Amplify Desmos Math CALIFORNIA

Grade 6

Volume 1: Units 1–4

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

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Dear Student,

Welcome to Amplify Desmos Math California! We are excited to be partnering with you this year. You play an essential role in math class, so we wanted to reach out to introduce ourselves and tell you a bit about who we are.

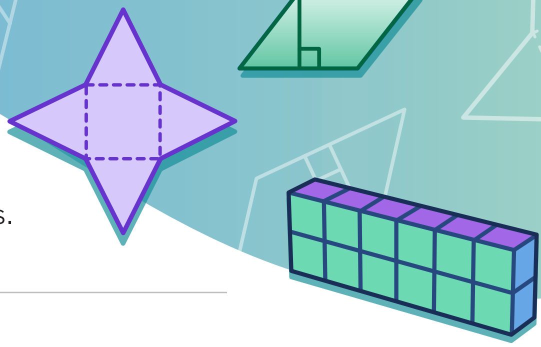
Amplify Desmos Math California is a team of math educators on a mission to support you and your classmates in learning math. We hope each lesson inspires you to use your creativity, ask questions, and discover connections between math concepts and the world around us.

Here is what you can expect this year:

- A blend of learning on both paper and devices.
- Interactive lessons that encourage you to ask questions, explore, settle disputes, create challenges for your classmates, and more!
- Digital activities that show what your ideas mean with plenty of chances for you to revise.
- Opportunities for you to engage with interesting and important big ideas in mathematics.

–The Amplify Desmos Math California Team

Unit 1 Area and Surface Area

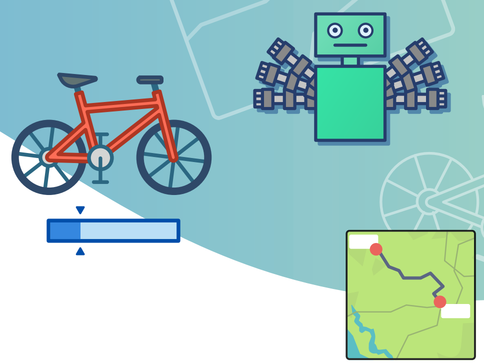


In this unit, you will learn to calculate areas of polygons. You will also represent polyhedra with nets and calculate their surface areas.

Unit 2 Introducing Ratios

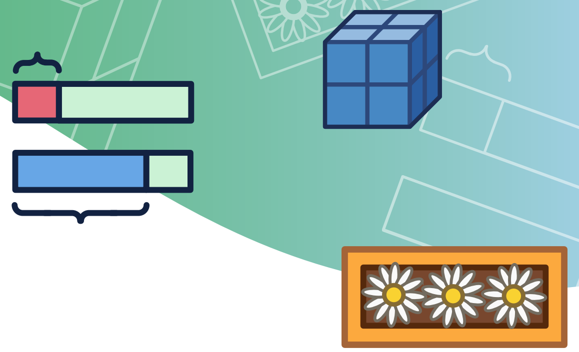
In this unit, you will be introduced to the concept of ratios. You will also represent ratios using double number lines, tables, and tape diagrams, and use ratio reasoning to solve problems.

Unit 3 Unit Rates and Percentages



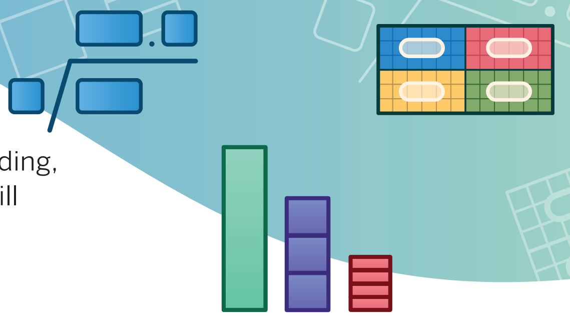
In this unit, you will apply ratio reasoning from Unit 2 to unit rates and recognize that equivalent ratios have the same unit rates. You will also use a variety of strategies and representations of percentages to determine missing percentages, parts, and wholes.

Unit 4 Dividing Fractions



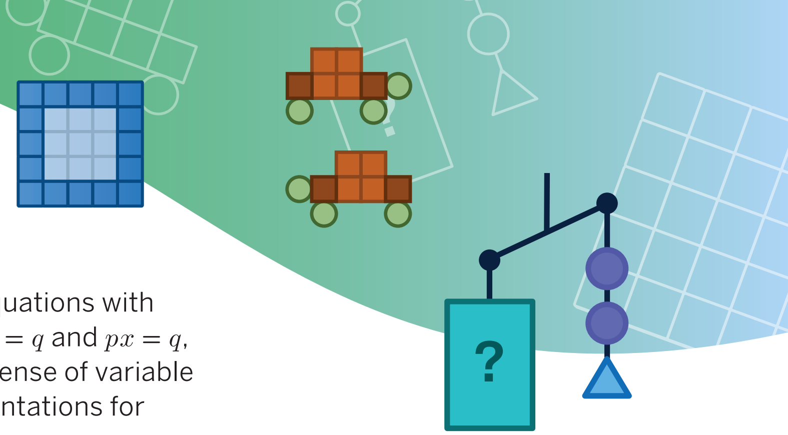
In this unit, you will extend what you learned about dividing whole numbers to divide fractions by fractions. You will learn a variety of strategies, including making tape diagrams, creating common denominators, and rewriting equivalent multiplication problems using the reciprocal.

Unit 5 Decimal Arithmetic



In this unit, you will develop and use a variety of strategies for adding, subtracting, multiplying, and dividing multi-digit decimals. You will also explore strategies for dividing whole numbers, including the standard algorithm.

Unit 6 Expressions and Equations



In this unit, you will reason about expressions and equations with variables. You will solve equations of the forms $x + p = q$ and $px = q$, write equivalent expressions using variables, make sense of variable expressions involving exponents, and create representations for relationships between two variables.

Unit 7 Positive and Negative Numbers

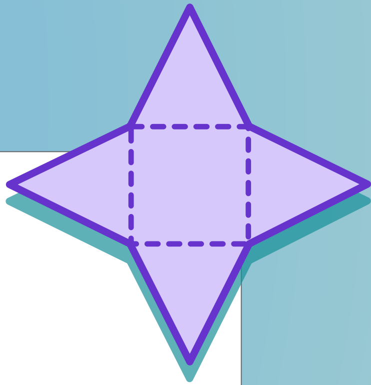
In this unit, you will explore positive and negative numbers in several contexts: on a number line, with inequalities, and on the coordinate plane.

Unit 8 Describing Data

In this unit, you will visualize data using dot plots, histograms, and box plots. You will also calculate measures of center and spread.

Unit 1

Area and Surface Area



Big Ideas in This Unit

CC2 Generalizing With Multiple Representations

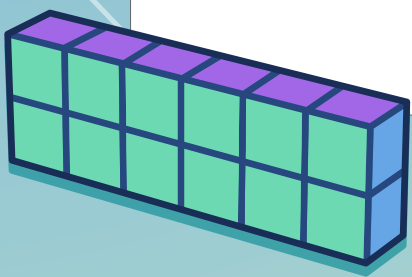
CC3 Nets and Surface Area

CC4 Graphing Shapes

Questions for Investigation

- What does it mean for two shapes to have the same area?
- How are the areas of rectangles, parallelograms, and triangles related?
- How are the surface area of polyhedra and the area of polygons related?

Explore: Tangram Paradoxes
How can you use tangram pieces to create tangram paradoxes?



Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!

Not yet — Almost — I got it!

I can . . .	Before	After
Explain what area is.		
Describe strategies for determining the area of a non-rectangular shape.		
Describe the characteristics of a quadrilateral.		
Use different strategies to determine the area of a parallelogram.		
Identify the base and height of a parallelogram.		
Calculate the area of any parallelogram using its base and height.		
Calculate the area of any triangle using its base and height.		
Use different strategies to determine the area of a trapezoid.		
Describe the characteristics of a polygon.		
Calculate the area of a polygon.		

I can . . .	Before	After
Explain what surface area is.		
Compare and contrast prisms and pyramids.		
Draw the net of a prism or a pyramid.		
Match a polyhedron with its net.		
Calculate the surface area of a prism or pyramid using their nets.		

Sub-Unit

1

Area

Name: ________ Date: ________ Period: ________

Nets and Surface Area | Graphing Shapes | Building Toward 6.G.1, SMP.1, SMP.6, SMP.7

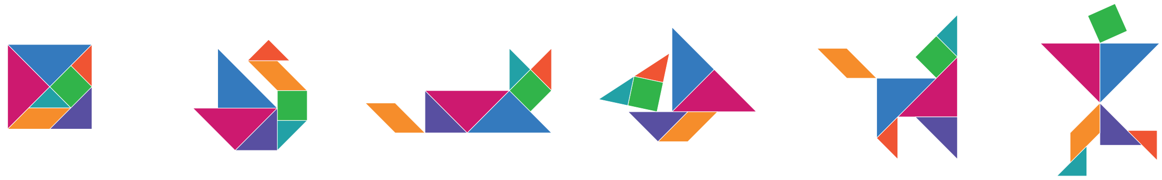
Explore: Tangram Paradoxes

How can you use tangram pieces to create tangram paradoxes?

Warm-Up

1. Tangram is a puzzle game that was invented in China around the 1700s. It consists of a square cut into seven pieces, rearranged to make all sorts of other shapes.

   Here are some examples of the different shapes you can make using just seven simple tangram pieces.



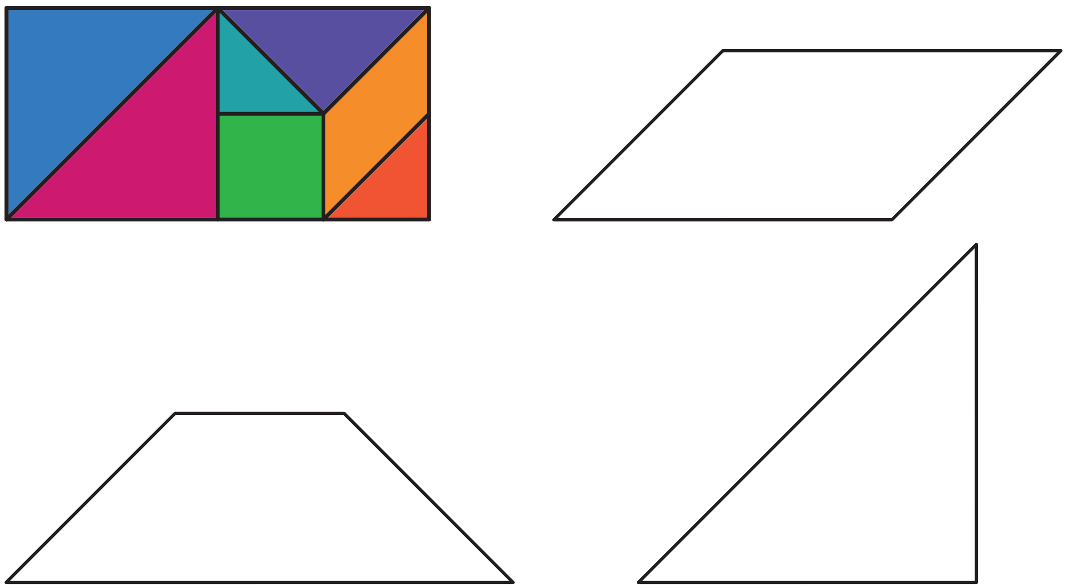
2. Using two or more of the tangram pieces, create as many different-sized squares as you can.

Activity

Name: ________ Date: ________ Period: ________

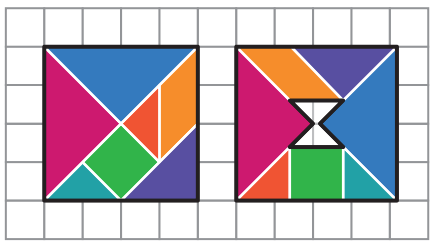
Tangram Paradoxes

3 **a** A rectangle made up of tangram pieces is given. Work with your partner to create the figures shown here using the arrangement of all seven of your tangram pieces in the rectangle.



b How are these figures alike? How are they different?

4 Here are two figures that can be created using all seven tangram pieces, but one has a blank space in the middle.



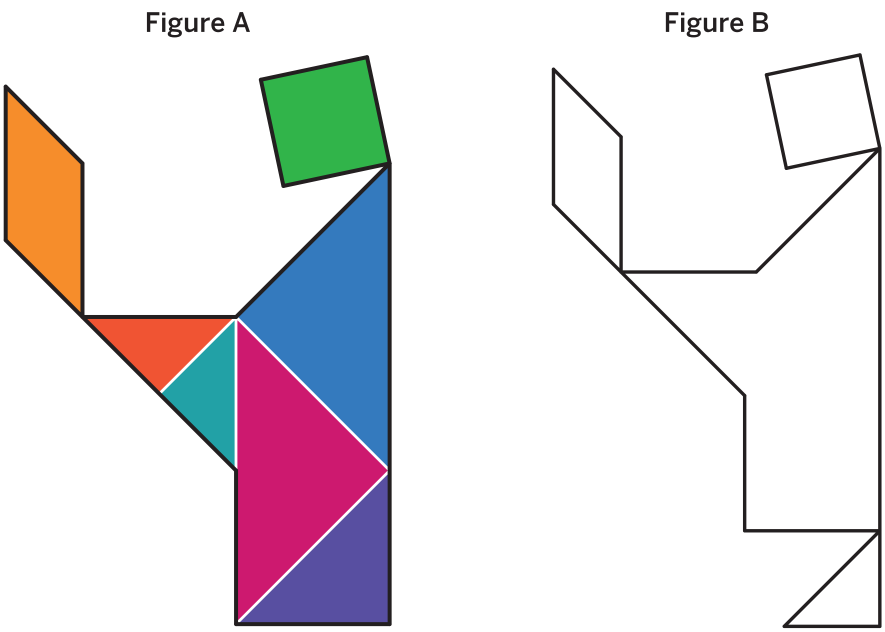
Discuss: How is it possible for one of these squares to have a hole in it when the other does not?

Activity

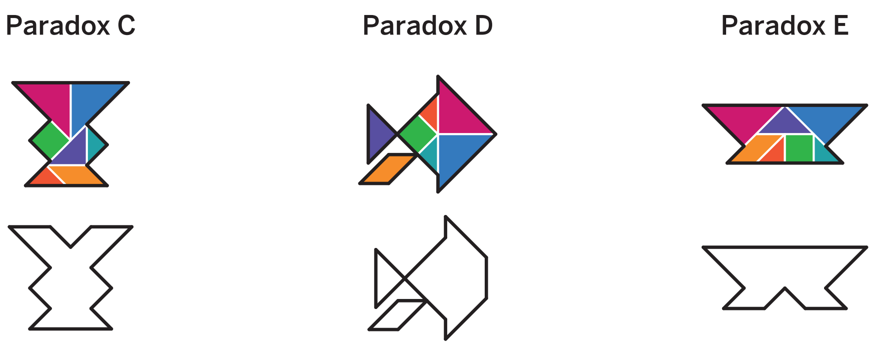
Name: .... Date: .... Period: ....

Tangram Paradoxes (continued)

5 Figures A and B are both side views of a person created using all seven tangram pieces, but Figure B has feet! Complete Figure B using the tangram pieces.



6 Here are more tangram paradoxes. Select one and try to create the pair.



Activity

Name: .... Date: .... Period: ....

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet — Almost — I got it!

I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem.

Not yet — Almost — I got it!

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet — Almost — I got it!

I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed.

Not yet — Almost — I got it!

I can select an appropriate tool to help me solve problems.

Not yet — Almost — I got it!

I can communicate my thinking and solutions clearly to others.

Not yet — Almost — I got it!

I can look for structure or patterns to help me solve problems.

Not yet — Almost — I got it!

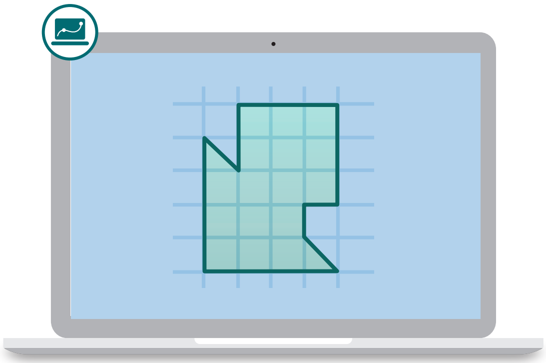
I can look for repeated calculations and other repeated steps to make generalizations.

Not yet — Almost — I got it!

Unit 1
Lesson
1

Name: .... Date: .... Period: ....

Nets and Surface Area | Graphing Shapes | 6.G.1, SMP.7



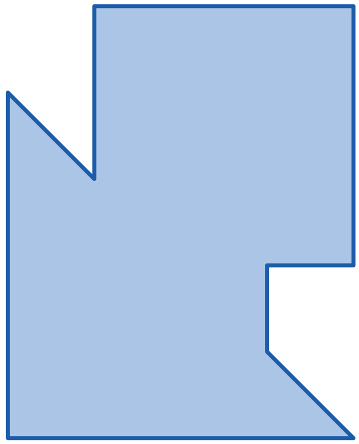
Shapes on a Plane

Let’s play with shapes and find their areas.

Warm-Up

1 Trevon made a shape with triangles, a square and a rectangle.

Describe what the shape reminds you of.

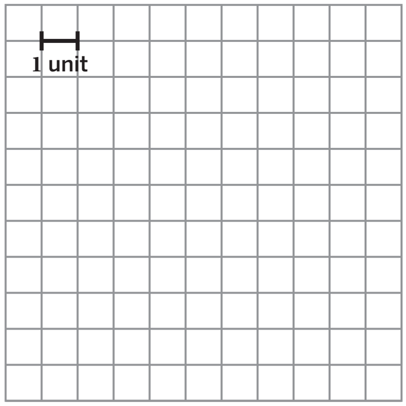


Activity 1

Name: .... Date: .... Period: ....

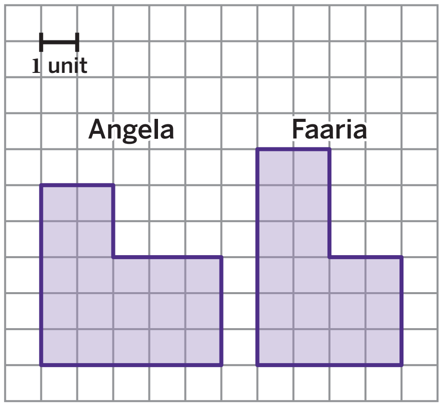
Areas of Non-Rectangular Shapes

2 **a** You will use a set of shape cutouts to make your own fun shape.



b Describe your shape and what it reminds you of.

3 Angela and Faaria both made shapes that look like boots.



Whose boot shape is larger? Circle one.

Angela Faaria I'm not sure

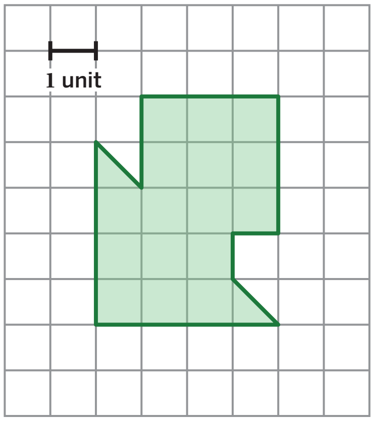
Explain your thinking.

Name: .... Date: .... Period: ....

Areas of Non-Rectangular Shapes (continued)

4 The **area** of a shape is one way to describe its size.

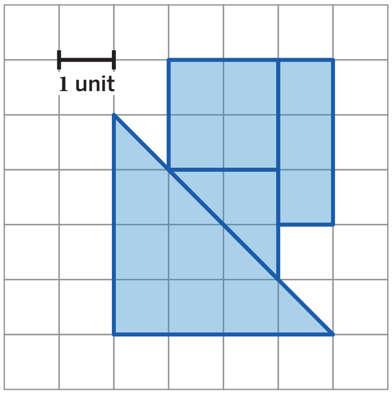
Determine the area of Trevon's shape.



5 There is often more than one way to determine area.

a Draw another way to determine the area of Trevon's shape.

b Describe your strategy.

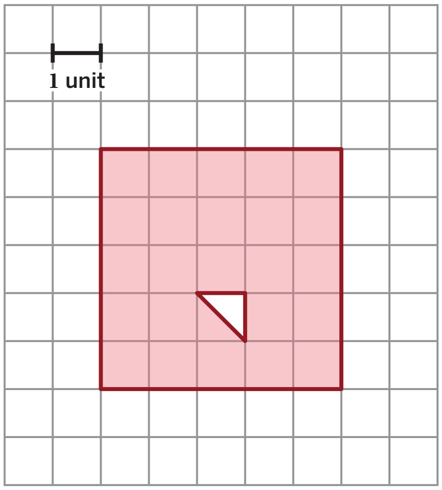


Activity 2

Name: .... Date: .... Period: ....

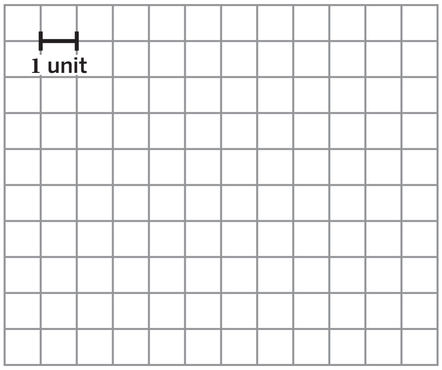
Area Challenges

6 **a** **Discuss:** What does Laila's shape remind you of?



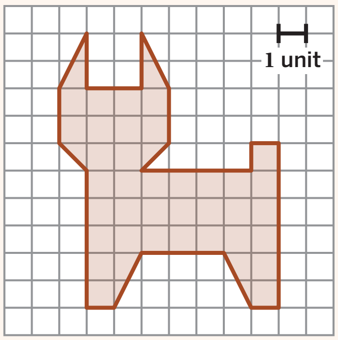
b Determine the area of Laila's shape. Draw on the shape if it helps with your thinking.

7 How many different areas can you make with your shape cutouts, if you arrange them without overlapping? Explain your thinking.



You're invited to explore more.

8 Determine the area of the dog.



Explain your thinking.

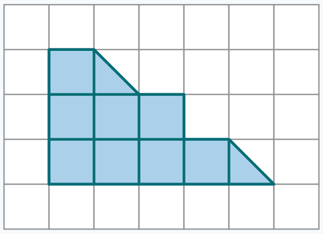
9 Synthesis

Discuss:

- What's something you learned today?
- What do you want to learn more about?

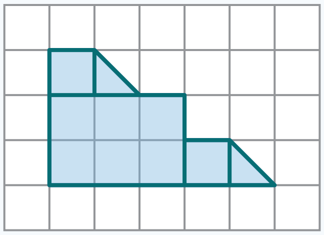
12 Summary 1.01

There are many strategies that can help you determine the area of shape on a grid.



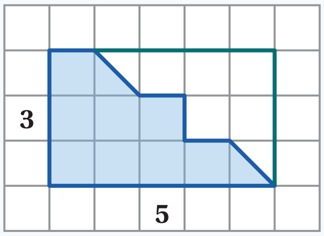
Count the number of unit squares that make up the shape.

There are 8 unit squares and two half unit squares.
$8 + 0.5 + 0.5 = 9$ square units.



Break the shape into non-overlapping rectangles and triangles.

We can break this shape into a 2-by-3 rectangle, two unit squares, and two triangles to calculate an area of $6 + 2 + 0.5 + 0.5 = 9$ square units.



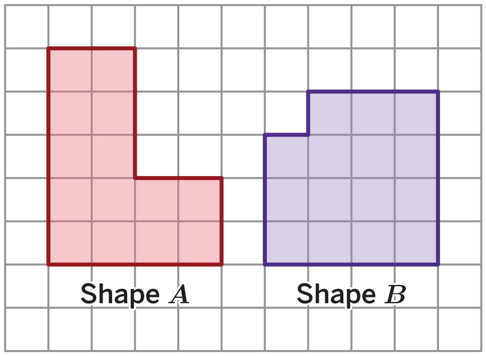
Draw a rectangle around the shape and subtract the empty space.

We can draw a 3-by-5 rectangle around this shape and subtract the empty squares to calculate an area of $15 - 6 = 9$ square units.

area A measure of the space inside a two-dimensional figure. It is expressed in square units.

Name: ________________ Date: ________ Period: ________

1. Which shape has a greater area? Show or explain your thinking.

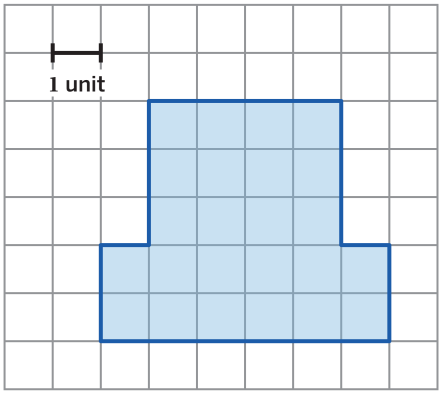


   A. Shape A

   B. Shape B

   C. They have the same area

Problems 2–4: Here is a new shape.



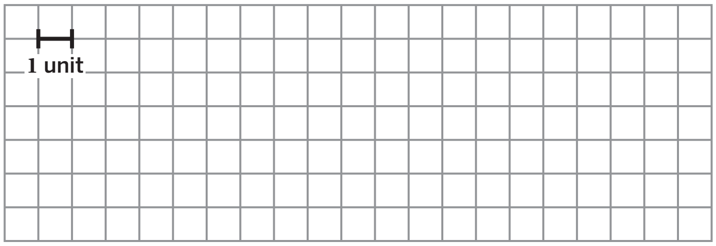
2. Determine the area of the shape. Show or explain your thinking.

3. Show or describe another way to determine the area of this shape.

4. Show or describe how you could change this shape so it has an area of 26 square units.

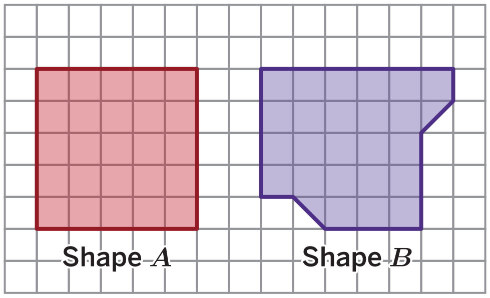
Name: ______________ Date: ________ Period: ________

5. Draw *three* different quadrilaterals, each with an area of 12 square units. Each square in this grid has an area of 1 square unit.



6. Which shape has a greater area?

Explain your thinking.



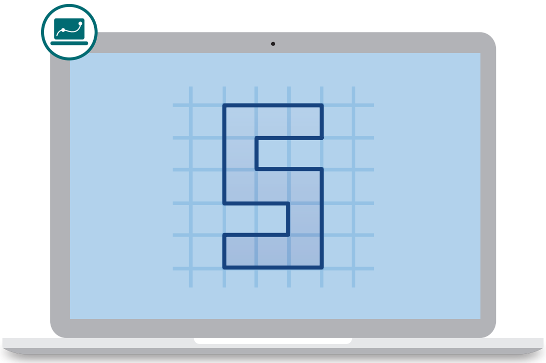
Spiral Review

7. Select *all* the numbers that are equivalent to 12.

- ☐ **A.** 4×3
- ☐ **B.** $2 + 6$
- ☐ **C.** $24 \times \frac{1}{2}$
- ☐ **D.** 24×2
- ☐ **E.** $4 + 4 \times 2$

Name: ________ Date: ________ Period: ________

Nets and Surface Area Graphing Shapes 6.G.1, SMP.3, SMP.6, SMP.7



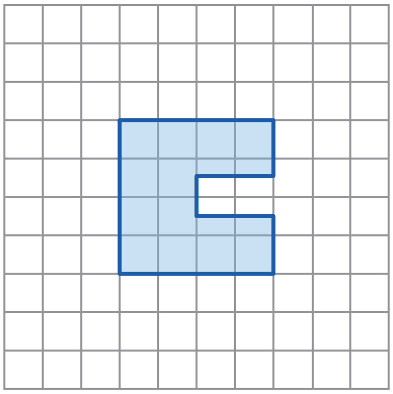
Letters

Let's explore the area of shapes.

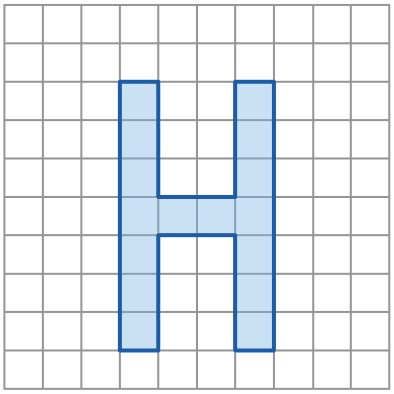
Warm-Up

1 Which figure doesn't belong? Explain your thinking.

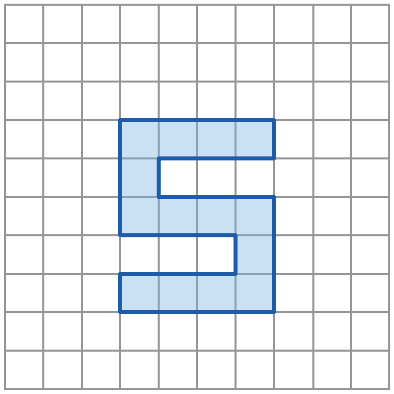
A.



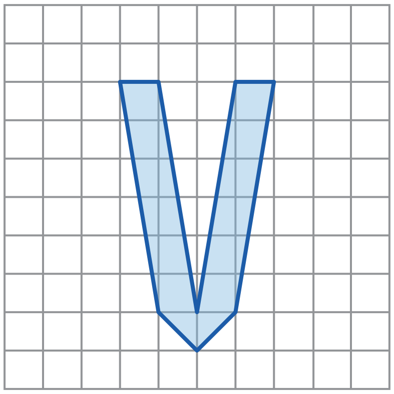
B.



C.



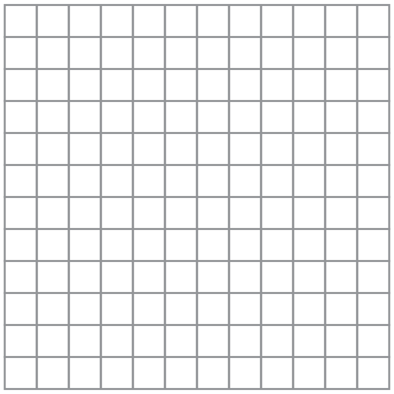
D.



Name: ______________ Date: ______ Period: ______

Rearranging Shapes

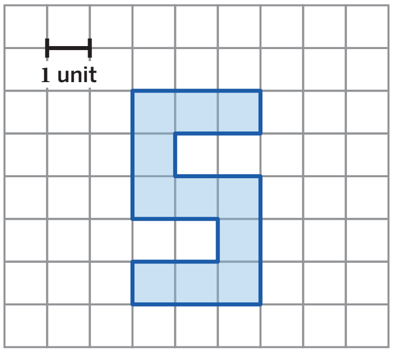
2 a Draw the first letter of your name on the grid.



b Tell a story about your name.

3 Saanvi colored in the “S” that she drew.

What is the area of the shape she colored?

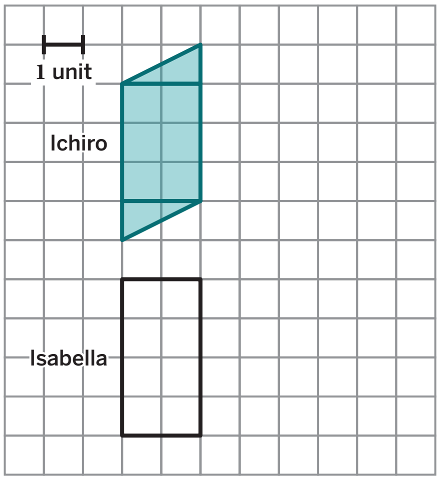


4 Ichiro and Isabella each drew an “I”.

Whose letter has a greater area? Circle one.

Ichiro   Isabella   They are the same

Explain your thinking.



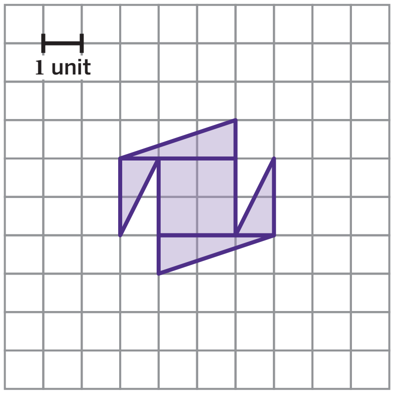
Name: .... Date: .... Period: ....

Rearranging Shapes (continued)

5 Zahra also cut up her "Z" to see how much area it covered.

What is the area of the shape she colored?

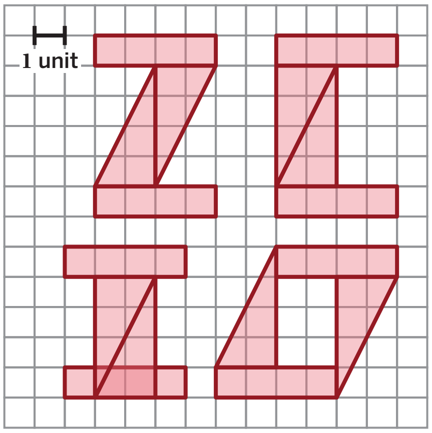
Use arrows to show how you could rearrange the pieces, if it helps with your thinking.



6 Zola cut a "Z" into pieces and rearrange it to make new letters.

Select *all* the new letters that have the same area as the "Z".

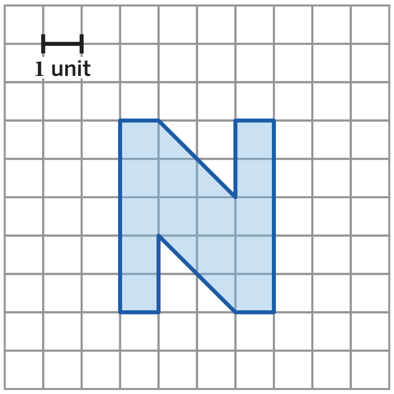
- ☐ C
- ☐ I
- ☐ O



7 Nathan made an "N".

a What is the area of the shape he colored? Sketch on the grid if it helps to show your thinking.

b **Discuss:** What strategy did you use?



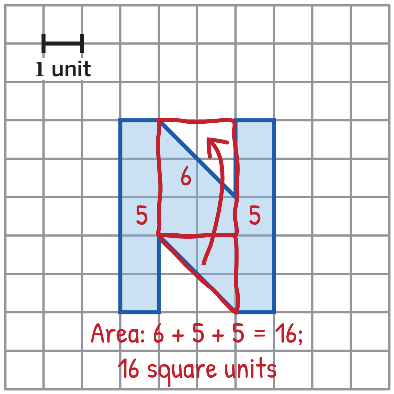
Activity 2

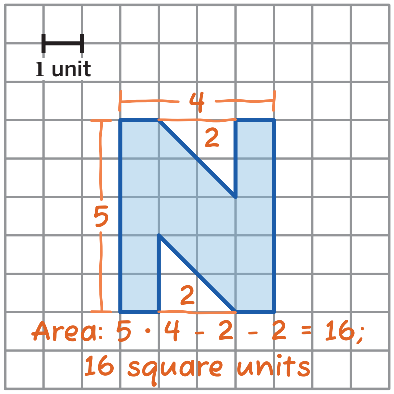
Name: .... Date: .... Period: ....

Area Strategies

8 Omar and Jayla used different strategies to determine the area of N.

a Take a look at each student's work.

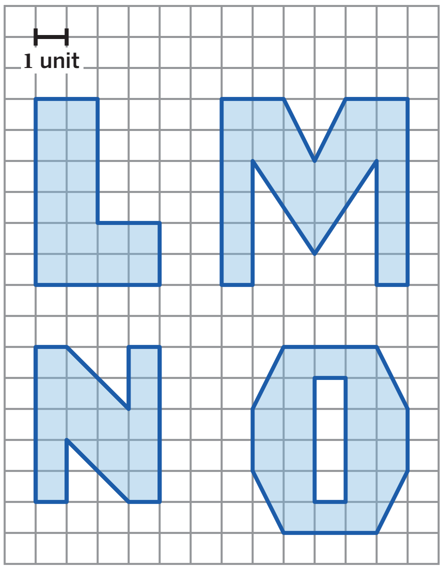




b Pick one student and explain how you think they determined the area.

9 Complete the table.

Letter	Area (sq. units)
L	
M	
N	16
O	

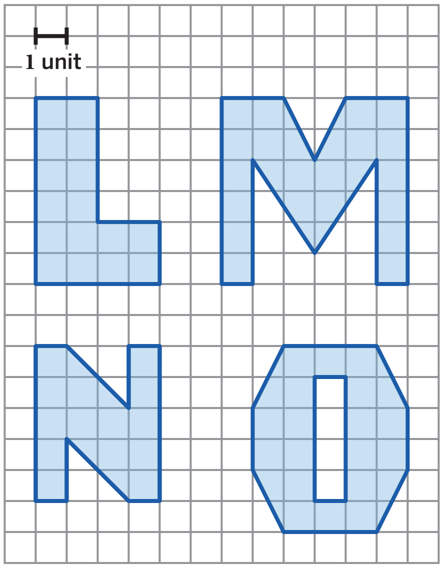


10 Synthesis

a Which area calculation are you most proud of? Circle one.

L M N O

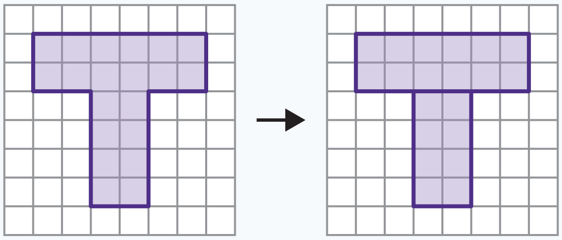
b Write some advice for someone determining this area.



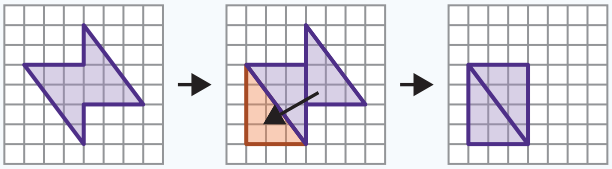
13 Summary 1.02

We can use shapes like rectangles, squares, and triangles to help us determine the area of more complex shapes. Here's how!

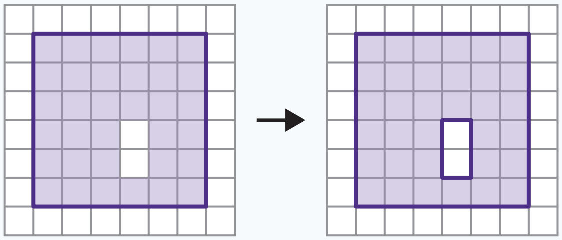
- Decompose the shape into two or more smaller shapes that have areas you know how to calculate.
- Add the smaller areas together.



- Decompose the shape and rearrange the pieces to form one or more other shapes that have areas you know how to calculate.
- Calculate the area of the new, simpler shape(s).



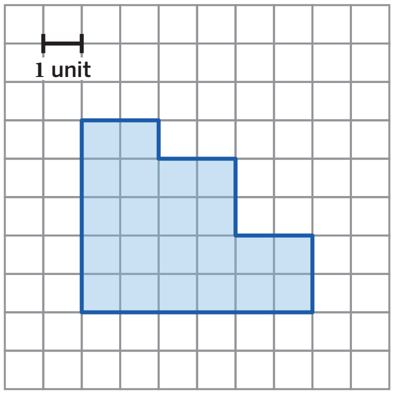
- If your shape has empty areas in it, determine its area as if it were a solid shape.
- Calculate the area of the empty space and subtract it from the total area.



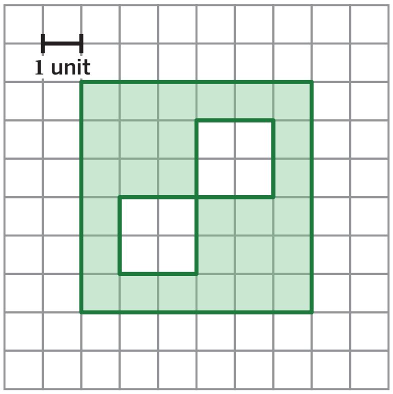
Name: ______ Date: ______ Period: ______

Problems 1–4: Determine the total area of each shaded region.

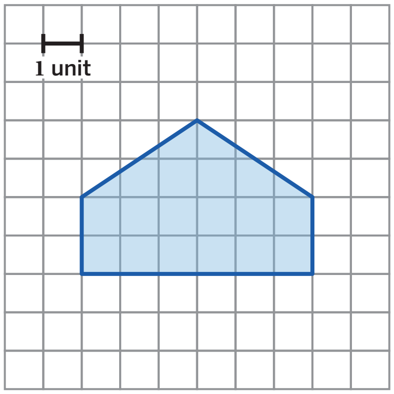
1.



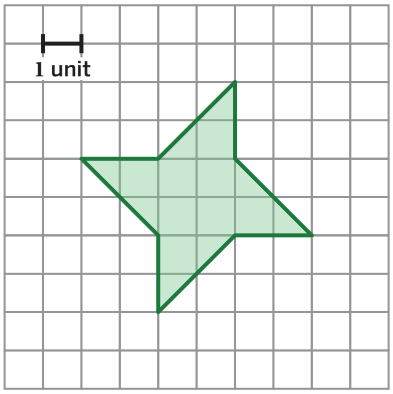
2.



3.



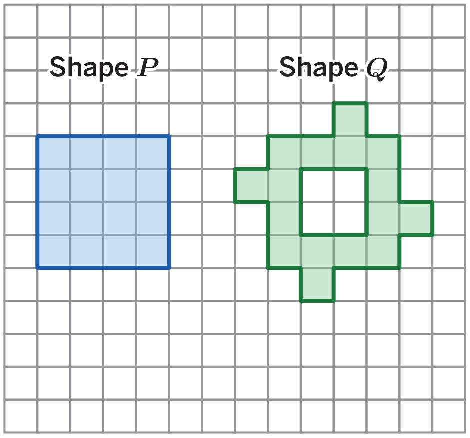
4.



5. Which shape has a greater area? Circle one.

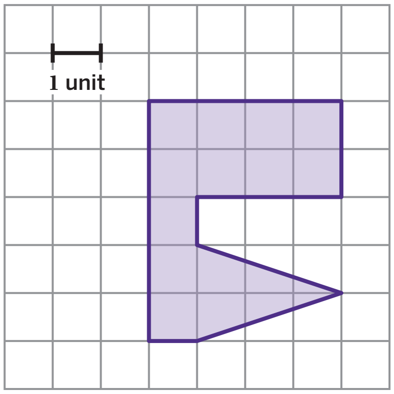
Shape P Shape Q They have the same area.

Show or explain how you know.



Name: .... Date: .... Period: ....

6. Jasmyn drew this shape. Determine its area.



A. 17 square units

B. 16 square units

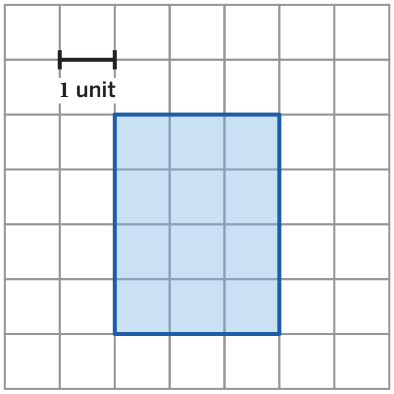
C. 14 square units

D. 11 square units

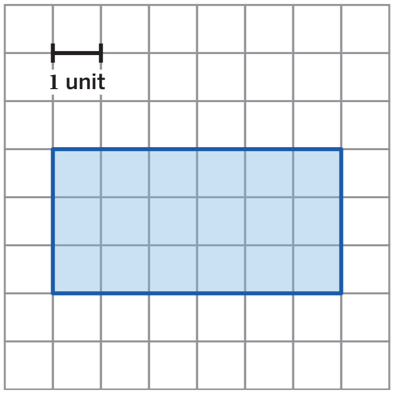
Spiral Review

7. Select *all* the rectangles with an area of 12 square units.

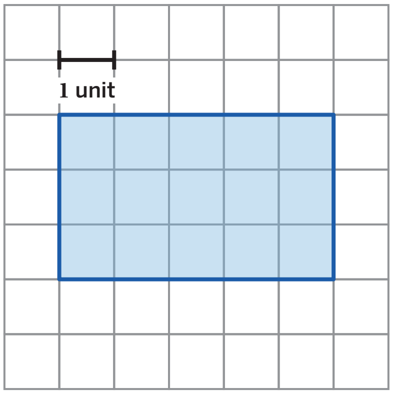
☐ **A.**



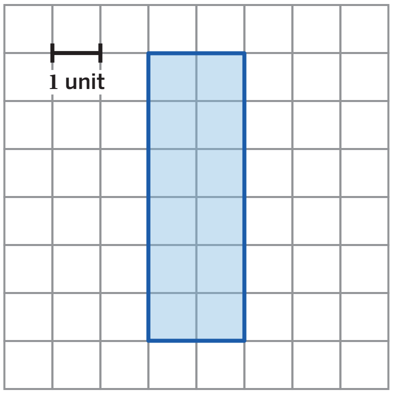
☐ **B.**



☐ **C.**



☐ **D.**



8. Select *all* the expressions that are equivalent to 16.

☐ **A.** $\frac{16 \cdot 2}{2}$

☐ **B.** $\frac{4 \cdot 8}{2}$

☐ **C.** $\frac{1}{2} \cdot 4 \cdot 8$

☐ **D.** $\frac{4 \cdot 4}{2}$

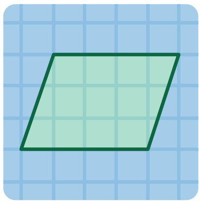
☐ **E.** $\frac{1}{4 \cdot 4}$

Unit 1
Lesson
3

Name: .... Date: .... Period: ....

Nets and Surface Area | Graphing Shapes | 6.G.1, SMP.3, SMP.6, SMP.7

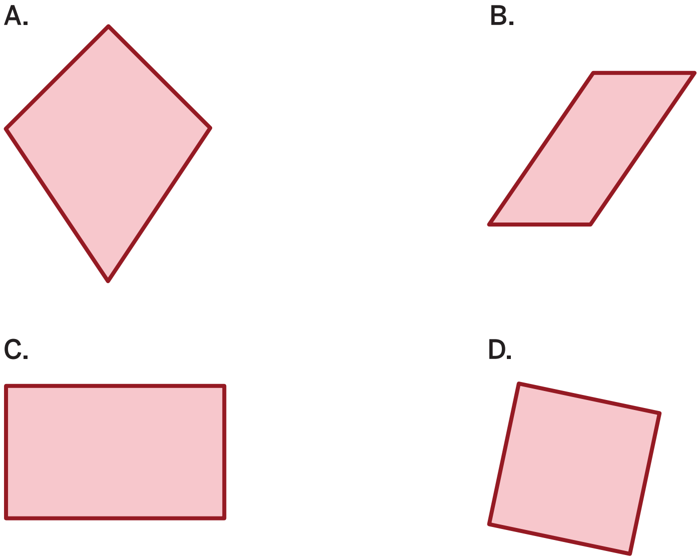
Exploring Parallelograms, Part 1



Let's investigate features of parallelograms.

Warm-Up

1. Which one doesn't belong? Explain your thinking.

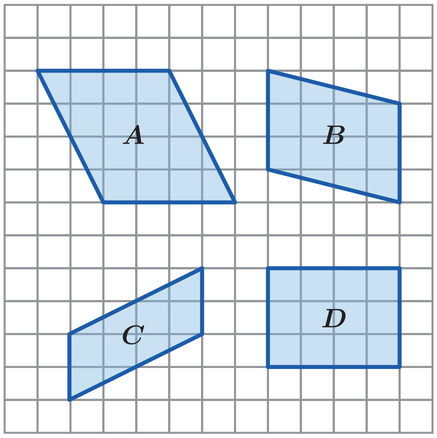


Name: Date: Period:

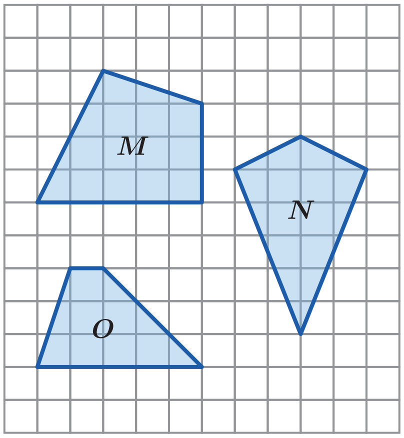
Parallelograms

2. Figures A, B, C, and D are **parallelograms**. Figures M, N, and O are quadrilaterals that are *not* parallelograms. What do you notice? What do you wonder?

Parallelograms



Not Parallelograms



I notice:

I wonder:

3. What do you think makes a shape a parallelogram?

a Write a first draft of your definition.

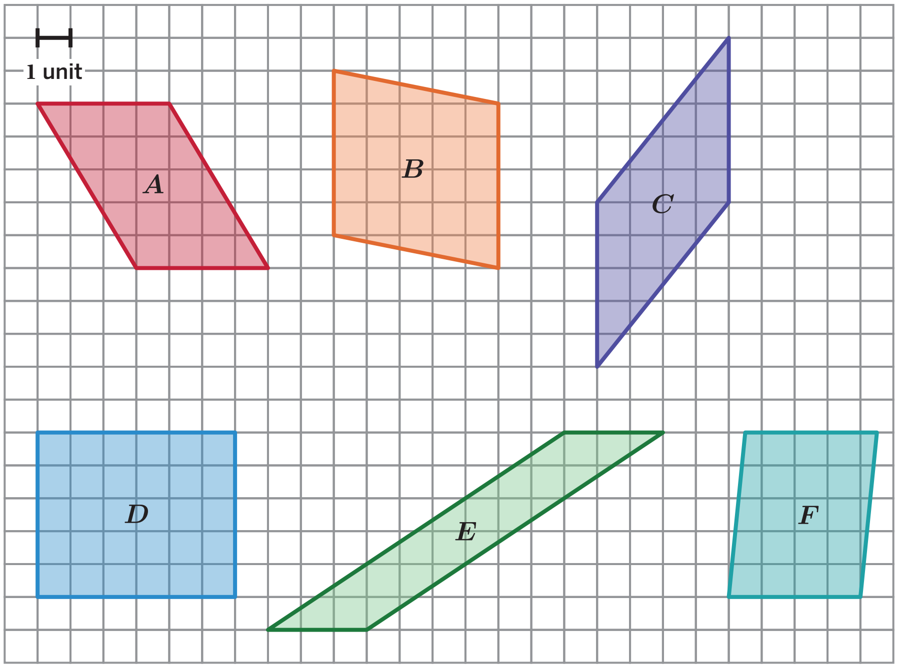
b Meet with a partner to discuss your first drafts. Use the questions on the screen to help you provide feedback to each other.

c Write a second draft that is stronger and clearer.

Name: ______ Date: ______ Period: ______

Area Strategies

4. Use any strategy to determine the area of these parallelograms.



Parallelogram	A	B	C	D	E	F
Area (sq. units)						

5. Describe your strategy for determining the area of parallelogram C.

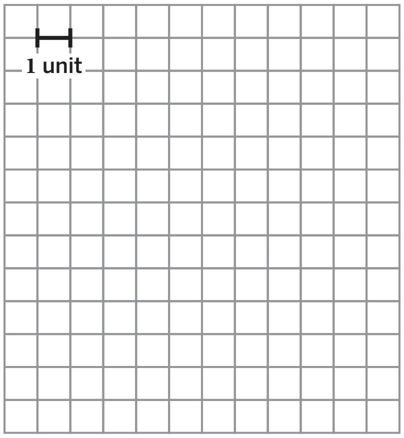
6. What other parallelograms would your strategy work for? Explain your thinking.

Name: .... Date: .... Period: ....

Area Strategies (continued)

7. Show or describe a classmate's strategy that was different from your own.

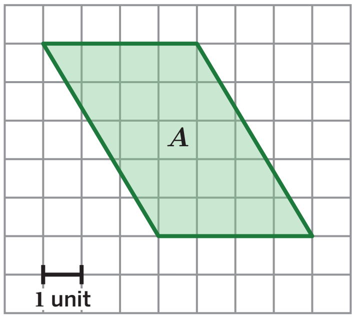
8. Draw a parallelogram with an area of 36 square units that is *not* a rectangle.



9. Explain how you know your parallelogram has an area of 36 square units.

Synthesis

10. Show or describe a strategy for calculating the area of a parallelogram. Use parallelogram A if it helps with your thinking.



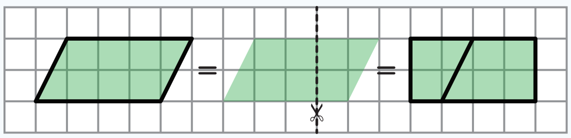
Summary 1.03

A *quadrilateral* is any shape that has four sides. A **parallelogram** is a type of quadrilateral that has two pairs of parallel sides that are the same length, such as rectangles and squares.

We can use different strategies to determine the area of a parallelogram.

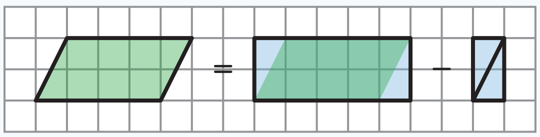
Cut the parallelogram into two pieces and rearrange the pieces to form a rectangle.

The parallelogram's area is equal to the area of the rectangle.



Draw a rectangle around the parallelogram so that it includes two right triangles. Rearrange the two triangles to form a smaller rectangle.

The parallelogram's area is equal to the difference between the areas of the larger rectangle and the smaller rectangle.



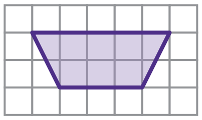
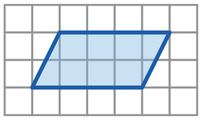
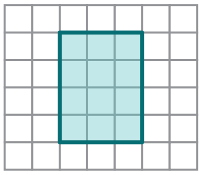
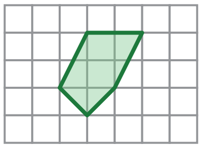
parallelogram A quadrilateral that has two pairs of parallel sides. Opposite, or parallel, sides of a parallelogram are the same length.

quadrilateral A polygon that has four sides. Squares, rectangles and parallelograms are examples of quadrilaterals.

Practice
1.03

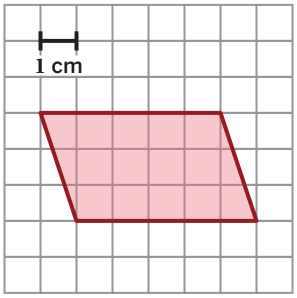
Name: ______________________ Date: __________ Period: __________

Problems 1–4: Determine whether each figure is a parallelogram. For figures that are not parallelograms, explain how you know.

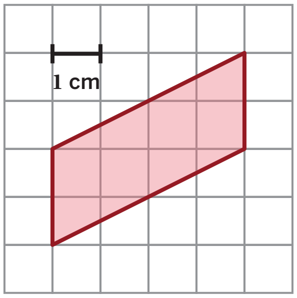
	Figure	Parallelogram (Yes / No)	If not a parallelogram, how do you know?
1.			
2.			
3.			
4.			

Problems 5–7: Use any strategy to determine the area of the parallelograms.

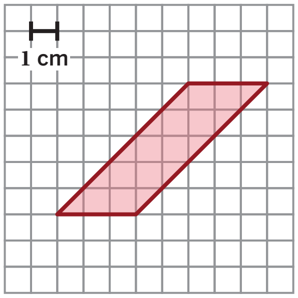
5.



6.

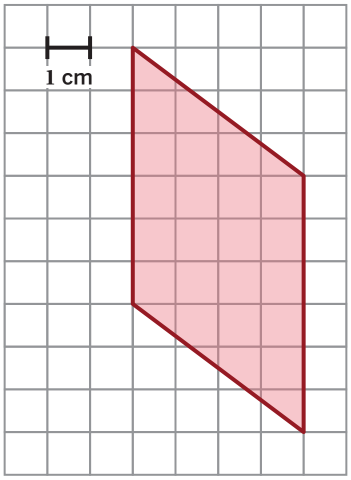


7.



Name: ________ Date: ________ Period: ________

Problems 8–9: Here is another parallelogram.

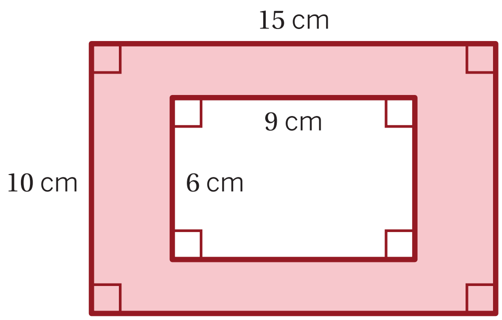


8. Determine its area. Explain your thinking.

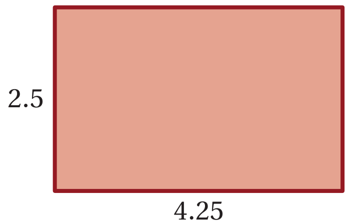
9. Show or describe another way to determine the area.

Spiral Review

10. Calculate the area of the shaded region. Show or explain your thinking.



11. A rectangle has a length of 4.25 inches and width of 2.5 inches. Select *all* the expressions that can be used to calculate the perimeter of the rectangle.



- ☐ **A.** $2 \cdot 4.25 + 2 \cdot 2.5$
- ☐ **B.** $4.25 \cdot 2.5$
- ☐ **C.** $2 \cdot (4.25 + 2.5)$
- ☐ **D.** $2 \cdot 4.25 \cdot 2.5$
- ☐ **E.** $4.25 + 2.5 + 4.25 + 2.5$

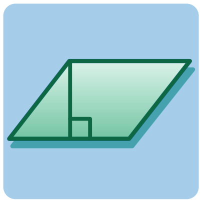
Unit 1
Lesson
4

Name: ______ Date: ______ Period: ______

Generalizing With Multiple Representations | Nets and Surface Area | Graphing Shapes

6.G.1, 6.EE.2.a, 6.EE.2.c, SMP.1, SMP.6, SMP.8

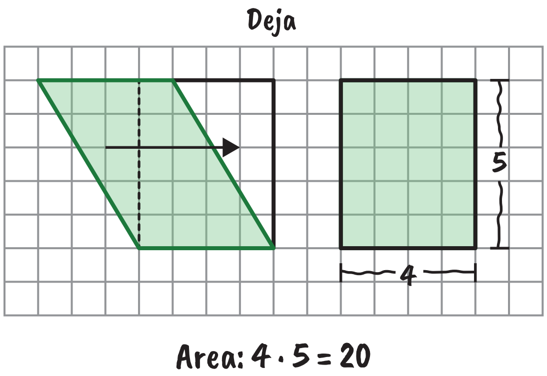
Exploring Parallelograms, Part 2

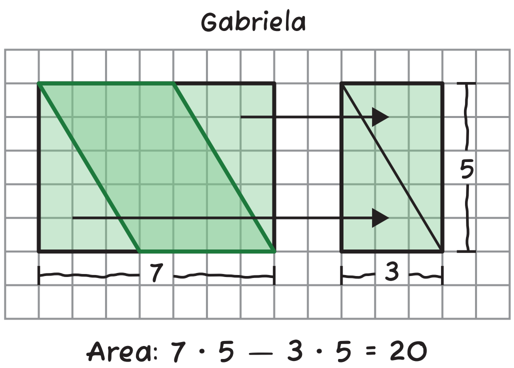


Let's determine the area of parallelograms.

