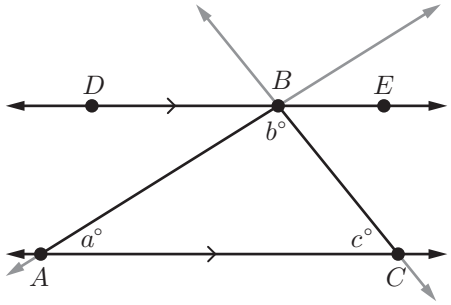
13 Here is a diagram with three pairs of parallel lines. Is it possible to determine all of the angle measures with the given information?

Show or explain your thinking.



14 Synthesis

Discuss What are some angle relationships that can help you determine unknown angle measures?

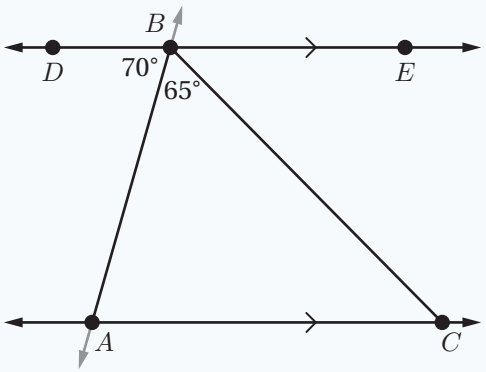


17 Summary 1.12

There are several strategies you can use to determine unknown angle measures in a diagram.

- Use translations to show that corresponding angles on parallel lines are congruent.
- Use rotations to show that vertical angles are always congruent.
- Use the fact that the sum of three interior angles in any triangle is 180 degrees.
- Use the fact that any straight line has an angle measure of 180°.

Here is an example. The measure of angle DBA is 70° and the measure of angle ABC is 65°. Let's determine the remaining angle measures.

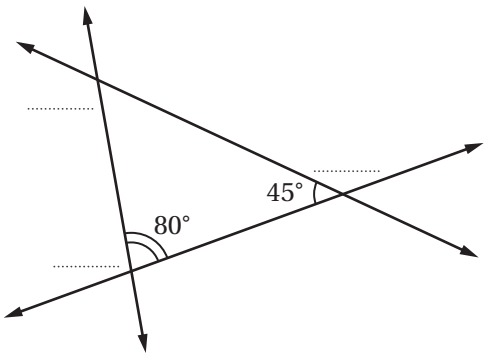


- The measure of $\angle BAC$ must also be 70° because you can translate and then rotate $\angle DBA$ 180° to fit exactly on top of $\angle BAC$.
- $\angle EBC$ must be 45° because it makes a straight line with $\angle DBA$ and $\angle ABC$, and $70 + 65 + 45 = 180$.
- $\angle BCA$ must also be 45° because you can translate and rotate $\angle EBC$ exactly on top of it and also because it is part of a triangle with the 70° and 65° angles.

Practice
1.12

Name: ..... Date: ..... Period: .....

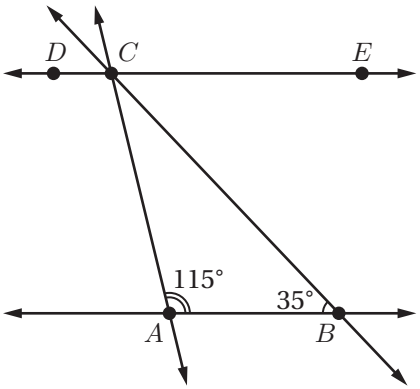
1. Here are three intersecting lines. Determine the missing angle measurements.



Problems 2–4: Line AB is parallel to line DE.

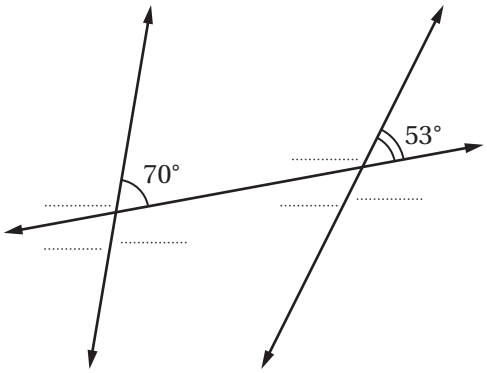
2. What is the measure of $\angle ACD$?

3. What is the measure of $\angle ECB$?



4. What is the measure of $\angle ACB$?

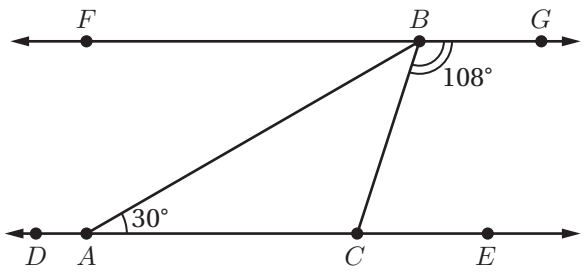
5. This diagram shows three lines with some missing angle measures. Complete the diagram by writing in all of the missing angle measures.



Name: ..... Date: ..... Period: .....

6. Line DE is parallel to line FG.

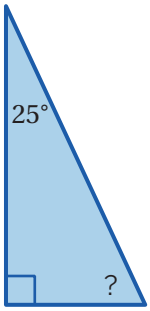
Is it possible to determine all five missing angle measures with the given information? Show or explain your thinking.



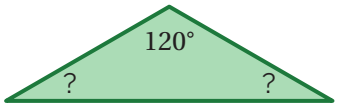
Spiral Review

Problems 7–10: Determine the missing angle measure(s) in each triangle. Show or explain your thinking.

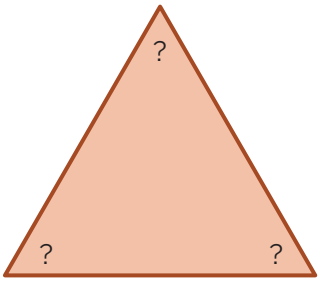
7. Right triangle:



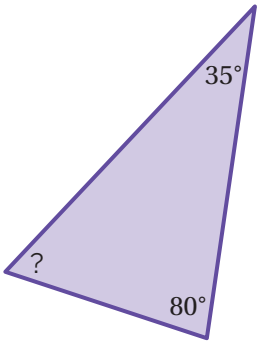
8. Isosceles triangle:



9. Equilateral triangle:

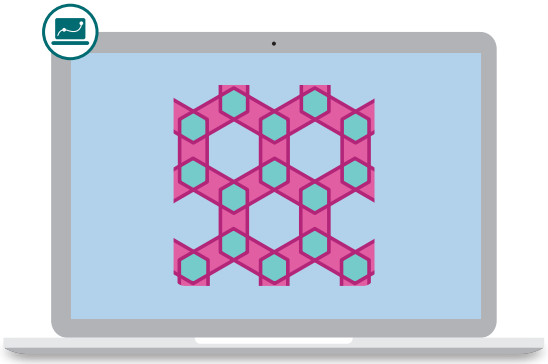


10. Acute triangle:



Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.1, 8.G.2, SMP.7



Tessellate

Let's use rigid transformations to create tessellations.

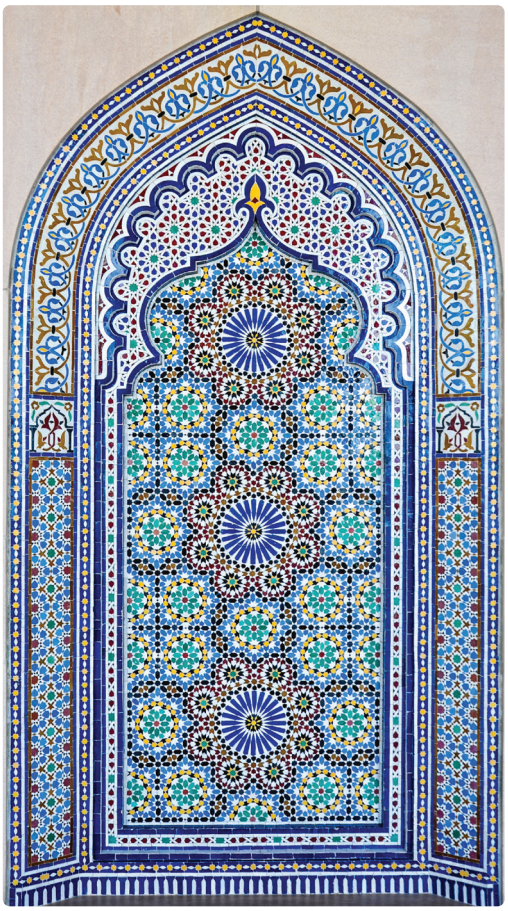
Warm-Up

1 The image shows a wall in the Sultan Qaboos Grand Mosque in Muscat, Oman.

What do you notice? What do you wonder?

I notice:

I wonder:



Hussain Warraich/Shutterstock.com

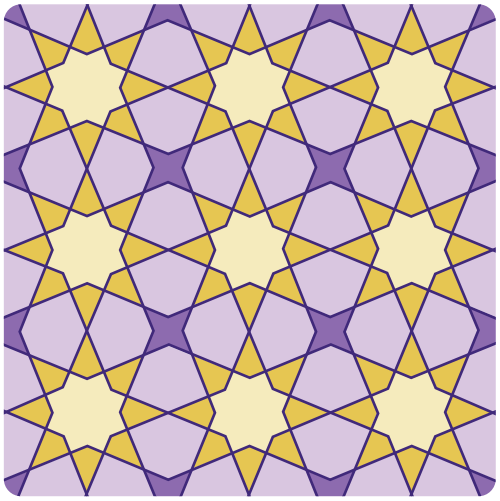
Activity 1

Name: ..... Date: ..... Period: .....

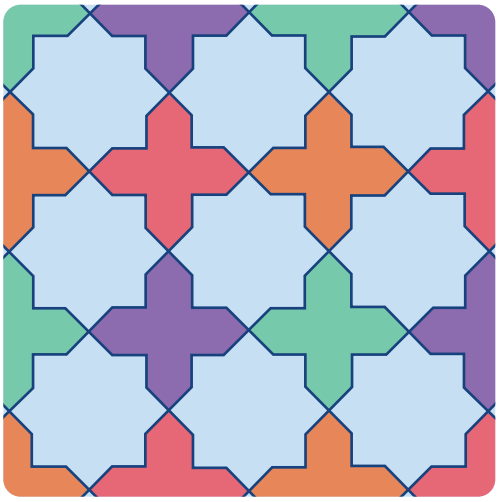
Tessellations and Islamic Art

For over a thousand years, Islamic art and architecture has used tessellations. A *tessellation* is any repeating pattern that can fill an entire plane without gaps.

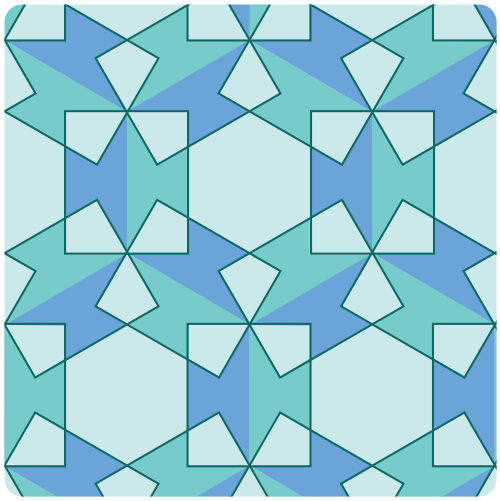
2 Take a look at several examples of tessellations.



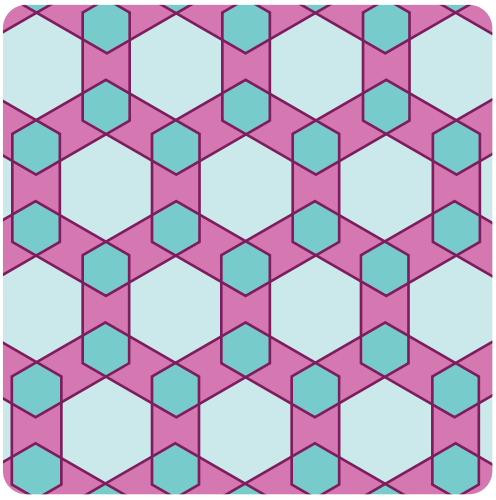
From the Great Mosque of Cordoba, Spain.



From Capella Palatina, Italy.



From Abd al-Samad Complex, Iran.



From the Mosque of Ibn Tulun, Egypt.

Discuss: What are some repeating patterns or figures you see in these designs?

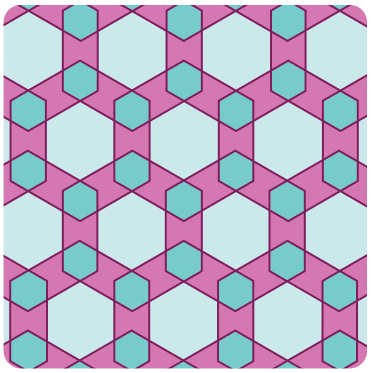
Name: ..... Date: ..... Period: .....

Tessellations and Islamic Art (continued)

3 Let's examine the pattern from the Mosque of Ibn Tulun in Egypt.

Circle one transformation and show or explain where you see that transformation.

Translation Rotation Reflection

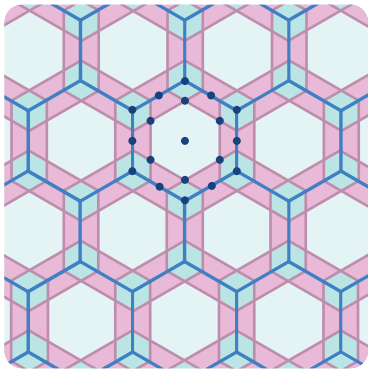


From the Mosque of Ibn Tulun, Egypt.

4 Islamic art and architecture traditionally uses circles and lines to create a shape and then a design within that shape.

Take a look at the construction of this tesselation to see how it started.

How might the dots be useful in creating a design like this?

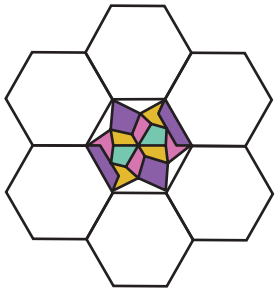


From the Mosque of Ibn Tulun, Egypt.

Name: ..... Date: ..... Period: .....

Hexagon Tessellations

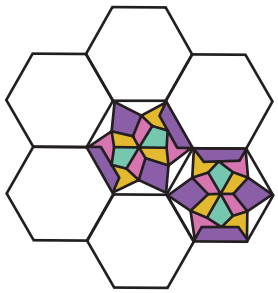
5 Here is a new design within a hexagon.



a Let's copy and translate the center design to make the basis of our tessellation.

b **Discuss:** What happened when we clicked "Tessellate"?

6 Neena performed a translation and then a rotation on the center hexagon.

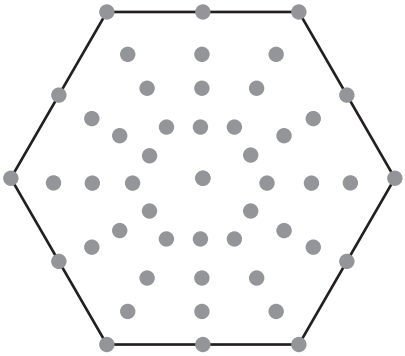


Could she use a *single* transformation to get the same result? Circle one.

Yes No I'm not sure

Explain your thinking.

7 **a** Sketch a design that could be used for a new tesselation.



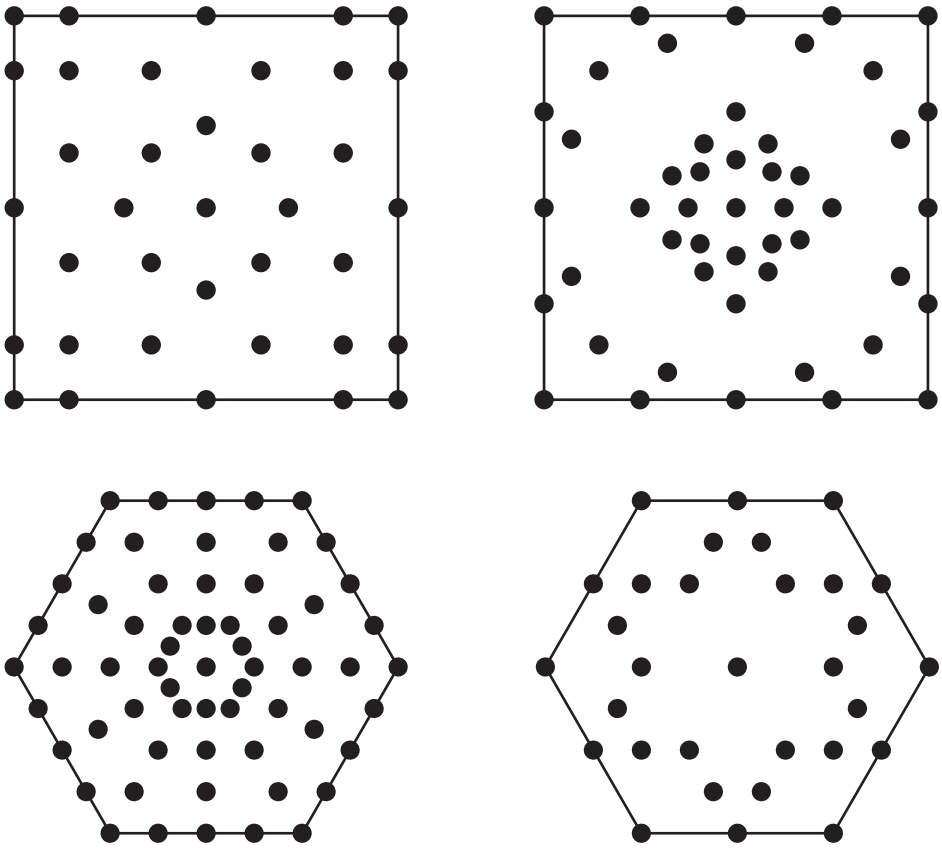
b Imagine what a tessellation of this hexagon might look like.

Name: ..... Date: ..... Period: .....

Make Your Own

8 Follow these steps to create your own tessellation.

a Choose *one* of the four dot patterns for your design.



b Using a straightedge, draw lines to create a pattern like the ones we've explored. Consider starting with a pencil and then going back over the lines with a marker.

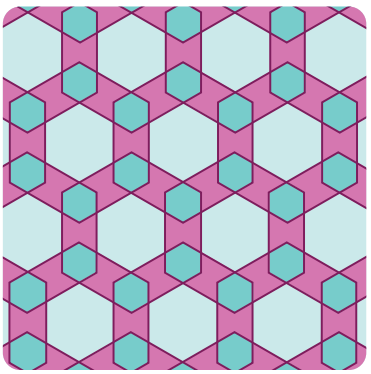
c Tessellate your design onto the Activity 3 Sheet that matches the shape you chose. Place the sheet over your design and use tape to keep it in place. Trace one copy, and then add more copies using translations, rotations, and reflections of the original.

d Add color to your design.

e Write clear, detailed instructions describing how to create your tessellation using transformations.

9 Synthesis

Explain 2–3 things you know about creating tessellations using rigid transformations.



From the Mosque of Ibn Tulun, Egypt.

12 Summary 1.13

You can use rigid transformations to create interesting repeating patterns of figures, such as *tessellations*. A tessellation is any repeating pattern that can fill an entire plane without gaps.

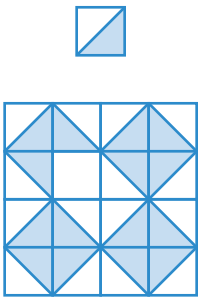
We see tessellations in many creations from around the world, but they are especially common in Islamic art and architecture. You can also see these repeating geometric patterns in floors, ceilings, wall art, and even on clothing designs.

Practice
1.13

Name: ..... Date: ..... Period: .....

1. Here is the last square Alexis needs to finish the tessellation.

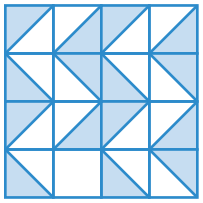
Describe how Alexis can transform the square so that it fits in the missing spot and completes the tessellation.



Problems 2–4: Alexis created a new tessellation using the same squares.



2. What transformation can Alexis use on the square to complete the tessellation?



3. Complete the tessellation.

4. Describe or show how you see a translation, rotation, and reflection in the tessellation.

Translation:

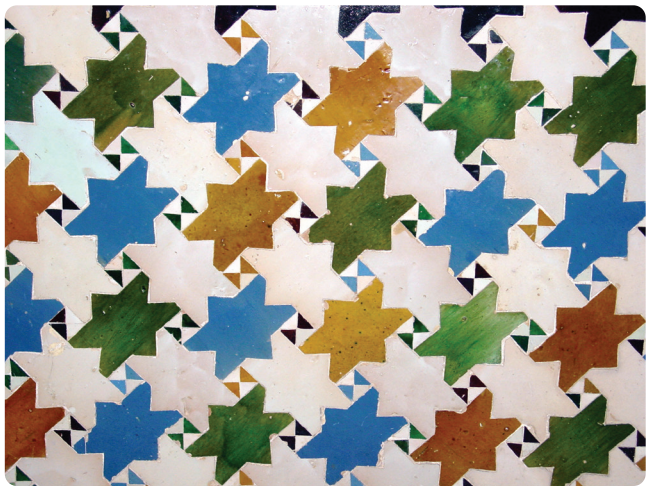
Rotation:

Reflection:

5. Draw a triangle and create a tessellation by arranging 3 copies of your triangle. Label the angles for each triangle as a, b, or c to show which angles have the same measure.

Name: ..... Date: ..... Period: .....

6. Here is one example of the many complex tile patterns found in the famous 14th century Moorish palace of Alhambra in Spain. Describe one transformation that you notice.



Vladimir Korostyshevskiy/Shutterstock.com

7. Are there any triangles that would *not* work for creating tessellations? Explain your thinking.

Spiral Review

Problems 8–9: Use this list of angles.

$60°$	$20°$	$100°$	$40°$	$110°$	$50°$	$30°$

8. Write one possible combination of three angle measures that can form a triangle.

9. Write *all* the remaining possible combinations of three angle measures that can form a triangle.

10. Line segment XY has a length of 5 units. Segment XY is rotated $90°$ clockwise around the origin and then translated 2 units left and 1 unit down to form segment $X'Y'$. What is the length, in units, of line segment $X'Y'$?

Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.1, 8.G.1.a, 8.G.1.b, 8.G.2, 8.G.3, 8.G.5, SMP.5, SMP.7

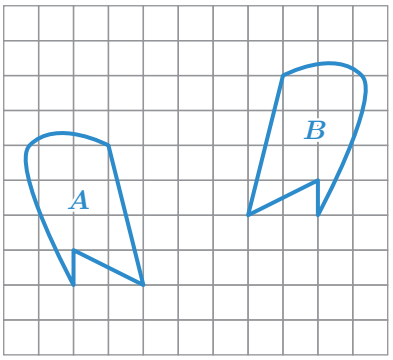
Practice Day 2



Let's practice what you've learned so far in this unit!

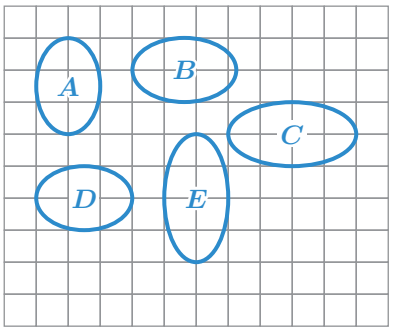
1. Is figure A congruent to figure B?

Explain your thinking.



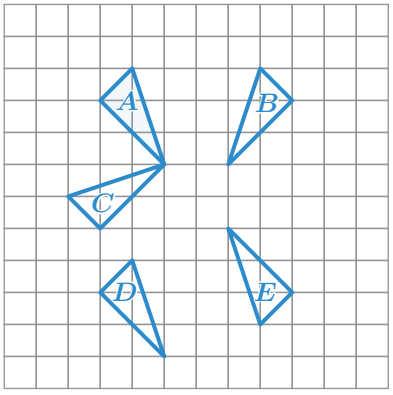
2. Are any of these figures congruent to each other?

Explain how you know.



3. Select *all* the triangle(s) that triangle A can map onto using a single rotation.

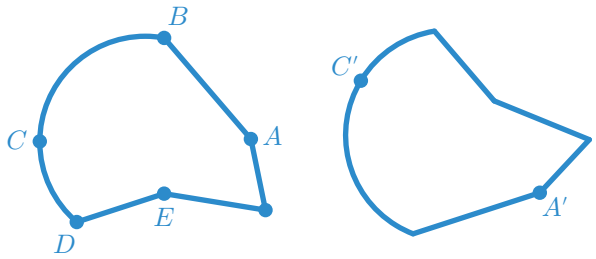
- ☐ **A.** Triangle A
- ☐ **B.** Triangle B
- ☐ **C.** Triangle C
- ☐ **D.** Triangle D
- ☐ **E.** Triangle E



Name: ..... Date: ..... Period: .....

Practice Day 2

4. Here are two congruent figures with some corresponding points labeled.

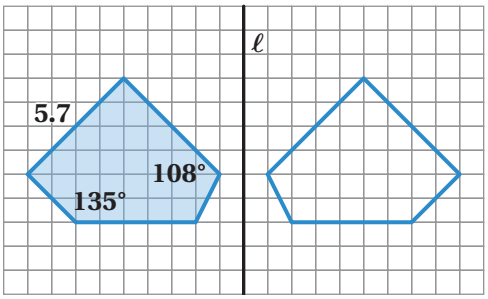


a Draw the points corresponding to B, D, and E, and label them B', D', and E'.

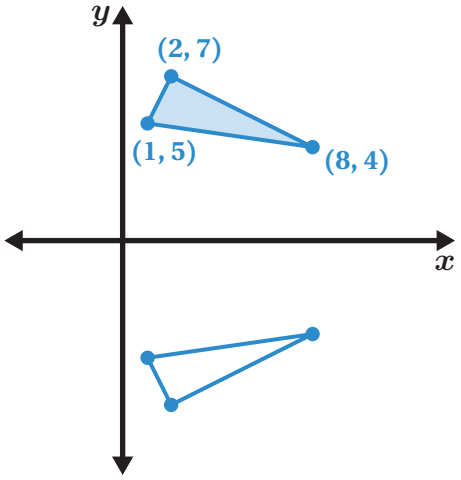
b Draw line segments AD and $A'D'$ and compare their lengths. Repeat for segments BC and $B'C'$ and for segments AE and $A'E'$. What do you notice?

c Can these figures have a pair of corresponding segments with different lengths?
Explain your thinking.

5. The shaded pre-image is reflected over line ℓ to create the unshaded image. Use the information in the pre-image to label the measures of the corresponding angles and corresponding sides in the image.



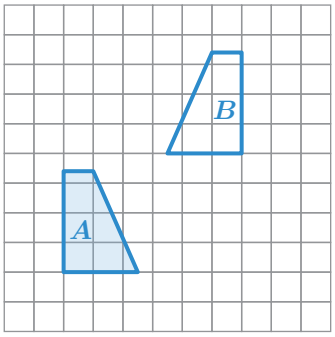
6. The shaded triangle is reflected over the x-axis. Label the coordinates of the corresponding points on the image.



Name: _____ Date: _____ Period: _____

Practice Day 2

7. Select *all* the sequences of transformations that could map figure A onto figure B.



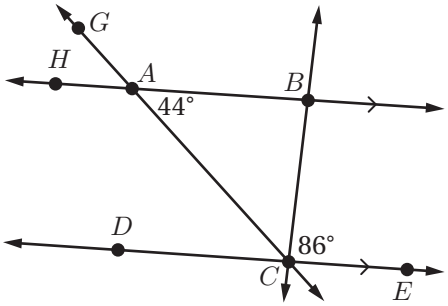
A. A single reflection.

B. A rotation, then a translation.

C. A reflection, then a translation.

D. A reflection, then another reflection.

8. Here is a pair of parallel lines cut by two transversals.



a Explain why the measure of angle GAH is 44°.

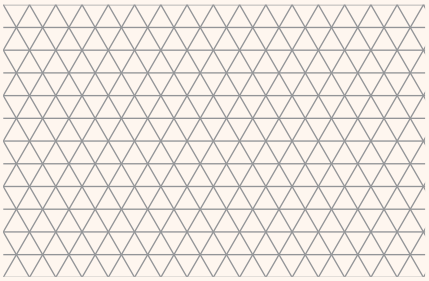
b What is the measure of angle ABC?

c What is the measure of angle ACB?

You're invited to explore more.

9. Sketch a figure with sides along the grid lines. Label the figure A.

10. Use rigid transformations on figure A to create an interesting design made from copies of figure A.

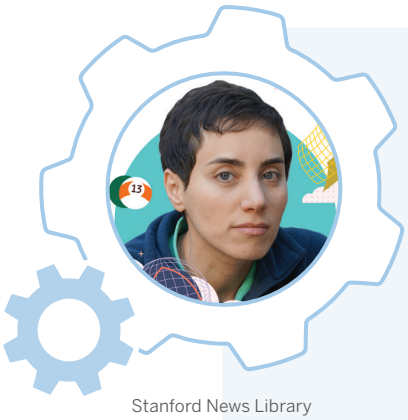


Career Connection

Have you ever played a game against a robot? Starting in 1770, the Mechanical Turk, which was presented as the world's first chess-playing robot, dazzled audiences for 84 years.

It was later revealed as a hoax because humans operated it the entire time. But, this didn't stop the development of robots that could *actually* play chess. In 1912, the world's first real chess robot, El Ajedrecista, used mechanical arms to move chess pieces. Fast forward to the 1980s, the computer Deep Blue won the first timed chess game against a world champion human chess player! More recently, robots have been developed to play pool and table tennis!

Robotics engineers use transformations to design robots to perform complex movements. Rotations allow robotic arms to pivot around joints, mimicking how human joints work. Robotics engineers work in many types of industries.



Stanford News Library

Meet Maryam Mirzakhani

Maryam Mirzakhani, who grew up in Iran, was a mathematician and Professor at Princeton University and Stanford University. In 2014, she was awarded the Fields Medal, one of the highest honors in math! She was the first woman and the first Iranian to earn this honor. Along with her colleagues, she used transformations to prove properties about certain quadrilateral shapes, such as billiard tables.

Are you interested in studying mathematics further or robotics? What can you do to learn more?

Math in the World

A pool table is similar to a billiards table, except it has pockets. What do you think will happen when the white ball reaches the larger group of balls? How might reflections be used?



Corepics VOF/Shutterstock.com

Math Mindset

How did you select and use both *tools* and *words* to help describe and perform transformations during this unit?

Unit 2

Dilations, Similarity, and Slope

Big Ideas in This Unit

CC2 Multiple Representations of Functions

CC4 Transformational Geometry

Questions for Investigation

- What does it mean to dilate a figure?
- How can transformations be used to decide whether two figures are similar?
- How can similar triangles be used to determine the slope of a line?

Explore: Standard Paper Sizes
Are all rectangles scaled copies of each other?

Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!

Not yet — Almost — I got it!

I can . . .	Before	After
Describe how a dilation affects a figure.		
Dilate a figure on a coordinate plane.		
Identify the center and scale factor of a dilation.		
Describe the effects of scale factors that are greater than 1, between 0 and 1, or equal to 1 when dilating a figure.		
Describe a sequence of transformations that maps a pre-image onto an image.		
Use a sequence of transformations to justify whether two figures are similar.		
Determine whether two figures are similar using corresponding angles and side lengths.		
Justify that two triangles are similar using their angle measures.		
Determine missing side lengths in similar triangles.		
Use similar triangles to determine the slope of a line.		

I can . . .	Before	After
Explain how to calculate the slope of a line.		
Use similar triangles to explain why the slope is the same between any two points on a non-vertical line on the coordinate plane.		

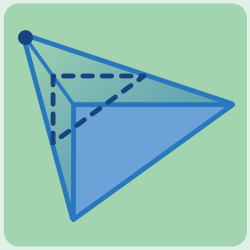
Sub-Unit

1

Dilations



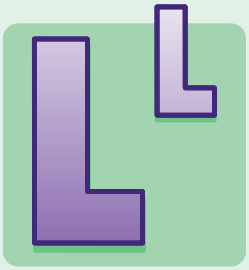
Explore
Standard Paper Sizes



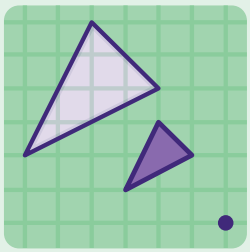
Lesson 1
Sketchy Dilations



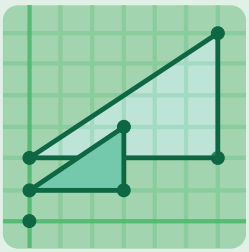
Lesson 2
Dilation Mini Golf



Lesson 3
Transformation Targets With Dilations



Lesson 4
Match My Dilation



Lesson 5
Dilations on a Plane

Name: ..... Date: ..... Period: .....

Transformational Geometry | Building Toward 8.G.4, SMP.2, SMP.7

Explore: Standard Paper Sizes



Are all rectangles *scaled copies* of each other?

Warm-Up

You will be given two sheets of paper. What do you notice? What do you wonder?

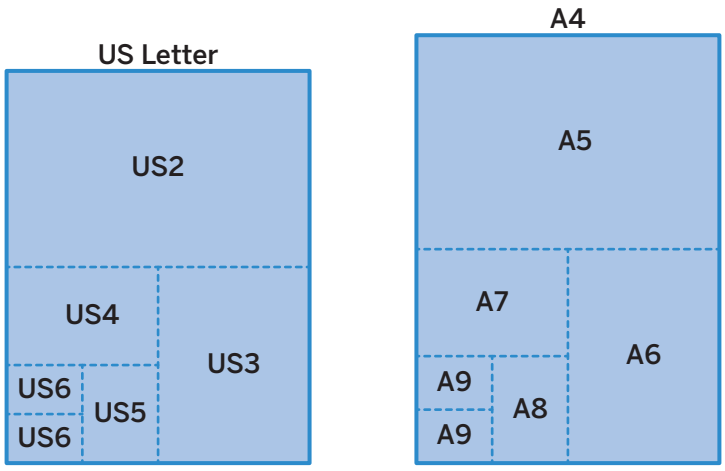
1. I notice . . .

2. I wonder . . .

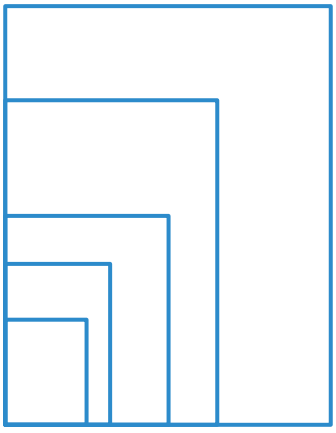
Name: ..... Date: ..... Period: .....

Sorting Rectangles

3. Cut each sheet of paper in half. Then cut each sheet in half again. Continue cutting each sheet in half, as illustrated in the diagram. Label each rectangle as shown.



4. For each sheet of paper, stack the rectangles so that they all line up at a corner, as shown in the diagram. You will have two stacks of rectangles. On each stack of rectangles, draw a line from the bottom left corner to the top right corner of the biggest rectangle. What do you notice?



Name: Date: Period:

Sorting Rectangles (continued)

5. Determine the *ratio* of the long side to the short side for each rectangle created from the US4 paper. Round to the nearest tenth.

Rectangle	Length of short side (mm)	Length of long side (mm)	Ratio of long side to short side
Full Sheet	216	280	1.3
US2	140	216	
US3	108	140	
US4	70	108	
US5	54	70	
US6	35	54	

6. Determine the ratio of the long side to the short side for each rectangle created from the A4 paper. Round to the nearest tenth.

Rectangle	Length of short side (mm)	Length of long side (mm)	Ratio of long side to short side
Full Sheet	210	297	1.4
A5	149	210	
A6	105	149	
A7	74	105	
A8	53	74	
A9	37	53	

7. **Discuss:** What do you notice about the ratios for the rectangles?

Name: ..... Date: ..... Period: .....

Building Math habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet Almost I got it!

I can represent real-world problems and interpret their solutions within the context of the problem.

Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed.

Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

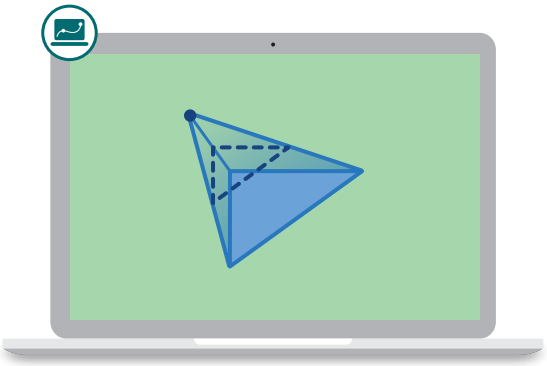
Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

Not yet Almost I got it!

Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.3, SMP.3

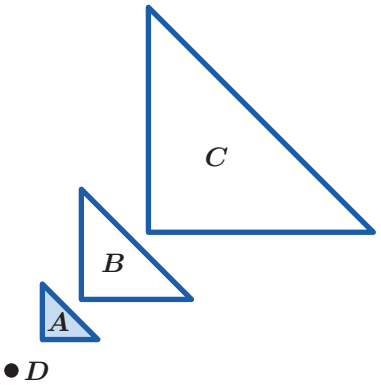


Sketchy Dilations

Let's explore scaled copies and dilations.

Warm-Up

1 Triangles B and C are images of triangle A.



What do you notice? What do you wonder?

I notice:

I wonder:

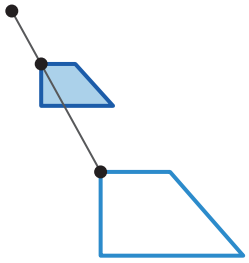
Activity 1

Name: ..... Date: ..... Period: .....

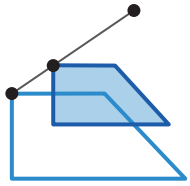
Dilations and Scaled Copies

2 **a** Take a look at these sketches, each created by a different stretching machine. The pre-image is shaded. The image is unshaded.

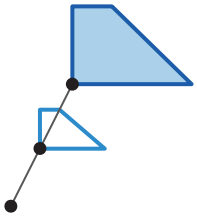
Machine #1



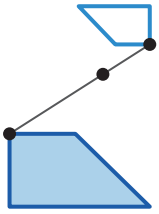
Machine #2



Machine #3

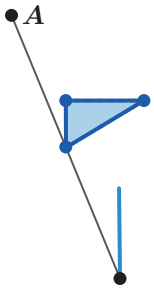


Machine #4



b **Discuss:** How does each stretching machine work?

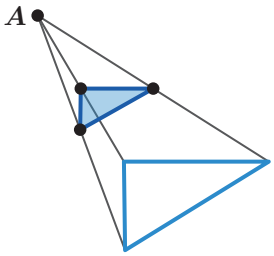
3 Here is a *pre-image* and part of its *image* in a stretching machine.

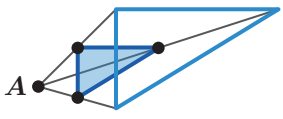


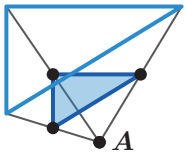
a Sketch the rest of its image.

b How would the image be different if point A were closer to the triangle?

4 Stretching machines create **dilations**. A dilation is a new type of transformation that creates a scaled copy.







Does moving point A change the size of the image, its location, or both? Circle one.

Size Location Both

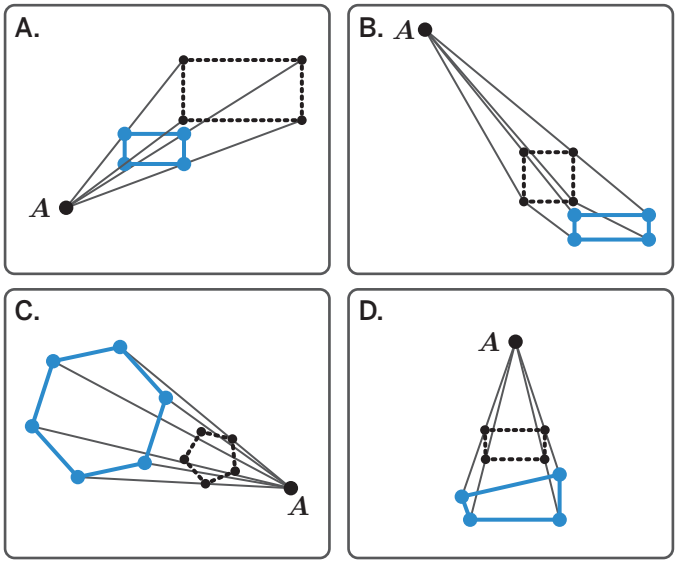
Explain your thinking.

Name: ..... Date: ..... Period: .....

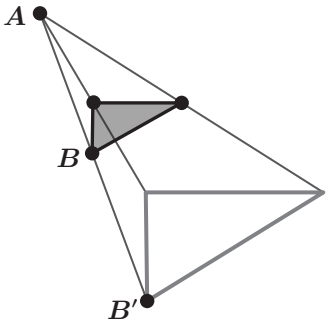
Dilation Play

5 Circle one image that is *not* a dilation (in other words, it could not have been created using a stretching machine).

Explain your thinking.



6 Let's watch what happens when points A and B' move.



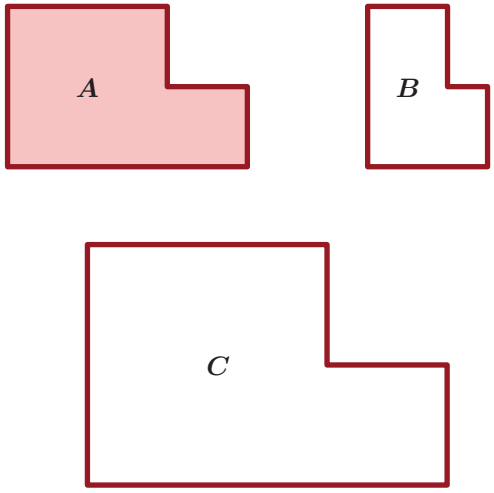
Here are several dilation challenges. Select *all* the ones you think are possible.

- ☐ **A.** Make the image smaller than the pre-image.
- ☐ **B.** Make the image the same size as the pre-image, but in a different location.
- ☐ **C.** Make the distance between A and B twice the distance between B and B'.
- ☐ **D.** Make points A, B, and B' form a triangle.

7 Which figure could be a *dilation* of figure A? Circle one.

Figure B Figure C Both Neither

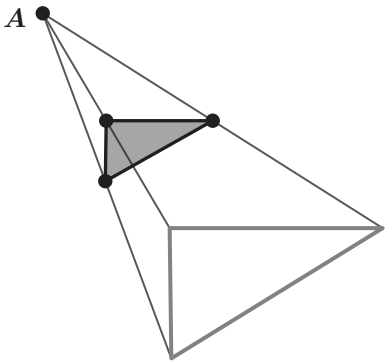
Explain your thinking.



8 Synthesis

Describe how dilations work.

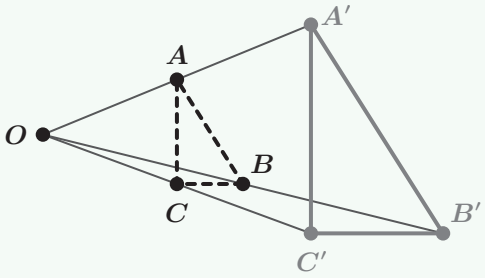
Use this example if it helps with your thinking.



11 Summary 2.01

A **dilation** is a type of transformation that creates *scaled copies*. Dilating a figure means moving each of its vertices along a line that's extended from the **center of dilation**. The original distance from the center of dilation to each vertex on the *pre-image* is multiplied by the same number to create the dilated image.

For example, Triangle ABC was dilated from the center of dilation, point O, to create triangle $A'B'C'$.



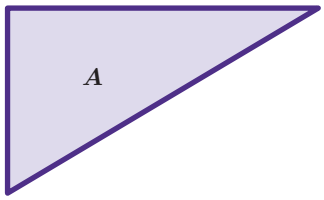
center of dilation The point from which we measure distances in a dilation.

dilation A type of transformation that creates scaled copies. A dilation moves every point in a figure away from a center of dilation by the distance of a scale factor.

Practice
2.01

Name: ..... Date: ..... Period: .....

Problems 1–2: Here is triangle A.



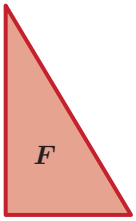
1. Draw a dilation of triangle A where the image has a larger area.

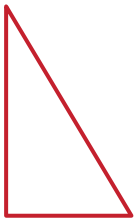
2. Draw a dilation of triangle A where the image has a smaller area.

3. Rectangle A is 12 centimeters by 3 centimeters. Rectangle B is a scaled copy of rectangle A. Select *all* the pairs that could be the dimensions of rectangle B.

- ☐ **A.** 6 centimeters by 1.5 centimeters
- ☐ **B.** 10 centimeters by 1 centimeter
- ☐ **C.** 18 centimeters by 4.5 centimeters
- ☐ **D.** 6 centimeters by 1 centimeter
- ☐ **E.** 80 centimeters by 20 centimeters

4. Select *all* the figures that could be a scaled copy of triangle F.

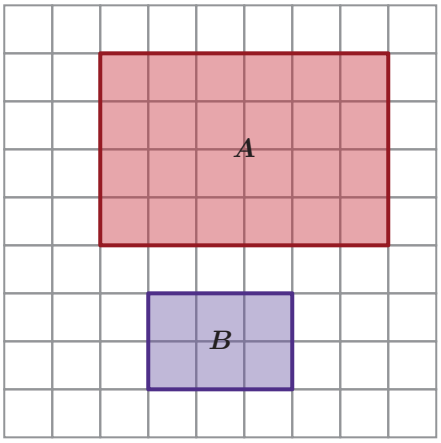


- ☐ **A.** 
- ☐ **B.** 
- ☐ **C.** 
- ☐ **D.** 
- ☐ **E.** 

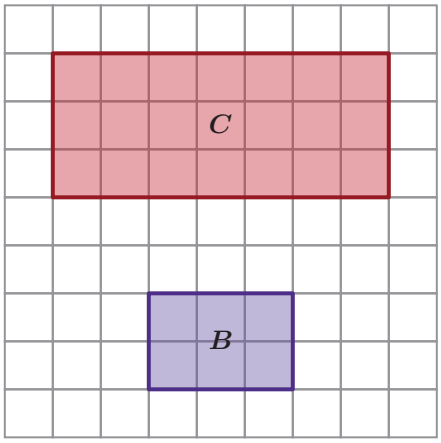
Name: ..... Date: ..... Period: .....

Problems 5–6: Determine whether each pair of figures represents an original figure and its scaled copy. Explain your thinking.

5.



6.



Spiral Review

7. Which of these sets of angle measures could be the three interior angle measures of a triangle?

A. $40°, 50°, 60°$

B. $50°, 60°, 70°$

C. $60°, 70°, 80°$

D. $70°, 80°, 90°$

8. What makes two figures congruent? Explain your thinking.

9. The point $(-2, 7)$ is translated 1 unit to the left and 6 units up. What are the coordinates of the image?

A. $(-1, 13)$

B. $(4, 8)$

C. $(8, 8)$

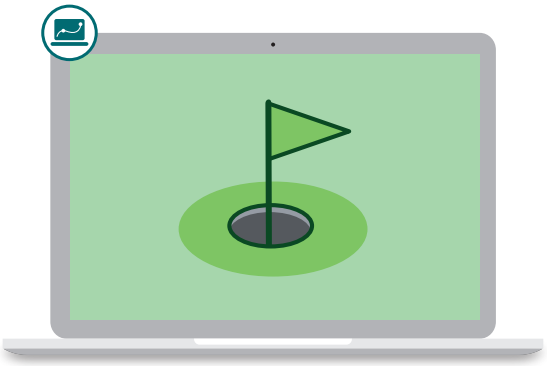
D. $(-3, 13)$

Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.3, SMP.6

Dilation Mini Golf

Let's dilate points using measurement tools.



Warm-Up

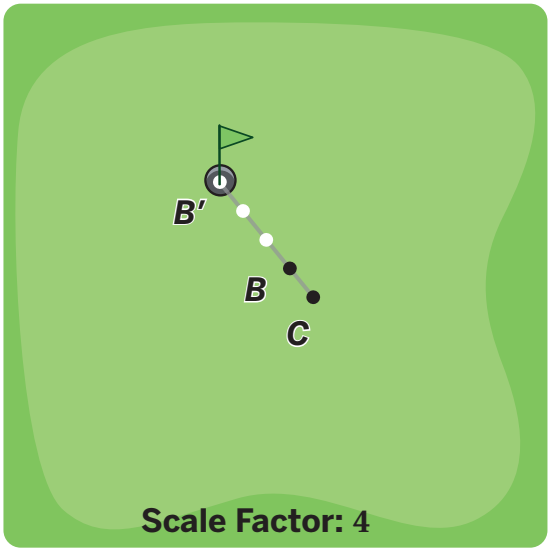
1 Welcome to Dilation Mini Golf.

Your goal is to get the ball into the hole by dilating point B.

a. Let's watch an animation to see what we mean.

b. **Discuss:**

- In this situation, what is the pre-image? What is the image?
- What do you think *scale factor:* 4 means?

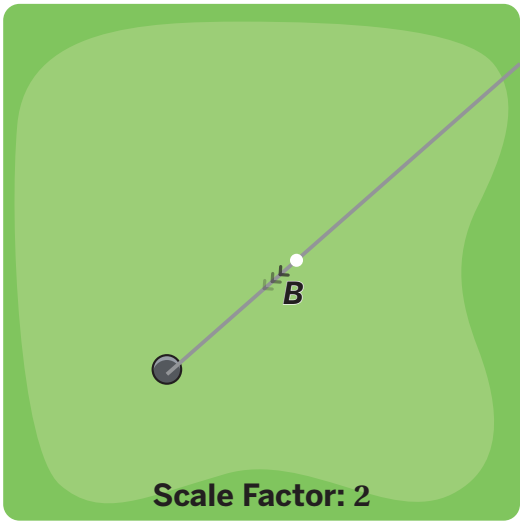


Name: ..... Date: ..... Period: .....

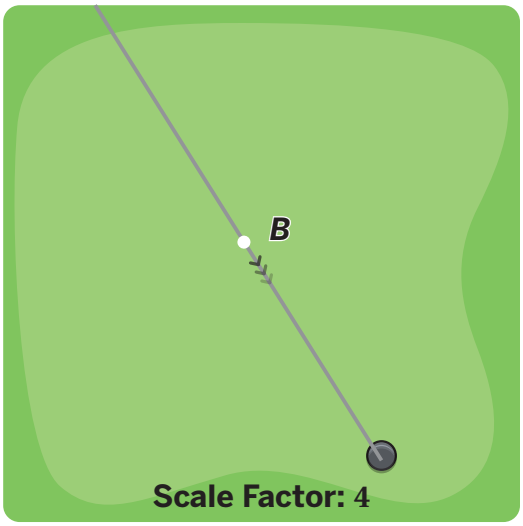
Rounds of Mini Golf

2 Point B will be dilated using point C as the center of dilation and a scale factor of 2.

Mark where point C should be so that the image of point B lands in the hole.



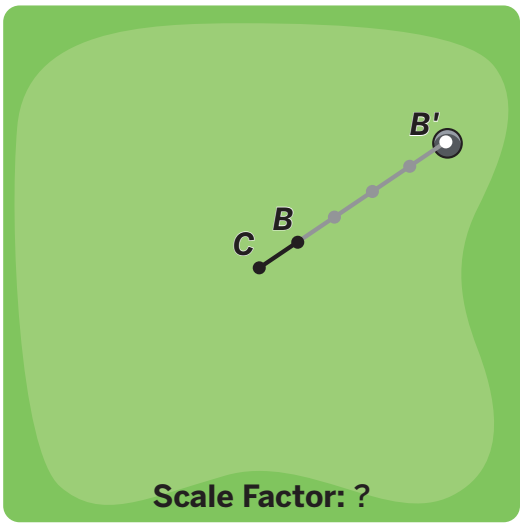
3 Point B will be dilated using point C as the center of dilation and a scale factor of 4. Mark where point C should be so that the image of point B lands in the hole.



4 Point B was dilated using point C as the center of dilation.

What is the scale factor?

Explain your thinking.



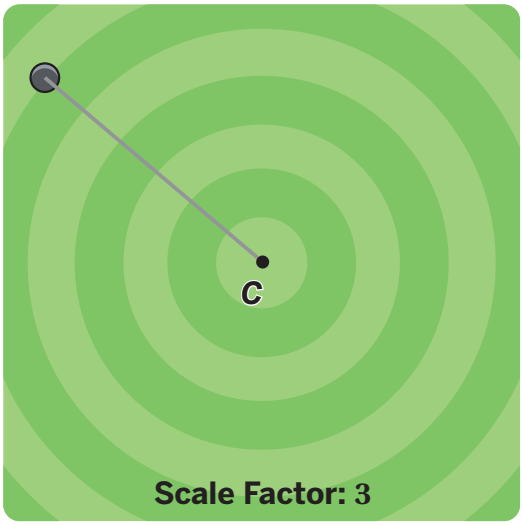
Name: Date: Period:

Dilation Distances

5 Now let's place the ball (point B). This will be your pre-image.

Point B will be dilated using point C as the center of dilation and a scale factor of 3.

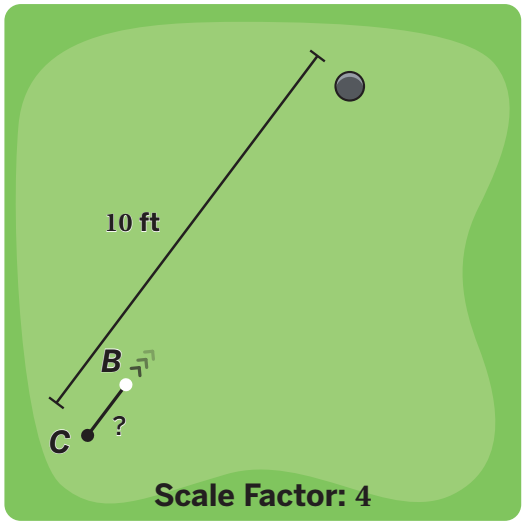
Mark point B so that its image lands in the hole.



6 Point B will be dilated using point C as the center of dilation and a scale factor of 4.

What should the distance between point C and point B be so that the image is in the hole?

Explain your thinking.

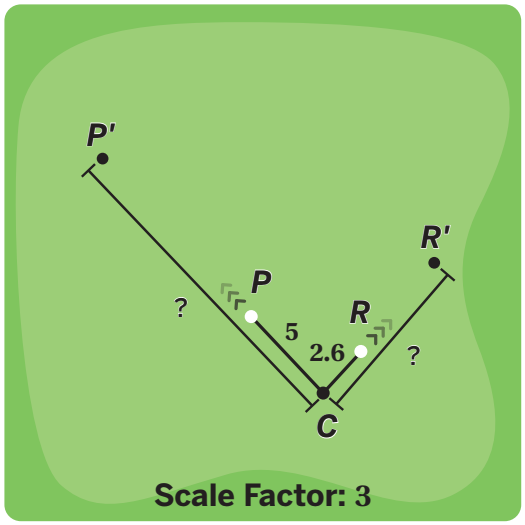


7 We can dilate more than one point at a time!

Points P and R will be dilated using point C as the center of dilation and a scale factor of 3.

What is the distance between point C and the image of point P?

What is the distance between point C and the image of point R?

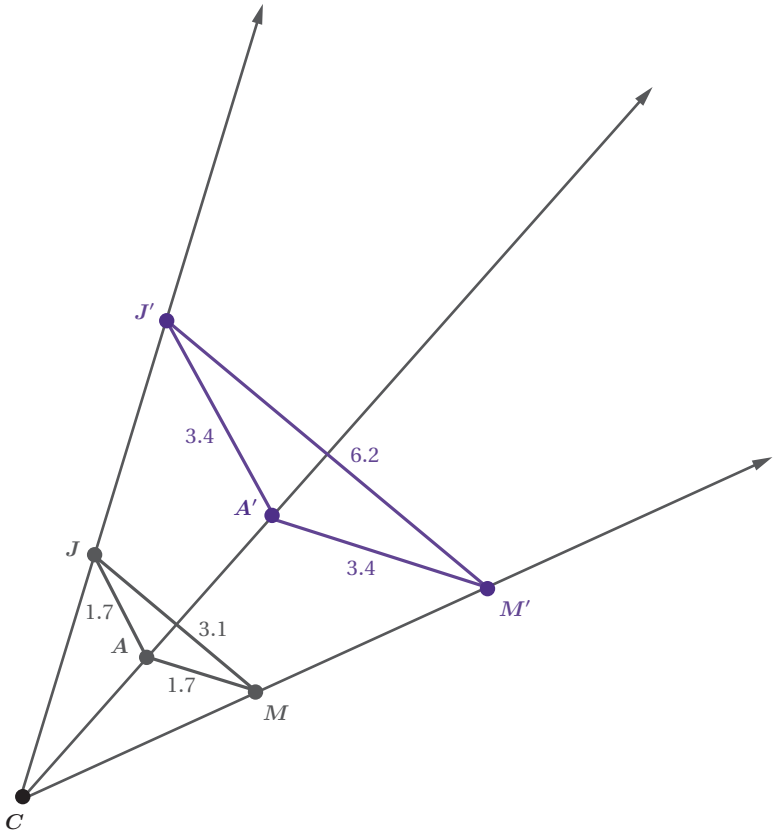


Activity 3

Name: ..... Date: ..... Period: .....

Dilating a Triangle

8 Let's watch an animation to see how triangle JAM is dilated using point C as the center of dilation and a scale factor of 2.



a Describe all of the ways you see a scale factor of 2 represented in this diagram.

b Select at least one more scale factor.

3 1.5 $\frac{3}{4}$ Other: .....

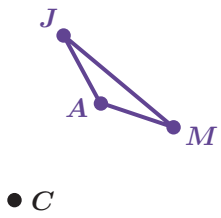
c On the same diagram, dilate triangle JAM using center C and each scale factor you chose.

d List everything that's alike about triangle JAM and its dilations.

9 Synthesis

Describe how to dilate a point or a figure given a center of dilation and a scale factor.

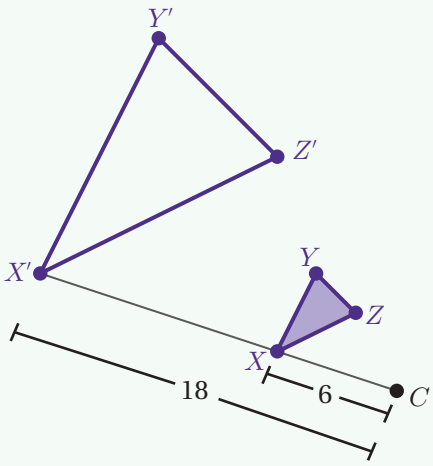
Use the example if it helps you with your thinking.



Scale Factor: 2

12 Summary 2.02

One strategy for dilating a figure is to measure the distance between the center of dilation and one of the pre-image points, multiply that distance by the scale factor, then place the image point that distance away from the center of dilation along the same line. Repeat this strategy with all the other points in the pre-image.



Practice
2.02

Name: ..... Date: ..... Period: .....

Problems 1–2: Segment AB is 3 centimeters long. Point O is the center of dilation.

1. How long is the image of segment AB after a dilation with a scale factor of 5?

2. How long is the image of segment AB after a dilation with a scale factor of 3.7?

Problems 3–6: Here are points A and B. Plot the points after each dilation.



3. Point C is the image of point B using point A as the center of dilation and a scale factor of 2.

4. Point D is the image of point A using point B as the center of dilation and a scale factor of 2.

5. Point E is the image of point B using point A as the center of dilation and a scale factor of $\frac{1}{2}$.

6. Point F is the image of point A using point B as the center of dilation and a scale factor of $\frac{1}{2}$.

Problems 7–8: Here is a diagram.



7. Point H is the image of point G using point F as the center of dilation. What is the scale factor?

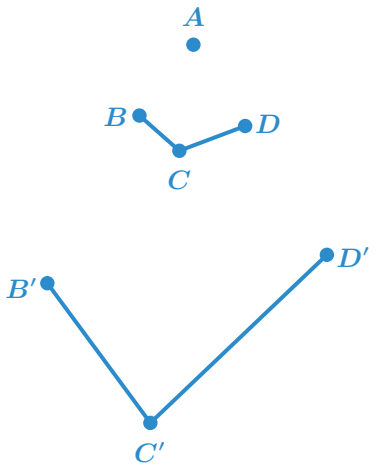
8. If the distance from point F to point H is 9 units, what is the distance from point F to point G?

Show or explain your thinking.

Name: ..... Date: ..... Period: .....

9. Isaiah claims that figure $B'C'D'$ is a dilation of figure BCD using point A as the center of dilation. Is Isaiah's claim correct?

Explain your thinking.

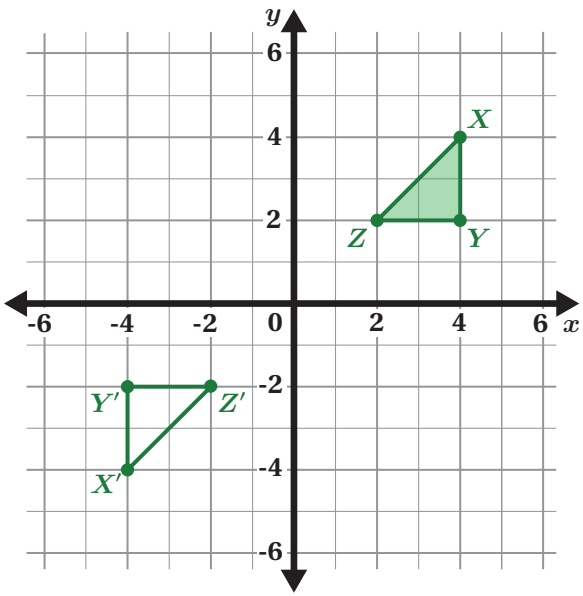


Spiral Review

Problems 10–11: Refer to triangle XYZ and triangle $X'Y'Z'$.

10. Determine whether each transformation maps triangle XYZ onto triangle $X'Y'Z'$. Select *True* or *False* for each statement.

	True	False
A reflection over the y-axis, followed by a reflection over the x-axis.		
A reflection over the x-axis followed by a reflection across the y-axis.		
A reflection across the x-axis followed by a translation 3 units left.		



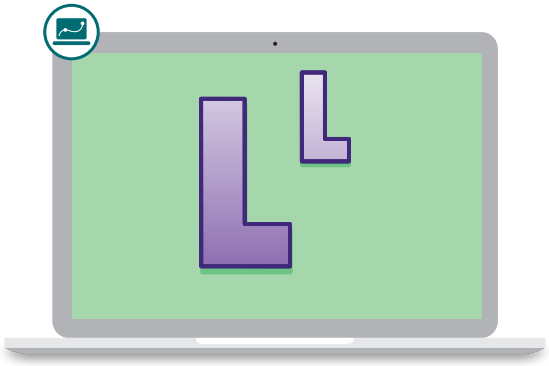
11. Is triangle XYZ congruent to triangle $X'Y'Z'$? Explain your thinking.

Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.1, 8.G.3, 8.G.4, SMP.1, SMP.6

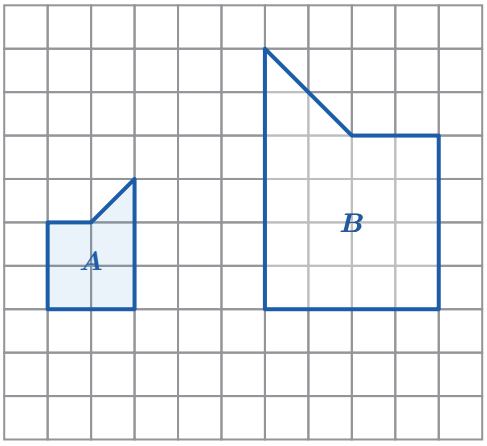
Transformation Targets With Dilations

Let's transform figures using dilations.



Warm-Up

1 Describe a *sequence of transformations* that could map polygon A onto polygon B.



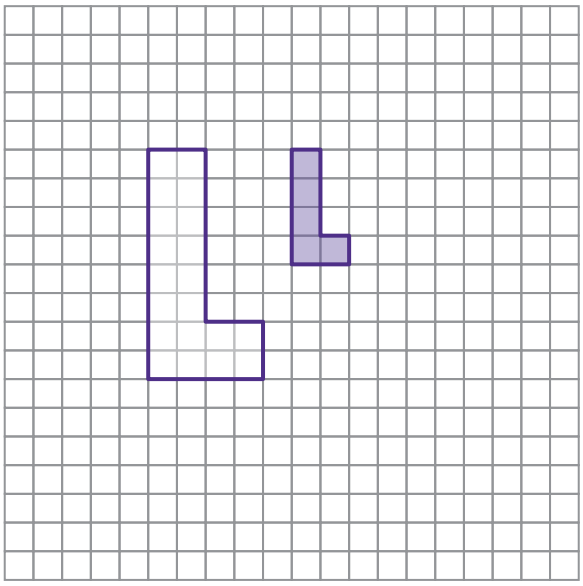
Activity 1

Name: ..... Date: ..... Period: .....

Transformation Targets

2 **Challenge #1:** Show or describe a sequence of transformations to map the pre-image (shaded) onto the image.

Draw any points or lines that you use in your sequence of transformations.



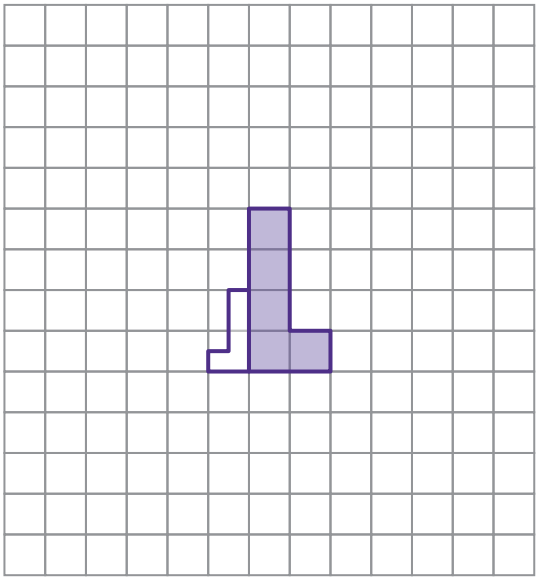
3 Aditi says you can complete the previous challenge with a dilation and a translation. Emiliano says you can complete this challenge with only a dilation. Whose claim is correct?

Aditi's Emiliano's Both Neither

Explain your thinking.

4 **Challenge #2:** Show or describe a sequence of transformations to map the pre-image (shaded) onto the image.

Draw any points or lines that you use in your sequence of transformations.



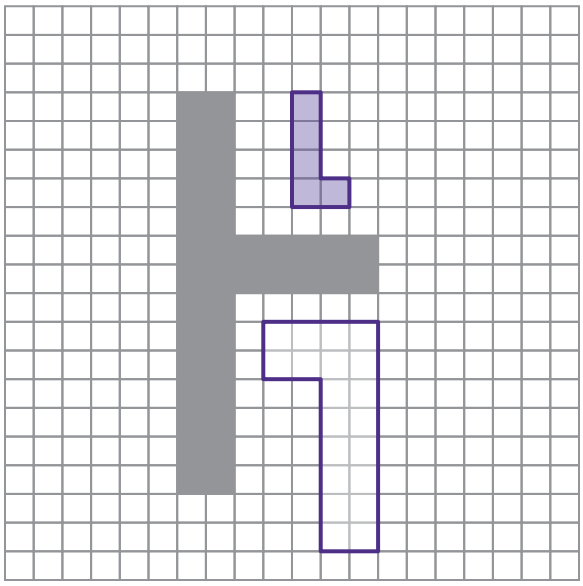
Name: ..... Date: ..... Period: .....

Now With Obstacles!

5 **Challenge #3:** Show or describe a sequence of transformations to map the pre-image (shaded) onto the image.

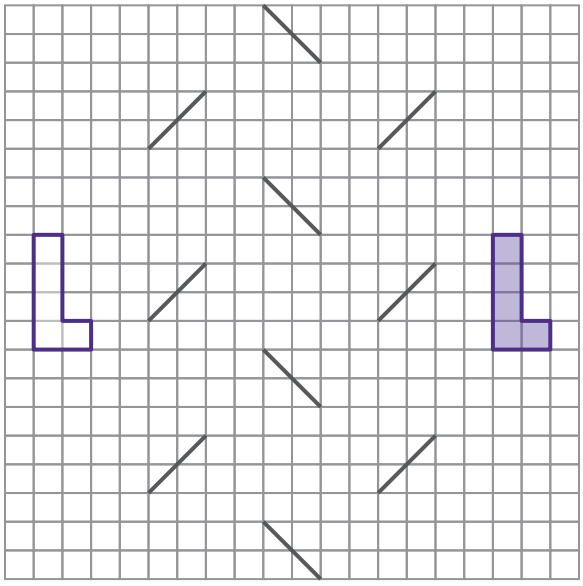
Avoid the obstacles!

Draw any points or lines that you use in your sequence of transformations.



6 Describe a sequence of transformations that you think will map the pre-image (shaded) onto the image.

Avoid the obstacles!



7 Sketch the result of each transformation you described in the previous problem on the grid.

Draw any points or lines that you use in your sequence of transformations.

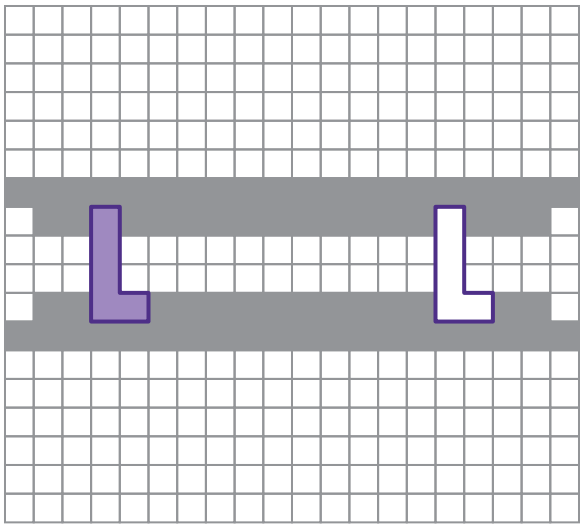
Name: ..... Date: ..... Period: .....

Now With Obstacles! (continued)

8 **Challenge #4:** Describe a sequence of transformations to transform the pre-image (shaded) onto the image.

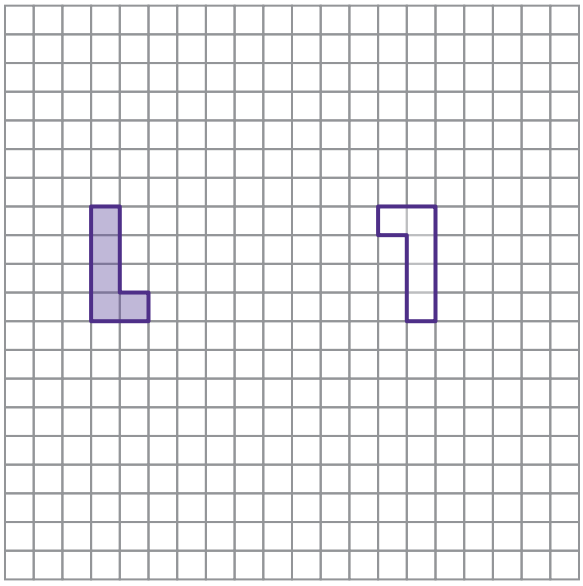
Avoid the obstacles!

Draw any points or lines that you use in your sequence of transformations.



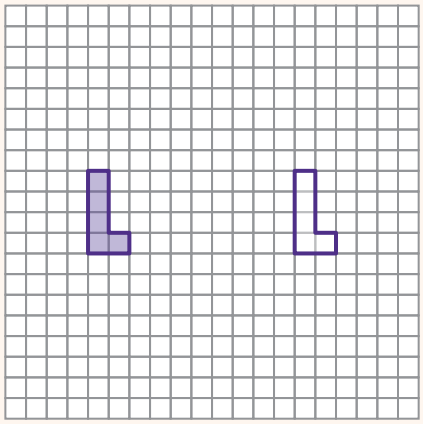
9 **Challenge #5:** Describe a sequence of transformations that uses *only* dilations to map the pre-image (shaded) onto the image!

Draw the center of dilation and the result of each dilation.



You're invited to explore more.

