Amplify Desmos Math CALIFORNIA

Grade 7

Volume 2: Units 5–8

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

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Dear Student,

Welcome to Amplify Desmos Math California! We are excited to be partnering with you this year. You play an essential role in math class, so we wanted to reach out to introduce ourselves and tell you a bit about who we are.

Amplify Desmos Math California is a team of math educators on a mission to support you and your classmates in learning math. We hope each lesson inspires you to use your creativity, ask questions, and discover connections between math concepts and the world around us.

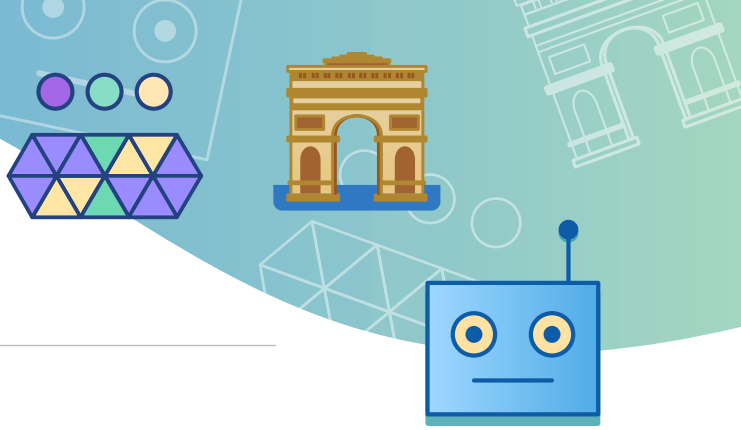
Here is what you can expect this year:

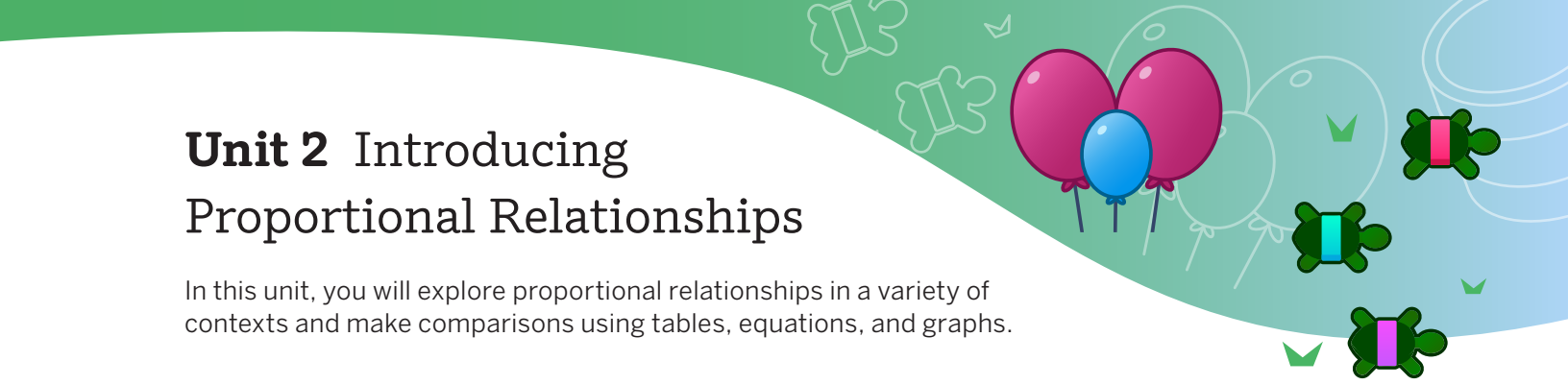
- A blend of learning on both paper and devices.
- Interactive lessons that encourage you to ask questions, explore, settle disputes, create challenges for your classmates, and more!
- Digital activities that show what your ideas mean with plenty of chances for you to revise.
- Opportunities for you to engage with interesting and important big ideas in mathematics.

–The Amplify Desmos Math California Team

Unit 1 Scale Drawings

In this unit, you will examine and create scaled copies of figures, as well as explore scale drawings of real-life objects.



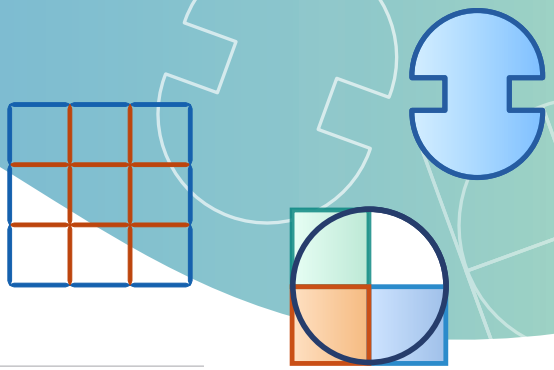


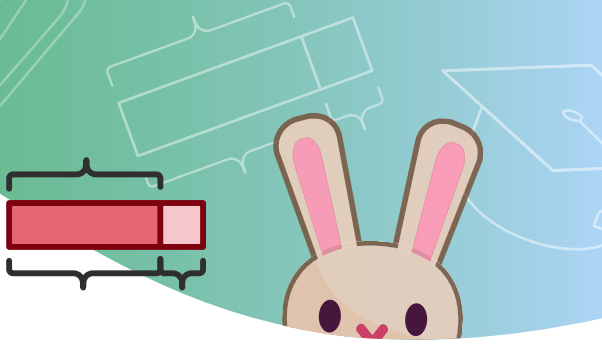
Unit 2 Introducing Proportional Relationships

In this unit, you will explore proportional relationships in a variety of contexts and make comparisons using tables, equations, and graphs.

Unit 3 Measuring Circles

In this unit, you will explore the relationships between the radius, diameter, circumference, and area of a circle. You will also practice writing and using equations to calculate missing measurements.





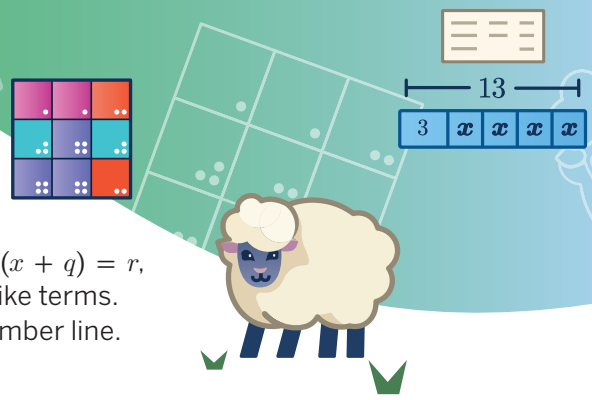
Unit 4 Proportional Relationships and Percentages

In this unit, you will apply what you have learned about proportional relationships to solve problems involving percent change and fractional quantities.

Unit 5 Operations with Positive and Negative Numbers

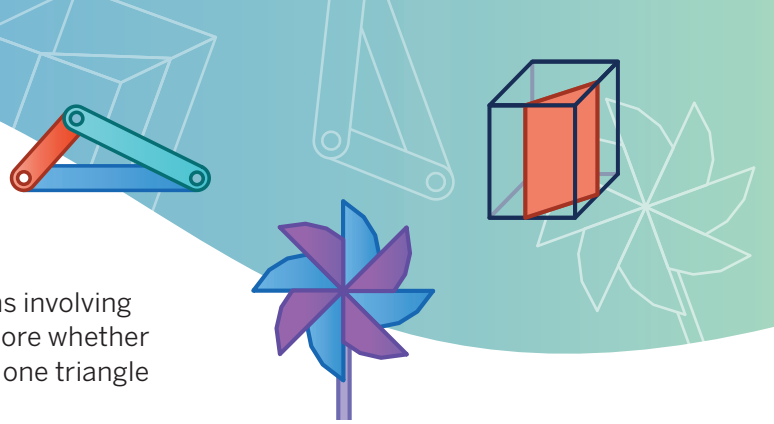
In this unit, you will develop and use a variety of strategies for adding, subtracting, multiplying, and dividing positive and negative numbers.

Unit 6 Expressions, Equations, and Inequalities

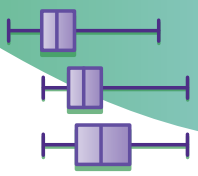
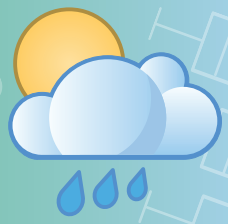


In this unit, you will solve equations of the form $px + q = r$ and $p(x + q) = r$, and equations that include expanding, factoring, or combining like terms. You will also solve inequalities and graph their solutions on a number line.

Unit 7 Angles, Triangles, and Prisms



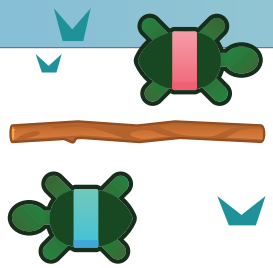
In this unit, you will solve real-life and mathematical problems involving angle measures, volume, and surface area. You will also explore whether it is possible to draw no triangles, one triangle, or more than one triangle given three measures of sides or angles.

Unit 8 Probability and Sampling

In this unit, you will learn about probability as a way to describe the likelihood of unknown events and use simulations to estimate the probability of real-world situations. You will also use samples to draw conclusions about and compare populations.

Unit 5



Operations With Positive and Negative Numbers

Big Ideas in This Unit

CC2 Proportional Relationships | Unit Rates in the World

NS Generalized Numbers Leading to Algebra | Number Line Understanding

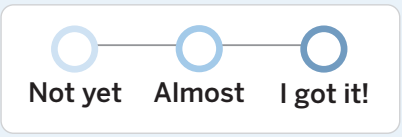
Questions for Investigation

- How do you represent addition, subtraction, multiplication, or division of numbers on a number line?
- How is solving problems with fractions or decimals the same or different from solving problems with only whole numbers?
- How can positive and negative numbers be used to represent real-world situations with equations, including those involving proportional relationships?

Explore: Target Zero
How can I get to zero in a card game?

Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



I can . . .	Before	After
Represent addition and subtraction on a horizontal or vertical number line diagram.		
Add and subtract rational numbers.		
Recognize that a number and its opposite have a sum of 0.		
Understand subtraction of rational numbers as adding the additive inverse $p - q = p + (\text{-}q)$.		
Show that the distance between two rational numbers on the number line is the absolute value of their difference.		
Apply properties of operations as strategies to add and subtract rational numbers.		
Apply properties of operations to calculate numbers in any form.		
Convert between different forms of numbers such as decimals to fractions.		
Multiply and divide rational numbers.		
Understand that integers can be divided if the divisor is not zero.		

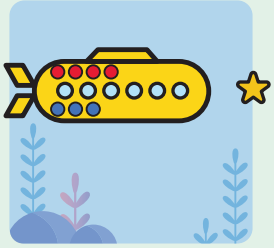
I can . . .	Before	After
Apply properties of operations as strategies to multiply and divide rational numbers.		
Solve real-world and mathematical problems with positive and negative numbers including whole numbers, fractions, and decimals.		
Solve real-world and mathematical problems involving the four operations with rational numbers.		

Sub-Unit

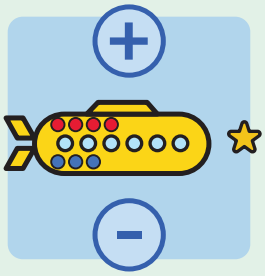
1

Adding and Subtracting

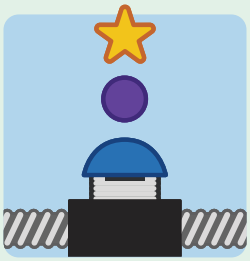
Explore
Target Zero



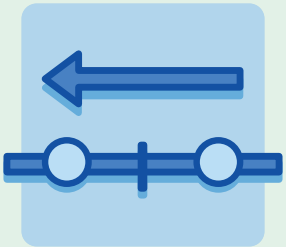
Lesson 1
Floats and Anchors



Lesson 2
More Floats and Anchors



Lesson 3
Bumpers



Lesson 4
Draw Your Own

Lesson 5
Number Puzzles

Name: ..... Date: ..... Period: .....

Number Line Understanding | Building Toward 7.NS.1, 7.NS.1.a, SMP.3, SMP.7

Explore:
Target Zero

How can I get to zero in a card game?

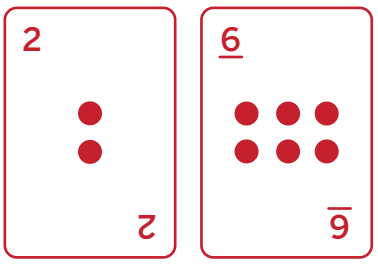
Warm-Up

1. Examine each set of cards and the score shown.

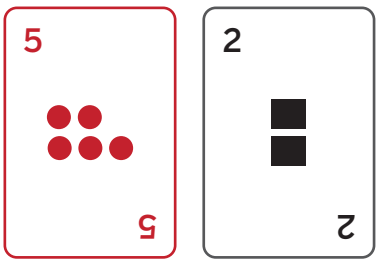
a Describe what you think is the rule for scoring each card set.

b Predict the score of Set 4.

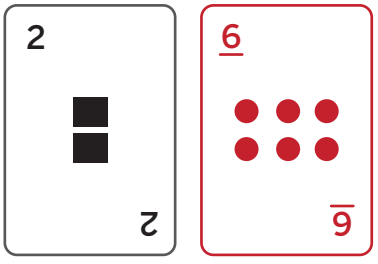
Set 1
Score: 8



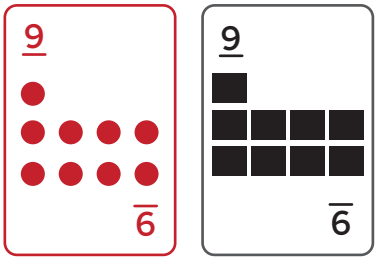
Set 2
Score: 3



Set 3
Score: 4



Set 4
Score: .....



Name: Date: Period:

Playing Strategically

2. Your group will be given a set of cards to play a game.The goal is to combine your card values to produce a value as close to zero as possible.

Getting ready:

- Shuffle the set of cards and place them in a pile in the middle of the group.
- Choose one player to start the game.

For each round:

- Each player takes two cards from the pile and places them, face up, in front of them.
- When all players have their cards, the first player decides whether they want to take one additional card from the pile, or pass (not do anything). This continues until all players have had a turn. **Note:** No player should have more than 3 cards.
- Complete the table for yourself at the end of each round, and compare your score to the other players. The player with the score closest to zero receives a check mark for winning the round.
- Reshuffle the cards for the next round.

	Cards	Score	Closest to 0?
Round 1			
Round 2			

3. How did you determine who was closest to zero after each round?

4. Consider the times when you decided to take an additional card.

Discuss

- What thinking helped you make your decision?
- If you had to change your strategy for the next round, how would you change it?

Name: Date: Period:

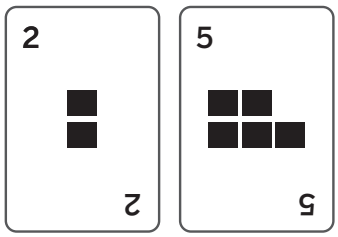
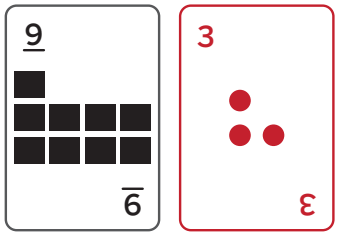
Playing Strategically (continued)

5. Continue playing the game with these updated rules.

- Each player will have three chances to take an additional card.
- Make sure your cards are not visible to the other players until the end of the round.
- After each player has taken all the cards they wish to take (a maximum of five cards), calculate your score.
- Show your cards and help each other confirm all scores are accurate. The player with the score closest to zero receives a check mark for winning the round.

	Cards	Score	Closest to 0?
Round 1			
Round 2			
Round 3			
Round 4			
Round 5			

6. Given each round, draw the ideal new card(s) to add to the current cards to win the game.

	Current Cards	New Card(s)
Round 1		
Round 2		

Name: Date: Period:

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet Almost I got it!

I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem.

Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed.

Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

Not yet Almost I got it!

Unit 5
Lesson
1

Name: ..... Date: ..... Period: .....

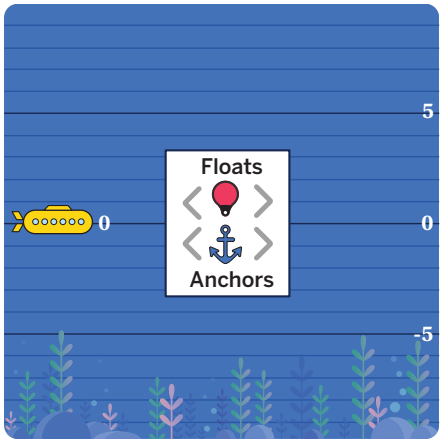
Number Line Understanding 7.NS.1, 7.NS.1.a, 7.NS.1.b, SMP.2, SMP.3, SMP.8

Floats and Anchors

Let's use floats and anchors to represent values on a number line.

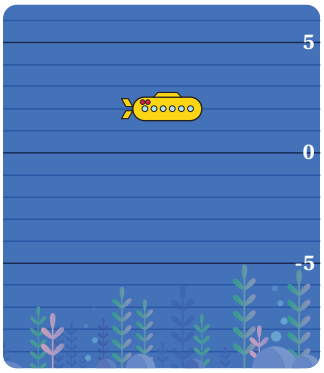
Warm-Up

1 This submarine is controlled by floats and anchors.

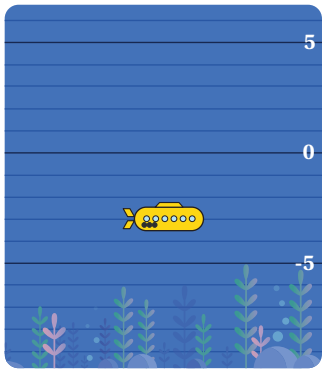


a Take a look at these different combinations.

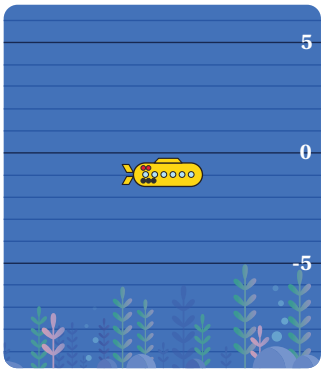
Add 2 floats



Add 3 anchors



Add 2 floats, Add 3 anchors



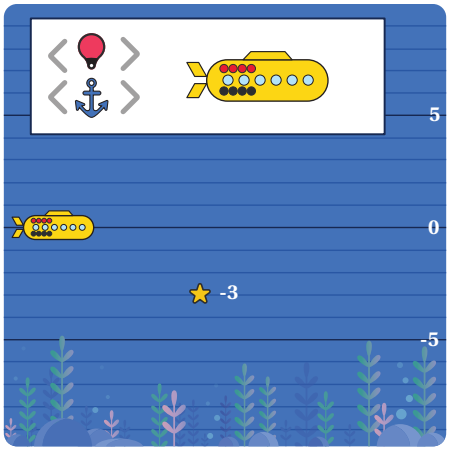
b **Discuss:** What do you notice? What do you wonder?

Activity 1

Name: ..... Date: ..... Period: .....

Collect the Star

2-3 This submarine starts with 4 floats and 4 anchors which represent **additive inverses**.



a **Discuss:** Why do you think this submarine's current position is at 0 units?

b The table shows one way to collect the star at -3 units. Write three more actions to get the submarine to -3 units.

Start	Action	Final
0	Remove 1 float Add 2 anchors	-3
0		-3
0		-3
0		-3

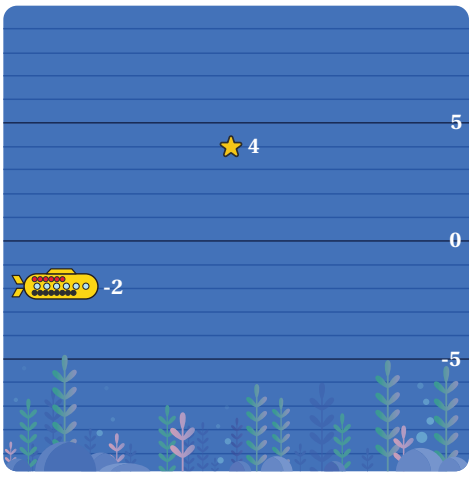
c Compare your answers with a partner's answers.

Discuss: Why do each of these actions result in the same final position?

4 This submarine starts with 6 floats and 8 anchors.

The submarine has space for up to 10 floats and up to 10 anchors.

Write an action that could move the submarine to 4 units to collect the star.



Start	Action	Final
-2		4

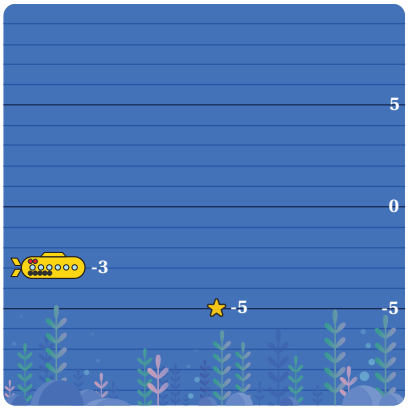
Name: ..... Date: ..... Period: .....

Collect the Star (continued)

5 For the previous problem where the submarine starts at -2 units, Ramon says to remove 2 anchors first and then add 4 floats to collect the star at 4 units.

Discuss: Why do you think Ramon says to remove 2 anchors first?

6 This submarine starts at -3 units.



Select *all* the actions that would move it to -5 units.

- ☐ **A.** Add 2 floats
- ☐ **B.** Add 2 anchors
- ☐ **C.** Remove 1 float and add 1 anchor
- ☐ **D.** Add 3 floats and add 5 anchors
- ☐ **E.** Remove 2 floats and add 4 anchors

7 Imagine a new submarine. For each action, put a check for whether the submarine would go up, go down, or stay in the same position.

Action	Up	Same Position	Down
Add 3 floats Add 4 Anchors			
Remove 10 anchors			
Remove 5 floats Remove 5 anchors			
Add 8 floats Remove 8 anchors			
Add 6 floats Add 2 anchors			
Remove 7 floats Add 3 anchors			
Add 6 anchors Add 6 floats			

Activity 2

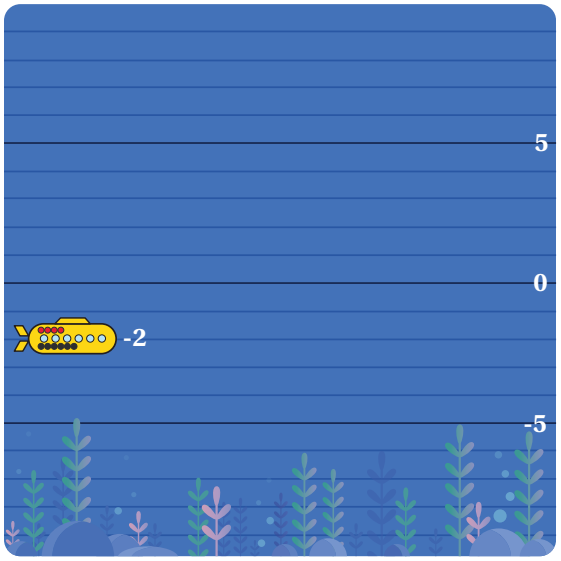
Name: Date: Period:

Sea-king Stars

8 The table shows the submarine's starting position and the action that will change its position.

What will be the submarine's final position?

Start	Action	Final
-2	Add 3 floats Remove 5 anchors	



9 Leo and Mai wrote expressions to answer the previous question.

Leo's expression: $-2 + 3 - 5$

Mai's expression: $-2 + 3 - (-5)$

Who wrote a correct expression? Circle one.

Leo Mai Both Neither

Explain your thinking.

Activity 3

Name: Date: Period:

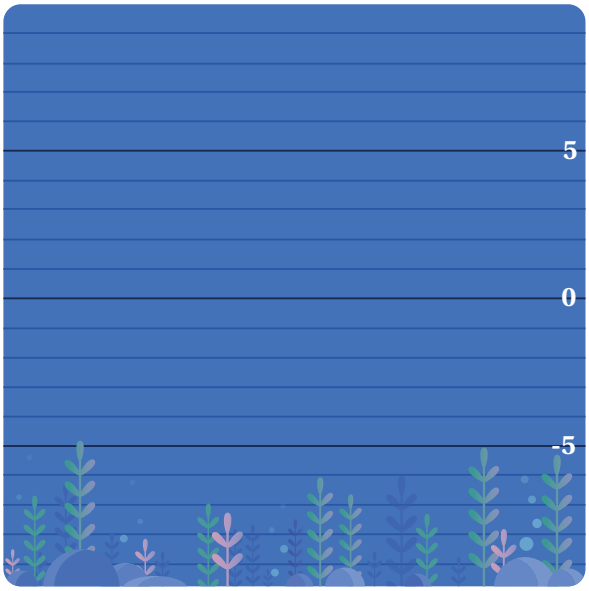
Captain's Challenge

10 A submarine is at 0 units. 4 floats and 4 anchors are added.

a What will be the submarine's final position?

b Select *all* the statements that would combine to a value of 0

- ☐ **A.** A submarine is at 0 units. 3 floats and 3 anchors are added.
- ☐ **B.** It is -5 degrees outside. The temperature rises 5 degrees.
- ☐ **C.** An account has a balance of $8. Another $8 is added.
- ☐ **D.** A diver is at a depth of -7 feet and then ascends 7 feet.



- ☐ **E.** A piggy bank contains $2.50 in the morning. Later that day, $2 is removed.

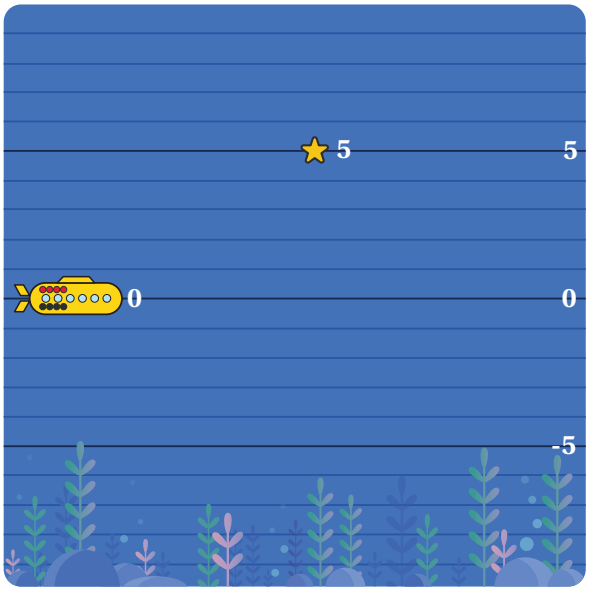
11 What is the final position of each submarine? Complete as many challenges as you have time for. Use the image if it helps with your thinking.

Start	Action	Final
0	Add 3 floats Add 7 anchors	
-9	Add 8 floats Remove 6 anchors	
1	Remove 3 floats Add 4 anchors	
-3	Remove 1 float Add 3 anchors	
-4	Remove 1 float	
5	Remove 8 floats Remove 3 anchors	

12 Synthesis

Describe a set of actions that would allow this submarine to collect the star at 5 units.

Try to come up with something none of your classmates will.



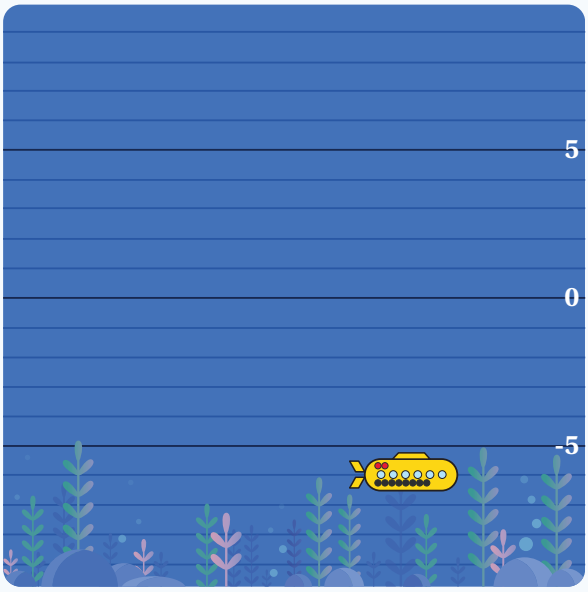
15 Summary 5.01

Using models such as floats and anchors on a vertical number line can be useful when representing addition and subtraction of positive and negative numbers.

For example, imagine a submarine whose position is at -6 units. The submarine will move from its position as 3 floats are added and 2 anchors are removed.

- 3 floats being added represents moving up 3 units or $+3$.
- 2 anchors being removed represents moving up 2 units or $-(-2) = +2$.

The submarine's new position would be $-6 + 3 + 2 = -1$ units.



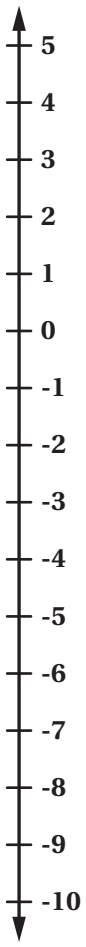
To move the submarine to 0 units from -1 units, 1 float can be added, represented by $+1$. -1 and $+1$ are *opposites*. When added together, they add to 0 because they are *additive inverses*.

additive inverse The additive inverse of a number a is the number that, when added to a, gives a sum of zero. It is the number's opposite.

Practice
5.01

Name: ..... Date: ..... Period: .....

Problems 1–5: One moment in December, it was -8°C in Harbin, China, and -2°C in Beijing, China. Use the number line if it helps with your thinking.



1. Which city was colder?

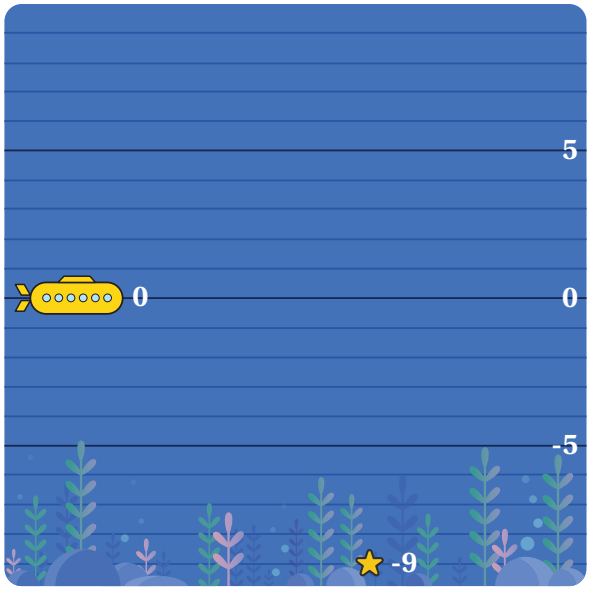
2. At the same moment, it was 7 degrees warmer in Shanghai than it was in Beijing. What was the temperature in Shanghai?

3. How many degrees warmer was it in Shanghai than in Harbin?

4. Later in the day, Beijing's temperature reached 0°C. Did the temperature rise or fall to reach 0°C? By how much?

5. Later in the day, Harbin's temperature was 1°C. By how much did Harbin's temperature change? Explain your thinking.

6. This submarine has 0 floats and 0 anchors. The submarine can hold up to 10 floats and 10 anchors.



a List *all* the combinations of floats and anchors that could collect the star at -9 units.

b How do you know there are no other combinations?

Practice
5.01

Name: ..... Date: ..... Period: .....

Spiral Review

7. Fill in each blank using the symbols $>$, $<$, or $=$.

3		-3
12		24
-12		-24
7		7.2
-7		-7.2

Problems 8–10: A color of green paint is made by mixing 2 cups of yellow paint with 3.5 cups of blue paint.

8. Complete the table to show how many cups of yellow and blue paint will make the same color of green but in a smaller amount.

Yellow Paint (cups)	Blue Paint (cups)

9. Complete the table to show how many cups of yellow and blue paint will make the same color of green but in a larger amount.

Yellow Paint (cups)	Blue Paint (cups)

10. Will a mixture that is 3 cups of yellow and 4.5 cups of blue be more blue, more yellow, or the same color of green as the original mixture? Explain your thinking.

11. Remy orders a meal that costs \$20. There is no tax on the meal, and Remy decides to tip 18%. Select the expression that represents the total cost, in dollars, including the tip.

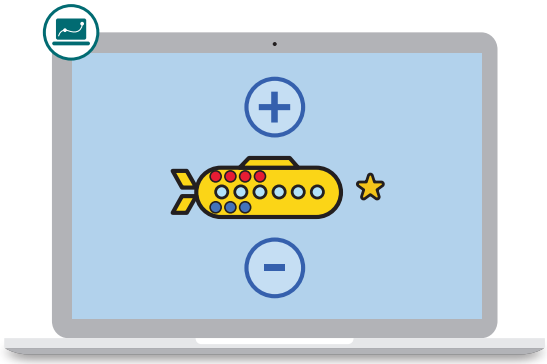
A. $20 + 0.18$ **B.** $20 + 1.18$ **C.** $20 \cdot 0.18$ **D.** $20 \cdot 1.18$

Name: ..... Date: ..... Period: .....

Number Line Understanding 7.NS.1, 7.NS.1.c, 7.NS.3, SMP.1, SMP.3, SMP.7

More Floats and Anchors

Let's use floats and anchors to reason about adding and subtracting positive and negative numbers.



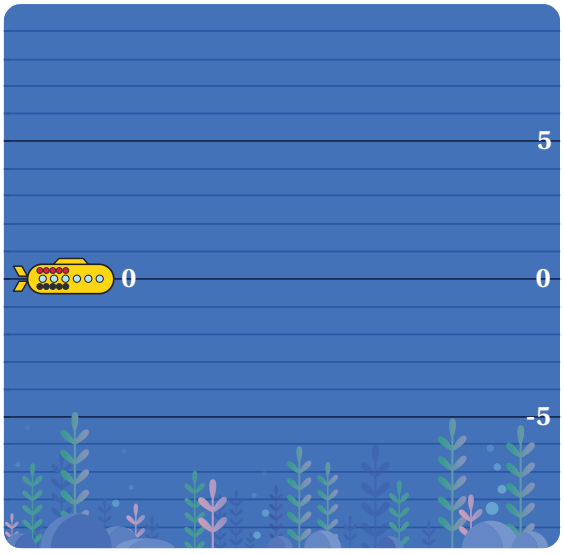
Warm-up

1 *If you add some floats and remove some anchors, the submarine will go up.*

Is this statement always, sometimes, or never true? Circle one.

Always Sometimes Never

Explain your thinking.

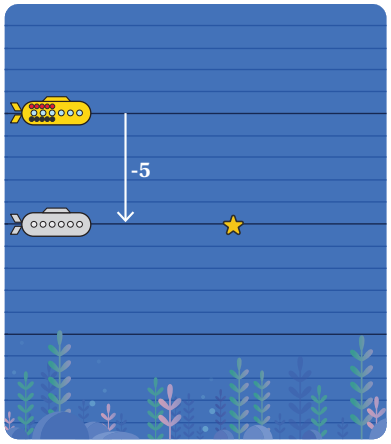


Activity 1

Name: ..... Date: ..... Period: .....

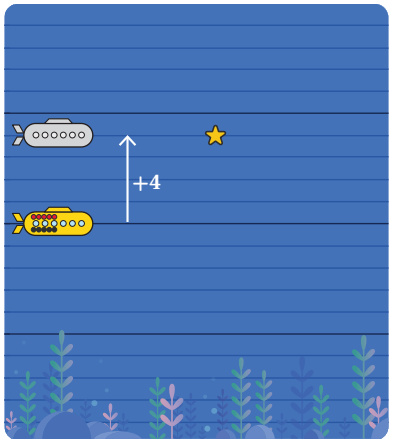
Ups and Downs

2 Select *all* the actions that would make this submarine go down 5 units.



- ☐ **A.** Add 5 floats
- ☐ **B.** Remove 5 floats
- ☐ **C.** Add 5 Anchors
- ☐ **D.** Remove 5 anchors
- ☐ **E.** Add 3 anchors and add 2 floats

3 Select *all* the actions that would make this submarine go up 4 units.



- ☐ **A.** Add 4 floats
- ☐ **B.** Remove 4 floats
- ☐ **C.** Add 4 Anchors
- ☐ **D.** Remove 4 anchors
- ☐ **E.** Add 3 floats and remove 1 anchor

4-5 Here are the details for four new submarine scenarios.

Complete the table.

Start	Action	Final Expression	Final Value
3	Add 7 anchors	$3 + (-7)$	
	Remove 7 floats	$3 - 7$	
-2		$-2 + 8$	
		$-2 - (-8)$	

Discuss: What do you notice? What do you wonder?

Name: ..... Date: ..... Period: .....

Depths of Understanding

6 Marc and Naoki are trying to evaluate $3 - (-2)$

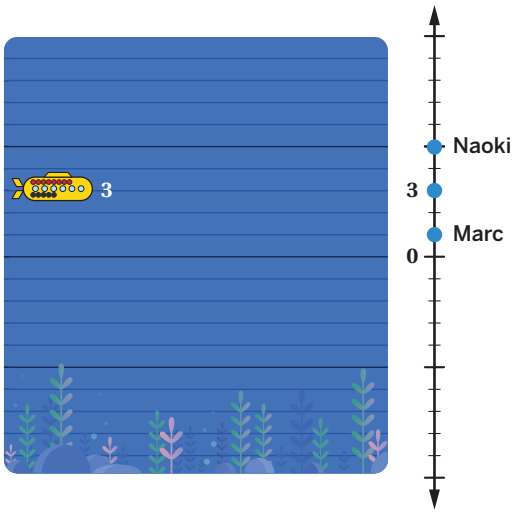
Marc says: *This is like adding 2 anchors, so the submarine goes down to 1.*

Naoki claims: *This is like removing 2 anchors, so the submarine goes up to 5.*

Whose thinking is correct? Circle one.

Marc (1) Naoki (5) Both Neither

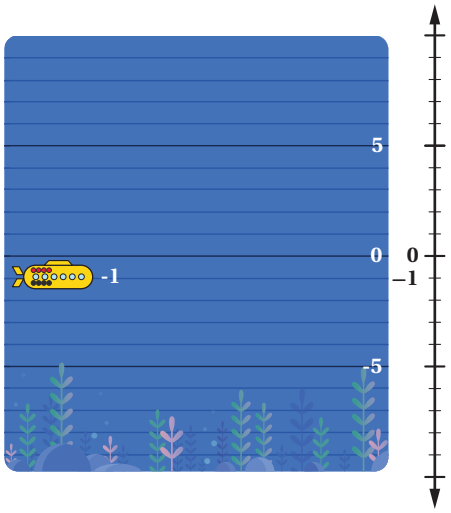
Explain your thinking.



7 What is the value of $-1 + (-4)$?

What is the value of $-1 - (-4)$?

Show or explain your thinking.



Name: ..... Date: ..... Period: .....

Depths of Understanding (continued)

8 Group the expressions into pairs that have the same value.

$-4 - (-10)$	$4 + (-10)$	$-4 + 10$	$-4 - 10$
$-4 + (-10)$	$4 + 10$	$4 - (-10)$	$4 - 10$

Pair 1	Pair 2
Pair 3	Pair 4

You're invited to explore more.

9 **a** Determine the value of each expression.

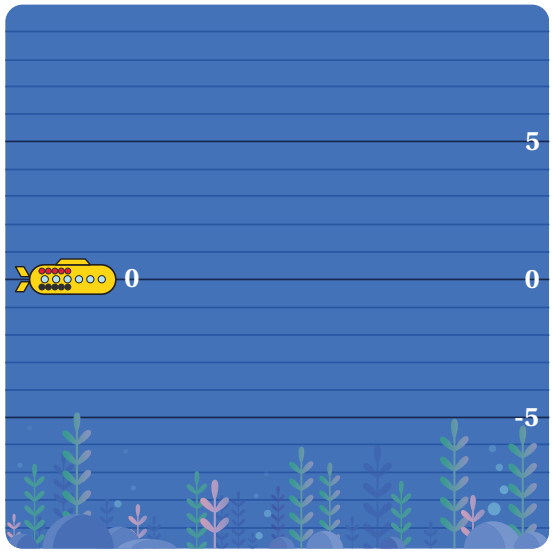
Expression	Value
1	
$1 - 2$	
$1 - 2 + 3$	
$1 - 2 + 3 - 4$	
$1 - 2 + 3 - 4 + 5$	

b Describe any patterns you notice.

c What is the value of the next expression? The 10th expression? The 100th expression?

10 Synthesis

Discuss: How can you use floats and anchors to explain why $0 - (-5)$ is equivalent to $0 + 5$?



13 Summary 5.02

Different combinations of floats and anchors can give you the same result. Here are some examples:

- If a submarine starts at 4 units, adding 2 floats or removing 2 anchors will both result in the submarine moving up to 6 units. So adding a positive number is the same as subtracting a negative number.

Adding Floats	Removing Anchors
$4 + 2 = 6$	$4 - (-2) = 6$

- If a submarine starts at 5 units, removing 1 float or adding 1 anchor will both result in the submarine moving down to 4 units. So subtracting a number is the same as adding its opposite.

Removing Floats	Adding Anchors
$5 - 1 = 4$	$5 + (-1) = 4$

Name: ..... Date: ..... Period: .....

Problems 1–4: Determine the value of each expression.

1. $5 + (-3)$

2. $-5 + 3$

3. $-5 - 3$

4. $-5 - (-3)$

5. The temperature was 13°F and then dropped 5 degrees. What was the final temperature?

6. The temperature was -13°F and then dropped 5 degrees. What was the final temperature?

7. The temperature was -13°F and then rose to 5°F. What was the *change* in temperature? Explain your thinking.

8. A swimmer was 8 feet underwater. Then he swam 3 feet deeper. Riku wrote the expression $-8 - 3$. Charlie wrote the expression $-8 + (-3)$. Explain why both Riku and Charlie are correct.

Name: _____ Date: _____ Period: _____

Problems 9–10: The table shows eight expressions.

9. Determine the value of each expression.

	Expression	Value
Expression 1	$1 + 2 - 3$	
Expression 2	$1 + 2 - 3 + 4$	
Expression 3	$1 + 2 - 3 + 4 - 5$	
Expression 4	$1 + 2 - 3 + 4 - 5 + 6$	
Expression 5	$1 + 2 - 3 + 4 - 5 + 6 - 7$	
Expression 6	$1 + 2 - 3 + 4 - 5 + 6 - 7 + 8$	
Expression 7	$1 + 2 - 3 + 4 - 5 + 6 - 7 + 8 - 9$	
Expression 8	$1 + 2 - 3 + 4 - 5 + 6 - 7 + 8 - 9 + 10$	

10. What is the value of the next expression? The 10th expression? The 20th expression?

Spiral Review

Problems 11–14: Complete each statement with a value that makes the statement true.

11. _____ < 13

12. _____ < -0.1

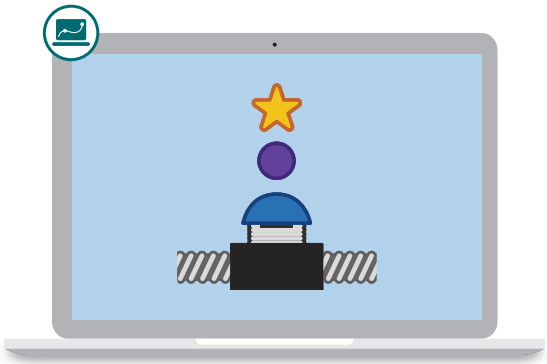
13. _____ > -2

14. $\frac{1}{2} >$ _____

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra | Number Line Understanding

7.NS.1.b, 7.NS.1.d, 7.NS.3, SMP.5, SMP.8



Bumpers

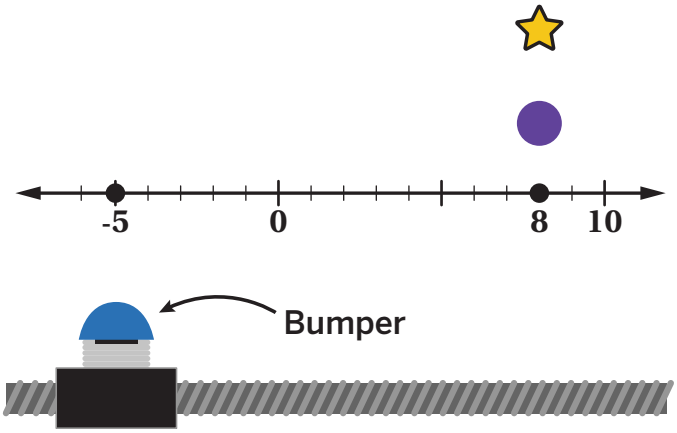
Let's add numbers including decimals and fractions on a number line.

Warm-up

1 Hitting the ball with the bumper collects the star.

The bumper is at -5 units and the ball is at 8 units.

Write the number that would move the bumper to be directly below the purple ball to hit it.



2 $-5 + x = 8$ is an equation that represents this situation.

a What does each part of the equation represent?

-5 represents . . .

x represents . . .

8 represents . . .

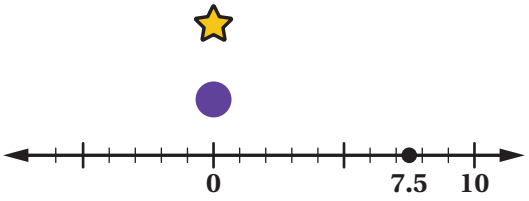
b **Discuss:** How might you solve this equation?

Name: ..... Date: ..... Period: .....

Things Could Get Bumpy

3 The bumper is at 7.5 units and the ball is at 0 units.

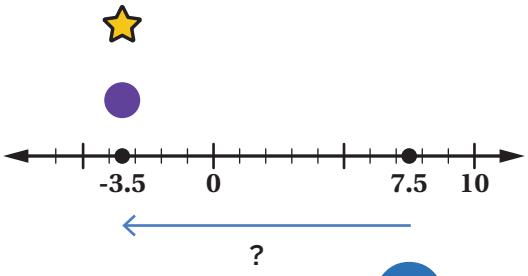
Write the number that would move the bumper and hit the ball.

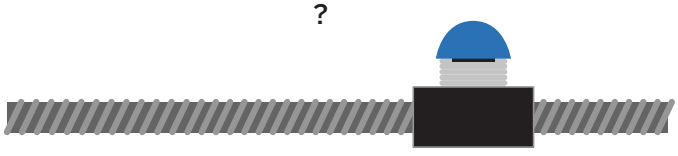


$7.5 + x = 0$

4 The bumper is at 7.5 units and the ball is at -3.5 units.

a **Discuss:** How can you make the bumper move to the left?





$7.5 + x = -3.5$

b Write the number that would move the bumper and hit the ball.

c How can you use additive inverses to explain your thinking?

5 Dalia says that the expression $7.5 + 3.5$ can be used to know how to move the bumper.

Do you agree? Circle one.

Agree Disagree I'm not sure

Explain your thinking.

Activity 2

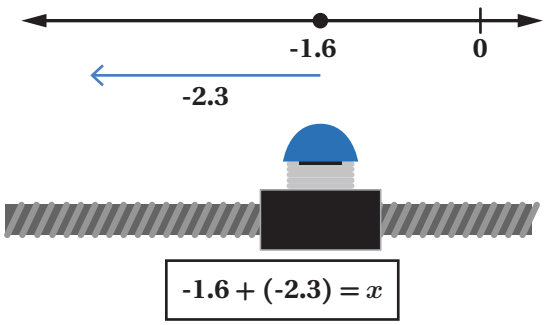
Name: Date: Period:

Carefully Placed

6 This bumper is at -1.6 units.

It's programmed to move -2.3 units.

Where should the ball be placed so that the bumper hits it?

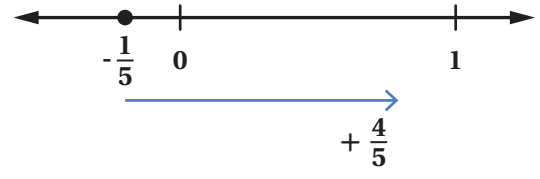


$-1.6 + (-2.3) = x$

7 This bumper is at $-\frac{1}{5}$ units.

It's programmed to move $\frac{4}{5}$ units.

Where should the ball be placed?

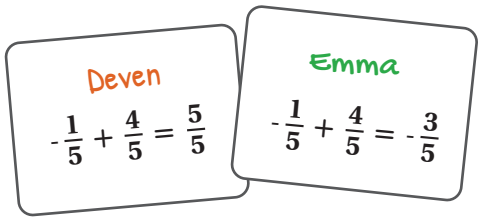


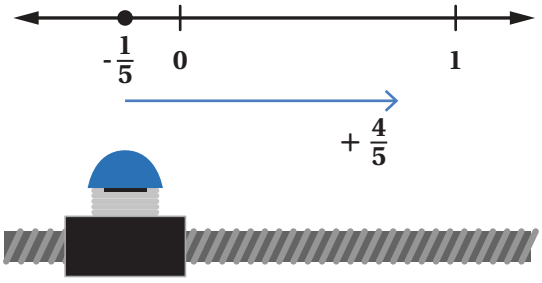
$\left(-\frac{1}{5}\right) + \frac{4}{5} = x$

8 Two students made mistakes on the previous challenge.

Circle the card with your favorite mistake.

What could you say to help the student understand their mistake?





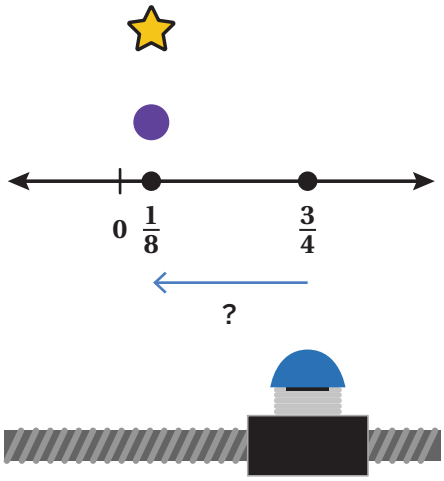
Activity 3

Name: ..... Date: ..... Period: .....

Bumper Challenge

9 What is the value of x that makes this equation true?

$\frac{3}{4} + x = \frac{1}{8}$



10 What is the value of x that makes each equation true? Solve as many challenges as you have time for.

a $4 + x = 10$

b $7 + x = 2$

c $-1.3 + x = 7.2$

d $\frac{9}{5} + x = \frac{3}{5}$

e $10 + (-2.5) = x$

f $8.1 + x = -1$

g $x + 8.4 = -4.2$

h $x + \frac{1}{6} = -\frac{2}{3}$

i $x + \left(-\frac{9}{2}\right) = -1$

j $x + (-13.2) = -7.6$

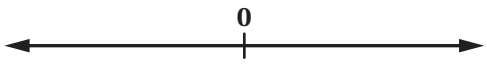
k $\frac{1}{2} + 2 = x$

l $5.7 + 0.4 = x$

11 Synthesis

Discuss: How can you use a number line to determine the value of x in an equation like $3.1 + x = -2$?

Use the number line if it helps you show your thinking.

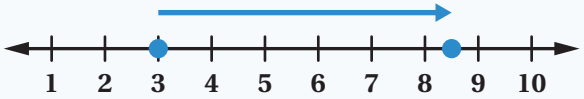


14 Summary 5.03

When you add positive and negative decimals and fractions, it might help to use a number line and think of each equation as representing *start* + *change* = *end*.

For example, in the equation $3 + (-5.5) = x$, 3 represents the starting location, -5.5 represents the change (moving a distance of 5.5 units to the left), and x represents the end location.

In the equation $3 + (5.5) = x$, 3 represents the starting location, 5.5 represents the change (moving a distance of 5.5 units to the right), and x represents the end location.



Practice
5.03

Name: ..... Date: ..... Period: .....

Problems 1–3: Determine the value of the variable that makes each equation true.

1. $40 + a = 30$

2. $-3.5 + c = 4.5$

3. $d + 2.4 = 0.9$

Problems 4–5: Evaluate each expression.

4. $2 - 3$

5. $-2 - 3$

6. Select the equation that is represented by this number line.

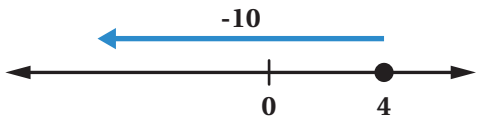
A. $-1 + x = 4$

B. $4 + x = -1$

C. $-1 + 4 = x$

D. $4 + (-1) = x$

7. Select the equation that is represented by this number line.



A. $4 + x = -10$

B. $-10 + x = 4$

C. $4 + (-10) = x$

D. $x + (-10) = 4$

Practice
5.03

Name: ..... Date: ..... Period: .....

Spiral Review

8. Last week, it rained g inches. This week, the amount of rain decreased by 5%. Which expressions represent the amount of rain that fell this week? Select *all* that apply.

- ☐ **A.** $g - 0.05$
- ☐ **B.** $g - 0.05g$
- ☐ **C.** $0.95g$
- ☐ **D.** $0.05g$
- ☐ **E.** $(1 - 0.05)g$

Problems 9–11: Determine whether each equation represents a proportional relationship. Explain your thinking.

9. Volume measured in cups, c, and the same volume measured in ounces, z. $c = \frac{1}{8}z$

10. Length, l, and width, w, for a rectangle whose area is 60 square units. $l = \frac{60}{w}$

11. Distance in miles, d, traveled by a car going at a constant speed after t hours. $d = 45t$

12. Tiam bought a train ticket online. The original price of the train ticket was $\$83.00$. Tiam used a coupon to receive a 20% discount. A sales tax of 9% was applied after the discount. How much did Tiam end up paying?

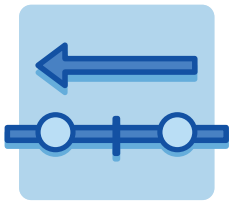
A. $\$18.09$ **B.** $\$56.44$ **C.** $\$72.38$ **D.** $\$75.00$

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra | Number Line Understanding

7.NS.1, 7.NS.1.b, 7.NS.1.c, 7.NS.1.d, 7.NS.3, SMP.7

Draw Your Own

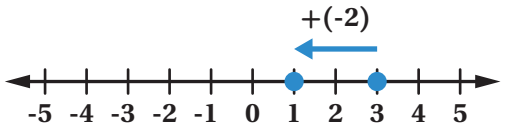


Let's use number lines to reason about addition and subtraction.

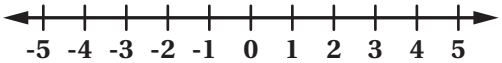
Warm-up

1. Renata drew a number line for $3 + (-2)$. Use her thinking to complete a number line diagram to determine the value of $-2 + 3$.

$3 + (-2) = 1$



$-2 + 3 =$ .....



Name: ..... Date: ..... Period: .....

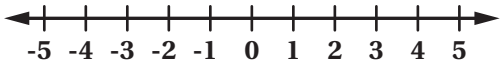
Pluses and Minuses

2. Renata says her number line for $3 + (-2)$ could also represent $3 - 2$ because adding two anchors has the same effect as removing two floats. Do you agree? Circle one.

Yes No I'm not sure

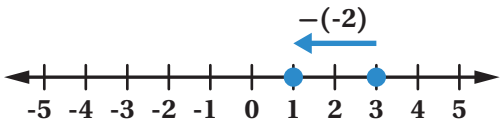
Explain your thinking.

3. Use your thinking from the previous problem to represent $-1 - 4$ on this number line.

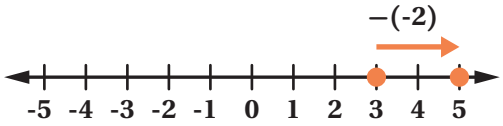


4. Irelle and Haruto each drew a number line diagram for $3 - (-2)$.

Irelle



Haruto



Whose number line diagram is correct? Circle one.

Irelle Haruto Both Neither

Explain your thinking.

5. Match each addition expression to a subtraction expression with the same value.

Subtraction Expression	Addition Expression
a. $9 - 7$	..... $-9 + (-7)$
b. $-9 - 7$	..... $-9 + 7$
c. $9 - (-7)$	..... $9 + 7$
d. $-9 - (-7)$	..... $9 + (-7)$

6. **Discuss:** How can addition be helpful when representing subtraction on a number line? Use the expressions from the previous problem if they help with your thinking.

Activity 2

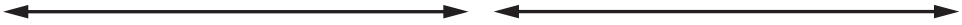
Name: _____ Date: _____ Period: _____

Draw Your Own Diagram

7. For each pair of expressions, complete the number line diagrams and determine the value of each expression.

a $7 - 4 =$ _____ $4 - 7 =$ _____

b $3 - (-9) =$ _____ $-9 - 3 =$ _____



c $-3.4 - (-2) =$ _____ $-2 - (-3.4) =$ _____

d $-3.4 + (-2) =$ _____ $-2 + (-3.4) =$ _____

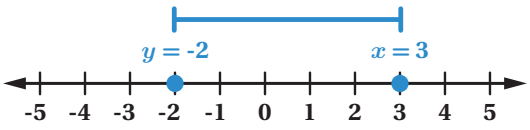
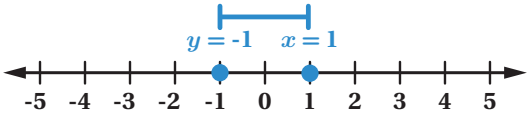
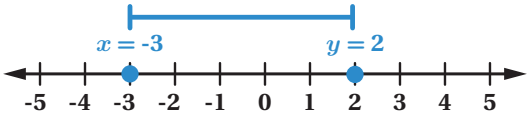
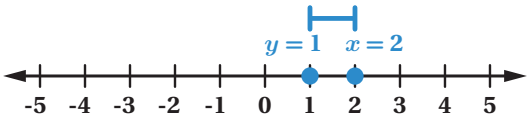
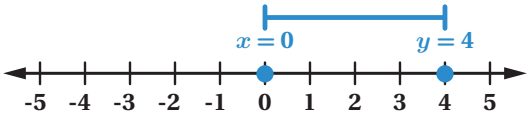
8. Describe 2–3 patterns you notice from the previous problem about the number line diagrams or their values.

Activity 3

Name: Date: Period:

Draw Your Own Conclusion

9. Lukas says: *The distance between x and y on a number line is $x - y$.* Complete the table below to explore this idea.

	Distance	$x - y$	Are these values the same?
			yes no
			yes no
			yes no
			yes no
			yes no

10. **Discuss:** Lukas's statement is *sometimes* true. How can you adjust his idea to make it so that it is *always* true?

You're invited to explore more.

11. Select one statement. Explain whether it is always, sometimes, or never true. Use examples, words, or number line diagrams to support your claim.

Statement A

$x - y$ is the opposite of $y - x$.

Statement B

x is less than $x + y$.

Statement C

$x - y$ is greater than $x + y$.

Synthesis

12. What happens to the value of a subtraction expression when you rearrange the order of the numbers? Use the examples if they help with your thinking.

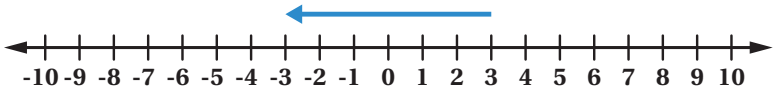
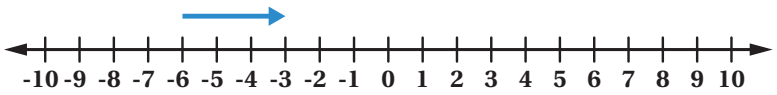
$7 - 4$ and $4 - 7$

$-2 - (-3.5)$ and $-3.5 - (-2)$

Summary 5.04

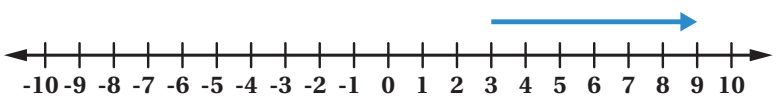
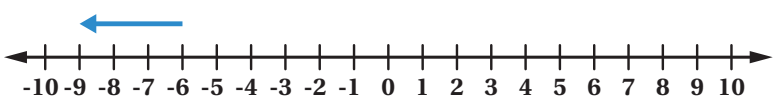
When determining the *sum* of terms in an expression, the order of the values does not change the final result. This is an example of the *commutative property*.

To represent addition on a number line, start at one of the values and use the other value for the direction and distance of the change.

$3 + (-6)$	
$-6 + 3$	

When determining the *difference*, the order of the values in an expression *does* affect the final result.

To represent subtraction on a number line, start at the first value and then use the second value to determine the direction and distance of the change. If subtracting a positive number, move to the left; if subtracting a negative number, move to the right.

$3 - (-6)$	
$-6 - 3$	

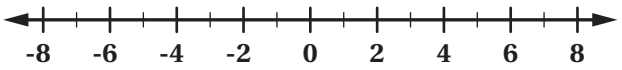
The distance between two values on a number line is the *absolute value* of their difference. For example, the distance between 3 and -6 on the number line is 9. $3 - (-6) = 9$ and $-6 - 3 = -9$. The absolute value of 9 and -9 is 9.

Practice
5.04

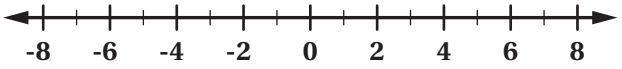
Name: ..... Date: ..... Period: .....

Problems 1–3: Use the number lines to determine the value of each expression.

1. $4 - 6$



2. $-3 + (-2)$



3. $-8 - (-3)$

Problems 4–9: Complete the tables and answer the follow-up questions.

4.

Expression	Value
$2 + (-3)$	
$-8 + 4$	
$-2 + (-7)$	-9
$-3.6 + (-2.6)$	

5.

Expression	Value
$-3 + 2$	-1
$4 + (-8)$	-4
$-7 + (-2)$	
$-2.6 + (-3.6)$	

6. Use your work from the previous tables to describe any patterns you notice.

7.

Expression	Value
$3 - 2$	1
$5 - (-9)$	14
$-6 - (-3)$	
$-1.5 - (-4.7)$	

8.

Expression	Value
$2 - 3$	
$-9 - 5$	
$-3 - (-6)$	3
$-4.7 - (-1.5)$	

9. Use your work from the previous tables to describe any patterns you notice.

Practice
5.04

Name: ..... Date: ..... Period: .....

Spiral Review

10. The current temperature of water on a stove is 198 degrees Fahrenheit. How much should the temperature increase to reach boiling point? (Water boils at approximately 212 degrees Fahrenheit).

11. According to the USDA, hamburger patties should be cooked until the internal temperature is 160°F, for safety. Valeria takes out a frozen patty to cook. Its current temperature is -7°F. How much does its temperature need to increase to reach 160°F?

Problems 12–14: Bettie's Boutique is having a 20% off sale.

12. Complete the table to determine the sale price for each item.

Item	Original Price	Sale Price
Hat	$15	
Tie	$25	
Scarf	$35	

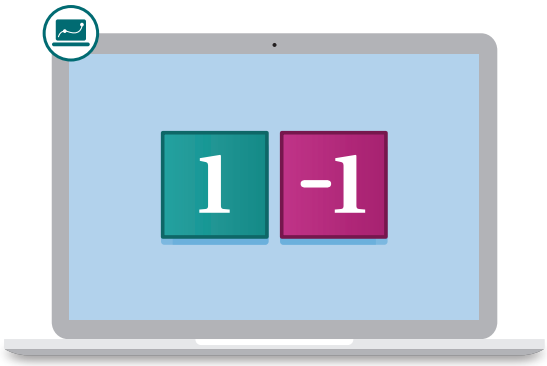
13. Explain why the relationship between original price and sale price is proportional.

14. Select *all* the equations that represent the relationship between the original price, x, and the sale price, y.

- ☐ **A.** $y = 0.8x$
- ☐ **B.** $y = x - 0.2$
- ☐ **C.** $y = x - 0.2x$
- ☐ **D.** $y = 0.2x$
- ☐ **E.** $y = (1 - 0.2)x$

Name: ..... Date: ..... Period: .....

Number Line Understanding 7.NS.1, 7.NS.1.c, 7.NS.1.d, 7.NS.3, 7.EE.3, SMP.1



Number Puzzles

Let's solve some puzzles involving positive and negative numbers.

Warm-up

1 The equation $1 + 2 = 3 + 4$ is false.

a Circle one or more numbers and change them to the opposite number to make the equation true.

b Explain how you know your equation is true.

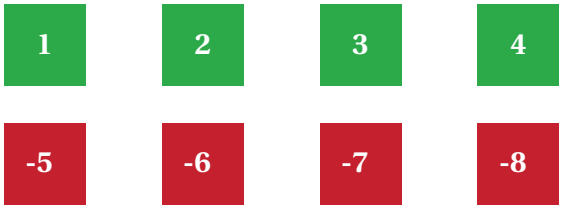
Activity 1

Name: ..... Date: ..... Period: .....

Number Puzzles

For each number puzzle:

- Make true equation(s) by using the numbers to fill in the blanks.
- Use each number only once per puzzle.
- Circle a number and change it to its opposite. Try to circle as few numbers as possible.



2 ☐ + ☐ = ☐ − ☐

3 ☐ + 5 = ☐

☐ − ☐ = 9

4 Do you think it's possible to solve the previous puzzle using 0 circles?

Yes No I'm not sure.

Explain your thinking.

Name: ..... Date: ..... Period: .....

Number Puzzles (continued)

For each number puzzle:

- Make true equation(s) by using the numbers to fill in the blanks.
- Use each number only once per puzzle.
- Circle a number and change it to its opposite. Try to circle as few numbers as possible.

1	2	3	4
-5	-6	-7	-8

5 ☐ + ☐ + ☐ = 0

6 ☐ + ☐ + ☐ = 0

☐ − ☐ − ☐ = 9

Activity 2

Name: ..... Date: ..... Period: .....

More Number Puzzles

The previous puzzles used only *integers*. These puzzles do not. For each number puzzle:

- Make true equation(s) by using the numbers to fill in the blanks.
- Use each number only once per puzzle.
- Circle a number and change it to its opposite. Try to circle as few numbers as possible.

7

-0.5	1	-1.5	2
-2.5	3	-3.5	4

$\square + 5 + \square = \square$

$-4.5 - \square - \square = \square$

8

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{8}$
$-\frac{3}{8}$	$-\frac{5}{8}$	$-\frac{7}{8}$	-1

$\square + \square - \square = 0$

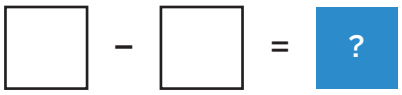
$\square - \square + \square = 1$

You're invited to explore more.

9 Use the You're Invited to Explore More Sheet to explore another number puzzle.

10 Synthesis

Imagine subtracting a pair of numbers.



Discuss: How can you tell whether the result will be positive or negative?

13 Summary 5.05

When adding and subtracting *integers*, fractions, and decimals, there are multiple paths to the same value. Here are some strategies for adding and subtracting positive and negative numbers:

- Imagine the problem as floats and anchors or think about it on a number line.
  For example, $(\text{-}3) + (\text{-}4)$ is like starting with $(\text{-}3)$ and adding 4 anchors or moving 4 to the left.
- Rewrite subtraction as addition.
  For example, $\text{-}3 - 4$ can be rewritten as $\text{-}3 + (\text{-}4)$, which is $\text{-}7$.
- Combine numbers that add or subtract to make 0.
  For example, when adding $\text{-}5$ and 6, you can break 6 into $5 + 1$. Using properties of operations, we can add $\text{-}5 + 5 + 1$ in pieces. The $\text{-}5 + 5$ portion of the expression adds to 0 and $0 + 1 = 1$, so the final value is 1.

Practice
5.05

Name: ..... Date: ..... Period: .....

Problems 1–2: Order the expressions from *least* to *greatest*.

1.

$-5 + (-4)$	$4 + 5$	$-4 + 5$	$4 + (-5)$

Least **Greatest**

2.

$-5 - (-4)$	$-4 - (-5)$	$-4 - 5$	$4 + (-5)$

Least **Greatest**

3. Is the solution to $-2.7 + x = -3.5$ positive or negative? Explain your thinking.

4. Which equation has the same solution as $-2.7 + x = -3.5$?

A. $-3.5 + 2.7 = x$

B. $3.5 - 2.7 = x$

C. $-3.5 + -2.7 = x$

D. $-3.5 - 2.7 = x$

Problems 5–8: Determine the value of the variable that makes each equation true.

5. $33 + a = -33.8$

6. $-9 - \mathrm{b} = 3.5$

7. $c = \left(-\frac{3}{4}\right) + \frac{3}{2}$

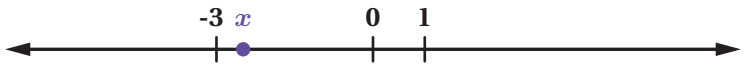
8. $d + 0.7 = -4$

Practice
5.05

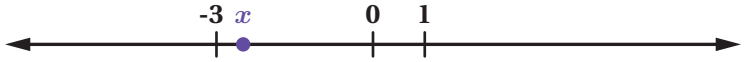
Name: ..... Date: ..... Period: .....

Spiral Review

9. Place the approximate location of $x + 4$ on the number line.



10. Place the approximate location of $4 - x$ on the number line.



11. Aditi's Attic is having a $5 off sale. Is the relationship between original price and sale price proportional? Explain your thinking.

Original Price	Sale Price
$15	$10
$25	$20
$35	$30

12. Select the expression that is equivalent to $2\frac{3}{4} - \left(-\frac{7}{8}\right)$.

A. $2\frac{3}{4} - \left(\frac{7}{8}\right)$ B. $2\frac{3}{4} + \frac{7}{8}$ C. $2\frac{3}{4} + \left(-\frac{7}{8}\right)$ D. $-2\frac{3}{4} + \left(-\frac{7}{8}\right)$

13. A snail is traveling at a rate of $\frac{1}{4}$ of a foot per $\frac{1}{3}$ of an hour. Select *all* the values that represent the snail's rate in feet per hour.

- ☐ A. $\frac{\frac{1}{4}}{\frac{1}{3}}$
- ☐ B. $\frac{4}{3}$
- ☐ C. $\frac{3}{4}$
- ☐ D. 0.75
- ☐ E. 1.3

Name: _____ Date: _____ Period: _____

Generalized Numbers Leading to Algebra | Number Line Understanding

7.EE.3, 7.NS.1, 7.NS.1.a, 7.NS.1.b, 7.NS.1.c, 7.NS.1.d, 7.NS.3, SMP.3

Practice Day 1

Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Variable Puzzles

1. $a =$ _____

$b =$ _____

$c =$ _____

$d =$ _____

2. $e =$ _____

$f =$ _____

$g =$ _____

$h =$ _____

Task B: Accounting

1.

Week	Starting Balance ($)	Final Balance ($)	Expression for Difference Between Final and Start	Value of Expression
1	24	8	$8 - 24$	-16
2	8	-18		
3	-18			-12
4		0		

2. Emika means . . .

Name: _____ Date: _____ Period: _____

Practice Day 1 (continued)

Task C: Temperature Check

1.

Day	Air Temperature (°C)	Wind Chill (°C)	Feels Like (°C)
Friday	9.1	-5.3	3.8
Saturday	5.5	-5.5	
Sunday		-15.5	-11.1
Monday	-2.3		-8.8
Tuesday	6.8	-8.7	

2. The day that felt the coldest was _____. **3.** Wednesday wind chill: _____

Task D: X Marks the Spot

1. $x =$ _____

Explanation:

2. $x =$ _____

Explanation:

3. $x =$ _____

Explanation:

You're invited to explore more.

Sum to Zero

	-12	0		5
0			-18	25
25		-18	5	-12
-12				-18
	-18	25	-12	

Sum to Zero Again

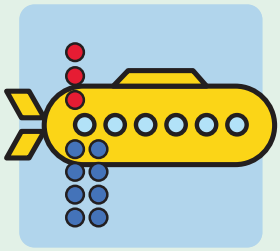
	-12	0		5
0			-18	25
25		-18	5	-12
-12				-18
	-18	25	-12	

Notes:

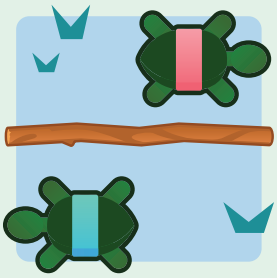
Sub-Unit

2

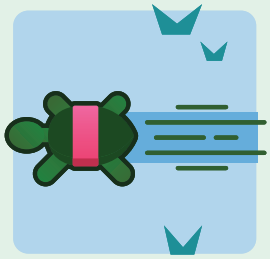
Multiplying and Dividing



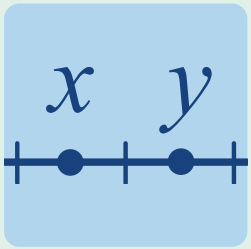
Lesson 6
Floating in Groups



Lesson 7
Back in Time



Lesson 8
Speeding Turtles

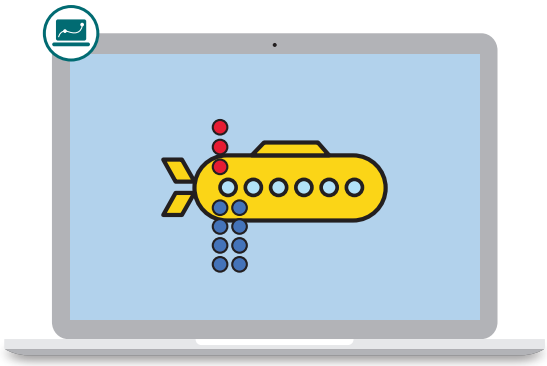


Lesson 9
Expressions

Lesson 10
Integer Puzzles

Name: ..... Date: ..... Period: .....

Number Line Understanding 7.NS.2, 7.NS.3, SMP.2

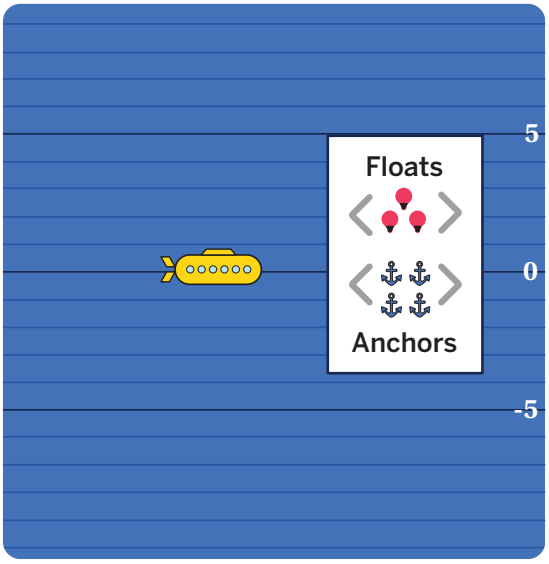


Floating in Groups

Let's use floats and anchors to make sense of multiplying integers.

Warm-up

1 This submarine is controlled by *groups* of floats and anchors.

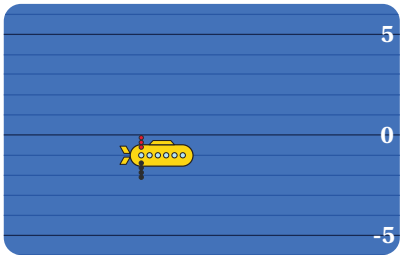


a Take a look at these different combinations.

Add 1 group of 3 floats

Add 1 group of 4 anchors

Add 1 group of 3 floats
Add 1 group of 4 anchors



b Choose a number. How can you add groups of 3 floats and 4 anchors to get the submarine to that number?

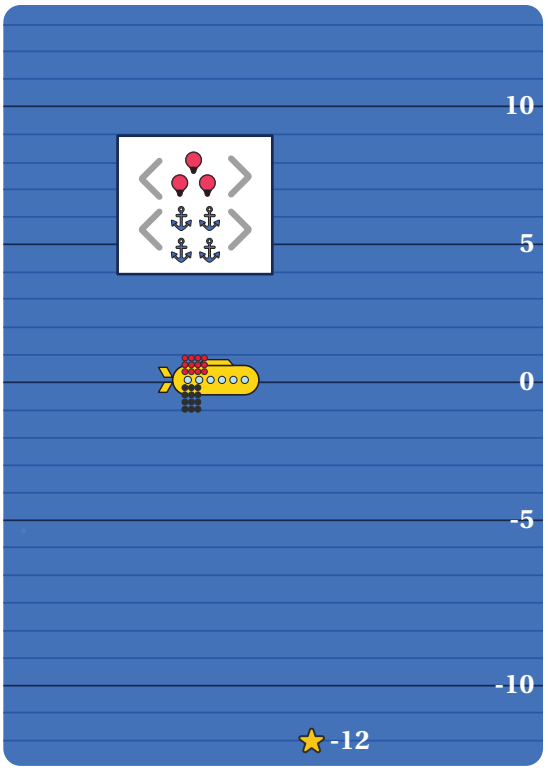
Name: Date: Period:

Star Power

2–3 This submarine starts at an elevation of 0 units. Floats can be added or removed in groups of 3. Anchors can be added or removed in groups of 4.

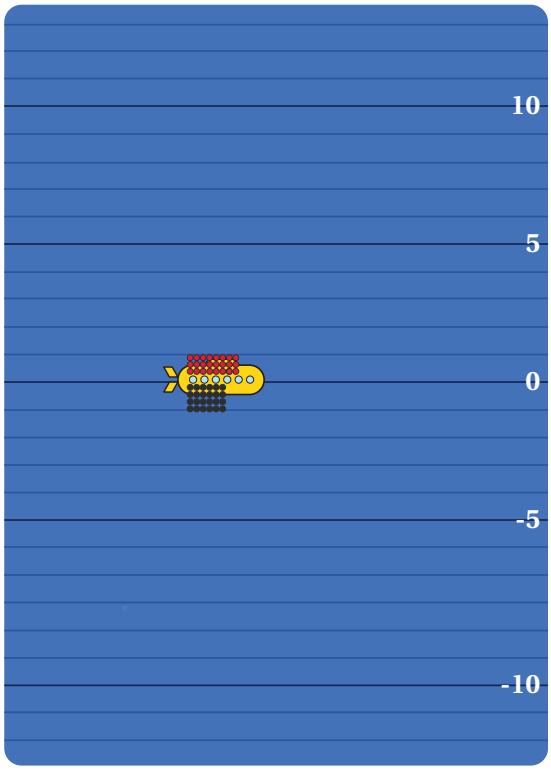
The table shows one way to get the submarine to -12 to collect the star. Write two more actions to get the submarine to -12.

Start	Action	Final
0	Add 3 groups of 4 anchors	-12
0		-12
0		-12



4 This submarine starts at 0 units.

Where will it end up after removing 6 groups of 3 floats?

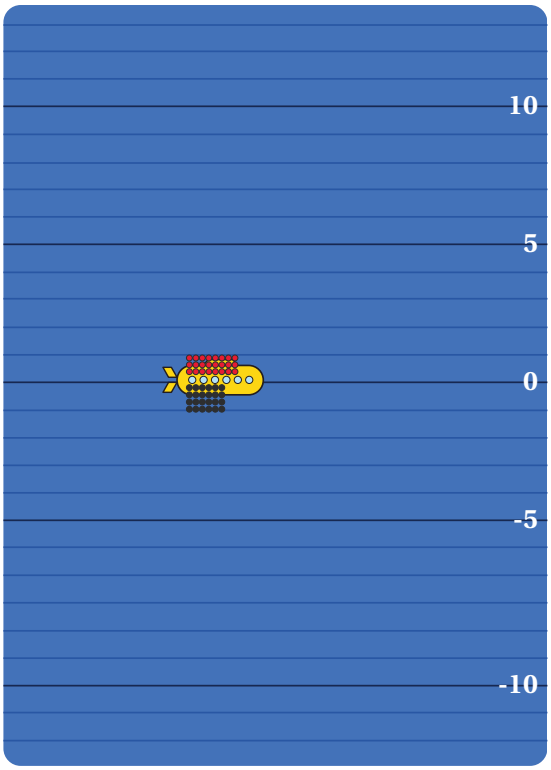


Name: ..... Date: ..... Period: .....

Star Power (continued)

5 This submarine starts at 0 units.

Where will it end up after removing 5 groups of 4 anchors?



Activity 2

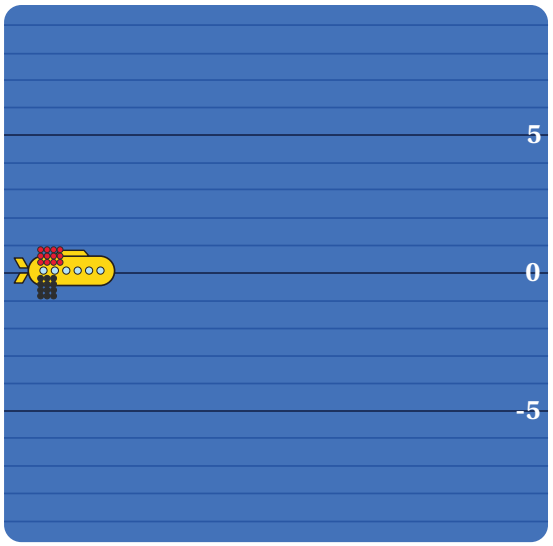
Name: ..... Date: ..... Period: .....

Underwater Expressions

6 Demetrius says the value of the expression $2 \cdot (-4)$ is -8.

Do you agree?

Explain your thinking. Use the submarine if it helps with your thinking.



7 Group the expressions and values that match with each action.

$-6 \cdot (-4)$	$4 \cdot 3$	$-3 \cdot 4$	-12
-24	12	24	$8 \cdot (-3)$

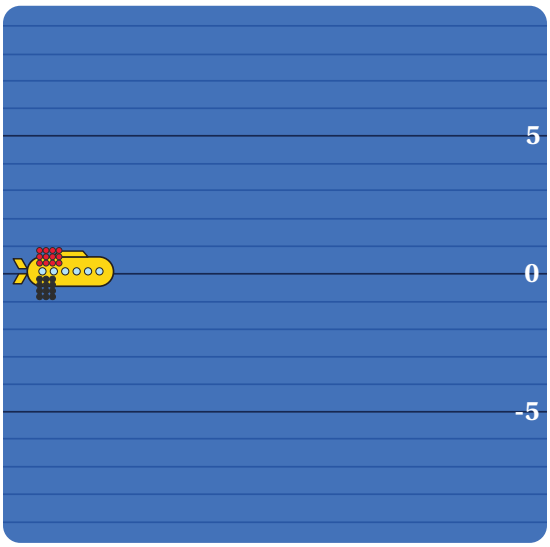
Remove 3 groups of 4 floats	Remove 6 groups of 4 anchors
Add 8 groups of 3 anchors	**Add 4 groups of 3 floats**

You're invited to explore more.

8 Use the You're Invited to Explore More Sheet to collect as many stars as you can.

9 Synthesis

Discuss: How can you use the floats and anchors scenario to explain why it makes sense that $-2 \cdot (-4)$ is positive?



12 Summary 5.06

We can use models, such as floats and anchors, to make sense of multiplying integers. Here are some examples for a submarine that starts at 0 units.

Action	Representation	Submarine's Direction	Final Value
Adding 2 groups of 3 floats	$2 \cdot 3$	Up	6
Removing 2 groups of 3 floats	$-2 \cdot 3$	Down	-6
Adding 2 groups of 4 anchors	$2 \cdot (-4)$	Down	-8
Removing 2 groups of 4 anchors	$-2 \cdot (-4)$	Up	8

As a general rule:

- When multiplying two negative numbers, the product is positive.
- When multiplying a negative number and a positive number, the product is negative.

Practice
5.06

Name: ..... Date: ..... Period: .....

Problems 1–3: Determine the value of the variable that makes each equation true.

1. $3 \cdot a = 12$

2. $-3 \cdot 4 = b$

3. $-3 \cdot c = 12$

Problems 4–7: A weather station on top of a mountain reports that the temperature is currently 0°C and has been decreasing at a constant rate of 3 degrees per hour.

4. What will the temperature be in 30 minutes?

5. What will the temperature be in 5 hours?

6. What was the temperature 1 hour ago?

7. What was the temperature 3 hours ago?

Problems 8–10: For each equation, check the box to show whether it is true or false. If the equation is false, change one value of the equation to make it true, and write the revised equation on the line.