Warm-Up

Deja and Gabriela used different strategies to determine the area of parallelogram A from Lesson 3.





1. Describe each student's strategy.

Deja's strategy:

Gabriela's strategy:

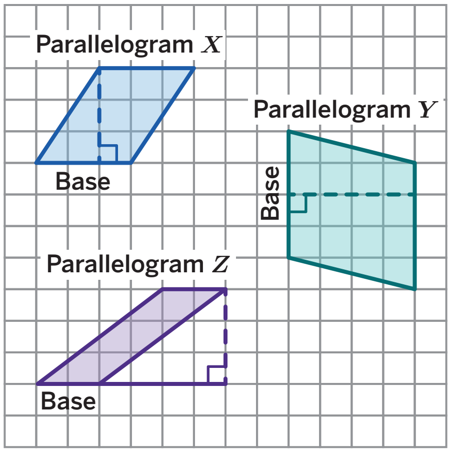
2. **Discuss:** How are Deja's and Gabriela's strategies alike? How are they different?

Name: ______________ Date: ______ Period: ______

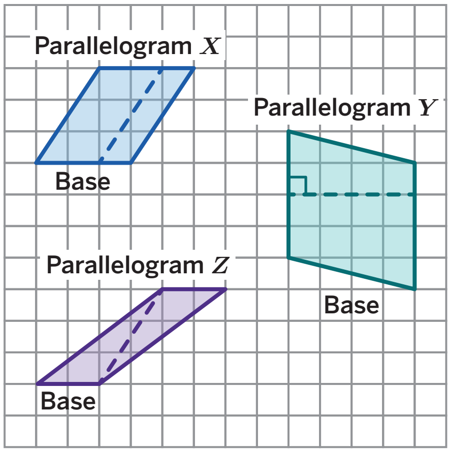
All About That Height

Here are some parallelograms with a few of their parts labeled.

Heights



Not Heights

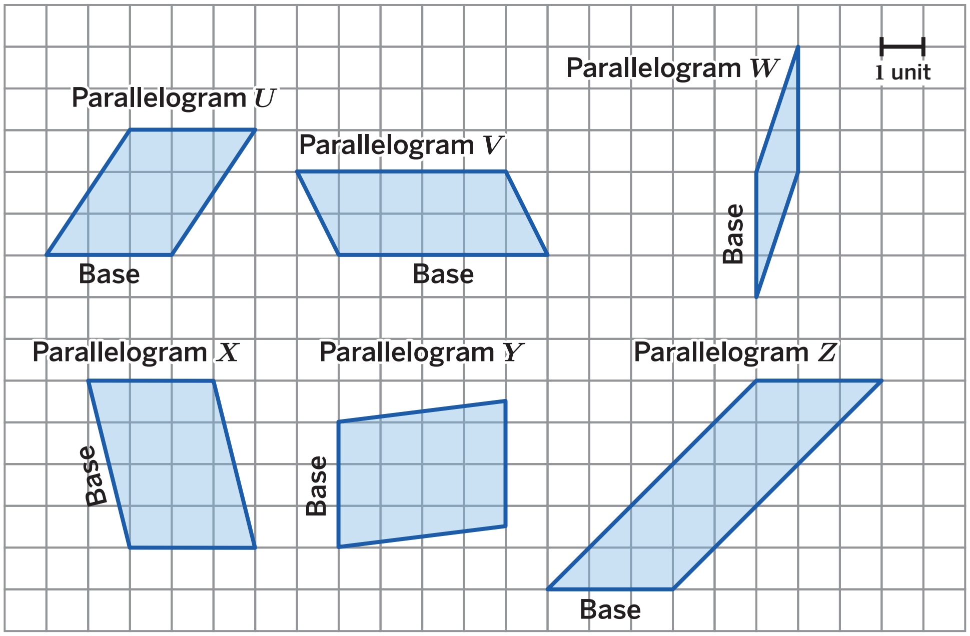


3. What do you notice? What do you wonder?

I notice:

I wonder:

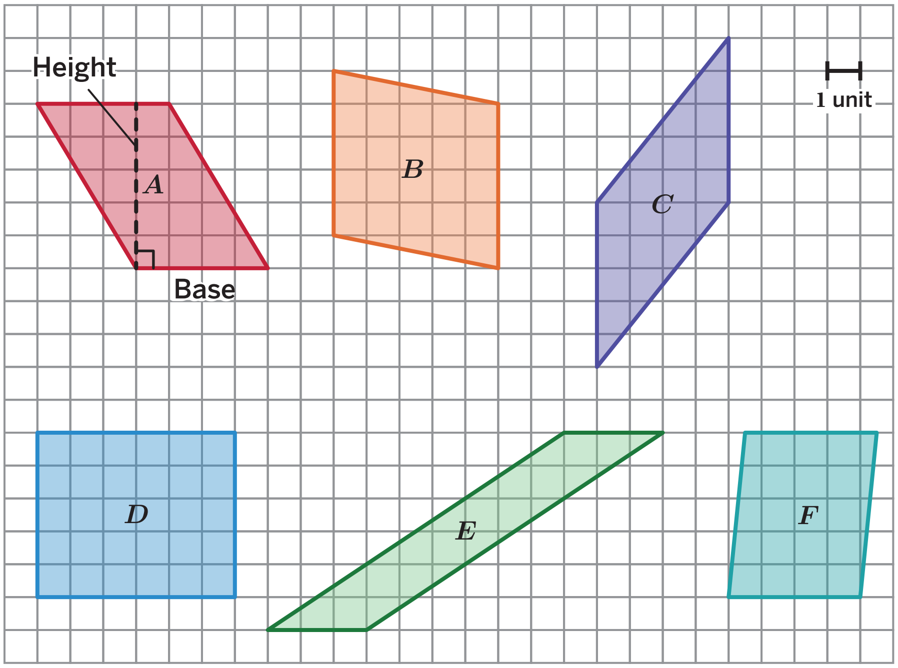
4. Draw the **heights** of these parallelograms for their matching **bases**.



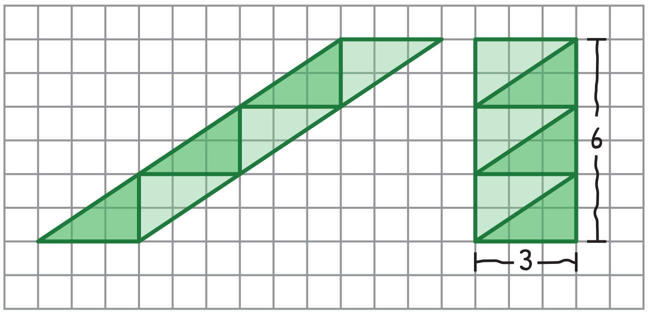
Name: .... Date: .... Period: ....

Area Formula

Here are the parallelograms from the previous lesson.



5. Here is how Andrea calculated the area of parallelogram E. Describe her strategy.



6. Deja noticed the area of the parallelogram A is $4 \cdot 5$ and the area of the parallelogram E is $3 \cdot 6$. What do you think these numbers come from?

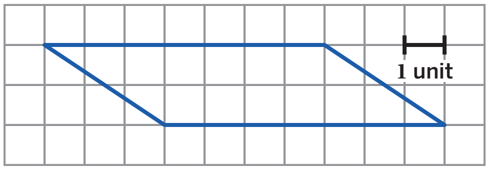
Name: .... Date: .... Period: ....

Area Formula (continued)

7. For each parallelogram, write the measurements of the base, height, and area in the table. In the last row, write an expression to calculate the area of *any* parallelogram.

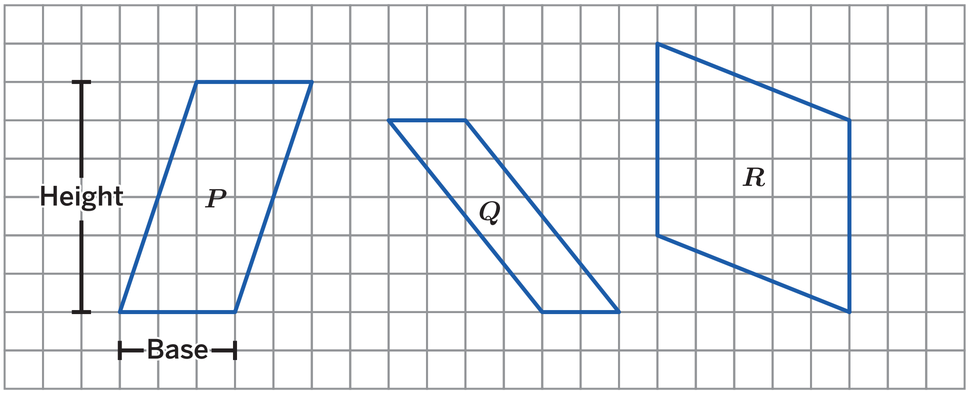
Parallelogram	Base (units)	Height (units)	Area (sq. units)
A	4	5	$4 \cdot 5 = 20$
B			______ $= 25$
C			______ $= 20$
D			______ $= 30$
E			
F			
Any parallelogram	b	h	

8. Use your formula to calculate the area of this parallelogram.



Synthesis

9. **a** Draw and label the bases and heights for parallelograms Q and R.

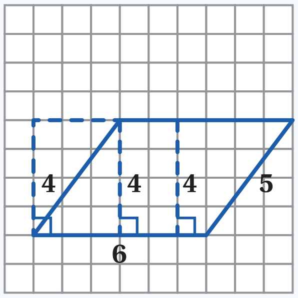


b Explain how you can use the base and height of a parallelogram to calculate its area.

Summary 1.04

We can determine the area of a parallelogram by multiplying the length of its **base** by the length of its **height**. We can choose any side of a parallelogram to be its base.

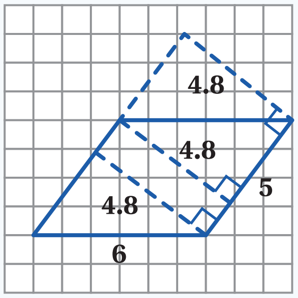
Here's an example of the same parallelogram with different sides selected as the base and different points used to measure the height. Each set of measurements will produce the same area.



Area = base • height

$A = 6 \cdot 4$

$A = 24$ square units



Area = base • height

$A = 5 \cdot 4.8$

$A = 24$ square units

base (of a parallelogram) The base is one side of a parallelogram. We can choose any side to be the base.

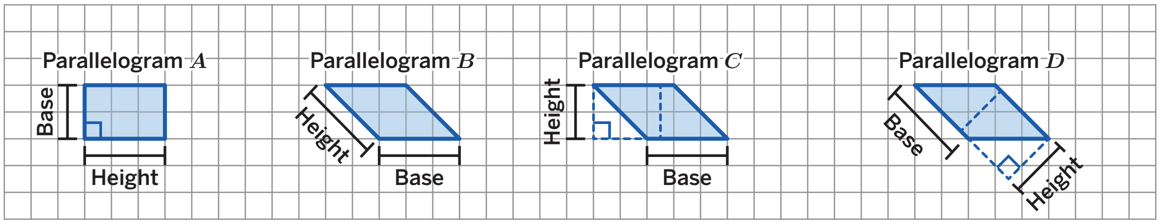
formula A rule or equation representing a mathematical relationship. For example the formula to calculate the area of a rectangle, A, is $A = \ell \cdot w$ where ℓ is the length and w is the width of the rectangle.

height (of a parallelogram) The height of a parallelogram is the shortest distance between a base and its opposite side. Sometimes the height falls outside the shape. The height is always perpendicular to the base.

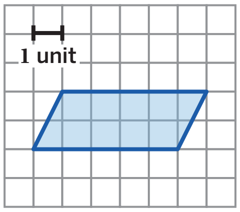
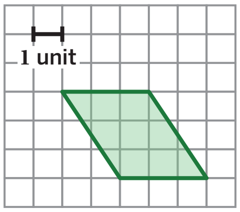
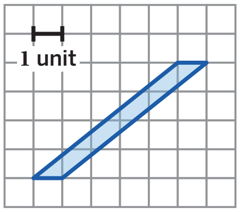
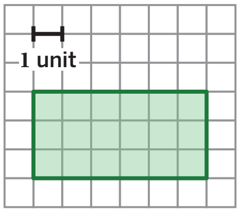
Practice
1.04

Name: .... Date: .... Period: ....

1. Circle *all* the parallelograms that have a correct height labeled for the labeled base.

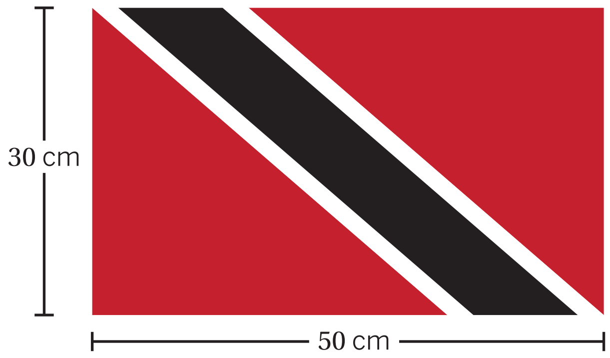


Problems 2–5: Determine the base, height, and area of these parallelograms.

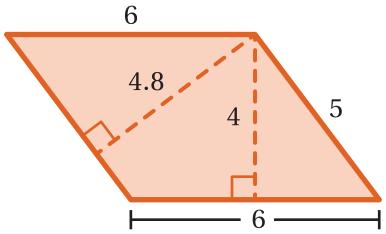
	Parallelogram	Base	Height	Area
2.				
3.				
4.				
5.				

Name: ______________________ Date: __________ Period: __________

6. Here is the flag of Trinidad and Tobago. Anthony wants to color the flag on a 50-by-30 centimeter card. If the base length of the black diagonal stripe is 10 centimeters, use the formula $A = b \cdot h$ to determine the area Anthony needs to color in black.



7. If the base of this parallelogram is the side that is 6 units long, what is the length of the matching height?



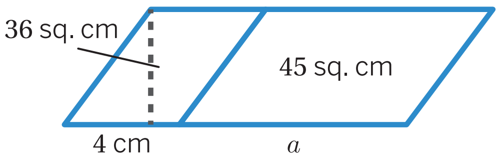
A. 6 units

B. 4.8 units

C. 4 units

D. 5 units

8. Here are two adjacent parallelograms. Determine the value of a.



Spiral Review

Problems 9–12: Determine the product in each of these expressions.

9. $0.5 \cdot 12 \cdot 4$

10. $\frac{1}{2} \cdot 4 \cdot 15$

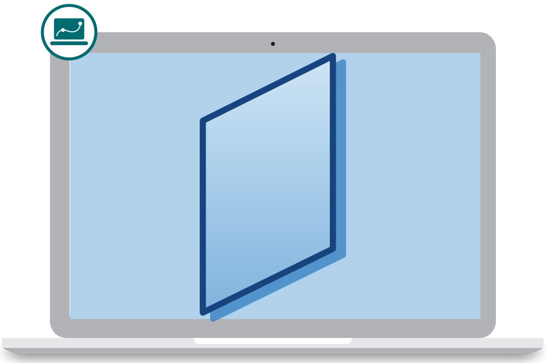
11. $\frac{1}{2} \cdot 7 \cdot 6$

12. $\frac{1}{2} \cdot 4.2 \cdot 5$

Unit 1
Lesson
5

Name: .... Date: .... Period: ....

Nets and Surface Area Graphing Shapes 6.G.1, SMP.3, SMP.6



Off the Grid, Part 1

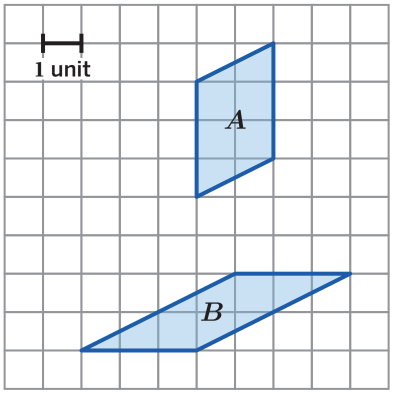
Let's practice determining the area of parallelograms.

Warm-Up

1 Which parallelogram has a greater area? Circle one.

A B They have the same area

Show or explain your thinking.

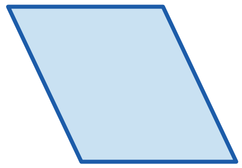


Activity 1

Name: ________ Date: ________ Period: ________

Measuring to Determine Area

2 In this lesson, you'll measure different parts of parallelograms.

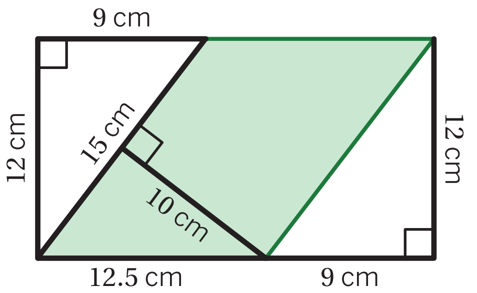


a. Let's watch how the measuring tool works.

b. Label as many measurements on this parallelogram as you want.

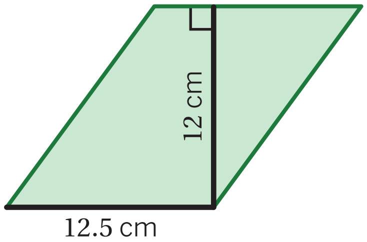
3 What is the area of this parallelogram?

Use as many measurements as you need to calculate the area.

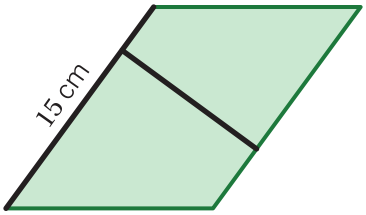


4 Here are some measurements that Alma and Ebony took. Sketch a line that Ebony can measure next to help her calculate the area.

Alma



Ebony

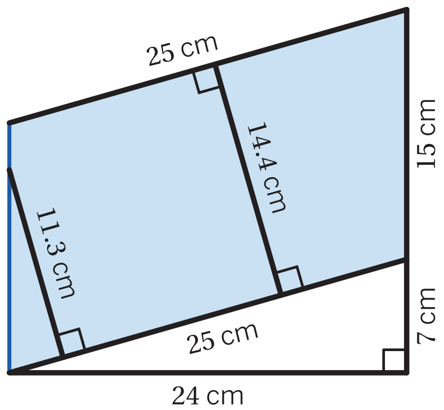


Name: .... Date: .... Period: ....

Measuring to Determine Area (continued)

5 What is the area of this parallelogram?

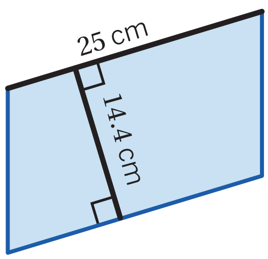
Use as many measurements as you need to help with your thinking.



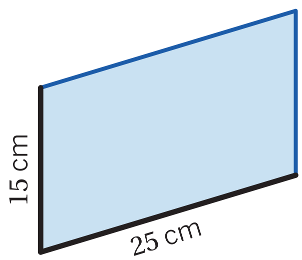
6 Here are some measurements taken by four different students.

Select *all* the parallelograms with measurements that can be used as a base and height pair.

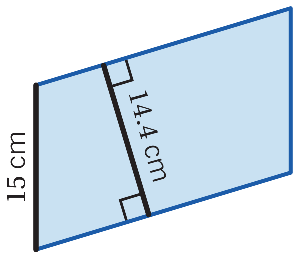
☐ **A.**



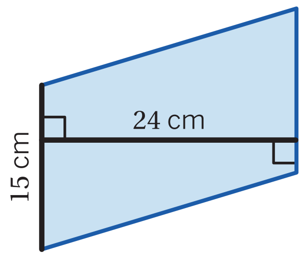
☐ **B.**



☐ **C.**



☐ **D.**

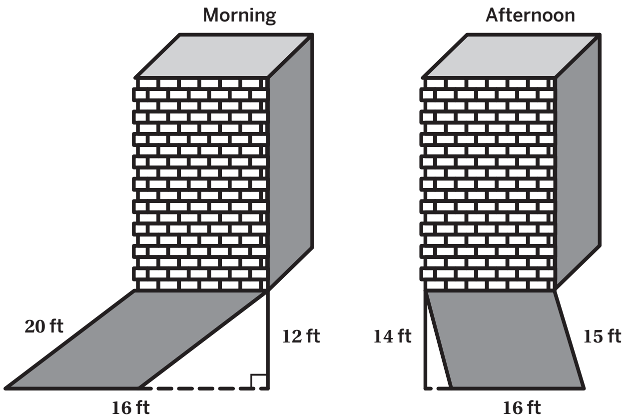


Activity 2

Name: .... Date: .... Period: ....

More Parallelograms

7 The diagram shows a building and a shadow it casts in the morning compared to a shadow it casts in the afternoon. The measurements are given for each shadow. Which shadow has a greater area?



Use as few measurements as you can to help you decide.

Morning Afternoon

They have the same area

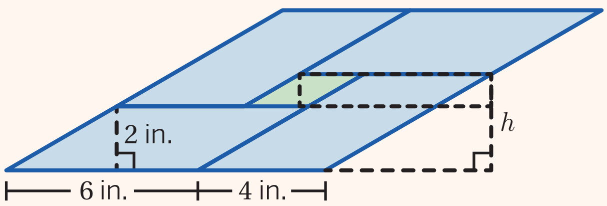
Show or explain your thinking.

8 A local charity organization has placed drop boxes for donations around town, such as the one shown here. Each drop box has a logo with an area of 375 square centimeters. The base of the logo on the drop box measures approximately 25 centimeters.

What is the approximate height of the logo on the side of a drop box?

You're invited to explore more.

9 The shaded region in this diagram is composed of four identical parallelograms and a smaller parallelogram.



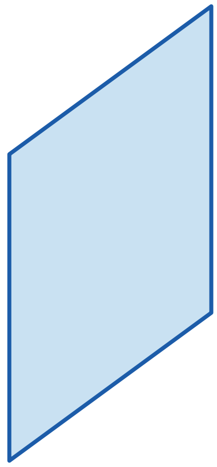
a What is the value of h?

b What is the total shaded area?

10 Synthesis

Discuss: How can you determine the area of any parallelogram?

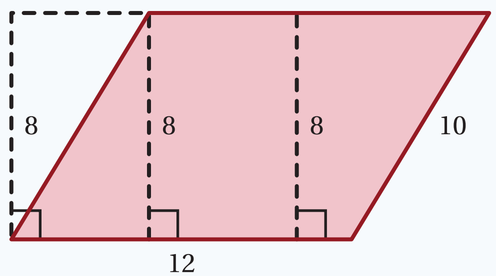
Draw on this image if it helps to show your thinking.



13 Summary 1.05

We can use a ruler to determine the lengths of the base and height of a parallelogram when it is not presented on a grid with lengths that we can count. No matter which side of a parallelogram you choose as the base, its area will be equal to the product of the length of the base and the length of its matching height.

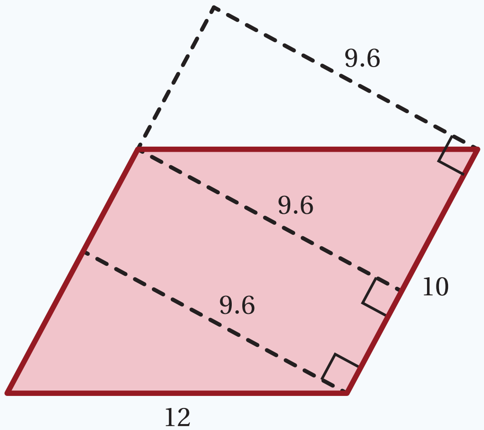
Here's an example of the same parallelogram with different sides selected as the base and different points used to measure the height. Each set of measurements will produce the same area.



Area = base · height

$A = 12 \cdot 8$

$A = 96$ square units



Area = base · height

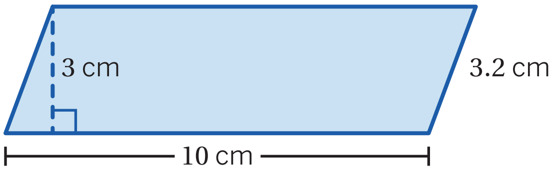
$A = 10 \cdot 9.6$

$A = 96$ square units

Name: .... Date: .... Period: ....

Problems 1–4: Determine the base, height, and area of each parallelogram.

1.

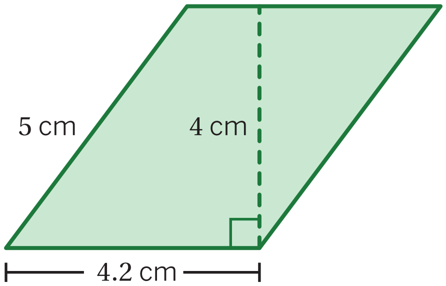


Base: ....

Height: ....

Area: ....

2.

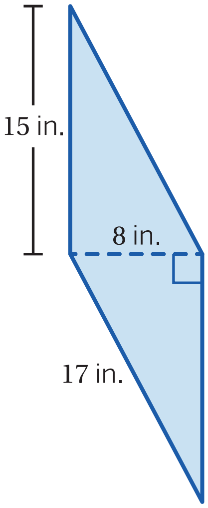


Base: ....

Height: ....

Area: ....

3.

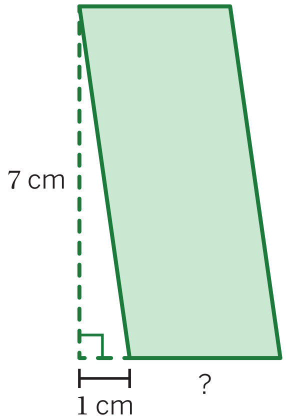


Base: ....

Height: ....

Area: ....

4.

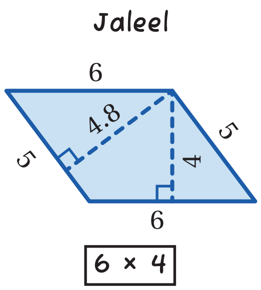


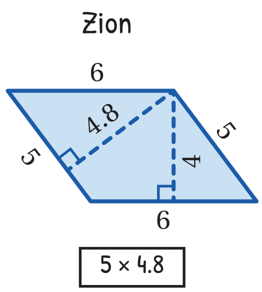
Base: ....

Height: ....

Area: 21 square centimeters

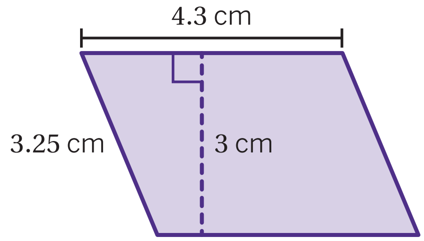
5. Jaleel and Zion each calculated the area of a parallelogram. Whose calculation is correct? Explain your thinking.





Name: .... Date: .... Period: ....

6. What is the area of this parallelogram in square centimeters?



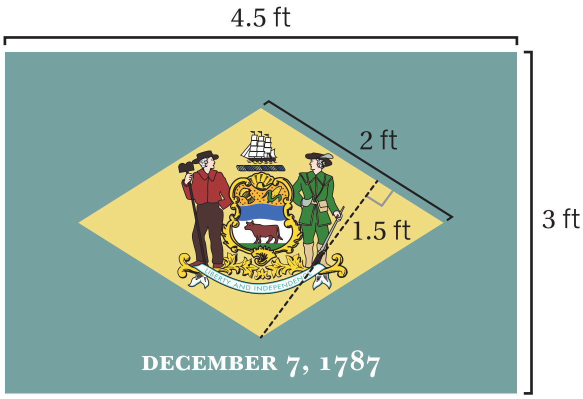
A. 9.75

B. 10.55

C. 12.9

D. 13.97

For Problems 7 and 8, use the Delaware state flag shown. The official colors — colonial blue and buff yellow — represent a Revolutionary War uniform worn by General George Washington. The state's coat of arms, reading "Liberty and Independence," is displayed on top of a diamond (parallelogram) because Delaware was once nicknamed the Diamond State.



7. Use the formula $A = b \cdot h$ to determine how much yellow fabric is used to make the center yellow parallelogram

8. Use the formula $A = l \cdot w$ to determine how much blue fabric is used to make the outer blue rectangle

Spiral Review

Problems 9–12: Determine each product.

9. $4321 \cdot 2$

10. $6534 \cdot 5$

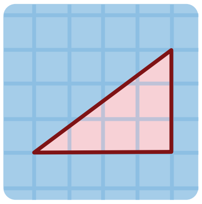
11. $42 \cdot 21$

12. $38 \cdot 57$

Name: ______ Date: ______ Period: ______

Graphing Shapes 6.G.1, SMP.7

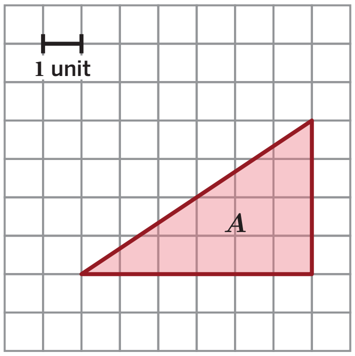
Exploring Triangles



Let's explore the area of triangles.

Warm-Up

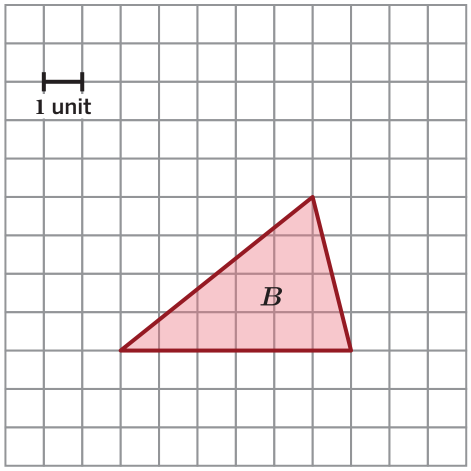
1. Determine the area of triangle A. Show or describe your thinking.



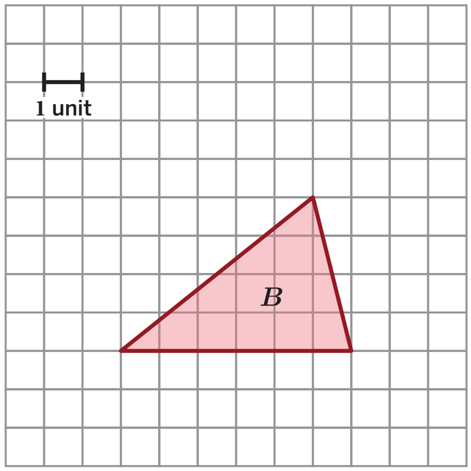
Name: ____________ Date: ________ Period: ________

Area Strategies

2. Determine the area of triangle B. Show or describe your thinking.



3. Find a classmate who calculated the area of triangle B using a different strategy. Show or describe how your partner calculated the area.



4. Let's look at two strategies for calculating the area of triangle B.

Discuss: How are these two strategies alike? How are they different?

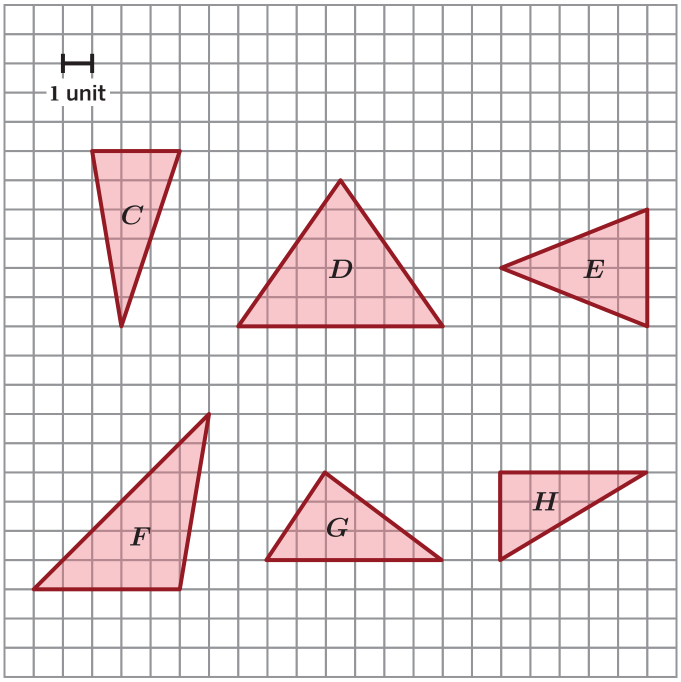
Activity 2

Name: ______ Date: ______ Period: ______

Lots of Triangles

5. Determine the area of as many of these triangles as you can.

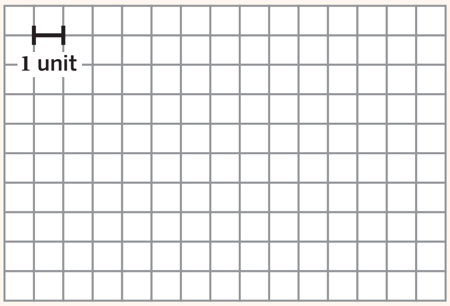
Triangle	Area (sq. units)
C	
D	
E	
F	
G	
H	

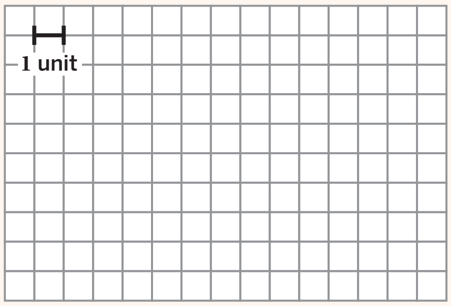


6. Describe the strategy that was most helpful to you. Did this strategy work for *all* the triangles?

You're invited to explore more.

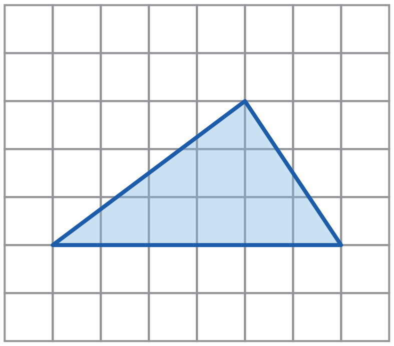
7. Draw two different triangles that both have an area of 18 square units.





Synthesis

8. Describe a strategy to determine the area of a triangle. Use the example if it helps with your thinking.

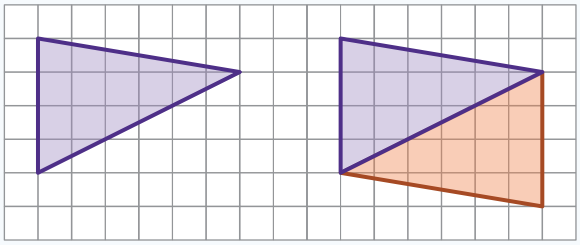


Summary 1.06

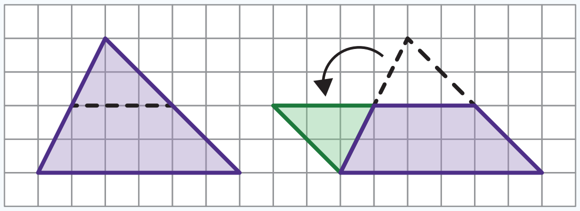
You can use what you know about the area of quadrilaterals to help you determine the area of triangles.

Here are three ways you can use a quadrilateral to help you determine the area of a triangle on a grid.

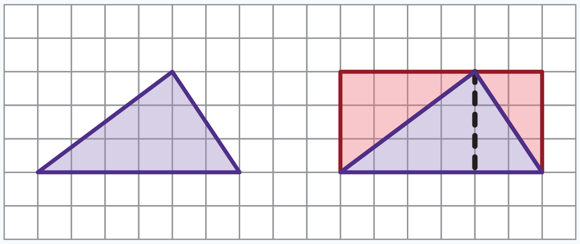
- Make a copy of the triangle and rearrange the two identical triangles to form a parallelogram.
  - Because the two triangles have the same area, each triangle has an area that is exactly half the area of the parallelogram.



- Cut the triangle and rearrange the pieces to form a parallelogram.
  - Because the triangle and the parallelogram are made up of exactly the same pieces, their areas are equal.



- Enclose the triangle in a large rectangle that can be cut into two smaller rectangles.
  - This also cuts the triangle into two smaller triangles.
  - Each of these smaller triangles has half the area of its enclosed rectangle.
  - The sum of the two smaller triangles' areas is equal to the area of the original triangle.

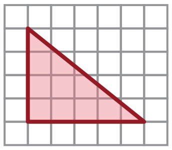


Name: ________________ Date: ________ Period: ________

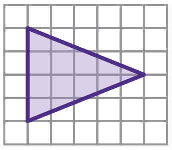
Problems 1–4: Determine the area of each triangle.

Each small square in the grid has an area of 1 square unit.

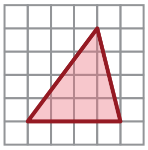
1.



2.



3.



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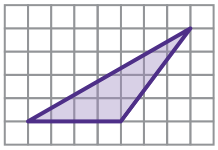
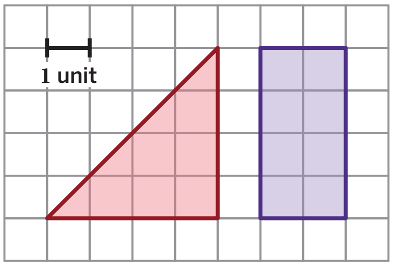
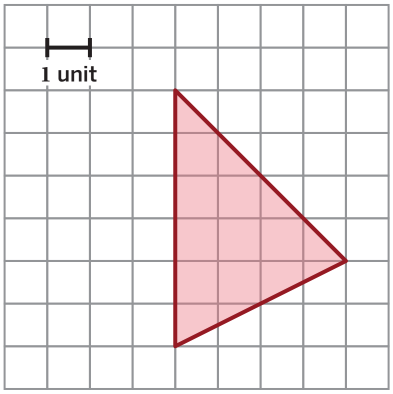


5. Aki thinks that these two shapes have the same area. Is Aki's thinking correct?



6. Determine the area of this triangle. Show or explain your thinking.



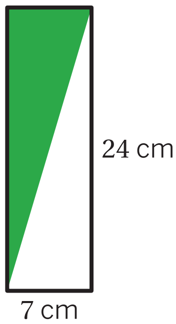
7. What is the area, in square centimeters, of the shaded part of this rectangle?

A. 15.5

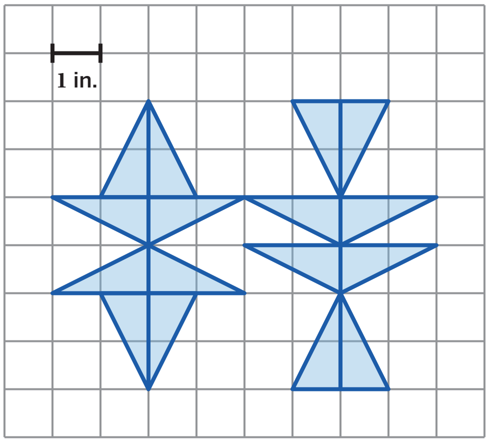
B. 62

C. 84

D. 168



8. Alice used triangle tiles to make a design on an 8-by-8 inch area. Determine the total area, in square inches, that is covered by the triangle tiles in her design.



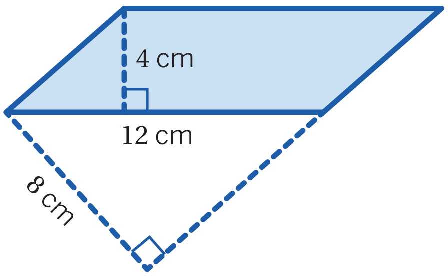
Spiral Review

9. Select *all* of the expressions that have the same value as $8 \div 2$.

- ☐ A. $\frac{8}{2}$
- ☐ B. $8 \cdot 2$
- ☐ C. $2 \div 8$
- ☐ D. $\frac{1}{2} \cdot 8$
- ☐ E. $\frac{2}{8}$

10. A parallelogram has an area of 20.4 square units. If the height is 4 units, what is the length of the base? Show your thinking.

11. Determine the perimeter of this parallelogram. Show or describe your thinking.



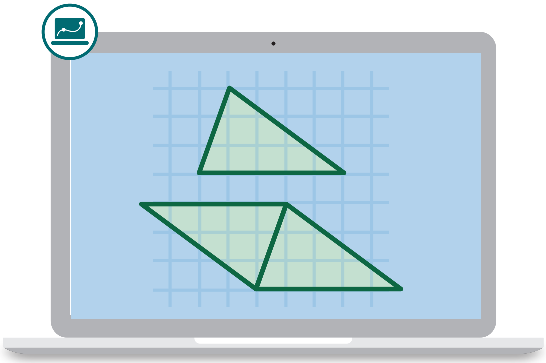
Unit 1
Lesson
7

Name: ____ Date: ____ Period: ____

Generalizing With Multiple Representations | Graphing Shapes | 6.G.1, 6.EE.2.a, SMP.8

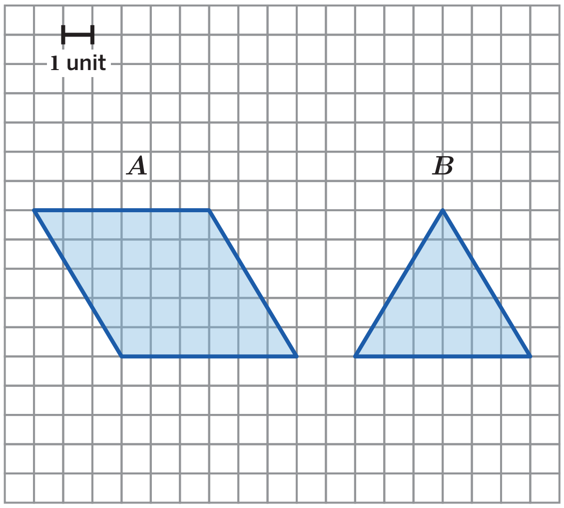
Triangles and Parallelograms

Let's explore the relationship between triangles and parallelograms.



Warm-Up

1 List two things that are the same about these figures.



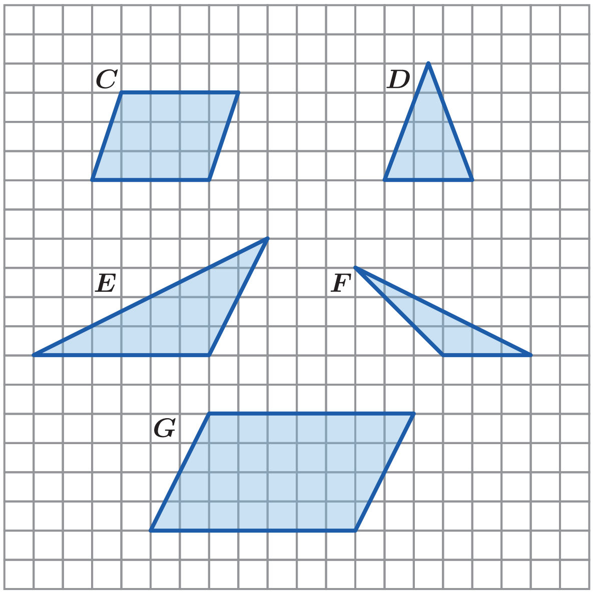
List two things that are different.

Name: .... Date: .... Period: ....

Triangles and Parallelograms

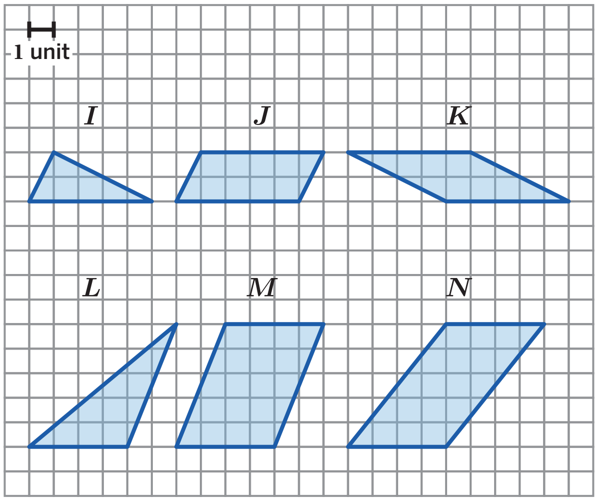
2 You can determine the heights of triangles, just like you can with parallelograms.

Name all the shapes that have the same heights.



3 **a** Determine the base, height, and area of each shape.

Shape	Base (units)	Height (units)	Area (sq. units)
I			
J			
K			
L			
M			
N			

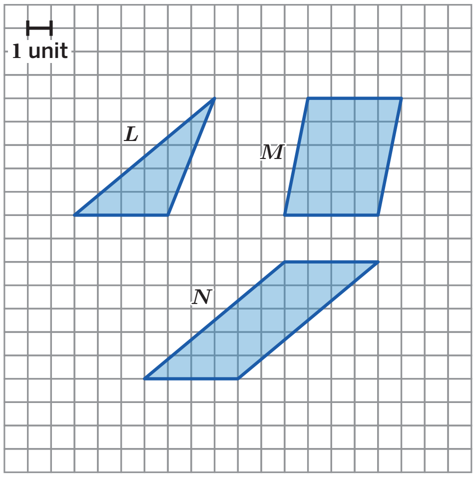


b **Discuss:** What patterns do you notice?

Name: .... Date: .... Period: ....

Triangles and Parallelograms (continued)

4 Here is a triangle and two parallelograms from the previous problem.

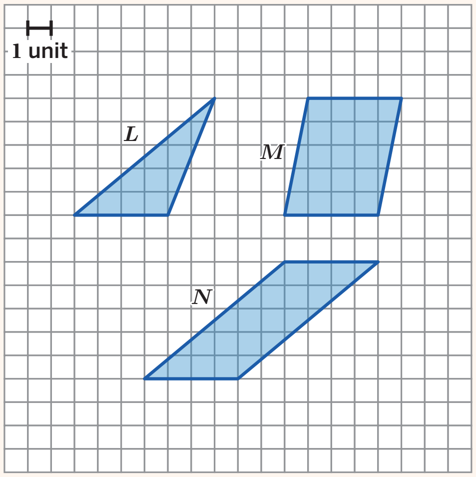


a Trace triangle L. Then compare the triangle you traced to each of the shapes.

b What is the relationship between the areas of these three shapes?

You're invited to explore more.

5 Draw a different triangle that has the same area as triangle L and is related to either parallelogram M or parallelogram N.

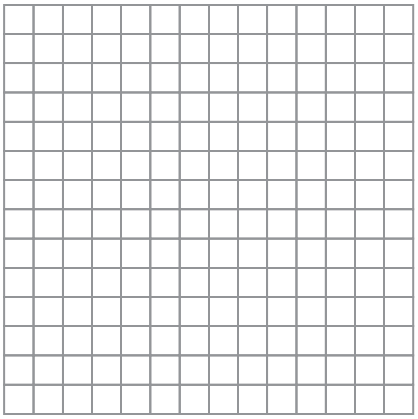


Activity 2

Name: .... Date: .... Period: ....

Generalizing Triangle Area

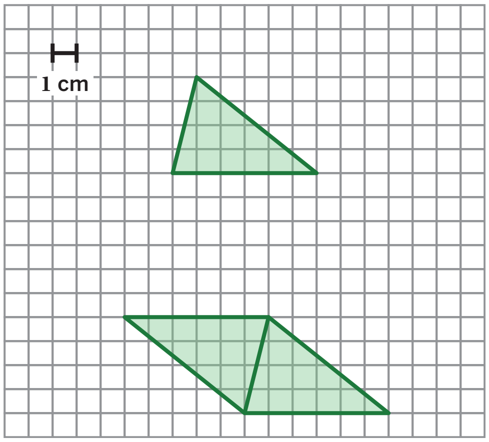
6 Let's see if we can always combine two copies of a triangle to form a parallelogram.



a. Create a triangle in the grid provided. Then trace the triangle.

b. **Discuss:** How many different parallelograms can you create?

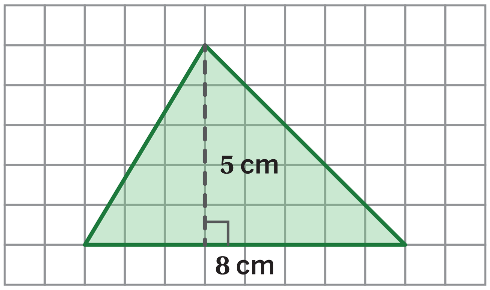
7 Here is a triangle and a parallelogram.



a. What is the area of the triangle?

b. What is the area of the parallelogram?

8 Here's the expression that Alisha entered to calculate the area of her triangle.

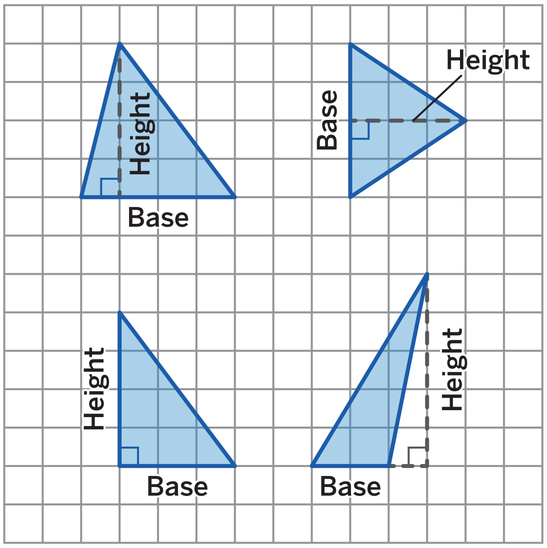


$8 \cdot \frac{5}{2}$

Explain what each number represents in the expression.

9 Synthesis

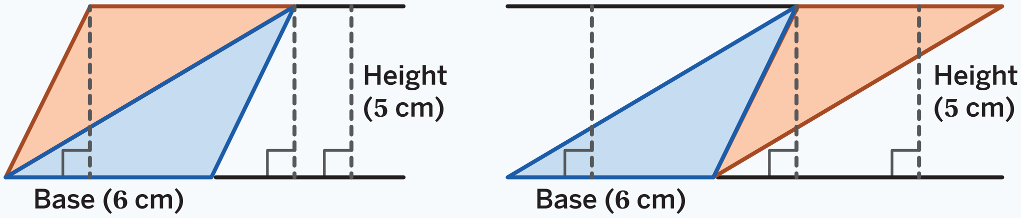
Discuss: How can you use the base and height of any triangle to calculate its area?



12 Summary 1.07

You can arrange two identical copies of any triangle in several ways to create a parallelogram with the same base and height measurements. This shows us that the area of a triangle is equal to half the area of its related parallelogram.

Here are two ways to form a parallelogram using two identical triangles with a base of 6 centimeters and a height of 5 centimeters.



The area of the parallelogram is $A = 6 \cdot 5 = 30$ square centimeters. Since the area of the triangle is half the area of the parallelogram, we can determine that the area of the triangle is 15 square centimeters.

Practice
1.07

Name: .... Date: .... Period: ....

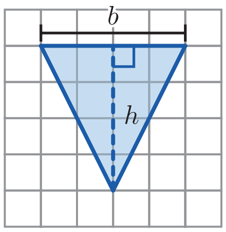
1. Determine whether each statement is true or false.

Statement	True	False
Any side of a triangle can be the base.		
The height of a triangle must always be one of its sides.		
A height that matches the base of a triangle can be drawn at any angle to the base.		
You can only draw one possible height for a chosen base.		

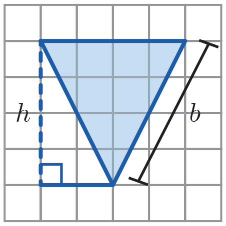
2. Choose one of the false statements in Problem 1 and explain why it is false.

3. Which triangle incorrectly identifies a height, h, and its matching base, b?

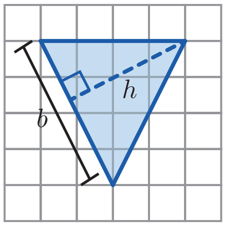
A. Triangle A



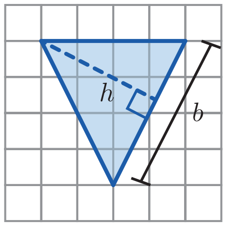
B. Triangle B



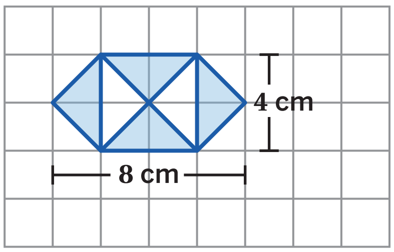
C. Triangle C

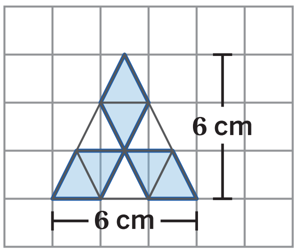


D. Triangle D



4. Ali and Haruto are drawing logos for a "Guess the Logo" competition. They added a bonus round where contestants determine the shaded area of each logo. Help them create the answer key for the bonus round.



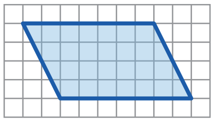
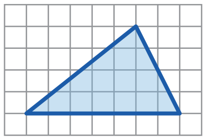
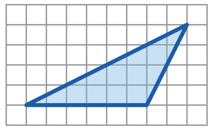
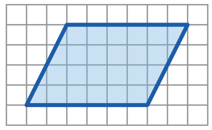


Practice
1.07

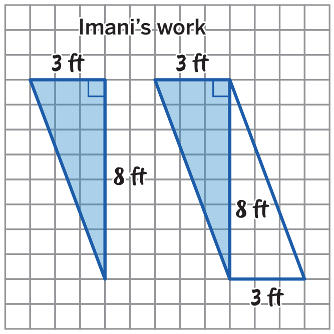
Name: ________ Date: ________ Period: ________

Problems 5–8: Determine the base, height, and area of each figure.

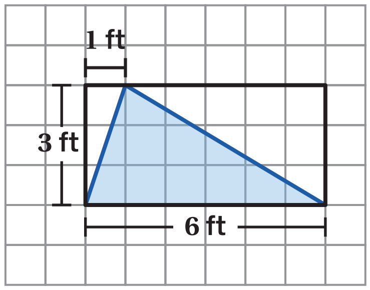
Each small square in the grid has an area of 1 square unit.

	5. 	**6.** 	**7.** 	**8.** 
Base				
Height				
Area				

9. Imani wants to determine the area of the triangle. Explain how Imani might use the parallelogram to calculate the area of the triangle.



10. Habib painted this triangle on his game room wall. What is the area, in square feet, of the triangle that Habib painted?



A. 3

B. 6

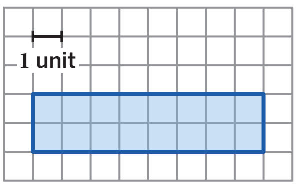
C. 9

D. 18

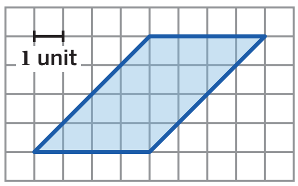
Spiral Review

Problems 11–13: Determine the area of each parallelogram.

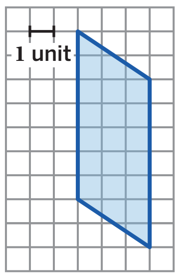
11.



12.

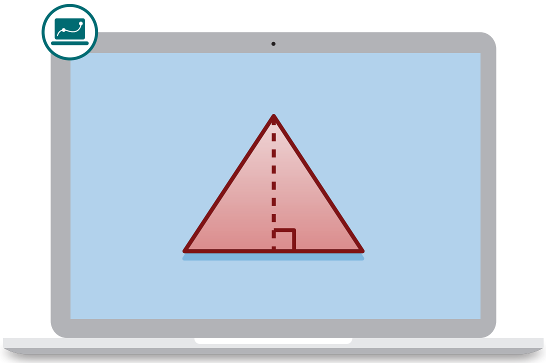


13.



Name: .... Date: .... Period: ....

Generalizing With Multiple Representations | Graphing Shapes | 6.G.1, 6.EE.2.a, 6.EE.2.c, SMP.8



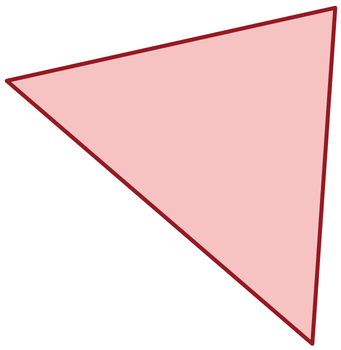
Off the Grid, Part 2

Let's practice calculating the area of triangles.

Warm-Up

1 Let's look at an animation showing the different sides of a triangle.

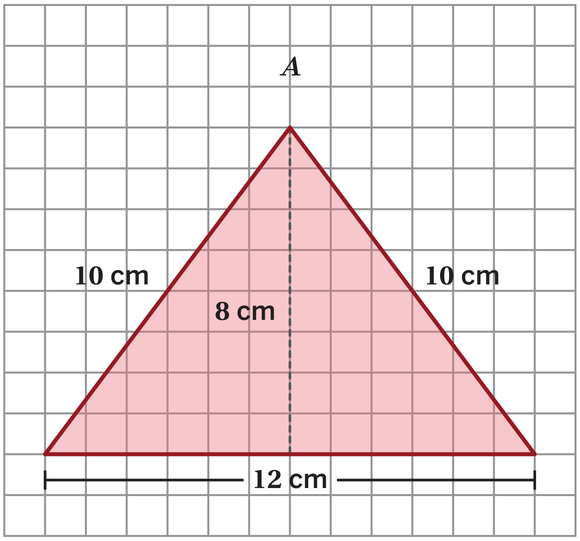
What is one thing that changes? What is one thing that stays the same?



Name: .... Date: .... Period: ....

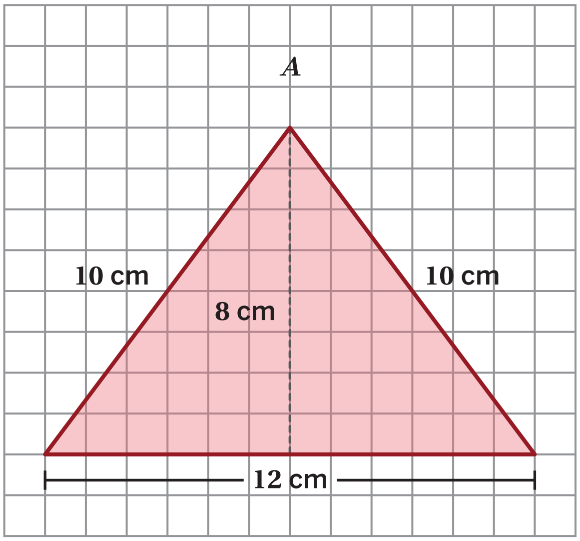
Base, Height, and Area

2 Use any strategy to determine the area of triangle A.

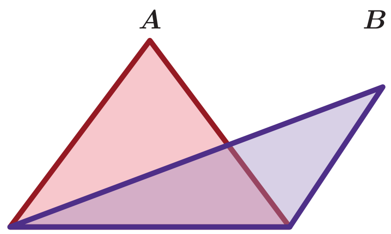


3 Select *all* the expressions that could represent the area of this triangle. Draw on the triangle if it helps with your thinking.