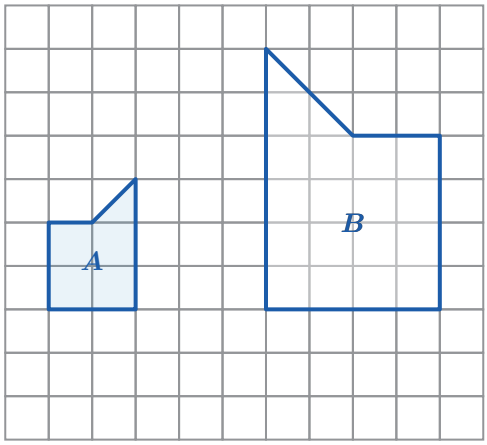
10 Use only dilations to transform the pre-image (shaded) onto the image!



11 Synthesis

Discuss: What are some strategies for determining a sequence of transformations that maps a pre-image onto an image?

Use the example if it helps you with your thinking.



14 Summary 2.03

Dilations can be combined with other *sequences of transformations*.

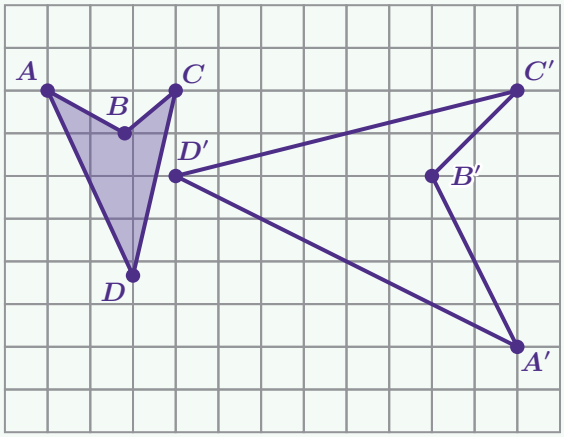
Here is a sequence of transformations that maps figure $ABCD$ onto figure $A'B'C'D'$

Step 1: Dilate figure $ABCD$ using point D as the center of dilation and a scale factor of 2.

Step 2: Translate the image after Step 1 so that point D maps onto point D'.

Step 3: Rotate the new image 90° clockwise around point D'.

Step 4: Reflect the new image across a horizontal line that contains points D' and B'.

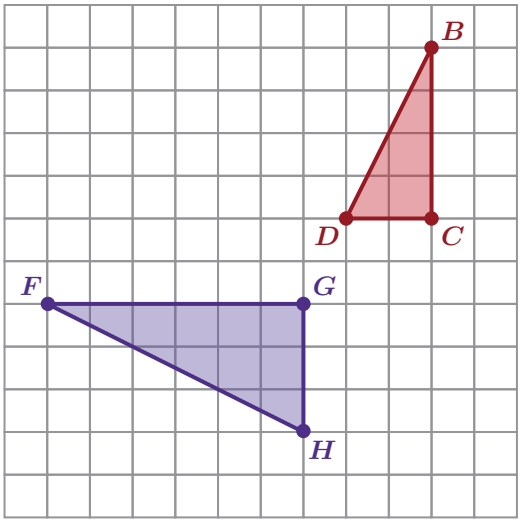


Practice
2.03

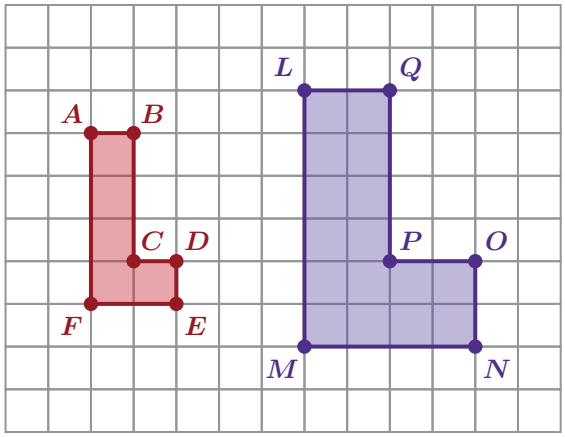
Name: ..... Date: ..... Period: .....

Problems 1–4: Is there a transformation or sequence of translations, rotations, reflections, or dilations that maps one figure onto the other? (Write *yes* or *no*.) If you wrote *yes*, describe the sequence of transformations. If you wrote *no*, describe how you know it's not possible.

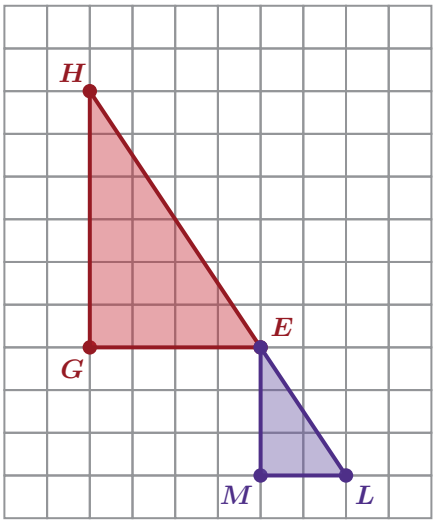
1.



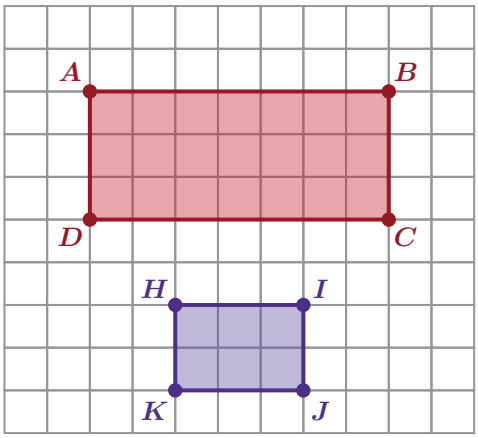
2.



3.



4.



Name: _____ Date: _____ Period: _____

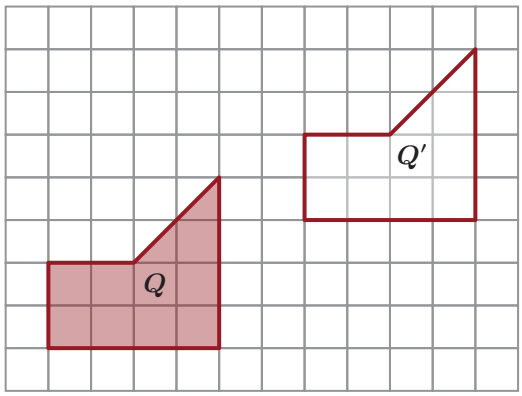
5. Which transformation will map the pre-image onto the image?

A. Translating right and then up.

B. Rotating $180°$.

C. Dilating by a scale factor of 4.

D. Reflecting across a horizontal line.



Spiral Review

Problems 6–9: Point G is located at $(5, -2)$. Determine the coordinates of the image after each transformation.

6. A reflection over the y-axis.

7. A reflection over the x-axis.

8. A $180°$ rotation around the origin.

9. A translation 2 units to the left and 1 unit up.

10. Triangle ABC is reflected over the x-axis and then translated 4 units up. Which statement is true?

A. Triangle $A'B'C'$ is not congruent to triangle ABC.

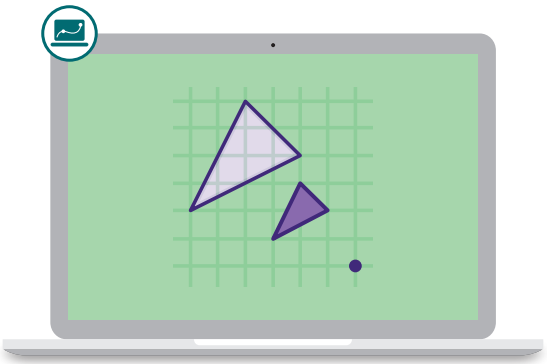
B. The area of triangle ABC is not equal to the area of triangle $A'B'C'$.

C. The angle measures of triangle ABC are congruent to the corresponding angle measures of triangle $A'B'C'$.

D. The perimeter of triangle ABC is greater than the perimeter of triangle $A'B'C'$.

Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.3, SMP.6, SMP.7



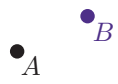
Match My Dilation

Let's dilate figures on a square grid.

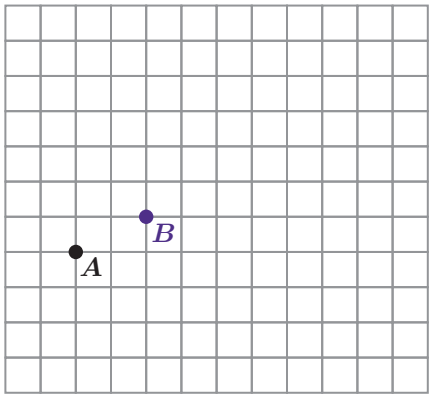
Warm-Up

Plot a point C that is the image of point B dilated using point A as the center of dilation and a scale factor of 3.

1 Try it *without* a grid.



2 Try it *with* a grid.



Describe your strategy for dilating *with* a grid.

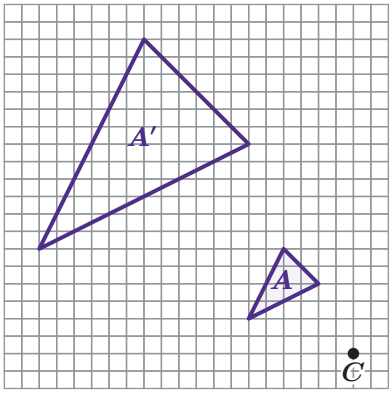
Activity 1

Name: ..... Date: ..... Period: .....

Dilation Challenges

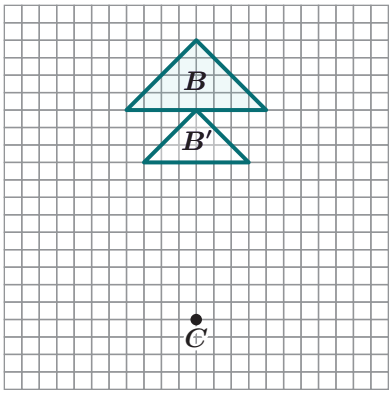
3 Triangle A' is the image of triangle A dilated using point C as the center of dilation.

What was the scale factor used in the dilation?

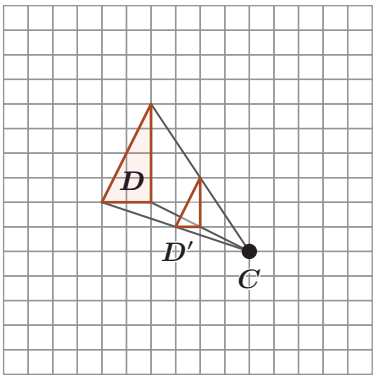


4 Triangle B' is the image of triangle B dilated using point C as the center of dilation.

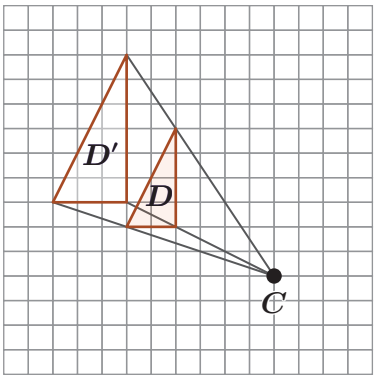
What was the scale factor used in the dilation?



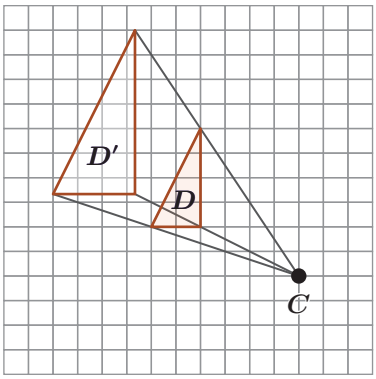
5 **a** Take a look at these dilations of triangle D.



Scale factor: 0.5



Scale factor: 1.5



Scale factor: 1.8

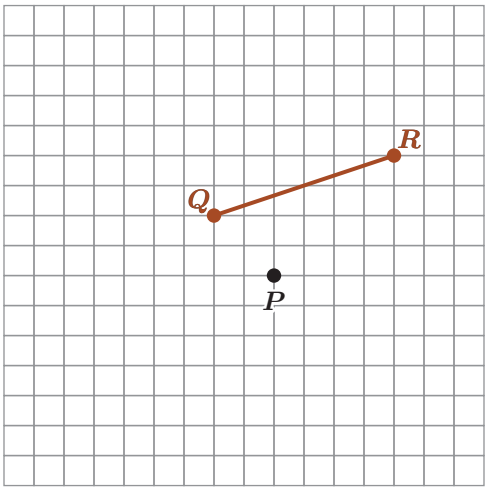
b How does the scale factor affect the size of the image? How does it affect the location of the image?

Activity 2

Name: ..... Date: ..... Period: .....

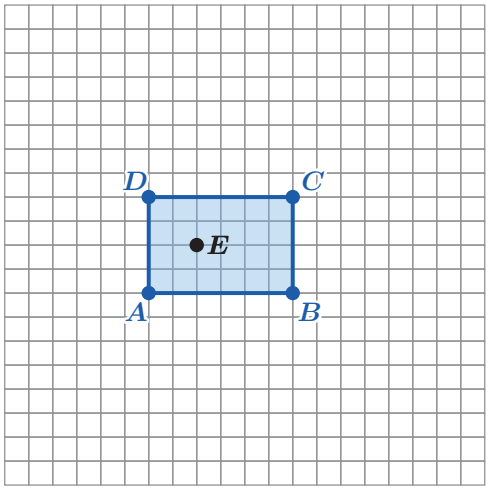
Match My Dilation

6 Show or explain how to dilate segment QR using point P as the center of dilation and a scale factor of $\frac{1}{2}$.



7 **a** Dilate quadrilateral $ABCD$ using point E as the center of dilation and a scale factor of 2.

b **Discuss:** How would the size and location of the image change if point E were in a different location?



Activity 3

Name: ..... Date: ..... Period: .....

Challenge Creator

8 You will use the Activity 3 Sheet to complete this activity.

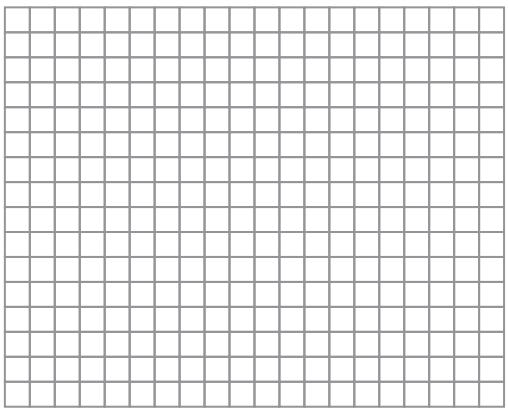
a. **Make It!** On the Activity 3 Sheet, create a dilation challenge.

b. **Solve It!** On this page, redraw your pre-image. Then draw your image using the center of dilation and scale factor you chose. Label the vertices A', B', and C'.

c. **Swap It!** Swap your challenge with one or more partners. Draw your partners' pre-images. Then draw each image using the center of dilation and scale factor.

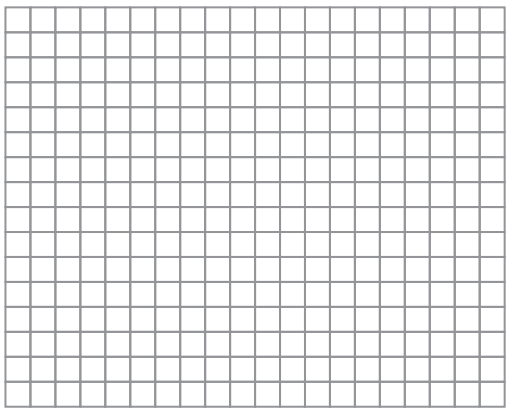
My Dilation Challenge

Scale Factor: .....



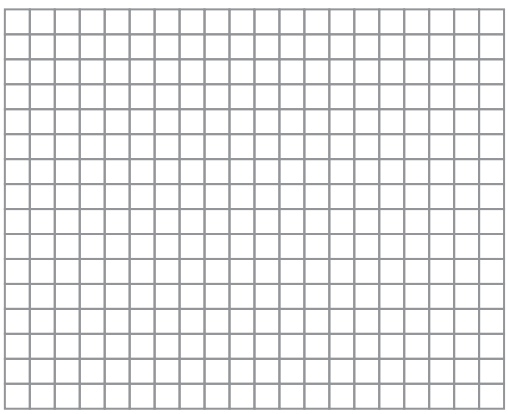
Partner 1

Scale Factor: .....



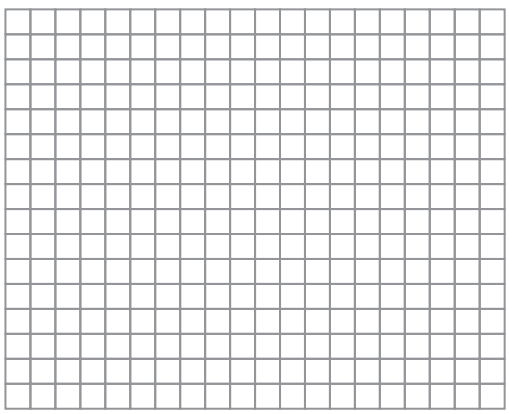
Partner 2

Scale Factor: .....



Partner 3

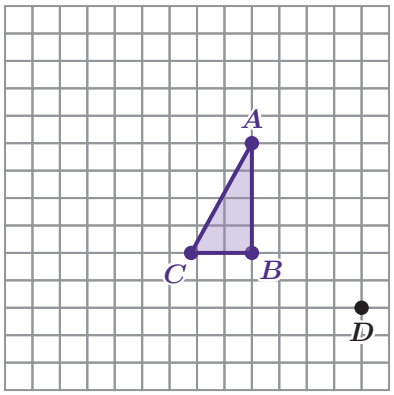
Scale Factor: .....



9 Synthesis

Discuss: What is important to remember when dilating a figure using a center of dilation and a scale factor?

Use the example if it helps you with your thinking.



12 Summary 2.04

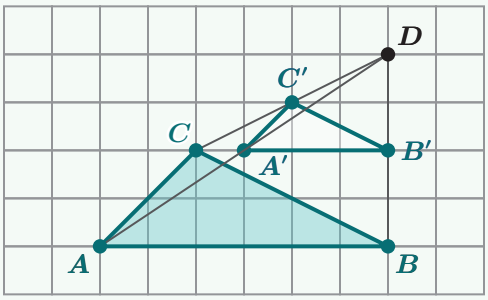
Another strategy for dilating a figure is to use a grid. Count the vertical and horizontal grid squares between the center of dilation and the pre-image, multiply each value by the scale factor, then count that number of grid squares away from the center to get the image.

If the scale factor of the dilation is:

- Greater than 1, the image will be *larger* than the pre-image and further from the center of dilation.
- Equal to 1, the image will be *the same size* as the pre-image and just as far from the center of dilation.
- Between 0 than 1, the image will be *smaller* than the pre-image and closer to the center of dilation.

For example, triangle ABC is dilated using point D as the center of dilation and a scale factor of $\frac{1}{2}$.

Since the scale factor is less than 1, the image is smaller than the pre-image and closer to the center of dilation.



Practice
2.04

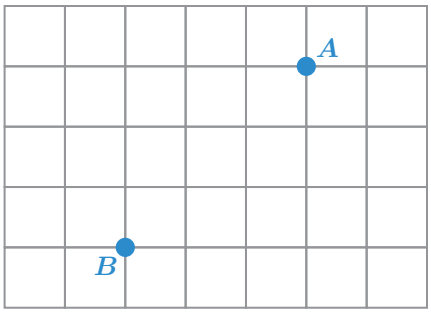
Name: ..... Date: ..... Period: .....

Problems 1–2: Segment AB measures 3 centimeters. Point O is the center of dilation.

1. How long is the image of AB after a dilation with a scale factor of $\frac{1}{5}$?

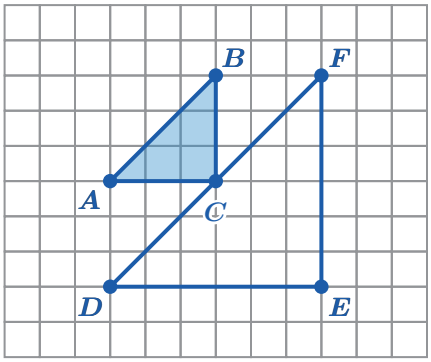
2. How long is the image of AB after a dilation with a scale factor of s?

3. Here are points A and B. Point F is the image of point B using point A as the center of dilation and a scale factor of $\frac{1}{3}$. Plot point F.



4. Triangle ABC was transformed into triangle DFE using a dilation.

Label the center of dilation P. Then determine the scale factor.

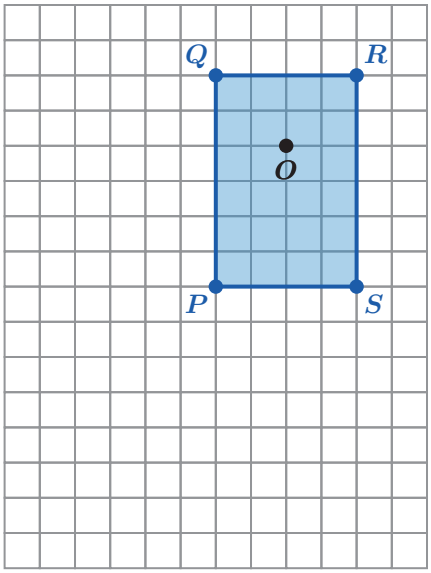


Problems 5–6: Here is rectangle $PQRS$.

Draw the image of rectangle $PQRS$ after each dilation.

5. A dilation using point R as the center of dilation and a scale factor of 2.

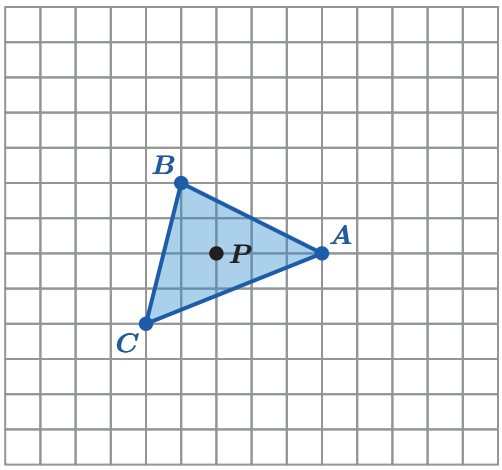
6. A dilation using point O as the center of dilation and a scale factor of $\frac{1}{2}$.



Practice
2.04

Name: ..... Date: ..... Period: .....

Problems 7–9: Here is triangle ABC.



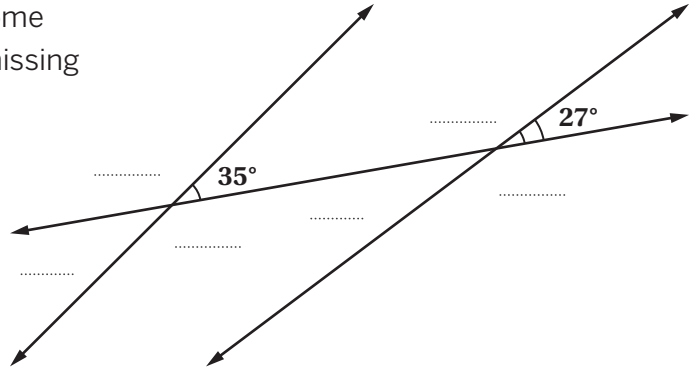
7. Dilate each vertex of triangle ABC using point P as the center of dilation and a scale factor of 2. Draw the image and label the vertices $A'B'C'$.

8. If the length of side AB is 4.5 units and the length of side BC is 4.1 units, what are the lengths of side $A'B'$ and side $B'C'$?

9. How would the size and location of the image change if point P were in a different location?

Spiral Review

10. Here are three lines, along with some angle measures. Determine the missing angle measures.



11. The point $(-4, 9)$ is translated r units to the right and d units down. Which rule describes the image after this translation?

A. $(-4 + r, 9 + d)$

B. $(-4 + r, 9 - d)$

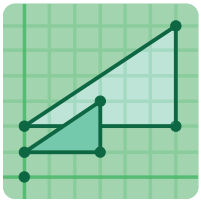
C. $(-4 - r, 9 + d)$

D. $(-4 - r, 9 - d)$

Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.3, SMP.6, SMP.7

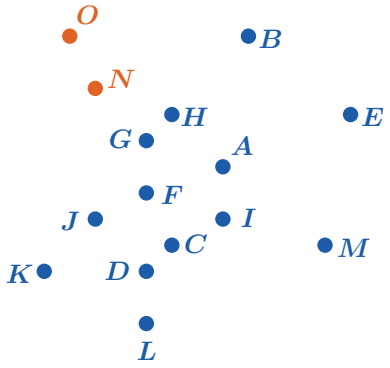
Dilations on a Plane



Let's look at dilations on the coordinate plane.

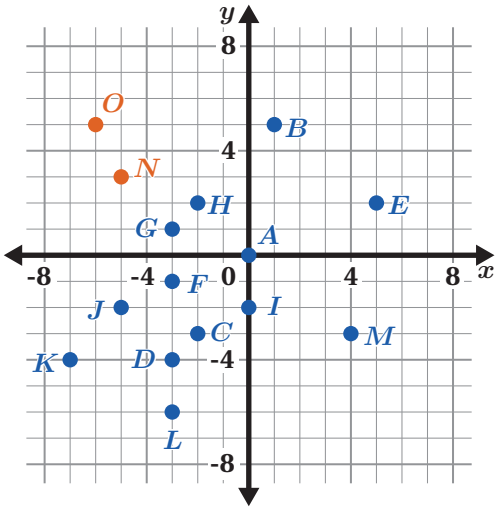
Warm-Up

1. Determine which point is a dilation of point N using point O as the center of dilation and a scale factor of 3.



2. Here is the same task, but with a coordinate plane.

Determine which point is a dilation of point N using point O as the center of dilation and a scale factor of 3.



Activity 1

Name: ..... Date: ..... Period: .....

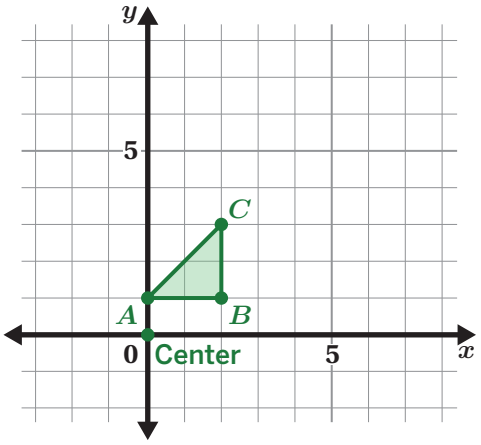
Dilate It!

Use any strategy or tool to perform each dilation. Label the corresponding points in the image with prime notation.

3. **a** Dilate triangle ABC using $(0, 0)$ as the center of dilation and a scale factor of 2.

b Write the image coordinates in the table.

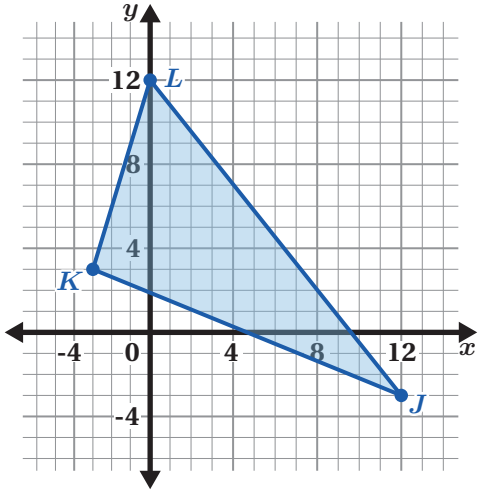
Pre-Image Coordinates	Image Coordinates
(0, 1)	
(2, 1)	
(2, 3)	



4. **a** Dilate triangle JKL using $(0, 0)$ as the center of dilation and a scale factor of $\frac{1}{3}$.

b Write the image coordinates in the table.

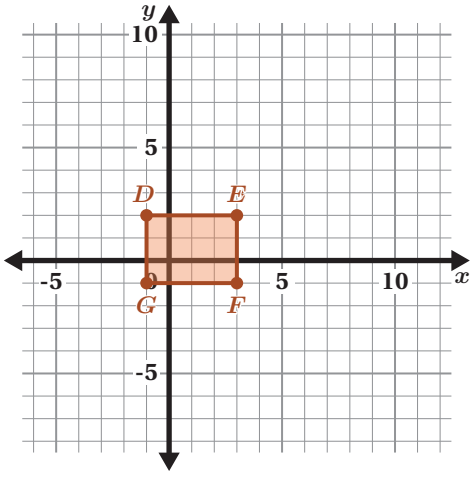
Pre-Image Coordinates	Image Coordinates
(12, -3)	
(-3, 3)	
(0, 12)	



5. **a** Dilate rectangle $DEFG$ using $(0, 0)$ as the center of dilation and a scale factor of 3.

b Write the image coordinates in the table.

Pre-Image Coordinates	Image Coordinates
(-1, 2)	
(3, 2)	
(3, -1)	
(-1, -1)	



Name: ..... Date: ..... Period: .....

Dilate It! (continued)

6. **a** In Problems 3–5, how is the scale factor applied to the pre-image coordinates in each table?

b If point (x, y) is dilated by a scale factor of k, what are the generalized coordinates of the image?

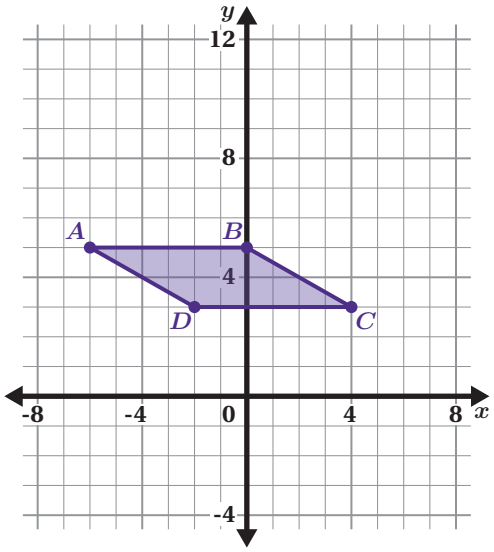
$$(x, y) \longrightarrow$$

Use any strategy or tool to perform the dilation. Label the corresponding points in the image with prime notation.

7. **a** Dilate parallelogram $ABCD$ using $(-6, 5)$ as the center of dilation and a scale factor of $\frac{1}{2}$.

b Write the image coordinates in the table.

Pre-Image Coordinates	Image Coordinates
$(-6, 5)$	
$(0, 5)$	
$(4, 3)$	
$(-2, 3)$	



8. **Discuss**:

a Can the rule you wrote for Problem 6 be used to determine the image coordinates if the center of dilation is not $(0, 0)$? Explain your thinking.

b What sequence of transformations can be used to dilate a figure when the center of dilation is not at the origin?

Activity 2

Name: Date: Period:

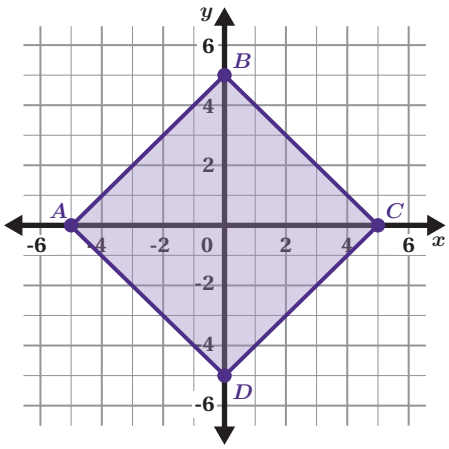
Dilation Information

You and your partner will get a set of dilation cards. Place them grid-side down without looking at them.

- Decide who will describe the dilation on a card and who will sketch the image. Start with Card 1.
- **Describer:** Give enough information about the dilation so that the Sketcher can sketch it.
- **Sketcher:** Pause after sketching and share what you think the dilation is.
- **Together:** Compare the card with the sketch and make adjustments as needed. Write a precise description of the dilation.
- Switch roles for Card 2 and repeat. Then do the same for Cards 3 and 4.

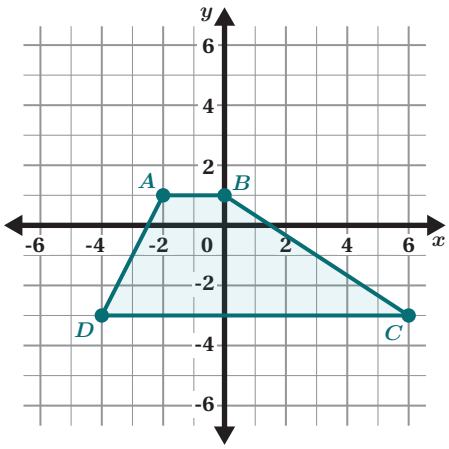
9. Sketch 1: Card 1 or Card 2 (Circle one.)

Card 1



Description of Transformation:
Dilate figure $ABCD$ by . . .

Card 2



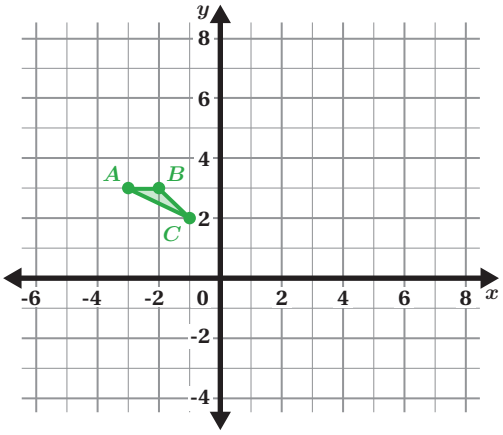
Description of Transformation:
Dilate figure $ABCD$ by . . .

Name: ..... Date: ..... Period: .....

Dilation Information (continued)

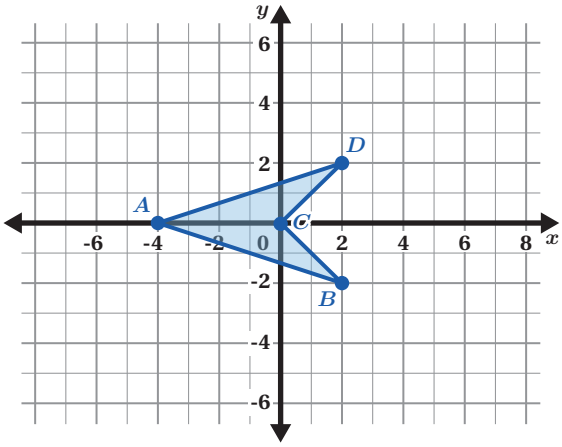
10. Sketch 2: Card 3 or Card 4 (Circle one.)

Card 3



Description of Transformation:
Dilate figure ABC by . . .

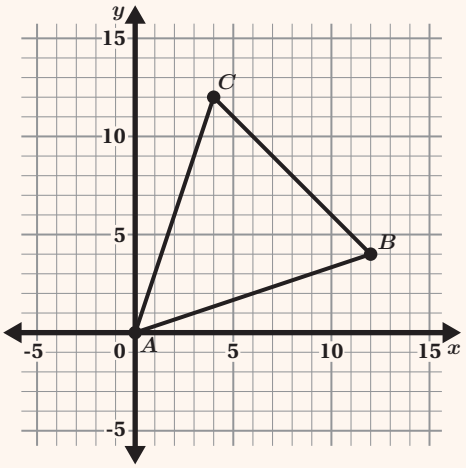
Card 4



Description of Transformation:
Dilate figure $ABCD$ by . . .

You're invited to explore more.

11. Here is triangle ABC.



a Dilate triangle ABC using center $(0, 0)$ and a scale factor of $\frac{3}{4}$. Label the vertices $A'B'C'$.

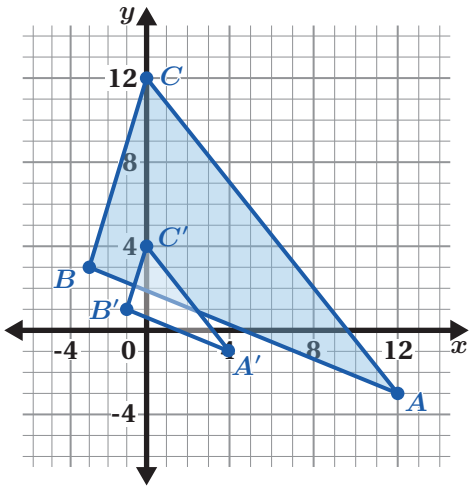
b Dilate triangle ABC using center $(12, 4)$ and a scale factor of $\frac{1}{4}$. Label the vertices $A''B''C''$.

c Explain why A'' and B' must be at the same coordinates.

Synthesis

12. **Discuss:** What is important to remember when dilating figures on a coordinate plane?

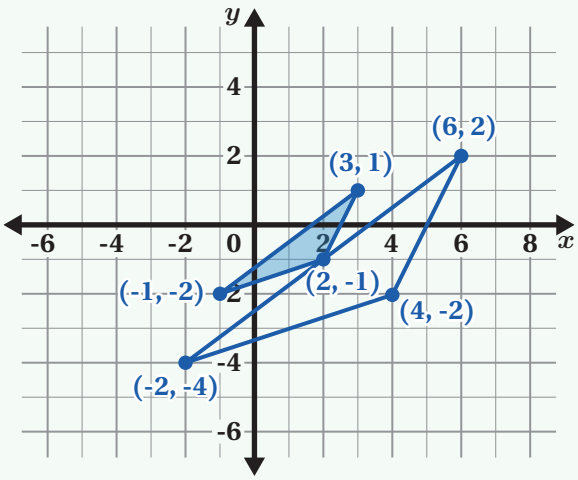
Use the example if it helps with your thinking.



Summary 2.05

Coordinates can help us communicate geometric information, such as dilations.

Let's say you're dilating this shaded pre-image using the center of dilation $(0, 0)$ and a scale factor of 2.



You can use the coordinate plane to measure the horizontal and vertical distances of each point from the center of dilation. Then you can multiply those distances by the scale factor to get the distances between the center of dilation and the points on the image.

You can use a rule to determine the coordinates of an image when the center of dilation is $(0, 0)$ and the scale factor is k.

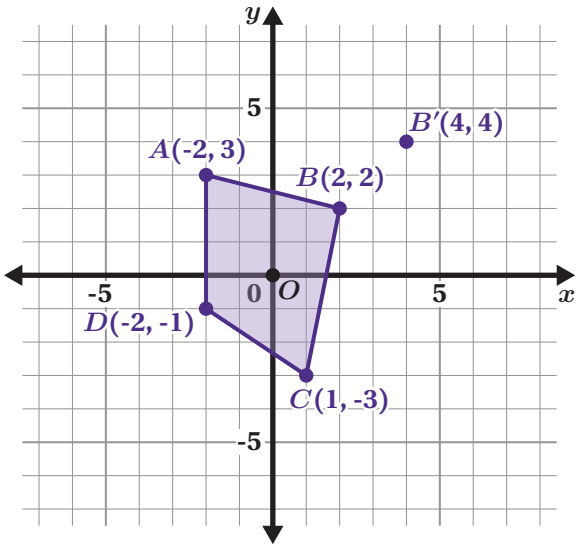
$$(x, y) \rightarrow (kx, ky)$$

In this example, because the center of dilation is $(0, 0)$, you can use the rule to multiply the pre-image coordinates by the scale factor to get the image coordinates.

Practice
2.05

Name: ..... Date: ..... Period: .....

Problems 1–3: Quadrilateral $ABCD$ is dilated with the origin as the center of dilation, moving point B onto point B'.

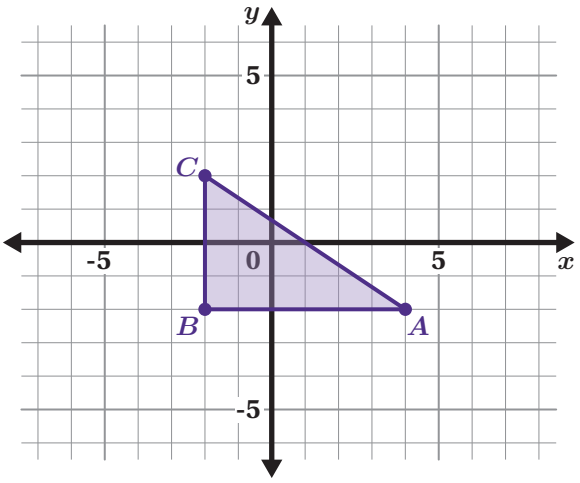


1. What is the scale factor of the dilation?

2. Draw quadrilateral $A'B'C'D'$.

3. Label the coordinate points for A', C', and D'.

Problems 4–6: Here is triangle ABC on the coordinate plane.



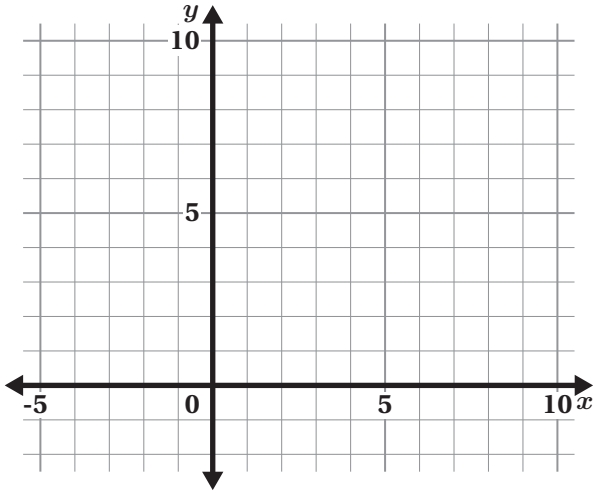
4. Using the origin as the center and a scale factor of 2, draw the dilation of triangle ABC. Label the image triangle DEF.

5. Using the origin as the center and a scale factor of $\frac{1}{2}$, draw the dilation of triangle ABC. Label the image triangle GHI.

6. Triangle GHI is a dilation of triangle DEF. Identify the center of dilation and the scale factor.

Name: _____ Date: _____ Period: _____

Problems 7–8: Triangle ABC has vertices located at $A(-3, 3)$, $B(-2, 3)$, and $C(-1, 2)$. Triangle $A'B'C'$ is the result of dilating triangle ABC using point $O(-5, 1)$ as the center of dilation and a scale factor of 3.



7. Draw triangle ABC and its image on the coordinate plane.

8. What are the coordinates of triangle $A'B'C'$?

9. Here are the pre-image and image coordinates of points on a graph. Describe the transformation.

Pre-Image Coordinates	Image Coordinates
(0, 4)	(0, 12)
(-1, 3)	(-3, 9)
(5, -6)	(15, -18)

Spiral Review

Problems 10–12: Use what you know about the interior angle measures of triangles to complete these problems.

10. Triangle JKL is a right triangle, and the measure of angle J is 28°. What are the measures of the other two angles?

11. Triangle PQR is an obtuse triangle, and the measure of angle Q is 72°. What are possible measures of the other two angles?

12. Triangle STU is an acute triangle, and the measure of angle U is 43°. What are possible measures of the other two angles?

Name: _____ Date: _____ Period: _____

Transformational Geometry | 8.G.1, 8.G.3, SMP.6, SMP.7

Practice Day 1

Let's practice what you've learned so far in this unit!

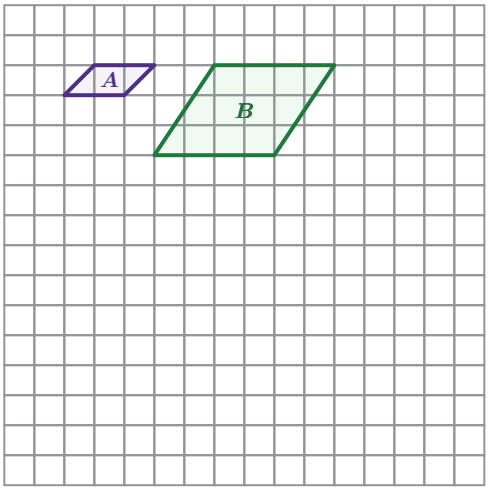


You will use problem cards for this Practice Day. Record all of your responses here.

Card 1

a. Explanation:

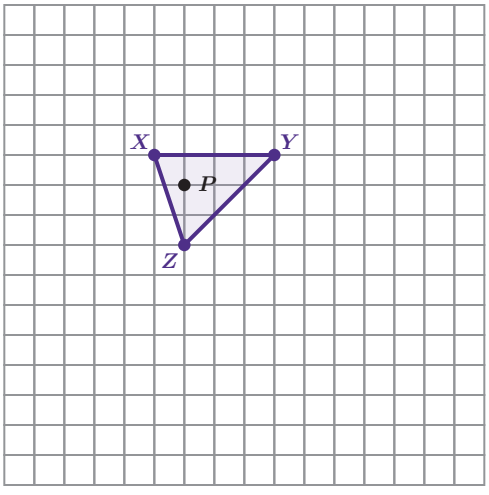
b.



Card 2

Description:

Card 3



Card 4

a. Center of dilation and explanation:

b. Scale factor: _____

Name: ..... Date: ..... Period: .....

Practice Day 1

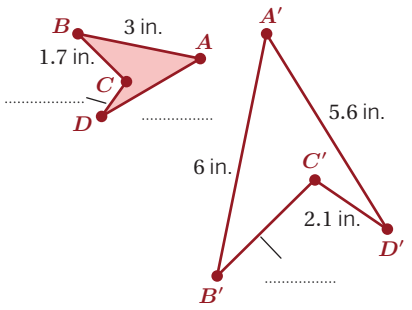
Card 5

Description:

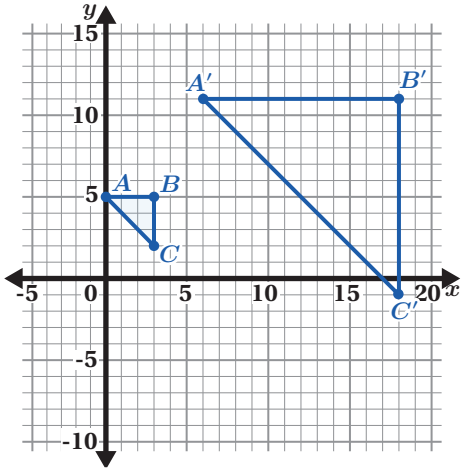
Card 6

a Scale factor: .....

b



Card 7



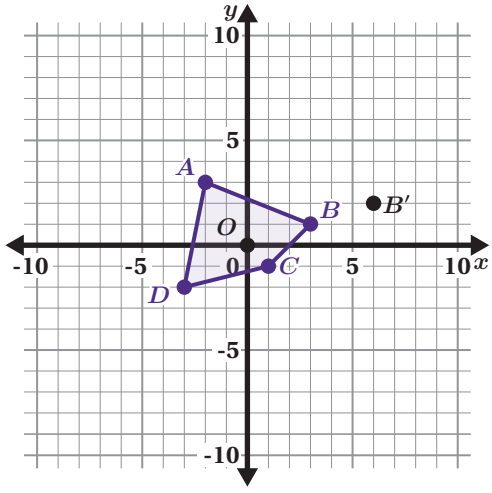
a Coordinates:

b Scale factor: .....

Card 8

a Scale factor: .....

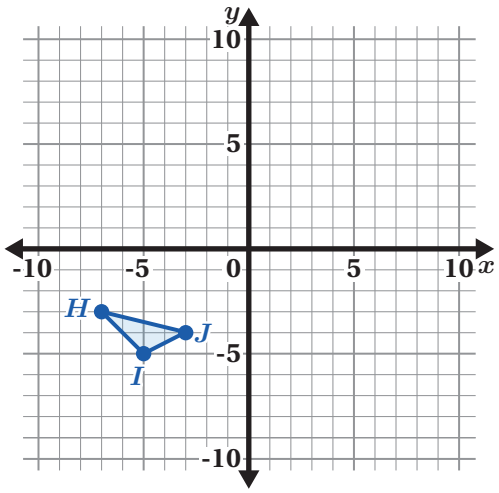
b



Name: ..... Date: ..... Period: .....

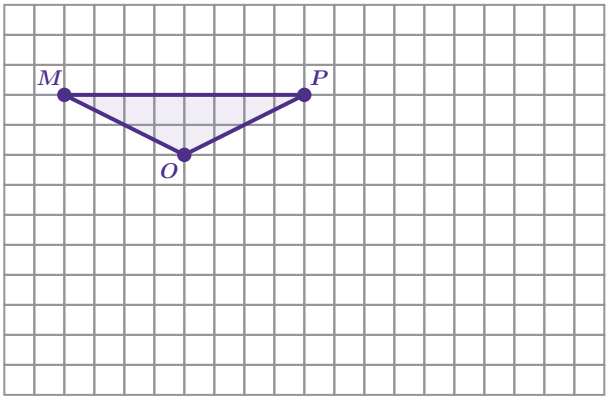
Practice Day 1

Card 9



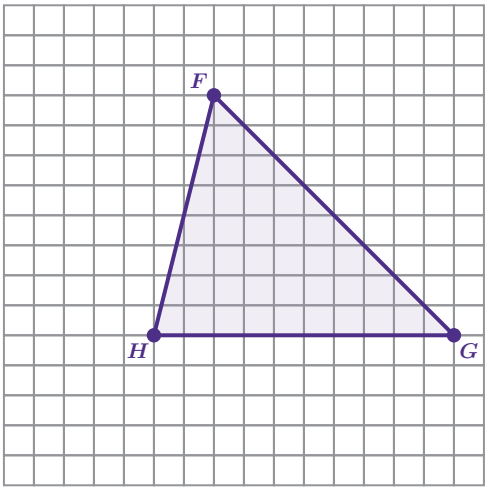
Card 10

a – b

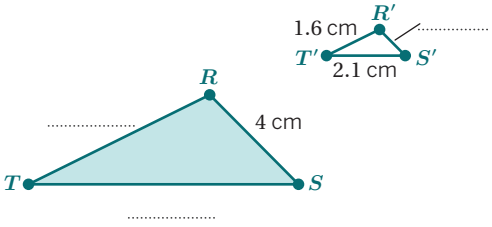


c Description:

Card 11



Card 12

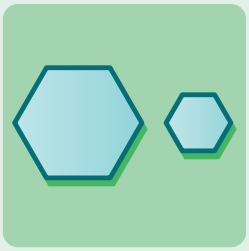


Notes:

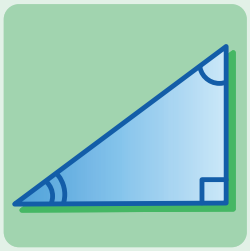
Sub-Unit

2

Similarity



Lesson 6
Social Scavenger Hunt



Lesson 7
Are Angles Enough?

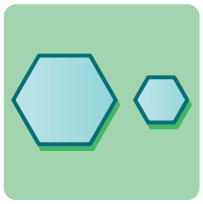


Lesson 8
Shadows

Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.2, 8.G.4, SMP.3, SMP.6

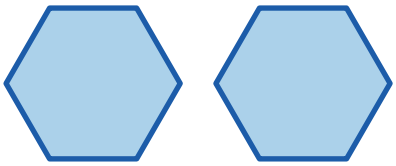
Social Scavenger Hunt

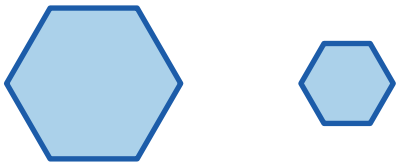


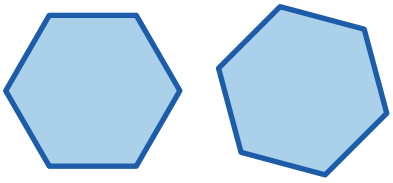
Let's explore what makes two figures similar.

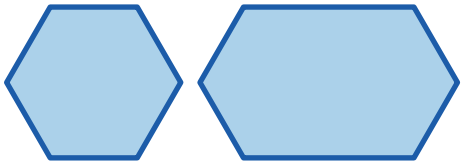
Warm-Up

1. Which one doesn't belong?

A. 

B. 

C. 

D. 

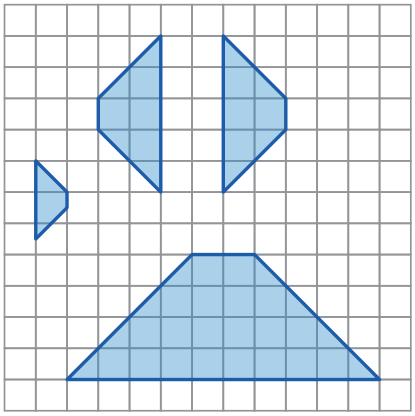
Explain your thinking.

Name: ..... Date: ..... Period: .....

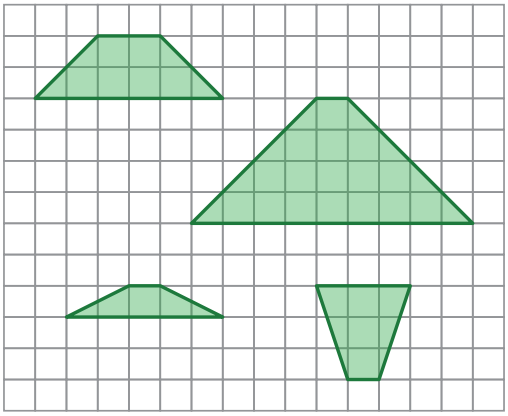
What Are Similar Figures?

2. Here are figures that are **similar** and not similar.

Similar

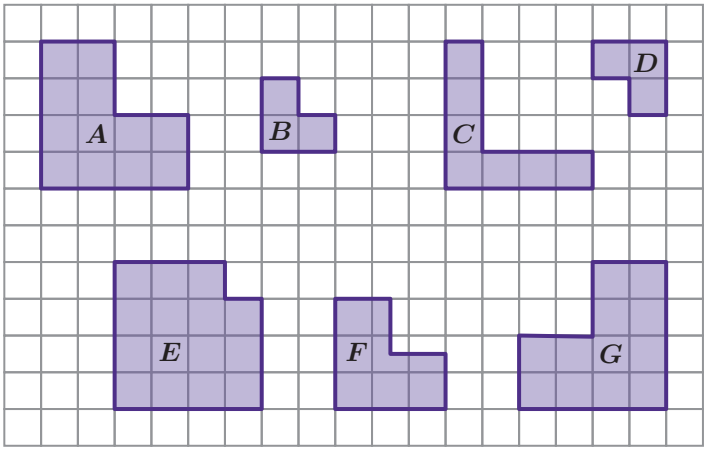


Not Similar



What do you think makes figures *similar*? How is this related to transformations?

3. Here are seven figures.



Choose two figures that fit each category. Explain your thinking.

Similar	Congruent	Not Similar

Activity 2

Name: ..... Date: ..... Period: .....

Social Scavenger Hunt

4. You will be given a card. For each round, you will have instructions for finding a partner. Once you have a partner, compare your cards and answer each question.

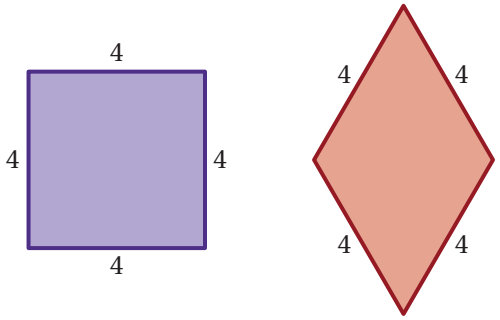
Round 1

Questions	Yes	No
Can a sequence of transformations map one figure onto the other?		
Are the figures similar?		
Are the figures congruent?		
Are the corresponding sides congruent?		
Are the corresponding angles congruent?		

Round 2

Questions	Yes	No
Can a sequence of transformations map one figure onto the other?		
Are the figures similar?		
Are the figures congruent?		
Are the corresponding sides congruent?		
Are the corresponding angles congruent?		

5. Here are two figures. Decide whether each statement is true or false.



a Their corresponding sides are congruent.

b Their corresponding angles are congruent.

c The figures are similar.

d The figures are congruent.

Name: ..... Date: ..... Period: .....

Social Scavenger Hunt (continued)

6. Here are two new rounds of the scavenger hunt. With each new partner, compare your cards and answer each question.

Round 3

Questions	Yes	No
Can a sequence of transformations map one figure onto the other?		
Are the figures similar?		
Are the figures congruent?		
Are the corresponding sides congruent?		
Are the corresponding angles congruent?		

Round 4

Questions	Yes	No
Can a sequence of transformations map one figure onto the other?		
Are the figures similar?		
Are the figures congruent?		
Are the corresponding sides congruent?		
Are the corresponding angles congruent?		

7. Decide whether each statement is *always*, *sometimes*, or *never* true.

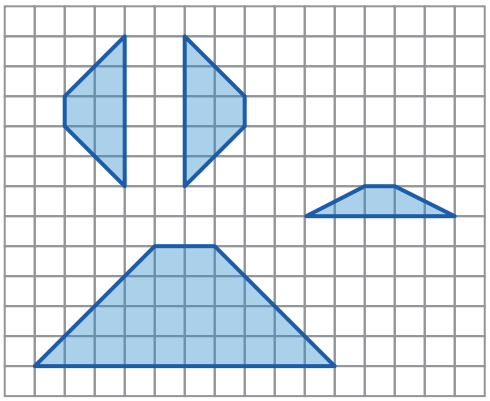
Statement	Always True	Sometimes True	Never True
a If figures are similar, they are also congruent.			
b If figures are similar, their corresponding angles are congruent.			
c If figures are similar, their corresponding sides are congruent.			
d If figures are similar, their corresponding side lengths have a common scale factor.			

Choose one statement and explain your thinking.

Synthesis

8. **Discuss:** What does it mean for two figures to be similar? What does it tell you about their corresponding side lengths and angle measures?

   Use the example if it helps with your thinking.



Summary 2.06

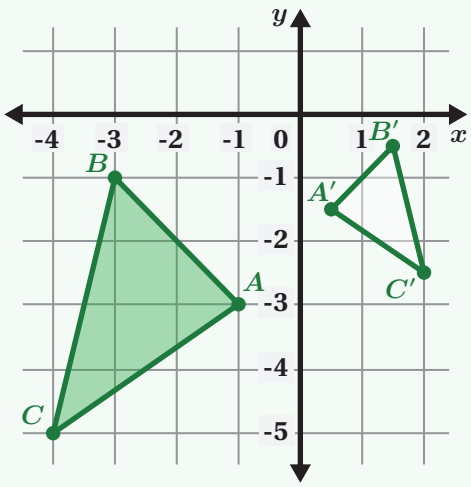
Two figures are *congruent* if one figure can be mapped exactly onto the other using a sequence of rigid transformations. Two figures are **similar** if one figure can be mapped to the other using a sequence of transformations.

We use the symbol ~ to say that two figures are similar. For example, triangle ABC ~ triangle $A'B'C'$.

In congruent figures, the *corresponding* angles and corresponding sides are congruent.

In similar figures, the corresponding angles are congruent and the corresponding side lengths are proportional but *not* always congruent.

For example, $\angle A$ is congruent to $\angle A'$, but segment AB is not the same length as segment $A'B'$. Segment $A'B'$ is twice as long as segment AB because the scale factor from triangle ABC to triangle $A'B'C'$ is 2.



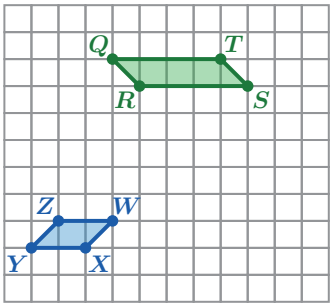
similar Two figures are similar to each other if one figure can be mapped with translations, rotations, reflections, and dilations. Two figures are similar if one is a scaled copy of the other.

Practice
2.06

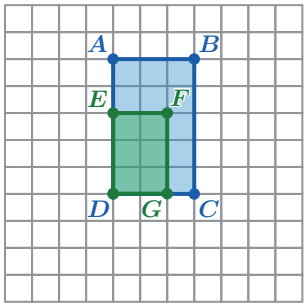
Name: ..... Date: ..... Period: .....

1. Which graph shows a pair of similar figures?

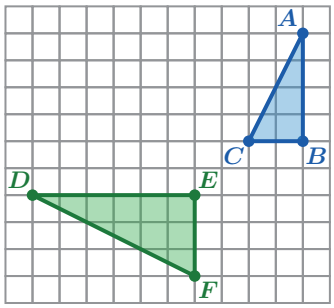
A.



B.

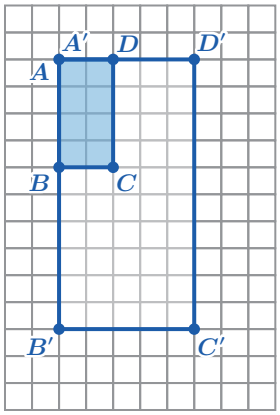


C.

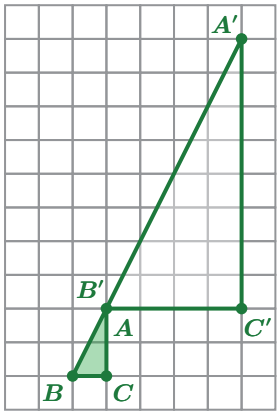


Problems 2–3: Show that the figures are similar by describing a sequence of transformations that maps the shaded pre-image onto the unshaded image.

2.



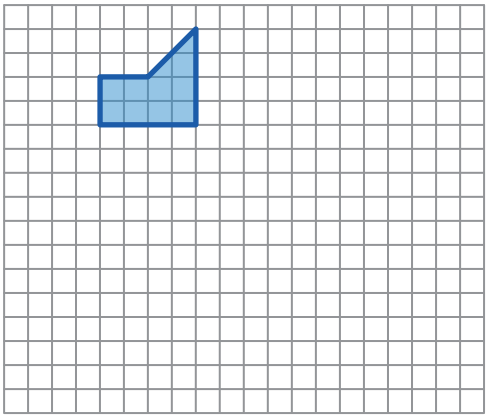
3.



Problems 4–5: Here is a polygon.

4. Draw a similar polygon that could be mistaken for being *not* similar.

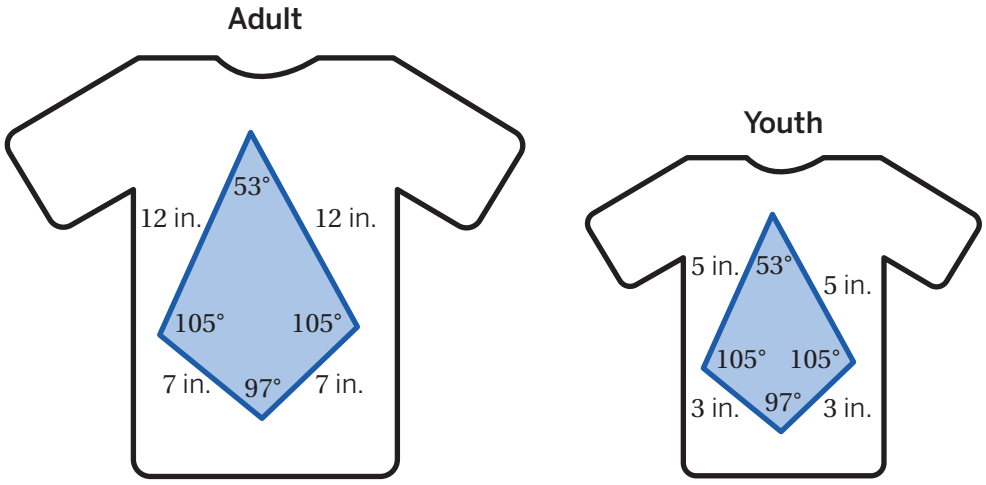
5. Draw a polygon that is not similar, but could be mistaken for a similar polygon.



Name: ..... Date: ..... Period: .....

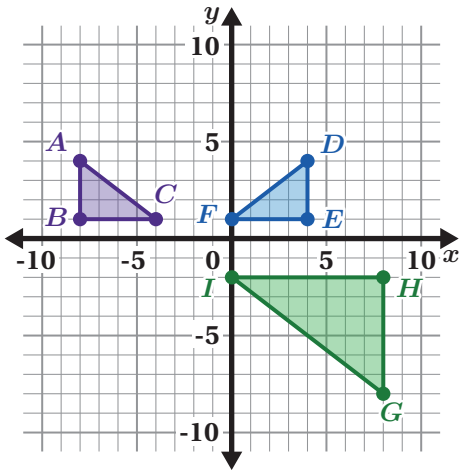
6. A fashion designer is working with a supplier to print her design on different-sized shirts. She checks the quality of the shirts by using congruence and similarity.

Based on the measurements, do you think the designer should continue to work with the supplier? Explain your thinking using congruence and similarity.



Spiral Review

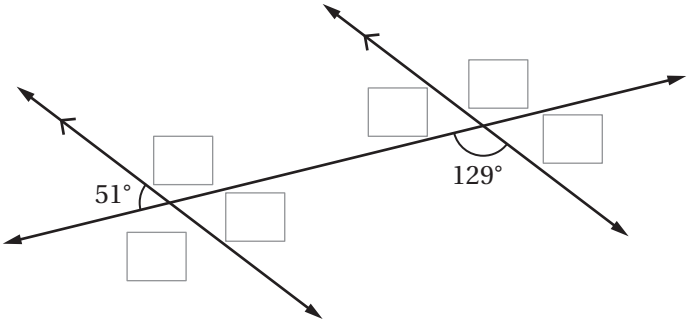
Problems 7–8: Here are some triangles on a graph.



7. Describe a single transformation that shows that triangle DEF is congruent to triangle ABC.

8. Describe a sequence of transformations that shows that triangle DEF is similar to triangle GHI.

9. Here are two parallel lines cut by a transversal and some angle measures. Determine the missing angle measures.

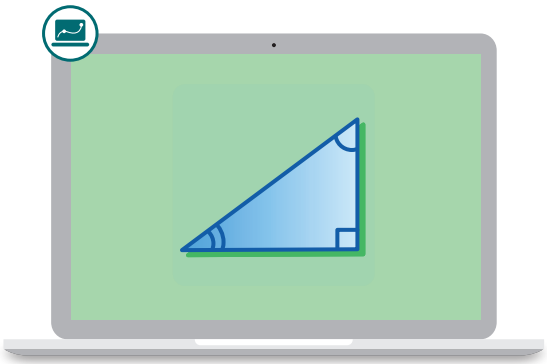


Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.5, SMP.3, SMP.8

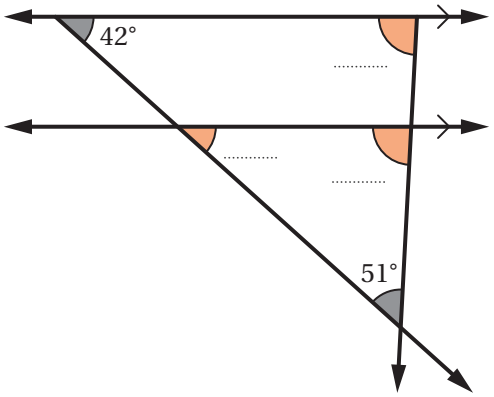
Are Angles Enough?

Let's determine whether congruent corresponding angles are enough to know whether triangles are similar.



Warm-Up

1 Determine the measure for each missing angle.

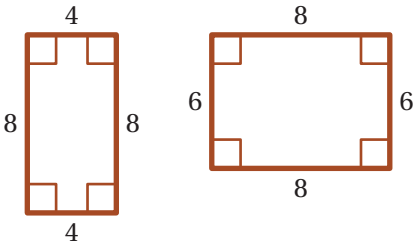


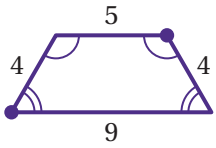
Activity 1

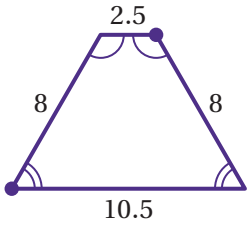
Name: ..... Date: ..... Period: .....

Are Angles Enough?

2 Take a look at these pairs of figures that have congruent angles.

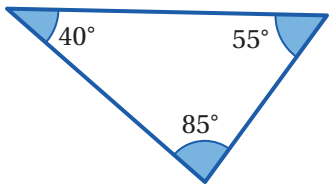






Discuss: How do you know that the figures in each pair are *not* similar?

3 **a** Take a look at this triangle with angle measures 40°, 55°, and 85°.



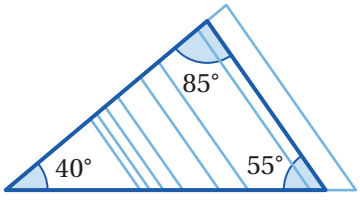
b If all of your classmates made triangles with the same angle measures as this one, would all the triangles be similar? Circle one.

Yes No I'm not sure

Explain your thinking.

4 Here are some triangles that all have 40°, 85°, and 55° angles.

a Let's watch an animation of these triangles.

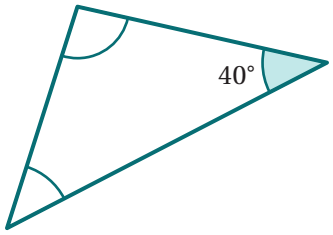


b **Discuss:** Are these triangles all similar?

Name: ..... Date: ..... Period: .....

Are Angles Enough? (continued)

5 **a** Take a look at this triangle with one angle measuring $40°$.



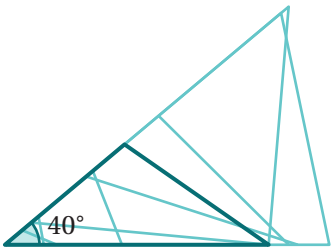
b If all of your classmates made triangles with one angle measuring $40°$, would all the triangles be similar? Circle one.

Yes No I'm not sure

Explain your thinking.

6 Here are some triangles that all have a $40°$ angle.

a Let's watch an animation of these triangles.



b **Discuss:** Are these triangles all similar?

7 How many pairs of congruent corresponding angles are needed to show that two triangles are similar? Explain your thinking.

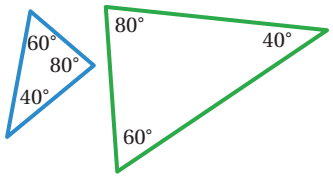
Activity 2

Name: ..... Date: ..... Period: .....

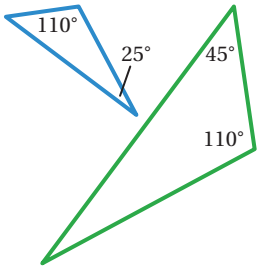
Similar Sort

8 Sort the pairs of figures into three groups. (Images are not to scale.)

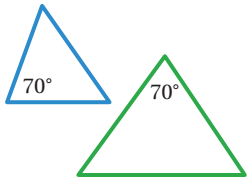
A.



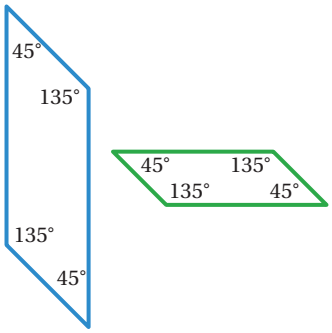
B.



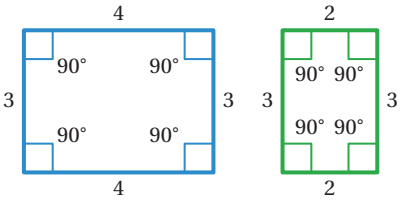
C.



D.



E.

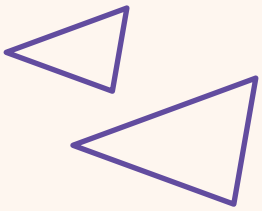


Similar	Not Similar	Not Enough Information

You're invited to explore more.

9 Here are two similar triangles. Santino says that in similar triangles, if you match up two pairs of sides at a vertex, then the third sides are always parallel. Is Santino's claim correct?

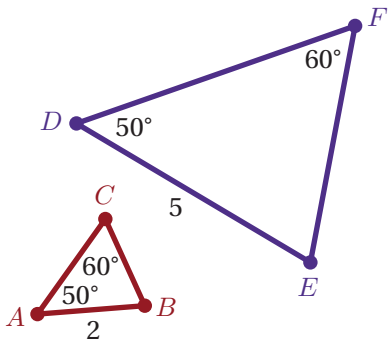
Explain your thinking.



10 Synthesis

Describe how you can use angles to determine whether triangles are similar.

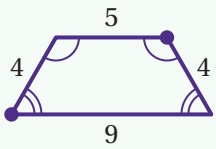
Use the example if it helps you with your thinking.



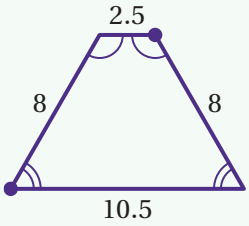
13 Summary 2.07

Similar figures have congruent corresponding angles. Is knowing that the corresponding angles in two figures are congruent enough to show that the figures are also similar? It depends!

Here are two figures that have congruent corresponding angles but are not similar figures.