		True	False	Revised Equation
8.	$3 \cdot (-6) = -18$			
9.	$5 \cdot (-2) = 10$			
10.	$-4 \cdot (-1) = -4$			

Practice
5.06

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 11–13: The sales tax rate in Tyler's state is 6.6%.

11. Write an equation to represent how the cost of an item changes after sales tax is added. Let x represent an original cost and y represent the cost after sales tax.

12. Tyler orders a meal that costs $\$15$. What is the after-tax amount?

13. If Tyler leaves an 18% tip on the after-tax amount, how much will he end up paying?

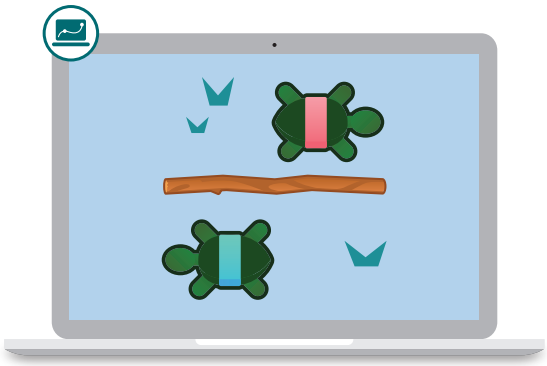
14. If Tyler pays with a $\$20$ bill and gets no change, what percent tip did he leave?

15. Zoe reads $8\frac{1}{2}$ pages in 15 minutes. Select *all* the values that represent Zoe's reading rate in pages per minute.

- ☐ A. $8\frac{1}{2}$
- ☐ B. $\frac{17}{30}$
- ☐ C. $\frac{8\frac{1}{2}}{15}$
- ☐ D. $\frac{17}{2} \cdot \frac{1}{15}$
- ☐ E. $\frac{15}{17}$

Name: ..... Date: ..... Period: .....

Proportional Relationships | Unit Rates in the World | 7.NS.2, 7.NS.2.a, 7.NS.3, 7.RP.2, SMP.1

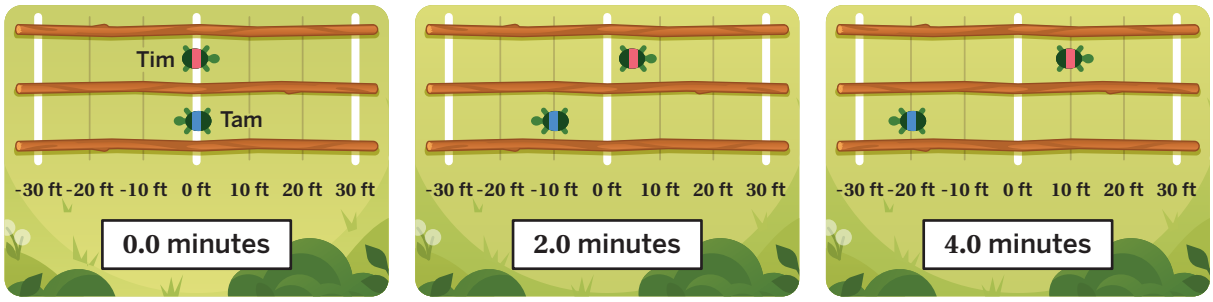


Back in Time

Let's use position, rate, and time to represent multiplying positive and negative numbers.

Warm-up

1 Here are a few moments from an animation.

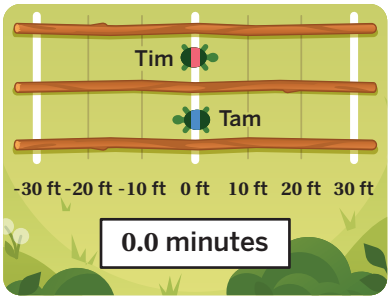


Write a story about Tim and Tam.

Name: ..... Date: ..... Period: .....

Time for Turtles

2 Tim and Tam each walk at a constant rate. Complete the last row of the table.



Time (min)	Tim's Position (ft)	Tam's Position (ft)
0	0	0
1	2.5	-5
2	5	-10
3	7.5	-15
. . .	. . .	. . .
6		

3 Tim's walking rate is 2.5 feet per minute.

What is Tam's walking rate?

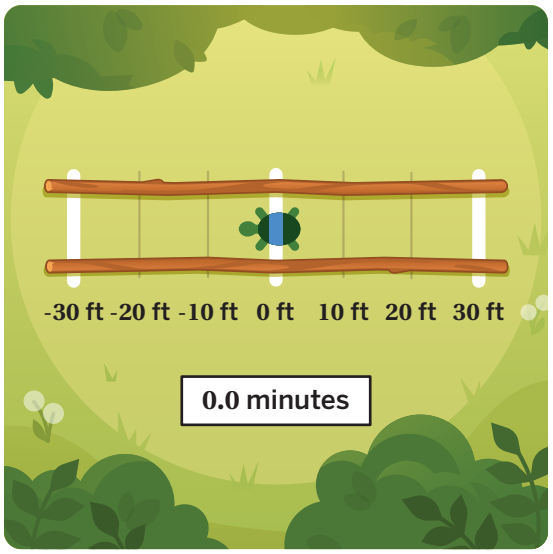
Explain your thinking.

Name: ..... Date: ..... Period: .....

Time for Turtles (continued)

Use the value you calculated for Tam's walking rate for the following problems.

4 What will Tam's position be in 3.2 minutes?



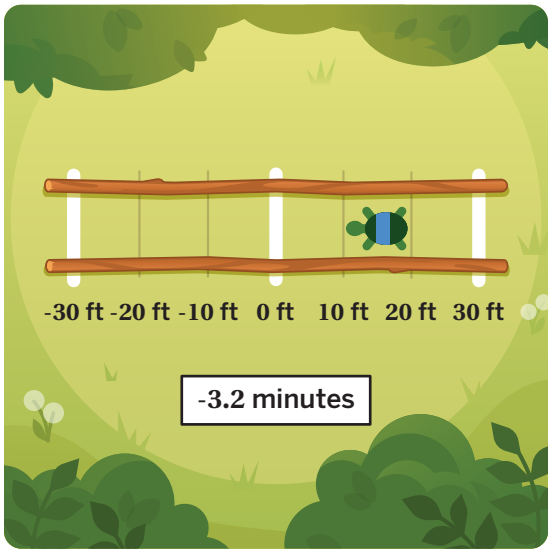
5 What was Tam's position 3.2 minutes ago?

6 One student wrote this equation to determine Tam's position on the previous problem:

$(-5)(-3.2) = 16$

Explain what each number represents in the situation.

-5 represents . . .



-3.2 represents . . .

16 represents . . .

Name: ..... Date: ..... Period: .....

Positives and Negatives

7 For each expression, put a check for whether it has a positive or negative value.

Expression	Positive	Negative
$(-7) \cdot (-8)$		
$(-7) \cdot 8$		
$3.5 \cdot (-12)$		
$(-3.5) \cdot (-12)$		
$\left(\frac{1}{2}\right)(-20)$		
$\left(-\frac{1}{2}\right)(-20)$		

Activity 3

Name: ..... Date: ..... Period: .....

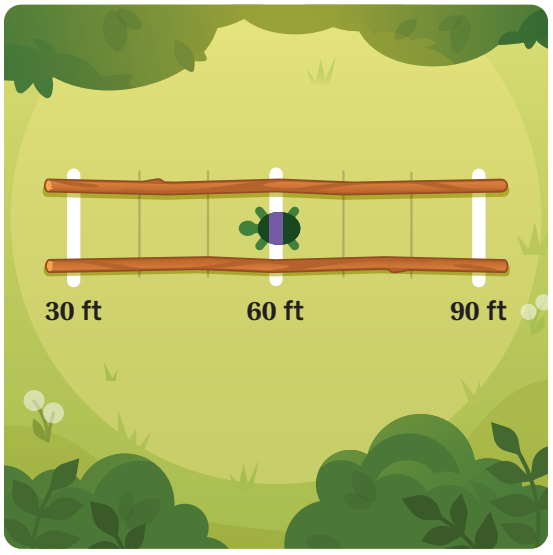
Turtles Through Time

8 Tommy walks at a rate of -2 feet per minute.

Right now he is at 60 feet.

Where was Tommy 10 minutes ago?

Explain how you know.



9 Select *all* the expressions that describe where Tommy was 10 minutes ago.

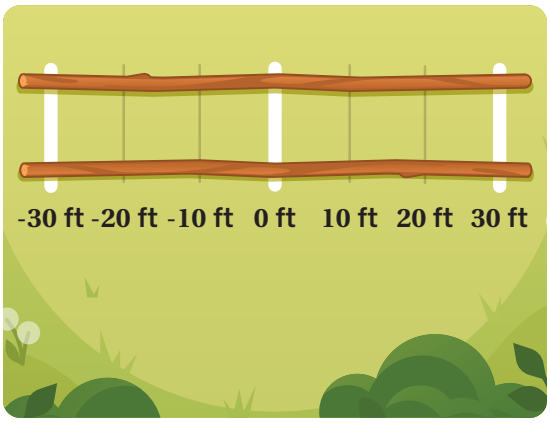
- ☐ **A.** $60 + 2 \cdot (-10)$
- ☐ **B.** $60 + (-2)(-10)$
- ☐ **C.** $60 - 20 \cdot 10$
- ☐ **D.** $60 - 20$
- ☐ **E.** $60 + 20$

You're invited to explore more.

10 Use the You're Invited to Explore More Sheet to explore another turtle puzzle.

11 Synthesis

Use a situation with turtles starting at 0 to explain why $5 \cdot (-2)$ is negative and $(-5) \cdot (-2)$ is positive.

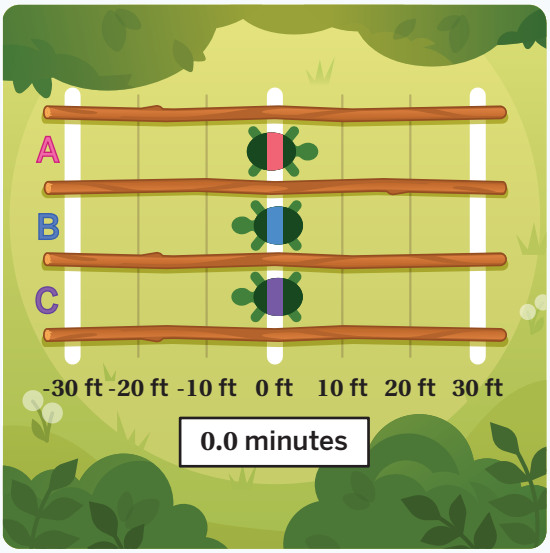


14 Summary 5.07

Multiplying positive and negative numbers can help represent position, rate, and time. The position of an object is equal to the walking rate multiplied by the time or *rate* • *time* = *position*.

Here are three turtles. They are all together at 0 feet.

- Turtle A walks to the right 6 feet per minute. 3 minutes ago, Turtle A was at -18 feet because $6 \cdot (-3) = -18$.
- Turtle B walks to the left 5 feet per minute. 6 minutes ago, Turtle B was at 30 feet because $(-5) \cdot (-6) = 30$.
- Turtle C walks to the left 3 feet per minute. In 4 minutes, Turtle C will be at -12 feet because $(-3) \cdot 4 = -12$.



Practice
5.07

Name: ..... Date: ..... Period: .....

Problems 1–4: Determine the value of each expression.

1. $5(-3)$

2. $-5 \cdot 3$

3. $(-5)(-3)$

4. $-5 \cdot (-0.3)$

Problems 5–8: Anton the Ant can travel up and down an anthill. Currently, his crawling rate is -7 centimeters per second, which means he's crawling down the anthill. Anton passes ground level at position 0.

5. Which equation represents Anton's position 8 seconds *after* he passes ground level?

A. $-7 \cdot 8 = 56$ **B.** $-7 \cdot 8 = -56$ **C.** $-7 \cdot -8 = 56$ **D.** $-7 \cdot -8 = -56$

6. Which equation represents Anton's position 8 seconds *before* he passes ground level?

A. $-7 \cdot 8 = 56$ **B.** $-7 \cdot 8 = -56$ **C.** $-7 \cdot -8 = 56$ **D.** $-7 \cdot -8 = -56$

7. Write an ant story that could be represented by the equation $5 \cdot (-6) = -30$.

8. Anton has friends in nearby hills named Anto and Anty. For each description, write an equation in the table that represents it. Assume each ant starts at position 0.

Description	Equation
Anto crawls for 28 seconds at -2 centimeters per second.	
Anty crawls for 3 seconds at -8 centimeters per second.	

Name: ..... Date: ..... Period: .....

Problems 9–10: Shawn and Jada are walking in the same direction through the park. They walk by a fountain at the same time. When they pass the fountain, Jada is walking at a speed of 5 feet per second, and Shawn is walking at a speed of 3.5 feet per second.

9. If 0 represents the location of the fountain, what values represent the location of each person in 5 seconds?

10. How many feet is Jada from Shawn 8 seconds before arriving at the fountain?

Spiral Review

Problems 11–12: A tank of water breaks and begins draining at a constant rate.

11. Complete the table.

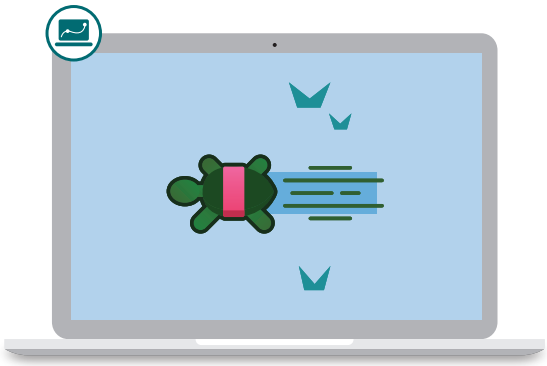
Time (min)	Water in the Tank (L)
0	770
1	756
2	
. . .	. . .
5	

12. Is there a proportional relationship between time and the amount of water left in the tank? Explain your thinking.

13. Angela buys a pair of shoes. The shoes cost $40. The store is having a 20% off sale on everything in the store. How much money does Angela save from the sale?

Name: ..... Date: ..... Period: .....

Proportional Relationships | Unit Rates in the World | 7.NS.2, 7.NS.2.b, 7.NS.2.c, 7.NS.3, 7.RP.2, SMP.2



Speeding Turtles

Let's use position, rate, and time to represent dividing integers.

Warm-up

1 Which one doesn't belong? Circle one.

$3 \cdot 2$ $\quad -\frac{12}{2}$

$(-3)(-2)$ $\quad 12 \div 2$

Explain your thinking.

Name: ..... Date: ..... Period: .....

Return of the Turtles

2 Tim walks -8 feet per minute for 3.25 minutes.

Where will Tim end up?

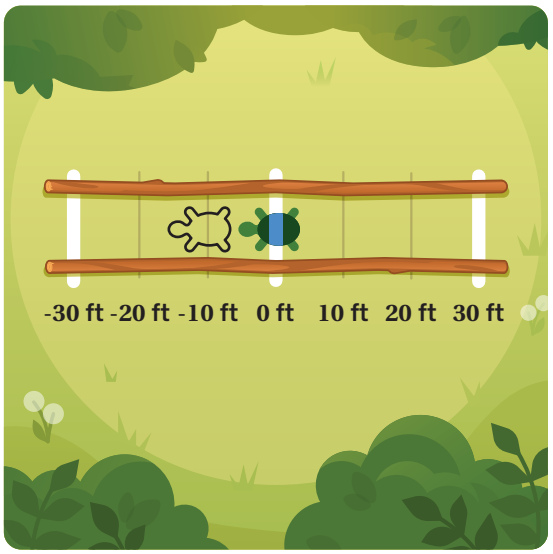
Rate (ft/min)	Time (min)	Position (ft)
-8	3.25	

3 Tam walked -4 feet per minute.

She began at 0 feet and ended up at -10 feet.

For how much time was Tam walking?

Rate (ft/min)	Time (min)	Position (ft)
-4		-10



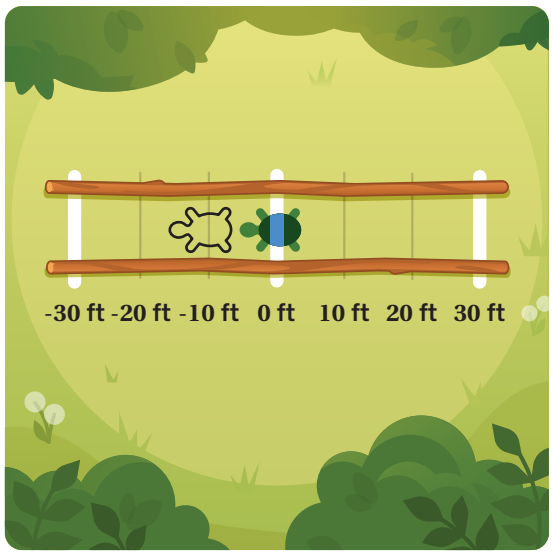
Name: ..... Date: ..... Period: .....

Return of the Turtles (continued)

4 Fabiana wrote this equation to determine the time from the previous problem:

$$\frac{-10}{-4} = t$$

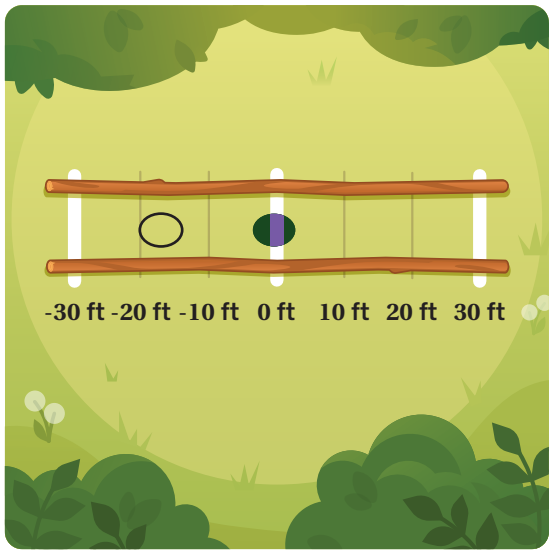
Explain why her equation makes sense.



5 Tommy walks -17 feet in 5 minutes.

What is Tommy's walking rate?

Rate (ft/min)	Time (min)	Position (ft)
	5	-17

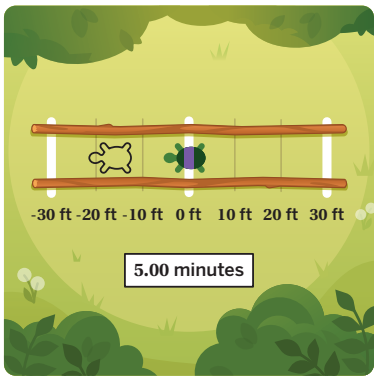


Activity 2

Name: ..... Date: ..... Period: .....

Turtle Express

6 Tommy walks -17 feet in 5 minutes.



a Select *all* the equations that represent this situation, where r represents Tommy's walking rate.

- ☐ **A.** $r \cdot 5 = -17$
- ☐ **B.** $(-17) \cdot 5 = r$
- ☐ **C.** $r = -\frac{17}{5}$
- ☐ **D.** $r = \frac{-17}{5}$
- ☐ **E.** $r = \frac{5}{-17}$

b **Discuss:** How do you know that these equations represent this situation?

7 Group the expressions that have the same value.

$6 \div 3$ $\quad -\frac{3}{6}$ $\quad -\frac{6}{3}$ $\quad \frac{-3}{6}$ $\quad \frac{-6}{3}$ $\quad \frac{-3}{-6}$ $\quad \frac{-6}{-3}$ $\quad 3 \div (-6)$ $\quad \frac{6}{-3}$ $\quad 3 \div 6$

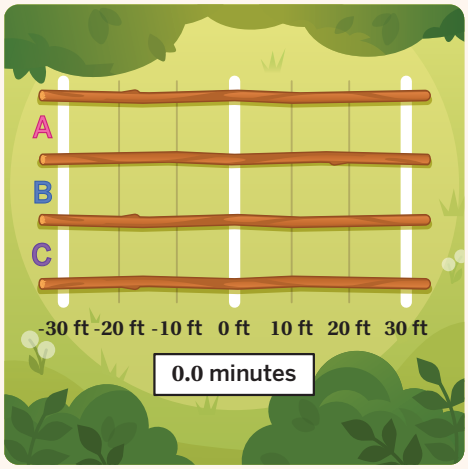
Group 1	Group 2	Group 3	Group 4

You're invited to explore more.

8 Determine a current position and rate for three new turtles so that in 5 minutes all three turtles will be in the same position.

No two turtles should have the same starting position and rate as each other.

Turtle	Position (ft)	Rate (ft/min)
A		
B		
C		



9 Synthesis

Explain why these expressions have the same value.

$-\frac{6}{3}$

$\frac{6}{-3}$

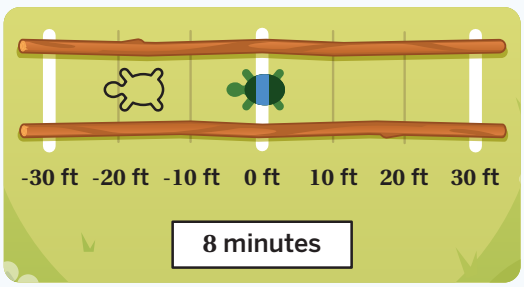
$\frac{-6}{3}$

12 Summary 5.08

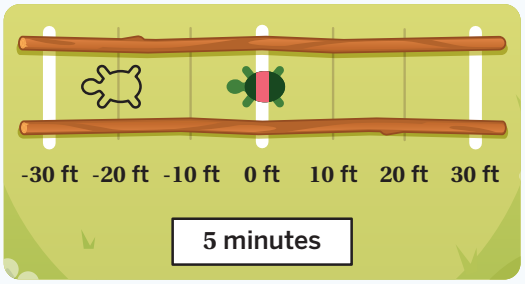
You can use your knowledge of position, rate, and time to multiply and divide positive and negative numbers.

Here are two examples.

Turtle A walks -2 feet per minute. If it starts at 0 feet, how long will it take to walk to -16 feet? Dividing position by walking rate tells us the time, so $\frac{-16}{-2} = 8$ minutes.



Turtle B takes 5 minutes to walk 20 feet to the left, or -20 feet. This means that each minute, Turtle B walks -4 feet. Dividing distance by time tells us the walking rate, so $\frac{-20}{5} = -4$.



Thinking about an expression in terms of position, rate, and time can help you determine whether the value is negative or positive.

Practice
5.08

Name: ..... Date: ..... Period: .....

1. Select *all* the expressions that have a negative value.

- ☐ **A.** $\frac{-15}{12}$
- ☐ **B.** $\frac{15}{-12}$
- ☐ **C.** $\frac{-15}{-12}$
- ☐ **D.** $-\frac{15}{12}$
- ☐ **E.** $\frac{15}{12}$

Problems 2–3: Determine the value of x that makes each equation true.

2. $-3x = 6.3$

3. $\frac{x}{-1.2} = -0.3$

Problems 4–5: A machine that drills holes for wells drilled at a constant rate to a depth of -72 feet in one day (24 hours).

4. How much did the depth change each hour? Make sure to show whether the change is positive or negative.

5. What was the depth after 15 hours?

6. Select *all* the values that are equivalent to $-\frac{12}{7}$.

- ☐ **A.** $-\frac{-12}{-7}$
- ☐ **B.** $-5\frac{1}{7}$
- ☐ **C.** $-1\frac{5}{7}$
- ☐ **D.** $1\frac{5}{7}$
- ☐ **E.** $\frac{-12}{-7}$

Name: ..... Date: ..... Period: .....

Problems 7–9: A biker is traveling across a bike trail. The bike trail runs east and west from the town monument. Positions east of the monument (mile 0) are positive and positions west of the monument are negative. The biker starts at mile -15.

7. The biker rides west and reaches mile -30 in 1.5 hours. What is the biker's average speed?

8. At mile -30, he begins biking east at a speed of 9 miles per hour. If he travels for 2.5 hours, where will he end up?

9. The biker slows down and heads back west at 6 miles per hour. How long will it take him to bike back to where he started (mile -15)?

Spiral Review

10. The equation $30 + (-30) = 0$ is an example of two numbers whose sum is 0. Write a different equation with two numbers whose sum is 0.

11. Write an equation with three numbers whose sum is 0.

12. Write an equation with *four* numbers, none of which are opposites, whose sum is 0. One example of opposites is -30 and 30.

13. Write an equation using *four* numbers that multiply to 0.

Unit 5
Lesson
9

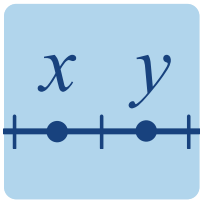
Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra | Number Line Understanding

7.NS.1, 7.NS.1.d, 7.NS.2, 7.NS.2.a, 7.NS.2.b, 7.NS.2.c, 7.NS.3, SMP.3, SMP.6, SMP.7

Expressions

Let's reason about variable expressions involving adding, subtracting, multiplying, and dividing positive and negative numbers.



Warm-up

1. Circle the expression with the greater value.

a	$5 - (-1)$	$-1 - 5$	They have the same value.
b	$5 + (-1)$	$-1 + 5$	They have the same value.
c	$5 \cdot (-1)$	$-1 \cdot 5$	They have the same value.
d	$\frac{5}{-1}$	$\frac{-1}{5}$	They have the same value.

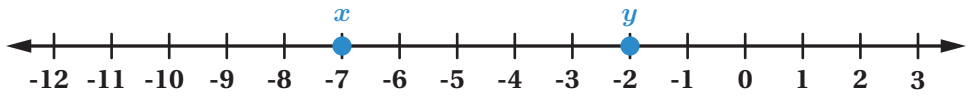
Activity 1

Name: Date: Period:

Ordering Expressions

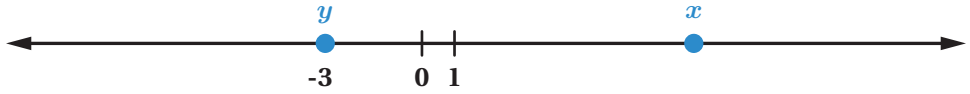
You will use a set of six expression cards. For each set of x- and y-values on the number line, sort the expression cards by value from *least* to *greatest*. Record your answers below. Use the number line if it helps you with your thinking.

2.



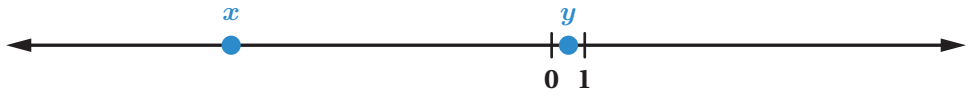
Least Greatest

3.



Least Greatest

4.



Least Greatest

5. Describe 2–3 patterns you noticed as you were ordering the expressions.

Activity 2

Name: ..... Date: ..... Period: .....

Always, Sometimes, Never

For Problems 6 and 7, select one statement. Explain whether it is always, sometimes, or never true. Use examples, words, or number line diagrams to support your claim.

6.

Statement A	Statement B	Statement C
$x + y$ is greater than $x - y$	$3 - x$ is less than $2 - x$	$\frac{-x}{x}$ is positive

a Statement ..... is (always / sometimes / never) true.

b My reasoning:

7.

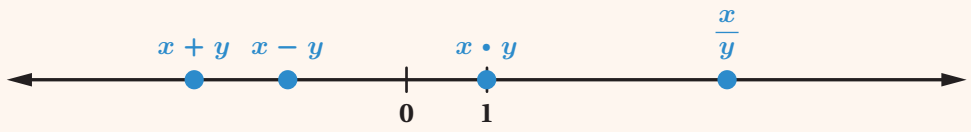
Statement D	Statement E	Statement F
$-x \cdot y$ is the opposite of $y \cdot (-x)$	$\frac{y}{2}$ is less than y	$x \cdot y$ is greater than $\frac{x}{y}$

a Statement ..... is (always / sometimes / never) true.

b My reasoning:

You're invited to explore more.

8. What must be true about the x- and y-values for this number line? Explain your thinking.



Activity 3

Name: Date: Period:

Distributive Property

Analyze both methods for evaluating each expression. Use what you know about multiplying signed numbers to determine each missing value.

9.

Order of operations	Distributive Property
$(14 + (-4)) \cdot (-3) = (10) \cdot (-3) = -30$	$(14 + (-4)) \cdot (-3) = 14(-3) + (-4) \cdot (-3) = -42 + \square$

a What is the sign of the missing value? Explain your thinking.

b What must the product of -4 and -3 be to make the last line true?

10.

Order of operations	Distributive Property
$-4 \cdot (-3 + 2) = -4 \cdot (-1) = 4$	$-4 \cdot (-3 + 2) = -4 \cdot (-3) + (-4) \cdot (2) = 12 + \square$

a What is the sign of the missing value? Explain your thinking.

b What must the product of -4 and 2 be to make the last line true?

Synthesis

11. **Discuss:** What are some strategies you can use to decide if a statement is always, sometimes, or never true?

Use the example statements if they help with your explanation.

$-\frac{x}{x}$ is positive

$\frac{y}{2}$ is less than y

Summary 5.09

Thinking flexibly will help you reason about the value of expressions that contain variables. Using number lines and testing with example values are two strategies that can help us make sense of these types of expressions.

Some things that are familiar from using positive numbers are not the same when working with negative numbers.

For example, when x is a positive number, $10 - x$ will be less than 10. But if x is a negative number, $10 - x$ will be greater than 10, since subtracting a number is the same as adding its opposite.

Practice
5.09

Name: ..... Date: ..... Period: .....

1. The value of x is -3. Order these expressions from *least* to *greatest*.

x	$1 - x$	$x - 1$	$\frac{-1}{x}$

Least **Greatest**

Problems 2–4: The table has six expressions and three sets of values for a and b.

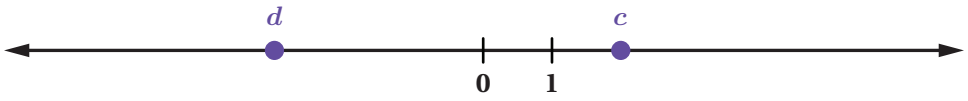
2. Complete the table.

	$-a$	ab	$-ab$	$a - b$	$b - a$	$-a + b$
$a = -\frac{1}{2}$, $b = 6$	$\frac{1}{2}$		3			
$a = \frac{1}{2}$, $b = -6$						
$a = -6$, $b = -\frac{1}{2}$						

3. Describe any patterns you notice in the table.

4. Create values for a and b so that ab is negative and $a - b$ is positive. Explain your thinking.

5. Select *all* the expressions that have a positive value based on the positions of c and d.



☐ **A.** $-d$ ☐ **B.** $c + d$ ☐ **C.** $c - d$

☐ **D.** $c \cdot d$ ☐ **E.** $-\frac{d}{c}$

Name: ..... Date: ..... Period: .....

6. Evaluate the expression $x \cdot y$ when $x = \frac{2}{5}$ and $y = -4$.

7. Which expression has the same value as the expression $-\frac{2}{7} - \frac{3}{7}$?

A. $\frac{2}{7} + \frac{3}{7}$

B. $-\frac{2}{7} + \frac{3}{7}$

C. $\frac{2}{7} + \left(-\frac{3}{7}\right)$

D. $-\frac{2}{7} + \left(-\frac{3}{7}\right)$

8. Fill in the missing values to simplify the expression $-7 \cdot (2 - 4)$ using the Distributive Property.

$-7 \cdot (2 - 4)$

$(-7) \cdot \square + (-7) \cdot \square$

$-14 + 28$

$\square$

Spiral Review

9. Use long division to divide 496 by 4. Show your work.

10. Use long division to divide 3.8 by 0.004. Show your work.

11. What is the same and what is different about the decimal representations $0.\overline{14}$ and $0.1\overline{4}$?

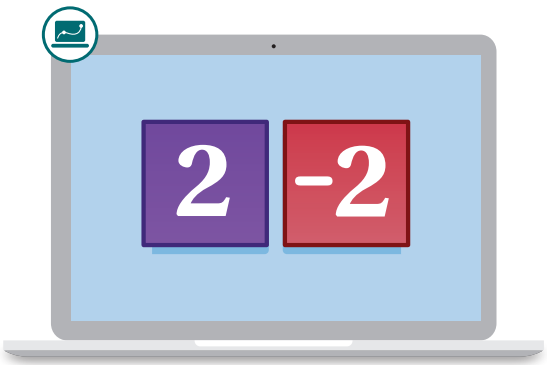
Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra | Number Line Understanding

7.EE.3, 7.NS.1.d, 7.NS.2, 7.NS.2.a, 7.NS.2.c, 7.NS.3, SMP.1, SMP.3

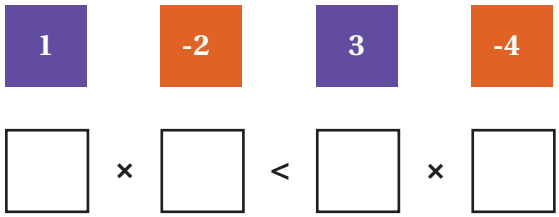
Integer Puzzles

Let's reason about the signs and values of integer expressions using the four operations.



Warm-up

1 a. Make a true inequality by using these numbers to fill in the blanks.



b. Explain how you know this inequality is true.

Activity 1

Name: ..... Date: ..... Period: .....

Puzzling Values

For each number puzzle:

- Fill in the blanks with these numbers.
- You may use each number once per puzzle.

-1	2	3	-4	-5	6	7	-8

2 Make this inequality true.

$\square(\square + \square) > 0$

3 **a** Make an expression.

$\square(\square + \square)$

b What is the value of your expression?

4 Josiah and Wey Wey made this expression. They used different strategies to determine the value of the expression.

$-5(2 + -4)$

Josiah

$(-5)(2) + (-5)(-4)$

$-10 + 20$

10

$-5(2 + -4)$

Wey Wey

$(-5)(2 - 4)$

$(-5)(-2)$

10

Discuss: What do you notice? What do you wonder?

Activity 2

Name: ..... Date: ..... Period: .....

Greatest Puzzling Values

For each number puzzle:

- Fill in the blanks with these numbers.
- You may use each number once per puzzle.

-1	2	3	-4	-5	6	7	-8

5 **a** Make an expression with the greatest possible value.

$\square \times \square + \square \times \square$

b What is the value of your expression?

6 Kiri created this expression which has a value of 26.

$7 \times 3 + 1 \times 5$

Write a hint or question to help her create an expression with an even greater value.

7 **a** Make an expression with the greatest possible value.

$$\frac{\square - \square}{\square + \square}$$

b What is the value of your expression?

Name: ..... Date: ..... Period: .....

Greatest Puzzling Values (continued)

8 Fill in the blanks with these numbers.

2	3	4	5	-2	-3	-4	-5

a Make an expression with the greatest possible value.

$$\frac{\square + \square \times \square}{\square - \square}$$

b What is the value of your expression?

You're invited to explore more.

9 Here are six copies of a new puzzle. Solve it in as many ways as you can.

-1	-2	-3	-4	5	6	7	8

$\square(\square+\square)=\square\times\square+\square$

$\square(\square+\square)=\square\times\square+\square$

$\square(\square+\square)=\square\times\square+\square$

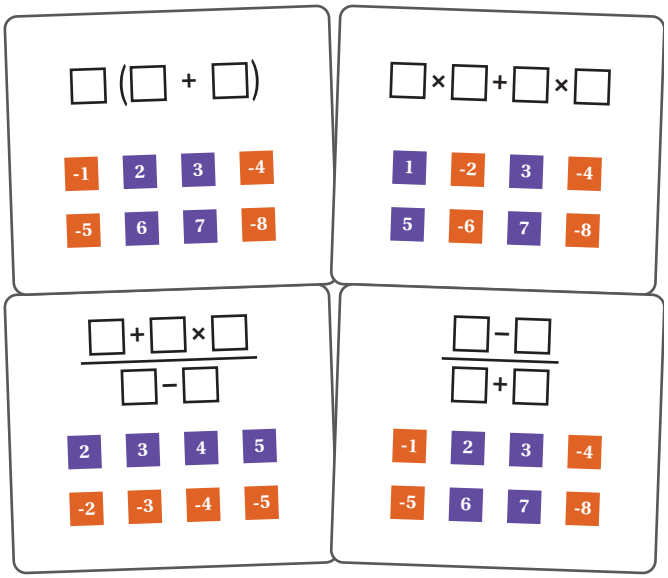
$\square(\square+\square)=\square\times\square+\square$

$\square(\square+\square)=\square\times\square+\square$

$\square(\square+\square)=\square\times\square+\square$

10 Synthesis

Discuss: What is something you learned today that might help someone else as they complete puzzles like these?



13 Summary 5.10

You can use properties of operations, order of operations, and multiplication and division of integers as strategies to solve integer puzzles.

For example, if we want to make this inequality true, it may be helpful to think about the signs of the sum inside the parentheses and the number outside of the parentheses.

$-8(-4 + 1) > 0$ is true because $-8(-3)$ is positive.

$-8(4 + 1) > 0$ is false because $-8(5)$ is negative.

Make the inequality true.

_____ (_____ + _____) > 0

Practice
5.10

Name: ..... Date: ..... Period: .....

Problems 1–4: Determine the value of the variable that makes each equation true.

1. $-22 + a = -5$

2. $-22 - 5 = b$

3. $-5c = -22$

4. $\frac{d}{-5} = 22$

5. Which expression has the greater value? Explain your thinking.

A. $(-22) - (-5)$ **B.** $(-5) - (-22)$ **C.** They have the same value

Problems 6–7: Let $x = -2$, $y = 4$, and $z = 2$.

6. Order these expressions from *least* to *greatest*.

$x - z$	$x - 2y$	$x \cdot y$	xyz

Least **Greatest**

7. Would your order be different if the value of x was 2 instead? Explain your thinking.

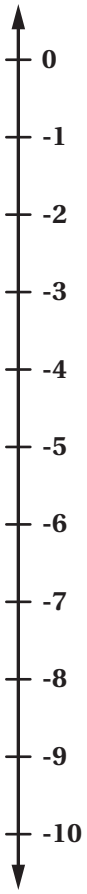
8. For the expressions $\frac{a}{b}$ and $a + b$, choose values for a and b so that $\frac{a}{b}$ is positive and $a + b$ is negative.

Practice
5.10

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 9–11: Lucia held a penny underwater in a pool and let it go. The penny traveled downward at a rate of 0.8 feet per second. Use the number line if it helps with your thinking.



9. How many seconds did it take for the penny to move -4.4 feet (4.4 feet downward)?

10. 3 seconds after letting go, the penny was at position -3.4 feet. What position was the penny let go from?

11. What was the position of the penny after it sank for 10 seconds? Explain your thinking.

12. A cake recipe uses $4\frac{2}{5}$ cups of strawberries to make a cake. How many cups of strawberries are needed to make 3 cakes?

Notes:

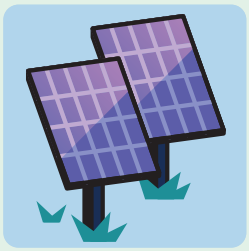
Sub-Unit

3

Applying Operations

Lesson 11
Temperatures Around the World

Lesson 12
Visiting the Arctic



Lesson 13
Solar Panels and More

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra 7.EE.3, 7.NS.1.b, 7.NS.3, SMP.1

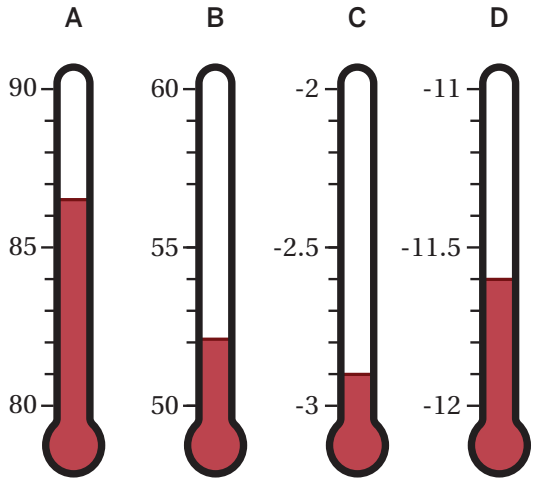
Temperatures Around the World

Let's add and subtract positive and negative numbers to compare temperatures and ice mass.

Warm-up

1 What is the approximate temperature on each thermometer?

Thermometer	Temperature (°F)
A	
B	
C	
D	



All temperatures are in °F.

2 These temperatures are the average annual temperatures of different locations as of 2014. Match each temperature to a location and record it on the map.

Discuss: your choices with a partner.

Name: ..... Date: ..... Period: .....

Changing Temperatures

3 How do you think the average annual temperatures of the places shown in the Warm-Up have changed over the last century? Circle one.

Gotten warmer Gotten colder Stayed the same

Explain your thinking.

4 This table shows how the average temperature in these places have changed from 1960 to 2014. Complete the table.

Location	Average Temperature in 1960 (˚F)	Average Temperature in 2014 (˚F)	Change From 1960 to 2014 (˚F)
NYC, USA	49.8	52.1	2.3
Khartoum, Sudan	84.6	86.5	
Greenland	-6	-2.9	
Antarctica		-11.6	1.3

5 Look at the column "Change From 1960 to 2014."

a What do you notice?

b What might be some impacts of these temperature changes?

Name: Date: Period:

Slippery Slope

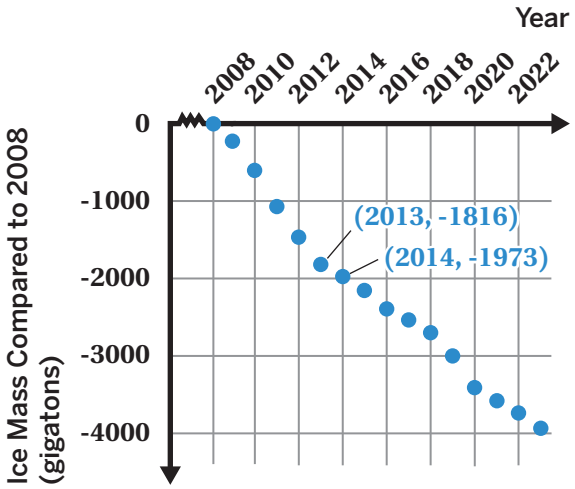
6 **Data Talk!** One impact of rising temperatures is that the amount of ice in the world is decreasing each year.

Greenland is mostly covered by ice.

This graph shows the mass of ice in Greenland in different years compared to the mass in 2008.

Tell a story about this graph.

Include the point (2013, -1816) in your story.



7 **Data Talk!** What is the approximate change in Greenland's ice mass from 2013 to 2014?

Show whether the change is positive or negative.

Name: ..... Date: ..... Period: .....

Slippery Slope (continued)

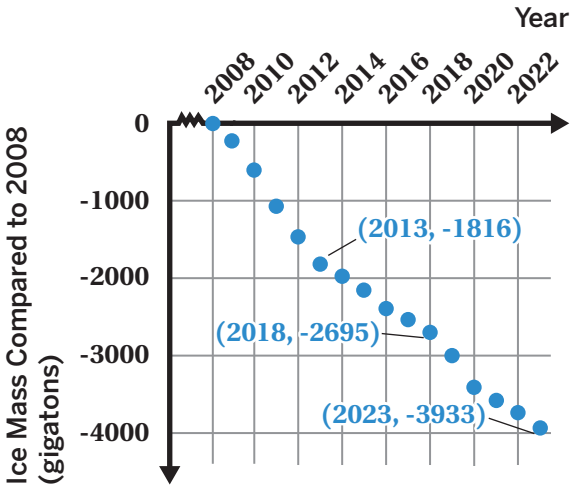
8 **Data Talk!** During which time period was ice melting fastest? Circle one.

2008 to 2013

2013 to 2018

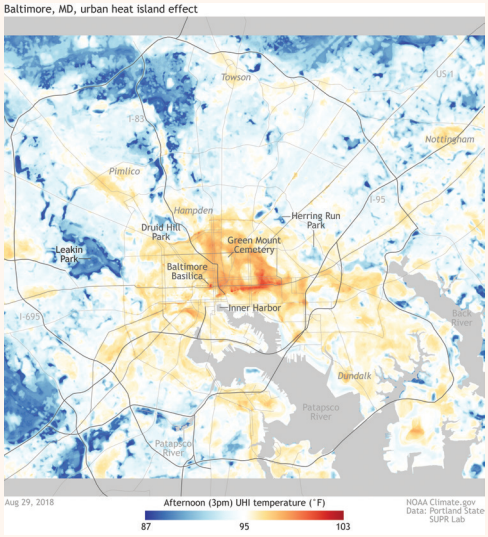
2018 to 2023

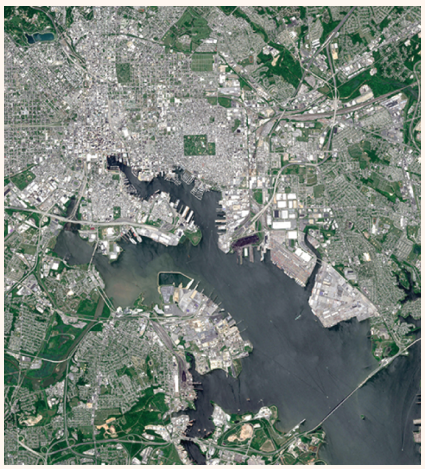
Explain your thinking.



You're invited to explore more.

9 In this activity, we've looked at average temperatures across entire regions. Temperatures can also vary between different parts of a small local area, even neighborhood to neighborhood. Here are two maps of Baltimore, Maryland. One shows the temperatures at 3 PM on August 29th, 2018 and the other shows the satellite view.





NOAA Climate.gov

What story might this data tell?

10 Synthesis

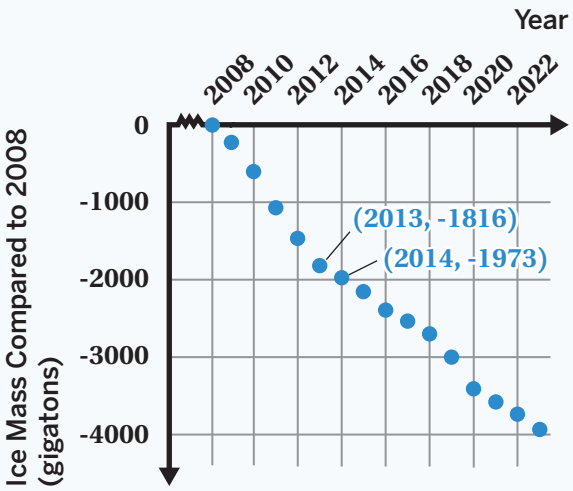
Discuss: Select one question:

- ☐ Why is it important to study the temperature in places both near and far from where you live?
- ☐ How can positive and negative numbers help us make sense of topics like changing temperatures and ice mass?
- ☐ What other questions might you ask to investigate these topics further?

13 Summary 5.11

Adding and subtracting positive and negative numbers can help you solve problems involving real-world situations.

You can tell from the graph that the ice mass in Greenland is decreasing each year. Subtraction lets you determine how much it decreased between any two years.



For example, to determine the change in the ice mass from 2013 to 2014, you need to find the difference between -1,973 and -1,816.

Since -1,973 is less than -1,816, the difference will be negative.

$-1973 - (-1816) = -1973 + 1816 = -157$

The change in ice mass from 2013 to 2014 is -157 gigatons. You can use this information to determine if the ice mass is continuing to change at the same rate in future years, or if the ice mass is changing at a faster or slower rate.

Practice
5.11

Name: Date: Period:

Problems 1–3: Determine the value of the variable that makes each equation true.

1. $3 \cdot (-2.5) = x$

2. $-3y = 33$

3. $-3 - \frac{1}{2} = z$

4. This table shows the transactions, in dollars, in a checking account for the month of January.

Determine the total of the transactions for the month. Show or explain your thinking.

January
-$38.50
$126.30
$429.40
-$265.00

Problems 5–7: On January 22, 1943, the town of Spearfish, South Dakota, set the record for the world's fastest temperature change.

- At 7:30 AM, the temperature was -4°F.
- By 7:32 AM, the temperature was 45°F.
- By 9:00 AM, the temperature was 54°F.
- By 9:27 AM, the temperature was -4°F.

5. What was the temperature change from 7:30 AM to 7:32 AM? Make sure to show whether the change was positive or negative.

6. What was the temperature change from 9:00 AM to 9:27 AM? Make sure to show whether the change was positive or negative.

7. Years later, the town of Bristol had a big temperature drop. Between 6:02 PM and 6:10 PM, the temperature went from 9°F to -11°F. How does this temperature change compare to Spearfish's big drop that started at 9 AM?

Practice
5.11

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 8–10: For each fraction, determine whether it terminates or repeats when written as a decimal.

8. $\frac{4}{9}$ Terminating Repeating

9. $\frac{5}{8}$ Terminating Repeating

10. $\frac{50}{75}$ Terminating Repeating

Problems 11–12: These are equation puzzles. Fill in the blanks so that each row and column makes a true equation.

11. Use numbers to complete the puzzle.

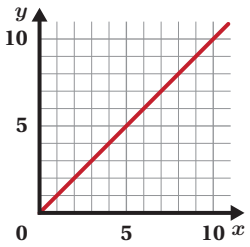
4	÷		×		=	0
×		+		+		
	×	2	−		=	0
+		−		+		
4	÷		+	2	=	0
=		=		=		
0		0		0		

12. Use the symbols +, −, ×, and ÷.

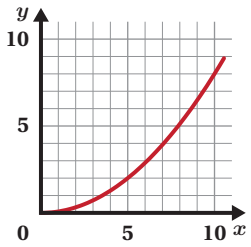
-4		-1		3	=	1
1		-3		-3	=	1
4		1		-2	=	1
=		=		=		
1		1		1		

13. Select *all* the graphs that don't represent a proportional relationship.

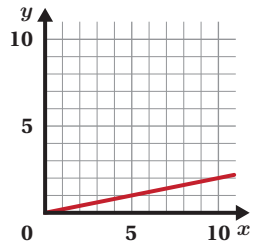
☐ A.



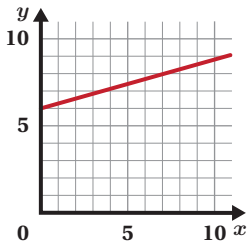
☐ B.



☐ C.



☐ D.



Name: ..... Date: ..... Period: .....

Unit Rates in the World | 7.EE.3, 7.NS.2, 7.NS.2.b, 7.NS.3, SMP.1, SMP.4

Visiting the Arctic

Let's use positive and negative rates to analyze our environment.

Warm-up

1. Let's watch a video. Here are two maps from the video.

What do you notice? What do you wonder?

I notice:

I wonder:

Activity 1

Name: ..... Date: ..... Period: .....

Arctic Sea Ice

This table shows the amount of sea ice in the Arctic during the summer for various years.

Year	Summer Sea Ice (cu. km)
1980	
1990	13,815
2000	11,084
2010	
2020	

2. Estimate the amount of sea ice for 1980, 2010, and 2020. Record your estimates in the table.

3. **Discuss:** How did you determine your estimates?

4. **Data Talk!** Let's look at the real data. How do your estimates compare with the real data?

5. Choose two years of *actual data*. Determine the average change in summer ice per year between those years. (Note: The average change per year is equal to the change in the summer sea ice divided by the number of years.)

6. Some scientists predict that there will be no summer sea ice in the Arctic by 2050. Do you think this prediction is reasonable? Explain your thinking.

You're invited to explore more.

7. Switzerland is taking action to reduce the rate of sea ice melt. Each year, a group of residents cover up part of the Rhône Glacier with huge white blankets. These blankets can help reduce seasonal melting by up to 70%. Imagine if a similar approach could be applied to the Arctic sea ice. Estimate when the summer sea ice would reach zero under these conditions.

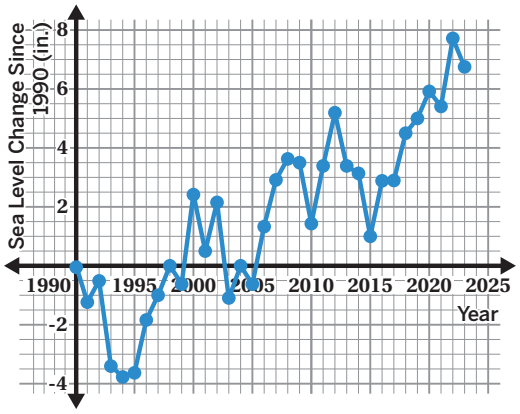
Activity 2

Name: ..... Date: ..... Period: .....

Ice Impact

The sea level and high tide is rising globally, in part due to melting ice. According to NASA, "high-tide flooding is an increasingly frequent occurrence in coastal areas around the Pacific."

Data Talk! Tonga is a country in the southwestern Pacific Ocean, made up of about 170 different islands.
This graph and table show the change in sea level in Nuku'alofa, Tonga, since 1990.



Year	Sea Level Change Since 1990 (in.)
1990	0
1994	–3.72
1998	0
2002	2.16
2006	1.32
2010	1.44
2014	3.12
2018	4.44
2022	7.68

8. The point (1994, -3.72) is on the graph. What does -3.72 represent in this situation?

9. Use the data to predict the sea level change in Nuku'alofa by the year 2100 (compared to 1990). Explain your thinking.

10. What do you think the impact of the rising sea level might be on . . .

a **. . . Nuku'alofa, Tonga?**

b **. . . You or your community?**

Synthesis

11. **Discuss:** How can you use the rate of ice melt to make predictions about future amounts of summer Arctic sea ice?

Use the example in the table if it helps with your explanation.

Year	Summer Sea Ice (cu. km)
1980	16,316
1990	13,815
2000	11,084
2010	4,742
2020	4,158

Summary 5.12

Analyzing data about sea ice and sea levels can help us make predictions about how these levels might change in the future. We can see patterns and make predictions by subtracting data values to get a rate and then extending that rate into the future using multiplication.

To calculate the average change in summer sea ice per year from 2010 to 2020, subtract the sea ice amounts for these years and then divide by 10.

$\frac{4158 - 4742}{10} = \frac{-584}{10} = -58.4$ cubic kilometers per year

To make a prediction about the amount of summer sea ice in 2040, multiply this rate by 20 and add that value to the sea ice amount from 2020.

$-58.4 \cdot 20 = -1168$

$4158 + (-1168) = 2990$ cubic kilometers

Making predictions like this can help us prepare for the future impact that changing sea ice and sea levels may have.

Year	Summer Sea Ice (cu. km)
1980	16,316
1990	13,815
2000	11,084
2010	4,742
2020	4,158

Practice
5.12

Name: Date: Period:

1. Order these expressions from *least* to *greatest*.

$-15.6 \div (-3)$	$-5.2 \cdot (-3)$	$-5.2 - (-3)$	$-5.2 + (-3)$	$15.6 \div (-3)$

Least **Greatest**

Problems 2–3: A bank charges a service fee of $\$7.50$ per month for a checking account. A checking account has a balance of $\$85.00$. No money is withdrawn or deposited, except the service fee.

2. How much money will be in the account after 5 months? Show or explain your thinking.

3. How many months until the account balance is negative? Show or explain your thinking.

4. This table shows the extreme elevations of four countries. Complete the table.

Country	Lowest Elevation (m)	Highest Elevation (m)	Difference (m)
China	-154	8,849	
Kazakhstan	-132	7,010	7,142
Jordan		1,854	2,285
Bolivia	90	6,542	

5. Jalen makes a cup of tea and lets it cool. The table shows the tea's temperature at different times since he made it. Jalen predicts that the tea's temperature will be 17.2°F after 60 minutes.

Do you agree with this prediction? Explain your thinking.

Time (min)	Temperature (°F)
5	153.1
10	136.2
15	124.3

Practice
5.12

Name: ..... Date: ..... Period: .....

Spiral Review

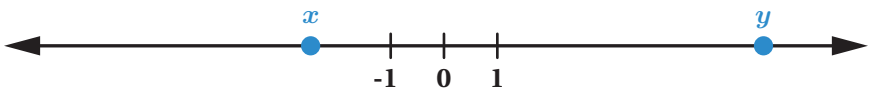
Problems 6–8: Determine the value of the variable that makes each equation true.

6. $5a = -1$

7. $\frac{b}{-3} = -15$

8. $-14 + c = -12.5$

9. This number line shows the positions of x and y. Select *all* the expressions that have a negative value.



☐ **A.** $2x$ ☐ **B.** $\frac{y}{10}$ ☐ **C.** xy ☐ **D.** $-\frac{x}{y}$ ☐ **E.** $y - x$

10. Here is an equation puzzle.

Fill in the blanks so that each row and column is a true equation.

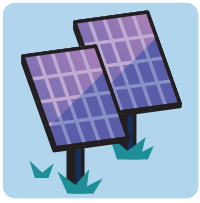
$\frac{1}{2}$	×		+	$-\frac{3}{2}$	=	0
×		×		×		
8	×		+		=	0
+		−		+		
	×	$\frac{3}{4}$	−		=	0
=		=		=		
0		0		0		

11. A chef needs 2 cups of flour for a recipe, but uses 2.1 cups. What is the percent error?

Name: ..... Date: ..... Period: .....

Proportional Relationships 7.EE.3, 7.NS.2, 7.NS.2.a, 7.NS.3, SMP.3, SMP.4

Solar Panels and More



Let's use operations with positive and negative numbers to explore carbon emissions.

Warm-up

Many of our daily activities emit carbon dioxide, which negatively impacts our planet. This table shows some actions that reduce carbon emissions.

Recycling waste	Using public transportation	Washing your clothes with cold water
Eating less meat	Biking instead of driving	Using solar panels

1. What do you think it means to reduce carbon emissions?

2. What might impact someone's decision to take an action or not?

Activity 1

Name: Date: Period:

Solar Panels

Sunny Side Up Café is researching solar panels as a way to reduce carbon emissions and save money on electricity. Each month, Sunny Side Up Café will get a credit on their electric bill for the electricity that is generated by the solar panels.

3. Two numbers on their April bill are smudged.

Determine the smudged values on the bill.

April			
	Kilowatt Hours (kWh)	$ per kWh	Total
Electricity Used	2,083	$0.13	$270.79
Electricity Generated	[illegible]	−$0.11	−$213.18
		Total Due:	[illegible]

4. How many kilowatt-hours of electricity would Sunny Side Up Café need to generate to have their total due for January be $0?

5. The solar panels cost $24,500. The owner of Sunny Side Up says it's not worth it to buy solar panels if they plan to stay in their building for only one more year. Do you agree?

6. Do you think buying solar panels is worth it for Sunny Side Up Café? Use the data to support your argument.

Activity 2

Name: Date: Period:

Carbon Footprint

Data Talk! Schools A, B, and C have received a grant to help pay for projects that will reduce their carbon emissions. On the Activity 2 Sheet, find the "Schools" table that shows their current carbon footprint and some information about each school.

7. What do you notice? What do you wonder?

I notice: I wonder:

8. Mar is a student at School 3. Mar thinks the school should use its grant to buy electric school buses and replace the windows with energy efficient windows. Mar wrote this expression to represent the school's new carbon footprint if they implement this plan:

$$1250 + 8(-13.9) + 15(-0.7)$$

a Explain what each value in Mar's expression represents.

b What will School 3's carbon footprint be if they use Mar's suggestions?

Name: ..... Date: ..... Period: .....

Carbon Footprint (continued)

9. Each school wants to make a plan that will reduce their carbon footprint. Choose one school from the Activity 2 Sheet and develop a plan that includes at least three actions they could take to reduce their carbon footprint.

10. Write an expression that represents the school's new carbon footprint based on your plan.

11. Use your expression to calculate the new carbon footprint for the school you chose.

You're invited to explore more.

13. The clothing industry produces as much as 10% of all of humanity's carbon emissions. In 2019, the clothing industry produced more than 3.6 billion tons of carbon. Using solar panels reduces carbon emissions by -4.4 tons per home. Which of the following would have a greater impact: the clothing industry reducing their carbon footprint by 1% or each of the $6{,}337{,}929$ households in Florida installing solar panels? Explain your thinking.

Synthesis

14. What new questions do you have about carbon emissions after exploring this lesson?

Summary 5.13

Performing operations with positive and negative numbers can help us represent and solve real-world problems.

Imagine a family of 5 whose current carbon footprint is 60.5 tons of carbon dioxide per year. This family decides to find ways to reduce this carbon footprint including:

- Installing solar panels (-4.4 tons of carbon)
- Recycling waste (-0.6 tons of carbon per person)
- Composting food waste (-0.5 tons of carbon per person)

We can represent the changes with this expression:

$$60.5 + (-4.4) + 5(-0.6) + 5(-0.5)$$

By making these changes, this family will bring their carbon footprint down to 50.6 tons per year.

Practice
5.13

Name: ..... Date: ..... Period: .....

Problems 1–3: Dylan's school lunch account had a balance of $56. After buying 8 lunches, the balance is $40.

1. How much does the balance change with each lunch bought? Show whether the change is positive or negative.

2. How many more lunches can Dylan buy before running out of money?

3. At Dylan's school, every student starts with a balance of $80 for school lunches. How many lunches were bought when Dylan first checked the balance?

Problems 4–5: The temperature was -12°C and increased to 2°C. What was the change in temperature?

4. Select the equation that matches the situation.

A. $-12 + x = 2$ **B.** $-2x = -12$ **C.** $-12 + 2 = x$ **D.** $2 + x = -12$

5. Write the answer in a sentence.

Problems 6–7: Felipe's family has solar panels on the roof of their house. They get a credit each month for the electricity generated by the solar panels. The bill each month is equal to the charge for electricity used plus the credit for electricity generated.

6. Complete the table.

7. How much was billed to Felipe's family in total from April to August?

Month	Charge ($)	Credit ($)	Bill ($)
April	71.59	-42.11	29.48
May	65.32	-67.80	-2.48
June	82.92	-71.10	
July	57.70	-68.02	
August		-54.34	7.50

Name: ..... Date: ..... Period: .....

8. If the value of x is $\frac{1}{4}$, order these expressions from *least* to *greatest*.

x	$1 - x$	$x - 1$	$-2x$

Least **Greatest**

9. If the value of x is $-\frac{1}{4}$, order these expressions from *least* to *greatest*.

x	$1 - x$	$x - 1$	$-2x$

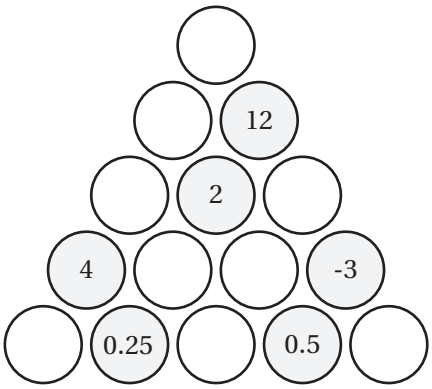
Least **Greatest**

Spiral Review

10. Fill in each blank with a number so that every row, column, and diagonal adds up to the same total.

9		-4	6
	4		
2		-1	
-3	7	8	-6

11. Fill in each blank with a number so that every number is the product of the two numbers directly below it.



Name: _____ Date: _____ Period: _____

Generalized Numbers Leading to Algebra | Number Line Understanding

7.EE.3, 7.NS.1.d, 7.NS.2, 7.NS.2.a, 7.NS.2.b, 7.NS.2.c, 7.NS.3, SMP.3

Practice Day 2

Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Match It!

1.

Value	Expression #1	Expression #2	Expression #3

2. Two Expressions: _____ and _____

Explanation:

Task B: Solve It!

1. $a =$ _____ **2.** $b =$ _____ **3.** $c =$ _____

4. $d =$ _____ **5.** $e =$ _____ **6.** $f =$ _____

Name: _____ Date: _____ Period: _____

Practice Day 2 (continued)

Task C: Positive, Negative, or Zero?

1. Positive: _____ Negative: _____ Zero: _____

2. Solution:

Task D: Changing Temperatures

1. Equation: _____ **2.** Equation: _____ **3.** Equation: _____ **4.** Equation: _____

5. $x =$ _____

Explanation:

6. Temperature: _____

You're invited to explore more.

1. Second expression: _____ Third expression: _____ Fourth expression: _____

2. Patterns:

3. 10th expression: _____

Notes:

Career Connection

Not all deserts are hot. Antarctica is a frozen desert, with an average winter temperature of -34.4°C.

Oleksandr Matsibura/Shutterstock.com

Many scientists study Antarctica because of its extreme conditions. Some NASA scientists and engineers have even tested robots in Antarctica that later landed on Mars because the desert conditions on both Mars and Antarctica are a lot alike. In fact, the average temperature of Mars is around -65°C.

One of the NASA scientists and engineers working to study Antarctica's ice levels is Sheila Wall. She was the lead structural analyst on the Ice, Cloud, and Land Elevation Satellite-2 (ICESat-2) mission, which launched in 2018. The satellite measures the elevations of Earth's ice-covered regions. It also measures the heights of other features, such as trees, lakes, and buildings.

Aerospace engineers design, develop, and maintain aircraft, spacecraft, and satellites. They might calculate with positive and negative numbers to determine changes in temperature or elevation.

Meet Sheila M. Wall

As an aerospace engineer, Sheila M. Wall uses math to create models of proposed designs and evaluate their structural integrity. As she described, "Creating my mathematical models, 3D numerical representations of the instruments, is one of my favorite aspects of my job." Along with working on the ICESat-2 mission, Sheila M. Wall has also worked on the Lunar Reconnaissance Orbiter mission to research the moon and the Lucy mission to explore the Jupiter Trojan asteroids.

Are you interested in studying aerospace engineering? What can you do to learn more?

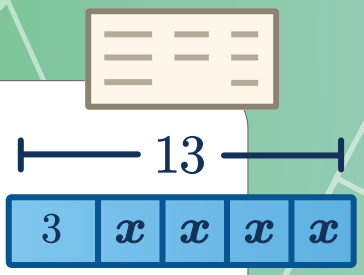
Math in the World

Antarctica holds the record for the coldest temperature recorded on Earth, -89.2°C, measured on July 21, 1983 at the Vostok research station. The highest temperature recorded in Antarctica was 18°C. How can you compare these temperatures?

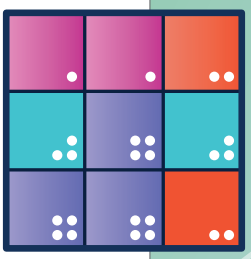
Math Mindset

Describe one strategy you can use to subtract -5 from -6.

Unit 6



Expressions, Equations, Inequalities



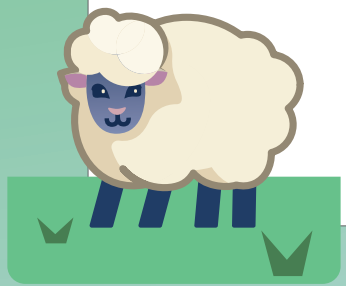
Big Ideas in This Unit

CC2 Unit Rates in the World

NS See Generalized Numbers Leading to Algebra

Questions for Investigation

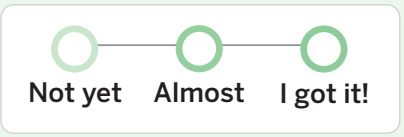
- Which representations help you make sense of different mathematical and algebraic situations?
- How can recognizing unit-rates help to solve equations and inequalities in real-world situations?
- Which strategies for solving simple equations or inequalities can help you solve more complex ones?



Explore: Keeping the Balance
How can we use a dog-walker model to help us make sense of balance and equality?

Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



I can . . .	Before	After
Use properties of operations to write equivalent expressions.		
Factor and expand expressions.		
Solve problems with positives, negatives, decimals, fractions, and whole numbers.		
Check my answers using mental math and estimation.		
Use tape diagrams and hanger diagrams to solve problems.		
Recognize and solve problems involving a unit rate.		
Use variables to represent numbers in a problem when writing equations or inequalities.		
Solve word problems by writing or using equations.		
Make meaning of the solutions of an equation in a real-world situation.		
Solve word problems by writing or using inequalities.		

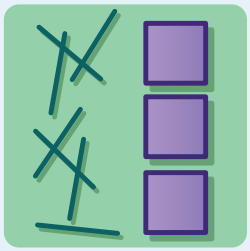
I can . . .	Before	After
Graph the solutions of an inequality on a number line.		
Make meaning of the solutions of an inequality in a real-world situation.		
Analyze different strategies for solving the same equation or inequality.		

Sub-Unit

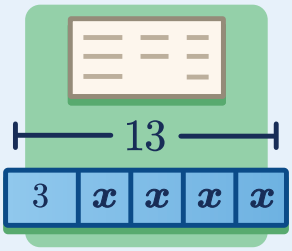
1

Equations and Tape Diagrams

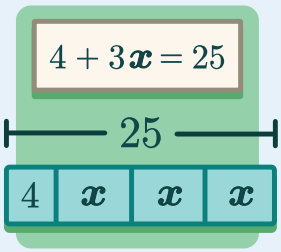
Explore
Keeping the Balance



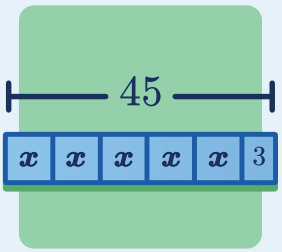
Lesson 1
Toothpicks and Tiles



Lesson 2
Smudged Receipts



Lesson 3
Equations



Lesson 4
Seeing Structure

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra | Building Toward 7.EE.4, SMP.1, SMP.2, SMP.4

Explore: Keeping the Balance

How can we use a dog-walker model to help us make sense of balance and equality?

Warm-Up

1. Here are two diagrams. What do you notice? What do you wonder?

Diagram A

Diagram B

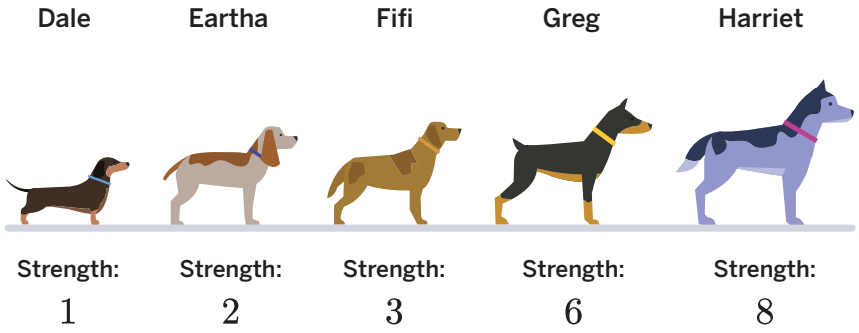
I notice. . .

I wonder. . .

Name: ..... Date: ..... Period: .....

Walking Dogs Like a Pro

Welcome to Pawston University! We have produced some of the nation's finest dog walkers. Your first challenge is to position the dogs on their leashes so that the dog walker is balanced.

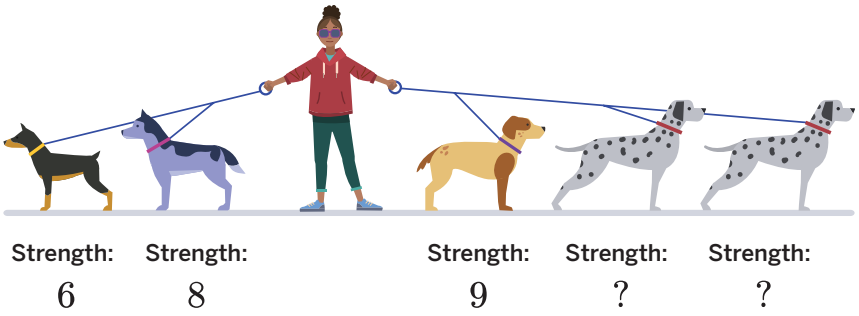


2. How can you balance three dogs on two leashes, holding one leash in each hand? Find as many answers as you can.

3. How can you balance four dogs on two leashes, holding one leash in each hand? Find as many answers as you can.

4. How can you balance all five dogs on two leashes, holding one leash in each hand? Find as many answers as you can.

Your next challenge is to figure out the strength of the two new dogs in the diagram — whose strengths have not been labeled. Assume that the dog walker feels an equal pull in both directions, and that the dogs who look the same have the same strength.



5. What is the strength of each new dog? Explain your thinking.

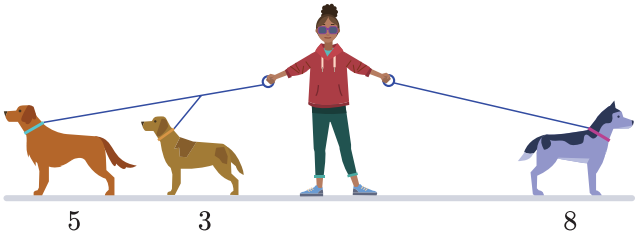
Name: Date: Period:

Walking Dogs Like a Pro

6. Rename the dog-walker's balancing techniques using language that makes the most sense to you.

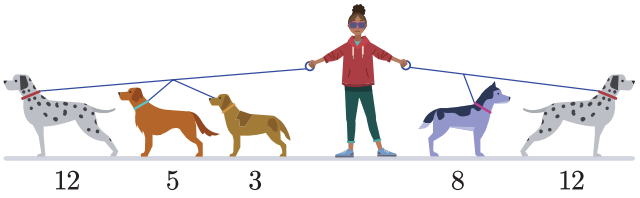
Old name: Addition Property of Equality

New name:



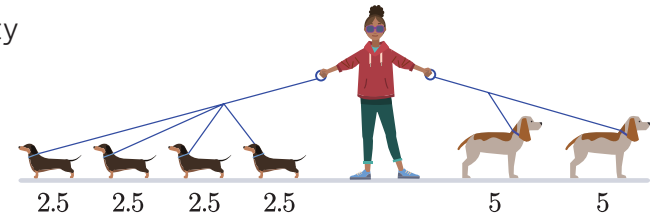
Old name: Subtraction Property of Equality

New name:



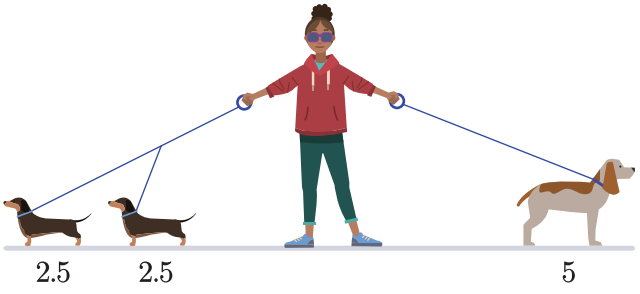
Old name: Multiplication Property of Equality

New name:



Old name: Division Property of Equality

New name:



Name: ..... Date: ..... Period: .....

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet Almost I got it!

I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem.

Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, making assumptions and revising my thinking as needed.

Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

Not yet Almost I got it!

Unit 6
Lesson
1

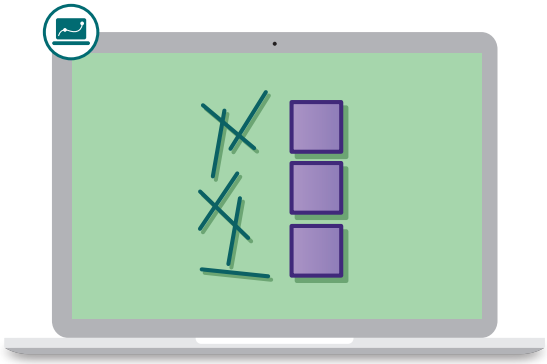
Name: ..... Date: ..... Period: .....

Unit Rates in the World

Building Toward 7.EE.4.a, SMP.3, SMP.8

Toothpicks and Tiles

Let's make predictions about relationships.

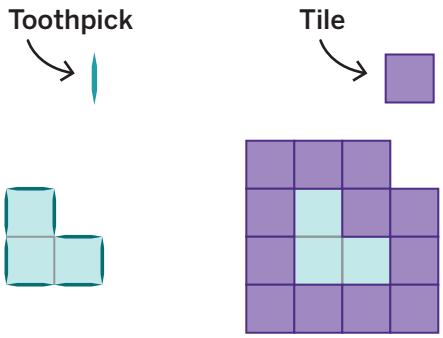


Warm-Up

1 Here are two identical copies of the same shape. One has a border of toothpicks around it. The other has a border of tiles.

How many toothpicks are used?

How many tiles?



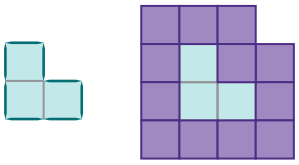
Activity 1

Name: Date: Period:

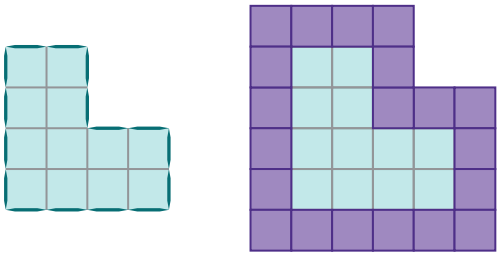
Patterns and Predictions

2 Stage 1 shows two figures. Stages 2 and 3 are *scaled copies* of Stage 1. Stage x is a scaled copy of Stage 1 with a *scale factor* of x.

Stage 1



Stage 2



How many border toothpicks and tiles are in Stages 2 and 3?

Stage	Border Toothpicks	Border Tiles
1	8	12
2		
3		

3 Will there ever be a stage with exactly 100 toothpicks? Explain your thinking.

4 There is a stage that uses 100 border tiles. Which stage? Explain your thinking.

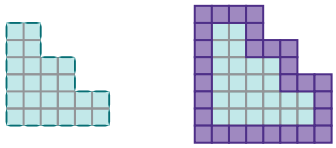
Name: ..... Date: ..... Period: .....

Patterns and Predictions (continued)

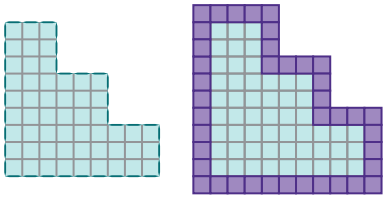
5 Here is a new design.

Stage 1

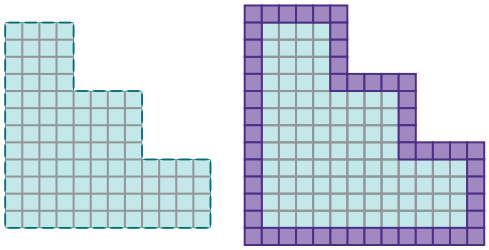
Stage 2



Stage 3



Stage 4



How many border tiles will there be in Stage 5? Explain your thinking.

Stage	Border Toothpicks	Border Tiles
1	12	16
2	24	28
3	36	40
4	48	52

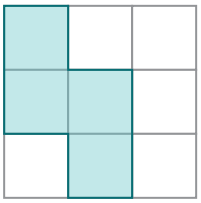
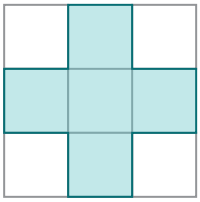
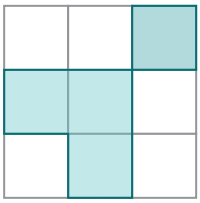
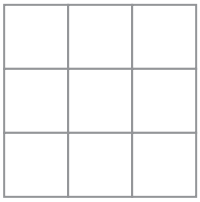
6 Which stage uses 100 border tiles?

Activity 2

Name: ..... Date: ..... Period: .....

Unique Designs

7 Choose one of these Stage 1 designs or create your own.

Design A	Design B	Design C	Create Your Own
			

a Use the Activity 2 Sheet to determine the number of border toothpicks and tiles for Stages 1–3 of the design you chose.

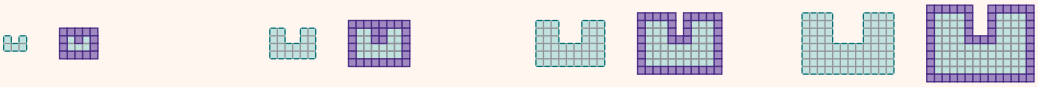
Stage	Border Toothpicks	Border Tiles
1		
2		
3		

b Predict how many border tiles are used in Stage 4.

You're invited to explore more.

8 There's something unusual about the number of border tiles in Stage 1 of this design.

Stage 1 **Stage 2** **Stage 3** **Stage 4**



Why is Stage 1 different from Stages 2 and beyond?

Stage	Border Toothpicks	Border Tiles
1	12	15
2	24	28
3	36	40
4	48	52

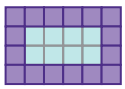
9 Synthesis

Here is a new pattern.

Stage 1

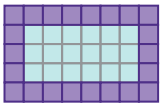
Border Tiles: 10

Stage 2



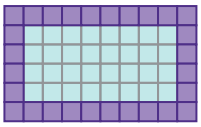
Border Tiles: 16

Stage 3



Border Tiles: 22

Stage 4

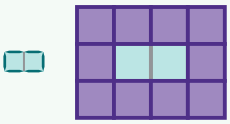
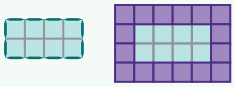
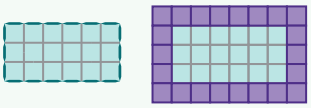


Border Tiles: 28

Describe how you can determine the number of border tiles at any stage.

12 Summary 6.01

Analyzing shape patterns can help you understand number patterns. Here is a design made with border toothpicks and border tiles.

Stage 1	Stage 2	Stage 3
		

In Stage 1, there are 6 toothpicks. In Stage 2, there are 12 while in Stage 3, there are 18. The number of toothpicks increases by 6 each time. The table shows that the number of tiles is always 4 more than the number of toothpicks.

We can extend these rules to make predictions about any stage of the pattern. For example, in Stage 5 there will be 30 toothpicks and the number of border tiles will be 4 greater, 34.

Stage	Border Toothpicks	Border Tiles
1	6	10
2	12	16
3	18	22

Practice
6.01

Name: ..... Date: ..... Period: .....

Problems 1–4: A sandwich store charges $20 to have 3 subs delivered and $26 to have 4 subs delivered. This includes the delivery fee.

1. How much does the store charge for each additional sub?

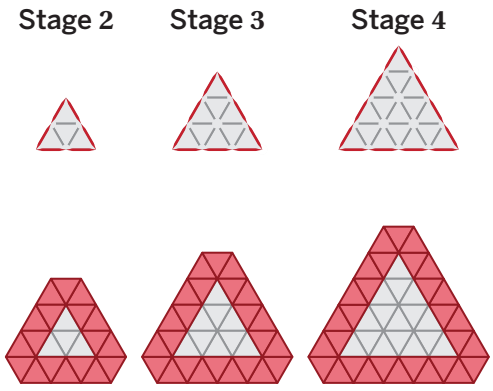
2. How much is the delivery fee?

3. Is the relationship between the number of subs delivered and the total amount charged proportional? Explain your thinking.

4. If the total charge is $56, how many subs are in the order? Explain your thinking.

5. Here are scaled copies of a figure. The top three have a toothpick border and the bottom three have a tile border. Complete the table to show the number of toothpicks and tiles for different stages.

Stage	Border Toothpicks	Border Tiles
2		
3		
4		
5		
6		



Practice
6.01

Name: ..... Date: ..... Period: .....

Spiral Review

6. Make each equation true by writing the correct operation.

$48 \square (-8) = -6$ $\quad$ $(-40) \square 8 = -5$

$12 \square (-2) = 14$ $\quad$ $18 \square (-12) = 6$

7. Select *all* of the statements that are true.

- ☐ **A.** $9 - 5 = -2 \cdot (-2)$
- ☐ **B.** $-10 \div \left(-\frac{1}{10}\right) = -1$
- ☐ **C.** $0.5 - 1 + 2.5 = 2$
- ☐ **D.** $-\frac{2}{3} > 2 - 3$
- ☐ **E.** $7 + 7 < -14$

Problems 8–9: Maneli and Santiago are trying to solve the equation $\frac{2}{3} + x = \frac{1}{3}$.