- ☐ A. $\frac{1}{2} \cdot 12 \cdot 8$
- ☐ B. $\frac{12 \cdot 10}{2}$
- ☐ C. $12 \cdot 8 \div 2$
- ☐ D. $6 \cdot 8$
- ☐ E. $6 \cdot 4$



4 Here is triangle A from the previous problem, along with a new triangle. Which triangle has the greater area? Circle one.



Triangle A Triangle B

They are the same Not enough information

Explain your thinking.

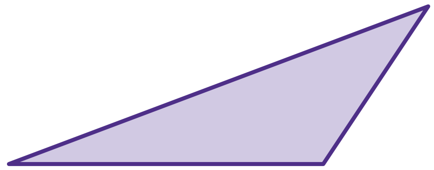
Activity 2

Name: ______________ Date: ______________ Period: ______________

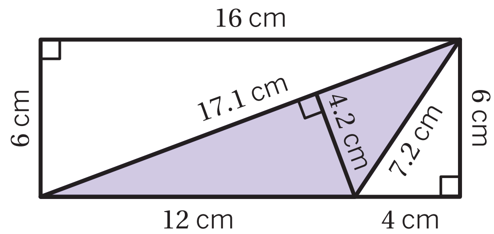
Choose Your Measurements

5 Ishaan wants to calculate the area of this triangle, but the measurements are not labeled.

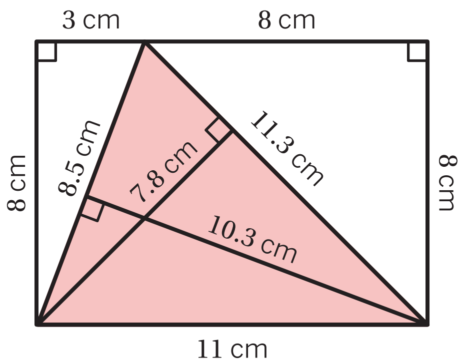
Draw on the triangle to show Ishaan what he should measure to calculate the area.



6 Use as many measurements as you want to calculate the area of the triangle.



7 Use as many measurements as you want to calculate the area of the triangle.



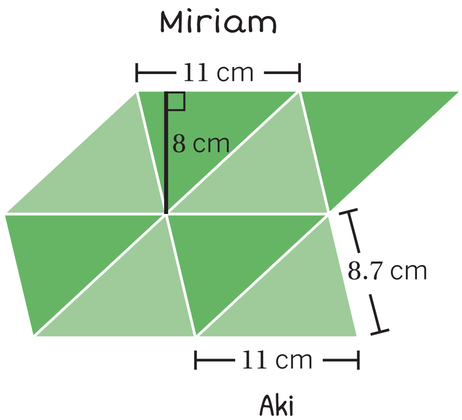
Name: Date: Period:

Choose Your Measurements (continued)

8 Aki is helping Miriam tile her bathroom wall using triangular tiles. All the tiles are identical in size. Aki and Miriam took the measurements of two different tiles and found different areas.

Here are the measurements they took.

Whose measurements lead to the correct area? Circle one.



Aki's　　Miriam's　　Both　　Neither

Explain your thinking.

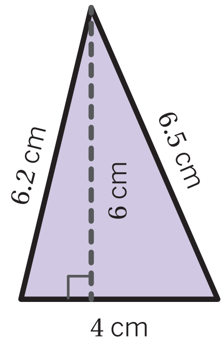
9 A pennant flag banner is used to decorate an event hall for a party. The area of a single pennant is 24 square inches. Sketch as many different triangles as you can with the same area as the pennant.

Name: .... Date: .... Period: ....

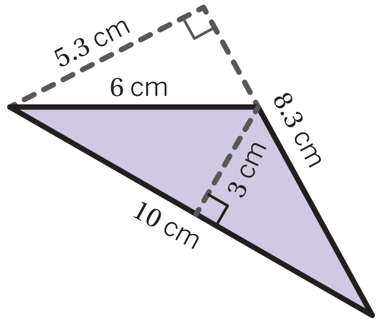
Repeated Challenges

10 Calculate the area of each triangle. Use as many measurements as you need.

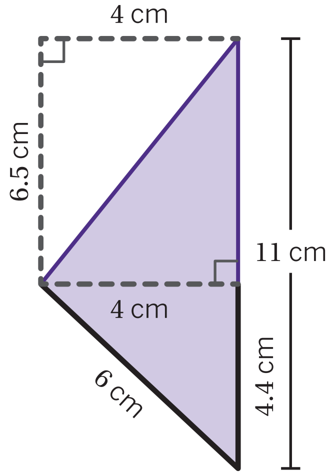
a



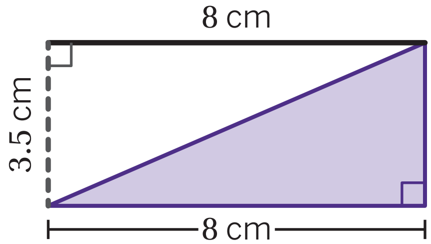
b



c

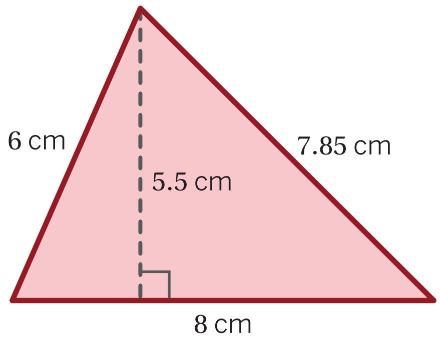


d



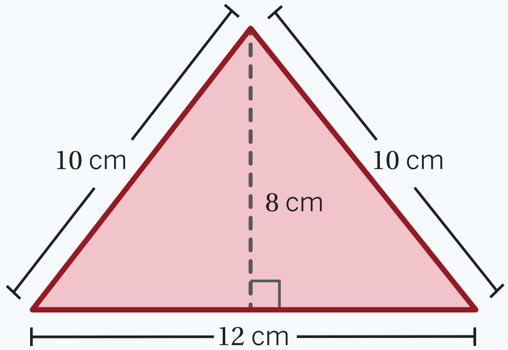
11 Synthesis

Describe how to calculate the area of a triangle. Use this example if it helps with your thinking.



14 Summary 1.08

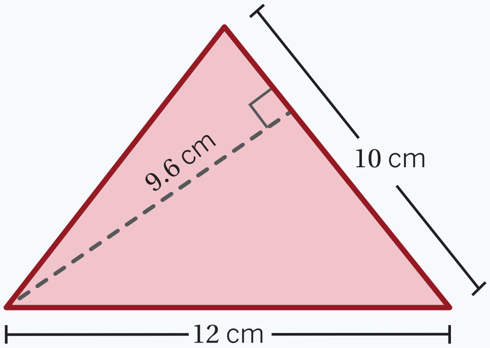
The area of any triangle is equal to half of the product of its base and height. You can select any side of the triangle to be the base. The height of a triangle is the perpendicular distance between a point on the base and the opposite corner of the triangle. The height is often shown with a dotted line.



$A = \frac{1}{2}bh$

$A = \frac{1}{2}(12)(8)$

$A = 48$ square centimeters



$A = \frac{1}{2}bh$

$A = \frac{1}{2}(10)(9.6)$

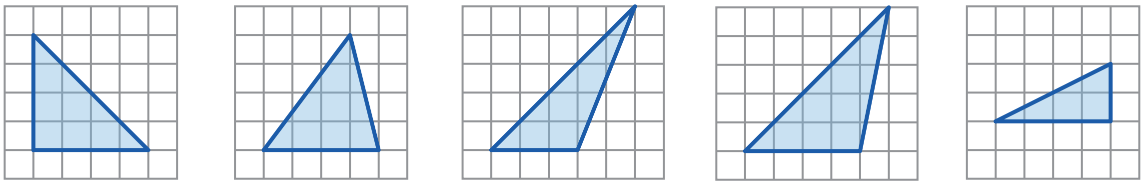
$A = 48$ square centimeters

All sides of the triangle can be a base, but some base-height pairs are easier to measure and calculate with.

Name: .... Date: .... Period: ....

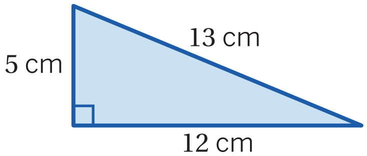
1. Select *all* of the triangles that have an area of 8 square units. Each small square in the grid has an area of 1 square unit.

☐ A. ☐ B. ☐ C. ☐ D. ☐ E.

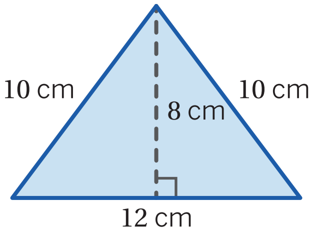


Problems 2–4: Determine the area of each triangle in square centimeters.

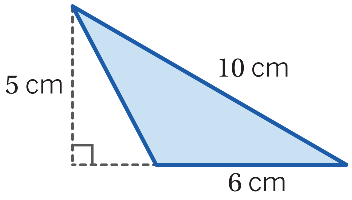
2.



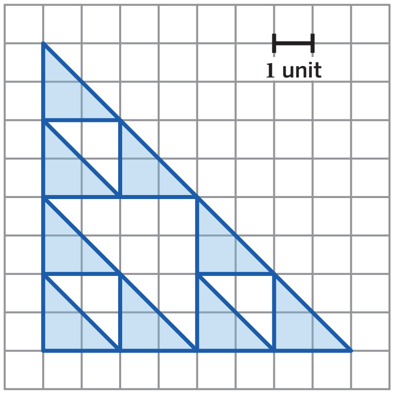
3.



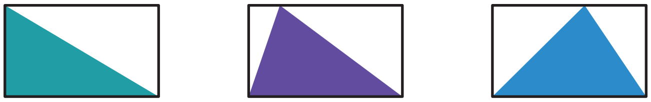
4.



5. Motion capture is a technology that records an actor's movements and transfers them onto a computer-generated character. Actors wear special suits covered in a specific pattern of sensors. One common pattern is called the Sierpiński triangle. Take a look at this Sierpiński triangle. Determine the area of the shaded region, and explain your thinking.

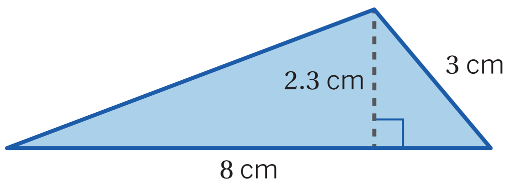


6. Here are three triangles drawn inside three identical rectangles. Mariam states that all three triangles have the same area. Is Mariam correct? Explain your thinking.

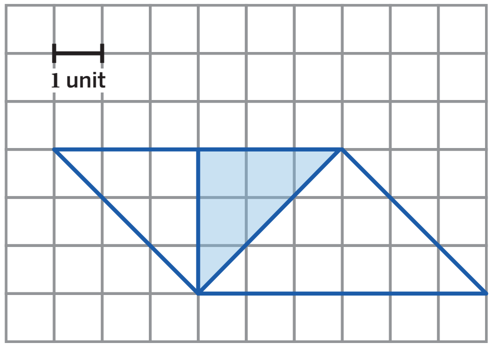


Name: .... Date: .... Period: ....

7. Determine the area of the triangle shown. Show or explain your thinking.



8. A company is designing a new logo that consists of a shaded triangle inside of a parallelogram. What fraction of the parallelogram's area is shaded?



Spiral Review

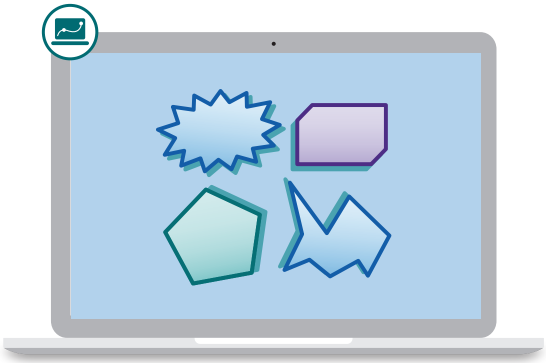
9. A parallelogram has an area of 12 square units and a base of 9 units. What is the matching height?

10. A parallelogram has an area of 7 square units and a height of $\frac{1}{4}$ units. What is the matching base?

Unit 1
Lesson
9

Name: ________ Date: ________ Period: ________

Generalizing With Multiple Representations | Graphing Shapes | 6.EE.2.c, 6.G.1, SMP.3, SMP.6



Pile of Polygons

Let’s play with polygons.

Warm-Up

1 Play a few rounds of Polygraph with your classmates!

You will use the Warm-Up Sheet with shapes for four rounds. For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a shape from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating shapes until you’re ready to guess which shape the Picker chose.

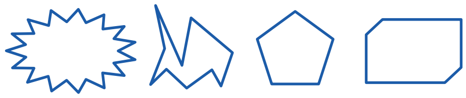
Record helpful questions from each round in this workspace:

Name: .... Date: .... Period: ....

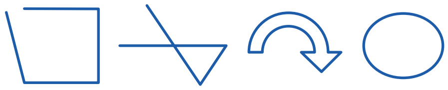
Polygons and Not Polygons

2 How are **polygons** different from shapes that are not polygons?

Polygon



Not Polygons



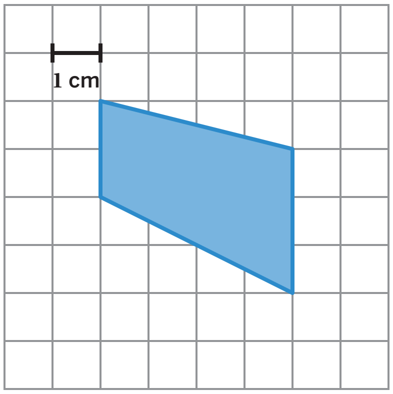
3 Sketch a shape that is a polygon and a shape that is not a polygon.

Polygon	**Not a Polygon**

Name: .... Date: .... Period: ....

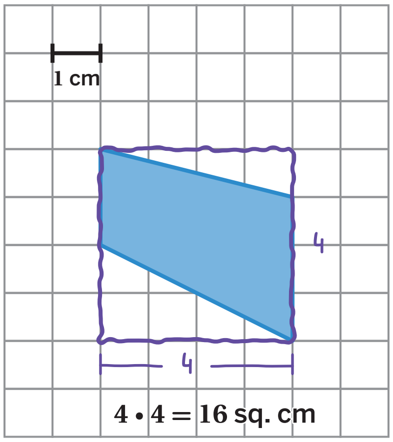
What is the Area?

4 Use any strategy to determine the area of this polygon.

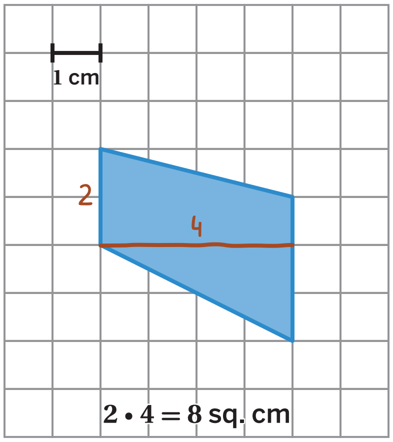


5 Jasmine and Callen both made mistakes when they calculated the area for this polygon.

Jasmine



Callen



a Choose your favorite mistake. Circle one.

Jasmine Callen

b Explain what you think is incorrect about the work you chose.

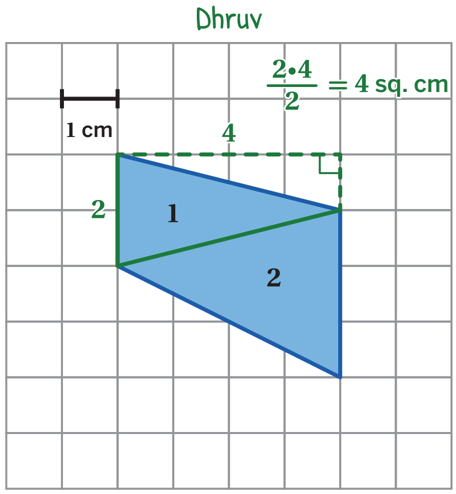
Name: .... Date: .... Period: ....

What is the Area? (continued)

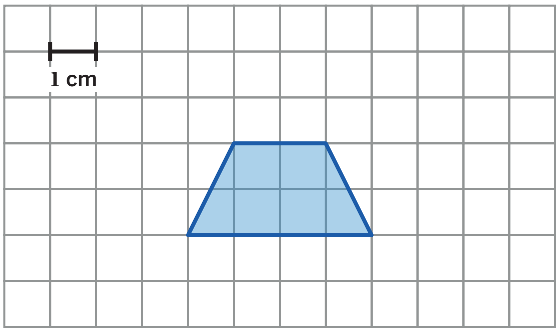
6 Dhruv decomposed a trapezoid into two triangles to calculate its area. Part of his work is shown.

- Calculate the area of the second triangle.
- Complete Dhruv's work to determine the area of the trapezoid.

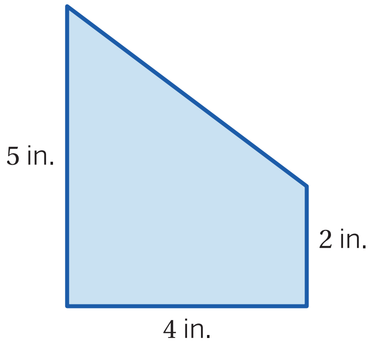
Show or explain your thinking.



7 Use Dhruv's strategy to determine the area of this trapezoid.



8 Determine the area of this polygon.



Activity 3

Name: ______ Date: ______ Period: ______

Challenge Creator

9 You will use the Challenge Creator Sheet to create your own area challenge.

a **Make It!** On the Challenge Creator Sheet, create your own area challenge.

b **Solve It!** On this page, determine the area of your polygon.

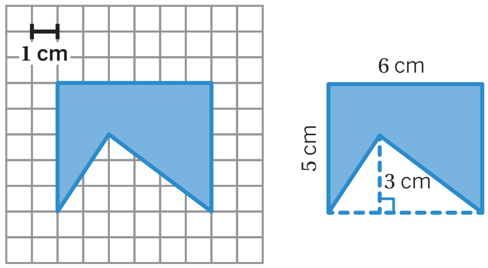
My Area

c **Swap It!** Swap your challenge with one or more partners. Determine the area of each partner's polygon.

	Partners' Areas
Partner 1	
Partner 2	
Partner 3	

10 Synthesis

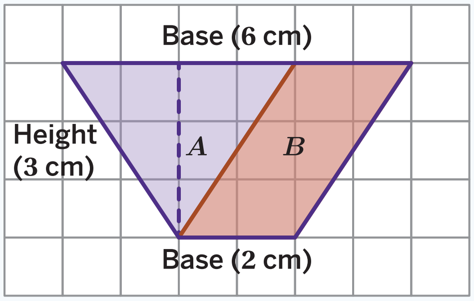
Describe a strategy for calculating the area of this polygon.



13 Summary 1.09

To help you determine the area of a **polygon**, such as a **trapezoid**, you can use shapes that have areas you already know how to calculate, like triangles and parallelograms.

Here's an example of two ways a polygon can be cut into triangles and parallelograms to help determine its area.



Area of Triangle A

$A = \frac{1}{2} \cdot 4 \cdot 3$

$A = 6$

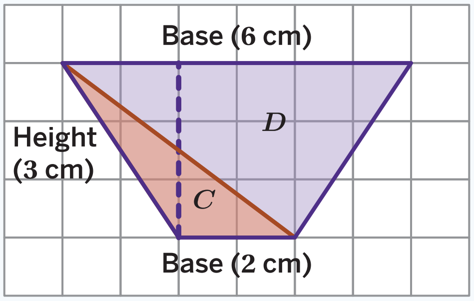
Area of Parallelogram B

$A = 2 \cdot 3$

$A = 6$

Area $= 6 + 6$

Area $= 12$ square centimeters



Area of Triangle C

$A = \frac{1}{2} \cdot 2 \cdot 3$

$A = 3$

Area of Triangle D

$A = \frac{1}{2} \cdot 6 \cdot 3$

$A = 9$

Area $= 3 + 9$

Area $= 12$ square centimeters

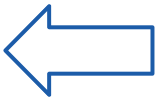
polygon A closed two-dimensional shape with straight sides that do not cross each other.

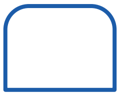
trapezoid A quadrilateral with at least one pair of parallel sides.

Practice
1.09

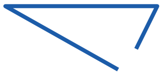
Name: ________________ Date: ________ Period: ________

1. Select *all* the polygons.

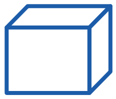
☐ **A.** 

☐ **B.** 

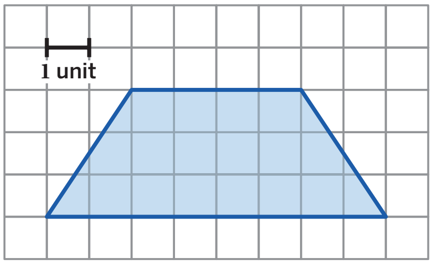
☐ **C.** 

☐ **D.** 

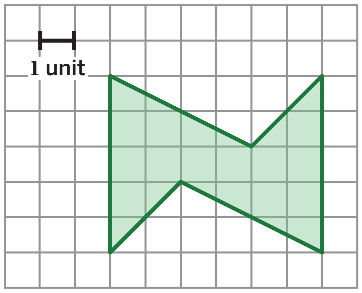
☐ **E.** 

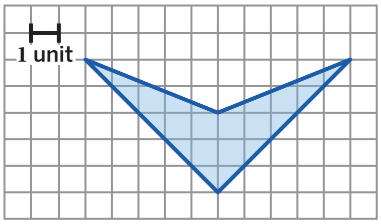
☐ **F.** 

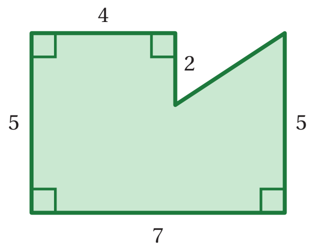
2. Determine the area of this polygon. Each square has an area of 1 square unit.



Problems 3–5: Determine the area of each polygon. Show your thinking.

3. 

4. 

5. 

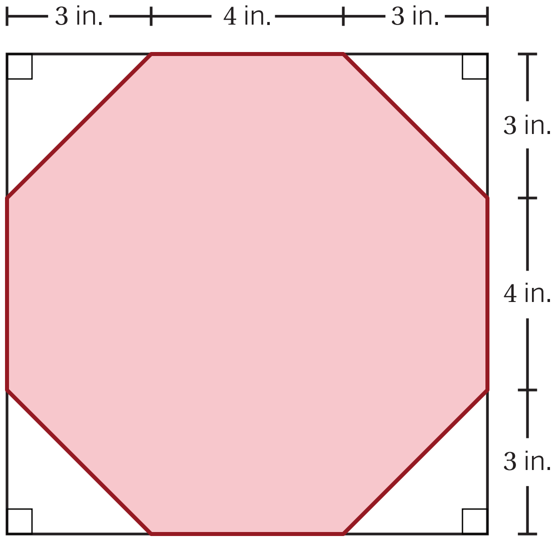
Practice
1.09

Name: .... Date: .... Period: ....

6. What is the total area of the shaded figure in square inches?

A. 16 B. 64

C. 82 D. 100



Spiral Review

Problems 7–10: Determine each product.

7. $\begin{array}{r} 126 \\ \times \quad 31 \\ \hline \end{array}$

8. $\begin{array}{r} 5126 \\ \times \quad 15 \\ \hline \end{array}$

9. $\begin{array}{r} 875 \\ \times \quad 463 \\ \hline \end{array}$

10. $\begin{array}{r} 980 \\ \times \quad 346 \\ \hline \end{array}$

Name: ______________________ Date: __________ Period: __________

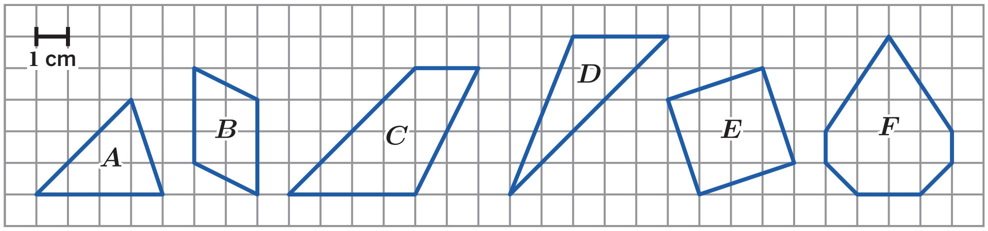
Generalizing With Multiple Representations | Nets and Surface Area | Graphing Shapes

6.G.1, 6.EE.2.a, 6.EE.2.c

Practice Day 1, Set A

Let's practice what you've learned so far in this unit!

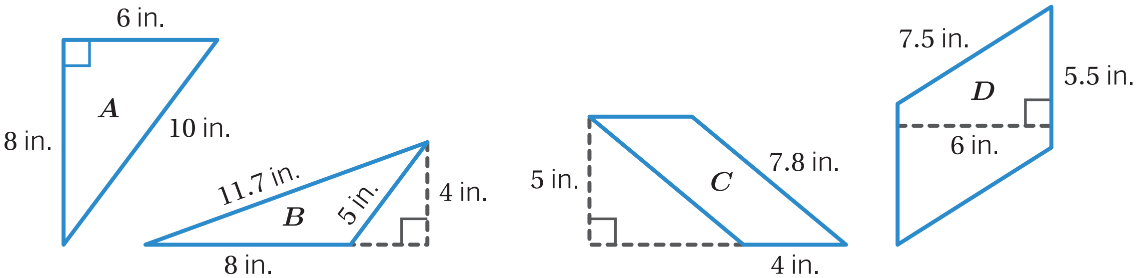
1. Here is a set of polygons.



Determine the areas of as many polygons as you can. Record each area in this table.

A	B	C	D	E	F

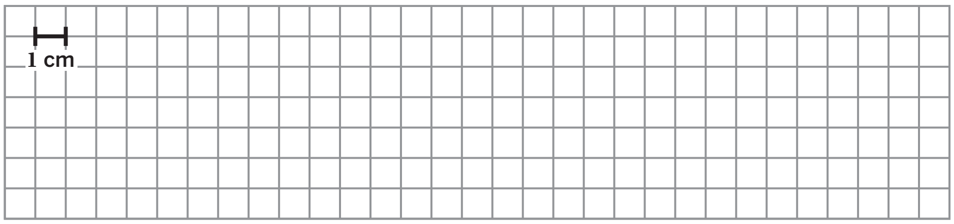
2. Here are shapes A, B, C, and D.



Order the shapes from *smallest* to *largest* area.

Smallest area **Largest area**

3. Draw two different triangles on the grid so that each has an area of 8 square centimeters.

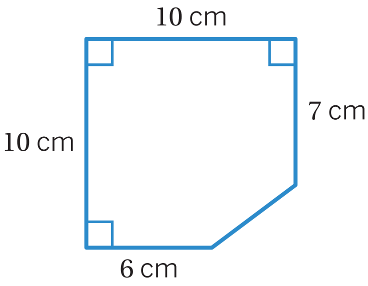


Name: .... Date: .... Period: ....

Practice Day 1, Set A

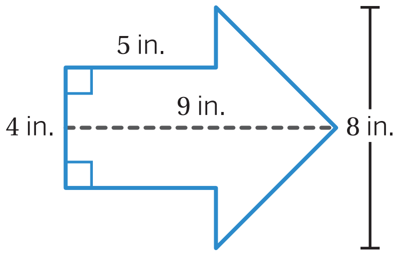
Calculate the area of each polygon.

4.



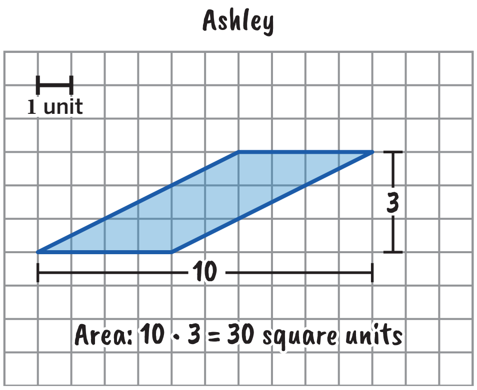
Area: .... square centimeters

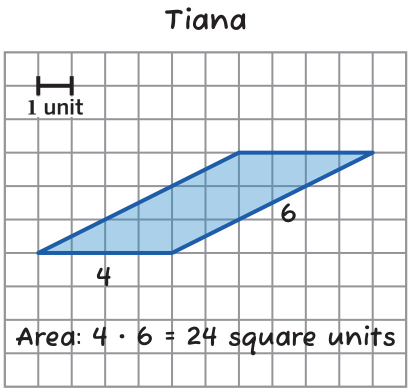
5.



Area: .... square inches

Here is the work that two students did to find the area of the same parallelogram.





6. Explain the error Ashley made.

7. Explain the error Tiana made.

8. Determine the correct area.

Name: .... Date: .... Period: ....

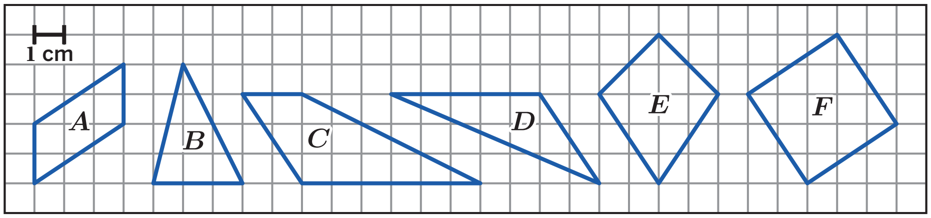
Generalizing With Multiple Representations | Nets and Surface Area | Graphing Shapes

6.G.1, 6.EE.2.a, 6.EE.2.c

Practice Day 1, Set B

Let's practice what you've learned so far in this unit!

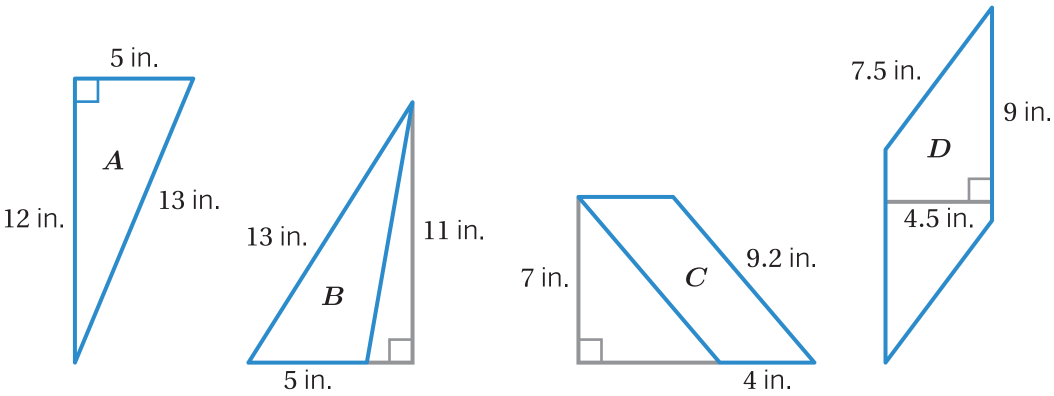
1. Here is a set of polygons.



Determine the areas of as many polygons as you can. Record each area in this table.

A	B	C	D	E	F

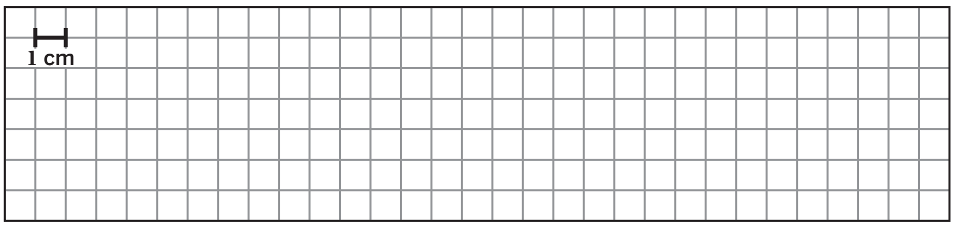
2. Here are shapes A, B, C, and D.



Order the shapes from *smallest* to *largest* area.

Smallest area **Largest area**

3. Draw two different triangles on the grid so that each has an area of 8 square centimeters.

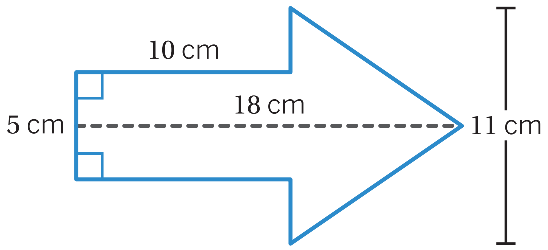


Name: .... Date: .... Period: ....

Practice Day 1, Set B

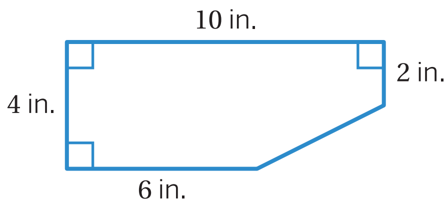
Calculate the area of each polygon.

4.



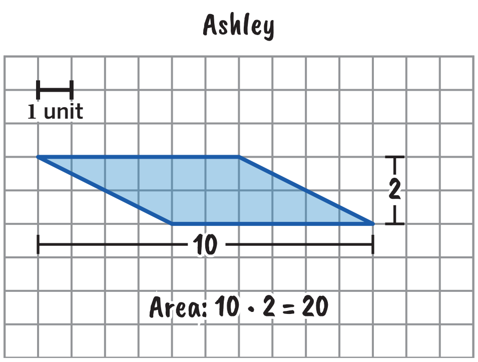
Area: .... square centimeters

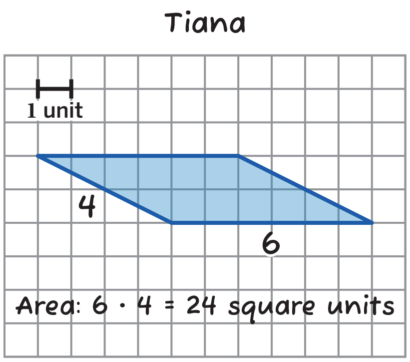
5.



Area: .... square inches

Here is the work that two students did to find the area of the same parallelogram.





6. Explain the error Ashley made.

7. Explain the error Tiana made.

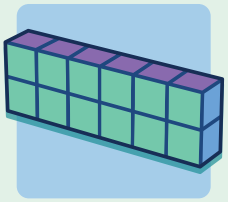
8. Determine the correct area.

Notes:

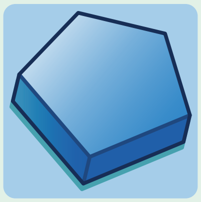
Sub-Unit

2

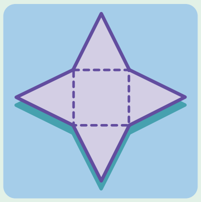
Surface Area



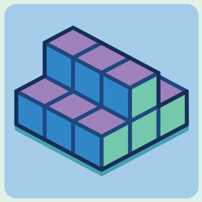
Lesson 10
Renata's Stickers



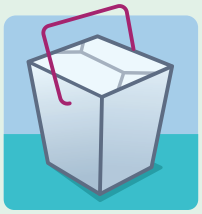
Lesson 11
Plenty of Polyhedra



Lesson 12
Nothing But Nets



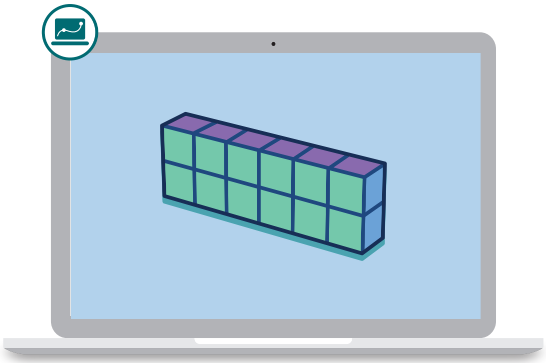
Lesson 13
Face Value



Lesson 14
Take It To Go

Name: ______ Date: ______ Period: ______

Nets and Surface Area | Building Toward 6.G.4, SMP.1, SMP.7



Renata's Stickers

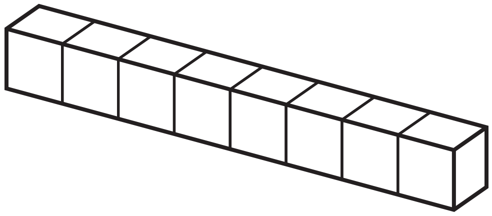
Let's cover rectangular prisms with stickers.

Warm-Up

1 Renata loves to cover things with stickers.

a. Let's watch her add stickers.

0 stickers



b. If Renata bought a pack of 30 stickers, would she have enough to cover this figure? Circle one.

Yes No I'm not sure

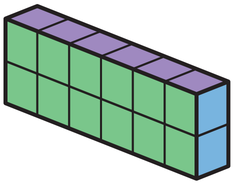
Explain your thinking.

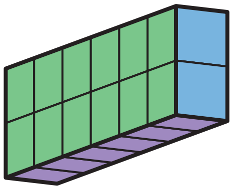
Activity 1

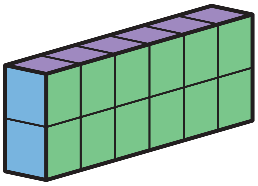
Name: ______ Date: ______ Period: ______

Exploring Surface Area

2 Here are different views of a figure. How many stickers will cover this figure?







3 Each of these figures is made with 8 cubes.

Which figure needs more stickers to cover it? Circle one.

Figure A Figure B Both need the same number

Explain your thinking.

Figure A

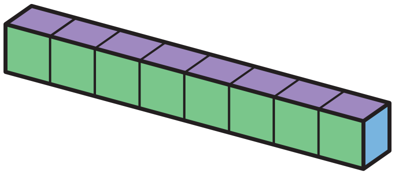
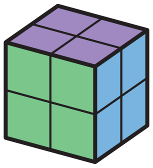


Figure B



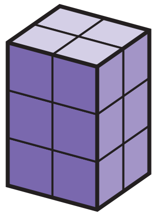
Name: ________ Date: ________ Period: ________

Exploring Surface Area (continued)

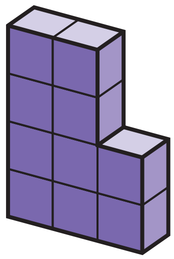
4 This rectangular prism is made of 12 cubic units.

Its **surface area** is the number of square units that cover all the faces of a three-dimensional shape, like Renata's stickers do.

Determine the surface area of this rectangular prism.



5 Determine the surface area of this prism.



Name: Date: Period:

Surface Area Strategies

6 Jaleel determined the surface area of a new prism.

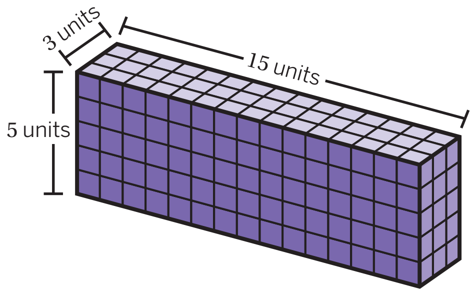
Jaleel

$15 \cdot 5 \cdot 2 = 150$

$3 \cdot 5 \cdot 2 = 30$

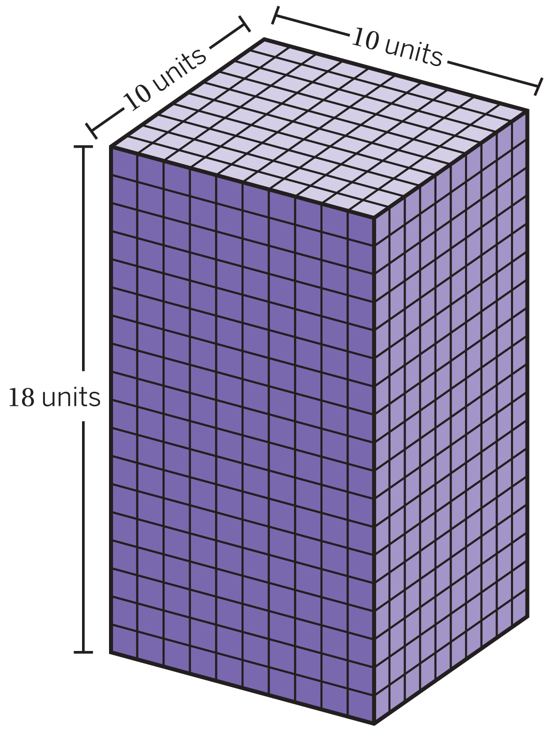
$15 \cdot 3 \cdot 2 = 90$

$150 + 30 + 90 = 270$ square units



Discuss: What did Jaleel do to determine the surface area?

7 Determine the surface area of this rectangular prism. Draw on the figure if it helps with your thinking.



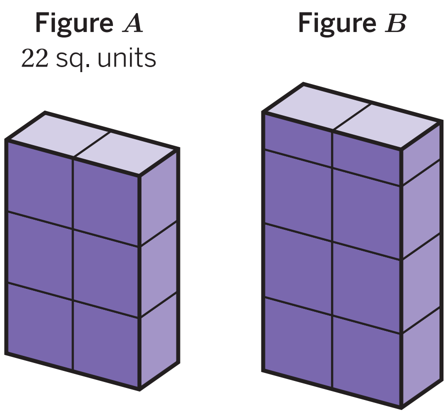
Name: ____________ Date: ________ Period: ________

Surface Area Strategies (continued)

8 The surface area of figure A is 22 square units.

Figure B is a *half unit* taller than figure A.

What is the surface area of figure B? Explain your thinking.

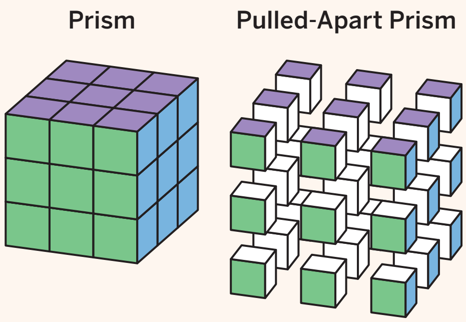


You're invited to explore more.

9 Here is a prism made with 27 cubes that Renata covered with stickers.

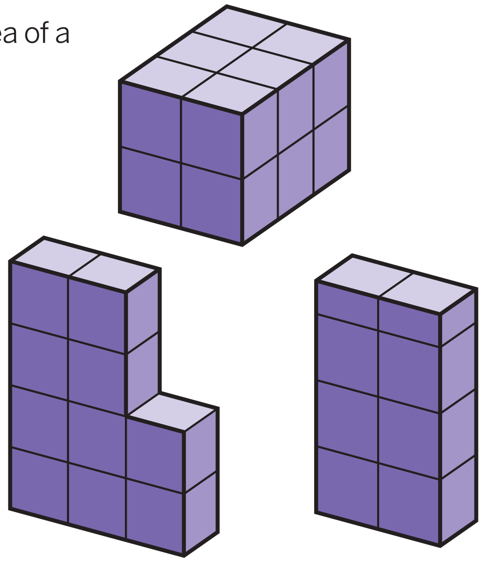
How many of the cubes have 0, 1, 2, and 3 stickers?

Stickers	Number of Cubes
0	
1	
2	
3	



10 Synthesis

Discuss: How do you determine the surface area of a rectangular prism?



13 Summary 1.10

The **surface area** of a rectangular prism is the sum of the areas of its surface. The **volume** of a rectangular prism measures the number of unit cubes that can be packed inside it without gaps or overlaps. Because volume is a three-dimensional measurement, it's measured in cubic units.

Here is a rectangular prism with a surface area of 52 square units and a volume of 24 cubic units.

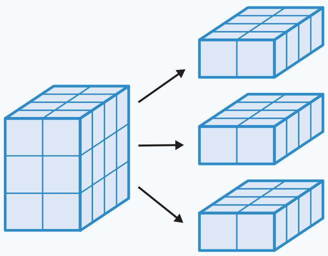
Surface Area

$(2 \cdot 3) \cdot 2 = 12$

$(4 \cdot 3) \cdot 2 = 24$

$(2 \cdot 4) \cdot 2 = 16$

$12 + 24 + 16 = 52$ square units



Volume

$8 + 8 + 8 = 24$

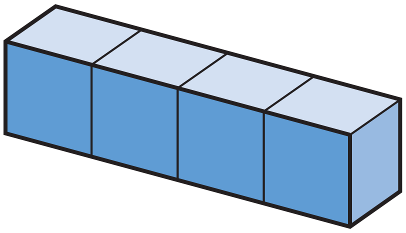
24 cubes = 24 cubic units

surface area The sum of the areas of the faces of a polyhedron.

Name: .... Date: .... Period: ....

Problems 1–2: Determine the surface area and volume of each rectangular prism.

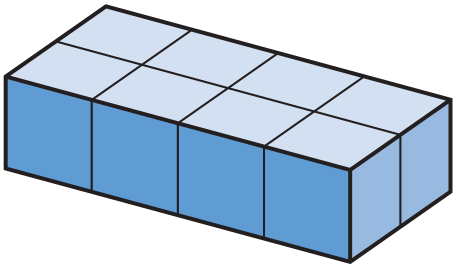
1.



Surface area: .... square units

Volume: .... cubic units

2.

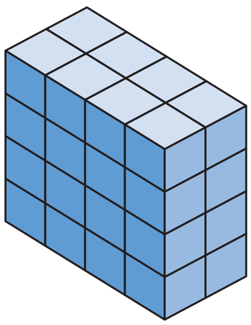


Surface area: .... square units

Volume: .... cubic units

3. Explain the difference between *volume* and *surface area*.

4. This rectangular prism is 4 units high, 4 units wide, and 2 units long. What is its surface area?



A. 16 square units

B. 32 square units

C. 48 square units

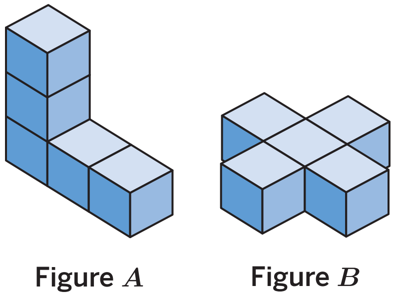
D. 64 square units

5. Select *all* of the calculations that surface area might help with.

- ☐ **A.** How many blocks fit in a container.
- ☐ **B.** How much wrapping paper a gift will need.
- ☐ **C.** How heavy a gift is.
- ☐ **D.** How much milk fits in a container.
- ☐ **E.** How much cardboard it takes to make a milk container.

Name: .... Date: .... Period: ....

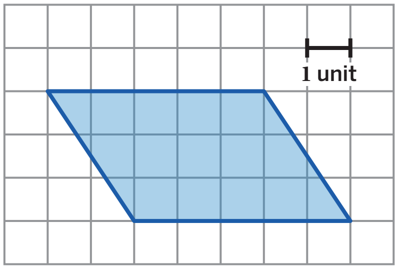
6. Compare the surface areas of figure A and figure B. Explain your thinking.



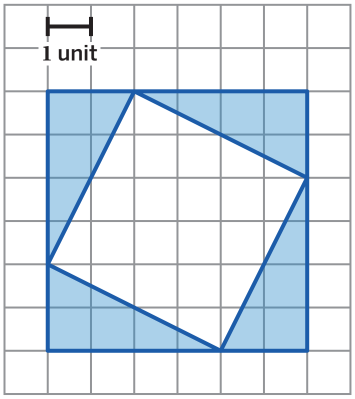
Spiral Review

Problems 7–8: Determine the area of each shaded figure.

7.



8.



Problems 9–10: Calculate each product.

9. $\frac{1}{2} \cdot 8 \cdot 10$

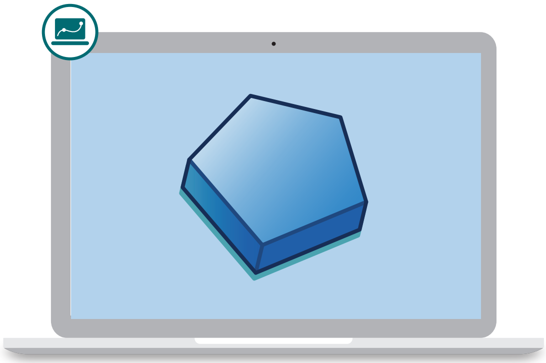
10. $5 \cdot \frac{1}{2} \cdot 7$

Unit 1
Lesson
11

Name: ________ Date: ________ Period: ________

Nets and Surface Area 6.G.4, SMP.6

Plenty of Polyhedra



Let's play with polyhedra.

Warm-Up

1 Play a few rounds of Polygraph with your classmates!

You will use the Warm-Up Sheet with 3-D shapes to play for four rounds. For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a 3-D shape from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating 3-D shapes until you're ready to guess which shape the Picker chose.

Record helpful questions from each round in this workspace:

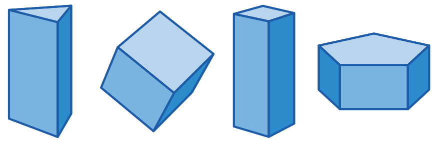
Name: .... Date: .... Period: ....

Prisms and Pyramids

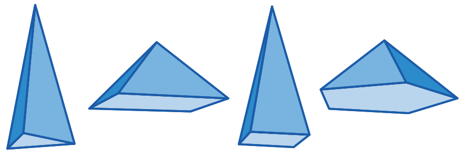
2 **Polyhedra** are 3-D shapes with flat sides. **Prisms** and **pyramids** are two types of polyhedra.

How are prisms and pyramids alike? How are they different?

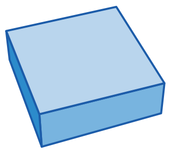
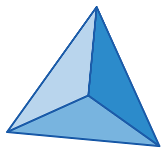
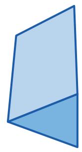
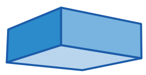
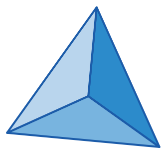
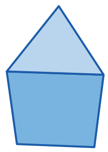
Prisms



Pyramids



3 Each flat side of a polyhedron is called a face. Complete the table for each polyhedron.

	Figure A	Figure B	Figure C
			
			
Number of Rectangular Faces			
Number of Triangular Faces			
Total Number of Faces			

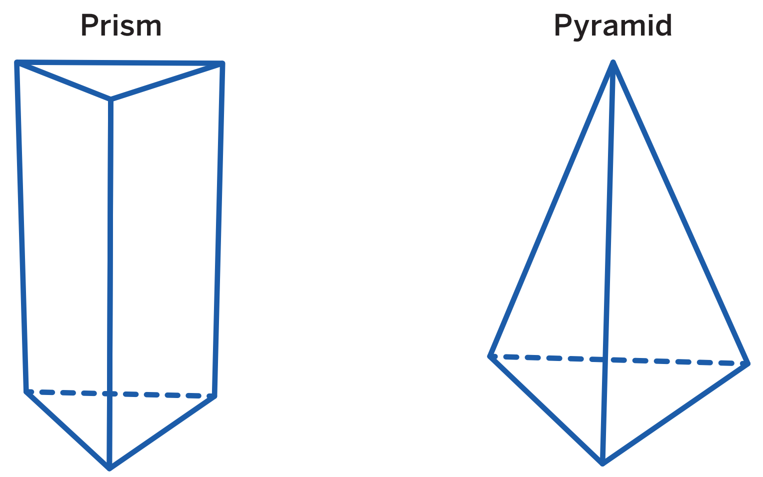
Name: .... Date: .... Period: ....

Prisms and Pyramids (continued)

4 Some of the faces of polyhedra are called **bases**.

- A prism has two identical bases that are parallel.
- A pyramid has one base.

Shade the base of the pyramid.



5 The shape of the base is what gives a polyhedron its name. For example, a prism with a rectangular base is called a rectangular prism.

Discuss: How would you name the figures in Problem 4?

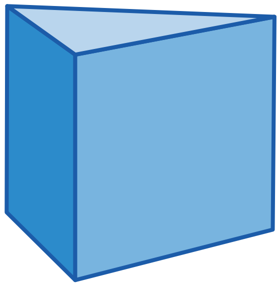
Activity
2

Name: .... Date: .... Period: ....

Nets of Prisms and Pyramids

6 When you unfold a polyhedron, you get a **net**.

Here is a triangular prism.

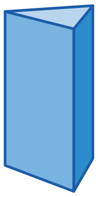


a Let's watch the prism unfold into its net.

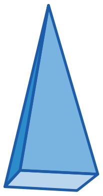
b **Discuss:** What you notice? What do you wonder?

7 Which solid will this net form when it's folded? Circle one.

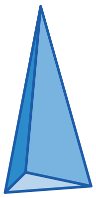
A.



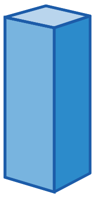
B.



C.



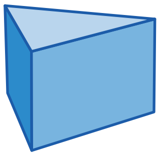
D.

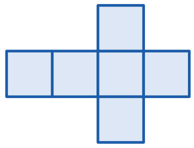


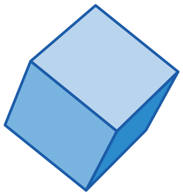
Name: .... Date: .... Period: ....

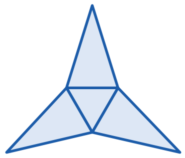
Nets of Prisms and Pyramids (continued)

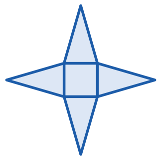
8 Match each polyhedron with its net. One net will have no match.

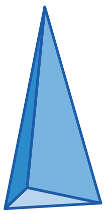
a 

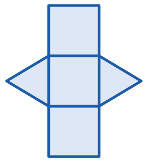
.... 

b 

.... 

.... 

c 

.... 

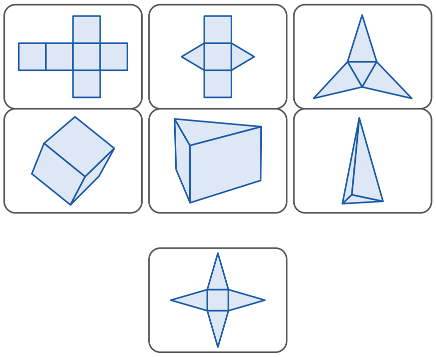
You're invited to explore more.

9 Kira claims she can create a prism with 10 faces.

What must be the shape of the base of her prism? Explain your thinking.

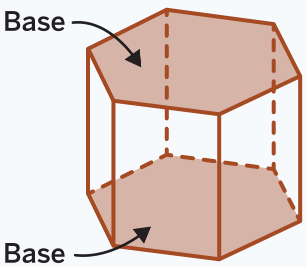
10 Synthesis

Discuss: How can you decide whether a shape is a prism, a pyramid, or neither?

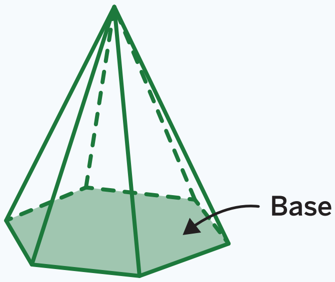


13 Summary 1.11

Prisms and **pyramids** are types of **polyhedra**. Each flat side of a prism or pyramid is a *face*. The face that gives the polyhedron its name is called the **base**.



Prism



Pyramid

Nets show us what a polyhedron would look like if it was "unfolded" and allow us to see each face at the same time.

base (of a pyramid or prism) The face that gives the solid its name.

net Two-dimensional representation of a three-dimensional shape.

polyhedron A closed three-dimensional shape with flat sides. The plural of polyhedron is polyhedra.

prism A three-dimensional shape, or polyhedron, with two bases that are identical copies.

pyramid A three-dimensional shape, or polyhedron, that has one base. All of the other faces are triangles that meet at a single vertex.

Practice
1.11

Name: ________________ Date: ________ Period: ________

Problems 1–8: Here is a set of polyhedra.

Figure A

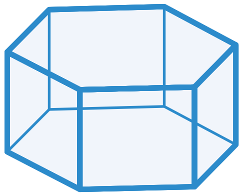


Figure B

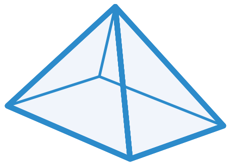


Figure C

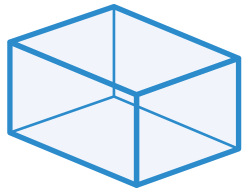


Figure D

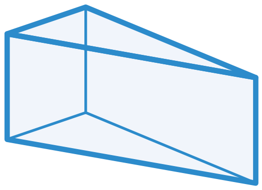
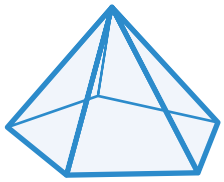


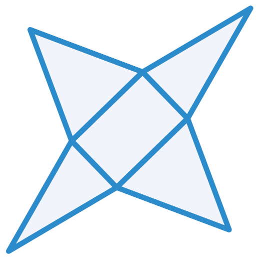
Figure E



1. Which polyhedra are prisms?

2. Which polyhedra are pyramids?

3. Which of these polyhedra could be created from this net?



4. What type of polyhedron is figure A?

5. What type of polyhedron is figure B?

6. What type of polyhedron is figure C?

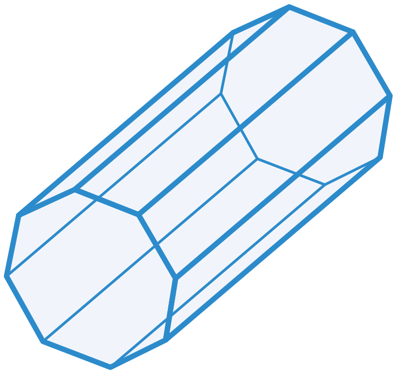
7. What type of polyhedron is figure D?

8. What type of polyhedron is figure E?

Name: ______________ Date: ______________ Period: ______________

Problems 9–10: Use the polyhedron shown.

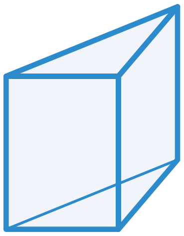
9. Is this polyhedron a prism, a pyramid, or neither? Explain your thinking.



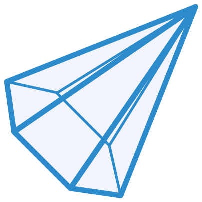
10. What is the name of this type of polyhedron?

Problems 11–12: What polygons make up the faces of these three-dimensional figures?

11.



12.



Spiral Review

Problems 13–14: Evaluate each expression. Show your thinking.

13. $5 + (4 \div 2)$

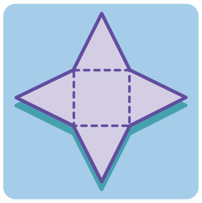
14. $(3 - 2) \cdot (4 + 1)$

Unit 1
Lesson
12

Name: ______ Date: ______ Period: ______

Nets and Surface Area 6.G.4, SMP.3, SMP.6

Nothing But Nets

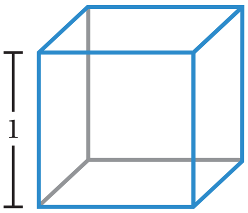


Let's make connections between polyhedra and their nets.

Warm-Up

Here is a polyhedron.

1. What could you call this type of polyhedron?



2. Draw its net.

Activity 1

Name: ______ Date: ______ Period: ______

Nets and Polyhedra

3. You will use a set of cards for this activity. Match each polyhedron to its net. Record your matches in the table below and circle the name of the polyhedron.