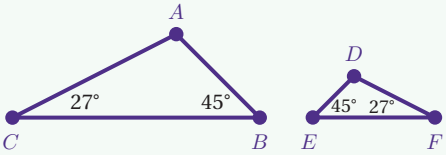


For triangles, knowing that the corresponding angles are congruent is enough to know that the triangles are similar.



This is even true if you only know two angle measures, because we can use the fact that the sum of the interior angles of any triangle is $180°$ to figure out the third angle measure This is known as the angle-angle criterion for similarity of triangles.

For example, the unknown angles in triangles CAB and FDE are each $108°$. All the corresponding angles are congruent, which means triangle CAB is similar to triangle FDE.



Practice
2.07

Name: ..... Date: ..... Period: .....

Problems 1–3: Here are some angle measures for different pairs of triangles. Determine whether each pair is *similar*, *not similar*, or if there is *not enough information* to know.

1. Triangle A: 53°, 71°
Triangle B: 53°, 71°

2. Triangle E: 63°, 45°
Triangle F: 14°, 71°

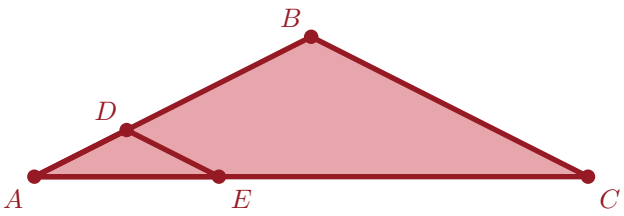
3. Triangle G: 100°
Triangle H: 70°

4. Draw two equilateral triangles to fit each category in the table. If it's not possible, write "Not possible."

Not Congruent	Not Similar

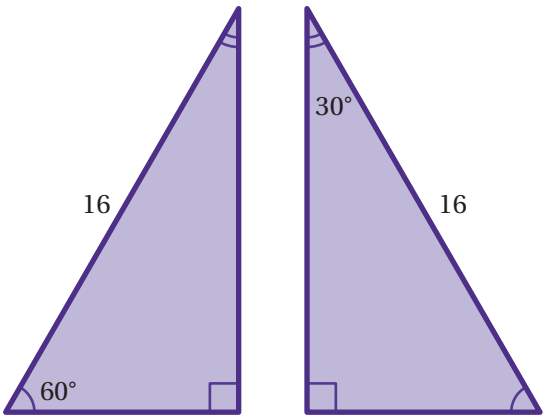
5. Do you think two equilateral triangles will be *always*, *sometimes*, or *never similar*? Explain your thinking.

6. In this figure, segment BC is parallel to segment DE. How do you know that triangle ABC is similar to triangle ADE?



Name: ..... Date: ..... Period: .....

7. Which of the following *best* describes the triangles shown?



A. Congruent but not similar

B. Similar but not congruent

C. Both similar and congruent

D. Neither similar nor congruent

8. Triangle A has two angles that measure $90°$ and $33°$. Triangle B has two angles that measure $90°$ and $57°$. Determine whether the triangles are *similar* or *not similar*. Explain your thinking.

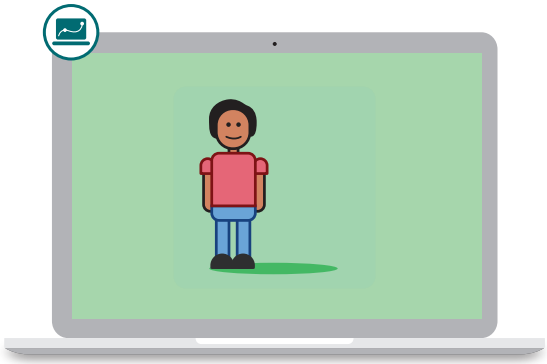
Spiral Review

9. A figure is translated 3 units to the right and then dilated by a scale factor of 3. Determine whether the pre-image and image are *congruent*, *similar*, or *neither*.

10. A figure is reflected, rotated, and dilated. After this sequence of transformations, the pre-image and image are congruent. What would the scale factor have to be for this to be possible?

Name: ..... Date: ..... Period: .....

Transformational Geometry 8.G.4, SMP.2, SMP.4

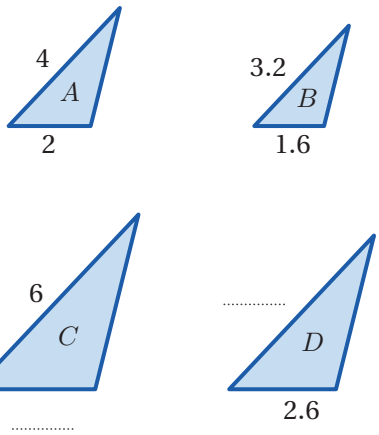


Shadows

Let's use what we know about similar triangles to determine missing side lengths.

Warm-Up

1 Here are four similar triangles.



a Examine the given side lengths. Then write in the missing values.

b **Discuss:** How did you determine these values?

Activity 1

Name: ..... Date: ..... Period: .....

Similar Triangles in Shadows

2 Let's watch a slider control the time of day.

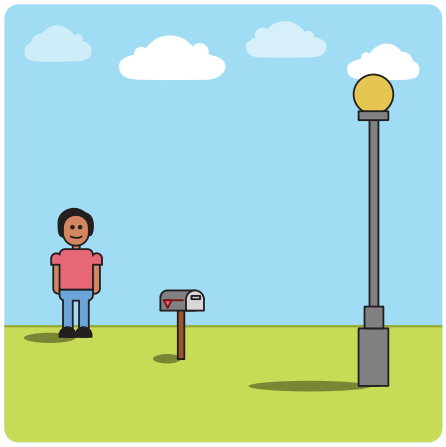
What do you notice? What do you wonder?

I notice:

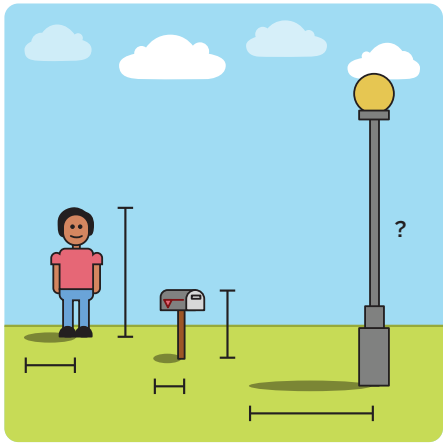
I wonder:

3 Kayla noticed she could form similar triangles using the shadows of these figures.

Where do you think she saw triangles? Why might she think these triangles are similar? Draw on the picture if it helps to show your thinking.



4 Your task is to determine the height of this lamppost.



a Decide which of these measurements you need and request them from your teacher:

- ☐ Height of the person
- ☐ Length of the person's shadow
- ☐ Height of the mailbox
- ☐ Length of the mailbox's shadow
- ☐ Length of the lamppost's shadow

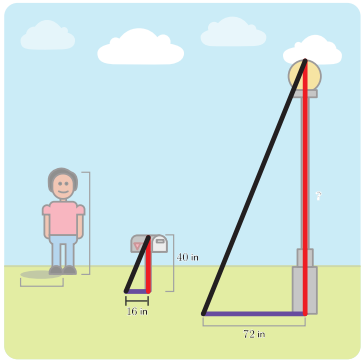
b Once you have enough information, determine the height of the lamppost.

Name: ..... Date: ..... Period: .....

Similar Triangles in Shadows (continued)

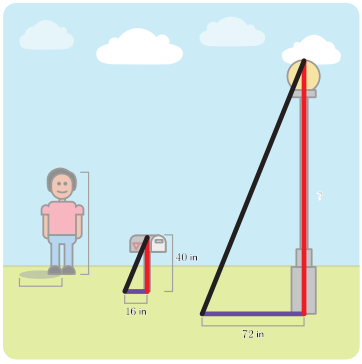
5 Here are Ama's and Neena's strategies for determining the height of the lamppost.

Ama



$\frac{40}{?} = \frac{16}{72}$

Neena



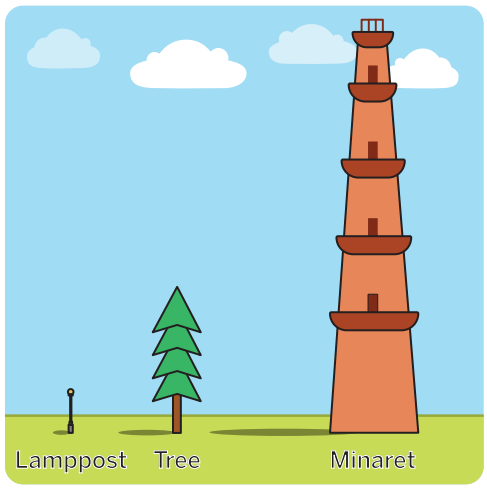
$\frac{40}{16} = \frac{?}{72}$

Discuss: How are their strategies alike? How are they different?

6 Here is the lamppost from the previous problem, as well as two new objects.

Determine the missing heights.

	Lamppost	Tree	Minaret
Height (ft)	15		
Shadow Length (ft)	6	20	56



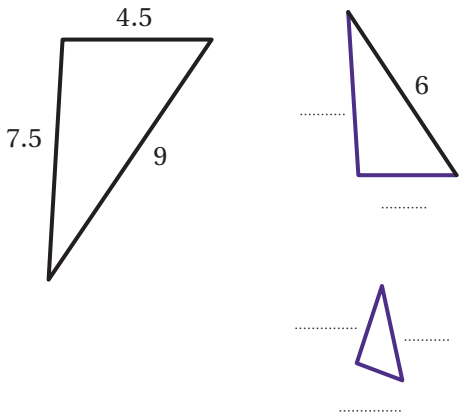
Activity 2

Name: ..... Date: ..... Period: .....

Similar Triangles Puzzle

7 Here are three similar triangles.

a Determine all the side lengths using as few hints as you can. You can ask for the measure of up to two side lengths, if needed.

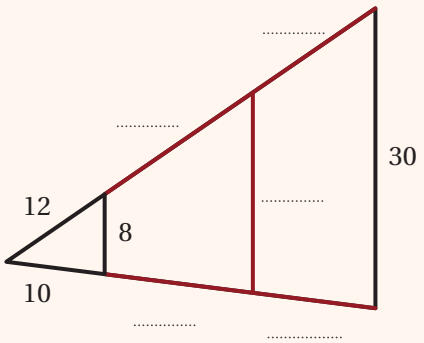


b What was your strategy?

You're invited to explore more.

8 Here are three similar triangles.

a Determine all the side lengths using as few hints as you can. You can ask for the measure of two or three additional side lengths, if needed.

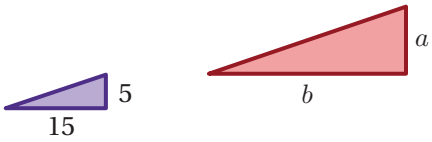


b What was your strategy?

9 Synthesis

Discuss: What are some strategies for determining unknown side lengths in similar triangles?

Use the example if it helps with your thinking.



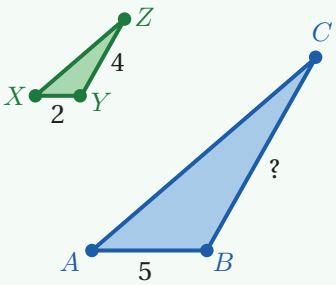
12 Summary 2.08

There are a variety of strategies for determining unknown side lengths of similar triangles.

You can use ratios that compare two triangles using the scale factor between them.

You can also determine the ratio of two side lengths in the same triangle, then apply that ratio to the corresponding sides in a similar triangle.

Here are two similar triangles, triangle ABC and triangle XYZ. You can use each of these strategies to determine the length of side BC.



Using the scale factor between the triangles.
Side XY and side AB are corresponding sides. The ratio of their side lengths is $\frac{5}{2}$, which means the scale factor is 2.5. To calculate the length of side BC, you can multiply the length of side YZ by the scale factor. $4 \cdot 2.5 = 10$, so side BC is 10 units long.

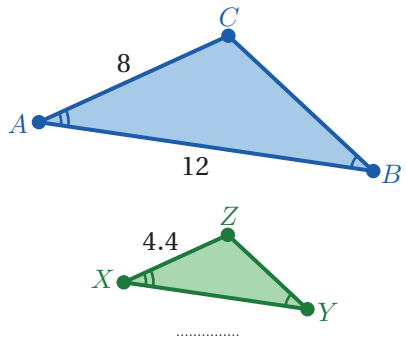
Using ratios of side lengths within one triangle.
The ratio of side YZ to side XY is $4 : 2$, or 2. That means the length of side BC is twice the length of side AB. $5 \cdot 2 = 10$, so side BC is 10 units long.

Practice
2.08

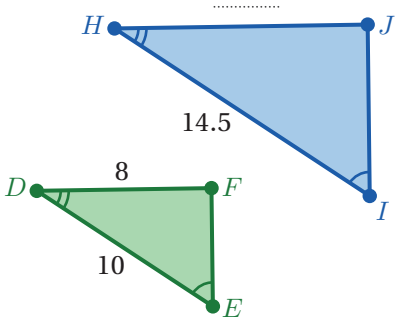
Name: ..... Date: ..... Period: .....

Problems 1–2: Triangle ABC is similar to triangle XYZ. Triangle DEF is similar to triangle HIJ. Determine the missing values. Explain your thinking. The figures may not be drawn to scale.

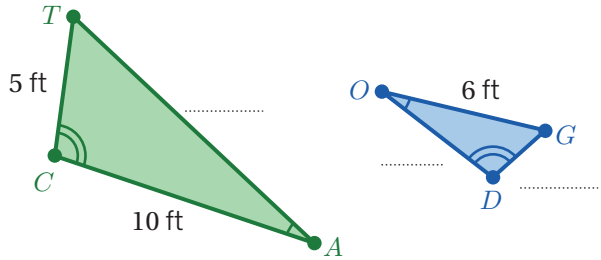
1.



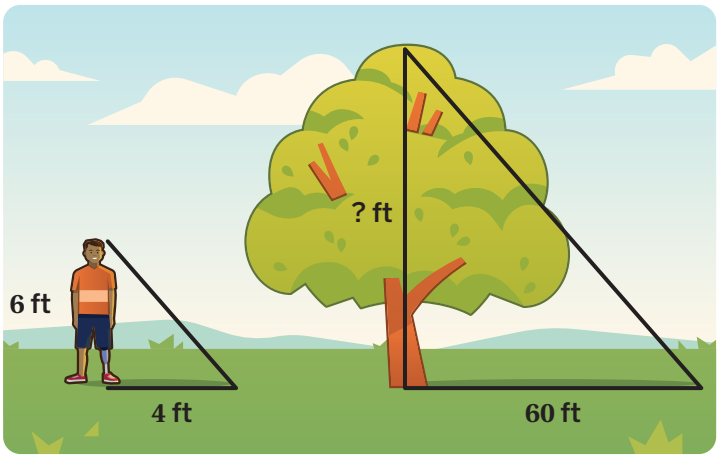
2.



3. Triangle CAT is similar to triangle DOG. Triangle CAT is dilated by a scale factor of $\frac{1}{2}$, creating triangle DOG. Determine the missing side lengths. Show or explain your thinking.



4. On a sunny day, a 6-foot-tall person has a shadow with a length of 4 feet. At the same time, a nearby tree has a shadow 60 feet long. What is the height of the tree?



A. 80 feet

B. 90 feet

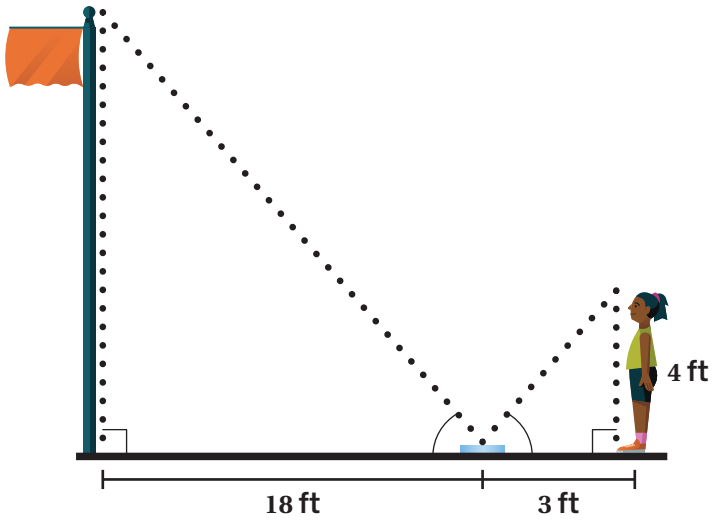
C. 120 feet

D. 180 feet

Practice
2.08

Name: ..... Date: ..... Period: .....

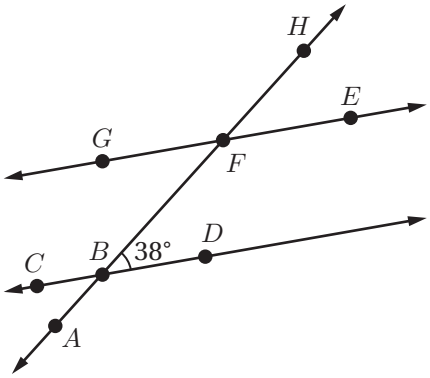
5. Zoe uses a mirror to measure the height of a flagpole. She places the mirror flat on the ground, then walks backward until she can see the flagpole in the mirror. As the diagram shows, the way that light reflects in the mirror creates a set of congruent angles. Now she can use her own height, the mirror's distance from the flagpole, and her distance from the mirror to calculate the height of the flagpole.



Use the measurements in the diagram to calculate the flagpole's height. Show or explain your thinking.

Spiral Review

Problems 6–11: Lines GE and CD are parallel. Determine the measures of each angle.



6. $\angle ABC$

7. $\angle ABD$

8. $\angle GFB$

9. $\angle BFE$

10. $\angle EFH$

11. $\angle GFH$

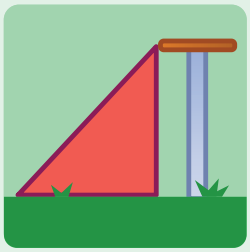
12. Angle E in triangle DEF is a right angle. Can angle D or angle F be a right angle?

Explain your thinking.

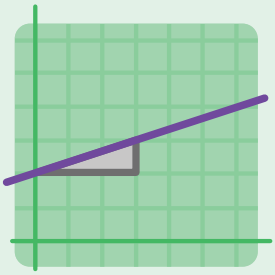
Sub-Unit

3

Slope



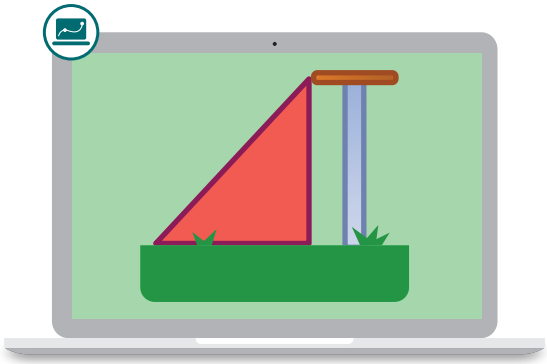
Lesson 9
Water Slide



Lesson 10
Slope Challenges

Name: ..... Date: ..... Period: .....

Multiple Representations of Functions | Transformational Geometry | 8.EE.6, 8.G.5, SMP.2, SMP.7



Water Slide

Let's look at similar triangles and lines.

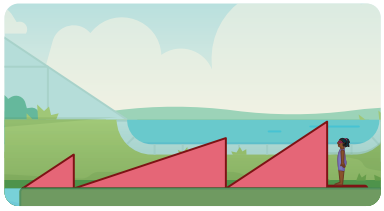
Warm-Up

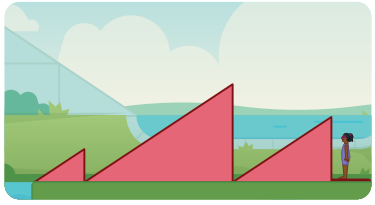
1 Let's watch someone try two different slides.

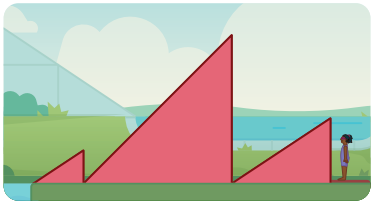
Discuss: What makes a smooth slide? What makes a bumpy slide?

2 Your goal is to create a smooth slide.

Which set of ramps do you think would make a smooth slide?

A. 

B. 

C. 

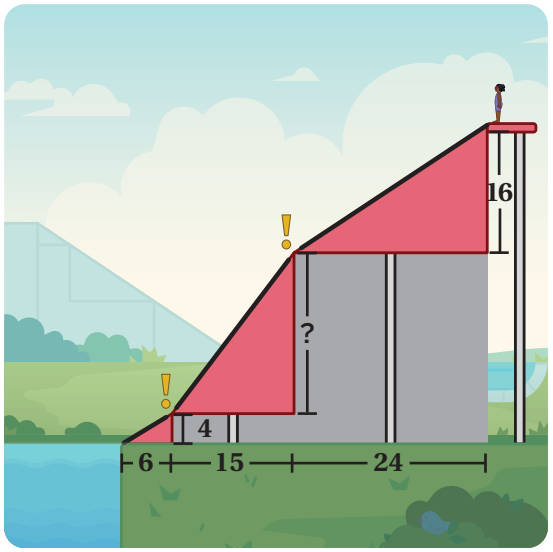
Name: Date: Period:

Smooth Slides

3 These ramps will make a bumpy slide!

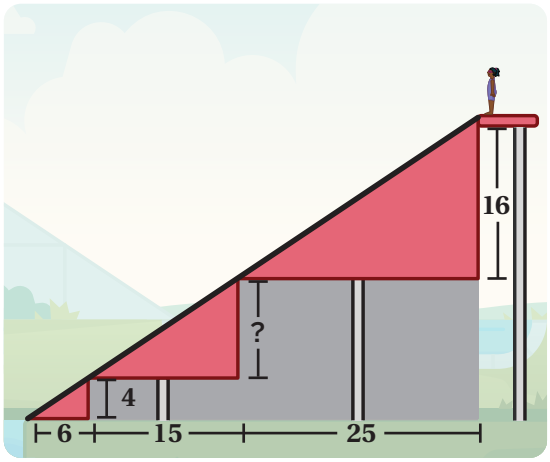
Fill in the height for Ramp 2 to make a smooth slide.

Ramp	Base (ft)	Height (ft)
Ramp 1	6	4
Ramp 2	15	
Ramp 3	24	16



4 Jada says: *The ramps are all similar triangles.*

How can Jada use the properties of similar triangles to find the height of the middle ramp?

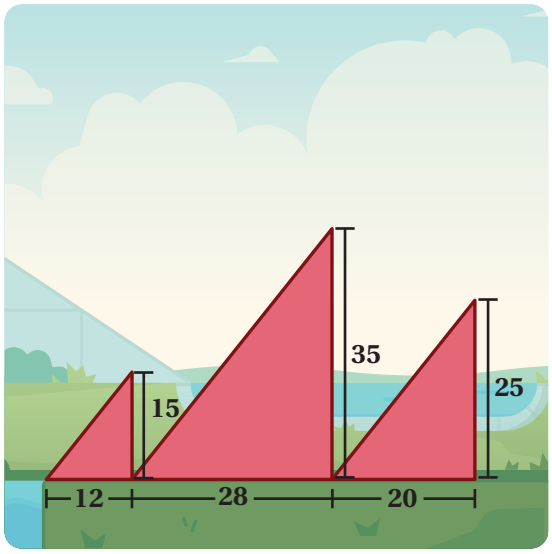


5 Here is a new set of ramps.

Will these ramps make a smooth slide? Circle one.

Yes No I'm not sure

Explain your thinking.



Activity 2

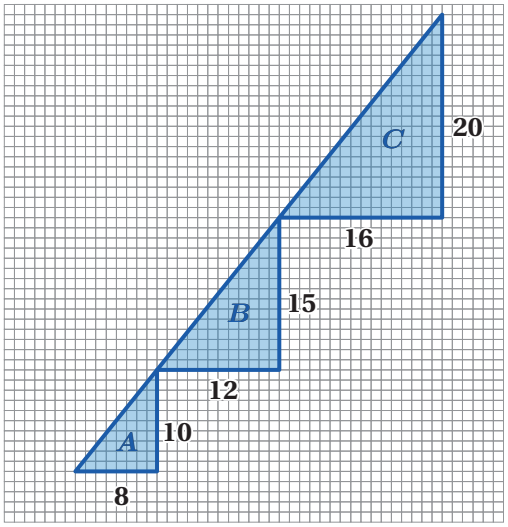
Name: ..... Date: ..... Period: .....

Introducing Slope

6 **Slope** measures the steepness of a line.

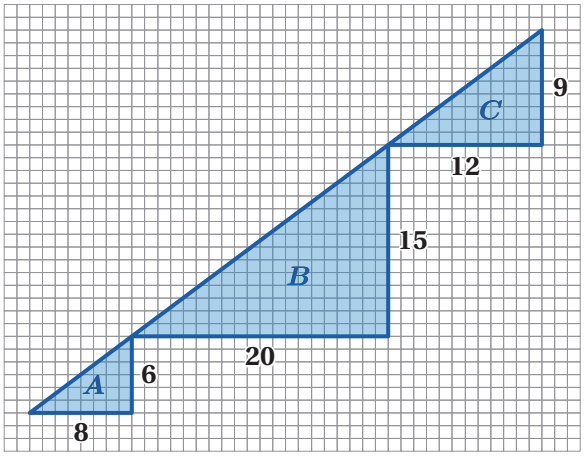
The slope of this line is $\frac{5}{4}$.

How do you think slope is calculated?



7 What is the slope of this line?

Explain your thinking.



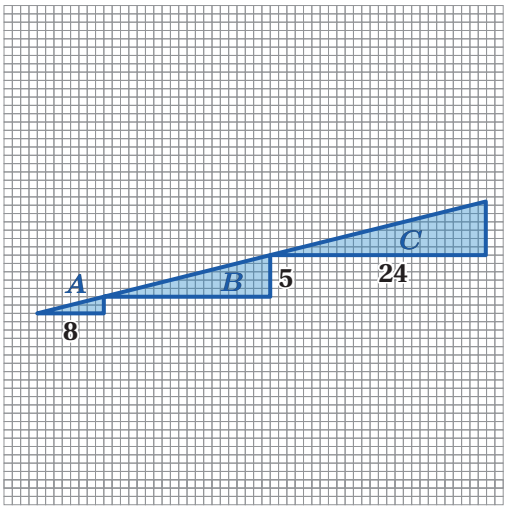
Name: ..... Date: ..... Period: .....

Introducing Slope (continued)

8 Here is a line.

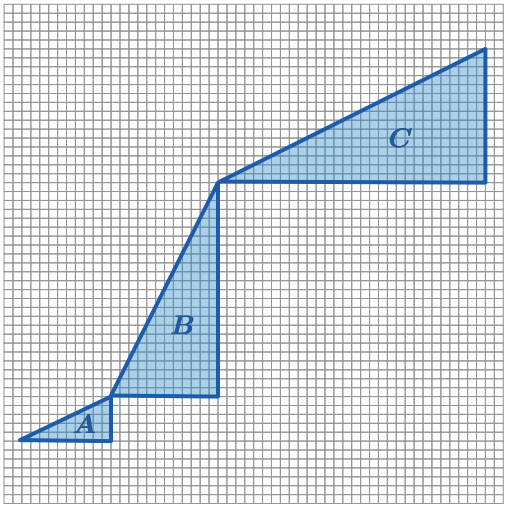
Determine the missing values for the slope triangles so that the line has a slope of $\frac{1}{4}$.

Triangle	Base (units)	Height (units)
A	8	
B		5
C	24	



9 Habib wants to create a line with a slope of $\frac{1}{2}$. He used slope triangles to create this line but made a mistake.

Triangle	Base (units)	Height (units)
A	10	5
B	12	24
C	30	15



a. **Discuss:** What do you think Habib did well?

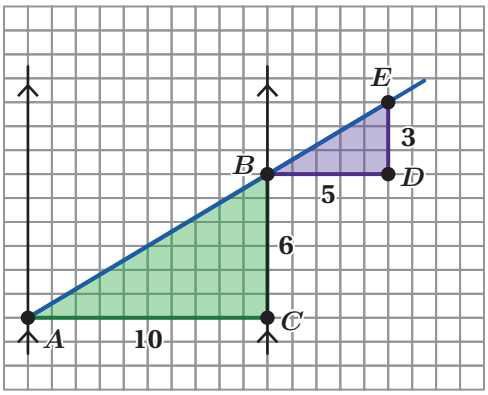
b. What would you recommend Habib change in his work?

Activity 3

Name: ..... Date: ..... Period: .....

Slope Between Two Points

10 The image shows two parallel lines and a transversal. The two triangles each have a side that is shared with the transversal.



a Which angle is congruent to angle BAC?

a Which angle is congruent to angle ABC?

11 **Discuss:** Are triangles ABC and BED similar? Explain your thinking.

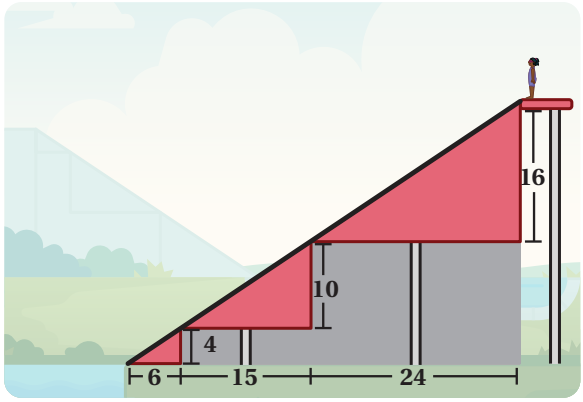
12 **a** What is the slope of AB? What is the slope of BE?

b What is the slope of the transversal?

c Why are the slopes between points A and B and points B and E the same?

13 Synthesis

Define *slope* in your own words and describe how to calculate it.

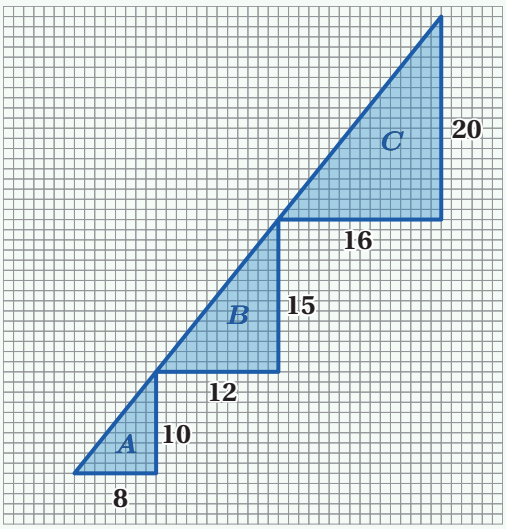


16 Summary 2.09

A non-vertical line can be made up of similar right triangles with proportional heights and bases.

Slope measures the steepness of a line and can be determined by the height-to-base ratio of the triangles on the line. There are infinite triangles on the same line that all have the same height-to-base ratio.

For example, the slope of this slide is $\frac{5}{4}$ because $\frac{15}{12} = \frac{35}{28} = \frac{25}{20} = \frac{5}{4}$.



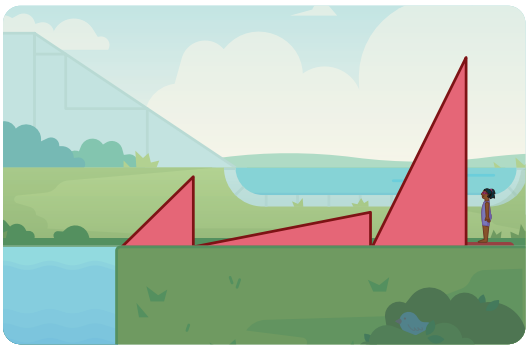
slope A number that describes the direction and steepness of a line. Slope represents the amount that y changes when x increases by 1. That's why the slope of a line is sometimes called a *rate of change*.

To calculate the slope, divide the vertical distance between any two points on the line by the horizontal distance between those points.

Practice
2.09

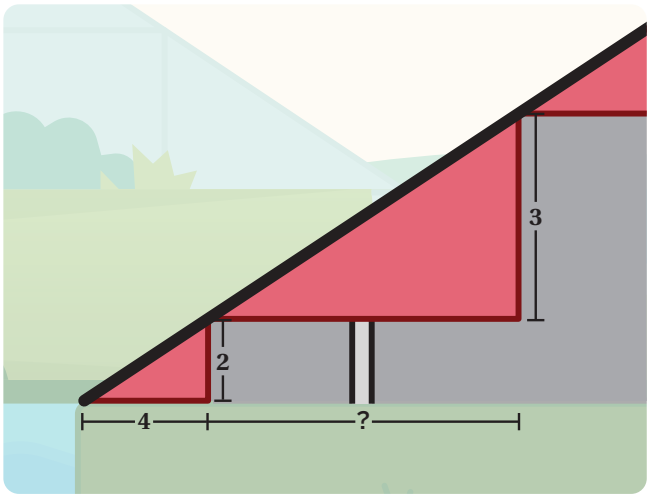
Name: ..... Date: ..... Period: .....

1. Here are three ramps. One ramp has a slope of 1, one ramp has a slope of 2, and one ramp has a slope of $\frac{1}{5}$. Label each ramp with its slope.



2. Jaylin created a slide with a slope of $\frac{1}{3}$. LaShawn created a slide with a slope of $\frac{1}{2}$. Whose slide was steeper? Explain your thinking.

Problems 3–5: Here are three ramps that make a smooth slide.



3. How long is the base of the second ramp?

4. Explain how you know the triangles that form the ramps are similar.

5. Which statement about the slopes of the ramps is true?

A. The slope of the first ramp is $\frac{2}{3}$ the slope of the second ramp because the height of the first ramp is $\frac{2}{3}$ the length of the second ramp.

B. The slope of the first ramp is the same as the slope of the second ramp because the triangles are similar.

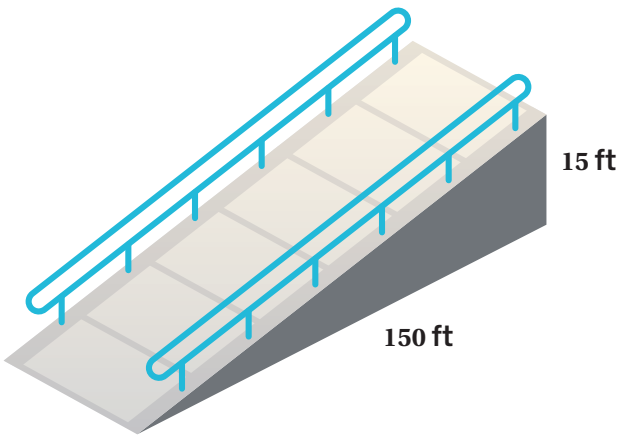
C. The slope of the first ramp is $\frac{4}{9}$ the slope of the second ramp because the area of the first ramp triangle is $\frac{4}{9}$ the area of the second ramp triangle.

D. The slope of the second ramp is 2 more than the slope of the first ramp because the difference between the bases of the ramps is 2.

Name: _____ Date: _____ Period: _____

6. The Americans With Disabilities Act of 1990, or ADA, is a civil rights act that protects Americans with disabilities from discrimination. It requires that public entities make their programs, services, and activities accessible. One requirement is that public wheelchair ramps must have a maximum slope of $\frac{1}{12}$

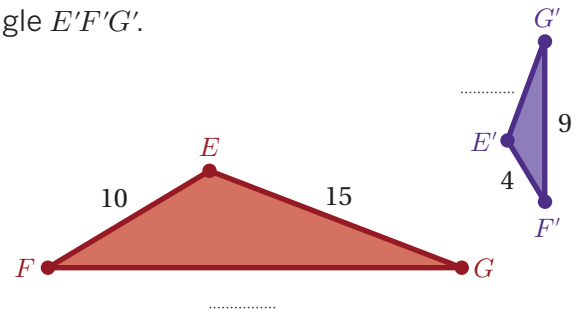
Does this ramp fit the requirement? Explain your thinking.



Spiral Review

Problems 7–8: Triangle EFG is similar to triangle $E'F'G'$.

7. Determine the missing values and explain your thinking.



8. Which transformation could map triangle EFG onto triangle $E'F'G'$?

A. A translation and a rotation.

B. A reflection and a rotation.

C. A translation and a dilation

D. A dilation and a rotation.

Name: ..... Date: ..... Period: .....

Multiple Representations of Functions 8.EE.6, SMP.3, SMP.8



Slope Challenges

Let's figure out the slopes of lines.

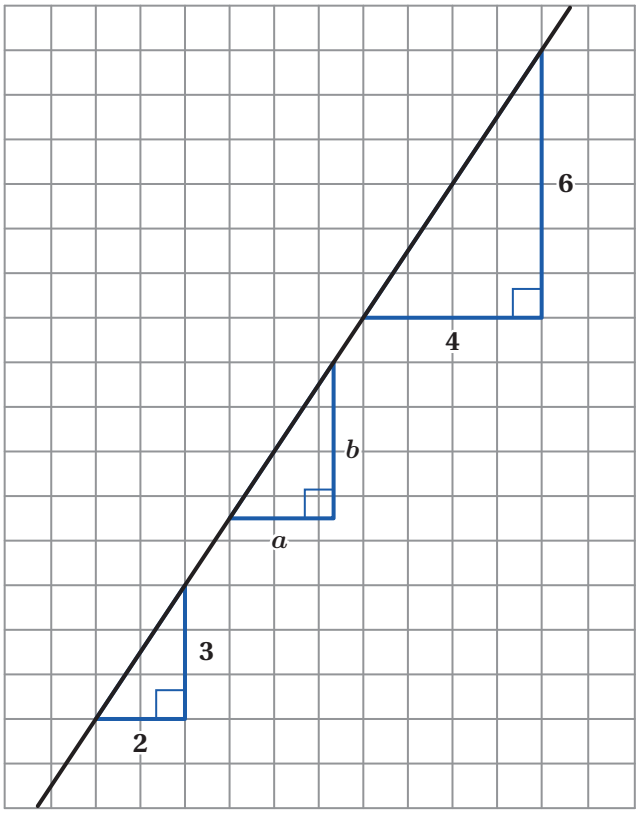
Warm-Up

1 Here are three **slope triangles**.

What do you notice? What do you wonder?

I notice:

I wonder:

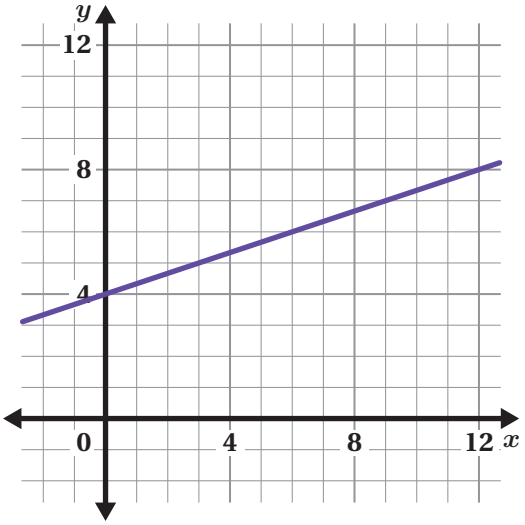


Activity 1

Name: ..... Date: ..... Period: .....

Determining Slope

2 Here is a line.

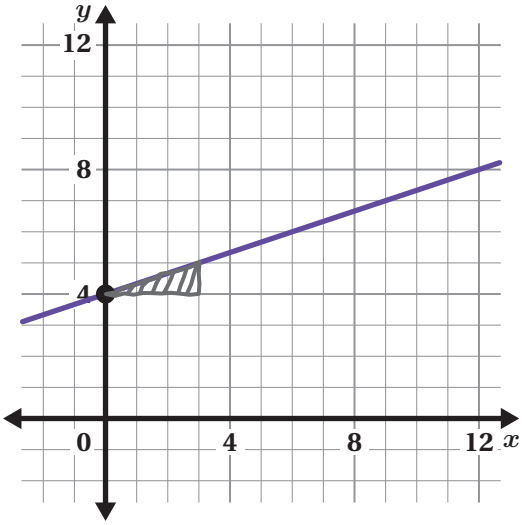


a Draw at least two *slope triangles* along the line.

b What is the slope of the line?

3 Liam says the slope of this line is 3.

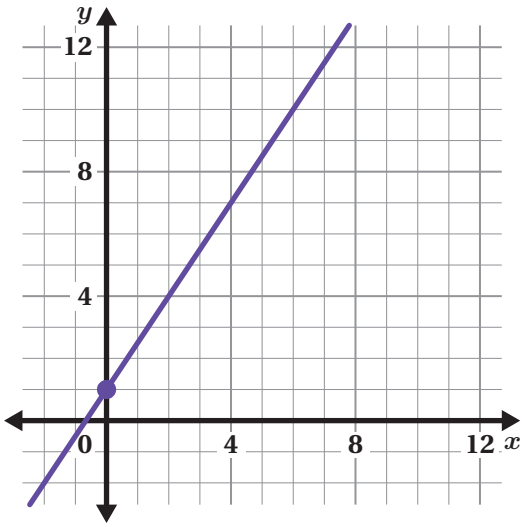
What would you say to Liam to help him understand that the slope is $\frac{1}{3}$?



Name: ..... Date: ..... Period: .....

Determining Slope (continued)

4 What is the slope of this line?



5 Kweku says the slope of the line in the previous problem is $\frac{6}{4}$.

Liam says the slope is $\frac{2}{3}$.

Whose thinking is correct? Circle one.

Kweku's Liam's Both Neither

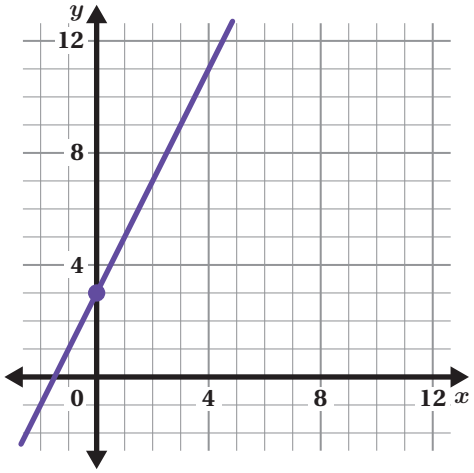
Explain your thinking.

Name: ..... Date: ..... Period: .....

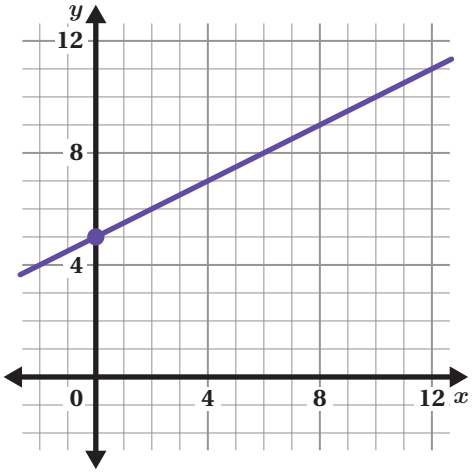
Repeated Challenges

6 What is the slope of each line? Solve as many challenges as you have time for. It's more important to make sense of the challenges than it is to work quickly!

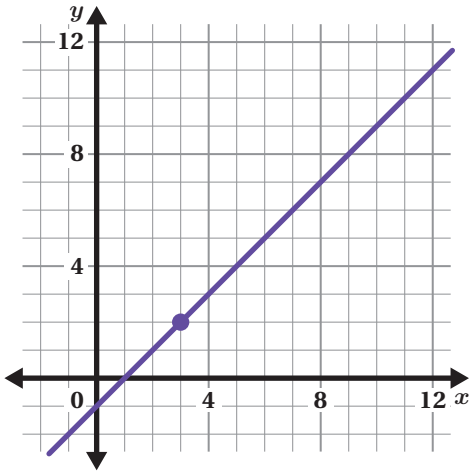
a



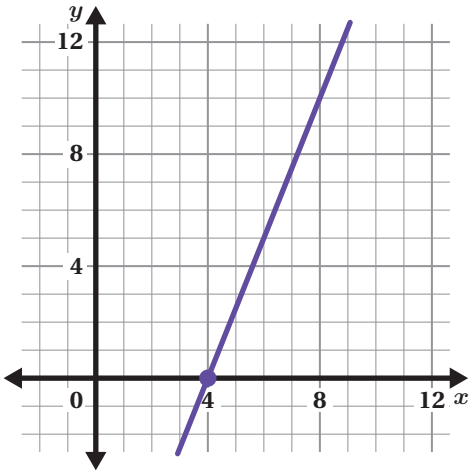
b



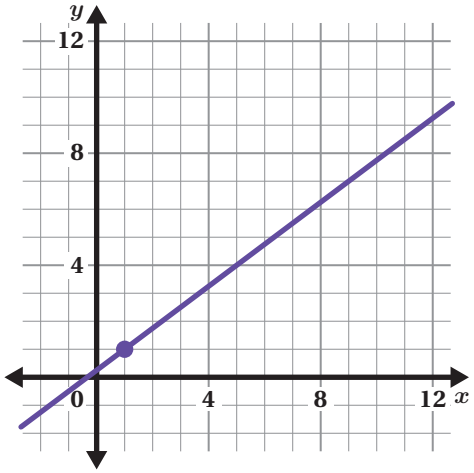
c



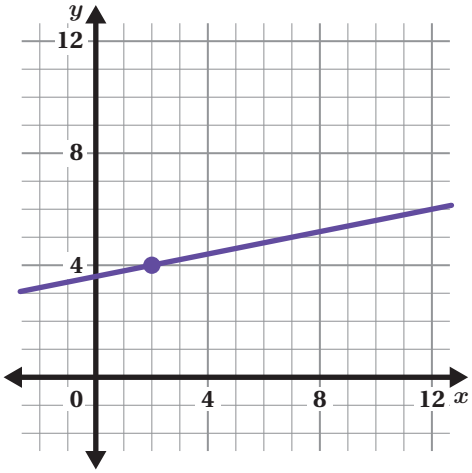
d



e



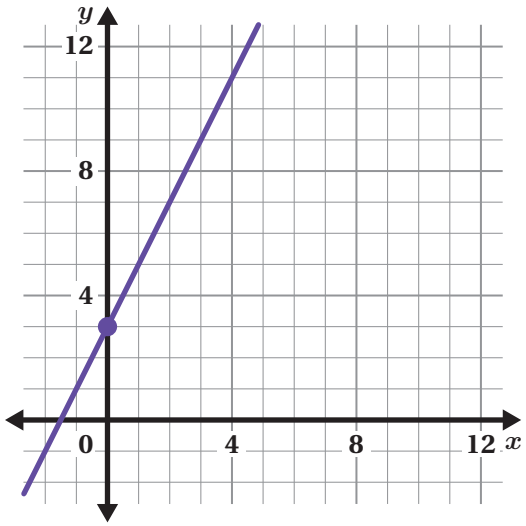
f



7 Synthesis

Discuss: What is one strategy you can use to determine the slope of any line?

Use the example if it helps you with your thinking.



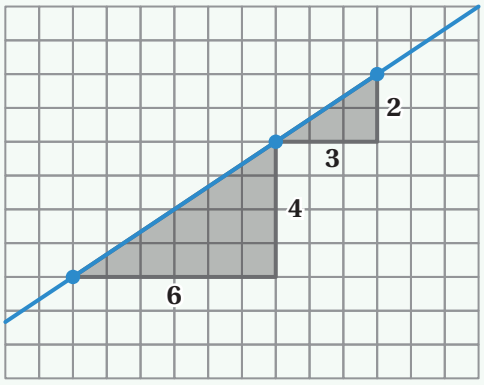
10 Summary 2.10

You can determine the slope of a non-vertical line by drawing similar right triangles, called **slope triangles**, between any two points on the line. The height of the slope triangle represents the vertical distance between the points, and the base of the triangle represents the horizontal distance between the points.

Slope is the ratio of the height to the base of a slope triangle.

Here's an example of two possible slope triangles that you could use to calculate the slope of this line.

The slope of this line is $\frac{4}{6}$, or $\frac{2}{3}$, or any equivalent value.



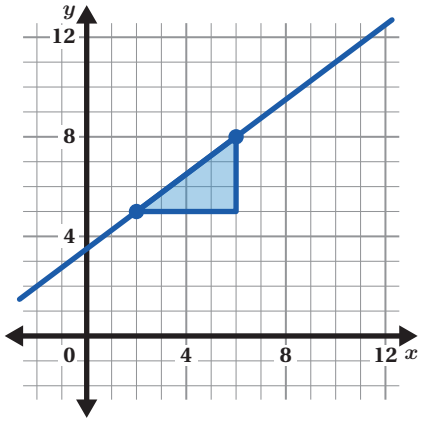
slope triangle A triangle drawn to include two points on a line in order to determine the slope of that line. The longest side lies on the line and the other two sides are vertical and horizontal. The height of the slope triangle represents the vertical distance between the points, and the base of the triangle represents the horizontal distance between the points.

Practice
2.10

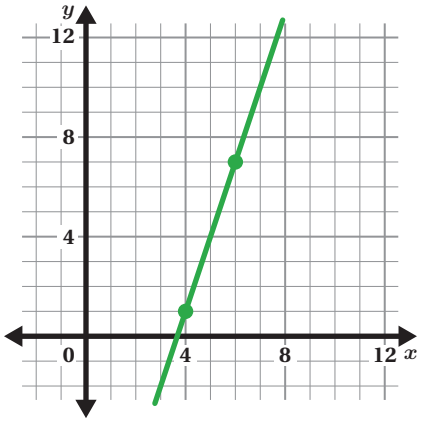
Name: ..... Date: ..... Period: .....

Problems 1–4: Determine the slope of each line.

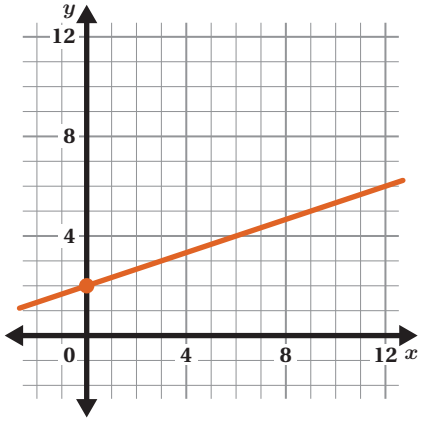
1. Slope: .....



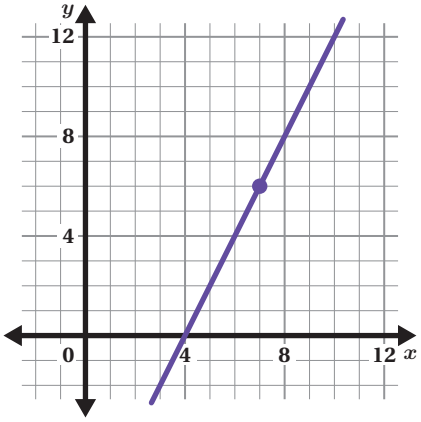
2. Slope: .....



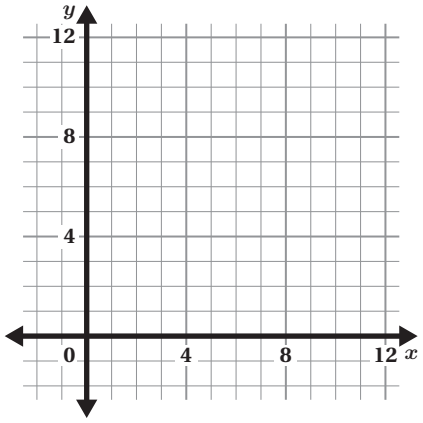
3. Slope: .....



4. Slope: .....

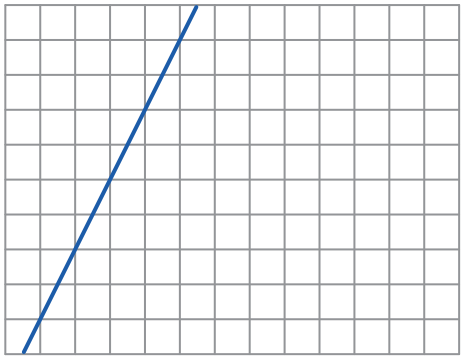


5. Draw a line with a slope of $\frac{2}{3}$.



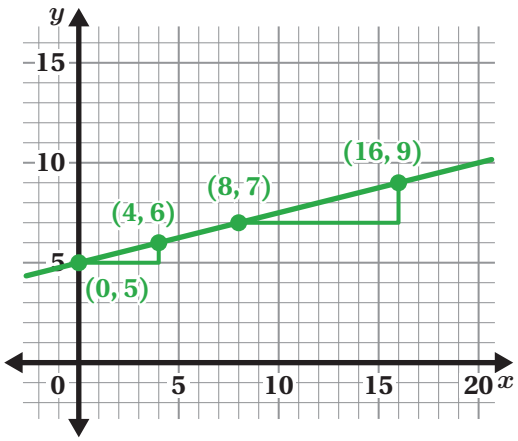
6. Here is a line. Draw a line that is parallel. What is the slope of each line?

Explain your thinking.



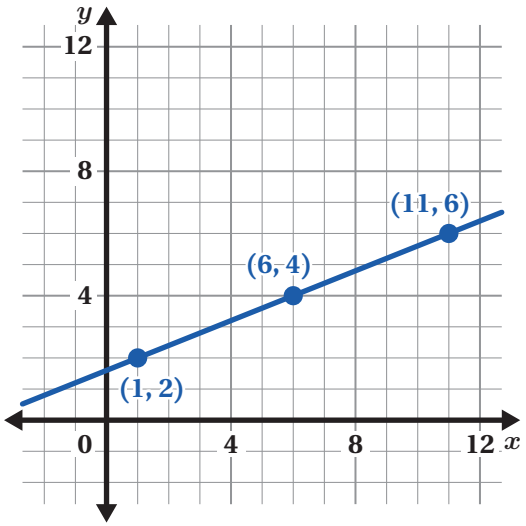
Name: ..... Date: ..... Period: .....

7. Select *all* the expressions that could represent the slope of this line.



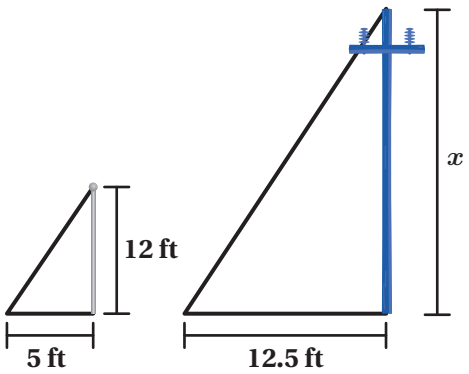
- ☐ A. $\frac{1}{4}$
- ☐ B. $\frac{4}{6}$
- ☐ C. 4
- ☐ D. $\frac{2}{8}$
- ☐ E. $\frac{8}{2}$

8. Fatima says the slope of this line is $\frac{2}{5}$. Daniela says the slope of this line is 2.5. Whose claim is correct? Explain your thinking.



Spiral Review

9. A 12-foot flagpole casts a shadow that is 5 feet long. At the same time, a nearby telephone pole casts a shadow that is 12.5 feet long. What is the height of the telephone pole, x?



- A. 5.2 feet
- B. 15 feet
- C. 18.5 feet
- D. 30 feet

10. Point $(4, -5)$ is dilated using $(0, 0)$ as the center of dilation and a scale factor of 3. What are the coordinates of the image?

- A. $(12, -15)$
- B. $(12, 15)$
- C. $(-15, 12)$
- D. $(15, 12)$

Name: _____ Date: _____ Period: _____

Transformational Geometry | Multiple Representations of Functions | 8.EE.6, 8.G.3, 8.G.4, SMP.6

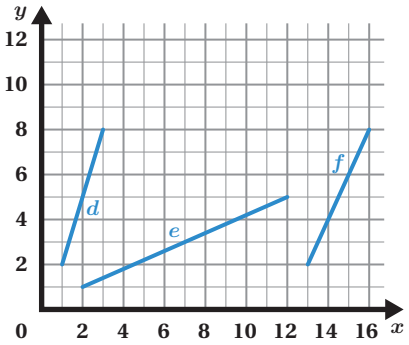
Practice Day 2, Set A



Let's practice what you've learned so far in this unit!

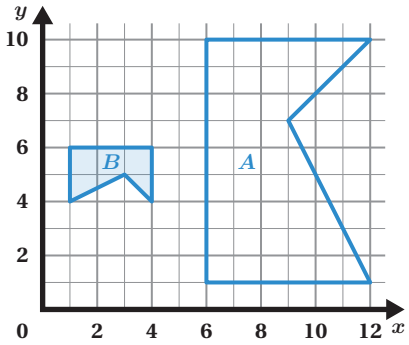
Record all of your responses here.

1. Here are three lines.



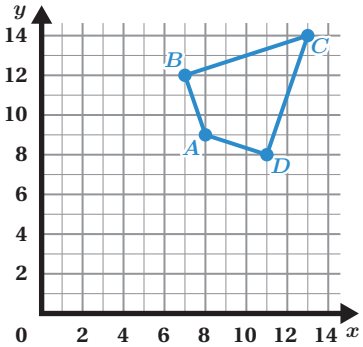
Which line is the steepest? Explain your thinking.

2. Polygon A and polygon B are similar.

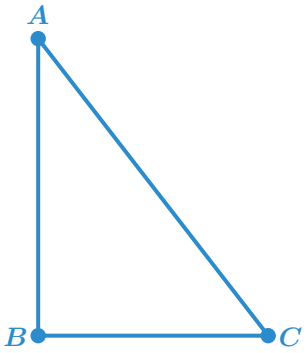


Describe a sequence of transformations that maps polygon A onto polygon B.

3. Quadrilateral $ABCD$ is dilated using point C as the center of dilation and a scale factor of 2. Draw quadrilateral $A'B'CD'$.



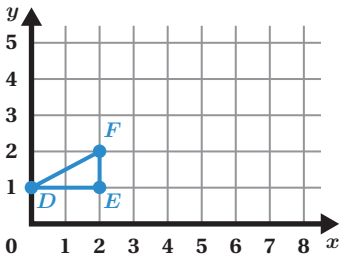
4. Here is triangle ABC. Draw a dilation of triangle ABC using point A as the center of dilation and a scale factor of $\frac{1}{2}$.



Name: Date: Period:

Practice Day 2

5. Here is triangle DEF.



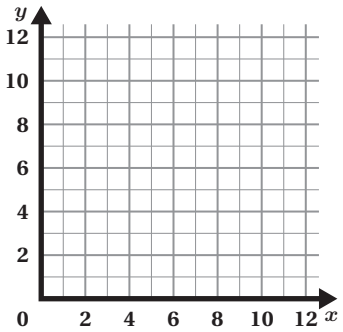
a Draw a dilation of triangle DEF with center $(0, 1)$ and a scale factor of 2.

b A new dilation with center $(0, 1)$ sends point F to $(9, 5.5)$. What is the scale factor?

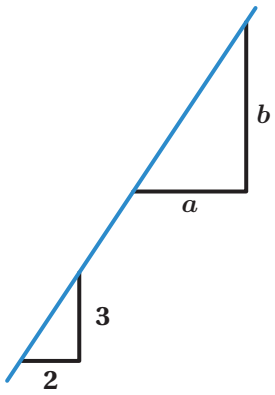
Explain how you know.

6. Triangle GHI has vertices located at $G(0, 5)$, $H(2, 0)$, and $I(5, 3)$. Triangle $G'H'I'$ is the result of dilating triangle GHI using the origin as the center of dilation and a scale factor of 2.

Draw triangle GHI and its image on the coordinate plane.



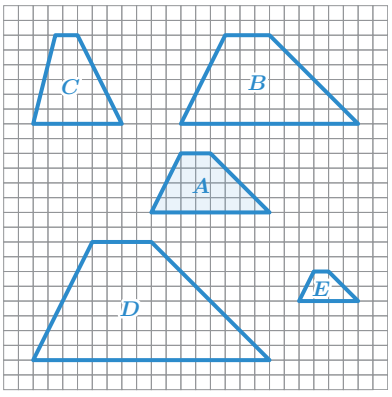
7. This graph shows two slope triangles.



a Determine a possible value for a and b.

b What is the slope of the line?

8. List the figures that are similar to figure A.



Name: ..... Date: ..... Period: .....

Transformational Geometry | Multiple Representations of Functions | 8.EE.6, 8.G.3, 8.G.4, SMP.6

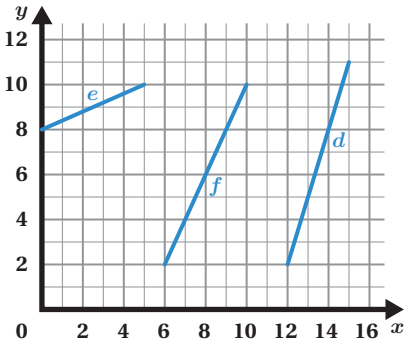
Practice Day 2, Set B



Let's practice what you've learned so far in this unit!

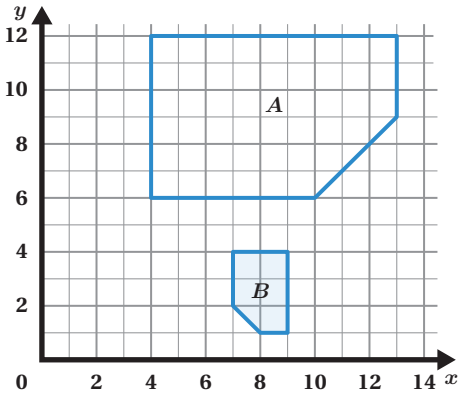
Record all of your responses here.

1. Here are three lines.



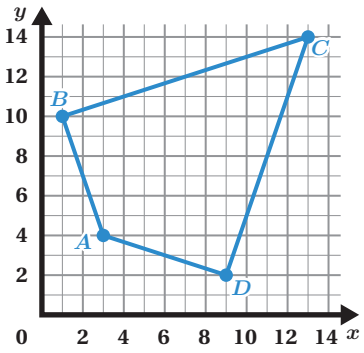
Which line is the steepest? Explain your thinking.

2. Polygon A and polygon B are similar.

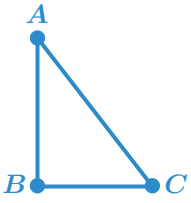


Describe a sequence of transformations that maps polygon A onto polygon B.

3. Quadrilateral $ABCD$ is dilated using point C as the center of dilation and a scale factor of $\frac{1}{2}$. Draw quadrilateral $A'B'CD'$.



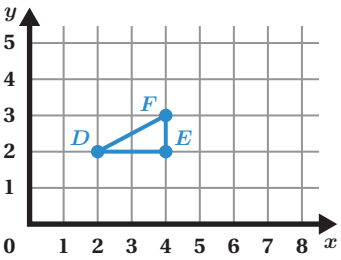
4. Here is triangle ABC. Draw a dilation of triangle ABC using point A as the center of dilation and a scale factor of 2.



Name: Date: Period:

Practice Day 2

5. Here is triangle DEF.



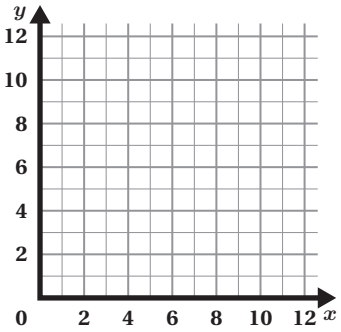
a Draw a dilation of triangle DEF using point $(4, 3)$ as the center of dilation and a scale factor of 2.

b A new dilation with using point $(2, 2)$ as the center of dilation sends point F to $(11, 6.5)$. What is the scale factor?

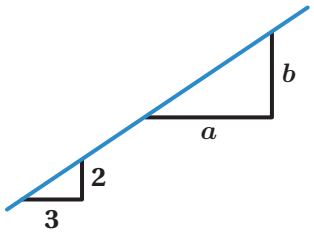
Explain how you know.

6. Triangle GHI has vertices located at $G(0, 3)$, $H(4, 5)$, and $I(3, 2)$. Triangle $G'H'I'$ is the result of dilating triangle GHI using the origin as the center of dilation and a scale factor of 2.

Draw triangle GHI and its image on the coordinate plane.



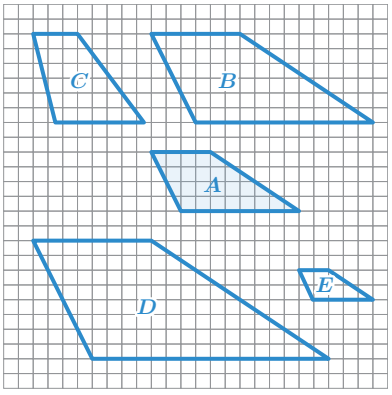
7. This graph shows two slope triangles.



a Determine a possible value for a and b.

b What is the slope of the line?

8. List the figures that are similar to figure A.



Career Connection

When an online map is zoomed in or out, what math is at work?

Dilations and similarity are among the key players! For different zoom levels, street lengths on a map remain proportional to their real life counterparts, related by a scale factor. Topographic maps, which also show elevation, use the concept of slope to describe how land surfaces change vertically over given distances.



NicoElNino/Shutterstock.com

Cartographers collect data about different places on Earth to create and maintain print and interactive online maps. They use scale factors and coordinates to represent Earth's curved surfaces on a flat map.



Meet Laura Escobar

Laura Escobar, from Bogotá, Colombia, is a professor of mathematics at Washington University in St. Louis, Missouri. Her research focuses on the connections between algebra and geometry. The concepts you learned in this unit (dilations, similarity, and slope) are just some of the many ways that algebra and geometry are connected.

Are you interested in studying more connections between algebra and geometry or exploring more about cartography? What can you do to learn more?

Math in the World

The circular centers of your eyes that appear black are called *pupils*. They allow light into your eyes. In human adults, a pupil's diameter is typically 2 to 4 millimeters in bright light and dilates to 4 to 8 millimeters in dim light. A dilated pupil is always similar to its non-dilated state. Why do you think that is?



MOUii/Shutterstock.com

Math Mindset

How do coordinates help communicate precise information about dilations? Why might that be important?

Unit 3

Proportional and Linear Relationships



Big Ideas in This Unit

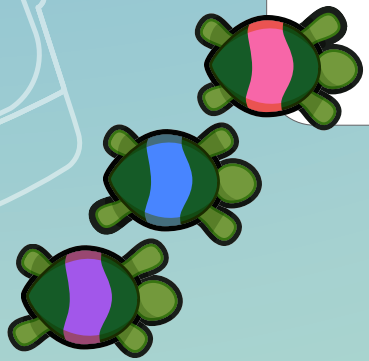
CC1 Data, Graphs, and Tables
CC2 Linear Equations
Multiple Representations of Functions
CC4 Transformational Geometry



Questions for Investigation

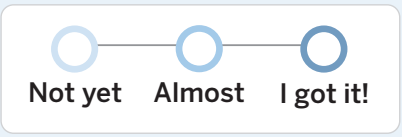
- What can proportional relationships and slope teach you about linear relationships?
- How do equations, tables, and graphs of linear relationships connect to one another?
- What does it mean for an ordered pair to be a solution to a linear equation?

Explore: Visual Patterns
What patterns do you notice in shapes and numbers?



Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



I can . . .	Before	After
Graph proportional relationships.		
Interpret the slope of a graph.		
Compare two different proportional relationships represented in different ways.		
Create a graph of a linear relationship.		
Describe the slope of a linear relationship in context.		
Describe the initial value of a linear relationship in context.		
Write an equation for a linear relationship in the form $y = mx + b$.		
Determine the slope of a linear relationship from a graph.		
Determine the initial value of a linear relationship from a graph.		

I can . . .	Before	After
Determine the slope of a linear relationship from a table.		
Determine the initial value of a linear relationship from a table.		
Write the equation of a line given two points on the line.		
Justify whether a point is a solution to an equation.		

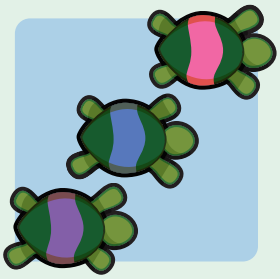
Sub-Unit

1

Proportionality Revisited



Explore
Visual Patterns



Lesson 1
Turtle Time Trials



Lesson 2
Water Tank



Lesson 3
Proportional Posters

Name: ..... Date: ..... Period: .....

Multiple Representations of Functions | Building Toward 8.EE.5, SMP.2, SMP.7

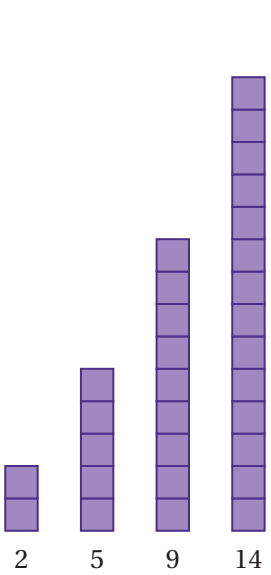
Explore: Visual Patterns

What patterns do you notice in shapes and numbers?

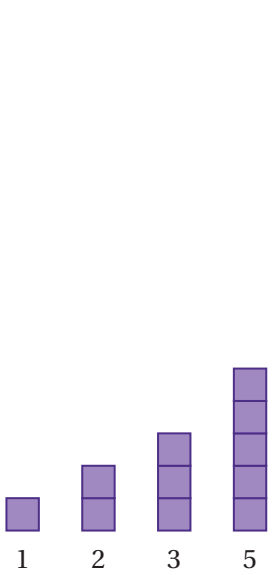
Warm-Up

1. Which one doesn't belong? Explain your thinking.

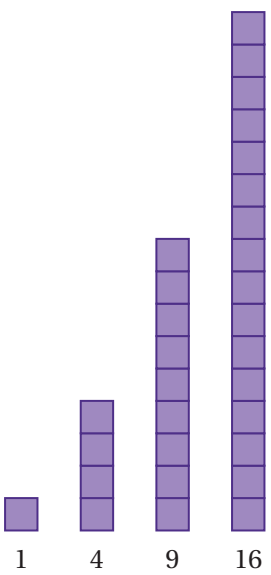
A.



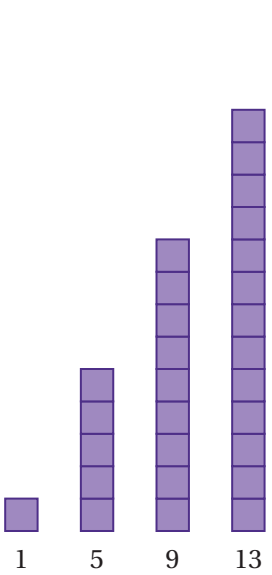
B.



C.



D.

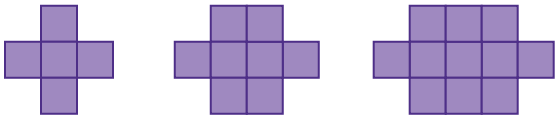


Name: ..... Date: ..... Period: .....

Sketchy Patterns

Here is a pattern.

Figure 1 **Figure 2** **Figure 3**



2. Study the pattern and draw a sketch of Figure 4.

3. Draw or describe what Figure 10 looks like.

4. **Discuss:** How the pattern is growing?

5. Complete the table to show the number of squares for each figure.

Figure	1	2	3	4	10	26
Number of squares						

6. Write an *expression* that represents the number of squares in Figure n.

Name: ..... Date: ..... Period: .....

Sketchy Patterns (continued)

You will use the Activity card to complete this activity.

7. Study the pattern and draw or describe what Figure 4 and Figure 10 will look like.

8. **Discuss:** How the pattern is growing?

9. Complete the table.

Figure	1	2	3	4	10	26	n
Number of objects							

Name: ..... Date: ..... Period: .....

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet Almost I got it!

I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem.

Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed.

Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

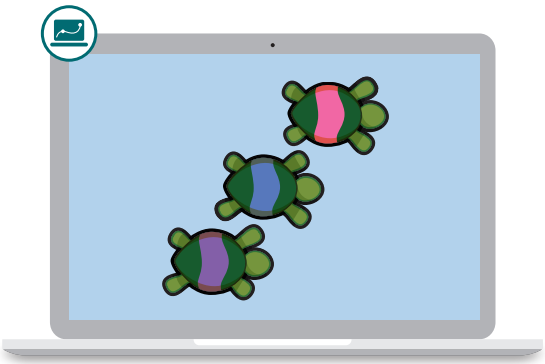
Not yet Almost I got it!

Unit 3
Lesson
1

Name: ..... Date: ..... Period: .....