- Maneli says: *I think we should multiply each side by $\frac{3}{2}$ because that is the reciprocal of $\frac{2}{3}$.*
- Santiago says: *I think we should add $-\frac{2}{3}$ to each side because that is the opposite of $\frac{2}{3}$.*

8. Which person's strategy should they use?

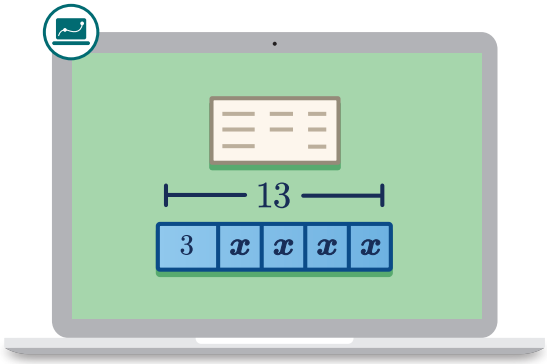
9. What is an equation that can be solved using the other person's strategy?

Problems 10–12: Rewrite each fraction in decimal form.

10. $\frac{13}{4}$ $\quad$ **11.** $\frac{15}{8}$ $\quad$ **12.** $\frac{22}{6}$

Name: ..... Date: ..... Period: .....

Unit Rates in the World 7.EE.3, 7.RP.2, SMP.1, SMP.2, SMP.3

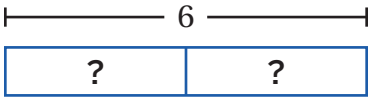


Smudged Receipts

Let's represent situations with equations.

Warm-Up

1 Here is a *tape diagram* that represents 2 sides of plain pasta that cost $6 in total at Pasta Express.



a What is the price of each side of plain pasta?

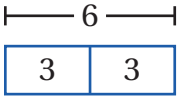
b **Discuss:** Where do you see each part of the situation in the tape diagram?

Name: ..... Date: ..... Period: .....

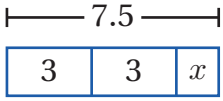
Smudged Receipts

2 Pasta Express charges for toppings on the pasta sides. Here are some receipts from the orders and the tape diagrams that represent them.

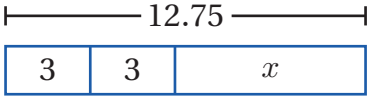
Pasta	(2 @ $3.00)	$6.00
Toppings		$0.00
Total:		$6.00



Pasta	(2 @ $3.00)	$6.00
Toppings		$1.50
Total:		$7.50



Pasta	(2 @ $3.00)	$6.00
Toppings		$6.75
Total:		$12.75



What does each part of the tape diagram represent?

a The 3s represent . . .

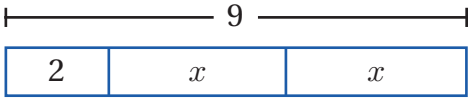
b The x represents . . .

c The length of the tape diagram represents . . .

3 Here is a new receipt with some smudged numbers.

Determine the price of a cantaloupe. Use the tape diagram if it helps with your thinking.

Honeydew	(1 @ $2.00)	$2.00
Canteloupe	(2 @ $)	$
Total:		$9.00



Explain your thinking.

Name: ..... Date: ..... Period: .....

Smudged Receipts (continued)

4 Match each receipt to the diagram that represents it. One diagram will have no match.

a.

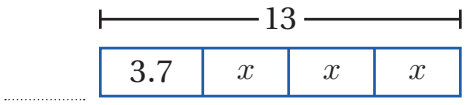
Juice	(4 @ $)	$
Total:		$13.00

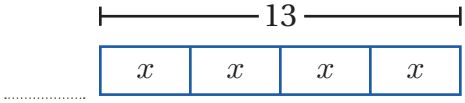
b.

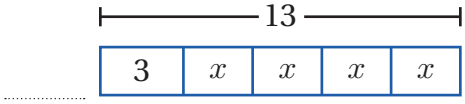
Chips	(3 @ $4.00)	$12.00
Salsa	(1 @ $)	$
Total:		$13.00

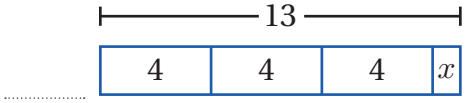
c.

Pineapple	(1 @ $3.70)	$3.70
Beans	(3 @ $)	$
Total:		$13.00









5 Let's take a closer look at the juice receipt. What is the price of one juice at this store?

Juice	(4 @ $)	$
Total:		$13.00

6 Let's take a closer look at the pineapple and beans receipt. What is the price of each can of beans? Explain your thinking.

Pineapple	(1 @ $3.70)	$3.70
Beans	(3 @ $)	$
Total:		$13.00

Activity 2

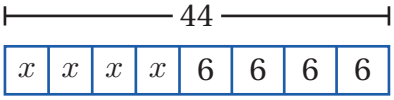
Name: Date: Period:

Ordering Food

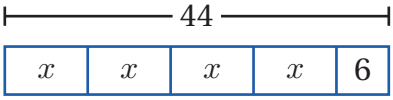
Raven and her three siblings were given $44 to order takeout food.

7 Burrito Express charges a $6 delivery fee. Each person can spend x dollars on their meal. Which diagram represents this situation?

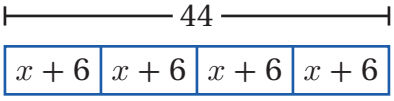
A.



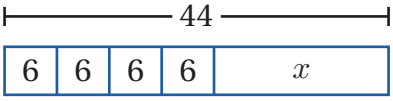
B.



C.

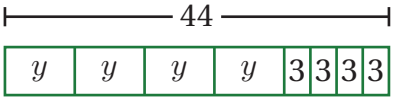


D.

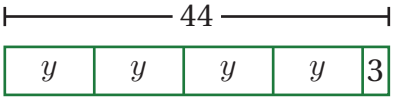


8 Salads-R-Us charges a $3 service fee for each meal. Circle a diagram that could help determine how much each sibling can spend on their meal.

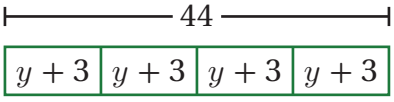
A.



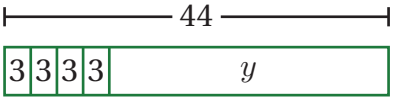
B.



C.



D.



Explain how this diagram represents the situation.

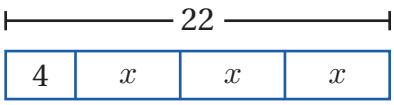
9 x represents the amount per meal at Burrito Express. y represents the amount per meal at Salads-R-Us.

a. Use the diagrams you selected to determine the values of x and y.

b. **Discuss:** Which restaurant should the siblings choose? Why?

10 Synthesis

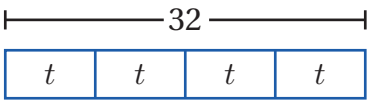
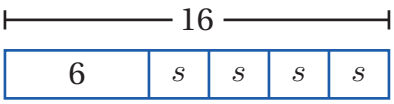
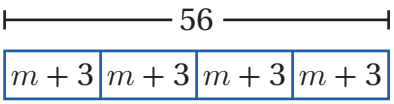
Write a situation that this diagram could represent.



13 Summary 6.02

Tape diagrams and expressions can be used to represent situations.

Here are three situations and the tape diagrams that represent them.

Situation A	Situation B	Situation C
4 friends visit an aquarium. It costs t dollars per person to enter. Their total cost is \$32.	4 pounds of strawberries cost s dollars each and 1 jar of raspberry jam costs \$6. Ama's total bill is \$16.	The food truck charges a set price of m dollars for a meal and \$3 for a beverage. The total bill for 4 friends is \$56.
		
$t = 8$	$s = 2.5$	$m = 11$

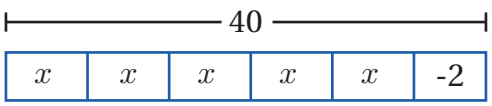
Practice
6.02

Name: ..... Date: ..... Period: .....

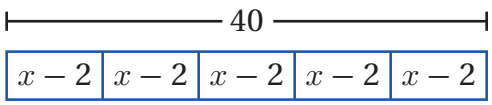
Problems 1–2: Axel wants to save $40 to buy a gift for his friend. Every week, his neighbor pays him to mow the lawn, and he donates $2 of what he earns to charity. Axel calculates that it will take him 5 weeks to earn enough for his friend's gift.

1. Which tape diagram represents this situation?

A.



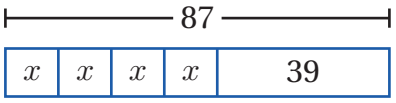
B.



Explain how the tape diagram represents the situation.

2. How much does Axel's neighbor pay him each week to mow the lawn?

3. Select *all* the situations that the tape diagram can represent.

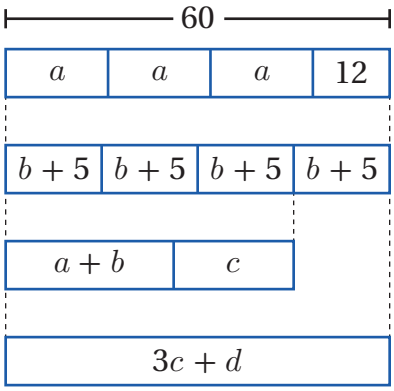


- ☐ **A.** There are 87 first graders in school. After 39 students are picked up, the teachers put the remaining students into 4 groups for an activity.
- ☐ **B.** Latifa buys a pack of 87 pencils. She gives 39 to her teacher and shares the remaining pencils between herself and 3 friends.
- ☐ **C.** Emiliano buys 4 packs of paper clips with 39 paper clips in each. Then he gives 87 paper clips to his teacher.
- ☐ **D.** Shanice's family buys 4 tickets to a fair and spends $39 on dinner. They spend $87 total.
- ☐ **E.** Daniela went shopping and spent $39 on 4 shirts. She then spent $87 on new shoes.

Name: _____ Date: _____ Period: _____

4. Determine the values of a, b, c, and d.

Variable	Value
a	
b	
c	
d	



Problems 5–6: Three slices of cheese pizza cost $9.75.

5. Describe what the tape diagram that represents this situation might look like.

6. Determine the cost of one slice of cheese pizza.

Spiral Review

Problems 7–14: Solve each equation.

7. $2x = 10$

8. $-3x = 21$

9. $\frac{1}{3}x = 6$

10. $-\frac{1}{2}x = -7$

11. $8.5 \cdot (-3) = a$

12. $(-7) + b = -11$

13. $c - (-3) = 15$

14. $d \cdot (-4) = 32$

15. What is the value of the expression $-20 \div 4 + 2(-8) + 10$?

A. 11

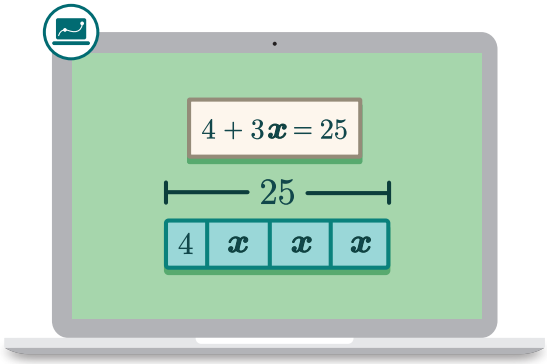
B. -11

C. -58

D. $-\frac{13}{3}$

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra 7.EE.3, 7.EE.4, SMP.1, SMP.2

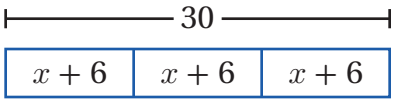


Equations

Let's connect representations of relationships.

Warm-Up

1 Here is a tape diagram.



a Which equation matches the tape diagram?

A. $3x + 6 = 30$

B. $3 + 6x = 30$

C. $3(x + 6) = 30$

b Draw a tape diagram for one of the equations that you *did not* select.

Activity 1

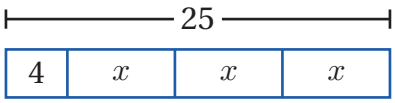
Name: Date: Period:

Connecting Representations

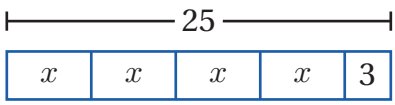
Liam plans to bake 25 cookies. He wants to keep 4 cookies for himself, and then split the rest evenly between his 3 friends.

2 Which equation and tape diagram match Liam's situation?

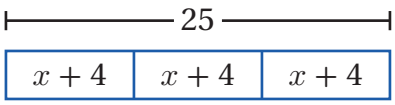
A. $4 + 3x = 25$



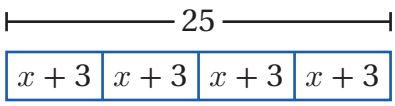
B. $4x + 3 = 25$



C. $3(x + 4) = 25$



D. $4(x + 3) = 25$



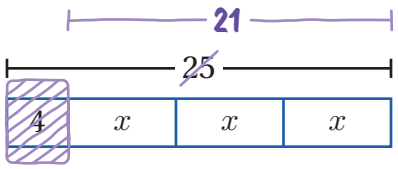
3 How many cookies should each of Liam's friends receive?

Name: ..... Date: ..... Period: .....

Connecting Representations (continued)

4 Liam used this tape diagram and equation to represent his situation. Here is his first step. What equation would represent his new tape diagram? Explain why this equation is helpful.

$$4 + 3x = 25$$



5 Liam ended up baking 33 cookies. He kept 5 for himself and split the rest evenly between 7 friends.

a Describe what a tape diagram for Liam's new situation would look like.

b Write an equation to represent Liam's new situation.

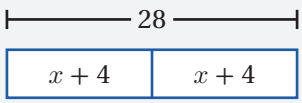
Activity 2

Name: Date: Period:

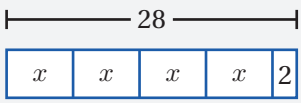
Missing Representations

6 Make groups that represent the same situation. Some groups will be missing a representation.

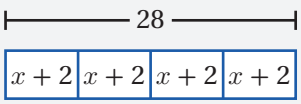
A.



B.



C.



D. A cake-baking kit says: 28 tablespoons of sugar is provided for 2 cakes. For each cake, save 4 tablespoons of sugar for frosting and put the rest in the batter.

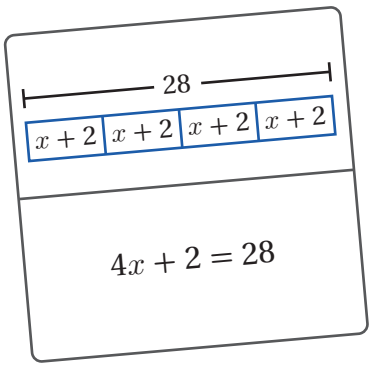
E. Riku's mom buys 4 cinnamon buns, one per family member. Each person also gets \$2 to spend on a beverage. The bill is \$28.

F. $2(x + 4) = 28$

G. $4x + 2 = 28$

Group 1	Group 2	Group 3
A	B	C

7 Irene incorrectly matched these two representations. What would you tell Irene to convince her that these don't match?

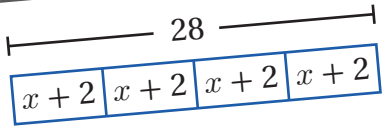


Name: Date: Period:

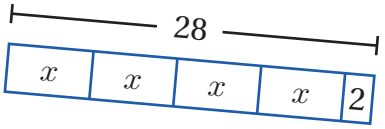
Missing Representations (continued)

8 One group did not have a matching equation. Write an equation that matches.

Riku's mom buys 4 cinnamon buns, one per family member. Each person also gets $2 to spend on a beverage. The bill is $28 total.



9 One group did not have a matching situation. Write a situation that matches.



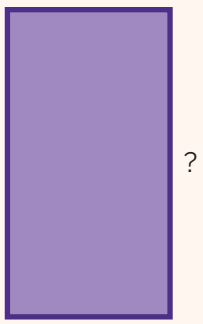
$4x + 2 = 28$

You're invited to explore more.

10 This rectangle has an unknown length, a width of 9 centimeters, and a perimeter of 52 centimeters.

a Write an equation or draw a tape diagram to represent this situation.

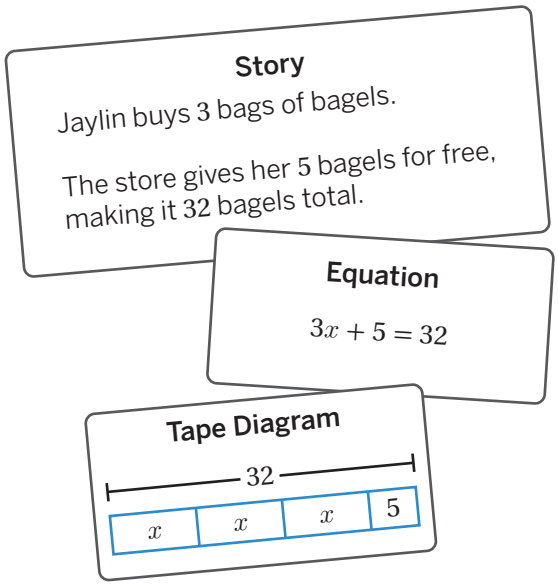
b Determine the length of the rectangle.



11 Synthesis

Here is a new situation.

Discuss: How is the number 9 important in each representation?



14 Summary 6.03

We can use tape diagrams and equations to make sense of stories and determine unknown amounts.

Here is a story with the equation and tape diagram that represent it.

Story	Equation	Tape Diagram
Two students go to the movie theater. They purchase two tickets and a \$5 popcorn to share. In total, they spend \$19.	$2x + 5 = 19$	19 x | x | 5

In the equation and tape diagram, x represents the unknown price of a movie ticket, 2 represents the number of tickets that were purchased, 5 represents the \$5 spent on popcorn, and 19 represents the total amount spent.

Practice
6.03

Name: ..... Date: ..... Period: .....

Problems 1–3: Here are two different situations.

1. Match each equation and situation with its tape diagram.

Equation/Situation

a. $3x + 6 = 27$

b. $6x + 3 = 27$

c. A family buys 6 tickets to a show. They also pay a $3 parking fee. They spend $27 total.

d. Diego has 27 ounces of juice. He pours equal amounts for each of his 3 friends and has 6 ounces left for himself.

Diagram A

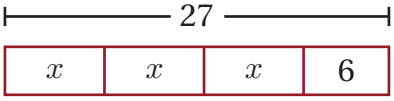
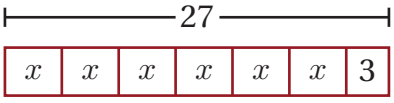


Diagram B



2. Using your responses from Problem 1, determine the value and meaning of x in Diagram A. Explain what x represents in context.

3. Using your responses from Problem 1, determine the value and meaning of x in Diagram B. Explain what x represents in context.

4. Match each equation with a tape diagram.

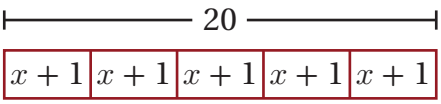
Equation

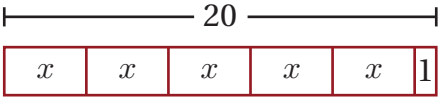
a. $5x + 1 = 20$

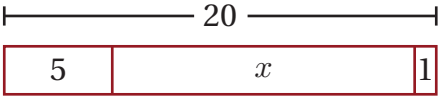
b. $5(x + 1) = 20$

c. $5 + x + 1 = 20$

Tape Diagram



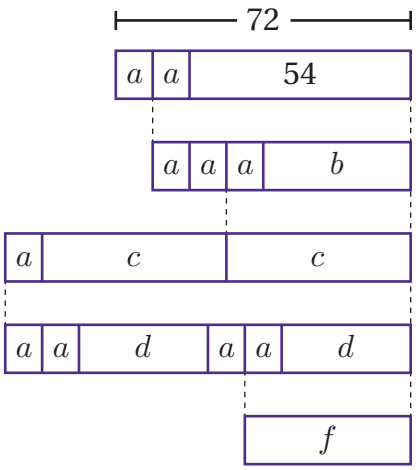




5. A family of 4 is ordering food for dinner. Each person orders a drink for \$2 and a meal. They spend a total of \$30. Write and solve an equation to determine the cost of each meal.

6. Determine the value of each variable.

Variable	Value
a	
b	
c	
d	
f	



Spiral Review

7. Determine the number of miles each car can travel in 1 hour assuming the operator drives at a constant speed.

Speed	Number of Miles in 1 Hour
135 miles in 3 hours	
22 miles in $\frac{1}{2}$ hour	
7.5 miles in $\frac{1}{4}$ hour	
$97\frac{1}{2}$ miles in $\frac{3}{2}$ hours	

Problems 8–11: Determine the value of each expression.

8. $100 \cdot (-0.09)$

9. $-7 \cdot (-1.1)$

10. $-7.3 \cdot 5$

11. $-0.2 \cdot (-0.3)$

12. Select *all the* expressions that have products that are positive.

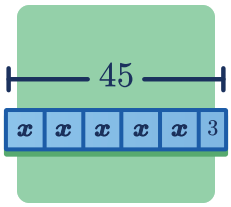
- ☐ **A.** $-\frac{5}{2} \cdot (-4) \cdot 1\frac{1}{2} \cdot 2$
- ☐ **B.** $-2\frac{1}{5} \cdot \left(-\frac{1}{6}\right) \cdot \left(-\frac{3}{4}\right) \cdot \left(-9\frac{7}{8}\right)$
- ☐ **C.** $-4 \cdot 0.3 \cdot (-8)$
- ☐ **D.** $-2.1 \cdot (-5.3) \cdot 7.2 \cdot (-0.4)$
- ☐ **E.** $\frac{4}{3} \cdot \frac{3}{4} \cdot \left(-\frac{1}{2}\right)$

Name: ..... Date: ..... Period: .....

Unit Rates in the World 7.EE.3, 7.EE.4, 7.RP.2, SMP.2, SMP.5, SMP.7

Seeing Structure

Let's analyze and solve questions in context.



Warm-Up

Here are eight equations.

Equation A	Equation B	Equation C	Equation D
$100 = 8(x + 9)$	$9(x + 7) = 100$	$100 = 5x$	$9x + 63 = 100$
Equation E	**Equation F**	**Equation G**	**Equation H**
$100 = 72 + 8x$	$(x + 7) \cdot 9 = 100$	$100 = 8x + 72$	$9x = 100$

1. Select two equations that have something in common. How are the two equations alike?

2. Create three groups so that the equations in each group have something in common. Then explain what your groups have in common.

Group 1	Group 2	Group 3
Equations:	Equations:	Equations: .....
All the equations in this group . . .	All the equations in this group . . .	All the equations in this group . . .

Activity 1

Name: Date: Period:

Which Diagram?

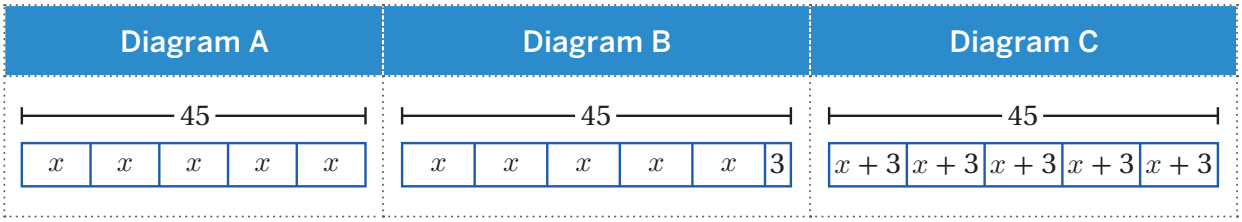
For each situation:

a. Choose a diagram that best represents it.

b. Write an equation.

c. Determine the solution.

d. Explain what the solution means in the situation.



3. A postal worker weighs 5 identical cardboard packages and a 3-pound plastic box. Everything weighs a total of 45pounds.

4. Tyani is making 5 gift bags. Tyani adds 3 more pencils to each bag. Altogether, the gift bags contain 45 pencils.

5. A national park charges $3 for each car that enters and also a fee for each person that enters. A family of 5 enters the park in 1 car and pays a total of $45.

6. A souvenir store sells hats for the same price. Ava buys 5 hats and pays a total of $45.

Activity 2

Name: ..... Date: ..... Period: .....

Questions and Answers

7. Natalia's family wants to inflate a total of 60 balloons for a party. Yesterday, they inflated 24 balloons. Today, they want to split the remaining balloons equally between 4 family members.

a Use this information to write a question that you don't already have the answer to.

b Write an equation or draw a tape diagram to represent the situation and your question. Be sure to define your variable.

c Solve the equation to answer your question.

d **Discuss:** How can you check if your solution makes sense?

8. An art class charges each student $15 to attend, plus a fee for supplies. The instructor hopes to collect $240 total from the 12 students who attend the class.

a Use this information to write a question that you don't already have the answer to.

b Write an equation or draw a tape diagram to represent the situation and your question. Be sure to define your variable.

c Solve the equation to answer your question.

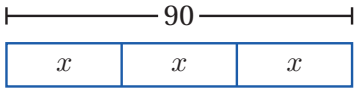
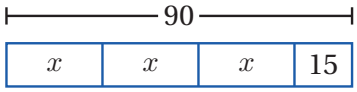
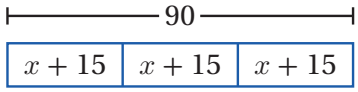
d **Discuss:** How can you check if your solution makes sense?

You're invited to explore more.

9. Write your own problem that can be solved with an equation or tape diagram. Then swap problems with a classmate and solve your classmate's problem.

Synthesis

10. Here are three equations and their tape diagrams.

$3x = 90$	$3x + 15 = 90$	$3(x + 15) = 90$
		

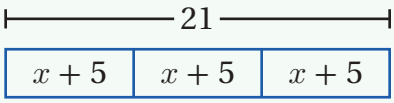
Discuss: How are the tape diagrams alike and different?

Summary 6.04

We can use a tape diagram to help us make sense of a situation and reason about an unknown value.

For example, Elena is training for a race. She trained 3 days this week for a total of 21 miles. On each training day, she ran several miles and biked 5 miles. If x represents the number of miles Elena ran, then this situation can be modeled using this equation and tape diagram.

$$3(x + 5) = 21$$

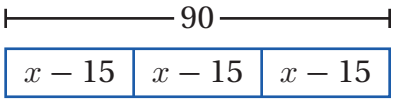


These representations allow us to determine that Elena ran 2 miles at each training day.

Practice
6.04

Name: ..... Date: ..... Period: .....

1. A school ordered 3 large boxes of markers. 3 teachers got 15 markers each, so there were 90 markers left. This tape diagram represents the situation. How many markers were originally in each box?



Problems 2–4: A family buys 4 tickets to a show. Each family member also spends \$2 on a snack. They spend \$24 total.

2. Which equation represents this situation?

A. $2(x + 4) = 24$

B. $4(x + 2) = 24$

3. What does x represent in the equation you chose?

4. Solve the equation you chose. What does the solution tell you about this situation?

Problems 5–7: Amir has 24 ounces of juice. He pours equal amounts for each of his 2 friends and then adds 4 more ounces for each person.

5. Which equation represents this situation?

A. $2(x + 4) = 24$

B. $4(x + 2) = 24$

6. What does x represent in the equation you chose?

7. Solve the equation you chose. What does the solution tell you about this situation?

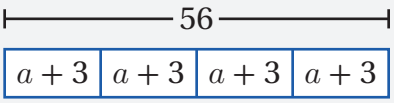
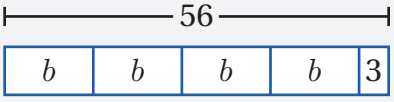
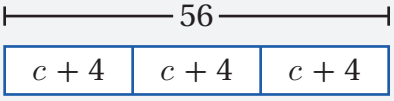
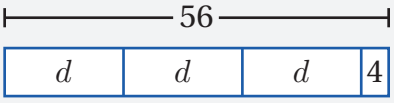
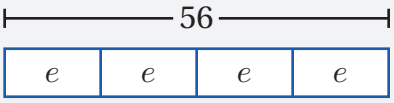
Problems 8–9: Draw a tape diagram and find the solution for each equation.

8. $6x + 11 = 21$

9. $6(x + 1) = 24$

Name: _____ Date: _____ Period: _____

10. Each tape diagram is the same length. Write an equation for each tape diagram and find the solution to the equation.

Diagram	Equation	Solution
		
		
		
		
		

Spiral Review

Problems 11–14: Determine the value of each expression.

11. $\frac{2}{3} \cdot \left(\frac{-4}{5}\right)$

12. $\left(\frac{-5}{7}\right) \cdot \left(\frac{7}{5}\right)$

13. $\left(\frac{-2}{39}\right) \cdot 39$

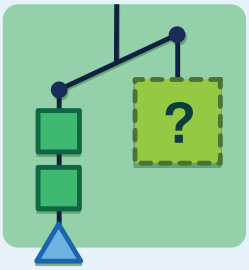
14. $\left(\frac{2}{5}\right) \cdot \left(\frac{-3}{4}\right)$

Notes:

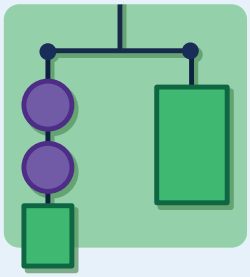
Sub-Unit

2

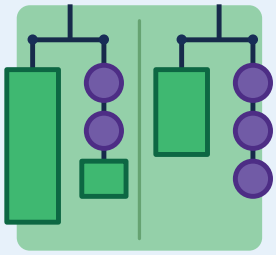
Solving Equations



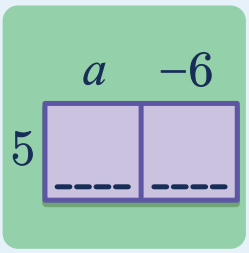
Lesson 5
Balancing Moves



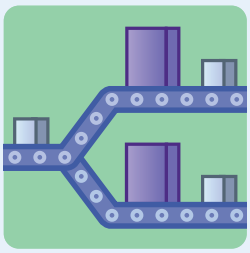
Lesson 6
Balancing Equations



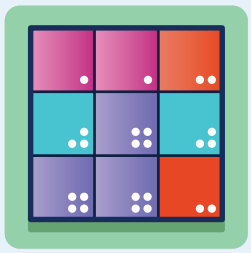
Lesson 7
Keeping It True



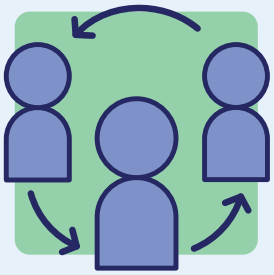
Lesson 8
Factoring and Expanding



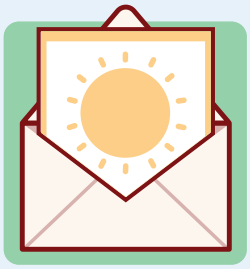
Lesson 9
Always-Equal Machines



Lesson 10
Collect the Squares



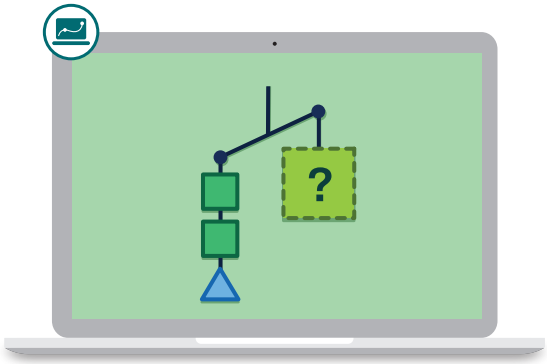
Lesson 11
Pass the Equation



Lesson 12
Community Day

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra 7.EE.3, SMP.3, SMP.6

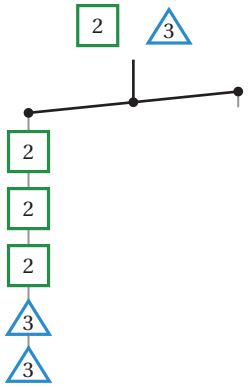


Balancing Moves

Let's see how hangers can represent balanced relationships.

Warm-Up

1 Balance the hanger by adding shapes to either side. Be sure to make the sides different.

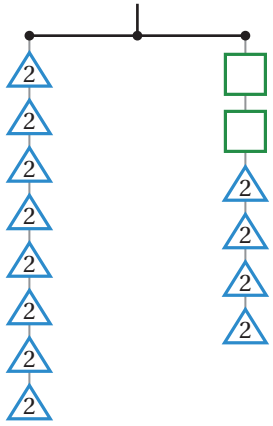


Name: ..... Date: ..... Period: .....

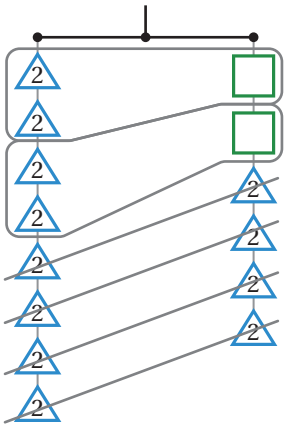
Unknown Weights

2 Determine the weight of the square so that the hanger stays balanced. Describe your strategy.

Weight of Triangle (lb)	Weight of Square (lb)
2	



3 This diagram shows Adnan's strategy for determining the weight of a square in the previous problem. Describe this strategy.

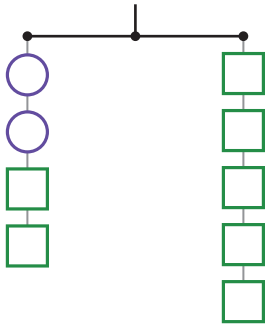


Name: ..... Date: ..... Period: .....

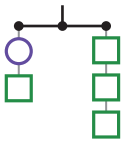
Unknown Weights (continued)

4 Hanger A is balanced. Select *all* the other hangers that must also be balanced.

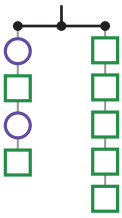
Hanger A



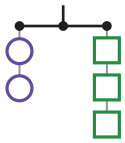
☐ Hanger B



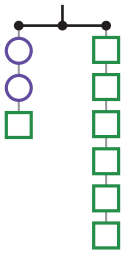
☐ Hanger C



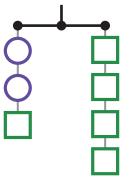
☐ Hanger D



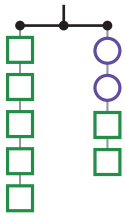
☐ Hanger E



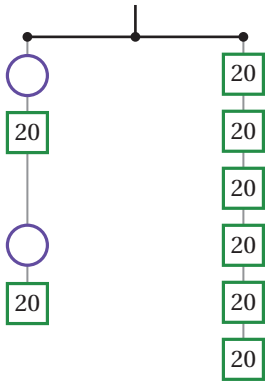
☐ Hanger F



☐ Hanger G



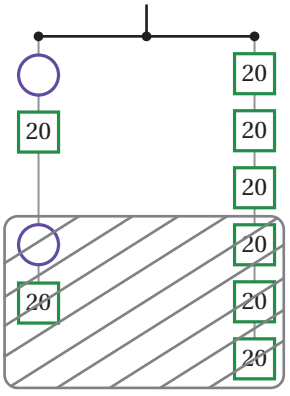
5 **a** Determine the weight of a circle so the hanger stays balanced.



Weight of Square (lb)	Weight of Circle (lb)
20	

b Describe your strategy.

6 This is how Zoe made a hanger with fewer objects. Will the new hanger be balanced? Explain your thinking.



Activity 2

Name: _____ Date: _____ Period: _____

Challenge Creator

7 Now it's your turn to create your own hanger diagram challenge.

a **Make It!** On the Activity 2 Sheet, design your challenge.

b **Solve It!** On this page, copy the weight of the first shape from the challenge you designed. Then determine the weight of the second shape.

Weight of _____	Weight of _____

c **Swap It!** Swap your challenge with one or more partners. Write the weight of your partner's first shape. Then determine the weight of your partner's second shape.

Partner 1: _____

Weight of _____	Weight of _____

Partner 2: _____

Weight of _____	Weight of _____

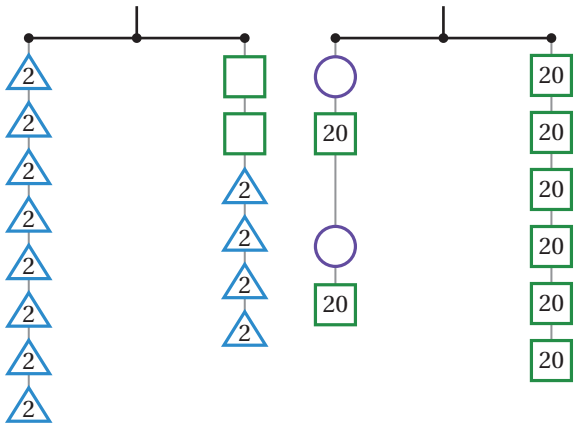
Partner 3: _____

Weight of _____	Weight of _____

8 Synthesis

Discuss: What strategies can be used for making a balanced hanger with fewer objects?

Use the diagrams if they help with your thinking.



11 Summary 6.05

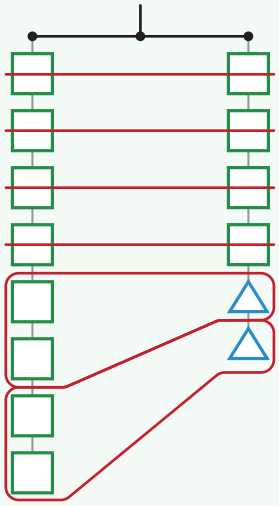
Hangers are balanced when the weight on both sides is the same. When you remove the same weight from both sides, the hanger will remain balanced.

Here are two examples of strategies you can use to determine an unknown value.

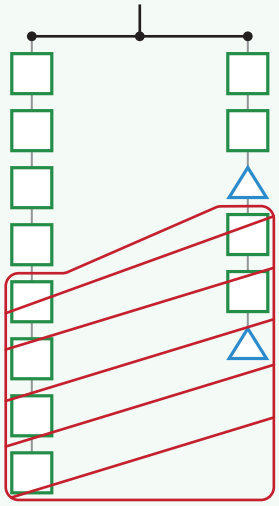
In Hanger A, you can cross off 4 squares on each side while still keeping the sides balanced. Then, 4 squares to the left are balanced with 2 triangles to the right. This means they have the same weight, so 1 triangle has the same weight as 2 squares.

In Hanger B, we can divide each side into two equal groups and remove one group on each side. This leaves 4 squares on the left and a cluster of 1 triangle and 2 squares on the right. Removing 2 more squares from each side leaves the weight of 1 triangle equal to the weight of 2 squares.

Hanger A



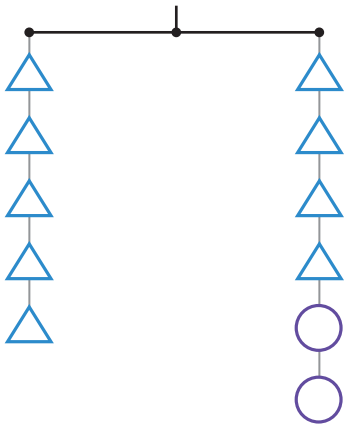
Hanger B



Practice
6.05

Name: ..... Date: ..... Period: .....

Problems 1–5: Determine the weight of a circle based on these different weights for a triangle.



1. 2 pounds

2. 1 pound

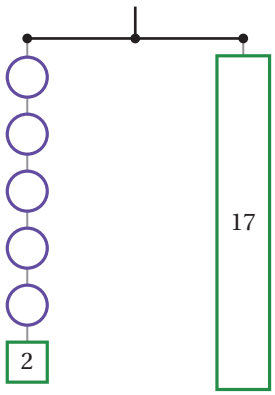
3. 0.5 pounds

4. 3 pounds

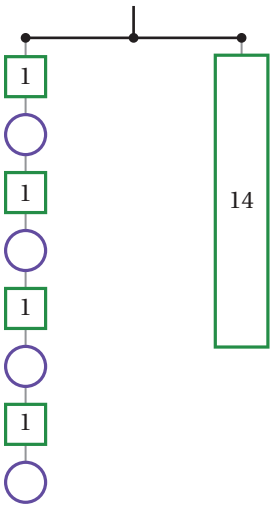
5. 100 pounds

Problems 6–7: Determine the weight of a circle so the hanger stays balanced.

6.

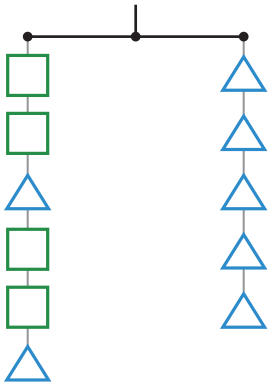


7.



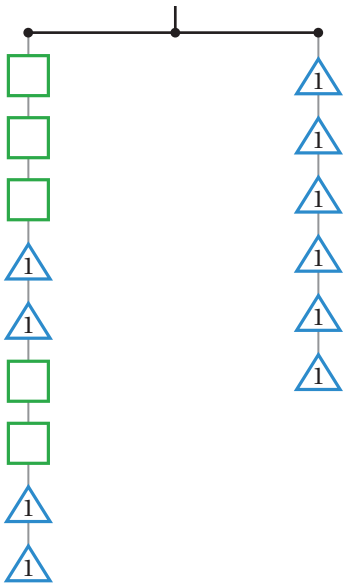
8. Here is a balanced hanger. Darryl says the weight of four squares is equal to the weight of three triangles.

Is Darryl correct? Explain your thinking.



Name: _____ Date: _____ Period: _____

9. Use this model to determine the value of a square.



A. 0.4

B. 0.5

C. 2.5

D. 4

Spiral Review

Problems 10–12: Solve each equation.

10. $8.5 \cdot 3 = a$

11. $c - 3 = 15$

12. $d + 2.1 = 7.3$

13. Select *all* the expressions that are equivalent to $2(x + 3)$.

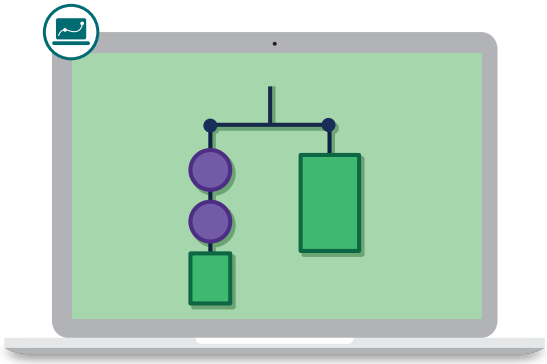
- ☐ **A.** $2 \cdot (x + 3)$
- ☐ **B.** $(x + 3) \cdot 2$
- ☐ **C.** $2 \cdot x + 2 \cdot 3$
- ☐ **D.** $2x + 3$
- ☐ **E.** $(2 \cdot x) + 3$

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra | 7.EE.3, 7.EE.4, 7.EE.4.a, SMP.3

Balancing Equations

Let's use hanger diagrams to help us solve equations.

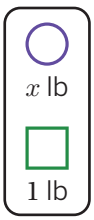


Warm-Up

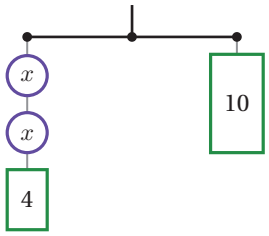
1 Hanger A is balanced.

Create a new balanced hanger that has the same weight for x.

New balanced hanger



Hanger A



$2x + 4 = 10$

Activity 1

Name: ..... Date: ..... Period: .....

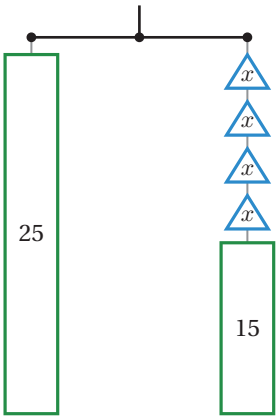
Connecting Hangers to Equations

2 The equation $25 = 4x + 15$ represents Hanger A.

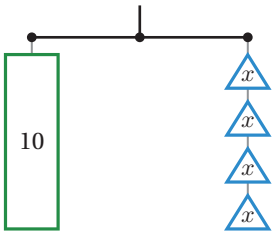
a Write an equation that represents Hanger B.

b What balanced move gets you from Hanger A to Hanger B?

Hanger A

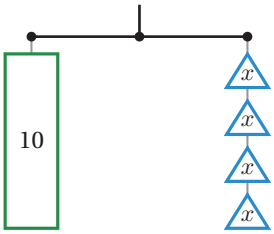


Hanger B

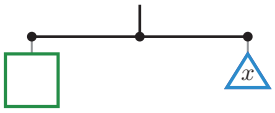


3 What is the weight of one triangle?

Hanger B



Hanger C



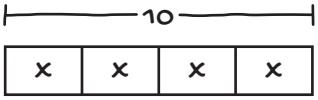
4 Here are Terrance's and Nikhil's strategies for determining the weight of one triangle on Hanger B.

Discuss: How are the two strategies alike? How are they different?

Terrance

$$\frac{10}{4} = \frac{4x}{4}$$

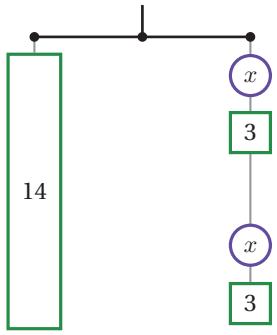
Nikhil



Name: ..... Date: ..... Period: .....

Connecting Hangers to Equations (continued)

5 Here is a new hanger. What is the value of x that will keep the hanger balanced?



6 Anand and Darius used equations to determine the value of x in the previous problem.

Darius wrote the equation $14 = 2x + 6$.
Anand wrote the equation $14 = 2(x + 3)$.

Who is correct? Circle one.

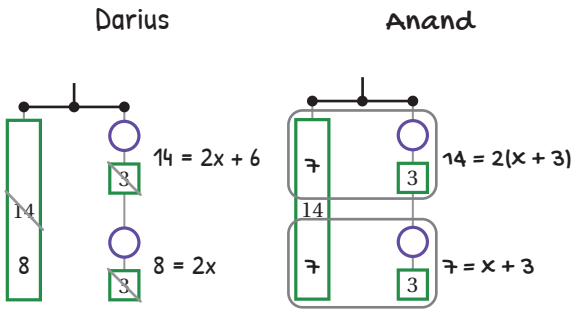
Darius Anand Both Neither

Explain your thinking.

7 Here are Darius's and Anand's work. Select a question to answer.

Why did Darius write $8 = 2x$?

Why did Anand write $7 = x + 3$?

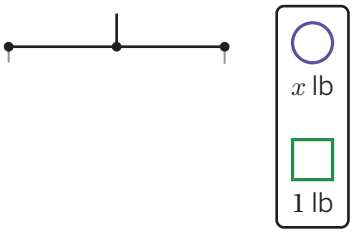


Activity 2

Name: ..... Date: ..... Period: .....

Solving Equations

8 **a** Create a hanger to represent $7 = 4x + 2$.



b What value of x makes the equation true?

9 **a** Determine the value of x that makes the equation $4(x + 2) = 40.4$ true.

b Describe the steps you used to determine the value of x.

10 What value of x makes each equation true? Solve as many challenges as you have time for.

a $3x + 1 = 7$

b $2(x + 5) = 16$

c $2x + 2.2 = 6.8$

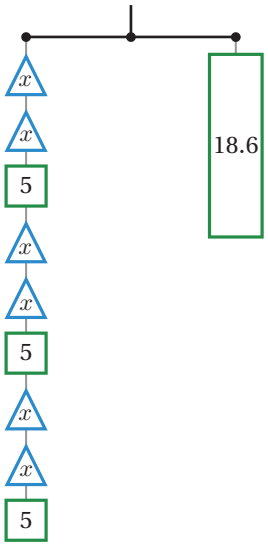
d $4(x + 1.1) = 20.8$

e $4x + \frac{3}{2} = \frac{17}{2}$

11 Synthesis

Discuss: How is solving an equation like solving for the weight of an object on a balanced hanger?

Use the diagram if it helps with your thinking.



$3(2x + 5) = 18.6$

14 Summary 6.06

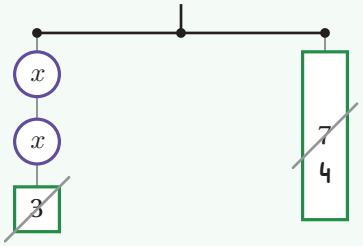
We can use a hanger diagram to represent an equation and help us understand how to find an unknown value in that equation. You can write the steps for finding an unknown value without using a hanger.

For example, the equation $2x + 3 = 7$ can be solved using these steps:

Subtract 3 from both sides.

$$\begin{aligned} 2x + 3 &= 7 \\ 2x + 3 - 3 &= 7 - 3 \\ 2x &= 4 \end{aligned}$$

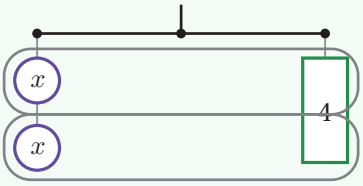
Remove 3 from both sides.



Divide both sides by 2.

$$\begin{aligned} 2x &= 4 \\ 2x \div 2 &= 4 \div 2 \\ x &= 2 \end{aligned}$$

Divide into two equal groups.



Practice
6.06

Name: ..... Date: ..... Period: .....

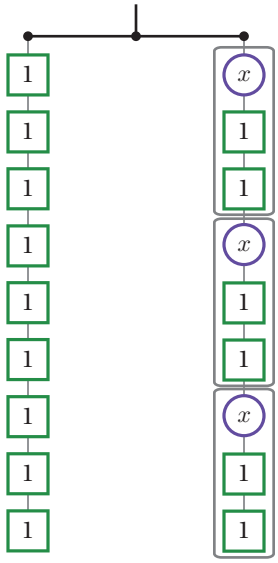
Problems 1–3: Solve each equation.

1. $x - 1 = 5$

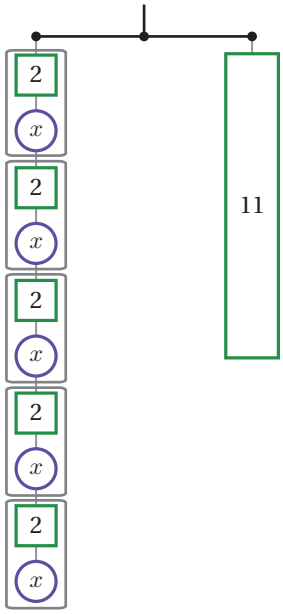
2. $2(x - 1) = 10$

3. $500 = 100(x - 1)$

4. Explain how each part of the equation $9 = 3(x + 2)$ is represented in the hanger.



Problems 5–7: Use the hanger.



5. Write an equation that represents this hanger.

6. Write another equation that represents the same hanger.

7. What is the value of x that makes the equations true?

Practice
6.06

Name: ..... Date: ..... Period: .....

Problems 8–9: Consider the equation $12.7 = 3x + 0.7$.

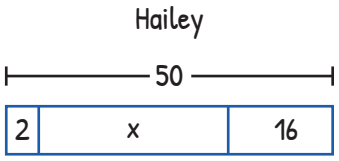
8. Draw a hanger to represent the equation in the space provided.

Your Diagram

9. What is the value of x that makes the equation true?

Spiral Review

Problems 10–12: Hailey drew a diagram to represent the equation $2x + 16 = 50$, but she made a mistake.



10. Explain the mistake Hailey made.

11. Draw a new tape diagram in the space provided to represent the equation $2x + 16 = 50$.

New Tape Diagram

12. Determine the value of x using your tape diagram from Problem 10.

13. Which value is equivalent to $-9 + 5 + 3 - (-8)$?

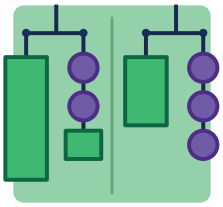
A. -19 **B.** -9 **C.** -1 **D.** 7

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra | 7.EE.3, 7.EE.4.a, SMP.7

Keeping It True

Let's solve equations with positive and negative numbers.



Warm-Up

Amanda solved some equations. Her work is shown.

$x + 4 = 6$	$2x = 4$
$x + 4 - 4 = 6 - 4$	$2x \div 2 = 4 \div 2$
$x = 2$	$x = 2$

1. **Discuss:** How do you think Amanda would solve these equations?

$x + 4 = -6$ $\quad\quad$ $-2x = 4$

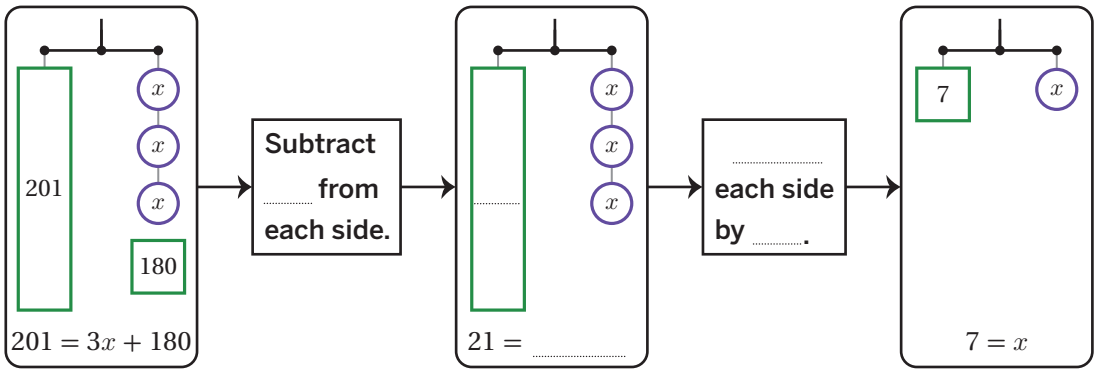
Activity 1

Name: _____ Date: _____ Period: _____

Keep It True

Solve each equation by completing the blanks in the hangers, equations, and descriptions.

2. **a**



b **Discuss:** What do you notice about the steps taken to solve the equation?

3.

Equation	Moves
$5 = 2x + 8$	**Step 1:** _____ from each side.
_____ $= 2x$	**Step 2:** _____ each side _____.
_____ $= x$	

4.

Equation	Moves
$2(x - 5) = -6$	**Step 1:** Divide each side _____.
$x - 5 = -3$	**Step 2:** _____.
_____ = _____	

5. How can you check that the values you calculated in Problems 2–4 are *solutions to the equations*?

Activity 2

Name: Date: Period:

Solve 'em

Here are two groups of equations.

Group A	Group B
$x - (-4) = -6$	$2(x - 1) = -200$
$50x + 200 = 1700$	$900 = -100(x - 3)$
$8.6 = 3x - 3.4$	$3(x + 4.5) = 36$

6. **Discuss:** How are the equations in each group alike or different?

7. Which group do the equations $-3x + \left(-\frac{1}{6}\right) = \frac{5}{6}$ and $-\frac{1}{2}(2x - 6) = -2$ belong to? Explain your thinking.

8. Choose two equations from *each* group to solve.

	Group A	Group B
Equation 1		
Equation 2		

Synthesis

9. **a** Write an equation that would belong in Group B.

Group B
$2(x - 1) = -200$
$900 = -100(x - 3)$
$3(x + 4.5) = 36$

b What advice would you give to help someone solve an equation like yours?

Summary 6.07

When solving an equation, the same, **inverse operation** should be applied to both sides of the equation at each step so that the equation remains true.

Here are two examples.

Equation 1	Equation 2
$3x - 6 = 9$ $3x - 6 + 6 = 9 + 6$ $3x = 15$ $3x \div 3 = 15 \div 3$ $x = 5$	$3(x - 6) = 9$ $3(x - 6) \div 3 = 9 \div 3$ $x - 6 = 3$ $x - 6 + 6 = 3 + 6$ $x = 9$

A *solution to an equation* is a value of a variable that makes the equation true. You can check your solution by substituting the value in for the variable and evaluating.

	Equation 1	Equation 2
Checking a Solution	$3(5) - 6 = 9$ $15 - 6 = 9$ $9 = 9$ ✅	$3(9 - 6) = 9$ $3(3) = 9$ $9 = 9$ ✅

inverse operations Two operations that "undo" each other. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.

Practice
6.07

Name: ..... Date: ..... Period: .....

Problems 1–3: Solve each equation by filling in the blanks.

1. $15x - 10 = 65$

$15x =$ .....

$x =$ .....

2. $3(x + 7) = -12$

$x + 7 =$ .....

$x =$ .....

3. $-100x - 100 = 0$

$-100x =$ .....

$x =$ .....

Problems 4–7: Solve each equation.

4. $-4x = -28$

5. $-4(x + 1) = -28$

6. $x - (-7) = -1$

7. $-3x + 7 = -1$

8. Here is an equation: $\frac{1}{4}(x + 8) = -6$. What value of x makes the equation true?

9. Using the numbers 0–9, complete the equations so that the solutions for x in both equations is the same. Use each number no more than once.

..... $x -$ ..... $=$ .....

..... $(x -$ .....$) =$ .....

Practice
6.07

Name: ..... Date: ..... Period: .....

Spiral Review

10. Match each situation to an equation.

Situation	Equation
a. Mariana has an 8-foot piece of ribbon. She cuts off a piece that is $\frac{1}{4}$ of a foot long and cuts the remainder into four equal pieces.	..... $8x + \frac{1}{4} = 4$
b. A baker uses 4 cups of flour. She uses $\frac{1}{4}$ cup to flour the counters and the rest to make 8 muffins.	..... $4 + \frac{1}{4}x = 8$
c. A stack of paper cups is 8 inches tall. The first cup is 4 inches tall and each of the rest of the cups adds $\frac{1}{4}$ inch to the height of the stack.	..... $\frac{1}{4} + 4x = 8$

Problems 11–13: There are 88 seats in a theater, and they are split into 4 identical sections. Each section has 14 red seats and an equal number of blue seats.

11. Draw a tape diagram or hanger to represent the situation.

12. Write an equation to represent the situation.

13. Determine the number of blue seats in each section.

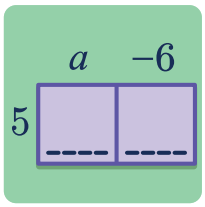
Unit 6
Lesson
8

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra 7.EE.1, 7.EE.2, 7.EE.3, 7.EE.4.a, 7.NS.2.c, SMP.3, SMP.7, SMP.8

Factoring and Expanding

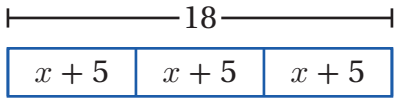
Let's think about efficient ways to solve equations with parentheses.



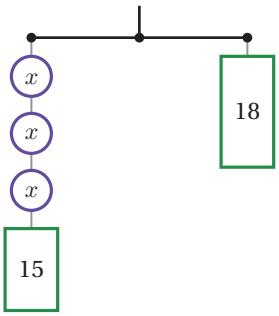
Warm-Up

1. Which one doesn't belong? Explain your thinking.

A.



B.



C. $3(x + 5) = 18$

D. $3x + 5 = 6$

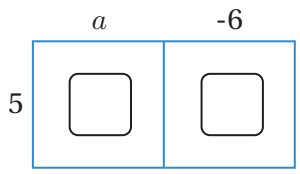
Activity 1

Name: ..... Date: ..... Period: .....

Factoring Puzzles

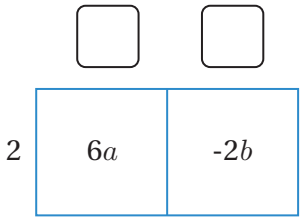
Complete the missing information in each puzzle.

2.



Factored	Expanded
$5(a-6)$	

3.



Factored	Expanded
	$6a-2b$

4.

Factored	Expanded
	$-25x+15$

5.

Factored	Expanded
$-(2c+3)$	

Name: Date: Period:

Step by Step by Step by Step

6. Here are Amir's and Sadia's first steps for solving $2(x - 9) = 10$.

Amir

$2(x - 9) = 10$
$2x - 18 = 10$

Sadia

$2(x - 9) = 10$
$x - 9 = 5$

a Are each of their first steps correct? Explain your thinking.

b Finish solving each equation. Show your thinking.

Amir

Sadia

Activity 3

Name: Date: Period:

Different First Steps

Solve these equations for x using both Amir's and Sadia's methods. Check the box when your solutions match.

7. $3(x + 2) = 21$

Expand first:

Divide first:

☐

8. $(x - 0.3)200 = 600$

Expand first:

Divide first:

☐

9. $-10\left(x - \frac{7}{10}\right) = -3$

Expand first:

Divide first:

☐

Synthesis

10. **a** What are two possible first steps you could use when solving an equation like $6(x + 4) = 30$?

b **Discuss:** What are some advantages to having different ways to solve an equation?

Summary 6.08

There are two ways to solve the equation $3(x + 1) = 9$. The first steps are different, but the value of x, the *solution to the equation*, is the same.

Expanding First (using the distributive property)

$$\begin{aligned} 3(x + 1) &= 9 \\ 3x + 3 &= 9 \\ 3x + 3 - 3 &= 9 - 3 \\ 3x &= 6 \\ 3x \div 3 &= 6 \div 3 \\ x &= 2 \end{aligned}$$

The distributive property allows us to multiply the factored term by each term on the inside of the parentheses.

Dividing First

$$\begin{aligned} 3(x + 1) &= 9 \\ 3(x + 1) \div 3 &= 9 \div 3 \\ x + 1 &= 3 \\ x + 1 - 1 &= 3 - 1 \\ x &= 2 \end{aligned}$$

Both sides of the equation are divided by the factored term that is outside of the parentheses.

expand Use the distributive property to multiply the factors in an expression and rewrite the expression as a sum. The new expression is equivalent to the original expression.

factor Use the distributive property to rewrite an expression as the product of two or more factors. The new expression is equivalent to the original expression.

Practice
6.08

Name: ..... Date: ..... Period: .....

Problems 1–3: Write each expression in expanded form.

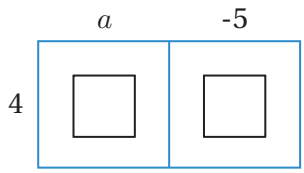
1. $-2(-6)$

2. $-2(-y)$

3. $-2(-6 - y)$

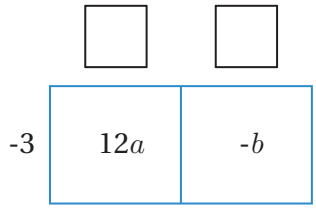
Problems 4–6: Complete the missing information in the puzzle and complete the table.

4.



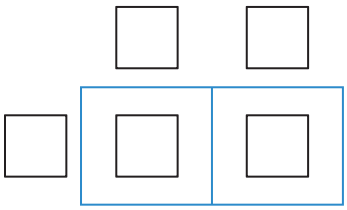
Factored	Expanded
$4(a - 5)$	

5.



Factored	Expanded
	$12a - b$

6.



Factored	Expanded
	$9x - 21$

Name: ..... Date: ..... Period: .....

Problems 7–9: Solve each equation. Show your thinking.

7. $2(x - 3) = 14$

8. $-5(x - 1) = 40$

9. $\frac{5}{7}(x - 9) = 25$

10. Emmanuel and Mauricio started solving the equation $7(x - 2) = 91$. Finish solving each equation.

Emmanuel

$7(x - 2) = 91$
$7x - 14 = 91$

Mauricio

$7(x - 2) = 91$
$x - 2 = 13$

Spiral Review

Problems 11–12: Use long division to write each fraction as a decimal.

11. $\frac{2}{5}$

12. $\frac{2}{3}$

13. Which equation is true when $x = 2$?

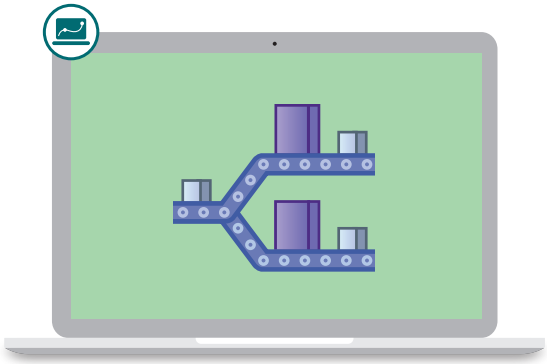
A. $3x + 4 = 8$ **B.** $5x - 3 = 12$ **C.** $6x + 4 = 16$ **D.** $4x + 5 = 17$

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra | 7.EE.1, SMP.3, SMP.6

Always-Equal Machines

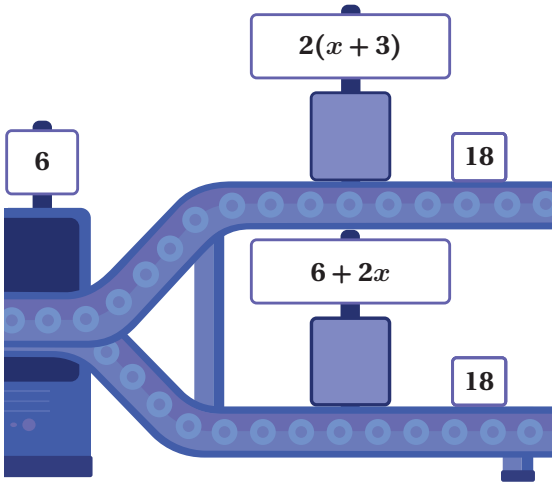
Let's explore equivalent expressions using always-equal machines.



Warm-Up

1 Here are two number machines. Let's watch what happens when an input goes into the machines. Record the results in the table.

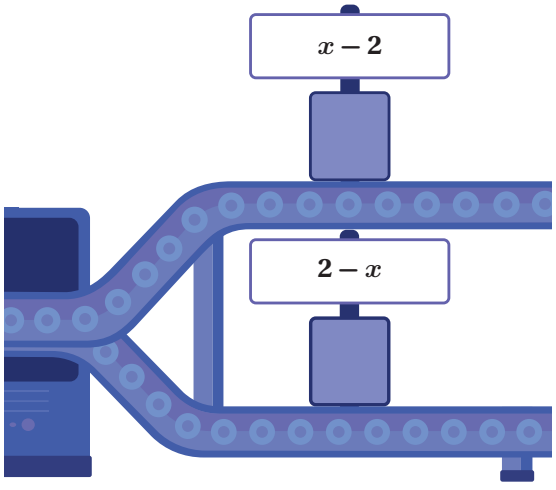
x	$2(x+3)$	$6+2x$



2 Here are two more number machines. When will these machines have equal outputs?

Always Sometimes Never

Explain your thinking.



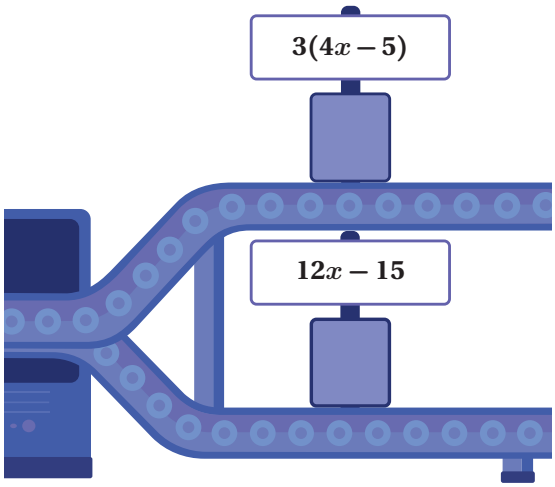
Activity 1

Name: ..... Date: ..... Period: .....

Equivalent Expressions

3 Expressions that are equal for every value of a variable are called **equivalent expressions**.

Are these equivalent expressions? Explain how you know.



4 **a** Which expression is equivalent to $2(10 - x)$?

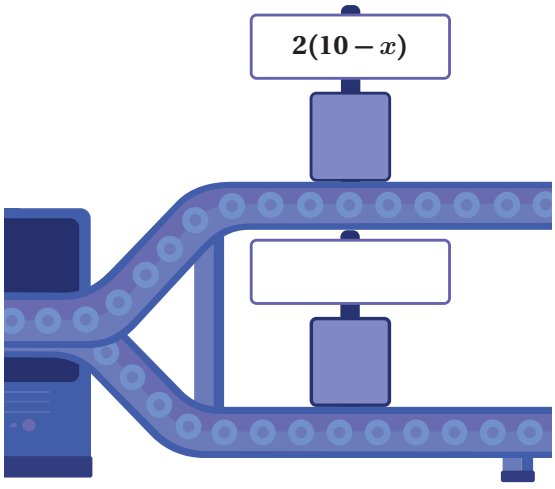
A. $20 - x$

B. $20 - 2x$

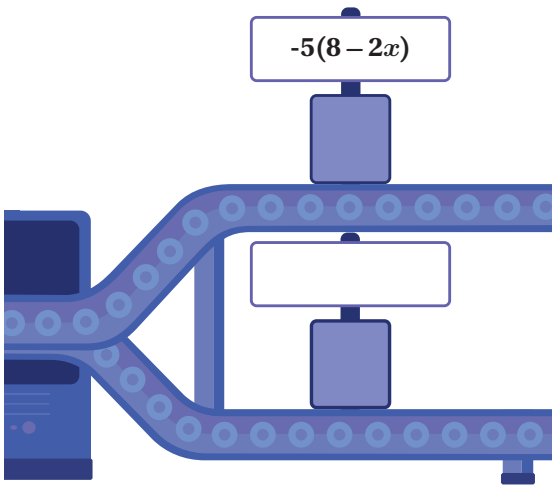
C. $2(x - 10)$

D. $2x - 20$

b Choose one input to test your expression.



5 Write an equivalent expression to $-5(8 - 2x)$.



Name: ..... Date: ..... Period: .....

Equivalent Expressions (continued)

6 Three students made mistakes writing an equivalent expression to $-5(8 - 2x)$.

Circle your favorite mistake.

$-5(8 - 2x)$

Zion	Juliana	Nickolas
$-5(2x - 8)$	$-40 - 10x$	$-40 + 2x$

Explain what you think is incorrect about the student's expression. Use the properties of operations to help correct the mistake.

7 **a** Select *all* of the expressions equivalent to $-15 + 6x$.

- ☐ **A.** $-3(5 - 2x)$
- ☐ **B.** $3(2x - 5)$
- ☐ **C.** $3(5 - 2x)$
- ☐ **D.** $6x + (-15)$
- ☐ **E.** $15 + (-6x)$

b Choose one of the equivalent expressions. Use the properties of operations to explain how you know it is equivalent to $-15 + 6x$.

Activity 2

Name: ..... Date: ..... Period: .....

More Than One Way

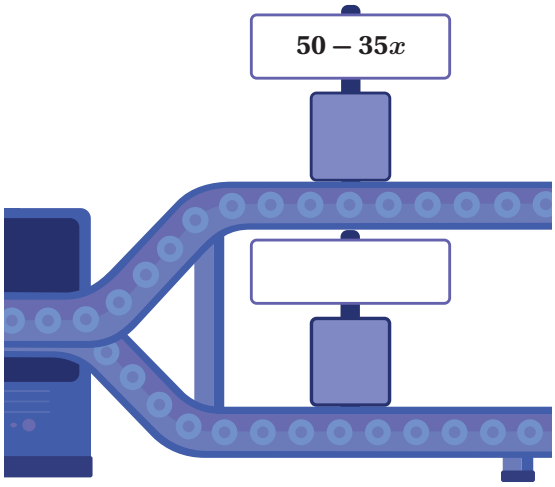
8 Group the equivalent expressions. One expression will have no group.

$8(x - 3)$	$-4(-6 + 2x)$	$-8(x - 24)$	$8x - 24$
$-24 + 8x$	$24 - 8x$	$\frac{1}{2}(16x - 48)$	

9 **Discuss:** How did you use the properties of operations to help group the equivalent expressions?

10 Write an equivalent expression to $50 - 35x$.

Try to write more than one.



Name: ..... Date: ..... Period: .....

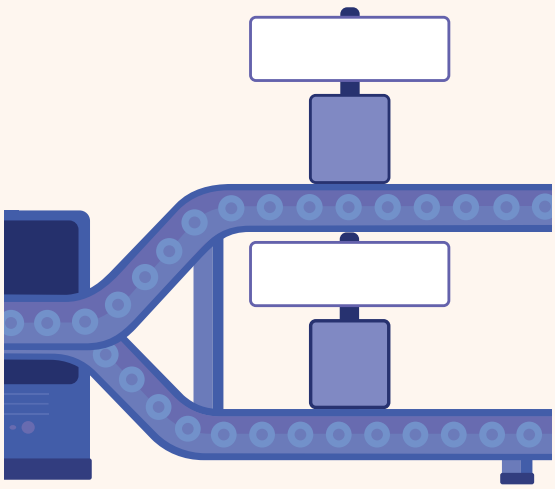
More Than One Way (continued)

11 Write at least three different equivalent expressions to $64x - 16$.

You're invited to explore more.

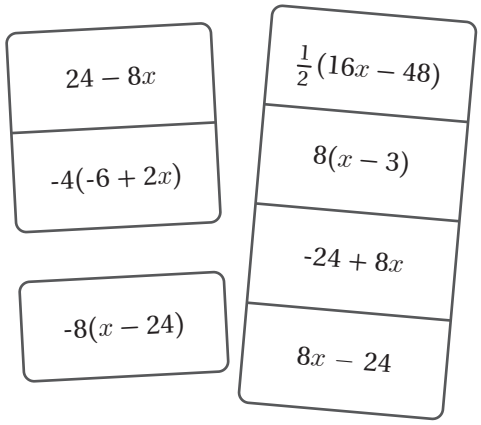
12 Make a never-equal machine that will never give the same output for any input. Write your expressions on the two machines.

Explain how you know these are never equal.



13 Synthesis

Discuss: How can you determine whether expressions are equivalent to each other? Use these examples if they help with your thinking.



16 Summary 6.09

Expressions that give the same output for every input are called **equivalent expressions**. To determine whether expressions are equivalent, you can test several inputs to see if they produce the same output.

In this example, $3x + 6$ and $3(x + 2)$ are equivalent because they give the same output for every input. You can test other values and they will always give matching outputs. $6(x + 3)$ is not equivalent because it doesn't give the same output for every input (it is only the same when $x = -4$).

x	$3x + 6$	$3(x + 2)$	$6(x + 3)$
10	36	36	78
7	27	27	60
-4	-6	-6	-6

You can also use properties of operations to determine if expressions are equivalent. In this example, you can use the distributive property to rewrite $3(x + 2)$ as $3x + 6$, so these are equivalent expressions. On the other hand, $6(x + 3)$ is rewritten $6x + 18$, so these expressions are not equivalent to $3x + 6$.

equivalent expressions Expressions that are equal for every value of a variable.

Practice
6.09

Name: ..... Date: ..... Period: .....

1. Alejandro says that $10x + 6$ and $5x + 11$ are equivalent because they equal 16 when x is 1. Do you agree with Alejandro? Explain your thinking.

2. Write at least three different expressions that are equivalent to $16a - 24$.

3. Write at least three different expressions that are equivalent to $\frac{-1}{2}(-12x + 30)$.

Problems 4–7: Write an equivalent expression in expanded form.

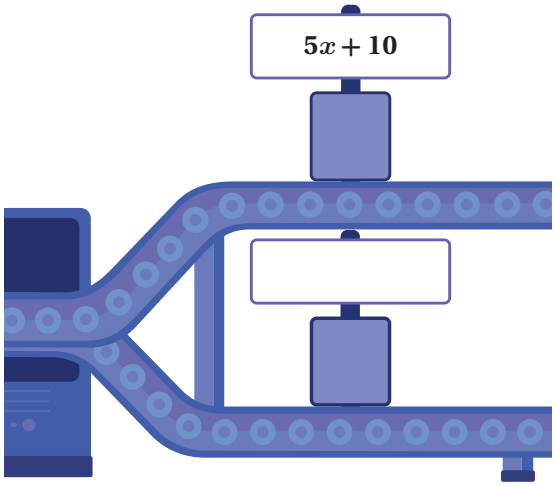
4. $8\left(-x + \frac{1}{4}\right)$

5. $-2(-6x - 1)$

6. $\frac{1}{5}(20y - 13)$

7. $9\left(4x + 3y + \frac{2}{3}\right)$

8. Write an expression that will always be equal to $5x + 10$ and an expression that will never be equal. Use x as the variable in each expression.



Name: ..... Date: ..... Period: .....

9. Here are three equivalent expressions: $-2(4 - 5x)$, $-8 + 10x$, and $10x - 8$. Explain how you know these are equivalent using properties of operations.

Spiral Review

Problems 10–12: The output from a coal power plant and a solar power plant in megawatts (MW) is shown in the tables.

10. Is the energy output of the coal power plant proportional to the number of days?

11. Write an equation showing the relationship. Use E to represent energy and d to represent the number of days.

Coal Power Plant

Energy (MWh)	Number of Days
1,200	2.4
1,800	3.6
4,000	8
10,000	20

12. Is the energy output of the solar power plant proportional to the number of days? Explain your thinking.

Solar Power Plant

Energy (MWh)	Number of Days
100	2.4
650	3.6
1,200	8
1,750	20

13. Match each expression to its value.

Expression	Value
a. $-30 \cdot (-10)$	..... -40
b. $-10 + (-30)$	 300
c. $-30 - 10$	 40
d. $10 - (-30)$	

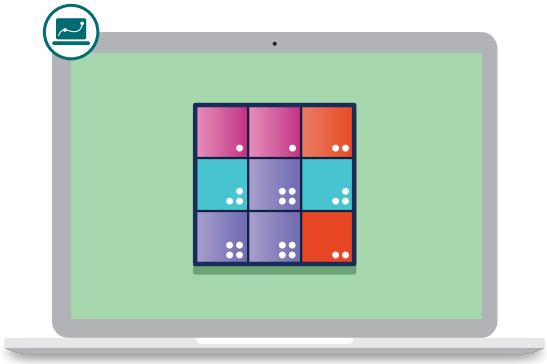
Unit 6
Lesson
10

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra | 7.EE.1, 7.EE.3, SMP.7

Collect the Squares

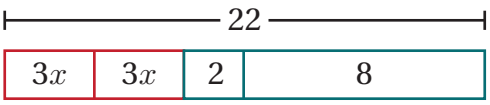
Let's write equivalent expressions with fewer terms.



Warm-up

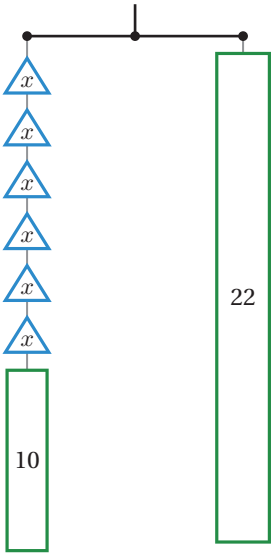
1 Here are some equivalent representations of the equation $6x + 10 = 22$.

A.

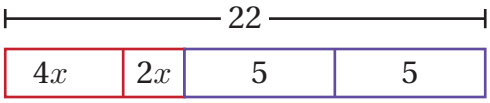


B. $5.5x + \frac{1}{2}x + 1 + 2 + 7 = 22$

C.



D.



Select one representation and explain how each term in the equation is represented.

Activity 1

Name: Date: Period:

Collect the Squares

2 Select two or more squares. Write their sum using the fewest number of *terms*. There is an example in the first row.

$4x + 1$	$9x$
$-2x$	$-10x$

Original Expression	Equivalent Expression
$4x + 1 + (-10x)$ $= 4x - 10x + 1$ $= -6x + 1$	$-6x + 1$

Name: ..... Date: ..... Period: .....

Collect the Squares (continued)

3 Select two or more squares. Write their sum using the fewest number of *terms*.

Original Expression	Equivalent Expression

$-2x - 2$	$8x + 8$
-3	$3(x + 1)$

4 Caleb combined these expressions. Some of his work is correct and some is incorrect.

Original Expression	Equivalent Expression
$-2x - 2 + 8x + 8$ $= -2x + 8x - 2 + 8$ $= 6x + 6$ $= 12x$	$12x$

$-2x - 2$	$8x + 8$
-3	$3(x + 1)$

Explain what you think is incorrect about Caleb's work.

Name: Date: Period:

Collect More Squares

5 Select two or more squares. Write their sum using the fewest number of terms. Repeat until you have collected all the squares.

Original Expression	Equivalent Expression

$3x + 6$	$x - 2$	$8(x - 3)$
$-4x - 4$	$x + 1$	$-2(x + 2)$
6	$7x$	$-8x$

Name: ..... Date: ..... Period: .....

Collect More Squares (continued)

6 Select two or more squares. Write their sum using the fewest number of terms. Repeat until you have collected all the squares.

Original Expression	Equivalent Expression

$-4(x - 5)$	$6.2(x + 1)$	$\frac{1}{4}x - 20$
$3.5x + 12$	$-1(x - 9)$	$-x - 12$
$8 - x$	$-2x$	$7(2x + 1)$

You're invited to explore more.

7 Use these squares to create expressions that are equivalent to $5x - 8$. Create as many different expressions as you can.

$-(3x + 2)$	$-3(x + 2)$	$8x - 6$
$-2x - 6$	$2(4x - 1)$	$-3x + 2$
$8x - 10$	$3x - 6$	$2x + 2 + x$

8 Synthesis

How does identifying **like terms** help to write equivalent expressions with the fewest number of terms?
Use this expression if it helps with your thinking.
$5x - 2(6x - 4)$

11 Summary 6.10

Here are two expressions.

$-\frac{1}{3}(3x - 6)$ $\quad\quad$ $2x - 4x + 5$

The sum of these expressions can be written as:

$-\frac{1}{3}(3x - 6) + 2x - 4x + 5$

By adding and subtracting **like terms** and expanding terms, we can write the sum using the fewest number of terms:

$-\frac{1}{3}(3x - 6) + 2x - 4x + 5$
$= -x + 2 - 2x + 5$
$= -3x + 7$

identity property of addition The sum of any number and 0 is the number itself.
like terms Terms with variables and exponents that are the same.

Practice
6.10

Name: _____ Date: _____ Period: _____

1. Select *all* of the expressions that are equivalent to $4x - 5 + 6$.

- ☐ **A.** $4x + (-5) + 6$
- ☐ **B.** $4x - 6 + 5$
- ☐ **C.** $4x + 1$
- ☐ **D.** $5x$
- ☐ **E.** $5 + 6 - 4x$

2. Fill in the blanks to make each equation true.

$6x +$ _____ $= 10x$

$6x +$ _____ $= 2x$

$6x +$ _____ $= -10x$

$6x +$ _____ $= 10x + 5$

$6x -$ _____ $= 2x$

$6x -$ _____ $= x$

$6x +$ _____ $= 10$

$6x - ($ _____ $) = 4x - 10$

3. Write an expression that is equivalent to $2(x + 5) - 3x$.

4. Fill in each blank with a number or expression such that each row and column has the same sum.

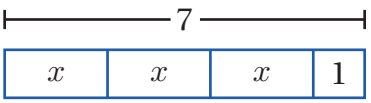
	$x + 2$	$2 - x$
$5 - x$	x	
	2	

Practice
6.10

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 5–7: This diagram can be represented by the equation $7 = 3x + 1$.



5. Explain where you can see the 3 in the diagram.

6. Determine the value of x.

7. Select *all* the situations that could be represented by this equation.

- ☐ **A.** Aaliyah is studying 7 hours this week for end-of-year exams. She spends 1 hour on English and an equal number of hours each on math, science, and history.
- ☐ **B.** Lan spends $\$3$ on 7 markers and $\$1$ on a pen.
- ☐ **C.** Sneha shares 7 grapes with 3 friends. She eats 1 grape and gives each friend the same number of grapes.
- ☐ **D.** Kiri read 1 book this week and has a goal to read 7 total books within the next 3 weeks. She will read the same number of books each week.
- ☐ **E.** Adriana runs 7 miles every day for 3 days and then runs 1 mile on the fourth day.

Problems 8–11: Solve each equation.

8. $5(n - 4) = -60$

9. $-3t + (-8) = 25$

10. $7p - 8 = -22$

11. $\frac{2}{5}(j + 40) = -4$

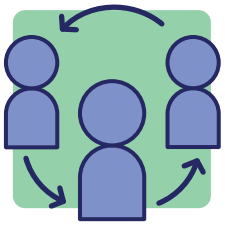
12. A small town had a population of 960 people last year. The population grew to $1{,}200$ people this year. By what percent did the population grow?

Unit 6
Lesson
11

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra 7.EE.3, 7.EE.4.a, SMP.7

Pass the Equation



Let's practice solving equations multiple ways.