Polyhedron	Net	Name		
		Triangular pyramid	Triangular prism	Rectangular prism
		Triangular pyramid	Triangular prism	Rectangular prism
		Triangular pyramid	Triangular prism	Rectangular prism
		Triangular pyramid	Triangular prism	Rectangular prism

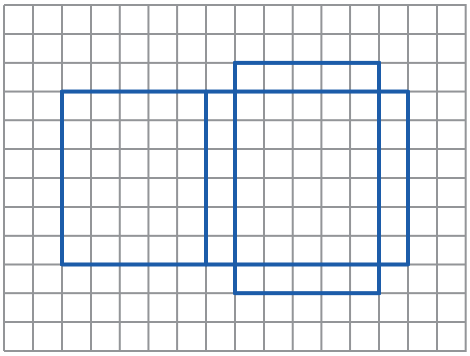
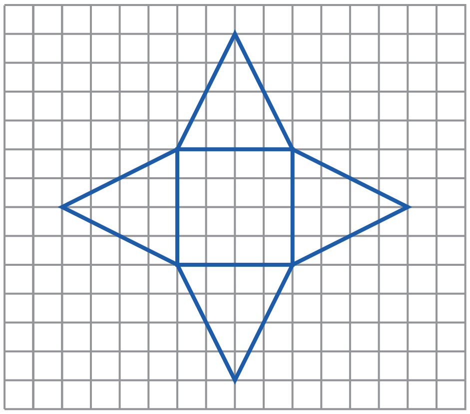
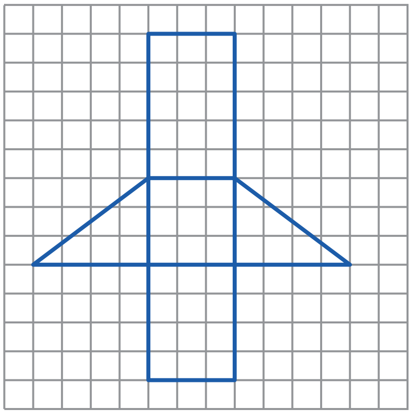
4. Sketch the missing net.

5. **Discuss:** How does the name of the polyhedron relate to its net?

Name: .... Date: .... Period: ....

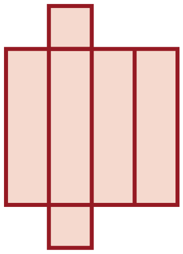
Make Polyhedra

6. You will use the Activity 2 Sheet for this activity. Take a look at the nets for polyhedra A, B, and C. Cut out, assemble, and name each polyhedron. Then calculate its surface area. Record your responses and show your thinking in the table below.

Net	Name	Surface Area
Polyhedron A 		
Polyhedron B 		
Polyhedron C 		

5 Synthesis

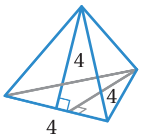
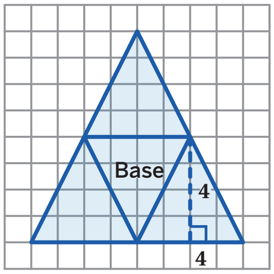
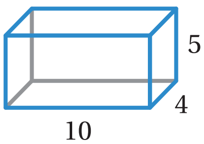
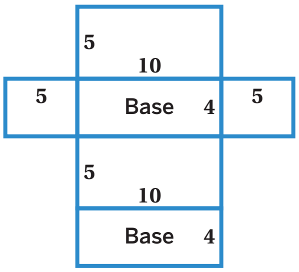
Discuss: How can a net help you calculate surface area?



Summary 1.12

We can draw a net to create a two-dimensional representation of a three-dimensional figure. We can use the net to help us determine the surface area of a polyhedron because it shows every face at once.

Here are some examples of how a net can help us find the surface area of a pyramid or a prism.

Polyhedron	Net	Surface Area
Triangular pyramid 		$4\left(\frac{1}{2} \cdot 4 \cdot 4\right) = 32$ square units
Rectangular prism 		$2(4 \cdot 10) + 2(4 \cdot 5) + 2(5 \cdot 10) = 220$ square units

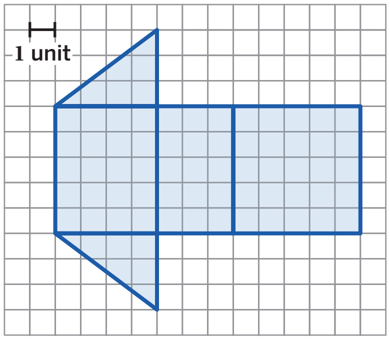
Practice
1.12

Name: .... Date: .... Period: ....

1. Select *all* the units that can be used to describe surface area.

- ☐ **A.** Square meters
- ☐ **B.** Feet
- ☐ **C.** Centimeters
- ☐ **D.** Cubic inches
- ☐ **E.** Square inches
- ☐ **F.** Square feet

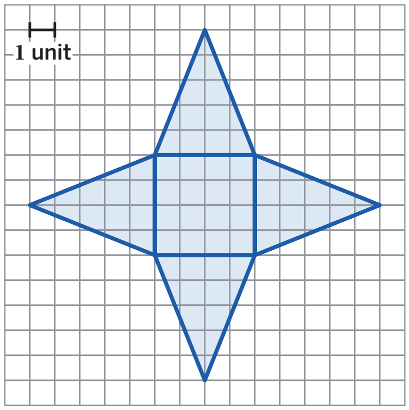
Problems 2–3: Here is a net.



2. Name the type of polyhedron that can be created from this net. Explain your thinking.

3. Determine the surface area of this polyhedron. Show or explain your thinking.

Problems 4–5: Here is a new net.



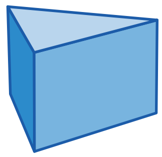
4. Name the type of polyhedron that can be created from this net. Show or explain your thinking.

5. Determine the surface area of this polyhedron. Show your thinking.

Practice
1.12

Name: ______________________ Date: __________ Period: __________

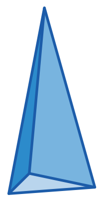
Problems 6–7: Here is an image of a polyhedron.



6. Name the polyhedron.

7. Sketch a net for this polyhedron.

Problems 8–9: Here is an image of a polyhedron.



8. Name the polyhedron.

9. Sketch a net for this polyhedron.

Spiral Review

Problems 10–14: Complete each equation.

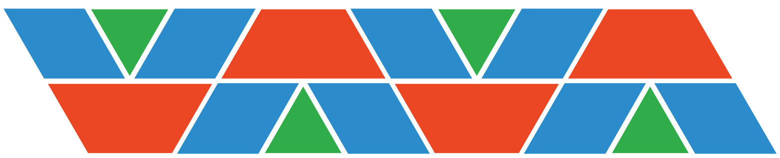
10. $3 \cdot \square = 15$

11. $4 \cdot \square = 24$

12. $15 \cdot \square = 5$

13. $24 \cdot \square = 4$

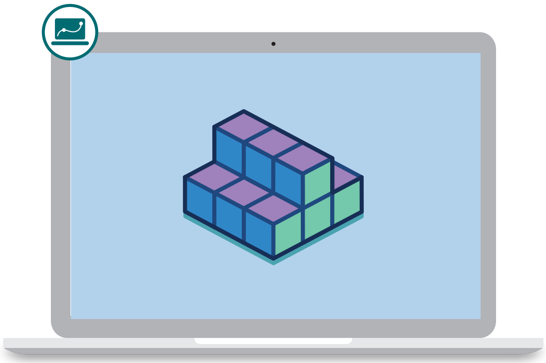
14. Take a look at the triangles in this pattern. What is the fewest number of these triangles needed to cover this pattern completely? Explain your thinking.



Unit 1
Lesson
13

Name: .... Date: .... Period: ....

Nets and Surface Area 6.G.4, SMP.3, SMP.6, SMP.7



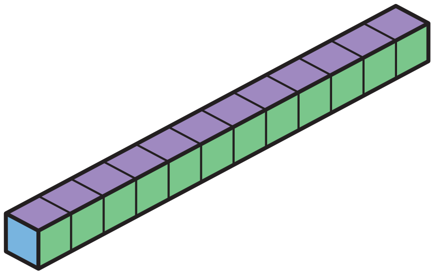
Face Value

Let’s determine the surface area of prisms and pyramids.

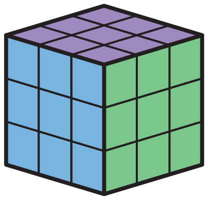
Warm-Up

1 Which one doesn’t belong? Explain your thinking.

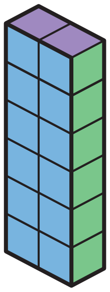
A.



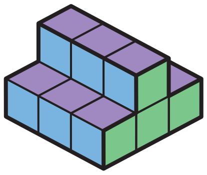
B.



C.



D.



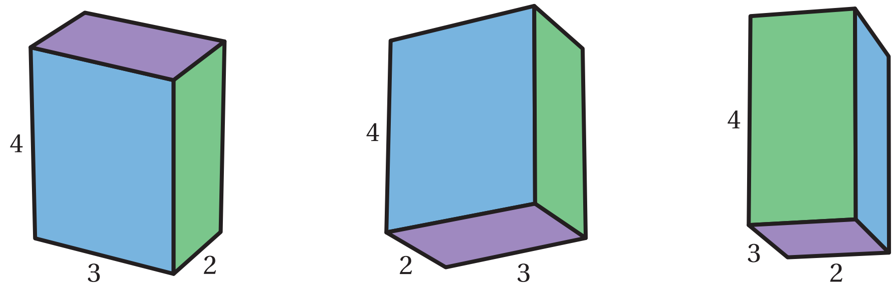
Activity 1

Name: ______ Date: ______ Period: ______

Surface Area Without a Grid

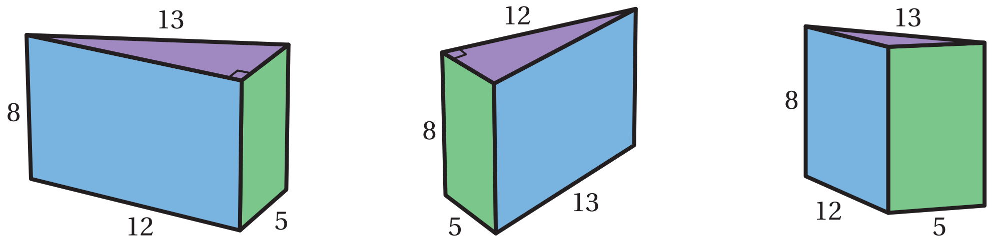
2 Note: All measurements in this lesson are in units.

a Take a look at all the faces of this rectangular prism in square units.



b Calculate its surface area.

3 Calculate the surface area of this triangular prism.

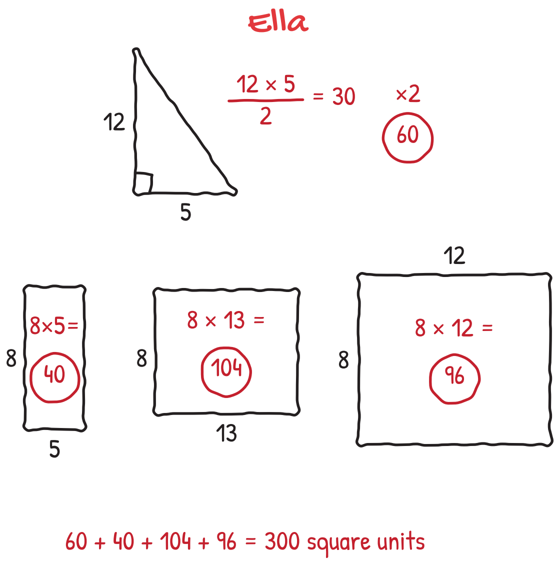


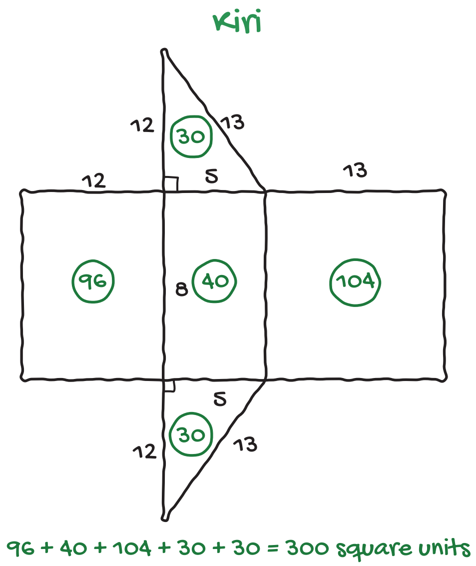
Name: ______ Date: ______ Period: ______

Surface Area Without a Grid (continued)

4 Here are two different strategies for finding the surface area of the prism from the previous problem.

Discuss: What did each student do? How are their strategies alike? How are they different?



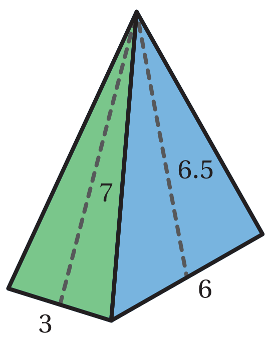


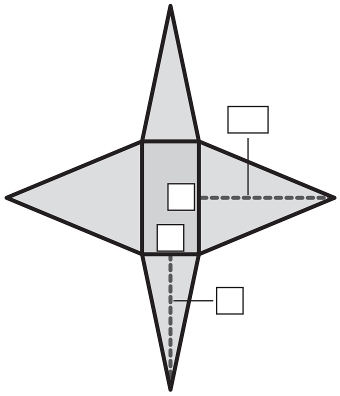
Name: .... Date: .... Period: ....

From Polyhedra to Nets

5 Here is a rectangular pyramid. The dotted lines represent the heights of the triangles.

Fill in the lengths to make a net that can be folded to create this pyramid.



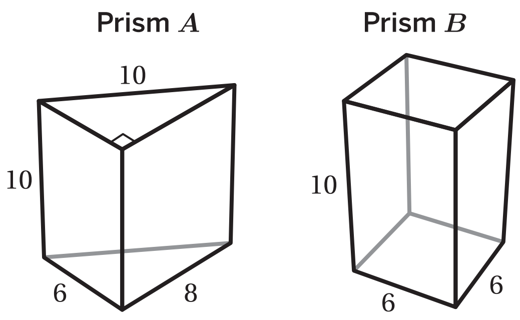


6 Calculate the surface area of the rectangular pyramid from the previous problem. Use the pyramid's net if it helps with your thinking.

7 Which of these polyhedra has a greater surface area? Circle one.

Prism A Prism B They are the same

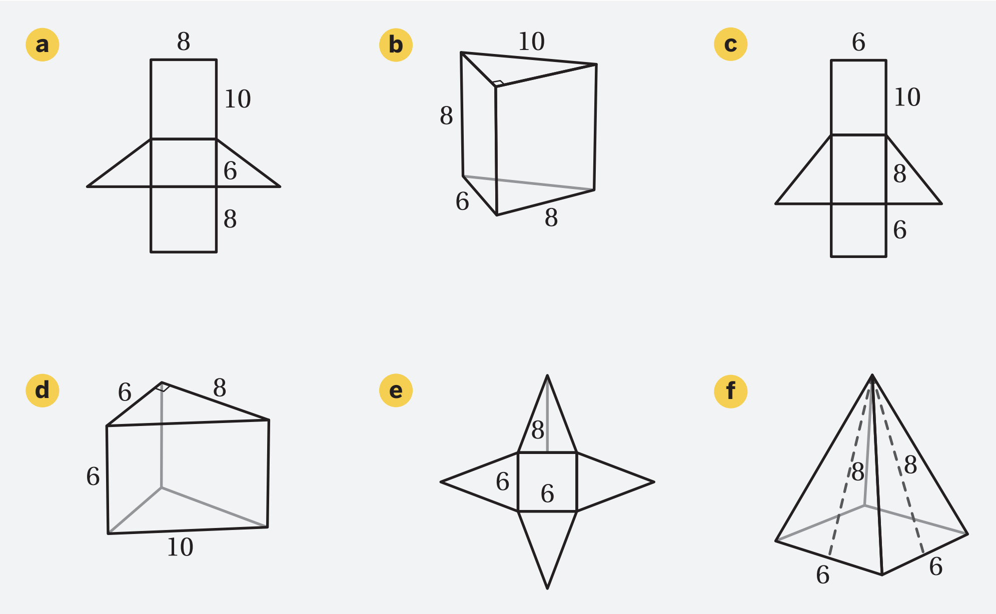
Explain your thinking.



Name: .... Date: .... Period: ....

From Polyhedra to Nets (continued)

8 Match each polyhedron and net to their surface area.



The surface area is 132 square units.	The surface area is 240 square units.	The surface area is 192 square units.

You're invited to explore more.

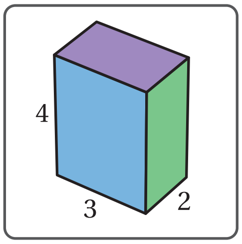
9 Select one question and write your response.

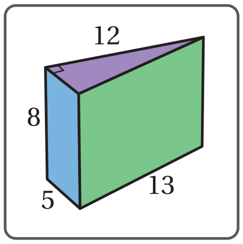
a What are the dimensions of two prisms that have the same surface area but different volumes?

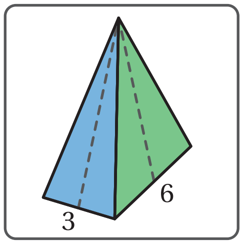
b What are the dimensions of two prisms that have the same volume but different surface areas?

10 Synthesis

Discuss: How can you calculate the surface area of a prism or a pyramid from a picture?





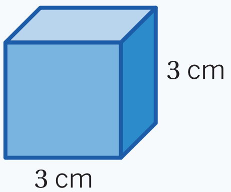


13 Summary 1.13

The surface area of any polyhedron is the total area of all the individual faces. Drawing a net or sketching individual faces can help us make sense of and keep track of calculations.

We can group identical faces together to reduce the number of steps in our calculations. For example, a cube is made of 6 identical faces, so we can determine the area of one face and multiply by 6 to determine the total surface area.

Here's an example.



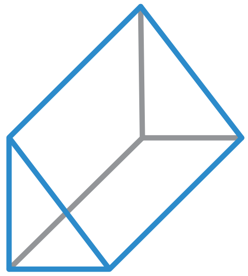
One Face
$3 \cdot 3 = 9$

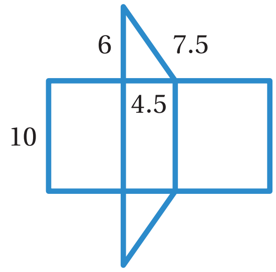
All Faces
$9 \cdot 6 = 54$

Surface Area
54 square centimeters

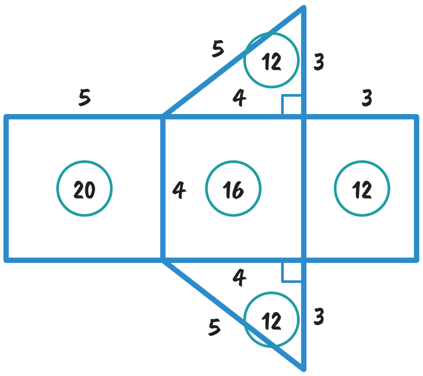
Name: ______________ Date: ______ Period: ______

1. Label *all* the edges of this polyhedron so that the lengths match the net.

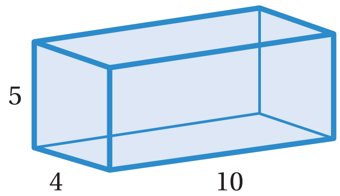




2. Takeshi made some mistakes when calculating the surface area of the triangular prism shown. Describe his mistakes and correct them.

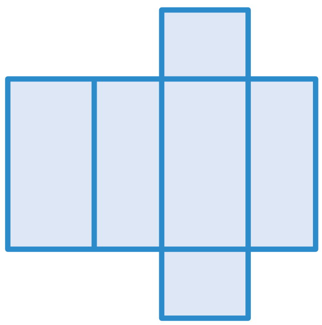


Problems 3–5: Here is a polyhedron and its matching net.



3. What is the name of this type of polyhedron?

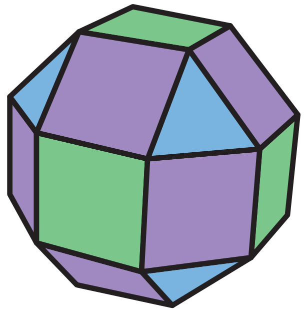
4. Use the polyhedron to label all the lengths in this net.



5. Use the net to calculate the surface area in square units. Show or explain your thinking.

Name: ________ Date: ________ Period: ________

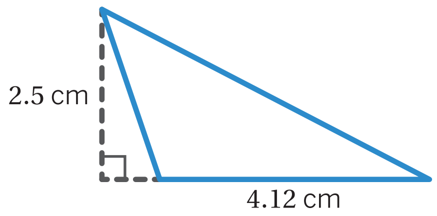
6. A *rhombicuboctahedron* is a polyhedron composed of 18 squares and 8 triangles. Here's an example where each square face has an edge length of 24 inches and each triangular face has a height of about 20.8 inches. Calculate the surface area. Show or explain your thinking.



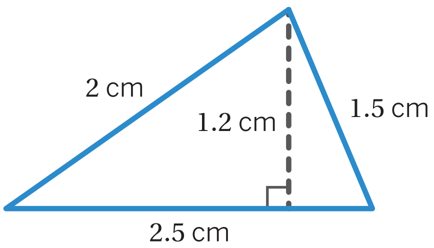
Spiral Review

Problems 7–8: Determine the area of each triangle.

7.



8.



Problems 9–12: Determine each product.

9. $3418 \cdot 7$

10. $82 \cdot 13$

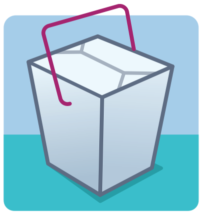
11. $2020 \cdot 8$

12. $96 \cdot 5$

Name: .... Date: .... Period: ....

Nets and Surface Area 6.G.1, 6.G.4, SMP.1, SMP.4

Take It To Go

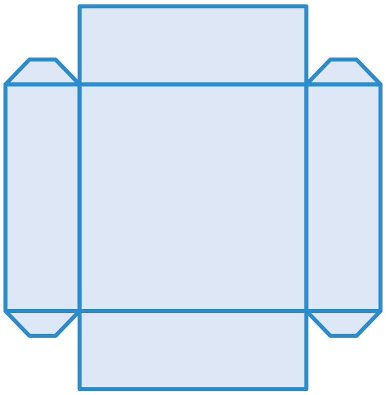


Let's design a to-go container.

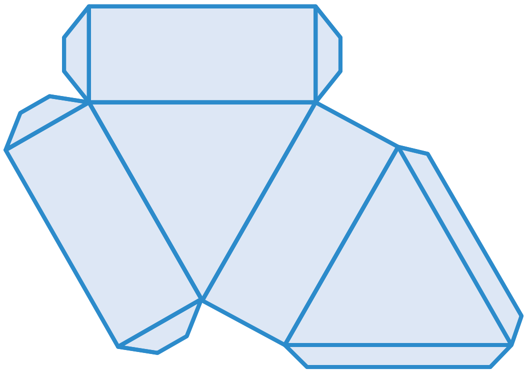
Warm-Up

1. DeAndre is opening a new restaurant. He is making patterns that can be folded into to-go containers for the different foods he will sell.

Pattern A



Pattern B



Discuss:

a How are the patterns alike? How are they different?

b Which container would you prefer to hold a bagel?

c Which pattern do you think requires more material?

Activity 1

Name: .... Date: .... Period: ....

Design a Container

DeAndre's restaurant will serve sandwiches, salads, and single slices of pizza. He needs to design to-go containers for each item.

Your task is to design a to-go container for one of the food items and calculate the amount of material you need to make it.

Use this information to help you create your design.

- A sandwich is roughly 4 inches by 4 inches by 2 inches.
- A salad is roughly 120 cubic inches.
- A slice is roughly the shape of a triangle with a height of 8 inches and a base of 5 inches.

2. Which food item are you designing a container for? Circle one.

Sandwich   Salad   Slice of Pizza

3. What is the shape of the base of your container?

4. How many faces does your container have?

5. Draw or describe how you want your container to look. Be sure to include all the necessary measurements.

6. Calculate how much material you need to make your container.

Name: ______ Date: ______ Period: ______

Make It!

7. Share your design with a partner. Discuss how you might improve your design and write down what adjustments you want to make.

8. Draw your revised pattern on blank paper using the measurements you designed.

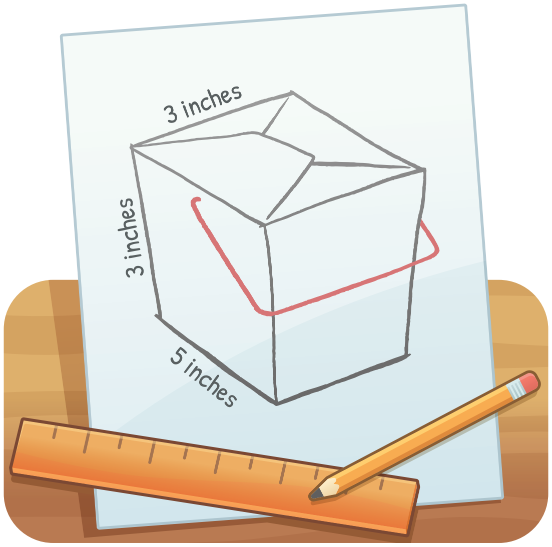
9. Cut out and fold your pattern to create your container.

You're invited to explore more.

10. To-go containers can be made out of different materials, like cardboard, foam, or aluminum. Research two different types of materials. Then write a pitch to DeAndre about which material(s) he should use for his containers, and why.

Synthesis

11. **a** How was surface area related to the work you did today?



b Now that you have seen your classmates' designs, what would you have done differently if you had more time?

Summary 1.14

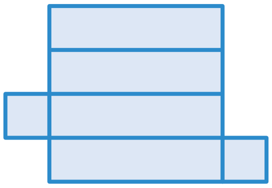
To-go containers and reusable plastic food containers are examples of polyhedra that we see in everyday life.

Mathematical modeling can help us design everyday objects, such as to-go containers. To do this, we need to:

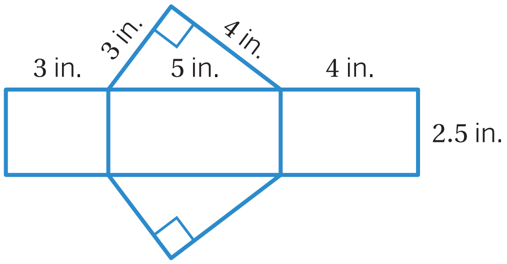
- Know the size and shape of the food item that will be placed in the container.
- Decide on the shape of the container.
- Make sure that the container will be big enough to hold the food item, without being too big.
- Know how much material we need to make the container. That's where surface area comes in handy!

Name: ________ Date: ________ Period: ________

1. Taylor says that this cannot be a net for a square prism because not all the faces are squares. Is Taylor correct? Explain your thinking.



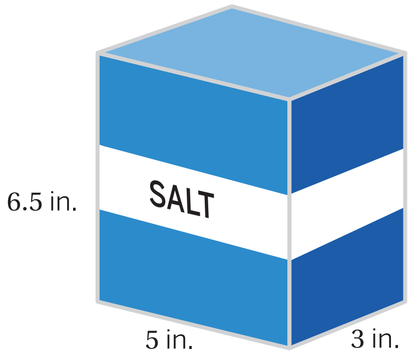
Problems 2–3: Here is a net.



2. Can you create a three-dimensional figure from this net? If so, which one?

3. What is the surface area of this figure? Explain your thinking.

Problems 4–5: This box of salt measures 5 inches by 3 inches by 6.5 inches.



4. Estimate how much cardboard the box uses. Explain your thinking.

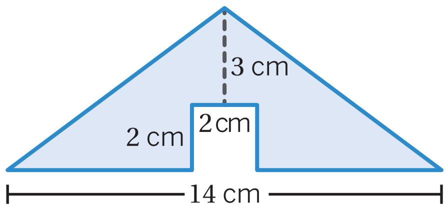
5. Estimate how much salt the box can hold. Explain your thinking.

Name: ______________ Date: ________ Period: ________

6. In 2011, a marketing agency built a giant tipped-over cereal box outside the Vancouver Art Gallery. It measured 6 meters tall, 4 meters long, and 1.6 meters wide. Determine the amount of cardboard they used to build it.

Spiral Review

7. Calculate the area of the shaded polygon. Explain your thinking.



8. Select *all* the values that are equivalent to 7.32.

- ☐ **A.** Seven and thirty-two tenths
- ☐ **B.** $7 + 0.3 + 0.02$
- ☐ **C.** 732 tenths
- ☐ **D.** 732 hundredths
- ☐ **E.** 7 ones + 3 tenths + 2 hundredths

9. A parallelogram has an area of 8 square units and a base of $\frac{1}{2}$ units. What is the matching height?

Name: ________ Date: ________ Period: ________

Generalizing With Multiple Representations | Nets and Surface Area | Graphing Shapes

6.G.1, 6.G.4,, 6.EE.2.a, 6.EE.2.c

Practice Day 2

Let's practice what you've learned so far in this unit!

Start with any of the scavenger hunt sheets.

- Record the sheet shape, solve the problem, and write your answer.
- Look for your answer at the top of another scavenger hunt sheet. Solve that problem.
- Repeat until you make it back to your starting sheet.

Sheet: ________ **Answer** →	**Sheet:** ________ ↙ **Answer**
Sheet: ________ **Answer** →	**Sheet:** ________ ↙ **Answer**

Name: .... Date: .... Period: ....

Practice Day 2

Sheet: .... Answer	Sheet: .... Answer
Sheet: .... Answer	Sheet: .... Answer
Sheet: .... Answer	Sheet: .... Answer

You're invited to explore more.

1. List the measurements of a rectangular prism that has a surface area of 96 square units.

2. Draw or describe two prisms that have the same surface area but different volumes.

Notes:

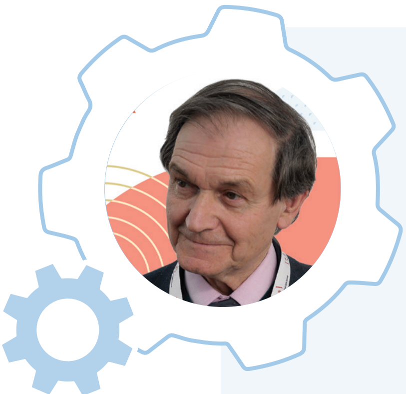
Career Connection

What might be impossible about this triangle?
What do you notice?

Peter Hermes Furian/Shutterstock.com

The triangle shown here is called the “impossible triangle” or the Penrose Triangle. It is an optional illusion and appears to contain features that cannot actually be realized by any solid, three-dimensional object. If you look closely, the bottom part of the triangle looks like it is both in front of and behind the topmost part of the triangle.

Graphic designers create visual designs, such as company logos, product packaging, advertisements, and more. These designs are intended to communicate ideas or information. They might use optical illusions as visual elements of designs they create to add a sense of mystery and help viewers engage with their designs.



“Roger Penrose” by Cirone-Musi, courtesy of Flickr

Meet Sir Roger Penrose

In 1958, English mathematician and physicist Sir Roger Penrose and his father Lionel Sharples Penrose published a paper on an “impossible triangle.” This triangle was first created by Swedish graphic artist Oscar Reutersvärd, possibly in 1934. Now commonly known as the Penrose Triangle, the triangle appears to contain a combination of properties that cannot be realized by any three-dimensional object.

Are you interested in studying graphics design or learning about other optical illusions? What can you do to learn more?

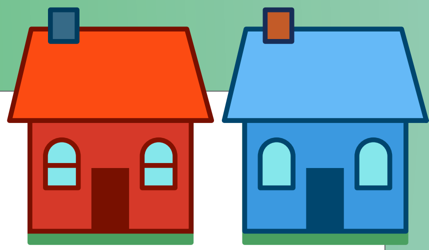
Community Connection

Research a visual design that uses one or more geometric shapes that you explored in this unit. This design can be a company logo, an advertisement, or a logo on product packaging. Share the visual design with someone at home or in your communities. Describe the geometric shapes that are included in the design.

Math Mindset

How does knowing how to determine the areas of parallelograms help you determine the areas of triangles and trapezoids?

Unit 2



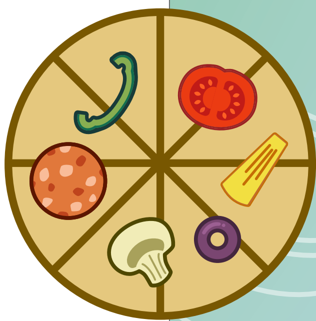
Introducing Ratios

Big Ideas in This Unit

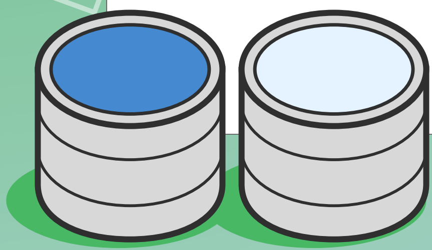
CC2 Fraction Relationships | Generalizing With Multiple Representations | Patterns Inside Numbers | CC3 Model the World

Questions for Investigation

- What does a ratio say about the relationship between quantities?
- How can ratios help you get the same taste, texture, or color every time you make a recipe?
- How can ratios help us consider issues of fairness?



Explore: Fermi Problems
How does ratio reasoning help solve estimation problems?



Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!

Not yet — Almost — I got it!

I can . . .	Before	After
Describe the relationship between two quantities		
Use words, numbers, and the symbol (:) to describe a ratio relationship.		
Identify equivalent ratios.		
Generate equivalent ratios by multiplying or dividing both values by the same number.		
Create a double number line to represent a situation.		
Use a double number line to solve problems involving equivalent ratios.		
Calculate a ratio for 1 of an item to solve problems.		
Determine the least common multiple of two numbers.		
Determine the greatest common factor of two numbers.		

I can . . .	Before	After
Compare two ratios.		
Use a ratio table to solve problems.		
Compare strategies for solving ratio relationships.		
Determine if ratio reasoning can be used to solve a problem.		
Use tape diagrams to solve problems involving ratio relationships.		
Solve problems involving part-part-whole relationships.		
Apply ratio reasoning to answer questions about a real-world situation.		

Sub-Unit

1

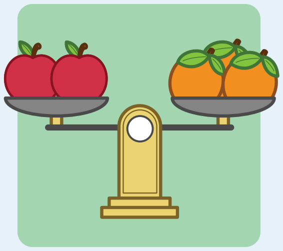
Ratios

Explore
Fermi Problems

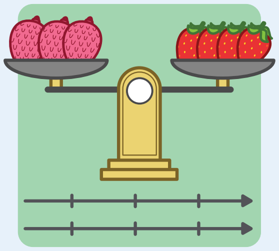
Lesson 1
Pizza Maker

Lesson 2
Ratio Relationships

Lesson 3
Rice Ratios



Lesson 4
Fruit Lab



Lesson 5
Balancing Act

Lesson 6
Product Prices

Unit 2
Lesson

Name: .... Date: .... Period: ....

Generalizing With Multiple Representations | Model the World

Building Toward 6.RP.1, SMP.1, SMP.4

Explore: Fermi Problems

How does ratio reasoning help solve estimation problems?

Warm-Up

1. Describe how you could make a rough estimate to solve this problem:
How many times does your heart beat in a year?
Include any information you would need to know.

Activity

Name: ____________ Date: ____________ Period: ____________

Fermi Problems

Fermi Problems use estimations and approximations to quickly "guesstimate" solutions to seemingly impossible-to-answer mathematical problems. They are named after Enrico Fermi who was an Italian scientist born in 1901. Fermi was known for his uncanny ability to quickly solve challenging problems like the ones you are about to read.

2. You will use a card with three Fermi problems. What are some questions you can ask about each problem to help you answer them? Work with your group to generate questions that could help identify the information someone would need to solve each problem.

ELD.PI.6.2.Em, Ex, Br, ELD.PI.6.6.Em, Ex, Br, ELD.PI.6.10.Em, Ex, Br

Fermi problem 1: ________________________________

Fermi problem 2: ________________________________

Fermi problem 3: ________________________________

Activity

Name: ________ Date: ________ Period: ________

Fermi Problems

3. Choose one of the Fermi problems from Problem 2 to solve as a group. Identify the questions, answers, and assumptions that are helpful. Work together to come up with a way you might solve your problem, adding more assumptions and related questions as necessary.

You will use a separate sheet to illustrate how your group interpreted the information to solve the Fermi problem for others to see. Be sure to include diagrams (or pictures), numbers, and words. **ELD.PI.6.2.Em, Ex, Br, ELD.PI.6.10.Em, Ex, Br**

You're invited to explore more.

4. Think about information that would be needed to work through this Fermi problem. Provide a plan for how you would solve the problem.

Some research has shown that it takes 10,000 hours of practice for a person to achieve the highest level of performance in any field — sports, music, art, chess, programming, etc. If you aspire to be a top performer in a field you love, such as Michael Jordan in basketball, Frida Khalo in painting, or Maya Angelou in poetry, how many years would it take you to meet that 10,000-hour benchmark if you start now? How old would you be?

Activity

Name: .... Date: .... Period: ....

Building Math Habits of Mind

Discuss:

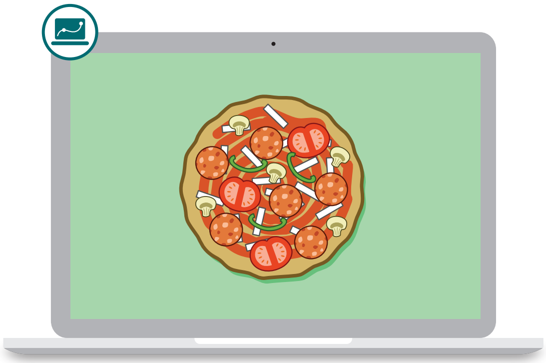
- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it. Not yet — Almost — I got it!	I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem. Not yet — Almost — I got it!
I can justify my thinking and ask questions to help me understand the thinking of others. Not yet — Almost — I got it!	I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed. Not yet — Almost — I got it!
I can select an appropriate tool to help me solve problems. Not yet — Almost — I got it!	I can communicate my thinking and solutions clearly to others. Not yet — Almost — I got it!
I can look for structure or patterns to help me solve problems. Not yet — Almost — I got it!	I can look for repeated calculations and other repeated steps to make generalizations. Not yet — Almost — I got it!

Unit 2
Lesson
1

Name: .... Date: .... Period: ....

Generalizing With Multiple Representations | Model the World | 6.RP.1, SMP.3



Pizza Maker

Let's create pizzas and describe how quantities relate to each other.

Warm-Up

1 You're making a pizza. What are some things you need?

Activity 1

Name: ________ Date: ________ Period: ________

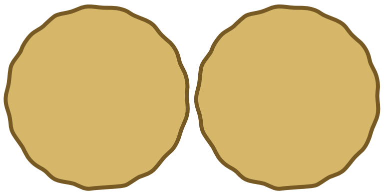
Your Pizza

2 You will use the Activity 1 Sheet and Cards to create your own pizza.

Add as many topping cards as you want to your pizza template. Then record how much sauce you used. (Each sauce card equals 1 ounce of sauce.)

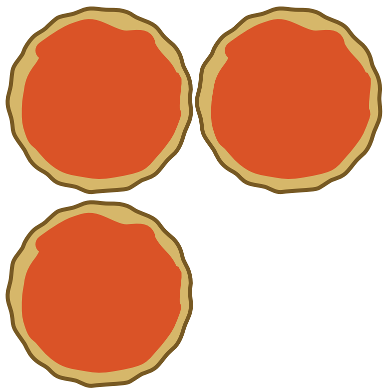
Sauce: ________ ounces

3 How much sauce do you need to make 2 of your pizzas?



4 A student used 14 ounces of cheese to make 2 pizzas.

How much cheese would they need for 3 pizzas?



Name: .... Date: .... Period: ....

Your Pizza (continued)

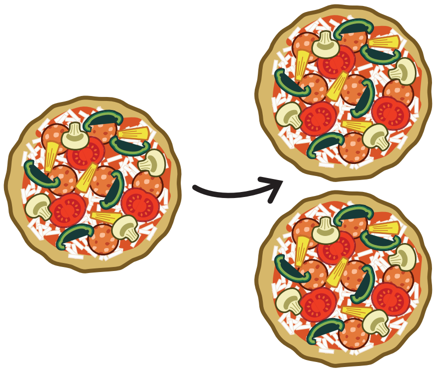
5 2 pizzas bake at 800°F.

What should the oven temperature be for 4 pizzas? Explain your thinking.

6 If we double the number of pizzas, it makes sense to double the amount of sauce, too.

Select *all* the items that it also makes sense to double.

- ☐ **A.** Amount of cheese
- ☐ **B.** Amount of each topping
- ☐ **C.** Oven temperature
- ☐ **D.** Bake time
- ☐ **E.** Total price of pizzas
- ☐ **F.** Delivery fee



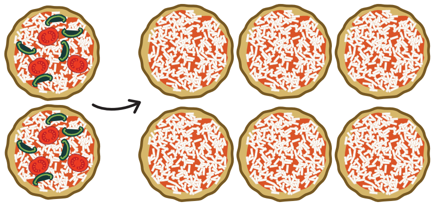
Activity 2

Name: ______ Date: ______ Period: ______

Ivan's Pizza

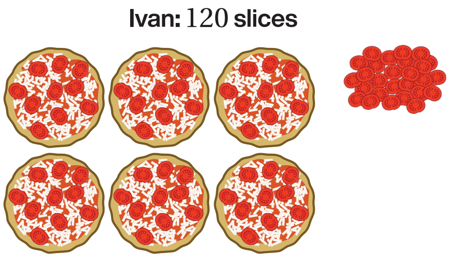
7 Ivan designed a vegetarian pizza.

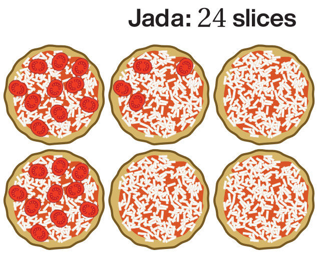
The table shows what he needs to make 2 pizzas. What does he need to make 6 pizzas?



Ingredients	2 Pizzas	6 Pizzas
Sauce	6 ounces	18 ounces
Tomato	20 slices	
Onion	30 slices	
Pepper	24 slices	

8 It takes 20 tomato slices to make 2 of Ivan's pizzas.



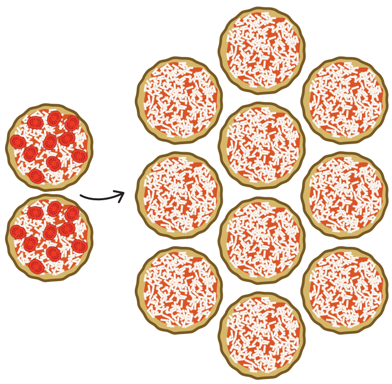


Ivan and Jada both made a mistake while making 6 of Ivan's pizzas. Circle your favorite mistake.

Ivan (120 tomato slices) Jada (24 tomato slices)

What would you recommend this person change about their work?

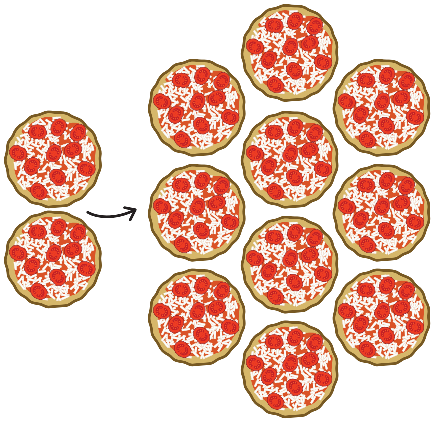
9 How many tomato slices does it take to make 10 of Ivan's pizzas?



10 Synthesis

Describe a strategy for solving problems like the previous one:

How many tomato slices does it take to make 10 *of Ivan's pizzas?*



13 Summary 2.01

You can represent the relationship between different quantities, such as the ingredients in a pizza recipe. Once you create a recipe with your favorite ingredients, you can use the relationship between those ingredients to make multiples of the same pizza.

For example, if your recipe uses 4 mushrooms to make 1 pizza, then you would need 12 mushrooms to make 3 pizzas.

Some quantities have no relationship between them. For example, if you bake one pizza at 800°F in a pizza oven, then you'll need to bake 2 pizzas at about the same temperature.

Name: .... Date: .... Period: ....

1. Pablo's Pizza Place wants to double the number of customers they have next year. Select *all* the quantities that are likely to double if they reach their goal.

   ☐ **A.** The number of pizzas they make.

   ☐ **B.** The hours the store is open.

   ☐ **C.** The amount of cheese they buy.

   ☐ **D.** The number of items on the menu.

   ☐ **E.** The amount of money they earn.

Problems 2–5: Pablo uses 2 bags of cheese and 3 bags of pepperoni to make 6 pizzas.

2. How many bags of cheese will he need to make 12 pizzas?

3. How many bags of cheese will he need to make 18 pizzas?

4. How many pizzas can Pablo make using 10 bags of cheese? Explain your thinking.

5. Pablo says he needs 9 bags of pepperoni to make 12 pepperoni pizzas. Is he correct? Explain your thinking.

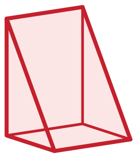
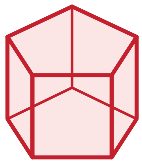
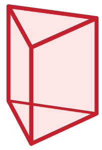
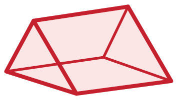
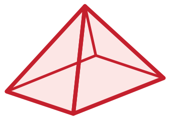
6. You can cook eggs to your liking by boiling them for different amounts of time. Pick your favorite type of boiled egg from the picture. How long would it take to cook 4 of these eggs in the same pot?

Practice
2.01

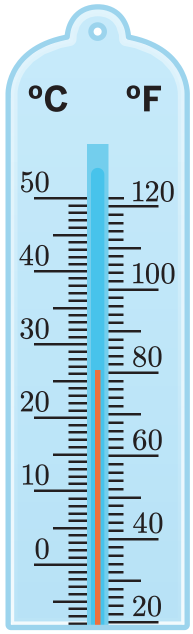
Name: ____________ Date: ____________ Period: ____________

Spiral Review

7. Select *all* of the triangular prisms.

- ☐ A. 
- ☐ B. 
- ☐ C. 
- ☐ D. 
- ☐ E. 

Problems 8–9: This thermometer shows the temperature in degrees Fahrenheit and Celsius.



8. What is the temperature in °C to the nearest whole number?

9. What is the temperature in °F to the nearest whole number?

10. A parallelogram has an area of 32 square units and a height of 4 units. What is the length of its base?

11. A parallelogram has an area of 12 square units and a base of $\frac{1}{2}$ units. What is the length of its height?

Name: ________ Date: ________ Period: ________

Generalizing With Multiple Representations | Model the World | 6.RP.1, SMP.6

Ratio Relationships

Let's describe how to compare pizza toppings.

Warm-Up

Evaluate each expression mentally.

1. $2 \cdot 15$

2. $4 \cdot 15$

3. $6 \cdot 15$

4. $12 \cdot 15$

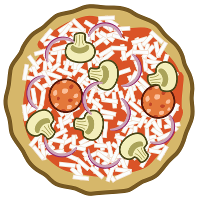
Activity 1

Name: ______ Date: ______ Period: ______

Pizza Ratios

Refer to the pizza with mushrooms, peppers and pepperoni.

5. Record the number of each pizza topping used to make this pizza.



Pepperoni: ______

Mushrooms: ______

Onions: ______

6. Complete each statement comparing the number of pepperonis to mushrooms.

a. The ratio of pepperoni to mushrooms is ______ to ______.

b. There are ______ pepperoni for every ______ mushrooms.

c. What is the ratio of mushrooms to pepperoni? ______ : ______

d. Write another sentence that describes the ratio of mushrooms to pepperoni.

ELD.PI.6.10a.Em, Ex, Br, ELD.PI.6.12a.Em, Ex, Br

7. Complete each statement comparing the number of onions to mushrooms.

a. The ratio of onions to mushrooms is ______ to ______.

b. There are ______ onions for every ______ mushrooms.

c. What is the ratio of mushrooms to onions? ______ : ______

d. Write another sentence that describes the ratio of mushrooms to onions.

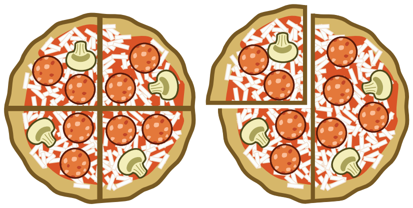
ELD.PI.6.10a.Em, Ex, Br, ELD.PI.6.12a.Em, Ex, Br

Activity 2

Name: ________ Date: ________ Period: ________

Two Truths and a Lie

8. Which statement is false? **ELD.PI.6.6.Em, Ex, Br**

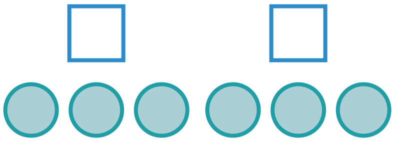


A. The ratio of mushrooms to pepperoni is $2 : 1$.

B. For every 4 mushrooms, there are 8 pepperoni.

C. The ratio of pepperoni to mushrooms is 12 to 6.

9. Which statement is false?

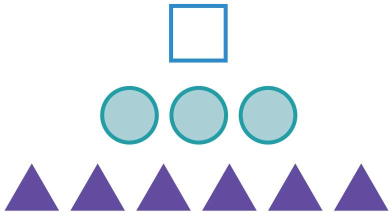


A. The ratio of circles to squares is $1 : 3$.

B. There are 2 squares for every 6 circles.

C. For every square, there are 3 circles.

10. Which statement is false?



A. For every circle, there are 2 triangles.

B. The ratio of circles to squares is 3 to 1.

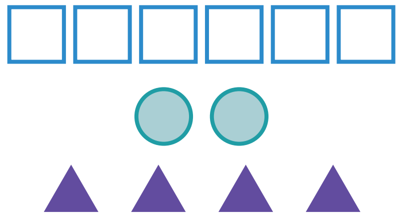
C. The ratio of squares to triangles is $1 : 2$.

Name: ________ Date: ________ Period: ________

Two Truths and a Lie (continued)

11. Here is another set of shapes. **ELD.PI.6.6.Em, Ex, Br, ELD.PI.6.10.Em, Ex, Br**

a Write three statements about these shapes: two that are true and one that is false.



b Trade your statements with a classmate. Which of their statements is false?

12. Now create your own challenge!

a Draw your own set of shapes.

b Write three statements about your drawing: two that are true and one that is false.

c Trade your challenge with a classmate. Which of their statements is false?

Synthesis

13. **a** Describe the ratio between these moons and stars in as many different ways as you can.

ELD.PI.6.12a.Em, Ex, Br

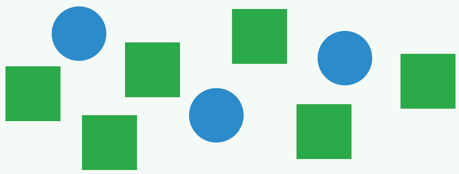
b Which way of describing a ratio is your favorite? Explain your reasoning.

Summary 2.02

One way to write a **ratio** is $a : b$ which means for every a of the first quantity, there are b of the second quantity.

There are many ways to describe a ratio in words.

For example, here are some ways you can describe the ratio between circles and squares in this diagram.



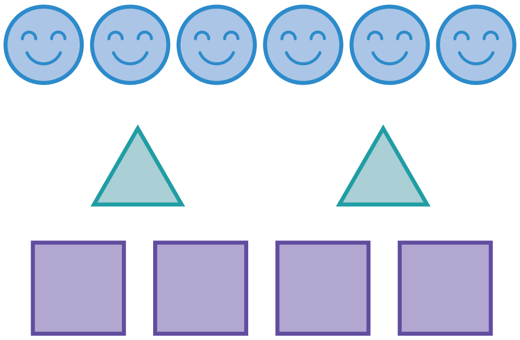
- The ratio of circles to squares is 3 to 6.
- There are 6 squares for every 3 circles.
- There are 2 times as many squares as there are circles.
- For every 1 circle, there are 2 squares.

ratio A relationship between two quantities, $a : b$, in which for every a of the first quantity, there are b of the second quantity.

Practice
2.02

Name: ________ Date: ________ Period: ________

Problems 1–4: Here is a set of smiley faces, triangles, and squares.



1. The ratio of smiley faces to triangles is

________ to ________.

2. The ratio of squares to triangles is

________ : ________.

3. For every 2 triangles, there are ________ squares.

4. Which statement is false?

A. The ratio of smiley faces to squares is $4 : 6$.

B. The ratio of squares to triangles is $4 : 2$.

C. There are 3 smiley faces for every 1 triangle.

Problems 5–8: There are 9 bananas, 4 apples, and 3 plums in a fruit basket.

5. The ratio of bananas to apples is ________ : ________.

6. The ratio of plums to apples is ________ to ________.

7. For every ________ apples, there are ________ plums.

8. For every 3 bananas, there is ________ plum.

Name: ______________________ Date: __________ Period: __________

Problems 9–10: There are 8 zebras, 10 giraffes, and 5 lions at a safari park.

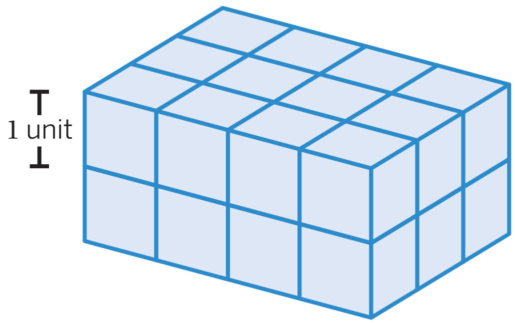
9. Write two ways to show a ratio of zebras to lions.

10. Write two way to show a ratio of giraffes to lions.

Spiral Review

Problems 11–12: Here is a rectangular prism.

11. Determine the volume of the prism. Show or explain your thinking.



12. Determine the surface area of the prism. Show or explain your thinking.

Unit 2
Lesson
3

Name: ________ Date: ________ Period: ________

Generalizing With Multiple Representations | Model the World | 6.RP.1, SMP.7

Rice Ratios

Let's explore ratios in recipes.

Warm-Up

Evaluate each expression mentally.

1. $4 \cdot 8$

2. $4 \cdot 10$

3. $4 \cdot 18$

4. $4 \cdot 30$

5. $4 \cdot 38$

Activity
1

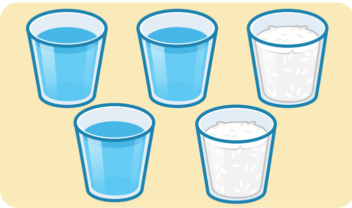
Name: ________ Date: ________ Period: ________

Rice Advice

6. Here are the cooking instructions for three different bags of basmati rice.
ELD.PI.6.6.Em, Ex, Br, ELD.PI.6.8.Em, Ex, Br, ELD.PI.6.10.Em, Ex, Br

Bag A

Boil 3 cups of water for every 2 cups of rice.



5 servings

Bag B

Boil $1\frac{1}{2}$ cups of water for every 1 cup of rice.

2.5 servings

Bag C

Boil 4 cups of water for every 2 cups of rice.

6 servings

a The ratios for Bag A and Bag B are called **equivalent ratios**. Why do you think they're called that? **ELD.PI.6.10a.Em, Ex, Br, ELD.PI.6.12a.Em, Ex, Br**

b **Discuss:** Could bag C also be called an equivalent ratio to Bag A or Bag B?

c Marco wants to follow the directions for Bag B, but he wants to use 3 cups of rice. What is the amount of water that he will need to use? Explain your thinking.

d What ratio of water to rice would Marco use to feed 15 people using Bag A recipe?

e Marco is hosting a party for 30 people (including himself). Which recipe could he use? Explain your thinking. **ELD.PI.6.10a.Em, Ex, Br**

Activity 2

Name: ____ Date: ____ Period: ____

Rice Around the World

Here are the recipes for four rice dishes from around the world. ELD.PI.6.6.Em, Ex, Br

7. Jamar invited a friend over for dinner. How much of each ingredient does he need to make 2 large bowls of jollof rice?

____ cups of rice

____ tablespoons of tomato paste

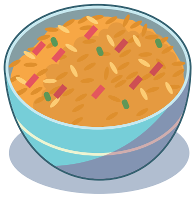
____ bell peppers

____ tomatoes

____ onions

____ cups of oil

Jollof Rice



Jollof rice is a tomato-based rice dish from Senegal, Ghana, and Nigeria.

Ingredients

Makes one large bowl

- 4 cups of rice
- 3 tablespoons of tomato paste
- 1 bell pepper
- 5 tomatoes
- 2 onions
- $\frac{1}{3}$ cup of oil

8. Nia wants to cook arroz con leche for 12 people.

a How much of each ingredient does she need?

____ cups of rice

____ cups of milk

____ cups of sugar

____ handfuls of raisins

____ cinnamon sticks

b Valeria wrote that Nia needs 9 cinnamon sticks. Why might Valeria think this? ELD.PI.6.6.Em, Ex, Br

c What advice would you give Valeria? ELD.PI.6.11.Em, Ex, Br

Arroz Con Leche

Arroz con leche is a creamy dessert from Mexico and Spain.

Ingredients

Serves 4 people

- 2 cups of rice
- 4 cups of milk
- $\frac{1}{3}$ cup of sugar
- 1 handful of raisins
- 1 cinnamon stick

Name: .... Date: .... Period: ....

Rice Around the World (continued)

9. Julian has 1 cup of sugar and wants to use all of it to make champorado.

a How much of the other ingredients does he need?

.... cups of rice

.... cups of water

.... cans of coconut milk

.... cups of cocoa powder

b How many people will Julian's champorado serve?

Champorado

Champorado is a chocolate rice porridge eaten in the Philippines.

Ingredients

Serves 4 people

- 1 cup of rice
- 4 cups of water
- 2 cans of coconut milk
- $\frac{1}{2}$ cup of cocoa powder
- 2 cups of sugar

10. Ariana says this recipe makes too much risotto.

a How much of each ingredient could she use to make a smaller amount of risotto?

.... cups of rice

.... cups of chicken broth

.... tablespoons of olive oil

.... tablespoons of butter

.... ounces of Parmesan cheese

b How many people will this serve?

Risotto

Risotto is an Italian rice dish that uses broth to create a creamy texture.

Ingredients

Serves 8 people

- 3 cups of rice
- 10 cups of chicken broth
- 4 tablespoons of olive oil
- 2 tablespoons of butter
- 8 ounces of Parmesan cheese

Synthesis

11. The cooking instructions on Bag A and Bag B call for equivalent ratios of water to rice.

Bag A

Boil 4 cups of water for every 2 cups of rice.

Bag B

Boil 2 cups of water for every 1 cup of rice.

a Explain what equivalent ratios are in your own words. **ELD.PI.6.12.Em, Ex, Br**

b Create a new ratio of water to rice that is equivalent to the ratios for Bag A and Bag B.

Summary 2.03

Each recipe calls for a specific ratio of ingredients but you can make more or less of the recipe if you use **equivalent ratios**. You can multiply or divide the values in a ratio by the same number to get the values in an equivalent ratio.