Multiple Representations of Functions | Linear Equations | Slopes and Intercepts

8.EE.5, SMP.1, SMP.4

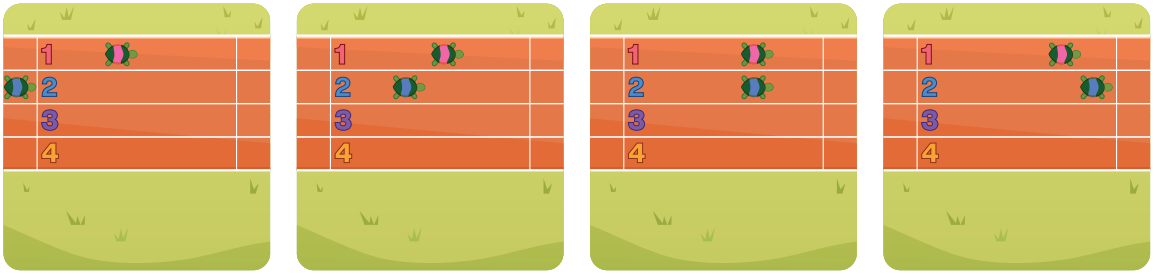


Turtle Time Trials

Let's explore a turtle race with multiple representations.

Warm-Up

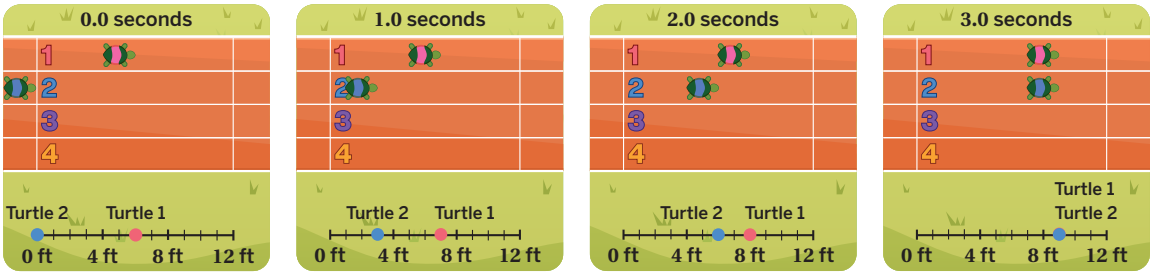
1 Let's watch a short animation. Write a story about what you see.



Name: ..... Date: ..... Period: .....

Turtle Race

2 Let's watch the same animation from the Warm-Up, but with additional information.



Complete the table.

Time (sec)	Distance of Turtle 1 (ft)	Distance of Turtle 2 (ft)
0	6	0
1	7	3
2	8	
3	9	

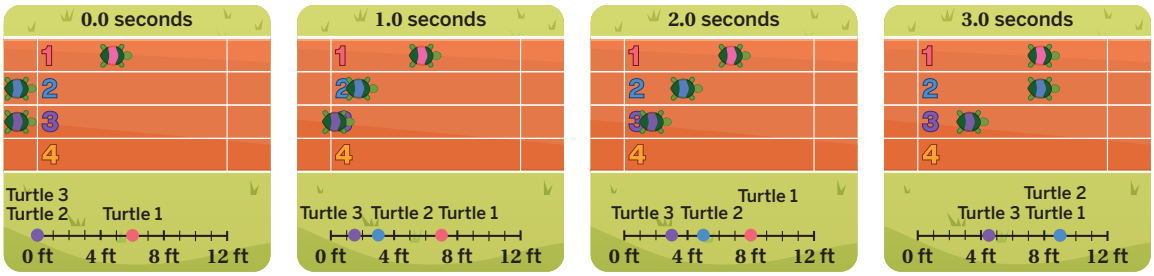
3 What is Turtle 2's speed as a *unit rate*? Explain your thinking.

4 Let's watch an animation showing Turtle 3 running in the same race.

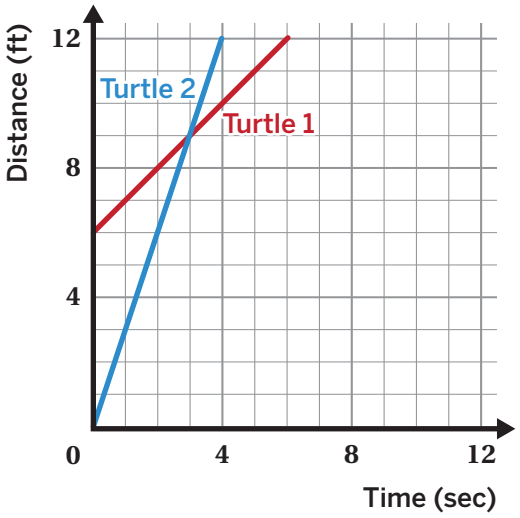
Name: ..... Date: ..... Period: .....

Turtle Race (continued)

5 Here is some information from the race with Turtle 3.



Graph the relationship between distance and time for Turtle 3.



6 Evan says that the relationship between distance and time is *proportional* for all three turtles.

Is Evan's claim correct?

Yes No I'm not sure

Explain your thinking.

7 **a** What is Turtle 3's speed as a *unit rate*? Explain your thinking.

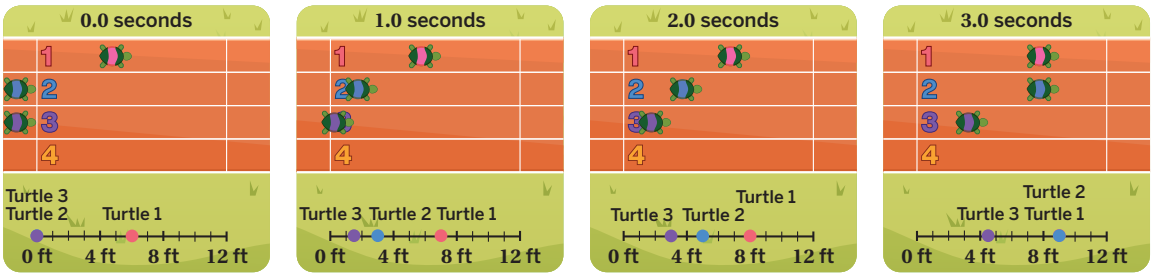
b How does this unit rate relate to the *slope* of Turtle 3's line?

Activity 2

Name: ..... Date: ..... Period: .....

Ready Turtle Four

8 Here is some information from the race with three turtles. In Lane 4, another turtle is running in the same race.



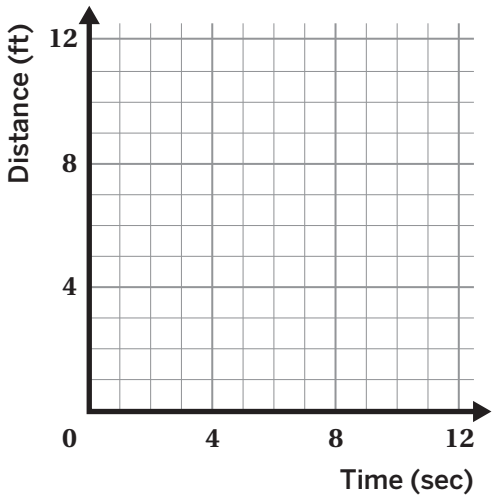
a Write an equation for Turtle 4 to make it finish in whatever place you want.

b **Discuss:** Based on your equation, how does Turtle 4's race compare to the other turtles?

Turtle	Equation
1	$d = 6 + 1t$
2	$d = 3t$
3	$d = 1.5t$
4	

9 **a** Create your own turtle race by sketching lines for up to four turtles. At least one line should be proportional. Use a different color for each turtle.

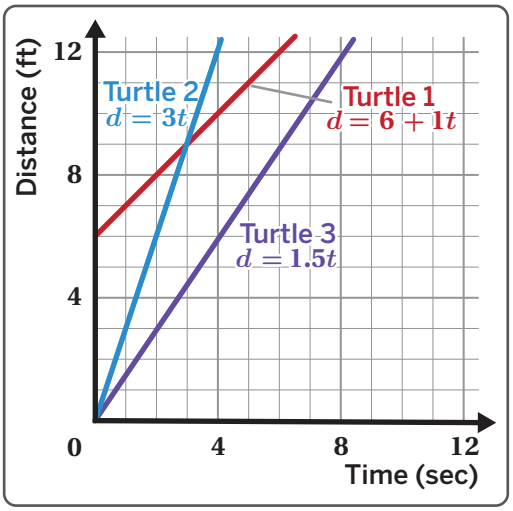
b **Discuss:** Using the information in your graph, tell a story about your turtle race.



10 Synthesis

Discuss: How do you use tables, graphs, and equations to compare the turtles in a race?

Use the example if it helps with your thinking.

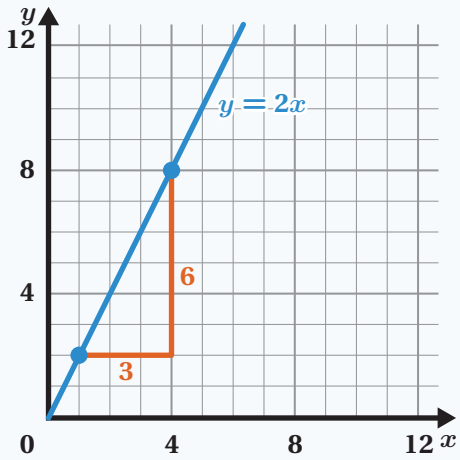


Time (sec)	Distance of Turtle 1 (ft)	Distance of Turtle 2 (ft)	Distance of Turtle 3 (ft)
0	6	0	0
1	7	3	1.5
2	8	6	3

13 Summary 3.01

A line that passes through the origin, $(0, 0)$, represents a *proportional relationship*. The *slope* of the line represents a *unit rate* for this relationship.

Here is a graph of the equation $y = 2x$. The slope of the line is $\frac{6}{3}$, or 2.

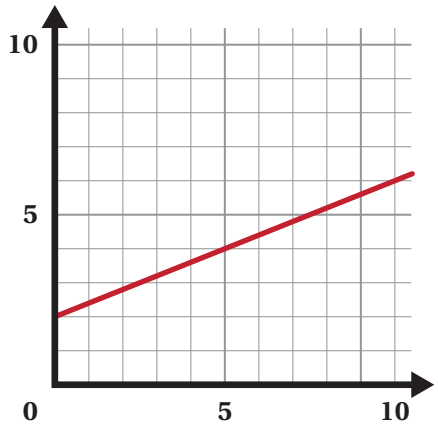


Practice
3.01

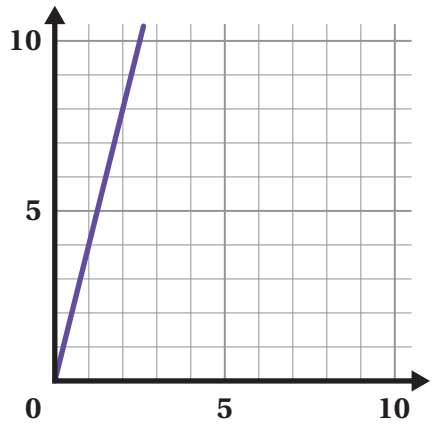
Name: ..... Date: ..... Period: .....

Problems 1–4: Determine whether each graph represents a proportional or non-proportional relationship. Then determine the slope of each line.

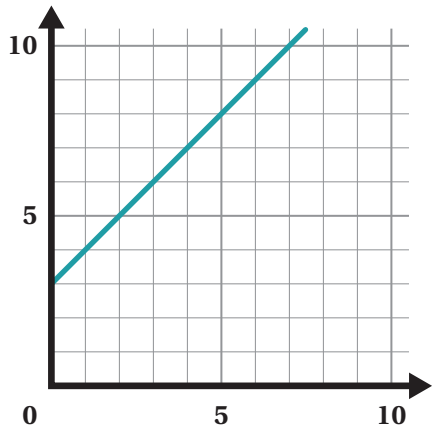
1.



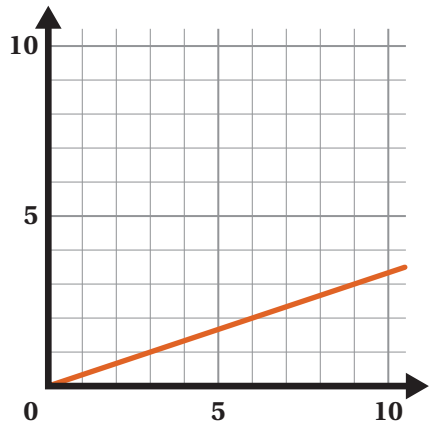
2.



3.



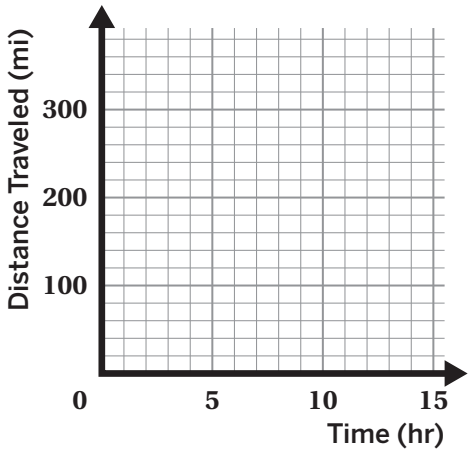
4.



Problems 5–6: From rest, a car travels at a constant rate. After 2 hours, the car has traveled 80 miles.

5. Graph the line showing the relationship between the car's distance traveled and time.

6. What is the slope of the line and what does it represent in context?



Practice
3.01

Name: ..... Date: ..... Period: .....

7. An ant and a ladybug compete in a 15-centimeter race. Write a story that represents the two lines on the graph.

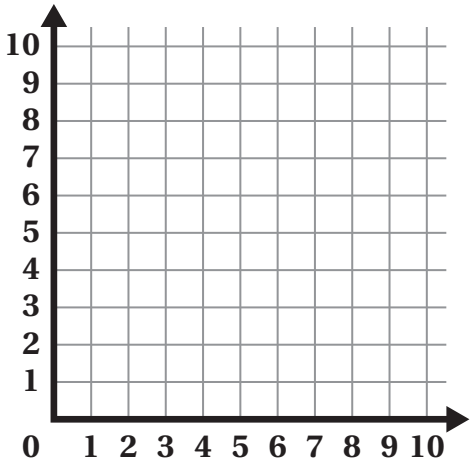


Spiral Review

Problems 8–9: Draw a line for each slope. Then label the line.

8. Line a has a slope of $\frac{3}{5}$.

9. Line c has a slope of 5.



Problems 10–11: This table shows a proportional relationship between x and y.

10. What is the constant of proportionality for the relationship between x and y?

A. 0.25

B. 0.50

C. 1.50

D. 1.75

x	y
0.25	0.375
1.5	2.250
2.75	4.125
6.50	9.75

11. Write an equation that represents the relationship between x and y.

Name: ..... Date: ..... Period: .....

Linear Equations | Multiple Representations of Functions | Slopes and Intercepts

8.EE.5, 8.EE.6, SMP.1, SMP.2, SMP.3



Water Tank

Let's analyze graphs of proportional equations.

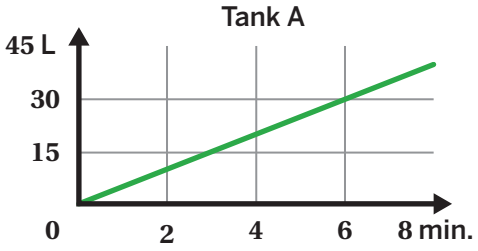
Warm-Up

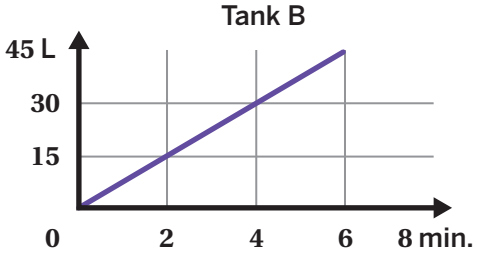
1 Water is essential to life. People, plants, and wildlife depend on it. Natural water systems, like rivers, lakes, and groundwater, help to support communities.

Discuss: What are some ways in which people depend on water?

2 People might fill water in bottles or tanks to use it for drinking, cleaning, or cooking. Let's watch two tanks fill up with water.

Discuss: How are the graphs alike? How are they different?





Activity 1

Name: ..... Date: ..... Period: .....

Two Tanks

3 Ella and Jamal both calculated slopes.

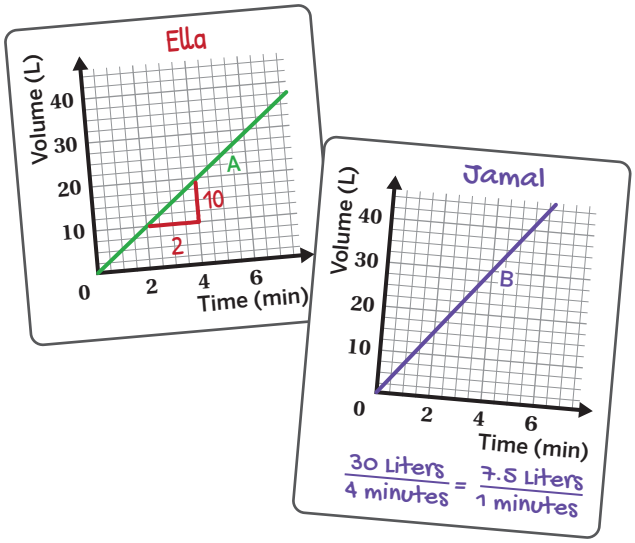
Ella says the slope of Tank A's line is $\frac{10}{2}$.

Jamal says the slope of Tank B's line is 7.5.

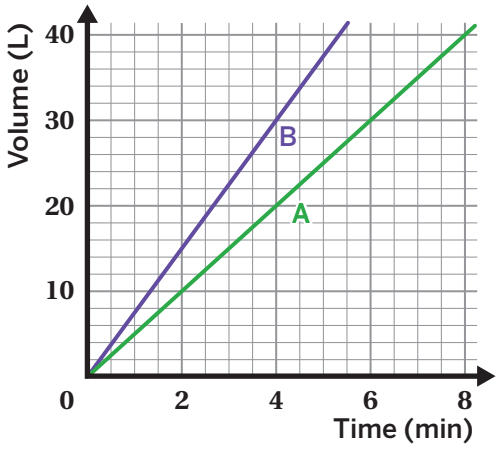
Whose claim is correct?

Ella's Jamal's Both Neither

Explain your thinking.



4 Here are the graphs of Tank A and Tank B, shown on the same coordinate plane.



a Are these relationships proportional? Explain your thinking.

b Explain what the slopes of each line represent for Tank A and Tank B.

5 For Tank A, the equation $V = 5t$ represents the volume, V, after t minutes.

a For Tank B, write an equation to represent the volume, V, after t minutes.

b In proportional relationships, how are the unit rate, constant of proportionality, and slope related?

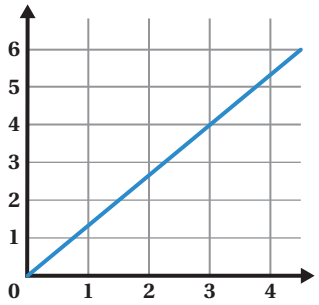
Activity 2

Name: ..... Date: ..... Period: .....

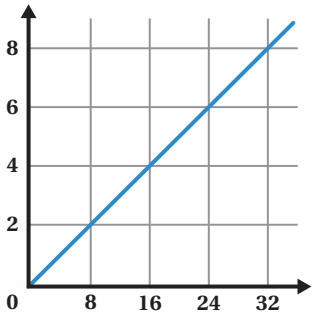
Pairing Graphs and Equations

6 Match each graph to the equation that represents the same proportional relationship.

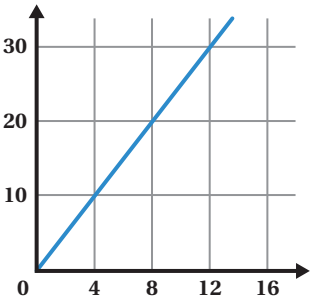
A.



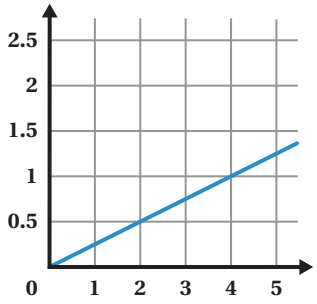
B.



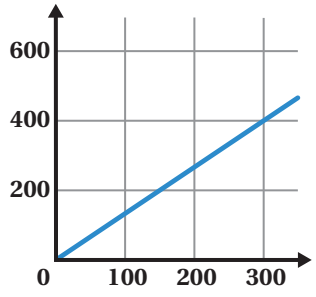
C.



D.



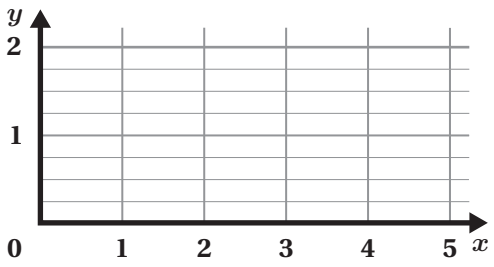
E.

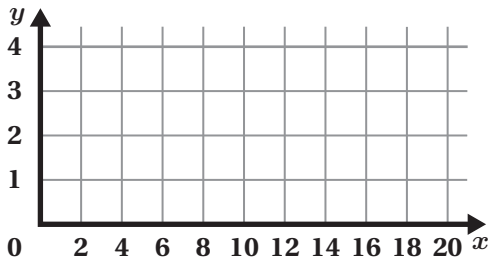


Equation	$y = \frac{4}{3}x$	$y = \frac{1}{4}x$	$y = 2.5x$
Graph			

7 **a** Draw the graph of $y = 0.75x$ on each set of axes.

b Explain how you decided where to draw the lines.

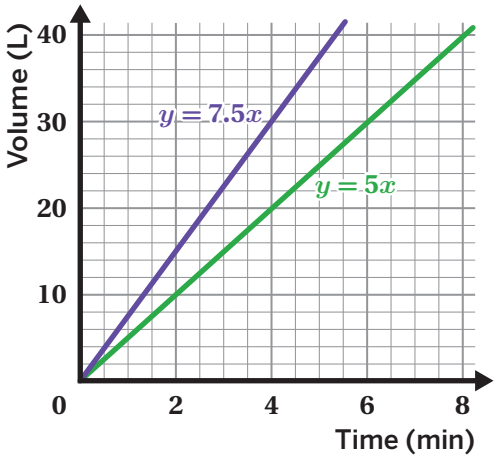




8 Synthesis

How are the equations of proportional relationships related to their graphs?

Use the examples if it helps with your thinking.

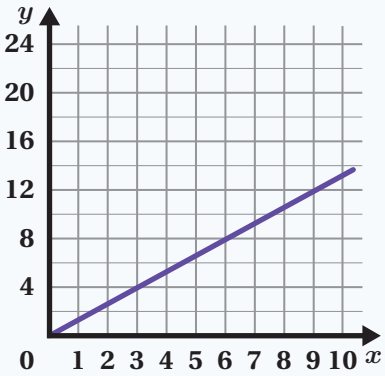


11 Summary 3.02

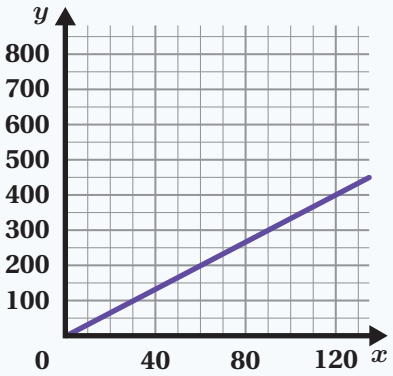
We can represent a proportional relationship using the equation $y = mx$, where m represents both a unit rate and the slope of the line.

Generally, you can derive the equation $y = mx$ to represent a proportional relationship, where m is the slope of the line and also represents the unit rate for the situation. The graph of this line passes through the origin.

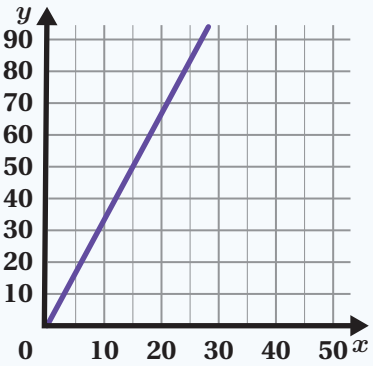
Graph A



Graph B



Graph C



- Graph A slope: $\frac{200}{150} = \frac{4}{3}$
- Graph B slope: $\frac{100}{30} = \frac{10}{3}$
- Graph C slope: $\frac{50}{15} = \frac{10}{3}$

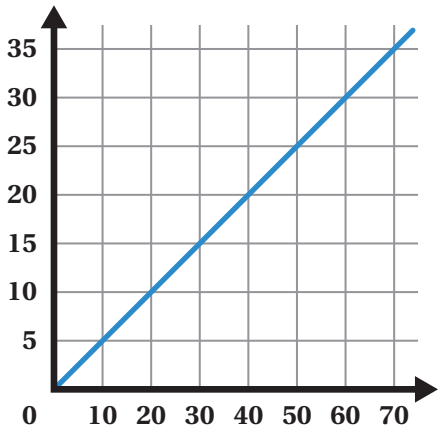
You can see that the slopes for Graph B and Graph C are equivalent. This means they have the same proportional relationship, even though the lines may look like they don't have the same steepness.

Practice
3.02

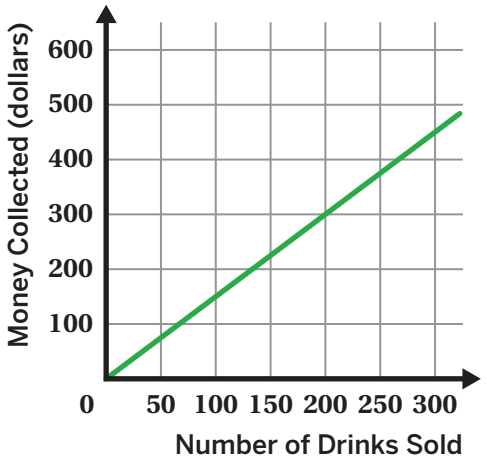
Name: ..... Date: ..... Period: .....

1. Write an equation for the graph of a proportional relationship that passes through the point $(25, 15)$. Explain your thinking.

2. Write an equation of the line graphed. Use $y = mx$ form where m represents the slope of the line.



Problems 3–5: Here is the graph of a proportional relationship.



3. Write an equation to represent this relationship, where x is the number of drinks sold at a concession stand and y is the amount of money collected in dollars.

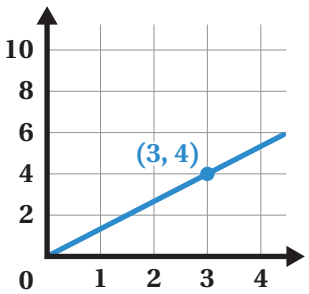
4. What is the slope of the line?

5. Use your equation to complete this table.

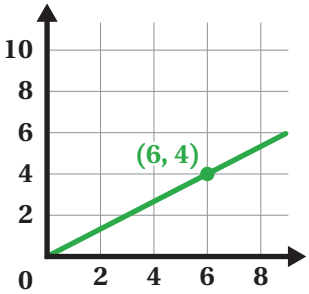
Drinks Sold	220		1
Money (Dollars)		1500	

Problems 6–7: Here are three graphs.

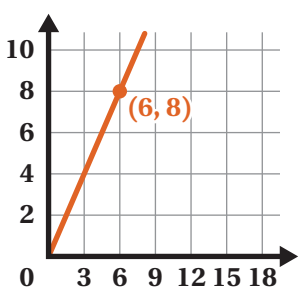
Graph 1



Graph 2



Graph 3



Name: ..... Date: ..... Period: .....

6. Does Graph 2 represent the same relationship as Graph 1? Explain your thinking.

7. Does Graph 3 represent the same relationship as Graph 1? Explain your thinking.

Spiral Review

8. Write an equation for the proportional relationship represented in this table.

x	y
25	2000
40	3200

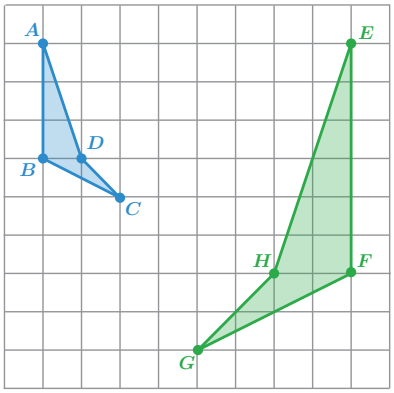
Problems 9–10: This table shows a proportional relationship between time, in minutes, and distance walked, in miles.

Time (min)	Distance (miles)
90	5
	12
162	
	1

9. Complete the table.

10. Describe the scales you could use on the x- and y-axes of a coordinate grid to show all the times and distances in the table.

11. Describe a sequence of rotations, reflections, translations, and dilations to show that one figure is similar to the other.



Name: ..... Date: ..... Period: .....

Linear Equations | Multiple Representations of Functions | Slopes and Intercepts

8.EE.5, 8.EE.6, SMP.1, SMP.3

Proportional Posters



Let's compare proportional relationships.

Warm-Up

1. The table, graph, and equation all represent the same situation.

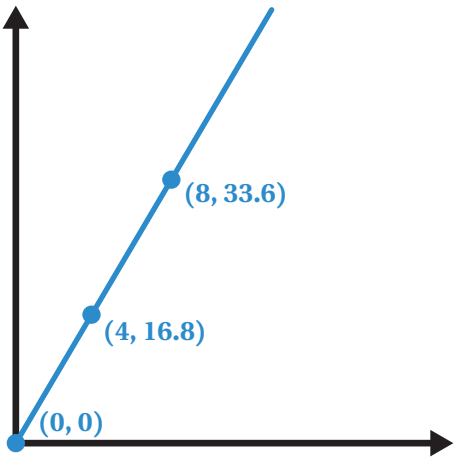
Table

x	y
0	0
4	16.8
8	33.6

Equation

$y = 4.2x$

Graph



a Describe a situation that could be represented by the equation, table, and graph.

b What does x and y represent in your situation?

c Label the axes of the graph to match your situation.

Name: ..... Date: ..... Period: .....

Comparing Two Different Representations

2. You will use the Activity 1 Sheet to select a situation to explore.

Situation: .....

3. Create a poster. Here is what your poster should include:

- ☐ Your names.
- ☐ The name of the situation you chose.
- ☐ Your answers to the questions about the situation.
- ☐ Explanations or calculations that show your reasoning for each of the answers.
- ☐ At least two new mathematical representations that allow you to compare the proportional relationships in your situation. Representations can include graphs, equations, and/or tables.

You're invited to explore more.

Situation A: Naoki and Peter

4. Naoki and Peter earned a total of $210. They each worked the same number of hours. Who earned more money? How much more?

5. Naoki and Peter earned a total of $315. They each earned the same amount of money. Who worked more hours? How many more?

Situation B: Ahmed's Lemonade

4. Ahmed used 30 cups of water to make some lemonade. According to Recipe 1, how much lemonade mix should he use?

5. Ahmed tried to follow Recipe 2 by using 4 cups of water and 20 cups of lemonade mix. What was the mistake? How can this be fixed?

Name: Date: Period:

Gallery Tour

As a class, you will take a gallery tour.

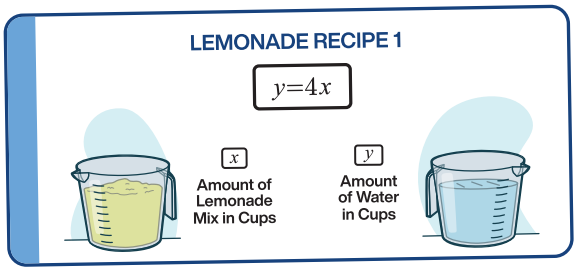
6. What features of your classmates' posters helped you understand their thinking?

7. Describe something you would change about your poster now that you have seen other groups' work.

Synthesis

8. **Discuss:** How can you compare proportional relationships when they are represented differently?

   Use the example if it helps with your thinking.



LEMONADE RECIPE 2

Lemonade Mix (cups)	Water (cups)
10	50
13	65
21	105

Summary 3.03

Using different representations is helpful when comparing proportional relationships, such as equations, tables, and graphs.

You can represent proportional relationships with the equation $y = mx$, where m is the slope of a line and also represents a unit rate for the situation. You can identify the slope or unit rate using all of the different representations, or by using slope triangles within a graph.

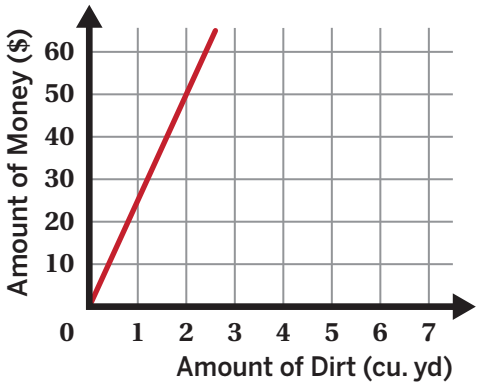
Practice
3.03

Name: ..... Date: ..... Period: .....

1. A recipe calls for 3 cups of flour for every 2 cups of water. Write an equation for the recipe, where x represents the number of cups of flour and y represents the number of cups of water.

Problems 2–4: A contractor must haul a large amount of dirt to a work site. She collected cost information from two companies.

Happy Hauling Service



EZ Excavation

Dirt (cu. yd)	Cost ($)
8	196
20	490
26	637

2. Calculate the rate of change for Happy Hauling Service.

3. Calculate the rate of change for EZ Excavation.

4. If the contractor has 40 cubic yards of dirt to haul and only cares about price, which company should she hire? Explain your thinking.

Problems 5–6: Students are selling raffle tickets for a school fundraiser. They collect $\$24$ for every 10 raffle tickets they sell. Suppose M is the amount of money the students collect for selling r raffle tickets.

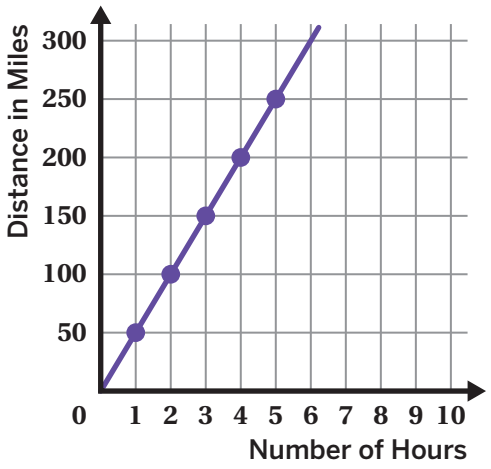
5. Complete the table.

Number of Tickets, r	Amount of Money, M
10	
20	
300	
1000	

6. Write an equation that reflects the relationship between M and r.

Name: _____ Date: _____ Period: _____

7. The graph shows a proportional relationship between the distance traveled on a road trip and the number of hours of travel. Which statement correctly identifies both the slope and the representation of slope for the situation?



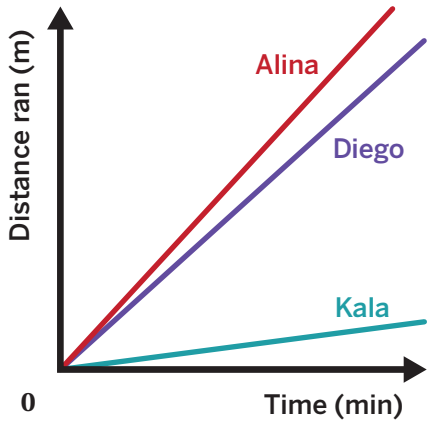
A. The slope is 50, so the distance traveled is 1 mile every 50 hours.

B. The slope is $\frac{1}{50}$, so the distance traveled is 1 mile every 50 hours.

C. The slope is 50, so the distance traveled is 50 miles every hour.

D. The slope is $\frac{1}{50}$, so the distance traveled is 50 miles every hour.

8. Alina runs twice as fast as Diego. Diego runs twice as fast as Kala. Could this graph represent the speeds of Alina, Diego, and Kala? Explain your thinking.



Spiral Review

9. The formula for converting temperature in degrees Celsius to degrees Fahrenheit is $C = \frac{5}{9}(F - 32)$. Use this formula to complete the table.

Temperature (°F)	Temperature (°C)
77	25
32	
	-18
50	

10. Select *all* of the expressions that have the same value.

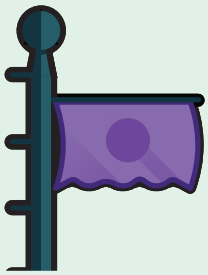
☐ **A.** $\frac{24}{-2}$ ☐ **B.** $-2 \cdot -12$ ☐ **C.** $-36 \cdot \frac{1}{3}$

☐ **D.** $-28 - (-16)$ ☐ **E.** $-8 + 20$

Sub-Unit

2

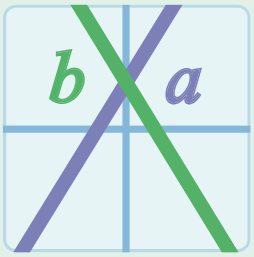
Linear Relationships



Lesson 4
Flags



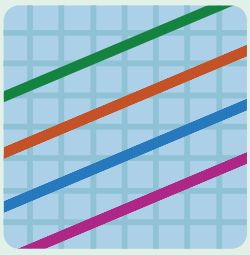
Lesson 5
Water Cooler



Lesson 6
Ups and Downs



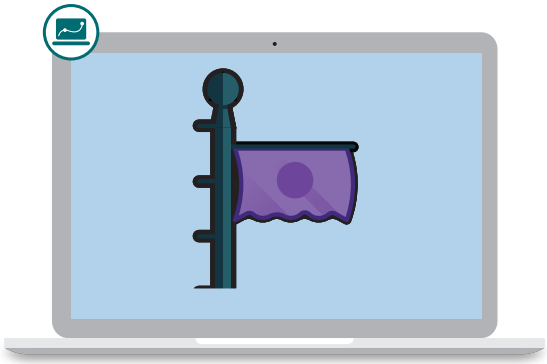
Lesson 7
Stacking Cups



Lesson 8
Translations

Name: ..... Date: ..... Period: .....

Data, Graphs, and Tables | Linear Equations | 8.EE.6, 8.F.3, 8.F.4, SMP.1, SMP.2, SMP.6



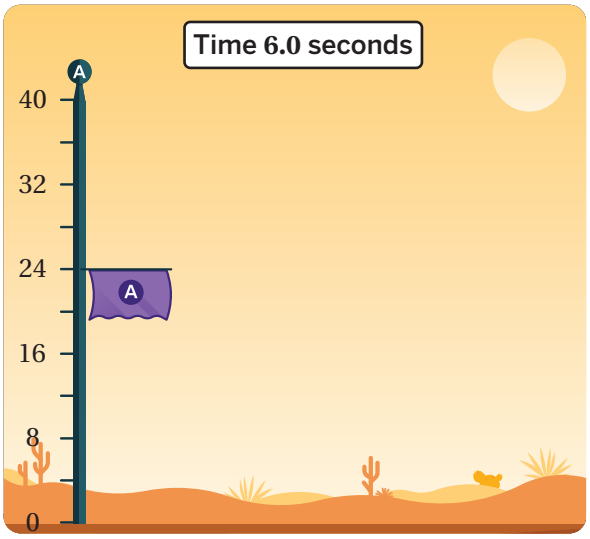
Flags

Let's explore non-proportional linear relationships.

Warm-Up

1 Let's watch an animation of a flag and its graph.

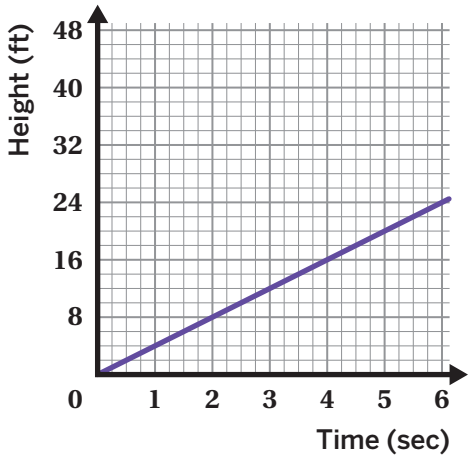
Discuss: What do you notice? What do you wonder?



Name: ..... Date: ..... Period: .....

Forming an Equation

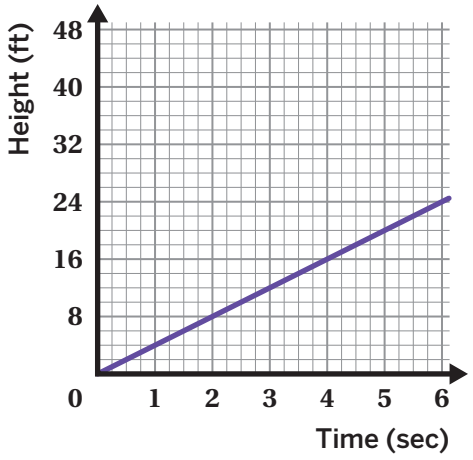
2 This line represents the relationship between height and time for the flag.



a What is the slope of the line?

b What does this number represent in the situation?

3 Write an equation for the flag's height, h, after t seconds.



Activity 2

Name: Date: Period:

Comparing Two Flags

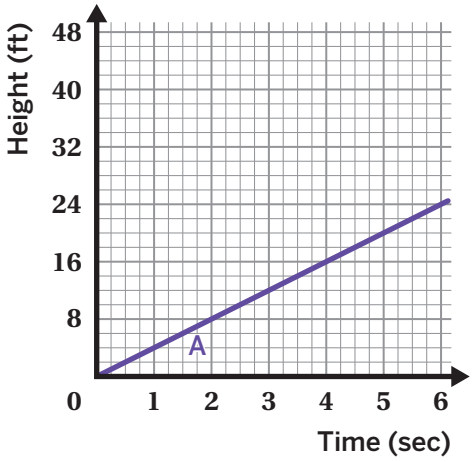
Let's explore the relationship between height and time for a new flag, Flag B.

4 Let's watch an animation of Flag A and Flag B.

Describe the behavior of Flag B.

5 Here is a graph representing the height of Flag A over time.

Draw a graph representing the height of Flag B over time.

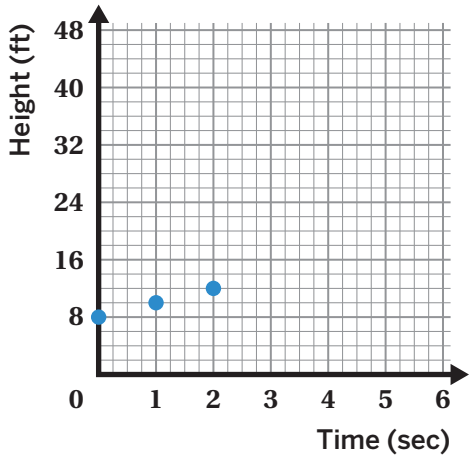


6 Here is a graph and a table with information about Flag B.

Cho claims: *Flag B rises at 2 feet per second, so its equation is* $h = 2t$.

Part of Cho's claim is correct and part of it is incorrect.

- What about Cho's claim is correct?
- What is incorrect?



Time (sec)	0	1	2
Height (ft)	8	10	12

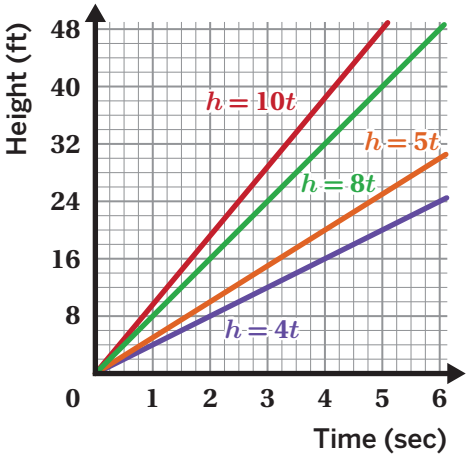
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Exploring Linear Relationships

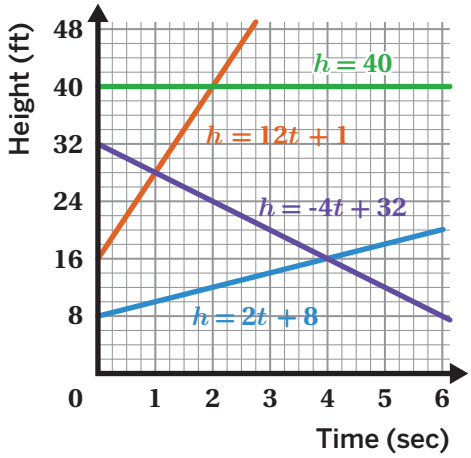
7 A relationship between two quantities is linear if there is a constant rate of change. It's called a **linear** relationship because its graph is a line.

Some linear relationships are proportional and some are non-proportional. Here are some examples.

Proportional Linear Relationships



Non-Proportional Linear Relationships



What are some differences between proportional and non-proportional linear relationships?

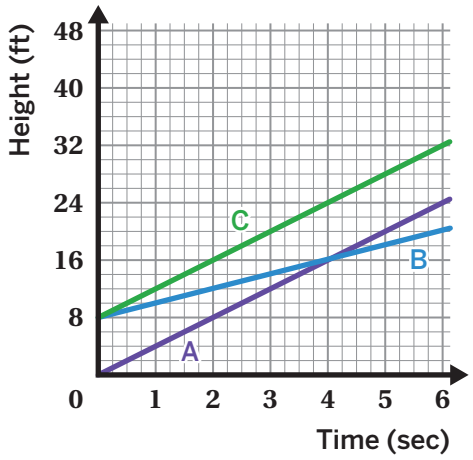
Name: Date: Period:

Exploring Linear Relationships (continued)

Now let's explore the relationship between height and time for a new flag, Flag C.

8 Based on the graph, select *all* the true statements.

- ☐ **A.** Flag C's graph is proportional.
- ☐ **B.** Flag C's graph is linear.
- ☐ **C.** The slope of Flag C's line is 8.
- ☐ **D.** Flag C rises at 8 feet per second.
- ☐ **E.** Flag C rises at 4 feet per second.



9 Write an equation for the height of Flag C.

Flag	Equation
A	$h = 4t$
B	$h = 2t + 8$
C	

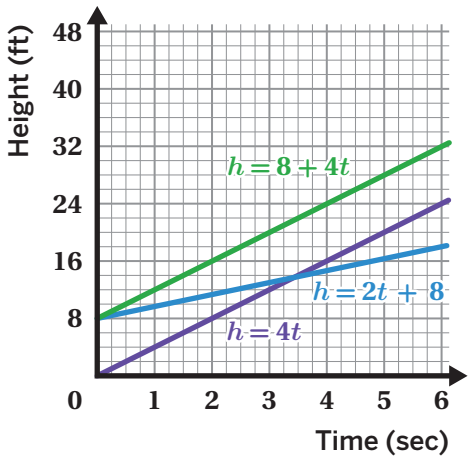
10 Can you write a new equation for Flag C so that the flag:

- Starts high?
- Starts low?
- Moves fast?
- Moves slow?

11 Synthesis

Discuss: How can you identify a non-proportional linear relationship in a graph and in an equation?

Use the example if it helps with your thinking.



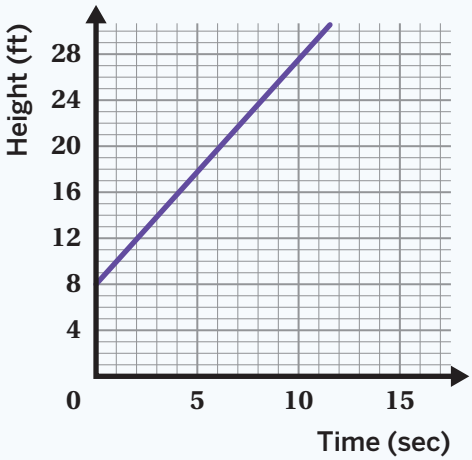
14 Summary 3.04

Linear relationships are represented by graphs that are lines. Some linear relationships are proportional relationships and some are not. Proportional relationships have an **initial value** of zero, while non-proportional linear relationships have an initial value that is not zero.

For example, this graph represents a flag's height, in feet, over time.

- The line starts at $(0, 8)$, which means the flag is at first 8 feet off the ground.
- The slope is 2, which represents the number of feet the flag rises each second.
- The equation $y = 8 + 2x$ represents the height of the flag y after x seconds.

This relationship is linear, but it's non-proportional because the line does not pass through the origin.



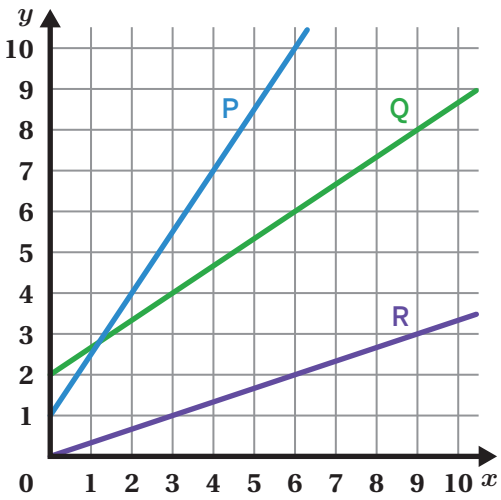
initial value The starting amount in a context.

linear A relationship is called linear when its graph is a line.

Practice
3.04

Name: ..... Date: ..... Period: .....

1. Match each equation with the graph of its line.

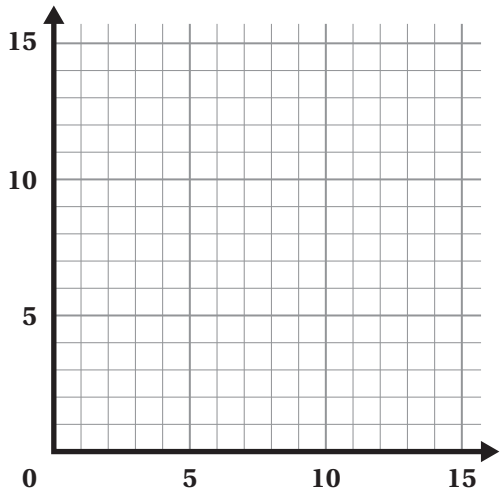


a $y = \frac{1}{3}x$ .....

b $y = \frac{3}{2}x + 1$ .....

c $y = \frac{2}{3}x + 2$ .....

2. Create a graph that shows a non-proportional linear relationship with a slope of $\frac{1}{4}$. Then write an equation that represents the graph.



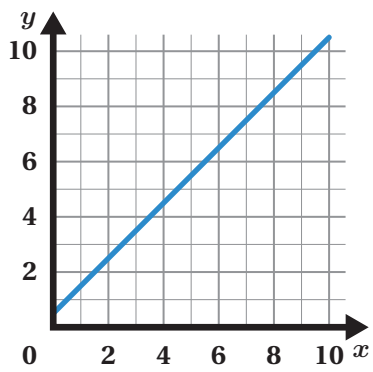
Problems 3–8: Determine whether each of these linear relationships is *proportional* or *non-proportional*.

3. From their initial step, a student walks at a constant speed of 4 miles per hour.

4. $y = 2x$

5. A giraffe at birth is 3 feet tall and grows 161 inches every month for a year.

6.



7.

x	y
2	4
3	5
5	7

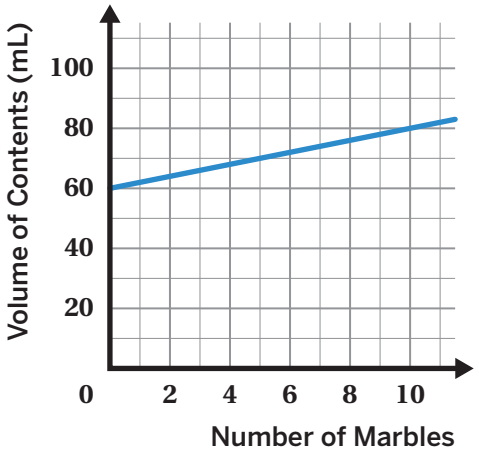
8. $y = 20 + 4x$

Practice
3.04

Name: ..... Date: ..... Period: .....

Problems 9–11: In Aesop's fable "The Crow and the Pitcher," a thirsty crow discovers a pitcher of water but is unable to reach the water with its beak. So the crow adds pebbles to the pitcher to make the water rise higher and higher until it reaches the top. Then it can drink the water!

Thiago recreates this situation using a cylinder containing water and a bunch of marbles. The line represents the volume of the cylinder's contents, y, after x marbles are added.



9. What is the slope of the line?

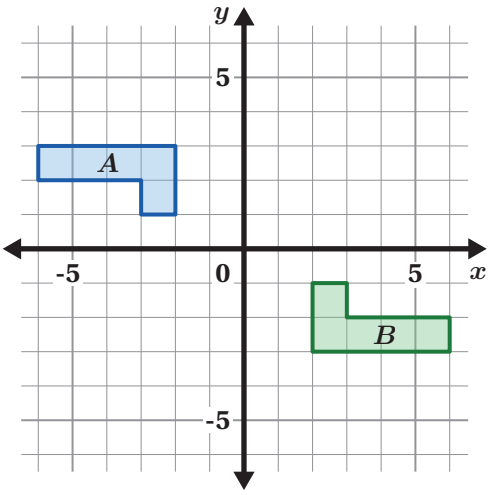
10. What does the slope represent in this situation?

11. Write an equation representing the relationship between x and y.

Spiral Review

Problems 12–13: Here are figures A and B.

12. Determine a sequence of transformations that maps figure A onto figure B.



13. Which of the following best describes figure A and figure B?

A. Congruent but not similar

B. Similar but not congruent

C. Both similar and congruent

D. Neither similar nor congruent

Name: ..... Date: ..... Period: .....

Data, Graphs, and Tables | Linear Equations | 8.EE.6, 8.F.3, 8.F.4, SMP.1, SMP.2, SMP.8

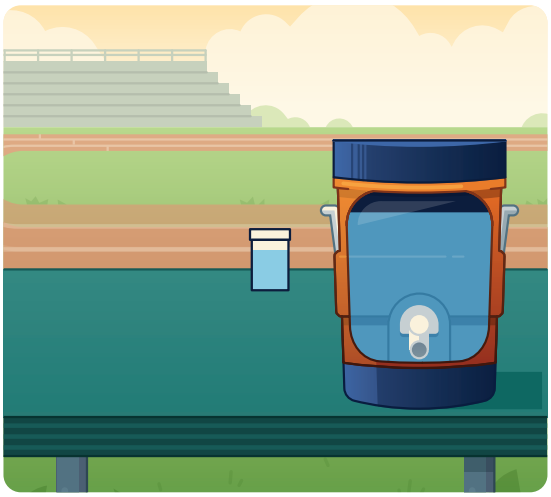


Water Cooler

Let's explore lines with different slopes.

Warm-Up

1 Let's watch a short animation about a water cooler.



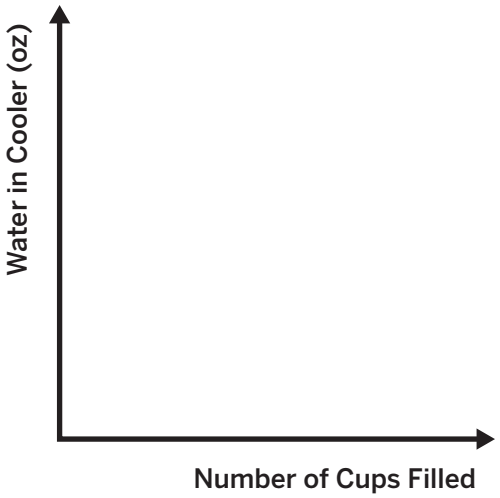
a **Discuss:** What do you notice? What do you wonder?

b Make an initial prediction. After how many cups do you think the cooler will run out of water?

Name: ..... Date: ..... Period: .....

Cooler Cups

2 **a** Sketch the relationship between the ounces of water remaining in the cooler and number of cups filled.



b **Discuss:** How can you see this relationship in your sketch?

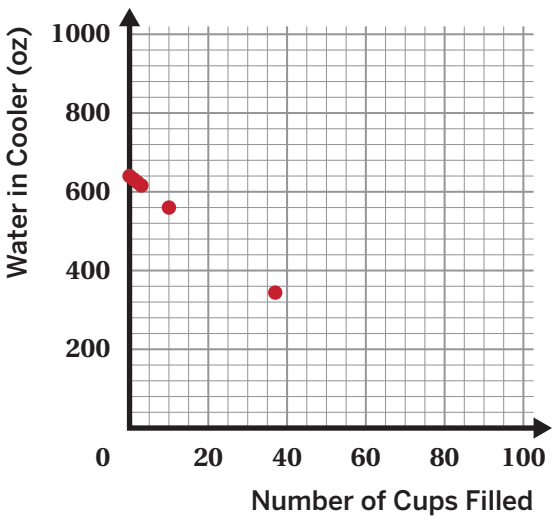
3 The table shows the amount of water remaining in the cooler after 0, 1, and 2 cups have been filled.

Determine the missing values.

Number of Cups Filled	0	1	2	3	. . .	10	37
Water in Cooler (oz)	640	632	624		. . .		

4 A classmate plotted their points from the table onto a graph.

Write an equation representing the amount of water in the cooler, y, after filling x cups.



Name: ..... Date: ..... Period: .....

Cooler Cups (continued)

5 Make a final prediction:

After how many cups will the cooler run out of water?

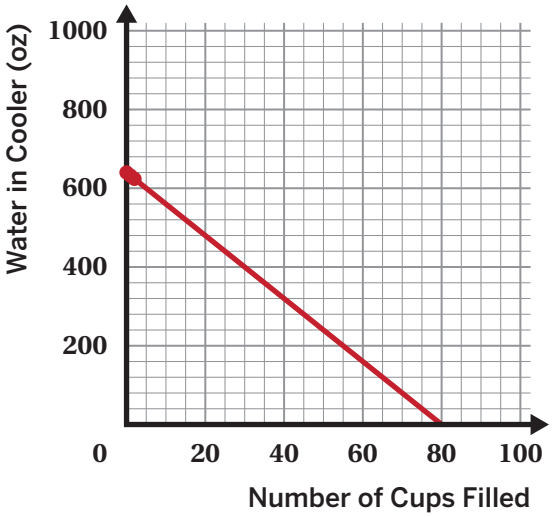
Explain your thinking.

6 Let's watch an animation to see how many cups the cooler actually fills!

7 Here is the equation that a classmate wrote for the relationship between the water in the cooler and the number of cups:

$y = 640 - 8x$

The **vertical intercept** is the point where the graph of a line crosses the vertical axis.



a What is the vertical intercept of this graph, and what does it represent in this situation?

b What is the slope of the line, and what does it represent in this situation?

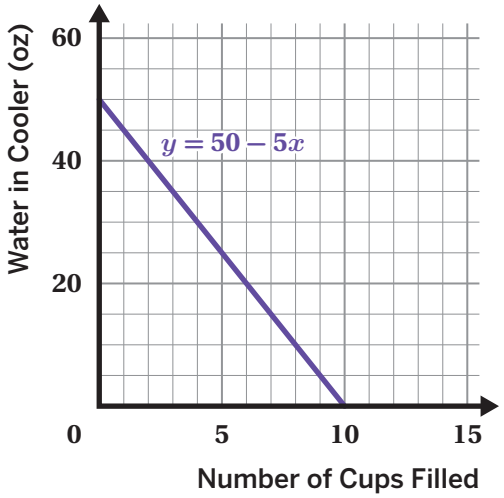
Activity 2

Name: ..... Date: ..... Period: .....

A New Water Cooler

The line $y = 50 - 5x$ represents a different water cooler situation.

8 Write a story that this graph could represent.



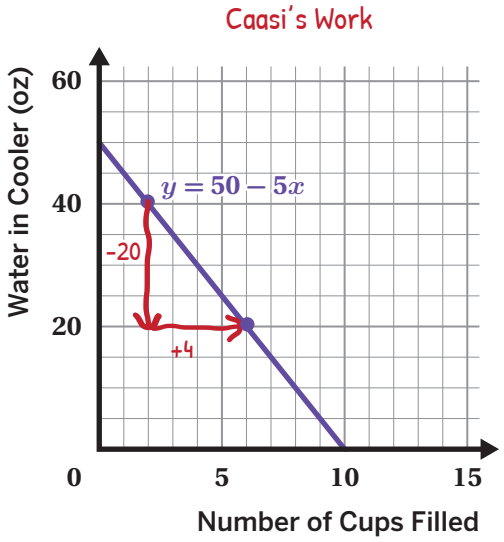
9 Caasi says that the water in the cooler decreases by 20 ounces for every 4 cups filled.

Jamar says that the water in the cooler decreases by 5 ounces for every 1 cup filled.

Whose claim is correct?

Caasi's Jamar's Both Neither

Explain your thinking.



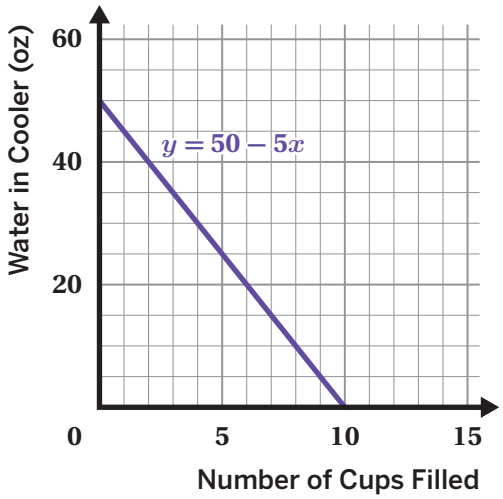
10 Look at the graph of $y = 50 - 5x$ from the previous problems. The line crosses the horizontal axis at the point $(10, 0)$. This point is called the **horizontal intercept**.

What does this intercept represent in this situation?

Name: ..... Date: ..... Period: .....

A New Water Cooler (continued)

11 Revise your original story so that it is stronger and clearer.

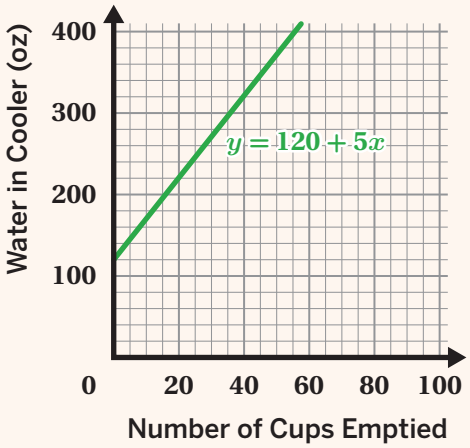


You're invited to explore more.

12 The line $y = 120 + 5x$ represents a third water cooler situation.

a Write a story that this graph could represent.

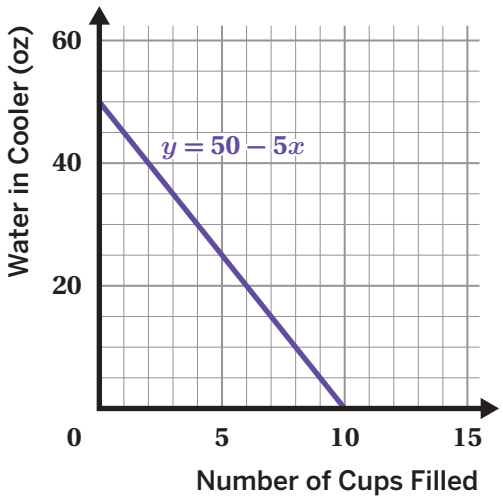
b How is this situation like the other two water cooler situations? How is it different?



13 Synthesis

Discuss: What are 2–3 things you can determine about a situation from its graph?

Use the graph if it helps with your thinking.

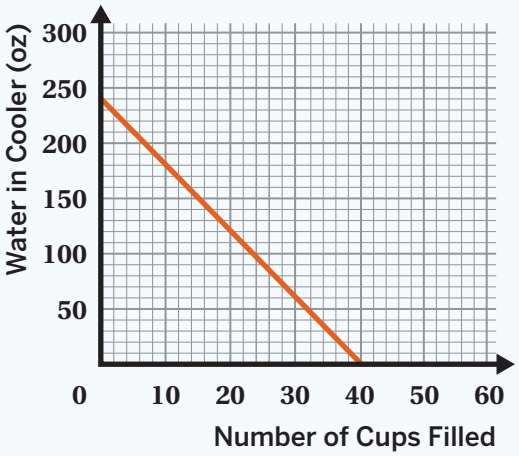


16 Summary 3.05

When a *linear relationship* has a negative slope, this means that as the x-values increase, the y-values decrease at a constant rate.

Let's say the equation $y = 240 - 6x$ represents the amount of water in a cooler, y, after x cups have been filled.

- The **vertical intercept**, also called the *y-intercept*, is $(0, 240)$. In this situation, the vertical intercept represents the starting amount of water in the cooler.
- The slope is -6. This means that the amount of water decreases by 6 ounces for each cup filled. Because the amount of water decreases each time, the slope is negative.
- The **horizontal intercept**, also called the *x-intercept*, is $(40, 0)$. In this situation, the horizontal intercept represents how many cups can be filled before the cooler runs out of water.



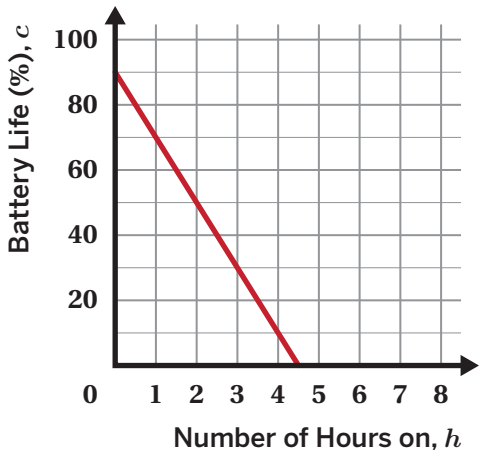
horizontal intercept (x-intercept) The point where the graph of a line crosses the horizontal axis or when $y = 0$.

vertical intercept (y-intercept) The point where the graph of a line crosses the vertical axis or when $x = 0$.

Practice
3.05

Name: ..... Date: ..... Period: .....

Problems 1–4: Tameeka is monitoring her computer's battery life. When the computer is on, the battery loses energy at a constant rate. This graph shows the percentage of battery life remaining, c, after the computer has been on for h hours.



1. What percent of battery life was left when Tameeka turned on the computer?

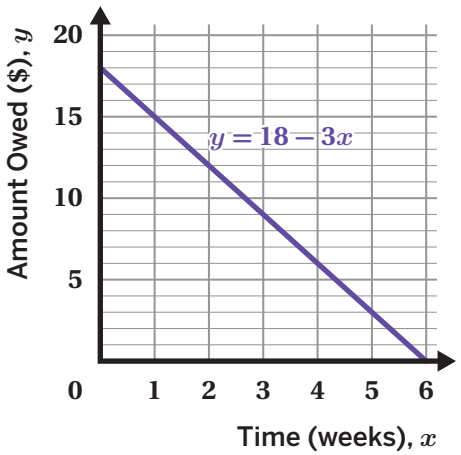
2. Complete the table.

Number of Hours On	0	1	2	. . .	3.5
Battery Life, %	90	70		. . .	

3. Write an equation that represents the percentage of battery life remaining, c, after the computer has been on for h hours.

4. After how many hours of being on will Tameeka's computer's battery life run out?

Problems 5–6: Juliana borrowed some money from her brother. She pays him back by giving him the same amount every week. The graph shows how much she owes him, y, after each week, x.



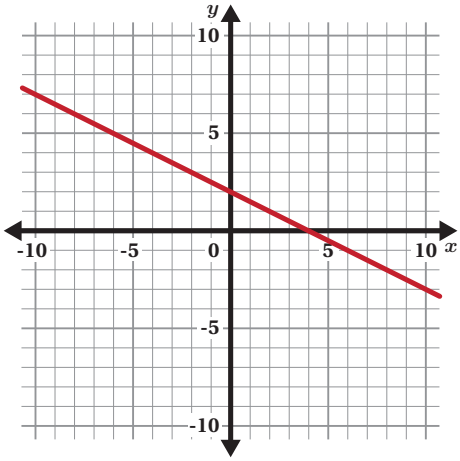
5. What is the vertical intercept and what does it represent in this situation?

6. What is the horizontal intercept and what does it represent in this situation?

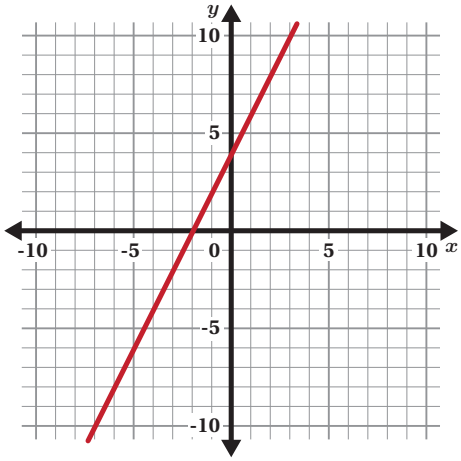
Name: _____ Date: _____ Period: _____

7. Which graph has a horizontal intercept of $(2, 0)$?

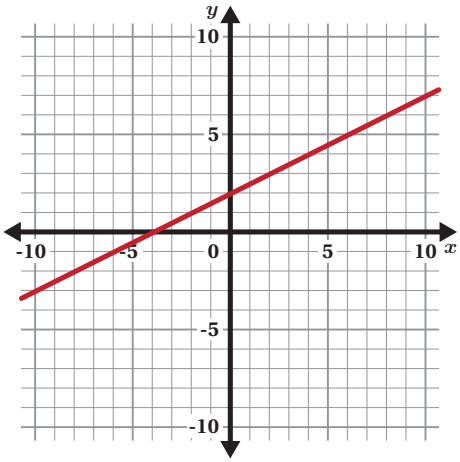
A.



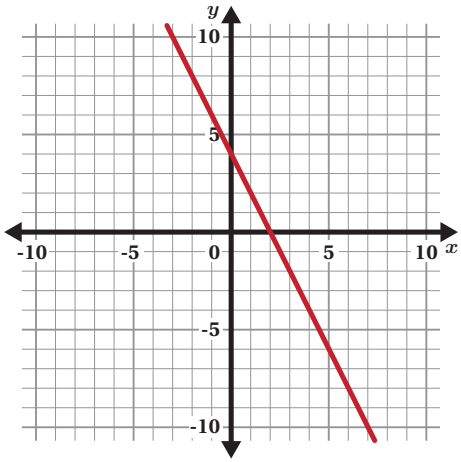
B.



C.

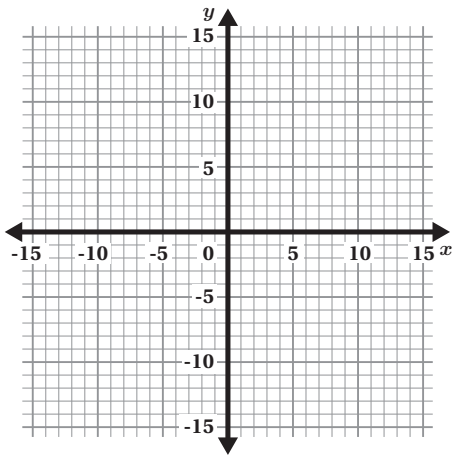


D.



Spiral Review

Problems 8–10: Determine the coordinates of the image after the point is dilated using the origin as the center of dilation, and a scale factor of 2.5.



8. $(1, 0)$

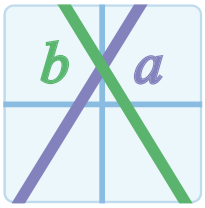
9. $(-3, 6)$

10. $(2, -5)$

Name: ..... Date: ..... Period: .....

Data, Graphs, and Tables | Linear Equations | 8.EE.6, 8.F.3, 8.F.4, SMP.2, SMP.6

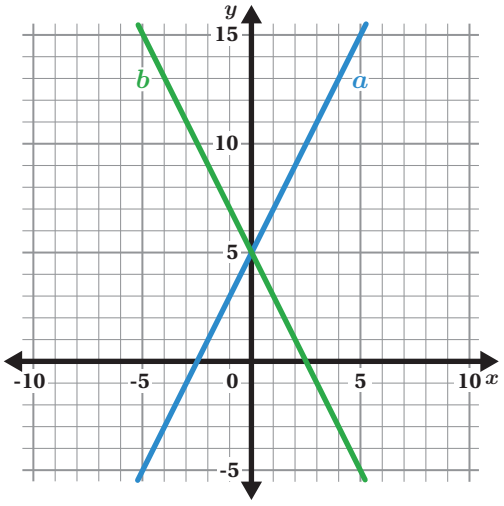
Ups and Downs



Let's explore more linear relationships in context.

Warm-Up

1. How are the two lines alike?



2. How are they different?

Name: Date: Period:

Two Transportation Cards

Two students have different cards they use to pay for public transportation.

3. Complete the table to show the amount of money remaining on the card for each student after several rides.

Student 1: The transportation card starts with $30. Each ride costs $2.

Number of Rides, x	Amount Remaining on Card ($), y
0	30
1	
2	
3	
4	

Student 2: The transportation card starts with $50. Each ride costs $3.

Number of Rides, x	Amount Remaining on Card ($), y
0	50
1	
2	
3	
4	

4. Write an equation that represents the amount on the card y, in dollars, after x rides.

Student 1	
Student 2	

5. **Discuss:**

- Are the slopes in each equation positive or negative?
- What does this represent in each situation?