Warm-Up

1. Explain some possible first steps you could take to solve the equation.

$2(4x - 3) = 30$

Name: Date: Period:

Equation Roundtable, Round 1

2. Follow the instructions on the Presentation Screen to solve these equations with your group.

$2x - 18 = 10$	$3(2 - x) = -8$
$5(x - 1) = 45$	$-6 = \frac{1}{2}(x - 8)$

Equation 1	Equation 2
Equation: Check:	Equation: Check:
Equation 3	**Equation 4**
Equation: Check:	Equation: Check:

3. What do you think is important to remember when solving these types of equations?

Activity 2

Name: Date: Period:

Equation Roundtable, Round 2

4. Here are four new equations. Solve them using the same instructions as Activity 1.

$8x - (-6x) - 18 = 10$

$55 = 5(x - 1) + 10$

$-10.5 = 6\left(x + \frac{1}{4}\right)$

$-2.8(x - 3) = 9\frac{4}{5}$

Equation 1	Equation 2
Equation: Check:	Equation: Check:
Equation 3	**Equation 4**
Equation: Check:	Equation: Check:

5. **Discuss:** How is your strategy alike and different from others?

Synthesis

6. There are different ways to solve the equation $2(-3 + 8x) = -10$.

a List two different first steps you could take to solve this equation.

b Which first step do you prefer? Explain your thinking.

Summary 6.11

You can solve an equation in many different ways.

$-6(3x - 5) = 75$	Rewrite subtracting 5 as adding (-5).
$-6(3x + (-5)) = 75$	Distribute the -6 to the $3x$ and -5.
$-18x + 30 = 75$	Subtract 30 from both sides.
$-18x = 45$	Divide each side by -18.
$x = -2.5$	

Here is a different strategy for solving the same equation.

$-6(3x - 5) = 75$	Divide both sides of the equation by -6.
$3x - 5 = -12.5$	Add 5 to both sides.
$3x = -7.5$	Divide each side by 3.
$x = -2.5$	

Practice
6.11

Name: ..... Date: ..... Period: .....

Problems 1–3: Kwame and Maia each solved the same equation but didn't show their work for getting to the next step.

Kwame	Maia
$3.3(x - 10) = 66$	$3.3(x - 10) = 66$
$3.3x - 33 = 66$	$x - 10 = 20$
$3.3x = 99$	$x = 30$
$x = 30$	

1. How did Kwame begin? Maia?

2. Show what they might have done to get from one step to another. Complete the missing steps for each student.

3. Which method do you prefer to use for this equation? Why?

Problems 4–7: Solve each equation. Show your thinking.

4. $2(x - 3) = 14$

5. $-5(x - 1) = 40$

6. $\frac{5}{7}(x - 9) = 25$

7. $\frac{1}{6}(x + 6) = 11$

Practice
6.11

Name: ..... Date: ..... Period: .....

Problems 8–9: Alina and Naoki are each solving the equation $7(x + 2) = 91$.
Alina starts by using the distributive property.
Naoki starts by dividing each side by 7.

8. Show what Alina's and Naoki's full solution methods might look like.

Alina **Naoki**

9. What is the same and what is different about their methods?

10. Solve the equation.
$-5(4 - 2x) - 2x = 12$

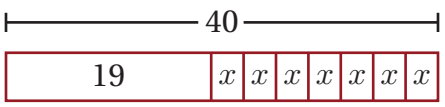
Spiral Review

11. Vicente and Zwena are trying to write $9x - 2x + 4x$ using fewer terms.

- Vicente says that $9x - 2x + 4x = 3x$ because the subtraction sign tells us to subtract everything that comes after $9x$.
- Zwena says that $9x - 2x + 4x = 11x$ because the subtraction only applies to $2x$.

Who is correct? Explain your thinking.

12. Write three different equations that represent the tape diagram. Then determine the value of x.



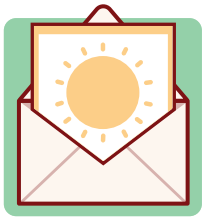
13. Choose the expression that is equivalent to $(9x + 5) + (-7x + 2)$.

A. $-2x + 10$ **B.** $-63x + 10$ **C.** $16x + 7$ **D.** $2x + 7$

Name: ..... Date: ..... Period: .....

Unit Rates in the World 7.EE.3, 7.EE.4, 7.EE.4.a, SMP.1, SMP.2

Community Day



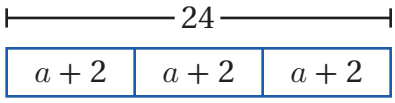
Let's represent and answer questions about situations in context.

Warm-Up

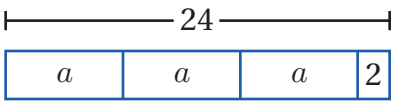
A baker put an equal number of cookies into 3 boxes. Then they put 2 more cookies in each box. They used 24 cookies total.

1. Which tape diagram best represents this situation?

A.



B.



2. Write an equation that represents the tape diagram you chose.

Name: Date: Period:

Three Reads

3. Here is a situation. Let's make sense of it together as a class.

Kyrie is making _____ invitations to Kyrie's school's Community Day.

Kyrie has already made _____ invitations and wants to finish the rest of them within a week.

Kyrie plans to spread out the remaining work so that Kyrie makes the same number of invitations each day.

a. **Discuss:** What is this situation about?

b. **Discuss:** How are these quantities related?

c. How many invitations should Kyrie make each day? Describe your strategy.

d. **Discuss:** How did your strategy compare with your partner's?

e. The deadline for sending out the invitations was moved up, so now Kyrie only has 3 days to make the rest of the invitations. How many invitations does Kyrie have to make each day?

f. Is it reasonable for Kyrie to make this many invitations each day?

Name: _____ Date: _____ Period: _____

Similar Problems

4. You will use the Activity 2 Sheet to explore a set of situations. Choose Set 1, Set 2, or Set 3.

Set _____

5. Create a poster. Here is what your poster should include *for each situation* in your set:

- ☐ The situation in words.
- ☐ A visual representation of the situation (tape diagram, hanger, etc.).
- ☐ An equation that represents the situation, along with your work for solving it.
- ☐ A check for the solution to the equation.
- ☐ The answer (with units) to the question in the situation.
- ☐ Connections between the visual representation, equation, and situation.

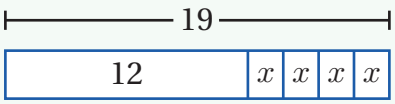
Synthesis

6. **Discuss:** What do you think is important to remember when solving problems using visual representations and equations?

Summary 6.12

You can use a visual representation or an equation to answer questions about a situation.

For example, Zahra buys 4 pens and a binder for $12. She pays a total of $19. This tape diagram represents Zahra's situation.



Zahra can determine the price of each pen algebraically by writing and solving the equation $12 + 4x = 19$:

$$12 + 4x = 19$$

$$4x = 7$$

$$x = 1.75$$

Zahra pays $1.75 for each pen.

Arithmetically, Zahra can reason that $12 can be initially deducted from $19, leaving $7 for four pens. 7 divided into 4 parts is $\frac{7}{4}$ or $1.75.

Practice
6.12

Name: ..... Date: ..... Period: .....

Problems 1–4: Select an equation to represent the situation. Then solve the equation and explain the solution's meaning in context.

$5x - 7 = 3$ $7 = 3(5 - x)$ $3x + 5 = -7$ $x + 7 = 3 \cdot 5$

	Situation	Equation	Solution	Solution's Meaning
1.	The temperature outside is currently -7°C. Since midnight, the temperature tripled and then rose 5 degrees. What was the temperature at midnight?			
2.	Ama has 7 pink roses plus some white roses. She gives all of her roses away by giving 5 roses to each of her 3 favorite teachers. How many white roses does Ama give away?			
3.	A family of 3 goes to a fair. Tickets cost $5 each, but each person has a coupon. They pay $7 altogether. How much money does each person save on buying their ticket?			
4.	A club puts its members into 5 groups for an activity. 7 students leave early, so there are only 3 students left to finish the activity. How many students were in each group?			

5. 6 soccer teams are practicing on a field. Each team has the same number of players. A coach asks 2 players from each team to leave the field to help move some equipment. Now there are 78 players on the field. Write and solve an equation whose solution is the number of players on each team.

6. One of the soccer teams enters a tournament! They play 2 games each day and then the championship game on the last day. They will play a total of 9 games. Write and solve an equation to determine how many days, x, the tournament lasts. Explain what your solution means in context.

Equation: Solution:

Explanation:

Spiral Review

7. Change the position of the parentheses to create an expression that is equivalent to $(8x - 9 - 12 + 5)$. Then explain how you know the two expressions are equivalent.

8. Change the position of the parentheses to create an expression that is *not* equivalent to $(8x - 9 - 12 + 5)$.

Problems 9–12: Determine the value of the variable that makes each equation true.

9. $a \cdot 3 = -30$ **10.** $-9 \cdot b = -45$ **11.** $-89 \cdot 12 = c$ **12.** $d \cdot 88 = -88000$

13. Select *all* the expressions that show x increased by 35%.

- ☐ **A.** $1.35x$
- ☐ **B.** $\frac{35}{100}x$
- ☐ **C.** $x + \frac{35}{100}x$
- ☐ **D.** $(1 + 0.35)x$
- ☐ **E.** $(100 + 35)x$
- ☐ **F.** $\frac{100 + 35}{100}x$

14. Alma has a loan of $12,589. This loan has a simple interest rate of 2.1% per year. No payments will be made on the loan until the end of one year. About how much interest will Alma pay on this loan at the end of the year?

A. $2,644 **B.** $264 **C.** $15,233 **D.** $12,853

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra | 7.EE.1, 7.EE.3, 7.EE.4, 7.EE.4.a, SMP.2

Practice Day 1, Set A

Let's practice what you've learned so far in this unit!

1. Write an expression equivalent to $7x + 1 - (3x - 1)$ that only has two terms.

2. Write an expression with two terms that represents the diagram.

$x + 6$	$x + 6$	$x + 6$	32

3. Tameeka's sunflower is currently 58 millimeters. She notices that the sunflower grows 8 millimeters each day. Write an expression to represent this situation.

Problems 4–5: Solve each equation.

4. $8x + 11 = 3$

$x =$ .....

5. $2(11 - x) = 40$

$x =$ .....

Problems 6–7: A bowling alley charges each person $\$6$ to bowl, plus a shoe rental fee. A group of 4 friends paid $\$36$ in total.

6. Draw a tape diagram to represent this situation.

7. How much was the shoe rental fee for each friend?

Name: ..... Date: ..... Period: .....

Practice Day 1 (continued)

Problems 8–9: Kiana delivers sandwiches for a restaurant. Kiana charge $\$8.25$ for each sandwich, plus a $\$10$ delivery fee. Kiana has an order that totals $\$59.50$.

8. Write an equation to represent this situation.

9. How many sandwiches are included in this order?

Problems 10–11: Solve each equation.

10. $\frac{1}{3}(x - 12) = -6$

$x =$ .....

11. $12 - 3(x + 10) = 18$

$x =$ .....

Problems 12–13: A store is having a sale, and shoes are 20% off! Arturo has a coupon for $\$3$ off and buys a pair of shoes. The store applies the coupon first and then takes 20% off. Arturo pays a total of $\$13.60$.

12. Write an equation to represent this situation.

13. What was the original price of the shoes before the sale and the coupon?

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra | 7.EE.1, 7.EE.3, 7.EE.4, 7.EE.4.a, SMP.2

Practice Day 1, Set B

Let's practice what you've learned so far in this unit!

1. Write an expression equivalent to $11x - 3 - (7x - 5)$ that only has two terms.

2. Write an expression with two terms that represents the diagram.

$x + 10$	$x + 10$	$x + 10$	20

3. Neena is saving money. She currently has $\$58$ saved and earns $\$8$ for every lawn she mows. Write an expression to represent this situation.

Problems 4–5: Solve each equation.

4. $14x + 20 = 6$

$x =$ .....

5. $6(3 - x) = 72$

$x =$ .....

Problems 6–7: A workout class charges each student $\$12$ to attend, plus a towel rental fee. Today, 5 students paid $\$75$ to attend the class.

6. Draw a tape diagram to represent this situation.

7. How much was the towel rental fee for each student?

Name: ..... Date: ..... Period: .....

Practice Day 1 (continued)

Problems 8–9: Jamar delivers sandwiches for a restaurant. Jamar charge $\$9.50$ for each sandwich, plus an $\$8$ delivery fee. Jamar has an order that totals $\$65$.

8. Write an equation to represent this situation.

9. How many sandwiches are included in this order?

Problems 10–11: Solve each equation.

10. $\frac{1}{4}(x - 6) = -3$

$x =$ .....

11. $5 - 2(x - 1) = 31$

$x =$ .....

Problems 12–13: A store is having a sale, and shoes are 15% off! Xavier has a coupon for $\$2$ off and buys a pair of shoes. The store applies the coupon first and then takes 15% off. Xavier pays a total of $\$15.30$.

12. Write an equation to represent this situation.

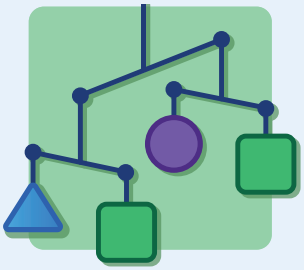
13. What was the original price of the shoes before the sale and the coupon?

Sub-Unit

3

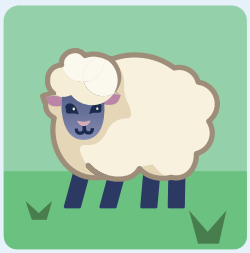
Inequalities

Lesson 13
I Saw the Signs



Lesson 14
Unbalanced Hangers

Lesson 15
Budgeting



Lesson 16
Shira the Sheep

Lesson 17
Write Them and Solve Them

Name: ..... Date: ..... Period: .....

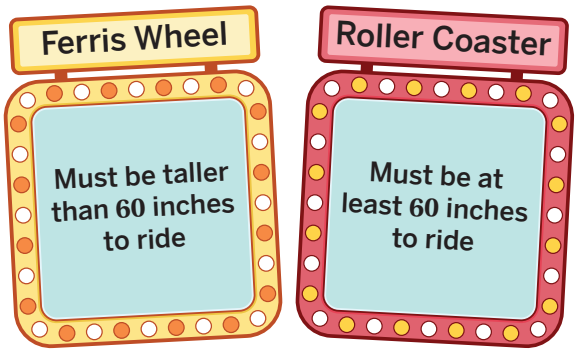
Generalized Numbers Leading to Algebra | 7.EE.4, SMP.2, SMP.3, SMP.6

I Saw the Signs

Let's work with inequalities.

Warm-Up

1 Here are two signs for two different rides at an amusement park.



Habib is exactly 60 inches tall. Which ride can he go on?

A. Ferris wheel **B.** Roller coaster **C.** Both **D.** Neither

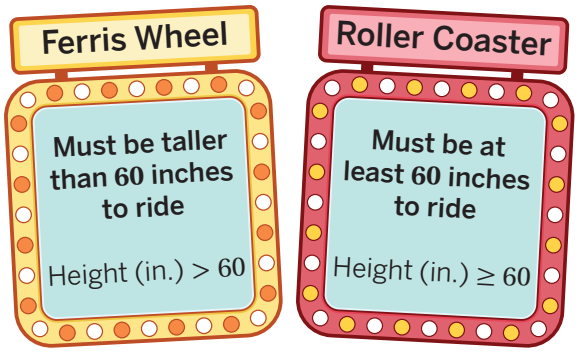
Explain your thinking.

Activity 1

Name: Date: Period:

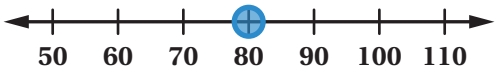
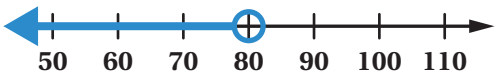
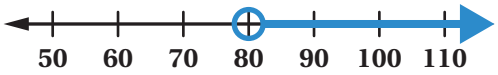
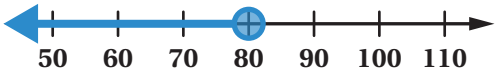
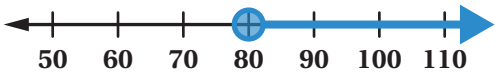
Riding the Rides

The park added symbols to make the signs clearer. The symbol $>$ means greater than. The symbol $\geq$ means **greater than or equal to**.



2 What is the shortest height Makayla can be and still ride both rides?

3 Here are some equations and inequalities, along with their graphs on a number line.

Equation/Inequality	Graph
$x = 80$	
$x < 80$	
$x > 80$	
$x \leq 80$	
$x \geq 80$	

Discuss: What do you notice?

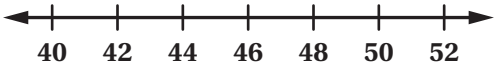
Activity 2

Name: ..... Date: ..... Period: .....

Riding More Rides

4 You are in charge of determining the height restriction for your ride. Create a graph and complete the sign for your ride.

5 You must be 48 inches or shorter to ride the kiddie swings. Make a graph on the number line to represent the possible heights of the riders.



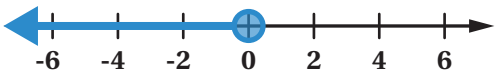
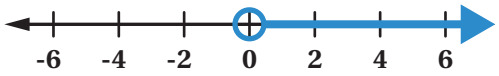
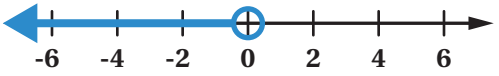
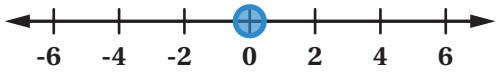
6 Luis is allowed to ride the kiddie swings. Omar is 6 inches shorter than Luis. Can Omar ride this ride? Explain your thinking.

Activity 3

Name: ..... Date: ..... Period: .....

Inequalities Out of Context

7 Match each inequality or description with the graph that represents it. A graph may have more than one match.

Graph	Inequality/Description
a. 	..... $x < 0$
b. 	..... $0 < x$
	..... $x \leq 0$
c. 	..... The temperature of a freezer must be less than 0°F.
d. 	..... The temperature of a freezer must be 0°F.

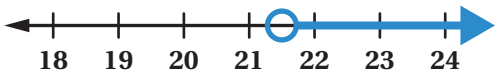
8 Fix the inequality $x > 23$ so that it represents the graph.

Name: ..... Date: ..... Period: .....

Inequalities Out of Context (continued)

9 Write an inequality that represents this graph.

10 To represent this graph, Tiara wrote the inequality $21.5 < x$. Devon wrote the inequality $x < 21.5$.



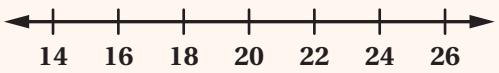
Whose inequality is correct? Circle one.

Tiara Devon Both Neither

Explain your thinking.

You're invited to explore more.

11 Create a graph that represents all the values that make the inequality $x + 10 \leq 25$ true.

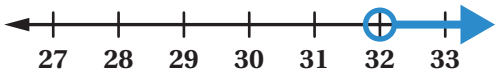


Explain your thinking.

12 Synthesis

Discuss: How can you tell from each representation that 32 is *not* included in the inequality?

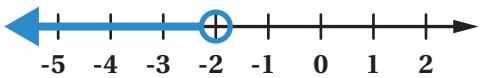
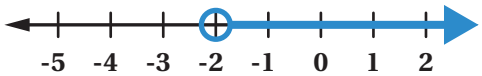
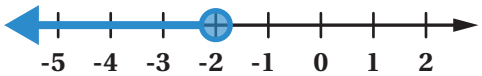
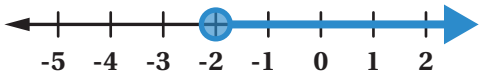
$x > 32$



15 Summary 6.13

You can use an inequality to describe a range of values. When graphing, we fill in the circle to represent including the boundary point. An open circle means the boundary point is not included.

Here are the symbols used to write inequalities.

Symbol	Name	Meaning	Inequality	Example
$<$	Less than	lower, smaller	$x < -2$	
$>$	Greater than	more than, higher, larger	$x > -2$	
$\leq$	**Less than or equal to**	no more than, the maximum amount	$x \leq -2$	
$\geq$	**Greater than or equal to**	at least, the minimum amount	$x \geq -2$	

≥ (greater than or equal to) $x \geq a$, x is greater than a or x is equal to a.

≤ (less than or equal to) $x \leq a$, x is less than a or x is equal to a.

Practice
6.13

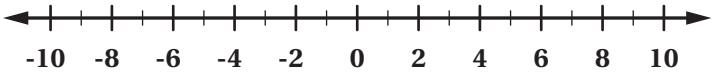
Name: ..... Date: ..... Period: .....

1. Select *all* the inequalities that are true when $x = 4$.

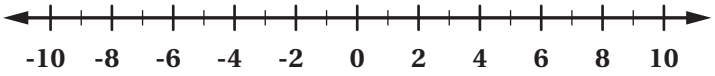
- ☐ **A.** $x < 2$
- ☐ **B.** $x < 10$
- ☐ **C.** $x < 4$
- ☐ **D.** $x \geq 4$
- ☐ **E.** $x \geq 8$

Problems 2–4: Make a graph on the number line that represents each inequality.

2. $x \leq 5$



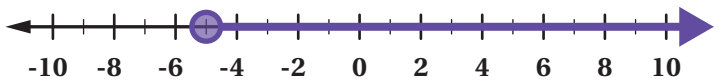
3. $x < \frac{5}{2}$



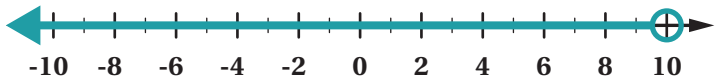
4. $x \geq -6.5$

Problems 5–6: Write an inequality that represents each graph.

5.



6.



Problems 7–8: Express each statement as an inequality, and write two values that make the inequality true.

7. a is greater than or equal to 0.1.

8. r is no more than 12.

Name: ..... Date: ..... Period: .....

Problems 9–10: Write an inequality that represents each verbal description.

9. Roller coasters have height requirements for safety reasons. The minimum height requirement on a roller coaster at a local theme park, h, is 54 inches.

10. Water freezes at any temperature at or below 32 degrees Fahrenheit.

11. Write a story that can be represented by the inequality $x < 10$.

Spiral Review

Problems 12–15: Here are two stories.

Story A

This year's freshman class is 10% smaller than last year's class. But during the first week of classes, 20 more students join. There are now 830 students in the freshman class.

Story B

A store reduces the price of a computer by $\$20$. Then during a 10% off sale, a customer pays $\$830$.

12. Determine which story the equations $0.9x + 20 = 830$ and $0.9(x - 20) = 830$ represent.

13. Explain why one equation has parentheses and the other doesn't.

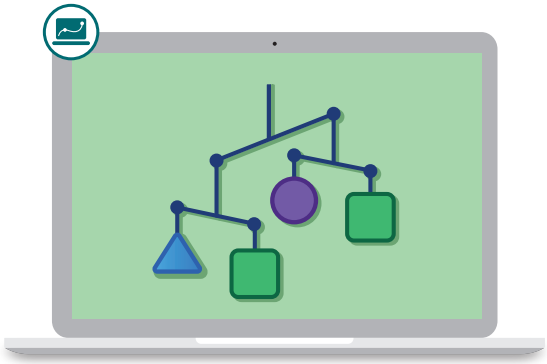
Problems 14–15: Solve each equation and interpret its meaning.

14. $0.9x + 20 = 830$

15. $0.9(x - 20) = 830$

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra 7.EE.4, 7.EE.4.b, SMP.3

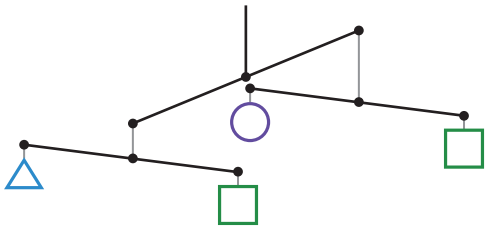


Unbalanced Hangers

Let's solve inequalities using hangers.

Warm-Up

1 Order the shapes in the hanger from *lightest* to *heaviest*.



Lightest

Heaviest

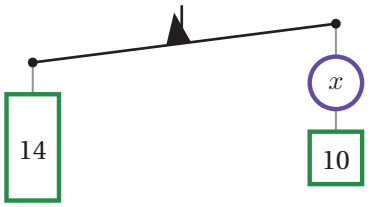
Explain how you decided which shape was lightest.

Activity 1

Name: Date: Period:

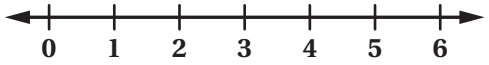
Unbalanced Hangers

2 Here is a hanger that is not balanced. What is one possible value of x?



3 Plot your response from the previous problem on the number line. Determine at least two more possible weights and plot those on the number line.

$14 > x + 10$



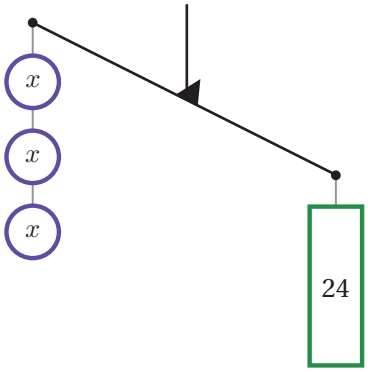
4 Describe *all* of the possible values of x that keep the right side lighter.

Name: ..... Date: ..... Period: .....

Unbalanced Hangers (continued)

The **solutions to an inequality** include all of the possible values that make an inequality true.

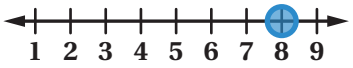
5 This hanger represents the inequality $3x < 24$. What are the solutions to this inequality?



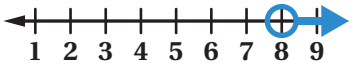
$3x < 24$

A. $x < 8$

B. $x = 8$

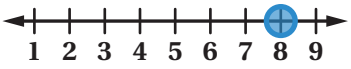


C. $x > 8$



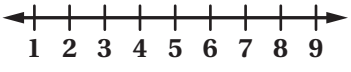
6 Here is the graph of the solution to the equation $3x = 24$.

$3x = 24$



a Graph what you think the solutions to $3x \leq 24$ look like.

$3x \leq 24$

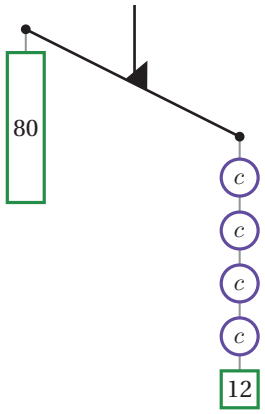


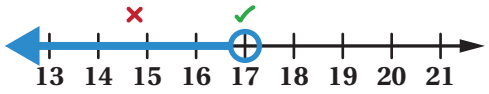
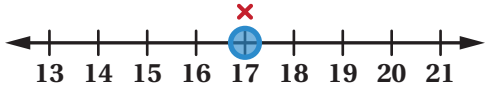
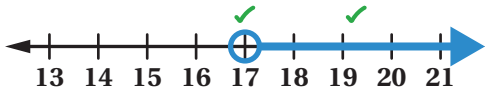
b **Discuss:** How are the graphs of the solutions to $3x < 24$, $3x = 24$, and $3x \leq 24$ alike? How are they different?

Name: Date: Period:

Solving Inequalities

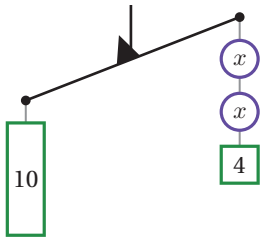
7 Here are three possible solutions to the inequality $80 < 4c + 12$ and their graphs.



Possible Solution	Graph
$17 > c$	
$17 = c$	
$17 < c$	

Discuss: What do the checks and x's in the graphs mean?

8 What are the solutions to the inequality $10 \geq 2x + 4$? Explain your thinking. Use the hanger if it helps with your thinking.



9 Jasmine and Terrance solve the inequality $10 \geq 2x + 4$. Jasmine says the solutions are $x \leq 3$. Terrance says the solutions are $3 \geq x$. Who is correct? Circle one.

Jasmine Terrance Both Neither

Explain your thinking.

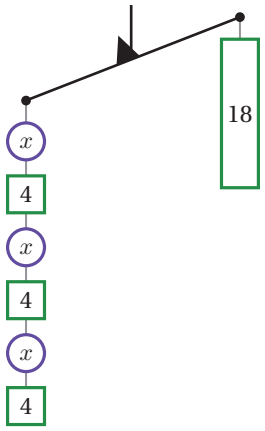
Activity 3

Name: ..... Date: ..... Period: .....

Repeated Challenges

10 What are the solutions to the inequality $3(x + 4) \geq 18$?

Use the hanger if it helps with your thinking.

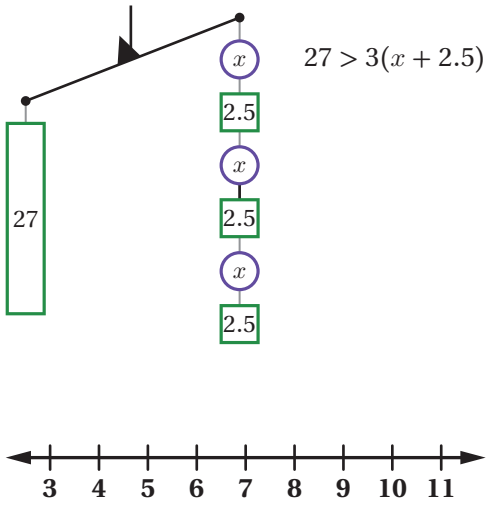


11
- Decide with a partner who will complete Column A and who will complete Column B.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.
- Solve as many inequalities as you have time for. Sense-making is more important than speed.

Column A	Column B
$4x + 2 \leq 10$	$6x + 4 \leq 16$
$12 > 2(x + 1)$	$24 > 3(x + 3)$
$10.4 \leq 2(x + 2.2)$	$8(x + 1.1) \geq 32.8$
$2x + \frac{3}{2} > \frac{17}{2}$	$4x + \frac{2}{3} > \frac{44}{3}$

12 Synthesis

Describe a process you can use to determine the solutions to an inequality.
Use the hanger if it helps show your thinking.



15 Summary 6.14

You can use many of the same strategies you use for solving equations to solve inequalities. The values of x that make the inequality true are known as the **solutions to an inequality**. You can test values by substituting them into the inequality.

For example, consider the inequality $27 > 3(x + 2.5)$.

- We can determine the value of x that balances the hanger, or the boundary point, by solving the equation $27 = 3(x + 2.5)$.
- When $x = 6.5$, the hanger is balanced, which means 6.5 is the *boundary point*. All values less than 6.5 will make the inequality true.

The solutions shown on the graph mean that all values of x *less than* 6.5 will make the inequality true.

To check the solution, substitute any value less than 6.5 into the original inequality.

$27 > 3(x + 2.5)$

$27 > 3(0 + 2.5)$

$27 > 3(2.5)$

$27 > 7.5$ ✓

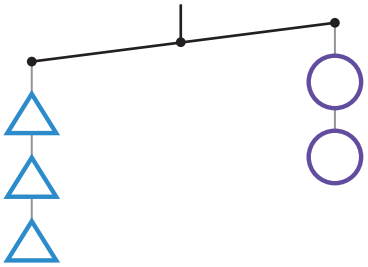
solutions to an inequality All values of a variable that make the inequality true.

Practice
6.14

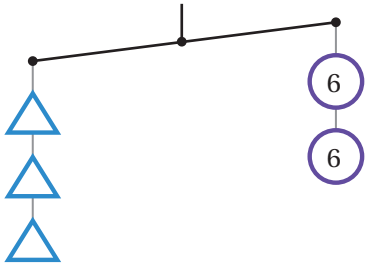
Name: ..... Date: ..... Period: .....

Problems 1–2: Here are two unbalanced hangers. Write an inequality to represent the relationship between the weights on each hanger. Use t to represent the weight of the triangle in grams. Use c to represent the weight of the circle in grams.

1.

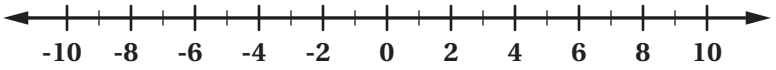


2.

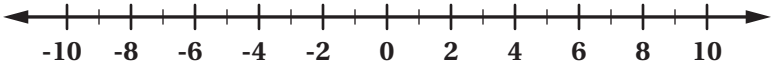


Problems 3–5: Use the number line diagrams to graph possible solutions for each inequality.

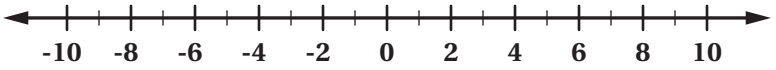
3. $5x \leq -20$



4. $11 > 2x + 1$

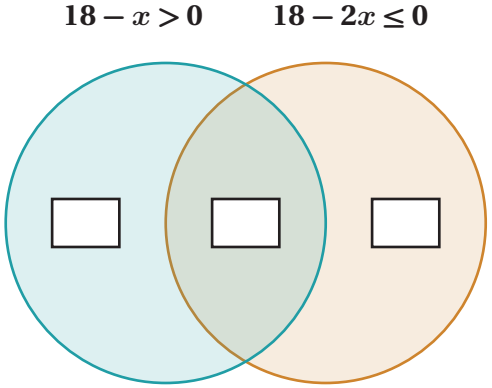


5. $2(x + 3) > 18$



6. List three values for x that would make $5x \leq -20$ true.

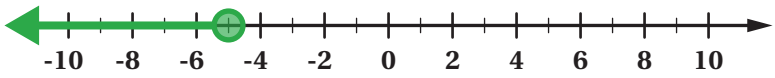
7. Write a value in each region that makes the inequality or inequalities true.



Name: ..... Date: ..... Period: .....

8. Which number line represents the solutions to the inequality $3x - 8 \leq 7$?

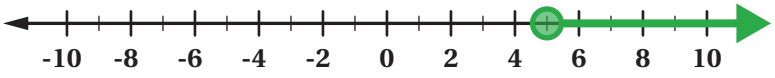
A.



B.

C.

D.



Spiral Review

9. Select *all* the values that are solutions to $x \leq -4$.

- ☐ **A.** 4
- ☐ **B.** -4
- ☐ **C.** -3.99
- ☐ **D.** -4.01
- ☐ **E.** 0

10. Match each fraction to its decimal equivalent.

Fraction	Decimal
a $\frac{11}{45}$	..... $0.\overline{24}$
b $\frac{6}{25}$	..... $0.2\overline{4}$
c $\frac{8}{33}$	..... 0.24

11. Finish writing $\frac{5}{8}$ as a decimal.

$$\begin{array}{r} 0.6 \\ 8\overline{)5.00} \\ -48 \\ \hline \end{array}$$

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra 7.EE.4, 7.EE.4.b, SMP.1, SMP.3

Budgeting

Let's solve problems about budgeting and spending money.

Warm-Up

1. Here is a situation with hidden information. Let's make sense of it together as a class.

Maia is selling magazine subscriptions. She earns _____ per week, plus _____ for every subscription she sells. She plans to buy soccer equipment with the money she earns.

This week, Maia wants to buy a new ball. The cheapest ball she wants costs _____.

a. **Discuss:** What is this situation about?

b. Choose a value for each blank that could make sense.

c. For the values you chose, how many magazines could Maia sell in order to buy the ball?

Activity 1

Name: ..... Date: ..... Period: .....

Maia's Magazines

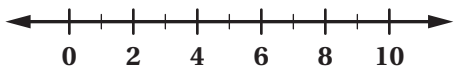
2. Maia is selling magazine subscriptions. She earns $19 per week, plus $3 for every subscription she sells. She plans to buy soccer equipment with the money she earns.

This week, Maia wants to buy a new ball. The cheapest ball she wants costs $43.

a Write and solve an equation to determine how many magazine subscriptions Maia needs to sell to make $43.

b List other numbers of magazine subscriptions Maia could sell and still buy the ball.

c Write and graph an inequality to represent *all* the number of subscriptions Maia could sell and still buy the ball.



3. The next week, Maia earns $37. She wants to use it to buy soccer shorts and 5 pairs of socks. The shorts she wants each cost $22.05.

a What do each pair of socks cost if Maia spends exactly $37 on the socks and shorts? (In Maia's city, there is no sales tax.) Write and solve an equation if it helps you with your thinking.

b Write an inequality to represent *all* the sock prices that Maia could afford with $37.

Activity 2

Name: Date: Period:

Bao's Budgeting

4. Bao has $175 saved in his bank account. He wants to know how much money he can take out each month and still have at least $25 in the account a year from now.

a Which inequality represents Bao's situation?

A. $175 - 12x \leq 25$

B. $175 + 12x \leq 25$

C. $175 - 12x \geq 25$

D. $175 + 12x \geq 25$

b What does 12 represent?

c What does x represent?

d Bao and his friend try to solve the inequality. Bao's solutions start with $x \leq$. His friend's solutions start with $x \geq$. Which symbol makes sense for this situation? Explain your thinking.

e Solve the inequality you chose and explain what the solutions mean in Bao's situation.

5. Bao is considering getting a part-time job. Instead of taking money out of his account each month, he would put money in. His account still has $175, and his goal is to have at least $1,000 in the account a year from now.

a Write an inequality where x represents the amount of money Bao should put in each month to reach his goal.

b Solve the inequality you wrote and explain what the solutions mean in Bao's situation.

Synthesis

6. Tay has a $\$30$ gift card to Tea Time Cafe. Tay spends $\$2.50$ on a tasty beverage every school day. Tay wants to know how many beverages Tay can buy using the gift card. Explain how the inequality $30 - 2.50x \geq 0$ represents Tay's situation.

Summary 6.15

Inequalities can be used in real-world situations, like budgeting for a project.

For example, Aditi has $\$5$ and sells homemade greeting cards for $\$1.50$ each. Her goal is to have $\$20$ total.

The solution to the equation $1.50x + 5 = 20$ represents the number of greeting cards, x, that she needs to sell in order to have exactly $\$20$. If she sells 10 greeting cards, she will have $\$20$ because $1.50(10) + 5 = 20$.

What if Aditi wants to have more than $\$20$? The inequality $1.50x + 5 > 20$ represents this situation.

While the solution to the equation was $x = 10$, the solution to the inequality is $x > 10$. Aditi will need to sell *more than* 10 cards to have *more than* $\$20$ total.

Aditi

$1.50x + 5 = 20$

$1.50x + 5 - 5 = 20 - 5$

$1.50x = 15$

$x = 10$

10 cards makes $20

$x > 10$ represents making more than $20

Practice
6.15

Name: Date: Period:

Problems 1–3: Alina donates x dollars out of every $\$9$ that she earns. This happens 7 times. If Alina has exactly $\$42$ remaining for herself, her situation can be represented with the equation $7(9 - x) = 42$.

1. Solve the equation.

2. Alina wants to have *at least* $\$42$ remaining for herself. Write an inequality to represent this situation.

3. Solve the inequality and explain what the solutions mean in the situation.

Problems 4–6: Jamir buys a candle that is 9 inches tall and burns down 0.25 inches per minute. He wonders how many minutes it will take until the candle is exactly 6 inches tall, so he writes the equation $9 - 0.25x = 6$.

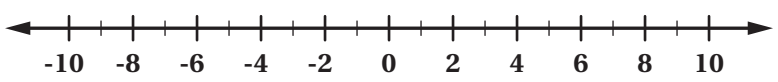
4. Solve the equation.

5. Jamir wants to know how many minutes until the candle's height is 6 inches *or less*. Write an inequality to represent this situation.

6. Solve the inequality and explain what the solutions mean in the situation.

Problems 7–8: Use the number line to graph the solution to each inequality.

7. $2x \geq -7$



8. $3x + 1 < 4$

Practice
6.15

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 9–12: Evaluate each expression.

9. $\frac{2}{5} \cdot (-10)$

10. $-8\left(-\frac{3}{2}\right)$

11. $\frac{10}{6} \cdot 0.6$

12. $-\frac{100}{37} \cdot (-0.37)$

Problems 13–16: Here are some prices customers paid for different items at a farmer's market. Determine the cost for 1 pound of each item.

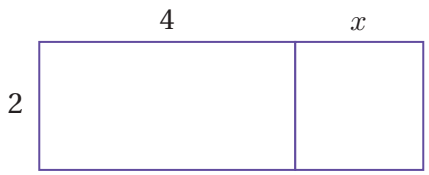
13. \$5 for 4 pounds of apples

14. \$3.50 for $\frac{1}{2}$ pounds of cheese

15. \$8.25 for $1\frac{1}{2}$ pounds of coffee beans

16. \$6.75 for $\frac{3}{4}$ pounds of fudge

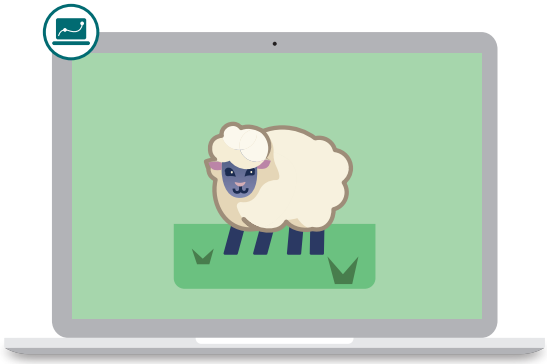
17. Select *all* the expressions that represent the area of the largest rectangle.



- ☐ **A.** $2 \cdot 4 + 2 \cdot x$
- ☐ **B.** $8 + x$
- ☐ **C.** $8 + 2x$
- ☐ **D.** $2(4 + x)$
- ☐ **E.** $2 + 4 + x$

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra 7.EE.4.b, SMP.3



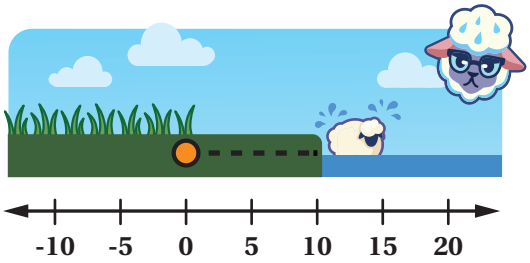
Shira the Sheep

Let's practice solving inequalities with positive and negative coefficients.

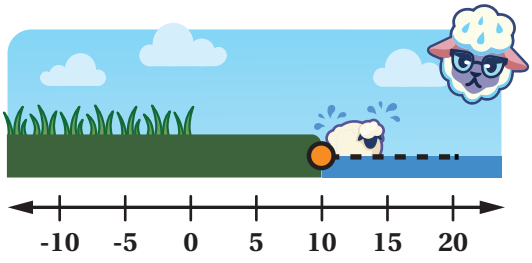
Warm-Up

1 a Shira the Sheep loves eating grass. She does not like water. Here are the graphs and results of different inequalities.

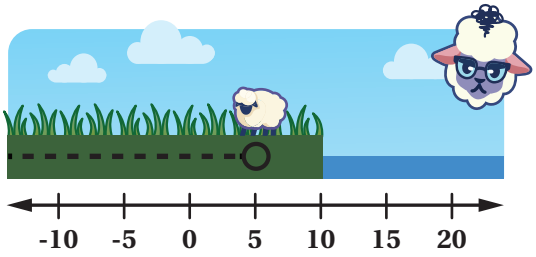
$x \geq 0$



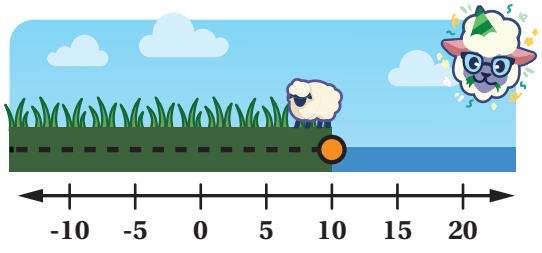
$x \geq 10$



$x < 5$



$x \leq 10$



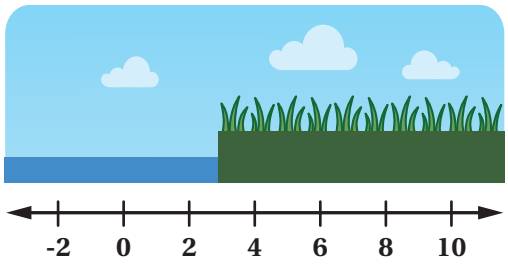
b **Discuss:** What do you notice?

Activity 1

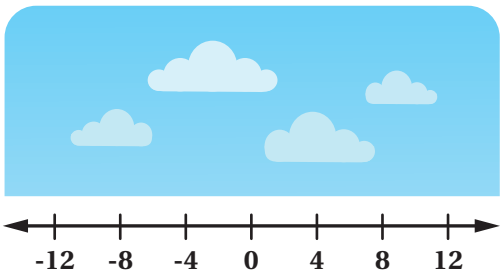
Name: ..... Date: ..... Period: .....

Shira the Sheep

2 The grass is represented by the inequality $5x > 15$. Solve the inequality to help Shira eat all the grass without falling in the water.

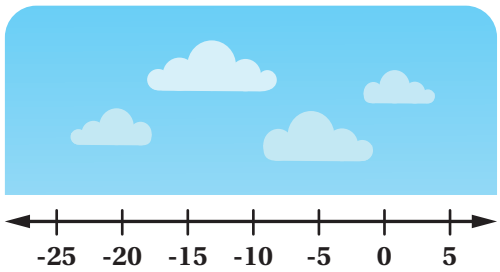


3 **a** Write the solutions to this inequality to help Shira eat all the grass.
$11 \geq 2x - 5$



b Sketch the solutions to this inequality on the number line.

4 **a** Write the solutions to this inequality to help Shira eat all the grass.
$10 - 6x < 70$



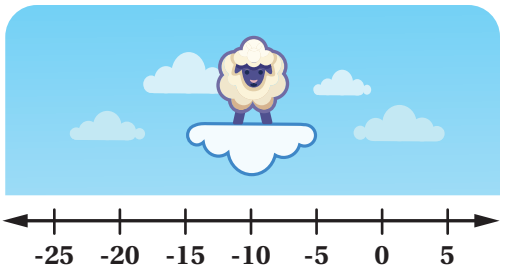
b Sketch the solutions to this inequality on the number line.

Name: ..... Date: ..... Period: .....

Shira the Sheep (continued)

5 Alma was solving the previous inequality, $10 - 6x < 70$. She knew the sheep needed to land at -10, but didn't know if the grass was to the right or left.

She wrote $10 - 6(0) < 70$. How might Alma's inequality help her decide where the grass is?



6 Solve this inequality to help Shira eat all the grass.

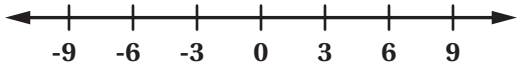
$-\frac{1}{2}x + 2 \leq 3$

Activity 2

Name: ..... Date: ..... Period: .....

Help Shira and Chloe

7 **a** Solve $25 - 4x < 1$.



b Graph its solutions.

8 Chloe made a mistake solving the inequality $25 - 4x < 1$ and wrote $x < 6$. Explain what you think is incorrect about her work.

Chloe

$25 - 4x < 1$

$x < 6$

9 Solve as many inequalities as you have time for to help Shira eat all the grass.

a $8 \geq 3x - 13$

b $-6x - 3 > 15$

c $\frac{2}{3}x + 9 \leq 15$

d $-89 \geq -12x - 5$

e $3x - 5 > 4$

f $0.2x + 0.6 \geq 0.8$

10 Synthesis

Explain how you solve and graph the solutions to any inequality. Use the inequality $-3x + 6 < 18$ if it helps you with your thinking.

13 Summary 6.16

When solving an inequality, it can help to start by solving a related equation. The solution to the equation tells you the boundary point of the graph of the solutions to the inequality. Once you determine the boundary point, you still need to decide whether the solutions include values greater or less than the boundary point. This can be done by testing a value to the right or left of the boundary point on the number line.

Let's solve the inequality $-3x + 6 < 18$ by starting with the related equation.

Equation	Explanation
$-3x + 6 = 18$	Write the inequality as an equation.
$-3x + 6 - 6 = 18 - 6$	Subtract 6 on both sides.
$-3x \div (-3) = 12 \div (-3)$	Divide by -3 on both sides.
$x = -4$	This is the boundary point.

You can show the solution on a number line by drawing an open circle on -4, the boundary point. To determine if the solutions to the inequality are to the right or left of -4, choose a value, such as 0, to test in the original inequality.

In this case, 0 is to the right of -4 on the number line. $-3(0) + 6 < 18$ is true, which means all values to the right of -4 are solutions to this inequality. The solutions to the inequality are $x > -4$.

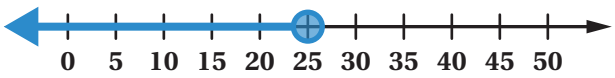
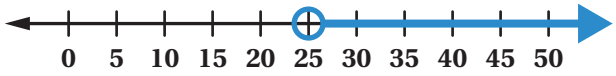
Practice
6.16

Name: ..... Date: ..... Period: .....

1. Select *all* the values of x that make the inequality $-x + 6 \geq 10$ true.

- ☐ **A.** -3.9
- ☐ **B.** 4
- ☐ **C.** -4.01
- ☐ **D.** -4
- ☐ **E.** 4.01
- ☐ **F.** 3.9
- ☐ **G.** 0
- ☐ **H.** -7

2. A library is having a party for any student who read at least 25 books over the summer. Match each situation or number line graph with an inequality.

Situation/Graph	Inequality
a. Ricardo read x books and was invited to the party.	..... $x < 25$
b. Prisha read x books but was not invited to the party.	..... $x > 25$
c. 	..... $x \leq 25$
d. 	..... $x \geq 25$

Problems 3–4: Use the inequality $100 - 3x \geq -50$.

3. Select *all* the values of x that make the inequality $100 - 3x \geq -50$ true.

- ☐ **A.** 0
- ☐ **B.** 50
- ☐ **C.** -50
- ☐ **D.** 49.9
- ☐ **E.** 50.1

4. In order to solve the inequality $100 - 3x \geq -50$, Makalya solved the equation $100 - 3x = -50$ and got $x = 50$. What is the solution to the inequality?

5. Diego is solving the inequality $-3x \geq 45$. He solves the equation $-3x = 45$ to determine $x = -15$. What is the solution to the inequality?

- **A.** $x < -15$
- **B.** $x > -15$
- **C.** $x \leq -15$
- **D.** $x \geq -15$

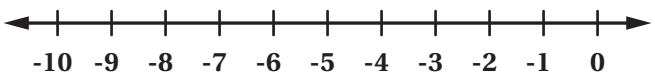
Practice
6.16

Name: ..... Date: ..... Period: .....

Problems 6–7: Complete the table to determine the solutions to each inequality. Write the solutions as an inequality and graph them on the number line.

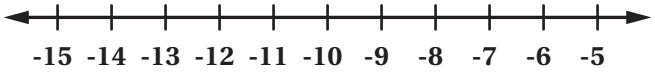
6. $-6x > 30$

x	-7	-6	-5	-4	-3	-2	-1	0
$-6x$								



7. $\frac{3}{4}x < -\frac{15}{2}$

x	-16	-14	-12	-10	-8	-6	-4	-2
$\frac{3}{4}x$								



Problems 8–9: Solve each inequality.

8. $4x + 8 < 20$.

9. $-3x + 4 \leq 19$

Spiral Review

10. Alma makes 5 cups of her favorite color of purple paint by mixing 3 cups of blue paint, $1\frac{1}{2}$ cups of red paint, and $\frac{1}{2}$ a cup of white paint. Alma has 2 cups of white paint. How much blue paint and red paint will Alma need to use with the 2 cups of white paint?

11. A backpack normally costs $25, but is on sale for $21. What percent is the discount? Show your thinking.

Name: ..... Date: ..... Period: .....

Generalized Numbers Leading to Algebra | 7.EE.4, 7.EE.4.b, SMP.2

Write Them and Solve Them

Let's write and solve inequalities and examine what the solutions to those inequalities mean in context.

Warm-Up

1. Jamal volunteers to pass out sandwiches to people who are hungry in his community. He raised $\$85$ and is trying to determine how many sandwiches he can buy for $\$6.25$ each.

He writes the inequality $6.25x \leq 85$.

Then he solves the inequality and gets $x \leq 13.6$.

Select *all* the statements that are true about this situation.

- ☐ **A.** He can buy 13.6 sandwiches.
- ☐ **B.** He can buy 14 sandwiches.
- ☐ **C.** He can buy 12 sandwiches.
- ☐ **D.** He can buy 10 sandwiches.
- ☐ **E.** He can buy -4 sandwiches.

Activity 1

Name: Date: Period:

Orange Juice and Donuts

Kiandra wanted to surprise some friends before school with orange juice and donuts. At the store, an orange juice costs $2.15 and a donut costs $0.75. There is no sales tax. The store has a $10 purchase minimum for credit cards. Kiandra paid with her credit card.

2. Write an inequality that describes Kiandra's situation.

3. Solve the inequality you wrote. Show your thinking.

4. What is the meaning of the solutions to your inequality in this situation?

Name: Date: Period:

Solve It!

5. You will use four sets of cards for this activity. Each set has a situation card and a corresponding support card.

- Decide with a partner who will have the situation card and who will have the support card.
- Switch roles after each round.

Situation Card Instructions

- Read the situation aloud.
- Write an inequality that represents the situation.
- Solve the inequality you wrote.
- Graph the solutions on a number line.
- Answer the question on the card using your solutions.

You may use this page for workspace.

Support Card Instructions

- Help your partner by asking the questions on the card or other questions you think will support them.

Synthesis

6. Sahana works at the pet store and gets paid $9.50 per hour. She needs to make at least $235 each week in order to pay her bills. Describe how to write an inequality that represents Sahana's situation.

Summary 6.17

You can use inequalities to represent and solve real-world problems. When you write an inequality, it can be helpful to first decide what quantity the variable represents. Then, write an inequality based on the relationships between the quantities in the situation.

Once you have solved your inequality, you will need to interpret the solution to ensure it makes sense for the situation. Some quantities only make sense with whole number values (e.g., number of people, number of buses, items that can be purchased, etc.), while other solutions can include decimal or fractional values (e.g., height of a roller-coaster rider, weight of a package, etc.).

Practice
6.17

Name: Date: Period:

Problems 1–3: A store sold $\frac{2}{5}$ of the shirts on display and brought out another 30. The store likes to keep at least 150 shirts on display. The manager wrote the inequality $\frac{3}{5}x + 30 \geq 150$ to describe the situation.

1. Explain what $\frac{3}{5}$ means in the manager's inequality.

2. Solve the inequality.

3. What do the solutions to the inequality mean in this situation?

Problems 4–6: Camila has up to $\$100$ to spend on her birthday party at a city swimming pool. She will invite 15 friends total. She also plans to spend $\$38.50$ on pizza.

4. Write an inequality to represent the situation.

5. Solve the inequality.

6. What do the solutions to your inequality mean in this situation?

Problems 7–8: Write an inequality to represent each situation.

7. In the cafeteria, there is one large 10-seat table and many smaller 4-seat tables. There are enough tables to fit at most 210 students. Write an inequality whose solution is the possible number of 4-seat tables in the cafeteria.

8. 5 barrels catch rainwater in the schoolyard. 4 barrels are the same size, and the fifth barrel holds 10 liters of water. The Environmental Club is hoping that the 5 barrels can catch at least 210 liters of water to use to water the school's garden. Write an inequality whose solution is the possible size of each of the 4 barrels.

Name: ..... Date: ..... Period: .....

9. How would the graph of the inequality you wrote in Problem 7 compare to the graph of the inequality you wrote in Problem 8?

Spiral Review

10. Select *all* the inequalities that have the same solutions as $-4x < 20$.

- ☐ **A.** $-x < 5$
- ☐ **B.** $4x > -20$
- ☐ **C.** $4x < -20$
- ☐ **D.** $x < -5$
- ☐ **E.** $x > w-5$

For Problems 11–14: Solve each equation.

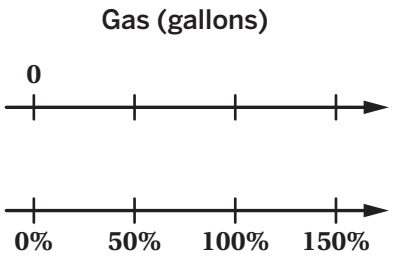
11. $-1d - 4 = -3$

12. $-\frac{1}{4}m + 5 = 16$

13. $10b + (-45) = -43$

14. $-8(y - 1.25) = 4$

15. The gas tank of a truck holds 30 gallons. The gas tank of a passenger car holds 50% less. How many gallons does the car's tank hold? Use the double number line if it helps with your thinking.



Name: _____ Date: _____ Period: _____

Generalized Numbers Leading to Algebra | 7.EE.1, 7.EE.3, 7.EE.4, 7.EE.4.a, 7.EE.4.b, SMP.1, SMP.6

Practice Day 2

Let's practice what you've learned so far in this unit!

You will use problem cards for this Practice Day. Record all of your responses here.

1. a _____ b ⟷

2. a _____ b ⟷

3. a _____ b _____

4. a _____ b _____

5. a _____ b _____

6. a _____ b ⟷

Name: _____ Date: _____ Period: _____

Practice Day 2 (continued)

7. **a** Equation: _____ **b** _____

Solution: _____

8. **a** _____ **b** _____

9. **a** _____ **b** _____

10. **a** Inequality: _____ **b** Explanation:

Solution: _____

11. **a** _____ **b** _____

12. **a** _____ **b** _____

Career Connection

How would you explain a concept without using words?

Have you ever wondered where the symbols $=$, $<$, $>$, $\leq$, and $\geq$ came from? The first use of the equal sign, $=$, is attributed to Robert Recorde, a Welsh physician and mathematician. He intentionally used two parallel lines to represent equality because they are always the same distance apart. The works of British mathematician Thomas Harriot included the first inequality symbols, $<$ and $>$. They were actually introduced by his book's editor, who altered the original triangular symbols Harriot used.

Logo designers create visually appealing graphics that include text, images, and symbols to identify businesses and other organizations. Logo designers use equations involving ratios to ensure their logo can be scaled to different sizes.

"Giuseppe Peano" Public Domain

Meet Giuseppe Peano

Giuseppe Peano was an Italian mathematician who was a founder of mathematical logic and set theory. He introduced symbols to represent certain phrases, such as "there exists" and "belongs to the set of." He created new symbols to represent his brand new ideas in mathematical logic, such as $\cup$, $\cap$, $\ni$, $\mathbb{N}$, or $\mathbb{Q}$, which you might encounter a bit later in your math studies.

Are you interested in a career in logo design or using symbols to represent concepts? What can you do to learn more?

Community Connection

Design your own math symbol or logo to express a mathematical concept. Share your design with 5 other people in your community and ask them what they think your design means.

Math Mindset

Why do you think symbols are sometimes used to express math ideas instead of words?

Unit 7

Angles, Triangles, and Prisms

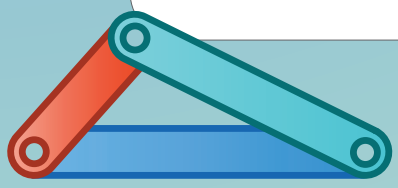
Big Ideas in This Unit

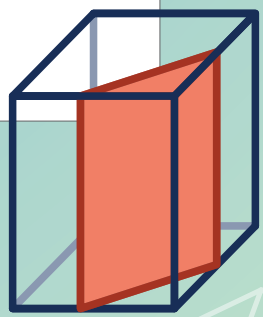
CC3 2-D and 3-D Connections | Angle Relationships | CC4 Shapes in the World | Scale Drawings

Questions for Investigation

- What strategies are helpful for determining unknown angle measures?
- How many unique polygons are possible with different sets of side lengths and angle measurements?
- What different shapes can you make by slicing through a three-dimensional figure?

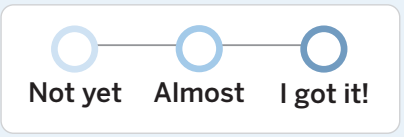
Explore: Shaping Up
How can we construct polygons according to their properties?





Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



I can . . .	Before	After
Determine angle measures around a vertex.		
Use supplementary, complementary, and adjacent angles to determine unknown angles in a figure.		
Write and solve simple equations for an unknown angle in a figure.		
Use vertical angles to write and solve simple equations for an unknown angle in a figure.		
Solve multistep problems involving complementary, supplementary, and vertical angles.		
Construct triangles from three measures of angles or sides.		
Determine whether conditions will form a unique triangle, more than one triangle, or no triangle.		
Justify whether three segments form a triangle.		
Determine if shapes are identical copies.		
Describe the two-dimensional figures that result from slicing three-dimensional figures.		

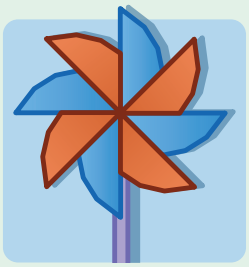
I can . . .	Before	After
Solve real-world and mathematical problems involving areas of two–dimensional objects.		
Solve real-world and mathematical problems involving surface area of three–dimensional objects.		
Solve real-world and mathematical problems involving volume of three–dimensional objects.		

Sub-Unit

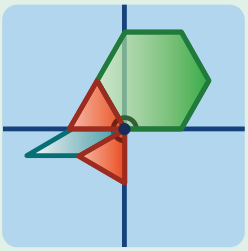
1

Angle Relationships

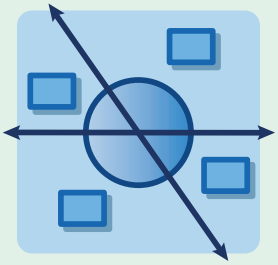
Explore
Shaping Up



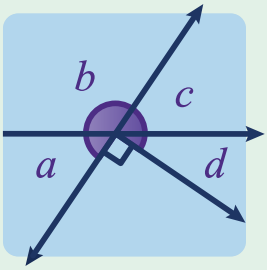
Lesson 1
Pinwheels



Lesson 2
Friendly Angles



Lesson 3
Angle Diagrams



Lesson 4
Missing Measures

Name: ..... Date: ..... Period: .....

2-D and 3-D Connections | Shapes in the World | Building Toward 7.G.2, 7.G.3, SMP.3, SMP.5, SMP.6

Explore: Shaping Up

How can we construct polygons according to their properties?

Warm-Up

1. Study the figures. Which one doesn't belong? Explain your thinking.

Figure A

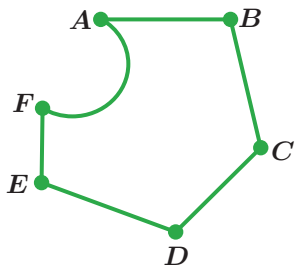


Figure B

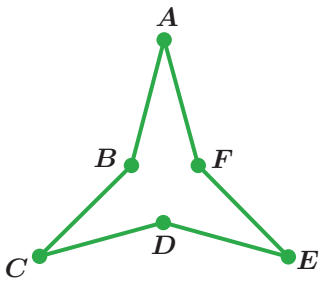


Figure C

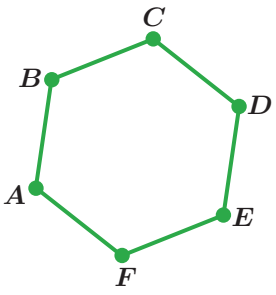
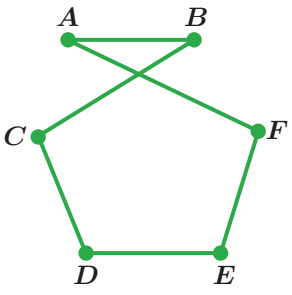


Figure D



Name: ..... Date: ..... Period: .....

Building Polygons

2. Write what you already know about each of the listed polygons.

Rhombus	Square

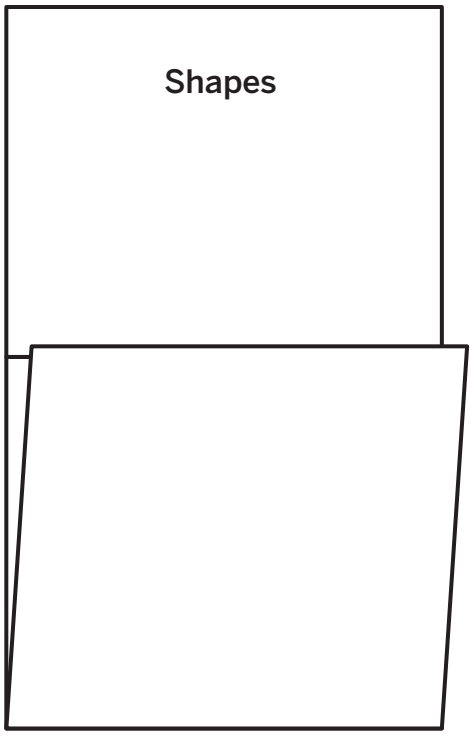
Rectangle	Parallelogram

Hexagon	Regular Hexagon

Name: ..... Date: ..... Period: .....

Building Polygons (continued)

3. You will use a set of cards and string for this activity.



a With your group, choose one card, looking only at the name of the shape.

Shape: .....

b Use the string to build that shape.

c **Discuss** How do you know the shape you built has the specific properties needed to define it?

Name: ..... Date: ..... Period: .....

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet Almost I got it!

I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem.

Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, making assumptions and revising my thinking as needed.

Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

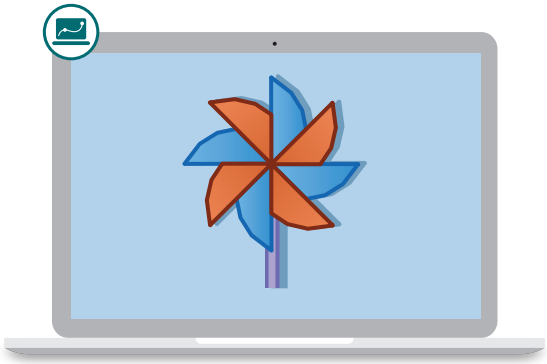
Not yet Almost I got it!

Name: ..... Date: ..... Period: .....

Angle Relationships | Shapes in the World | Building Toward 7.G.5, SMP.1, SMP.6, SMP.8

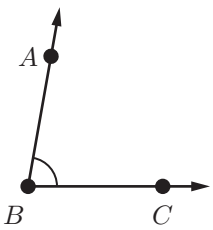
Pinwheels

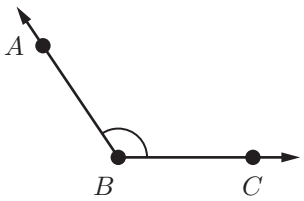
Let's explore angles and estimate angle measures.

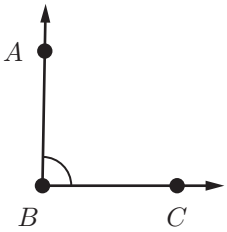


Warm-Up

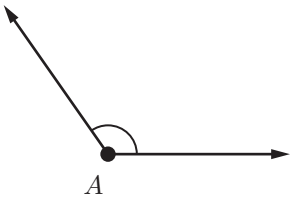
1 For each challenge, estimate the measure of $\angle ABC$.

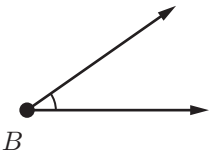


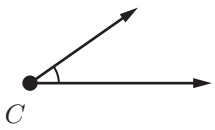




2 **a** Estimate the measure of each angle.





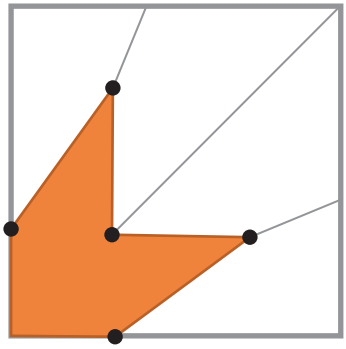


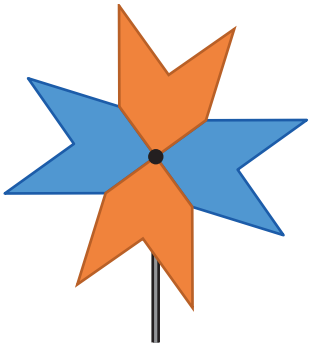
b Explain your strategy for estimating each measure.

Name: ..... Date: ..... Period: .....

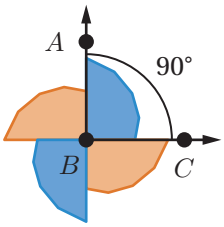
Pinwheels

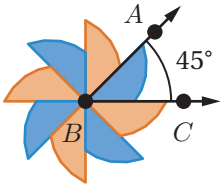
3 **Discuss:** How did this design become a pinwheel?

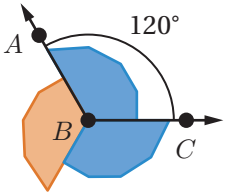


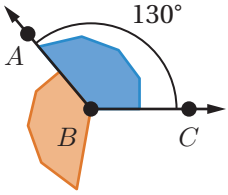


4 Here are four pinwheel designs, each with a different measure for $\angle ABC$.









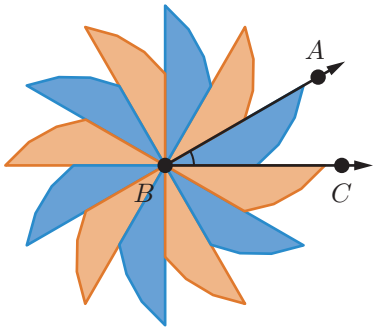
Discuss: What do you notice? What do you wonder?

Name: ..... Date: ..... Period: .....

Pinwheels (continued)

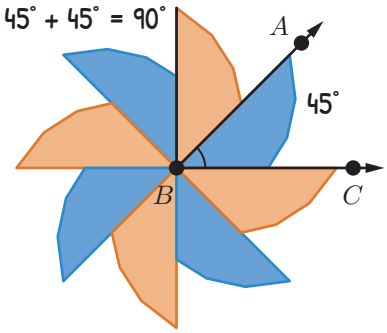
5 Here is one student's pinwheel design.
What is the measure of $\angle ABC$?

Show or explain your thinking.

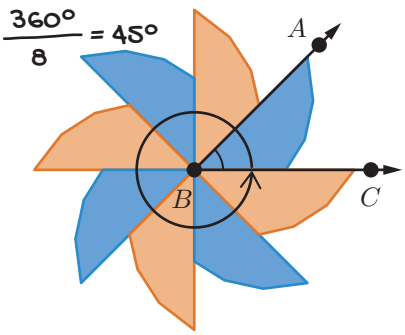


6 Callen and Duri both determined the measure of $\angle ABC$ in a new pinwheel.

Callen



Duri

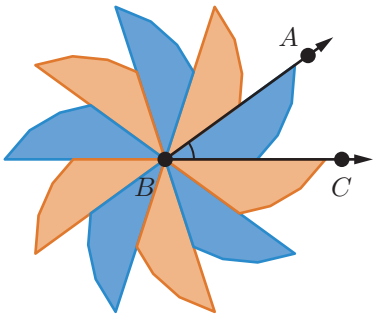


How are Callen's and Duri's strategies alike?

How are they different?

7 Here is another student's pinwheel design.

What angle did they use?

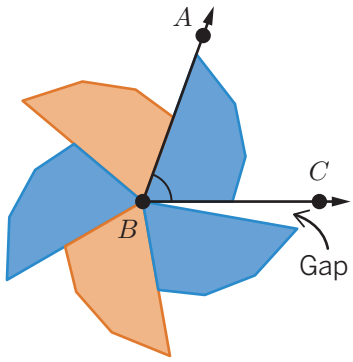


Activity 2

Name: ..... Date: ..... Period: .....

More Pinwheel Designs

8 Here is a new pinwheel with a gap.



Which of these could be the measure of $\angle ABC$?

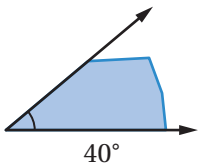
A. $18°$ **B.** $60°$

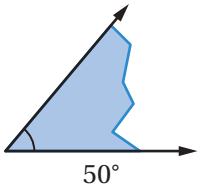
C. $70°$ **D.** $108°$

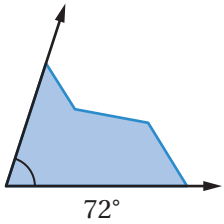
Explain your thinking.

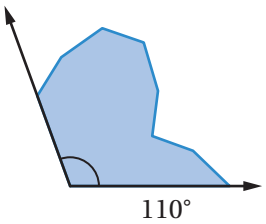
9 Here are the angle measures for some new pinwheel designs. Two pinwheels will have a gap and two will not.

Select one angle that will create a pinwheel with a gap. Explain your thinking.









You're invited to explore more.

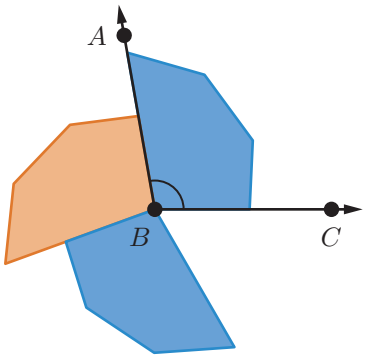
10 Experiment with different angles to create:

- **a** A pinwheel with a $10°$ gap.
- **b** A pinwheel with a $40°$ gap.
- **c** A pinwheel made up of four pieces, with the largest possible gap.

11 Synthesis

Describe how to estimate the measure of an angle in a pinwheel.

Use this example if it helps with your thinking.



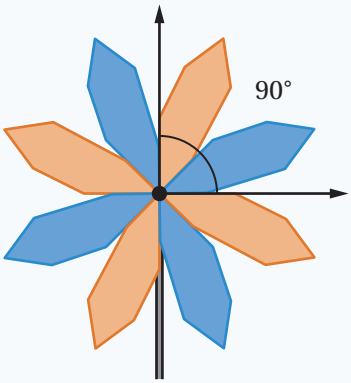
14 Summary 7.01

One strategy for determining the measures of unknown angles is to compare them to angle measures you know.

Here are three familiar angles and how they can be used to determine the measure of an angle in a pinwheel made of identical shapes.

Right Angle (90°)

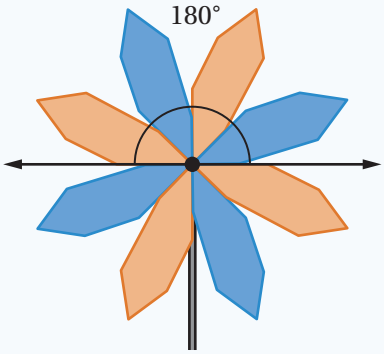
A *right angle* is half of a straight angle. It measures 90°.



$\frac{90^\circ}{2} = 45^\circ$

Straight Angle (180°)

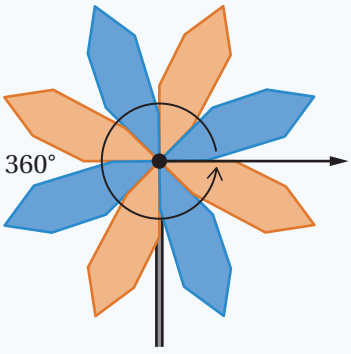
A *straight angle* forms a straight line. It measures 180°.



$\frac{180^\circ}{4} = 45^\circ$

Circle (360°)

The measure around a circle is 360°.



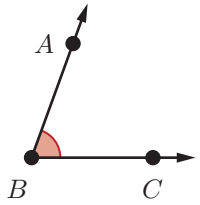
$\frac{360^\circ}{8} = 45^\circ$

Practice
7.01

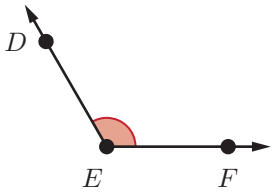
Name: ..... Date: ..... Period: .....

Problems 1–3: Estimate the measure of each angle (in degrees).

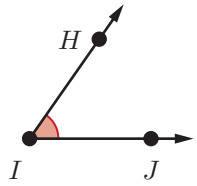
1. $\angle ABC$



2. $\angle DEF$



3. $\angle HIJ$



4. Sketch a right angle.

5. What is a right angle's measure in degrees?

6. Sketch a $180°$ angle.

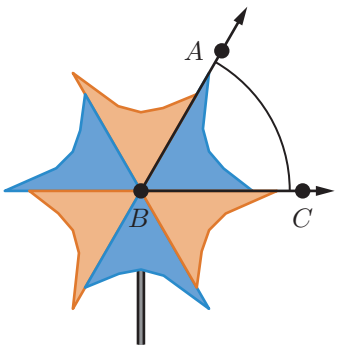
7. Explain why you think a $180°$ angle is called a straight angle.

Practice
7.01

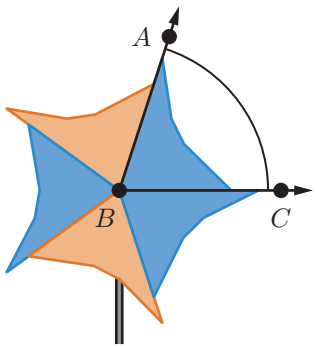
Name: ..... Date: ..... Period: .....

Problems 8–10: Determine the measure of $\angle ABC$ in each pinwheel. Explain your thinking.

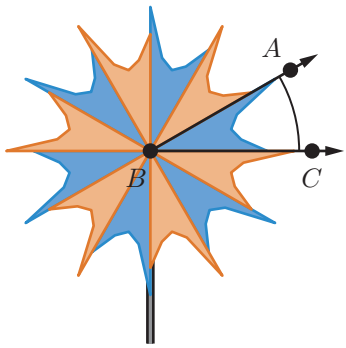
8.



9.



10.



11. A new pinwheel is made with 18 equal parts. What angle is used to make the pinwheel design?

Spiral Review

Problems 12–14: For each expression, write an equivalent expression in expanded form.

12. $-3(2x - 4)$

13. $0.1(-90 + 50a)$

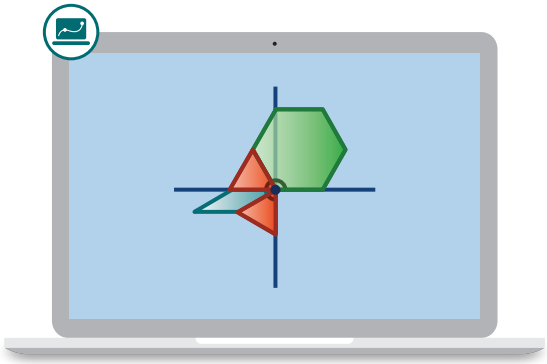
14. $-7(9 - x)$

15. Is the relationship between x and y proportional? If so, what is the constant of proportionality?

x	y
2	$\frac{5}{2}$
4	5
6	$\frac{15}{2}$
12	15

Name: ..... Date: ..... Period: .....

Angle Relationships 7.G.5, 7.EE.4, SMP.6, SMP.7

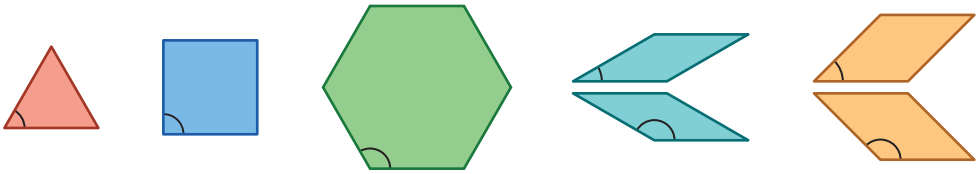


Friendly Angles

Let's explore complementary and supplementary angles.

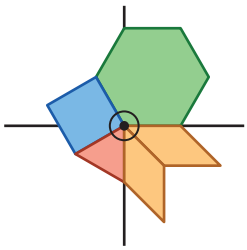
Warm-Up

1 These shapes were used to create four 360° designs.

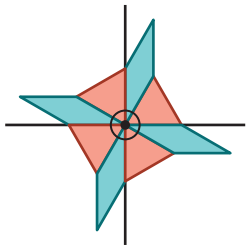


a. Pick a design that you like.

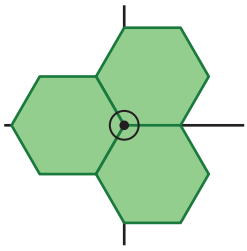
A.



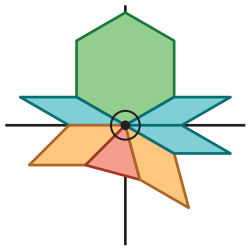
B.



C.



D.



b. **Discuss:** What do you like about the design you chose?

Activity 1

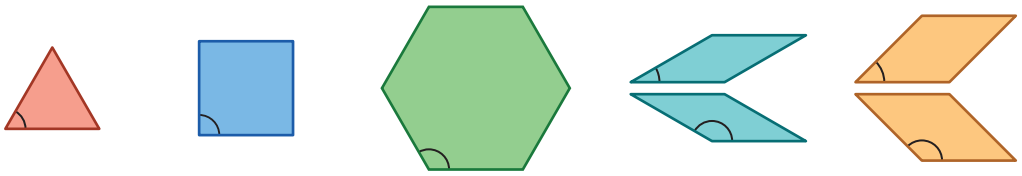
Name: ..... Date: ..... Period: .....

Mystery Measures

2 You and your partner will use a set of shapes or the digital screen for this activity.

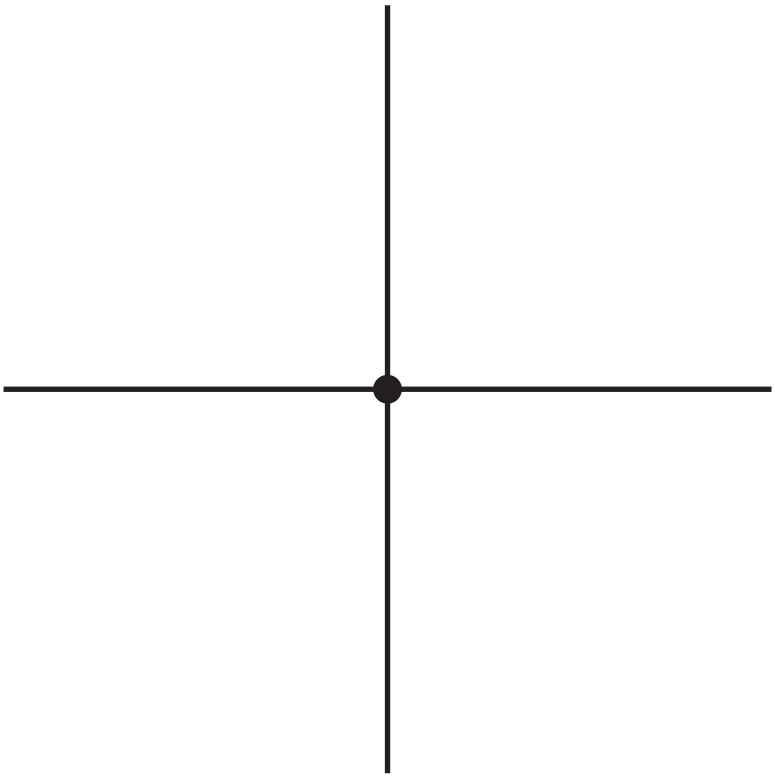
Each shape has at least one unknown angle measure.

a Determine as many angle measures as you can by creating designs with your shapes. Label each shape with its angle measure.



....

Workspace:

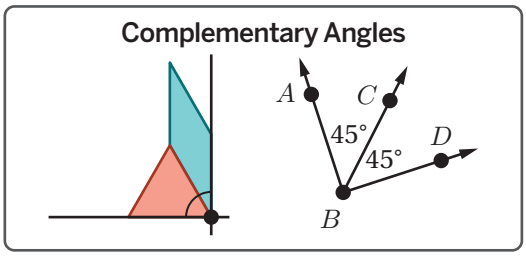


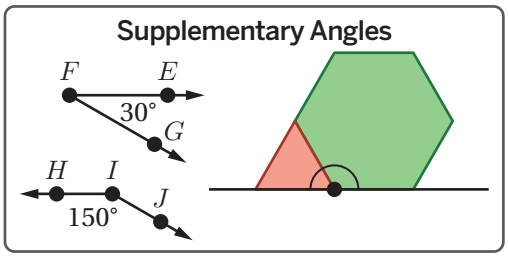
b Choose two angles. Explain how you determined their measures.

Name: ..... Date: ..... Period: .....