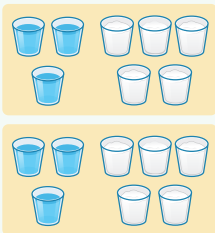
Original Recipe

3 to 2

Multiply

Divide

Doubled Recipe



6 to 4

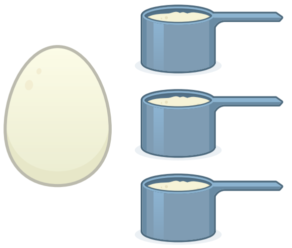
These are equivalent ratios because you can multiply the values in the first ratio by 2 to get the second ratio. You also can divide the values in the second ratio by 2 to get the first ratio.

equivalent ratios Two ratios are equivalent if you can multiply each of the values in the first ratio by the same number to get the values in the second ratio.

Practice
2.03

Name: ______________ Date: ________ Period: ________

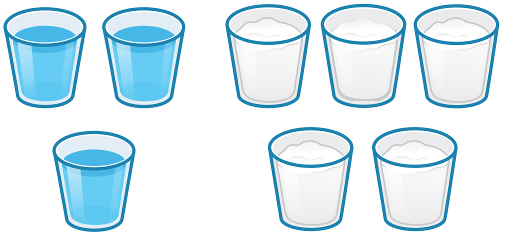
Problems 1–2: There are many recipes for pasta. Some suggest a ratio of 1 egg for every 3 ounces of flour.



1. Draw a picture that shows how many ounces of flour you would need for 2 eggs. Then write the ratio of eggs to flour that represents your drawing.

2. Write two equivalent ratios that represent your drawing from Problem 1.

3. A bakery uses a ratio of 3 cups of water for every 5 cups of flour to bake bread. List 2 other ratios of water to flour they could use to bake the same bread.



Problems 4–7: Koharu's pie dough recipe uses 6 ounces of flour, 4 ounces of butter, and 2 ounces of water. Complete the sentences to describe the ratios in her recipe.

4. For every 2 ounces of ______________, there are 6 ounces of ______________.

5. The ratio of ______________ to ______________ is $6 : 2$.

6. The ratio of ______________ to ______________ is $2 : 3$.

7. The ratio of ______________ to ______________ is $3 : 2$.

Practice
2.03

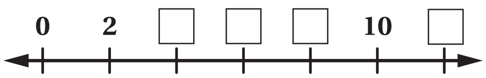
Name: .... Date: .... Period: ....

8. Koharu made a new batch of pie dough with 3 ounces of flour, 2 ounces of butter, and 1 ounce of water. Will her pie dough taste the same as the original recipe? Explain your thinking.

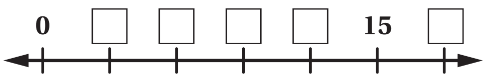
Spiral Review

Problems 9–10: Fill in the blanks on each number line.

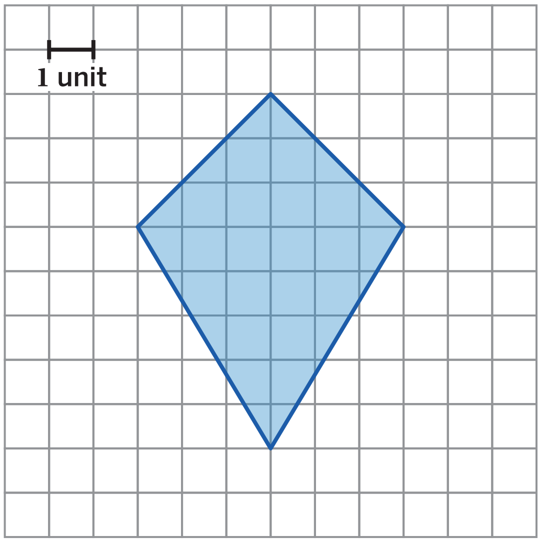
9.



10.



11. Determine the area of this polygon. Show or explain your thinking.



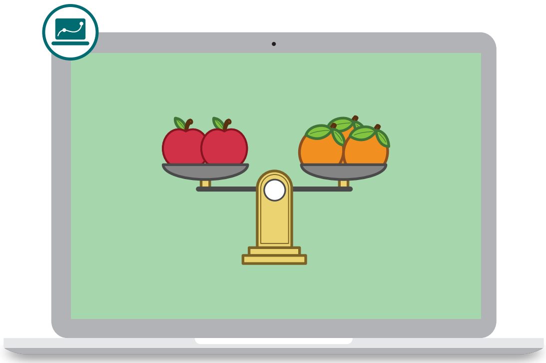
12. Marco makes trail mix using 3 bags of chocolate chips, 4 bags of walnuts, and 2 bags of peanuts. This recipe makes 12 cups of trail mix.

How many bags of walnuts will he need to make 6 cups of trail mix?

Name: .... Date: .... Period: ....

Generalizing With Multiple Representations | Patterns Inside Numbers | Model the World

6.RP.3.a, SMP.1, SMP.3, SMP.7

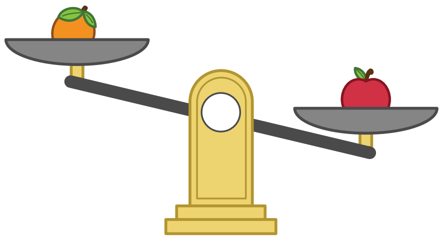


Fruit Lab

Let's investigate equivalent ratios by balancing fruit on scales.

Warm-Up

1 Let's watch apples and oranges balance on a scale.



a When the scale balances, record the values in the table.

b Find as many ways as you can to balance the scale.

Number of Oranges	Number of Apples

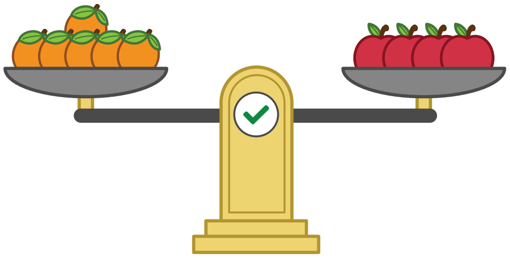
Activity
1

Name: .... Date: .... Period: ....

Apples to Oranges

2 Here is Victor's **ratio table** from the Warm-Up. What do you notice about the table? What do you wonder?

Number of Oranges	Number of Apples
15	10
3	2
6	4



I notice:

I wonder:

3 Write another equivalent ratio in the last row. Try to find one that you think no one else will think of.

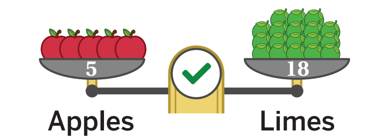
Activity

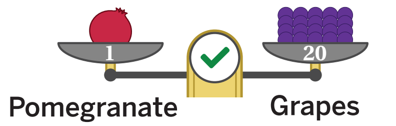
2

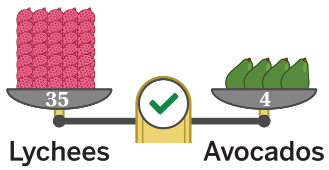
Name: ________________ Date: ________ Period: ________

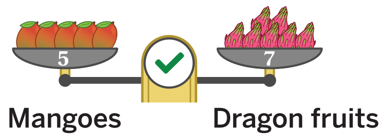
Fruit Lab

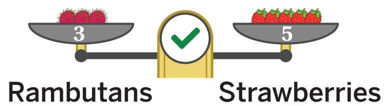
4 You will use the Activity 2 Sheet to complete this activity. Choose a pair of fruits to see how they balance. Then record several equivalent ratios for that pair of fruits. Repeat with different combinations of fruits.

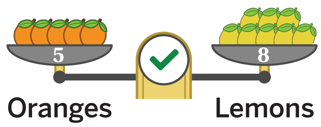


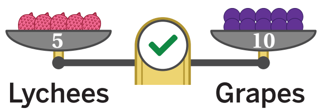


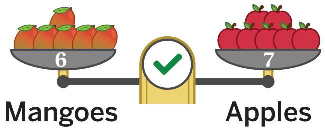




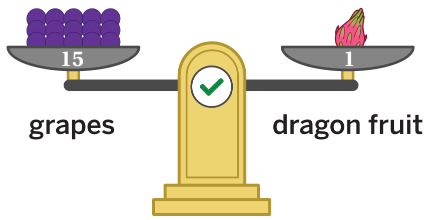








5 Ella knows that 15 grapes balance with 1 dragon fruit. She says 16 grapes will balance with 2 dragon fruits. Will this 16 : 2 ratio balance the scale? Circle one.



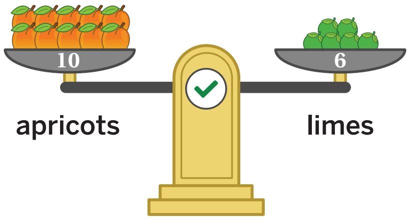
Yes No I'm not sure

Explain your thinking.

Name: .... Date: .... Period: ....

Fruit Lab (continued)

6 The scale balances with a ratio of 10 apricots to 6 limes. Select *all* of the equivalent ratios.



- ☐ **A.** 20 apricots to 16 limes
- ☐ **B.** 50 apricots to 30 limes
- ☐ **C.** 7 apricots to 3 limes
- ☐ **D.** 5 apricots to 3 limes
- ☐ **E.** 11 apricots to 7 limes

7 The table shows some ratios of limes to lychees that balance the scale. Dyani says that 22 limes will balance with 55 lychees. Will the 22 : 55 ratio balance? Circle one.

Number of Limes	Number of Lychees
2	5
20	50

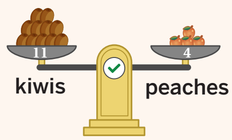
Yes No I'm not sure

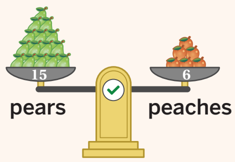
Explain your thinking.

You're invited to explore more.

8 A ratio of 11 kiwis : 4 peaches balances. So does a ratio of 15 pears : 6 peaches.

Write a ratio of kiwis to pears that would balance. Explain your thinking.

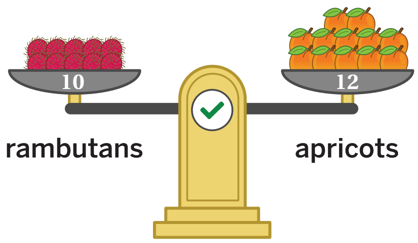




9 Synthesis

Discuss: When you know a ratio balances a scale, how can you create equivalent ratios that also balance the scale?

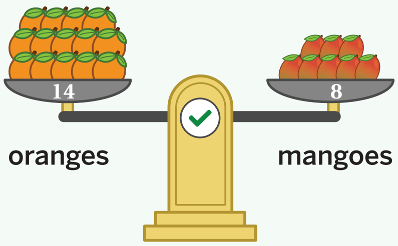
Use the example if it helps with your thinking.



12 Summary 2.04

We can use balance scales to help us understand equivalent ratios. When both quantities in a ratio are multiplied or divided by the same amount, the ratio relationship remains the same, and the scale stays balanced.

For example, the ratio of oranges to mangoes on this scale is $14 : 8$. The **ratio table** represents different numbers of oranges and mangoes that are in equivalent ratios and will also balance the scale.



Number of Oranges	Number of Mangoes
14	8
7	4
21	12

$\div 2$

$\cdot 3$

ratio table A table of values organized in columns and rows that contains equivalent ratios.

Practice
2.04

Name: .... Date: .... Period: ....

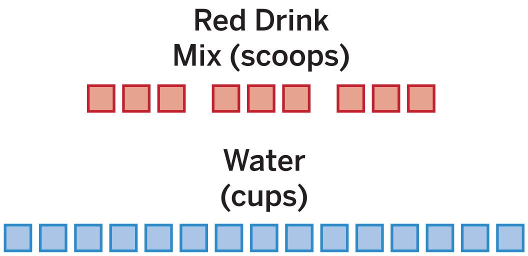
Problems 1–3: A package of red drink mix says to combine 3 scoops of red drink mix and 5 cups of water.

1. Complete the table with several ratios of red drink mix to water that are equivalent to the package instructions.

Red Drink Mix (scoops)	Water (cups)
3	5

2. Choose one of your ratios and explain how you know it's equivalent. Draw a diagram if it helps with your thinking.

3. Jaylin drew this diagram for one of the ratios. Will this mix taste the same as the original? Show or explain your thinking.



4. Select *all* of the ratios that are equivalent to 4 : 5.

☐ **A.** 3 : 4 ☐ **B.** 8 : 10

☐ **C.** 1 : 2.5 ☐ **D.** 9 : 10

☐ **E.** 20 : 25

5. Write a different ratio that is equivalent to 4 : 5.

6. Koharu has a new recipe for pie dough. This recipe needs 12 ounces of flour, 10 ounces of butter, and 3 ounces of cold water. Which ratio of flour to butter could he use if he wanted to make a smaller pie that tastes the same?

☐ **A.** 6 : 6 ☐ **B.** 4 : 1

☐ **C.** 6 : 5 ☐ **D.** 1 : 4

Name: ____________ Date: ________ Period: ________

Problems 7–8: You can make a certain color of green paint by mixing 10 ounces of green paint with 2 gallons of white paint.

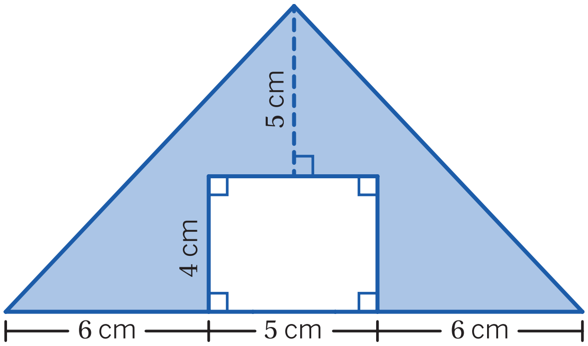
7. Draw a diagram to represent this ratio.

8. Select *all* the true statements.

- ☐ **A.** For every 5 ounces of green paint, you need 1 gallon of white paint.
- ☐ **B.** The ratio of green paint to white paint is $1 : 5$.
- ☐ **C.** For every gallon of white paint, you need 5 ounces of green paint.
- ☐ **D.** For every ounce of green paint, you need 5 gallons of white paint.
- ☐ **E.** The ratio of white paint to green paint is $10 : 2$.

Spiral Review

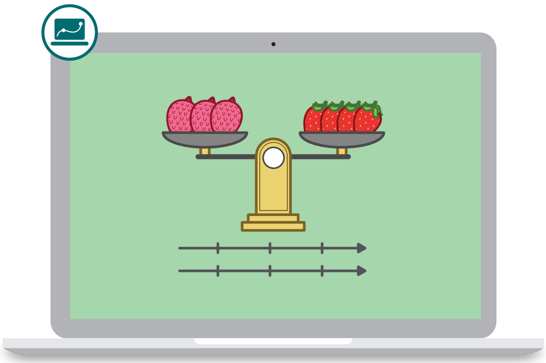
9. Determine the area of the shaded region. Explain your thinking.



Name: ________________ Date: ________ Period: ________

Fraction Relationships | Generalizing With Multiple Representations

6.RP.3, 6.RP.3.a, SMP.3, SMP.6

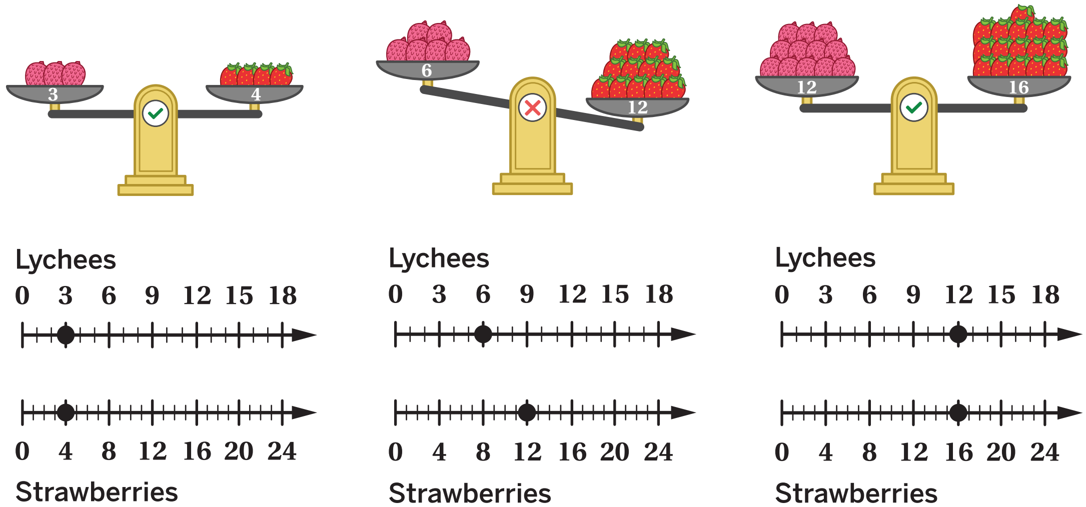


Balancing Act

Let's use double number lines to represent equivalent ratios.

Warm-Up

1 Here are some **double number lines** that represent lychees and strawberries on a scale. What do you notice? What do you wonder?



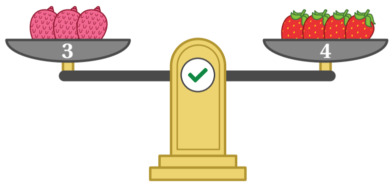
I notice . . .

I wonder . . .

Name: .... Date: .... Period: ....

Double Number Lines

2 This scale balances with a ratio of 3 lychees to 4 strawberries.



a Record several equivalent ratios in the table. Try to find ones that none of your classmates will.

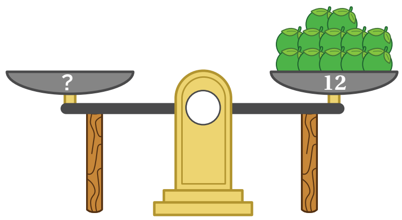
Number of Lychees	Number of Strawberries
3	4

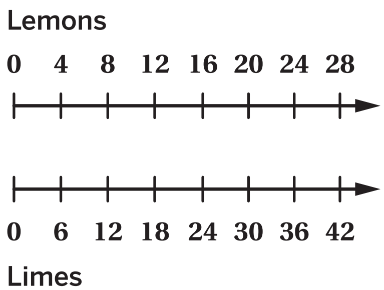
b **Discuss:** How can you use the double number line to determine the ratios that balance?

3 This scale balances with a ratio of 4 lemons to 6 limes.

How many lemons will balance with 12 limes?

Use the double number line if it helps to show your thinking.



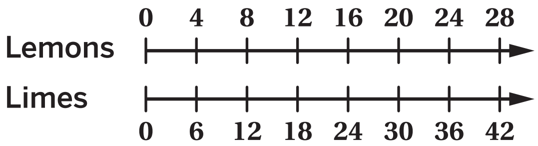


Activity 1

Name: .... Date: .... Period: ....

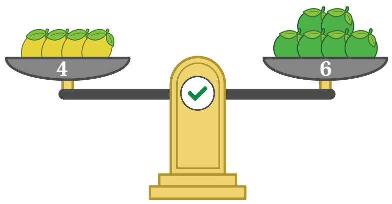
Double Number Lines (continued)

4 Complete the table. Use the double number line if it helps to show your thinking.



Number of Lemons	Number of Limes
4	6
20	
	42
32	

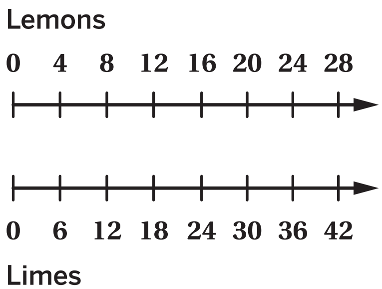
5 Kiri says that 16 lemons will balance with 24 limes. Lola says that 36 lemons will balance with 24 limes.



Whose thinking is correct? Circle one.

Kiri's Lola's Both Neither

Explain your thinking.

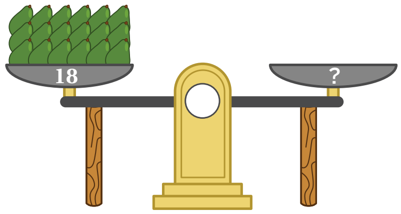


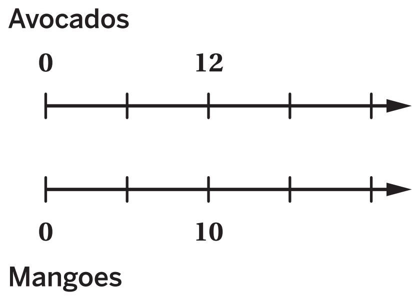
Name: .... Date: .... Period: ....

More Double Number Lines

6 A ratio of 12 avocados : 10 mangoes balances this scale. How many mangoes will balance with 18 avocados?

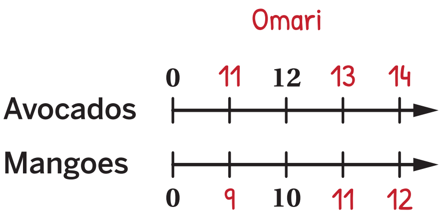
Use the double number line if it helps to show your thinking.

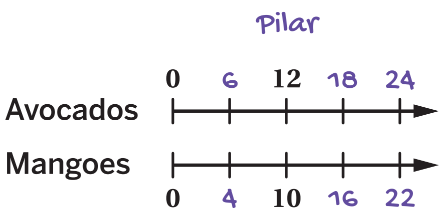




7 Omari and Pilar both labeled the rest of the diagram from Problem 6. They each made a mistake.

a Circle the person who made your favorite mistake.





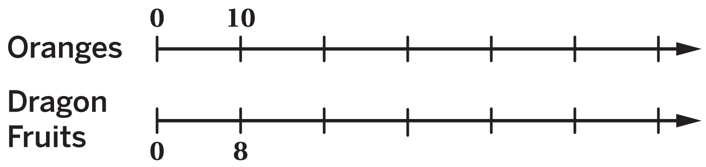
b What is something they did well? What would you recommend this person change about their work?

Name: .... Date: .... Period: ....

More Double Number Lines (continued)

8 10 oranges balance with 8 dragon fruits.

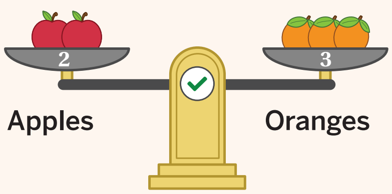
Complete the table. Use the double number line if it helps to show your thinking.



Number of Oranges	Number of Dragon Fruits
10	8
	24
5	
	20

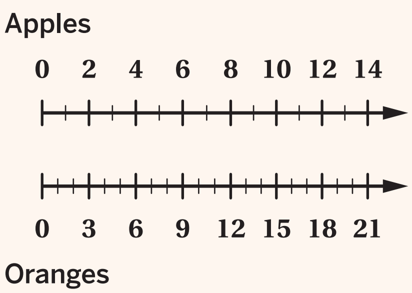
You're invited to explore more.

9 2 apples balance with 3 oranges.



a. How many oranges will balance with 101 apples?

b. How many apples will balance with $25\frac{1}{2}$ oranges?

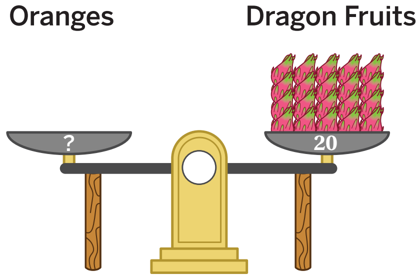


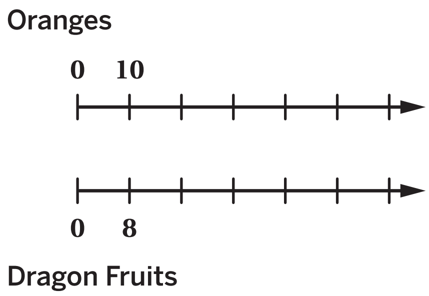
c. Create your own problem involving equivalent ratios of apples and oranges. Then trade problems with a classmate and solve them.

10 Synthesis

Discuss: How can you use a double number line to solve problems with ratios?

Use the example if it helps to show your thinking.

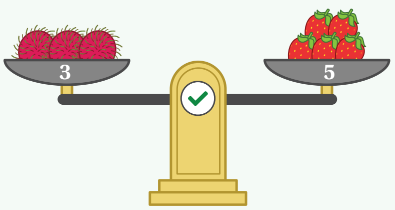


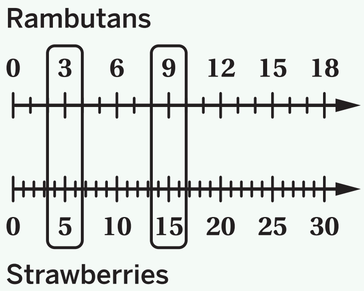


13 Summary 2.05

A **double number line** is another way to represent equivalent ratios. Each double number line is made up of a pair of parallel number lines. The tick marks are labeled so that the numbers that line up vertically make equivalent ratios.

For example, if a ratio of 3 rambutans to 5 strawberries will balance on a scale, you can use a double number line to determine how many strawberries will balance with 9 rambutans.





To represent a ratio of 3 : 5, you begin with 3 and 5 in the same location on each number line and then count up by units of 3 on one number line and units of 5 on the other line. In this example, you can determine that it will take 15 strawberries to balance 9 rambutans. Each pair of matching values represents an equivalent ratio to 3 : 5.

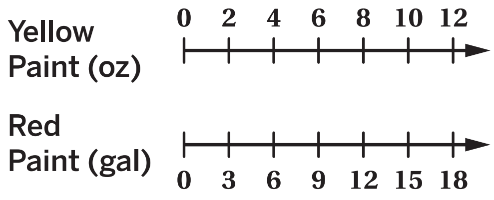
double number line A pair of parallel number lines showing equivalent ratios. The tick marks are labeled so that the marks that line up vertically are equivalent ratios.

Practice
2.05

Name: .... Date: .... Period: ....

1. You can make a certain orange paint by mixing 2 ounces of yellow paint with 3 gallons of red paint.

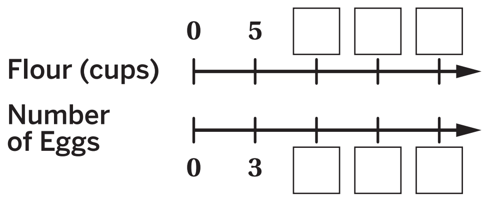
List two other combinations of yellow paint and red paint that can create this shade of orange.



Yellow Paint (oz)	Red Paint (gal)

Problems 2–5: This double number line represents the amount of flour and eggs needed for a cookie recipe.

2. Complete the double number line.



3. What is the ratio of cups of flour to number of eggs?

4. How much flour do you need for 12 eggs?

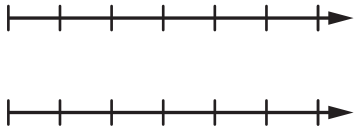
5. How many eggs do you need for 15 cups of flour?

Practice
2.05

Name: .... Date: .... Period: ....

Problems 6–8: Metropolis Elementary recommends 2 adults join for every 15 students on a field trip.

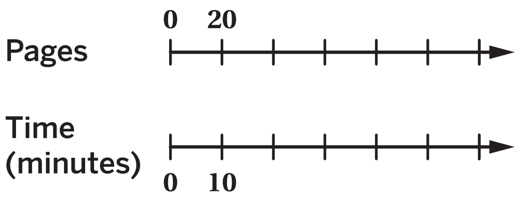
6. Draw a double number line to represent this situation.



7. How many adults need to go on a field trip with 75 students?

8. How many adults need to go on a trip with 50 students? Explain your thinking.

9. Neo can read 20 pages in 10 minutes. Use the double number line to determine how many pages Neo can read in 1 hour.



Spiral Review

10. Diego estimates that there will need to be 3 pizzas for every 7 kids at his party. Select *all* the statements that represent this ratio.

- ☐ **A.** The ratio of kids to pizzas is 7 : 3.
- ☐ **B.** The ratio of pizzas to kids is 3 : 7.
- ☐ **C.** The ratio of kids to pizzas is 3 : 7.
- ☐ **D.** The ratio of pizzas to kids is 7 : 3.
- ☐ **E.** For every 7 kids, there needs to be 3 pizzas.

Name: ______ Date: ______ Period: ______

Fraction Relationships Model the World 6.RP.3, SMP.8

Product Prices

Let’s calculate the prices of different amounts of items.

Warm-Up

Juana asks the store clerk if she can buy just one tomato. The store clerk says she can for $0.50.

1. Does this seem fair? Explain your thinking.

Activity
1

Name: ______ Date: ______ Period: ______

How Much for One?

You will use a set of product cards to complete this activity.

Imagine you're at a store buying the products on Cards A, B, and C for your family.

2. Sort the products on Cards A, B, and C from *least* expensive to *most* expensive.

Least Expensive **Most Expensive**

3. What strategy did you use to sort the cards?

Mayra and Roberto were comparing the prices on the cards. **ELD.PI.6.6.Em, Ex, Br, ELD.PI.6.11.Em, Ex, Br**

Mayra ordered the cards:

C	A	B

Least Expensive **Most Expensive**

Roberto ordered the cards:

C	B	A

Least Expensive **Most Expensive**

4. Explain what Mayra might have been thinking when she ordered the cards.

5. Explain how Roberto's order represents the price for each item.

Activity
2

Name: .... Date: .... Period: ....

How Much for Many?

Now imagine you're browsing through the aisles looking at the products on Cards D, E, and F. **ELD.PI.6.1.Em, Ex, Br, ELD.PI.6.6.Em, Ex, Br, ELD.PI.6.11.Em, Ex, Br**

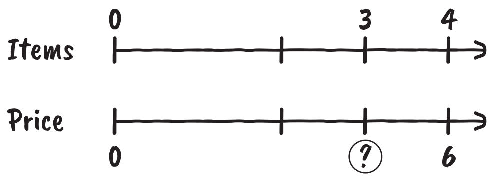
6. Answer the questions on Cards D, E, and F. Show or explain your thinking.

Card D	Card E	Card F

7. Compare your answers and strategies with a classmate.

Discuss: How are your strategies alike? How are they different?

8. Mariana drew this double number line to help her answer the question on a card.



Which card do you think she was working on? Circle one.

Card D Card E Card F

Explain your thinking.

9. Look through Cards G–L. Which question would you answer next, and why?

Name: .... Date: .... Period: ....

How Much for Many? (continued)

10. Select *three* cards from Cards G–L to answer. Show or explain your thinking.

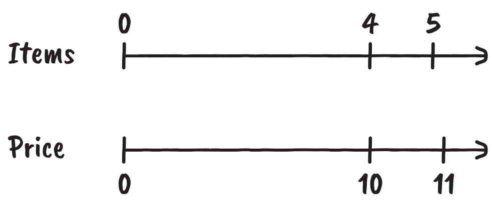
Card	Card	Card ....

Two students made a mistake when solving Card K.

ELD.PI.6.6.Em, Ex, Br, ELD.PI.6.11.Em, Ex, Br

11. Circle the student who made your favorite mistake.

Mariana



Naoki

$\$10 \div 5 = \$2$

$\$2 \cdot 4 = \$8$

12. What did this student do well?

13. What would you recommend this student change in their work?

You're invited to explore more.

14. You have \$20 to spend on a mix of products from Cards A–L. You can purchase up to the number of products shown on each card.

Create a list of items that will cost as close to \$20 as you can get, without going over.

Synthesis

15. Describe a strategy for calculating the cost of 5 items when given the cost for 2 items.

$7

Summary 2.06

When you're spending your money, you can use the price per item to decide if something is a good deal, or to determine how much different amounts of a product will cost. The word *per* means "for each."

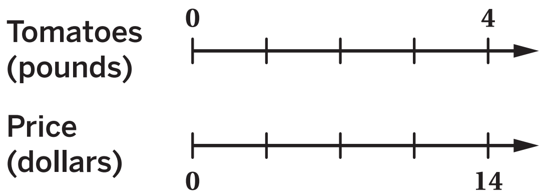
For example, if 3 bottles of juice cost $\$7.50$, you can determine how much 8 bottles of juice will cost.

- The price per bottle is $\$2.50$, because $7.50 \div 3 = 2.50$.
- The price of 8 bottles is 8 times the price per bottle, or $8 \cdot 2.50 = \$20$.

$7.50

Name: .... Date: .... Period: ....

Problems 1–4: The double number line shows that 4 pounds of tomatoes cost \$14.



1. Draw and label tick marks showing the price of 1 pound of tomatoes.

2. Draw and label tick marks showing the price of 2 pounds of tomatoes.

3. Draw and label tick marks showing the price of 3 pounds of tomatoes.

4. Ariel needs 6 pounds of tomatoes to make sauce. How much would that cost? Show your work.

Problems 5–7: Calculate the price for each item.

5. 12 eggs for \$3

6. 3 bags of rice for \$7.50

7. 10 apples for \$3.50

Problems 8–10: Use the prices for each item that you calculated in Problems 5–7 to determine the total cost of these items.

8. 6 eggs

9. 4 bags of rice

10. 7 apples

Name: ______________ Date: ________ Period: ________

Problems 11–14: A package of 3 bags of Brand A fish crackers costs $\$2.40$.

11. Draw a double number line to represent this situation.

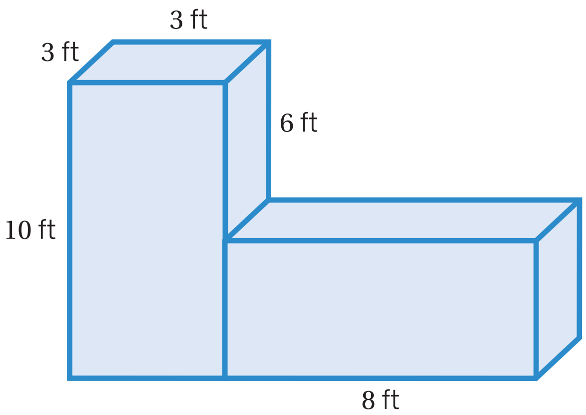
12. How much would 18 bags of fish crackers cost?

13. How many bags of fish crackers can Noaki buy for $\$10$?

14. A package of 5 bags of Brand B fish crackers costs $\$3.80$. Which brand would you recommend Noaki buy if he wants to buy 18 bags of fish crackers?

Spiral Review

15. This figure is made up of two boxes. What is the total volume of sand you would need to fill both boxes?



Name: ________ Date: ________ Period: ________

Generalizing With Multiple Representations | Patterns Inside Numbers | Fraction Relationships | Model the World | Number Line Understanding | Ratios, Percents, and Proportional Relationships

6.RP.1, 6.RP.3, 6.RP.3.a, SMP.2

Practice Day 1

Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Rainy Roof

1. Solution: ________ ounces of water

2. Solution: ________ ounces of water

3. Circle one: Yes or No

Explanation:

4. Solution: ________ hours

Use the table if it helps with your thinking.

Time (min)	Water in Bucket (oz)
5	2

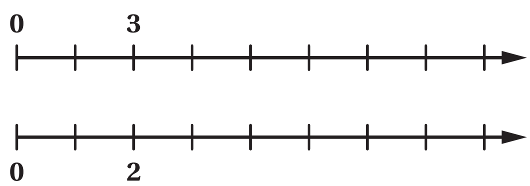
Task B: Potato Price

1. Solution: ________ for 1 pound

2.

____

____



3. Circle one: Yes or No

Explanation:

4. Circle one: Yes or No

Explanation:

Name: .... Date: .... Period: ....

Practice Day 1

Task C: Not-So-Simple Symbols

1. **a** The ratio of squares to circles is .... : 2.

b For every 1 square, there are .... triangles.

2. Circle *all* equivalent ratios: A B C D E

3. Your drawing:

4. Your ratios:

Task D: Baking Biscuits

1. Solution: .... cup(s) of butter

.... cup(s) of milk

2. Solution: .... cup(s) of butter

.... cup(s) of milk

3. Circle one: Yes or No

Explanation:

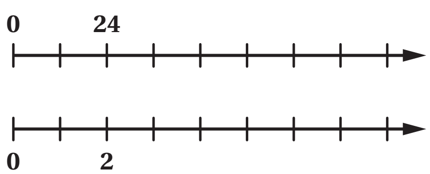
4. Solution: .... cup(s) of flour

.... cup(s) of butter

Task E: Museum Monitors

1. ....

....



2. Solution: .... adults

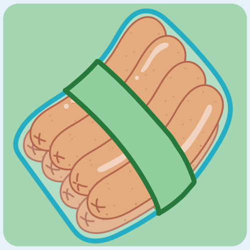
3. Your question:

4. Your answer:

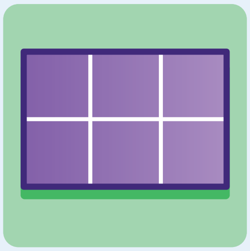
Sub-Unit

2

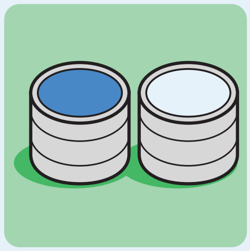
Common Factors and Multiples



Lesson 7
Common Multiples



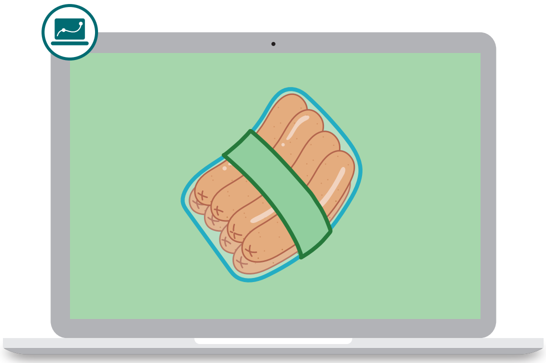
Lesson 8
Common Factors



Lesson 9
Mixing Paint, Part 1

Name: ________ Date: ________ Period: ________

Patterns Inside Numbers 6.NS.4, SMP.1, SMP.3



Common Multiples

Let's learn more about multiples.

Warm-Up

1 Abdel is grilling tofu dogs for his friends. His favorite tofu dogs come in packs of 8. His favorite buns come in packs of 6.

What advice would you give to Abdel on how many packs to purchase?

2 Abdel bought 48 tofu dogs and 48 buns.

What else could he buy if he only bought whole packs? Select *all* that apply.

- ☐ **A.** 2 tofu dogs and 2 buns
- ☐ **B.** 12 tofu dogs and 12 buns
- ☐ **C.** 24 tofu dogs and 24 buns
- ☐ **D.** 32 tofu dogs and 32 buns
- ☐ **E.** 72 tofu dogs and 72 buns

Activity 1

Name: ________ Date: ________ Period: ________

Least Common Multiples

3 24 and 48 are **common multiples** of 8 and 6.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

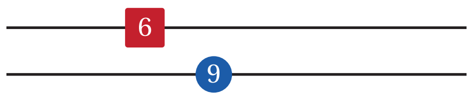
a Take a look at these other common multiples.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

b List some common multiples of 6 and 9.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Name: .... Date: .... Period: ....

Least Common Multiples (continued)

4 The **least common multiple (LCM)** is the smallest number that is a common multiple of two numbers.

Multiples of 6: 6, 12, 18, 24, 30, 36, . . .

Multiples of 9: 9, 18, 27, 36, 45, 54, . . .

What is the least common multiple of 6 and 9?

5 **a** Circle the multiples of 8 and 10.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

b Determine the least common multiple of 8 and 10

6 Determine the least common multiple of 4 and 12.

Explain your thinking,

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Activity 2

Name: ______ Date: ______ Period: ______

More Multiples

7 Determine the least common multiple for each pair of numbers in the table. Use the grid if it helps with your thinking.

Numbers	Least Common Multiple
10 and 15	
10 and 4	
6 and 4	
6 and 7	
28 and 7	
9 and 12	

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

8 *To find the least common multiple of two numbers, just multiply them together.*

Is this statement always, sometimes, or never true? Circle one.

Always Sometimes Never

Explain your thinking.

9 *The least common multiple of two numbers is the larger number.*

Is this statement always, sometimes, or never true? Circle one.

Always Sometimes Never

Explain your thinking.

Activity 2

Name: .... Date: .... Period: ....

More Multiples (continued)

10 Abdel wants to get some dessert to go with his tofu dogs. He wants to buy an equal number of FreezyPops and DinoPops.

- FreezyPops come in packs of 9.
- DinoPops come in packs of 12.

a What is the least number of packages he will need to buy to have an equal number of each dessert?

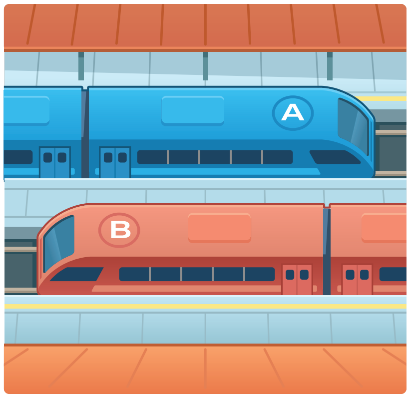
FreezyPops: .... packages

DinoPops: .... packages

b **Discuss:** How did the LCM of 9 and 12 help you calculate the number of packages Abdel needs to buy?

11 Train A leaves the station every 5 hours, and Train B leaves the station every 8 hours.

If both trains leave the station at the same time, after how many hours will they be at the station together?



You're invited to explore more.

12 What are two numbers that have a least common multiple of 100?

List as many pairs as you can.

13 Synthesis

Describe a strategy for determining the least common multiple of two numbers, such as 8 and 12.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

16 Summary 2.07

Some **common multiples** of 2 and 3 are 6, 12, and 18.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

The **least common multiple (LCM)** of 2 and 3 is 6. It's helpful to determine the LCM when solving problems like how many packages of tofu dogs and buns you need to buy or how often two trains stop at the same station. For example, if Train A stops at the station every 2 hours, and Train B stops at the station every 3 hours, then both trains will be at the station every 6 hours.

Sometimes one of the numbers itself is a common multiple of two given numbers. For example, 12 is a common multiple of 6 and 12.

Sometimes the least common multiple of two numbers is equal to their product. For example, 15 is the least common multiple of 3 and 5.

common multiple When two numbers have the same multiple, we call that a common multiple.

least common multiple (LCM) The smallest number that is a common multiple of two numbers.

Practice
2.07

Name: ______________________ Date: __________ Period: __________

Problems 1–3: Here is a grid.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1. Circle the multiples of 12.

2. Draw a square around the multiples of 15.

3. What is the least common multiple of 12 and 15?

Problems 4–5: Lucia wants to buy the same number of cups and plates, but cups are sold in packs of 8 and plates are sold in packs of 12.

4. List *at least* three combinations of packs of cups and plates that Lucia could buy so she has the same number of each.

5. What is the fewest number of cups she could buy while also buying an equal number of plates?

6. A green light blinks every 4 seconds and a yellow light blinks every 5 seconds. When will both lights first blink at the same time?

7. A red light blinks every 12 seconds and a blue light blinks every 9 seconds. When will both lights first blink at the same time?

Name: ______ Date: ______ Period: ______

8. For Problems 8–9: Here are the numbers 4, 6, 8, 12, 18, and 24.

- ☐ **A.** 4 and 6
- ☐ **B.** 4 and 8
- ☐ **C.** 6 and 8
- ☐ **D.** 6 and 18
- ☐ **E.** 12 and 24.

9. What is the least common multiple of the pairs of numbers you chose?

10. Here is a number chart.

- Multiples of 5 will be painted red,
- Multiples of 6 will be painted blue,
- Multiples of 15 will be painted green.

How many numbers will be painted with all three colors?

Explain your thinking.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Spiral Review

11. 10 bananas cost $4.50. What is the price for one banana?

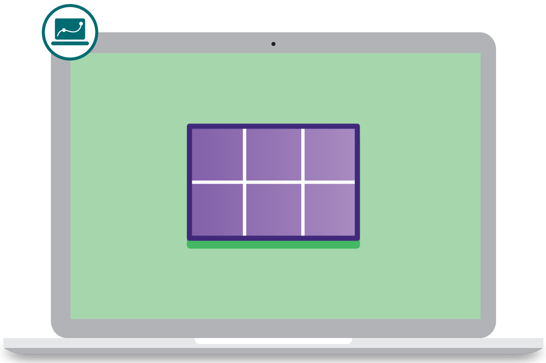
12. If 20 cookies cost $5, what is the price for 25 cookies?

13. 4.5 pounds of potatoes cost $12. What is the price for 3 pounds of potatoes?

Unit 2
Lesson
8

Name: ______ Date: ______ Period: ______

Patterns Inside Numbers 6.NS.4, SMP.3, SMP.6



Common Factors

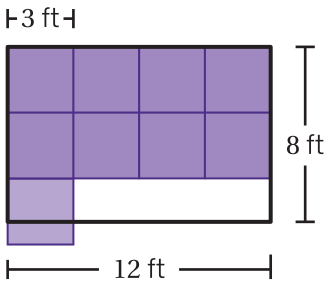
Let's explore factors.

Warm-Up

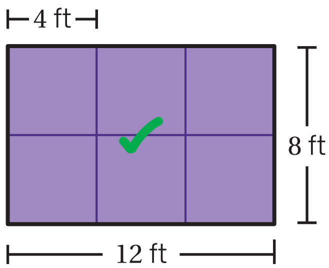
1 Here's the floor of a 8-by-12 foot room.

There are square tiles available in different sizes. For example, A 4-by-4 foot square will tile the floor of this 8-by-12 foot room.

3-by-3 foot tiles



4-by-4 foot tiles



a Find other square sizes that tile the room.

b **Discuss:** What does it mean to tile?

Activity 1

Name: ________ Date: ________ Period: ________

Greatest Common Factors

2 A 3-by-3 centimeter square will tile this 12-by-18 centimeter rectangle. This means that 3 is a **common factor** of 12 and 18.

Select *all* the other common factors of 12 and 18.

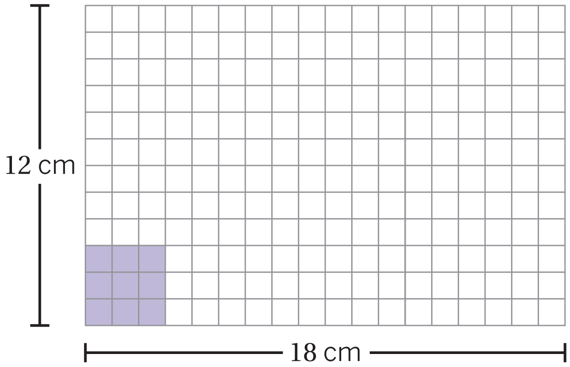
☐ **A.** 1

☐ **B.** 2

☐ **C.** 4

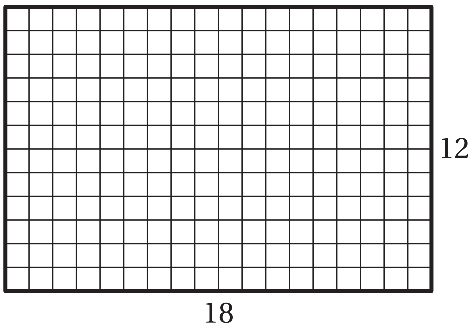
☐ **D.** 6

☐ **E.** 12



3 The **greatest common factor (GCF)** is the greatest number that is a common factor of two numbers.

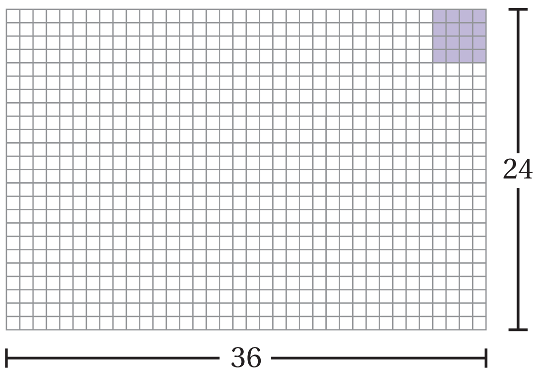
What is the greatest common factor of 12 and 18?



4 4 is a common factor of 24 and 36.

a Determine as many common factors of 24 and 36 as you can.

b Determine the greatest common factor of 24 and 36.



Activity 1

Name: .... Date: .... Period: ....

Greatest Common Factors (continued)

5 Here is Tameeka's strategy for determining the greatest common factor of 24 and 36.

Tameeka

Factors of 24: 1, 2, 3, 4, 6, 8, . . .

Factors of 36: 1, 2, 3, 4, 6 . . .

6 is the GCF.

a What is something Tameeka did well?

b What change would you recommend for Tameeka's work?

6 What is the greatest common factor of 27 and 36? Explain your thinking.

Activity 2

Name: ______ Date: ______ Period: ______

Common Factors and Multiples

7 The greatest common factor of 20 and 15 is 5.

What is the *least common multiple (LCM)* of 20 and 15?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

8 How are *GCF* and *LCM* alike? How are they different?

Alike:

Different:

9 Determine the least common multiple or greatest common factor. Use a 100-grid or draw a diagram if it helps with your thinking.

a What is the least common multiple of 6 and 4?

b What is the greatest common factor of 9 and 12?

c What is the least common multiple of 10 and 6?

d What is the least common multiple of 2 and 16?

e What is the greatest common factor of 16 and 2?

f What is the greatest common factor of 18 and 27?

Activity 3

Name: .... Date: .... Period: ....

GCF and LCM

10 Tameeka made a table showing the GCF and LCM of several pairs of numbers.

a Complete Tameeka's work by determining the product of the numbers and the product of the LCM and GCF.

Number pairs	GCF	LCM	Product of the Numbers	Product of the LCM and GCF
6, 4	2	12		
5, 20	5	20		
3, 7	1	21		
12, 10	2	60		

b What do you notice?

c Tameeka wrote two numbers that have the least common multiple of 30 and the greatest common factor of 3. If one of the numbers is 15, determine the other number.

11 Sort the sum and product expressions. One sum expression can be matched with multiple product expressions.

Sum expressions

$12 + 15$ ....

$36 + 48$ ....

$12 + 54$ ....

Product Expressions

A. $6 \cdot (2 + 9)$

B. $12 \cdot (3 + 4)$

C. $3 \cdot (4 + 5)$

D. $6 \cdot (6 + 8)$

E. $3 \cdot (4 + 18)$

12 Use the common factor of the addends to make true equations.

Use each number, 2, 3, 4, 6, 7, 8, and 9 only once to fill in the blanks.

$15 + 27 = \boxed{3} \cdot (\boxed{5} + \boxed{})$

$14 + 8 = \boxed{} \cdot (\boxed{} + \boxed{})$

$48 + 18 = \boxed{} \cdot (\boxed{} + \boxed{})$

13 Synthesis

Discuss:

- What does the greatest common factor mean?
- Why do you think we don't study the least common factor?
- Does every pair of numbers have a GCF and a LCM?

16 Summary 2.08

A *factor* of a number is a whole number that divides evenly into the given number (with no remainder).

For example, to determine the **common factors** of 12 and 8, list the factors of each and circle the common ones.

Factors of 12: $1, 2, 3, 4, 6, 12$

Factors of 8: $1, 2, 4, 8$

The **greatest common factor (GCF)** of 8 and 12 is 4.

When two numbers have no common factors other than 1, their greatest common factor (GCF) is 1. For example, the GCF of 8 and 15 is 1.

If one number is a multiple of the other, such as 5 and 15, the least common multiple (LCM) is the larger number, 15, and the GCF is the smaller number, 5.

For any pair of numbers, the product of the numbers is equal to the product of their LCM and GCF.

The sum of two whole numbers with a *common factor* can be expressed as a *product*. For example, $15 + 18 = 3(5 + 6)$.

common factor When two numbers have the same factor, we call that a common factor.

greatest common factor The largest number that is a common factor of two numbers.

Practice
2.08

Name: .... Date: .... Period: ....

1. What is the greatest common factor of 12 and 44?

2. What is the greatest common factor of 4 and 6?

3. What is the least common multiple of 4 and 6?

Problems 4–6: Jayla's parents are replacing their bathroom floor with square tiles. The tiles will be laid side by side to cover the entire floor with no gaps, and none of the tiles can be cut. The floor is a rectangle that measures 48-by-60 inches.

4. What is the side length of the largest possible tile Jayla's parents could use?

5. How many of these tiles do they need?

6. List *three* other whole-number tile sizes that could cover the bathroom floor.

Problems 7–8: There are 90 sixth graders and 75 seventh graders in a school chorus. The music director wants to make groups of performers, with the same combination of sixth graders and seventh graders in each group. She wants to form as many groups as possible.

7. What is the greatest number of groups that could be formed? Show your thinking.

8. Using your answer from the previous problem, determine how many students of each grade would be in each group.

Name: ______________ Date: ______ Period: ______

9. Two numbers have the least common multiple of 45 and the greatest common factor of 3. If one of the numbers is 15, determine the other number.

Problems 10–12: For each scenario, determine whether you would use common factors or common multiples to determine the solution.

	Scenario	Common factors	Common multiples
10.	Fatima and Jalen both did laundry today. Fatima does laundry every 9 days and Jalen does laundry every 6 days. What is the next day they will do laundry on the same day?	☐	☐
11.	Kai is planting 40 blueberry bushes and 48 flower bushes in rows. Each row will have the same number of bushes. How many rows can Kai plant?	☐	☐
12.	The local grocery store is giving away door prizes. Every 12th customer wins $\$100$ and every 8th customer wins free groceries for a month. Zahra was the first to win both prizes. What number customer was she?	☐	☐

Spiral Review

Problems 13–15: Circle the expression that has the greater value.

13. $5 \cdot 0.4$ $500 \cdot 0.04$ They have the same value.

14. $14.2 - 2.35$ $142 - 23.5$ They have the same value.

15. $1.82 + 33.3$ $18.2 + 3.33$ They have the same value.

Unit 2
Lesson
9

Name: ____ Date: ____ Period: ____

Fraction Relationships | Model the World | 6.RP.3, 6.RP.3.a, SMP.2, SMP.3, SMP.7

Mixing Paint, Part 1

Let's see how mixing colors relates to ratios.

Warm-Up

Mentally determine the missing value that makes each pair of fractions equivalent.

1. $\frac{1}{5} = \frac{\square}{10}$

2. $\frac{2}{5} = \frac{6}{\square}$

3. $\frac{4}{\square} = \frac{12}{15}$

4. $\frac{\square}{8} = \frac{15}{12}$

Activity 1

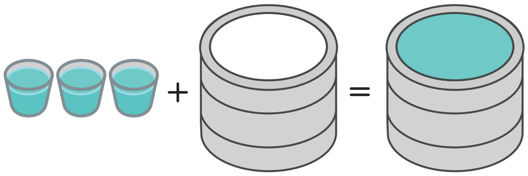
Name: .... Date: .... Period: ....

Comparing Ratios

5 Paint stores create different colors by using different ratios of white paint to tint.

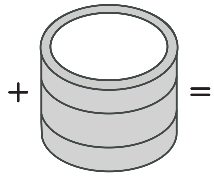
a Choose *one* tint color.

Example Paint Mix



b Circle the amount of tint you want to add to 2 gallons of white paint.

My Paint Mix

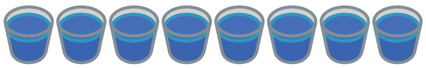


Use the digital activity to see your paint mix.

Red 

Teal 

Purple 

Blue 

6 Write the ratio you created.

.... ounces tint : 2 gallons white paint

a Can you find two different ways to make a *darker* color? Use the digital activity to check your work.

.... ounces tint : .... gallons white

.... ounces tint : .... gallons white

b Can you find two different ways to make a *lighter* color? Use the digital activity to check your work.

.... ounces tint : .... gallons white

.... ounces tint : .... gallons white

Activity **1**

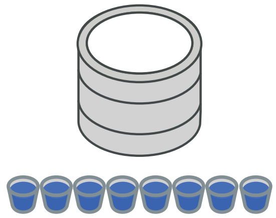
Name: ________________ Date: ________ Period: ________

Comparing Ratios (continued)

7 Here are Luca's and Marc's ratios. Which will make a darker blue? Circle one.

Luca's ratio Marc's ratio They'll make the same blue

Luca's Ratio
6 ounces blue
2 gallons white



Marc's Ratio
8 ounces blue
4 gallons white

Explain your thinking.

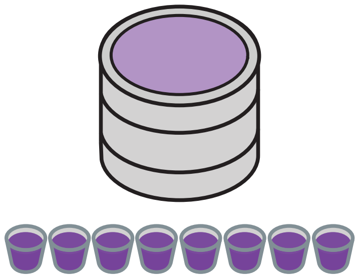
8 **a** Let's watch to see which ratio makes a darker blue.

b **Discuss:** What is a different strategy you could use to compare the ratios?

9 Here is Amoli's ratio:

8 ounces purple : 4 gallons white

Select *all* of the choices that will result in a darker purple.



- ☐ **A.** Mixing 4 ounces purple and 2 gallons white
- ☐ **B.** Mixing 2 ounces of purple per gallon of white
- ☐ **C.** Mixing 9 ounces purple and 4 gallons white
- ☐ **D.** Adding 2 ounces of purple and 2 gallons of white
- ☐ **E.** Adding 3 ounces of purple and 1 gallon of white

Activity
2

Name: ____________ Date: ________ Period: ________

Lighter or Darker Paint

10 Order the ratios from *darkest* blue to *lightest* blue.

A. 5 ounces blue : 4 gallons white

B. 4 ounces blue : 3 gallons white

C. 10 ounces blue : 6 gallons white

D. 9 ounces blue : 6 gallons white

Darkest Blue **Lightest Blue**

11 Here is Luca's work from the previous screen.

Discuss:

- What is an advantage of using Luca's strategy to order the ratios?
- What is a different strategy you could use to order the ratios?

The least common multiple of number of gallons of white paint is 12 ounces.

10 ounces blue : 6 gallons white 20 : 12

9 ounces blue : 6 gallons white 18 : 12

4 ounces blue : 3 gallons white 16 : 12

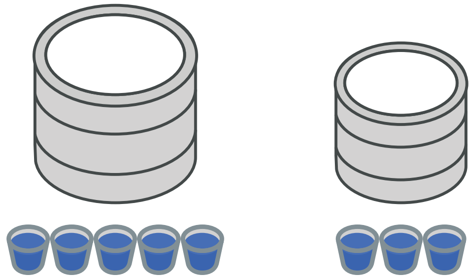
5 ounces blue : 4 gallons white 15 : 12

12 Solve all six challenges. For each pair of ratios, choose which ratio makes a darker blue.

Ratio A	Ratio B	Ratio A	Ratio B	They make the same blue
2 oz blue : 4 gal white	3 oz blue : 4 gal white			
4 oz blue : 3 gal white	4 oz blue : 5 gal white			
3 oz blue : 2 gal white	5 oz blue : 4 gal white			
5 oz blue : 2 gal white	15 oz blue : 6 gal white			
7 oz blue : 3 gal white	5 oz blue : 2 gal white			
5 oz blue : 4 gal white	9 oz blue : 7 gal white			

13 Synthesis

Describe a strategy for comparing two ratios. Use the example if it helps with your thinking.



Ratio A	Ratio B
5 ounces blue 4 gallons white	3 ounces blue 2 gallons white

16 Summary 2.09

You can use different strategies to compare two ratios.

Let's compare the ratios of two cans of paint to see which will make a lighter shade of gray.

Ratio A	Ratio B
5 ounces black paint 3 gallons white paint	7 ounces black paint 4 gallons white paint

Strategy 1: Change both ratios so that they share one quantity.