6. How much money will *each* student have on their card after 10 rides? Show your thinking.

Activity 2

Name: ..... Date: ..... Period: .....

Matchmaker

7. You will get a set of graphs. Match each graph with its corresponding situation.

	Situation	Graph
a	A bike rental company charges a one-time fee of $\$30$ and then $\$5$ per hour to use a bike. Let y represent the total amount a customer pays in dollars after x hours.	
b	Chef Kawbena purchases 30 gallons of tomato sauce for her restaurant. She uses 5 gallons of tomato sauce each day. Let y represent the number of gallons of tomato sauce remaining after x days.	
c	A candle is 30 centimeters tall and burns at a constant rate of 1 centimeter every 5 minutes. Let y represent the height of the candle in centimeters after x minutes.	
d	Ayaan starts with $\$5$ in his savings account. He adds $\$30$ to his account each month and doesn't take out any money. Let y represent the total amount of money in Ayaan's savings account after x months.	

Name: ..... Date: ..... Period: .....

Matchmaker (continued)

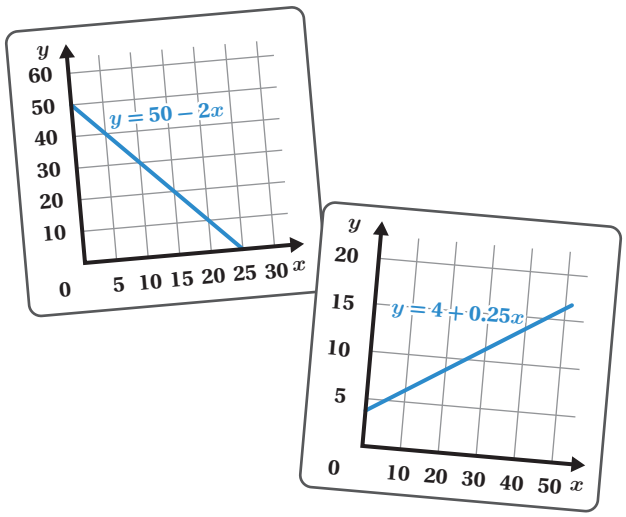
8. Complete the table.

	What is the slope? What does it represent in the situation?	What is the vertical intercept? What does it represent in the situation?	Write an equation that represents the situation.
a			
b			
c			
d			

Synthesis

9. **Discuss:** How can you determine whether a situation has a positive or negative slope?

Use the example if it helps with your thinking.



Summary 3.06

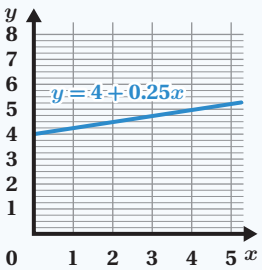
Generally, you can derive the equation $y = mx + b$ to represent any linear relationship, where m represents the slope and b represents the vertical intercept.

For linear relationships with a *positive* slope, the y-values increase at a constant rate as the x-values increase. For linear relationships with a *negative* slope, the y-values decrease at a constant rate as the x-values increase.

Here are two examples.

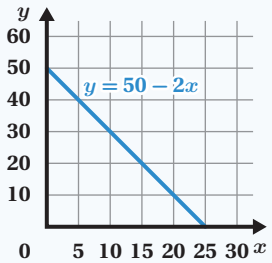
Positive slope: A medium-sized frozen yogurt costs \$4, plus \$0.25 per topping.

Let y represent the total cost of the frozen yogurt after adding x toppings.



Negative slope: A student loads an arcade game card with \$50. Every time she plays a game, \$2 is subtracted from the amount available on the game card.

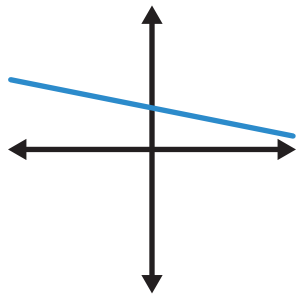
Let y represent the amount in dollars on the card after the student plays x games.



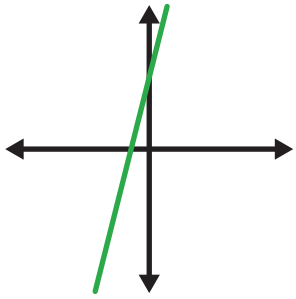
Name: ..... Date: ..... Period: .....

Problems 1–3: Determine whether the slope of each line is *positive* or *negative*.

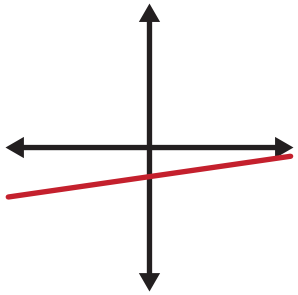
1.



2.



3.



Problems 4–5: The elevation of an airplane and its time since takeoff can be modeled by a linear equation. The horizontal axis represents time in minutes and the vertical axis represents elevation in feet. For each situation, determine whether the slope is positive or negative.

4. The plane descends at a rate of $1{,}000$ feet per minute.

5. The plane ascends at a rate of $2{,}000$ feet per minute.

Problems 6–7: Ella borrowed $\$40$ from her brother. She pays him back by giving him $\$5$ each week.

6. Complete the table to show the amount Ella owes each week.

Time (weeks)	Amount Owed ($)
0	40
1	
2	
3	

7. Write an equation that represents the amount that Ella owes, in dollars, after x weeks.

Name: ..... Date: ..... Period: .....

Problems 8–10: An acorn falls 20 feet from a tree to the ground at a rate of 4 feet every 2 seconds. Determine whether each statement is *true* or *false*.

8. The initial height of the acorn is 20 feet.

9. The acorn falls at a rate of $\frac{1}{2}$ foot every second.

10. The acorn is 5 feet above the ground after 10 seconds.

11. y is the total cost of grapes after purchasing x ounces of grapes.

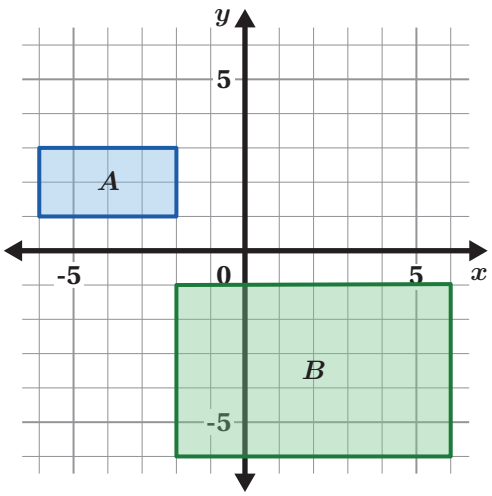
Sketch a graph that could represent this situation.



Spiral Review

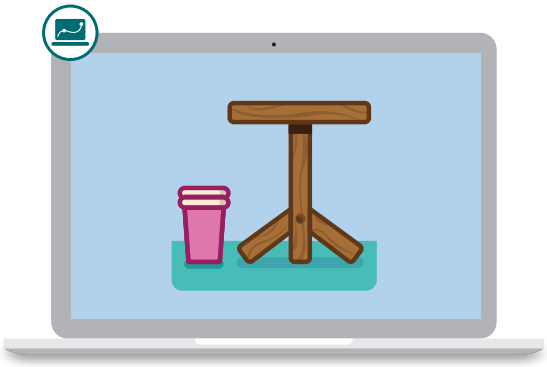
12. The vertices of triangle ABC are $A(12,6)$, $B(1, 3)$, and $C(18, 0)$. Triangle ABC is dilated by a scale factor of $\frac{1}{3}$ using point $(0,0)$ as the center of dilation, resulting in triangle JKL. What are the coordinates of the vertices of triangle JKL?

13. Are figures A and B similar? Explain your thinking.



Name: ..... Date: ..... Period: .....

Data, Graphs, and Tables | Linear Equations | 8.EE.6, 8.F.4, SMP.4

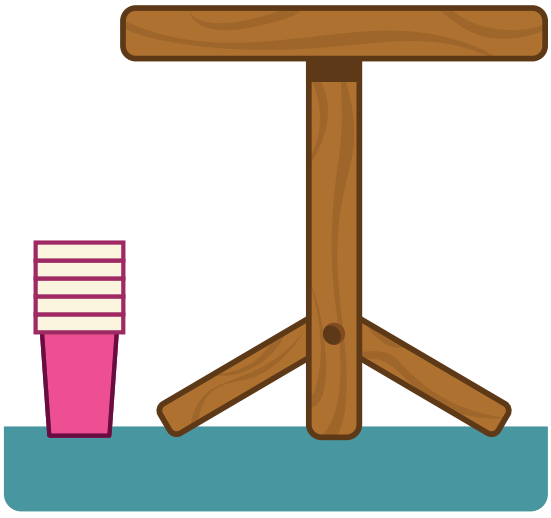


Stacking Cups

Let's use a linear relationship to make a prediction

Warm-Up

1 a. How many stacked cups do you think you need to reach the top of this table?

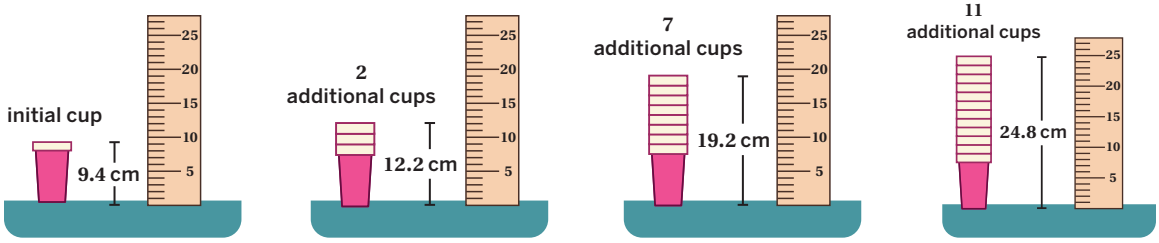


b. What information would help you make a more precise prediction?

Name: ..... Date: ..... Period: .....

Stacking Cups

2 Let's look at different stacks of cups. The initial cup has a height of 9.4 centimeters.



Record *at least two* data points.

Number of Additional Cups	Height (cm)
0	
	12.2
7	

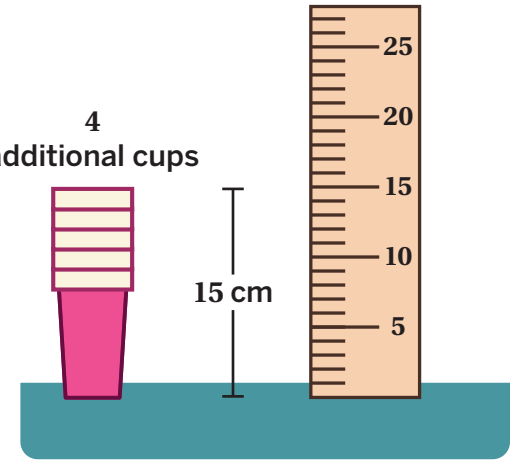
3 Sylvia found that a stack of 4 additional cups has a height of 15 centimeters.

She thinks that a stack of 8 additional cups will have a height of 30 centimeters.

Is she correct? Circle one.

Yes No

Explain your thinking.



Name: ..... Date: ..... Period: .....

Stacking Cups (continued)

4 The height from the floor to the top of the table is 50 centimeters.

Previously, you predicted a number of cups to reach the top of the table.

Now that you have more information, calculate the exact number of stacked cups you need.



5 Let's see how many cups you actually need to reach the top of the table!

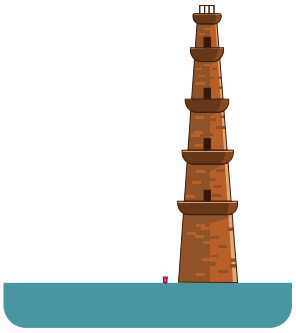
Activity 2

Name: ..... Date: ..... Period: .....

More and More Cups

6 This minaret is 42.5 meters (or 4,250 centimeters) tall.

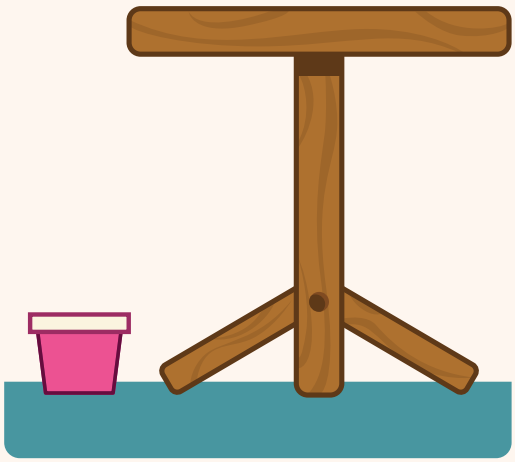
How many stacked cups do you need to reach the top of the minaret?

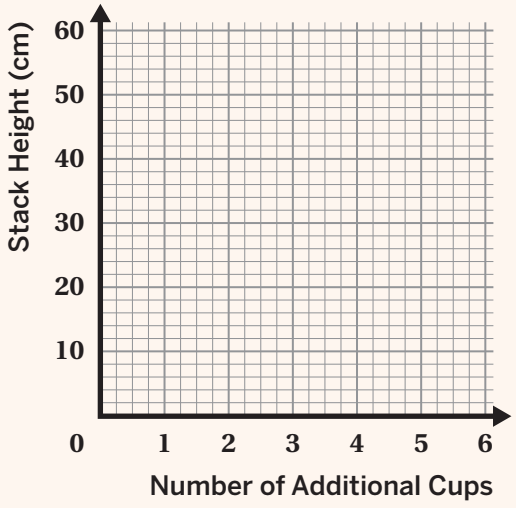


7 Let's see how many cups you actually need to reach the top of the minaret!

You're invited to explore more.

8 What cup and lip heights would you use so that 5 additional cups will reach the exact height of the table (50 centimeters)? Use the graph if it helps with your thinking.





Activity 3

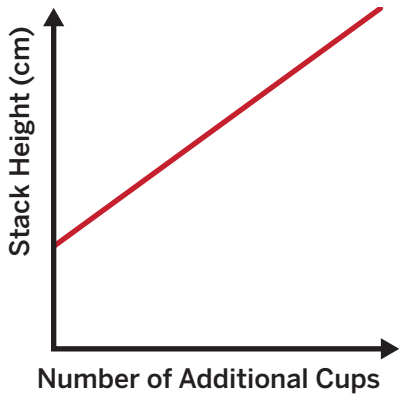
Name: ..... Date: ..... Period: .....

Modeling the Relationship

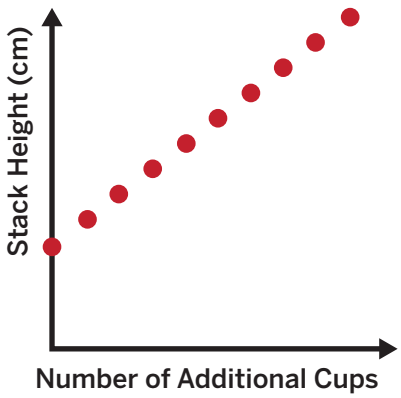
9 Graphs can help us understand a relationship better.

Karima and Vicente each made a graph.

Karima



Vicente

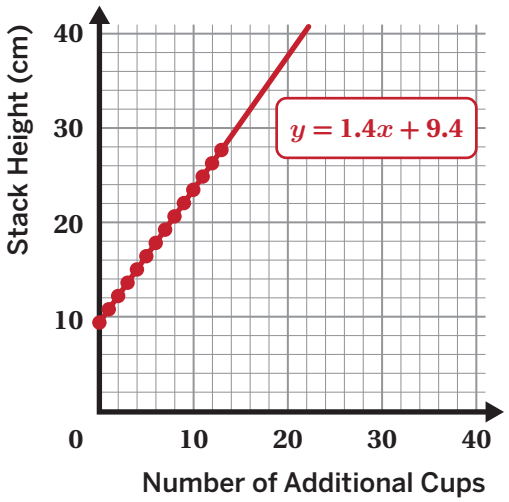


When might Karima's graph be useful? When might Vicente's graph be useful?

10 Here is a graph with a line and an equation that represent the relationship between number of cups and stack height.

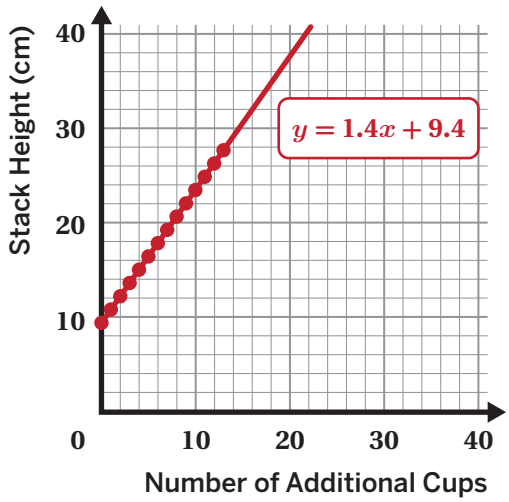
Discuss:

- What is the slope of the line? What does it represent in this situation?
- What is the y-intercept (vertical intercept) of the line? What does it represent in this situation?



11 Synthesis

Discuss: How can you use a linear relationship to model a situation and make predictions?



14 Summary 3.07

Linear relationships can help us make predictions. We can use the graph or the equation of a linear relationship to determine the value of one variable when we're given the other variable.

Let's say we want to know how many stacked cups will have a height of 30 centimeters. We can look at a graph and determine the number of additional cups, x, that correspond to $y = 30$. In this situation, the slope of the line tells us how many centimeters the height of the stacked cups increases by every time we add a cup to the stack.

We might think of the y-intercept as the stack height when there are 0 cups, but this doesn't make sense in this situation. Instead, the y-intercept tells us the height of the initial cup. Unlike a proportional relationship, the graph of this linear relationship doesn't pass through the origin.

Practice
3.07

Name: ..... Date: ..... Period: .....

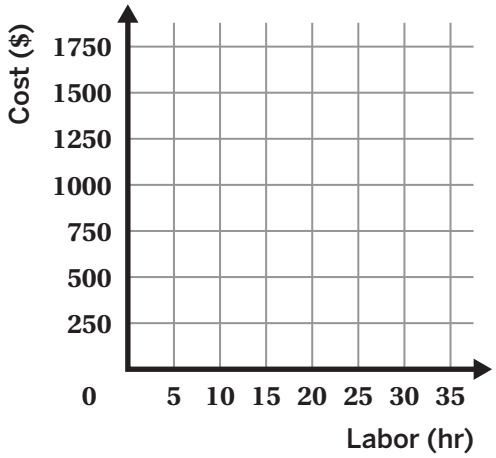
Problems 1–4: To paint a house, a painting company charges a flat rate of $500 for supplies, plus $50 for each hour of labor.

1. How much would the painting company charge to paint a house that needs 20 hours of labor? 50 hours of labor? Write your answers in the table.

Labor (hr)	Cost ($)
20	
50	

2. Draw a line representing the relationship between the number of hours of labor needed to paint the house and the total cost of paint.

3. Plot a point to show the cost of 20 hours of labor.



4. What is the slope of the line? What does it represent?

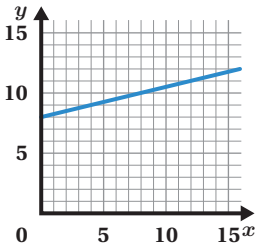
5. At a carnival, y represents the amount of money in a cash box after x tickets are purchased for carnival games. The line representing the relationship between x and y has a slope of $\frac{1}{4}$ and a y-intercept of 8. Explain what the slope and y-intercept represent in this situation.

Practice
3.07

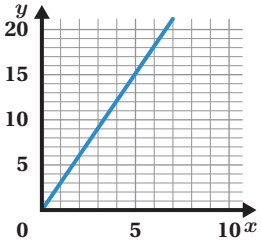
Name: ..... Date: ..... Period: .....

Problems 6–9: For each real-world situation, choose the graph that best represents it.

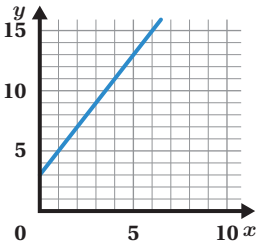
Graph A



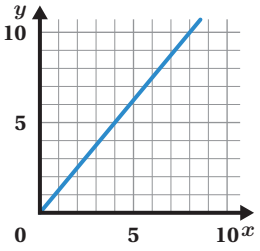
Graph B



Graph C



Graph D



6. y represents the perimeter of an equilateral triangle and x represents its side length. The slope of the line representing the relationship between x and y is 3.

7. y represents the amount of money collected after x raffle tickets are purchased. The slope of the line representing the relationship between x and y is 0.25.

8. y represents the number of chapters a student has read after x days. The slope of the line representing the relationship between x and y is $\frac{5}{4}$.

9. y represents the total cost, in dollars, of x blueberry muffins, including a delivery fee. The slope of the line representing the relationship between x and y is 2.

Spiral Review

10. Here is a table showing the values of a linear relationship. Write an equation to represent this relationship.

x	0	1	2
y	17	12	7

11. Write the equation of a line that has a slope of 2 and an initial value of $(0, 8)$.

12. Write the equation of a line that has a slope of -3 and an initial value of $(0, -12)$.

Name: ..... Date: ..... Period: .....

Multiple Representations of Functions | Transformational Geometry | 8.EE.6, 8.G.1, SMP.1, SMP.7



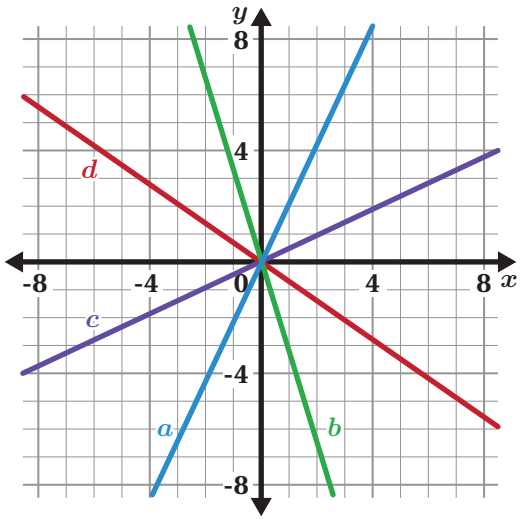
Translations

Let's see what happens to the equations of translated lines.

Warm-Up

1-2 Write equations to match the lines on the graph.

Line	Equation
a	
b	
c	
d	



Activity 1

Name: ..... Date: ..... Period: .....

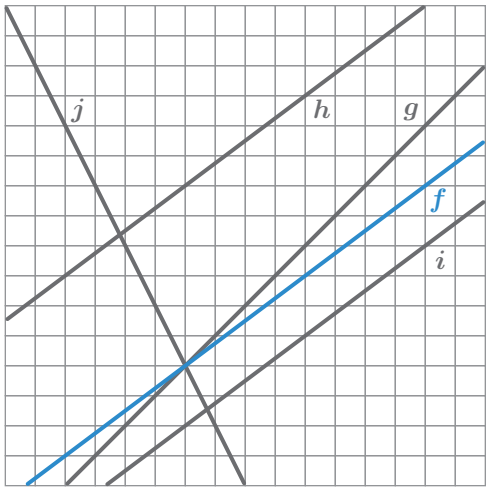
Translating Lines

3 Here are several lines.

You can only see part of them, but they actually continue forever in both directions.

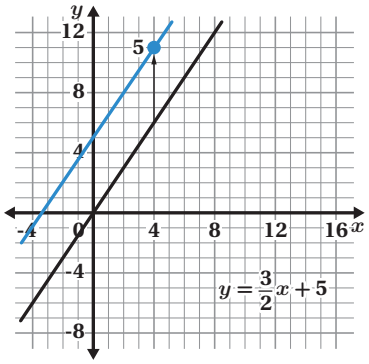
Which lines are images of line f after a *translation*? Select *all* that apply.

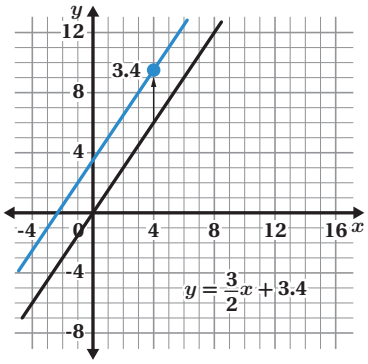
- ☐ Line g
- ☐ Line h
- ☐ Line i
- ☐ Line j

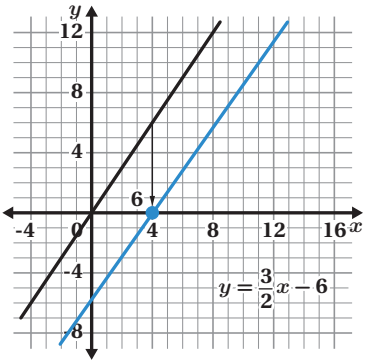


4 Describe a translation that maps line f onto each of the lines that are images of line f.

5 Take a look at equations for three different translations of the line $y = \frac{3}{2}x$.







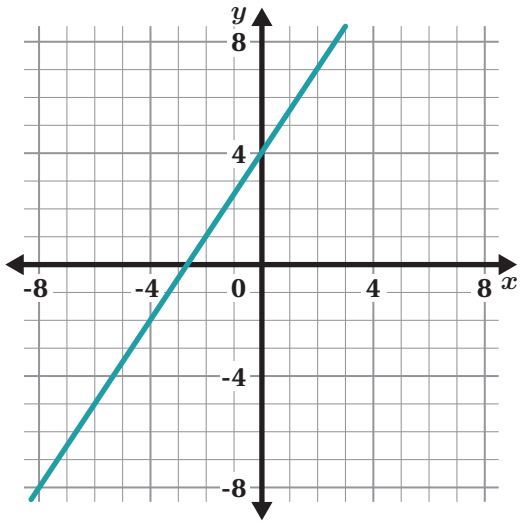
What do you notice about the relationship between the y-intercept (vertical intercept) of the translated line and its equation?

Activity 2

Name: ..... Date: ..... Period: .....

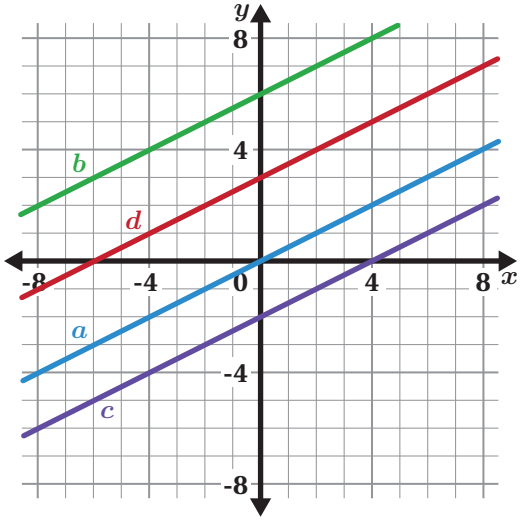
Equations of Lines

6 Adjust one number in the equation $y = \frac{3}{2}x + 1$ to match the line.



7 Write equations to match the lines on the graph. The first one has been done for you.

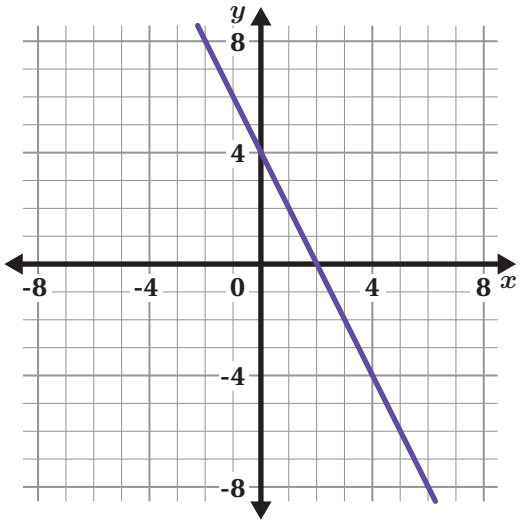
Line	Equation
a	$y = \frac{1}{2}x$
b	
c	
d	



Name: ..... Date: ..... Period: .....

Equations of Lines (continued)

8 What is the equation of this line?



A. $y = -\frac{1}{2}x + 4$

B. $y = 4x - 2$

C. $y = -x + 4$

D. $y = -2x + 4$

Explain your thinking.

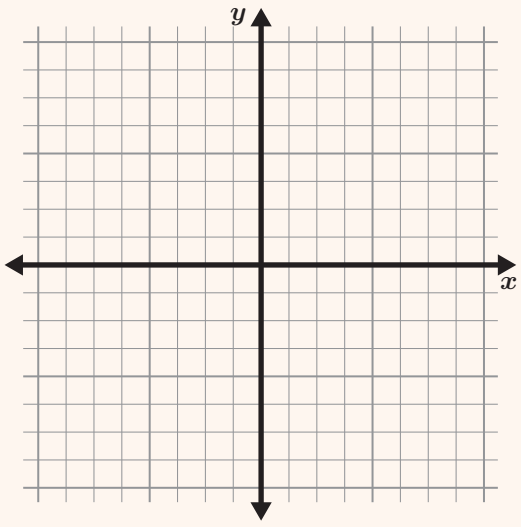
You're invited to explore more.

9 Zoe says that the graph of $y = 3(x + 4)$ is the same as the graph of $y = 3x$, but translated up 4 units.

Is Zoe's claim correct?

Yes No

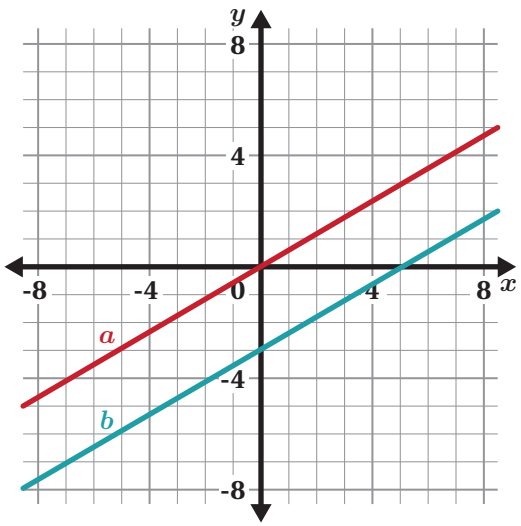
Show or explain your thinking.



10 Synthesis

Here is a graph of two equations.

How are the equations of line a and line b alike and different?



13 Summary 3.08

A *translation* of a line that represents a proportional relationship creates a line that is parallel to the pre-image, but changes the location of the vertical intercept, also known as the y-intercept.

The equation $y = mx$ represents a line that passes through the origin. The equation $y = mx + b$ represents a vertical translation of line $y = mx$ by b units.

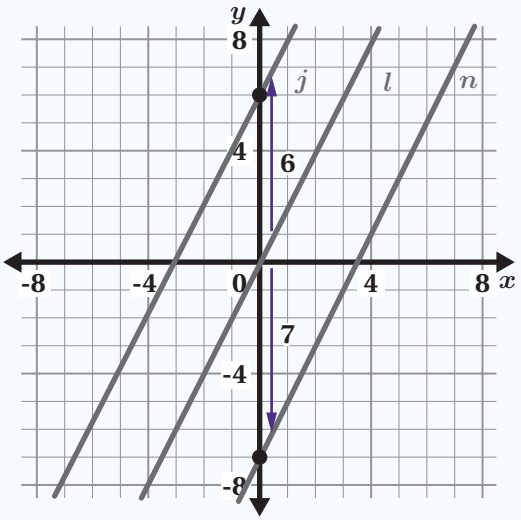
If $b > 0$, the line is translated up.

If $b < 0$, the line is translated down.

For example, the equation of line l is $y = 2x$.

If line l is translated 6 units up to produce line j, the equation of line j is $y = 2x + 6$.

If line l is translated 7 units down to produce line n, the equation of line n is $y = 2x - 7$.



Practice
3.08

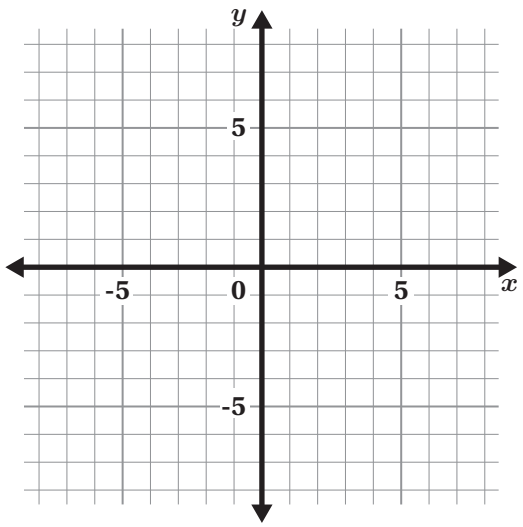
Name: _____ Date: _____ Period: _____

1. Select *all* the equations whose graphs have the same y-intercept.

- ☐ **A.** $y = 3x - 8$
- ☐ **B.** $y = -8x + 3$
- ☐ **C.** $y = 3x + 8$
- ☐ **D.** $y = 5x - 8$
- ☐ **E.** $y = 2x - 8$
- ☐ **F.** $y = 13x - 8$

Problems 2–3: Here is a coordinate plane.

2. Graph the equations $y = \frac{1}{4}x$ and $y = \frac{1}{4}x - 3$.



3. How are the graphs alike? How are they different?

Problems 4–5: A cable company charges existing customers $\$70$ per month for cable service. For new customers, there is an additional one-time fee of $\$100$.

4. Write a linear equation representing the relationship between x, the number of months of service, and y, the total amount paid in dollars by a customer.

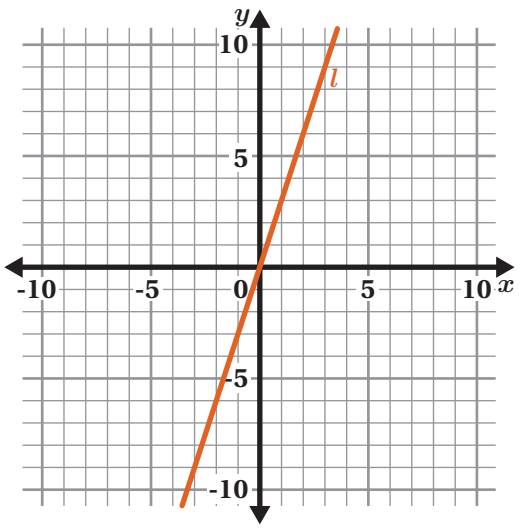
Existing customer: _____

New customer: _____

5. When the two equations are graphed on the coordinate plane, how are the graphs similar?

Name: ..... Date: ..... Period: .....

6. Here is the graph of line l. Which equation represents a line that is a translation of l?

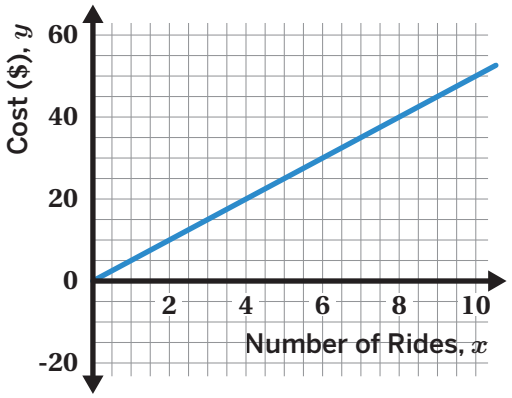


A. $y = \frac{1}{3}x - 7$ **B.** $y = 2x + 5$

C. $y = 3x - 8$ **D.** $y = -3x + 1$

Problems 7–9: Jasmine goes to an amusement park. The amusement park charges $5 for each ride. Here is a graph that represents the cost, y, after x rides.

7. Write an equation that represents the cost, y, after Jasmine goes on x rides.



8. Mio goes to the same amusement park and has a coupon that can be used for 2 free rides. On the same coordinate plane, graph the relationship that represents the amount of money, y, that Mio would spend after x rides, if she uses the coupon.

9. Write an equation for the relationship you graphed in the previous problem

Spiral Review

Problems 10–12: Determine the coordinates of the image after the point is translated 4 units left and 3 units down.

10. $(3, 6)$ **11.** $(-1, 5)$ **12.** $(7, -7)$

Name: ..... Date: ..... Period: .....

Linear Equations | Multiple Representations of Functions | Slopes and Intercepts

8.EE.5, 8.EE.6, 8.F.4, SMP.2, SMP.7

Practice Day 1



Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Parking Lots

1. Story about Lot A, Lot B, and Lot C:

2. **a** Circle one: True False
Explanation:

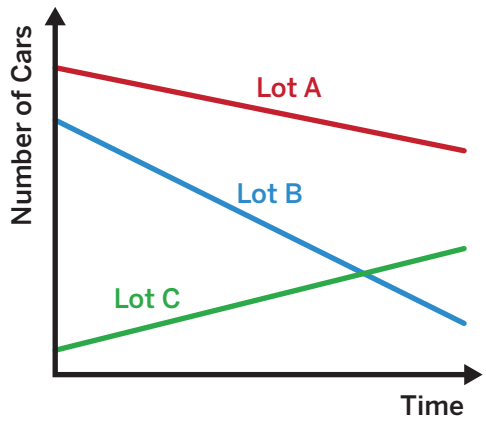
b Circle one: True False
Explanation:

c Circle one: True False
Explanation:

d Circle one: True False
Explanation:

e Circle one: True False
Explanation:

3.



Name: _____ Date: _____ Period: _____

Practice Day 1 (continued)

Task B: Bakery

1. Unit rate: _____

2. Equation for flour: _____

Equation for sugar: _____

3. Flour: _____

Sugar: _____

4. Ingredient: _____

Explain your thinking:

Task C: Clothing Store

1. Group each table/equation/graph:

Group 1: _____ Group 2: _____ Group 3: _____

2. Cost of pants: _____ Cost of shirts: _____ Cost of hats: _____

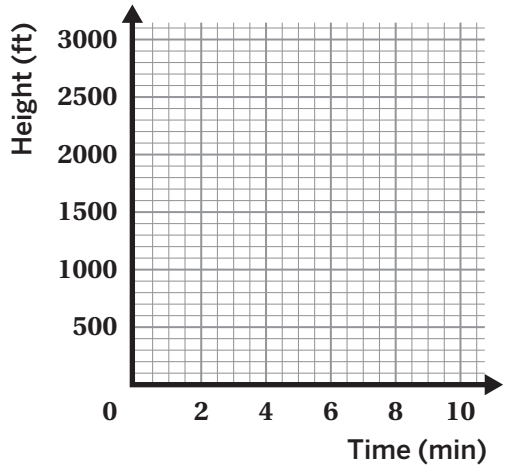
Explain your thinking:

Name: ..... Date: ..... Period: .....

Practice Day 1 (continued)

Task D: Hot Air Balloon

1.



2. The horizontal intercept represents:

3. Complete the table.

Time (min), t	Height of Red Hot Air Balloon, h	Height of Blue Hot Air Balloon, h
1		
	2000	
4		

4. Equation for red hot air balloon:

.....

Equation for blue hot air balloon:

.....

Task E: Function Translation

1. Equation of line m: .....

Equation of line r: .....

Equation of line q: .....

Equation of line n: .....

Equation of line p: .....

2. **a** Translate line a onto line b:

.....

Translate line a onto line c:

.....

Translate line a onto line d:

.....

b Equation of line b:

.....

Equation of line c:

.....

Equation of line d:

.....

Sub-Unit

3

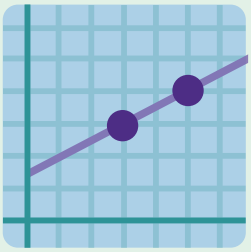
Linear Equations and Points



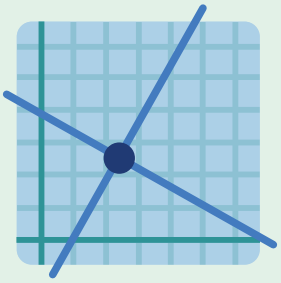
Lesson 9
Landing Planes



Lesson 10
Coin Capture



Lesson 11
Why Intercepts?



Lesson 12
Solutions



Lesson 13
Pennies and Quarters

Name: ..... Date: ..... Period: .....

Multiple Representations of Functions 8.EE.6, SMP.7, SMP.8

Landing Planes

Let's think about strategies for calculating slope.



Warm-Up

1 Determine possible values for each variable to make the equations true.

$$\frac{a}{b} = -2$$

$$\frac{m}{n} = 2$$

$$q - r = -2$$

Name: ..... Date: ..... Period: .....

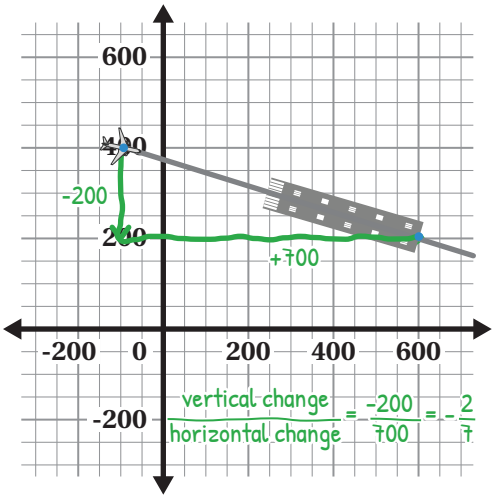
Planes, Lines, and Slopes

2 Predict whether the slope of the line through each set of points is *positive*, *negative*, or *zero*.

Points	Positive Slope	Negative Slope	Zero Slope
(600, 500) and (400, 500)			
(7, 1) and (12, 7)			
(10, 40) and (30, 20)			

3 Here are two students' strategies for calculating the slope of the line that goes through (-100, 400) and (600, 200).

Kwame's work



Wey Wey's work

x	y
-100	400
600	200

+700 (x) -200 (y)

$$\frac{\text{change in } y}{\text{change in } x} = \frac{-200}{700} = -\frac{2}{7}$$

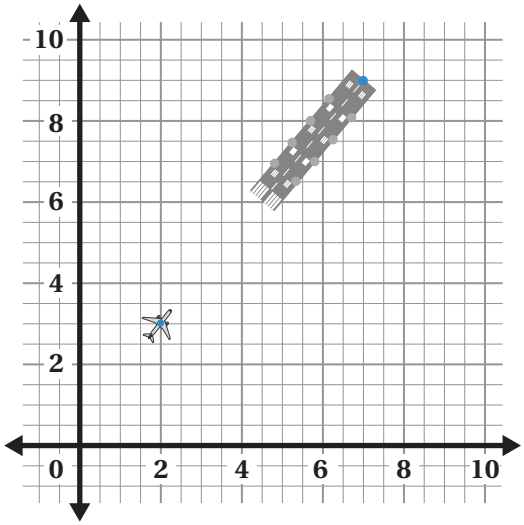
Discuss: How are their strategies alike? How are they different?

Name: ..... Date: ..... Period: .....

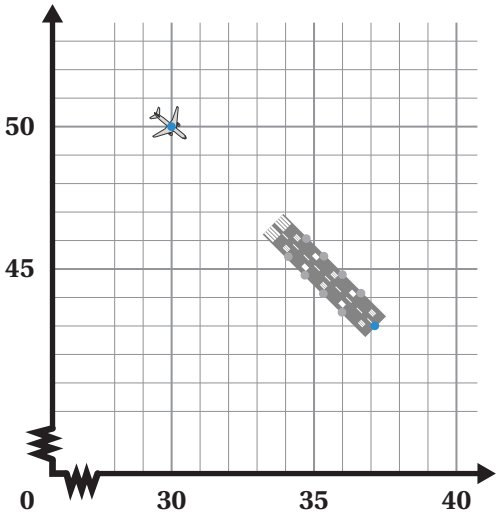
Strategies for Calculating Slope

4 Your task is to land the plane.

To do that, calculate the slope of the line that goes through $(2, 3)$ and $(7, 9)$.



5 To land the plane, calculate the slope of the line that goes through $(30, 50)$ and $(37, 43)$.



Activity 3

Name: _____ Date: _____ Period: _____

Challenge Creator

6 **a** **Make It!** Create your own plane-landing challenge.

- Choose a pair of coordinates to represent the starting position (Point 1) and ending position (Point 2) of your plane. Record the points on this page.
- On graph paper, choose a scale and label your axes, then plot your two points. Label the points with their coordinates.

b **Solve It!** On this page, calculate the slope that passes through your two points.

c **Swap It!** Trade graphs with one or more partners. On this page, calculate the slope of the line that passes through their two points.

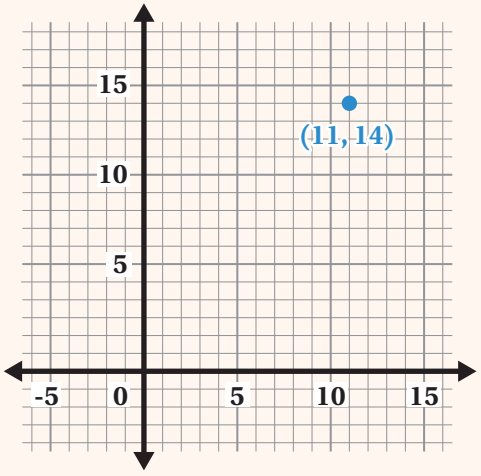
d With each partner, compare your strategies for calculating the slope of a line that passes through two given points.

My Challenge	Partner 1's Challenge	Partner 2's Challenge
Point 1: _____	Point 1: _____	Point 1: _____
Point 2: _____	Point 2: _____	Point 2: _____
Slope: _____	Slope: _____	Slope: _____

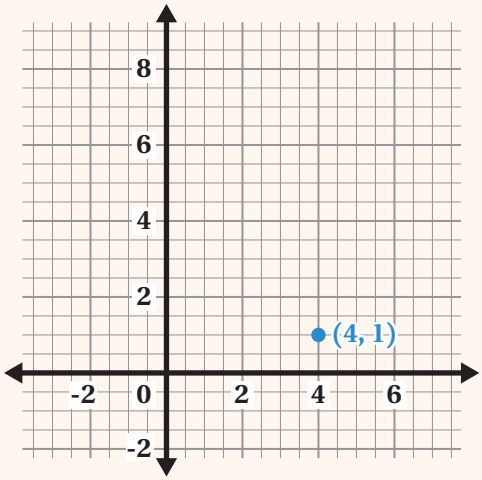
You're invited to explore more.

7 We know the slope of the path of each of these planes, but not their starting positions. Determine the value of p so that the line passing through the points has the indicated slope.

a Plane A starts at $(p, 2)$ and stops at $(11, 14)$. Its path has a slope of 2.



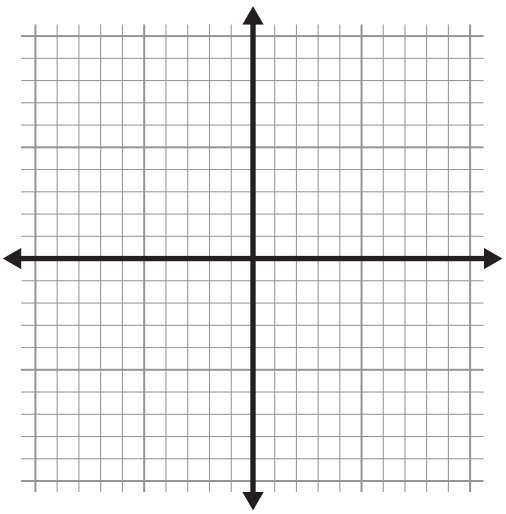
b Plane B starts at $(1, p)$ and stops at $(4, 1)$. Its path has a slope of -2.



8 Synthesis

What are some strategies for finding the slope of a line that goes through two given points?

Use the graph if it helps to show your thinking.

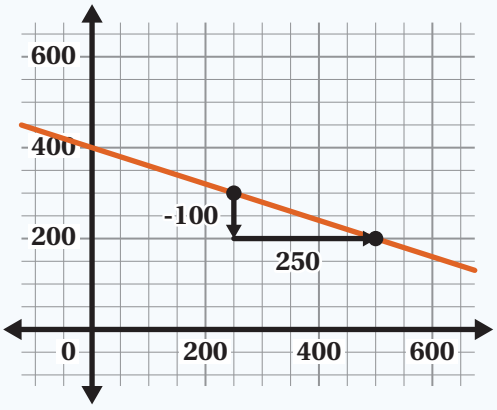


11 Summary 3.09

You can determine the slope of a line using two points on that line. Lines with positive slopes increase in height from left to right, while lines with negative slopes decrease in height from left to right.

You can use slope triangles to calculate the vertical change and horizontal change between two points on a coordinate plane.

Using Slope Triangles



$$\text{Slope} = \frac{\text{vertical change}}{\text{horizontal change}} = -\frac{100}{250} = -\frac{2}{5}$$

You can also calculate the slope by listing the coordinates in a table and then determining the difference between the y-coordinates (the vertical change) and the difference between the x-coordinates (the horizontal change).

Using Coordinates in a Table

x	y
250	300
500	200

+250 (from 250 to 500) -100 (from 300 to 200)

$$\frac{\text{change in } y}{\text{change in } x} = \frac{-100}{250} = -\frac{2}{5}$$

The slope is the ratio of the vertical change to the horizontal change.

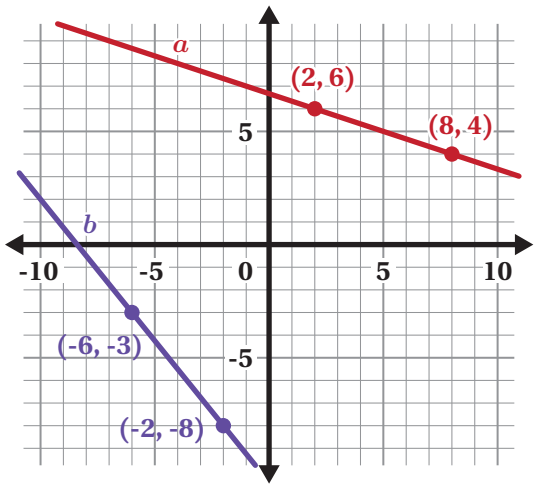
Practice
3.09

Name: ..... Date: ..... Period: .....

1. Calculate the slope of each line.

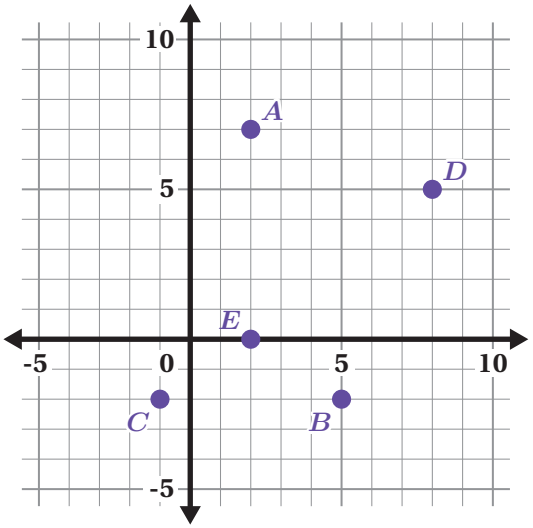
line a: .....

line b: .....

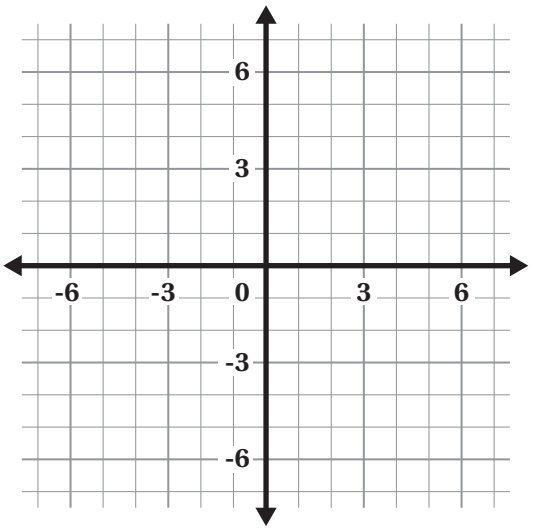


2. Draw a line with a slope of $-\frac{1}{3}$ that passes through point A.

What other point lies on that line?



Problems 3–4: Here is a blank graph.



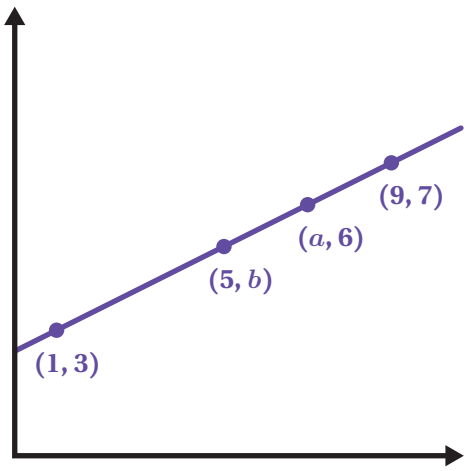
3. Draw a line with a slope of 4 and a negative y-intercept.

4. Explain how you know the slope of your line is 4.

Practice
3.09

Name: ..... Date: ..... Period: .....

Problems 5–7: All the points in this graph are on the same line.



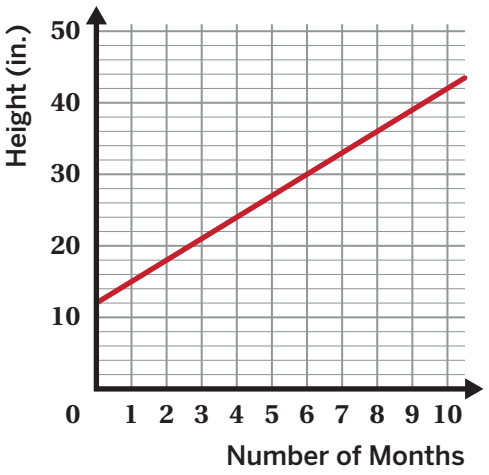
5. What is the slope of the line? Explain your thinking.

6. What are the values for a and b?

7. What is the x-value when $y = 0$?

Spiral Review

Problems 8–10: The graph shows the height of a bamboo plant, h, n months after it has been planted.



8. Write an equation that gives the bamboo's height, h, after n months.

9. How many inches tall will the bamboo plant be after 18 months? Show your thinking.

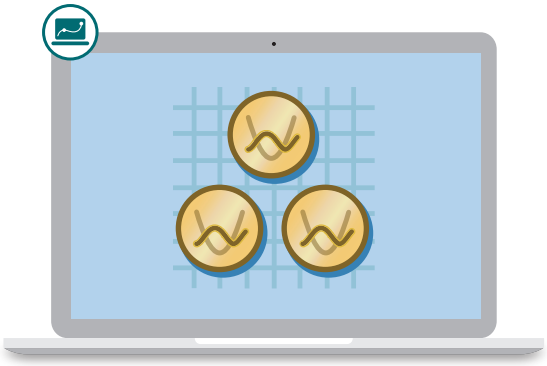
10. How many months will it take for the bamboo plant to reach 72 inches? Show your thinking.

Name: ..... Date: ..... Period: .....

Multiple Representations of Functions 8.EE.6, SMP.3, SMP.6, SMP.7

Coin Capture

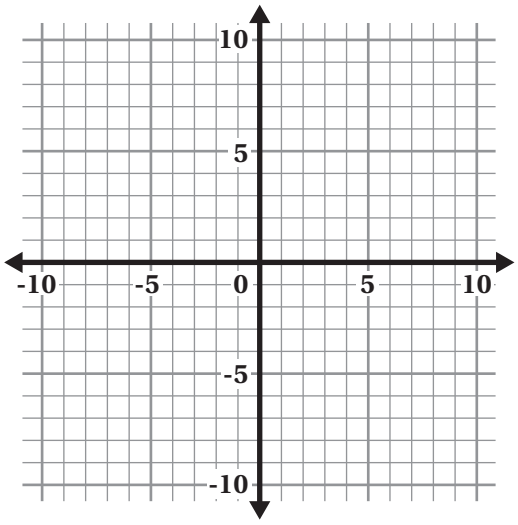
Let's write equations for vertical and horizontal lines.



Warm-Up

1 **a** Plot four points in different locations. The x-coordinate of each point should be 7.

b **Discuss:** What would your and your classmates' points look like if they were all on one graph?



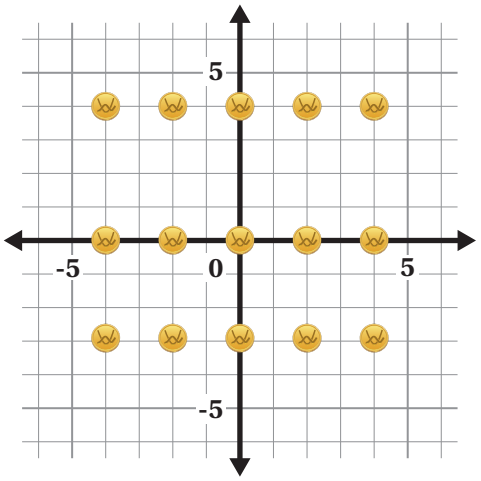
2 Let's look at the points that some other students graphed. Write an equation to represent all the points with an x-coordinate of 7.

Activity 1

Name: ..... Date: ..... Period: .....

Capture the Coins

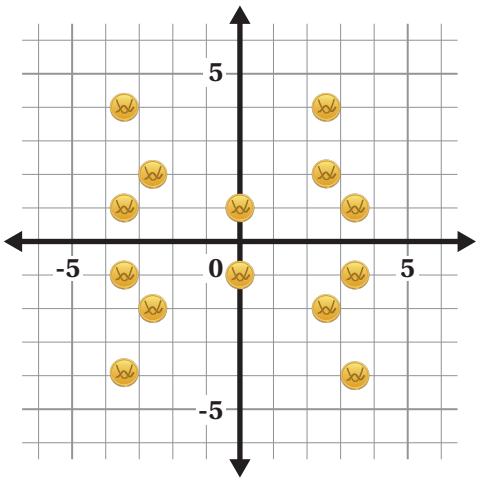
3 **a** Draw lines through the coins to "capture" them. Try to draw as few lines as possible.



b Write an equation for each line you drew.

Equations:

4 **a** Draw lines through the coins to "capture" them. Try to draw as few lines as possible.



b Write an equation for each line you drew.

Equations:

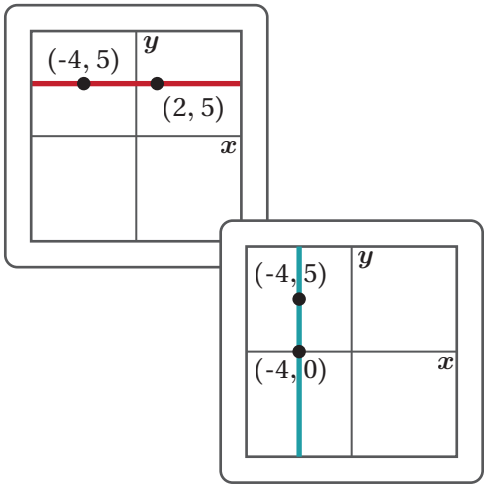
5 Juana says that vertical lines have a slope of zero.

Nekeisha says that horizontal lines have a slope of zero.

Whose claim is correct?

Juana's Nekeisha's Both Neither

Explain your thinking.



Activity 2

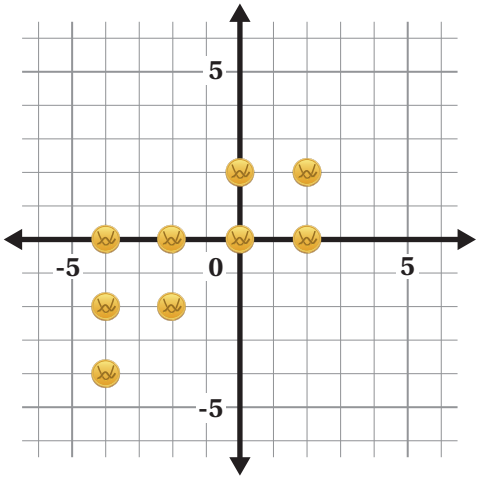
Name: ..... Date: ..... Period: .....

Challenge Creator

6 **a** Draw lines through the coins to "capture" them. Try to draw as few lines as possible.

b Write an equation for each line you drew.

Equations:



7 Create your own Coin Capture challenge!

a **Make It!** Use the Activity 2 Sheet to create your challenge.

b **Solve It!** On this page, write equations for the lines you would use to capture all the coins in your challenge. Try to use as few lines as you can.

c **Swap It!** Trade graphs with a partner and solve each other's challenges.

My Challenge

Equations:

Partner 1's Challenge

Equations:

Partner 2's Challenge

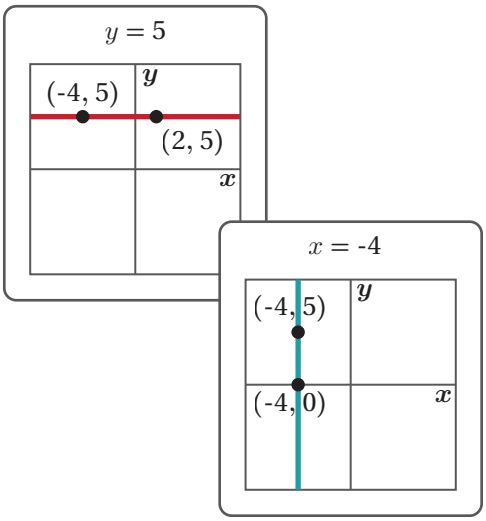
Equations:

Partner 3's Challenge

Equations:

8 Synthesis

How can you tell from looking at a linear equation if its graph is a horizontal or vertical line?



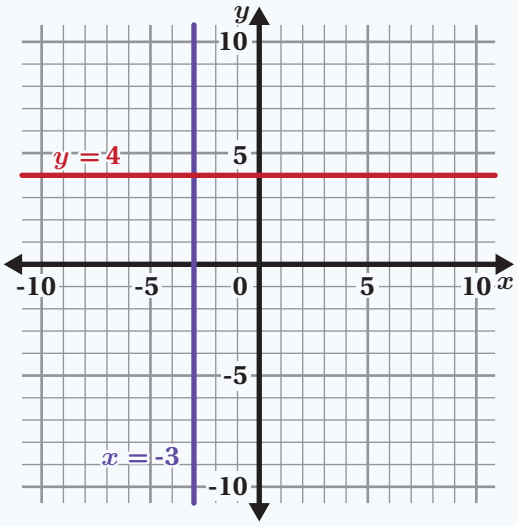
11 Summary 3.10

On the coordinate plane:

- Horizontal lines represent situations where the y-value is constant and the x-values change. Horizontal lines have a slope of 0. A horizontal equation is of the form $y = mx + b$, where the slope is zero.
- Vertical lines represent situations where the x-value is constant and the y-values change. Vertical lines have an *undefined* slope.

For example, the equation $y = 4$ represents the horizontal line because every point on the line has the same y-coordinate, 4.

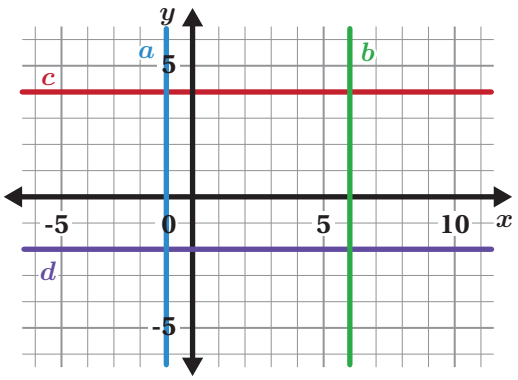
The equation $x = -3$ represents the vertical line because every point on the line has the same x-coordinate, -3.



Practice
3.10

Name: ..... Date: ..... Period: .....

Problems 1–4: Here are four lines on a coordinate plane. Write an equation for each line.



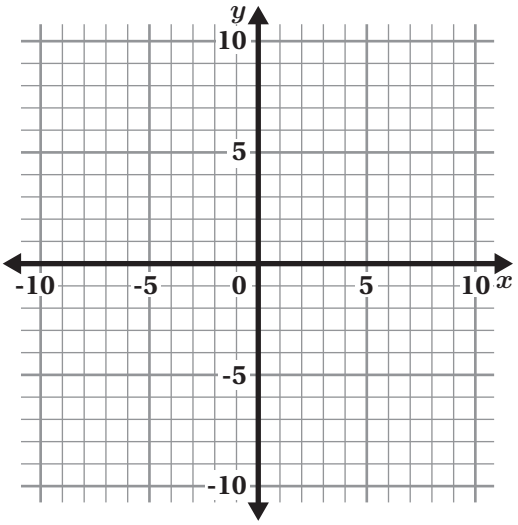
1. line a: .....

2. line b: .....

3. line c: .....

4. line d: .....

Problems 5–6: Use the coordinate plane to draw lines that meet the given slopes and y-intercepts. Then write an equation for each line.



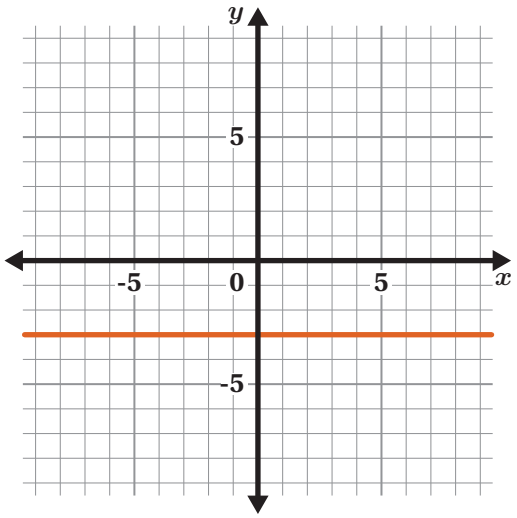
5. Slope: 0
y-intercept: -2

Equation: .....

6. Slope: 2
y-intercept: -1

Equation: .....

7. This graph represents a linear relationship. Choose the equation it matches.



A. $x = 3$

B. $x = -3$

C. $y = 3$

D. $y = -3$

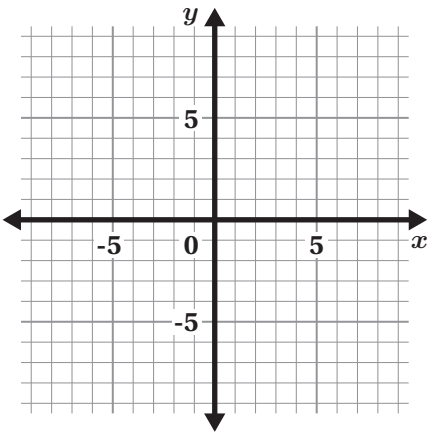
Name: ..... Date: ..... Period: .....

8. Write an equation for the line that passes through the points $(4, 3)$ and $(4, 15)$.

9. Write an equation for the line that passes through the points $(1, -6)$ and $(-6, -6)$.

Spiral Review

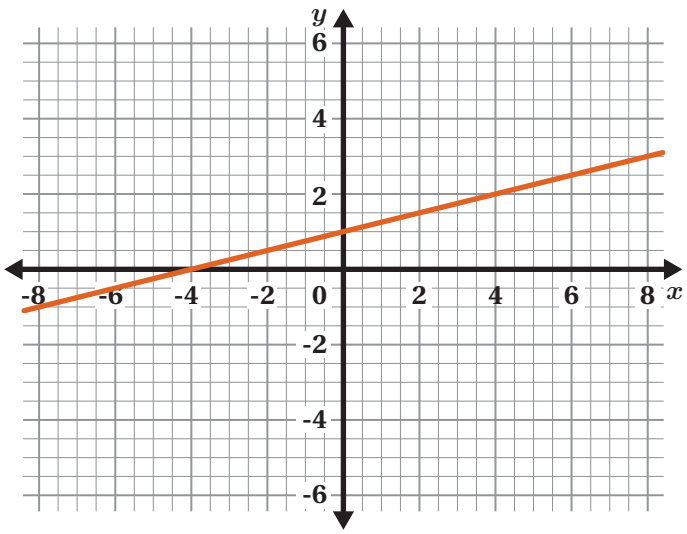
10. Graph the equation $y = -x - 5$.



11. Select *all* the pairs of points that have lines passing through them with a slope of $\frac{2}{3}$.

- ☐ **A.** $(0, 0)$ and $(2, 3)$
- ☐ **B.** $(0, 0)$ and $(3, 2)$
- ☐ **C.** $(1, 5)$ and $(4, 2)$
- ☐ **D.** $(-2, -2)$ and $(4, 2)$
- ☐ **E.** $(20, 30)$ and $(-20, -30)$

12. What is the slope of this line?



Name: ..... Date: ..... Period: .....

Linear Equations 8.F.4 , SMP.1, SMP.3, SMP.6

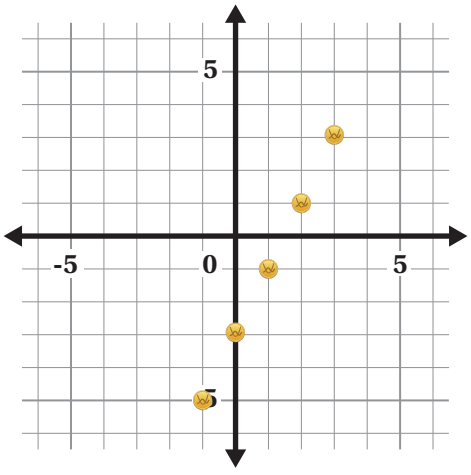


Why Intercepts?

Let's write an equation for a line that passes through two given points.

Warm-Up

1 Darryl wants to "capture" these coins with just one line.



a What slope could Darryl use?

b What y-intercept could Darryl use?

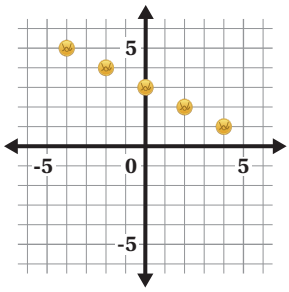
c Write an equation for the line Darryl could use.

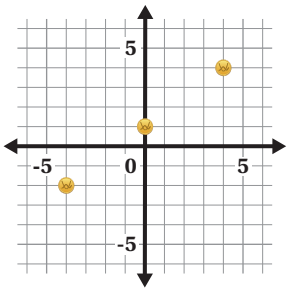
Activity 1

Name: ..... Date: ..... Period: .....

Determining the Equation

2 Write a single linear equation to capture all the coins for each challenge.

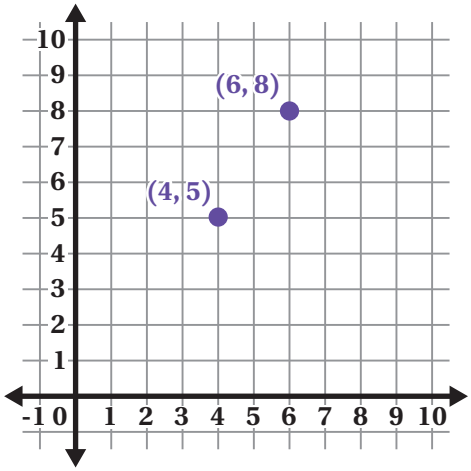




3 The points $(4, 5)$ and $(6, 8)$ represent the location of two coins.

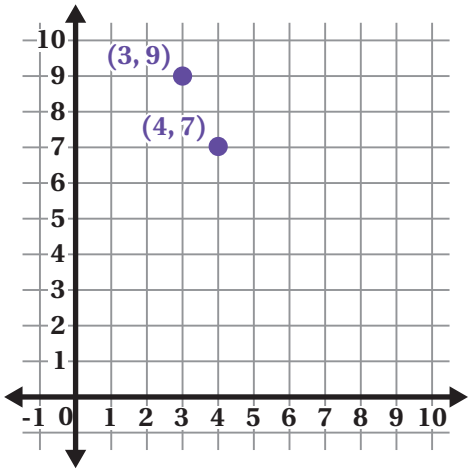
a Write an equation of the line that goes through both points.

b What was your strategy for determining the y-intercept?



4 Here are two new points: $(3, 9)$ and $(4, 7)$.

Describe how you could determine the y-intercept of the line going through these two points.



Name: ..... Date: ..... Period: .....

Determining the Equation (continued)

5 Here are two students' strategies for determining the y-intercept in the previous problem.

Tariq

x	y
3	9
4	7

+1 ↓ −2

x	y	
0	15	
1	13	+2
2	11	+2
3	9	+2
4	7	+2
−1		
−1		
−1		
−1		

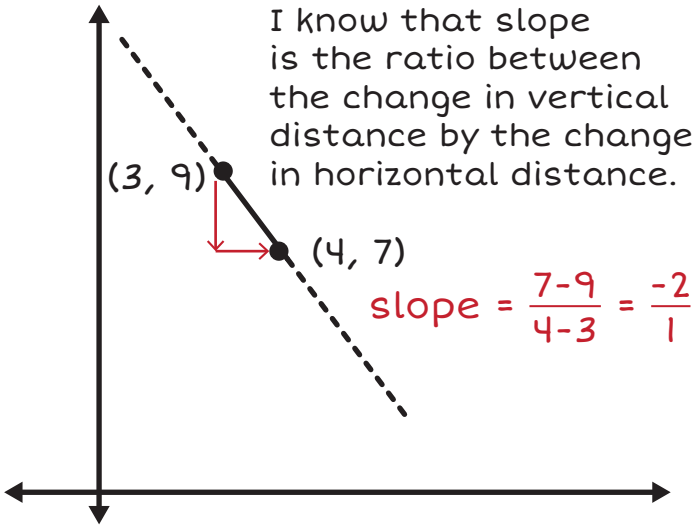
slope: $\frac{-2}{1}$ y-intercept = (0,15)

I can place the values in a table and work backwards to find the initial value.

y = mx + b, where m is the slope and b is the y-intercept.