Relationships and Equations

3 The terms **complementary** and **supplementary** describe special pairs of angles.





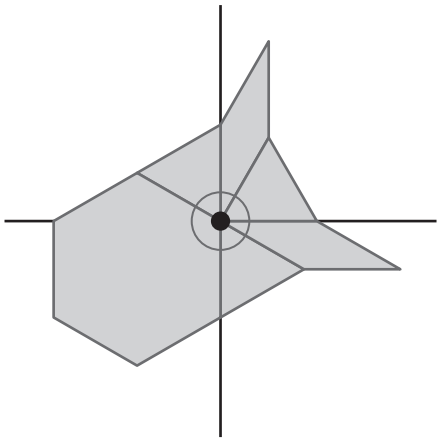
Describe what you think these terms mean.

Complementary angles . . .

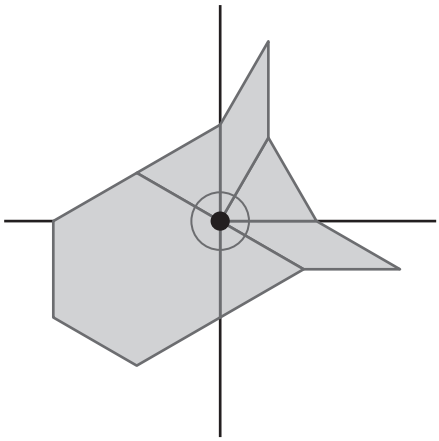
Supplementary angles . . .

4 Here is a new design.

a Shade in a pair of *complementary* angles.



b Shade in a pair of *supplementary* angles.

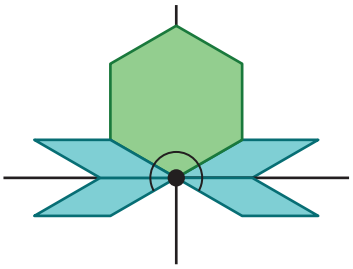


Name: ..... Date: ..... Period: .....

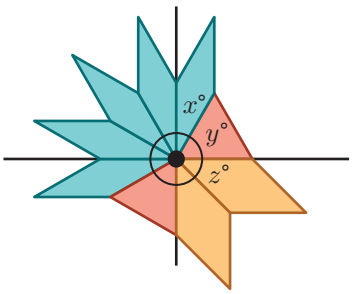
Relationships and Equations (continued)

5 Ivory used the equation $2x + 120 = 180$ to determine one angle measure in this diagram.

Explain or show what each part of Ivory's equation represents in the diagram.



6 Here is a new diagram.



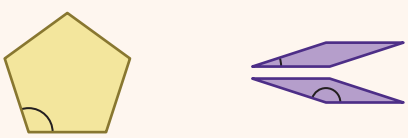
a Select *all* the true equations.

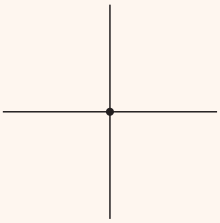
- ☐ **A.** $3x = 90$
- ☐ **B.** $x + y = 90$
- ☐ **C.** $5x = 180$
- ☐ **D.** $x + y + 2z = 180$
- ☐ **E.** $x + y + z = 360$

b **Discuss:** How did you decide which equations are true?

You're invited to explore more.

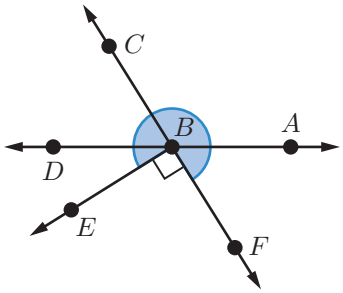
7 Use the You're Invited to Explore More Sheet to determine as many of these unknown angle measures as you can by creating designs in the workspace provided. Record each angle measure below its shape.





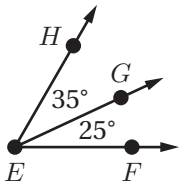
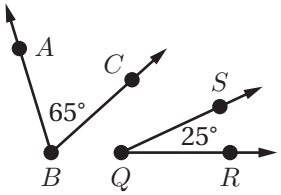
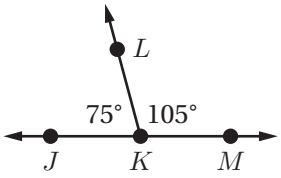
8 Synthesis

Discuss: Where do you see **complementary** and **supplementary** angles in the diagram?



11 Summary 7.02

Here are three angle relationships that can help you determine missing angle measures.

Adjacent Angles	Complementary Angles	Supplementary Angles
		
$\angle HEG$ and $\angle FEG$ are adjacent angles.	$\angle ABC$ and $\angle RQS$ are complementary angles.	$\angle JKL$ and $\angle MKL$ are supplementary angles.

adjacent angles Angles that share a side and a vertex.

complementary angles Two angles whose measures add up to 90°.

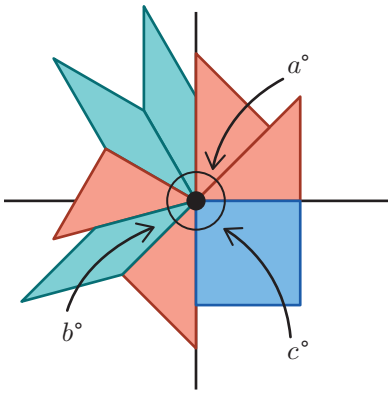
supplementary angles Two angles whose measures add up to 180°.

Practice
7.02

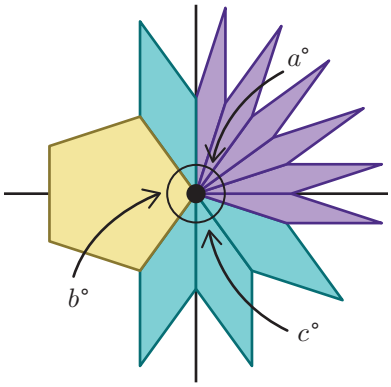
Name: ..... Date: ..... Period: .....

Problems 1–2: Determine the values of a, b, and c in each diagram.

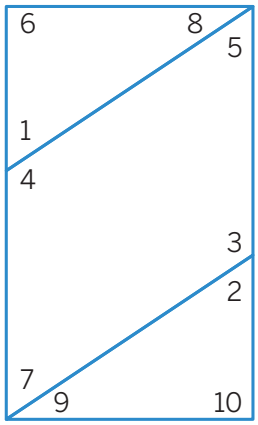
1.



2.



Problems 3–6: Here is a rectangle.



3. List a pair of angles that are complementary.

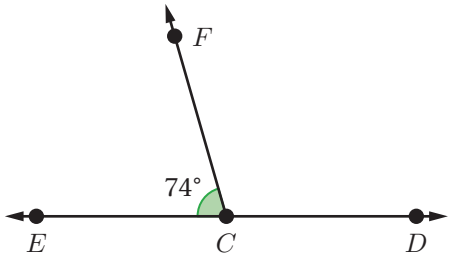
4. List a pair of angles that are supplementary.

5. If angle 7 measures 56°, determine the value of one other angle.

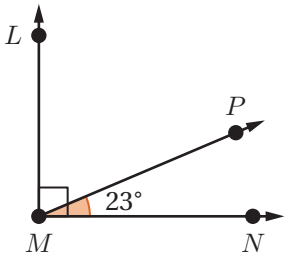
6. If angle 1 measures 56°, determine the value of one other angle.

Problems 7–8: Determine the missing angle measures.

7. $\angle FCD$



8. $\angle LMP$

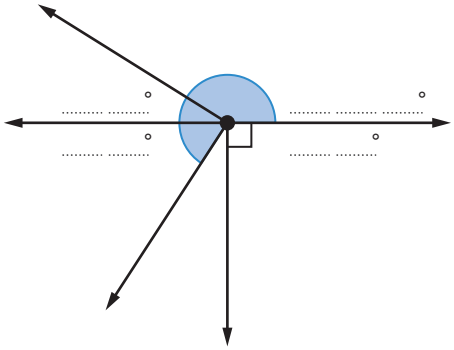


Name: ..... Date: ..... Period: .....

9. For each angle, determine the measure of its complementary angle *(or complement)* and its supplementary angle *(or supplement)*.

Angle (°)	Complement (°)	Supplement (°)
43		
78		
22		

10. Fill in each blank using the numbers 0 to 9 only once. One number will not be used.



Spiral Review

Problems 11–14: Solve each equation.

11. $x + 40 = 180$ **12.** $x + 40 = 90$ **13.** $2x + 40 = 180$ **14.** $2(x + 40) = 180$

Problems 15–17: A small dog gets fed $\frac{3}{4}$ of a cup of dog food twice a day.

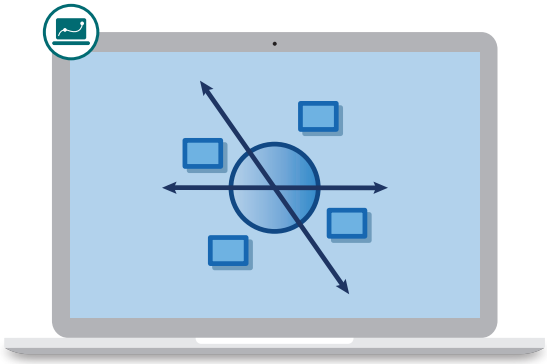
15. Write an equation representing the relationship between the number of days, d, and the number of cups of food, f.

16. How many days will a large bag of dog food last if a new bag contains 210 cups of food?

17. There are only 101 cups of food left in a bag of dog food. Write an inequality and interpret the solution to determine the maximum number of days the food will last.

Name: ..... Date: ..... Period: .....

Angle Relationships 7.G.5, 7.EE.4, SMP.2, SMP.3, SMP.7

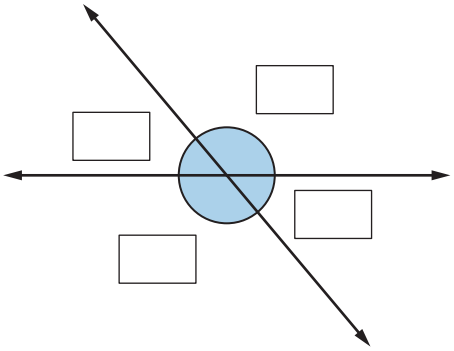


Angle Diagrams

Let's explore vertical angles.

Warm-Up

1 a Estimate each angle measure.

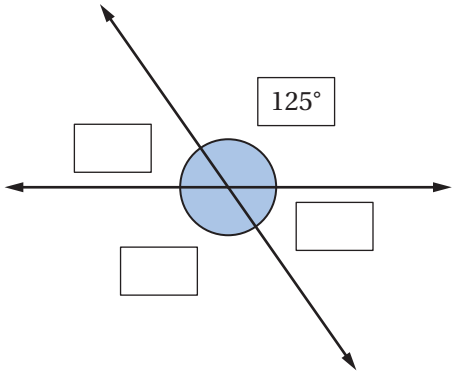


b **Discuss:** How did you choose your estimates?

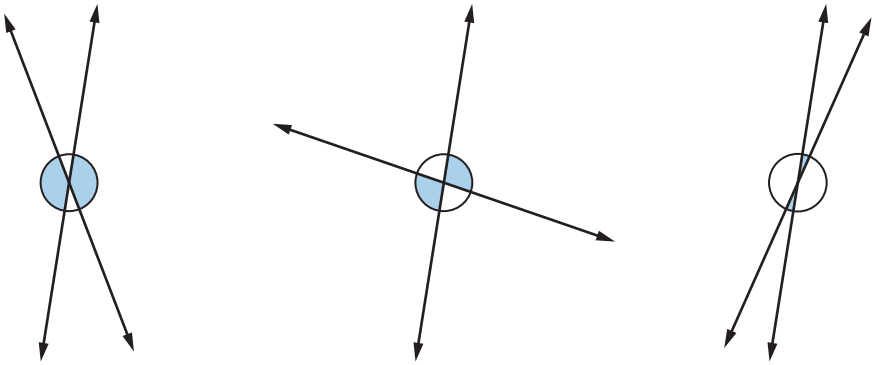
Name: ..... Date: ..... Period: .....

Vertical Angles

2 Here is an angle puzzle. Use the given angle measure to determine all the angle measures.



3 Lola noticed that when two lines cross, the angles that are opposite each other have the same measure. These angles are called **vertical angles**.



Are the measures of vertical angles always, sometimes, or never the same? Circle one.

Always Sometimes Never

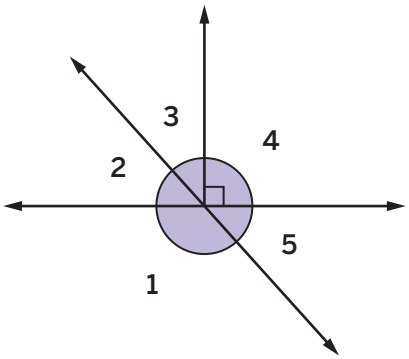
Explain your thinking.

Activity 2

Name: Date: Period:

Vertical Angle Puzzle

4 Here is a new angle puzzle. Which of these is a pair of vertical angles?



A. 1 and 4

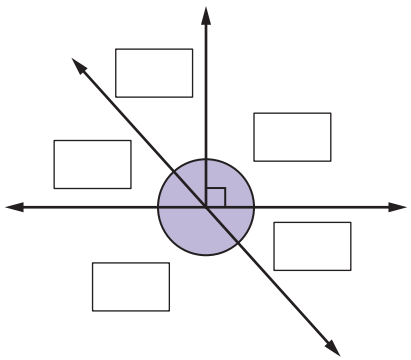
B. 2 and 3

C. 2 and 5

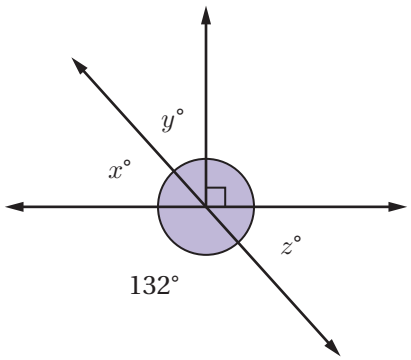
D. 3 and 5

Discuss: Choose one of the other pairs. How do you know they are *not* vertical angles?

5 Here is a new angle puzzle. You can ask for the measure of an angle. Determine all the angle measures using as few hints as you can.



6 Kwasi and Lola wrote equations to help them solve the previous angle puzzle.



Kwasi's equation: $x + 132 = 180$

Lola's equation: $132 + z = 180$

Whose equation is correct? Circle one.

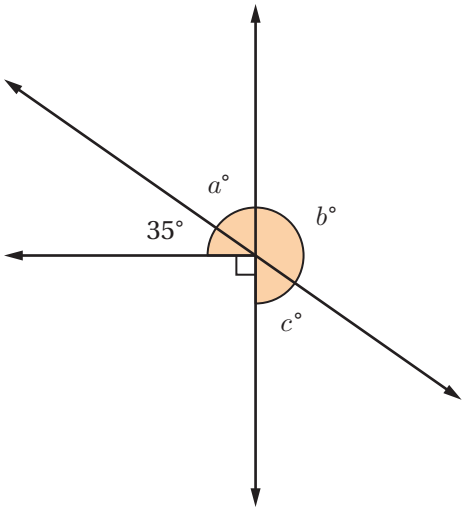
Kwasi Lola Both Neither

Explain your thinking.

Name: Date: Period:

Writing and Using Equations

Use the diagram for Screens 7 and 8.



7 Write a true equation based on this angle puzzle. Try to write an equation none of your classmates will.

8 Determine the values of a, b, and c.

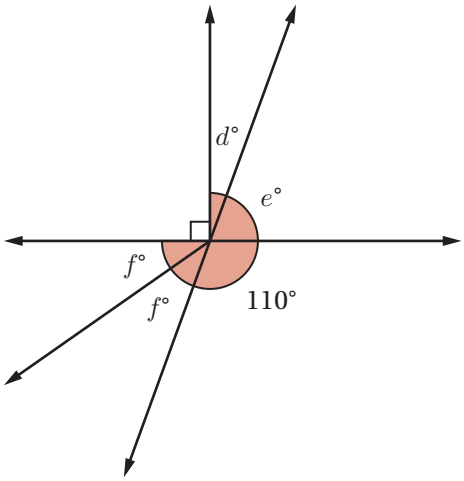
Discuss: How can equations help to determine unknown values?

Name: Date: Period:

Writing and Using Equations (continued)

9 Here is a new angle puzzle.

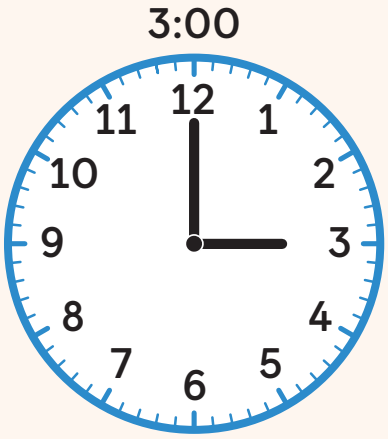
Determine the values of d, e, and f.



10 Kwasi wrote the equation $f + 110 = 180$ for the previous puzzle. Change Kwasi's equation to make it true.

You're invited to explore more.

11 Here is a clock.



a What is the angle between the clock's hour hand and minute hand at 3:00?

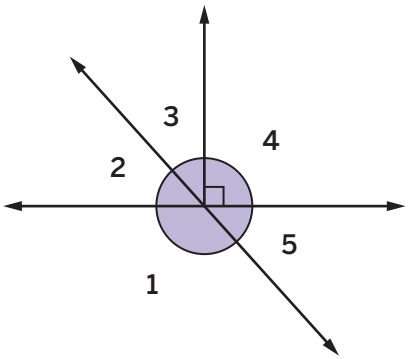
b What is the angle between the clock hands at 2:20? (Hint: It is not $60°$)

c What is one time when the angle between the clock hands is $40°$?

12 Synthesis

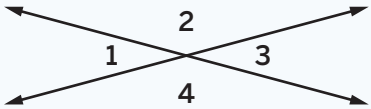
Discuss: What are **vertical angles**?

Use the example if it helps with your thinking.



15 Summary 7.03

When two lines cross, the angles that are opposite each other have the same measure. These angles are called **vertical angles**.



$\angle 1$ and $\angle 3$ are a pair of vertical angles. Another pair is $\angle 2$ and $\angle 4$.

Using vertical angles can help you determine unknown angle measures.

For example, if the measure of $\angle 1$ is $30°$, then:

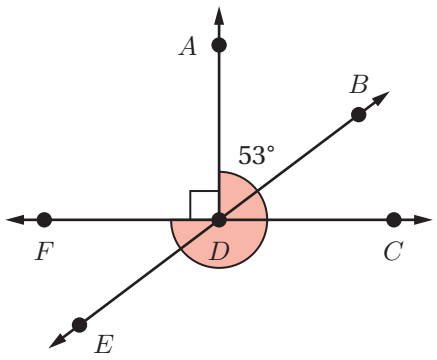
- The measure of $\angle 3$ is $30°$ because $\angle 1$ and $\angle 3$ are *vertical angles*.
- The measure of $\angle 2$ is $150°$ because $\angle 1$ and $\angle 2$ are *supplementary angles*.
- The measure of $\angle 4$ is $150°$ because $\angle 2$ and $\angle 4$ are *vertical angles*.

vertical angles Angles that are opposite each other when two lines cross. Vertical angles have the same measure.

Practice
7.03

Name: Date: Period:

Problems 1–3: Here is a diagram.



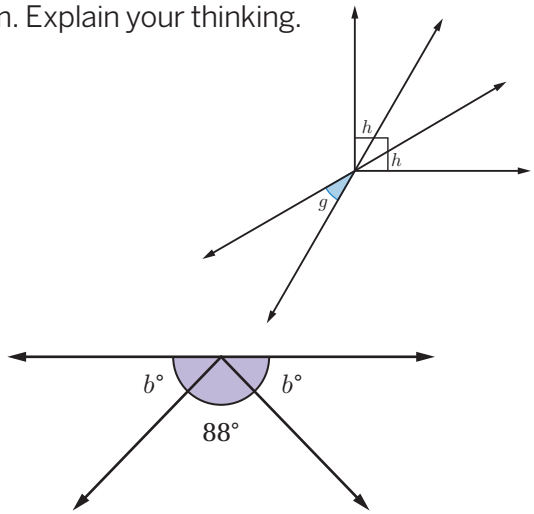
1. Determine the measure of each angle.

Angle	Measure (degrees)
ADB	53
BDC	
CDE	
FDE	
FDA	

2. Identify two angles that form a pair of vertical angles in the diagram.

3. Explain how you know they are vertical angles.

4. Write an equation that represents the angle diagram. Explain your thinking.



Problems 5–6: Here is a new diagram.

5. Which equation represents the relationship between the angles in the figure?

A. $88 + b = 90$

B. $88 + b = 180$

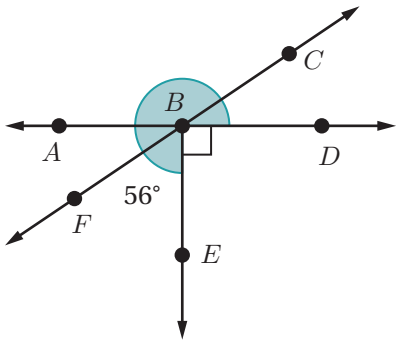
C. $2b + 88 = 90$

D. $2b + 88 = 180$

6. Dakota says that the angles marked b are vertical angles. Eva disagrees. Who is correct? Explain your thinking.

Name: ..... Date: ..... Period: .....

7. This diagram includes supplementary, complementary, and vertical angles. Determine each of the missing angle measures.



Angle	Measure (degrees)
ABC	
CBD	
DBE	
FBA	

Spiral Review

8. Select *all* of the equations that are equivalent to $3x + 45 = 180$.

- ☐ **A.** $3(x + 45) = 180$
- ☐ **B.** $3(x + 15) = 180$
- ☐ **C.** $3(x + 15) = 60$
- ☐ **D.** $x + 15 = 60$
- ☐ **E.** $3x = 135$

Problems 9–11: Eva is solving the equation $4\left(x + \frac{3}{2}\right) = 8$.

She says, "I can subtract $\frac{3}{2}$ from each side to get $4x = \frac{13}{2}$ and then divide by 4 to get $x = \frac{13}{8}$." Dakota says, "I think you made a mistake."

9. How can Dakota determine that Eva's solution is incorrect?

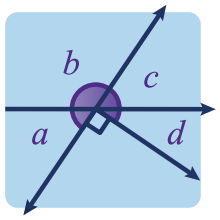
10. Describe the error that Eva might have made.

11. Determine the correct solution for x.

Name: ..... Date: ..... Period: .....

Angle Relationships 7.G.5, 7.EE.4, SMP.1, SMP.6

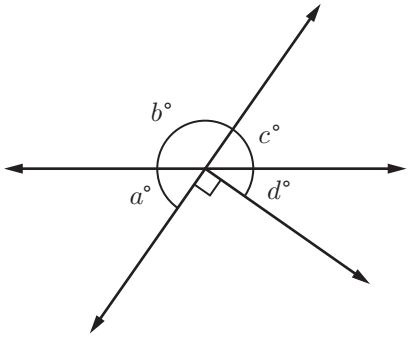
Missing Measures



Let's determine missing angle measures.

Warm-Up

1. Write as many true equations based on the diagram as you can.



Activity 1

Name: ..... Date: ..... Period: .....

Solving Challenges

2. You will use six sets of cards for this activity. Each set has a challenge card and a corresponding support card.

- Decide who will be Partner A and who will be Partner B.
- For Challenge A1, Partner A will have the challenge card and Partner B will have the support card.
- For Challenge B1, Partner B will have the challenge card and Partner A will have the support card.
- Complete as many challenges as you have time for.

Challenge Card Instructions

- Use the diagram to estimate each measure.
- Write at least one equation based on the angle relationships.
- Ask your partner for the missing measure. Then determine every other measure.

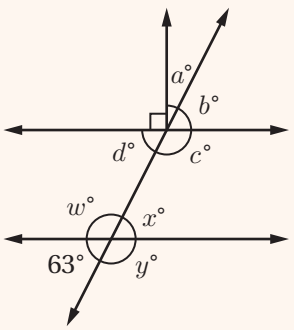
Support Card Instructions

- Help your partner by asking the questions on the card or other questions you think will support them.
- Provide the missing measure for your partner.

Challenge	Measure Estimates	Equation(s)	Missing Measures (degrees)
1			
2			
3			

You're invited to explore more.

3. The angles marked d and w are supplementary angles. Determine the unknown angle measures.



Name: ..... Date: ..... Period: .....

Trading Challenges

4. You will now create and trade challenges.

a Create your own challenge.

- On a separate sheet of paper, use a ruler or straightedge to draw a diagram with supplementary, complementary, and/or vertical angles.
- Use a protractor to measure and label one angle.
- Label the remaining angles with variables.

b Solve your challenge.

- Without using a protractor, determine as many missing angle measures as you can in your own diagram.
- Show all of your work in the table. (Do not write directly on your diagram!)

c Trade and solve.

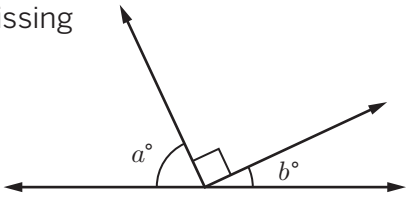
- Trade challenges with a partner.
- Without using a protractor, determine as many missing angle measures as you can in your partner's diagram.
- Show all of your work in the table.
- Repeat up to three more times.

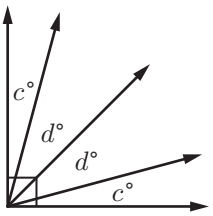
My Challenge	Challenge 1
Given angle measure: ..... Missing angle measures:	Partner 2's name: ..... Given angle measure: ..... Missing angle measures:
Challenge 2	**Challenge 3**
Partner 3's name: ..... Given angle measure: ..... Missing angle measures:	Partner 4's name: ..... Given angle measure: ..... Missing angle measures:

Synthesis

5. What is important to remember when determining missing angle measures in a diagram?

Use the examples if they help with your thinking.



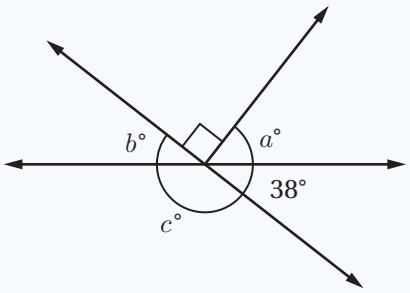


Summary 7.04

Angle relationships can help you write equations to determine unknown angle measures.

Here are several equations that represent the angle relationships in this diagram.

Equation	Relationship
$a + 38 = 90$	complementary angles
$c + 38 = 180$	supplementary angles
$b + \mathrm{c} = 180$	supplementary angles
$b = 38$	vertical angles
$a + b + c + 90 + 38 = 360$	circle



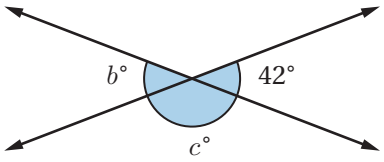
You can solve these equations to determine the values of a, b, and c.

In this diagram, $a = 52°$, $b = 38°$, and $c = 142°$.

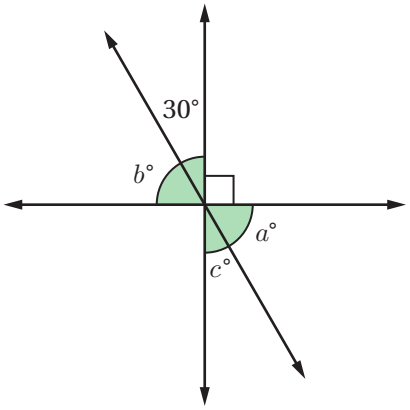
Practice
7.04

Name: ..... Date: ..... Period: .....

1. Determine the values of b and c.



Problems 2–3: Here is a diagram.

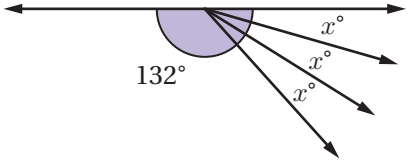


2. Select *all* of the true statements.

- ☐ **A.** $b = c$
- ☐ **B.** $30 + b = 180$
- ☐ **C.** $a + b + c + 30 = 180$
- ☐ **D.** $a = b$
- ☐ **E.** $a + c = 90$

3. Determine the values of a, b, and c.

Problems 4–7: Here is a new diagram.

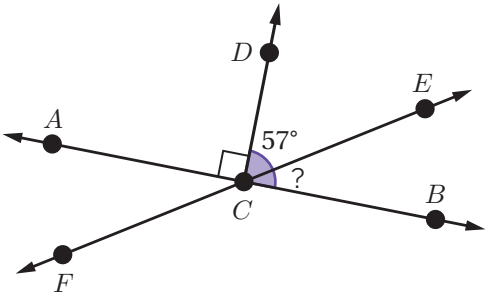


4. Write an equation that represents the angle relationships in this diagram.

5. Explain what each part of your equation means in the diagram.

6. Solve your equation.

7. Here is a diagram. Determine the measure of $\angle BCE$.



Name: ..... Date: ..... Period: .....

8. Draw a diagram that includes two angles that are both vertical and complementary.

9. Draw a diagram that includes two angles that are both vertical and supplementary.

Spiral Review

10. Select *all* of the equations that are equivalent to $4x + 20 = 90$.

- ☐ **A.** $4(x + 5) = 90$
- ☐ **B.** $x + 5 = 22.5$
- ☐ **C.** $4(x + 20) = 90$
- ☐ **D.** $x + 20 = 22.5$
- ☐ **E.** $4x = 70$

Problems 11–12: Solve each inequality.

11. $-24 > -6(x - 0.5)$

12. $8x + 6 > -30$

Problems 13–15: A runner ran $\frac{2}{3}$ of a race in 21 minutes. He ran the entire race at a constant speed.

13. Is the relationship between time and distance proportional? Explain your thinking.

14. How long did the runner take to run the entire race?

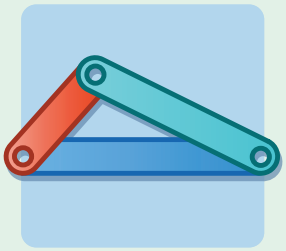
15. If the race was 4 miles, what was the runner's pace (minutes per mile)?

Notes:

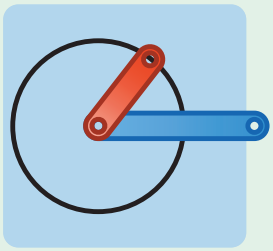
Sub-Unit

2

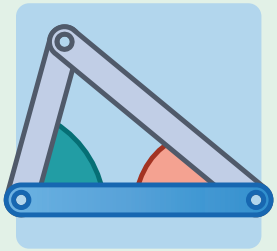
Drawing Triangles



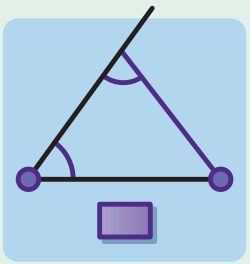
Lesson 5
Can You Build It?



Lesson 6
Is It Enough?



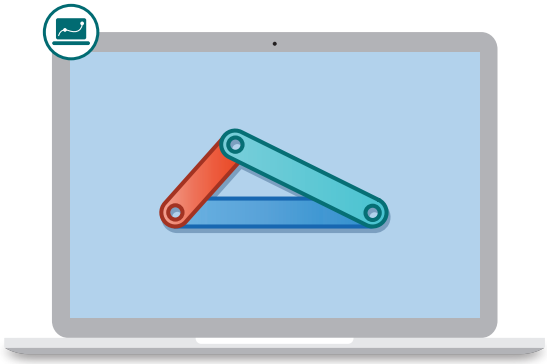
Lesson 7
More Than One?



Lesson 8
Can You Draw It?

Name: ..... Date: ..... Period: .....

2-D and 3-D Connections 7.G.2, SMP.1, SMP.3, SMP.8



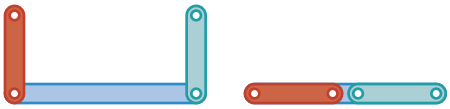
Can You Build It?

Let's explore what combinations of three line segments form a triangle.

Warm-Up

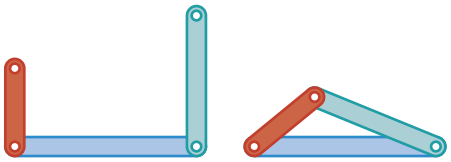
1 Here are two sets of line segments that were used to try to form a triangle.

Set 1



Before After

Set 2



Before After

What do you notice? What do you wonder?

I notice:

I wonder:

Activity 1

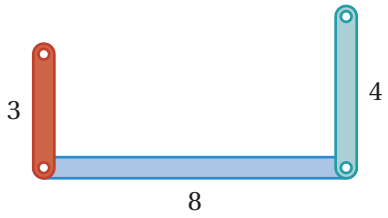
Name: Date: Period:

Length of the Third Side

You will use a set of line segments to complete this activity.

2 Will these three line segments form a triangle? Circle one.

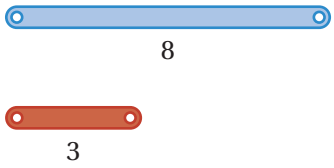
Yes No I'm not sure



Explain your thinking.

3 Set 1 and Set 2 from the Warm-Up each have one line segment that is 8 units long and one that is 3 units long. Set 1 does not form a triangle. Set 2 does.

Try other lengths for the third segment. Try to find several that do and do not form a triangle.



Forms a Triangle	Does Not Form a Triangle
10 units	2 units

4 Here is a graph of other lengths that students tried. The lengths that form a triangle are represented by green dots.

Describe what you notice about those lengths.

Activity 2

Name: ..... Date: ..... Period: .....

Not Too Long, Not Too Short

You will use a set of line segments to complete this activity.

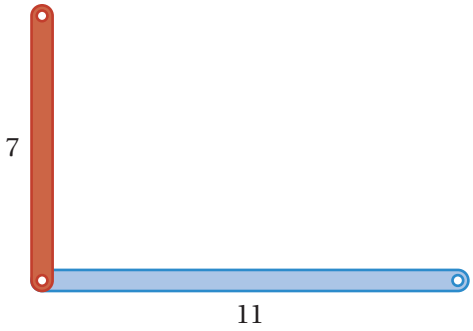
5 Diamond is convinced that a third segment that is 19 units long will form a triangle.

Mohamed thinks that 19 units is too long.

Whose claim is correct? Circle one.

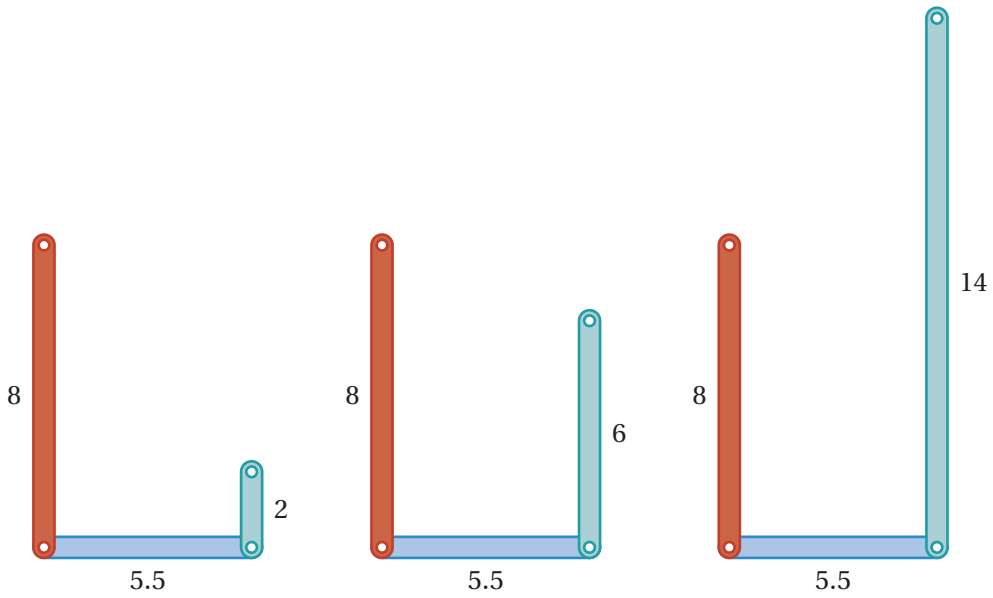
Diamond Mohamed

Explain your thinking.



6 Which of these lengths will form a triangle with side lengths 5.5 and 8 units? Circle one.

2 units 6 units 14 units



Name: ..... Date: ..... Period: .....

Not Too Long, Not Too Short (continued)

7 You will use a challenge card to create your own triangle challenge.

- Choose two line segments of any length.
- Determine three multiple choice options. One option should form a triangle with your other line segments, and two options should not.
- Swap your triangle challenge with one or more partners.
- Determine the lengths that will form a triangle for each of your partners' challenges.
- Complete as many challenges as you have time for.

	Given Length	Given Length	Correct Third Length
..... 's Challenge			
..... 's Challenge			
..... 's Challenge			
..... 's Challenge			
..... 's Challenge			

Activity 3

Name: Date: Period:

Does It Add Up

You will use a set of line segments to complete this activity.

8 A triangle has a perimeter of 24 units.

Perimeter = 24 units

What are three possible lengths for the sides of this triangle?

Length 1	Length 2	Length 3

9 Abena made her first side 12 units long.

12

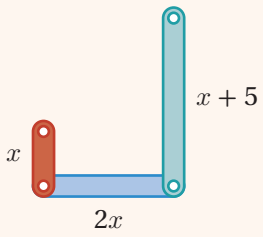
Perimeter = 24 units

Will she be able to form a triangle with a perimeter of 24 units? Explain your thinking.

You're invited to explore more.

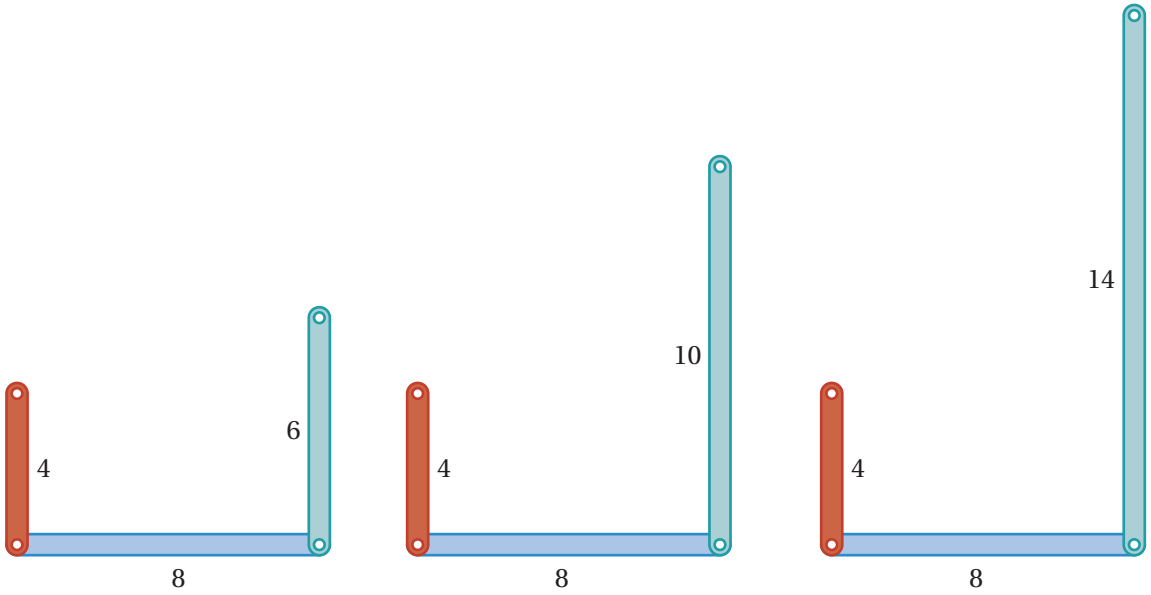
10 A triangle has sides that are x, $2x$, and $x + 5$ units.

What are some values of x that would form a triangle?



11 Synthesis

Discuss: How can you determine whether three line segments will form a triangle?

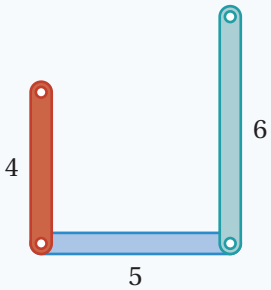


14 Summary 7.05

Three line segments do not always make a triangle.

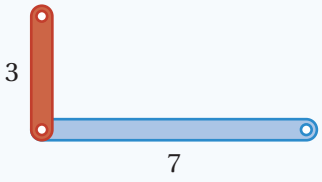
In order for three line segments to form a triangle, the sum of the two shorter segments' lengths must be greater than the third segment's length.

For example, $4 + 5 > 6$, so these three line segments would make a triangle.



You can use this relationship to determine what possible lengths would create a triangle.

For example, if two side lengths of a triangle are 3 units and 7 units, then the third side must be greater than 4 and less than 10 units.



Practice
7.05

Name: _____ Date: _____ Period: _____

1. Select *all* the sets of line segments that will form a triangle.

- ☐ **A.** 3, 4, and 8 units
- ☐ **B.** 7, 6, and 12 units
- ☐ **C.** 5, 13, and 11 units
- ☐ **D.** 12, 6, and 4 units
- ☐ **E.** 4, 6, and 10 units

2. Choose one of the other sets of segments from Problem 1 and explain why it will *not* form a triangle.

3. One side of a triangle is 5.5 inches long and another side is 10.5 inches long. Select *all* possible side lengths for the third side.

- ☐ **A.** 3 inches
- ☐ **B.** 5 inches
- ☐ **C.** 7 inches
- ☐ **D.** 10 inches
- ☐ **E.** 12 inches
- ☐ **F.** 20 inches

Problems 4–5: A triangle has one side that is 4 centimeters long and another side that is 9 centimeters long. The third side is a whole number of centimeters.

4. What is the shortest possible length of the third side? _____ centimeters

5. What is the longest possible length of the third side? _____ centimeters

6. The direct distance between Airport A and Airport B is 120 miles. The direct distance between Airport B and Airport C is 240 miles. Which of these could be the direct distance between Airport A and Airport C?

A. 370 miles **B.** 300 miles **C.** 120 miles **D.** 110 miles

7. Create a set of four segment lengths so that:

- Each length is different.
- Each length is a whole number (in centimeters).
- No matter which of the three lengths you choose, you will *always* be able to form a triangle.

Explain your thinking.

Segment	Length (cm)
A	
B	
C	
D	

Practice
7.05

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 8–13: Evaluate each expression.

8. $5 - 8$

9. $5 + (-8)$

10. $-5 + 8$

11. $-5 + (-8)$

12. $-5 - (-8)$

13. $-5 - 8$

Problems 14–15: Solve each equation.

14. $1.5 = 0.6(w + 0.3)$

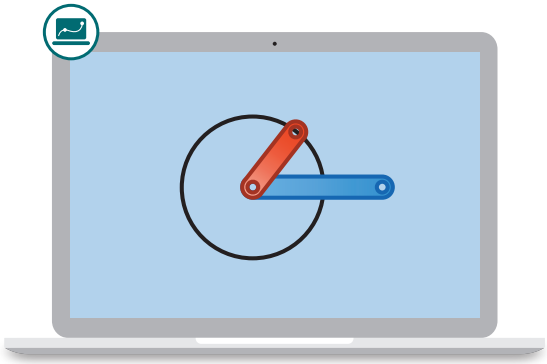
15. $1.5 = 0.6w + 0.3$

16. Arjun says that because $\frac{1}{12}$ and $\frac{2}{12}$ produce repeating decimal values, any fraction with a denominator of 12 will also produce a repeating decimal.

Is Arjun correct? Explain your thinking.

Name: ..... Date: ..... Period: .....

2-D and 3-D Connections 7.G.2, SMP.3, SMP.5



Is It Enough?

Let's explore connections between line segments and circles.

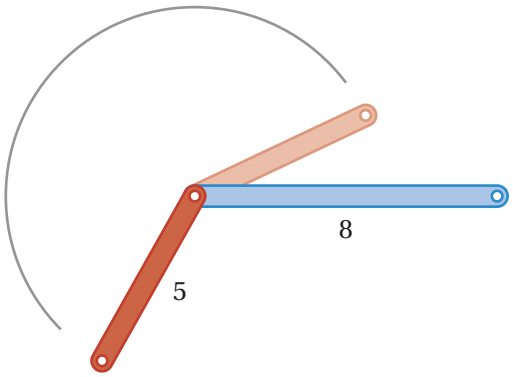
Warm-Up

1 Let's watch a line segment rotate around.

What shape does its path create? Explain why this makes sense.

The shape the path makes is . . .

This makes sense because . . .



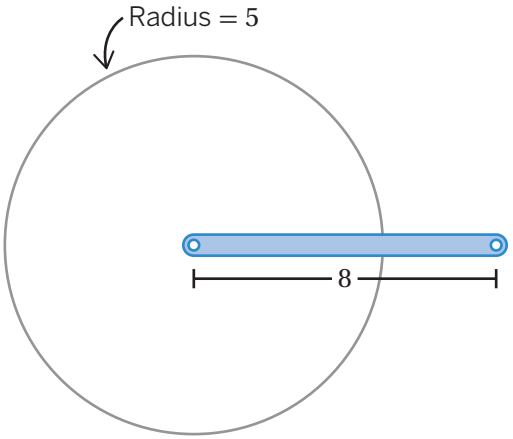
Activity 1

Name: ..... Date: ..... Period: .....

Building Triangles

You will use a set of line segments to help with your thinking.

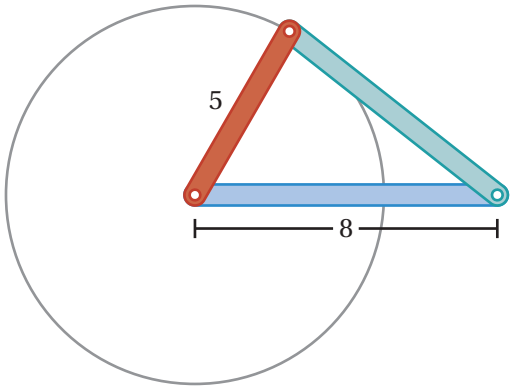
2 Use the circle and the segments to create different triangles.

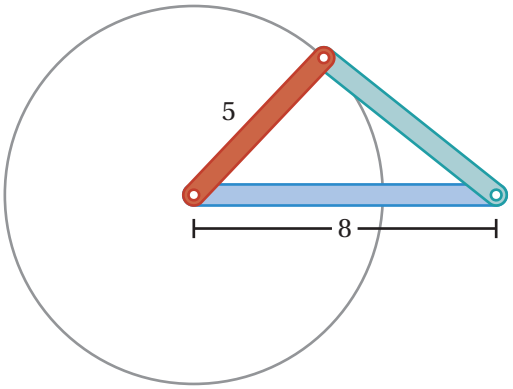


Select each triangle you're able to make.

- ☐ A triangle with a very long third side
- ☐ A triangle with a short third side
- ☐ A triangle with a 90° angle
- ☐ A triangle with all acute angles
- ☐ An isosceles triangle

3 A student made these two triangles.

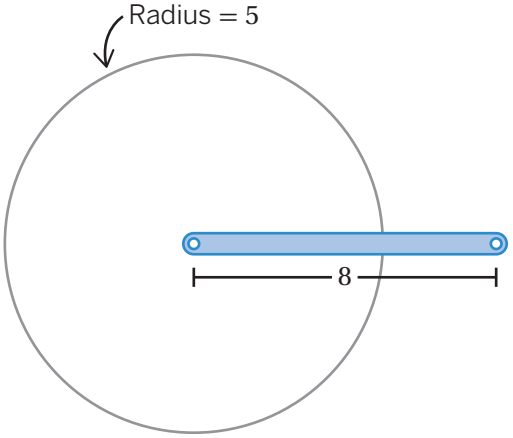




Discuss: How can you tell these triangles are not the same?

4 Angel wants to draw a triangle with sides that are 5, 8, and 9 units long.

Describe or show how Angel might draw a third side that is 9 units long.

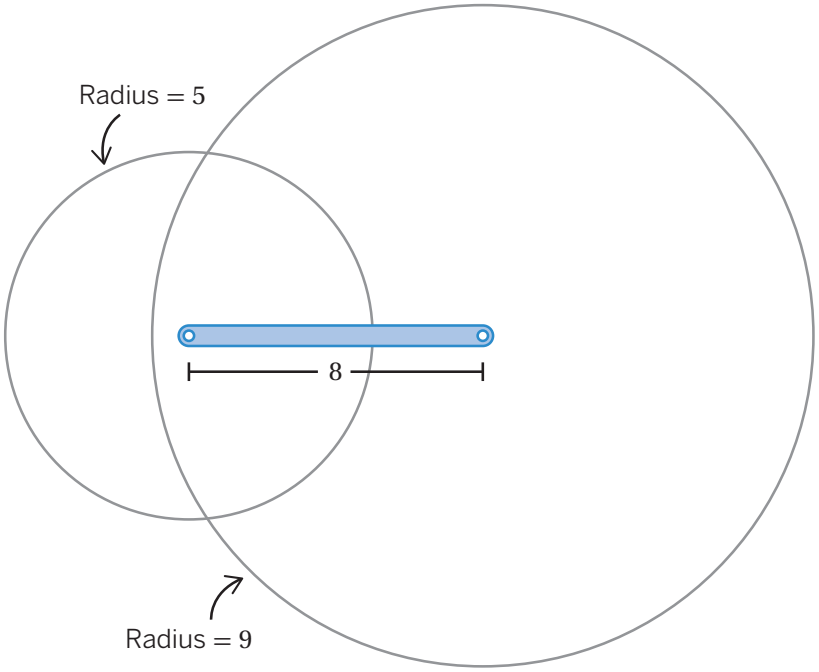


Name: ..... Date: ..... Period: .....

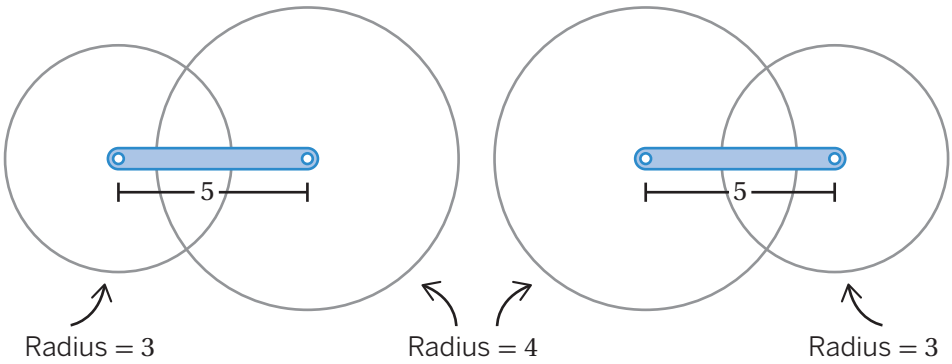
Building Triangles (continued)

5 Here is Angel's strategy.

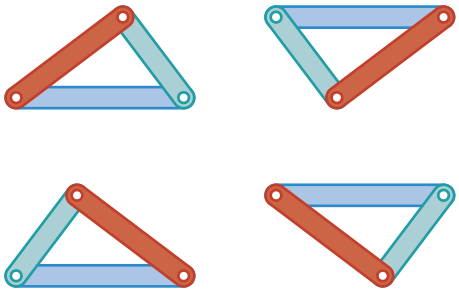
Draw a triangle that Angel could have drawn with side lengths that are 5, 8, and 9 units.



6 Use Angel's strategy to create two triangles with sides that are 5, 3, and 4 units long.



7 Here are four possible triangles with side lengths of 5, 3, and 4 units. Angel thinks these triangles are **identical copies**.



Do you agree? Circle one.

Yes No I'm not sure

Explain your thinking.

Activity 2

Name: ..... Date: ..... Period: .....

Uniqueness

You will use a set of line segments to help with your thinking.

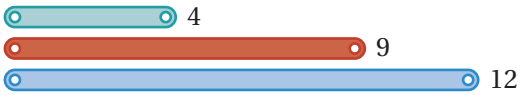
8 **a** Create two different triangles with the same side lengths. Then draw your triangles.

Triangle A **Triangle B**

b **Discuss:** Will these three side lengths always create *identical copies*?

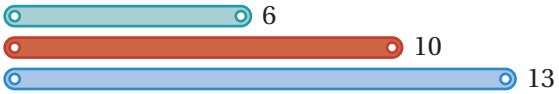
9 Circle the number of non-identical polygons that can be made using each set of line segments.

a



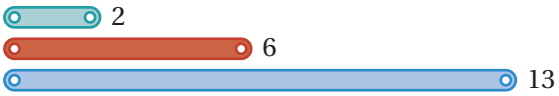
Zero One More than one

b



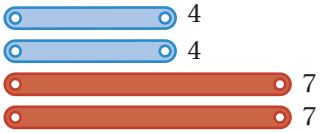
Zero One More than one

c



Zero One More than one

d



Zero One More than one

Activity 3

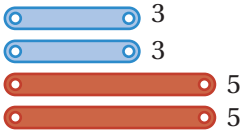
Name: ..... Date: ..... Period: .....

Quadrilaterals

You will use a set of line segments to help with your thinking.

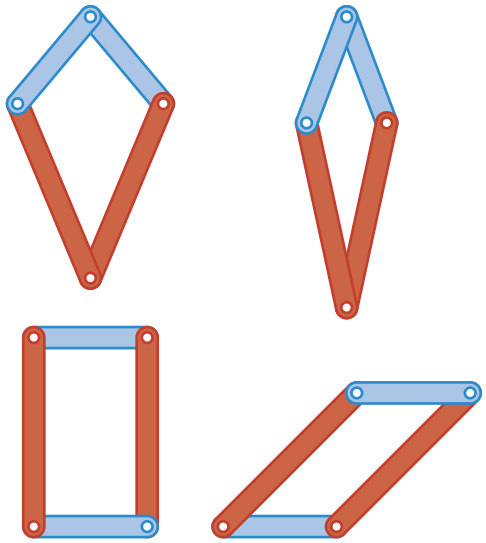
10 Lukas made a quadrilateral with sides that are 3, 3, 5, and 5 units long.

Draw the shape he might have made.



11 Here are four quadrilaterals that Lukas made with the sides that are 3, 3, 5, and 5 units long.

Describe why it's possible for Lukas to create quadrilaterals that are not *identical copies*.



You're invited to explore more.

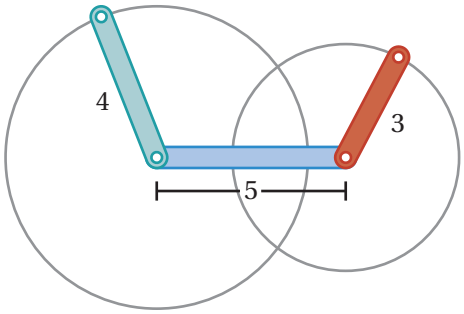
12 In the diagram, 9 toothpicks are used to make three equilateral triangles.

Draw or describe how you can move only 3 of the toothpicks to make a diagram that has exactly 5 equilateral triangles.

13 Synthesis

a. Describe how to create a triangle given three side lengths.

b. Explain why there will be only one possible triangle.



16 Summary 7.06

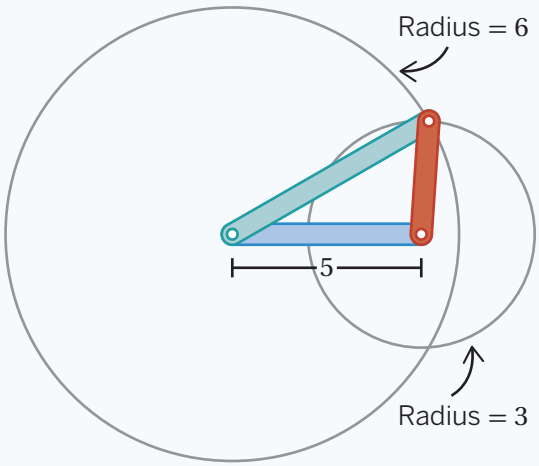
You can use circles to draw triangles.

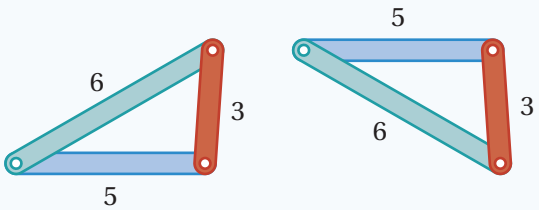
Here are the steps to draw a triangle with sides that are 5, 3, and 6 units long:

- Draw a line segment that is 5 units long.
- Draw a circle of *radius* 3 units centered at one end point.
- Draw a circle of radius 6 units centered at the other end point.
- Draw line segments connecting the end points of the 5 unit segment to a point where the two circles intersect

All the triangles whose sides are 5, 3, and 6 units long will be **identical copies** because they have the same shape and size.

In fact, it is only possible to create one unique triangle if you know its three side lengths (unless you can't make a triangle at all).





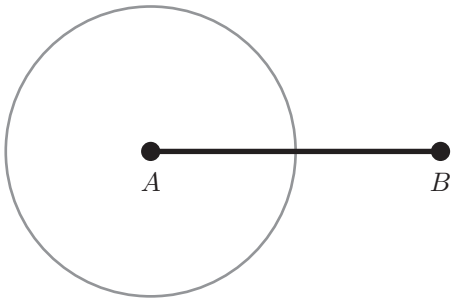
identical copy A copy of a figure that has the same shape and size as the original.

Practice
7.06

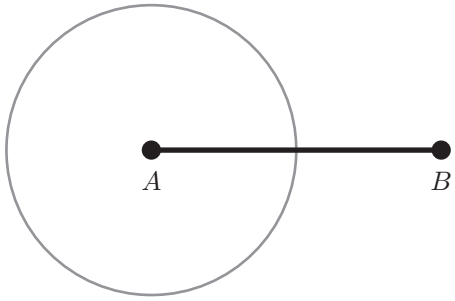
Name: ..... Date: ..... Period: .....

1. Segment AB is 6 units long and the radius of circle A is 3 units. Draw two different triangles where one side is 6 units long and another side is 3 units long.

Triangle 1



Triangle 2

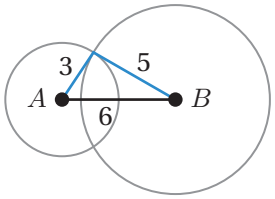


2. Faith drew two triangles with side lengths of 3, 5, and 6 units.

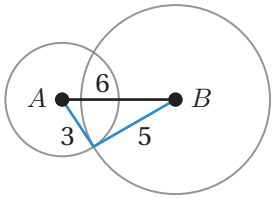
Are the triangles identical?

Explain your thinking.

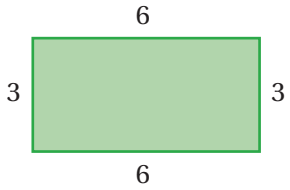
Triangle 1



Triangle 2



3. Antwon drew this rectangle. Write an argument to convince him that this is not the only quadrilateral he can draw with these side lengths.



4. A box contains wood planks of several different lengths. There is one 3-foot plank, one 4-foot plank, one 5-foot plank, and one 7-foot plank. What is the maximum number of different triangles that can be made using these planks as sides?

A. 1

B. 2

C. 3

D. 4

Name: ..... Date: ..... Period: .....

5. Create a set of five side lengths so that:

- Each length is different.
- Each length is a whole number (in inches).
- No matter which three you choose, you will never be able to make a triangle.

Explain how you know that your set of lengths meet all the requirements.

Side	Length (in.)
A	
B	
C	
D	
E	

Spiral Review

6. Anya states that a triangle with segment lengths 7 units, 10 units, and 17 units is possible. Is she correct? Explain your thinking.

7. A triangle has one side that is 6 units long and another side that is 3 units long. Select *all* the possible lengths for the third side.

- ☐ **A.** 2 units
- ☐ **B.** 3 units
- ☐ **C.** 4 units
- ☐ **D.** 6 units
- ☐ **E.** 8 units
- ☐ **F.** 10 units

Problems 8–11: Evaluate each expression.

8. $4 - 9$

9. $-4 - 9$

10. $4 - (-9)$

11. $-4 - (-9)$

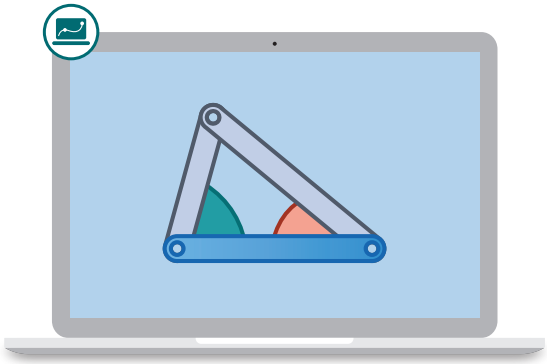
Problems 12–13: If you deposit \$300 in an account and it grows by 6% each year, how much money will be in your account after . . .

12. 1 year?

13. 2 years?

Name: ..... Date: ..... Period: .....

2-D and 3-D Connections | Angle Relationships | 7.G.2, 7.NS.3, SMP.3, SMP.5



More Than One?

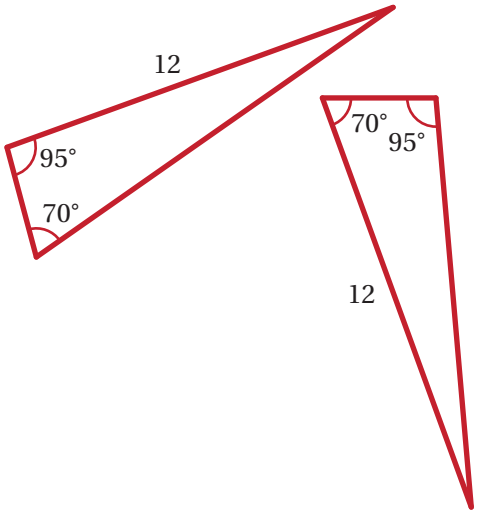
Let's build and compare triangles.

Warm-Up

1 Are these triangles identical? Circle one.

Yes No I'm not sure

Explain your thinking.



Activity 1

Name: ..... Date: ..... Period: .....

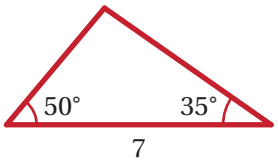
Two Angles, One Side

You will use a set of line segments and angles to help you with your thinking.

2 Luca made a triangle with these measurements:

- A side length of 7 units
- A 35° angle
- A 50° angle

Luca's Triangle



Roberto wants to make a triangle with the same measurements. Will Roberto's triangle be identical to Luca's? Circle one.

Definitely Maybe No way

Explain your thinking.

3 Create triangles with the same measurements as Luca's triangle.

Try to create more than one non-identical triangle.

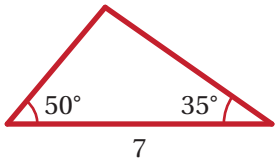
Name: ..... Date: ..... Period: .....

Two Angles, One Side (continued)

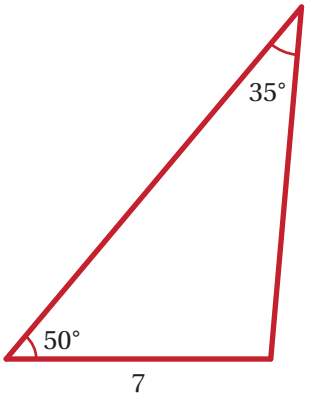
4 Here are the triangles that Luca and Roberto created.

What information would you give Roberto so that he can create a triangle identical to Luca's?

Luca



Roberto



Activity 2

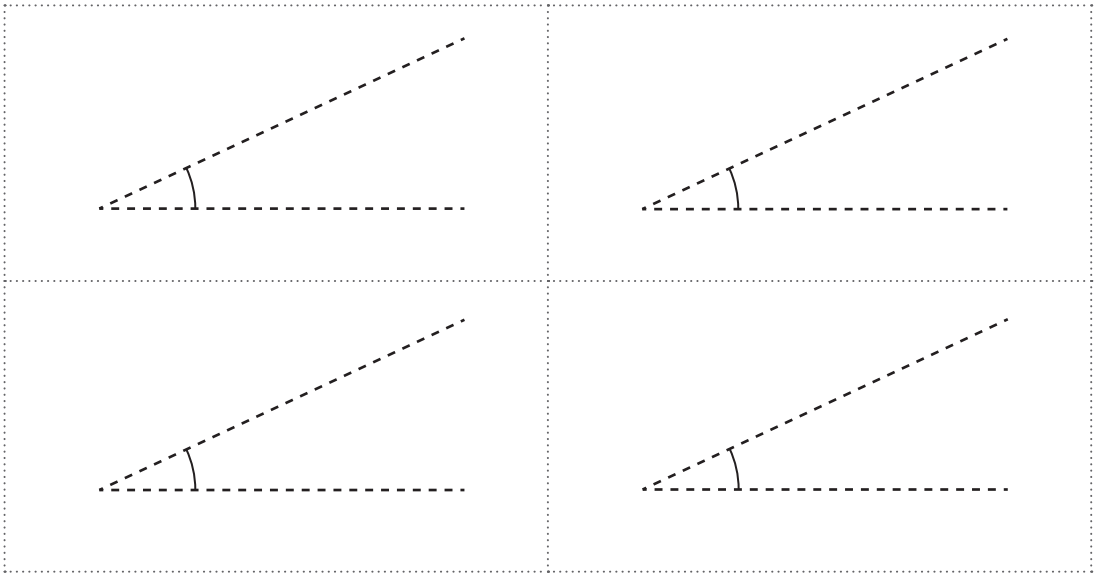
Name: Date: Period:

Side Angle Side

You will use a set of line segments and angles to help you with your thinking.

5 **a** Create as many non-identical triangles as you can with these measurements:

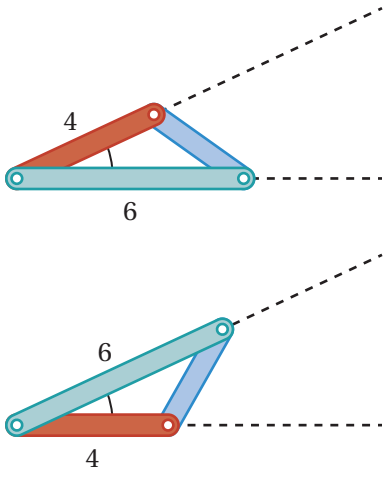
- Side lengths of 4 and 6 units
- One angle of 25°



b **Discuss:** What was your strategy for making non-identical triangles?

6 Malik claims he will always get *identical copies* if the 25° angle is between the side lengths of 4 and 6 units.

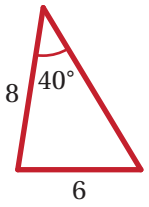
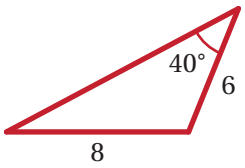
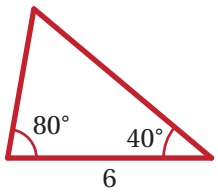
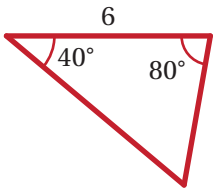
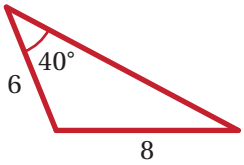
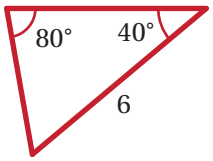
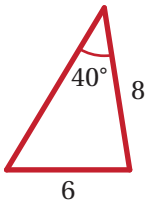
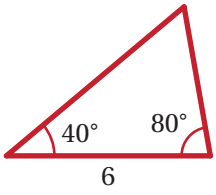
Is Malik's claim correct? Explain your thinking.



Name: Date: Period:

Side Angle Side (continued)

7 Make groups of *identical copies*. One triangle will not have a group.

Triangle A	Triangle B	Triangle C	Triangle D
			
Triangle E	**Triangle F**	**Triangle G**	**Triangle H**
			

Group 1	Group 2	Group 3

8 Isabelle claims Triangles A and E are identical copies and belong in the same group. Is Isabelle's claim correct? Circle one.

Yes, they're identical

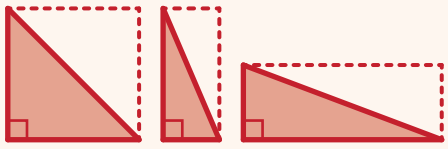
No, they're not identical

Explain your thinking.

You're invited to explore more.

9 Fatima thinks that in a right triangle, the other two angles are *complementary* (add up to 90°).

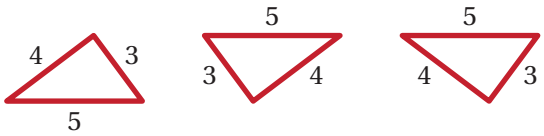
Is Fatima correct? Explain your thinking.



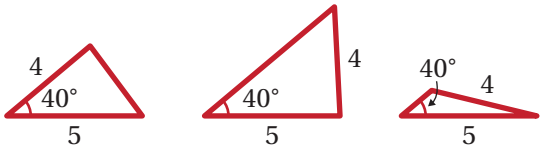
10 Synthesis

Discuss: Why are triangles with three of the same side lengths identical copies, but triangles with two of the same side lengths and one angle measure not always identical copies?

Three Sides



Two Sides, One Angle



13 Summary 7.07

If three side lengths make a triangle, all triangles with those side lengths are identical copies. But what about a combination of side lengths and angles? Some combinations of measurements can make more than one unique triangle.

Here are three triangles: A, B, and C.

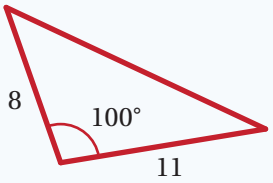
Each triangle has three of the same measurements: one side that is 8 units long, one side that is 11 units long, and one 100° angle.

Only triangles A and C are *identical copies* because the 100° angle is in between the side lengths of 8 and 11 units.

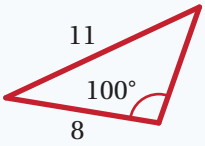
This means that there is more than one unique triangle that can be made with these measurements.

In general, knowing the order or placement of the sides and angles can help you determine whether two triangles are *identical copies* and how many unique triangles there are.

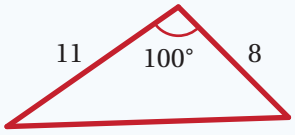
Triangle A



Triangle B



Triangle C



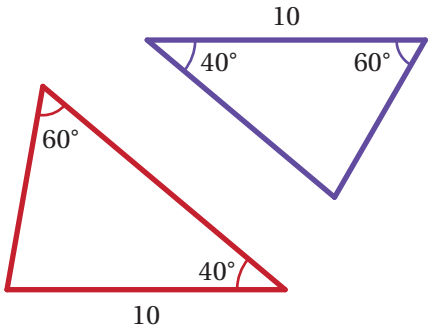
Practice
7.07

Name: ..... Date: ..... Period: .....

1. Are these two triangles identical? Circle one.

Yes No Not enough information

Explain your thinking.

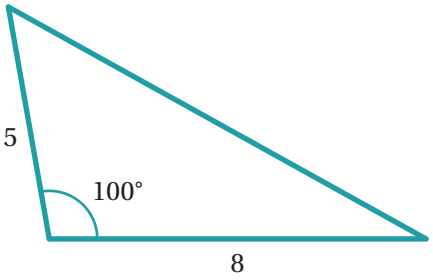


2. This triangle has a side length of 5 units, a side length of 8 units, and a 100° angle.

Is this the only triangle that can be created with these three measurements? Circle one.

Yes No Not enough information

Explain your thinking.

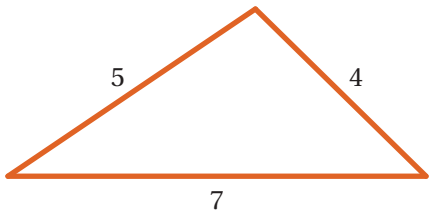


3. This triangle has side lengths of 7, 4, and 5 units.

Is this the only triangle that can be created with these three measurements? Circle one.

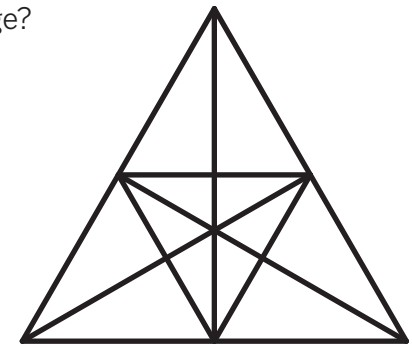
Yes No Not enough information

Explain your thinking.



Name: ..... Date: ..... Period: .....

4. How many non-identical triangles are there in this image? Show or explain your thinking.



Spiral Review

5. Angle A and angle B are complementary angles. Angle A measures $57°$. What is the measure of angle B?

6. Angle H and angle K are supplementary angles.

- The measure of angle H is $63°$.
- The measure of angle K is $(2x)°$.

Which equation can be used to determine the value of x?

A. $63 = 2x$ **B.** $63 + 2x = 180$ **C.** $63 + 2x = 90$ **D.** $63 + 2x = 360$

Problems 7–10: For each given angle, determine the measure of the supplementary angle.

7. $80°$ angle

8. $25°$ angle

9. $119°$ angle

10. $x°$ angle

Problems 11–12: Two months ago, the price of a cell phone was c dollars. Last month, the price of the phone increased by 10%.

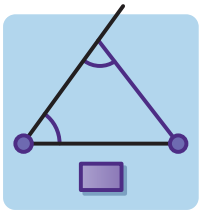
11. Write an expression for the price of the phone last month.

12. This month, the price of the phone decreased by 10%. Is the price of the phone this month the same as it was two months ago? Explain your thinking.

Name: ..... Date: ..... Period: .....

2-D and 3-D Connections | Angle Relationships | 7.G.2, 7.NS.3, SMP.1, SMP.6, SMP.7

Can You Draw It?



Let's draw triangles.

Warm-Up

1. Sketch a shape or figure that includes:

- A line segment with a length of 3 centimeters.
- A 45° angle.

a Without the use of a ruler or a protractor.

b With the use of a ruler and protractor.

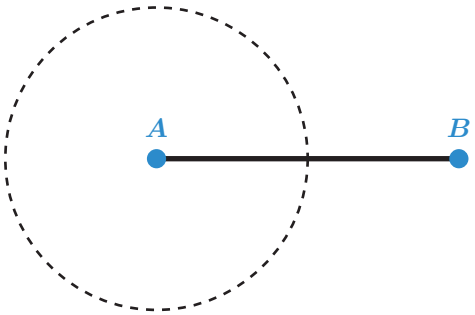
Name: Date: Period:

Complete the Triangles

2. Sadia is trying to draw a triangle with sides measuring 3 centimeters, 2 centimeters, and 4 centimeters. Here are the steps she took so far:

- Step 1: Draw a 4-centimeter line segment AB.
- Step 2: Use a compass to draw a circle around point A with a radius of 2 centimeters.

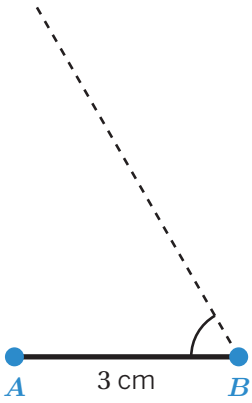
Describe or show how Sadia can finish drawing this triangle.



3. Nekeisha is trying to draw a triangle with one 3-centimeter side, one 4-centimeter side, and one 60° angle. Here are the steps she took so far.

- Step 1: Draw a 3-centimeter line segment.
- Step 2: Use a protractor to draw a 60° angle at one end of the line segment.

Describe or show how Nekeisha can finish her drawing.



Name: Date: Period:

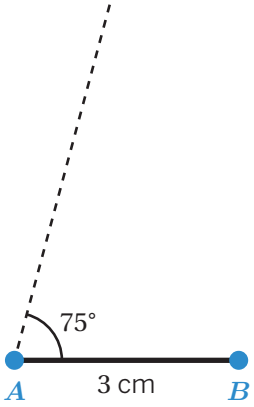
Complete the Triangles (continued)

4. Ahmed is trying to draw a triangle with one 45° angle, one 75° angle, and one 3-centimeter side.

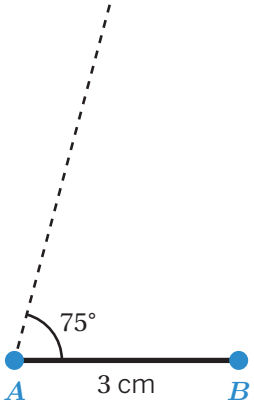
Here are the steps he took so far:

- Step 1: Draw a 3-centimeter line segment.
- Step 2: Use a protractor to draw a 75° angle.

Describe or show how Ahmed can finish his drawing.



5. Is it possible for Ahmed to draw another non-identical triangle that matches this description? Use the diagram to show or explain your thinking.



Activity 2

Name: Date: Period:

Drawing Challenges

For each description:

- Draw as many non-identical triangles as you can.
- Determine how many unique triangles can be made with these measurements.
- **Discuss:** How do you know if non-identical triangles are possible?

6. Two 3-centimeter sides and one $50°$ angle.

How many unique triangles can be made with these measurements? Circle one.

Zero One More than one

7. Two 3-centimeter sides with a $50°$ angle between them.

How many unique triangles can be made with these measurements? Circle one.

Zero One More than one

8. One $30°$ angle, one $60°$ angle, and one $90°$ angle.

How many unique triangles can be made with these measurements? Circle one.

Zero One More than one

Name: ..... Date: ..... Period: .....

Drawing Challenges (continued)

9. One $60°$ angle, one $90°$ angle, and one 3-centimeter side.

How many unique triangles can be made with these measurements? Circle one.

Zero One More than one

10. Two $90°$ angles and one 3-centimeter side.

How many unique triangles can be made with these measurements? Circle one.

Zero One More than one

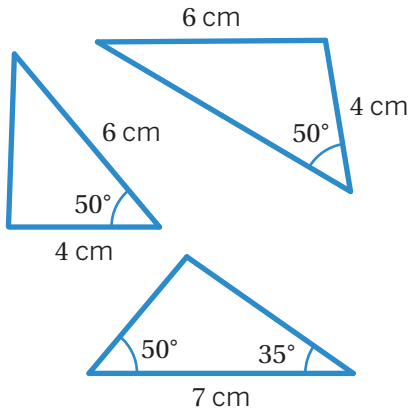
You're invited to explore more.

11. What is the greatest number of non-identical triangles possible when you know the measurements of two sides and one angle?

Show or explain your thinking.

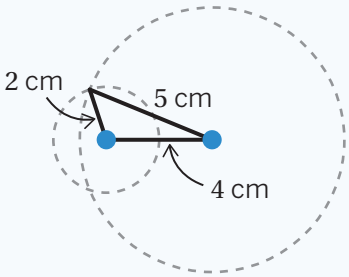
Synthesis

12. Describe how many non-identical triangles can be made with different combinations of measurements.



Summary 7.08

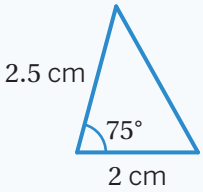
You can use a protractor, ruler, and compass to draw triangles with given measurements.



For example, when given three side lengths, using a ruler and a compass to draw circles with radiuses that match the given lengths can help you draw a triangle.

There is only one unique triangle that can be drawn when given three side lengths. When given two side lengths and an angle measure, multiple non-identical triangles can be drawn.

For example, here is one strategy for drawing a triangle that has a 75° angle, a side length of 2 centimeters, and a side length of 2.5 centimeters.

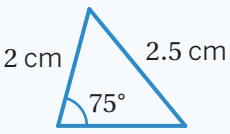


Step 1: Draw a 2-centimeter line segment.

Step 2: Draw a 75° angle using a protractor.

Step 3: Measure 2.5 centimeters along the other ray of the angle and connect the triangle.

Here is another non-identical triangle that has the same three measurements. The 75° angle can be positioned in different places in relation to the 2 and 2.5 centimeter sides.



Practice
7.08

Name: ..... Date: ..... Period: .....

1. A triangle has a $90°$ angle, a $60°$ angle, and one 6-centimeter side. Select True or False for each statement about this triangle.

	True	False
The triangle contains two angles that are complementary.		
The triangle contains two angles that are supplementary.		
Many non-identical triangles can be made using these measurements.		

Problems 2–4: For each set of three measurements, decide whether you can create zero triangles, one triangle, or more than one non-identical triangle. Circle your choice, then explain your thinking.

2. One 4-centimeter side, one 6-centimeter side, and one $50°$ angle.

Zero One More than one

3. One 4-centimeter side, one 5-centimeter side, and one 6-centimeter side.

Zero One More than one

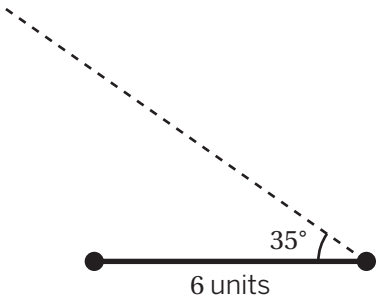
4. One $90°$ angle, one $100°$ angle, and one $30°$ angle.

Zero One More than one

5. A triangle has a $90°$ angle, a $35°$ angle, and a side that is 6 units long.

The 6-unit side is in between the $90°$ and $35°$ angles.

Complete the diagram and label your diagram with the given measurements.



Name: ..... Date: ..... Period: .....

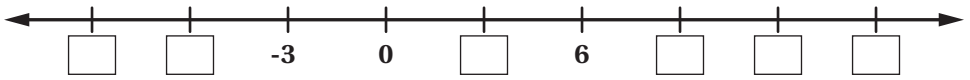
6. Write one side length and two angles so that only one unique triangle is possible.

Explain your thinking.

Side Length	Angle	Angle

Spiral Review

Problems 7–8: Use this number line.



7. Complete the number line. The markings on the number line are equally spaced.

8. Put a star over the point on the number line that represents the value of $(-3)(-2)$.

Problems 9–12: Calculate each value.

9. $\frac{24}{-4}$ **10.** $\frac{-24}{-4}$ **11.** $\frac{-24}{4}$ **12.** $\frac{-24 - 4}{-4}$

13. A factory produces 3 bottles of sparkling water for every 7 bottles of plain water. If those are the only two products they produce, what percent of their production is sparkling water?

Name: _____ Date: _____ Period: _____

2-D and 3-D Connections | Angle Relationships | 7.G.2, 7.G.5, 7.EE.4, SMP.3, SMP.5

Practice Day 1

Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Missing Measures

1. Equation(s):

2. $a =$ _____

3. $x =$ _____

$y =$ _____

$z =$ _____

Explanation:

You're invited to explore more.

Draw a diagram where these equations are true: $x + y + 50 = 180$ and $2x = 90$. Determine the values of x and y.

Task B: Draw It

1.

2.

3.

4.

You're invited to explore more.

Draw a diagram with two angles that are both vertical and supplementary, or explain why it is impossible. If you are able to draw a diagram, provide the angle measures of your two angles.

Name: Date: Period:

Practice Day 1 (continued)

Task C: Convince a Skeptic

1. Circle one: Yes No

Explanation:

2. Circle one: Yes No

Explanation:

You're invited to explore more.

Are three angles and one side length enough to determine a unique triangle? Convince a skeptic of your answer.

Task D: Two Truths and a Lie

1. Statement that is a lie:

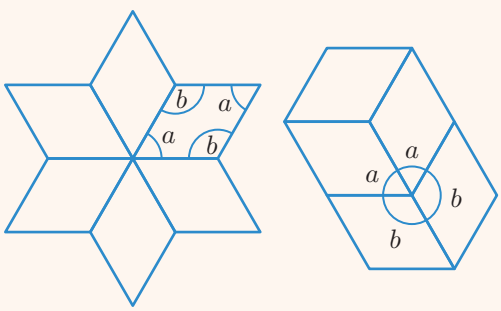
Explanation:

2. Statement that is a lie:

Explanation:

You're invited to explore more.

Write two truths and a lie based on both of these diagrams. Trade with a classmate and ask them to determine the lie.