- Multiply both ratios so they each have the same amount of black paint.
- The *LCM* for the number of ounces of black paint for both ratios is 35.
- Multiply Ratio A by 7 to get 35 ounces of black paint and 21 gallons of white paint.
- Multiply Ratio B by 5 to get 35 ounces of black paint and 20 gallons of white paint.

When both ratios have the same amount of black paint, Ratio A has more gallons of white paint, which means it will be a lighter shade of gray.

Strategy 2: Calculate how much each ratio is per 1 quantity.

- Calculate the number of ounces of black paint per gallon of white paint.
- Ratio A has $\frac{5}{3} = 1\frac{2}{3}$ ounces of black paint for every gallon of white paint.
- Ratio B has $\frac{7}{4} = 1\frac{3}{4}$ ounces of black paint for every gallon of white paint.

Ratio A has less black paint for 1 gallon of white paint, which means it will be a lighter shade of gray.

Name: .... Date: .... Period: ....

Problems 1–3: To make 1 can of sky blue paint, Ama mixes 2 ounces of blue tint with 3 gallons of white paint.

1. Write a ratio of blue tint to white paint that would make the *same* color blue.

2. Write a ratio of blue tint to white paint that would make a *darker* blue.

3. Write a ratio of blue tint to white paint that would make a *lighter* blue.

4. If you blend 2 scoops of chocolate frozen yogurt with 1 cup of milk, you will make a milkshake with a stronger chocolate flavor than if you blended 3 scoops of chocolate frozen yogurt with 2 cups of milk. Show or explain why this is true.

5. There are two mixtures of light purple paint.

- Mixture A is made with 5 cups of purple paint and 2 cups of white paint.
- Mixture B is made with 15 cups of purple paint and 8 cups of white paint.

Which mixture makes a lighter shade of purple? Explain your thinking.

Name: ____________ Date: ________ Period: ________

Problems 6–7: Here are mixtures to create different colors of green.

6. Order these mixtures from *lightest* green to *darkest* green.

A. 2 gallons white : 4 ounces green

B. 3 gallons white : 5 ounces green

C. 5 gallons white : 8 ounces green

Lightest green **Darkest green**

7. Determine the amount of green paint you would need to mix with 5 gallons of white paint to create a darker green than the darkest green.

Spiral Review

Problems 8–10: Here are two recipes for lemonade.

- **Recipe A:** Mix 3 cups of lemon juice with 2 cups of water.
- **Recipe B:** Mix 3 cups of lemon juice with 3 cups of water.

8. What fraction of Recipe A is lemon juice?

9. What fraction of Recipe B is lemon juice?

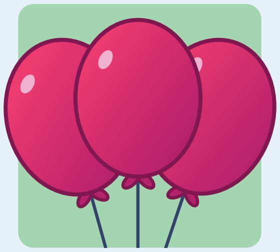
10. Which recipe has a stronger lemon flavor? Explain your thinking.

Sub-Unit

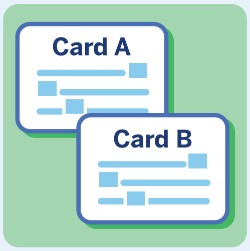
3

Solving Problems With Ratios

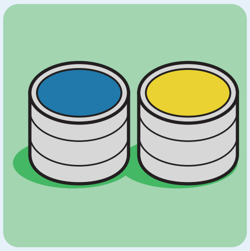
Lesson 10
Disaster Preparation



Lesson 11
Balloons



Lesson 12
Community Life



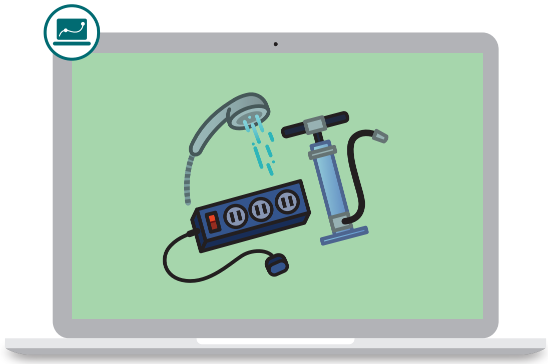
Lesson 13
Mixing Paint, Part 2

Lesson 14
City Planning

Lesson 15
Lunch Waste

Name: ______ Date: ______ Period: ______

Patterns Inside Numbers | Model the World | 6.RP.3.a, SMP.2, SMP.4, SMP.8



Disaster Preparation

Let's use ratio tables to help prepare for disasters.

Warm-Up

1 Cities need to prepare for possible disasters.

What are three things a city should have for its people in case of a disaster?

1.

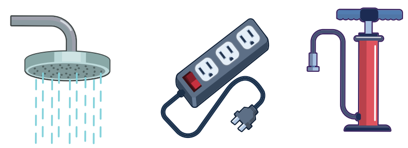
2.

3.

2 The Federal Emergency Management Agency, or FEMA, has a list of items that cities should stock up on in case of a disaster. Here are three of those items.

How many of each item do you think a city with a population of 100 should have?

Handheld Shower | **Power Strip** | **Air Pump**



Population	Handheld Showers	Power Strips	Air Pumps
100			

Activity 1

Name: Date: Period:

Shower, Power, and Air

3 Here are the FEMA recommendations for a city of 100 people.

How many of each item would you recommend for Lucas, Wisconsin?

	Population	Handheld Showers	Power Strips	Air Pumps
FEMA Recommendations	100	4	5	1
Lucas, Wisconsin	700			

4 How many of each item would you recommend for Blue Ridge, Georgia?

	Population	Handheld Showers	Power Strips	Air Pumps
FEMA Recommendations	100	4	5	1
Blue Ridge, Georgia	1,200			

5 How many of each item would you recommend for Hamlin City, Kansas?

	Population	Handheld Showers	Power Strips	Air Pumps
FEMA Recommendations	100	4	5	1
Hamlin City, Kansas	25			

Name: ________ Date: ________ Period: ________

Shower, Power, and Air (continued)

6 Taylor recommended that Hamlin City, Kansas should buy 1 handheld shower, 1 power strip, and 1 air pump.

	Population	Handheld Showers	Power Strips	Air Pumps
FEMA Recommendations	100	4	5	1
Lucas, Wisconsin	700	28	35	7
Blue Ridge, Georgia	1,200	48	60	12
Hamlin City, Kansas	25	1	1	1

a. **Discuss:** Why do you think Taylor made this recommendation?

b. What do you agree with about Taylor's recommendations? What do you disagree with?

7 Here are FEMA's actual recommendations for Lucas, Wisconsin; Blue Ridge, Georgia; and Hamlin City, Kansas.

Discuss: What is FEMA's strategy for calculating the number of each item? Are there any recommendations you disagree with?

	Population	Handheld Showers	Power Strips	Air Pumps
Lucas, Wisconsin	700	28	35	7
Blue Ridge, Georgia	1,200	48	60	12
Hamlin City, Kansas	25	1	2	1

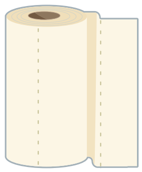
Name: ________ Date: ________ Period: ________

FEMA Poster

8 FEMA provides guidance about other items to stock up on in case of disaster. Here are some examples.

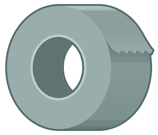
Paper Towels

For every 5 people, have 1 roll of paper towels.



Duct Tape

Have 3 rolls of duct tape for every 25 people.



Magnifying Glass

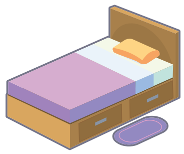
Have 1 magnifying glass for every 50 people.

Cotton Balls

For every 100 people, have 4 bags of 50 cotton balls each.

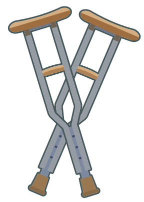
Beds

Have 1 bed for each person, plus 10 extra beds for volunteers.



Crutches

Have 6 pairs of crutches.



a Use FEMA's guidance to make a recommendation for preparing these 3 cities for a disaster.

	Population	Rolls of Paper Towels	Magnifying Glasses	Cotton Balls	Pairs of Crutches
Branch City, Arkansas	300				
Bennington City, Nebraska	2,000				
Harrisburg, Pennsylvania	50,200				

b Is there anything you disagree with about FEMA's recommendations? If so, explain which numbers you think should change and why. If not, explain why not.

Name: ________ Date: ________ Period: ________

FEMA Poster (continued)

c Complete these steps and make a poster of your work:

ELD.PI.6.2.Em, Ex, Br

- ☐ Choose a city or town that is meaningful to you and look up its population.

City, State	Population (to the nearest 10 people)
________	________

- ☐ Make recommendations about items that this city should stock up on. Choose *at least* four different supplies from the FEMA list. Then determine how many of each item the city should have on hand in case of a disaster.

Item 1:	**Item 2:**
Item 3:	**Item 4:**

- ☐ Show or explain how you determined the amount of each item your city will need.

- ☐ Explain *at least* two changes or additions you think FEMA should make to its guidance.

9 Synthesis

Explain how to use a table of equivalent ratios to determine unknown values. Use the example if it helps with your thinking.

	Population	Handheld Showers	Power Strips
FEMA Recommendations	100	4	5
Lucas, Wisconsin	700	28	35
Blue Ridge, Georgia	1,200	48	60

12 Summary 2.10

We can use ratio tables to help make plans for situations that we haven't experienced yet.

Here are some supply recommendations for a 50 person taco party:

- 10 pounds of carnitas
- 15 cups of pinto beans
- 125 tortillas

People	Carnitas (lb)	Pinto Beans (cups)	Tortillas
50	10	15	125
200	40	60	500
10	2	3	25

× 4

÷ 5

Let's use a table to determine the different amounts of each ingredient we might need for different-sized parties. For example, if we only had 10 people coming to the taco party, we would only need 2 pounds of carnitas, 3 cups of pinto beans, and 25 tortillas. We just have to multiply or divide all of the values in each row by the same number to preserve each ratio relationship.

Practice
2.10

Name: ______________ Date: ______ Period: ______

Problems 1–3: A recipe for tropical fruit juice says to combine 4 cups of pineapple juice with 5 cups of orange juice.

1. Complete the table to determine how much of each type of juice you need for 1, 2, 3, and 4 batches of the recipe.

Batches	Pineapple Juice (cups)	Orange Juice (cups)
1	4	5
2		
3		
4		

2. The recipe also calls for $\frac{1}{3}$ cup of lime juice for every 5 cups of orange juice. Add an additional column of values to the table to represent the amount of lime juice for 1, 2, 3, and 4 batches of the recipe.

3. If you use 12 cups of pineapple juice with 20 cups of orange juice, will the recipe taste the same? Explain your reasoning.

Problems 4–5: It takes about 9 kilograms of olives to make 2 liters of olive oil.

4. Complete the table to determine how much olive oil each orchard made.

	Olives (kg)	Olive Oil (L)
Ratio	9	2
Orchard A	9,000	
Orchard B	5,400	

5. Afia claims that to make 4 liters of olive oil, you need 11 kilograms of olives. Is she correct? Explain your thinking.

Name: ________ Date: ________ Period: ________

6. Determine the unknown values in the table.

Number of Loaves	Bananas	Butter (cups)	Sugar (cups)	Eggs	Flour (cups)
4	12	2	3	8	6
2					

Spiral Review

Problems 7–10: Determine each product. Show your thinking.

7.
$$\begin{array}{r} 735 \\ \times\ 352 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 680 \\ \times\ 502 \\ \hline \end{array}$$

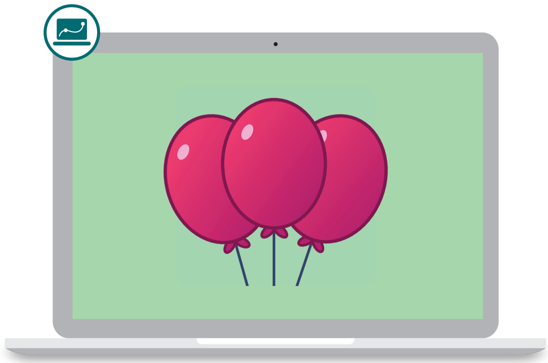
9.
$$\begin{array}{r} 347 \\ \times\ 802 \\ \hline \end{array}$$

10.
$$\begin{array}{r} 401 \\ \times\ 285 \\ \hline \end{array}$$

Unit 2
Lesson
11

Name: ____ Date: ____ Period: ____

Patterns Inside Numbers Model the World 6.RP.3, 6.RP.3.a, SMP.2, SMP.5, SMP.7



Balloons

Let's develop and use tools to solve problems involving equivalent ratios.

Warm-Up

Evaluate each expression mentally.

1. $2 \cdot 31$

2. $8 \cdot 31$

3. $9 \cdot 31$

4. $11 \cdot 31$

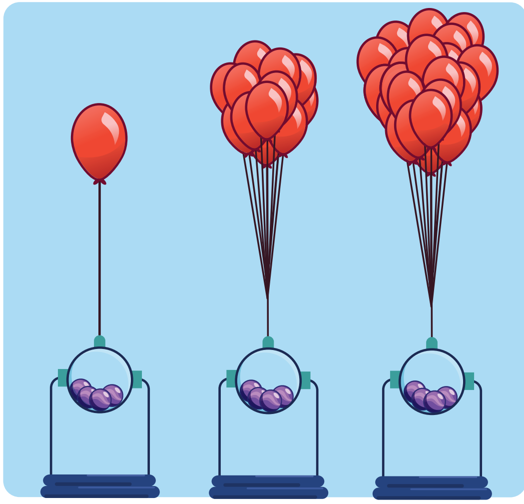
Activity
1

Name: ______ Date: ______ Period: ______

Balloon Float

5 Helium balloons can make objects float, but too many balloons will make objects fly away!

Let's watch an animation to see the *middle* container float.

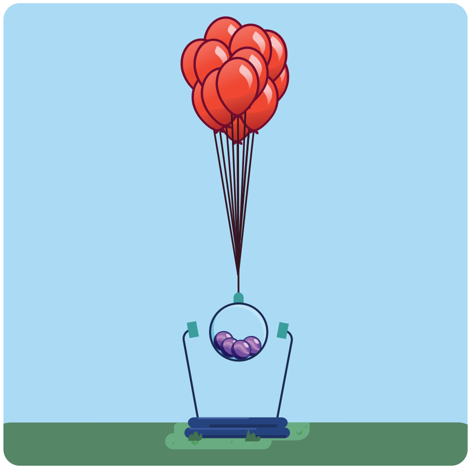


6 Red balloons float purple marbles at a ratio of $12 : 4$.

What will happen to the marbles if we add 1 balloon and 1 marble? Circle one.

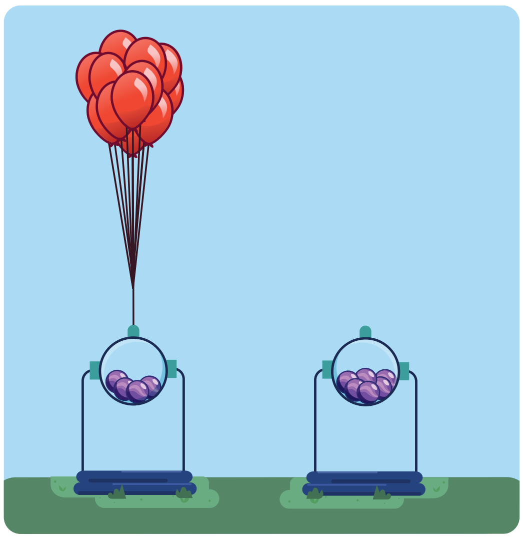
Sink down Float in place Fly up

Explain your thinking.



7 Red balloons float purple marbles at a ratio of $12 : 4$.

How many red balloons will float 6 purple marbles?

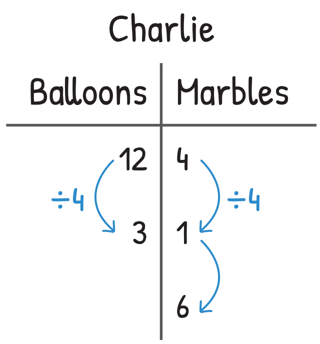


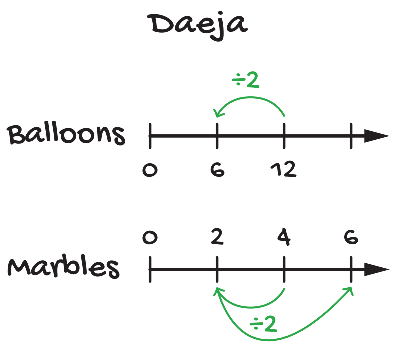
Name: .... Date: .... Period: ....

Balloon Float (continued)

8 Here are Charlie's and Daeja's strategies for determining how many red balloons will float 6 purple marbles.

a Look at each student's strategy.

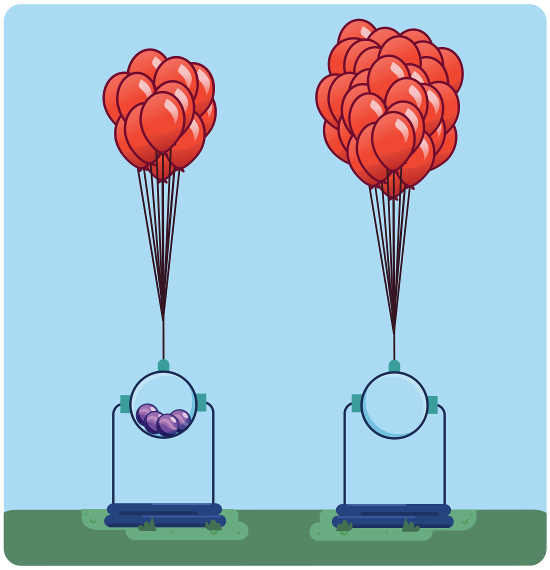




b Select one student by circling their name, then explain how they could finish their strategy to solve the problem.

9 Red balloons float purple marbles at a ratio of $12 : 4$.

How many purple marbles will 30 red balloons float?



Name: Date: Period:

Marble Float

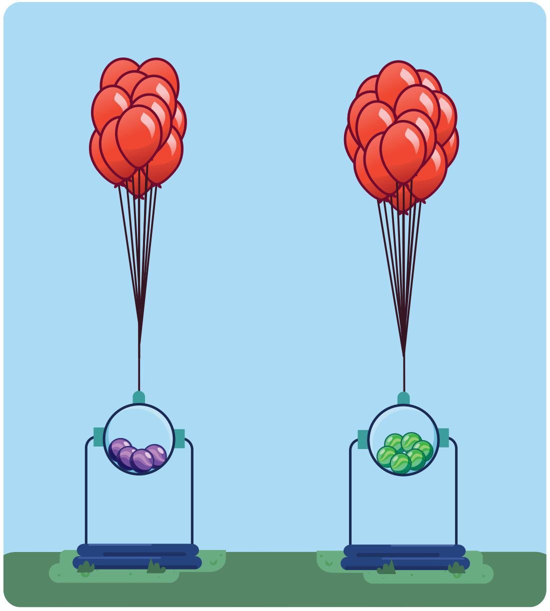
10 Red balloons float purple marbles at a ratio of $12 : 4$.

Red balloons float green marbles at a ratio of $15 : 6$.

Which is heavier: a purple marble or a green marble?

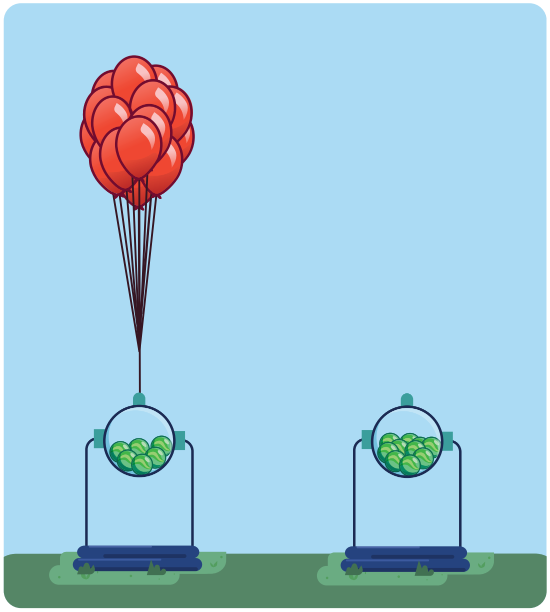
Purple Green They're the same

Explain your thinking.



11 Red balloons float green marbles at a ratio of $15 : 6$.

How many red balloons will float 10 green marbles?



Name: .... Date: .... Period: ....

Marble Float (continued)

12 Here are Charlie's and Daeja's strategies for determining how many red balloons will float 10 green marbles.

Discuss: How are their strategies alike? How are they different?

Charlie

	Balloons	Marbles	
	15	6	
$\times\frac{1}{6}$	2.5	1	$\times\frac{1}{6}$
$\times 10$	25	10	$\times 10$

Daeja

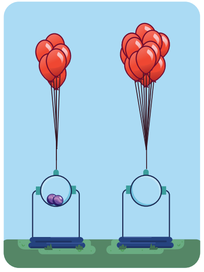
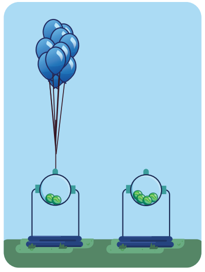
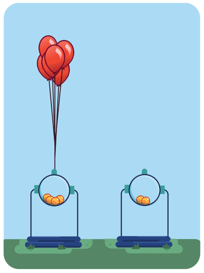
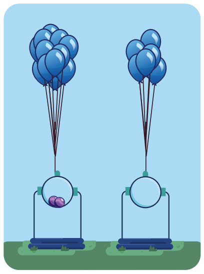
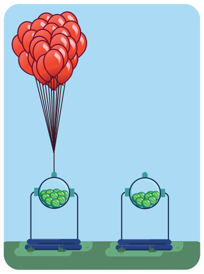
	Balloons	Marbles	
	15	6	
$\times 5$	75	30	$\times 5$
$\div 3$	25	10	$\div 3$

Activity 3

Name: ______ Date: ______ Period: ______

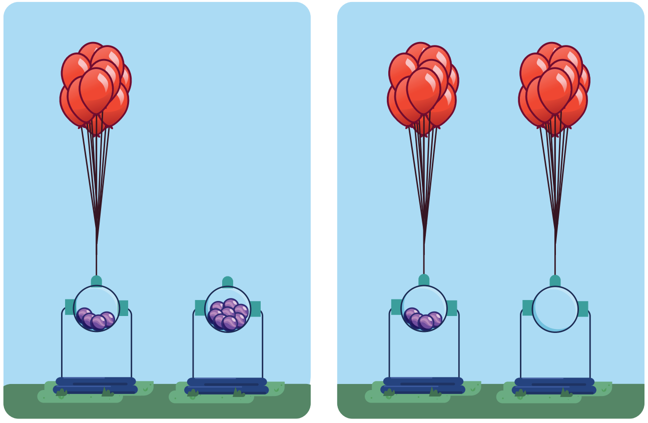
Repeated Challenges

13 For each ratio, create an equivalent ratio to make the balloons float.

	Ratio	Number of Balloons	Number of Marbles
	Red balloons float purple marbles at a ratio of 6 : 2.	12 red balloons	
	Blue balloons float green marbles at a ratio of 10 : 2.		4 green marbles
	Red balloons float orange marbles at a ratio of 6 : 4.		2 orange marbles
	Blue balloons float purple marbles at a ratio of 12 : 2.	6 blue balloons	
	Red balloons float green marbles at a ratio of 25 : 10.		8 green marbles

14 Synthesis

Describe a strategy for determining missing values in equivalent ratios, like an unknown number of balloons or marbles.



17 Summary 2.11

There are a few helpful strategies you can use to determine missing values in equivalent ratios. One strategy is to determine a new ratio where one of the quantities is equal to 1.

For example, if 6 balloons can make 3 marbles float, you can use the ratio $6 : 3$ and equivalent ratios to solve different problems.

To determine the number of balloons that can float 8 marbles:

- Determine the number of balloons that float 1 marble.
- Then you can multiply that ratio by 8 to determine that 16 balloons float 8 marbles.

	Number of Balloons	Number of Marbles	
	6	3	
$\div 3$	2	1	$\div 3$
$\times 8$	16	8	$\times 8$

To determine the number of marbles that 4 balloons can float:

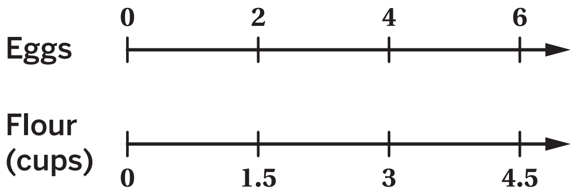
- Determine the number of marbles that 1 balloon can float.
- Then you can multiply that ratio by 4 to determine that 4 balloons float 2 marbles.

	Number of Balloons	Number of Marbles	
	6	3	
$\div 6$	1	0.5	$\div 6$
$\times 4$	4	2	$\times 4$

Practice
2.11

Name: ________ Date: ________ Period: ________

Problems 1–2: Here is a double number line showing the ratio of eggs to flour for different-sized cakes.



1. How much flour do you need for each egg in this recipe?

2. How many eggs would you need for 18 cups of flour?

Problems 3–5: The same cake recipe uses 2 cups of sugar for every 3 cups of flour.

3. Draw a double number line to represent this situation.

4. How much sugar do you need for 18 cups of flour?

5. Which representation do you prefer to help you answer the previous question: a table, a double number line, or some other tool? Explain your thinking.

6. Raven and Tiana are both training for a swimming competition in the same pool. Raven can swim 6 laps in 3 minutes. Tiana can swim 3 laps in 2 minutes. If both swimmers maintain their pace, which statement is *not* true?

A. Raven can swim 2 laps per minute.

B. Tiana can swim 1.5 laps in one minute.

C. In 6 minutes, Raven can swim 3 more laps than Tiana.

D. In 12 minutes, Tiana swims 8 fewer laps than Raven.

Practice
2.11

Name: ______ Date: ______ Period: ______

Problems 7–8: Inola is making personal pizzas for her birthday party.

7. For 4 pizzas, she uses 10 ounces of cheese. Complete the table using this ratio.

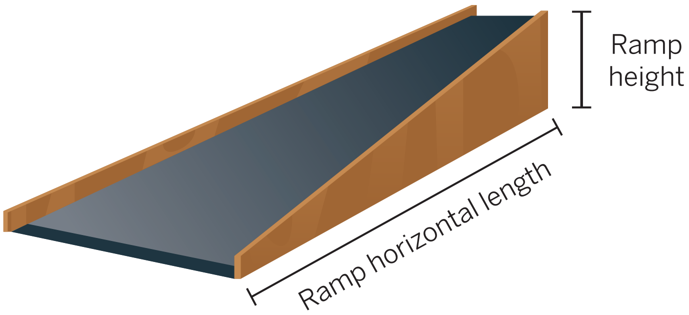
Number of Pizzas	Cheese (oz)
12	
22	
11	

8. Inola went to the farmers market to get ingredients. Determine the price per item of each vegetabvle.

Purchase	Price per item ($)
6 onions for $1.80	
12 mushrooms for $3	
5 peppers for $5.50	

9. The Americans with Disabilities Act (ADA) states that the maximum height-to-length ratio of a curb ramp is 1 : 12. That means for every 1 inch of ramp height, there must be at least 12 inches of ramp length.

Chloe measured the height of this ramp as 30 inches. What's the minimum horizontal length of the ramp?



Spiral Review

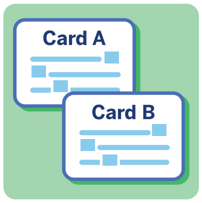
10. Fill in each blank using the numbers 1 to 12 only once to make each expression true.

$\frac{\square}{\square} < \frac{\square}{\square}$ $\frac{\square}{\square} = \frac{\square}{\square}$ $\frac{\square}{\square} > \frac{\square}{\square}$

Unit 2
Lesson
12

Name: Date: Period:

Patterns Inside Numbers Model the World 6.RP.3, SMP.1, SMP.5



Community Life

Let's practice solving ratio problems in context.

Warm-Up

A chef is making jollof rice for families at the Metropolis Community Center. Their recipe requires pounds of rice. The store sells pounds of rice for . Customers can buy any amount of rice.

ELD.PI.6.1.Em, Ex, Br, ELD.PI.6.6.Em, Ex, Br, ELD.PI.6.12.Em, Ex, Br

1. **Discuss:** What is this situation about?

2. Given the values, create a table or a double number line to represent the situation.

3. What question might you ask about the situation?

4. Use your diagram to answer this question.

Name: ______ Date: ______ Period: ______

Sort ’em

You will use the Activity 1 Sheet to complete this activity.

5. For each problem:

- Determine whether you plan to use equivalent ratios to solve. Do not attempt to solve.
- Explain or show your thinking.

Problem A

Yes No I’m not sure.

Problem B

Yes No I’m not sure.

Problem C

Yes No I’m not sure.

Problem D

Yes No I’m not sure.

Problem E

Yes No I’m not sure.

Problem F

Yes No I’m not sure.

Activity 2

Name: ______ Date: ______ Period: ______

Closer Look

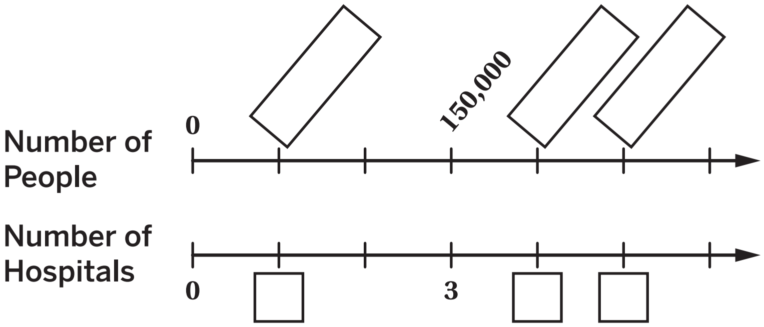
You will use the Activities 2 & 3 Sheet, which shows the problems from the Activity 1 Sheet with the numbers filled in. **ELD.PI.6.1.Em, Ex, Br, ELD.PI.6.10.Em, Ex, Br**

6. Antwon and Riku started working on Problem D. Work with a partner to complete the missing pieces of their work.

Antwon

Number of People	Number of Hospitals
0	0
150,000	3

Riku



7. **Discuss:** How are the two representations similar? How are they different?

8. Which representation do you prefer for the situation in Problem D? Explain your thinking.

9. Answer Problem D: *If the population grows to* 250,000 *people, would you recommend that the city plan to have* 6 *hospitals?* Explain your thinking.

Activity 3

Name: .... Date: .... Period: ....

Solve ’em

10. Select *two* problems from the Activities 2 & 3 Sheet (other than Problem D) to solve. Show or explain your reasoning.

Problem	**Problem** ....

You’re invited to explore more.

11. Write your own situation and problem that involves equivalent ratios. Trade problems with a partner and try to solve each other’s problems.

Synthesis

12. A shower uses 5 gallons of water every 2 minutes. How many gallons will it use during an 11-minute shower?

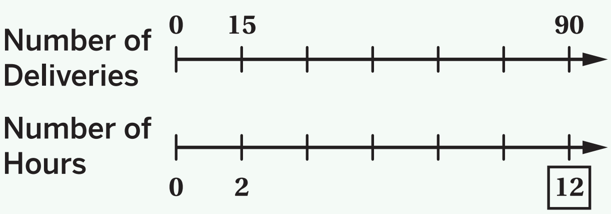
Without solving, describe your favorite strategy for solving a problem like this.

Summary 2.12

We can use equivalent ratios to help solve real-world problems that involve at least two quantities. When working with real-world problems, we may need to round numbers or think about the circumstances of the situation when determining which solutions make sense.

Let's say the Metropolis Delivery Service makes 15 deliveries every 2 hours. They need to make 90 deliveries tomorrow.

You can use a double number line using the values that you know (15 deliveries in 2 hours) to determine the values that you don't (90 deliveries in ? hours).



You can also use a ratio table to determine that it will take 12 hours to make all 100 deliveries.

Number of Deliveries	Number of Hours
15	2
30	4
90	12

Practice
2.12

Name: ________ Date: ________ Period: ________

Problems 1–2: Julian is paid $\$90$ for 8 hours of work at a restaurant.

1. Complete the table to determine the amount of money Julian is paid.

Hours of Work	Amount Paid ($)
24	
10	
3	

2. Wey Wey is paid $\$56$ for 5 hours of work. Is she being paid the same as Julian? Show or explain your thinking.

Problems 3–6: A chef needs 15 gallons of vinegar to make pickles. A store sells 2 gallons of vinegar for $\$3$ and allows customers to buy any amount of vinegar they want.

3. Make a table to represent this situation.

4. Draw a double number line to represent this situation.

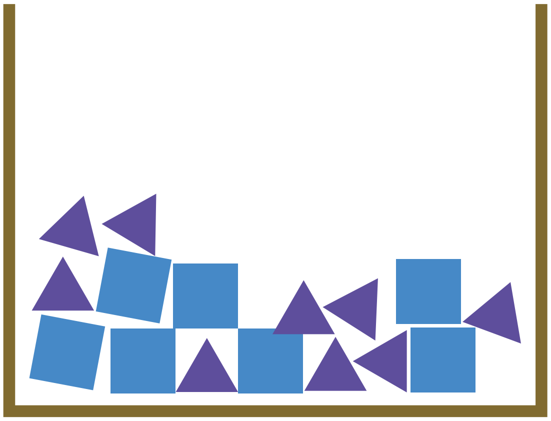
5. Use any representation to determine how much the 15 gallons of vinegar will cost.

6. Which representation did you prefer in this situation? Explain your thinking.

Name: .... Date: .... Period: ....

7. The garlic bread at a restaurant uses 4 cloves of garlic for every 3 loaves. How many loaves can you make with 10 cloves of garlic?

8. Tyler has a box with square and triangle building blocks in it. How many blocks of each kind would Tyler need to add so that the ratio of square blocks to triangle blocks is 5 : 7?



Spiral Review

Problems 9–12: Write in a number to make each equation true.

9. $4 \cdot \boxed{} = 24$

10. $24 \cdot \boxed{} = 4$

11. $\frac{1}{6} \cdot 6 = \boxed{}$

12. $6 \cdot \boxed{} = 1$

Unit 2
Lesson
13

Name: ________ Date: ________ Period: ________

Patterns Inside Numbers | Model the World | 6.RP.3, SMP.1, SMP.6

Mixing Paint, Part 2

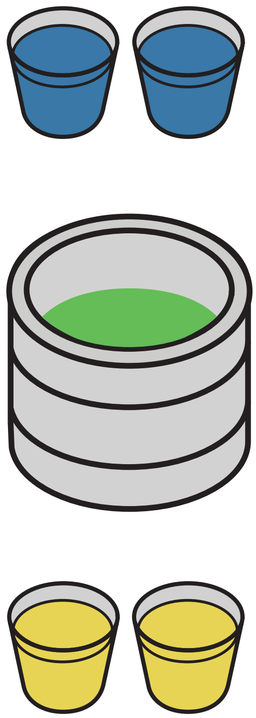
Let's use tape diagrams to represent ratios.

Warm-Up

1 Let's watch how to make a new color.

What do you notice? What do you wonder?

I notice:



I wonder:

Activity 1

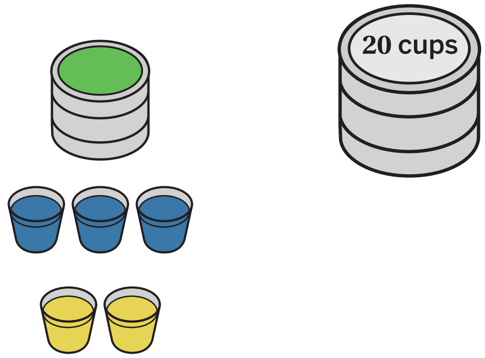
Name: .... Date: .... Period: ....

How Much of Each?

2 Tyrone makes a green paint by mixing 3 cups of blue with 2 cups of yellow.

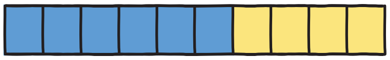
He needs 20 more cups of green paint to finish painting a mural.

How much of each color should he mix?



Blue (cups)	Yellow (cups)	Total (cups)
		20

3 Tyrone drew a **tape diagram** to help determine that he needs 12 cups of blue and 8 cups of yellow to make 20 cups of green paint.



Where do you see the 3 : 2 ratio, 20, and 12 represented in Tyrone's diagram?

The 3 : 2 ratio is shown by . . .

The 20 total cups are shown by . . .

The 12 cups of blue are shown by . . .

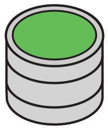
Name: .... Date: .... Period: ....

How Much of Each? (continued)

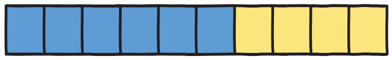
4 Kayla needs 35 gallons of the same green paint.

She used this tape diagram to determine how much of each paint color she needs.

How many gallons should go into each box in the tape diagram?



5 Amir's containers hold a half-gallon of paint.



He drew a tape diagram to help determine that he needs 6 containers of blue paint and 4 containers of yellow paint to make the same green paint as Tyrone.

Will his tape diagram result in the same shade of green paint? Explain your thinking.

Activity
2

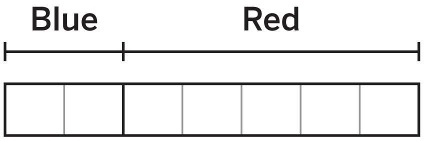
Name: ______ Date: ______ Period: ______

Same Color?

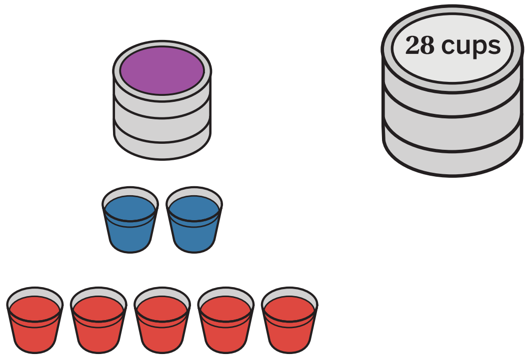
6 Sai makes purple paint by mixing 2 cups of blue and 5 cups of red.

How much of each color should Sai mix to get 28 cups of purple paint?

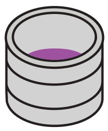
Use the tape diagram if it helps with your thinking.



Blue (cups)	Red (cups)	Total (cups)
		28



7 Select *all* of the combinations that would make the same color.



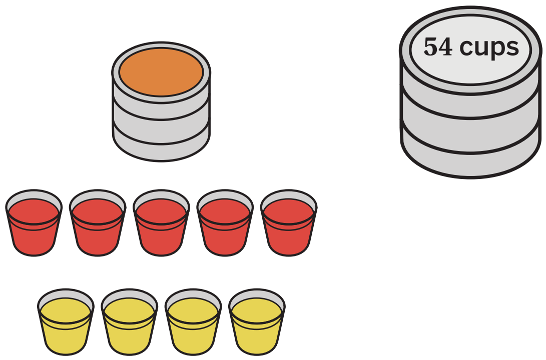
- ☐ **A.** 1 cup blue and 2.5 cups red
- ☐ **B.** 2.5 cups blue and 1 cup red
- ☐ **C.** 2 quarts blue and 5 quarts red
- ☐ **D.** 2 cups blue and 5 gallons red
- ☐ **E.** 1 gallon blue and 1 cup red

8 Dylan has a recipe for orange paint that mixes 5 parts red paint and 4 parts yellow paint.

How much of each color should Dylan mix to get 54 cups of orange paint?

Draw your own tape diagram if it helps with your thinking.

Red (cups)	Yellow (cups)	Total (cups)
		54



Name: ______________________ Date: __________ Period: __________

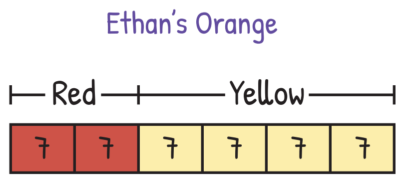
Same Color? (continued)

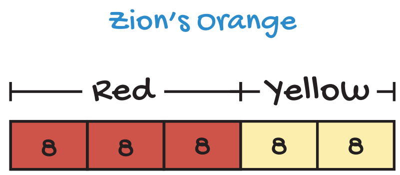
9 Here are Ethan's and Zion's recipes for orange paint.

Which student made more paint? Circle one.

Ethan   Zion   Same amount

Explain your thinking.





10 Kadeem's recipe for purple paint calls for 3 parts blue, 4 parts red, and 1 part white paint.

He needs 24 liters of purple paint to paint his mural. How much of each color will he need?

Draw your own tape diagram if it helps with your thinking.

Blue (L)	Red (L)	White (L)	Total (L)
			24

You're invited to explore more.

11 Create *three* equivalent ratios by filling in each blank using the numbers 0 to 9 only once.

□ : □

□ : □

□ : 1□

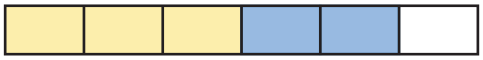
12 Synthesis

Here are the ingredients for a mango lassi drink.

Discuss: How does the tape diagram represent this situation?

Ingredients for Mango Lassi

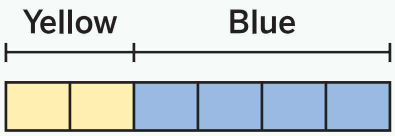
- 6 cups of mango
- 4 cups of yogurt
- 2 cups of milk



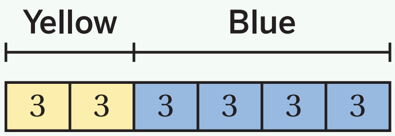
15 Summary 2.13

We can use **tape diagrams** to represent things like the ratio of different paints in a mixture.

For example, when 2 cups of yellow paint are mixed with 4 cups of blue paint, it creates 6 cups of green paint. Here is a tape diagram representing that ratio, where each part represents 1 cup of paint.



But if each part represented 3 cups of paint, there would be 6 cups of yellow paint, 12 cups of blue paint, and a total of 18 cups of green paint. This is a way to see a ratio that is equivalent to the original ratio.



tape diagram A way to represent relationships between quantities (like ratios) as lengths of a tape. The entire tape diagram represents the whole and is divided to represent the parts.

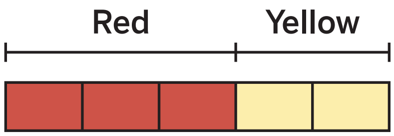
Name: .... Date: .... Period: ....

Problems 1–2: The ratio of coaches to players at practice is $2 : 5$. There are 21 people at practice.

1. How many coaches are at practice?

2. How many players are at practice?

Problems 3–4: Here is a tape diagram representing the ratio of red paint to yellow paint in a mixture of orange paint.



3. What is the ratio of red paint to yellow paint?

4. Complete the table below to show the amount of yellow and red paint needed to make each quantity of orange paint.

Orange (gal)	Red (gal)	Yellow (gal)
25		
30		

5. The ratio of cats to dogs at a shelter is $4 : 5$. In total, there are 27 dogs and cats.

How many dogs are there? Show your thinking.

Practice 2.13

Name: ______ Date: ______ Period: ______

6. Last month, there were 4 sunny days for every rainy day. If there were 30 days in the month, how many days were rainy? Show your thinking.

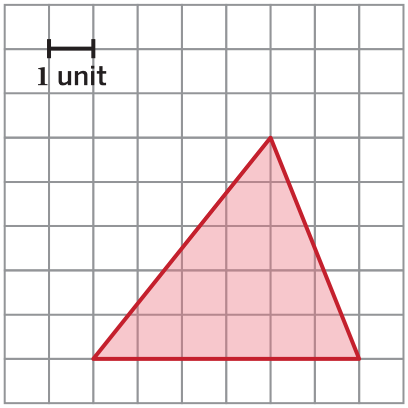
Spiral Review

7. What is $\frac{1}{2}$ of 12?

8. What is $\frac{1}{4}$ of 12?

9. What is $\frac{3}{4}$ of 12?

10. Determine the area of this triangle. Show or explain your thinking.



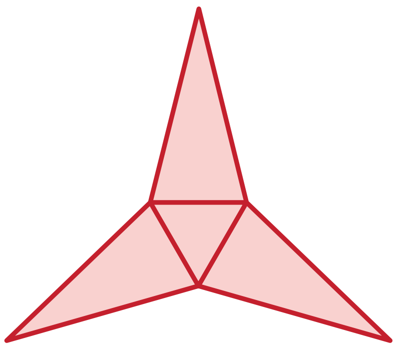
11. What type of polyhedron can you assemble from this net?

A. Triangular pyramid

B. Trapezoidal pyramid

C. Rectangular pyramid

D. Triangular prism



Unit 2
Lesson
14

Name: .... Date: .... Period: ....

Patterns Inside Numbers | Model the World | 6.RP.3, SMP.2, SMP.4, SMP.6

City Planning

Let's explore city planning using ratios.

Warm-Up

1 Imagine that you're moving to a new city.

What would be important to you when looking for a place to live?

Activity 1

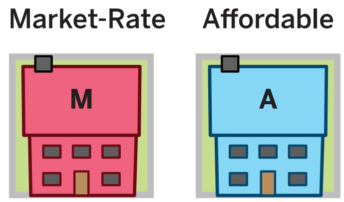
Name: .... Date: .... Period: ....

Affordable and Market-Rate Housing

2 Many cities have a shortage of housing affordable enough for residents to have money left over for other necessities, like food and healthcare.

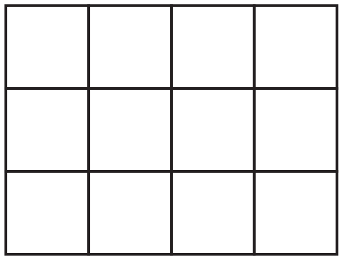
One approach cities use is to create *affordable housing units* that have cost limits.

Housing with no cost limit is called *market-rate*.

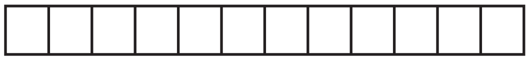


Imagine you are a city planner. Design a neighborhood.

Write **M** in each square that you want to represent a market-rate house and **A** in each square that you want to represent an affordable house.



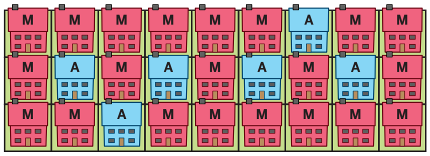
Create a tape diagram to represent your neighborhood.

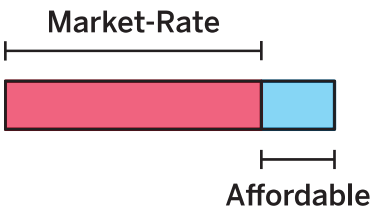


3 Metropolis requires a 7 : 2 ratio of market-rate housing units to affordable housing units.

How does your neighborhood compare to Metropolis's requirement?

Metropolis Neighborhood





Name: .... Date: .... Period: ....

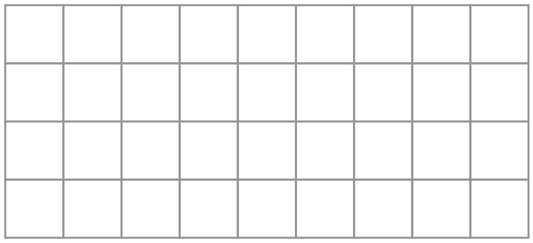
Affordable and Market-Rate Housing (continued)

4 Imagine you are a community planner. Your task is to make sure each neighborhood has a 7 : 2 ratio of market-rate units to affordable units.

A city is developing a new neighborhood with 36 units of land.

How many units of each type of housing should the city plan for?

Use the tape diagram or the grid if it helps with your thinking.



Market-Rate	Affordable	Total
		36

5 Metropolis requires a 7 : 2 ratio of market-rate units to affordable units.

Market-Rate	Affordable	Total
62	10	72

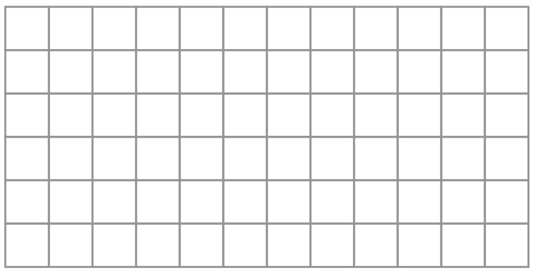
Does this neighborhood meet the requirement? Circle one.

Yes No I'm not sure

Explain your thinking. If you're not sure, what would help you be more sure?

6 How many units of each type of housing does this neighborhood need to meet Metropolis's requirement?

Use the tape diagram or the grid if it helps with your thinking.



Market-Rate	Affordable	Total
		72

Name: Date: Period:

Green Space

7 Urban green spaces, such as parks and gardens, give people space for physical activity, relaxation, peace, and an escape from the heat.

Here are two neighborhoods in Evergreen City.

Where would you prefer to live? Circle one.

Neighborhood A Neighborhood B

Explain your thinking.

Neighborhood A

Neighborhood B

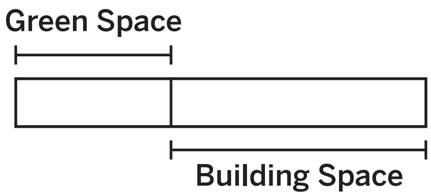
8 Evergreen City requires a $3 : 5$ ratio of units of green space to units of building space.

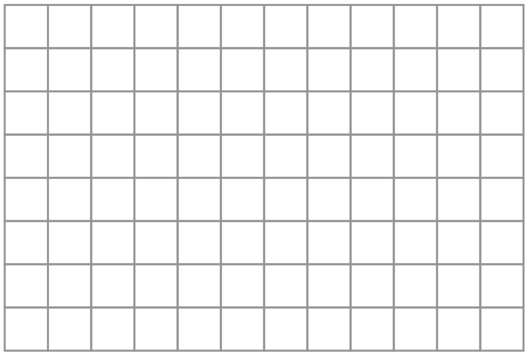
The city is developing 96 units of land for a new neighborhood.

How many of each type of space should the city plan for?

Use the tape diagram or the grid if it helps with your thinking.

Green Space	Building Space	Total
		96





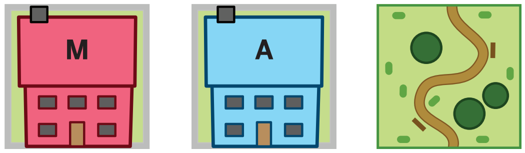
Name: ______ Date: ______ Period: ______

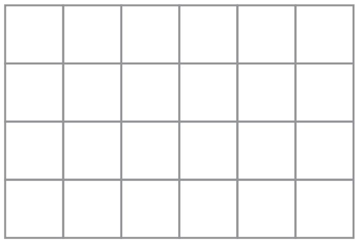
Green Space (continued)

9 Overall, Evergreen City requires a $4:1:3$ ratio of market-rate housing to affordable housing to green space.

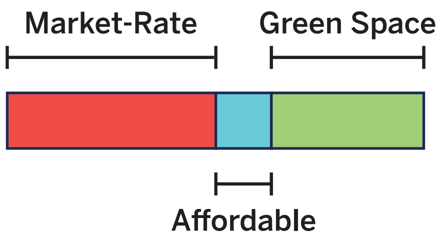
Here are 24 units of land.

Design a neighborhood that meets Evergreen City's requirements.
Check your work using the digital activity.



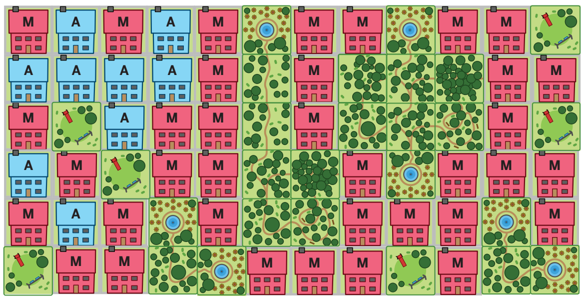


10 This neighborhood in Evergreen City meets the requirement of a $4:1:3$ ratio of market-rate housing to affordable housing to green space.



However, residents claim the neighborhood is not fair.

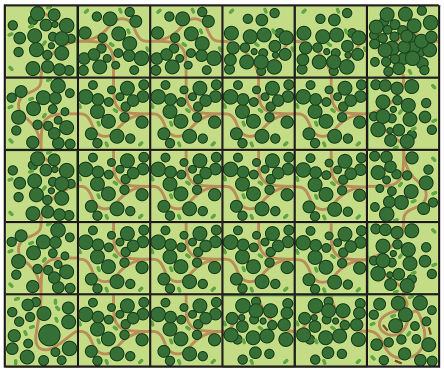
a Why might the residents feel it's not fair?



b What changes do you think should be made?

11 Synthesis

Des-Town requires a 3 : 2 ratio of building space to green space.



Discuss: How might a city planner determine how many units of building space can be developed in this neighborhood?

Draw on the diagram if it helps with your thinking.

14 Summary 2.14

Ratio tables, tape diagrams, and models can help us determine unknown amounts, which can help us solve real-world problems.

For example, Metropolis has requirements for the ratio of green space to building space in each new neighborhood development. The requirements say that there should be 2 units of green space for every 5 units of building space.

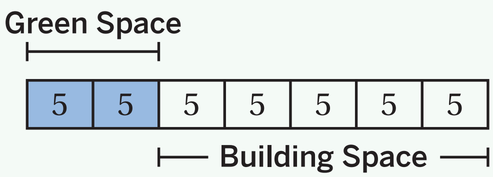
A new development has 35 units of land. Let's use both a ratio table and a tape diagram to determine how many units of building space can they build.

Ratio Table

Green Space	Building Space	Total
2	5	7
10	25	35

$35 \div 7 = 5$

$5 \cdot 5 = 25$



$7 \cdot 5 = 35$

$5 \cdot 5 = 25$

So for 35 total units of land, Metropolis will have 25 units of building space.

Name: .... Date: .... Period: ....

Problems 1–2: Pasta is made from 3 parts water and 5 parts flour. Sora is making 32 ounces of pasta for a party.

1. How much water does Sora need to make 32 ounces of pasta?

2. How much flour does Sora need to make 32 ounces of pasta?

Problems 3–4: Sora's salad dressing recipe uses 6 teaspoons of vinegar for every 15 teaspoons of olive oil.

3. Complete the table to show the amount of vinegar and olive oil needed to make each amount of salad dressing.

Vinegar (tsp)	Oil (tsp)	Salad Dressing (tsp)
		42
		14

4. Ariel used a recipe that makes 7 teaspoons of salad dressing by combining oil and vinegar. She used 3 teaspoons of vinegar. Could Ariel be using the same recipe as Sora? Explain your thinking.

5. A teacher is planning a class trip to the aquarium. The aquarium requires 2 adults to join for every 15 students. If the teacher orders 85 tickets, how many tickets are for adults and how many are for students? Show or explain your thinking.

Practice
2.14

Name: ______ Date: ______ Period: ______

Spiral Review

Problems 6–11: Determine each product mentally.

6. $34 \cdot 10$

7. $34 \cdot 100$

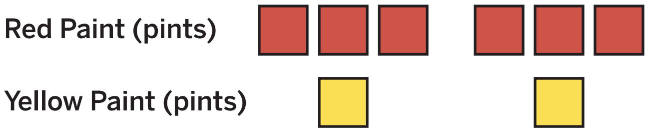
8. $340 \cdot 100$

9. $3.4 \cdot 10$

10. $3.4 \cdot 100$

11. $0.34 \cdot 100$

12. This diagram represents the pints of red and yellow paint in a mixture. Select *all* the statements that accurately describe the diagram.



- ☐ **A.** The ratio of pints of yellow paint to pints of red paint is 2 to 6.
- ☐ **B.** For every 3 pints of red paint, there is 1 pint of yellow paint.
- ☐ **C.** For every pint of yellow paint, there are 3 pints of red paint.
- ☐ **D.** For every pint of yellow paint, there are 6 pints of red paint.
- ☐ **E.** The ratio of pints of red paint to pints of yellow paint is 6 : 2.

Name: ______ Date: ______ Period: ______

Patterns Inside Numbers | Model the World | 6.RP.3, SMP.3, SMP.4, SMP.5

Lunch Waste

Let's use ratios to solve problems about lunch waste.

Warm-Up

1. What gets thrown away during lunch time at your school?

2. Estimate how much lunch trash your school creates each day.

Activity 1

Name: ____ Date: ____ Period: ____

How Much Waste?

ELD.PI.6.6.Em, Ex, Br, ELD.PI.6.10.Em, Ex, Br

Maria and Hoang noticed that their school creates a lot of trash during lunch time. After lunch one day, they weighed the trash from all the students in their class. They determined that 25 students threw away 10 pounds of trash. Of that trash:

- 1 pound was disposable lunch trays.
- 6 pounds were food.
- 3 pounds were other types of trash (wrappers, milk cartons, etc.).

Use these values to calculate the following:

3. How much lunch trash would the 400 students in their school create in a day? Show or explain your thinking.

4. Most schools have 180 school days each year. How much trash would the school generate in a school year? Show or explain your thinking.

5. How much of the yearly trash is disposable lunch trays, food, and other trash? Show or explain your thinking.

Disposable Lunch Trays	Food	Other Trash
____ pounds	____ pounds	____ pounds

Name: ______ Date: ______ Period: ______

Cutting Waste

Maria and Hoang want to explore different ways to reduce the amount of trash their school creates.

6. One way to reduce trash is to switch from disposable lunch trays to reusable trays.

How much yearly trash could the school reduce by switching from disposable lunch trays to reusable trays?

7. Composting is another way to reduce food trash. Maria and Hoang want to cut the amount of food that gets thrown away each year by 5,000 pounds.

How many students need to compost their food trash at lunch every day to meet this goal? Show or explain your thinking. **ELD.PI.6.10.Em, Ex, Br**

Maria and Hoang want to set a goal for reducing total yearly trash at the school.

8. What do you think is a reasonable goal? Explain your thinking. **ELD.PI.6.11.Em, Ex, Br**

9. Write a plan for how you think Maria and Hoang can achieve this goal. Include specific details about who they should share the recommendation with and how the school can achieve the goal.

You're invited to explore more.

10. A stack of 125 disposable lunch trays is approximately 24 inches tall. 400 students use disposable lunch trays at lunch every day. After how many days could the students make a 21-foot tall stack of trays?

Synthesis

11. Write a plan that includes the following:

a Possible ways to study the amount of lunch trash your school generates each year.

b How you could reduce that trash.

c Who you would share your recommendation with. Explain your thinking.

Summary 2.15

We can use ratios to understand issues in our lives and help develop solutions.

For example, let's say Maria decided to do a food waste experiment at home. She determined that her family threw away 9 pounds of trash in 5 days. Of that 9 pounds of trash, 3 pounds were plastic, 4 pounds were food, and 2 pounds were other waste.

You can use ratios to learn more about her family's waste habits.

- You can determine how much trash Maria's family would throw out in a year (365 days).

  365 days is 73 times as long as her experiment.

  $9 \cdot 73 = 657$ pounds of trash

- Then you can determine how much of that yearly trash would be plastic, food, and other waste.

  Plastics: $\frac{3}{9} \cdot 657 = 219$ pounds

  Food: $\frac{4}{9} \cdot 657 = 292$ pounds

  Other: $\frac{2}{9} \cdot 657 = 146$ pounds

Practice
2.15

Name: ______ Date: ______ Period: ______

Spiral Review

Problems 1–3: Oscar wants to both spend and save the money he earns. For every $\$7$ he puts in his wallet, he puts $\$3$ in savings.