The equation is y = -2x + 15.

Nia



I know that slope is the ratio between the change in vertical distance by the change in horizontal distance.

slope = $\frac{7-9}{4-3} = \frac{-2}{1}$

y = -2x + b

I'll substitute (3, 9) in for x and y

9 = -2(3) + b

9 = -6 + b

9 + 6 = b

15 = b

The equation is y = -2x + 15

a Choose a student and explain their strategy to a classmate.

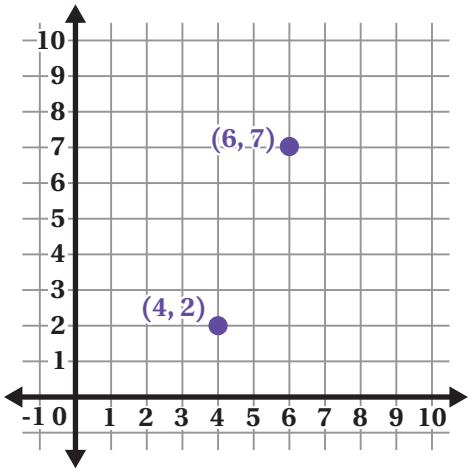
b **Discuss:** How are the students' strategies alike? How are they different?

Name: ..... Date: ..... Period: .....

Critiquing, Correcting, and Clarifying

6 Here are two new points: $(4, 2)$ and $(6, 7)$.

Write an equation of the line that goes through both points.



7 Victor made a mistake while writing the equation for the line that goes through $(4, 2)$ and $(6, 7)$.

a **Discuss:** What did Victor do well?

b Explain why Victor's work is incorrect.

Victor

x	y
4	2
6	7

+2 +5 slope: $\frac{5}{2}$

$y = \frac{5}{2}x + b$

$4 = \frac{5}{2}(2) + b$

$4 = 5 + b$

$-1 = b$

$y = \frac{5}{2}x - 1$

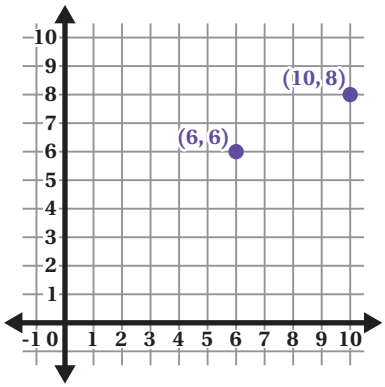
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Repeated Challenges

8 For each problem, write an equation of the line that goes through both points.

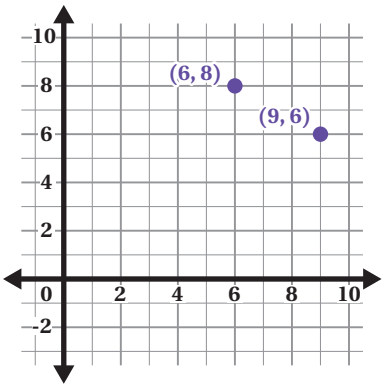
1. Points: (6, 6) and (10, 8).

Equation: .....



2. Points: (6, 8) and (9, 6).

Equation: .....



3. Points: (-1, 5) and (2, 2).

Equation: .....

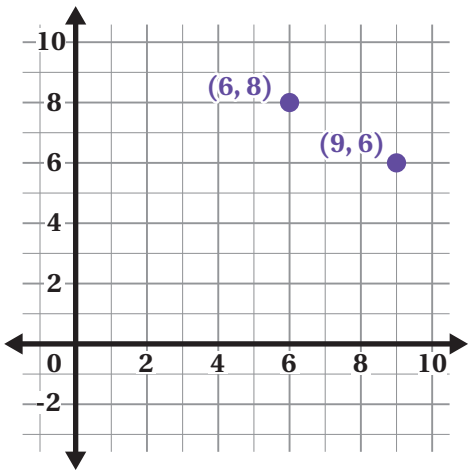
4. Points: (-4, -1) and (4, 3).

Equation: .....

9 Synthesis

Describe how to write an equation of a line that goes through two points.

Use the example if it helps with your thinking.



12 Summary 3.11

Here are two different strategies for writing the equation of a line using two given points.

Strategy Using a Table

First, calculate the slope using a table. Next, work backwards until the x-value is 0 to determine the initial value or y-intercept. Then write the equation in the form $y = mx + b$.

Tariq

x	y
3	9
4	7

+1 (→ 4), −2 (→ 7)

x	y	
0	15	
1	13	+2
2	11	+2
3	9	+2
4	7	+2

−1, −1, −1, −1

slope: $\frac{-2}{1}$ y-intercept = (0,15)

I can place the values in a table and work backwards to find the initial value.

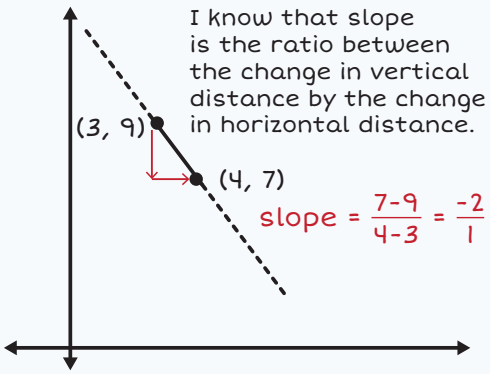
y = mx + b, where m is the slope and b is the y-intercept.

The equation is y = -2x + 15.

Strategy Using Slope Triangles

First, calculate the slope by drawing a slope triangle to determine the ratio between the change in the vertical distance by the change in the horizontal distance. Next, substitute the x- and y-value of one of the ordered pairs. Then write the equation in the form $y = mx + b$.

Nia



y = -2x + b

I'll substitute (3, 9) in for x and y

9 = -2(3) + b

9 = -6 + b

9 + 6 = b

15 = b

The equation is y = -2x + 15

Practice
3.11

Name: ..... Date: ..... Period: .....

1. Bao and Maia are each writing an equation of the line that passes through the points $(2, 9)$ and $(12, 14)$. They both calculate the slope as $\frac{1}{2}$.

- Bao substitutes the point $(2, 9)$ to determine the y-intercept.
- Maia substitutes the point $(12, 14)$ to determine the y-intercept.

Here is each student's work and their solutions for the y-intercept. Determine any mistakes in each student's work and explain how you would fix them.

Bao

$y = \frac{1}{2}x + b$

$9 = \frac{1}{2}(2) + b$

$9 = 1 + b$

$b = 8$

Maia

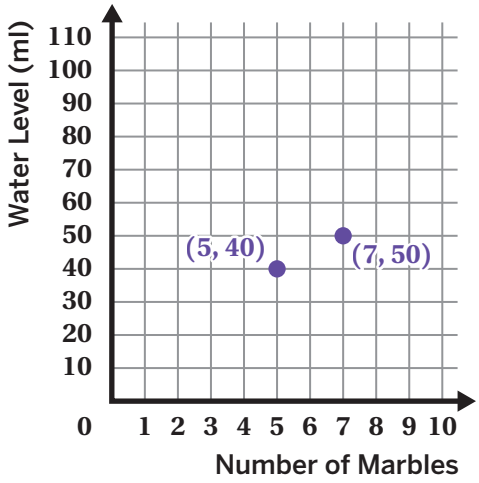
$y = \frac{1}{2}x + b$

$12 = \frac{1}{2}(14) + b$

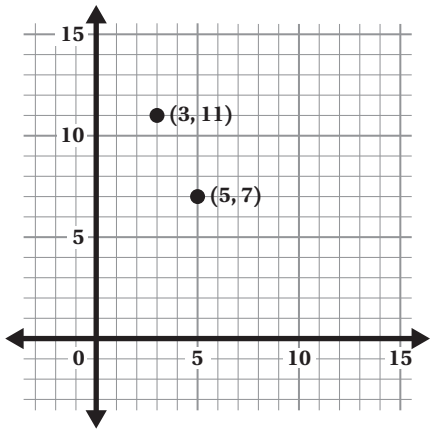
$12 = 7 + b$

$b = 5$

2. Chloe added marbles to a container of water. When she added 5 marbles, the water level was 40 millimeters. When she added 7 marbles, the water level was 50 millimeters. Write an equation for the water level, y, after x marbles are added. Show or explain your thinking.



3. Here is a graph showing the points $(3, 11)$ and $(5, 7)$. What is the y-intercept of the line that passes through these points?

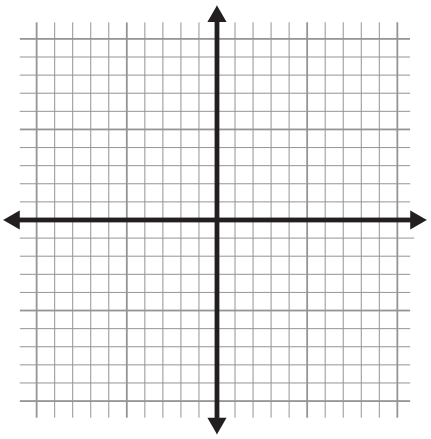


Practice
3.11

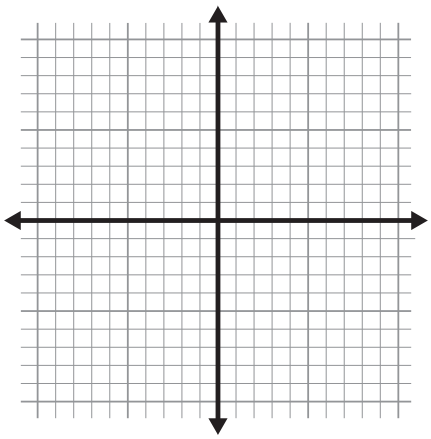
Name: ..... Date: ..... Period: .....

Problems 4–5: Write the equation of the line that passes through each pair of points. Show your work, and use the coordinate plane if it helps with your thinking.

4. $(2, 14)$ and $(6, 26)$



5. $(-5, 7)$ and $(1, 1)$



6. A line has a slope of 2 and passes through the point $(-6, 1)$. Which is the equation of that line?

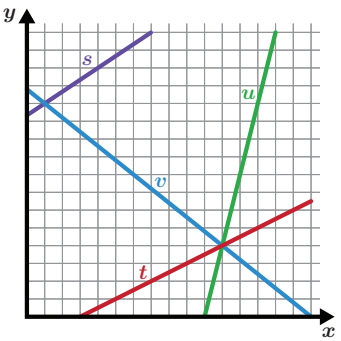
A. $y = 2x + 4$ **B.** $y = 2x + 13$ **C.** $y = 2x + 8$ **D.** $y = 2x - 11$

Spiral Review

Problems 7–10: Determine if the slope of each line is positive or negative.

7. Line s **8.** Line t

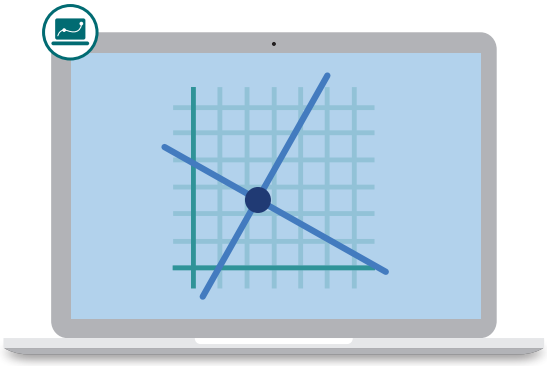
9. Line u **10.** Line v



11. Leah-James orders pictures from a photographer. Each picture costs $\$8.50$. A one-time shipping fee of $\$4.75$ is added to the cost of the order. The total cost of Leah-James' order before tax is $\$89.75$. How many pictures did Leah-James order? Explain your thinking.

Name: ..... Date: ..... Period: .....

Linear Equations 8.F.4, SMP.3, SMP.6



Solutions

Let's think about solutions to two-variable linear equations.

Warm-Up

1 Write *two* ordered pairs (x, y) that make the equation $x + 2y = 10$ true.

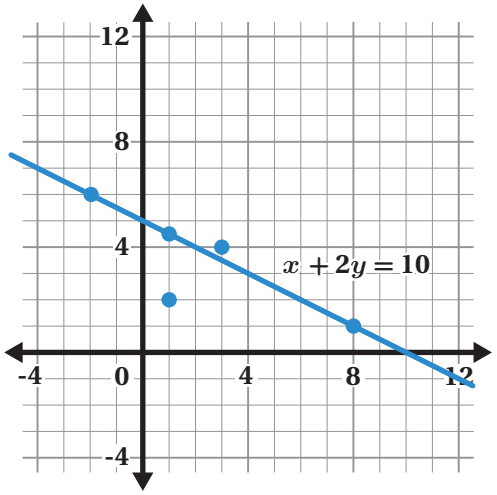
x	y

Name: ..... Date: ..... Period: .....

Solutions to Linear Equations

2 This graph shows the line $x + 2y = 10$, as well as some points that are **solutions** to the equation and some that are not.

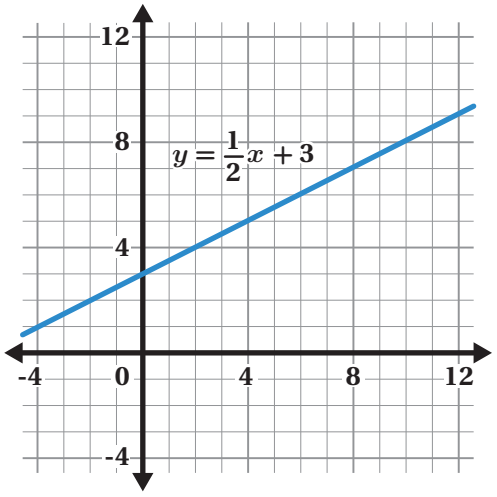
Show or explain how you can tell from the graph if a point is *not* a solution to the equation.



3 This graph shows the line $y = \frac{1}{2}x + 3$.

Complete the table so each point is a solution to the equation.

x	y
4	
	20

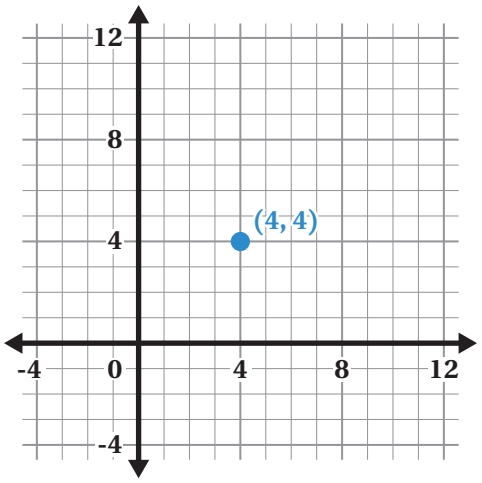


4 Describe your strategy for finding the solutions you wrote in the table.

Name: ..... Date: ..... Period: .....

More Solutions

5 Write an equation for a line that has a solution of $(4, 4)$.

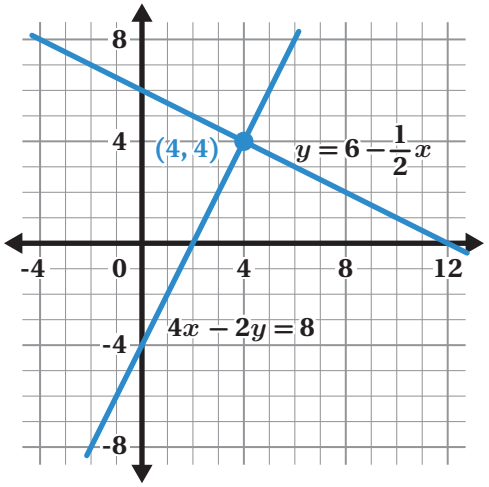


6 Here are the equations Hamza and Neo wrote for the previous problem:

Hamza: $4x - 2y = 8$

Neo: $y = 6 - \frac{1}{2}x$

Select *all* the statements that are true.



- ☐ $(4, 4)$ is a solution for both lines.
- ☐ $(7, 2.5)$ is a solution to $y = 6 - \frac{1}{2}x$.
- ☐ The lines have more than one solution in common.
- ☐ The line $4x - 2y = 8$ has more than one solution.

Name: ..... Date: ..... Period: .....

Challenge Creator

7 Create your own solution challenge!

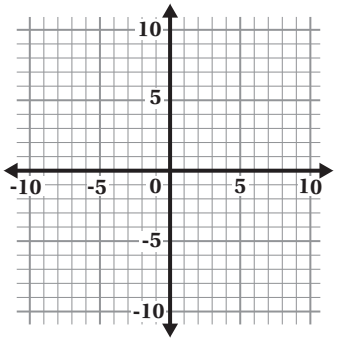
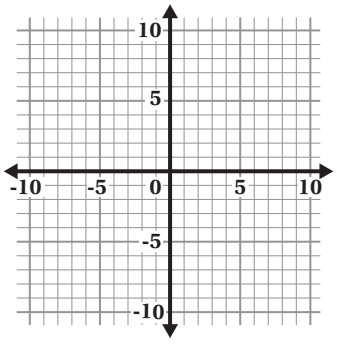
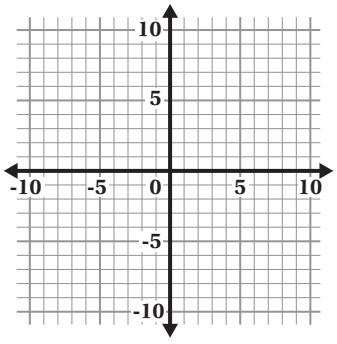
a **Make It!** Plot a point on the Activity 3 Sheet. Then write the equation of a line that the point is a solution for.

b **Solve It!** Then write another solution to your equation here.

Another solution: .....

c **Swap It!** Swap your challenge with one or more partners.

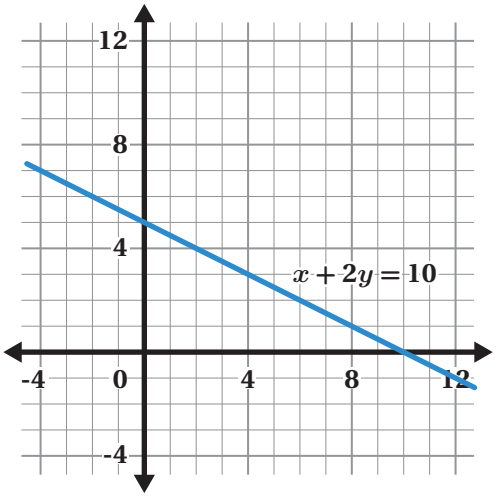
- Verify that the point plotted on your partner's graph is a solution to their equation. If it's not, allow your partner to revise their equation.
- Determine another solution to their equation. Use the graph if it helps with your thinking.

	Equation	Solutions	Graph
Partner 1		x | y	
Partner 2		x | y	
Partner 3		x | y	

8 Synthesis

Discuss: How can you determine whether a point is a solution to an equation?

Use the example if it helps with your thinking.



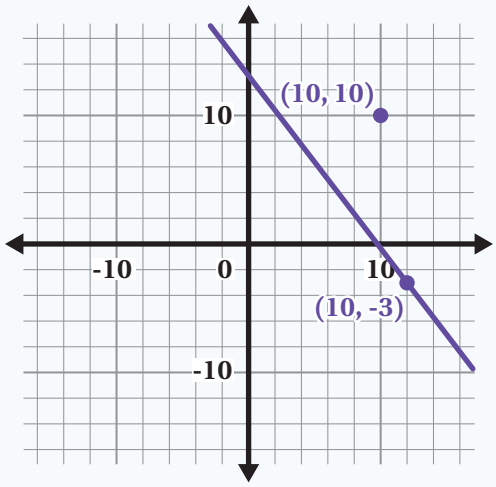
11 Summary 3.12

A **solution to an equation** with two variables is an ordered pair (x, y) that makes the equation true.

Every point that lies on a line is a solution to that equation. Points that do not lie on the line are *not* solutions to the equation.

Here is a graph of the linear equation $3x + 2y = 24$.

- $(10, -3)$ is a solution to the equation $3x + 2y = 24$ because the point is on the graph of the line, and because $3(10) + 2(-3) = 24$.
- $(10, 10)$ is *not* a solution because the point is not on the line, and because $3(10) + 2(10) = 50$, not 24.
- Although we can't see it on the graph, $(-10, 27)$ is also a solution because $3(-10) + 2(27) = 24$.



solution to an equation The value that makes the equation true. A solution to an equation with two variables is a pair of values that makes the equation true, often written as an ordered pair, (x, y)

Practice
3.12

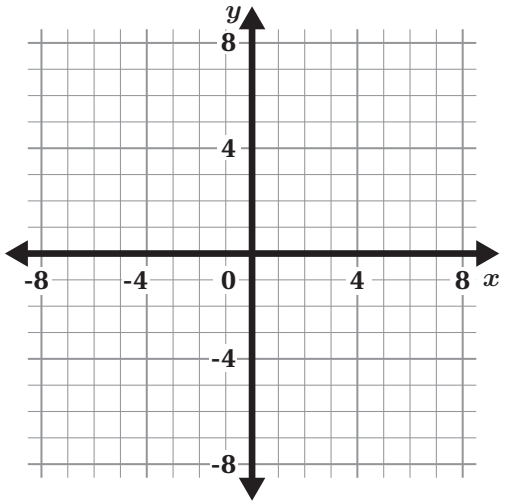
Name: ..... Date: ..... Period: .....

1. Select all of the ordered pairs (x, y) that are solutions to the linear equation $2x + 3y = 6$.

- ☐ **A.** $(0, 2)$
- ☐ **B.** $(0, 6)$
- ☐ **C.** $(2, 3)$
- ☐ **D.** $(3, -2)$
- ☐ **E.** $(3, 0)$
- ☐ **F.** $(6, -2)$

2. The graph of a linear equation passes through the points $(-4, 1)$ and $(4, 6)$. Select *all* the points that are also solutions to this equation. Use the graph if it helps with your thinking.

- ☐ **A.** $(0, 3.5)$
- ☐ **B.** $(8, 5)$
- ☐ **C.** $(12, 11)$
- ☐ **D.** $(-6, 0)$
- ☐ **E.** $(5, 6)$



3. Match each equation with its three solutions.

Equation	Solutions
a $2x + 3y = 7$	..... $(-3, -7), (0, -4), (-1, -5)$
b $3x = \frac{y}{2}$	..... $\left(3\frac{1}{2}, 0\right), (-1, 3), \left(0, 2\frac{1}{3}\right)$
c $x - y = 4$	..... $(14, 21), (2, 3), (8, 12)$
d $y = -x + 1$	..... $(0.5, 3), (1, 6), (1.2, 7.2)$
e $y = 1.5x$	..... $\left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{4}, \frac{3}{4}\right), \left(\frac{1}{8}, \frac{7}{8}\right)$

4. Determine whether the following statement is *true* or *false*. Explain your thinking.

The ordered pairs $(6, 13)$, $(21, 33)$, and $(99, 137)$ all lie on the line whose equation is $y = \frac{4}{3}x + 5$.

Name: ..... Date: ..... Period: .....

5. Here is a linear equation: $y = \frac{1}{4}x + \frac{5}{4}$.

Are $(1, 1.5)$ and $(12, 4)$ both solutions to this equation? Explain your thinking.

Problems 6–7: Complete each table.

6. $y = \frac{2}{3}x$

x	y
-3	
	2

7. $y + x = 5$

x	y
-3	
	2

Spiral Review

8. This table represents a linear relationship.

Write an equation in the form $y = mx + b$ that represents this relationship.

x	y
2	20
4	15
8	5

Problems 9–10: Deja adds $20 to her bus card. Every time she rides the bus, $2 is subtracted from the amount available on her card. After x rides, there are y dollars left on the card.

9. Write an equation to represent this relationship.

10. Use your equation to determine how much is left on Deja's bus card after she rides the bus three times.

Name: ..... Date: ..... Period: .....

Linear Equations 8.F.4, SMP.4, SMP.7

Pennies and Quarters

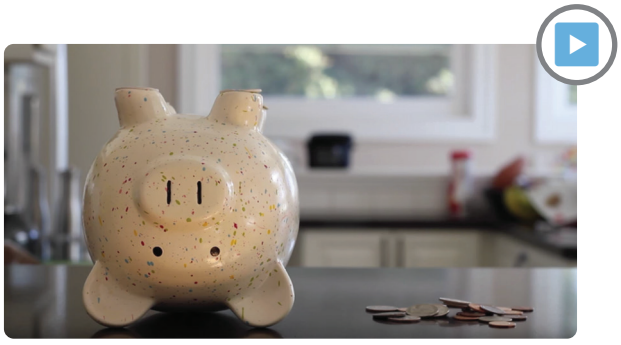
Let's determine solutions to real-world linear relationships.



Warm-Up

1 Let's watch a video.

What are some questions you could ask about this situation?



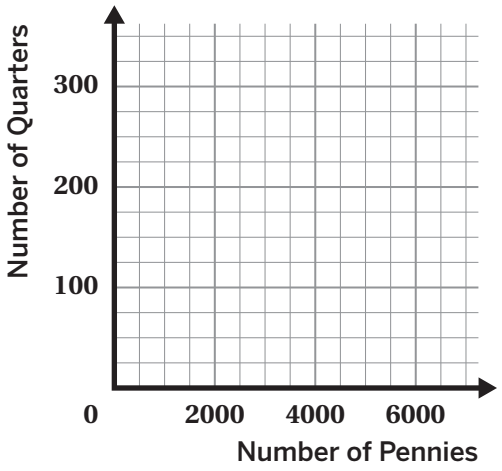
Name: ..... Date: ..... Period: .....

Pennies and Quarters

2 The piggy bank in the video was filled with pennies and quarters worth a total of $62.00.

Write *three* possible combinations of pennies and quarters that are worth $62.00.

Number of Pennies	Number of Quarters



Then graph the combinations of pennies and quarters you wrote.

3 Describe how you can tell if a combination of pennies and quarters is worth $62.00.

4 Let's look at the graph of some students' combinations from Problem 2.

Write an equation that describes *all* the combinations of pennies, p, and quarters, q, that are worth $62.00.

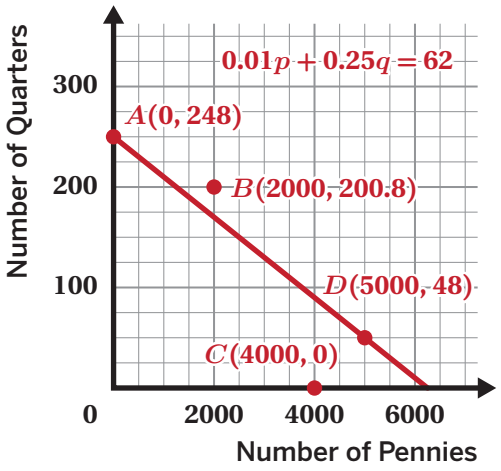
Name: ..... Date: ..... Period: .....

Pennies and Quarters (continued)

5 Ava wrote the equation $0.01p + 0.25q = 62$ to represent all the combinations of pennies, p, and quarters, q, that are worth \$62.00.

Select *all* the points that are solutions to the equation.

- ☐ **A.** Point A
- ☐ **B.** Point B
- ☐ **C.** Point C
- ☐ **D.** Point D



Discuss: How did you determine which points were solutions to the equation?

6 If there are 200 quarters, how many pennies do you need for a total of \$62.00?

Use the graph if it helps with your thinking.

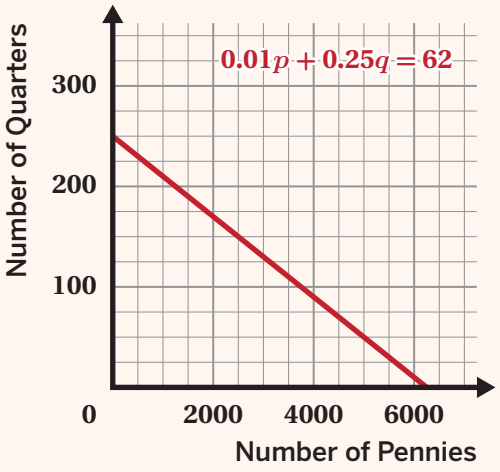
You're invited to explore more.

7 Here is a line and an equation describing all the combinations of pennies and quarters that are worth \$62.00.

A different pile of pennies and quarters is worth \$30.

a Draw the graph of this situation on the same plane.

b Describe the strategy you used.



Name: ..... Date: ..... Period: .....

Situation Sort

8 **a** Read each situation.

b **Discuss:**

- What is each situation about?
- What quantities or relationships do you see in each situation?

Situation 1

Sydney plans to buy 120 beverages for a picnic. Seltzers are sold in packs of 6. Waters are sold in packs of 8. x represents the number of packs of seltzers and y represents the number of packs of waters.

Situation 2

A coach has a $120 budget to buy lunch for their team. They are ordering from a restaurant that charges $8 per sandwich, plus a $6 delivery fee. x represents the number of sandwiches and y represents the total cost of the lunch.

9 Match each representation and possible solution with Situation 1 or Situation 2 from the previous problem. One representation has no match.

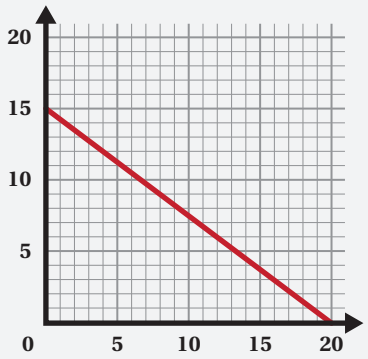
A

x	y
2	22
3	30
4	38

B

$120 = 6x + 8y$

C



D

$y = 8x + 6$

E

$y = 8x + 120$

F

$(5, 46)$

G

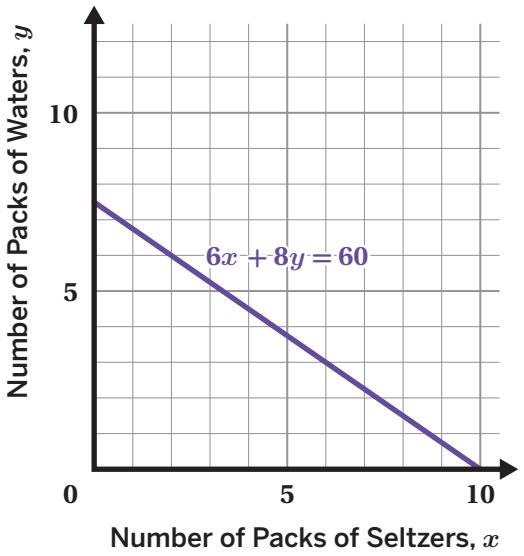
$(8, 9)$

Situation 1	Situation 2

10 Synthesis

Describe how you could use a graph or equation to determine whether a point is a solution to a linear relationship.

Use the example if it helps with your thinking.

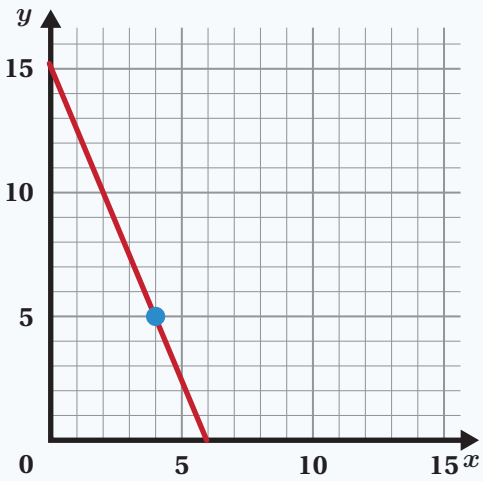


13 Summary 3.13

The four representations of a linear relationship — table, graph, equation, and verbal description — are all useful when solving real-world problems.

Let's say a coach has a $120 budget to buy dinner for their team. Pizzas cost $20 and sandwiches cost $8. x represents the number of pizzas bought and y represents the number of sandwiches bought.

This situation can be modeled by the linear relationship $20x + 8y = 120$.



Here are two ways to show that 4 pizzas and 5 sandwiches is one solution to the equation:

- The values $x = 4$ and $y = 5$ make the equation true.

  $20(4) + 8(5) = 120$

  $80 + 40 = 120$

  $120 = 120$

- The point $(4, 5)$ is on the graph of the linear relationship.

Practice
3.13

Name: Date: Period:

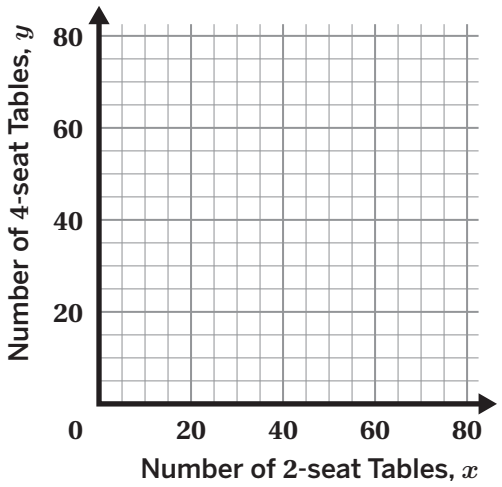
Problems 1–5: The owner of a new restaurant is ordering tables and chairs. She wants to have only tables for 2 and tables for 4. The total number of people that can be seated in the restaurant is 120.

1. Complete the table to show some possible combinations of 2-seat tables and 4-seat tables that will seat 120 customers.

Number of 2-Seat Tables	Number of 4-Seat Tables

2. Write an equation that represents the number of 2-seat tables, x, and the number of 4-seat tables, y, that the owner can order.

3. Graph all the possible combinations of 2-seat and 4-seat tables that will seat 120 customers.



4. What is the slope of the line? What does it tell you about the situation?

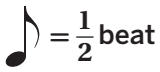
5. What are the x- and y-intercepts of the line? What do they represent in the situation?

Practice
3.13

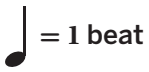
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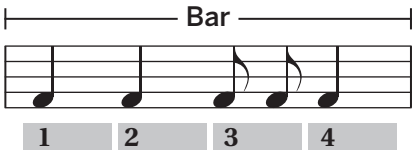
Problems 6–7: A bar is a section of music made up of notes. A quarter note is 1 beat long. An eighth note is $\frac{1}{2}$ a beat long. A music composer wants to create a bar that's 4 beats long.

Eighth Note



Quarter Note



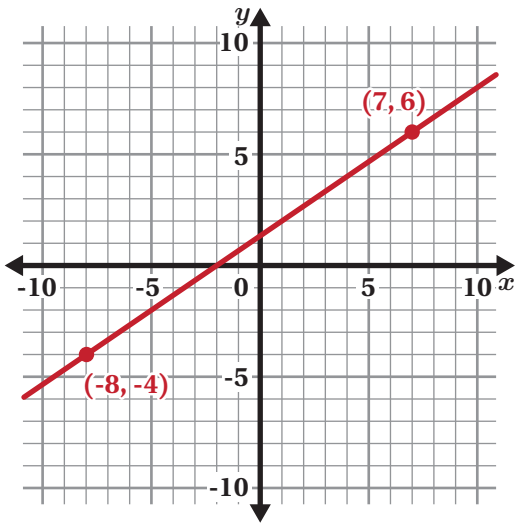


6. What are two possible combinations of eighth notes ($\frac{1}{2}$ a beat long) and quarter notes (1 beat long) that they could use to make a bar that's 4 beats long? Use a graph if it helps with your thinking.

7. Write an equation to represent the number of eighth notes, x, and number of quarter notes, y, in a bar that has a total of 4 beats.

Spiral Review

Problems 8–11: Here is the graph of a linear relationship.



8. What is the y-intercept of the line?

A. $\frac{4}{3}$ B. $\frac{3}{4}$

C. $\frac{2}{3}$ D. $\frac{3}{2}$

9. What is the slope of the line?

A. $\frac{4}{3}$ B. $\frac{3}{4}$

C. $\frac{2}{3}$ D. $\frac{3}{2}$

10. Write an equation that represents the line.

11. What is one possible solution to the equation?

Name: _____ Date: _____ Period: _____

Linear Equations | Multiple Representations of Functions | 8.EE.5, 8.EE.6, 8.F.4, SMP.2, SMP.7

Practice Day 2

Let's practice what you've learned so far in this unit!



You will use problem cards for this Practice Day. Record all of your responses here.

Card 1

Select *all* that apply: A B C D E

Card 2

Select *all* that apply: A B C

Card 3

Line g: _____

Line h: _____

Card 4

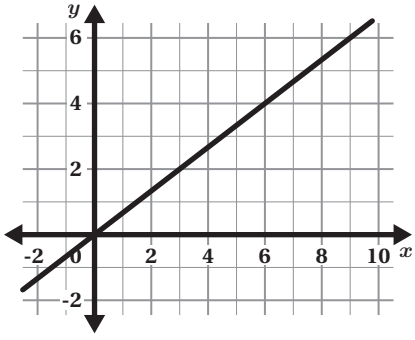
a. Slope: _____

b. y-intercept: _____

c. Equation: _____

Card 5

a. Graph:



b. Equation: _____

Equation: _____

c. Equation: _____

Explanation:

Card 6

a. Equation: _____

b. _____ dollars

Card 7

Circle one: Arjun or Kyrie

Explanation:

Name: _____ Date: _____ Period: _____

Practice Day 2 (continued)

Card 8

Circle one: Sol Axel Troy

Explanation:

Card 9

Statement: _____

Card 10

Line f: _____

Line g: _____

Card 11

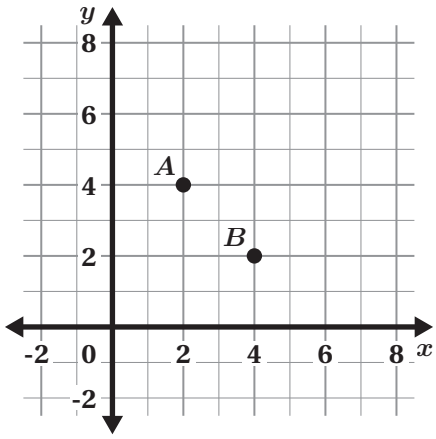
Equation: _____

Card 12

Number of Drinks	Number of Burgers
2	
	4
8	

Card 13

a and b



c Equation: _____

Equation: _____

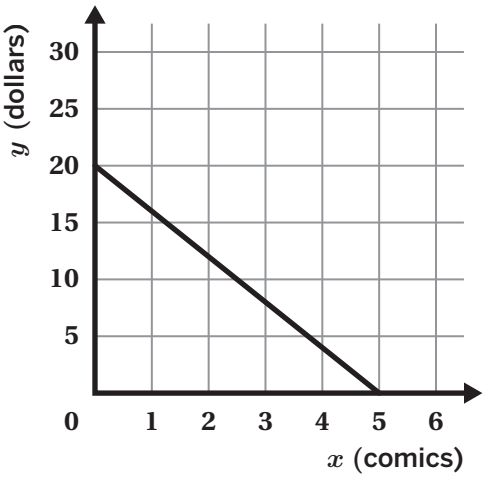
Name: _____ Date: _____ Period: _____

Practice Day 2 (continued)

Card 14

a Slope: _____

b



They started with _____ dollars.

Card 15

_____ gallons

Card 16

Circle one: line m line p line r

Explanation:

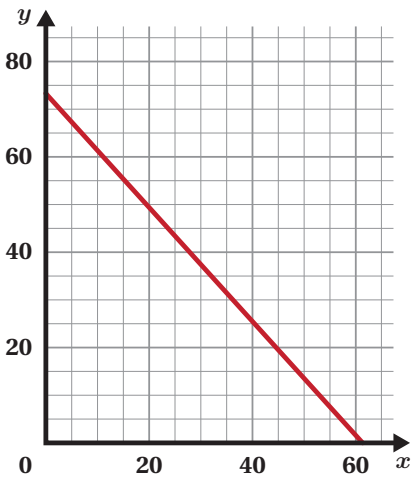
Notes:

Career Connection

Solutions to equations that model real-world scenarios need to make sense.

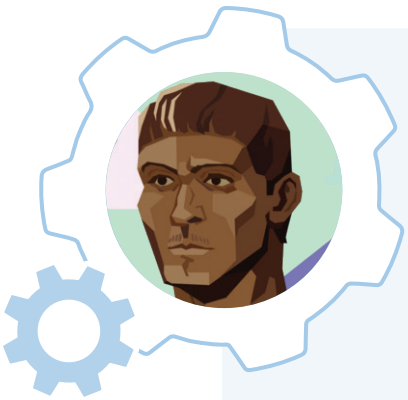
Here is a real-world scenario involving a budget constraint, which is an example of *economics*, the study of how a society uses their resources.

A school store has a budget of $\$365$ to spend on x t-shirts and y hats. Each t-shirt costs $\$6$ and each hat costs $\$5$. The equation $6x + 5y = 365$ models this scenario. What are some solutions for the number of t-shirts and hats the store can order and spend their entire budgeted amount?



The graph of this equation shows that *all* the ordered pairs that lie on this line are solutions, yet only positive integer solutions make sense. Why?

Economists study how a society uses its resources (economics). They collect and analyze data and advise on economic issues, such as budget constraints or supply and demand. They might use linear equations to help solve these problems. Sometimes, the solutions need to be integers.



Meet Diophantus

A Diophantine equation is an equation involving two or more variables where the desired solutions are pairs of integers. The term *Diophantine* comes from Diophantus of Alexandria, a 3rd century mathematician, who was known for his published work *Arithmetica*, in which he described the solutions to algebraic problems. Algebraic symbols and variables had not yet been invented, so Diophantus used words to describe the "first unknown" or "second unknown."

Are you interested in studying economics or exploring Diophantine equations further? What can you do to learn more?

Community Connection

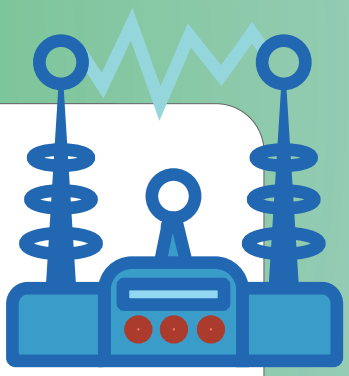
Create a real-world problem in which the solutions are integers and that can be modeled and solved using a Diophantine equation. Share your problem with at least 3 people in your home or communities and invite them to solve the problem.

Math Mindset

Here is an equation.
$3(x+2) - 4x = \frac{2}{3}(6x + 1)$
A friend says you can multiply each term by 3 as a first step.
Do you agree? Explain.

Unit 4

Linear Equations and Linear Systems



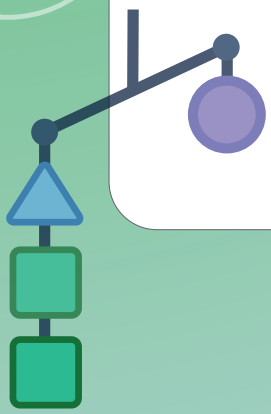
Big Ideas in This Unit

CC2 Linear Equations | Multiple Representations of Functions

Questions for Investigation

- How can you solve an equation with variables on both sides of the equal sign?
- How can systems of equations be used to represent situations and solve problems?
- What does it mean for an equation or system of equations to have *no*, *one*, or *infinitely many solutions?*

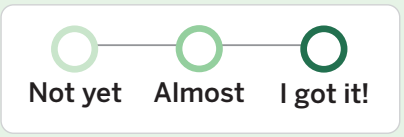
Explore: Equation Puzzles
How is an equation like a puzzle?





Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



I can . . .	Before	After
Solve any linear equation with one variable.		
Determine when a linear equation has one solution.		
Determine when a linear equation has no solution.		
Determine when a linear equation has infinitely many solutions.		
Describe what the point of intersection of two lines represents.		
Solve problems that represent two linear relationships in context and determine the solution.		
Estimate the solution to a system of linear equations based on their graphs.		
Solve a system of linear equations without using a graph.		
Analyze the equations in a system of linear equations to determine if the system has one solution.		

I can . . .	Before	After
Analyze the equations in a system of linear equations to determine if the system has no solution.		
Analyze the equations in a system of linear equations to determine if the system has infinitely many solutions.		
Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions.		

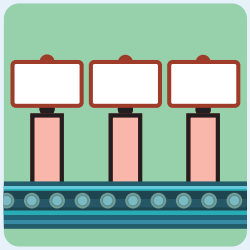
Sub-Unit

1

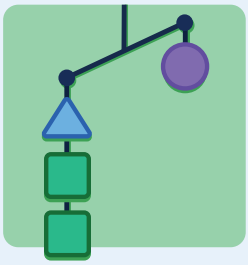
Solving Linear Equations



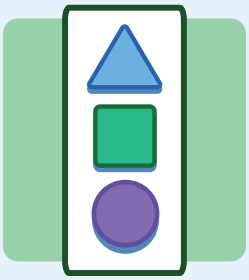
Explore
Equation Puzzles



Lesson 1
Number Machines



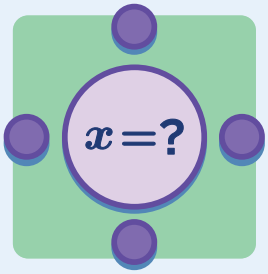
Lesson 2
Keep It Balanced



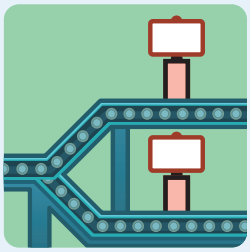
Lesson 3
Balanced Moves



Lesson 4
More Balanced Moves



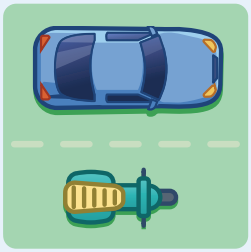
Lesson 5
Equation Roundtable



Lesson 6
All, Some, or None? Part 1



Lesson 7
Strategic Solving, Part 1



Lesson 8
When Will They Meet?

Name: ..... Date: ..... Period: .....

Linear Equations | Building Toward 8.EE.8, SMP.1, SMP.2, SMP.7

Explore:
Equation Puzzles

How is an equation like a puzzle?



Warm-Up

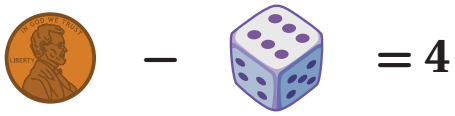
1. Here are two puzzles.

What do you notice? What do you wonder?

Equation A



Equation B



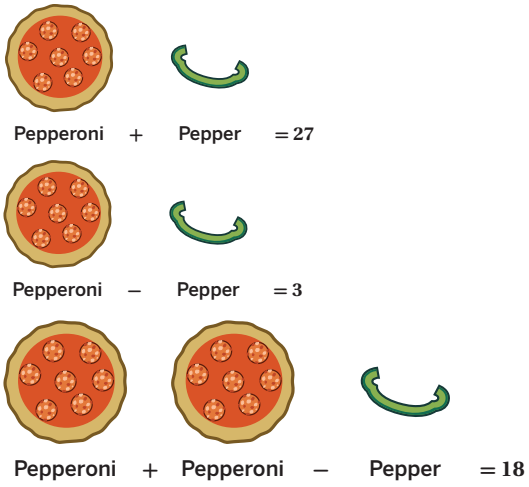
I notice:

I wonder:

Name: Date: Period:

Decode the Puzzle

2. Let's look at another puzzle. Amara claims the value of the pepperoni is 15.



Is Amara correct? Explain your thinking.

Discuss: What are some different strategies you can use to solve the puzzle equation?

3. You will use the Activity Sheet to complete this problem. Solve each puzzle.

Choose one puzzle. **Discuss:**

- Which shape did you choose to determine the value of first? Why?
- How did you determine the remaining values?
- What equation could be used to represent the puzzle? Use variables to represent the shapes.

Name: Date: Period:

Decode the Puzzle (continued)

4. Now it's your turn!

a Make it! Create your own equation puzzle with at least two equations. Make sure you know the solution!

b Swap it! Swap your challenge with one or more partners. Record your solution to your partner's equations.

Name: ..... Date: ..... Period: .....

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet Almost I got it!

I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem.

Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed.

Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

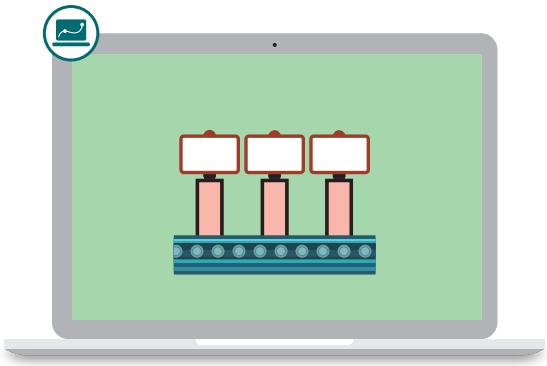
Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

Not yet Almost I got it!

Name: ..... Date: ..... Period: .....

Linear Equations | Building Toward 8.EE.7, 8.F.1, SMP.7



Number Machines

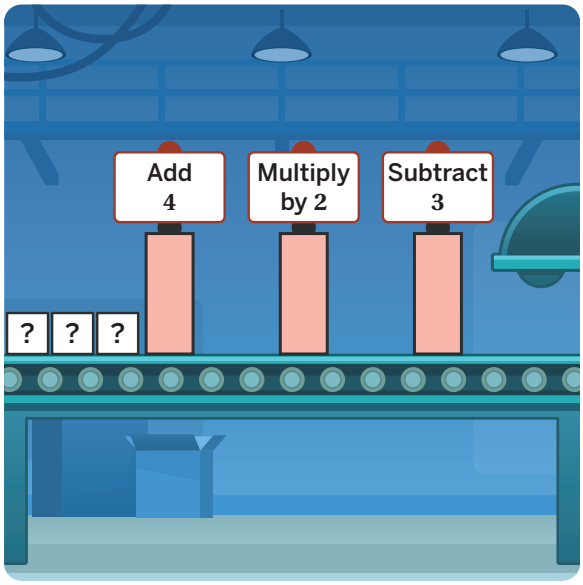
Let's investigate some number machines.

Warm-Up

1 Let's watch a machine that follows these instructions:

- Add 4
- Multiply by 2
- Subtract 3

Discuss: What are some questions you can ask about this machine?



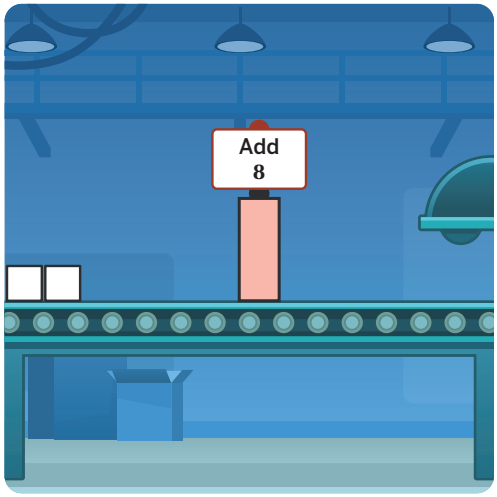
2 What will come out if we put 10 into this machine?

Name: ..... Date: ..... Period: .....

Number Machines

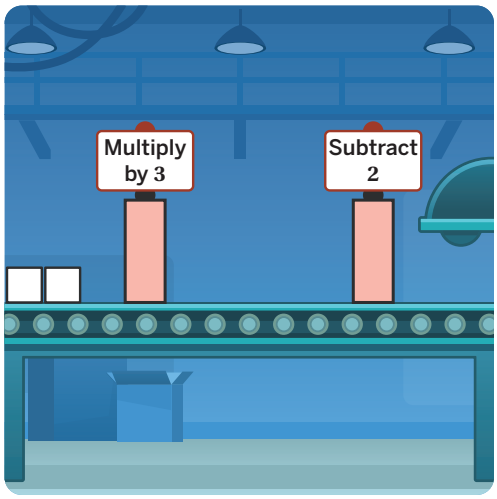
3 Alina put a number into this machine, and 13 was the output.

What was the input?



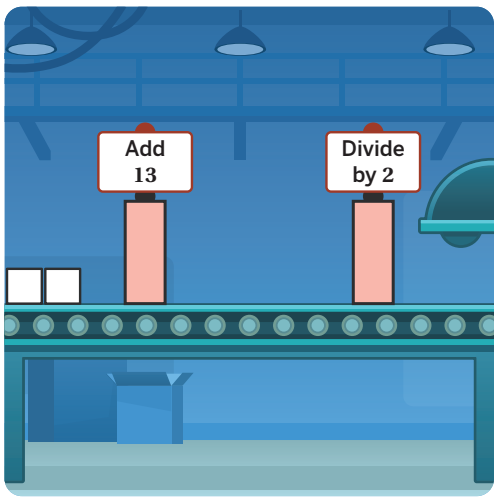
4 LaShawn put a number into this machine, and 40 was the output.

What was the input?



5 Nathan put a number into the machine, and 15 was the output.

What was the input?



Name: Date: Period:

Number Machines (continued)

6 Felipe tried the machine from the previous problem.

He said: *If you take the output, then subtract 13 and multiply by 2, you will get the input.*

Is his claim correct? Circle one.

Yes No I'm not sure

Explain your thinking.

Activity 2

Name: ..... Date: ..... Period: .....

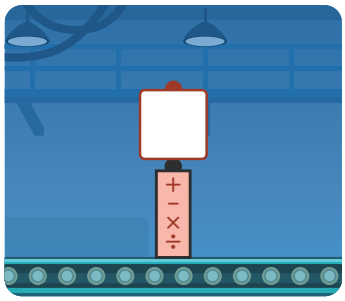
Challenge Creator

7 Now it's your turn to create your own number machine challenge.

a **Make it!**

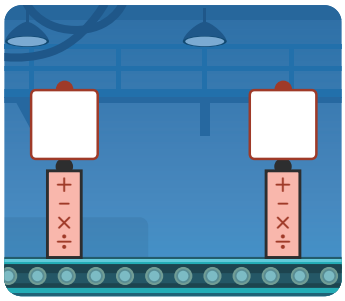
- Choose whether to make a one-, two-, or three-step machine.
- Circle the operation and write a value in the empty box for each step.
- Choose an input value and record the output value.

One-Step Machine



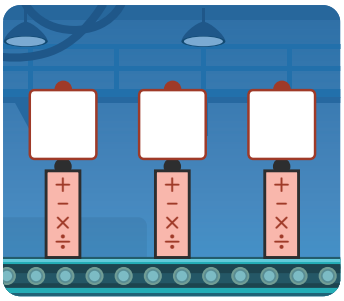
Output: .....

Two-Step Machine



Output: .....

Three-Step Machine



Output: .....

b **Swap it!**

- Share your number machine and its output with a partner. Keep your input a secret!
- Determine the input your partner used for their machine.

	Output	Steps	Input
Partner 1			
Partner 2			
Partner 3			
Partner 4			
Partner 5			
Partner 6			

Activity 3

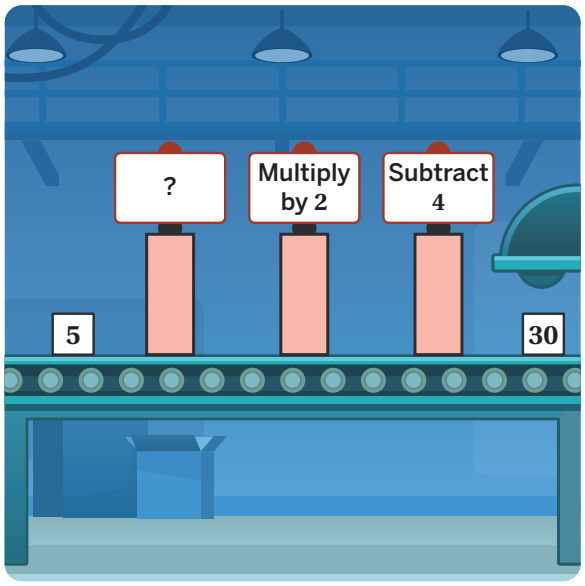
Name: ..... Date: ..... Period: .....

What's the Operation?

8 Here's another number machine. 5 was the input, and 30 was the output.

a What could the first step in the number machine be?

b Can you think of another possibility?



You're invited to explore more.

9 Here is a number puzzle.

Start with any number for this puzzle, and the output will *always* be 3.

a Try it!

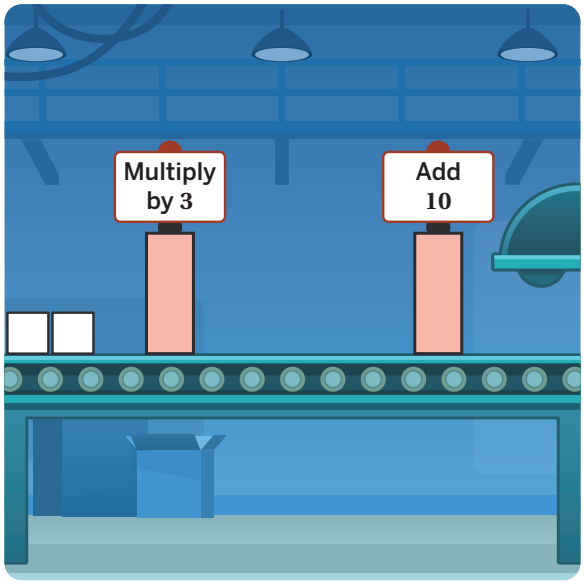
b Design a different number puzzle where the output will *always* be 5.

- Take your number and multiply by 2
- Then add 6
- Then divide by 2
- Then subtract the original number

10 Synthesis

Discuss: If you know a number machine's output, how can you determine the input?

Use the example if it helps with your thinking.



13 Summary 4.01

You can use a number machine to perform a series of operations on a number.

- If only the *input* (the number put into the machine) is provided, the *output* (the number that comes out of the machine) can be determined by performing the series of operations in order.
- If only the output is provided, the input can be determined by guessing and checking, by working backward, or by writing and solving an equation.

For example, a number machine performs this series of operations in order:

- Pick an input
- Multiply by 3
- Add 8
- Subtract 1

The output is 19. Here are two ways you can determine the input:

Work backward from the output

Start with 19, add 1, subtract 8, and then divide by 3. $(19 + 1 - 8) \div 3 = 4$.

These steps show that the input is 4.

Write and solve an equation

The equation $(3x + 8) - 1 = 19$ represents the number machine, where x is the input.

Solving this equation shows that $x = 4$.

Practice
4.01

Name: ..... Date: ..... Period: .....

1. Here is a number puzzle:

- Start with the number 5
- Multiply by 4
- Add 2

What is the output?

2. Chloe asks David to try this number puzzle:

- Pick a number
- Add 2
- Multiply by 3
- Subtract 6

David's output is 27. What number did he start with?

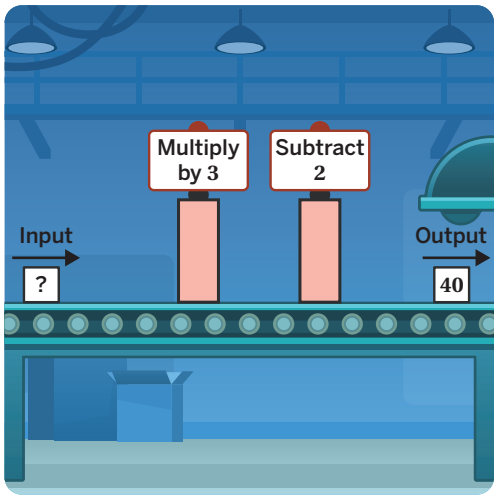
Explain your thinking.

3. In a basketball game, Aki scores twice as many points as Tyani, Tyani scores 4 points fewer than Nekeisha, and Nekeisha scores 3 times as many points as Mariana. If Mariana scores 5 points, how many points did Aki score?

Explain your thinking.

4. Diego put a number into the machine, and 40 was the output. What was the input?

Explain your thinking.



Name: ..... Date: ..... Period: .....

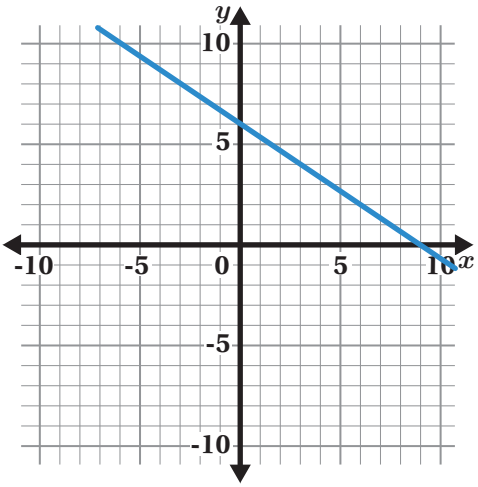
Problems 5–6: Clare inputs n into a number machine. The expression $\frac{3(n+1)}{3}$ represents her output.

5. Describe the possible steps of the number machine.

6. Describe how the output is related to the input.

Spiral Review

7. What is the equation of the line shown?



A. $y = -\frac{2}{3}x + 6$

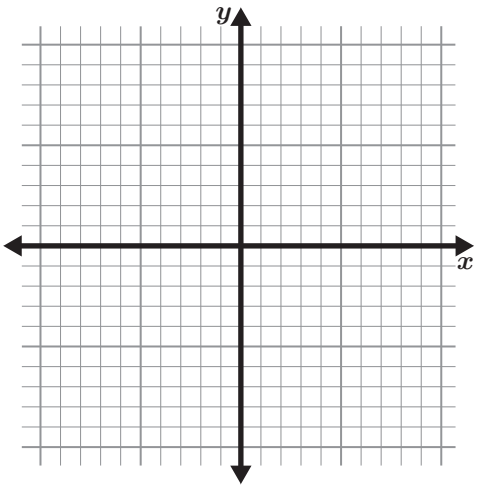
B. $y = -\frac{3}{2}x + 6$

C. $y = \frac{2}{3}x + 9$

D. $y = \frac{3}{2}x + 9$

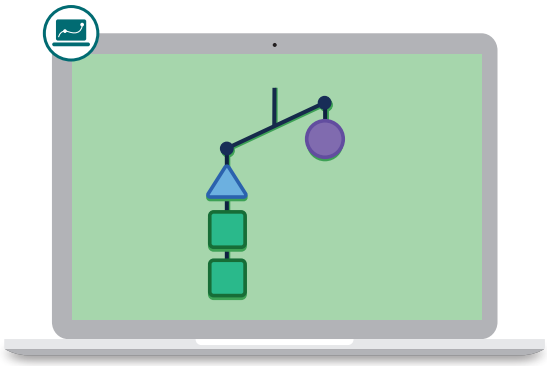
8. Figure $QRST$ is graphed on a coordinate plane. Determine the coordinates of the image $Q'R'S'T'$ after the figure is reflected across the y-axis. Use the coordinate plane if it helps with your thinking.

Pre-Image Coordinates	Image Coordinates
Q (3, -15)	
R (-6, 12)	
S (0, 9)	
T (-4, -3)	



Name: ..... Date: ..... Period: .....

Linear Equations | Building Toward 8.EE.7, 8.EE.7.b, SMP.2, SMP.3



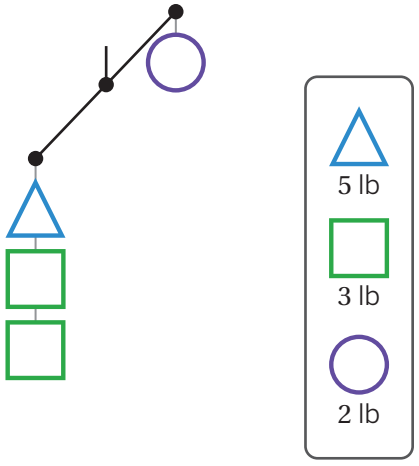
Keep It Balanced

Let's balance hanger diagrams.

Warm-Up

1 Here is a hanger diagram.

What shapes could be added to each side of the hanger so that it balances? Try to make the sides different!

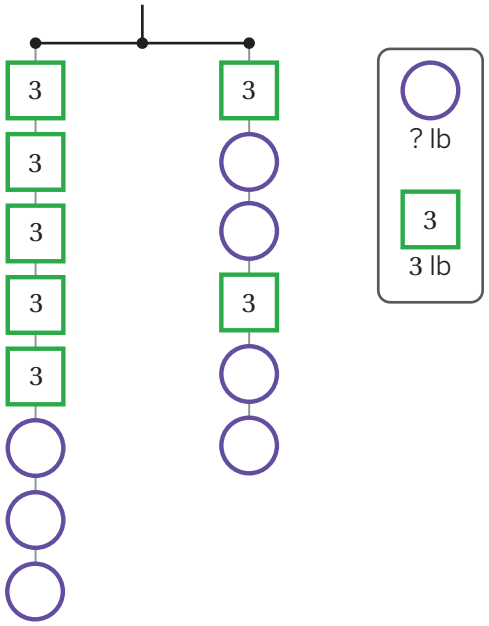


Activity 1

Name: ..... Date: ..... Period: .....

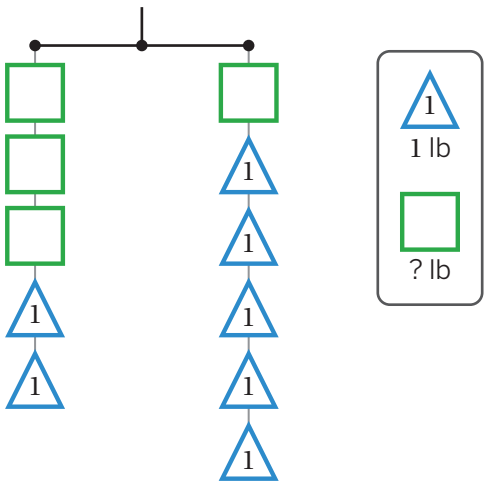
Solve It

2 **a** Determine the weight of the circle.



b Describe your strategy for finding the weight of the circle.

3 **a** Determine the weight of the square.



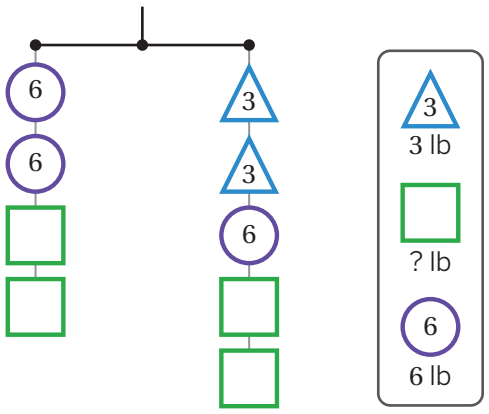
b Describe your strategy for finding the weight of the square.

4 Anand thinks the square weighs 5 pounds.
Darius thinks the square weighs 2 pounds.

Whose claim is correct? Circle one.

Aanand's Darius's Both Neither

Explain your thinking.



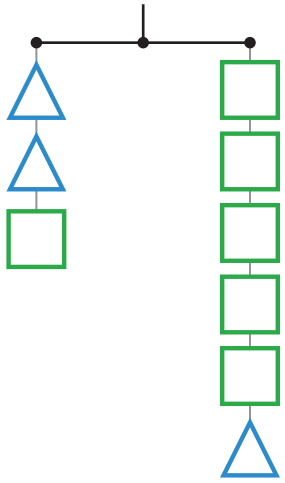
Name: ..... Date: ..... Period: .....

Solve It (continued)

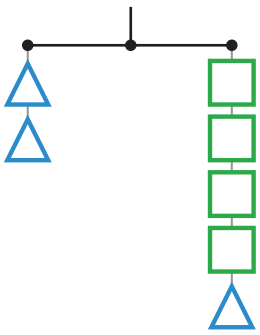
5 We can add or remove blocks from a hanger and keep the hanger balanced.

If Hanger A is balanced, which of these hangers are also balanced? Note: They are made with the same squares and triangles as Hanger A. Select *all* that apply.

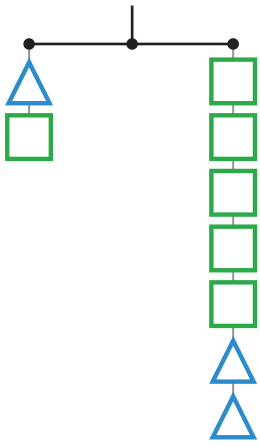
Hanger A



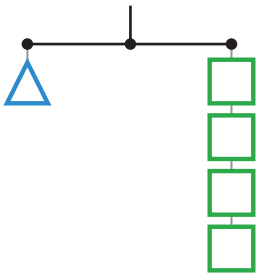
☐ Hanger B



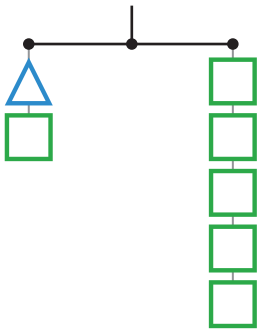
☐ Hanger C



☐ Hanger D



☐ Hanger E



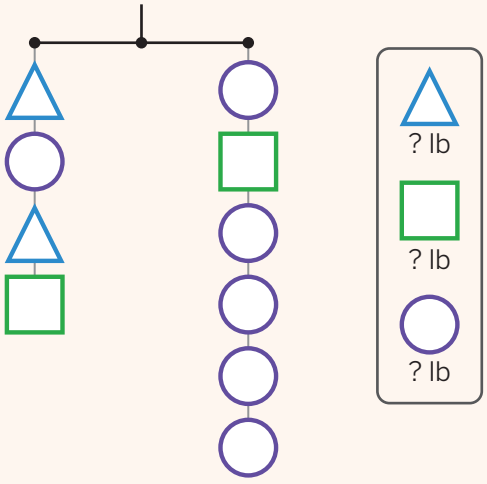
You're invited to explore more.

6 Determine the weights for all of the objects to make the hanger balance.

Triangle:

Square:

Circle:



Activity 2

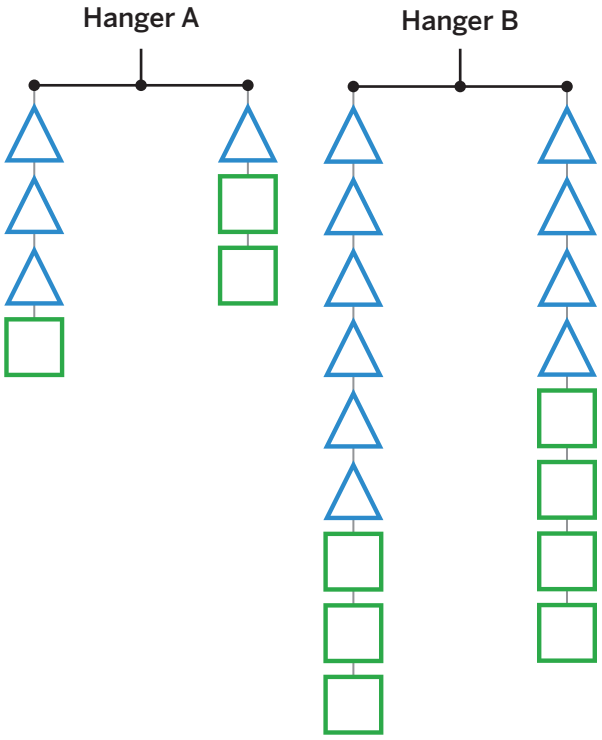
Name: ..... Date: ..... Period: .....

Hanger Equations

7 Hanger A balances. Will Hanger B also balance? Circle one.

Yes No

Explain your thinking.



8 Let's look at the two hangers and record some more information about them.

- x represents the weight of each triangle (in pounds).
- 5 is the weight of each square (in pounds).
- Hanger A can be represented by the equation $3x + 5 = x + 10$.

Write an equation to represent Hanger B.

Name: ..... Date: ..... Period: .....

Hanger Equations (continued)

9 **a** Determine weight of each triangle in Hanger A.

b Determine the weight of each triangle in Hanger B.

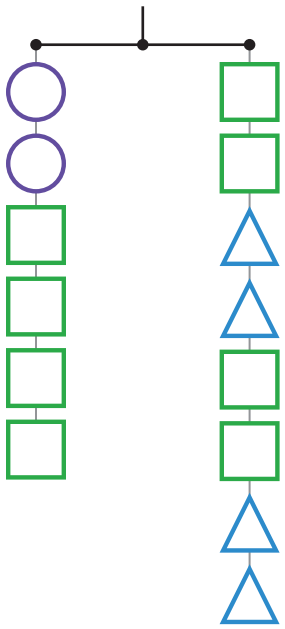
c What do you notice?

10 The equations that represent Hanger A and Hanger B are called **equivalent equations**. What do you think that means?

11 Synthesis

Discuss: Given a balanced hanger, how could you make another balanced hanger using shapes with the same weight?

Use the example if it helps with your thinking.