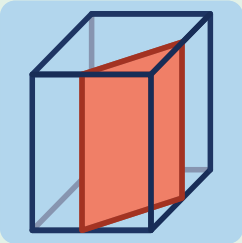


Notes:

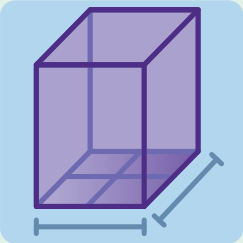
Sub-Unit

3

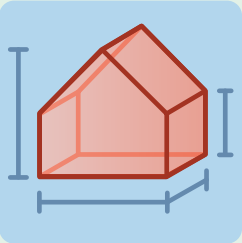
Solid Geometry



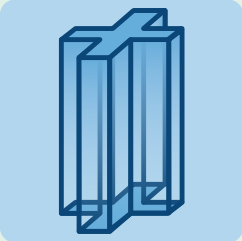
Lesson 9
Slicing Solids



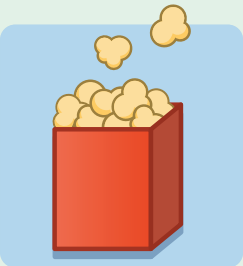
Lesson 10
Simple Prisms



Lesson 11
Complex Prisms



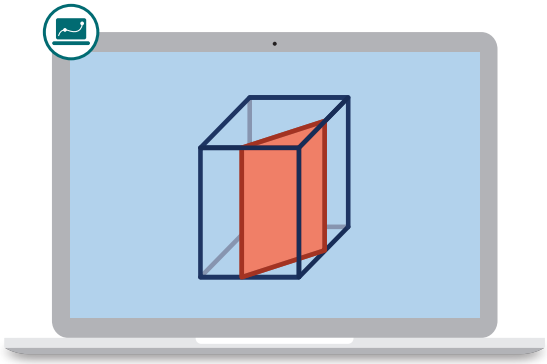
Lesson 12
Surface Area Strategies



Lesson 13
Popcorn Possibilities

Name: ..... Date: ..... Period: .....

2-D and 3-D Connections 7.G.3, SMP.3, SMP.6

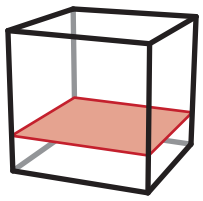
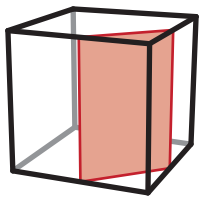
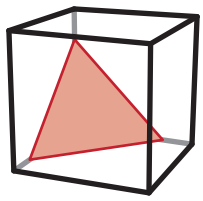
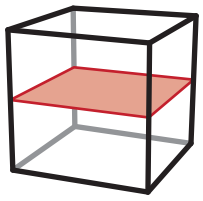
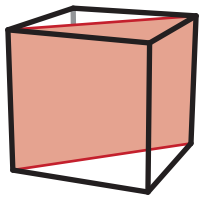
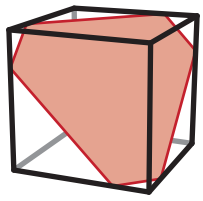


Slicing Solids

Let's explore and describe cross sections of solids.

Warm-Up

1 A plane cuts a cube into two pieces. When the plane cuts the cube parallel to the *base*, the **cross section** is a square. Here are different ways of cutting the cube and their cross sections.

Cut 1	Cut 2	Cut 3
		
		

What do you notice? What do you wonder?

I notice:

I wonder:

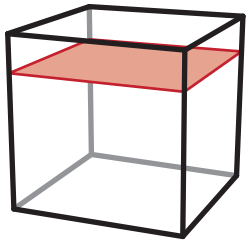
Activity 1

Name: ..... Date: ..... Period: .....

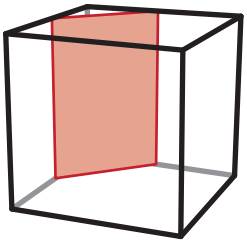
Creating Cross Sections

2 Different cuts create different cross sections.

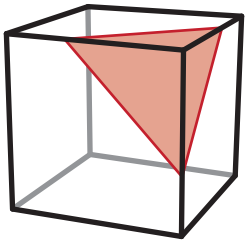
Cut 1



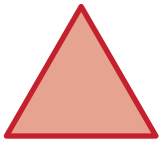
Cut 2

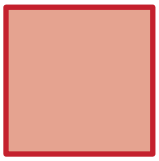


Cut 3



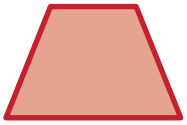
Select *all* of the cross sections you think you can make from a cube.

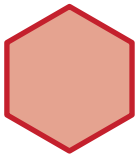
☐ A. 

☐ B. 

☐ C. 

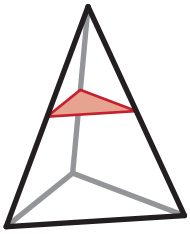
☐ D. 

☐ E. 

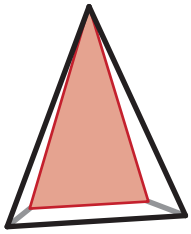
☐ F. 

3 Here is a triangular *pyramid*.

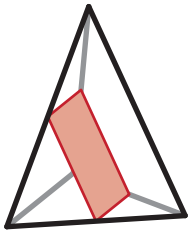
Cut 1



Cut 2



Cut 3



Select *all* of the shapes you think you can make.

☐ A. Equilateral triangle

☐ B. Isosceles triangle

☐ C. Rectangle

☐ D. Trapezoid

☐ E. Hexagon

Name: ..... Date: ..... Period: .....

Creating Cross Sections (continued)

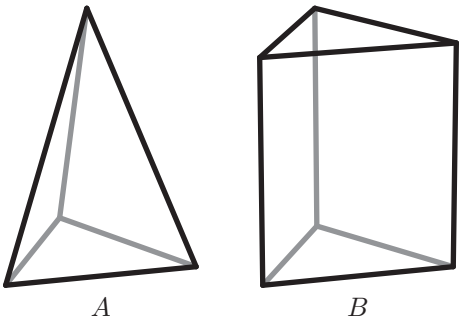
4 Amir claims he can cut a rectangular cross section of figure A.

Peter claims he can cut a rectangular cross section of figure B.

Whose claim is correct? Circle one.

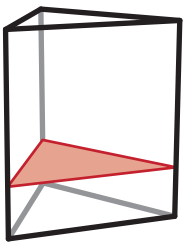
Amir Peter Both Neither

Show or explain your thinking.

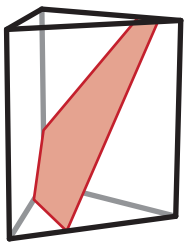


5 Here is a new *prism*.

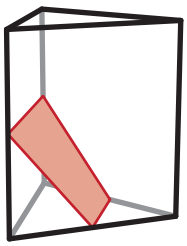
Cut 1



Cut 2



Cut 3



What is the greatest number of sides a cross section could have?

A. 4 **B.** 5 **C.** 6 **D.** More than 6

Explain your thinking.

Activity 2

Name: ..... Date: ..... Period: .....

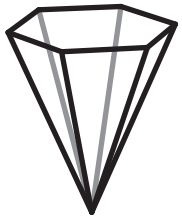
Prisms and Pyramids

6 Here is a pyramid and a prism. They have identical hexagon bases.

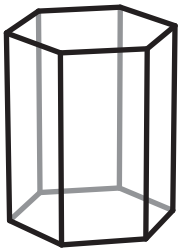
If you cut the solids parallel to their bases, how would the cross sections be alike?

How would they be different?

Pyramid



Prism

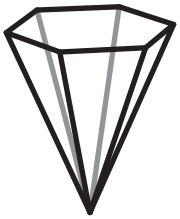


7 Imagine cutting each solid with a vertical cut.

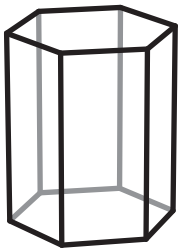
What might the cross section of the pyramid look like?

What might the cross section of the prism look like?

Pyramid

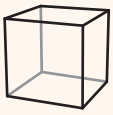
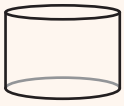
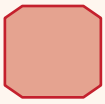
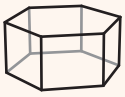


Prism



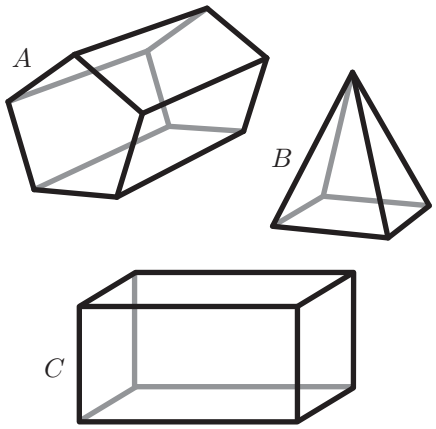
You're invited to explore more.

8 Match each solid with exactly one possible cross section.

Solid		Cross Section
a.		..... 
b.		..... 
c.		..... 
d.		..... 

9 Synthesis

Discuss: Why do different cuts of a solid create different possible cross sections?



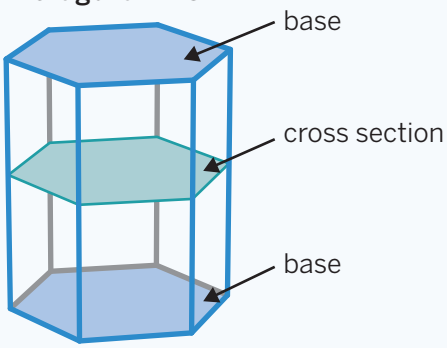
12 Summary 7.09

If you cut a hexagonal *prism* parallel to the *base*, the cross section is a hexagon that is the same size as the base. If you make a vertical cut instead, the cross section is a rectangle that is as tall as the prism.

If you cut a hexagonal *pyramid* parallel to the base, the cross section is a hexagon that is smaller than the base. If you make a vertical cut instead, the cross section is a triangle that is taller than it is wide.

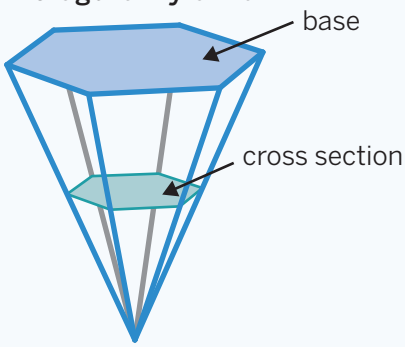
Prism

Hexagonal Prism



Pyramid

Hexagonal Pyramid

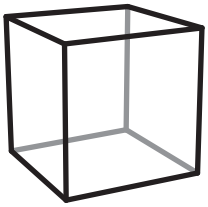


cross section The shape you see when you slice through a 3-D figure.

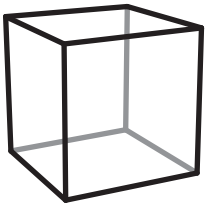
Name: ..... Date: ..... Period: .....

Problems 1–3: Show how to cut a cube to make each cross section.

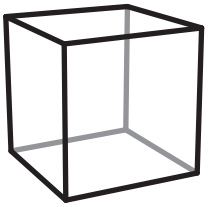
1. Square



2. Triangle

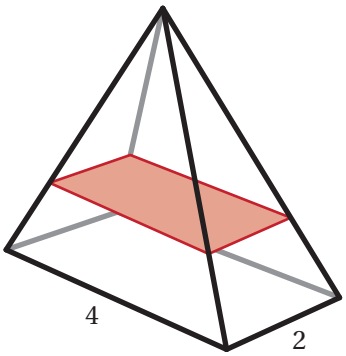


3. Rectangle



Problems 4–5: Determine how each cross section will change.

4. Here is a pyramid that has been cut parallel to its base. What will happen to the area of the cross section as the cut moves closer to the base?

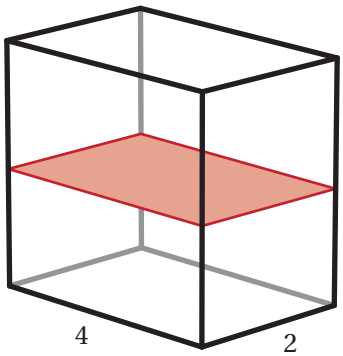


A. Increase

B. Decrease

C. Stay the same

5. Here is a rectangular prism that has been cut parallel to its base. What will happen to the area of the cross section as the cut moves closer to the base?



A. Increase

B. Decrease

C. Stay the same

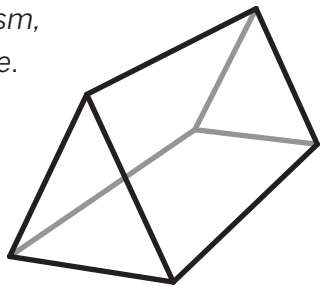
6. Cuts are made through a cube and a right square pyramid. One cut is vertical and one cut is horizontal. Select *all* the statements that are true about the two-dimensional cross sections that could result from one of these cuts.

- ☐ **A.** A triangle could result from one of these slices through the cube.
- ☐ **B.** A square could result from one of these slices through the cube.
- ☐ **C.** A rectangle, but not a square, could result from one of these slices through the cube.
- ☐ **D.** A triangle could result from one of these slices through the pyramid.
- ☐ **E.** A square could result from one of these slices through the pyramid.

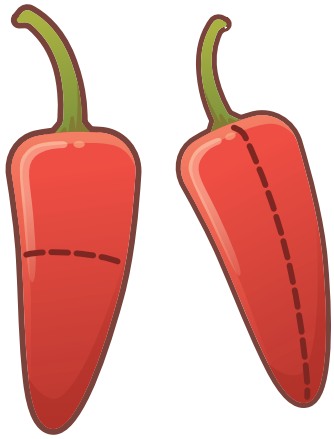
Practice
7.09

Name: ..... Date: ..... Period: .....

7. Rebecca says: *No matter which way you cut this triangular prism, the cross section will be a triangle*. Sydney says: *I'm not so sure*. Describe or show a cut that Sydney might be thinking of.



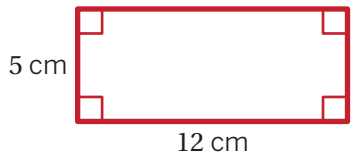
8. Here are two peppers. One is cut horizontally, and the other is cut vertically, producing different cross sections. Sketch the two cross sections.



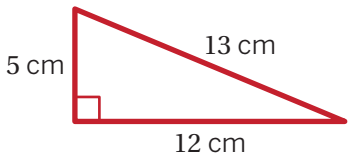
Spiral Review

Problems 9–11: Determine the area of each shape.

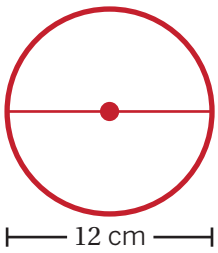
9.



10.



11.



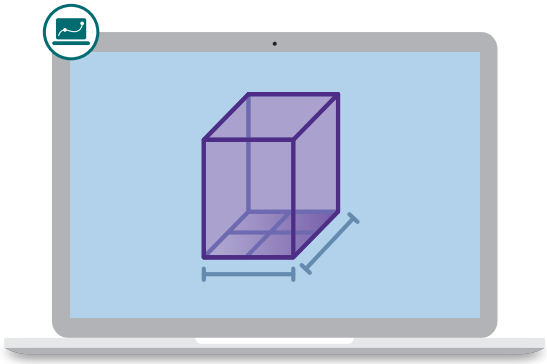
12. Select *all* the expressions that are equivalent to $3x - 4 + 2x - 6$.

- ☐ A. $x - 2$
- ☐ B. $5x - 2$
- ☐ C. $5(x - 2)$
- ☐ D. $5x + 10$
- ☐ E. $5x - 10$

13. Write an expression that is equivalent to $2(x + 4) - 7$ with the fewest number of terms.

Name: ..... Date: ..... Period: .....

2-D and 3-D Connections | 7.G.6, 7.NS.3, SMP.3

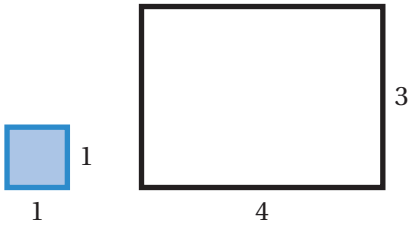


Simple Prisms

Let's calculate the volume of prisms.

Warm-Up

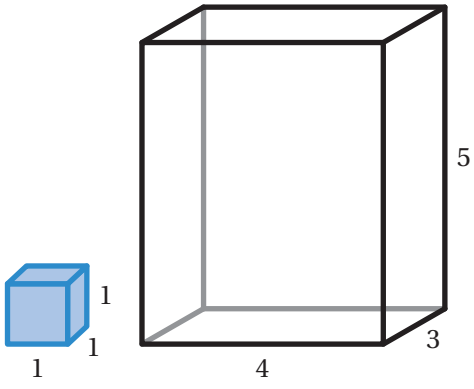
1 How many unit squares will it take to cover this rectangle?



2 Here is a rectangular prism.

a How many unit cubes will it take to fill this rectangular prism?

b **Discuss** What strategy did you use?

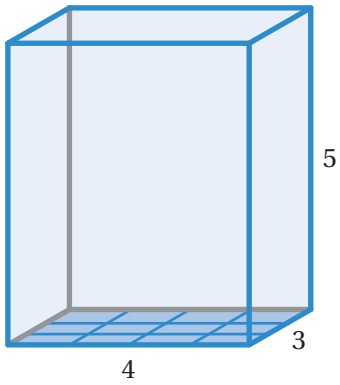


Name: ..... Date: ..... Period: .....

Volume of Prisms

3 Here is how Mio calculated the **volume** of the prism:

$V = 12 \cdot 5$



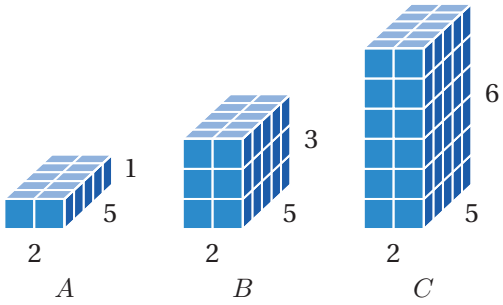
Explain what 12 and 5 represent.

12 represents . . .

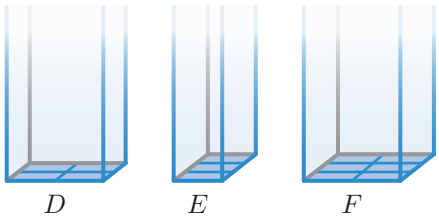
5 represents . . .

4 Here are three rectangular prisms: A, B, and C. They all have the same base.

Use Mio's strategy to determine the missing volumes.



5 Here are the bases of prisms D, E, and F. They all have the same volume.



Which prism has the greatest height? Circle one.

D E F Not enough information

Explain your thinking.

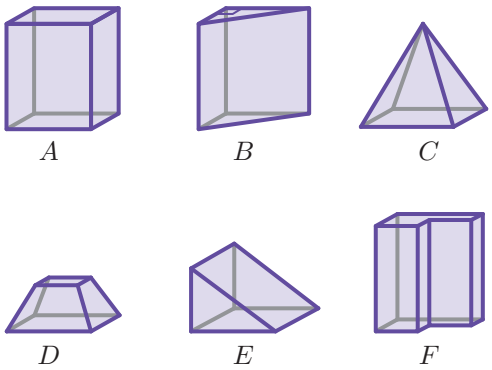
Activity 2

Name: ..... Date: ..... Period: .....

Volume Calculation Strategies

6 Here's how Mio described her strategy for determining the volume:

Step 1: Calculate the area of the base.
Step 2: Multiply that area by the height of the object.

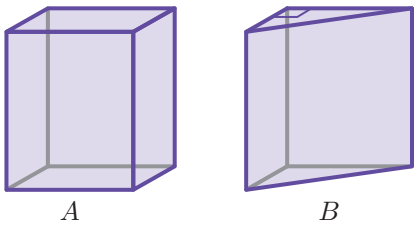


a Circle *all* of the objects that Mio's strategy will work for.

b **Discuss** How did you decide which objects to select?

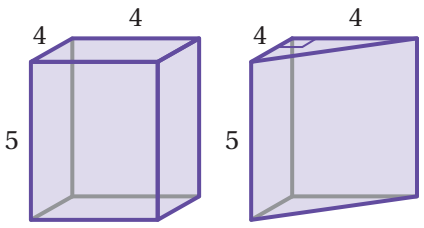
7 Here are two objects from the previous problem. One is a rectangular prism. The other is a triangular prism.

What information would you need to calculate each of their volumes?



8 Calculate the volume of each prism.

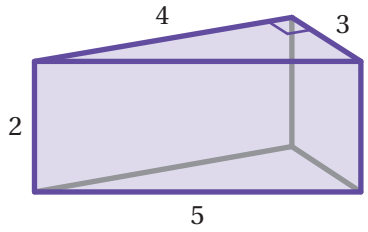
	Volume (cubic units)
Rectangular Prism	
Triangular Prism	



Name: ..... Date: ..... Period: .....

Volume Calculation Strategies (continued)

9 Here is a new triangular prism. Calculate its volume.



10 Nasir and Omari also calculated the volume of this triangular prism. They each made a mistake.

Nasir

$\frac{1}{2} \cdot 4 \cdot 3 \cdot 5 = 30$ cubic units

Omari

$4 \cdot 3 \cdot 2 = 24$ cubic units

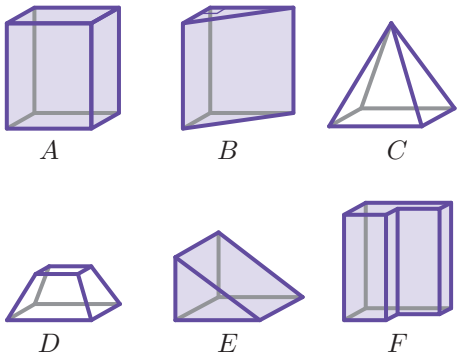
Choose one student and consider their calculations.

a. **Discuss** What did the student do well?

b. What would you recommend the student change about their calculations?

11 Synthesis

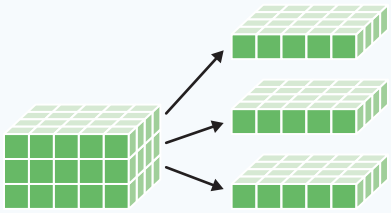
Discuss: How can you determine the volume of a prism?



14 Summary 7.10

Any cross section of a prism that is parallel to the base will be identical to the base. This means you can slice a prism into layers to help you calculate its **volume**.

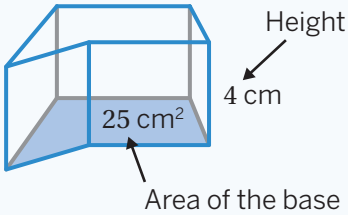
For example, if you have a rectangular prism that is 3 units tall and has a base that is 4 units by 5 units, you can imagine the prism as 3 layers of $4 \cdot 5$ cubic units.



That means the volume of this rectangular prism is $(4 \cdot 5) \cdot 3$ cubic units.

In general, you can calculate the volume of any prism by multiplying the area of its base by its height.

In other words, the volume of a prism is $V = B \cdot h$, where h is its height and B is the area of its base.



For example, this prism has a volume of 100 cubic centimeters because $25 \cdot 4 = 100$.

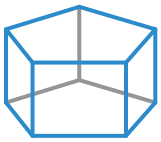
volume The number of cubic units that fill a 3-D region without any gaps or overlaps.

Practice
7.10

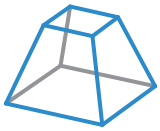
Name: ..... Date: ..... Period: .....

Problems 1–2: Here is a set of 3-D objects.

Object A

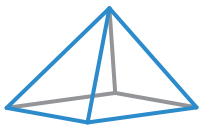


Object B



Object C

Object D



Object E

1. Circle all of the prisms.

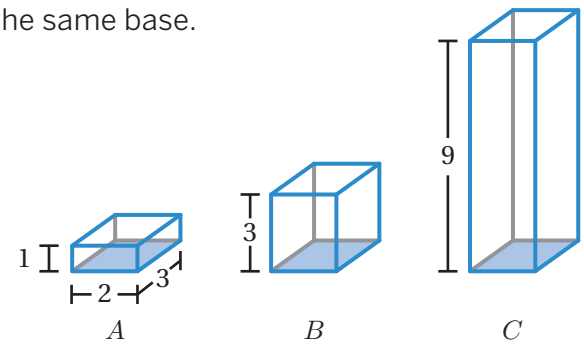
2. For each prism, shade one of the bases.

Problems 3–5: Here are three prisms with the same base.

3. Determine the volume of prism A.

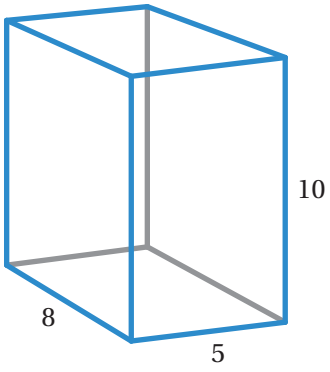
4. Determine the volume of prism B.

5. Determine the volume of prism C.

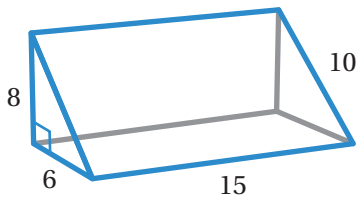


Problems 6–7: Determine the volume of each prism. Show or explain your thinking.

6.

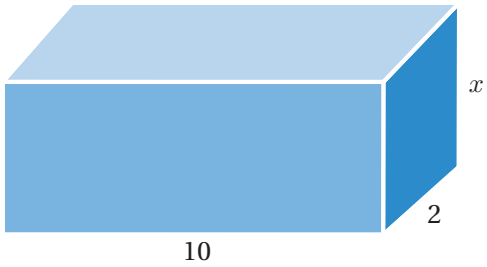


7.



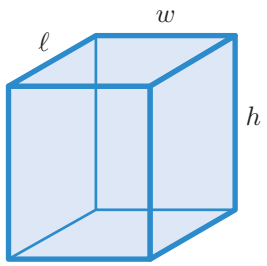
Name: ..... Date: ..... Period: .....

8. The volume of this rectangular prism is 60 cubic units. What is its height?



9. Use whole numbers between 1 and 9, without repeating, to create two prisms with the same volume.

	Prism 1 (units)	Prism 2 (units)
Length, ℓ		
Width, w		
Height, h		



Spiral Review

10. Select *all* the expressions that are equivalent to $3(x - 2) + 5$.

☐ **A.** $3x + 3$ ☐ **B.** $3(x - 1)$ ☐ **C.** $3x - 1$ ☐ **D.** $-1 + 3x$ ☐ **E.** $1 - 3x$

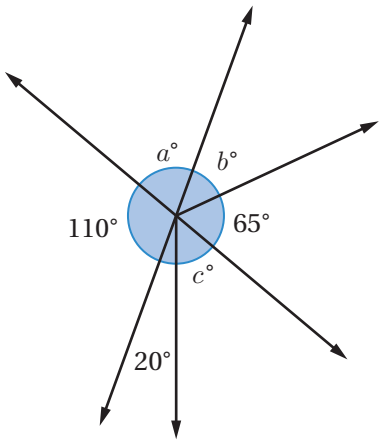
11. Mayra buys a $25 hat. The sales tax in her state is 6.25%. How much will Mayra spend in total, including the tax?

Problems 12–14: Determine the measure of each angle.

12. What is the value of a?

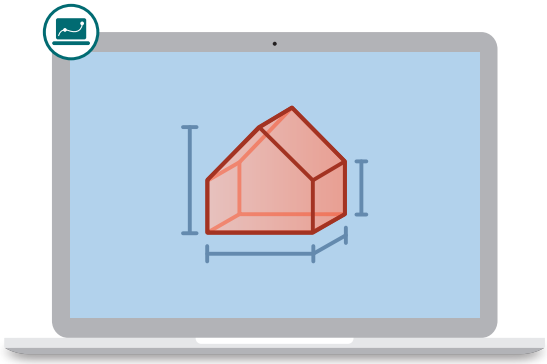
13. What is the value of b?

14. What is the value of c?



Name: ..... Date: ..... Period: .....

2-D and 3-D Connections | Shapes in the World | 7.G.6, 7.NS.3, SMP.1, SMP.3, SMP.7

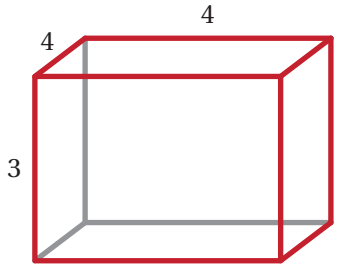


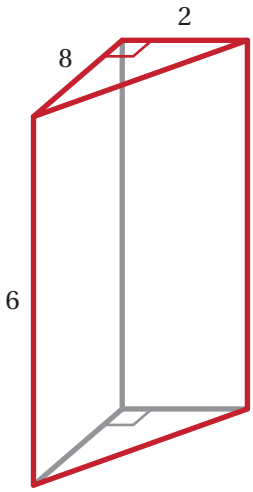
Complex Prisms

Let's determine the volume of prisms with other bases.

Warm-Up

1 Calculate the volume of each prism.



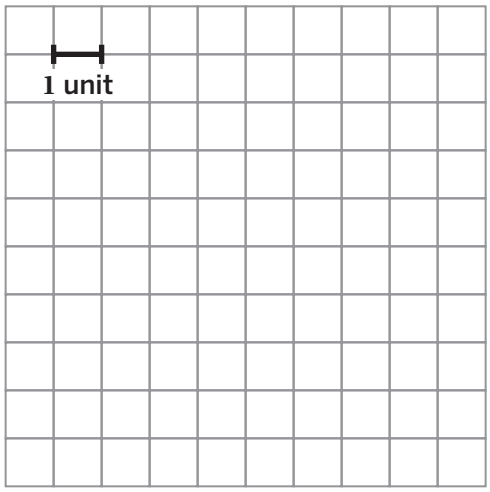


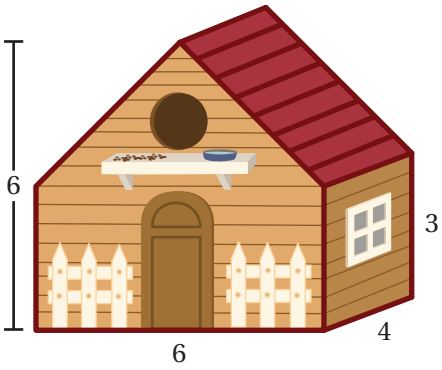
Activity 1

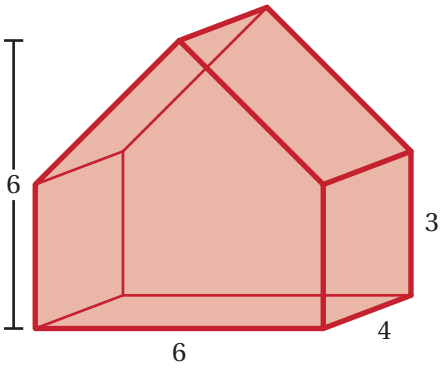
Name: ..... Date: ..... Period: .....

Using the Base to Calculate Volume

2 Sketch the base of this prism on the grid.







3 What is the area of the prism's base?

Show or explain how you determined the area.

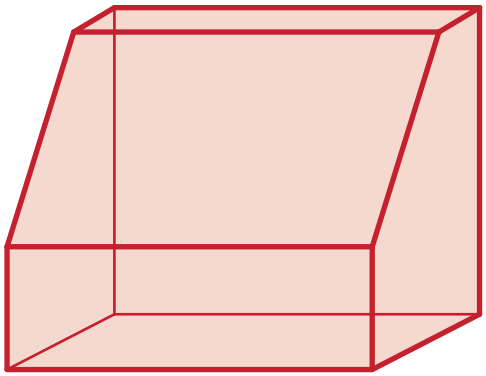
4 What is the volume of the prism?

Discuss: How does the shape of a prism's base relate to the volume of the prism?

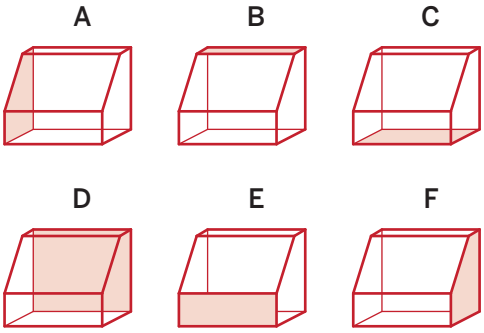
Name: ..... Date: ..... Period: .....

Prisms With Other Bases

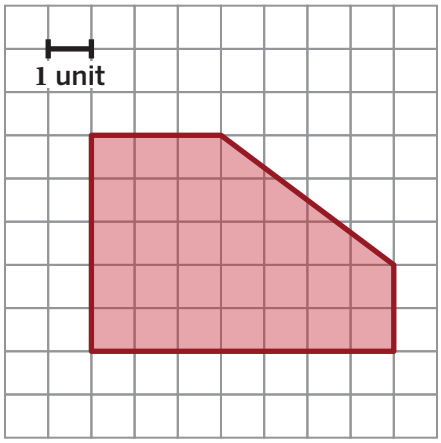
5 Here is a new prism.



Circle *all* the faces that are bases you could use to calculate the prism's volume.

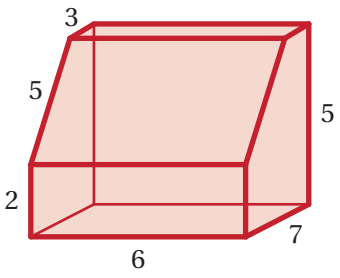


6 Here is one face of the prism.



What is its area? Show your thinking.

7 Jaylin says the volume of this prism is $29 \cdot 5$, or 145 cubic units.



Kimaya claims that it's $29 \cdot 6$, or 174 cubic units.

Whose claim is correct? Circle one.

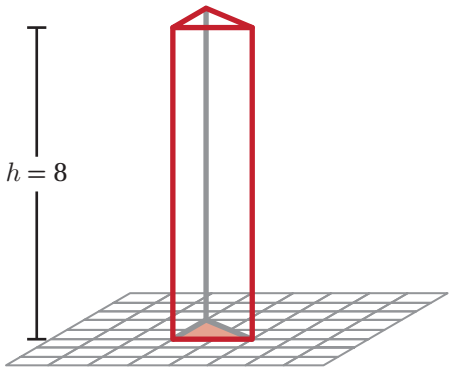
Jaylin Kimaya Both Neither

Explain your thinking.

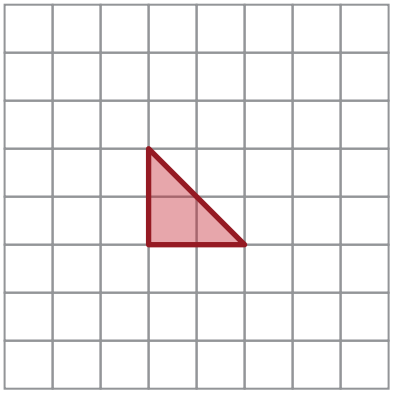
Name: ..... Date: ..... Period: .....

Prisms With Other Bases (continued)

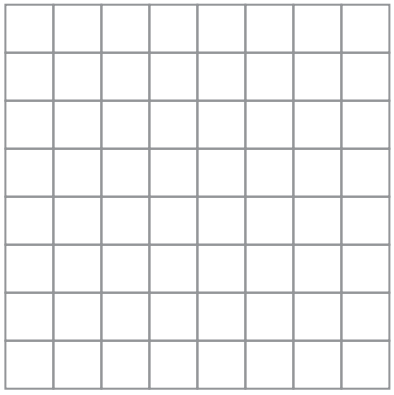
8 This triangular prism has a height of 8 units and a volume of 16 cubic units. Draw a new base so its volume is 48 cubic units.



Base



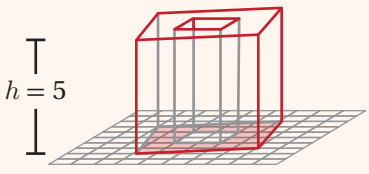
New Base



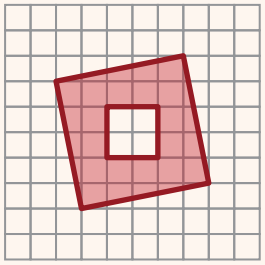
You're invited to explore more.

9 A prism has a height of 5 units and a volume of 110 cubic units. Its base is made of two squares. Draw a new base, made of two squares, that creates a prism with a volume of 80 cubic units.

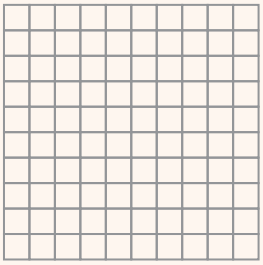
Prism



Base

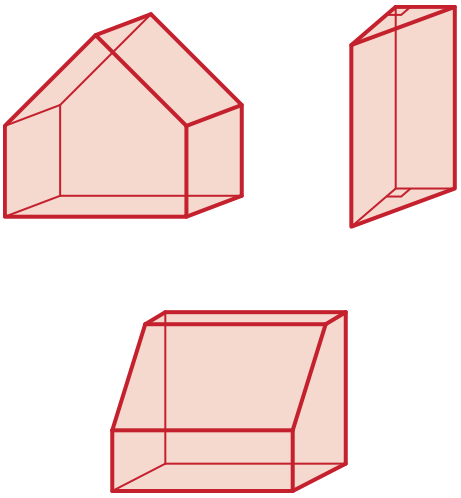


New Base



10 Synthesis

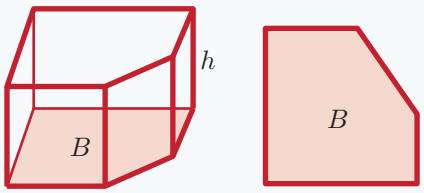
Here are several prisms you've seen in this lesson.



Discuss: What is a general strategy that you can use to determine the volume of any prism?

13 Summary 7.11

To calculate the volume of a prism, you can multiply the area of the base, B, by the height, h.

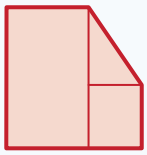


Sometimes the shape of a prism's base is a more complex *polygon*.

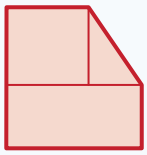
There are many strategies for calculating the area of a complex shape, including breaking it into rectangles and triangles, or surrounding the shape in a rectangle and subtracting the missing piece.

Here are three first steps you might take in calculating the area of this prism's base:

Example 1



Example 2



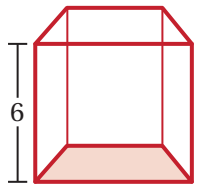
Example 3

Practice
7.11

Name: ..... Date: ..... Period: .....

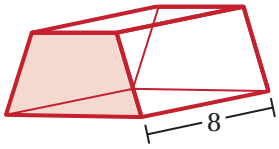
Problems 1–2: The volume of each prism is 24 cubic units. What is the area of each prism's base?

1.



Area of the base: .....

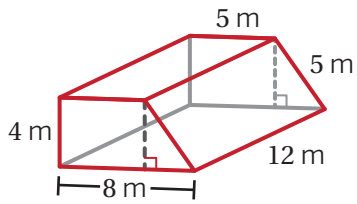
2.



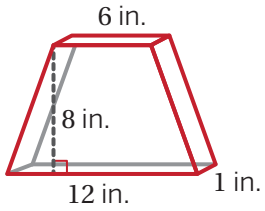
Area of the base: .....

Problems 3–5: Determine the volume of each prism. Show or explain your thinking.

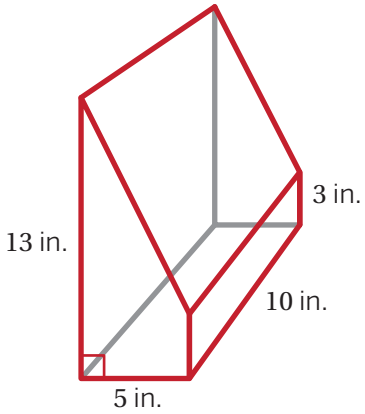
3.



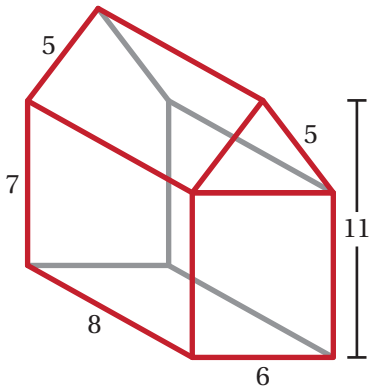
4.



5.



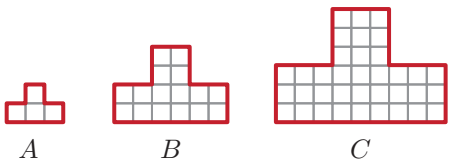
6. We can create a house-shaped prism by attaching a triangular prism on top of a rectangular prism. Select *all* the true statements about the house-shaped prism.



- ☐ **A.** The shape of the base is a triangle.
- ☐ **B.** The shape of the base is a pentagon.
- ☐ **C.** The area of the base is 54 square units.
- ☐ **D.** The area of the base is 42 square units.
- ☐ **E.** The volume of the prism is 432 cubic units.

Name: ..... Date: ..... Period: .....

7. Here are the bases of three different prisms. They all have the same volume. How tall could each prism be? Explain your thinking.



Spiral Review

Problems 8–11: Write each fraction as a decimal.

8. $\frac{1}{2}$

9. $\frac{1}{4}$

10. $\frac{3}{4}$

11. $\frac{1}{5}$

Problems 12–13: Two different triangles each have one side that is 5 centimeters long and one side that is 3 centimeters long.

12. Sketch two non-identical triangles with these measurements.

Triangle A

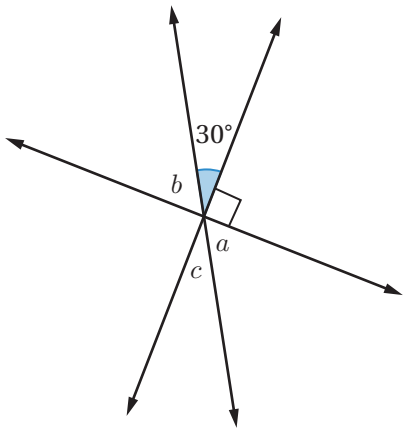
5 cm

Triangle B

5 cm

13. Explain how you can tell that your two triangles are not identical.

14. Select *all* the equations that represent a relationship between angles in this figure.

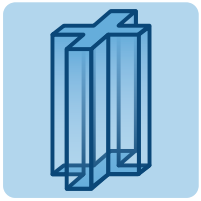


- ☐ **A.** $90 - 30 = b$
- ☐ **B.** $a + c + 30 + b = 180$
- ☐ **C.** $a + c = 30$
- ☐ **D.** $30 + b = a + c$
- ☐ **E.** $a = 30$
- ☐ **F.** $90 + a + c = 180$

Name: ..... Date: ..... Period: .....

2-D and 3-D Connections | Shapes In The World | Scale Drawings | 7.G.1, 7.G.6, 7.NS.3, SMP.5

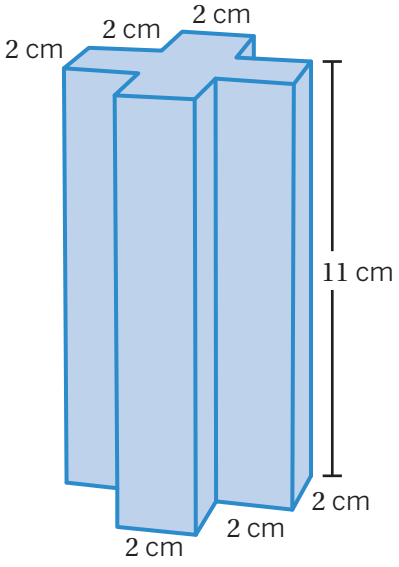
Surface Area Strategies



Let's calculate the surface area of different prisms.

Warm-Up

1. Here is a prism.



a How many *faces* does this prism have? Explain your thinking.

b Calculate the area of the prism's base.

Name: Date: Period:

Examining Different Strategies

2. Here are Amoli's, Nyanna's, and Miko's strategies for calculating the **surface area** of the prism from the Warm-Up.

Amoli	Nyanna	Miko
I have to draw each of the 14 different faces, find their areas, and add them up.	There are only two different shapes: the plus sign and the rectangle. I can find the area of each shape and use a calculator to multiply by the number there are of each shape.	I see another way! Imagine unfolding the prism into a net. I can use one large rectangle instead of 12 smaller ones.

a. **Discuss:** How would you describe each student's strategy in your own words?

b. How are Amoli's and Nyanna's strategies alike? How are they different?

3. Let's look at Miko's strategy more closely.

a. Sketch the "one large rectangle" Miko is talking about.

b. What are the dimensions of this rectangle? Show or explain your thinking.

4. Use any strategy to calculate the surface area of this solid. Organize your thinking and calculations so that others can follow them.

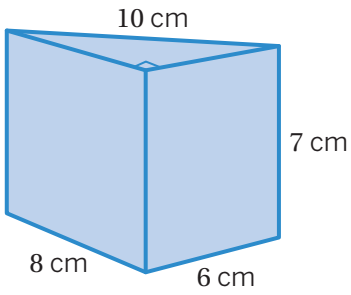
Name: ..... Date: ..... Period: .....

Calculating Surface Area

5. Here are three prisms. For each prism:

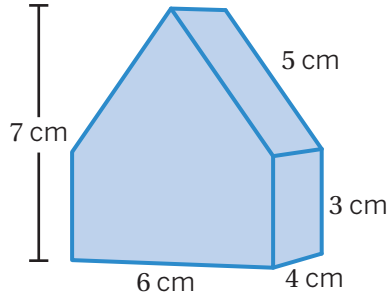
- Determine how many faces the prism has.
- Use any strategy to calculate the surface area. Organize your thinking and calculations so that others can follow them.
- Trade papers with your partner. Work together to reach an agreement.

a



Number of faces: Surface area:

b

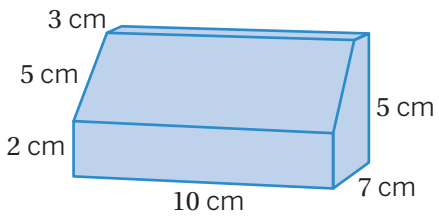


Number of faces: Surface area:

Name: Date: Period:

Calculating Surface Area (continued)

c.



Number of faces: Surface area:

6. **Discuss:** When is it useful to know the surface area of an object?

7. If the models in Problem 5 represent actual structures using a scale of 2 centimeters to 200 feet, what is the surface area of each structure?

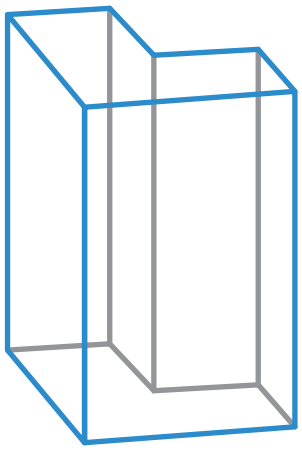
You're invited to explore more.

8. Sketch a prism with a surface area of 220 square units.

Synthesis

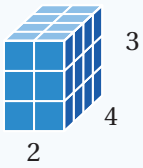
9. Describe your favorite method for calculating the surface area of a prism.

Use this prism if it helps you with your explanation.



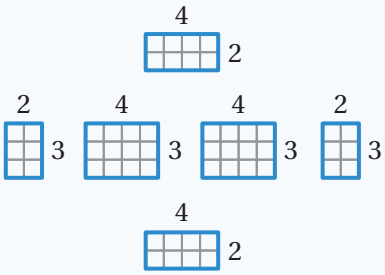
Summary 7.12

Here are two strategies for calculating the surface area of a rectangular prism.



Strategy 1

Calculate the area of each face separately and then add all of the areas.

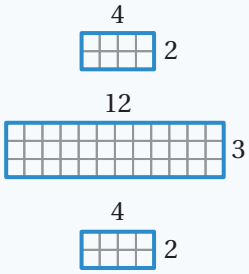


$8 + 6 + 12 + 12 + 6 + 8 = 52$

Surface area: 52 square units

Strategy 2

Break the prism into its two identical bases and unfold the sides into one long long rectangle. Add the three areas.



$8 + 36 + 8 = 52$

Surface area: 52 square units

Using either strategy to calculate the surface area or using equivalent calculations will result in the same total surface area.

surface area The number of square units that covers all the faces of a polyhedron, without any gaps or overlaps.

Practice
7.12

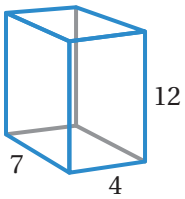
Name: ..... Date: ..... Period: .....

1. Select *all* the situations where knowing the surface area of an object would be useful.

- ☐ **A.** The amount of paint needed to paint a room.
- ☐ **B.** The amount of water needed to fill an aquarium.
- ☐ **C.** How much wrapping paper a gift will need.
- ☐ **D.** How many watermelons fit in a box for shipping.
- ☐ **E.** The amount of gasoline left in the tank of a vehicle.

Problems 2–4: Determine the volume and surface area of each prism.

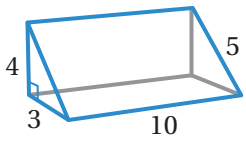
2.



Volume:

Surface area:

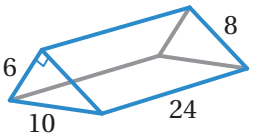
3.



Volume:

Surface area:

4.

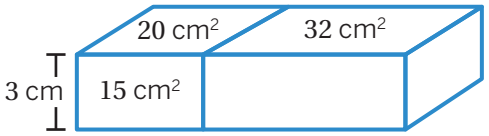


Volume:

Surface area:

5. Determine the surface area and volume of this shape.

Show or explain your thinking.



Surface area:

Volume:

Practice
7.12

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 6–9: Write each fraction as a percentage.

6. $\frac{1}{4}$

7. $\frac{1}{5}$

8. $\frac{3}{5}$

9. $\frac{3}{10}$

Problems 10–12: In a 4-by-6 foot Colorado state flag, the gold-colored circle has a 1-foot radius.

Public Domain

10. How much fabric is needed to create the flag?

11. How much of the fabric is gold?

12. What percent of the flag is gold?

13. Draw one or more diagrams that show complementary and supplementary angles.

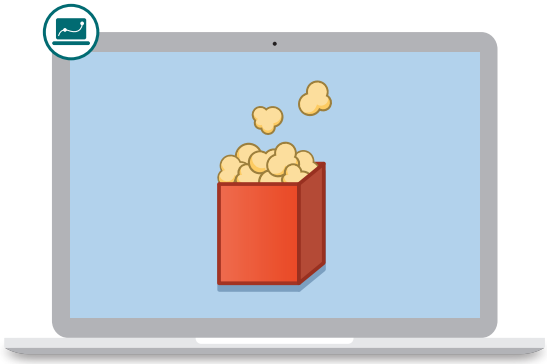
Unit 7
Lesson
13

Name: ..... Date: ..... Period: .....

2-D and 3-D Connections | Shapes In The World | 7.G.6, 7.NS.3, SMP.4

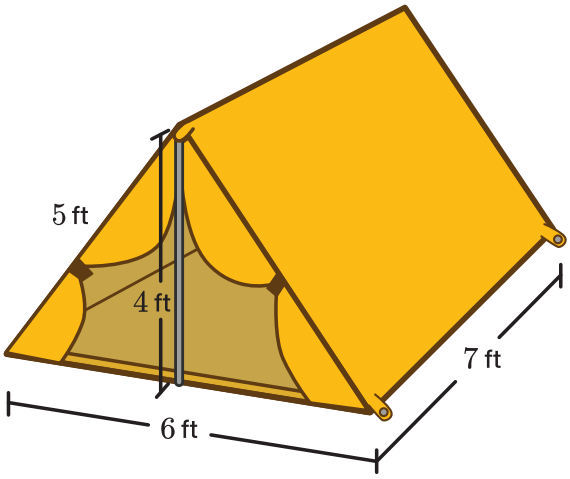
Popcorn Possibilities

Let's apply surface area and volume to real-world situations.



Warm-Up

1 Daeja is trying to determine how much fabric it took to create this tent.



a What would be more useful for her to calculate? Circle one.

Surface Area Volume

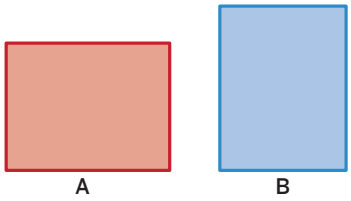
b Calculate the measurement you selected.

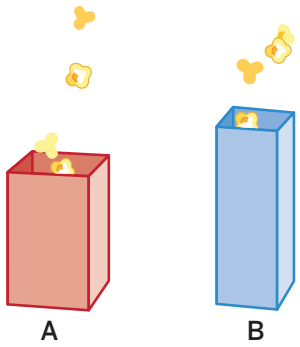
Activity 1

Name: ..... Date: ..... Period: .....

Which Holds More?

2 Let's watch two 8.5-by-11 inch sheets of paper fold into containers.





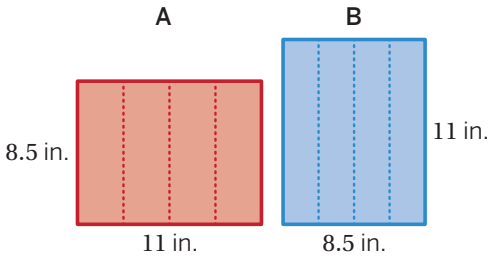
a Which container do you think will hold more popcorn? Circle one.

Container A Container B They will hold the same amount

b What information would help you know for sure?

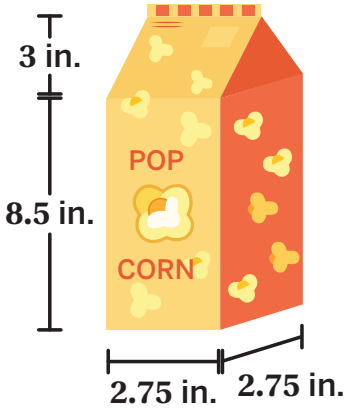
3 Determine the amount of popcorn each container can hold. Show your thinking.

	Width (in.)	Height (in.)	Popcorn (cu. in.)
A	11	8.5	
B	8.5	11	



4 Here is a to-go container for popcorn. Its rectangular part has the same dimensions as Container A.

Calculate the approximate amount of popcorn this container can hold. Show your thinking.

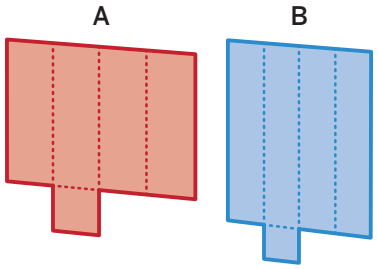


Activity 2

Name: ..... Date: ..... Period: .....

Which Uses More Paper?

5 An extra piece of paper has been added to the bottom of each container (so it can actually hold popcorn).



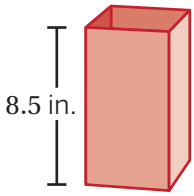
Which container do you think uses more paper? Circle one.

Container A Container B They use the same amount

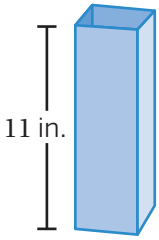
Explain your thinking.

6 Determine the amount of paper each container uses (including the bottom). Show your thinking.

Container A



Container B

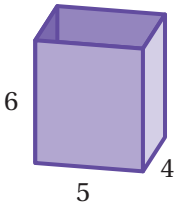


Activity 3

Name: ..... Date: ..... Period: .....

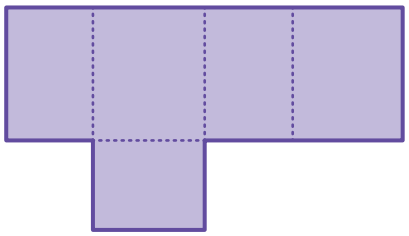
More Popcorn Containers

7 Here is a new container.



a How many cubic inches of popcorn can it hold?

b How many square inches of paper does it use?



8 **a** Draw a *different* container that can hold 120 cubic inches of popcorn.

b Calculate the amount of paper your container uses.

You're invited to explore more.

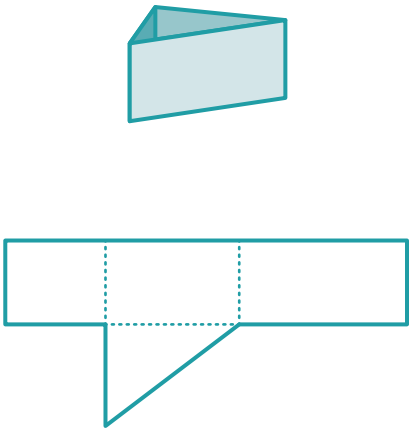
9 **a** Draw a new container that can hold approximately 120 cubic inches of popcorn using as little paper as possible. Label your container's dimensions.

Sketch

b Calculate the amount of paper your container uses.

10 Synthesis

Discuss: What is important to remember when calculating the surface area or volume of a prism? Use the example if it helps with your thinking.



13 Summary 7.13

Knowing when to calculate volume and surface area can be helpful in answering questions about situations in context.

Questions related to volume:

- How much water can a container hold?
- How much material did it take to build a solid object?

Questions related to surface area:

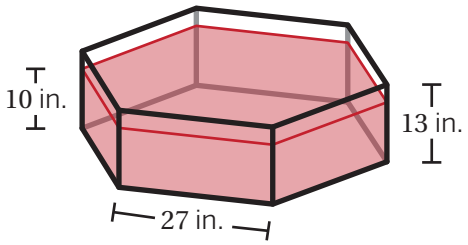
- How much fabric is needed to cover a surface?
- How much of an object needs to be painted?

One way to decide if a question is asking about volume or surface area is to think about the units of measure. Volume is measured in cubic units and surface area is measured in square units.

Practice
7.13

Name: ..... Date: ..... Period: .....

Problems 1–3: Here is a new sandbox Polina is designing for her local playground.



1. Polina knows she needs $18{,}940$ cubic inches of sand to fill the sandbox up 10 inches. What is the area of the sandbox's base?

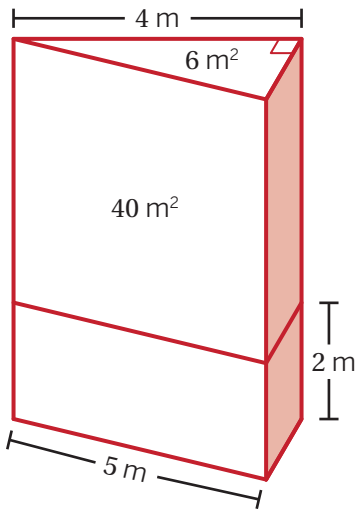
2. If Polina wanted to fill the sandbox up 3 more inches to the top, how much more sand would she need?

3. Polina wants to paint the entire outside of the sandbox (not including the bottom). How many square inches will she need to cover with paint? Explain your thinking.

4. There are two boxes of cereal in the shape of rectangular prisms. The first box has a height of 10 inches, a length of 8 inches, and a width of 3.5 inches. The second box has a height of 10 inches, a length of 7.5 inches, and a width of 2.5 inches. What is the difference in volume, in cubic inches, between the two boxes of cereal?

A. 467.5 **B.** 92.5 **C.** 280 **D.** 375

Problems 5–6: Here is a pair of stacked triangular prisms with bases of the same size.



5. Determine the surface area of this shape.

6. Determine the volume of this shape.

Practice
7.13

Name: ..... Date: ..... Period: .....

Spiral Review

Problems 7–10: Write each decimal or percentage as a fraction.

7. 0.25

8. 40%

9. 1%

10. 0.8

Problems 11–13: Dalia buys a winter jacket at a used clothing store that costs $\$30$. The sales tax is 6% where she lives.

11. How much does Dalia pay in total?

12. Write an equation that represents the total cost (including sales tax), c, of any item bought in this store with price p.

13. Dalia buys a backpack at a different store where she lives. It is on sale for 30% off. Dalia pays $\$33.39$ total (including sales tax). What was the original price of the backpack?

14. The surface area of a cube is 54 square centimeters. What is the volume of the cube?

Explain your thinking.

Name: _____ Date: _____ Period: _____

2-D and 3-D Connections | Shapes in the World | 7.G.6, 7.NS.3, SMP.2

Practice Day 2

Let's practice what you've learned so far in this unit!

You will use problem cards for this Practice Day. Record all of your responses here.

1. Group the cards based on whether the question is asking about surface area or volume. List the numbers of the cards you placed in each group.

Surface Area: _____ Volume: _____

2. Choose one surface area card to examine more closely.

Card: _____

Use this space to answer the question on the card you chose. You are finished with this problem when your paper shows:

- ☐ A sketch with labeled estimates of measurements you need to calculate the surface area.
- ☐ Your calculations, organized so that others can follow them.
- ☐ Your answer in a sentence.

Name: _____ Date: _____ Period: _____

Practice Day 2 (continued)

3. Choose one volume card to examine more closely.

Card: _____

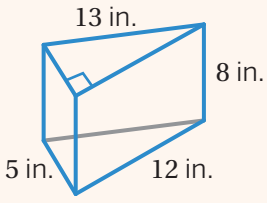
Use this space to answer the question on the card you chose. You are finished with this problem when your paper shows:

- ☐ A sketch with labeled estimates of measurements you need to calculate the volume.
- ☐ Your calculations, organized so that others can follow them.
- ☐ Your answer in a sentence.

You're invited to explore more.

A student made at least one error when calculating the surface area of each of these prisms. For each prism, explain the error(s). Then calculate the correct surface area.

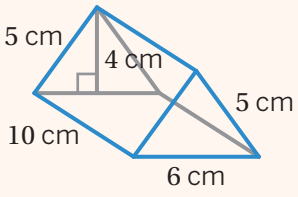
4.



The surface area is 240 square inches.

$$(12 + 13 + 5) \cdot 8 = (30) \cdot 8$$
$$= 240$$

5.

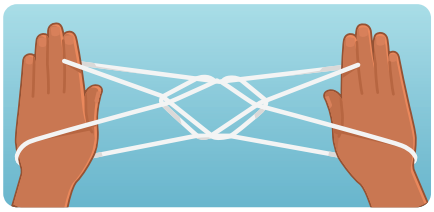


The surface area is 162 square centimeters.

$$\frac{1}{2}(6 \cdot 4) + 3(5 \cdot 10) = \frac{1}{2}(24) + 3(50)$$
$$= 12 + 3(50)$$
$$= 12 + 150$$
$$= 162$$

Career Connection

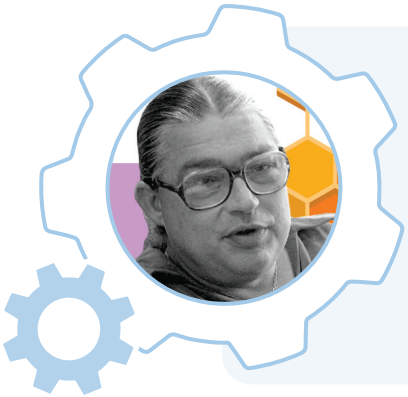
Where do you see the math of this unit in this string figure?



Bee

Indigenous peoples around the world have made string figures — patterns made from a loop of string stretched and woven with their hands — for thousands of years. This practice continues to this day in some societies. Mathematicians, including Thomas Storer, have studied these string games to better learn the complex mathematical ideas embedded in each of them.

Anthropologists study the characteristics of past and present human societies throughout the world. Mathematical anthropologists use math to identify patterns and make predictions about human behavior.



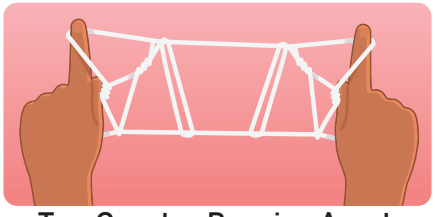
Meet Thomas Storer

Thomas Storer was a professor of mathematics at the University of Michigan, where he also taught the Ojibwe language (or Anishinaabemowin). A Navajo, he was one of the first Native Americans to earn a Ph.D. in mathematics. He is known for his studies of string figures which he first learned from his grandmother as a child.

Are you interested in studying anthropology? What can you do to learn more?

Math in the World

Here is another string figure.
What angle relationships do you see?
Are the triangles identical copies?
How can you tell?



Two Coyotes Running Apart

Math Mindset

Describe a time when you used patterns, shapes, angles, or other math concepts to create something – even if you weren't focused on thinking about math while creating it.

Unit 8

Probability and Sampling

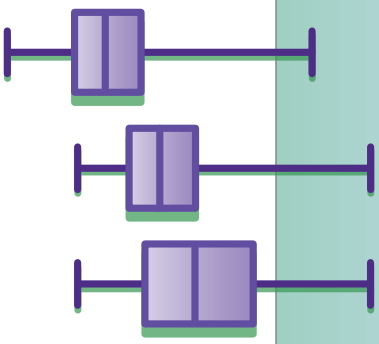
Big Ideas in This Unit

CC1 Populations and Samples | Visualize Populations | Probability Models

CC2 Proportional Relationships | Unit Rates in the World

Questions for Investigation

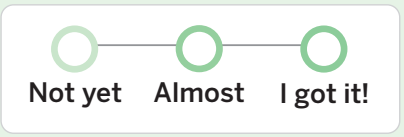
- How can we determine how likely an event is to happen?
- How can we simulate events in the world to make predictions?
- When is a sample representative of a population? How might this affect our analysis?



Explore: The Invention of Fairness
What makes a game "fair" or "unfair"?

Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!



I can . . .	Before	After
Understand that the probability of an event is a number between 0 and 1 that expresses the likelihood of the event occurring.		
Use the results from a repeated experiment to approximate the probability of an event.		
Identify the outcomes in the sample space which compose the event.		
Develop a probability model and use it to calculate probabilities of events.		
Compare probabilities from a model to observed frequencies.		
Predict the relative frequency given the results from a repeated experiment.		
Understand that the probability of a compound (multistep) event is the fraction of outcomes in the sample space for which the compound event occurs.		
Create a list, table, or tree diagram that represents the sample space of a compound (multistep) event.		

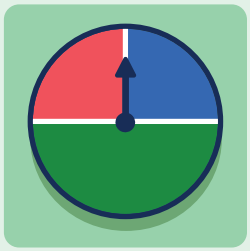
I can . . .	Before	After
Find probabilities of compound (multistep) events.		
Design and use a simulation to generate frequencies for compound events.		
Understand that statistics can be used to gain information about a population by examining a sample of the population		
Use measures of center and measures of variability to compare two populations.		
Make inferences about a population from a random sample.		
Decide if a sample is representative of a population.		
Generate multiple samples of the same size to determine the variation in estimates or predictions.		
Use a sample and proportional reasoning to estimate information about a population.		
Interpret box plots and dot plots that represent real-world situations.		

Sub-Unit

1

Probability

Explore
The Invention of Fairness



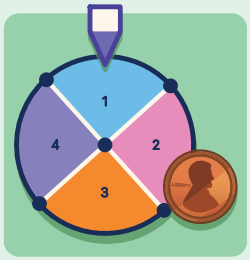
Lesson 1
How Likely?

Lesson 2
Prob-bear-bilities

Lesson 3
Mystery Bag

Lesson 4
Spin Class

Lesson 5
Is It Fair?



Lesson 6
Fair Games

Lesson 7
Weather or Not

Lesson 8
Simulate It!

Name: ..... Date: ..... Period: .....

Probability Models | Building Toward 7.SP.5, SMP.2, SMP.3

Explore: The Invention of Fairness

What makes a game "fair" or "unfair"?

Warm-Up

1. **Discuss:** How can you describe a game that is fair and one that is unfair? Explain your thinking.

Name: ..... Date: ..... Period: .....

Game Time

Data Talk! You will be given two number cubes to play the game.

2. Here are the rules for round 1. Each player rolls two number cubes 10 times, and then gives them to the next player. Players may win points on any roll — not just their own.

a. Play the game.

b. Track the results in the table.

	Wins if . . .	Reward (Points)	Win Tally	Total Points
Player A	The sum of the number cubes is 4.	1		
Player B	The sum of the number cubes is 7.	1		
Player C	The sum of the number cubes is 12.	1		

3. Does this game seem fair? Explain your thinking.

4. If you had the choice, which player would you choose to be the next time the game is played? Explain your thinking.

Name: ..... Date: ..... Period: .....

Game Time (continued)

5. There are new reward points for round 2.

a. **Discuss:** What reward should each player get to make the game more fair?

b. Record what you decide in the table.

c. Play as you did before.

d. Track the results in the table.

	Wins if . . .	Reward (Points)	Win Tally	Total Points
Player A	The sum of the number cubes is 4.			
Player B	The sum of the number cubes is 7.			
Player C	The sum of the number cubes is 12.			

6. Are the new rules more fair? Explain your thinking.

7. List all of the possible sums you can get from rolling two number cubes.

8. What sum(s) do you think will be most likely when rolling two number cubes? Least likely? Explain your thinking.

Name: ..... Date: ..... Period: .....

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet Almost I got it!

I can represent real-world problems and interpret their solutions within the context of the problem.

Not yet Almost I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet Almost I got it!

I can apply the math that I know to solve real-world problems, making assumptions and revising my thinking as needed.

Not yet Almost I got it!

I can select an appropriate tool to help me solve problems.

Not yet Almost I got it!

I can communicate my thinking and solutions clearly to others.

Not yet Almost I got it!

I can look for structure or patterns to help me solve problems.

Not yet Almost I got it!

I can look for repeated calculations and other repeated steps to make generalizations.

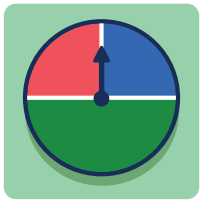
Not yet Almost I got it!

Name: ..... Date: ..... Period: .....

Probability Models | Building Toward 7.SP.5, SMP.3

How Likely?

Let's conduct experiments to determine the likelihood of events.



Warm-Up

1. Here are six events.

a Plot points on the line that represent the likelihood of each event happening.

Point A: A randomly chosen person is right-handed.

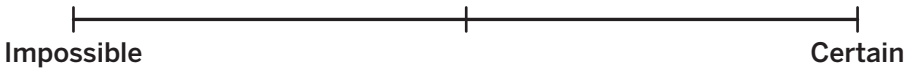
Point B: You guess a stranger's birthday on your first try.

Point C: A fair coin lands heads up.

Point D: A randomly chosen middle school student did not sleep enough last night.

Point E: You dial a random phone number. A famous movie star answers.

Point F: This sentence ends with a period.



b **Discuss:** How did you decide where to plot each point?

Activity 1

Name: ..... Date: ..... Period: .....

Experiment Stations

Number Cube

2. Roll a number cube and record the **outcome** with a tally mark in the table. Repeat this **experiment** 20 times.

1	2	3	4	5	6

3. Write the letter that matches the *likelihood* of each **event** happening on your next roll. Use the number cube to help you decide.

..... Rolling a 1.

..... Rolling a number less than 10.

..... Rolling an even number.

..... Rolling a number greater than 10.

a. Impossible

b. Unlikely

c. Equally likely as not

d. Likely

e. Certain

Coins

4. Flip a coin and record the outcome in the table. Repeat this experiment 10 times.

Heads	Tails

5. Flip two different coins and record the outcome in the table. Repeat this experiment 15 times.

Both Heads	Both Tails	1 Heads and 1 Tails

6. Write the letter that matches the likelihood of each event. Use the coin(s) to help you decide.

..... Flipping one coin and landing heads up.

..... Flipping two coins that both land heads up.

..... Flipping two coins. One lands heads up and one lands tails up.

a. Impossible

b. Unlikely

c. Equally likely as not

d. Likely

e. Certain

Name: ..... Date: ..... Period: .....

Experiment Stations (continued)

Random Letters

7. The letters TENNESSEE are in a bag. Pick a letter from the bag and record the outcome. Then put the letter back in the bag. Repeat this experiment 20 times.

T	E	N	S	P

8. Write the letter that matches the likelihood of each event happening on the next pick. Use the letters TENNESSEE to help you decide.

..... Picking the letter T from the bag.

..... Picking the letter E from the bag.

..... Picking the letters N or T from the bag.

..... Picking a consonant from the bag.

..... Picking the letter P from the bag.

a. Impossible

b. Unlikely

c. Equally likely as not

d. Likely

e. Certain

Spinner

9. Spin the spinner and record each outcome in the table. Repeat this experiment 20 times.

Green	Red	Yellow	Blue

10. Write the letter that matches the likelihood of each event happening on your next spin. Use the spinner to help you decide.

..... The spinner landing on green.

..... The spinner landing on red.

..... The spinner landing on yellow.

..... The spinner landing on blue.

a. Impossible

b. Unlikely

c. Equally likely as not

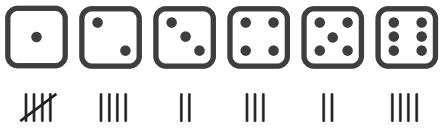
d. Likely

e. Certain

Synthesis

11. Here are the results of a number cube that was rolled 20 times.

Results

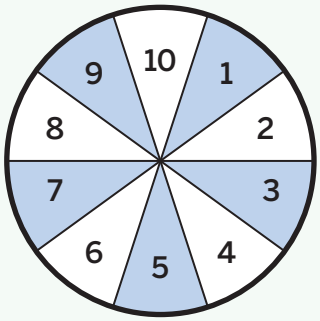


Discuss: How can you determine the likelihood of events? Use the example if it helps with your thinking.

Summary 8.01

When you spin a spinner with 10 equal sections labeled 1–10, one possible **outcome** is that the spinner will land on the number 5. You can describe the likelihood of events using these phrases: impossible, unlikely, equally likely as not, likely, or certain.

Event	How likely is it?
Lands on 5	Unlikely
Lands on an even number	Equally likely as not
Lands on a number that is *not* 5	Likely
Lands on a whole number	Certain
Lands on 12	Impossible



event A set of one or more outcomes in a chance experiment.
chance experiment An experiment that can be performed multiple times, in which the outcome may be different each time.
outcome One of the possible results of a chance experiment.
random Outcomes of a chance experiment are random if they are all equally likely to happen.

Practice
8.01

Name: ..... Date: ..... Period: .....

Problems 1–5: Write the letter that matches the likelihood of each event.

Use the image if it helps with your thinking.

1. ..... Rolling a 7.

2. ..... Rolling a number less than 5.

3. ..... Rolling a 3.

4. ..... Rolling an even number.

5. ..... Rolling a number greater than 0.

a. Impossible

b. Unlikely

c. Equally likely as not

d. Likely

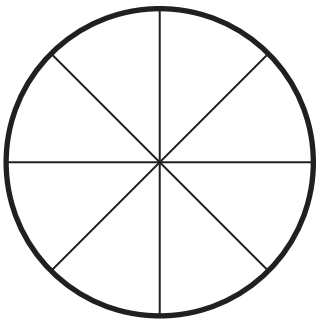
e. Certain

6. A bag contains 14 marbles. There are 6 blue, 7 yellow, and 1 red marble. One marble is selected at random. Which statements are true? Select *all* that apply.

- ☐ **A.** It's impossible that a green marble will be selected.
- ☐ **B.** It's unlikely that a yellow marble will be selected.
- ☐ **C.** It's certain that a blue marble will be selected.
- ☐ **D.** It's unlikely that a red marble will be selected.
- ☐ **E.** It's equally likely as not that a yellow marble will be selected.

7. Label the sections on the spinner so that it has all of these likelihoods in one spin.

- Equally likely as not to spin green (G)
- Impossible to spin blue (B)
- Very unlikely to spin yellow (Y)
- Unlikely to spin red (R)



Name: ..... Date: ..... Period: .....

8. Sahana is curious how many of her classmates watch her favorite T.V. show, so she starts asking around at lunch. She gets the following responses:

yes yes yes no no no no no no yes no no no yes

If she asks one more person randomly in the cafeteria, do you think they will say "yes" or "no"? Explain your thinking.

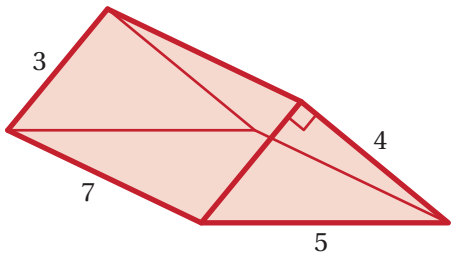
Spiral Review

9. Order these numbers from *least* to *greatest*.

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{5}$	0.6	0.3

Least **Greatest**

Problems 10–11: Here is a triangular prism. All of the measurements are in centimeters.

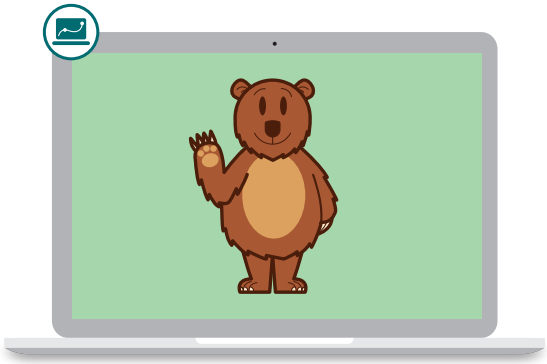


10. Calculate the volume of the prism.

11. Calculate the surface area of the prism.

Name: ..... Date: ..... Period: .....

Probability Models | Proportional Relationships | 7.SP.5, 7.EE.3, 7.RP.1, 7.SP.7.a, SMP.5, SMP.6



Prob-bear-bilities

Let's express probabilities as numbers.

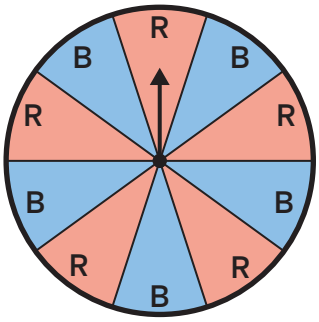
Warm-Up

1 Which game would you rather play?

A. Roll a number cube. Win if it lands on a number greater than 1.

B. Spin this spinner. Win if it lands on red.

Explain your thinking.



Activity 1

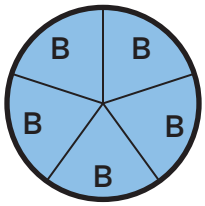
Name: ..... Date: ..... Period: .....

Intro to Probability

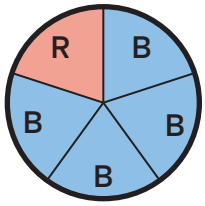
2 Here are some other spinners with different sections of red and blue.

Probability of Spinning Red

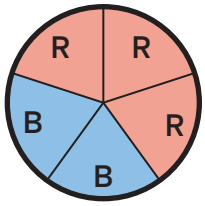
$\frac{0}{5}$ or 0%



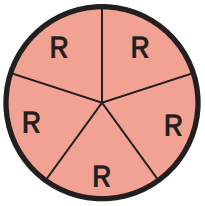
$\frac{1}{5}$ or 20%



$\frac{3}{5}$ or 60%

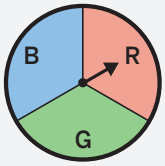


$\frac{5}{5}$ or 100%



Describe how you think **probability** is determined.

3 For each event, put a check mark under the probability that matches it.

		Probability		
Event		0.25	$\frac{1}{3}$	50%
Picking a purple block from this bag.				
Picking a purple block from this bag.				
Randomly selecting the letter E from this word.	D I C E			
A number cube lands on an even number.				
This spinner lands on red.				

Name: ..... Date: ..... Period: .....

Intro to Probability (continued)

4 For each challenge, order the events by likelihood. Complete as many challenges as you have time for.

a
- The probability of an event is 50%.
- A spinner with 5 equal sections, 2 of which are red, lands on red.
- The probability of an event is $\frac{3}{4}$.

Least Likely **Most Likely**

b
- Equally likely as not
- The probability of an event is $\frac{1}{4}$.
- A spinner with 5 equal sections, 4 of which are red, lands on red.

Least Likely **Most Likely**

c
- A spinner with 5 equal sections, 4 of which are red, lands on red.
- The probability of an event is 75%.
- A fair coin lands tails up.

Least Likely **Most Likely**

d
- You draw a red block from a bag with 6 green blocks and 2 purple blocks.
- The probability of an event is 20%.
- The outcome of an event is certain.

Least Likely **Most Likely**

Activity 2

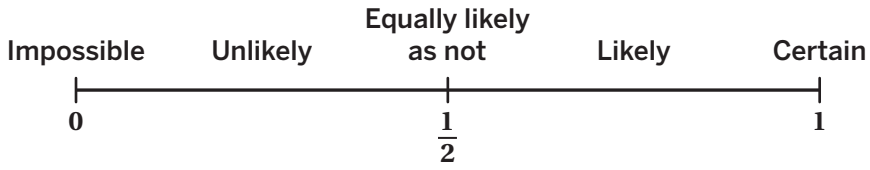
Name: ..... Date: ..... Period: .....

Probabilities and Sample Spaces

5 Here is a randomizer that chooses one character on each spin.

a Use the Activity 2 Sheet to try out the randomizer.

b Plot a point on the line to show how likely you think it is to get a bear on one spin.



6 A **sample space** is the collection of all possible outcomes of an experiment.

The sample space in the previous problem has 4 characters.

Explain why the probability of spinning a bear is $\frac{1}{4}$.

Activity 3

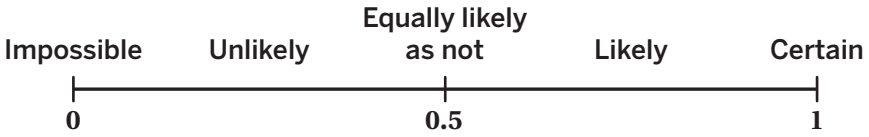
Name: ..... Date: ..... Period: .....

Prob-bear-ly Mismatched

7 Here is a different randomizer that chooses one head and one body on each spin.

a Use the Activity 3 Sheet to try out the randomizer.

b Plot a point on the line to show how likely you think it is to get a whole bear on one spin.



8 How many different characters are in this randomizer's sample space?

Explain your thinking.

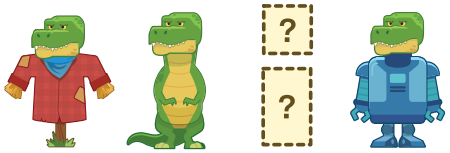
Name: ..... Date: ..... Period: .....

Prob-bear-ly Mismatched (continued)

9 This drawing of a sample space is missing one character.

Describe the head and body of the character that will complete the sample space.

Draw a sketch if you have time.



10 Use the completed sample space to answer:

a What is the probability of spinning a character that is at least part bear?

b What is the probability of spinning a character with mismatched parts?

You're invited to explore more.

11 Here is a different randomizer that chooses one head, one body, and one set of legs on each spin. Use the You're Invited to Explore More Sheet to try out the randomizer.

What is the probability of the randomizer choosing all three parts from the same character?

Explain your thinking.

12 Synthesis

Discuss: How can you use a sample space to help you determine the probability of an event?

15 Summary 8.02

Probabilities are numbers between 0 and 1 written as fractions, decimals, or *percentages*. A probability of 1 means the event will always happen. A probability of 0 means the event will never happen.

Here are some examples of events and their probabilities.

Example	Probability
Picking a green marble out of a bag that contains only red and yellow marbles.	0
Rolling a 1 on a number cube.	$\frac{1}{6}$ (or equivalent)
Flipping a coin and it landing heads up.	50% (or equivalent)
Picking a yellow marble from a bag of 10 marbles, where 8 of the marbles are yellow.	0.8 (or equivalent)
Picking a green marble in a bag that only contains green marbles.	1

probability A number that tells us how likely an event is to happen.

sample space The list of every possible outcome for a chance experiment.

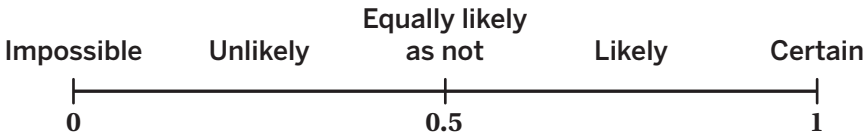
uniform probability model A model in which every outcome has an equal probability of occurring.

Practice
8.02

Name: ..... Date: ..... Period: .....