1. Draw a tape diagram to represent this situation.

2. If Oscar put $\$70$ in his wallet, how much money would he put into savings? Show your thinking.

3. If Oscar earns $\$70$ total, how much will be put into savings? Show your thinking.

4. The first floor of a doll house contains the living room, kitchen, and dining room. The combined area of these three rooms is 189 square inches. The areas of the living room, kitchen, and dining room are in the ratio $4 : 3 : 2$.

What is the area of each room? Show or explain your thinking.

Problems 5–7: Here are some pairs of equivalent ratios. Show or explain how you know they are equivalent.

5. $15 : 6$ and $10 : 4$

6. $40 : 15$ and $8 : 3$

7. $40 : 15$ and $200 : 75$

8. A class of 30 students shares a set of 150 crayons. Which statement is true on a day with 5 absent students?

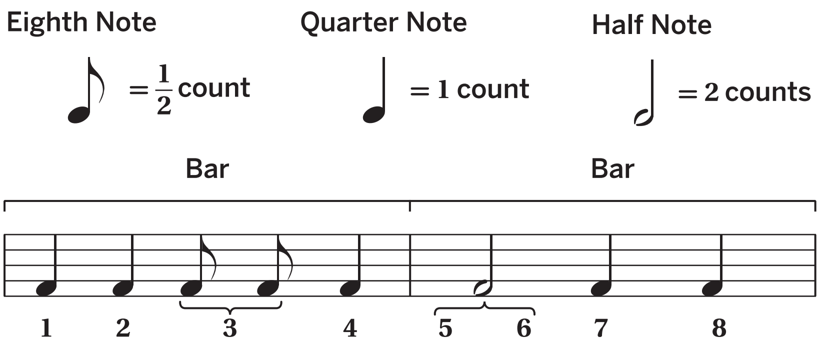
A. For every 5 students, there is 1 crayon.

B. For every 6 students, there is 1 crayon.

C. There are 5 crayons for each student.

D. There are 6 crayons for each student.

For Problems 9–11, use this information. Here are the notations for representing three types of notes in a musical composition. The composition of notes shown here has two sections, called bars.



9. How many counts are in each bar?

10. "Twinkle, Twinkle Little Star" has a total of 48 counts. If *Twinkle, Twinkle Little Star* has the same number of counts per bar as the composition shown, how many bars are in the entire song?

11. Traditionally, "Twinkle, Twinkle Little Star" is played at 100 counts, or beats, per minute. How long will it take, to the nearest second, to play the entire song?

Name: ________ Date: ________ Period: ________

Patterns Inside Numbers | Fraction Relationships | Model the World | 6.RP.3, 6.NS.4, SMP.7

Practice Day 2

Let's practice what you've learned so far in this unit!

You will use problem cards for this Practice Day. Record all of your responses here.

Card 1

________ van-accessible parking spaces

Card 2

________ van-accessible parking spaces

Card 3

________ kilograms of honey

Card 4

________ bees

Card 5

a ________ adults

b ________ tickets

Card 6

a ________ adults

b ________ tickets

Card 7

Circle one: Yes or No

Card 8

Circle one: Biking Swimming

Name: ________ Date: ________ Period: ________

Practice Day 2

Card 9

Circle one: Yes or No

Card 10

________ tacos

Card 11

________ cups of flour

Card 12

________ cups of chocolate chips

Card 13

Card 14

Card 15

Card 16

You're invited to explore more.

1. Josiah's recipe for sparkling orange juice uses 3 parts orange juice mixed with 4 parts soda water. Josiah made a mistake and added 5 cups of orange juice to 2 cups of soda water.

   How much of each ingredient could Josiah add to fix his drink mix?

Notes:

Career Connection

How can ratios help drummers "keep the beat"?


AleksKo/Shutterstock.com

Jazz drummer Clayton Cameron is known for using math ratios to refine the art of using drum brushes to play the drums, known as the "brush technique." He has even coined the term "a-rhythm-etic" to describe how the rhythm of a piece of music is based on ratios.

Music is measured by ratios called *time signatures*, such as $\frac{4}{4}$ or $\frac{3}{4}$. The first number tells how many beats to count. The second number tells the kind of note. If the second number is 4, the notes are quarter notes. A $\frac{4}{4}$ time signature would count $1, 2, 3, 4, 1, 2, 3, 4$, and so on. A $\frac{3}{4}$ measure of music would count $1, 2, 3, 1, 2, 3$, and so on.

Drummers keep the beat of a song by using ratios to maintain a steady rhythm, which helps other musicians playing with them. They can change the beat to give different feelings to the sound.

Clayton Cameron

Born in Los Angeles, California, Clayton Cameron is a lecturer on Global Jazz Studies at the UCLA Herb Alpert School of Music. After receiving a degree in music from California State University at Northridge, he became a rising star in the music industry, performing as a percussionist with many award-winning acts. He led a TED Talk called "A-rhythm-etic. The Math Behind the Beats" to explain the mathematical ratios behind his drumming techniques.

Are you interested in studying the relationship between math and music or becoming a musician? What can you do to learn more?

Community Connection

To play an octave on a piano, you would play two notes that span a distance of 8 white keys.

- The two notes have pitches — how high or low the sound is — in a ratio of $2 : 1$.
- Each octave doubles the pitch of a note.

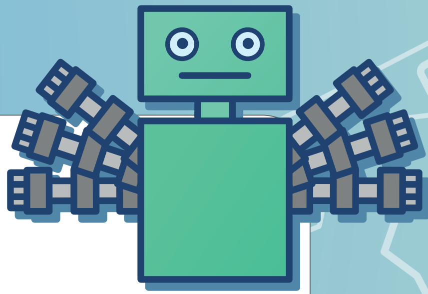
If two notes played on a piano have pitches in a ratio of $8 : 1$, how many octaves apart are they?


Lunyov Andrey/Shutterstock.com

Math Mindset

How can you use number lines, tables, or tape diagrams to help you think about equivalent ratios?

Unit 3



Unit Rates and Percentages

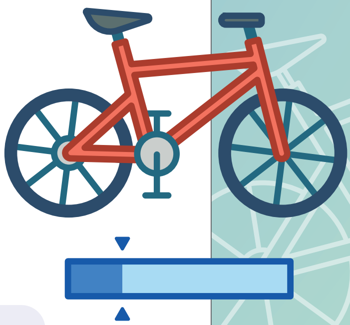
Big Ideas in This Unit

CC2 Generalizing With Multiple Representations | Fraction Relationships

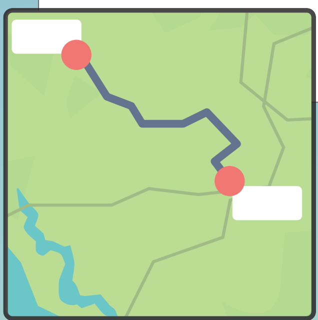
CC3 Model the World

Questions for Investigation

- How are the terms same rate, constant rate, and unit rate alike and different?
- What is the relationship between unit rates and percentages?
- How are percentages used to estimate and compare quantities?



Explore: Student Council Representation
How can we make sure everyone has fair representation?



Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!

Not yet Almost I got it!

I can . . .	Before	After
Compare different units of measure of length, volume, and mass/weight		
Explain why it takes more of a smaller unit to measure the same quantity.		
Convert measurements from one unit to another in the same system of measurement.		
Convert measurements from one unit to another in different measurement systems.		
Compare rates that are written in different units.		
Calculate and interpret the two unit rates for the same relationship.		
Choose which unit rate to use to solve a problem and explain my choice.		
Use unit rates to complete a table of equivalent ratios.		
Make comparisons and calculate unknown quantities using unit rates.		
Use the word percent and the symbol % to mean "for every 100."		

I can . . .	Before	After
Calculate 10%, 25%, 50%, or 75% of a number.		
Make connections between percentages and ratios		
Create tape diagrams, double number line diagrams, or tables to determine unknown parts, percentages, or wholes.		
Calculate any percentage of a number.		
Calculate an unknown percentage.		

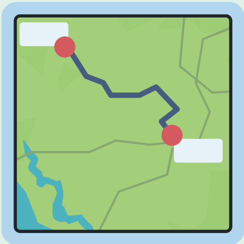
Sub-Unit 1

Units and Measurement

Explore
A Fair Vote

Lesson 1
Many Measurements

Lesson 2
Counting Classrooms



Lesson 3
Pen Pals

Name: Date: Period:

Generalizing With Multiple Representations | Model the World | 6.RP.3, SMP.1, SMP.2

Explore: A Fair Vote

How can we make sure everyone has fair representation?

Warm-Up

1. The students at a middle school are preparing to vote for the members of their Student Council. The school uses this table to determine who can be elected to each office.

Office	Grade
President	8
Vice President	8
Secretary	7
Treasurer	7
Historian	6
Spirit Commissioner	6

What do you notice? What do you wonder?

I notice:

I wonder:

Activity

Name: ________ Date: ________ Period: ________

Who Disagrees More?

Students in one class from each grade were asked whether they agreed or disagreed with the following statement: “The current structure of the Student Council is fair.”

ELD.PI.6.6.Em, Ex, Br

	Agree	Disagree
Class A (Grade 8)	14	10
Class B (Grade 7)	9	15
Class C (Grade 6)	12	18

2. Rank the classes in order of disagreement – the class that disagrees most strongly to the class that disagrees least strongly. Show or explain your thinking.

Class ________ Class ________ Class ________

Disagrees most strongly **Disagrees least strongly**

Refer to the table at the top of this page. Suppose the votes of each class (Class A, Class B, and Class C) represent the portion of the 100 votes from that grade who agree or disagree with the current structure.

3. Based on this, how many students in each grade will vote to change the structure? How many students will vote to keep the structure? Show or explain your thinking.

	Keep the structure	Change the structure
Grade 8	14	10
Grade 7	9	15
Grade 6	12	18

Name: Date: Period:

Who Disagrees More? (continued)

4. Based on this data, will the structure of the Student Council be changed? Explain your thinking. **ELD.PI.6.11.Em, Ex, Br**

The current Student Council and the school's administration met to determine how to restructure the Student Council for future elections. They decided to keep the six existing offices and add four more offices, each with the title of Representative. The table shows the school's current enrollment by grade.

Grade	Students
8	250
7	285
6	320

5. Any student can be elected to any of the 10 positions. How should the offices be filled so that they provide the fairest representation of the entire student population? Show or explain your thinking. **ELD.PI.6.11.Em, Ex, Br**

You're invited to explore more.

The principal reconsiders how many students will be given a vote. The same number of students from each grade will be given a vote. The class from each grade (Class A, Class B, or Class C) represents how all of these students from that grade will vote.
What is the fewest number of students who can vote from each grade so that the number of votes for keeping or changing the structure are whole numbers? Show or explain your thinking.

Activity

Name: ______________________ Date: ____________ Period: ____________

Building Math Habits of Mind

Discuss:

- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it.

Not yet **Almost** **I got it!**

I can represent real-world problems and interpret their solutions within the context of the problem.

Not yet **Almost** **I got it!**

I can justify my thinking and ask questions to help me understand the thinking of others.

Not yet **Almost** **I got it!**

I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed.

Not yet **Almost** **I got it!**

I can select an appropriate tool to help me solve problems.

Not yet **Almost** **I got it!**

I can communicate my thinking and solutions clearly to others.

Not yet **Almost** **I got it!**

I can look for structure or patterns to help me solve problems.

Not yet **Almost** **I got it!**

I can look for repeated calculations and other repeated steps to make generalizations.

Not yet **Almost** **I got it!**

Name: ________ Date: ________ Period: ________

Model the World Building Toward 6.RP.3.d, SMP.3, SMP.6

Many Measurements

Let's connect units of measure with everyday objects.

Warm-Up

1. Which do you think is taller? Circle one.

A coconut   A pineapple

2. Which do you think is larger? Circle one.

A grapefruit   A plum

3. Which do you think is heavier? Circle one.

A cherry   A grape

Activity 1

Name: ______ Date: ______ Period: ______

Describe It

4. **Discuss:** Use words, drawings, hand gestures, familiar objects, or other strategies to answer the question: *How much is* ______?

ELD.PI.6.1.Em, Ex, Br, ELD.PI.6.12.Em, Ex, Br

- ☐ 1 foot
- ☐ 1 meter
- ☐ 1 gallon
- ☐ 1 millimeter
- ☐ 1 cup
- ☐ 1 square foot
- ☐ 1 yard
- ☐ 1 pound

5. Which measurements were *less* complicated to describe? Which measurements were *more* complicated to describe?

Less Complicated	More Complicated

6. Sort the Activity 1 Cards based on whether they measure length, volume, or weight. There will be four cards in each group.

Length	Volume	Weight

7. Sort the measurements in each group from the *smallest* unit to the *largest* unit.

	Smallest Unit			Largest Unit
Length				
Volume				
Weight				

Name: .... Date: .... Period: ....

Match It

8. Match each Activity 2 Card with the unit of measurement that best represents it.

1 Kilogram Card	**1 Ounce** Card	**1 Millimeter** Card ....
1 Mile Card	**1 Liter** Card	**1 Gram** Card ....
1 Kilometer Card	**1 Pound** Card	**1 Cup** Card ....
1 Milliliter Card	**1 Gallon** Card	**1 Centimeter** Card ....

9. **Discuss:** Choose *one* of the measurements from Problem 8. What else could you measure with this unit of measurement? **ELD.PI.6.1.Em, Ex, Br**

You're invited to explore more.

10. Here are four unit conversions arranged by what they're measuring. Add any other unit conversions you can think of for each category.

Length	Weight
1 foot = 12 inches	1 kilogram = 1000 grams

Volume	Time
1 gallon = 4 quarts	1 hour = 60 minutes

Synthesis

11. **a** List several things you could measure about this can.

b What units would you use to measure each of those things?

ELD.PI.6.10.Em, Ex, Br

Summary 3.01

Units of measurement can be used to describe things like length, volume, and weight or mass. Certain units of measurement might be more appropriate to use than others, depending on what you're measuring. Here are some examples of units of measurement, arranged from the *smallest* unit to the *largest* unit.

Length	Volume	Weight
millimeter	milliliter	gram
centimeter	fluid ounce	ounce
inch	cup	pound
foot	quart	kilogram
yard	liter	ton
meter	gallon	
kilometer		
mile		

Practice
3.01

Name: .... Date: .... Period: ....

Problems 1–3: For each pair, circle the larger unit of measurement.

1. **A.** Meter
B. Kilometer

2. **A.** Yard
B. Foot

3. **A.** Pound
B. Ounce

4. Match each object with the unit you would most likely use to measure it.

- **a** The height of a building. .... Gallons
- **b** The length of a fingernail. .... Centimeters
- **c** The weight of a paper clip. ....Grams
- **d** The distance between two cities. .... Pounds
- **e** The weight of a bowling ball. .... Feet
- **f** The volume of a water cooler. .... Kilometers

5. Determine whether each unit of measurement measures length, volume, or weight.

Unit	Length	Volume	Weight
Yard			
Milliliter			
Fluid Ounce			
Pound			
Ounce			

Name: ________ Date: ________ Period: ________

Problems 6–8: Identify a unit that can be used to measure:

6. The length of a neighborhood road.

7. The volume of a car's gas tank.

8. The weight of a newborn baby.

Spiral Review

Problems 9–10: Determine each quotient. Show or explain your thinking.

9. $1275 \div 15$

10. $1500 \div 25$

11. In a jazz orchestra, there is a horn section and a rhythm section. The ratio of horn players to rhythm players is 13 to 4. What is the ratio of rhythm players to total players?

A. $4 : 13$

B. $13 : 17$

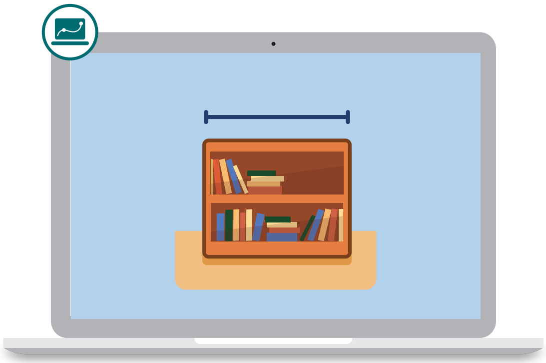
C. $4 : 9$

D. $4 : 17$

Unit 3
Lesson
2

Name: ______ Date: ______ Period: ______

Model the World 6.RP.3.d, SMP.2, SMP.3, SMP.6



Counting Classrooms

Let's measure using different units

Warm-Up

Evaluate each expression mentally. Try to think of more than one strategy.

1 $\frac{1}{3}$ of 15

2 $\frac{2}{3}$ of 15

3 $\frac{2}{3} \cdot 15$

4 $\frac{2}{5} \cdot 20$

Activity
1

Name: ______ Date: ______ Period: ______

Classroom Lengths

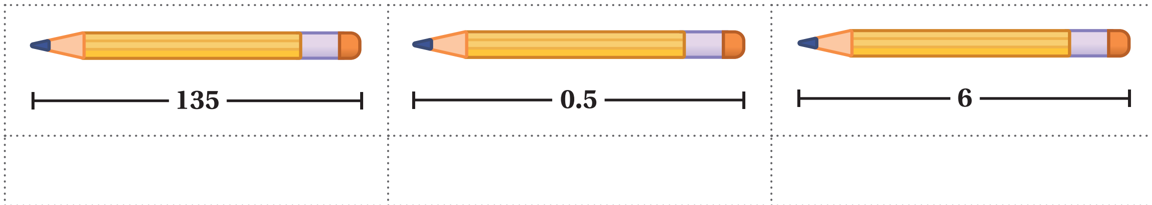
5 Complete the table by measuring different objects around your classroom.

Object	Inches	Feet	Millimeters	Meters
Width of doorway				
Height of waste bin				

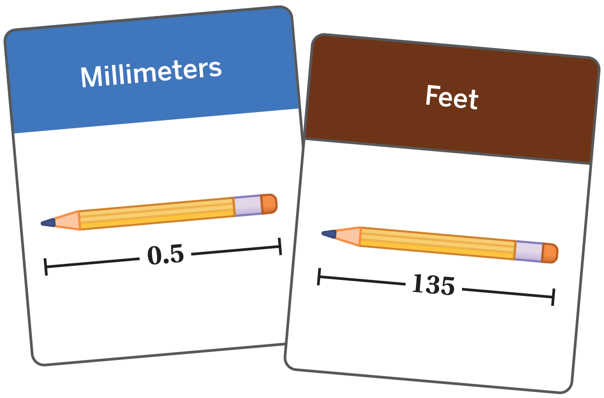
Discuss: Which units were most useful for measuring each object?

6 Here are some measurements for Sahana's pencil. Match each unit to the appropriate value. One unit will not have a match.

Feet	Meters	Millimeters	Inches



7 Amara incorrectly matched these two pairs of cards. What would you say to help her understand her mistake?



Name: .... Date: .... Period: ....

Same Amount, Different Units

8 Sahana's classroom is 6 yards wide.

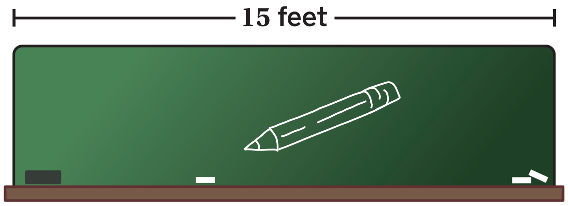
Her teacher wants to buy a new chalkboard that is 15 feet wide.

Will the new chalkboard fit on the wall? Circle one.

Yes No I'm not sure

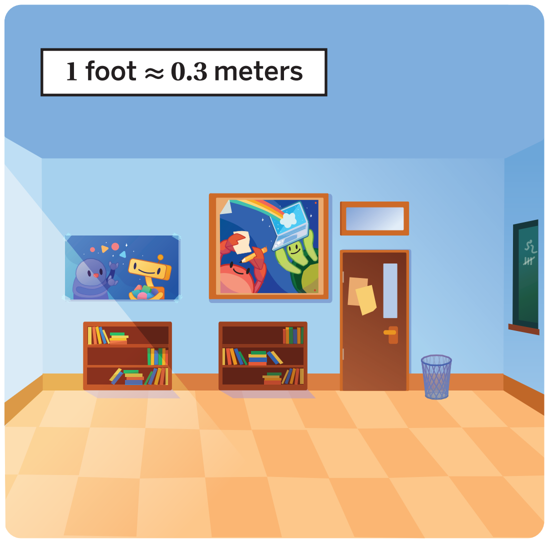
Explain your thinking.

1 yard = 3 feet



9 The classroom wall is 12 feet tall.

How many meters is that?



10 Ama tried to determine how many meters are in 12 feet and got stuck.

Discuss: What did Ama do? What might she do next?

Ama

ft	m
1	0.3
12	?

Activity 3

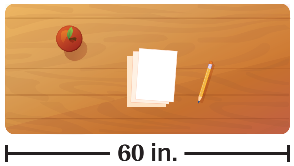
Name: .... Date: .... Period: ....

Repeated Challenges

11 Solve as many challenges as you have time for.

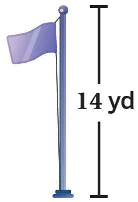
The desk is 60 inches wide. How many feet is that?

12 inches = 1 foot



The flagpole is 14 yards high. How many feet is that?

1 yard = 3 feet

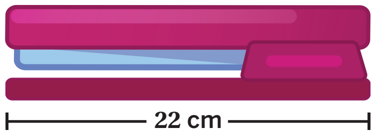


The mug holds 780 milliliters of liquid. How many cups is that?

240 milliliters = 1 cup

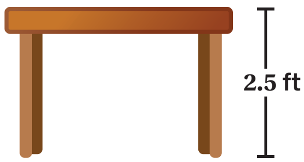
The stapler is 22 centimeters long. How many millimeters is that?

1 centimeter = 10 millimeters



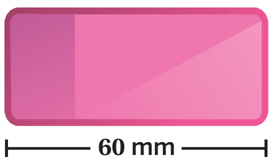
The desk is 2.5 feet tall. How many meters is that?

1 foot ≈ 0.3 meters



The eraser is 60 millimeters long. How many centimeters is that?

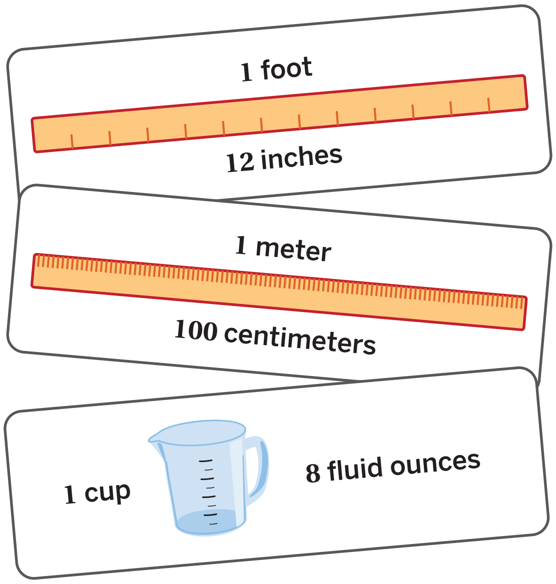
1 centimeter = 10 millimeters



12 Synthesis

Sahana says: *When you measure an object using different units, you need more of the smaller unit.*

Discuss: What do you think about Sahana's claim? Explain your reasoning.



15 Summary 3.02

When you're measuring the same quantity with different units, you need more of the smaller unit and fewer of the larger unit to describe the measurement. So a room that's 3 meters long will measure approximately 10 feet in length because a meter is longer than a foot.

The size of the object can help to determine the best unit of measurement.

For example, the length of the bottom edge of this notebook is 22 centimeters or 220 millimeters.

- It takes more millimeters to describe the length because millimeters are smaller than centimeters.
- You may choose to describe the length in centimeters instead of millimeters because of the size of the notebook.

Name: ______ Date: ______ Period: ______

1. Juana says: *This classroom is 11 meters long. A meter is longer than a yard, so if I measure the length of this classroom in yards, I will get less than 11 yards*. Is Juana correct? Explain your thinking.

Problems 2–6: Use the conversion rate that makes the most sense to determine the approximate value of each missing quantity. Show or explain your thinking.

1 pound ≈ 454 grams	1 liter ≈ 4.2 cups
1 meter ≈ 1.1 yards	1 inch = 2.54 centimeters
1 kilogram ≈ 35 ounces	1 kilogram ≈ 2.2 pounds

2. 4 pounds ≈ ______ grams

3. 5 inches = ______ centimeters

4. 110 yards ≈ ______ meters

5. 7 liters ≈ ______ cups

6. Tyron wants to mail a package that weighs $4\frac{1}{2}$ pounds. Which of the following could be the weight of the package in kilograms?

A. 2.04 kilograms

B. 4.5 kilograms

C. 9.92 kilograms

D. 4,500 kilograms

Practice
3.02

Name: ______ Date: ______ Period: ______

Spiral Review

7. Do each of these units measure length, volume, or weight?

Unit	Length	Volume	Weight
mile			
cup			
pound			
milliliter			
yard			
kilogram			
liter			
centimeter			

Problems 8–11: Use the conversion rate that makes the most sense to determine the approximate value of each missing quantity.

1 foot $= 12$ inches	1 yard $= 3$ feet
1 meter $= 100$ centimeters	1 yard $= 36$ inches

8. A room is 30 feet wide. How many yards is that?

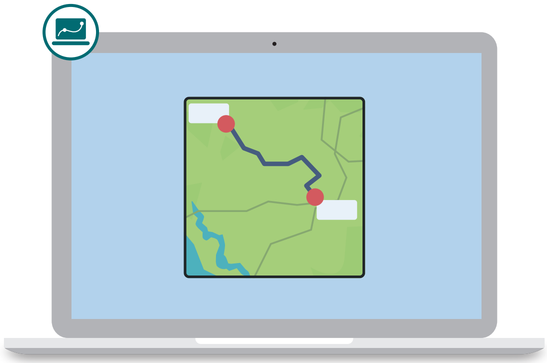
9. A door is 84 inches tall. How many feet is that?

10. A car is 4.5 meters long. How many centimeters is that?

11. A table is 72 inches wide. How many yards is that?

Name: .... Date: .... Period: ....

Model the World 6.RP.3.d, SMP.2, SMP.5, SMP.6



Pen Pals

Let’s compare measurements in different units.

Warm-Up

1 Four pen pals share letters with each other.

Discuss: What do you notice? What do you wonder?

Name	Eva	Ayaan	Thiago	Binta
Country	United States	India	Brazil	Liberia
Favorite Food	Bubble tea	Mango lassi	Quindim	Spaghetti
Favorite Animal	Horse	Dog	Horse	Bird
Favorite Sport	Football	Cricket	Futebol	Football

Activity
1

Name: ______________________ Date: __________ Period: __________

Traveling to School

2 The pen pals discuss how far they each live from school. Use your best estimates to order the pen pals from *closest* to *farthest* from school.

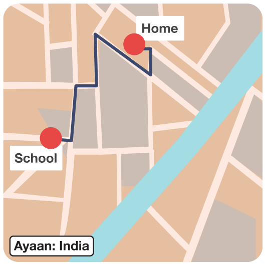
Thiago: Brazil

20 kilometers

Eva: United States

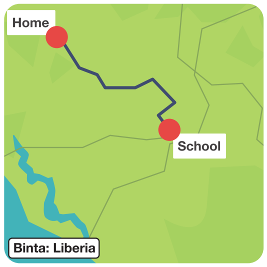
2,000 feet

Ayaan: India



900 meters

Binta: Liberia



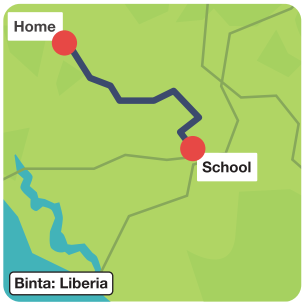
15 miles

Closest **Farthest**

3 Binta lives 15 miles from her school in Liberia. Thiago lives 20 kilometers from his school in Brazil.

Who lives closer to their school? Circle one and explain your thinking.

Binta Thiago About the same distance

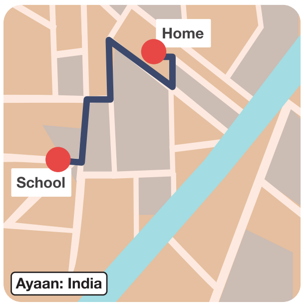


8 kilometers ≈ 5 miles

4 Ayaan lives 900 meters from his school in India. Eva lives 2,000 feet from her school in the United States.

Who lives closer to their school? Circle one and explain your thinking.

Ayaan Eva About the same distance



3 meters ≈ 10 feet

Activity
2

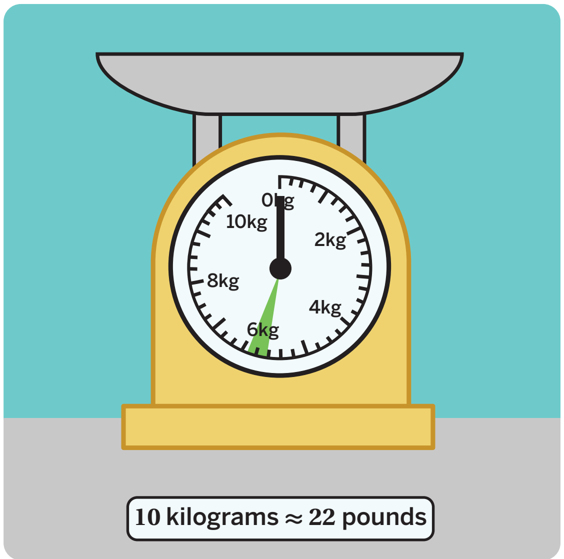
Name: .... Date: .... Period: ....

Weighing Strategies

5 Thiago’s horse eats about 6 kilograms of hay per day.

Eva wants to know how many pounds that is.

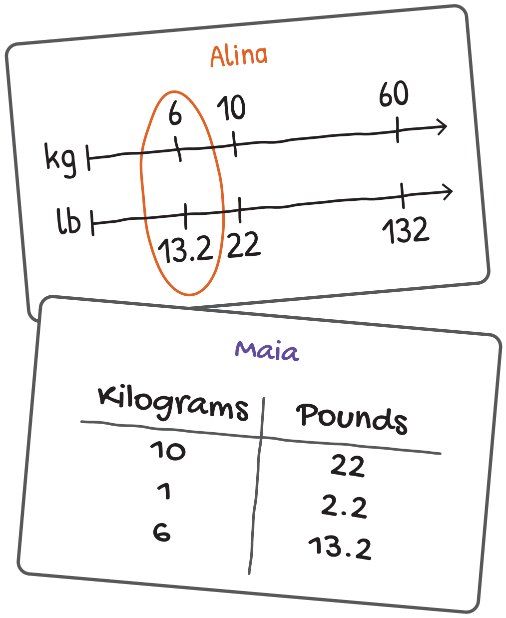
About how many pounds is 6 kilograms?



6 Alina and Maia both determined how many pounds of hay Thiago's horse eats per day.

Alina used a double number line and Maia used a table.

Choose a student and explain their thinking.



Activity
3

Name: .... Date: .... Period: ....

Favorite Things

7 Binta decides to make Ayaan's recipe for mango lassi.

The recipe calls for 135 milliliters of milk.

About how many tablespoons of milk should Binta use?

MANGO LASSI
Ingredients
Serves 2 people

- 250 milliliters mango pulp
- 240 milliliters yogurt
- 135 milliliters milk
- 20 grams sugar
- 1 gram cardamom powder

30 milliliters ≈ 2 tablespoons

8 Thiago decides to make Binta's recipe for spaghetti.

The recipe calls for 450 grams of bell pepper.

About how many ounces of bell pepper should Thiago use?

SPAGHETTI
Ingredients

- 1 box of spaghetti
- 450 grams of bell pepper
- 200 grams of tomato
- 150 grams of onion
- 3 bouillon cubes
- 400 grams of ground beef
- 800 grams of Italian sausage
- Habanero peppers, curry powder, ginger, salt, oil

200 grams ≈ 7 ounces

You're invited to explore more.

9 People have known for over 2,000 years that Earth is round, but it took a long time to discover how big it is.

A Greek mathematician named Eratosthenes was the first known person to calculate the distance around Earth's equator. In about 240 BCE, he calculated the distance around Earth's equator to be about 250,000 stadia using an estimated distance from Alexandria to Syene, along with the lengths of shadows.

The actual distance is about 24,901 miles.

What is the difference, in miles, between Eratosthenes's calculation and the actual distance? Explain your thinking.

10 Synthesis

Discuss: What is one strategy for converting a measurement from one unit to another?

Use the examples if they help you with your thinking.

8 kilometers ≈ 5 miles

200 grams ≈ 7 ounces

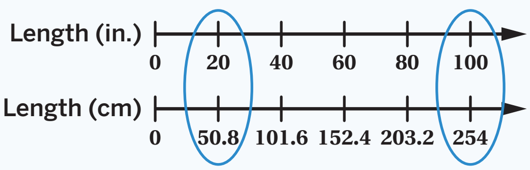
30 milliliters ≈ 2 tablespoons

13 Summary 3.03

You can use equivalent ratios to convert measurements from one unit to another.

For example, if you know that 100 inches = 254 centimeters, you can use a double number line or a table to convert 20 inches to centimeters, too.

Double Number Line



Ratio Table

Inches	Centimeters
100	254
1	2.54
20	50.8

÷ 100 ÷ 100

× 20 × 20

Practice
3.03

Name: .... Date: .... Period: ....

1. Malik is 57 inches tall. If 100 inches $= 254$ centimeters, which value is closest to his height in centimeters?

   A. 22.4 centimeters
   B. 57 centimeters
   C. 144.8 centimeters
   D. $3,551$ centimeters

Problems 2–5: Use the conversion rate that makes the most sense to determine the approximate value of each missing quantity. Show or explain your thinking.

1 kilogram $= 1000$ grams	3 ounces ≈ 85 grams
11 pounds ≈ 5 kilograms	4 kilograms ≈ 141 ounces

2. 15 ounces $\approx$ .... grams

3. 20 kilograms $\approx$ .... pounds

4. 2 kilograms $\approx$ .... ounces

5. 2500 grams $\approx$ .... pounds

6. A yard is equal to 3 feet, and there are $1,760$ yards in 1 mile. How many feet are there in 4 miles?

   A. $3,520$
   B. $5,280$
   C. $7,040$
   D. $21,120$

Practice
3.03

Name: ________________ Date: ________ Period: ________

7. Dhruv's family exchanged 250 dollars for 4,250 pesos. Complete the table to determine the conversions between pesos and dollars.

Dollars	Pesos
250	4,250
25	
1	
3	
	510

8. Here is a table showing how far some animals can sprint in 1 minute. Order these animals from longest sprint distance to shortest sprint distance.

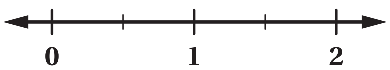
Note: 1 inch = 2.54 centimeters

Animal	Sprint distance
Cougar	1,408 yd
Antelope	1 mi
Hare	49,632 in.
Kangaroo	1,073 m
Ostrich	1.15 km
Coyote	3,773 ft

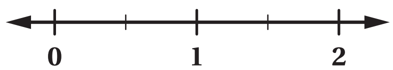
Spiral Review

Problems 9–12: Plot each value on the number line.

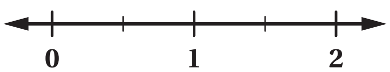
9. $\frac{2}{5}$



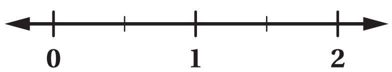
10. $1\frac{2}{5}$



11. $\frac{5}{8}$



12. $\frac{12}{8}$



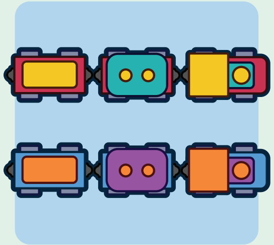
Notes:

Sub-Unit

2

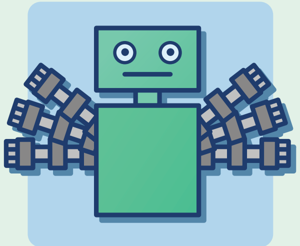
Unit Rates

Lesson 4
World Records

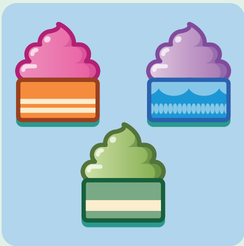


Lesson 5
Model Trains

Lesson 6
Soft Serve



Lesson 7
Welcome to the Robot Factory



Lesson 8
More Soft Serve

Name: Date: Period:

Generalizing With Multiple Representations Model the World

6.RP.2, 6.RP.3, 6.RP.3.a, 6.RP.3.b, SMP.3

World Records

Let's calculate unit rates and use them to compare speeds.

Warm-Up

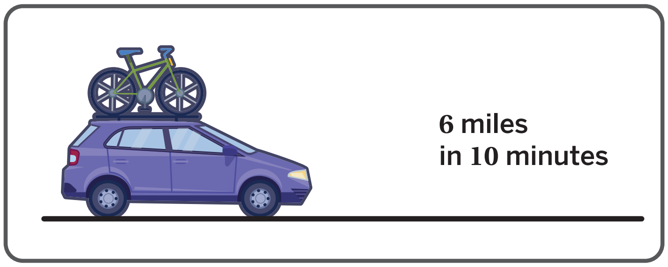
1. A car travels 6 miles in 10 minutes at a constant speed.

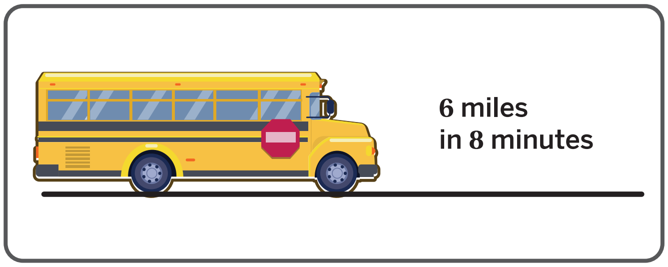
   A bus travels 6 miles in 8 minutes at a constant speed.

   Which travels faster? Circle one.

   Car Bus They travel the same speed.

   Explain your thinking. ELD.PI.6.6.Em, Ex, Br, ELD.PI.6.11.Em, Ex, Br





Activity 1

Name: ______ Date: ______ Period: ______

Moving 10 Meters

2. Let's look at some instructions for determining your walking speed.

Record your data.

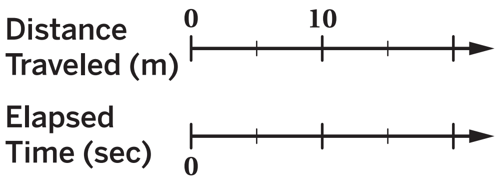
Distance: ______ meters Time: ______ seconds

3. A **unit rate** is a rate per 1. In this situation, the unit rate is the number of meters you can walk in 1 second.

Use your recorded data to calculate your unit rate and determine your walking speed.

Use the table or the double number line to help with your thinking.

Distance (m)	Time (sec)
10	
	1



4. At this rate, how long would it take you to walk 5 meters? 18 meters?

Activity
2

Name: .... Date: .... Period: ....

World Records

Here are three athletes who set records in different sports.

Danyil Boldyrev 15 m Climb	Keni Harrison 100 m Hurdle	César Cielo 50 m Swim
		
Fair Use	Fair Use	Fair Use

5. Predict these athletes' speeds from *slowest* to *fastest*. Include your 10 meter walk in the rankings.

Slowest **Fastest**

6. What information do you need to determine the actual order?

7. Let's look at some video clips from these athletes' competitions.

a Note the approximate times of each athlete.

Danyil Boldyrev	Keni Harrison	César Cielo
.... seconds	.... seconds	.... seconds

Name: ______ Date: ______ Period: ______

World Records (continued)

b Use the approximate times to revise your speed estimates. Show your thinking.

Slowest **Fastest**

c **Discuss:** Did your order change? **ELD.PI.6.3.Em, Ex, Br**

8. Let's look at the full-length videos from the athletes' competitions and the exact times for each record.

a Use the exact times to calculate the speed of each athlete.

	Event	Distance (m)	Exact Time (s)	Speed (m/s)
Danyil Boldyrev	Climbing	15	5.6	
Keni Harrison	Hurdling	100	12.2	
César Cielo	Swimming	50	20.91	
You	Walking	10		

b Determine the most accurate order. Show your thinking.

Slowest **Fastest**

Synthesis

9. Describe how unit rates can help you compare speeds. Use the tables if they help with your thinking. **ELD.PI.6.10.Em, Ex, Br**

Athlete 1

Distance (m)	Time (sec)
50	8

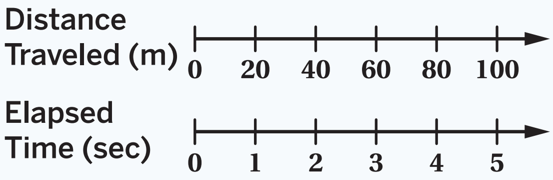
Athlete 2

Distance (m)	Time (sec)
60	10

Summary 3.04

A **rate** compares two quantities with different units. For example, 100 meters per 5 seconds is a rate that compares distance and time. Suppose there's a train traveling at this rate. You can use a table of equivalent ratios or a double number line to calculate the **unit rate**, which is 20 meters per 1 second.

Distance (m)	Time (sec)
100	5
20	1



Now you can use the unit rate to answer other questions about the train. For example, to determine how far the train travels in 30 seconds, you can just multiply the unit rate of 20 meters per second by 30 to get 600. That means the train travels 600 meters in 30 seconds.

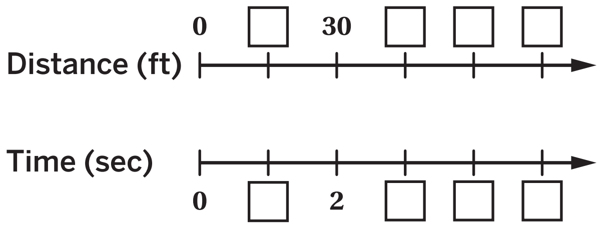
rate A comparison, or ratio, that describes how two quantities change together.

unit rate A rate that describes how one quantity changes when the other quantity changes by exactly 1 unit.

Name: .... Date: .... Period: ....

Problems 1–4: A person on a scooter travels 30 feet in 2 seconds at a constant rate.

1. Fill in the missing values on the double number line.



2. What is the speed of the scooter in feet per second?

3. At this rate, how long would it take the scooter to travel 105 feet?

4. A person on a skateboard travels 55 feet in 4 seconds. Is the skateboard traveling faster than, slower than, or at the same speed as the scooter?

A. Faster **B.** Slower **C.** The same speed

5. Order these animals from *slowest* to *fastest*.

Galapagos Tortoise

FOTOGRIN/Shutterstock.com

16 meters in 3 minutes

Garden Snail

Zebra-Studio/Shutterstock.com

8 meters in 5 minutes

Three-Toed Sloth

Nacho Such/Shutterstock.com

9 meters in 2 minutes

Slowest **Fastest**

6. Ariana gets paid $\$90$ for every 5 hours of work in her neighbor's garden. Last summer, Lucy got paid $\$36$ for every 2 hours of work in the same garden. Are they paid at the same rate? Show or explain your thinking.

7. Alisha buys 6 muffins. She spends $\$24$. If each muffin costs the same amount, how much does 1 muffin cost?

8. Fill in each blank using the numbers 0 to 9 only once so that Marc and Prisha have the same speed.

Marc

☐ **seconds**

☐☐ **meters**

Prisha

☐ **seconds**

☐☐ **meters**

Spiral Review

Problems 9–10: A recipe for pasta dough calls for 150 grams of flour per large egg.

9. How much flour do you need for 6 large eggs?

10. How many large eggs do you need for 450 grams of flour?

Problems 11–12: Mentally calculate each quotient.

11. $34 \div 10$

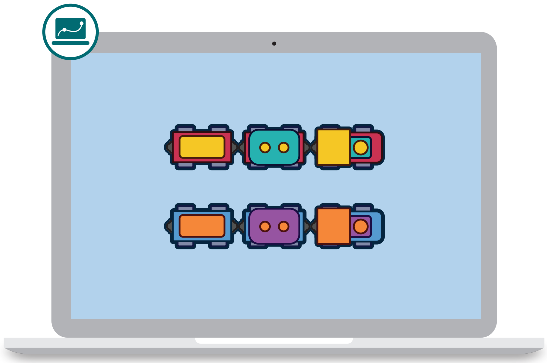
12. $3.4 \div 10$

Unit 3
Lesson
5

Name: .... Date: .... Period: ....

Generalizing with Multiple Representations | Model the World

6.RP.2, 6.RP.3.b, 6.RP.3.d, SMP.3, SMP.6



Model Trains

Let's use ratios to compare speeds.

Warm-Up

1 Which one doesn't belong? Circle one.

5 miles in 15 minutes	20 miles per 1 hour	3 minutes per mile	32 kilometers per 1 hour

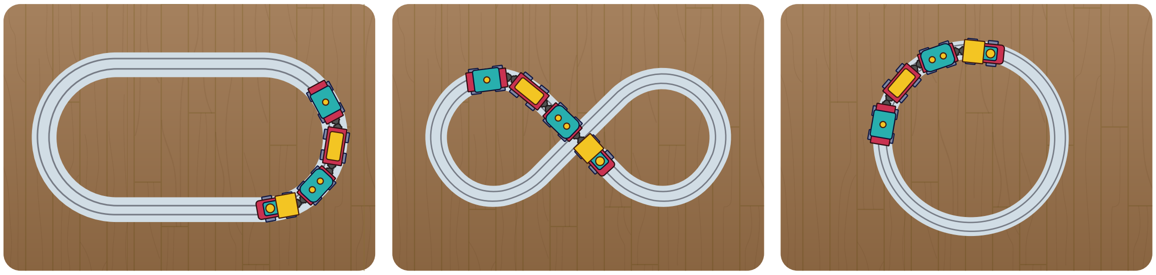
Explain your thinking.

Activity 1

Name: ______ Date: ______ Period: ______

How Fast?

2 A children's museum has three types of model train sets for students to build and play with. Let's watch how the train moves on each track.

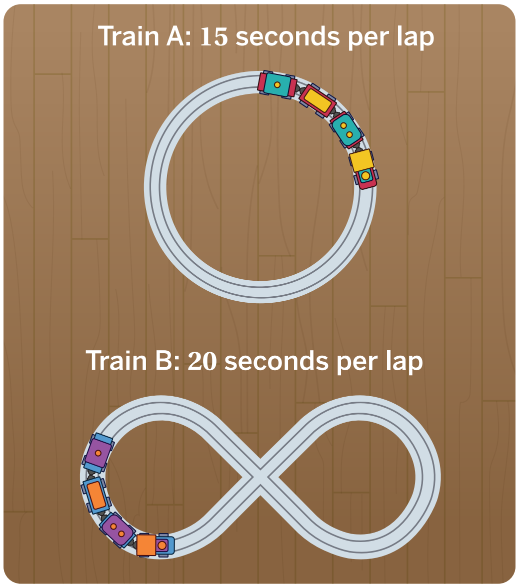


3 Here are trains from two students.

Which train is faster? Circle one.

Train A   Train B   Not enough information

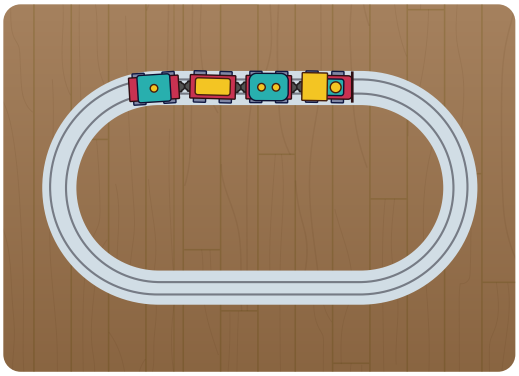
Explain your thinking. If you don't have enough information, what information would help you determine which train travels faster?



4 Here is a track. It is 325 centimeters long.

This train takes 10 seconds per lap.

What is its speed in centimeters per second?



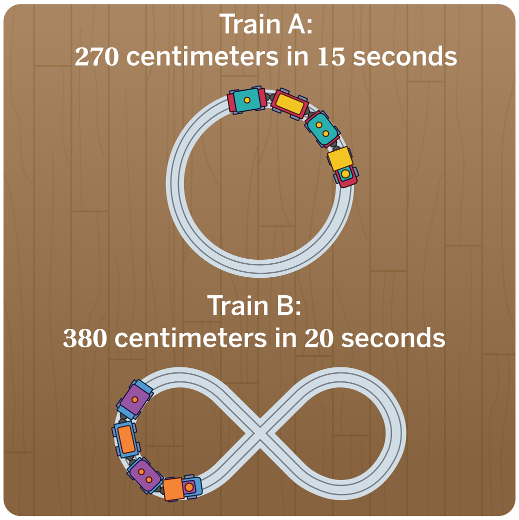
Name: .... Date: .... Period: ....

How Fast? (continued)

5 Which train is faster? Circle one.

Train A Train B They go the same speed

Explain your thinking.



6 Amoli and Tiam used different strategies to determine which train was faster.

Discuss: How are their strategies alike? How are they different?

Amoli

Train A

$270 \div 15 = 18$ cm per sec

Train B

$380 \div 20 = 19$ cm per sec

Train B is faster.

Tiam

Train A

cm	Sec
270	15
1080	60

Train B

cm	Sec
380	20
1140	60

Train B is faster.

Activity
2

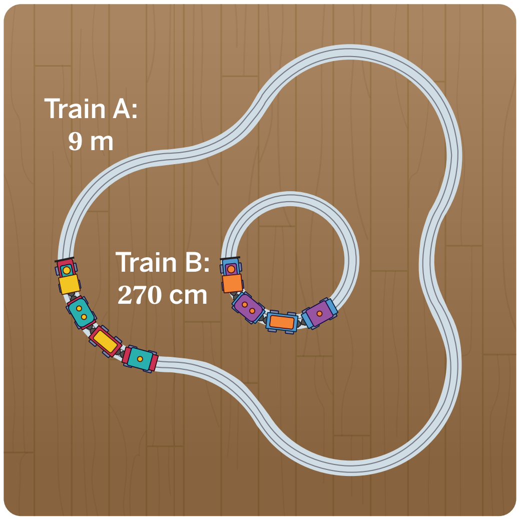
Name: Date: Period:

Which is Faster?

7 Here are two trains. They each complete a lap in 20 seconds.

What is each train's speed in centimeters per second?

	Speed (centimeters per second)
Train A	
Train B	



8 Here are distances and times for four model trains.

Order the trains by speed.

A. 3.25 meters in 1 minute

B. 3.25 meters in 20 seconds

C. 270 centimeters in 20 seconds

D. 325 centimeters in 30 seconds

1 meter = 100 centimeters

1 minute = 60 seconds

Slowest Fastest

You're invited to explore more.

9 A train's speed is 60 centimeters per second.

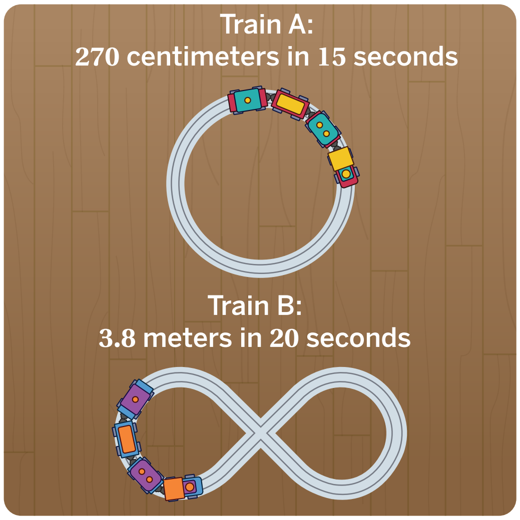
Write a track length. Then determine the number of laps the train can complete in 10 seconds.

Track Length (cm)	Laps in 10 Seconds

10 Synthesis

Describe *two* strategies for determining which of two trains is faster.

Use the examples if they help with your thinking.



13 Summary 3.05

When you're comparing different rates, like speeds, it's helpful to convert the rates to the same units of measurement. Then you can use equivalent ratios or unit rates to more accurately compare the rates.

Let's compare the speeds of two runners competing in different races.

- Runner A runs the 400-meter dash in 50 seconds.
- Runner B runs a 5-kilometer race in 20 minutes.

We can convert both of these speeds to meters per second knowing 5 kilometers equals 5,000 meters and 20 minutes equals 1,200 seconds.

Runner A

Seconds	Meters
50	400
1	8

8 meters per second

Runner B

Seconds	Meters
1,200	5,000
1	$4\frac{1}{6}$

$4\frac{1}{6}$ meters per second

Runner A runs at a faster rate because they ran a greater distance (8 meters) than Runner B ($4\frac{1}{6}$ meters) in the same amount of time (1 second).

Practice
3.05

Name: ________ Date: ________ Period: ________

Problems 1–3: Mia and Liam were trying out new remote control cars. Mia's car traveled 135 feet in 3 seconds. Liam's car traveled 228 feet in 6 seconds. Both cars traveled at a constant speed.

1. Determine the speed of each remote control car in feet per second.

Mia's Car's Speed	**Liam's Car's Speed**
________ feet per second	________ feet per second

2. Whose car traveled faster?

3. Deven says he has a remote control car that can travel 12 yards per second. Is his car faster or slower than the others' cars? Show your thinking.

4. Emmanuel types 208 words in 4 minutes. Vihaan types 342 words in 6 minutes. Both type at a constant rate. Who types faster? Explain your thinking.

5. Here are the approximate distances and times for four Olympic swimmers in different events. Order the swimmers from *slowest* to *fastest*.

Swimmer A: 800 meters in 8 minutes

Swimmer B: 100 meters in 50 seconds

Swimmer C: 1.5 kilometers in 15.5 minutes

Swimmer D: 50 meters in 20 seconds

Slowest **Fastest**

Practice
3.05

Name: .... Date: .... Period: ....

Problems 6–7: Penguin A walks 10 feet in 5 seconds. Penguin B walks 12 feet in 8 seconds. Each penguin keeps walking at those speeds.

6. How far does each penguin walk in 45 seconds?

7. If the two penguins start at the same place and walk in the same direction, how far apart will the two penguins be after 2 minutes? Show or explain your thinking.

Spiral Review

Problems 8–9: There are $3{,}785$ milliliters in 1 gallon, and there are 4 quarts in 1 gallon.

8. How many milliliters are in 3 gallons? Show or explain your thinking.

9. How many milliliters are in 1 quart? Show or explain your thinking.

Unit 3
Lesson
6

Name: ______ Date: ______ Period: ______

Generalizing With Multiple Representations | Model the World

6.RP.2, 6.RP.3, 6.RP.3.b, SMP.3, SMP.6

Soft Serve

Let's compare soft serve prices using unit rates.

Warm-Up

1 a. Take a look at the prices for different sizes of soft serve sold at a store.

S	M	L
\$2	\$2.80	\$3.60
5 oz	7 oz	9 oz

b. **Discuss:** Which size offers the best deal?

Activity 1

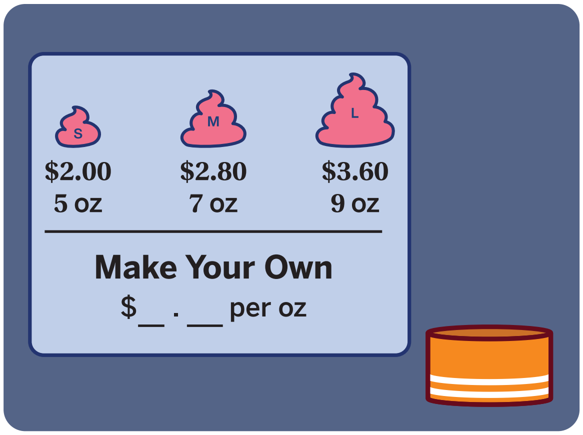
Name: .... Date: .... Period: ....

Two Unit Rates

2 Kala notices that soft serve costs the same per ounce no matter what size you get.

She suggests that the store put the rate on the menu.

How much does soft serve cost per ounce?



3 The store added the price per ounce, or **unit price**, to the menu.

A customer asks for 8 ounces of soft serve.

How much will this cost?

4 A new customer comes in with $3 and wants to spend it all on soft serve.

How many ounces can they get for $3?

Name: .... Date: .... Period: ....

Two Unit Rates (continued)

5 Here is how Neena figured out how much soft serve you can get for \$3.

	Cost (dollars)	Weight (ounces)	
	2	5	
÷2	1	2.5	÷2
×3	3	7.5	×3

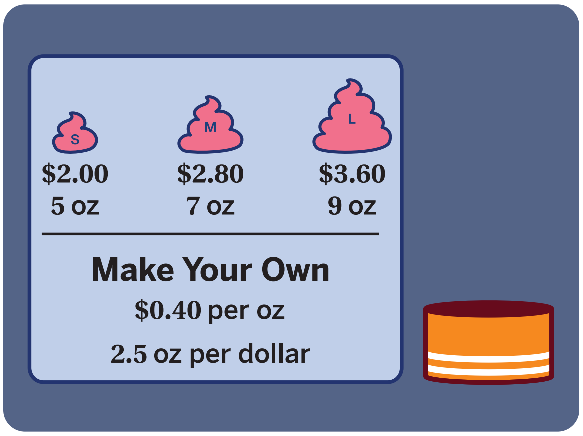
a **Discuss:** What was Neena's strategy?

b Explain or show where you can see *ounces per dollar* in Neena's work.

6 The store's menu now includes *both* unit rates.

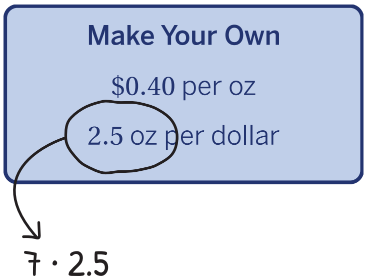
A new customer comes in with \$7 and wants to spend it all on soft serve.

How much soft serve can they get for \$7?



7 Here is how Jamal figured out how much soft serve you can get for \$7.

How do you think Jamal knew which unit rate to use?



Activity 2

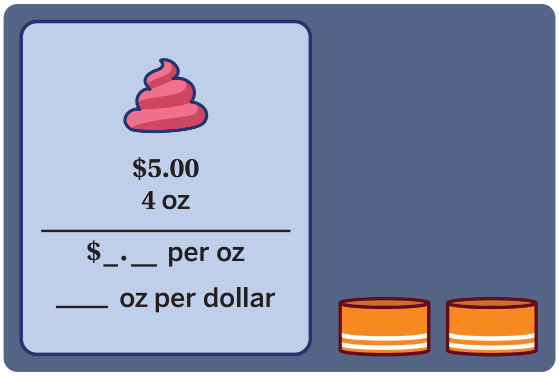
Name: .... Date: .... Period: ....

New Flavors

8 The store offers a new flavor, Swirl, with this pricing: $5 for every 4 ounces.

a How much does Swirl cost per ounce?

b How many ounces can you get per dollar?



9 How much does 7 ounces of Swirl cost?

Explain your thinking.