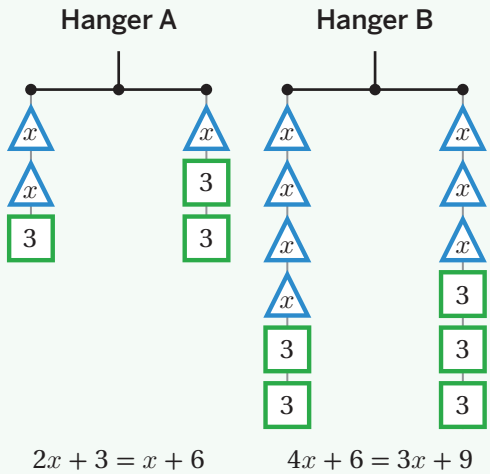
14 Summary 4.02

A hanger diagram is balanced when its left and right sides are equal in weight. Adding or removing equally weighted items from each side of a hanger diagram keeps the hanger balanced. Balanced hanger diagrams can represent equations.

For example, Hangers A and B represent **equivalent equations**.

These equations are equivalent because if you add $2x$ and 3 to each side of Hanger A, you create the also balanced Hanger B.

The solution to each equation is $x = 3$, which is the weight of each triangle on both hangers.

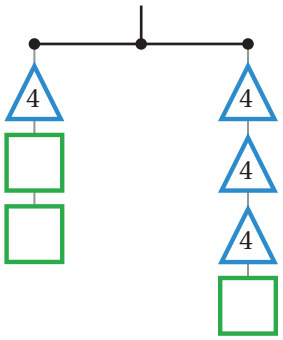


equivalent equations Equations that have the exact same solution(s).

Practice
4.02

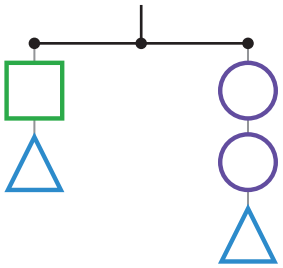
Name: ..... Date: ..... Period: .....

1. Here is a balanced hanger. What is the weight of one square if a triangle weighs 4 grams?



Show or explain your thinking.

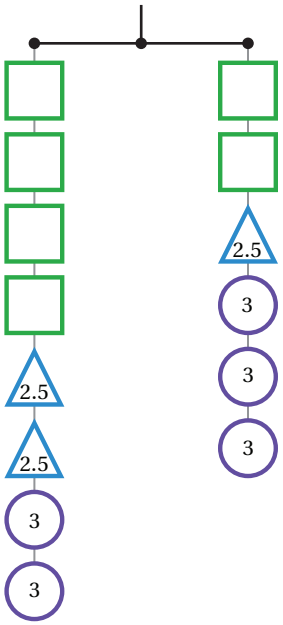
2. Which of these moves would keep the hanger in balance? Select *all* that apply.



- ☐ **A.** Add 2 circles to the left side and 1 square to the right side.
- ☐ **B.** Add 2 triangles to each side.
- ☐ **C.** Add 1 triangle to the left side and 1 square to the right side.
- ☐ **D.** Remove 2 circles from the right side and 1 square from the left side.
- ☐ **E.** Add 1 circle to the left side and 1 square to the right side.

3. Create a balanced hanger using triangles that weigh 2 pounds and squares that weigh 3 pounds.

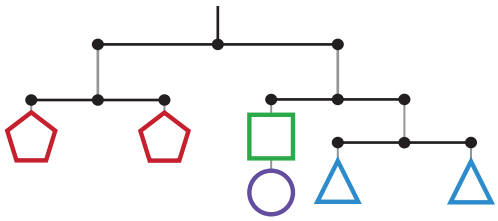
4. Here is a balanced hanger. Each triangle weighs 2.5 pounds, each circle weighs 3 pounds, and x represents the weight of each square. Select *all* the equations that are equivalent to this hanger.



- ☐ **A.** $x + x + x + x + 11 = x + 11.5$
- ☐ **B.** $2x = 0.5$
- ☐ **C.** $4x + 5 + 6 = 2x + 2.5 + 6$
- ☐ **D.** $2x + 2.5 = 3$
- ☐ **E.** $4x + 2.5 + 2.5 + 3 + 3 = 2x + 2.5 + 3 + 3 + 3$

Name: ..... Date: ..... Period: .....

5. Here is a balanced hanger. Each shape weighs 1, 2, 3, or 4 pounds. Determine the weight of the triangle.



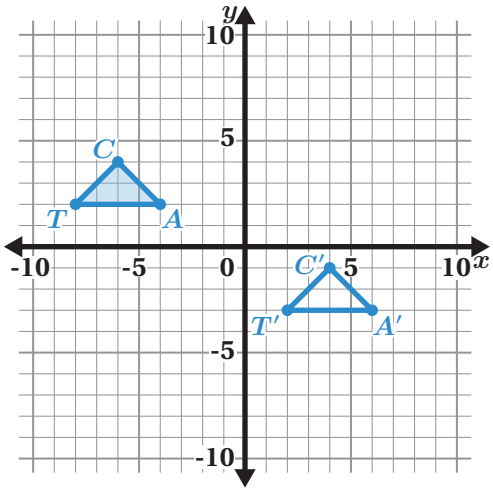
Spiral Review

Problems 6–10: Determine whether each statement is true or false.

6. Triangle $C'A'T'$ is congruent to triangle CAT because triangle CAT has been translated 10 units to the right and 5 units down.

7. Triangle $C'A'T'$ is congruent to triangle CAT because triangle CAT has been translated 10 units down and 5 units to the right.

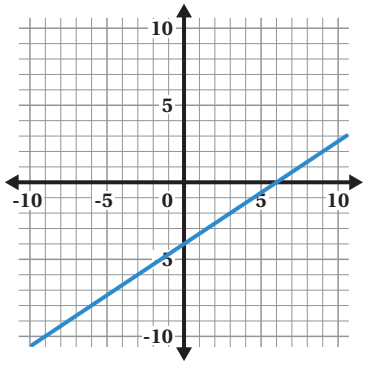
8. Triangle $C'A'T'$ is congruent to triangle CAT because triangle CAT has been reflected across the y-axis and translated 5 units to the right.



9. Triangle $C'A'T'$ is congruent to triangle CAT because triangle CAT has been translated 5 units down and 10 units to the right.

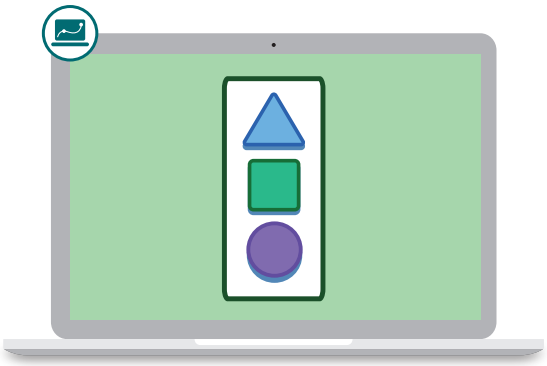
10. Triangle $C'A'T'$ is congruent to triangle CAT because triangle CAT has been rotated 180° around the origin.

11. Write the equation of this line.



Name: ..... Date: ..... Period: .....

Linear Equations 8.EE.7, 8.EE.7.b, SMP.2, SMP.3



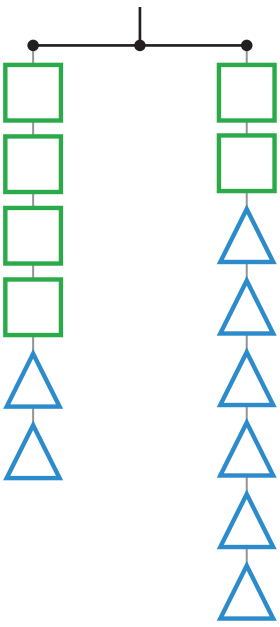
Balanced Moves

Let's rewrite equations while keeping the same solutions.

Warm-Up

1 Hanger A is balanced. Draw a different balanced hanger using the same shapes.

Hanger A



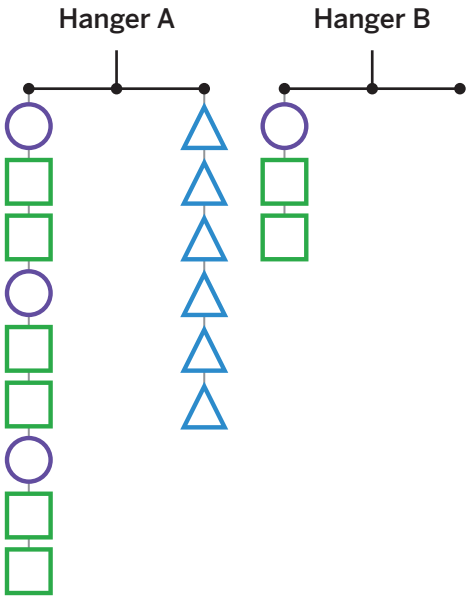


Name: ..... Date: ..... Period: .....

Multiplication and Division

2 We can balance hangers by adding or subtracting shapes from each side, or by multiplying or dividing.

If Hanger A is balanced, build the right side of Hanger B so that it also balances.

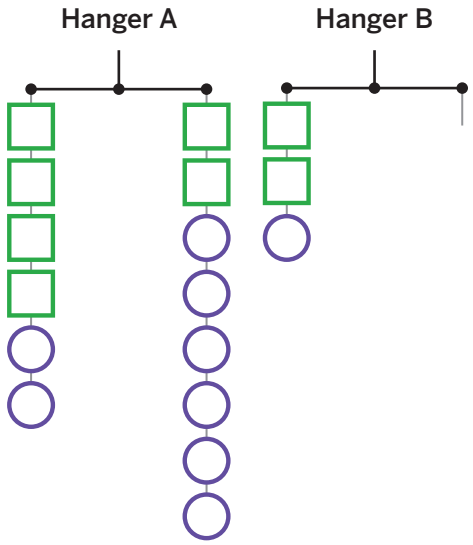


3 In this diagram:

- x represents the weight of each square.
- y represents the weight of each circle.

Ethan divided each side of Hanger A by 2 to make Hanger B.

Write an *expression* to represent the right side of Hanger B.



$4x + 2y = 2x + 6y$

$2x + y =$

Name: Date: Period:

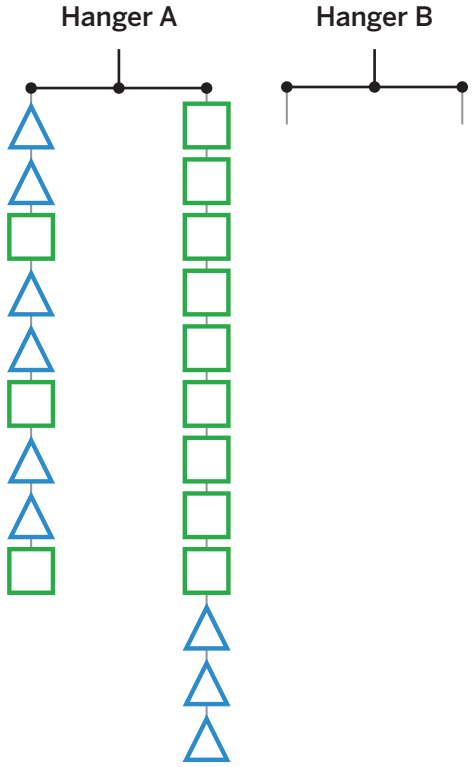
Multiplication and Division (continued)

4 In this diagram:

- x represents the weight of each triangle.
- y represents the weight of each square.

Jamir used balanced moves to change the number of shapes on each side of Hanger A to make Hanger B.

Write an equation that could represent a balanced Hanger B.



$3(2x + y) = 9y + 3x$

Activity 2

Name: ..... Date: ..... Period: .....

Equivalent Equations

5 In this diagram:

- x represents the weight of each triangle.
- 2 is the weight of each circle.

Select an equation that could represent a balanced hanger.

A. $12 = 2x + 4$

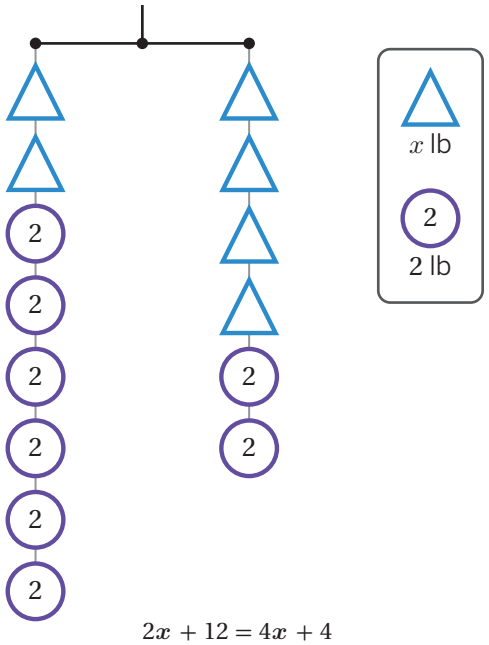
B. $x + 6 = 2x + 2$

C. $8 = 2x$

D. $2x + 8 = 4x$

E. $4x + 24 = 8x + 8$

Show or explain your thinking.



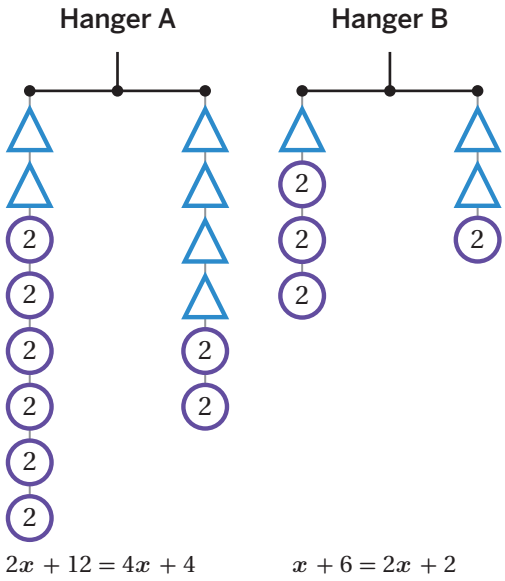
$2x + 12 = 4x + 4$

6 Here are two balanced hangers

a Determine the weight of the triangle in each hanger.

Weight of Circle (lb)	Weight of Triangle in Hanger A (lb)	Weight of Triangle in Hanger B (lb)
2		

b What does this tell you about the equations?



$2x + 12 = 4x + 4$ $x + 6 = 2x + 2$

Activity 3

Name: Date: Period:

Solving Equations

7 Here's the first step Dalia took to solve the equation $6x + 12 = 10x - 4$.

What did she do to both sides to create a simpler equivalent equation?

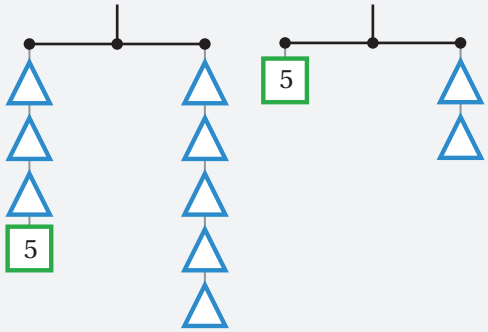
Dalia

$6x + 12 = 10x - 4$

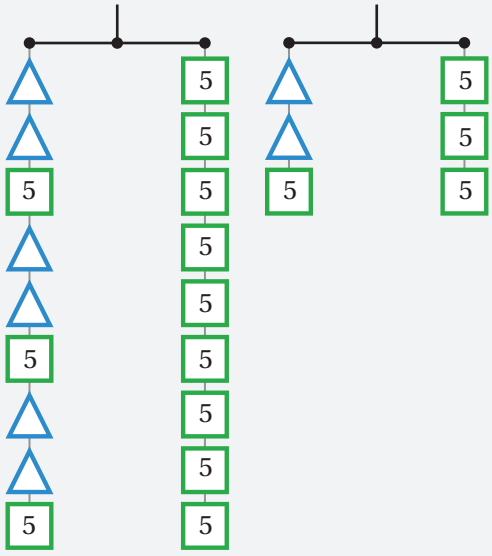
$12 = 4x - 4$

8 Match each hanger set or equation set with the balanced equation move that describes it.

Hanger Set A



Hanger Set B



Equation Set C

$\frac{5x}{-3} = \frac{12}{1}$

$5x = -36$

Equation Set D

$15 - 7x = 3 + 5x$

$12 - 7x = 5x$

Equation Set E

$3x + 5 = 5x$

$5 = 2x$

Equation Set F

$6x + 15 = 45$

$2x + 5 = 15$

Multiply each Side by -3	Divide each side by 3	Subtract 3 from each side	Subtract $3x$ from each side

Name: Date: Period:

Solving Equations (continued)

9 Jaylin solved this equation from the card sort: $15 - 7x = 3 + 5x$.

Is his solution correct? Circle one.

Yes No I'm not sure

Explain your thinking.

Jaylin

Equation Set D

$15 - 7x = 3 + 5x$

$12 - 7x = 5x$

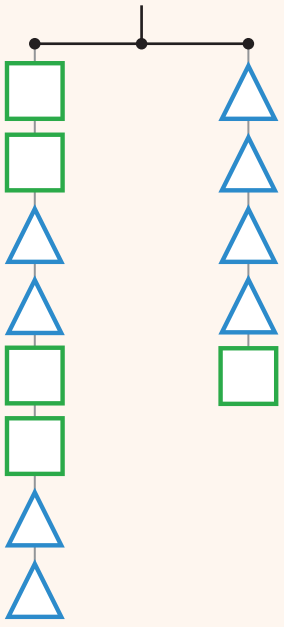
$12 = 12x$

$1 = x$

You're invited to explore more.

10 What is the weight (in pounds) of a square in this balanced hanger?

Explain your thinking.



11 Synthesis

How can balanced equation moves be helpful when solving equations?

Use the example if it helps with your thinking.

Jaylin

Equation Set D

$15 - 7x = 3 + 5x$

$12 - 7x = 5x$

$12 = 12x$

$1 = x$

14 Summary 4.03

You can use hanger diagrams to visualize and represent equations. Hanger diagrams can help you see how moves that keep the hanger balanced create *equivalent equations*, such as:

- Adding or subtracting the same term on each side of an equation.
- Multiplying or dividing the expressions on each side of an equation by the same number.

You can use these moves to solve equations for an unknown variable. Here is an example of using balanced moves to solve an equation:

Solving Step	Balanced Equation Move
$4x + 9 = -2x - 3$ $-9 \qquad\quad -9$	Subtract 9 from both sides.
$4x = -2x - 12$ $+2x \quad +2x$	Add $2x$ to both sides.
$\frac{6x}{6} = \frac{-12}{6}$ $x = -2$	Divide by 6 on both sides.

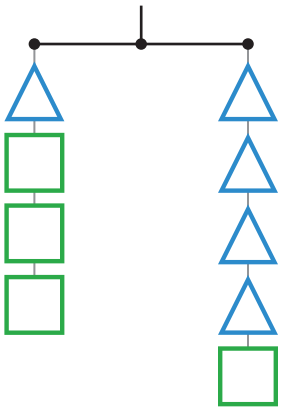
You can check if a value is a *solution* to an equation by substituting it into the original equation. If it makes the equation true then the solution is correct.

$$4(-2) + 9 = -2(-2) - 3$$
$$-8 + 9 = 4 - 3$$
$$1 = 1$$

Practice
4.03

Name: ..... Date: ..... Period: .....

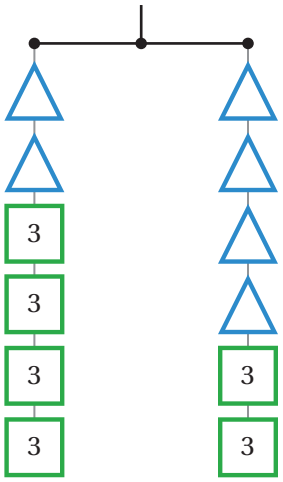
Problems 1–2: In this balanced hanger diagram, x represents the weight of the triangle and y represents the weight of the square.



1. Write an equation using x and y to represent the hanger diagram.

2. If $x = 6$, what is the value of y? Show or explain your thinking.

3. In the diagram shown, x represents the weight of each triangle, and 3 represents the weight of each square.



Select *all* the equations that could represent a balanced hanger.

- ☐ **A.** $6x = 6$
- ☐ **B.** $x + 6 = 2x + 3$
- ☐ **C.** $6 = 2x$
- ☐ **D.** $6 + 12x = 12 + 6x$
- ☐ **E.** $2x + 12 = 4x + 6$

4. Match each set of equations with a possible step that turns the first equation into the second equation.

Equations	Possible Steps
a $6x + 9 = 4x - 3$ $2x + 9 = -3$	..... Divide each side by -4.
b $-4(5x - 7) = -18$ $5x - 7 = 4.5$	..... Multiply each side by -4.
c $8 - 10x = 7 + 5x$ $4 - 10x = 3 + 5x$	..... Divide each side by 4.
d $-\frac{5}{4}x = 4$ $5x = -16$	..... Subtract $4x$ from each side.
e $12x + 4 = 20x + 24$ $3x + 1 = 5x + 6$	..... Subtract 4 from each side.

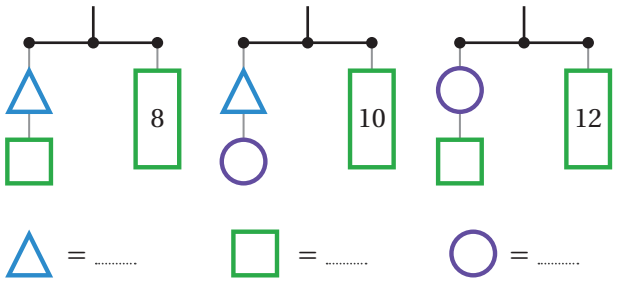
Name: ..... Date: ..... Period: .....

Problems 5–6: DeAndre and Diego are each trying to solve $2x + 6 = 3x - 8$.

5. The result of DeAndre's first step is $-x + 6 = -8$. Describe DeAndre's first step.

6. The result of Diego's first step is $6 = x - 8$. Describe Diego's first step.

7. Here are some balanced hanger diagrams. Determine the weight of each shape. Explain your thinking.



Spiral Review

Problems 8–10: Determine whether each point lies on the graph of the linear equation $4x - y = 3$. Write *yes* or *no*.

8. $(0, 3)$

9. $\left(\frac{3}{4}, 0\right)$

10. (-1, -7)

11. Triangle G is transformed, resulting in triangle H. Which sequence of transformations could be used to show that triangle H is similar but not congruent to triangle G?

A. A translation followed by a dilation

B. A rotation followed by a reflection

C. A reflection followed by a translation

D. A translation followed by a rotation

Name: ..... Date: ..... Period: .....

Linear Equations 8.EE.7, 8.EE.7.b, SMP.3, SMP.6, SMP.7

More Balanced Moves



Let's solve some equations.

Warm-Up

1. Here are several equations.

- Equation A: $12x + 3 = 3(5x + 9)$
- Equation B: $2x + 5x = x + 3$
- Equation C: $-3x + 12 = 9x - 4$
- Equation D: $x - 4 = \frac{1}{3}(6x - 54)$

a Draw a hanger diagram to represent one of the equations.



b **Discuss:** Why might it be useful to represent an equation with a hanger diagram? What limitations might this representation have?

Activity 1

Name: Date: Period:

Step by Step by Step by Step

Sadia and Amir started solving the same equation. Here is their work.

Sadia

$12x + 3 = 3(5x + 9)$

$4x + 1 = 5x + 9$

Amir

$12x + 3 = 3(5x + 9)$

$12x + 3 = 15x + 27$

2. In what ways are the steps they took alike and different?

Caleb and Roberto also solved the equation $12x + 3 = 3(5x + 9)$. Some of their work is correct and some of their work is incorrect.

Caleb

$12x + 3 = 3(5x + 9)$

$7x + 3 = 3(9)$

$7x + 3 = 27$

$7x = 24$

$x = \frac{24}{7}$

Roberto

$12x + 3 = 3(5x + 9)$

$12x + 3 = 15x + 27$

$27x + 3 = 27$

$27x = 24$

$x = \frac{24}{27}$

3. What are some moves they made that kept the equation balanced?

4. In each student's work, circle and explain the mistake you think they made.

Caleb's mistake:

Roberto's mistake:

Activity 2

Name: Date: Period:

Make Your Own Steps

5. Solve each equation for x. Show your thinking.

a $2x + 5x = x + 3$

b $-3x + 12 = 9x - 4$

c $-4(x - 3) = 12x - 4$

d $8x + 7 = 6x - 13$

e $x - 4 = \frac{1}{3}(6x - 54)$

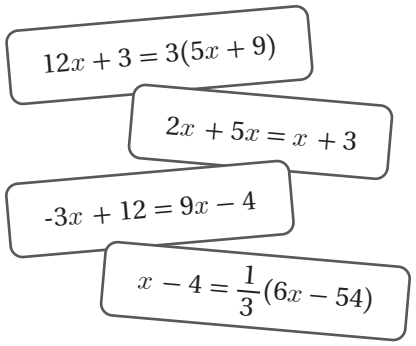
f $3(x - 2) + 2x = 25$

You're invited to explore more.

6. There are 24 pencils and 3 cups. The second cup holds one more pencil than the first cup. The third cup holds one more pencil than the second cup. How many pencils does each cup contain? Show or explain your thinking.

Synthesis

7. **Discuss:** What are some helpful moves when solving equations?



Summary 4.04

There are many ways to solve an equation using balanced equation moves. Generally, you want to perform steps that will move you closer to an equivalent equation where the variable is isolated, such as an equation of the form $\text{variable} = \text{number}$. Here are some examples of moves that can be helpful in solving an equation.

Applying the *distributive property*

$$2x + 4 = 3(2x + 4)$$
$$2x + 4 = 6x + 12$$

Collecting *like* terms on one side of an equation

$$-10x + 3x = 15$$
$$-7x = 15$$

Adding or subtracting the same term from both sides

$$-x - 8 = 4x + 7$$
$$-8 = 5x + 7$$

Multiplying or dividing both sides of the equation by the same value

$$-15x = 5(2x + 4)$$
$$-3x = 2x + 4$$

Practice
4.04

Name: ..... Date: ..... Period: .....

1. Here is Anushka's and Lukas's work solving the equation $\frac{2}{5}b + 1 = -11$.

Anushka

$\frac{2}{5}b + 1 = -11$

$\frac{2}{5}b = -10$

$b = -10 \cdot \frac{5}{2}$

$b = -25$

Lukas

$\frac{2}{5}b + 1 = -11$

$2b + 1 = -55$

$2b = -56$

$b = -28$

Whose solution is correct? Circle one.

Anushka's Lukas's Both Neither

Explain your thinking.

2. Solve the equation $3(x - 4) = 12x$. Show your thinking. Remember to check your solution.

3. Here is how Liam solved an equation. Circle his mistake and explain what he did wrong.

Liam

$-2(3x - 5) = 4(x + 3) + 8$

$-6x + 10 = 4x + 12 + 8$

$-6x + 10 = 4x + 20$

$10 = -2x + 20$

$-10 = -2x$

$5 = x$

What is the correct solution?

Name: ..... Date: ..... Period: .....

4. Elena solved the equation $2(-3x + 4) = 5x + 2$. Describe what she did in each step.

Elena

$2(-3x + 4) = 5x + 2$

$-6x + 8 = 5x + 2$

$8 = 11x + 2$

$6 = 11x$

$\frac{6}{11} = x$

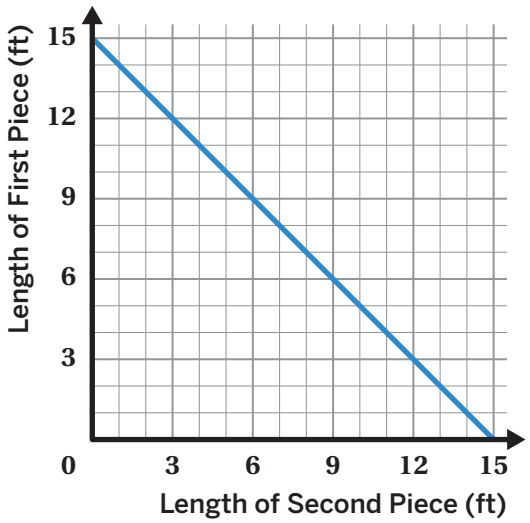
Problems 5–6: Determine whether $x = -3$ is a solution for each equation. Show or explain your thinking.

5. $-2(x + 2) = -10$

6. $8(x - 1) = 18x + 22$

Spiral Review

Problems 7–10: A length of ribbon is cut into two pieces to use in a craft project. The graph shows the length x of the second piece for each length y of the first piece.



7. How long is the ribbon? Explain your thinking.

8. What is the slope of the line?

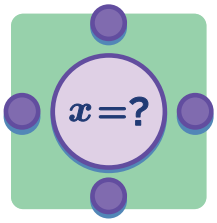
9. Explain what the slope of the line represents in this situation.

10. What does the point $(15, 0)$ represent in this situation?

Name: ..... Date: ..... Period: .....

Linear Equations 8.EE.7, 8.EE.7.b, SMP.3, SMP.6

Equation Roundtable



Let's analyze balanced and unbalanced equation moves.

Warm-Up

Solve each equation mentally. Be prepared to share your strategy.

1. $5 - x = 8$

2. $-1 = x - 2$

3. $-3x = 9$

4. $-10 = -5x$

Name: Date: Period:

Find and Fix

5. Jordan solved three equations. Check the work for each equation.

- If Jordan's work keeps the equation balanced, solve the equation a different way.
- If Jordan's work creates an unbalanced equation, explain the mistake. Then solve the equation correctly.

Equation 1:

$\frac{3}{4}x + \frac{1}{2} = \frac{1}{4}x - 3$

$\frac{3}{4}x + \frac{1}{2} = \frac{1}{4}x + -3$

$3x + 2 = 1x + -3$

$2x + 2 = -3$

$2x = -5$

$x = -\frac{5}{2}$

Equation 2:

$1.1(x - 3) = 0.1(2x - 6)$

$11(x - 3) = 1(2x - 6)$

$11x - 33 = 2x - 6$

$9x - 33 = -6$

$9x = 27$

$x = 3$

Equation 3:

$4 - 2(3x - 2) = 14 - x$

$2(3x - 2) = 14 - x$

$6x - 4 = 14 - x$

$6x - 4 = 14$

$6x = 18$

$x = \frac{18}{6}$

$x = 3$

Name: Date: Period:

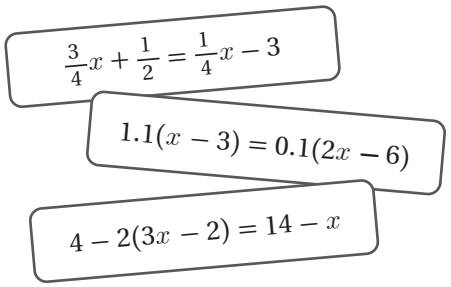
Equation Roundtable

6. Your group will get a set of cards with equations on them. Each person will choose a card to start with and then you will collaboratively solve the equation on that card. Use this space to check the solution to your card's equation.

7. **Discuss:** What are some strategies your group used to check the solutions?

Synthesis

8. **Discuss:** How might an equation become unbalanced?



Summary 4.05

You can use different steps to solve the same equation. For example, here are two ways to solve the equation $\frac{1}{3}(3x + 9) = 6x + 18$:

Start by distributing $\frac{1}{3}$ to $(3x + 9)$

$$\begin{aligned} \tfrac{1}{3}(3x + 9) &= 6x + 18 \\ x + 3 &= 6x + 18 \\ -5x + 3 &= 18 \\ -5x &= 15 \\ x &= -3 \end{aligned}$$

Start by multiplying both sides of the equation by 3

$$\begin{aligned} \tfrac{1}{3}(3x + 9) &= 6x + 18 \\ 3x + 9 &= 18x + 54 \\ 3x - 45 &= 18x \\ -45 &= 15x \\ -3 &= x \end{aligned}$$

Practice
4.05

Name: ..... Date: ..... Period: .....

Problems 1–3: Solve each equation. Show your thinking.

1. $2(x + 5) = 3x + 1$

2. $3y - 4 = 6 - 2y$

3. $3(n + 2) = 9(6 - n)$

4. Here is how Gabriela solved an equation. Circle Gabriela's mistake(s) and explain what she did wrong.

Gabriela

$12(5 + 2y) = 4y - (5 - 9y)$

$72 + 24y = 4y - 5 - 9y$

$72 + 24y = -5y - 5$

$24y = -5y - 77$

$29y = -77$

$y = -\frac{77}{29}$

What is the correct solution?

Problems 5–6: Solve each equation. Show your thinking.

5. $\frac{1}{9}(2m - 16) = \frac{1}{3}(2m + 4)$

6. $1.5(5 + 0.2y) = 0.4y - (0.6 - 0.9y)$

Name: _____ Date: _____ Period: _____

7. What is the solution to the equation $4.1(x + 2) = 8.1x - 1.8$?

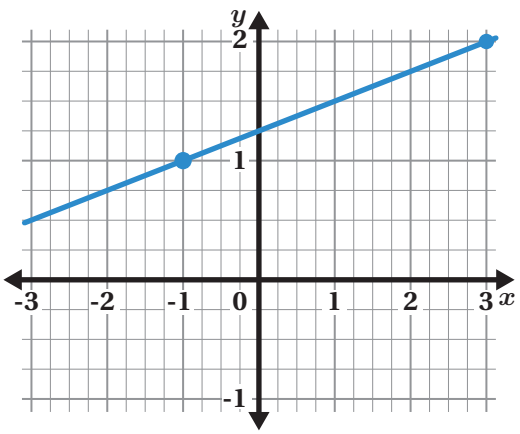
A. $x = 1.6$ **B.** $x = 2.5$ **C.** $x = 0.4$ **D.** $x = -0.3$

8. Complete the equation so that the solution is $n = 8$.

$-4\left(\frac{2}{5}n - 6.1\right) =$ _____

Spiral Review

9. Here is the graph of a linear equation. Select *all* the true statements about this line and its equation.



- ☐ **A.** One solution to the equation is the ordered pair $(3, 2)$.
- ☐ **B.** One solution to the equation is the ordered pair $(-1, 1)$.
- ☐ **C.** One solution to the equation is the ordered pair $\left(\frac{3}{2}, 1\right)$.
- ☐ **D.** There are only two solutions.
- ☐ **E.** The equation of the line is $y = \frac{1}{4}x + \frac{5}{4}$.

Problems 10–13: Evaluate each expression.

10. $8 + 5^2$

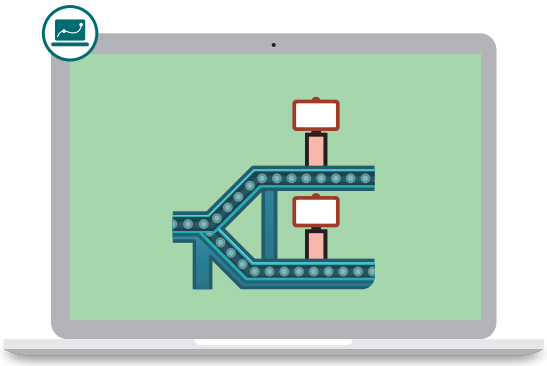
11. $(1 + 9)^3$

12. $3 \bullet 2^4$

13. $30 - 3^3$

Name: ..... Date: ..... Period: .....

Linear Equations 8.EE.7.a, SMP.3, SMP.7

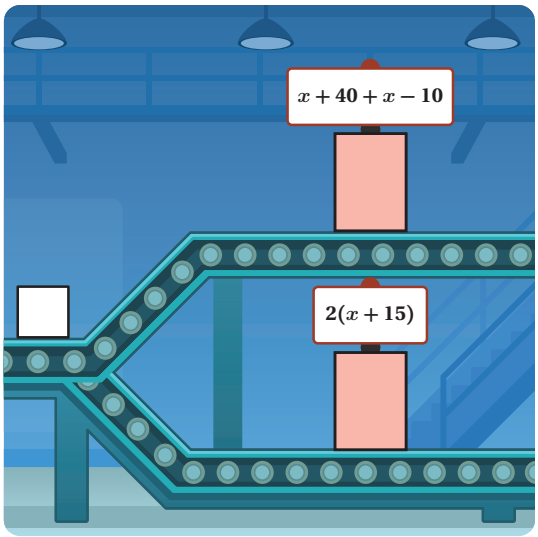


All, Some, or None? Part 1

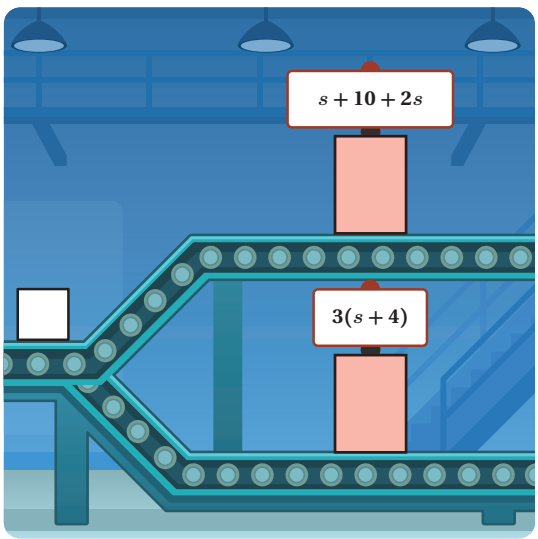
Let's think about how many solutions an equation can have.

Warm-Up

1 Here are two number machines. Tasia put a number into both machines, and the outputs were the same. What was Tasia's input?



2 Here are two new number machines. Try to find a number to put into both machines to get the same outputs.

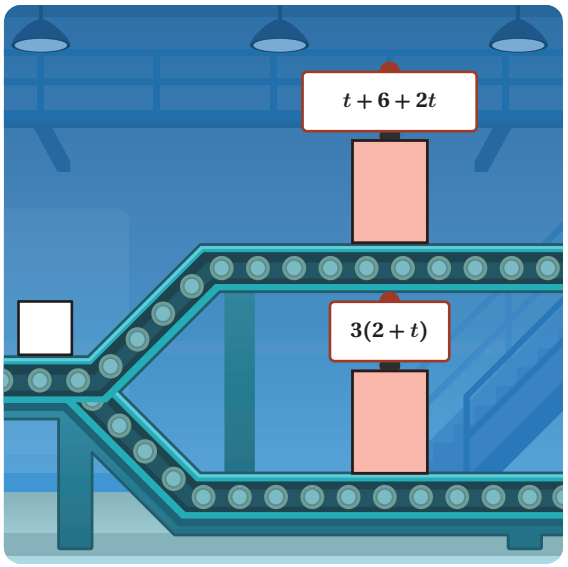


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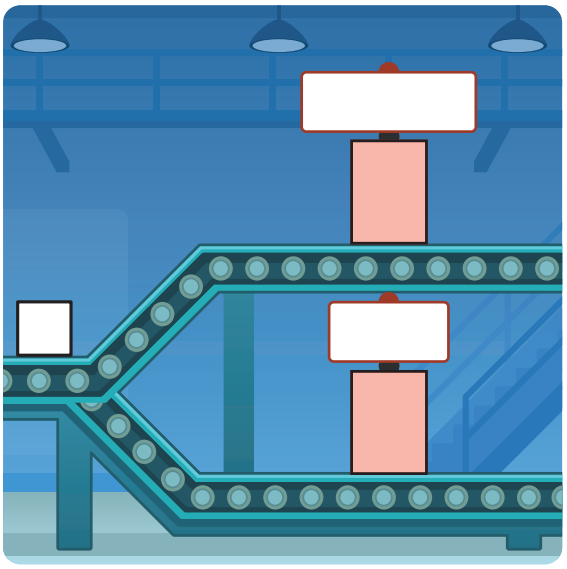
Always-Equal and Never-Equal

3 Let's look at two number machines that *always* give the same outputs for any input.

Use the two expressions to explain why both machines will give the same outputs for any input.



4 Write two expressions to create two new number machines that will give the same outputs for any input.

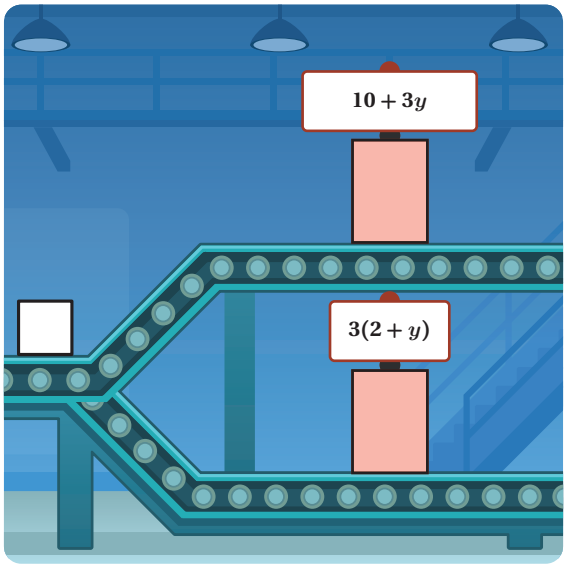


Name: ..... Date: ..... Period: .....

Always-Equal and Never-Equal (continued)

5 Let's look at two number machines that *never* give the same outputs for any input.

Use the two expressions to explain why both machines will *never* give the same outputs for any input.



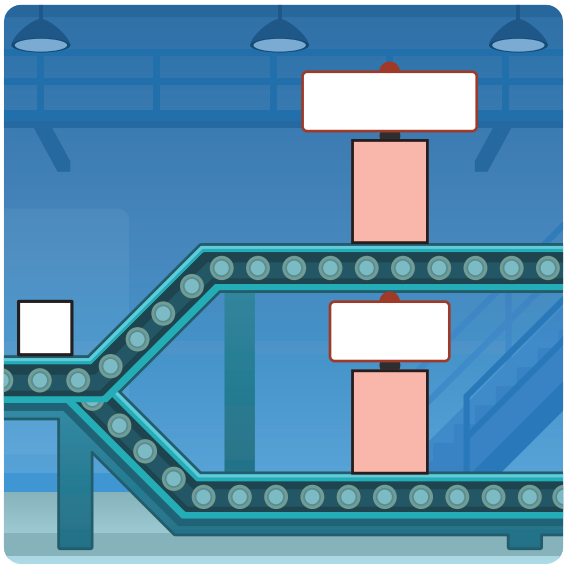
6 Which set of expressions will create number machines that will never give the same outputs for any input?

A. $2x + 3$ and $3 + 2x$

B. $2x + 3$ and $5 + 2x$

C. $2x + 3$ and $2x + 3$

D. $2x + 2$ and $3 + 3x$

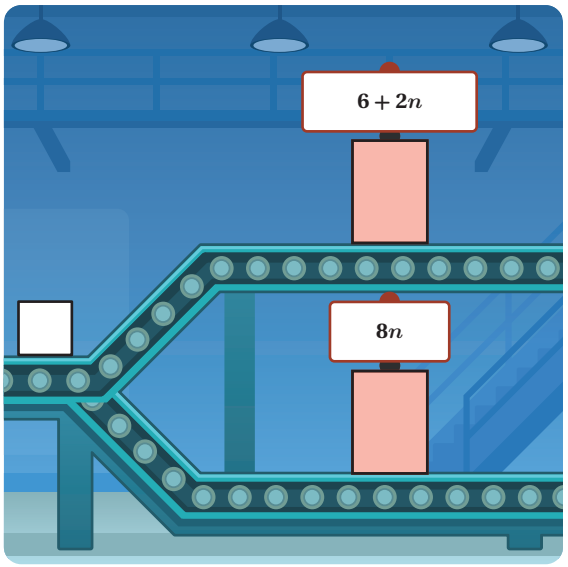


Activity 2

Name: ..... Date: ..... Period: .....

Number of Solutions

7 Here are two new number machines.



a Write an equation to find an input number that will produce the same outputs for each machine.

b For how many values of n will these machines produce the same output?

Circle one.

All values of n | One value of n | No values of n

8 Group the equations based on their number of solutions.

$v + 2 = v - 2$	$2n = n$	$7 - r = r - 7$	$\frac{1}{2} + x = \frac{1}{3} + x$
$y(-6) \cdot (-3) = 2 \cdot y \cdot 9$	$2t + 6 = 2(t + 3)$	$2n = 2n$	$3(n + 1) = 3n + 1$

No Solution (True for No Values)	One Solution (True for One Value)	Infinitely Many Solutions (True for All Values)

Name: Date: Period:

Number of Solutions (continued)

9 Polina solved an equation from each column. Here is her work.

No Solution (True for No Values)	One Solution (True for One Value)	Infinitely Many Solutions (True for All Values)
$v + 2 = v - 2$ $2 = -2$	$7 - r = r - 7$ $14 - r = r$ $14 = 2r$ $7 = r$	$2t + 6 = 2(t + 3)$ $2t + 6 = 2t + 6$ $6 = 6$

Discuss: What do you notice about the final statement for each equation?

10 Kiandra looked at this equation and, without writing anything, said it must have no solution. What might she have noticed to lead her to this conclusion?

No Solution (True for No Values)
$\frac{1}{2} + x = \frac{1}{3} + x$

11 Write an equation for each number of solutions.

No solution:

One solution:

Infinitely many solutions:

12 Synthesis

How can you determine whether an equation has no solution, one solution, or infinitely many solutions?

15 Summary 4.06

Equations can have one solution, no solution, or **infinitely many solutions**.

Here are some examples.

One Solution	No Solution	Infinitely Many Solutions
$3x + 8 = 6 + 2 - 3x$	$3(x + 4) = 3x + 7$	$10 - 3x = 8 - 3x + 2$
This equation is only true when $x = 0$.	This equation is never true for any value of x.	This equation is always true for any value of x.
A linear equation has *one solution* when the expressions on either side of the equation have one value for the variable that makes them equal.	A linear equation has *no solution* when the expressions on either side of the equation have no value for the variable that make them equal.	A linear equation has *infinitely many solutions* when the expressions on either side of the equation are *equivalent*: always equal no matter the value of the variable.

infinitely many solutions An equation has infinitely many solutions if it is true for any value of the variable.

Practice
4.06

Name: _____ Date: _____ Period: _____

Problems 1–4: Decide whether the given equation has *one solution*, *no solution*, or *infinitely many solutions*. Explain your thinking.

1. $x - 13 = x + 1$

2. $x + \frac{1}{2} = x - \frac{1}{2}$

3. $2(x + 3) = 5x + 6 - 3x$

4. $-(7 - 5x) = 6x - 3$

5. Ivory says that the equation $2x + 2 = x + 1$ has no solution because the left-hand side is double the right-hand side. Is Ivory's claim correct? Explain your thinking.

6. Write the other side of this equation so that it is true for all values of x.

$\frac{1}{2}(6x - 10) - x =$ _____

7. Write the other side of this equation so that it is true for no values of x.

$\frac{1}{2}(6x - 10) - x =$ _____

8. The equation $5\left(0.2x + \frac{1}{25}\right) = x + \frac{1}{5}$ has infinitely many solutions. Anya creates another equation, $10\left(0.2x + \frac{1}{25}\right) = 2x + \frac{2}{5}$, and says it also has infinitely many solutions. Why might she think that?

Name: ..... Date: ..... Period: .....

9. Here is the work that Elena did to solve the equation $3x + 10 = 5x - 4$.

Elena says that this equation has no solution because when she substituted 2, -6, and 0.5 for x, all of the equations were false.

$x = 2$	$x = -6$	$x = 0.5$
$3(2) + 10 = 5(2) - 4$	$3(-6) + 10 = 5(-6) - 4$	$3(0.5) + 10 = 5(0.5) - 4$
$6 + 10 = 10 - 4$	$-18 + 10 = -30 - 4$	$1.5 + 10 = 2.5 - 4$
$16 = 6$	$-8 = -34$	$11.5 = -1.5$
False	False	False

Do you agree that this equation has no solution? Explain your thinking.

Spiral Review

Problems 10–11: Solve each equation. Show your thinking.

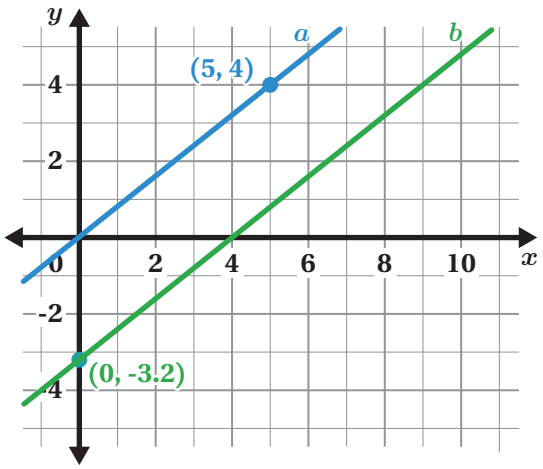
10. $-4(r + 2) = 4(2 - 2r)$

11. $1.3 + 6d = 2.7 - 8d$

12. These two lines are parallel. Write an equation for each line.

Line a: .....

Line b: .....



Name: ..... Date: ..... Period: .....

Linear Equations 8.EE.7, 8.EE.7.a, 8.EE.7.b, SMP.1, SMP.7

Strategic Solving, Part 1



Let's solve linear equations with no solution, one solution, and infinitely many solutions.

Warm-Up

1. Here are three equations:

- $13x = 3.25$
- $13x = 385x$
- $13x = 10x + 3x$

Choose one and write a situation that it could represent.

Name: Date: Period:

Predicting Solutions

2. Predict whether each equation has no solution, one solution, or infinitely many solutions. For equations with one solution, predict whether the solution will be *positive*, *negative*, or *zero*.

Equation	No Solution	One Solution			Infinitely Many Solutions
		x is Positive	x is Negative	$x = 0$	
$13x = 3.25$					
$13x = 385x$					
$13x = 10x + 3x$					
$13x + 42 = -584$					
$13x + 42 = 13x + -42$					

3. Choose one equation and explain how you made a prediction about its solution.

4. Why do you think it might be helpful to pause and try to predict the number of solutions or the sign of the solution before you start solving an equation?

Activity 2

Name: ..... Date: ..... Period: .....

What Happened?

5. Deven tried to solve the equation $13x + 42 = 13x + -42$, but thinks there is a mistake.

Do you agree? Circle one.

Yes No I'm not sure

Explain your thinking.

Deven

$13x + 42 = 13x + -42$

$42 = -42$

$0 = -84$

There is no solution because 0 is not equal to -84.

6. Write an equation with infinitely many solutions.

7. Why do you think this equation has infinitely many solutions?

Activity 3

Name: Date: Period:

The Choice Is Yours

Equation A

$2r + 49 = -8(-r - 5)$

Equation B

$\frac{n}{7} - 12 = 5n + 5$

Equation C

$\frac{4m - 16}{4} = \frac{-16 + 8m}{8}$

Equation D

$p - 5(p + 4) = p - (8 - p)$

Equation E

$3(c - 1) + 2(c - 1) = 5(c - 1)$

Equation F

$-\frac{1}{2}(t + 3) - 10 = -6.5$

Equation G

$\frac{10 - v}{4} = 2(v + 17)$

Equation H

$2(2q + 1.5) = 18 - q$

8. Examine these equations. Organize the equations into two or three groups based on the patterns you notice.

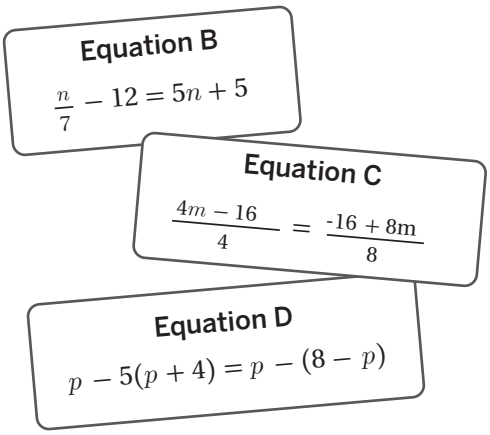
Group 1	Group 2	Group 3

9. **Discuss:** How did you group the equations?

10. Choose *three* equations to solve. (Choose at least one from each group.) Show your thinking.

Synthesis

11. **Discuss:** What are some strategies for solving equations like these?



Summary 4.07

Equations can have many different features, including fractions, decimals, negative values, grouping symbols, and multiple terms. Based on the features, it can be helpful to think about what steps might be most useful in solving the equation.

When solving an equation with one solution, the goal is to end up with the variable isolated on one side of the equation and its value on the other. But this doesn't happen when there is no solution or infinitely many solutions.

One Solution	No Solution	Infinitely Many Solutions
$3x + 8 = 6 + 2 - 3x$	$3(x + 4) = 3x + 7$	$10 - 3x = 8 - 3x + 2$
$3x + 8 = 8 - 3x$	$3x + 12 = 3x + 7$	$10 - 3x = 10 - 3x$
$6x + 8 = 8$	$12 = 7$	$10 = 10$
$6x = 0$		
$x = 0$		
This equation is only true when $x = 0$.	This equation is never true for any value of x.	This equation is always true for any value of x.

Practice
4.07

Name: ..... Date: ..... Period: .....

Problems 1–3: Identify whether each equation has a solution that is *positive*, *negative*, or *zero* without solving.

1. $7x = 3.25$

2. $-7x = 32.5$

3. $3x + 11 = 11$

Problems 4–7: Solve each equation. Show your thinking.

4. $2b + 8 - 5b + 3 = -13 + 8b - 5$

5. $2x + 7 - 5x + 8 = 3(5 + 6x) - 12x$

6. $3(3 - 3x) = 2(x + 3) - 30$

7. $\frac{1}{3}(z + 4) - 6 = \frac{2}{3}(5 - z)$

8. Kadeem is solving the linear equation $x - 3(2 - 3x) = 2(5x + 3)$. Here are his final two steps. Select the statement that correctly describes Kadeem's solution.

Kadeem
$-6 + 10x = 10x + 6$
$-6 = 6$

A. The solution is the ordered pair $(-6, 6)$.

B. The solution is $x = 0$.

C. There are infinitely many solutions since $-6 = 6$ is a false statement.

D. There is no solution because $-6 = 6$ is a false statement.

Name: ..... Date: ..... Period: .....

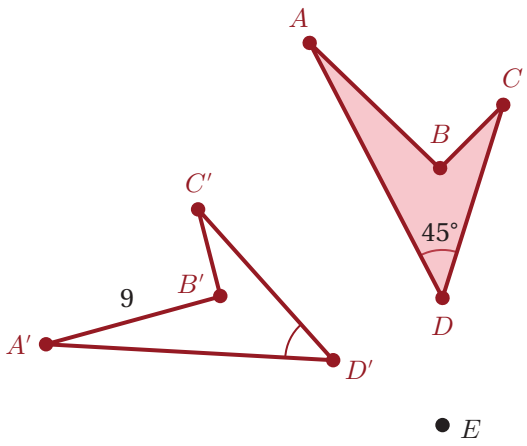
9. What is the solution to this equation?

$-\frac{1}{5}(5x - 15) + 18 = 6x$

A. $x = \frac{7}{20}$ **B.** $x = 2$ **C.** $x = \frac{20}{7}$ **D.** $x = 3$

Spiral Review

Problems 10–11: Figure $A'B'C'D'$ is the image of figure $ABCD$ after a rotation around point E.



10. What is the length of side AB?
Explain your thinking.

11. What is the measure of $\angle A'D'C'$?
Explain your thinking.

12. Select *all* of the ordered pairs (x, y) that are solutions to the linear equation $4x + 2y = 12$.

☐ **A.** $(3, 0)$ ☐ **B.** $(2, 4)$ ☐ **C.** $(0, 6)$

☐ **D.** $(2, 2)$ ☐ **E.** $(1, 4)$ ☐ **F.** $(2, 1)$

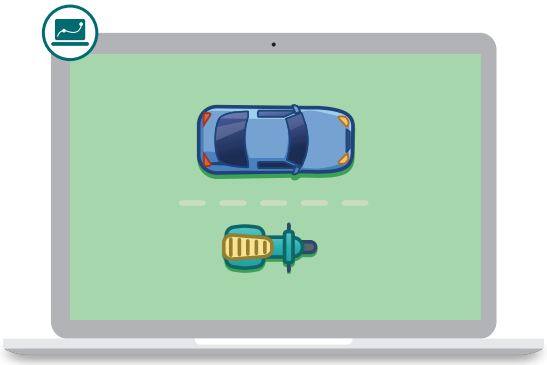
Unit 4
Lesson
8

Name: _____ Date: _____ Period: _____

Linear Equations | Multiple Representations of Functions | 8.EE.7, SMP.2

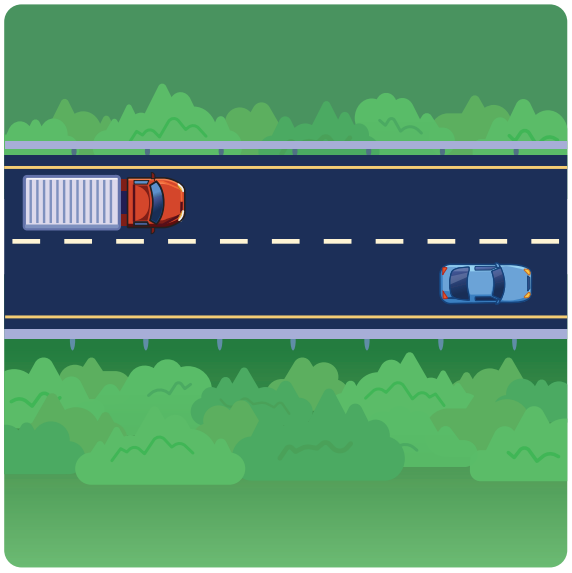
When Will They Meet?

Let's use equations to think about situations.



Warm-Up

1 Let's watch an animation of a truck and a car.



a Do you think the truck will meet the car? Circle one.

Yes No I'm not sure

b What information could help you prove your answer?

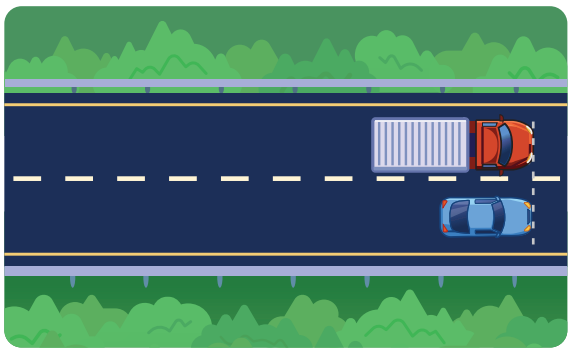
Name: ..... Date: ..... Period: .....

Distance and Time

2 The table shows each vehicle's position at certain times. The vehicles are moving at a constant *rate*. Fill in the missing information in the table.

Time (sec)	Truck Position (m)	Car Position (m)
0	0	18
1	15	29
2	30	40
3		
4		
. . .	. . .	. . .
t		

3 When will the truck meet the car?



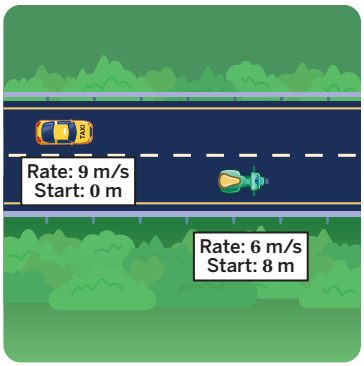
Activity 2

Name: ..... Date: ..... Period: .....

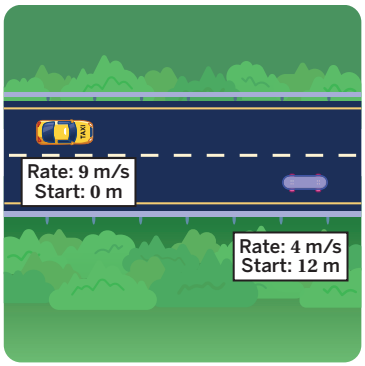
Choose Your Vehicle

4 Choose and circle a vehicle to compare to the taxi. The rate (in meters per second) and the starting point are displayed for each vehicle.

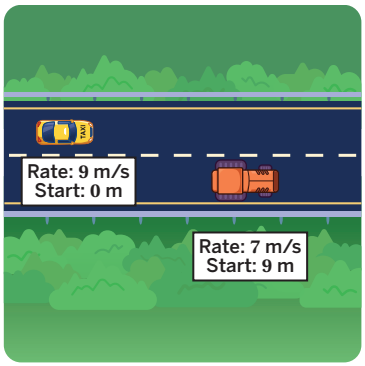
Scooter



Skateboard



Tractor



5 Let t represent time in seconds. For the vehicle you chose, which equation could you solve to determine when the two vehicles meet?

Scooter

A. $6 + 8t = 9t$

B. $8t + 6 = 9$

C. $6t + 8 = 9t$

Skateboard

A. $4t + 12 = 9t$

B. $12t + 4 = 9$

C. $4 + 12t = 9t$

Tractor

A. $7 + 9t = 9t$

B. $7t + 9 = 9t$

C. $9t + 7 = 9$

Explain your thinking.

6 When will the vehicle you chose meet the taxi?

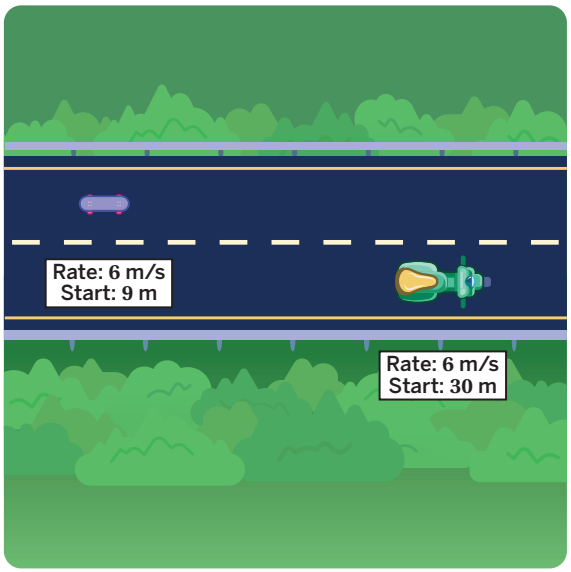
Name: ..... Date: ..... Period: .....

Meet Up

7 Demetrius wants to figure out when these vehicles will meet, so he wrote these expressions.

Skateboard Position (m)	Scooter Position (m)
$6t + 9$	$6t + 30$

Without solving an equation, Demetrius knew the vehicles would never meet. How might he have figured this out?



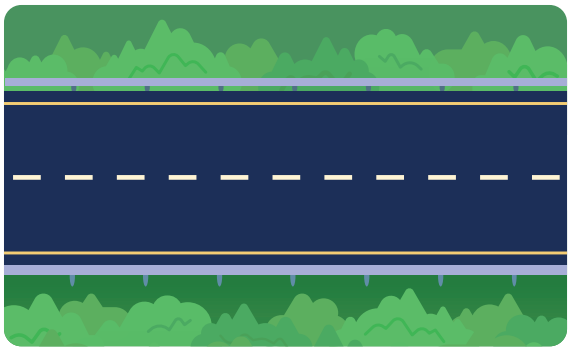
Name: _____ Date: _____ Period: _____

Meet Up (continued)

8 Write expressions, in terms of t, that could represent two vehicles traveling at different rates with different starting positions that will eventually meet.

Truck position expression: _____

Car position expression: _____



9 When will the two vehicles meet?

You're invited to explore more.

10 A tractor and a scooter are in a race. Write expressions, in terms of t, for each vehicle so that the vehicles start separated and meet at 10 seconds.

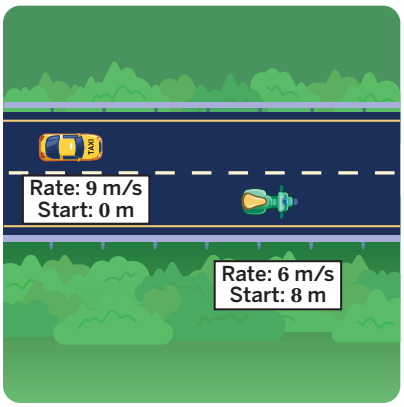
Tractor position expression: _____

Scooter position expression: _____

11 Synthesis

How can writing expressions to represent the position of vehicles at time t help you determine when they will meet?

Use the example if it helps with your thinking.



14 Summary 4.08

We can write two expressions in one variable and set them equal to each other to represent a scenario in which two conditions are equal. We can solve this equation to determine the unknown quantity.

For example, imagine two hikers walking in the same direction on a flat trail. The hikers will meet each other when they are at the same location on the trail at the same time.

To determine when this occurs, an expression can be used to represent the location and walking speed of each hiker.

	Location (ft)	Walking Speed (ft/s)	Expression
Hiker 1	30	4	$30 + 4t$
Hiker 2	10	7	$10 + 7t$

You can set these two expressions equal to each other to form one equation that can be solved.

$30 + 4t = 10 + 7t$

$20 = 3t$

$t = \frac{20}{3}$ or about 6.7 seconds

Practice
4.08

Name: Date: Period:

1. For what value of x do the expressions $\frac{2}{3}x + 2$ and $\frac{4}{3}x - 6$ have the same value?

2. Which story could the equation $3x + 6 = 2 + 4x$ represent?

A. A truck starts 6 meters down a road and is moving at a speed of 3 meters per second. A car starts 2 meters down a road and is moving at a speed of 4 meters per second. After x seconds, the car and the truck meet.

B. A motorcycle starts 3 meters down a road and is moving at a speed of 6 meters per second. A bicycle starts 4 meters down a road and is moving at a speed of 2 meters per second. After x seconds, the motorcycle and the bike meet.

Problems 3–6: Tiana and Miko are biking in the same direction on the same path.

3. Miko rides at a constant speed of 16 miles per hour. Write an expression that represents the number of miles Miko travels after t hours.

4. Tiana starts riding a half hour before Miko. If Miko has been riding for t hours, how long has Tiana been riding?

5. Tiana rides at a constant speed of 12 miles per hour. Write an expression that represents the number of miles Tiana travels after Miko has been riding for t hours.

6. Use your expressions to determine when Miko and Tiana meet. Show or explain your thinking.

Name: _____ Date: _____ Period: _____

7. Here are two cell phone plans.

- Plan A costs $70 per month and comes with a free phone worth $500.
- Plan B costs $50 per month, but doesn't come with a free phone.

If you choose Plan B and buy a $500 phone, after how many months will your total cost be the same as the cost of choosing Plan A? Show or explain your thinking.

Spiral Review

Problems 8–9: Solve each equation. Show your thinking.

8. $5(x + 2) = 30$

9. $5x + 2 = 30$

10. A triangle is graphed on a coordinate grid. Which transformation will result in a triangle that is *not* congruent to the original triangle?

A. A reflection over the y-axis.

B. A translation 5 units to the left.

C. A dilation by a scale factor of 4.

D. A 180° counterclockwise rotation around the origin.

11. Write the equation of a line that has a slope of 3 and a vertical intercept of -10.

Name: _____ Date: _____ Period: _____

Linear Equations | Multiple Representations of Functions | 8.EE.7, 8.EE.7.a, 8.EE.7.b

Practice Day 1



Let's practice what you've learned so far in this unit!

Start with any of the scavenger hunt sheets.

- Record the sheet shape, solve the problem, and write your answer.
- Look for your answer at the top of another scavenger hunt sheet. Solve that problem.
- Repeat until you make it back to your starting sheet.

Sheet: _____ **Answer:** →	**Sheet:** _____ ↙ **Answer:**
Sheet: _____ **Answer:** →	**Sheet:** _____ ↙ **Answer:**

Name: Date: Period:

Practice Day 1

Sheet:

Answer:

Sheet:

Answer:

Sheet:

Answer:

Sheet:

Answer:

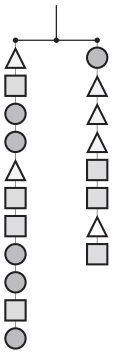
Sheet:

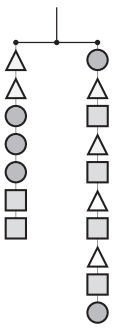
Answer:

Sheet:

Answer:

Use these hanger diagrams if they help with your thinking.





Notes:

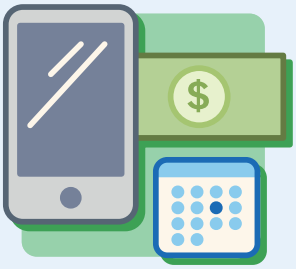
Sub-Unit

2

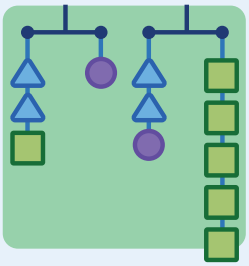
Systems of Linear Equations



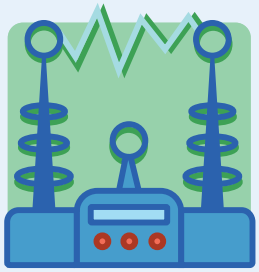
Lesson 9
On or Off the Line?



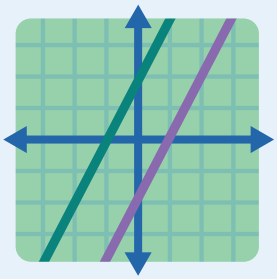
Lesson 10
On Both Lines



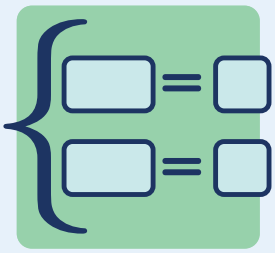
Lesson 11
Make Them Balance



Lesson 12
Line Zapper



Lesson 13
All, Some, or None? Part 2



Lesson 14
Strategic Solving, Part 2

Name: ..... Date: ..... Period: .....

Linear Equations 8.EE.8, 8.EE.8.a, 8.EE.8.c, SMP.2, SMP.3, SMP.6, SMP.7

On or Off the Line?

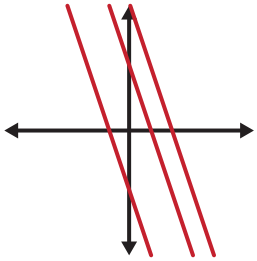
Let's interpret the meaning of points on and off lines.



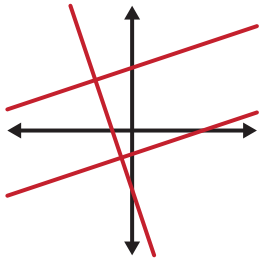
Warm-Up

1 Which one doesn't belong? Explain your thinking.

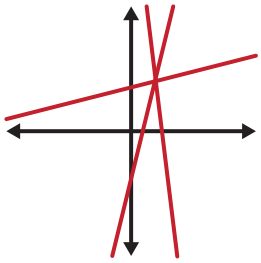
A.



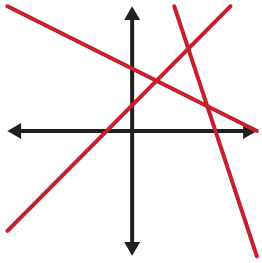
B.



C.



D.



Activity 1

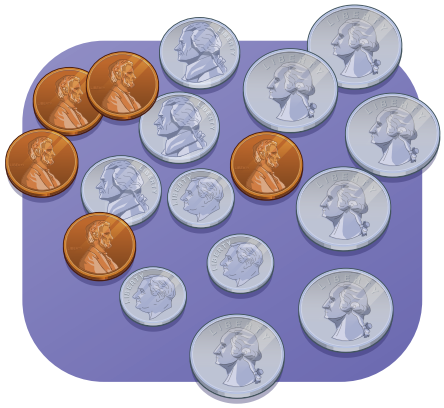
Name: ..... Date: ..... Period: .....

Two Dollars

2 I have $2 worth of coins in my pocket.

What is a combination of any coins that I could have?

Try to think of a combination that no one else in the class will write.

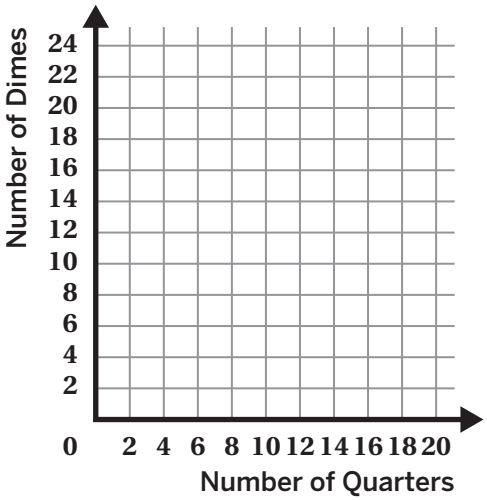


3 Here is more information about my coins:

- I only have quarters and dimes.

a Fill in three rows of possible combinations of quarters and dimes that are worth $2.

Number of Quarters	Number of Dimes



b Plot your points on the graph.

4 Here is some more information about my coins:

- I have a total of 17 coins.

a Fill in at least three rows of possible combinations of 17 coins.

Number of Quarters	Number of Dimes

b Use a different color to plot your points on the graph from the previous problem.