1. Place a point on the line to show each probability's likelihood from impossible to certain.

60% 8 out of 10 0.37 20% $\frac{5}{6}$



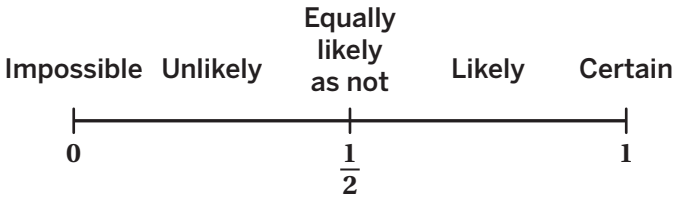
Problems 2–4: List the sample space for each experiment. For example, the sample space of flipping one fair coin is heads or tails.

2. Selecting a random season of the year.

3. Selecting a random day of the week.

4. Rolling a number cube.

Problems 5–10: Determine the probability of each event. Use the line if it helps with your thinking.



5. You read this sentence.

6. A fair coin lands heads up when flipped.

7. A weekend day is selected randomly from the days of the week.

8. You see a live unicorn outside today.

9. A spinner with equal parts red, yellow, and green lands on green on the next spin.

10. You pick a green block when you randomly pick one block from a bag with 7 green blocks and 3 blue blocks.

Problems 11–14: A computer randomly selects a letter from the English alphabet.

11. List the sample space.

12. How many different outcomes are possible?

13. What is the probability the computer selects a vowel (A, E, I, O, or U)?

14. What is the probability the computer selects the first letter of your first name?

15. This table shows the number of bags of different flavors of crackers in a large box. A bag will be selected at random from the box.

Flavor of Cracker	Number of Bags
Plain	12
Barbeque	16
Ranch	8
Cheddar Cheese	20

Which statement about the flavor of the crackers chosen is best supported by the information in the table?

A. Plain is the least likely flavor.

B. Choosing barbecue is twice as likely as choosing ranch.

C. There is an equal chance of choosing plain, barbeque, ranch, or cheddar cheese.

D. Choosing cheddar cheese is twice as likely as choosing ranch.

Spiral Review

Problems 16–17: e represents an object's weight on Earth and m represents that same object's weight on the Moon. The equation $m = \frac{1}{6}e$ represents the relationship between these quantities.

16. What does $\frac{1}{6}$ represent in this situation?

17. If a person weighs 24 pounds on the Moon, how many pounds would they weigh on Earth?

Name: ..... Date: ..... Period: .....

Probability Models | Proportional Relationships | Unit Rates in the World

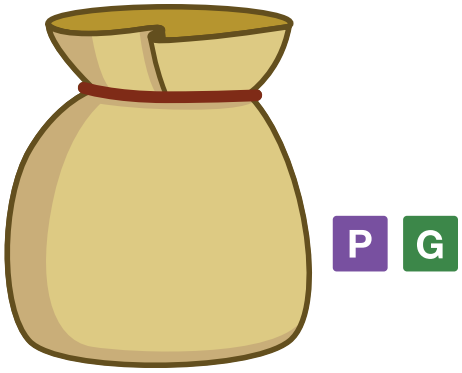
7.SP.5, 7.SP.6, 7.RP.2.b, SMP.1, SMP.3

Mystery Bag

Let's use repeated experiments and proportionality to predict what is inside a mystery bag.

Warm-Up

1 Here is a bag that you can put blocks into.



a Draw blocks in the bag so the probability of picking a green block (G) is 40%.

b Using your bag, select the statement you think is true.

If you pick a block and place it back in the bag after each pick . . .

- ☐ After 10 tries, you will always pick exactly 4 green blocks.
- ☐ After 100 tries, you will most likely pick 40% green blocks.

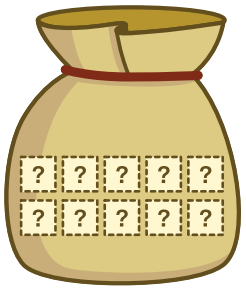
Activity 1

Name: ..... Date: ..... Period: .....

What's in the Bag?

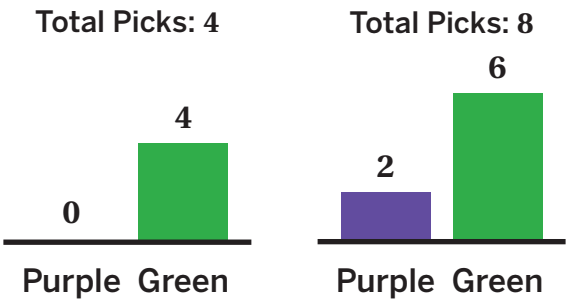
2 Here is Rishi's bag. It has 10 blocks.

Note: Each time he picks a block, it is put back into the bag before the next pick.



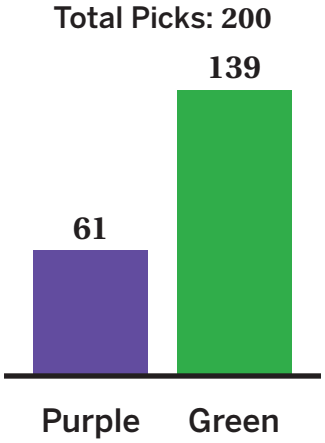
These are the results after 4 picks and after 8 picks.

How many of the 10 blocks do you think are green?



3 This graph shows the results from 200 picks.

Use these results to decide how many of the 10 blocks you think are green. Explain your thinking.



4 Here is how Nasir decided the number of green blocks in the bag.

Explain Nasir's strategy.

Nasir

g = number of green blocks

$$\frac{139}{200} = \frac{g}{10}$$

$$0.695 = \frac{g}{10}$$

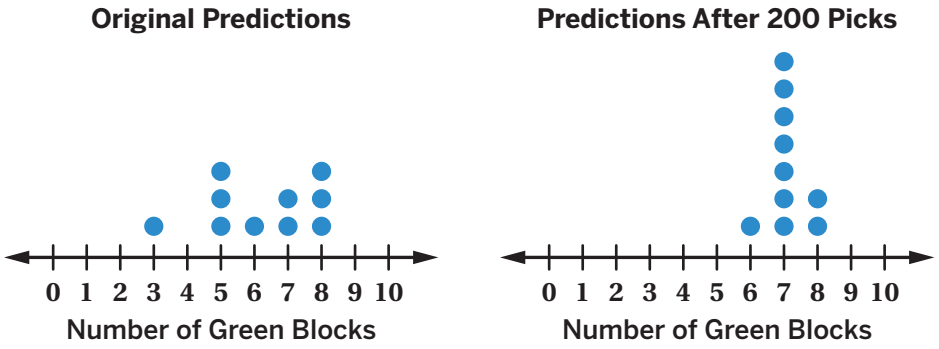
$$10 \cdot 0.695 = \frac{g}{10} \cdot 10$$

$$g \approx 7 \text{ blocks}$$

Name: ..... Date: ..... Period: .....

What's in the Bag? (continued)

5 These graphs show the original predictions and updated predictions from another class.



Discuss: What do you notice and wonder?

6 Let's reveal what's in the bag.

Discuss: How did this compare to what you predicted?

Activity 2

Name: Date: Period:

Blocks in the Bag

7 Brianna's bag has 8 blocks.

The graph shows the results from 20 picks.

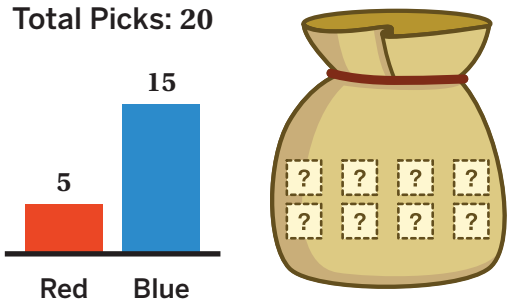
Order the contents of Brianna's bag from *least* likely to *most* likely.

a. 1 red, 7 blue

b. 2 red, 6 blue

c. 6 red, 2 blue

d. 0 red, 8 blue



Least Likely **Most Likely**

8 Keya and Ashley were trying to decide which color blocks were in Brianna's bag.

- Keya says there are 2 red and 6 blue blocks.
- Ashley says there is 1 red and 7 blue blocks.

Who do you agree with? Circle one.

Keya Ashley Neither

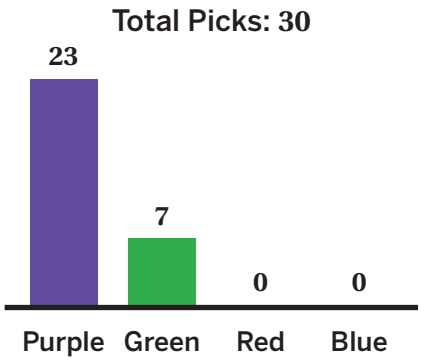
Explain your thinking.

Name: ..... Date: ..... Period: .....

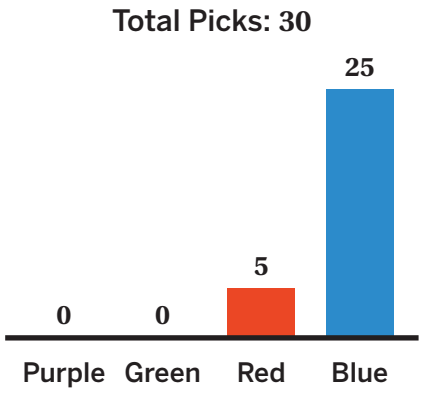
Blocks in the Bag (continued)

9 Use the results to determine how many blocks of one color are likely to be in each bag. Complete as many problems as you have time for.

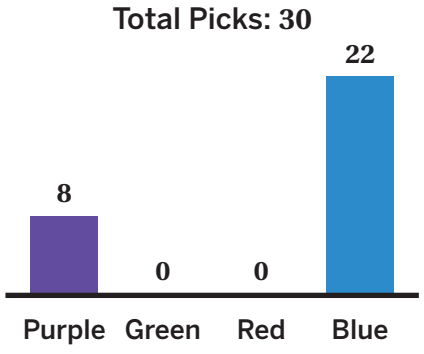
a There are 4 blocks in a bag. How many of the blocks are likely to be purple?



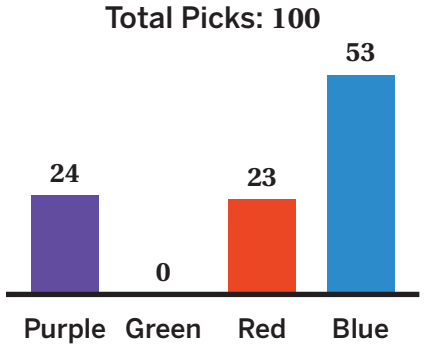
b There are 4 blocks in a bag. How many of the blocks are likely to be red?



c There are 6 blocks in a bag. How many of the blocks are likely to be purple?



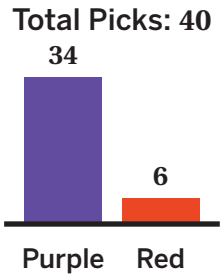
d There are 8 blocks in a bag. How many of the blocks are likely to be blue?

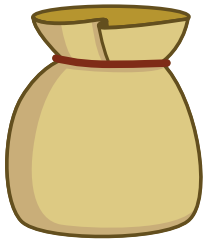


10 Synthesis

Describe how you can use results from a repeated experiment to make predictions.

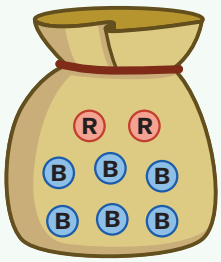
Use the results shown if that helps with your thinking.





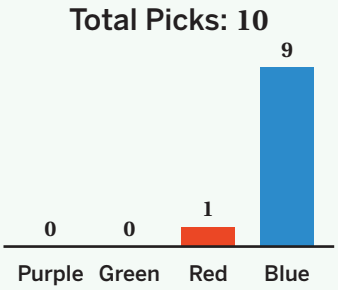
13 Summary 8.03

In situations where you don't know the sample space, you can use data from experiments and proportional reasoning to predict what the sample space is. Repeating an experiment more times can help make your prediction more accurate.



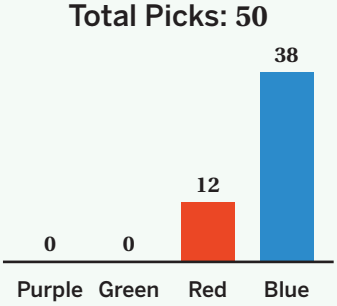
For example, here are the results from picking a marble out of a mystery bag 10 times. The bag has 8 marbles in it.

Only 1 out of the 10 marbles was red, so the *constant of proportionality* is 0.1. Multiplying 0.1 times the number of marbles in the bag (8), may lead someone to predict that there is only $0.1 \cdot 8 = 0.8$ or 1 red marble in the bag.



After 50 picks, the constant of proportionality (0.24) times the number of marbles in the bag is $0.24 \cdot 8 = 1.92$ which is close to 2.

This is a more accurate prediction of the number of red marbles in the bag.



Practice
8.03

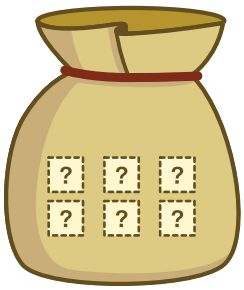
Name: ..... Date: ..... Period: .....

Problems 1–2: A month is chosen at random from the months of the year.

1. What is the probability of getting a month that starts with the letter "A"? Consider listing the sample space to show your thinking.

2. If you repeat this experiment 600 times, how many times do you expect to get a month that starts with "A"?

3. A bag has 6 blocks in it. Joel picks a block out of the bag 60 times. He gets a green block 43 times. Based on these results, how many blocks do you expect to be green? Explain your thinking.

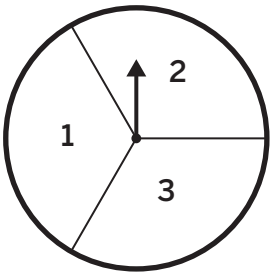


4. Miko spun a spinner with numbered sections 15 times. Here are the results. Which spinner is most likely the one Miko used?

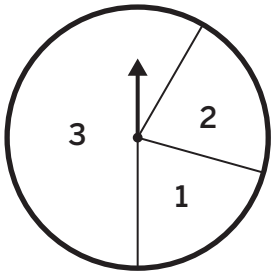
Miko's results:

3, 3, 3, 1, 3, 2, 2, 3, 3, 1, 3, 1, 3, 3, 2

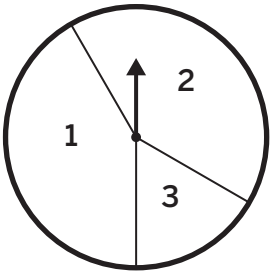
A.



B.



C.



Explain your thinking.

Problems 5–7: A textbook has 428 numbered pages, starting with 1. You are equally likely to stop on any of the pages if you flip through the book randomly.

5. What is the probability that you turn to page 45?

6. What is the probability that you turn to an even-numbered page?

7. If you repeat this experiment 50 times, about how many times do you expect to turn to an even-numbered page?

Name: ..... Date: ..... Period: .....

8. The table shows the number of different colors of crayons in a case. A 7th grader will randomly choose one crayon from the case.

Color	Number of Crayons
Blue	2
Red	8
Orange	4
Yellow	5

Based on the information in the table, which statement is true?

A. The crayon is least likely to be orange.

B. The crayon is 4 times as likely to be red as it is to be blue.

C. The crayon is equally likely to be orange or yellow.

D. The crayon is more likely to be red than all other colors combined.

Spiral Review

Problems 9–12: For each word, determine the probability of selecting the letter "A" at random.

9. LAMB

10. SAFETY

11. ALABAMA

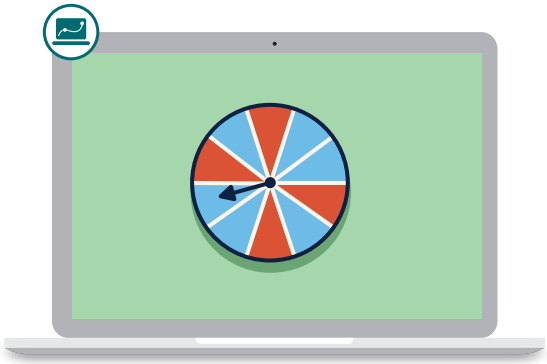
12. SCORE

13. Select *all* the situations in which the surface area would be useful to know.

- ☐ **A.** Ordering tiles to replace the roof of a house.
- ☐ **B.** Estimating how long it will take to clean the windows of a greenhouse.
- ☐ **C.** Deciding whether leftover soup will fit in a container.
- ☐ **D.** Estimating how long it will take to fill a swimming pool with a garden hose.
- ☐ **E.** Buying fabric to sew a couch cover.

Name: ..... Date: ..... Period: .....

Probability Models 7.SP.6, 7.SP.7, 7.SP.7.b, SMP.1, SMP.3, SMP.8

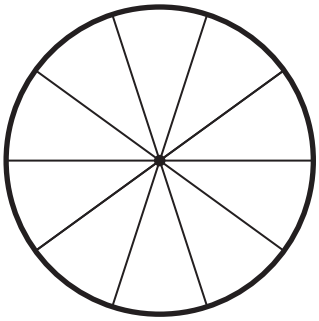


Spin Class

Let's explore what happens to results when you perform an experiment repeatedly.

Warm-Up

1 a Use the colors blue (B) and red (R) to create a spinner that has a 40% chance of spinning red.



b Spin your spinner several times or watch a spinner on the screen.

Record your results in a way that helps you keep track of the spins and outcomes.

Activity 1

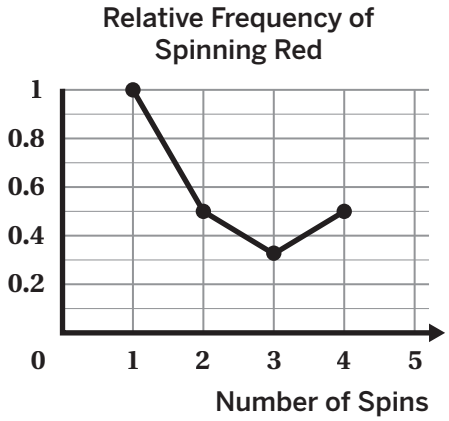
Name: ..... Date: ..... Period: .....

Graphs of Repeated Experiments

2 Amari and Nathan made a spinner together and spun it 4 times.

They recorded the fraction of spins that are red, which is called the **relative frequency**.

Discuss: What do you notice and wonder?



Spin	Outcome	Relative Frequency of Spinning Red
1	Red	$\frac{1}{1}$
2	Blue	$\frac{1}{2}$
3	Blue	$\frac{1}{3}$
4	Red	$\frac{2}{4}$

3 Amari and Nathan's fifth spin was red.

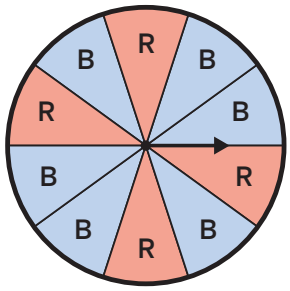
Use this information to complete the table and add a point to the graph from the previous problem.

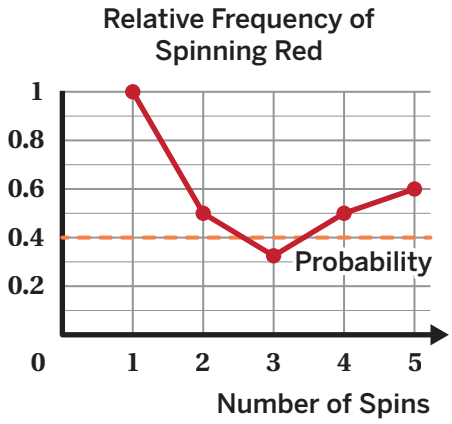
4 Here is the spinner that Amari and Nathan made.

Nathan drew a line on his graph showing the probability of spinning red.

He thinks at least one of the points should have been on the line.

Do you agree? Explain your thinking.

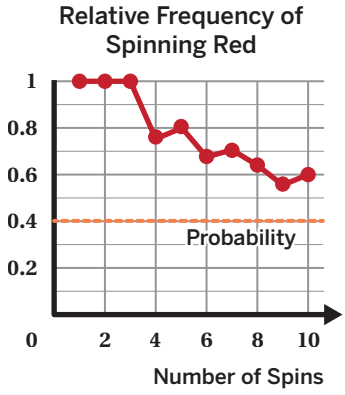


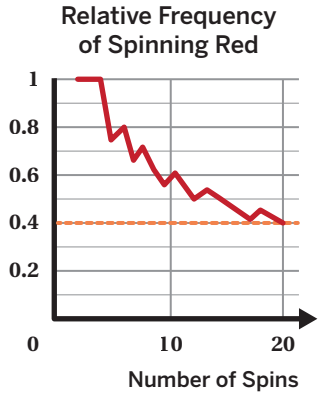


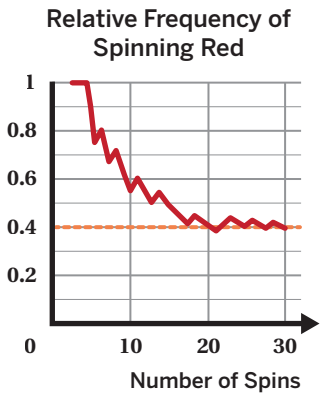
Name: ..... Date: ..... Period: .....

Graphs of Repeated Experiments (continued)

5 Here are the results from spinning Amari and Nathan's spinner 10, 20, and 30 times.

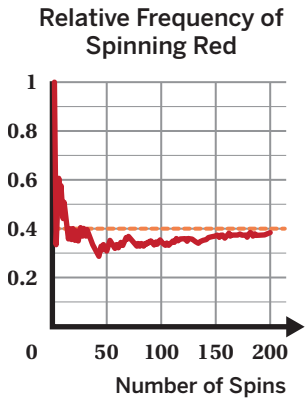


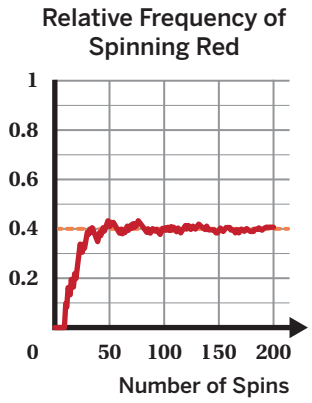


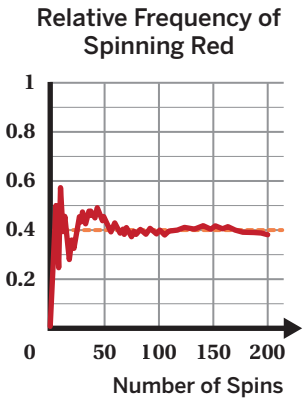


Discuss: What happens to the relative frequency of spinning red as the number of spins increases?

6 These graphs show the results of three different students spinning Amari and Nathan's spinner 200 times each.







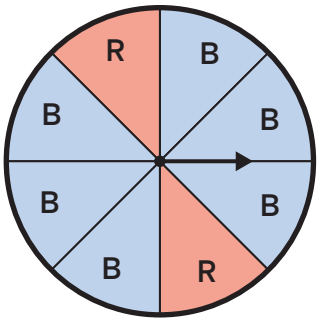
Describe how the graphs are alike and different.

Activity 2

Name: ..... Date: ..... Period: .....

Build a Spinner

7 Here is a new spinner. Order these results from *least* likely to *most* likely.



a. After 10 spins, at least half are red.

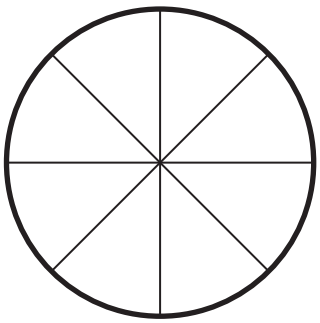
b. After 100 spins, at least half are blue.

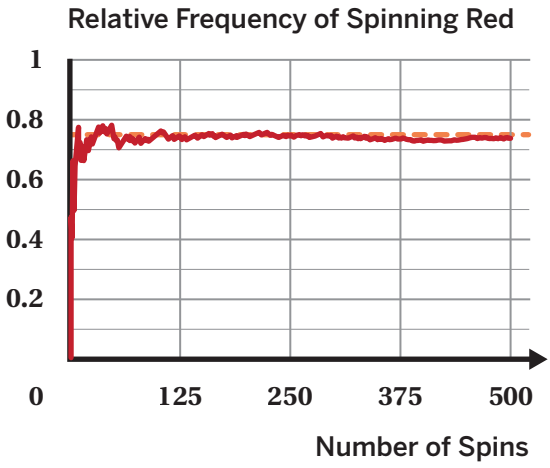
c. After 100 spins, at least half are red.

d. After 10 spins, at least half are blue.

Least Likely **Most Likely**

8 Create a red (R) and blue (B) spinner that could have produced this graph.





Activity 3

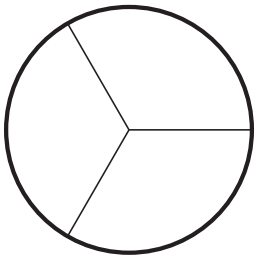
Name: ..... Date: ..... Period: .....

Challenge Creator

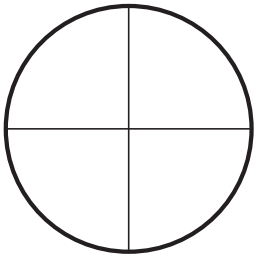
9 You will use the Activity 3 Sheet to create your own spinner challenge.

- On this page, choose *one* spinner. Label each section red (R) or blue (B). Determine the probability of spinning red on your spinner and record it.

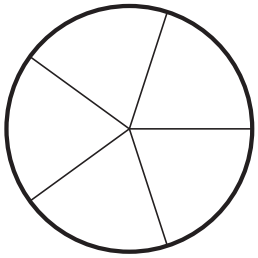
3-Section Spinner



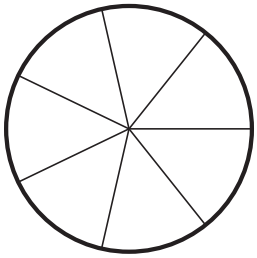
4-Section Spinner



5-Section Spinner



7-Section Spinner



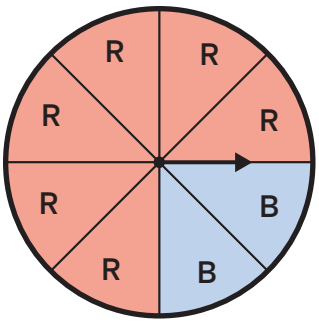
- On the Activity 3 Sheet, sketch a graph like the ones in Activity 2 based on the probability of spinning red (R) on your spinner. Include a dashed line that shows the probability.
- Swap your challenge with one or more partners. Take turns describing what the spinner that would produce the graph might look like.

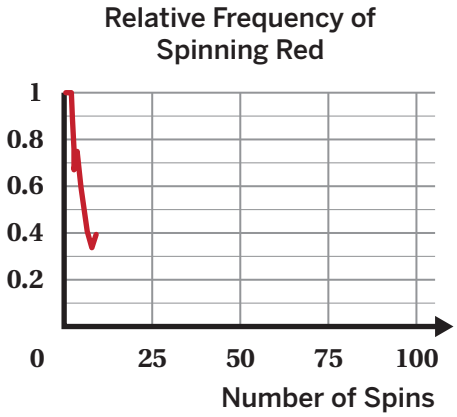
	Partner 1	Partner 2	Partner 3	Partner 4
Probability of Spinning Red (R)				
What Their Spinner Might Look Like				

10 Synthesis

Describe how this graph might look if you spin many more times.

Sketch the rest of the graph if it helps with your thinking.





13 Summary 8.04

Graphing the outcomes of a repeated experiment can help you make sense of how likely an event is to occur. The fraction of experiments that result in a certain outcome is called the **relative frequency** of that outcome.

When an experiment is only repeated a few times, the results may surprise you because they may be far from the probability you expected. As the number of experiments increases, the relative frequency of each outcome gets close to its probability.

For example, the probability of a flipped coin landing heads up is $\frac{1}{2}$ or 0.5. This means that if the coin is flipped many times, we expect it to land heads up about half of the time.

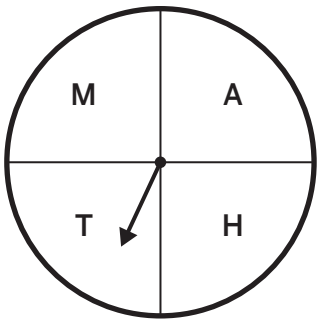
If the coin landed tails up three times in a row, this does not mean that the next flip is more likely to be heads up. The chances of landing heads up are the same on each flip regardless of the previous outcomes.

relative frequency The number of times an outcome happens divided by the number of repeated experiments. After several experiments, the relative frequency of an outcome gets close to its probability.

Practice
8.04

Name: Date: Period:

Problems 1–2: A spinner has four equal sections, with one letter from the word MATH in each section.



1. You spin the spinner 20 times. About how many times do you expect it will land on "A"?

   Explain your thinking.

2. You spin the spinner 80 times. About how many times do you expect it will land on something other than "A"?

   Explain your thinking.

Problems 3–5: A fair coin is flipped 100 times.

3. About how many times would you expect the coin to land heads up?

4. What is another number of times that would be likely for it to land heads up?

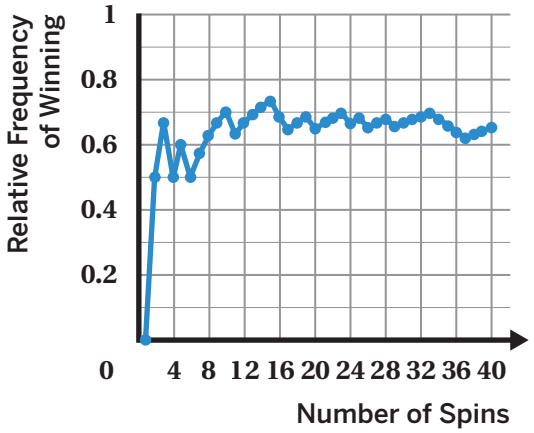
5. Adah says it's impossible for the coin to land heads up all 100 times. Basheera says it's possible, but unlikely. Who is correct?

   Explain your thinking.

6. A spinner is spun 40 times for a game. This graph shows the relative frequency of winning the game.

   Based on the graph, estimate the probability of winning this game.

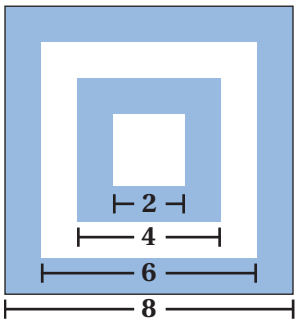
   Explain your thinking.



Practice
8.04

Name: ..... Date: ..... Period: .....

Problems 7–8: Here is a square dartboard.

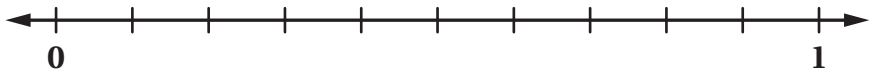


7. Is a dart thrown at random more likely to land in the shaded or the unshaded area? Show or explain your thinking.

8. A dart is thrown at the board 100 times at random. About how many times is the dart likely to land in the shaded area? Explain your thinking.

Spiral Review

9. Plot each number on the number line: 0.5, 0.75, 0.33, 0.67, 0.25



Problems 10–11: Solve each inequality.

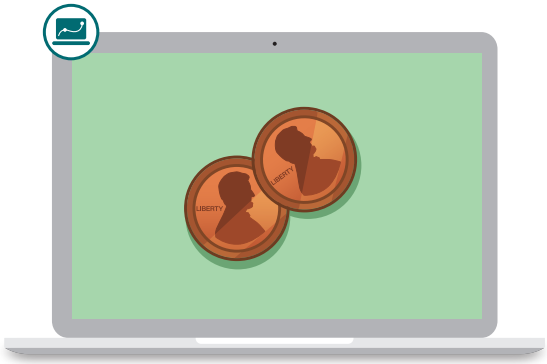
10. $12 < 14 - 3x$

11. $2x + 12 > -6$

Unit 8
Lesson
5

Name: ..... Date: ..... Period: .....

Probability Models | Proportional Relationships | 7.SP.7.b, 7.SP.6, 7.SP.7, 7.RP.3, SMP.2. SMP.6, SMP.8



Is It Fair?

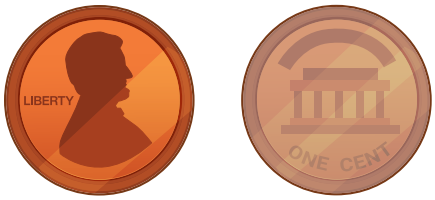
Let's decide what it means to be fair.

Warm-Up

1 Some of these coins are *unfair*.

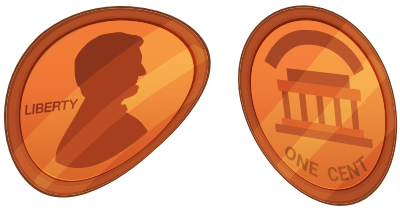
a Take a look at each coin.

Coin A



Coin B

Coin C



Coin D

b Select one coin and describe why you think it's unfair.

Activity 1

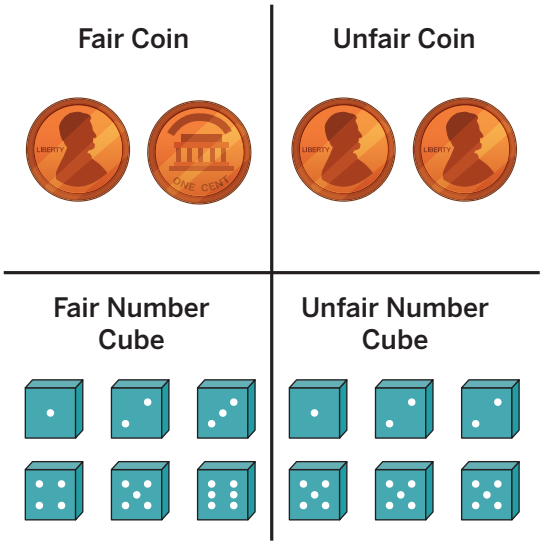
Name: ..... Date: ..... Period: .....

Fair or Not?

2 Let's investigate *fair* and *unfair* objects.

With fair objects, there is an equal probability that each outcome will occur.

Flip a fair coin 10 times and record your results.



a Did you get the results you expected?

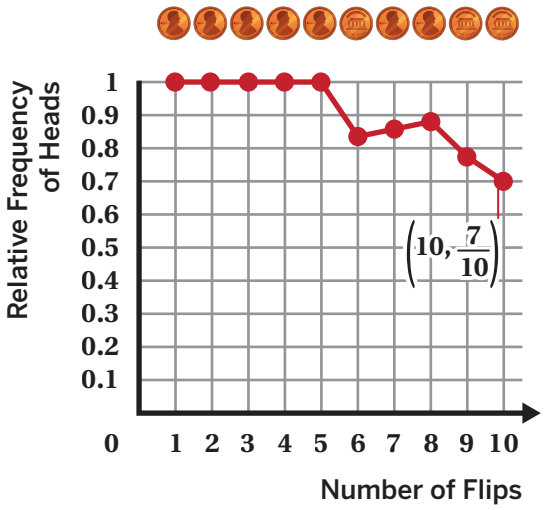
b Select *all* the results where you would suspect a coin is unfair.

- ☐ **A.** 5 heads, 5 tails
- ☐ **B.** 6 heads, 4 tails
- ☐ **C.** 8 heads, 2 tails
- ☐ **D.** 9 heads, 1 tails
- ☐ **E.** 10 heads, 0 tails

3 One coin was flipped 10 times. The image shows that the coin landed 7 times heads up and 3 times tails up.

Do you think this is a fair coin?

4 **Data Talk!** Here is a graph of the flips from the previous problem.



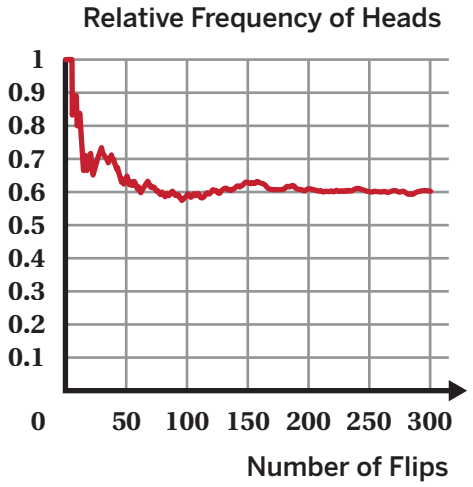
a What does the point $\left(10, \frac{7}{10}\right)$ mean?

b How might a graph help you determine if a coin is fair?

Name: ..... Date: ..... Period: .....

Fair or Not? (continued)

5 **Data Talk!** A coin was flipped 300 times. Based on the results, is the coin fair? Explain your thinking.



6 Based on these results, what is the probability of the next flip landing heads up?

Activity 2

Name: ..... Date: ..... Period: .....

How Likely?

7 Here is a game involving a number cube.

What do you think is the probability that Player 2 wins on any single roll?

Roll an odd number: Player 1 wins	Roll an even number: Player 2 wins

8 **a** Play a few rounds of the game using a number cube. Record your results in the table.

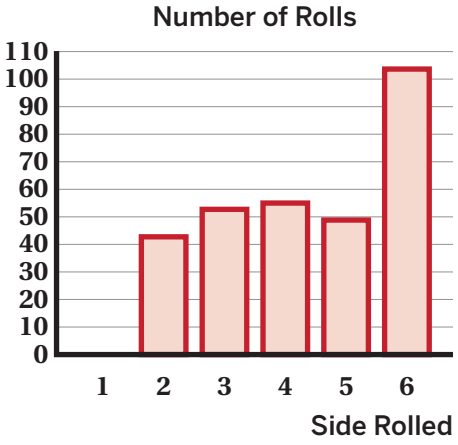
Player 1	Player 2

b Do you think this is a fair number cube? Explain your thinking.

9 Here are the results from two students who played the game and rolled a number cube 300 times.

a What do you think is the probability that Player 2 wins?

b **Discuss:** About how many times do you expect Player 2 to win if the number cube is rolled 600 times? Explain your thinking.



Name: ..... Date: ..... Period: .....

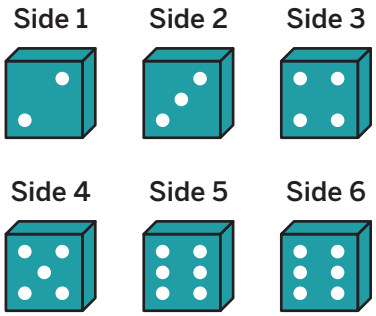
How Likely? (continued)

10 Here is the number cube used by the two students in the previous problem.

Create a game with this number cube that would be fair.

Player 1 wins if . . .

Player 2 wins if . . .



You're invited to explore more.

11 In this game, players will flip the same coin twice.

The rules are:

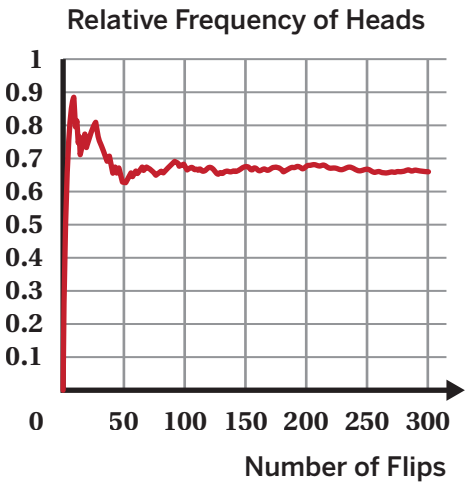
- Flip heads, then tails: Player 1 wins.
- Flip tails, then heads: Player 2 wins.
- If the results match, no one wins.

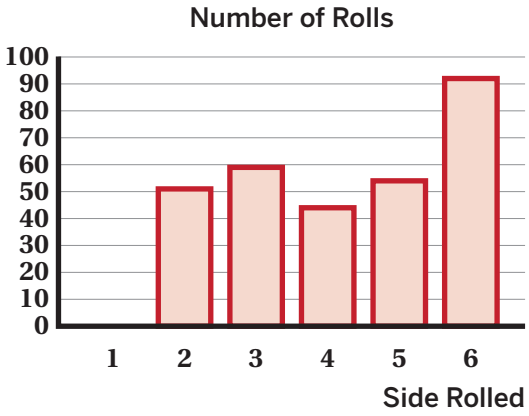
Explain why this game is fair even if the coin is unfair.

12 Synthesis

Discuss: How can you use a repeated experiment to decide whether an object is fair?

Use the results of these experiments if they help with your thinking.





15 Summary 8.05

Repeated experiments can help you decide if an object is fair.

If an experiment is repeated only a few times, the results may not be what you expect, even if the object is fair. The more times you repeat the experiment (i.e., hundreds or thousands of times), the closer the relative frequency should get to the probability. This allows you to make a better decision about whether the object is fair.

For example, here is a fair coin. The probability of this coin landing heads up is $\frac{1}{2}$.

If you flip the coin only 3 times, it may land heads up all 3 times. You may think the coin is unfair, but continuing to repeat the experiment can change your perspective.

If you flipped the coin 1,000 times, it would land heads up about half of the time because the sample space of this event is "heads" and "tails."

Practice
8.05

Name: ..... Date: ..... Period: .....

Problems 1–3: Deja has a six-sided number cube.

1. If this were a standard number cube, what would be the probability that the cube lands on a five?

2. Deja suspects the six-sided number cube is not standard.

- Deja rolled a five 40 times out of 100.
- Manuel rolled a five 21 times out 50.
- Santino rolled a five 11 times out of 30.

Based on these results, estimate the probability of rolling a five.

3. Is it likely this is a standard number cube? Explain your thinking.

Problems 4–5: Santino wants to know if his coin is fair, so he flips it 10 times. It lands heads up 3 times and tails up 7 times.

4. Are these results enough to determine if the coin is fair? Explain your thinking.

5. What could Santino do to be more sure of his results?

Problems 6–9: A game is played with two tetrahedral (4-sided) dice. The dice are in the shape of a pyramid with the numbers 1, 2, 3, and 4 written on each triangular side. After the dice are rolled, the numbers on the sides facing down are added together.

6. What are all of the possible sums for rolling two tetrahedral dice?

7. What sum do you think you will be most likely to get when rolling these dice? Least likely? Explain your thinking.

8. Create rules for a fair two-player game that uses these dice.

Player A wins if...

Player B wins if...

Name: _____ Date: _____ Period: _____

9. Explain why you think your game in Problem 8 is fair.

10. A random number generator selects a digit from 1 to 5. Troy uses the generator 1,500 times. Which statement best predicts how many times the digit 3 will appear among the 1,500 results?

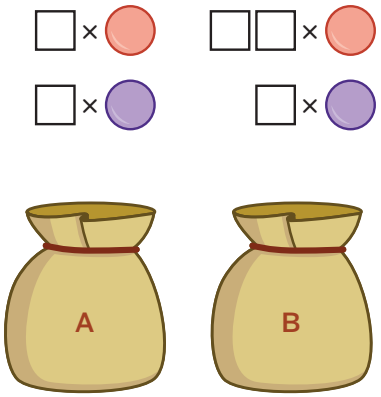
A. It will appear exactly 300 times.

B. It will appear close to 300 times but probably not exactly 300 times.

C. It will appear exactly 340 times.

D. It will appear close to 340 times but probably not exactly 340 times.

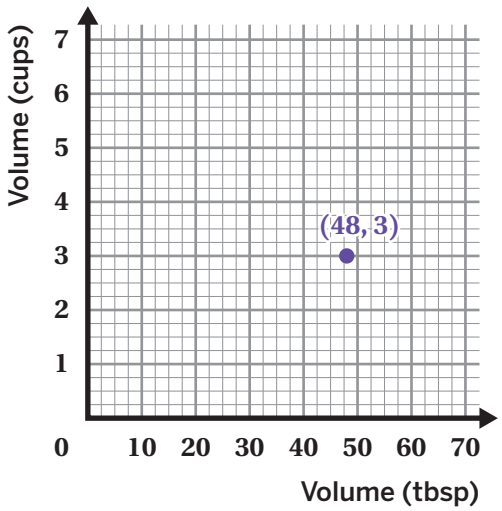
11. Fill in each blank using the digits 0 to 9 only once so that the probability of drawing a purple marble is the same for each bag.



Spiral Review

Problems 12–15: There is a proportional relationship between sugar measured in cups and sugar measured in tablespoons. For example, 48 tablespoons of sugar is equivalent to 3 cups of sugar.

12. Plot and list the coordinates of two more points that represent the relationship.



13. What is the constant of proportionality for this relationship?

14. Write an equation representing this relationship. Use c for cups and t for tablespoons.

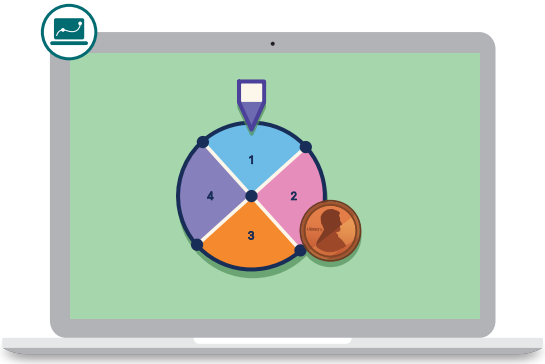
15. How many teaspoons of sugar are there for 4.5 cups of sugar?

Name: ..... Date: ..... Period: .....

Probability Models 7.EE.3, 7.SP.8, 7.SP.8.a, 7.SP.8.b, SMP.5

Fair Games

Let's determine whether or not games are fair.



Warm-Up

1 Here is a game involving two fair coins. Player 1 wins if both coins land heads up (HH) or both coins land tails up (TT). Otherwise, Player 2 wins.

Play a few rounds, then answer: *Which player would you rather be in the game?*

2 Charlie made an organized list and claims that the game is not fair because Player 1 is twice as likely to win as Player 2.

Is Charlie's claim correct? Circle one.

Yes No I'm not sure

Update the organized list as needed. Explain your thinking.

H H → Player 1 wins!

T T → Player 1 wins!

H T → Player 2 wins!

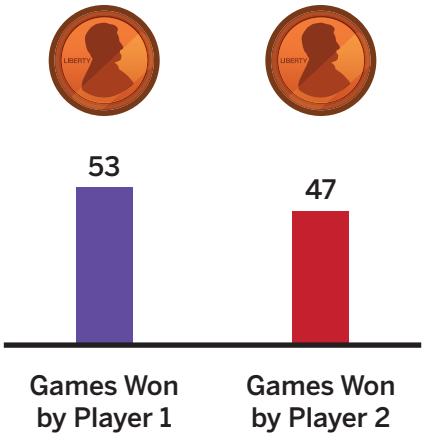
Activity 1

Name: ..... Date: ..... Period: .....

Coins and Tree Diagrams

3 The game from the Warm-Up was played 100 times. Here are the results.

Discuss: Why is this a fair game?

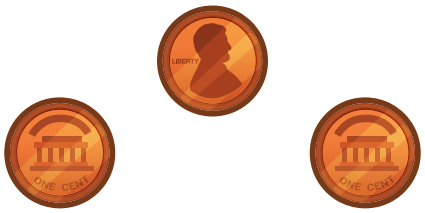


4 Here's a twist: there are 3 coins to flip.

Like before, Player 1 wins if the coins land all heads up or all tails up.

Otherwise, Player 2 wins.

Do you think the game is fair? Why or why not? Play the game and record your results in the scoreboard if it helps with your thinking.



Player 1	Player 2

5 Andrea is playing as Player 1 and wants to know her probability of winning.

a Create an organized list and circle the outcomes that represent Andrea winning.

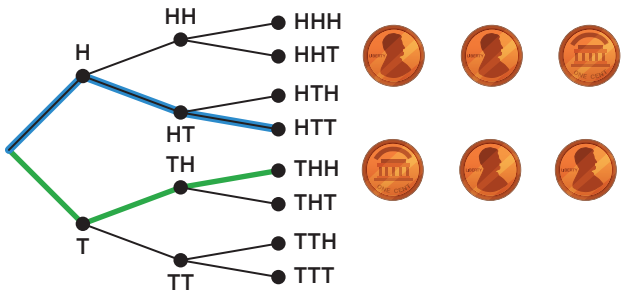
b What is the probability that Player 1 wins (all heads or all tails)?

Name: ..... Date: ..... Period: .....

Coins and Tree Diagrams (continued)

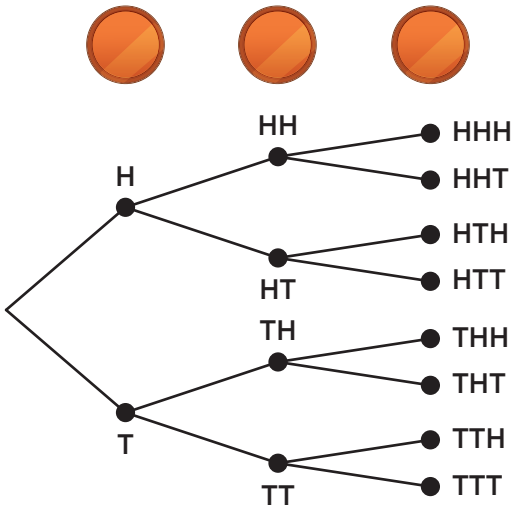
6 A **tree diagram** is another way to represent the sample space of **compound events**. This tree diagram represents flipping 3 coins.

a Explain why the probability Player 1 wins is $\frac{2}{8}$. Use the tree diagram to explain your thinking.



b **Discuss:** How does this compare to your conclusion in Screens 4–5? Do you still think the game is fair?

7 Use the tree diagram to help answer:



a What is the probability of *exactly* 1 coin landing tails up?

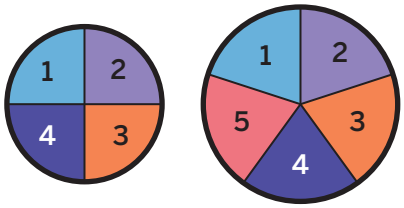
b What is the probability of *at least* 1 coin landing tails up?

Activity 2

Name: ..... Date: ..... Period: .....

Spinners and Tables

8 Here is a new game involving these two spinners. Player 1 wins if the numbers are *the same or one apart*. Otherwise, Player 2 wins.



a Use the Activity 2 Sheet to play a few rounds of the game with a partner. Record the results in the table.

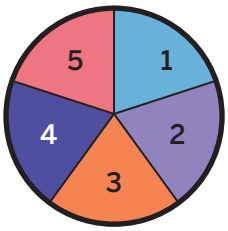
Player 1	Player 2

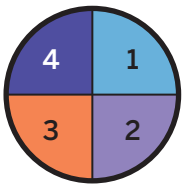
b Make a tree diagram to represent the sample space.

9 Here is a table representing the game. Each space in the table represents a possible outcome. Compare this table to the tree diagram you made.

How are they alike?

How are they different?





	1	2	3	4	5
1	1, 1	1, 2	1, 3	1, 4	1, 5
2	2, 1	2, 2	2, 3	2, 4	2, 5
3	3, 1	3, 2	3, 3	3, 4	3, 5
4	4, 1	4, 2	4, 3	4, 4	4, 5

Name: ..... Date: ..... Period: .....

Spinners and Tables (continued)

10 Player 1 wins if the numbers are *the same or one apart*. Otherwise, Player 2 wins. Use your tree diagram, the table, or another tool to help you answer:

a What is the probability of Player 1 winning?

b What is the probability of Player 2 winning?

	1	2	3	4	5
1	1, 1	1, 2	1, 3	1, 4	1, 5
2	2, 1	2, 2	2, 3	2, 4	2, 5
3	3, 1	3, 2	3, 3	3, 4	3, 5
4	4, 1	4, 2	4, 3	4, 4	4, 5

You're invited to explore more.

11 Here are some other games that you could play using the same two spinners. Get a feel for the games by playing each one with your partner using the Activity 2 Sheet.

Four Score

- *Multiply* the numbers.
- Player 1 wins if the result is a *multiple of* 4.
- Player 2 wins if it is anything else.

You Spin Sum, You Lose Sum

- *Add* the two numbers.
- Player 1 wins if the sum is 6 *or greater.*
- Player 2 wins if the sum is 5 *or less.*

Lucky 7

- *Multiply* the numbers.
- Player 1 wins if the result is *greater than* 7.
- Player 2 wins if the result is *less than* 7.

Big Wheel, Big Deal

- Player 1 wins if the *big wheel's number is greater* than the small wheel's number.
- Otherwise, Player 2 wins.

Select *all* the games that are fair (Player 1 and 2 have an equal chance of winning).

☐ **A.** Four Score

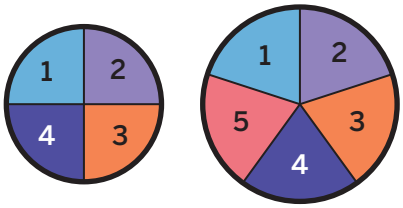
☐ **B.** You Spin Sum, You Lose Sum

☐ **C.** Lucky 7

☐ **D.** Big Wheel, Big Deal

12 Synthesis

Here are three representations of the sample space for this pair of spinners.
Choose one representation.



Discuss:

- What is an advantage or this representation?
- What is a disadvantage of this representation?

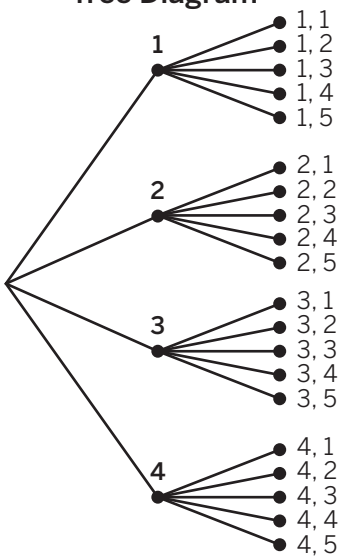
List

- Left: 1, Right: 1
- Left: 1, Right: 2
- Left: 1, Right: 3
- Left: 1, Right: 4
- Left: 1, Right: 5
- Left: 2, Right: 1
- Left: 2, Right: 2
- Left: 2, Right: 3
- Left: 2, Right: 4
- Left: 2, Right: 5
- Left: 3, Right: 1
- Left: 3, Right: 2
- Left: 3, Right: 3
- Left: 3, Right: 4
- Left: 3, Right: 5
- Left: 4, Right: 1
- Left: 4, Right: 2
- Left: 4, Right: 3
- Left: 4, Right: 4
- Left: 4, Right: 5

Table

	1	2	3	4	5
1	1, 1	1, 2	1, 3	1, 4	1, 5
2	2, 1	2, 2	2, 3	2, 4	2, 5
3	3, 1	3, 2	3, 3	3, 4	3, 5
4	4, 1	4, 2	4, 3	4, 4	4, 5

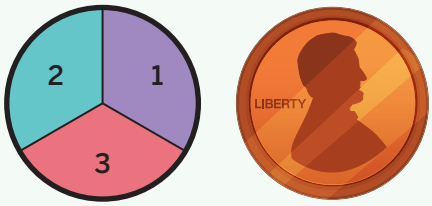
Tree Diagram



15 Summary 8.06

There are several different ways to make sense of **compound events**.

Here is one example: Let's spin a spinner and flip a fair coin. There are 6 outcomes in the *sample space* of this multistep event, which you can see in a list, a table, and a **tree diagram**.



List

1, Heads 1, Tails

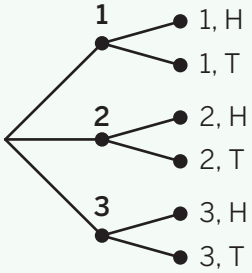
2, Heads 2, Tails

3, Heads 3, Tails

Table

	H	T
1	1, H	1, T
2	2, H	2, T
3	3, H	3, T

Tree Diagram

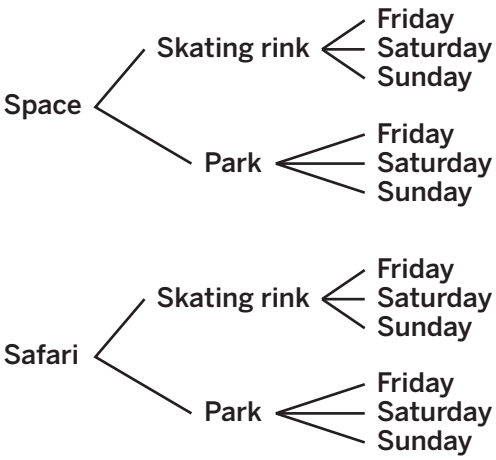


compound (multistep) event An experiment that has two or more events.

tree diagram A diagram that represents all the possible outcomes in an experiment. It is often used to represent compound events.

Name: ..... Date: ..... Period: .....

Problems 1–5: Farah made a tree diagram to help her choose a theme, location, and day of the week for her birthday party.



1. How many themes is Farah considering?

2. How many locations is Farah considering?

3. How many days is Farah considering?

4. One party Farah is considering is a space theme at the skating rink on Sunday. Write two other possible parties Farah could have.

5. If Farah lets her brother choose a theme, location, and day at random, what is the probability that Farah's birthday will be a safari at the park on Saturday? Explain your thinking.

Problems 6–7: Isabella selects one type of lettuce and one dressing to make a salad.

- Lettuce types: iceberg, romaine
- Dressings: ranch, Italian, Caesar

6. Create a list, table, or tree diagram to represent all the possible combinations of choices. Then determine the number of possible outcomes.

7. If Isabella picks a type of lettuce and dressing at random, is it more likely that she will get iceberg lettuce or Caesar dressing? Explain your thinking.

Name: ..... Date: ..... Period: .....

8. Twelve students play a game. Each student is assigned a number 1 to 12. Someone rolls two six-sided number cubes. The student whose number matches the sum gets a point. After 100 rounds, which student is most likely to have the most points? Explain your thinking.

Problems 9–11: Juan and Neo play a game. Each player holds out his hand at the same moment to represent a rock, paper, or scissors. They each choose one of the three items.

9. Create a list, table, or tree diagram to represent all the possible combinations of choices.

10. What is the probability that they both choose the same object?

11. Rock beats scissors, scissors beat paper, and paper beats rock. Two of the same object is considered a tie. Is this a fair game? Why or why not?

Spiral Review

Problems 12–13: $\frac{1}{3}$ produces a decimal that repeats every one digit. What fraction produces each of the following:

12. A decimal that repeats every two digits.

13. A decimal that terminates after two digits.

14. Select *all* the true equations.

- ☐ **A.** $8 = (8 + 8 + 8 + 8) \div 3$
- ☐ **B.** $(10 + 10 + 10 + 10 + 10) \div 5 = 10$
- ☐ **C.** $(6 + 4 + 6 + 4 + 6 + 4) \div 6 = 5$
- ☐ **D.** $4 = (4 + 2 + 4 + 2) \div 4$
- ☐ **E.** $(2 + 2 + 2 + 2 + 2) \div 2 = 2$

Name: ..... Date: ..... Period: .....

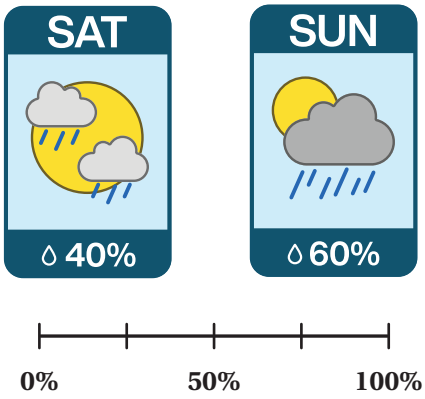
Probability Models | Population and Samples | 7.SP.8, 7.SP.8.c, 7.SP.2, SMP.2, SMP.3, SMP.4

Weather or Not

Let's use probability tools to simulate real-world events.

Warm-Up

1 Aniyah saw this forecast for the weekend's weather.



What do you think is the probability that it will rain *at least once* this weekend?

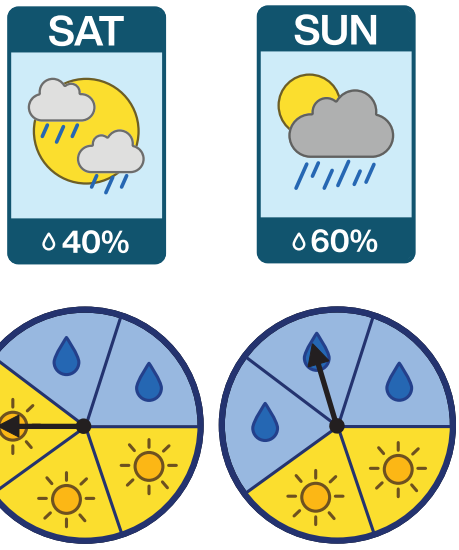
Plot a point on the line to show your guess. Explain your thinking.

Activity 1

Name: Date: Period:

Introducing Simulation

2 Aniyah wanted to run a **simulation**: an experiment designed to estimate the probability of a real-world event. She created these spinners to represent the weather forecast.



a Use the spinners on the Activity 1 Card or digital screen to run one or more experiments.

b Describe how Aniyah could use these spinners to estimate the probability that it will rain this weekend.

3 **Data Talk!** Aniyah ran 500 experiments. Here are the results.

Discuss: Choose a row and explain what it tells you about this situation.

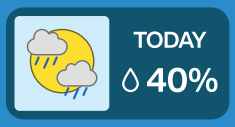
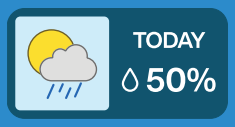
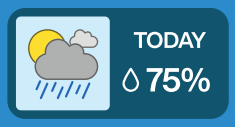
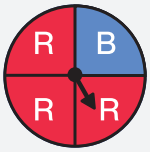
Experiments with:	Count	Relative Frequency
No rain	109	21.8%
1 day of rain	260	52%
2 days of rain	131	26.2%

4 What would you tell Aniyah about the probability of rain this weekend? Use evidence from the simulation to support your claim.

Name: ..... Date: ..... Period: .....

Make a Simulation

5 Match each simulation to the weather forecast it could represent.

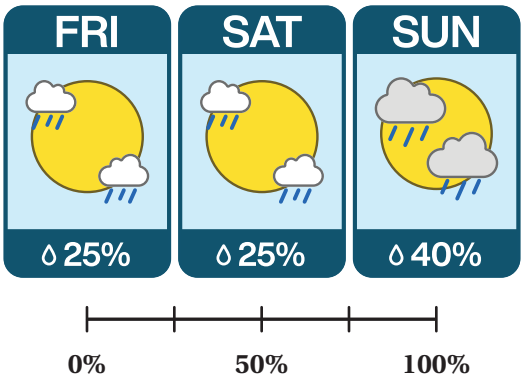
Simulation			
A fair coin lands heads up 			
A purple block is picked from this bag 			
A number cube lands on an odd number 			
The spinner lands on red 			
A green block is picked from this bag 			

Name: ..... Date: ..... Period: .....

Make a Simulation (continued)

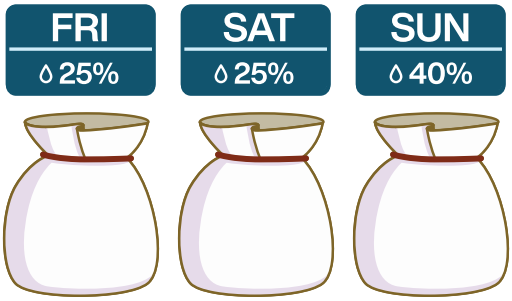
6 Ivan is planning a 3-day vacation. Here is the forecast at his destination.

What do you think is the probability that it will rain at least once during these 3 days? Plot a point on the line to show your guess.



7 Ivan wants to design a simulation to estimate the probability of rain during his vacation.

Draw blocks on each bag to match the probability of rain on each day.



8 **Data Talk!** Let's watch some simulated experiments on screen.

a Record the results of the simulation you watched in the table.

b Estimate the probability that it rains at least once on Ivan's vacation.

Experiments with:	Count	Relative Frequency
No rain		
1 day of rain		
2 days of rain		
3 days of rain		

Activity 3

Name: Date: Period:

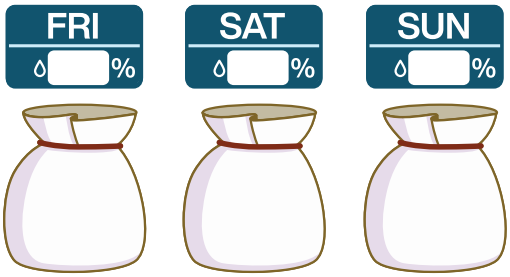
Meet the Criteria

For each challenge:

- Consider the given criteria for a 3-day period.
- Make a weather forecast for each day by writing a percentage in the white box that you think will meet the criteria.
- Draw blocks on the bag that match your forecast.
- Use the digital screen to run the simulation and test your forecast.

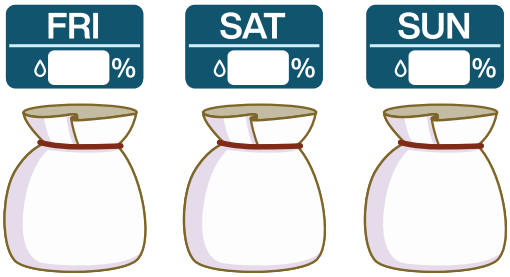
9 Criteria:

- The chance that it rains *all three days* is between 50% and 75%.



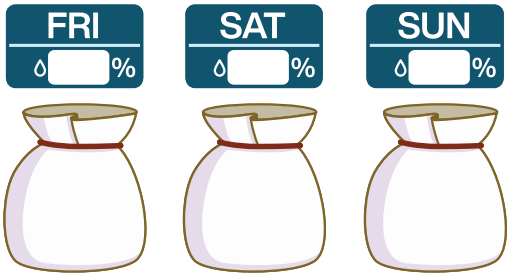
10 Criteria:

- The most likely weather is 2 *days of rain*.
- The next most likely weather is 1 *day of rain*.



11 Criteria:

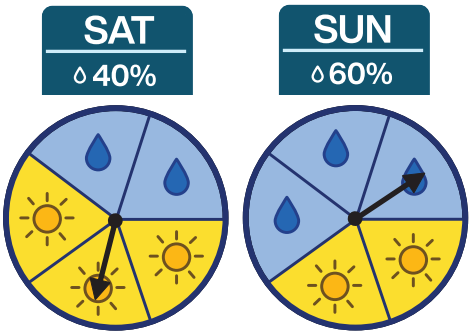
- The chance of *no rain at all* is about 25%.



12 Synthesis

Discuss: How can you design and use **simulations** to estimate the probability of a real-world event?

Use these spinners and table if they help with your thinking.



Experiments with:	Count	Relative Frequency
No rain	109	21.8%
1 day of rain	260	52%
2 days of rain	131	26.2%

15 Summary 8.07

Designing a good simulation starts with determining the probability that it will rain at least once over a three day period. In order to design a good simulation, first determine the probability of the individual events occuring.

For example, you could use a coin, number cube, or spinner to simulate a 50% probability of rain.

Flipping a Coin

Landing heads up

$\left(\frac{1}{2} = 50\%\right)$

Rolling a Number Cube

Rolling an even number

$\left(\frac{3}{6} = 50\%\right)$

Using a Spinner

Spinning a raindrop

$\left(\frac{5}{10} = 50\%\right)$

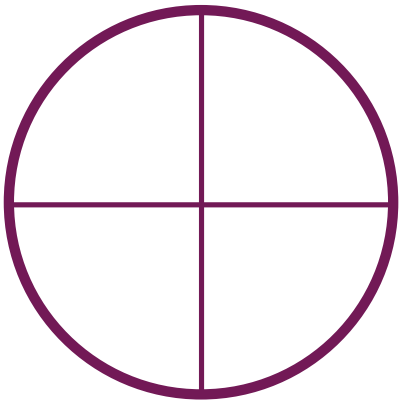
To simulate the probability of rain over three days where each day has a 50% chance of rain, you can use three coins, number cubes, or spinners and repeat the experiment many times.

simulation An experiment that is used to estimate the probability of a real-world event.

Name: ..... Date: ..... Period: .....

Problems 1–3: The weather forecast says there is a 75% chance it will rain today.

1. Design a spinner you could use to simulate a 75% chance of rain.



2. Explain why using a number cube to simulate this probability may be less useful than using a spinner.

3. Describe or draw a different way you could simulate this probability.

4. Esteban has 3 kittens. According to the vet, each kitten is born with blue eyes and there is a 50% chance they will change color once the kittens reach three months. Esteban designs a simulation using 3 coins, where heads represent the eyes changing color.

The table shows the results of 100 experiments.

Experiments with:	Count
No blue-eyed kittens	11
1 blue-eyed kitten	32
2 blue-eyed kittens	43
3 blue-eyed kittens	14

Estimate the probability that at least one of Esteban's kittens will still have blue eyes at three months old. Explain your thinking.

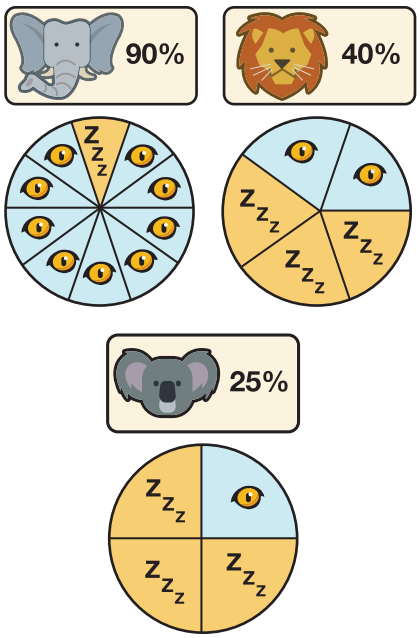
5. A coach can choose either Player 1 or Player 2 to take a penalty shot in a soccer game. Player 1 scored on 3 out of their previous 4 penalty shots. Player 2 scored on 20 out of their previous 30 penalty shots. Who would you advise that the coach choose? Explain your thinking.

Practice
8.07

Name: Date: Period:

Problems 6–7: Brianna designed and used a simulation to help her estimate the probability of seeing her three favorite animals awake when she visits the zoo. She records the results of 300 experiments.

Experiments with:	Count	Relative Frequency
No animals awake	12	4%
1 animal awake	171	57%
2 animals awake	105	35%
3 animals awake	12	4%

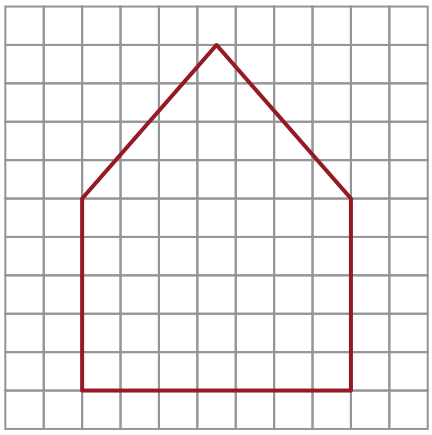


6. Estimate the probability that all 3 of her favorite animals will be awake when she visits the zoo.

7. Estimate the probability that at least 2 of her favorite animals will be awake when she visits the zoo.

Spiral Review

Problems 8–9: Here is a diagram of the base of a bird feeder. Each square on the grid represents 1 square inch.



8. What is the area of the base of the bird feeder?

9. The distance between the two bases is 8 inches. What is the volume of the bird feeder?

Problems 10–14: Write each fraction as a percent.

10. $\frac{3}{5}$ **11.** $\frac{1}{50}$ **12.** $\frac{9}{10}$ **13.** $\frac{9}{5}$ **14.** $\frac{18}{60}$

Name: ..... Date: ..... Period: .....

Probability Models 7.SP.8, 7.SP.8.c, SMP.1, SMP.4

Simulate It!

Let's use simulations to estimate probabilities.

Warm-Up

1. Here are two situations. Choose one and describe a simulation that could help answer the question.

A. Adrian has one sister.

Every night, either he or his sister is chosen at random to do the dishes.

Adrian wonders: What is the probability he can go 3 straight nights without having to do the dishes?

B. A school randomly assigns the incoming 7th graders between two teachers at the beginning of the year.

A group of three friends all want to be in Mr. Jordan's class.

They wonder: What is the probability they will *all* get assigned to Mr. Jordan's class?

Activity
1

Name: Date: Period:

Design a Simulation

For this activity, you will use the Activity 1 Sheet and either the Digital Companion or physical probability tools.

2. Choose one situation from your Activity 1 Sheet and circle it.

A. Animal Hospital

B. DinoSocks

C. Blood Types

D. Power the Flashlight

3. Design a simulation that you could use to answer the first question for your situation. Describe your simulation precisely enough that someone else could perform it.

Activity 2

Name: Date: Period:

Perform the Simulation

For this activity, you will use the Digital Companion or physical probability tools to run the simulation you designed in Activity 1 and make a recommendation based on your results.

4. Create a poster of your results and recommendations. Here is what your poster should include:

- ☐ A short description of your situation.
- ☐ A description of the simulation(s) you designed for the situation.
- ☐ A summary of the results from the simulation you performed.
- ☐ Answers to each of the two questions about your situation.
- ☐ Evidence that supports your answers, along with all of your work.
- ☐ Two new questions that you have about the situation.

Use this page as a draft space if it helps with your thinking.

Synthesis

5. 💬 **Discuss:** What do you think is important to consider when designing and using simulations to explore probabilities of real-world events?

Summary 8.08

You can use simulations to estimate the probability of an event. Simulations are especially useful for estimating the probability of compound events. The more simulations you perform, the closer the probability in the relative frequency of each outcome should be to the probability.

Many professionals such as scientists, computer programmers, financial analysts, and sports analysts create simulations to model the outcomes of complicated real-world events. Using computer software, they are able to perform thousands of simulations to answer questions about everyday situations. Similarly, you can create a simulation using items such as spinners or bags to determine the probability of a real-world question you may want to answer.

Name: ..... Date: ..... Period: .....

Problems 1–3: The probability of rolling two number cubes with the same number is $\frac{6}{36}$.

1. Where does 36 come from? Explain your thinking.

2. What is the probability that the numbers on the cubes are different?

3. What is the probability of rolling two number cubes and getting *at least* one 5?

4. Esteban's cat had kittens and all three of them were white kittens with blue eyes! About 80% of white cats with blue eyes become deaf. Describe a simulation that would help determine the probability that at least two of the kittens will become deaf.

Problems 5–6: When soccer games are tied at the end of overtime, they are decided by penalty kicks. Each team gets 5 kicks. A coach knows that her team typically makes a penalty kick about 60% of the time. She decides to run a simulation of this situation 100 times.

Penalty Kicks Made	Count
0 out of 5	1
1 out of 5	9
2 out of 5	30
3 out of 5	27
4 out of 5	24
5 out of 5	9

5. Based on these results, estimate the probability that the team makes 4 or more penalty kicks.

6. Based on these results, estimate the probability that the team makes 2 or fewer penalty kicks.

Name: ..... Date: ..... Period: .....

7. Ariana and Juliana play a new game to decide who will pay the bill for lunch. They roll two number cubes.

- If the larger of the two numbers is even, Ariana pays.
- If the larger of the two is odd, Juliana pays.
- If the numbers are the same and they are even, Ariana pays. Otherwise, Juliana pays.

Ariana thinks that it is more likely that she pays the bill. Is Ariana's claim true? Explain your thinking.

Spiral Review

Problems 8–11: Determine the value of each expression.

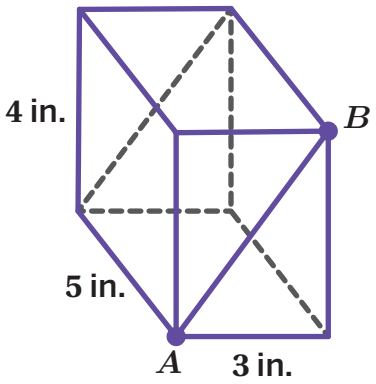
8. $(4.2 + 2.6) \div 2$

9. $(4.2 + 2.6 + 4) \div 3$

10. $(4.2 + 2.6 + 4 + 3.6) \div 4$

11. $(4.2 + 2.6 + 4 + 3.6 + 3.6) \div 5$

Problems 12–13: A rectangular prism is cut along a diagonal to create two triangular prisms. The distance between points A and B is 5 inches.



12. What is the surface area of the original rectangular prism? Explain your thinking.

13. What is the total surface area of the two triangular prisms combined?

Name: ..... Date: ..... Period: .....

Probability Models | Proportional Relationships | 7.SP.6, 7.SP.5, 7.SP.7, 7.SP.7.a, 7.SP.7.b, 7.SP.8.a, 7.SP.8.b, 7.SP.8.c, 7.RP.3, SMP.2, SMP.4

Practice Day 1

Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Carnival Games

1. Most likely to win a prize:

Explanation:

2. **Least Likely**

Most Likely

3. Event description:

Task B: Rubber Duck

1. ..... times

2. ..... ducks

Explanation:

3. Probability: .....

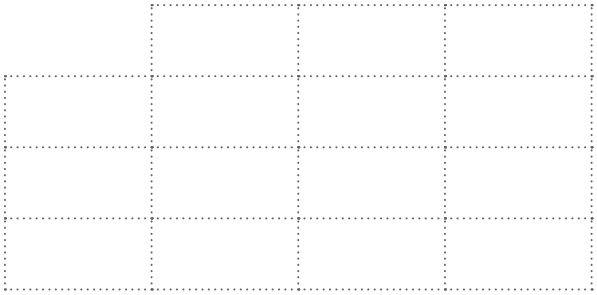
Explanation:

Name: _____ Date: _____ Period: _____

Practice Day 1 (continued)

Task C: Sure Spins-A-Lot

1.



2. Probability: _____

Explanation:

3. Game design:

Task D: Diamond Drop

1. Simulation design:

2. Probability: _____

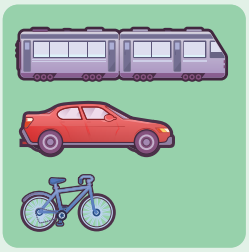
3. Probability: _____

You're invited to explore more.

Sub-Unit

2

Sampling

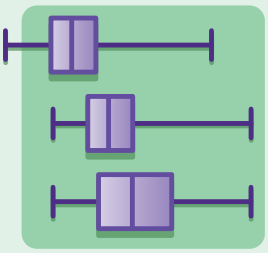


Lesson 9
Car, Bike, or Train?

Lesson 10
Crab Island

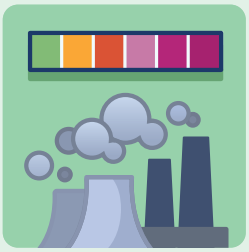
Lesson 11
Headlines

Lesson 12
Flower Power



Lesson 13
Plots and Samples

Lesson 14
School Newspaper

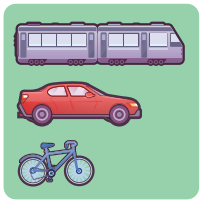


Lesson 15
Air Quality

Name: ..... Date: ..... Period: .....

Visualize Populations 7.SP.4, SMP.2, SMP.3, SMP.6

Car, Bike, or Train?



Let's analyze and compare sets of data using the mean and mean absolute deviation (MAD).

Warm-Up

Data Talk! Marco wants to know the best way to get to school.

For 15 days, he collected data about how many minutes it took him to get to school.

	Car	Bicycle	Train
Minutes	12, 62, 14, 16	25, 23, 22, 28, 27	11, 15, 30, 16, 27, 15

1. Which option should Marco choose? Why?

Activity 1

Name: ..... Date: ..... Period: .....

Marco's Mean and MAD

Data Talk! Here is the data from the previous problem about how long it took Marco to get to school over 15 days.

	Car	Bicycle	Train
Minutes	12, 62, 14, 16	25, 23, 22, 28, 27	11, 15, 30, 16, 27, 15

Marco wondered which option is best, so he checked with some calculations.

2. Explain what Marco did to calculate the *mean* of the car.

Mean of Car

$12 + 62 + 14 + 16$

$74 + 30$

104

$\frac{104}{4} =$ 26 minutes

3. What does the mean say about traveling by car?

4. Marco also wanted to know the consistency of the travel times for each mode of transportation, so he calculated the **mean absolute deviation (MAD)**.

Marco started by measuring how far each value was from the mean. Complete the missing calculations.

5. Then, Marco found the mean of the distances. Fill in the missing values to determine the **MAD**.

6. Why do you think Marco is using both the **mean** and the **MAD** to help him decide?

MAD of Car

car: 12 62 14 16

distance from the mean 26: 14 36

$\frac{14 + 36 + \square + \square}{\square}$

$= \frac{\square}{\square}$

mean absolute deviation: $\square$

Activity 2

Name: Date: Period:

Decisions

Data Talk! Here is the data from the previous activity.

	Car	Bicycle	Train
Minutes	12, 62, 14, 16	25, 23, 22, 28, 27	11, 15, 30, 16, 27, 15
Mean (min)			
MAD (min)			

7. One of these data sets has a mean of 26 minutes and another has a mean of 19 minutes. Write these statistics where they belong in the table and calculate the missing mean.

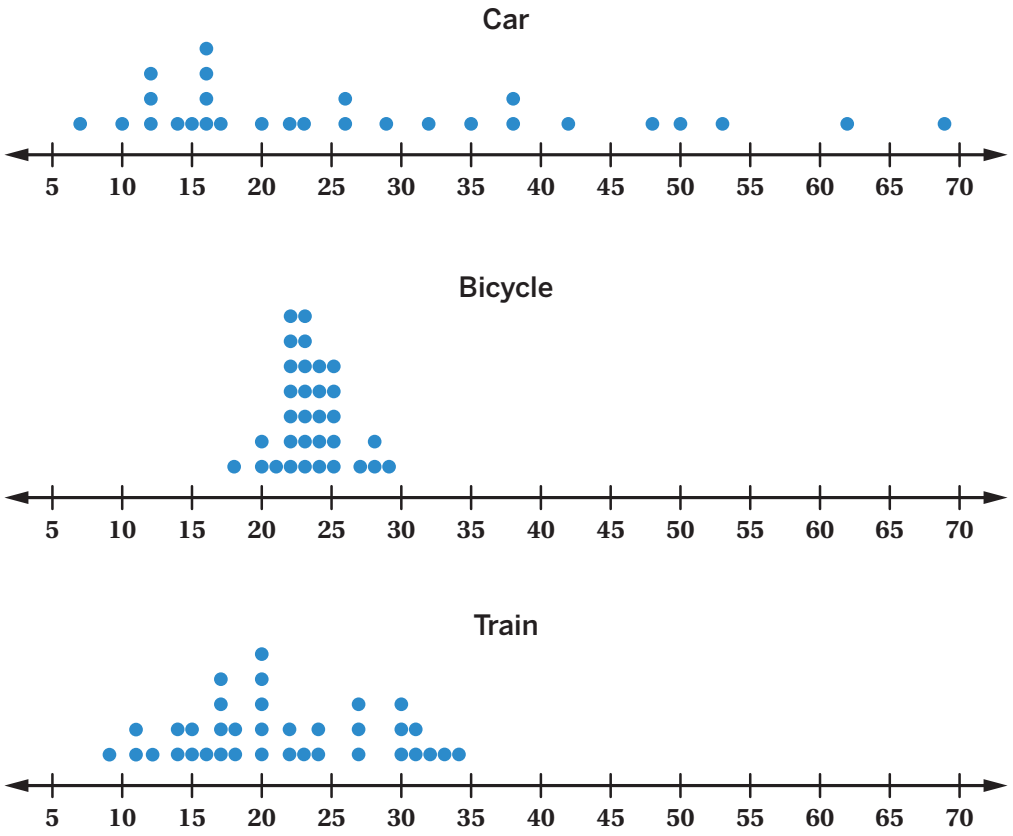
8. One of these data sets has a MAD of 2 minutes and another has a MAD of 18 minutes. Write these statistics where they belong in the table and calculate the missing MAD.

9. Do you think Marco should take a car, bicycle, or train to get to school? Use at least two different pieces of evidence to support your claim.

Name: Date: Period:

More Data

Data Talk! Marco decided to collect more data.



10. Does this data change your opinion about which option Marco should choose? Explain your thinking.

11. What considerations does Marco have to make to choose based on environmental factors? What questions might he ask to help his research?

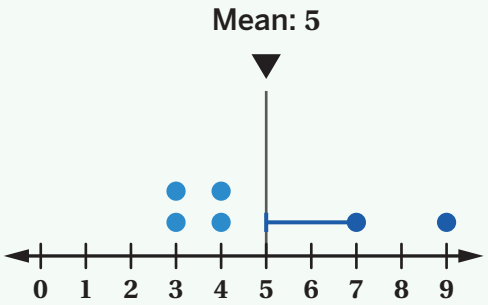
Synthesis

12. **Discuss:** When comparing sets of data, why is it important to consider both the mean and the MAD?

Summary 8.09

When comparing two sets of data, you can compare their centers, shapes, and spreads. Data sets can have the same center but very different shapes and spreads.

The **mean** is a measure of center. One way to measure the spread of data is to use the distances between each value in the data set and the mean. The average of those distances is called the **mean absolute deviation (MAD)**. In this example, the MAD is 2 units.



Data Point	3	3	4	4	7	9
Distance from Mean	2	2	1	1	2	4

$$\frac{2+2+1+1+2+4}{6} = 2$$

MAD: 2

mean A way to measure the center of a data set. If you equally distribute a set of items into different groups, the mean is the number of items in each group. It is also the balance point of a dot plot. To calculate the mean, you can add the values of all the data points, then divide by the number of data points.

mean absolute deviation (MAD) One way to measure how spread out a data set is from the mean. To calculate the MAD, add the distances between each data value and the mean. Then divide by the number of data values in the set.

Practice
8.09

Name: ..... Date: ..... Period: .....

Problems 1–3: A school's art club holds a bake sale on Fridays to raise money for art supplies. This table shows the number of cookies they sold each week in the fall and in the spring.

Fall	Spring
20	19
26	27
25	29
24	21
29	25
20	22
19	26
19	21
24	25
24	25

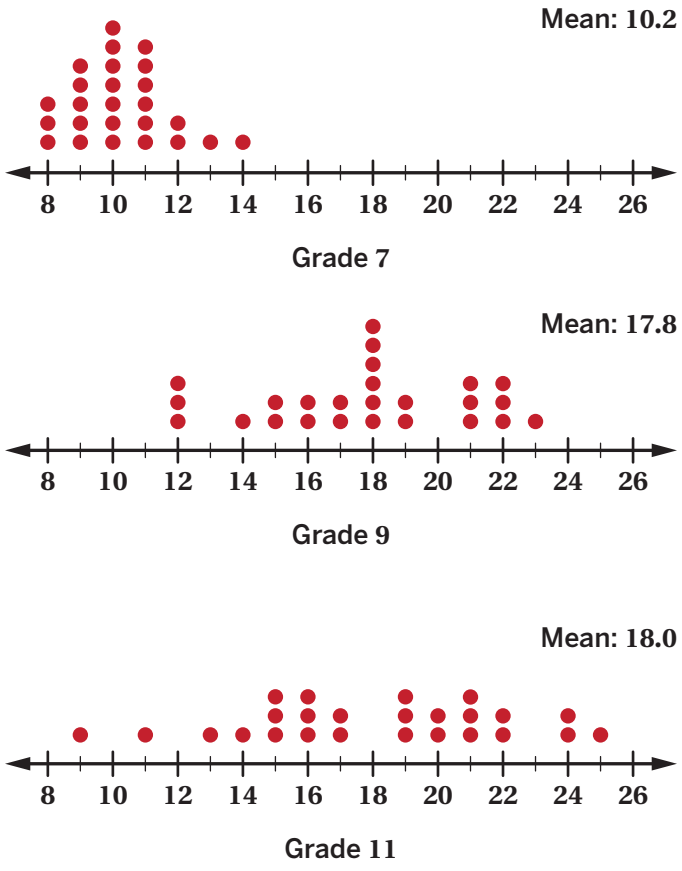
1. Calculate the mean number of cookies sold in the fall.

2. Calculate the mean number of cookies sold in the spring.

3. Based on this data, were sales generally higher in the spring than in the fall? Explain your thinking.

4. Compare the weights of the backpacks for students in these three grades. Include both the mean weights and their spread in your response.

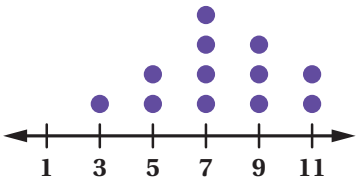
Backpack Weight by Grade Level (pounds)



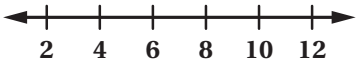
Name: ..... Date: ..... Period: .....

Problems 5–7: Consider the dot plot.

5. Create a different dot plot with the same mean as this dot plot. Your dot plot should have more than 6 but fewer than 20 data points.



6. Find the MAD of each dot plot.



7. Compare the MAD of the original dot plot to the MAD of your new dot plot.

Spiral Review

Problems 8–9: Evaluate each expression.

8. $\frac{(6-3)+(6-6)+(9-6)}{3}$

9. $\frac{(5-1)+(5-3)+(7-5)+(9-5)}{4}$

Problems 10–13: Rewrite each expression with fewer terms.

10. $4(5x-3y)-10x+6y$

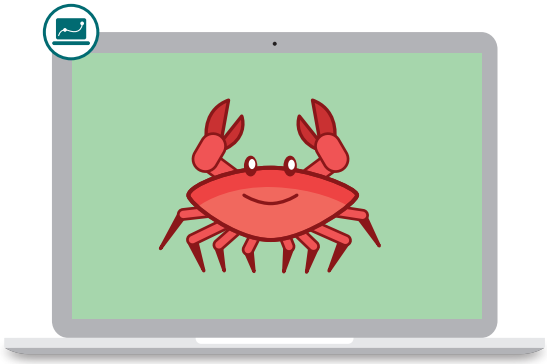
11. $2.5(2x+4y)-5(4y-x)$

12. $7.9(5x+3y)-4.2(5x+3y)-1.7(5x+3y)$

13. $6(x+2y)-2(y-2x)$

Name: ..... Date: ..... Period: .....

Populations and Samples | Visualize Populations | 7.SP.1, 7.SP.3, SMP.1, SMP.3, SMP.7



Crab Island

Let's compare large populations of data.

Warm-Up

1 Which one doesn't belong? Explain your thinking.

A. $\frac{1+5+9}{3}$

B. $\frac{1+2+3+4+5}{3}$

C. $\frac{2+4+5+6+8}{5}$

D. $\frac{3+5+13}{3}$

Activity 1

Name: ..... Date: ..... Period: .....

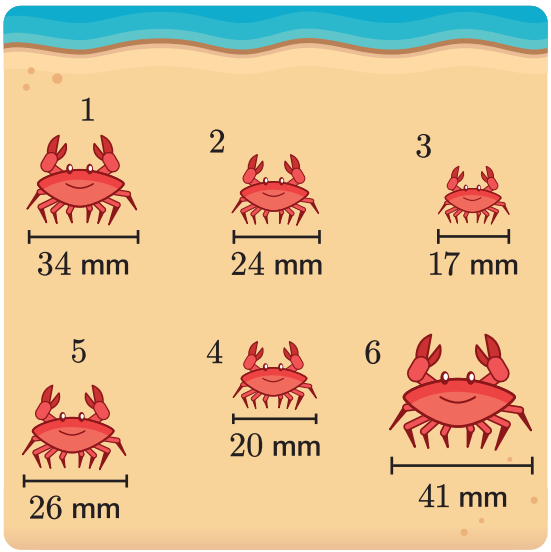
Sampling

2 Crab Island has a particular species of crabs.

What is a question about these crabs that we could collect data for?

3 **a** **Data Talk!** Take a look at this group of crabs Amari is studying.

Crab	Width (mm)
1	34
2	24
3	17
4	20
5	26
6	41



b **Discuss:** What question could this group of crabs help answer?

4 Calculate the mean of Amari's group of crabs.

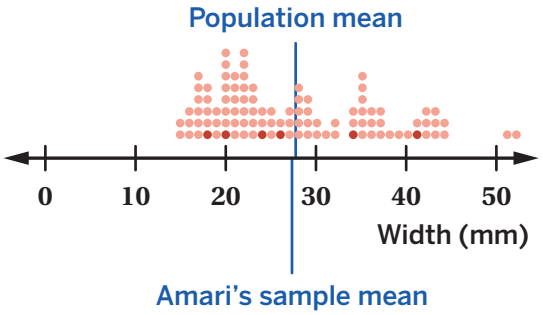
Name: ..... Date: ..... Period: .....

Sampling (continued)

5 Here is a dot plot of the widths of the crabs in the **population**.

The crabs Amari chose are highlighted. This is a **sample**.

What do you notice? What do you wonder?

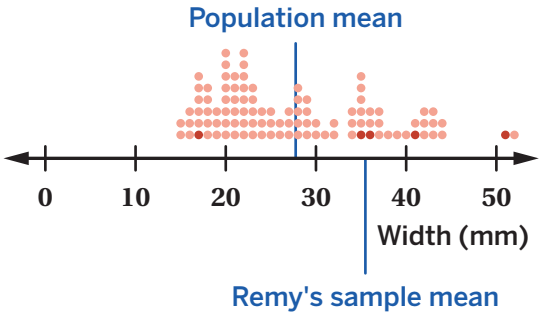


I notice:

I wonder:

6 **Data Talk!** Remy chose a different sample. The dot plot shows Remy's sample and sample mean.

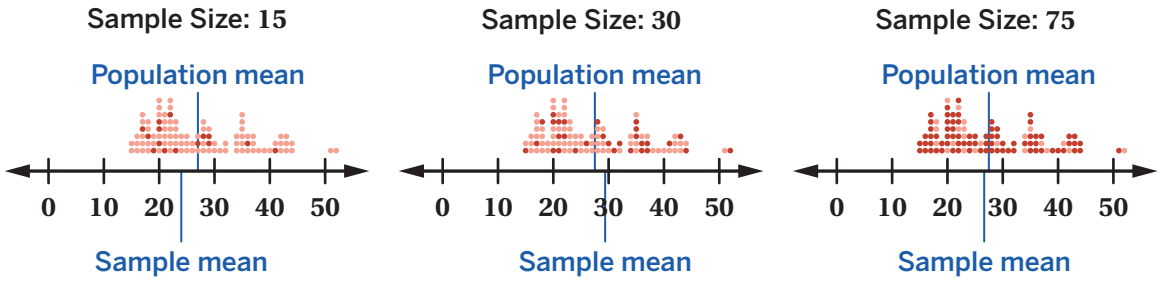
What might happen if someone used Remy's sample to study the crabs on Crab Island?



Name: ..... Date: ..... Period: .....

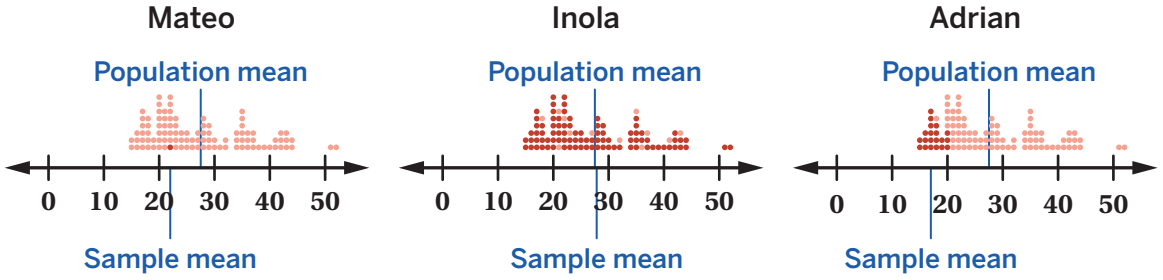
Explore More Samples

7 **Data Talk!** Here are three different samples of crab widths.



Discuss: What are advantages and disadvantages of using a large sample?

8 Mateo, Inola, and Adrian each collected different samples of crabs. Select a sample and explain what advice you would give to that person to improve their sample.



Activity 3

Name: _____ Date: _____ Period: _____

Populations and Samples

9 Alina wonders how students at the school feel about the cafeteria food. Alina asked 50 students. In this situation, what is the population and what is the sample?

Place a P next to the population and an S next to the sample.

_____ The students in the school

_____ The cafeteria food

_____ The head cook

_____ The 50 students

10 Complete each row of possible questions, populations, and samples.

Question	Population	Sample
What is the average circumference of a tree in my state?	All of the trees in the state	
	The eggs in San Diego	One egg from each store in the city
What is the average weight of an apple?		The apples in the school cafeteria

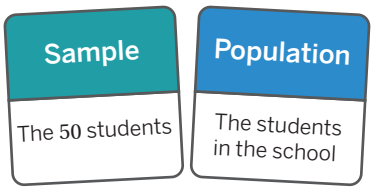
You're invited to explore more.

11 **a** What is a question you could ask where the population is all of the books in the school?

b What is a question you could ask where a sample is all of the books in the school?

12 Synthesis

Discuss: What are some advantages and disadvantages of using samples to answer a question about a population?



15 Summary 8.10

To answer a question about a population, it is sometimes not realistic to collect data from the entire **population**. Instead, you can collect data from a **sample** of the population.

The sample you choose should be large enough to be able to draw conclusions about the population.

Here are some examples of populations and samples.

Population	Sample
All of the people who watch basketball.	The people at a basketball game.
All 7th grade students in your school.	The 7th graders in your school who are in a band.
All oranges grown in the U.S.	The oranges in your local grocery store.

population A set of people or objects that are being studied.

sample A part of a population.

Name: ..... Date: ..... Period: .....

Problems 1–3: Aniyah wonders: How much time do 7th graders at my school spend outdoors on a typical day?

1. What is the population for Aniyah's question?

2. Select *all* possible samples for Aniyah's question.

- ☐ **A.** The 20 students in a 7th grade math class.
- ☐ **B.** The first 20 people to arrive at Aniyah's middle school on a particular day.
- ☐ **C.** The 7th graders participating in a science fair with students from four middle schools.
- ☐ **D.** The 10 7th graders on the school soccer team.
- ☐ **E.** The students on the high school debate team.

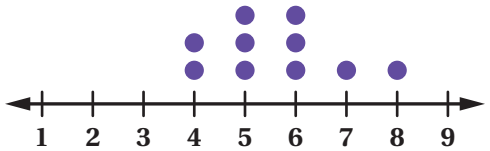
3. Select two samples and list a different possible population each sample could belong to.

Sample	Possible Population
The 20 students in a 7th grade math class.	
The first 20 people to arrive at Aniyah's middle school on a particular day.	
The 7th graders participating in a science fair with students from four middle schools.	
The 10 7th graders on the school soccer team.	
The students on the high school debate team.	

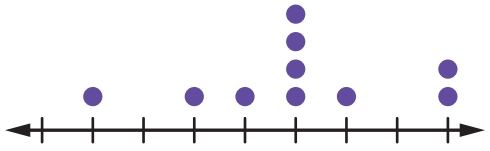
4. A popular shoe store released a new shoe model. At a local school, the average shoe size of a 7th grade student is 5.5. Each of these dot plots shows the shoe size for a sample of ten 7th grade students. Which sample is more representative of the population of 7th grade students at the school? Circle your response.

Sample A Sample B

Explain your thinking.



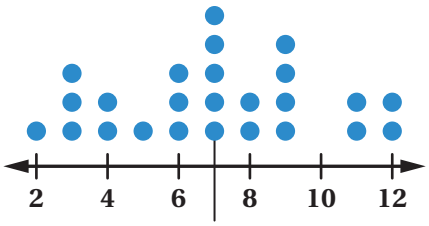
Sample A shoe sizes



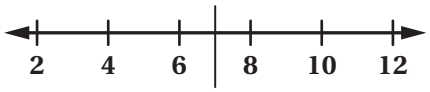
Sample B shoe sizes

Name: ..... Date: ..... Period: .....

5. Here is a dot plot of a population. Create a dot plot of a sample with a mean that's the same as the population mean. Your sample should have more than 6, but fewer than 20 data points.



Population mean: 7



Sample mean

Spiral Review

6. Select *all* the measures of center.

- ☐ **A.** Mean
- ☐ **B.** IQR (interquartile range)
- ☐ **C.** Range
- ☐ **D.** Median
- ☐ **E.** MAD (mean absolute deviation)

7. There are 50 marbles in a bag. Students picked a marble, recorded its color, and put it back in. Each student carried out a different number of experiments. The table shows their results. Estimate the probability of getting a green marble from this bag. Explain your thinking.

	Number of Experiments	Green Marbles
Student 1	4	1
Student 2	12	5
Student 3	9	3

Problems 8–10: Calculate the mean of each data set.

8. 8 , 9 , 9 , 9 , 10

9. 2 , 6 , 12 , 16

10. 5 , 6 , 12 , 13

Problems 11–12: A bookstore has a 15% discount on all books.

11. How much money is the 15% discount worth on a book that normally costs $18?

12. After the discount, how much would the book cost?

Name: ..... Date: ..... Period: .....

Populations and Samples | Visualize Populations | 7.SP.1, 7.SP.3, SMP.3

Headlines

Let's see what makes a good sample.

Warm-Up

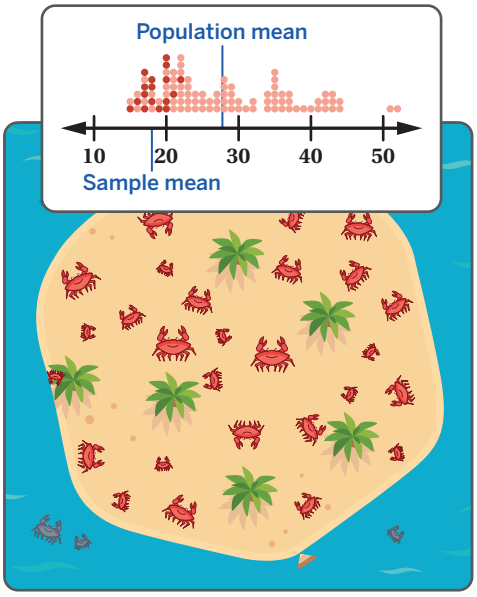
1 **Data Talk!** Adrian collected this sample of crabs and then wrote this headline:

"Crabs Are Smaller Than Ever!"

Do you agree with Adrian's headline? Circle one.

Yes No I'm not sure

Explain your thinking.



Name: ..... Date: ..... Period: .....

Graduation Rates

2 We are going to use your class as a sample to learn more about people in the United States.

Answer this question: *Have you graduated high school yet?* Circle one.

Yes No

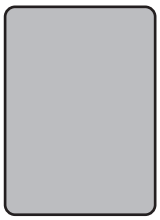
3 **Data Talk!** Read the news headline and the information about the sample that the newspaper used.

What do you notice about this sample?

Des-News Online

0% of People Have Graduated High School in the United States!

We sampled 30 middle school students in the United States and 0 said they graduated high school.



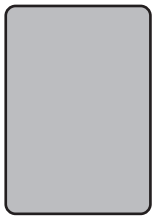
4 Read the news headline and the information about the sample that the newspaper used.

Why is this sample more likely to be **representative** of the population?

Des-News Online

90% of People Have Graduated High School in the United States!

We sampled 57,000 random census surveys from Americans 25 and older. 51,300 reported graduating high school.



Name: ..... Date: ..... Period: .....

Representative Sampling

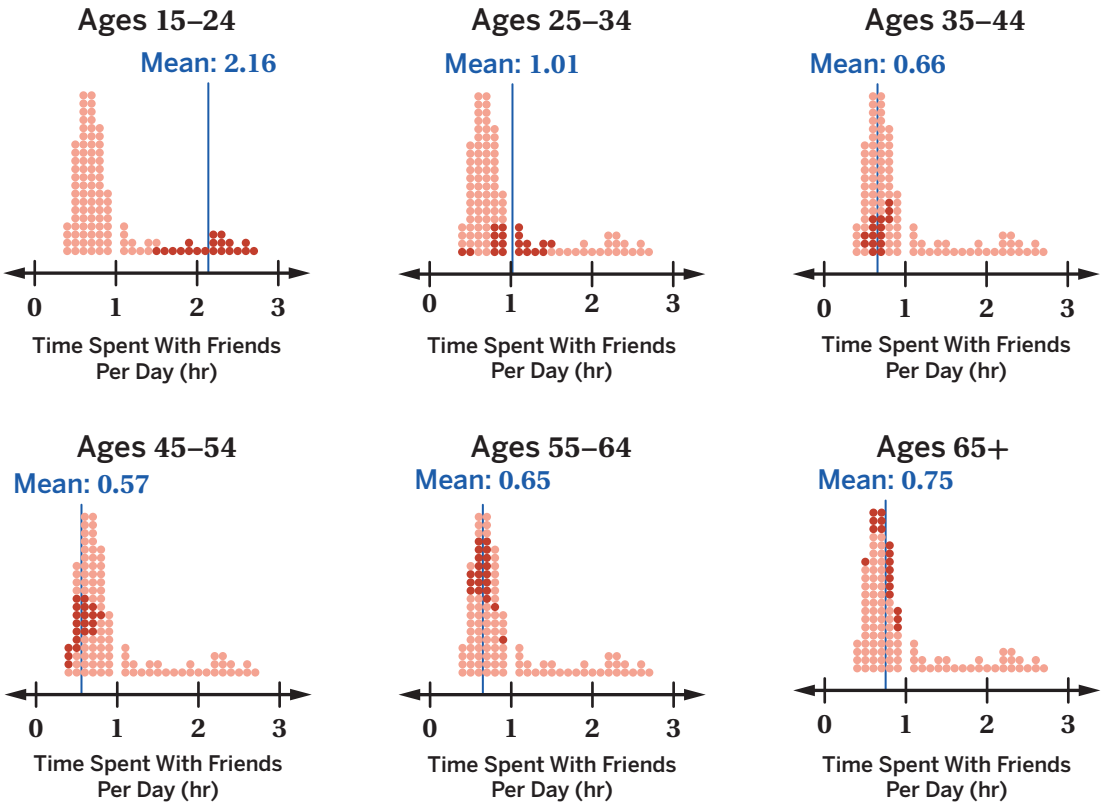
5 **Data Talk!** The American Time Use Survey asks about 9,400 randomly selected people how they use their time each year.

One of the questions they ask is: *How many hours do you spend with friends on average per day?*

Discuss: How do you think people of different ages might answer this question?

6 Here is a representative sample of 112 responses from the American Time Use Survey.

a Take a look at the results for different ages.



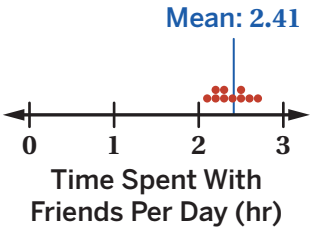
b Describe what you notice about the sample.

Name: ..... Date: ..... Period: .....

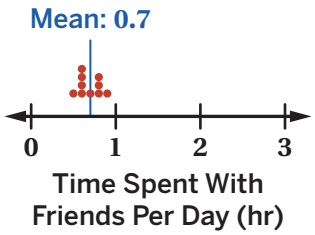
Representative Sampling (continued)

7 Here are three sampling methods and the samples that were collected.

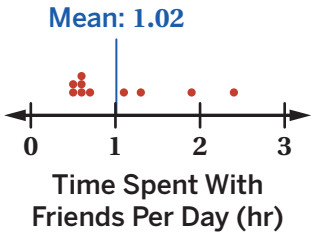
10 students in one high school class



10 random people at a nursing home



10 random phone numbers and ask whoever answers



Choose a sampling method and use the sample to write a headline about how much time people in the United States spend with their friends per day.

Your headline: .....

8 Ada wants to collect new data to see how people in the United States spend their time. Order the methods from *least* likely to *most* likely to produce a representative sample of the population.

Least Likely

Most Likely

a. Post a poll on social media and look at the first 20 responses.

b. Ask 20 random people at the grocery store.

c. Ask every single person in the country.

d. Call 20 phone numbers at random.

e. Ask 20 friends.

You're invited to explore more.

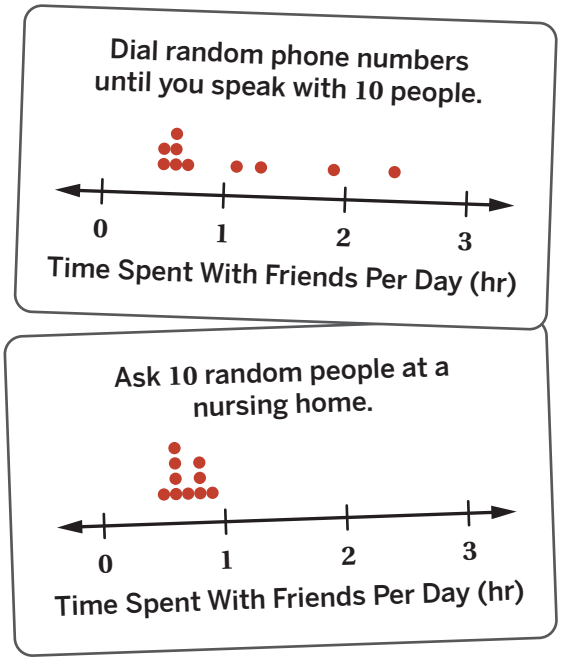
9 On the You're Invited to Explore More Sheet there are dot plots showing how much time people in the United States spend alone each day. Explore the mean response for different ages.

Describe an *incorrect* conclusion someone might make from an unrepresentative sample.

10 Synthesis

Explain how collecting a representative sample or an unrepresentative sample can affect someone's understanding of a population.

Use the examples if they help with your thinking.



13 Summary 8.11

Samples are useful when a population is too large to survey or measure. Depending on the strategy you use to sample, your sample might or might not be **representative** of the population. Some samples are not good representations of the population.

A representative sample has a distribution that closely resembles the distribution of the population. Representative samples are useful for making predictions about the whole population.

For example, if you were curious about all middle school students' favorite sport to play, the population would be all middle schoolers. A representative sample of this population might be randomly selecting 5 students from each class or 15 students from each grade to ask. A sample that is *not* representative of this population would be asking students in the tennis club because their responses might lead someone to believe that tennis is the favorite sport among all middle schoolers.

representative A sample is representative if its distribution resembles the population's distribution in center, shape, and spread.

Practice
8.11

Name: ..... Date: ..... Period: .....

1. Select *all* the reasons why random samples tend to produce a more representative sample.

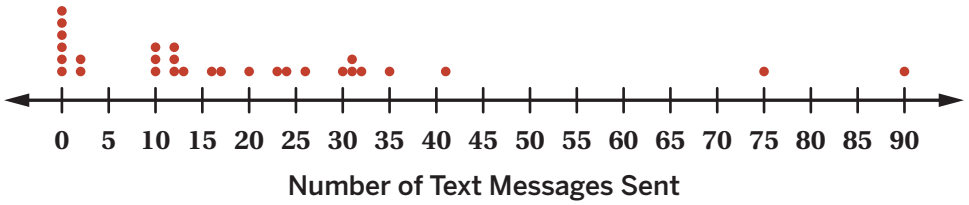
- ☐ **A.** You can determine how many people you want in the sample.
- ☐ **B.** It's the easiest way to select a sample from a population.
- ☐ **C.** It avoids the bias that can occur with other sampling methods.
- ☐ **D.** Each person in the population has an equal chance of being selected.
- ☐ **E.** The sample mean will always be the same as the population mean.

2. Jada wants to learn about the percentage of students who like the food in the cafeteria. Jada asks the first 25 students who purchase lunch at the cafeteria if they like the food.

Is Jada's method likely to produce a representative sample?

Explain your thinking.

Problems 3–4: This dot plot shows the number of text messages sent on one day for a sample of students at a high school. 29 random students were sampled.



3. What do the six dots at 0 on the dot plot represent?

4. Because this sample is representative of the population, describe what a dot plot for the entire high school might look like.

Name: ..... Date: ..... Period: .....

5. A random sample of 15 items were selected at a grocery store. The dot plot shows their prices. Is the mean of this sample likely to be representative of the population mean?

Yes No

Explain your thinking.

Problems 6–9: Think of a new situation.

6. Write a question you're interested in finding the answer to.

7. What is the population for your question?

8. Describe a strategy that is unlikely to produce a representative sample to answer your question.

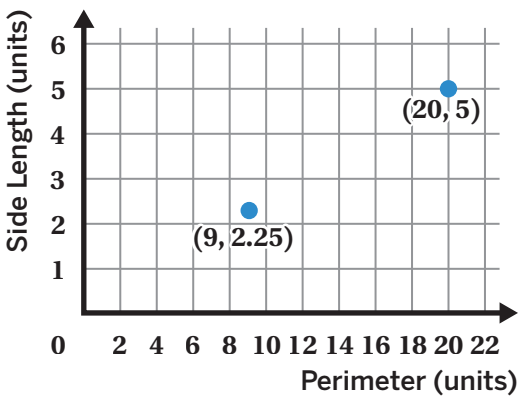
9. Describe a strategy to get a representative sample to answer your question.

Spiral Review

10. Based on a weather report, the probability that it will snow tomorrow is 0.9. Which word describes the likelihood that it will snow tomorrow?

A. Certain **B.** Impossible **C.** Likely **D.** Unlikely

Problems 11–12: This graph shows the side length of a square and its perimeter.



11. Plot and label two more points on the graph.

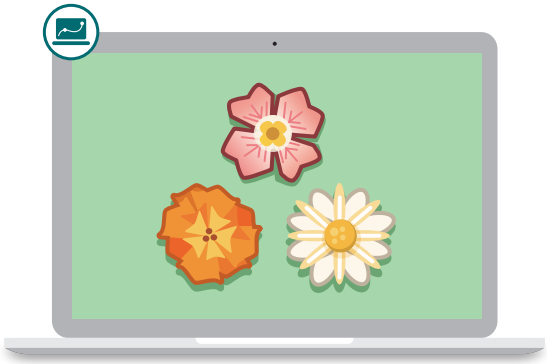
12. Is there a proportional relationship between the perimeter and side length?

Explain your thinking.

Name: ..... Date: ..... Period: .....

Probability Models | Populations and Samples | Proportional Relationships | Unit Rates in the World

7.SP.1, 7.RP.3, 7.SP.2, SMP.1, SMP.3



Flower Power

Let's use percentages to make predictions about populations.

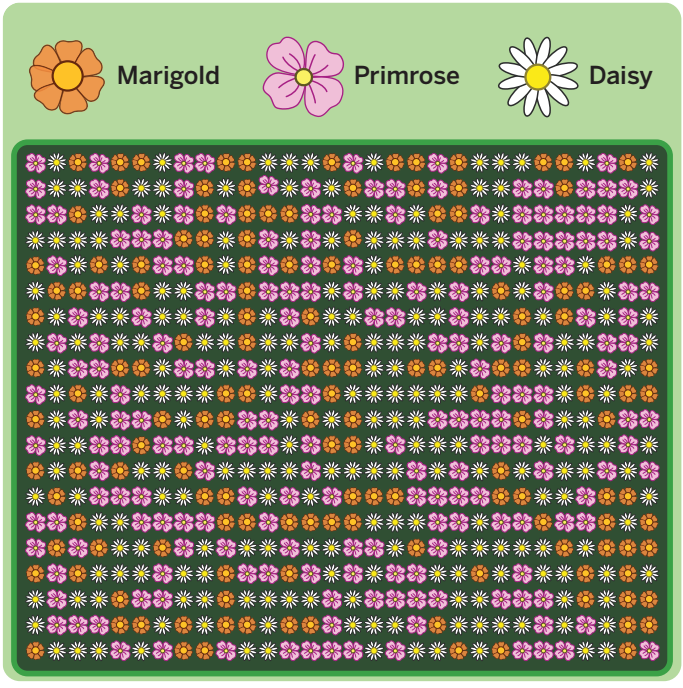
Warm-Up

1 A gardener planted some seeds from a bag of Wildflower Seed Mix.

What do you notice? What do you wonder?

I notice:

I wonder:



2 **Data Talk!** The gardener wonders how many of each flower type are in the garden. He says there are too many to count one by one.

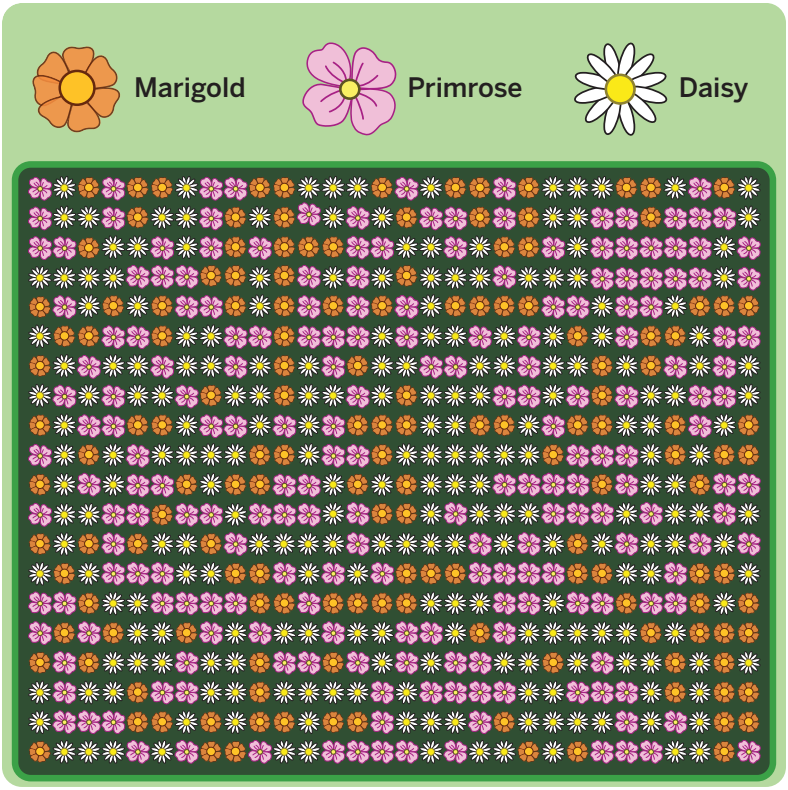
Describe a method he could use to estimate how many of each flower type there are.

Activity 1

Name: ..... Date: ..... Period: .....

Estimating the Marigolds

3 Here are all 600 flowers in the garden.



a Choose a sample of at least 25 flowers from the garden.

b Complete the table for your sample.

Flower Type	Count	Percentage
Marigold		
Primrose		
Daisy		
Total		100%

4 Use your table to help you estimate how many of the 600 flowers in the garden are marigolds.

Name: ..... Date: ..... Period: .....

Estimating the Marigolds (continued)

5 **Data Talk!** Eliza and Javier chose a 5-by-5 sample by Eliza closing her eyes and placing her finger on the image.

Here are their strategies for estimating the total number of marigolds.

Explain each student's strategy.

Eliza

Marigolds are 28% of the sample.

Marigolds are probably 28% of the garden.

28% of 600 is 168 marigolds.

Javier

In the sample, 7 of 25 are marigolds.

Marigolds	Total
7	25
168	600

.24 .24

6 Let's watch an animation to reveal how many marigolds are in the entire garden.

Discuss:

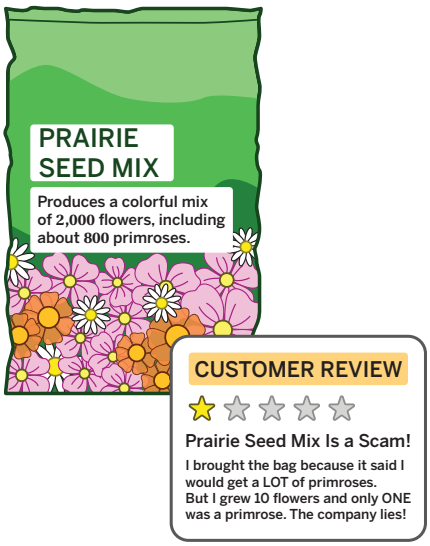
- How did the actual number of marigolds in the garden compare to the earlier estimates?
- What do you think could have improved the estimates?

Activity 2

Name: ..... Date: ..... Period: .....

The Bad Review

7 Here is a new bag of seeds and a customer review.



a **Discuss:**

- What does the company claim about the bag?
- Why did the customer leave a one-star review?

b How could you investigate the company's claim about primroses?

8 Let's grow a sample of up to 100 flowers from the Prairie Seed Mix.

a Write down the number of each type of flower in your sample in the table.

Flower Type	Marigold	Primrose	Daisy	Total
Count				

b How many of the 2,000 flowers do you expect to be primroses based on your sample? Explain your thinking.

9 Based on your sample, what do you think about the company's claim now?

10 Synthesis

Discuss: How can you use a sample and proportional reasoning to estimate information about a population?

Use this sample and the population if that helps with your thinking.

13 Summary 8.12

How can we use data from a sample to make claims about a population? One way is to use *proportional* reasoning.

Let's say someone is wondering how many students at their school might vote for a candidate for student council. It would be challenging and time consuming to ask all 500 students at the school, so they collect a sample of 25 students. It is important to gather the sample in a way that makes sure the sample is likely to be representative, like asking one student from each homeroom or asking 25 students at random.

If 10 out of the 25 students in the sample said they would vote for this candidate, there are several strategies for making a prediction about the population.

Strategy A

10 out of 25 is equal to $\frac{10}{25}$ or 40% of the sample.

40% of the population (500 students) would be $0.4 \cdot 500 = 200$ students.

Strategy B

The population is $\frac{500}{25} = 20$ times as large as the sample, so multiply the number of votes by 20 to determine the number of students in the population who would vote for them.

Votes	Total Students
10	25
200	500

Name: ..... Date: ..... Period: .....

Problems 1–3: Faaria and Ariel wondered what percentage of students at school would dye their hair blue. They each surveyed a different random sample of the students at school.

1. Faaria asked 10 students, and 2 of them said they would. Based on Faaria's sample, what percent of the students would dye their hair blue?

2. Ariel asked 100 students, and 17 of them said they would. Based on Ariel's sample, what percent of the students would dye their hair blue?

3. Whose percentage is likely to be closest to the percentage of all the students? Explain how you know.

Problems 4–6: In a school of 580 students, one class was asked which hand they write with. "L" means they use their left hand, and "R" means they use their right hand.

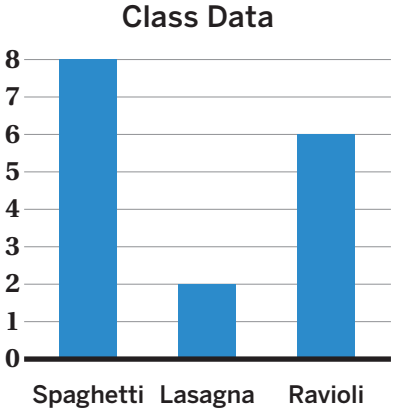
Here are the results: L, R, R, R, R, R, R, R, R, L, R, R, R, R, R

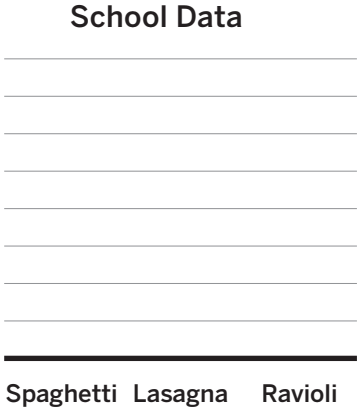
4. Based on this sample, estimate the percentage of students at the school who write with their left hand.

5. Estimate the number of students at the school who write with their left hand.

6. A different class of 18 students is surveyed. Estimate how many write with their left hand. Explain your thinking.

7. 16 students at a school were asked about their favorite pasta dish. The results are shown in the bar graph. Create a new bar graph showing the possible results for all 400 students in the school. Be sure to scale and label the vertical axis.





Practice
8.12

Name: ..... Date: ..... Period: .....

Spiral Review

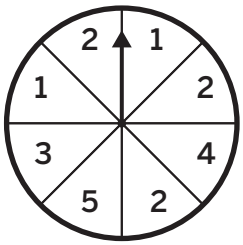
8. Match each expression in the first list with an equivalent expression from the second list.

Expression	Equivalent Expression
a. $(8x + 6y) - (2x + 4y)$	..... $10x - 10y$
b. $(8x + 6y) - (2x - 4y)$	..... $10x - 2y$
c. $8x - 6y - (-2x + 4y)$	..... $6x + 2y$
d. $8x - 6y - (-2x - 4y)$	..... $6x + 10y$

9. Complete the table so that each column has an equivalent fraction, decimal, and percent.

Fraction	$\frac{2}{5}$		$\frac{3}{10}$	
Decimal		0.75		0.125
Percent	40%	75%		

10. A spinner is divided into 8 equal sections.



If the arrow is spun only once, what is the probability that it will land on a number greater than 3?

A. $\frac{1}{4}$

B. $\frac{2}{3}$

C. $\frac{5}{8}$

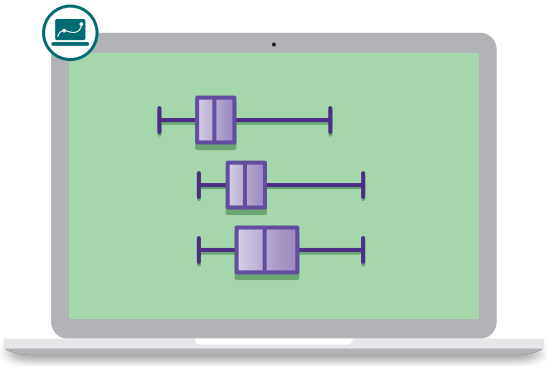
D. $\frac{1}{2}$

11. Alejandro says the volume of a cube with a surface area of 216 square millimeters is 216 cubic millimeters. Do you agree?

Explain your thinking.

Name: ..... Date: ..... Period: .....

Populations and Samples | Visualize Populations | 7.SP.2, 7.SP.3, 7.SP.4, SMP.3, SMP.6



Plots and Samples

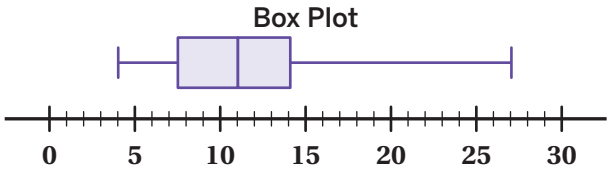
Let's use box plots and dot plots to make predictions.

Warm-Up

1 Tiana used this ordered list to create the box plot shown. What do you notice? What do you wonder?

Ordered List

4, 5, 7, | 8, 8, 10, | 12, 13, 13, | 15, 18, 27



I notice:

I wonder:

Name: ..... Date: ..... Period: .....

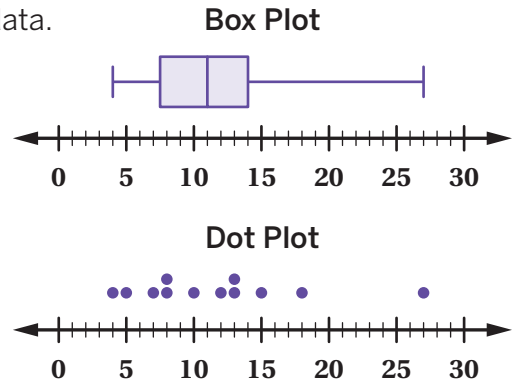
Gathering Samples

2 **Data Talk!** Here are two plots of the same data.

Write one fact about the data that you can see in each plot.

From the box plot, I can see . . .

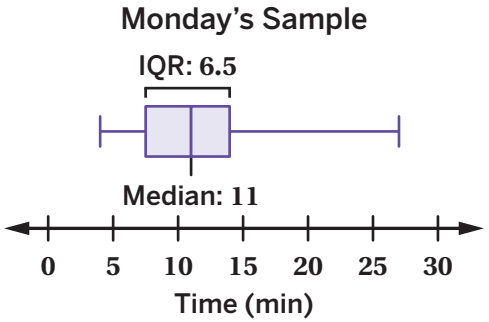
From the dot plot, I can see . . .



3 The previous data was a sample that Tiana collected on Monday.

She wants to know how long it takes students at her school to get to school by car. She asked 12 random students who came to school by car.

The box plot shows the **median** and **interquartile range (IQR)** of the data.

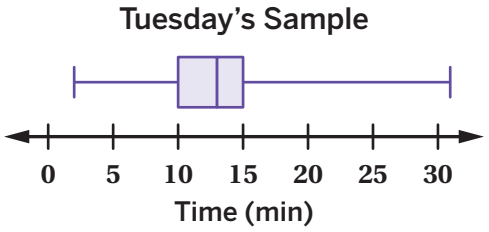


Explain what each measure tells you about the data.

Median:

IQR:

4 On Tuesday, Tiana asked 12 other random students who arrived by car how long it took them to get to school.



a Estimate the IQR of this sample.

b Estimate the median of this sample.

Name: ..... Date: ..... Period: .....

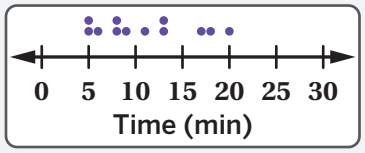
Gathering Samples (continued)

5 Tiana gathered more samples on Wednesday, Thursday, and Friday. Group the representations by day.

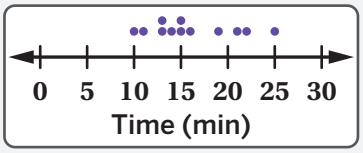
A. **Median:** 10 minutes
IQR: 8 minutes

B. **Median:** 15 minutes
IQR: 7 minutes

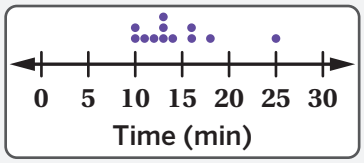
C.



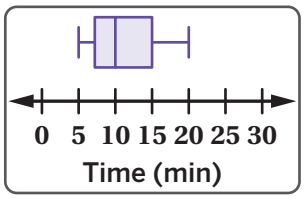
D.



E.

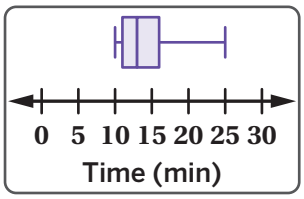


Wednesday's Sample



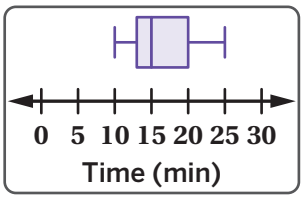
.....

Thursday's Sample



.....

Friday's Sample



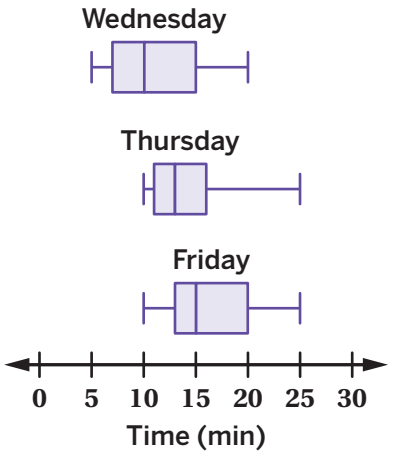
.....

6 Tiana wanted to compare her samples from Wednesday, Thursday, and Friday.

Each box plot represents the time it took for 12 random students to get to school by car.

How are these samples alike?

How are they different?

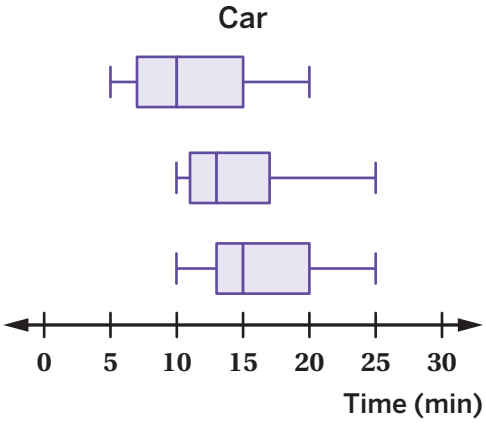


Activity 2

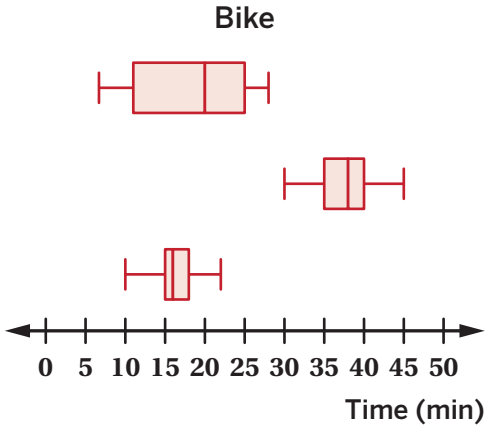
Name: ..... Date: ..... Period: .....

Making Predictions

7 Jayla collected samples from students who bike to school. Predict the median time it will take students to get to school by car and by bike.



Median prediction: .....



Median prediction: .....

8 **Data Talk!** You predicted two population medians in the previous problem. Which of the predictions were you more confident about? Circle one.

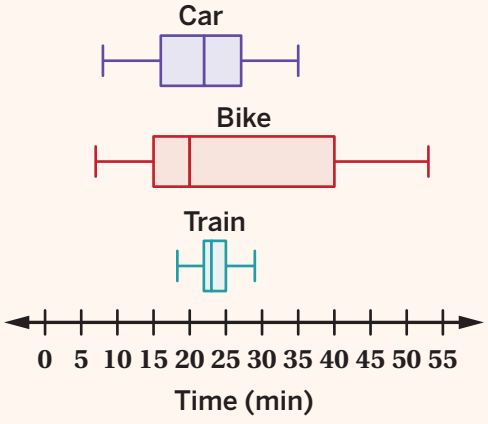
Car Bike The same

Explain your thinking.

You're invited to explore more.

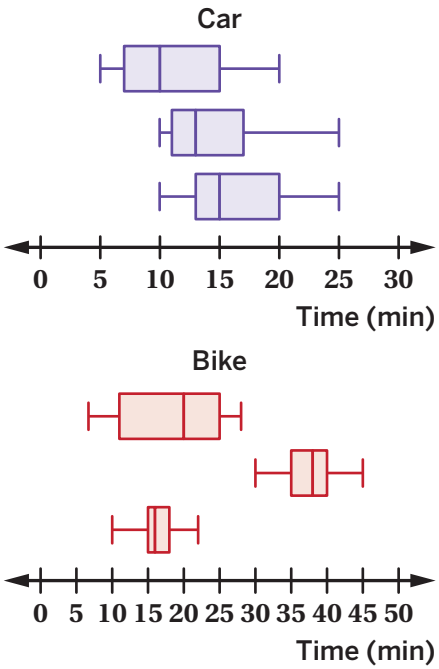
9 Here are the times it took for 2,000 students to get to school by car, bike, and train.

Write advice to a new student at this school describing an advantage of each vehicle.



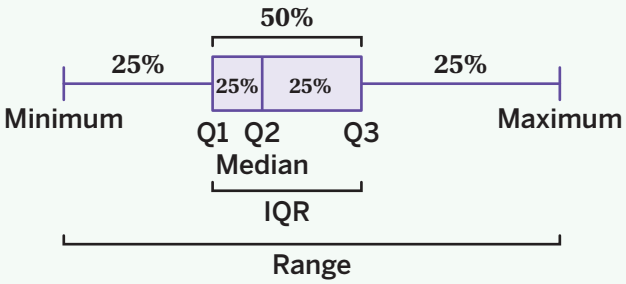
10 Synthesis

Describe how you can use medians and IQRs from multiple samples to make predictions about a population.

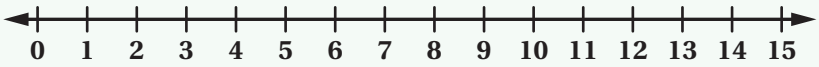


13 Summary 8.13

Box plots show the data split up into quartiles, with each section representing about 25% of the data in the data set.



The measure of center on a box plot is the **median** and the measure of spread is the **interquartile range (IQR)**, or the distance between the middle 50% of the data.



One way to make sense of data is to collect and compare multiple samples from the same population. In general, when samples have similar medians or interquartile ranges, predictions about the population are more likely to be accurate. If the medians for two samples are very different, you may be less confident that your predictions about the population are accurate.

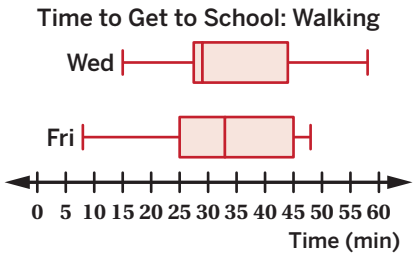
interquartile range (IQR) A way to measure how spread out a data set is. The IQR is calculated as the distance from Q1 to Q3.

median A measure of center. It is the middle value of a data set when the values are in numerical order. When there is an even number of data points, the median is the average of the two middle values.

Practice
8.13

Name: ..... Date: ..... Period: .....

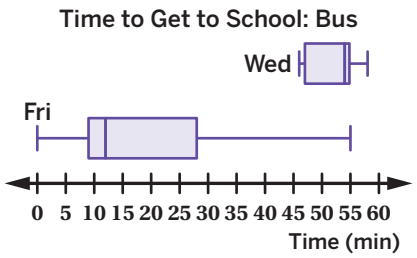
Problems 1–2: Kai wants to understand how long it takes for students to get to school using different methods. On Wednesday, Kai surveyed random samples of students who walked and students who took the bus. He repeated this on Friday.



1. Estimate the median time it takes to get to school by each method.

Walking:

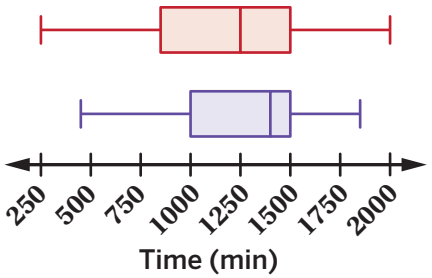
Bus:



2. Which estimate is more likely to be accurate?

Explain your thinking.

Problems 3–5: A video game developer wants to know how long it takes people to finish playing a new game. She surveyed two random samples of 20 players and asked how long it took each of them.

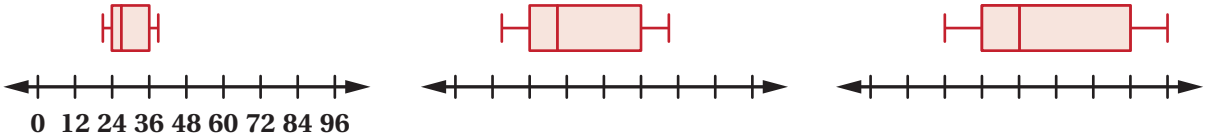


3. Estimate the median time it takes a player to finish the game.

4. How close do you think your estimate is to the actual median time?

5. How could the developer change her sampling strategy to make you more confident that the data is representative of the population?

Problems 6–7: Here are three box plots of the same data set on different axes.



6. Label the number lines under the other two box plots.

7. Determine the median of the data set.

Practice
8.13

Name: ..... Date: ..... Period: .....

Spiral Review

8. Complete the table so that each column has an equivalent fraction, decimal, and percent.

Fraction	$\frac{4}{5}$		$\frac{3}{8}$	
Decimal		$0.\overline{6}$		0.01
Percent	80%	$66.\overline{6}\%$		

9. The recipe for a signature pizza uses 3 cups of cheese, $\frac{3}{4}$ cup of olives, and $1\frac{1}{2}$ cups of onions. How many cups of onions are needed to make 10 signature pizzas?

A. 10 cups

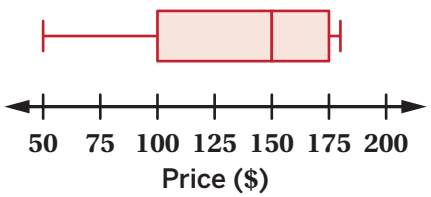
B. 30 cups

C. $7\frac{1}{2}$ cups

D. 15 cups

Problems 10–11: Determine the median and interquartile range (IQR) for each box plot.

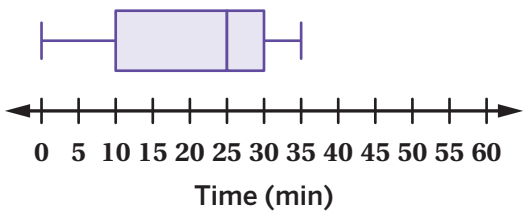
10.



Median:

IQR:

11.



Median:

IQR:

Problems 12–13: Solve each equation. Show your thinking.

12. $-6(x + 1) = 42$

13. $-5(2x - 3) = 25$

Name: ..... Date: ..... Period: .....

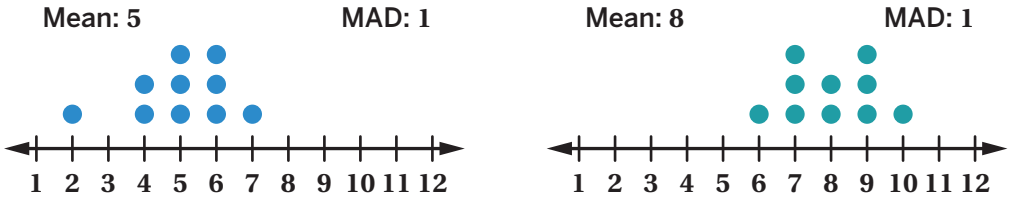
Visualize Populations 7.EE.3, 7.SP.3, 7.SP.4, SMP.1, SMP.6

School Newspaper

Let's compare populations by analyzing the difference between the measures of center and variability.

Warm-Up

Here are two dot plots.

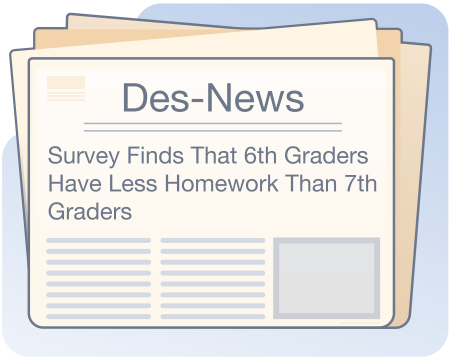


1. **Discuss** What do you notice? What do you wonder?

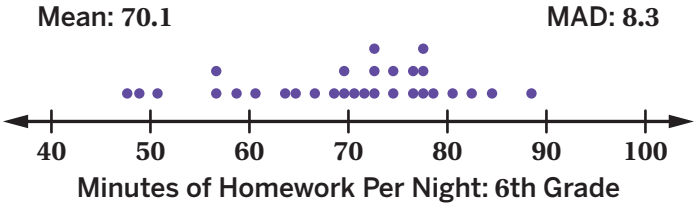
Name: ..... Date: ..... Period: .....

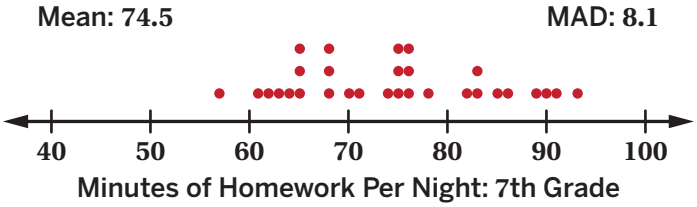
Homework Headline

2. **Data Talk!** Students at Median Middle School are investigating issues for the school newspaper.



They surveyed an equal number of randomly selected 6th and 7th graders out of the 500 students at their school. Here are the results of the survey and a headline based on the results.





a The mean of the 6th grader data set is 70.1 minutes. What does this statistic represent?

b The MAD of each data set is about 8 minutes. What does this statistic represent?

c Based on this data, why might someone believe this headline?

d Based on this data, why might someone not believe this headline?

Name: ..... Date: ..... Period: .....

Homework Headline (continued)

3. Wohali and Ama use different strategies to decide if they believe the headline.

Wohali

$74.5 - 70.1 = 4.4$

I believe the headline because the mean of the 6th graders' data is 4.4 minutes less than the mean of the 7th graders' data.

Ama

$74.5 - 70.1 = 4.4$

$$\frac{4.4}{8.3} \approx 0.53$$

I don't believe the headline. Even though the mean is 4.4 minutes less, 4.4 minutes isn't a big difference when looking at the spread of the data. The difference is not even 1 MAD, it's about half of that! There will be a lot of overlap between the data sets.

Whose claim do you agree with? Explain your thinking.

4. The survey results for 8th graders show a mean of 90 minutes and a MAD of 8 minutes.

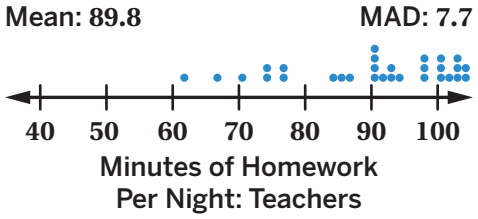
Let's use Ama's strategy to investigate whether there is a big difference in homework time for 7th and 8th graders.

a Calculate how many MADs apart the means are. Use the larger MAD in your calculation.

b What might Ama say about whether 8th graders have more homework than 7th graders?

5. Teachers use the word "prep" to describe the work they do at home. Here are some data and statistics about their homework time.

a Calculate how many MADs apart the means are for the time teachers work at home with the time 7th graders spend on homework.



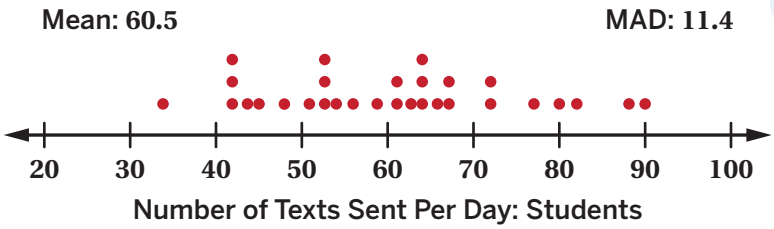
b How does the data representing the teachers' homework time compare to the 7th graders' data? Use your calculation in part a to support your thinking.

Name: ..... Date: ..... Period: .....

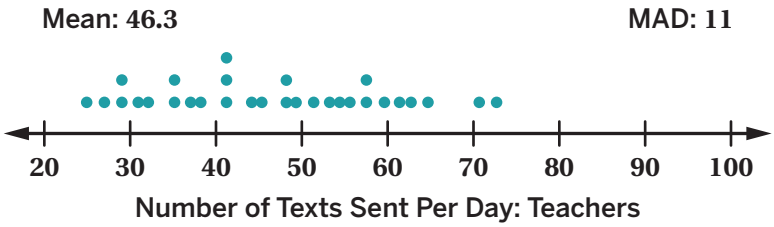
Texting Title

6. **Data Talk!** Students at Median Middle School surveyed a random sample of 30 students and 30 teachers.

Here are the survey results and the headline they wrote.



Number of Texts Sent Per Day: Students



Number of Texts Sent Per Day: Teachers

a Based on this data, why might someone believe this headline?

b Based on this data, why might someone *not* believe this headline?

7. Let's investigate further.

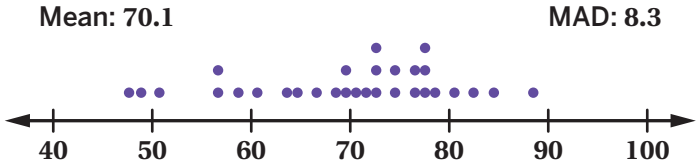
a How many MADs apart are the means? Show your thinking.

b Does that calculation make the headline more or less believable? Explain your thinking.

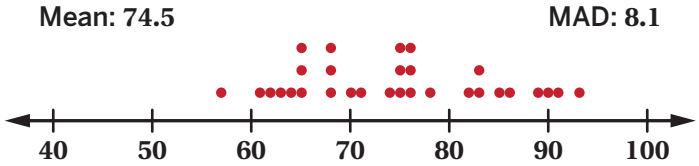
Synthesis

8. How can you use the MAD to determine how different two populations are?

Use the data from Median Middle School if it helps with your thinking.



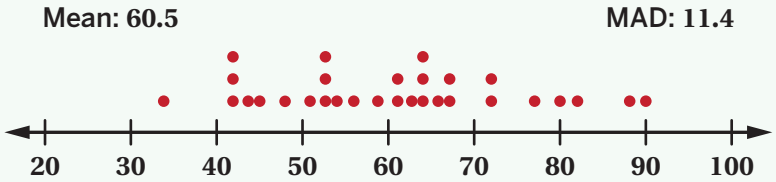
Minutes of Homework Per Night: 6th Grade



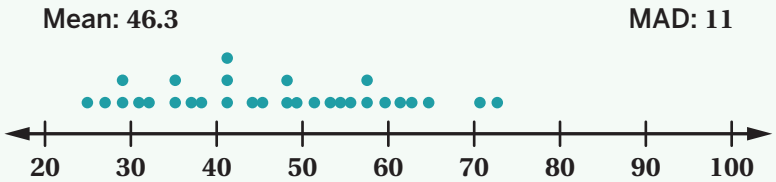
Minutes of Homework Per Night: 7th Grade

Summary 8.14

In general, it is easier to compare two individuals or objects than it is to compare two populations. For example, you can answer the question, "Which 7th grader is taller?" by measuring the heights of two 7th grade students and comparing them directly. However, to answer the question, "Do middle school students send more texts per day than their teachers?" you need to collect samples and analyze the measures of center and variability.



Number of Texts Sent Per Day: Students



Number of Texts Sent Per Day: Teachers

Here are the results of a random sample of 30 students and 30 teachers surveyed about the number of texts they send per day. To decide if the data sets are very different from each other, we can calculate the difference in their means and compare it to the larger MAD. The difference is $60.5 - 46.3 = 14.2$, which is about 1.25 times the MAD of 11.4. When the difference is more than 1 times the larger MAD, the data sets are very different. This suggests that students do send more texts than teachers.

Practice
8.14

Name: ..... Date: ..... Period: .....

Problems 1–3: A school's art club held a fundraiser to raise money for art supplies. This data shows the number of T-shirts sold each week during each season.

Fall

20 26 25 24 29
20 19 19 24 24

MAD: 2.8

Spring

19 27 29 21 25
22 26 21 25 25

MAD: 2.6

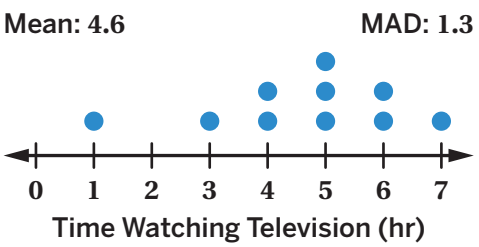
1. Determine the mean number of T-shirts sold in the fall and in the spring.

2. Calculate how many MADs apart the means are. Use the larger MAD in your calculation.

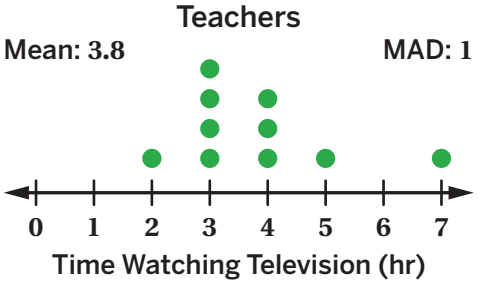
3. Based on this data, are sales generally higher in the spring than in the fall? Explain your thinking.

Problems 4–5: Abdullah compared the time that students and teachers spent watching television over the weekend. He took a random sample of 10 students and 10 teachers and made a dot plot of their responses.

Students



Teachers



4. Is there a big difference between the students' data and the teachers' data?

Explain your thinking.

5. Abdullah then took a random sample of 10 parents and found that they watched a mean of 2.5 hours of television, with a MAD of 1 hour.

Is there a big difference between how much television parents and students watch? Explain your thinking.

Name: ..... Date: ..... Period: .....

6. These dot plots represent the time that it took new runners to run a mile.

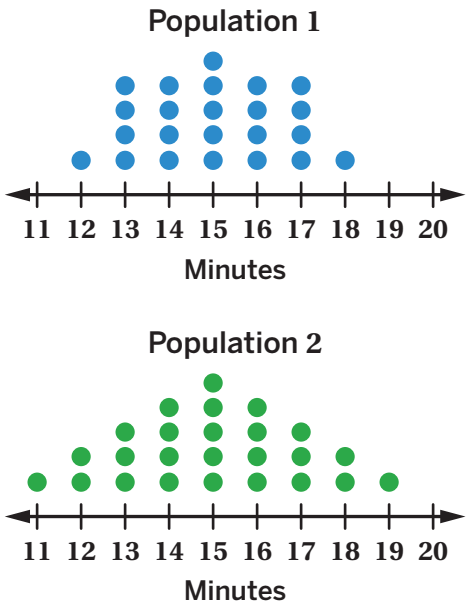
Which statement is *best* supported by the information in the dot plots?

A. The two populations have an equal number of runners.

B. The two populations have the same range.

C. The two populations have different medians.

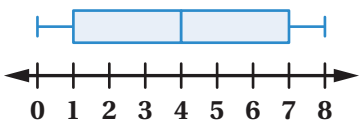
D. The two populations have the same mean.



Spiral Review

Problems 7–9: Determine the interquartile range (IQR) for each box plot.

7.



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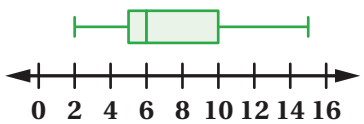

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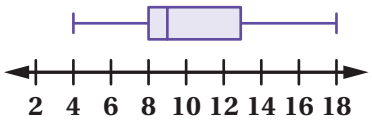


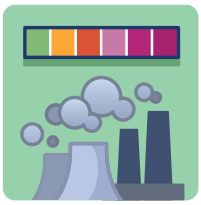
10. Sora says that 0.77 is a *repeating decimal* because both digits are the same. Is Sora's statement correct? Explain your thinking.

11. The area of a square is 25 square inches. The square is scaled by a scale factor of 3. What is the area of the scaled copy?

Name: ..... Date: ..... Period: .....

Populations and Samples | Visualize Populations | 7.EE.3, 7.SP.1, 7.SP.2, 7.SP.4, SMP.1, SMP.2, SMP.4

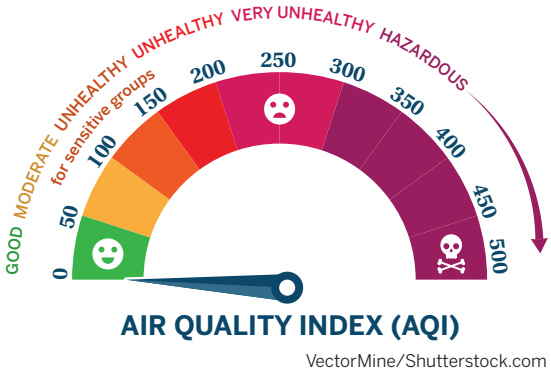
Air Quality



Let's analyze data about air quality.

Warm-Up

1. Here are two images related to air quality:



VectorMine/Shutterstock.com

a. **Discuss:** What do you notice? What do you wonder?

b. Why do you think the air quality in a place might be important?

c. Why might air quality be different in different places?

Activity 1

Name: ..... Date: ..... Period: .....

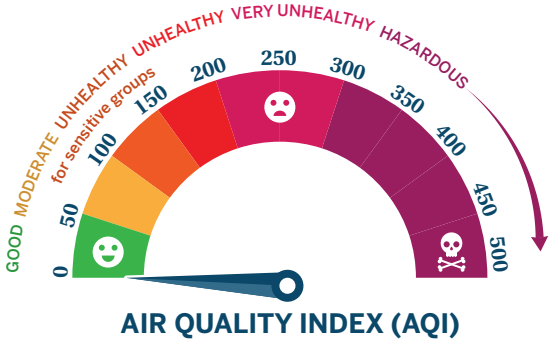
Air Quality Sampling

A group of students wondered: *What is the air quality like near sources of pollution, like freeways, factories, or airports?*

To investigate, they gathered data from sensors that recorded a value (0–500) on the air quality index (AQI) every hour during 2021 and 2022.

Pixel B/Shutterstock.com

2. What would a data point of 160 represent in this situation?



VectorMine/Shutterstock.com

3. Choose an air quality sensor you're interested in investigating. Circle your choice.

A. Sensor within a mile of a major airport in Dallas, Texas.

B. Sensor in a large city in the desert in Phoenix, Arizona.

C. Sensor in a heavily wooded area with occasional fires in Klamath Falls, Oregon.

D. Sensor in a valley, near a major trucking freeway in Bakersfield, California.

E. Sensor near several factories in Granite City, Illinois.

F. Sensor within a mile of a power plant in Dearborn, Michigan.

4. Describe a population and a possible sample that would help you investigate the air quality at the location you chose.

Population:

Sample:

5. Sora says that in order to analyze this data, we should use samples instead of the population. Why might this make sense?

Name: ..... Date: ..... Period: .....

Air Quality Sampling (continued)

6. Here are some methods we can use to sample from the air quality data. Circle one that is *unlikely* to produce data that is representative of the population.

A. Collect a sample of 500 random data points.

B. Collect two samples: 700 data points from February 2021 and 700 from July 2022.

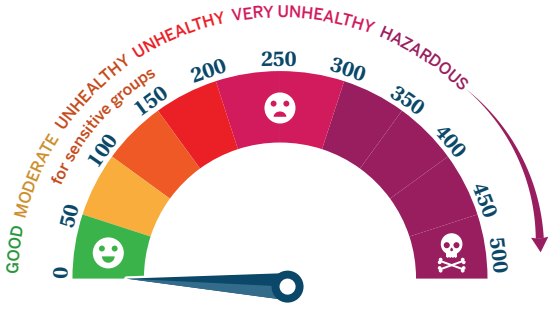
C. Collect two samples of 300 data points. Then decide whether to collect more samples.

D. Collect ten samples of 20 data points.

Explain your thinking.

7. **Data Talk!** Use the Digital Companion to select your location and generate sample(s) of the data.

a Follow the directions on the screen.



VectorMine/Shutterstock.com

b Based on your sample(s), describe the air quality at your location. Use statistics from the Digital Companion to support your description.

Activity 2

Name: ..... Date: ..... Period: .....

Comparing Locations

Data Talk! Now you'll analyze another location to see how the air quality compares to your first location.

8. Select a *different* location.

A. Sensor within a mile of a major airport in Dallas, Texas.

B. Sensor in a large city in the desert in Phoenix, Arizona.

C. Sensor in a heavily wooded area with occasional fires in Klamath Falls, Oregon.

D. Sensor in a valley, near a major trucking freeway in Bakersfield, California.

E. Sensor near several factories in Granite City, Illinois.

F. Sensor within a mile of a power plant in Dearborn, Michigan.

a Use the Digital Companion to sample the data for your new location. Follow the directions on the screen. Record any useful information in the space below.

b Compare the air quality at the two locations you chose. Use statistics from the Digital Companion to support your comparison.

9. **Discuss:** Think about your own community.

- What is the air quality like where you live or go to school?
- How do you think the air quality varies in your community?
- What questions do you have about air quality in your community?

Synthesis

10. **Discuss** Select one question to answer:

- ☐ How can statistics and sampling help us make sense of topics like air quality?
- ☐ What other questions might you ask to investigate this topic more?

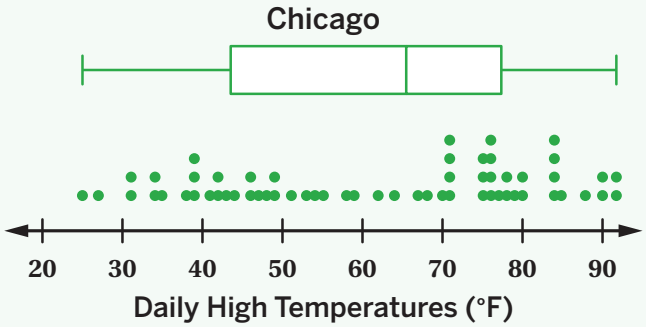
Summary 8.15

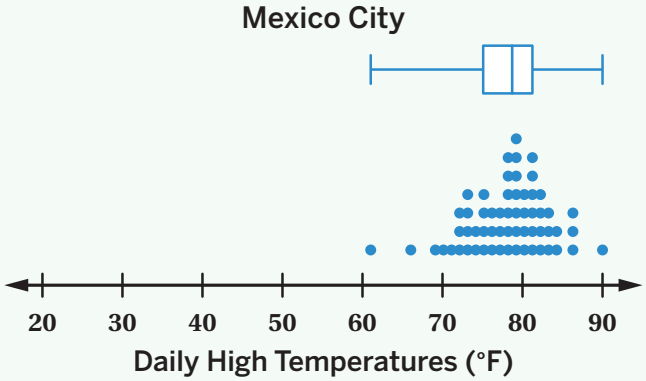
Statistics and sampling, along with visual representations of data, can help you to make sense of real-world topics. The more we understand the world around us, the more we can take action to improve it.

Let's say you want to compare the weather of Chicago and Mexico City. Here is a random sample of the daily high temperature for 60 days in 2023.

The box plots and dot plots let us see patterns in the data. We can also study measures of center and spread.

The mean and median temperatures are higher in Mexico City than in Chicago, so we might say it is generally hotter there. The IQR and MAD add more detail to our comparison. Chicago has greater measures of spread, so it has more variability in its temperature over the year.

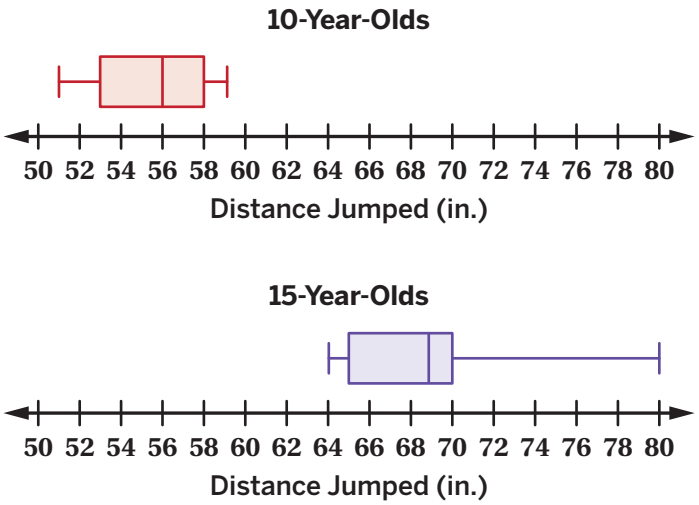




	Mean	Median	IQR	MAD
Chicago	61.5	65.5	34	17.4
Mexico City	77.9	78.5	6	3.9

Name: ..... Date: ..... Period: .....

Problems 1–2: These two box plots show the distances of standing jumps, in inches, for a random sample of 10-year-olds and a random sample of 15-year-olds at a school.



1. Based on the data, do 10-year-olds or 15-year-olds jump farther? Use data to support your answer.

2. For which age group do you think would be easier to predict the length of a students' jump? Use data to support your answer.

Problems 3–4: Here is a random sample of last month's high temperatures in two different cities.

Temperature (°F)

City A	75	72	68	71	79	76	75
City B	76	77	80	66	80	68	75

3. Why is it easier to use a sample to compare the temperatures in the two cities?

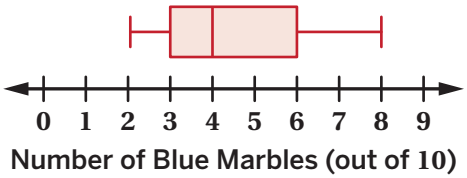
4. Based on this sample data, which city do you believe is warmer? Use statistics to support your explanation.

Practice
8.15

Name: ..... Date: ..... Period: .....

Spiral Review

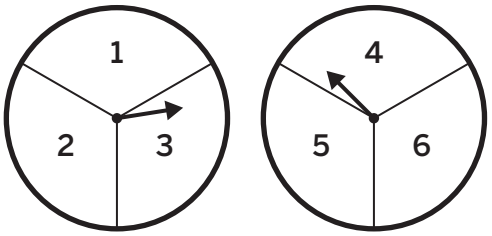
5. Marc is trying to determine how many blue marbles are in a bag of 1,000 marbles. He can only fit 10 marbles in his hand at a time. He takes 7 samples of 10 marbles, returning the marbles to the bag each time.



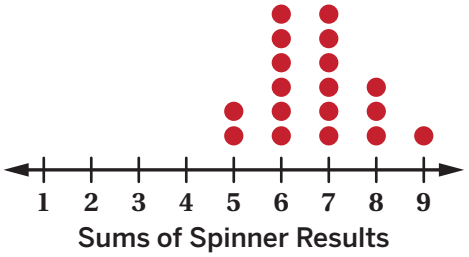
Number of Blue Marbles (out of 10)

Based on these results, how many of the 1,000 marbles do you expect are blue?

Problems 6–9: Here are two spinners with three equal sections. Each spinner is spun at the same time and their results are added together.



Here are the results of 18 spins.



Sums of Spinner Results

6. Based on these results, which sums are most likely?

7. Based on these results, which sums are least likely?

8. About how many times would you expect to spin a sum of 7 out of 100 spins?

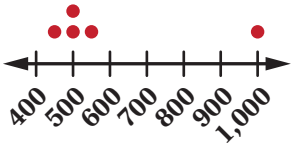
9. What is the probability of spinning a sum of 6? Make a tree diagram, table, or list if it helps with your thinking.

Problems 10–12: Calculate the mean of each data set.

10. 5, 8, 8, 10, 11, 13, 15

11. 7, 7.2, 7.4, 7.6, 7.8

12.



Name: ..... Date: ..... Period: .....

Populations and Samples | Visualize Populations | Proportional Relationships | 7.SP.4, 7.SP.7.a, 7.SP.1, 7.SP.2, 7.SP.3, 7.SP.5, 7.SP.6, 7.SP.7, 7.SP.8.a, 7.SP.8.b, 7.RP.3, SMP.1, SMP.2

Practice Day 2

Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: School Snacks

1. Probability: .....

2. ..... combinations
Explanation:

3. Probability: .....

4. Probability: .....

5. About ..... times
Explanation:

Task B: Raisin Brands

1. Population:

2. Median:

3. IQR:

4.

5.

Name: Date: Period:

Practice Day 2 (continued)

Task C: Oranges vs. Bananas

1. Circle one: A B C D

2. Circle one: Yes No
Explanation:

3. Circle one: Less confident More confident
Explanation:

Task D: Quality Control

1.

2. Circle *all* that apply: A B C D E

3. Circle one: Yes No
Explanation:

You're invited to explore more.

Notes:

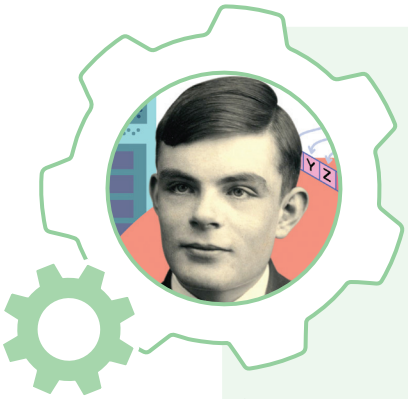
Career Connection

How would you crack a code with 150 million million million possibilities?

No, that's not a typo – that's how many possible combinations there were for each message encrypted by the Enigma ciphers used by the Nazis in World War II. (A *cipher* is a method for writing a code.) These ciphers used three rotating encryption components (rotors) and a reflector. Each rotor would give a different result each time, even if the same key was pressed! It took mathematicians, such as Alan Turing, and early computers to break these codes using patterns, probability, and logic.

Giorgio Rossi/Shutterstock.com

Cryptographers develop ciphers to encrypt and protect data for businesses and other organizations. They analyze ciphers to try to break them to ensure that the ciphers are strong.



Heritage Images/Getty Images

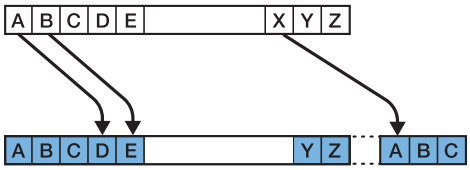
Meet Alan Turing

Alan Turing was an English mathematician, computer scientist, and pioneer in artificial intelligence. His invention of what today are called "Turing machines" explored the limits of what computers could do – decades before the invention of the personal computer. During World War II, Turing worked at Bletchley Park as part of Britain's code breaking efforts of German ciphers. It is estimated that these efforts shortened the war by 2 years and saved millions of lives.

Are you interested in studying cryptography? What can you do to learn more?

Community Connection

A Caesar cipher – named after Julius Caesar – is a type of substitution cipher. Each letter is substituted with another letter, e.g., "shift by 3 letters." Design your own encryption system and write a secret message. Share your message with at least 3 other people in your communities to see if they can crack your code!



Math Mindset

How do you think probability is related to code breaking? How would you look for patterns or use probability to help crack a code?