Name: Date: Period:

Two Dollars (continued)

5 Let's look at the graphs of both of these conditions on the same coordinate plane.

I have $2 in my pocket: I only have quarters and dimes, and I have a total of 17 coins.

In this situation, which conditions does the point $(12, 5)$ meet? Circle one.

I have $2 worth of coins in my pocket	I have a total of 17 coins	Both	Neither

Explain your thinking.

6 Let's look at the graphs of these conditions on the same coordinate plane.

I have $2 in my pocket: I only have quarters and dimes, and I have a total of 17 coins.

How many quarters and dimes must I have?

Number of Quarters	Number of Dimes

Explain your thinking.

Activity 2

Name: Date: Period:

Challenge Creator

7 You will use the Activity 2 Sheet to create your own challenge.

a. **Make it!** On the Activity 2 Sheet, choose a scenario and complete the statement.

b. **Solve it!** On this page, choose one scenario from Activity 2 sheet to focus on. Record the ordered pair that the statement represents and determine which line(s) your statement applies to.

Situation	Ordered Pair	Lines

c. **Swap it!** Swap your challenge with one or more partners. On this page, record the ordered pair that the statement represents and determine which line(s) their statement applies to.

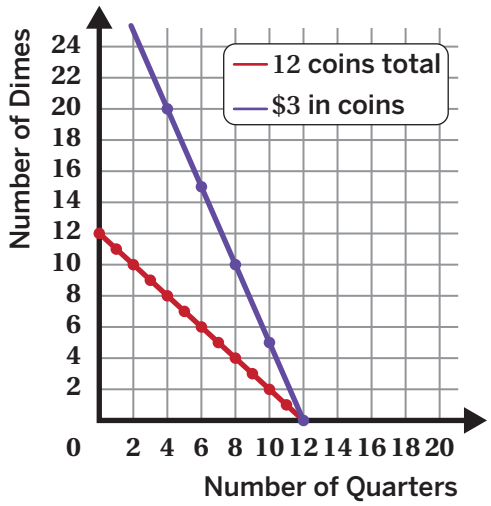
	Situation	Ordered Pair	Lines
Partner 1			
Partner 2			
Partner 3			
Partner 4			

8 Synthesis

Here is a graph that represents two conditions:

- I have $3 worth of coins in my pocket.
- I have a total of 12 coins.

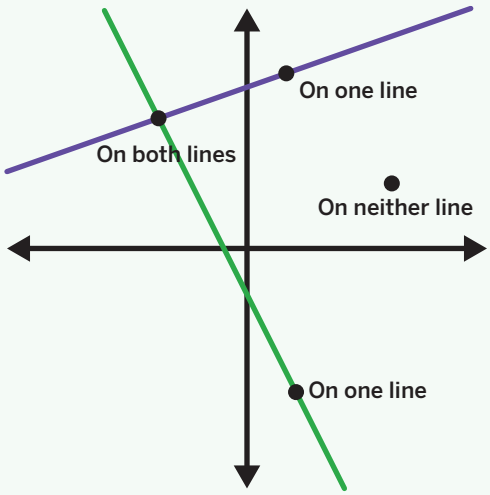
How can you use the graph to determine whether a combination of dimes and quarters meets both conditions, one condition, or neither condition?



11 Summary 4.09

Linear relationships can represent many situations. Lines graphed on the same coordinate plane can simultaneously represent multiple conditions or relationships involving the same variables.

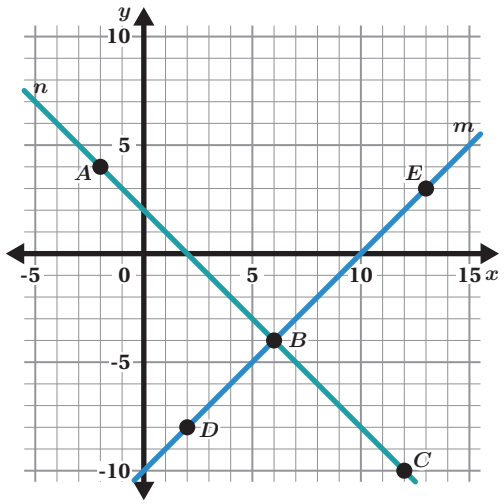
- The coordinates of a point that is on both lines make both relationships true.
- The coordinates of a point on only one line make only one relationship true.
- The coordinates of a point on neither line make neither relationship true.



Practice
4.09

Name: ..... Date: ..... Period: .....

Problems 1–5: Here is a coordinate plane. Determine which line represents each condition.



1. The coordinates of each point have a sum of 2.

2. The y-coordinate of each point is 10 less than the x-coordinate.

3. Select *all* the points whose coordinates have a sum of 2.

- ☐ Point A
- ☐ Point B
- ☐ Point C
- ☐ Point D
- ☐ Point E

4. Select *all* the points whose y-coordinate is 10 less than the x-coordinate.

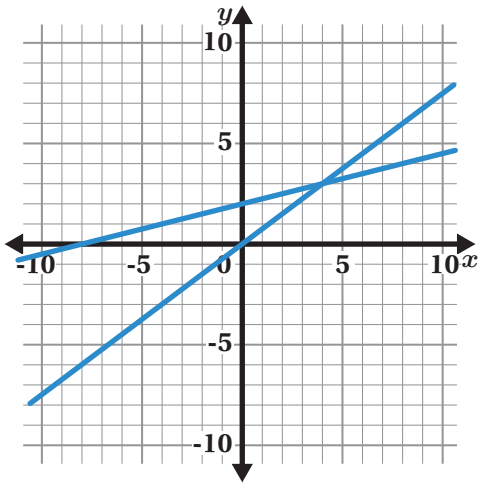
- ☐ Point A
- ☐ Point B
- ☐ Point C
- ☐ Point D
- ☐ Point E

5. Select *all* the points whose coordinates have a sum of 2 and the y-coordinate is 10 less than the x-coordinate.

- ☐ Point A
- ☐ Point B
- ☐ Point C
- ☐ Point D
- ☐ Point E

Name: _____ Date: _____ Period: _____

6. These two lines represent a system of equations. What is the y-coordinate of the point that makes both equations true?



A. 0

B. $\frac{1}{5}$

C. 3

D. 4

Spiral Review

Problems 7–9: Consider the equation $4x - 4 = 4x +$ _____. What value or expression could you write in the blank so that the equation is true for:

7. No values of x?

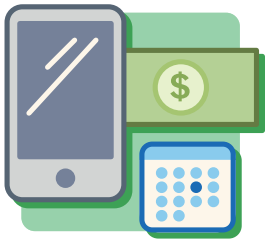
8. All values of x?

9. One value of x?

10. Solve the equation $4(2x + 5) = 44$. Show your thinking.

Name: _____ Date: _____ Period: _____

Linear Equations 8.EE.8, 8.EE.8.a, 8.EE.8.b, 8.EE.8.c, SMP.1, SMP.4, SMP.6



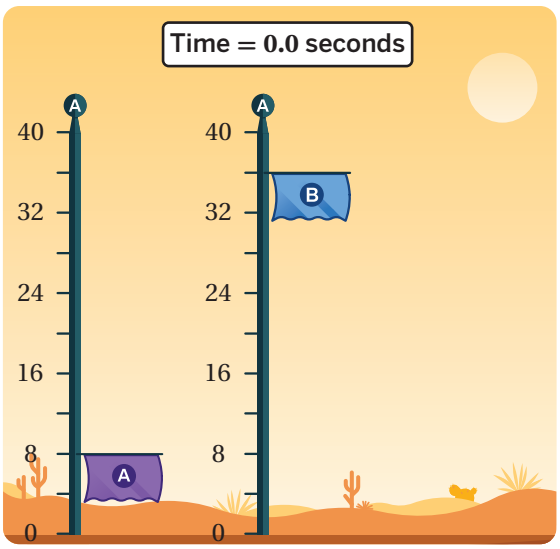
On Both Lines

Let's use lines to analyze real-world situations.

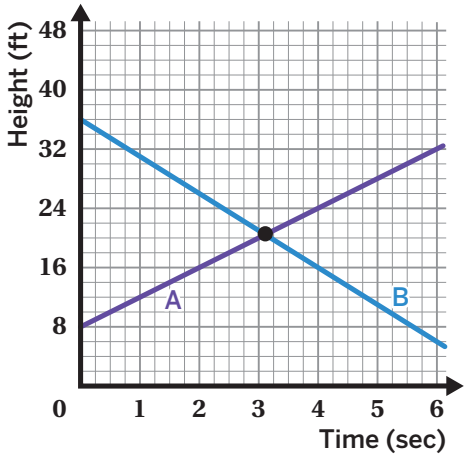
Warm-Up

Let's watch an animation with two flags.

1. **Discuss:** What do you notice? What do you wonder?



2. At what point do both lines meet? What does this point represent in this situation?

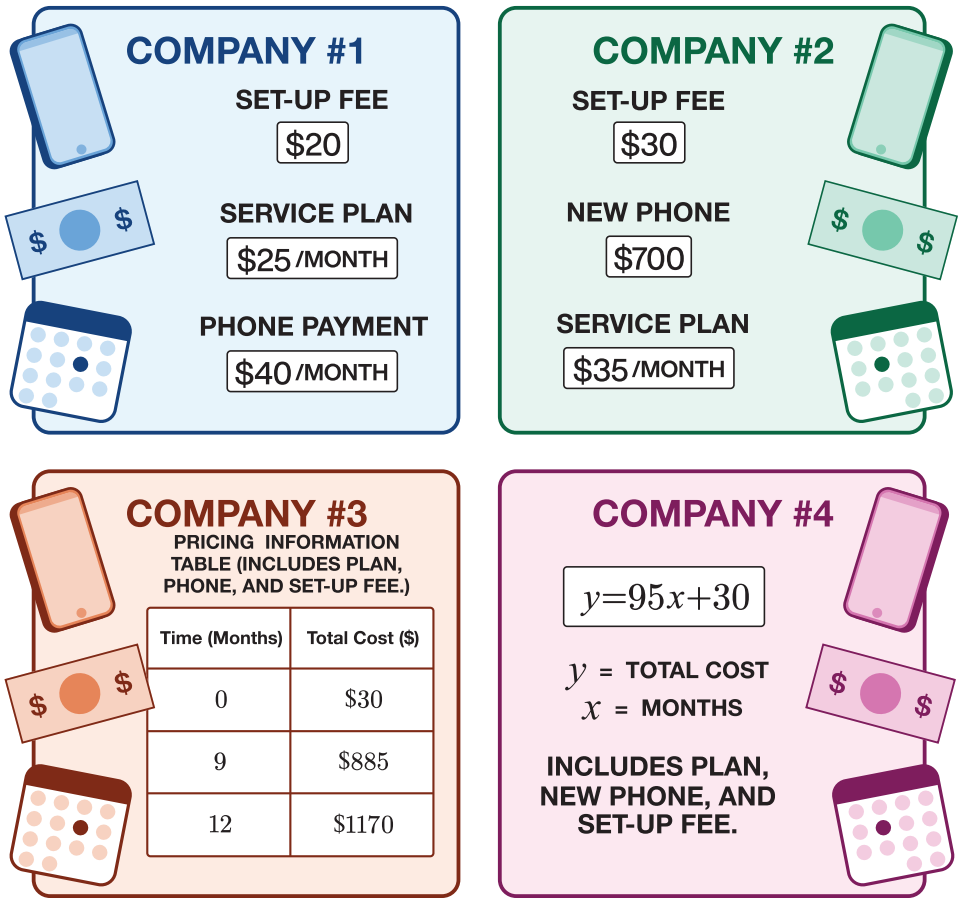


Activity 1

Name: ..... Date: ..... Period: .....

On Both Lines

You are shopping for a new cell phone and plan with unlimited data. Here are four companies and information about their set-up fee, phone cost, and service plan.



3. Choose *two* different companies and make a poster comparing them. Be sure to include these items in your poster:

- ☐ Your names.
- ☐ The names of your selected companies.
- ☐ A graph with both companies on the same set of axes.
- ☐ An equation to represent each scenario.
- ☐ Your conclusions about the situation.
- ☐ The meaning of the point of intersection.
- ☐ When each company has the better deal.

You're invited to explore more.

4. Select a third phone provider. On your poster, explain when this third plan would be a better and worse deal than your two previously selected plans.

Activity 2

Name: Date: Period:

Gallery Tour

You will take a gallery tour as a class.

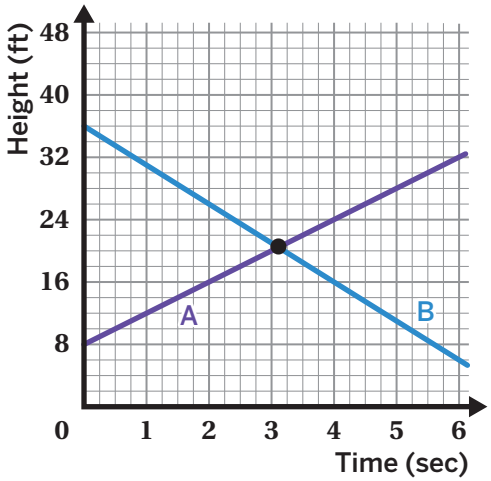
5. What features of your classmates' posters helped you understand their thinking?

6. Describe something you would change about your poster now that you have seen other groups' work.

Synthesis

7. Describe how a graph can help you compare different linear relationships.

Use the example if it helps with your thinking.

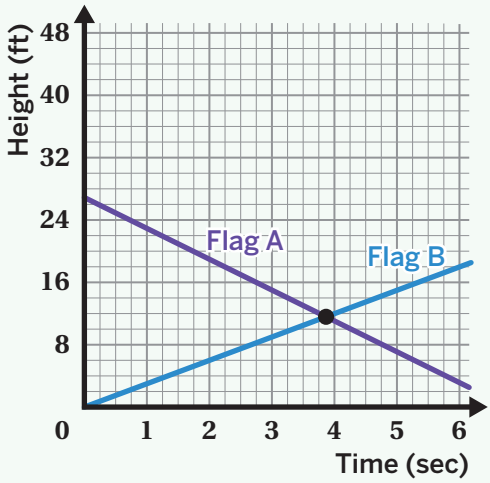


Summary 4.10

If there are two equations that share the same variables, you can find the solution that makes both equations true by locating the point of intersection.

For example, consider this graph.

Although you can't see the exact values of the point of intersection, you can tell that the flags are the same height, at about 11.5 feet, just before 4 seconds.

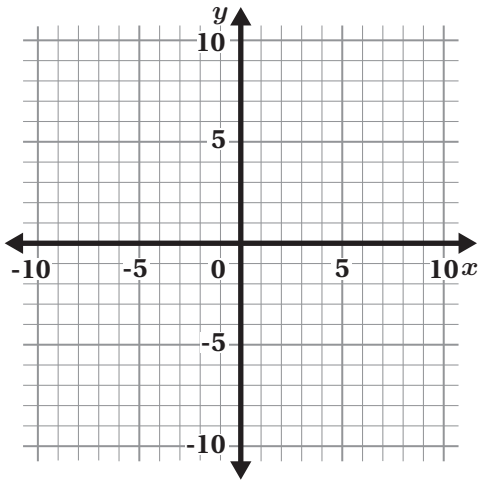


Practice
4.10

Name: ..... Date: ..... Period: .....

1. Jayden has $11 and begins saving $5 each week to buy a new phone. At the same time that Jayden begins saving, Aditi has $60 and begins spending $2 per week on supplies for her art class. Is there a week when they will have the same amount of money? Explain your thinking.

2. Draw a graph to determine x- and y-values that make both of the equations $y = -\frac{2}{3}x + 3$ and $y = x - 2$ true.



Problems 3–4: Prisha and Mia agree to go to the movies after they have earned the same amount of money for the same number of hours worked.

Mia earns $7 per hour mowing her neighbors' lawns. She also earned $14 one time for hauling away bags of recyclables.

Prisha babysits her neighbor's children. The table shows the amount of money, m, she earns in h hours.

h	m
1	$8.40
2	$16.80
3	$33.60

3. How many hours do they each have to work before they go to the movies?

4. How much will each of them have earned?

Name: ..... Date: ..... Period: .....

5. The point where the graphs of two equations intersect has a y-coordinate of 2. One equation is $y = -3x + 5$.

Determine the other equation if its graph has a slope of 1.

6. The graph of the equations $y = \square - x$ and $y = \square x - 3$ intersect at the point $(2, 1)$. Determine the missing values in the equations. Show or explain your thinking.

Spiral Review

Problems 7–9: Determine whether each equation is true for *One value*, *All values*, or *No values* of x.

7. $10 + 3x = -4.2x + 9$

8. $5(4x + 1) - x = 19x + 5$

9. $-2(3x - 7) = -6x + 12$

Problems 10–12: Determine whether $x = -1$ is a solution to each equation.

10. $1.5x + 2 = 8.5x - 4$

11. $\frac{1}{2} + 2x = \frac{1}{2}x - 1$

12. $3x + 2 = 5x - (-4)$

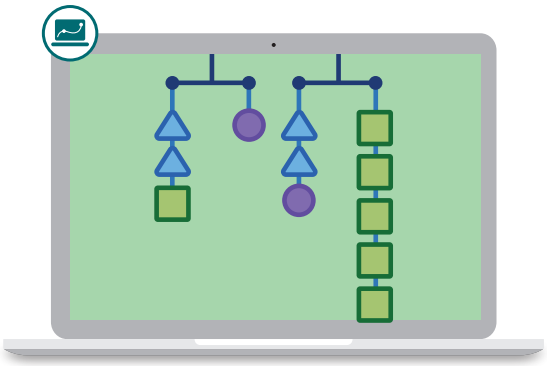
Unit 4
Lesson
11

Name: ..... Date: ..... Period: .....

Linear Equations 8.EE.8, 8.EE.8.a, 8.EE.8.b, 8.EE.8.c, SMP.7

Make Them Balance

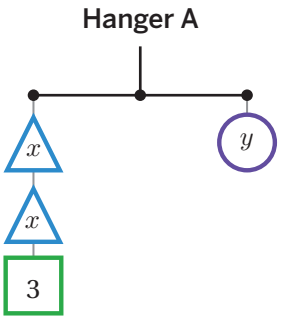
Let's explore solutions to more than one linear relationship.



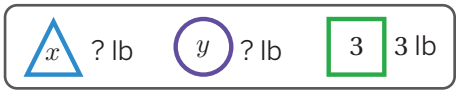
Warm-Up

1 Determine values for x and y that will balance the hanger.

x	y



$2x + 3 = y$



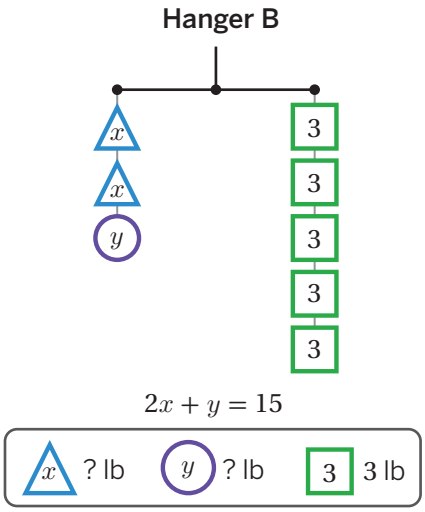
2 Let's look at a graph that shows points whose x- and y-values balance the hanger.

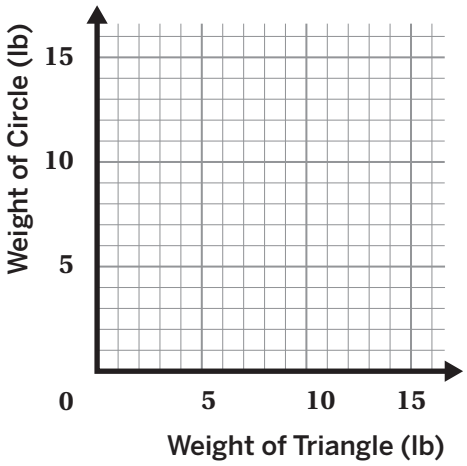
Discuss: What do you notice? What do you wonder?

Name: Date: Period:

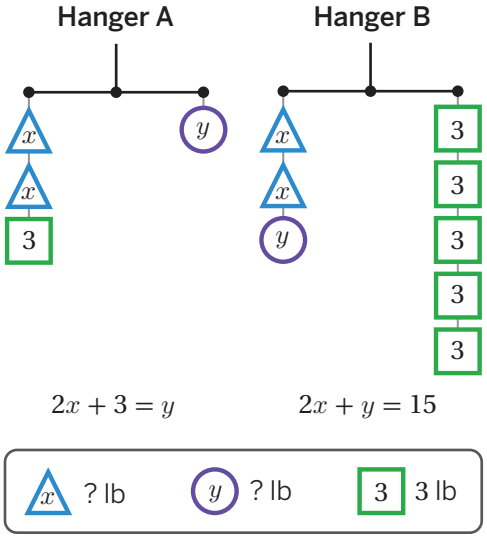
Two Hangers

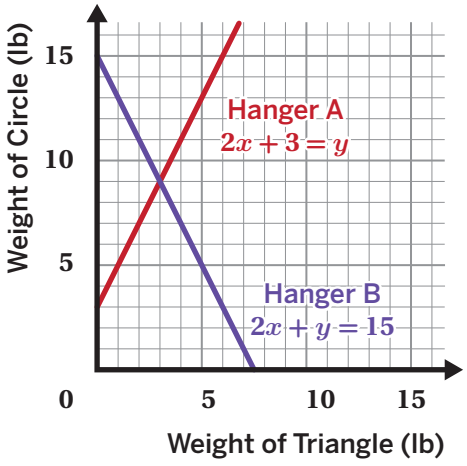
3 Here's another hanger. Plot three different pairs of x- and y-values that will balance this hanger.





4 Here are both hangers' equations graphed on the same coordinate plane.





Discuss: Can you identify a point that balances just Hanger A? Just Hanger B? Both? Neither?

Name: Date: Period:

Two Hangers (continued)

5 You have been experimenting with two representations of this **system of equations**:

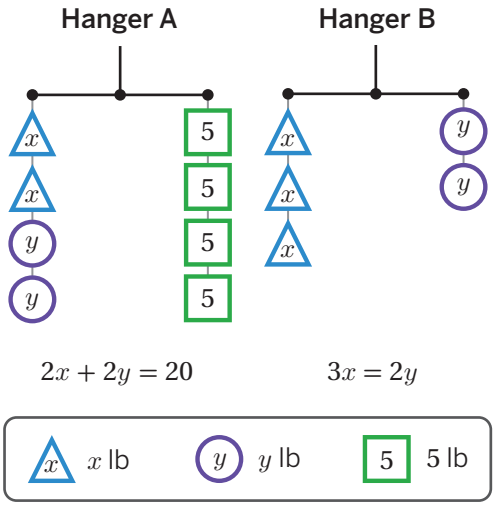
$$\begin{cases} 2x + 3 = y \\ 2x + y = 15 \end{cases}$$

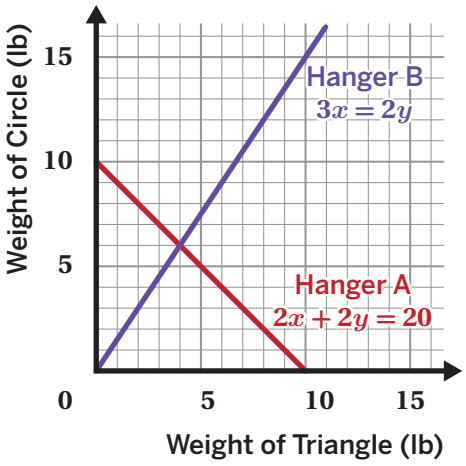
A **solution to a system of equations** is a set of values that makes both equations in that system true.

Write the solution to this system as an ordered pair. Explain your thinking.

6 This system represents another set of hangers:

$$\begin{cases} 2x + 2y = 20 \\ 3x = 2y \end{cases}$$





Which point is a solution to this system? Circle one.

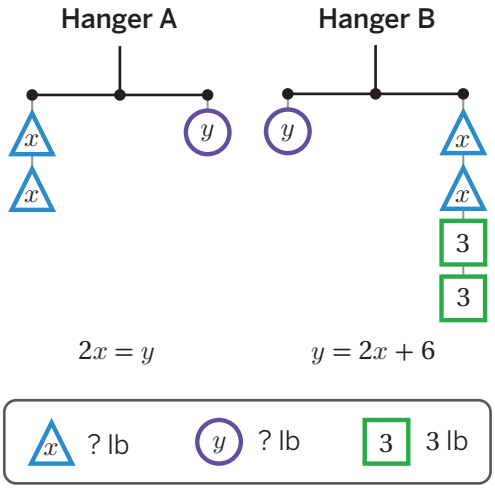
(1, 9) (4, 6) (6, 9) (6, 4)

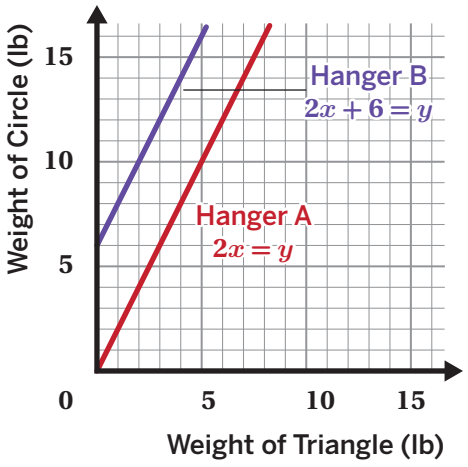
Explain your thinking.

Name: ..... Date: ..... Period: .....

Hanger Solutions

7 Here is one set of equations.





Discuss: Can you identify a point that balances just Hanger A? Just Hanger B? Both? Neither?

8 This system of equations from the previous problem has no solution:

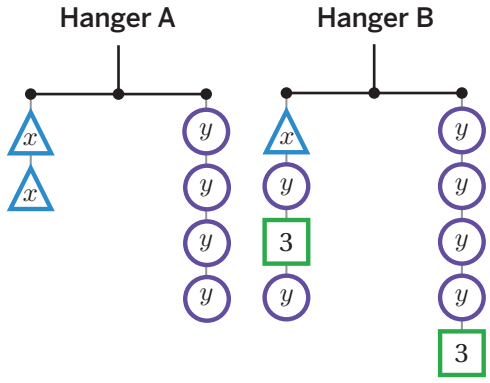
$$\begin{cases} 2x = y \\ y = 2x + 6 \end{cases}$$

How can you use the hangers, graphs, or equations to tell that this system of equations has no solution?

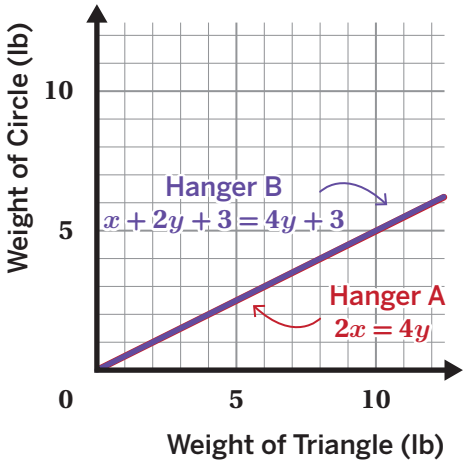
Name: Date: Period:

Hanger Solutions (continued)

9 Here is another set of equations.



$2x = 4y$ $x + 2y + 3 = 4y + 3$



x ? lb y ? lb 3 3 lb

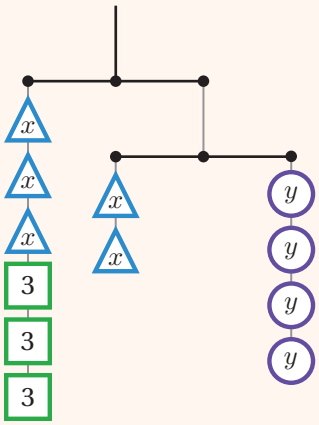
Discuss: How many solutions do you think this system has? How do you know?

$$\begin{cases} 2x = 4y \\ x + 2y + 3 = 4y + 3 \end{cases}$$

You're invited to explore more.

10 Determine values for x and y that will balance both hangers.

x	y



x ? lb y ? lb 3 3 lb

11 Synthesis

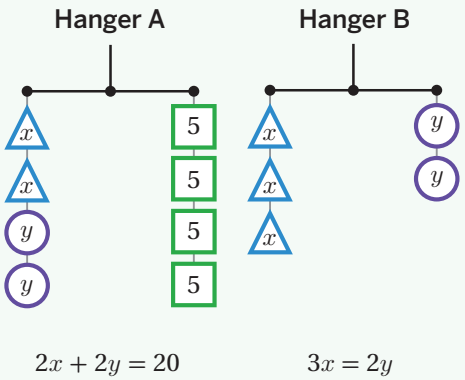
Discuss: How can you tell if an ordered pair is a solution to a system of linear equations? Use the examples from the summary if it helps your thinking.

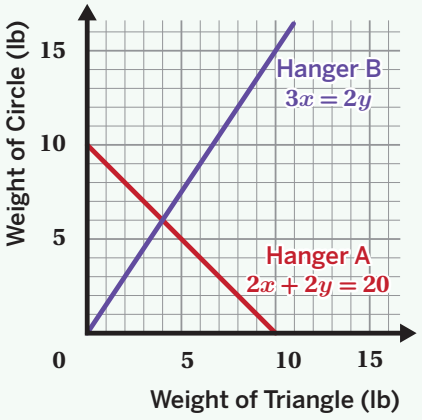
14 Summary 4.11

You can use hanger diagrams and graphs to determine the **solution to a system of equations**.

For example, here is a system of equations:

$$\begin{cases} 2x + 2y = 20 \\ 3x = 2y \end{cases}$$





The ordered pair $(4, 6)$ is the point of intersection, which means that it will make both equations true. Both hangers will balance when the triangles weigh 4 pounds and the circles weigh 6 pounds.

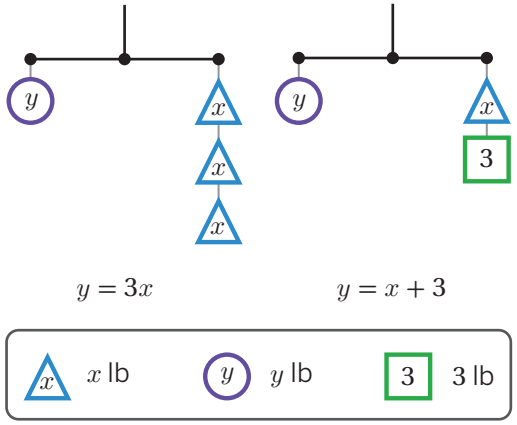
system of equations Two or more equations that represent the constraints on a shared set of variables.

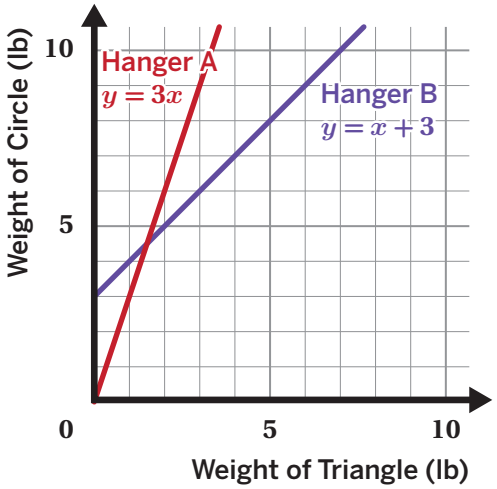
solution to a system of equations A set of values that makes all equations in that system true. When the equations are graphed, the solution to the system is the point of intersection.

Practice
4.11

Name: ..... Date: ..... Period: .....

Problems 1–2: The hangers and the graph represent the same system of equations.

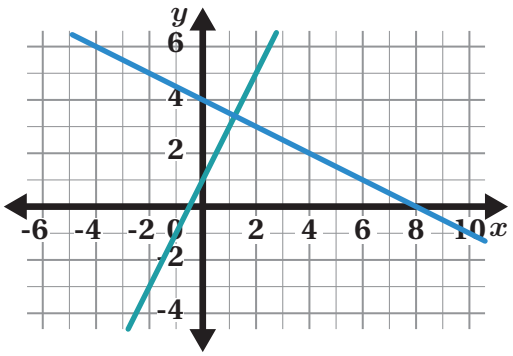




1. Determine the solution to the system of equations.

2. What does the solution tell you about the weight of a triangle and the weight of a circle that will balance the hanger?

Problems 3–4: Here is a graph.



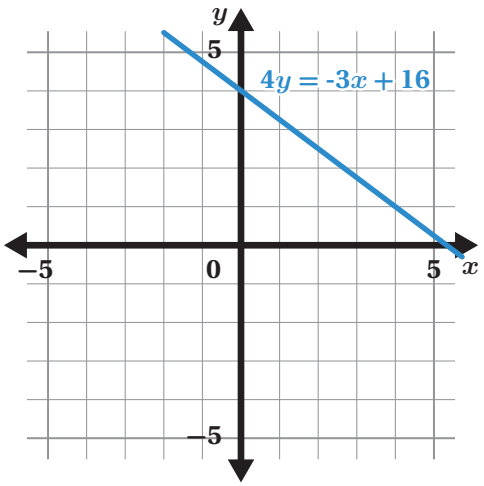
3. Write an equation that can represent each line.

4. Estimate the solution to the system.

Practice
4.11

Name: Date: Period:

Problems 5–7: Here is a graph that represents one equation in a system of equations.



5. Write a second equation for the system so that it has infinitely many solutions.

6. Write a second equation whose graph goes through $(0, 1)$ so that the system has no solution.

7. Write a second equation whose graph goes through $(0, 2)$ so that the system has one solution: $(4, 1)$.

Spiral Review

8. Select *all* the equations that have no solution.

- ☐ **A.** $2 + 4(4x + 5) = 8x + 2x - 11$
- ☐ **B.** $-x + 3x - 7 = 2(x - 7)$
- ☐ **C.** $7 - 5x(-3) = 5(3x - 2)$
- ☐ **D.** $6x + 3(2x - 1) = 5x - 4 + 7x + 1$

Problems 9–11: How many solutions does each equation have? Show or explain your thinking.

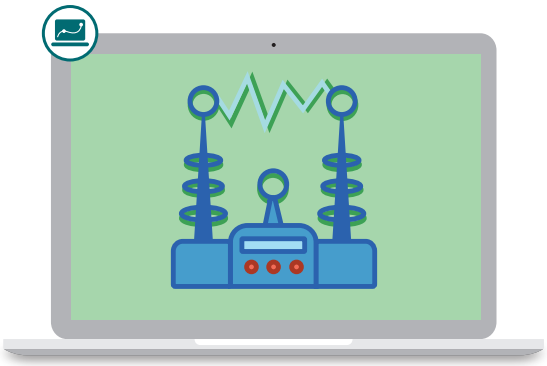
9. $\frac{15(x-3)}{5} = 3(2x - 3)$

10. $0.4(x + 7) = 0.2(x + 40) - 5.2 + 0.2x$

11. $12x - 14 = 2(7x + 8) + x + 9$

Name: ..... Date: ..... Period: .....

Linear Equations 8.EE.8, 8.EE.8.b, SMP.6, SMP.7



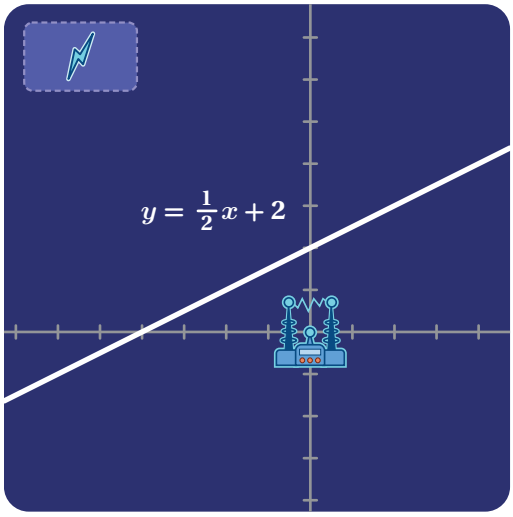
Line Zapper

Let's solve systems of linear equations.

Warm-Up

1 Let's capture the line by writing the coordinates for a point that's on the line.

Zap	Point
Zap 1	



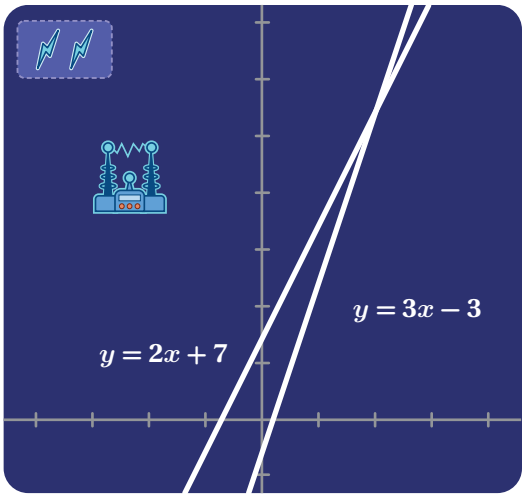
Name: Date: Period:

Two Lines, One Zap

2 Capture both of the lines by writing coordinates for points on these lines.

Use no more than two zaps!

Zap	Point
Zap 1	
Zap 2	



3 Cameron wanted to capture both lines with one zap.

Cameron solved an equation to identify the point of intersection and determined that $x = 10$.

How could Cameron determine the y-value of the point of intersection?

Cameron

$y = 2x + 7$
$y = 3x - 3$

$2x + 7 = 3x - 3$
$-2x \quad -2x$
$7 = 1x - 3$
$+3 \quad +3$
$10 = 1x$
$x = 10$

Activity 2

Name: ..... Date: ..... Period: .....

Line Zapper

4 The following lines are hidden in a graph:

$$\begin{cases} y = -x + 10 \\ y = 2x + 4 \end{cases}$$

Capture the lines by writing the coordinates for a point on both of these lines. You only have one zap!

Zap	Point
Zap 1	

5 **a** Select *all* the lines that would be captured if the point $(2, 4)$ were zapped.

- ☐ **A.** Line A: $y = \frac{1}{2}x$
- ☐ **B.** Line B: $y = 2x$
- ☐ **C.** Line C: $y = -x + 6$

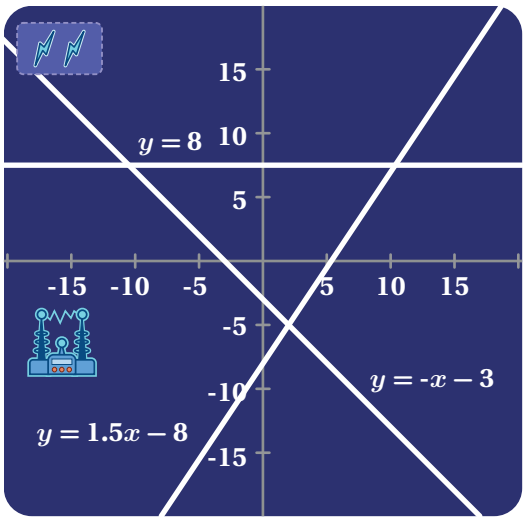
b Explain how you decided which lines to select.

Name: ..... Date: ..... Period: .....

Line Zapper (continued)

6 Capture all of the lines by writing coordinates for points on the lines. You only have two zaps!

Zap	Point
Zap 1	
Zap 2	



7 The following lines are hidden in a graph:

$$\begin{cases} y = -2x + 9 \\ y = 3x - 1 \\ y = \frac{1}{2}x - 1 \end{cases}$$

Zap	Point
Zap 1	
Zap 2	

Capture all of the lines by writing the coordinates for points on the lines. You only have two zaps!

You're invited to explore more.

8 The following lines are hidden in a graph:

$$\begin{cases} y = 3x + 6 \\ y = 3(x - 5) \\ y = -\frac{1}{2}x - 15 \\ -x + y = 12 \end{cases}$$

Capture all of the lines by writing coordinates for points on the lines. You only have three zaps!

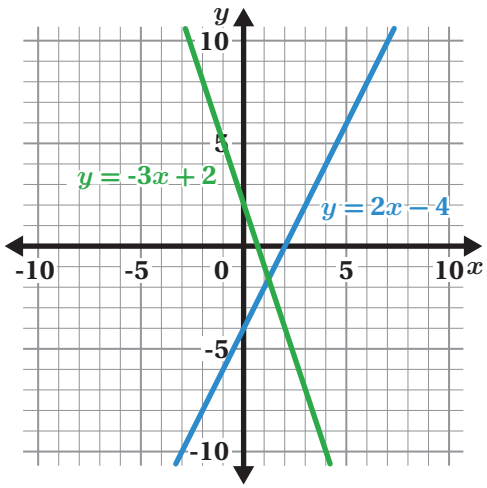
Zap	Point
Zap 1	
Zap 2	
Zap 3	

9 Synthesis

Here is the graph of this system of equations:

$\begin{cases} y = -3x + 2 \\ y = 2x - 4 \end{cases}$

How can you determine the *exact* solution to this system of equations?



12 Summary 4.12

For a point to be a solution to a system of equations, the x- and y-coordinates must make both of the equations true. This ordered pair is the *point of intersection* when the system is graphed.

For example, here is a system of equations:

$\begin{cases} y = 4x - 5 \\ y = -2x + 7 \end{cases}$

To determine the solution to the system, you can write a single equation by taking the two expressions that are equal to y and setting them equal to each other.

$$\begin{aligned} 4x - 5 &= -2x + 7 \\ 6x - 5 &= 7 \\ 6x &= 12 \\ x &= 2 \end{aligned}$$

You can then substitute the solution for x into either of the original equations to determine the value of y.

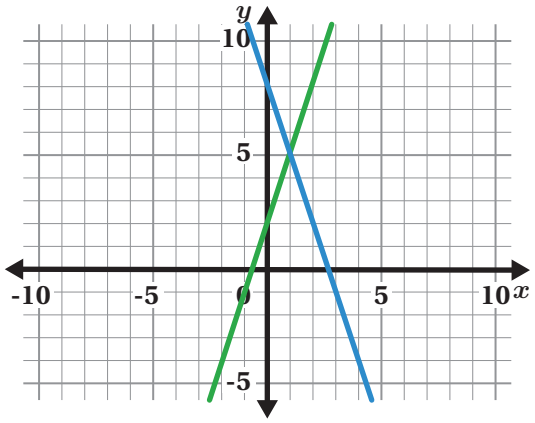
$$\begin{aligned} y &= 4x - 5 \\ y &= 4(2) - 5 \\ y &= 8 - 5 \\ y &= 3 \end{aligned}$$

The solution to this system of equations is the point $(2, 3)$.

Practice
4.12

Name: Date: Period:

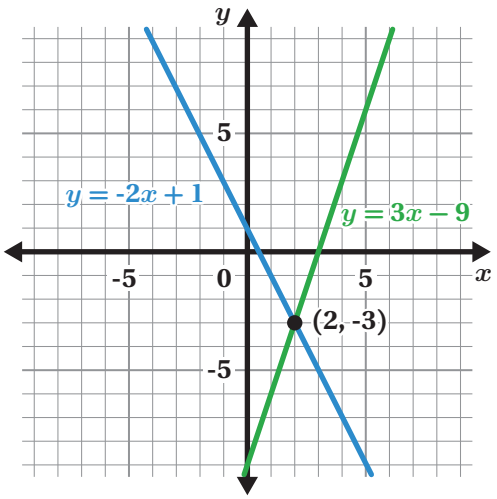
Problems 1–2: Here is the graph of this system of equations: $\begin{cases} y = -3x + 8 \\ y = 3x + 2 \end{cases}$



1. How can you determine the solution to this system of equations by looking at the graph?

2. What is the solution to the system of equations?

Problems 3–5: Use the lines in the graph to decide whether each statement is true or false.



3. The solution to the equation $-2x + 1 = 3x - 9$ is $x = 2$.

4. The point $(2, -3)$ is a solution to this system of equations: $\begin{cases} y = -2x + 1 \\ x = 2 \end{cases}$

5. The point $(0, 1)$ is a solution to the equation $y = -2x + 1$.

Practice
4.12

Name: ..... Date: ..... Period: .....

Problems 6–7: Solve each system of equations. Show or explain your thinking.

6. $\begin{cases} y = 4x + 8 \\ y = 2x + 16 \end{cases}$

7. $\begin{cases} y = -4x + 2 \\ y = 3x - 5 \end{cases}$

8. Here is how Haru tried to solve this system of equations. Did Haru solve the system of equations correctly? Explain your thinking.

Haru

$$\begin{cases} y = -3x + 1 \\ x = 4 \end{cases}$$

$-3x + 1 = 4$

$-3x = 3$

$x = -1$

$y = -3(-1) + 1$

$y = 4$

Solution: (-1, 4)

Spiral Review

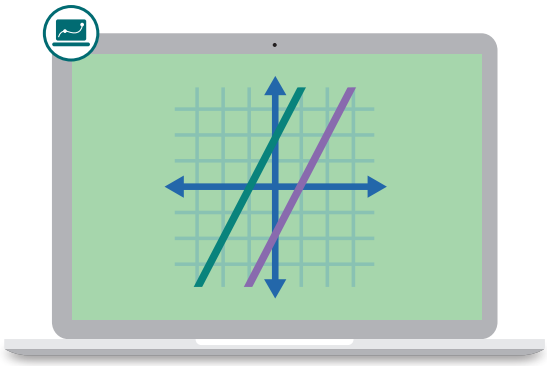
Problems 9–10: The temperature in degrees Fahrenheit, F, is related to the temperature in degrees Celsius, C, which is represented by the equation $F = \frac{9}{5}C + 32$.

9. There is one temperature where the degrees Fahrenheit and degrees Celsius are the same, so that $C = F$. What is that temperature? Show or explain your thinking.

10. What is -32 °C in Farenheit?

Name: ..... Date: ..... Period: .....

Linear Equations 8.EE.8, 8.EE.8.b, SMP.7



All, Some, or None? Part 2

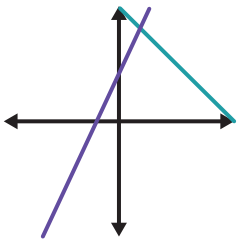
Let's solve systems with no solution and infinitely many solutions.

Warm-Up

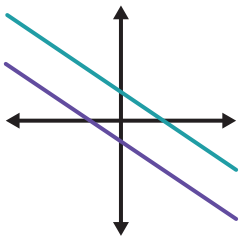
1 Which graph could represent this system of equations?

$$\begin{cases} y = 2x + 4 \\ y = -x + 10 \end{cases}$$

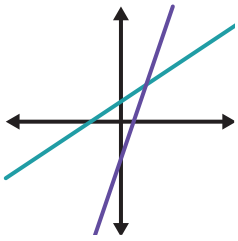
A.



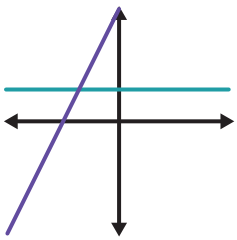
B.



C.



D.



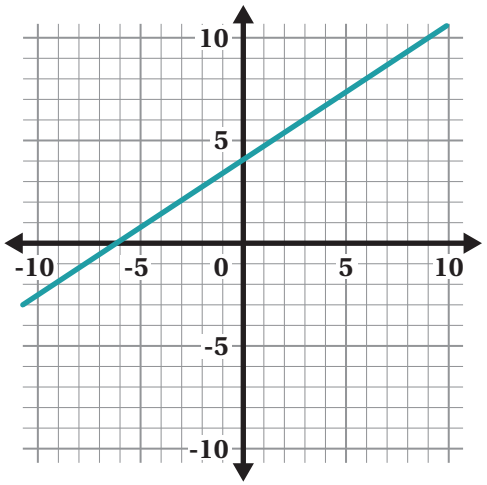
Explain your thinking.

Name: ..... Date: ..... Period: .....

Connecting Graphs and Equations

2 Here is the graph of $y = \frac{2}{3}x + 4$.

Graph another line to create a system of equations that has *no* solution.



3 Here are the equations for a system that has no solution:

$$\begin{cases} y = \frac{2}{3}x + 4 \\ y = \frac{2}{3}x - 5 \end{cases}$$

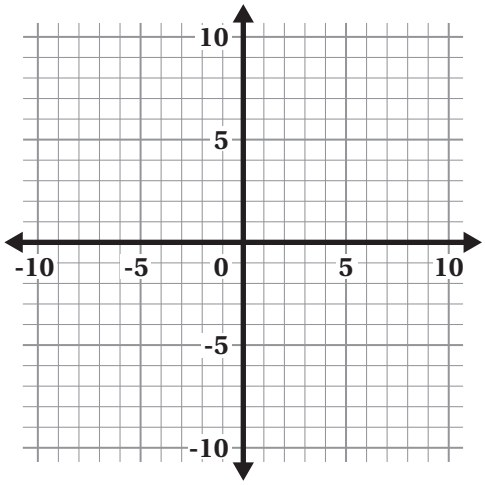
How can you determine from the equations that the system will have no solution?

Name: Date: Period:

Connecting Graphs and Equations (continued)

4 Here is the graph of $y = \frac{1}{2}(3x + 4)$.

Graph another line to create a system of equations that has *infinitely many* solutions.



5 Here are the equations for a system that has infinitely many solutions:

$$\begin{cases} y = \frac{1}{2}(3x + 4) \\ y = \frac{3}{2}x + 2 \end{cases}$$

How can you determine from the equations that the system will have infinitely many solutions?

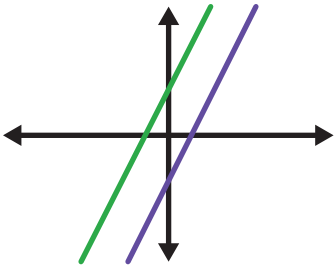
Activity 2

Name: ..... Date: ..... Period: .....

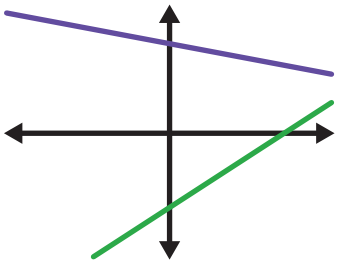
Sorting Systems

6 Group the systems of equations based on their number of solutions.

A.



B.



C. $\begin{cases} y = \frac{3}{4}x - 14 \\ y = -\frac{1}{4}x + 9 \end{cases}$

D. $\begin{cases} y = 2x + 1 \\ -2x + y = 1 \end{cases}$

E. $\begin{cases} x + y = 10 \\ y = -x - 3 \end{cases}$

F. $\begin{cases} y = x + 10 \\ y = x - 2 \end{cases}$

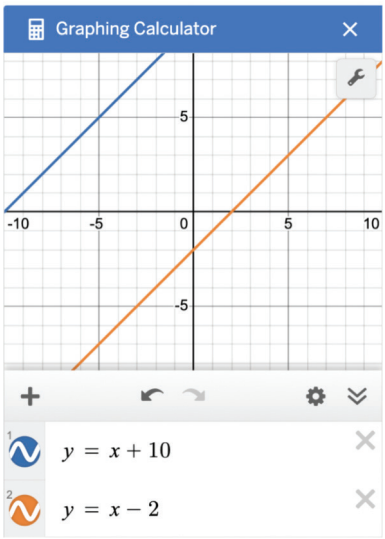
No Solution	One Solution	Infinitely Many Solutions

Name: ..... Date: ..... Period: .....

Sorting Systems (continued)

7 Here are Oscars's and Melanie's strategies to check whether System F goes in the "No Solution" group.

Oscar



Melanie

F $\begin{cases} y = x + 10 \\ y = x - 2 \end{cases}$

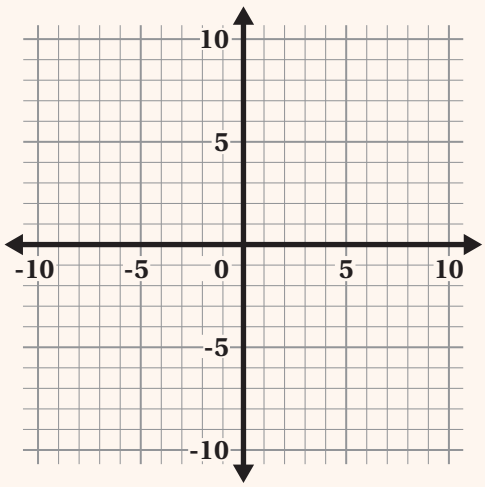
$x + 10 = x - 2$
$-x \qquad -x$
$10 = -2$

Discuss: How are their strategies alike? How are they different?

You're invited to explore more.

8 The graphs of the equations $Ax + By = 15$ and $Ax - By = 9$ intersect at $(2, 1)$. Determine the values of A and B.

Use the grid if it helps with your thinking.



9 Synthesis

Discuss: How can you determine the number of solutions for a system of equations?

C

$$\begin{cases} y = \frac{3}{4}x - 14 \\ y = -\frac{1}{4}x + 9 \end{cases}$$

D

$$\begin{cases} y = 2x + 1 \\ -2x + y = 1 \end{cases}$$

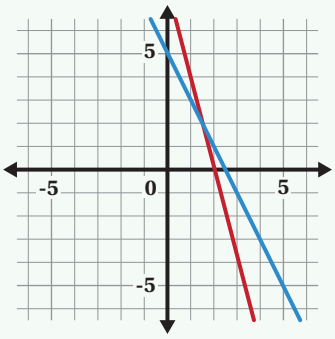
E

$$\begin{cases} x + y = 10 \\ y = -x - 3 \end{cases}$$

12 Summary 4.13

Systems of two linear equations can have one solution, no solution, or *infinitely many solutions*. You can determine the number of solutions to a system of equations by graphing, comparing the slopes and y-intercepts, or solving the system algebraically.

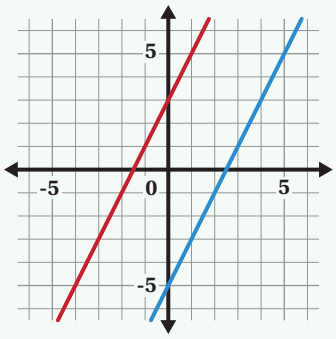
One solution:



$$\begin{cases} y = -4x + 8 \\ y = -2x + 5 \end{cases}$$

- Different slopes

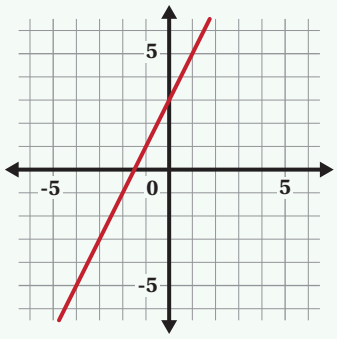
No solution:



$$\begin{cases} y = 2x + 3 \\ y = 2x - 5 \end{cases}$$

- Same slopes
- Different y-intercepts

Infinitely many solutions:



$$\begin{cases} y = 2x + 3 \\ y = 2x + 3 \end{cases}$$

- Same slopes
- Same y-intercepts

infinitely many solutions An equation has infinitely many solutions if it is true for any value of the variable. A system of equations has infinitely many solutions if the equations in the system are equivalent. Equivalent equations will create the same line on a graph, so every point on the line is a solution to each equation in the system.

Practice
4.13

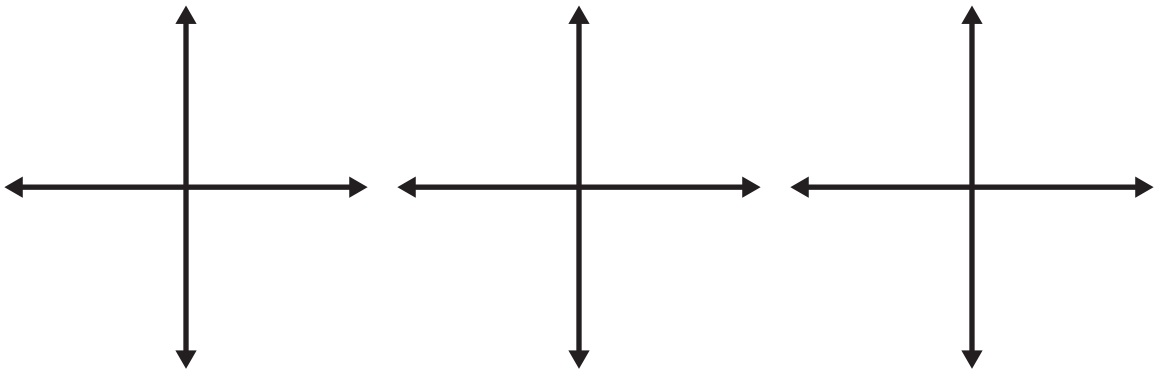
Name: ..... Date: ..... Period: .....

Problems 1–3: Sketch two lines that match each description. Then describe the number of solutions for each system of equations.

1. Two lines with the same slope and different y-intercepts.

2. Two lines with different slopes.

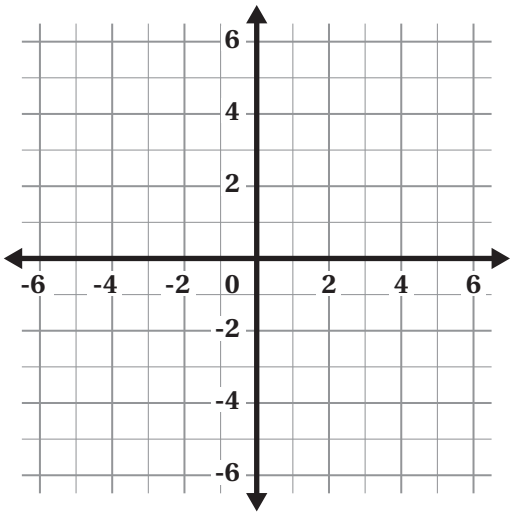
3. Two lines with the same slope and same y-intercept.



4. How many solutions does this system have?

$$\begin{cases} y = \frac{1}{4}x + 2 \\ y = 2x - 5 \end{cases}$$

Show or explain your thinking.



Problems 5–6: Consider this system of equations: $\begin{cases} x - 6y = 4 \\ 3x - 18y = 4 \end{cases}$

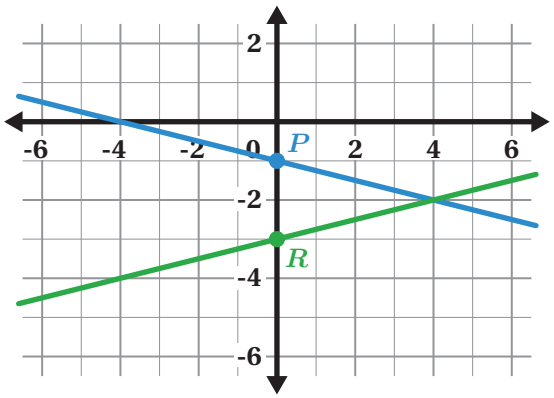
5. Change one number to make a new system with one solution.

6. Change one number to make a new system with an infinite number of solutions.

Name: ..... Date: ..... Period: .....

Problems 7–8: Ali graphed this system:

$$\begin{cases} y = -\frac{1}{4}x - 1 \\ y = \frac{1}{4}x - 3 \end{cases}$$



Ali marked its solutions with points P and R.

7. Which statement describes Ali's solutions?

A. The solutions are correct.

B. The y intercepts are marked instead of the intersection point of the two lines.

C. The y intercepts are marked instead of the $x-$intercepts

D. Only the $y-$intercepts are marked instead of the $x-$and $y-$intercepts.

8. What is the solution to the system of equations?

Spiral Review

Problems 9–11: Solve the equation. Show your thinking.

9. $\frac{3y - 6}{9} = \frac{4 - 2y}{-3}$

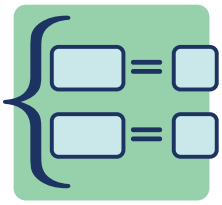
10. $0.3(x - 10) - 1.8 = 2.7x$

11. $-0.6x + 1.8(x + 5) = 0.3x$

Name: ..... Date: ..... Period: .....

Linear Equations 8.EE.8, 8.EE.8.b, SMP.3, SMP.7

Strategic Solving, Part 2



Let's explore strategies for solving systems of equations.

Warm-Up

Determine the solution of each system mentally. Be prepared to share your strategy.

1. $\begin{cases} x = 5 \\ y = x - 7 \end{cases}$

2. $\begin{cases} y = 4 \\ y = x + 3 \end{cases}$

3. $\begin{cases} x - y = 4 \\ x + y = 10 \end{cases}$

Activity 1

Name: Date: Period:

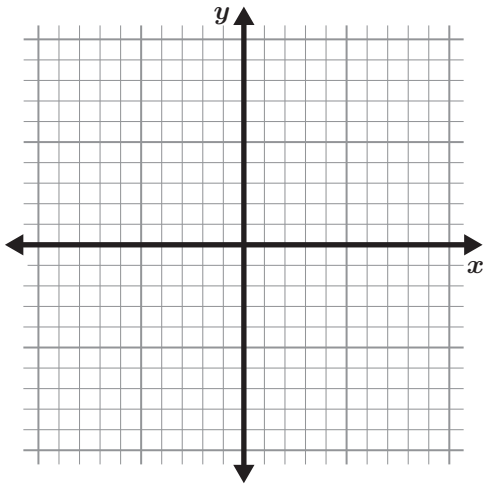
Thinking About Solutions

4. How many solutions does this system have? Circle one.

$\begin{cases} x + y = 5 \\ x + y = 7 \end{cases}$

One solution No solution Infinitely many solutions

Explain your thinking.



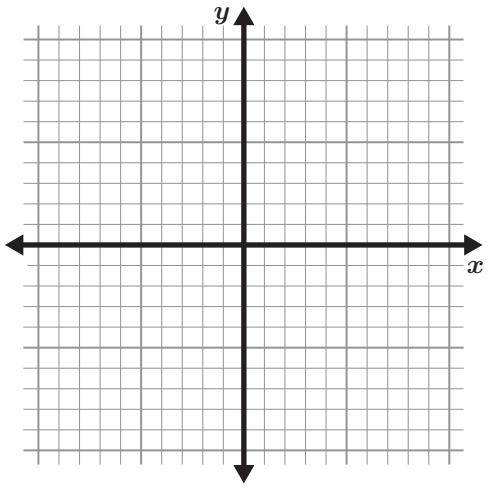
5. Martina says that (2, -4) is a solution to the system. Sai says there are infinitely many solutions.

$\begin{cases} y = 4(x - 3) \\ y = 4x - 12 \end{cases}$

Whose claim is correct? Circle one.

Martina's Sai's Both Neither

Explain your thinking.



Activity 2

Name: Date: Period:

The Choice Is Yours

System A

$$\begin{cases} y = 3x + 2 \\ 2x + y = 47 \end{cases}$$

System B

$$\begin{cases} y = 4 \\ y = -2 \end{cases}$$

System C

$$\begin{cases} y = \frac{1}{4}x + 7 \\ x = -4 \end{cases}$$

System D

$$\begin{cases} y = -3x + 10 \\ y = -2x + 6 \end{cases}$$

System E

$$\begin{cases} y = 3x + 5 \\ -3x + y = 5 \end{cases}$$

System F

$$\begin{cases} y = -3x - 5 \\ y = 4x + 30 \end{cases}$$

System G

$$\begin{cases} y = 3 \\ x = -2y + 56 \end{cases}$$

System H

$$\begin{cases} -4x + y = 30 \\ y = -3x - 5 \end{cases}$$

6. Examine these systems. Organize the systems of equations into two or three groups based on the patterns you notice.

Group 1	Group 2	Group 3

7. **Discuss:** How did you group the equations?

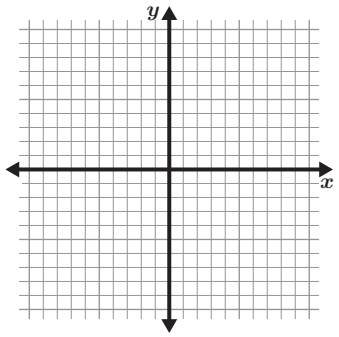
Name: ..... Date: ..... Period: .....

The Choice Is Yours (continued)

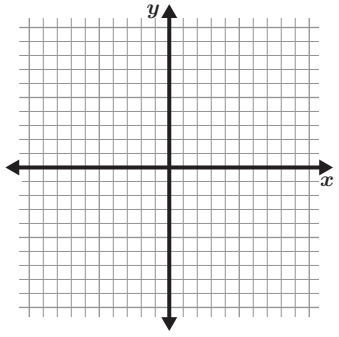
8. Choose *three* systems to solve. (Choose at least one from each group.)

Show your thinking and use the graphs if they help you to solve.

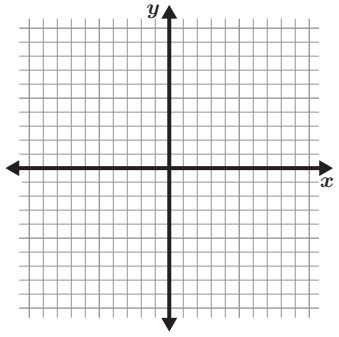
System 1:



System 2:

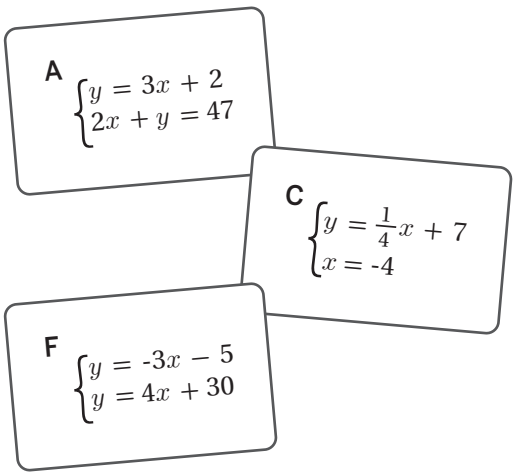


System 3:



Synthesis

9. What are some strategies for solving systems like these?



Summary 4.14

An ordered pair (x, y) is a solution to a system of equations if it makes both equations true. If you know the value of one variable in one of the equations, you can substitute it into the other equation to solve for the second variable.

Here is a system of equations:

$$\begin{cases} y = 5x \\ 2x - y = 9 \end{cases}$$

Since $y = 5x$, you can substitute $5x$ for y in $2x - y = 9$, and then solve for x.

$$\begin{aligned} 2x - (5x) &= 9 \\ -3x &= 9 \\ x &= -3 \end{aligned}$$

You can then substitute the solution for x into either of the original equations to determine the value of y.

$$\begin{aligned} y &= 5x \\ y &= 5(-3) \\ y &= -15 \end{aligned}$$

The ordered pair (-3, -15) is the solution to the system of equations.

Practice
4.14

Name: ..... Date: ..... Period: .....

Problems 1–2: Solve each system of equations.

1. $\begin{cases} y = 6x \\ 4x + y = 7 \end{cases}$

2. $\begin{cases} y = 3x \\ x = -2y + 70 \end{cases}$

Problems 3–6: Solve each system of equations. Show or explain your thinking.

3. $\begin{cases} y = 3x - 2 \\ y = -2x + 8 \end{cases}$

4. $\begin{cases} y = -3x - 5 \\ y = 4x + 30 \end{cases}$

5. $\begin{cases} y = 2x - 9 \\ y = 4 + 2x \end{cases}$

6. $\begin{cases} x = 2 \\ y = 3x - 1 \end{cases}$

7. Here is a system of equations: $\begin{cases} x = 14 \\ 2x - 5y = 13 \end{cases}$

In the solution (x, y), what is the value of y?

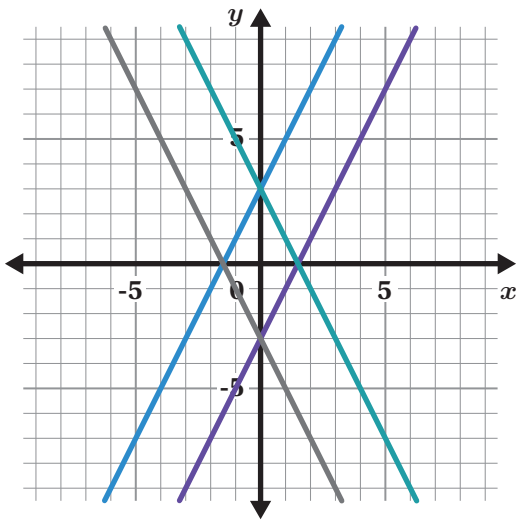
8. Here is an incomplete system of equations. Create a second equation so that the system has no solution.

$$\begin{cases} y = \frac{3}{4}x - 4 \\ \boxed{?} \end{cases}$$

Spiral Review

9. Label each line on the graph with its corresponding equation.

- Line a: $y = 2x + 3$
- Line b: $y = -2x + 3$
- Line c: $y = 2x - 3$
- Line d: $y = -2x - 3$



10. Solve the equation. Show your thinking.

$4\,(2a - 6) + 7 = -10 + 4a$

Name: _____ Date: _____ Period: _____

Linear Equations 8.EE.7, 8.EE.8, 8.EE.8.a, 8.EE.8.b, 8.EE.8.c, SMP.3, SMP.7

Practice Day 2

Let's practice what you've learned so far in this unit!



You will use task cards for this Practice Day. Record all of your responses here.

Task A: Second Draft

Equation A

1. Balanced move:

Unbalanced move:

2. Correct solution: _____

Work:

Equation B

1. Balanced move:

Unbalanced move:

2. Correct solution: _____

Work:

Task B: How Many Solutions?

1. a b c

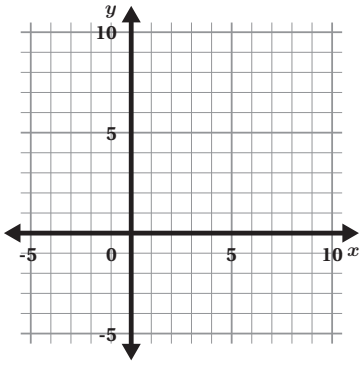
2. It means that:

3. Another equation: _____

Name: _____ Date: _____ Period: _____

Practice Day 2

Task C: More Than One Strategy

Strategy 1	Strategy 2	Can you find a third way?
		

2. I prefer this way because:

Task D: Solve It!

1. I chose System _____.

Solution: _____

2. I chose System _____.

Solution: _____

Name: ..... Date: ..... Period: .....

Practice Day 2

Task E: Paint It!

1.

Blue Paint (cups)	White Paint (cups)
1	4
5	
	12
18	

2. x represents:

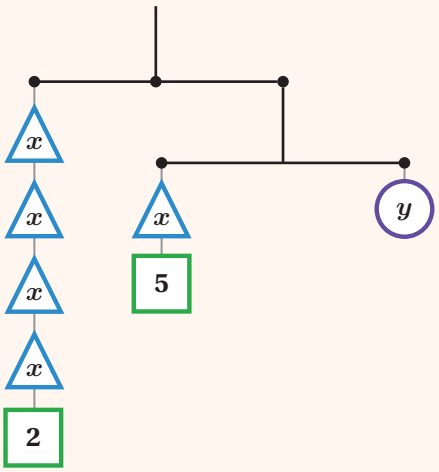
y represents:

3. Blue paint: ..... cups White paint: ..... cups

Work:

You're invited to explore more.

1. Determine values of x and y so that both hangers balance.



2. The graphs of the equations $Ax + By = 7$ and $Ax - By = 3$ intersect at the point $(1, -2)$. Determine the values of A and B.

Notes:

Career Connection

How can systems of linear equations help you outside of math class?

In Lesson 10, you used systems of equations to compare the plans of four different cell phone companies. Systems of linear equations can be used to help solve many real-world problems that involve constant rates of change, such as problems involving a constant hourly or monthly payment plan. Using systems of equations to model and solve real-world problems can be traced back thousands of years. Ancient Babylonians used the concept of systems of equations, even though they did not use algebraic symbols.

Educators work to invite and inspire others to grow their knowledge and understanding in one or more fields of study. Consider asking your math teacher why they chose to become an educator.



Meet Bob Moses

Born and raised in Harlem, Bob Moses studied philosophy at Hamilton College and Harvard University and was a high school math teacher before he became one of the most important leaders of the civil rights movement in the 1960s. From 1969 to 1976, he taught math in Tanzania. When he returned, he founded the Algebra Project to help American students gain the algebra skills they needed to qualify for honors math and science courses in high school and be ready for college. His work was motivated by the belief "that every child has a right to a quality education to succeed in this technology-based society and to exercise full citizenship."

Are you interested in becoming an educator or studying education? What can you do to learn more?

Community Connection

Research two companies that charge a monthly fee plus a one-time service fee. Write a system of equations to model the total cost based on the number of months of service. Determine the number of months for when the costs will be the same. Share the results with someone in your home or communities. Here are some ideas:

- Gym memberships that may charge a monthly fee plus a one-time joining fee.
- Streaming services that may charge a monthly fee plus a one-time setup fee.

Math Mindset

How can you determine the number of solutions to this system of linear equations, just by looking at the equations?

$$6x - y = 12$$
$$6x - y = 18$$