10 Match each rate with either chocolate or vanilla.

$2 per ounce	$0.50 per ounce	2 ounces per dollar	$\frac{1}{2}$ ounce per dollar	$9 for 4.5 ounces

Chocolate

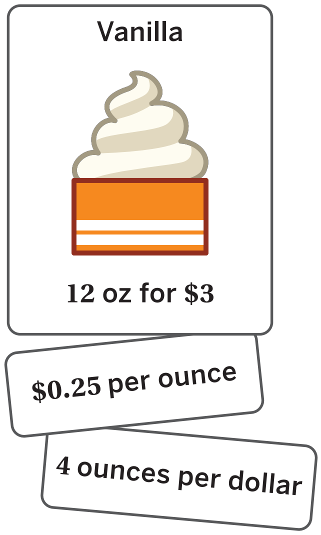
2 oz for $4

Vanilla

8 oz for $4

11 Synthesis

Discuss: How do you calculate the two unit rates for vanilla soft serve?



14 Summary 3.06

When two quantities are related in a ratio, you can describe the relationship using two different unit rates.

For example, the ratio $A : B$ can be represented as:

- The amount of Quantity A per 1 of Quantity B.
- The amount of Quantity B per 1 of Quantity A.

In situations that involve money, one of the two possible unit rates is the **unit price** (the price per unit of an item).

Let's say a store advertises 4 pounds of granola for $\$5$.

You can use a table to determine the two different unit rates.

- Price per 1 pound: $\$1.25$ per pound of granola. This is the unit price.
- Number of pounds per $\$1$: 0.8 pounds of granola per dollar.

Granola (lb)	Price ($)
4	5.00
1	1.25
0.8	1.00

unit price The price per unit of an item. You can count how many units by counting the items or the weight.

Practice
3.06

Name: ________ Date: ________ Period: ________

Problems 1–4: A copy machine can make 500 copies every 4 minutes.

1. How many copies can the copy machine make per minute?

2. How many minutes does it take per copy?

3. How many copies can the copy machine make in 10 minutes?

4. A teacher made 700 copies. How long did it take?

Problems 5–7: Jamar's class painted 50 square feet of a mural using 4 cans of paint.

5. How many square feet did they paint per can of paint?

6. How many cans did they use per square foot?

7. Jamar's class wants to paint a total of 310 square feet. Jamar calculated that they would need 3,875 cans of paint. Here is his work.

Jamar

12.5 square feet

$310 \cdot 12.5 = 3875$

Is Jamar correct? Circle one.

Yes No

Explain your thinking.

Name: .... Date: .... Period: ....

Problems 8–10: At the grocery store, Ethan bought 1.5 pounds of grapes for $\$3.75$.

8. How much do grapes cost per pound?

9. How many pounds of grapes does he get per dollar?

10. How many pounds of grapes could Ethan buy for $\$20$?

11. Here are the prices for cans of juice at different stores. The cans are the same brand and size. Which store offers the best deal? Explain your thinking.

Store A	Store B	Store C
4 cans for $\$2.48$	5 cans for $\$3.00$	$\$0.59$ per can

Spiral Review

Problems 12–15: Evaluate each expression.

12. $\frac{1}{4}$ of 60

13. $\frac{3}{4}$ of 60

14. $\frac{1}{4}$ of 30

15. $\frac{3}{4}$ of 30

Unit 3
Lesson
7

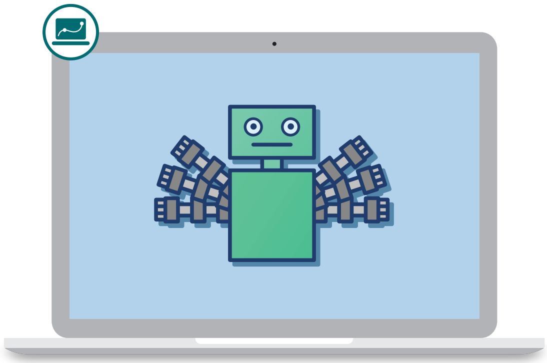
Name: Date: Period:

Generalizing With Multiple Representations | Model the World

6.RP.2, 6.RP.3, 6.RP.3.b, SMP.7

Welcome to the Robot Factory

Let's determine unknown values using unit rates.



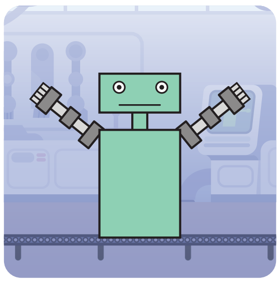
Warm-Up

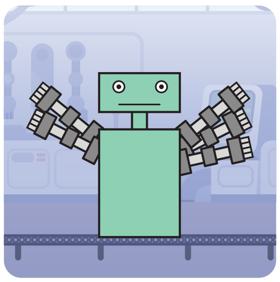
1 This table shows some lengths in both inches and feet.

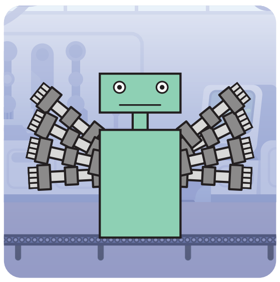
What are *three* things you notice about the table?

Length (ft)	Length (in.)
1	12
3	36
5	60
10	120

2 Welcome to the Robot Factory! Take a look at how many arms the robot has in each image.







Name: ______ Date: ______ Period: ______

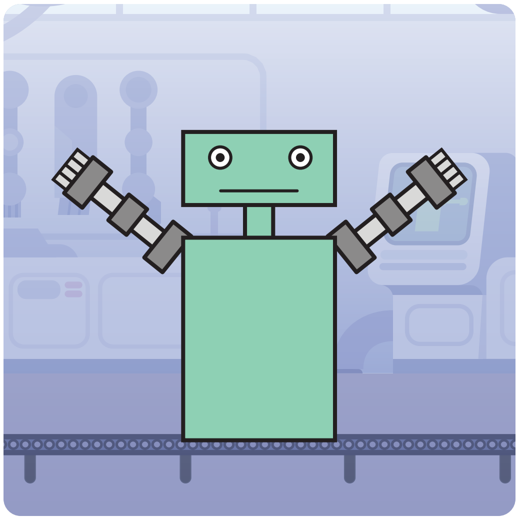
Arms and Fingers

3 This robot has 2 arms and 8 fingers.

Here are some other robots with different numbers of arms.

Complete the table to show the number of fingers on each robot.

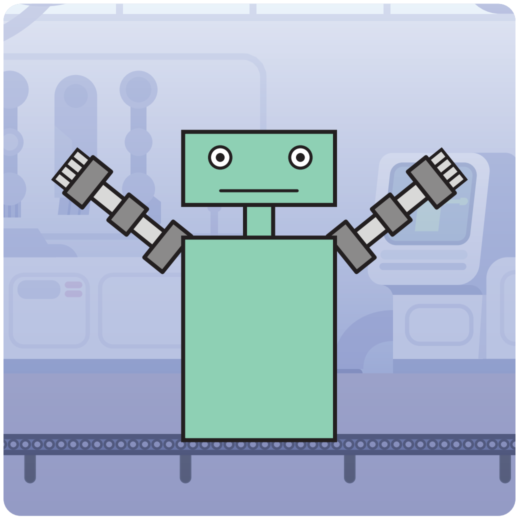
Arms	Fingers
2	8
7	
3	
9	



4 A new row has been added to the table.

How many arms go with this many fingers?

Arms	Fingers
2	8
	44



5 Choose *one* question and write your response.

- ☐ If you know the number of *fingers*, how can you determine the number of arms?
- ☐ If you know the number of *arms*, how can you determine the number of fingers?

Activity
2

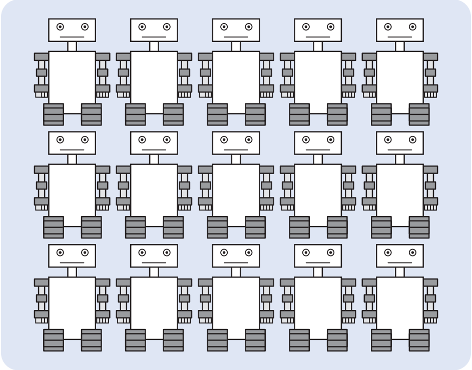
Name: ____________ Date: ________ Period: ________

Painting Robots

6-7 6 robots need 2 gallons of paint.

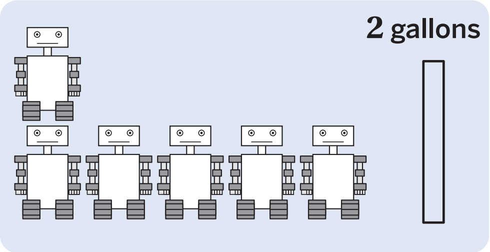
Complete the table.

Number of Robots	Amount of Paint (gal)
6	2
15	
21	
11	



8 Write instructions for how you could determine the amount of paint needed for *any* number of robots.

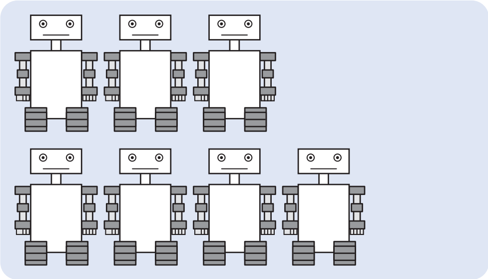
Use your table from the previous problem if it helps with your thinking.



Name: .... Date: .... Period: ....

Painting Robots (continued)

9 Here are some extra-large robots. 4 robots need 10 gallons of paint.



Complete the table.

Number of Robots	Amount of Paint (gal)
4	10
7	
9	
13	

You're invited to explore more.

10 Lisa wrote down the amount of paint and the painting time needed for different numbers of robots. Some of the values are missing. Complete the table.

Number of Robots	Amount of Paint (gal)	Painting Time (min)
5	2	
	5	10
15		12
	1	

11 Synthesis

Explain how you can use a table of equivalent ratios to determine unknown values, like the amount of paint needed for different numbers of robots.

Use this table if it helps with your thinking.

Number of Robots	Amount of Paint (gal)
1	$\frac{1}{3}$
6	2
33	11
18	6
9	3

14 Summary 3.07

Unit rates can help you determine missing values in a table.

For example, let's say 4 pounds of apples cost $\$10$.

That means the cost of 1 pound of apples is $\$2.50$. So you can calculate the cost of any amount of apples by multiplying the weight by 2.5.

That also means that for $\$1$, you can buy 0.4 pounds of apples. So you can calculate the number of pounds of apples you can buy for any amount of money by multiplying the amount of money by 0.4.

$\div 0.4$
$\cdot 2.50$

Pounds	Dollars
4	10
2	5
1	2.5
0.4	1

$\div 2.5$
$\cdot 0.4$

Name: .... Date: .... Period: ....

Problems 1–4: This table shows how many onions and tomatoes you need to make different-sized batches of a salsa recipe.

Onions	Tomatoes
2	16
4	32
6	48

1. How many onions do you need for 40 tomatoes?

2. How many tomatoes do you need for 3.5 onions?

3. One unit rate in this situation is 8. What does that represent?

4. Another unit rate is $\frac{1}{8}$. What does that represent?

Problems 5–6: It takes 10 pounds of potatoes to make 15 servings of mashed potatoes.

5. How many servings of mashed potatoes can you make with 15 pounds of potatoes? Use the table if it helps with your thinking.

Potatoes (lb)	Mashed Potatoes (servings)
10	15

6. How many pounds of potatoes do you need to make 45 servings of mashed potatoes? Use the table if it helps with your thinking.

Name: .... Date: .... Period: ....

7. Liam walks 1 mile in 20 minutes. At this rate, how many miles could Liam walk in 1 hour 30 minutes?

8. A train is traveling at a constant rate. Complete the table to show the relationship between the train's travel time and its distance traveled.

Time (hr)	Distance Traveled (mi)
2	110
1	
	27.5
$1\frac{1}{2}$	
	165

Spiral Review

9. A sandwich is placed on a digital scale. The scale reads 4.3. What could be the unit of measurement?

A. Milligrams **B.** Ounces **C.** Pounds **D.** Inches

10. Lola's family is planning to purchase a car that is 176.5 inches long. They have a parking space that is 16.25 feet long. Could this car fit in the parking space? Explain your thinking.

Problems 11–14: Evaluate each expression.

11. $\frac{25}{100}$ of 50

12. $\frac{20}{100}$ of 30

13. $\frac{5}{100}$ of 90

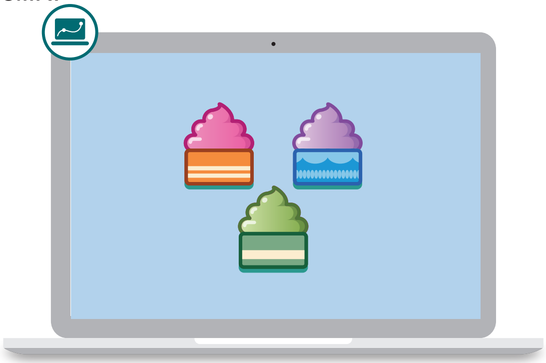
14. $\frac{75}{100}$ of 64

Name: ________ Date: ________ Period: ________

Generalizing With Multiple Representations | Model the World

6.RP.2, 6.RP.3, 6.RP.3.a, 6.RP.3.b, SMP.1, SMP.3, SMP.7

More Soft Serve



Let's compare ratios and calculate unknowns using unit rates.

Warm-Up

Determine the value of each expression mentally. Try to think of more than one strategy.

1. $\frac{1}{5} \cdot 30$

2. $\frac{3}{5} \cdot 30$

3. $\frac{3}{5} \cdot 15$

4. $\frac{3}{5} \cdot 3$

Activity 1

Name: ________ Date: ________ Period: ________

Missing Orders

5 Which soft serve shop has the best price per ounce? Circle one.

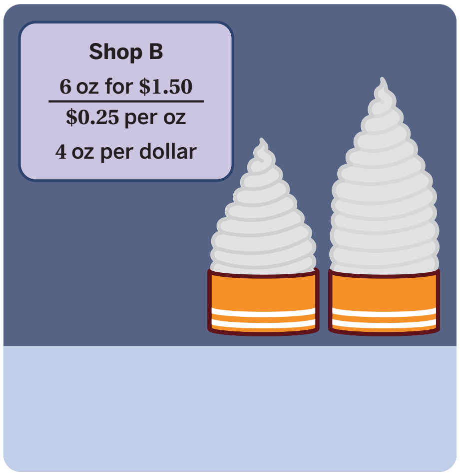
Shop A (5 oz for $2.00)   Shop B (6 oz for $1.50)   Shop C (4 oz for $1.20)

Explain your thinking.

6 Here are some new orders for Shop B.

Complete the table.

Weight (oz)	Cost (dollars)
6	1.50
8	
	3.50



Activity 1

Name: .... Date: .... Period: ....

Missing Orders (continued)

7 Riya completed the table in the previous problem by multiplying by the unit rates.

Explain how you could use Riya's strategy to calculate the cost of a 10.5-ounce soft serve.

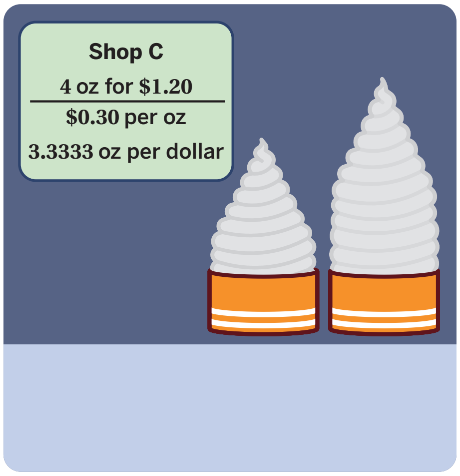
Shop B
6 oz for $1.50
$0.25 per oz
4 oz per dollars

Weight (oz)	Cost (dollars)
6	1.50
8 $\xrightarrow{\times 0.25}$	2.00
14 $\xleftarrow{\times 4}$	3.50

8 Here are two new orders for Shop C.

Use Riya's strategy to calculate the missing values.

Weight (oz)	Cost (dollars)
4	1.20
	2.85
11	



Name: ________ Date: ________ Period: ________

Challenge Creator

9 a **Make It!**

- Choose a soft serve flavor.
- Choose the weight and the cost for a small cup of soft serve. Record them on your Challenge Sheet.

How much does your soft serve cost *per ounce*?

________ dollars per ounce

How many ounces can you get *per dollar*?

________ ounces per dollar

- Choose the *weight* of a medium soft serve and the *cost* of a large soft serve. Record them on your Challenge Sheet.

b **Swap It!**

- Swap your challenge with one or more partners.
- Fill in these tables with the weight and cost of each medium soft serve and large soft serve.

Partner 1

	Weight (oz)	Cost (dollars)
Medium		
Large		

Partner 2

	Weight (oz)	Cost (dollars)
Medium		
Large		

Partner 3

	Weight (oz)	Cost (dollars)
Medium		
Large		

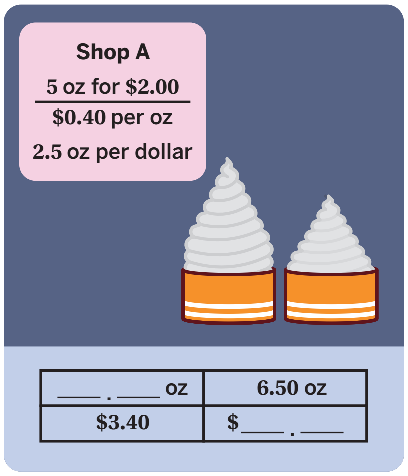
Partner 4

	Weight (oz)	Cost (dollars)
Medium		
Large		

10 Synthesis

Discuss: How can you calculate the unknown weights or costs of different soft serve orders?

Use the example if it helps with your thinking.



13 Summary 3.08

You can describe the relationship between the same two quantities using two different unit rates.

Let's say a shop charges $\$6.40$ for 8 ounces of soft serve. The unit rates in this situation are:

- Dollars per ounce: $\$0.80$ per ounce $(6.40 \div 8 = 0.80)$
- Ounces per dollar: 1.25 ounces per dollar $(8 \div 6.40 = 1.25)$

The unit rate you need for your calculations depends on what information you're given and what you want to determine.

So if you're given the number of ounces and want to determine the cost, you'll need to multiply by the dollars per ounce.

But if you're given the cost and want to determine the number of ounces you can get, you'll need to multiply by the ounces per dollar instead.

It's often helpful to determine *both* unit rates, so you can answer as many kinds of questions about the situation as possible!

Practice
3.08

Name: ______ Date: ______ Period: ______

Problems 1–2: A kangaroo hops 2 kilometers in 3 minutes.

1. How long does it take the kangaroo to hop 5 kilometers?

2. How far does the kangaroo hop in 2 minutes?

Problems 3–4: Neel buys 8 dog treats for $4.40.

3. What is the cost per dog treat?

4. Complete the table to show other numbers of dog treats he could buy at this rate.

Dog Treats	Cost ($)
8	4.40
18	
25	
	6.05

5. Haru and Victor are racing on scooters. Haru travels 15 meters in 6 seconds. Victor travels 22 meters in 10 seconds. Who is moving faster?

A. Haru **B.** Victor **C.** They are moving at the same speed

Explain your thinking.

6. Sothy plans to walk 10,000 steps. He starts his walk at 8:00 AM. At 8:23 AM, his phone tells him that he has taken 2,000 steps. If he continues at this rate, when will he reach 10,000 steps?

Name: ____________ Date: ________ Period: ________

7. A corn vendor at a farmers market is selling a bag of 8 ears of corn for $\$2.56$. Another vendor is selling a bag of 12 ears of corn for $\$4.32$. Which bag is the better deal? Show or explain your thinking.

Spiral Review

8. A soccer field is 105 meters long. A football field is 120 yards long. Which field is longer?

Note: 11 meters is approximately 12 yards.

A. soccer field

B. football field

C. They're the same length

Explain your thinking.

9. Select *all* the statements that represent a unit rate.

- ☐ **A.** Cucumbers cost $\$2.00$ per pound.
- ☐ **B.** Eric buys 3 pairs of shorts for $\$75.00$.
- ☐ **C.** It takes Jose 4 hours to drive 180 miles.
- ☐ **D.** A recipe calls for 3 eggs for every cup of flour.
- ☐ **E.** A teacher groups her students so there are 5 students per group.

Problems 10–11: Evaluate each expression.

10. $15 \div \frac{1}{5}$

11. $\frac{1}{4} \div 12$

Name: ________ Date: ________ Period: ________

Generalizing With Multiple Representations | Model the World

6.RP.2, 6.RP.3, 6.RP.3.b, 6.RP.3.d, SMP.2

Practice Day 1

Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Milk Containers

1. Measurements:

2. A: ________ B: ________ C: ________

3. Solution: ________ quarts

4. Solution: ________ quarts

5. Solution: ________ ounces

You're invited to explore more.

Ways to get 2 gallons:

Task B: The Better Deal

1. Circle one: Original Deal New Deal

Explanation:

2. Circle one: Original Deal New Deal

Explanation:

3. Solution: ________ dollars

Explanation:

You're invited to explore more.

Snacks:

Name: .... Date: .... Period: ....

Practice Day 1 (continued)

Task C: Pancake Breakfast

1. .... pancakes

2. .... cups

3. .... pancakes

Explanation:

4. .... cups

Work or Explanation:

You're invited to explore more.

Circle one: Yes or No

Explanation:

Task D: Apples and Cinnamon

1. Solution: .... cups

2. Solution: .... cups

3. Solution: .... pounds

4. Solution: .... pounds

5.

Least strong **Strongest**

Apples (lb)	Applesauce (cups)
4	5

You're invited to explore more.

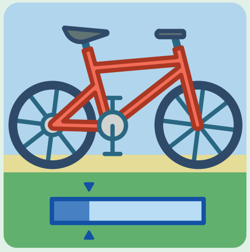
a .... minutes

b .... cups of applesauce

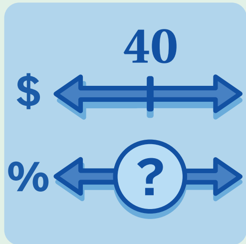
Sub-Unit 3

Percentages

Lesson 9
Lucky Duckies

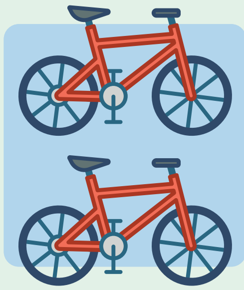


Lesson 10
Bicycle Goals



Lesson 11
What's Missing?

Lesson 12
Cost Breakdown



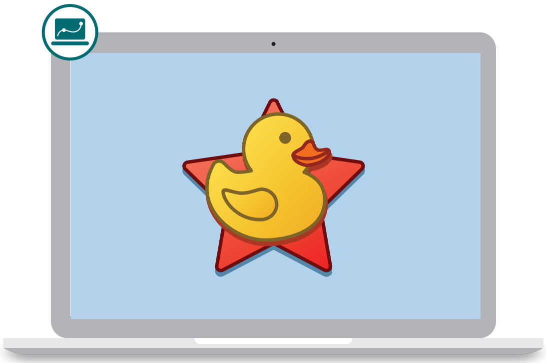
Lesson 13
More Bicycle Goals

Lesson 14
A Country as a Village

Unit 3
Lesson
9

Name: ________ Date: ________ Period: ________

Fraction Relationships | Model the World | 6.RP.3, 6.RP.3.c, SMP.1, SMP.3, SMP.8



Lucky Duckies

Let's learn about friendly percentages with rubber duckies.

Warm-Up

1-2 Here is a carnival game called *Duck, Duck, Choose*.

Players win a prize if they catch a rubber ducky with a star on the bottom.

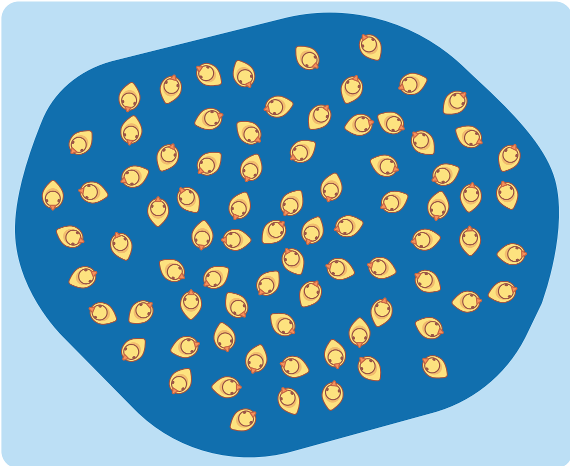
There are two games that both have 80 duckies. Which game has more duckies with stars?

A. The game where 50% of duckies have stars

B. The game where 50 duckies have stars

C. They are the same

Explain your thinking.



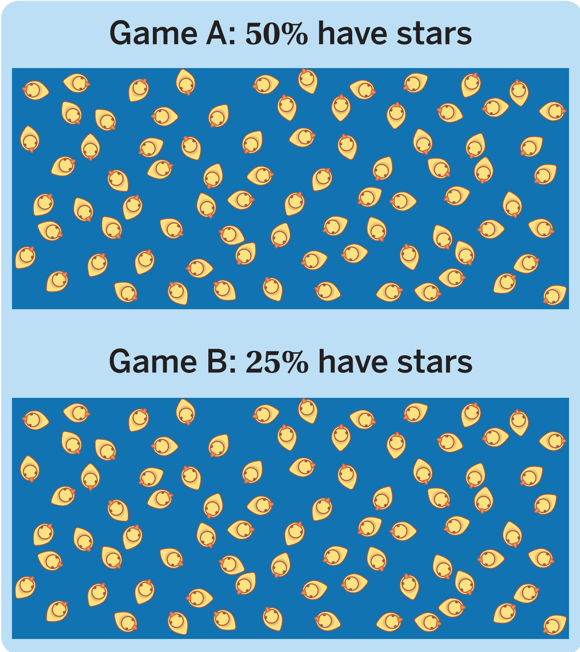
Activity 1

Name: ______ Date: ______ Period: ______

Ducky Game Design

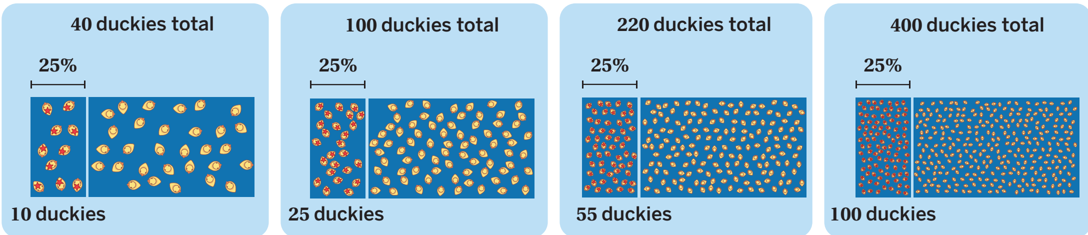
3 a. Use the digital activity to move the dividers so that each game has about the right number of duckies with stars.

b. **Discuss:** How did you decide where to place each divider?



4 Here are some games where 25% of the duckies have stars.

a. Take a look at the total number of duckies in each game.

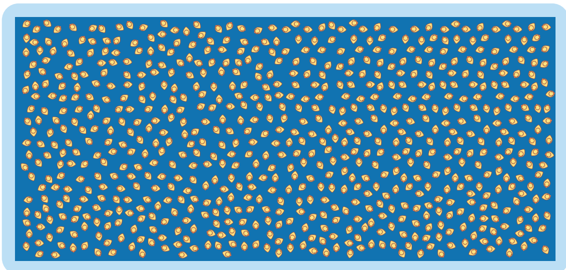


b. Describe what 25% of a number means.

5 10 percent (10%) means 10 for every 100.

This game has 800 duckies. 10% of them have stars.

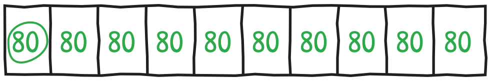
How many of the duckies have stars?



Name: .... Date: .... Period: ....

Ducky Game Design (continued)

6 Here is how Santiago figured out the number of duckies that are winners when 10% out of 800 duckies win.

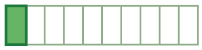


Show or explain what he may have been thinking.

7 Group these choices based on what percentage they represent. One choice will have no match.

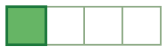
a 10% is shaded green.

b $\frac{3}{4}$ is shaded green.

c 

d $\frac{1}{2}$ is shaded green.

e 25% is shaded green.

f 

g 75% is shaded green.

h $\frac{1}{4}$ is shaded green.

Group 1	Group 2	Group 3

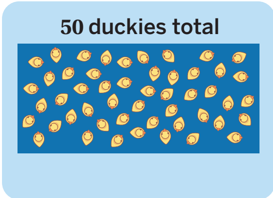
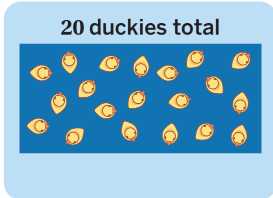
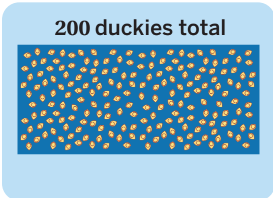
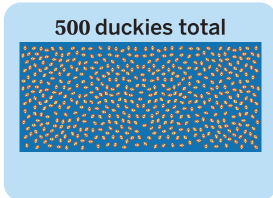
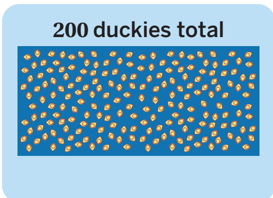
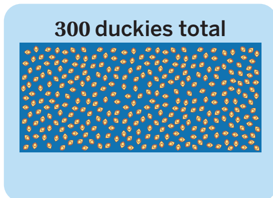
Activity 2

Name: .... Date: .... Period: ....

Repeated Challenges

8

- Pair up with a classmate. Decide who will complete Column A and who will complete Column B.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

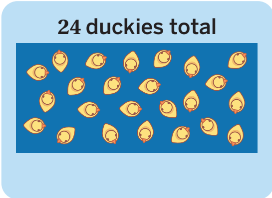
Column A	Column B
10% of 50 duckies have stars. How many duckies have stars? 	25% of 20 duckies have stars. How many duckies have stars? 
25% of 200 duckies have stars. How many duckies have stars? 	10% of 500 duckies have stars. How many duckies have stars? 
75% of 200 duckies have stars. How many duckies have stars? 	50% of 300 duckies have stars. How many duckies have stars? 

Activity
2

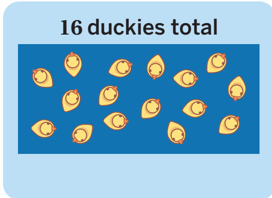
Name: .... Date: .... Period: ....

Repeated Challenges (continued)

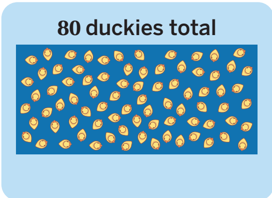
50% of 24 duckies have stars. How many duckies have stars?



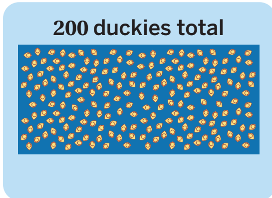
75% of 16 duckies have stars. How many duckies have stars?



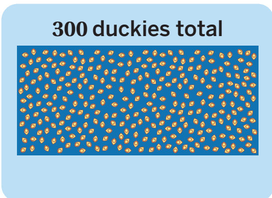
25% of 80 duckies have stars. How many duckies have stars?



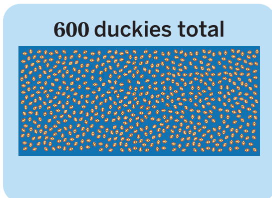
10% of 200 duckies have stars. How many duckies have stars?



50% of 300 duckies have stars. How many duckies have stars?



25% of 600 duckies have stars. How many duckies have stars?

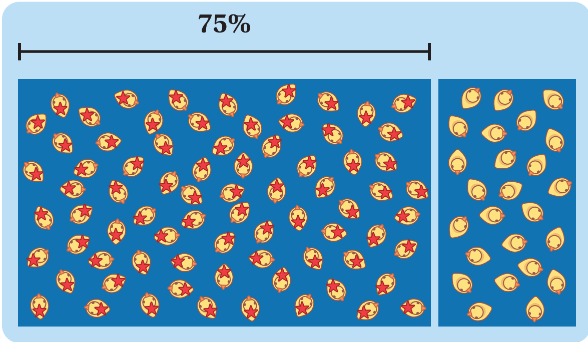


You're invited to explore more.

9 Now you design a Ducky Game! Suppose you have 30 winning duckies. If you want these duckies to represent 10% in the game, how many total duckies should you include? What about 25%? 50%? 75%?

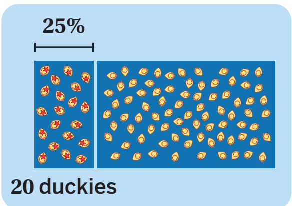
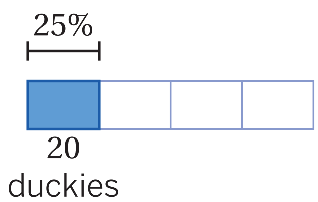
10 Synthesis

Discuss: What does 75% of a number mean?



13 Summary 3.09

Each of the different Ducky Games in this lesson had a certain **percentage** of ducks with stars: 10%, 25%, 50%, or 75%. Fractions and tape diagrams can help us interpret these percentage problems.

<table>
<tr><th>Example Problem</th><th>Using Fractions</th><th>Using Tape Diagrams</th></tr>
<tr>
<td>25% of the 80 duckies have stars.

</td>
<td>25% of something means $\frac{25}{100}$ or $\frac{1}{4}$.
$\frac{1}{4}$ of 80 duckies is 20 duckies.
80 total duckies

</td>
<td>There are four 25s in 100, so the tape diagram can be split into 4 pieces. The total number of duckies can also be split into 4 parts, so there are 20 duckies in each section.
80 total duckies
20 20 20 20</td>
</tr>
</table>

<u>**percent**</u> It is represented by the percent symbol: % and means for every 100. We use percentage to represent ratios and fractions.

Name: ______ Date: ______ Period: ______

1. Here are 24 stars. Circle 25% of these stars.

2. Evan made 40 muffins. 50% of the muffins are chocolate. How many muffins are chocolate?

3. Which is greater? Show or explain your thinking.

   A. 75% of 8

   B. 25% of 32

   C. They are the same.

Problems 4–5: Complete each statement. Make a tape diagram if it helps with your thinking.

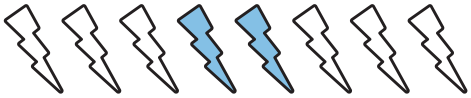
4. 10% of 20 is ______.

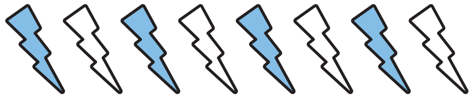
5. 25% of 60 is ______.

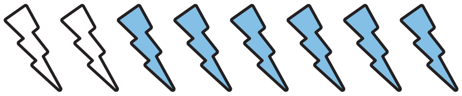
6. Explain how you could mentally calculate 10% of any number.

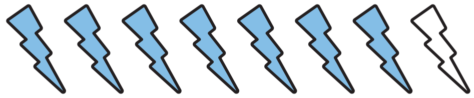
Name: ____________ Date: ________ Period: ________

7. Which group shows 75% of the lightning bolts shaded?

A. 

B. 

C. 

D. 

Spiral Review

8. Abdel paid \$13 for 3 books. Jayden bought 12 books priced at the same rate. How much did Jayden pay for the 12 books? Explain your thinking.

Problems 9–11: Determine whether each product will be *less than*, *greater than*, or *equal to* 40.

9. $\left(\frac{6}{4}\right) \cdot 40$

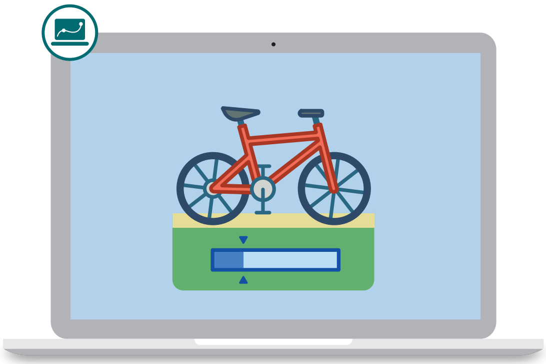
10. $\left(\frac{8}{8}\right) \cdot 40$

11. $\left(\frac{1}{2}\right) \cdot 40$

12. Jamar jogs 1 mile in 16 minutes. At this rate, how many miles could Jamar jog in 1 hour 20 minutes?

Name: ______ Date: ______ Period: ______

Fraction Relationships | Model the World | 6.RP.3, 6.RP.3.c, SMP.2, SMP.3, SMP.6

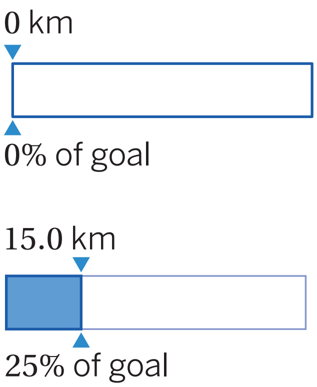


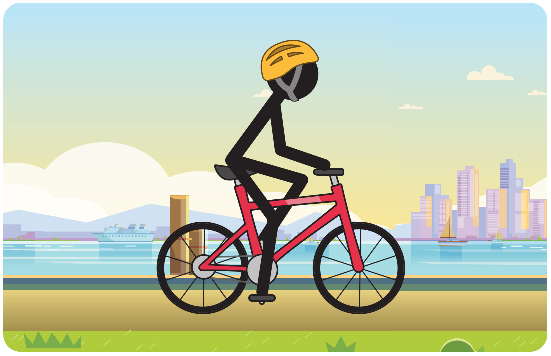
Bicycle Goals

Let's connect percentages and ratios.

Warm-Up

1 Let's watch the percentage change as the biker tries to beat their goal.





2 What is this biker's goal distance?

Explain your thinking.

Activity 1

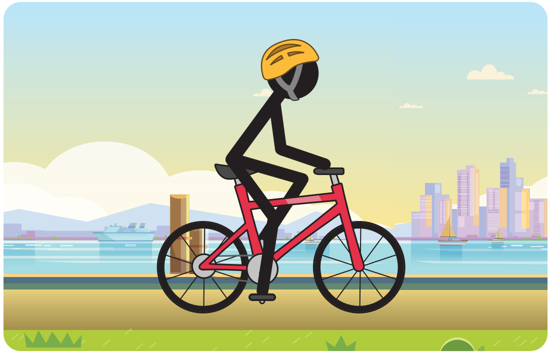
Name: .... Date: .... Period: ....

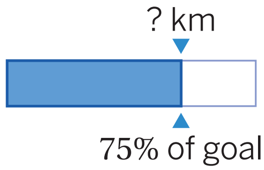
Bicycle Goals

3 Alejandro's goal was to ride 36 kilometers.

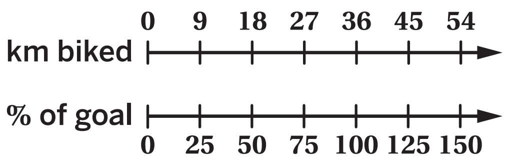
His app says he rode 75% of his goal.

How far did he ride?





4 a Take a look at the double number line that represents Alejandro's goal and progress.



b Describe how you can tell that the goal distance is 36 kilometers.

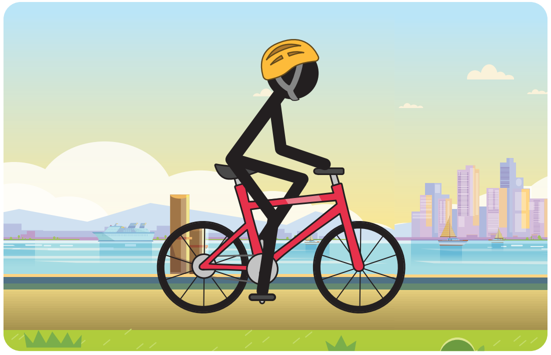
Name: ________ Date: ________ Period: ________

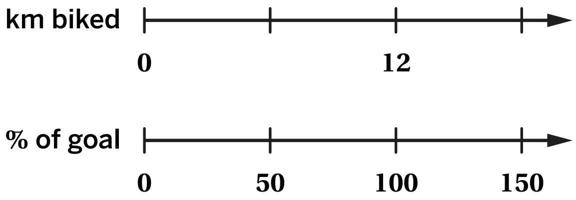
Bicycle Goals (continued)

5 Basheera's goal was to ride 12 kilometers.

Her app says she rode 150% of her goal.

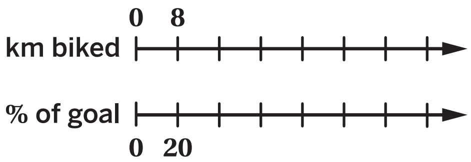
How far did she ride?





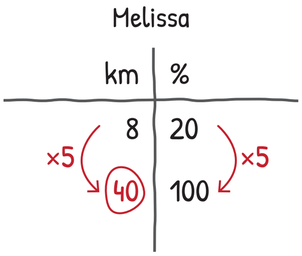
6 According to an app, Callen and Folami biked 8 kilometers, which is 20% of their goal.

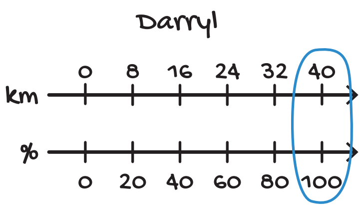
What was their goal distance?



7 Here are two different strategies for calculating the goal when 20% of the goal is 8 kilometers.

Discuss: How did each student use ratios to calculate the goal?





Name: .... Date: .... Period: ....

Percentages and Ratios

8 Match the strategies that represent the same percentage problem.

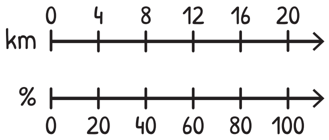
a

km	%
80	100
8	10
16	20

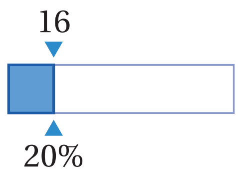
b

km	%
20	100
2	10
16	80

c



d



20% of 80 km	80% of 20 km

9 Miko's app says he biked 30 kilometers, which is 120% of his goal.

What was his goal distance? Use the double number line diagram to support your thinking.

You're invited to explore more.

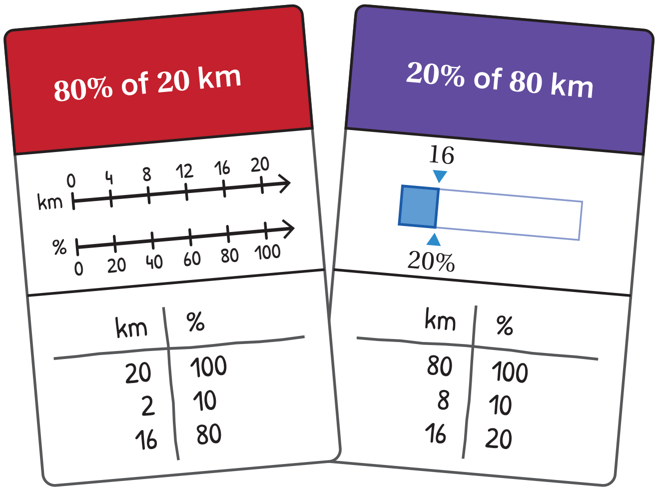
10 How many ways can you make a biker ride 10 kilometers?

Enter a goal and a percent of that goal for the biker to ride.

Goal				
Percent of Goal				

11 Synthesis

Describe how solving problems with percentages is like solving problems with ratios.



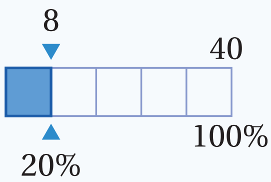
13 Summary 3.10

You can represent percentages using tape diagrams, double number lines, and tables. The strategies you've already used to solve ratio problems can help you think about and solve percentage problems, too!

Let's say a biker traveled 8 kilometers, which is 20% of their goal distance. What's their goal distance?

Here are three ways to represent and solve this percentage problem.

Tape Diagram

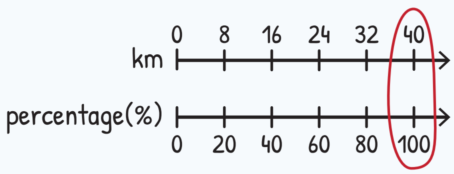


Table

km	%
8	20
40	100

×5 ×5

Double Number Line

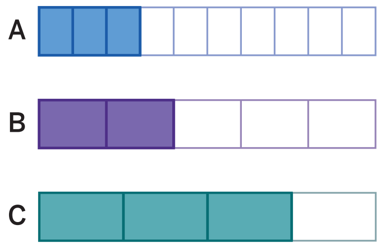


So the biker's goal distance is 40 kilometers.

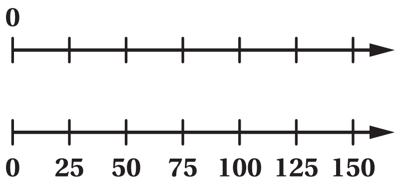
Name: ______________________ Date: __________ Period: __________

1. What percent of each figure is shaded? Record your answers in the table.

Figure A	
Figure B	
Figure C	



2. Martina shot 40 free throws at basketball practice. 25% of her free throws went into the basket. How many of them went into the basket? Use the double number line if it helps with your thinking.



3. On Tuesday, Parv made 12 cookies. On Wednesday, he made 150% as many cookies as he made on Tuesday. How many cookies did Parv make on Wednesday?

Problems 4–5: Leonardo works as a server in a restaurant. He gets tipped 20% of the cost of each order.

4. What tip will he get if the food costs $60?

5. Leonardo got an $18 tip. What was the cost of the food for this order?

6. Nikhil says that to determine 20% of a number, you can divide the number by 5. For example, 20% of 60 is 12 because $60 \div 5 = 12$. Does Nikhil's method always work? Explain your thinking.

Name: .... Date: .... Period: ....

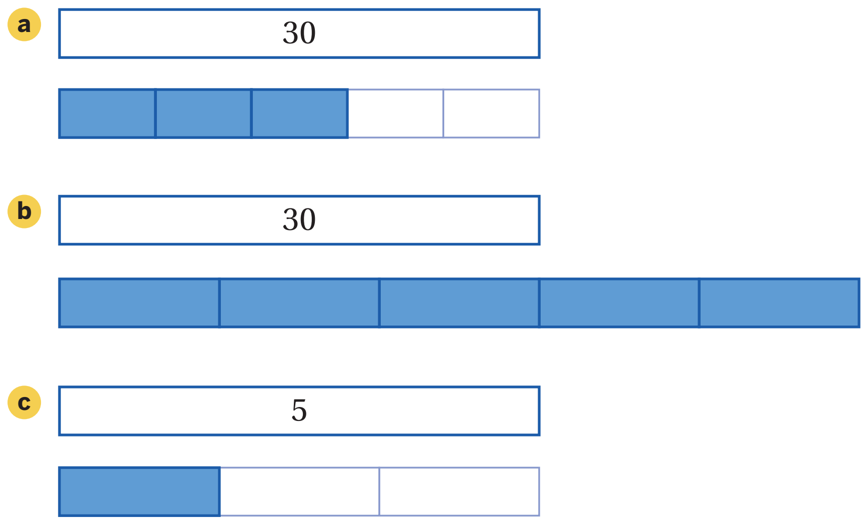
Spiral Review

Problems 7–8: Light travels about $180{,}000{,}000$ kilometers in 10 minutes.

7. How many kilometers per minute is that?

8. How many kilometers per *second* is that?

9. Match each expression with the tape diagram that represents it.



.... $\frac{3}{5} \cdot 30$

.... $\frac{1}{3} \cdot 5$

.... $\frac{5}{3} \cdot 30$

10. Emmanuel types 265 words in 5 minutes. Vihaan types 147 words in 3 minutes. Both type at a constant rate. Who types faster? Explain your thinking.

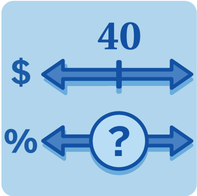
Unit 3
Lesson
11

Name: .... Date: .... Period: ....

Fraction Relationships | Generalizing With Multiple Representations

6.RP.3.c, 6.RP.3, SMP.7

What's Missing?



Let's use ratio reasoning to find unknown quantities.

Warm-Up

Evaluate each expression mentally.

1. $\frac{3}{10} \cdot 20$

2. $\frac{3}{10} \cdot 25$

3. $\frac{3}{10} \cdot 5$

4. $\frac{3}{10} \cdot \frac{5}{2}$

Activity 1

Name: Date: Period:

Card Sort: What's Missing

5. You will use a set of cards for this activity. Match each card to its place in the table. Then fill in all the empty spaces that remain. **ELD.PI.6.6.6.Em, Ex, Br, ELD.PI.6.8.Em, Ex, Br**

	Question	Representation	Answer
a	I have a 40% off coupon. If I use it to buy a shirt that costs $\$20$, how much would the shirt cost after the discount?		
b	I have a 20% off coupon. If I use it to buy a shirt and save $\$40$, what was the original price of the shirt?		
c	I paid $\$40$ for a jacket with an original price of $\$50$. What percent of the original price did I pay?		
d			

Activity 2

Name: ______ Date: ______ Period: ______

Sale Price and Original Price

6. Complete the table. **ELD.PI.6.6.6.Em, Ex, Br, ELD.PI.6.8.Em, Ex, Br**

	Question	Representation	Answer
a	Eliza bought a hat for \$21. The original price is \$30. What percent of the original price did she pay?		
b	A discount store sells items at 80% of the original price. If the original price of pants is \$55, what is the sale price?		
c	A discount store sells items at 80% of the original price. If the sale price of sneakers is \$96, what is the original price?		

You're invited to explore more.

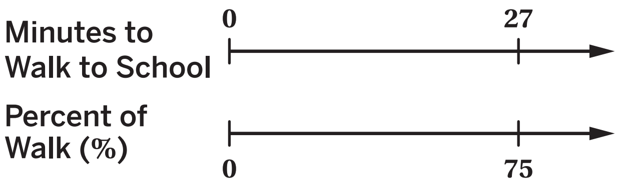
7. Precious biked 125% of her daily goal on Monday. What percent of her total weekly goal did she bike on Monday?

Precious's Biking Goals

Day	Su	M	T	W	Th	F	S
Goal (km)	0	8	4	10	0	8	20

Synthesis

8. **Discuss:** How can this double number line help you calculate the total time Eliza take to walk to school?



Summary 3.11

You can use tables, tape diagrams, and double number lines to solve percentage problems.

There are three main types of percentage problems.

- Determine the whole when you're given the part and the percentage.
- Determine the percentage when you're given the part and the whole.
- Determine the part when you're given the percentage and the whole.

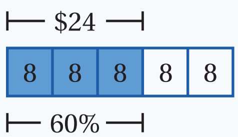
Let's say the original price of a sweater is $40. The sweater is on sale for 40% off. This means the sweater costs 60% of the original price. What is the price of the sweater after the discount?

Here are three representations that you can use to find the part (the price of the sweater after the discount) given the whole and the percentage.

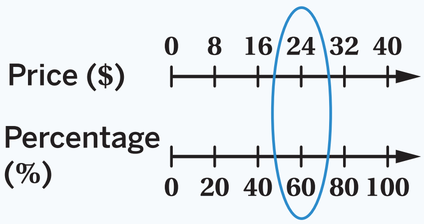
Table

Price ($)	Percentage
40	100%
8	20%
24	60%

Tape Diagram



Double Number Line



Practice
3.11

Name: ______ Date: ______ Period: ______

Problems 1–4: Evaluate each percentage problem.

1. 100% of 40

2. 50% of 40

3. 150% of 40

4. 10% of 40

Problems 5–6: A hardware store offers customers a coupon for $25 off.

5. The original price of a power drill is $125. If a customer uses the coupon, what percent will they save?

6. The original price of a ladder is $250. If a customer uses the coupon, what percent will they save?

Problems 7–8: Kiri is curious how many people think aliens exist. She asks 30 students in her class.

7. 12 students in Kiri's class say they think aliens exist. What percent of the class is that?

8. Kiri's older brother also asks the 25 students in his class. 11 students say they think aliens exist. Which class has a greater percent of students who think aliens exist?

A. Kiri's class **B.** Kiri's brother's class **C.** Same percent

Explain your thinking.

Practice
3.11

Name: ________ Date: ________ Period: ________

Spiral Review

Problems 9–10: Afia is 56 inches tall. Note: 100 inches $= 254$ centimeters.

9. What is Afia's height in centimeters?

10. What is her height in meters?

11. The recipe for a berry fruit smoothie says to combine 3 cups of blueberries, 7 cups of strawberries, and 2 cups of apple juice. Complete the table to determine how much of each type of berry you need for the different number of batches.

Batches	Blueberries (cups)	Strawberries (cups)	Apple juice (cups)
1	3	7	2
2			
3			
4			

Name: ______ Date: ______ Period: ______

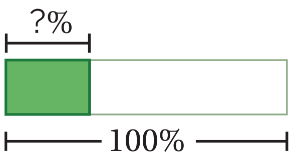
Fraction Relationships | Model the World | 6.RP.3, 6.RP.3.c, SMP.3, SMP.8

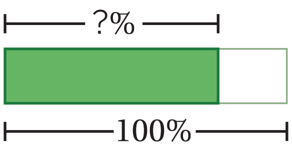
Cost Breakdown

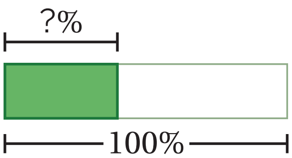
Let's calculate any percentage of a number.

Warm-Up

1 For each challenge, write your best estimate of the missing percent.







Activity 1

Name: .... Date: .... Period: ....

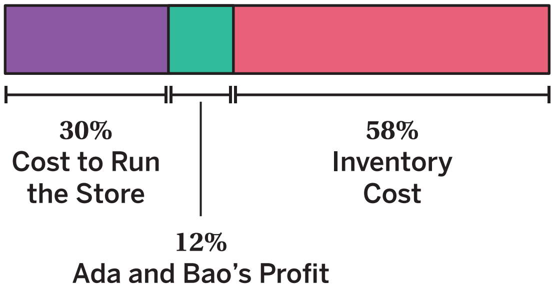
Break It Down

2 Ada and Bao run a clothing store.

The price of each item includes the profit that Ada and Bao make, the cost of inventory, and the cost to run the store.

The diagram shows where the money goes when Ada and Bao sell a t-shirt.

What do you notice? What do you wonder?



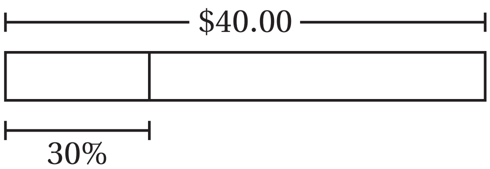
I notice:

I wonder:

3 30% of the price of each shirt goes to running the store.

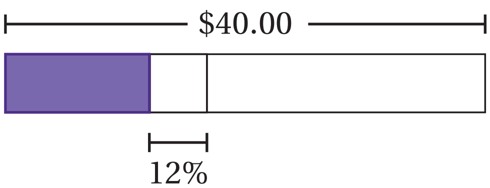
How much of a $40 shirt goes to running the store?

Use the tape diagram if it helps with your thinking.



4 Ada and Bao keep 12% of the price of each shirt as profit.

What is their profit on a $40 shirt?



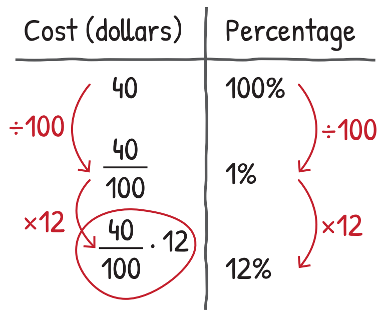
Activity 1

Name: ______ Date: ______ Period: ______

Break It Down (continued)

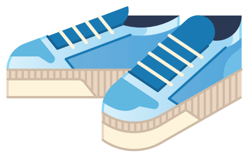
5 Here is how Bao calculated 12% of \$40.

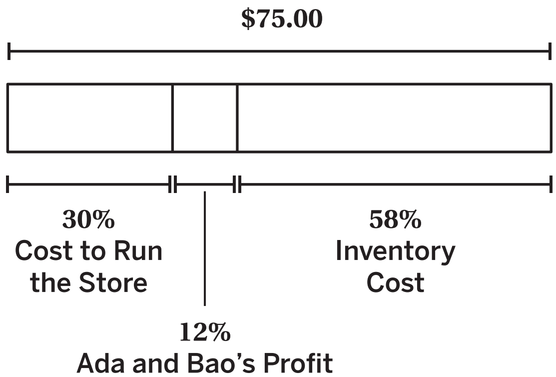
Explain how you could use Bao's strategy to calculate how much of the price of each shirt goes to inventory cost (58% of \$40).



6 Here is a \$75 pair of shoes. Calculate each value.

Cost to Run the Store	
Ada and Bao's Profit	
Inventory Cost	



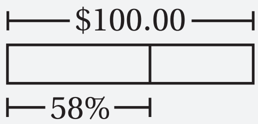
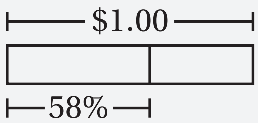
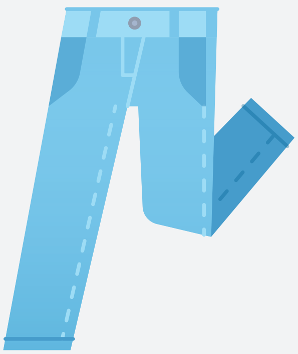
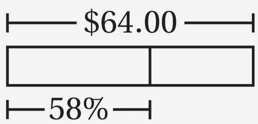
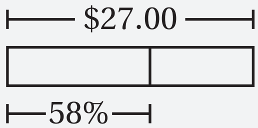


Activity 2

Name: ________ Date: ________ Period: ________

Another Strategy

7 Ada and Bao's inventory costs are 58% of the total cost. What is their inventory cost for each item in this table?

Item	Representation	Total Cost (dollars)	Inventory Cost (dollars)
Dress 		\$100	
Sticker 		\$1	
Jeans 		\$64	
Hat 		\$27	

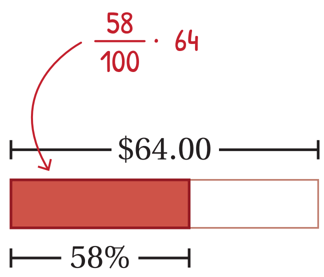
Activity 2

Name: ________ Date: ________ Period: ________

Another Strategy (continued)

8 Ada thinks of 58% as 58 cents for every dollar. So he writes $\frac{58}{100} \cdot 64$ to calculate 58% of $\$64$.

Describe how Ada might calculate 36% of $\$15$.



9 Match each expression with a question. One expression will have no match.

a $\frac{36}{100} \cdot 70$

b $\frac{15}{100} \cdot 36$

c $\frac{100}{36} \cdot 48$

d $\frac{36}{100} \cdot 15$

e $70 \div 100 \cdot 36$

f $\frac{36}{100} \cdot 48$

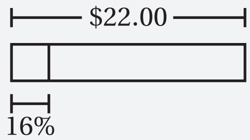
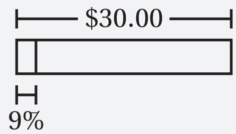
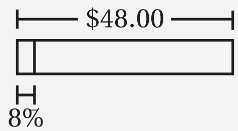
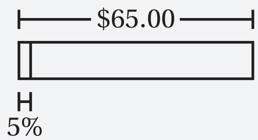
What is 36% of $\$15$?	What is 36% of $\$48$?	What is 36% of $\$70$?

Activity
3

Name: ______ Date: ______ Period: ______

Repeated Challenges

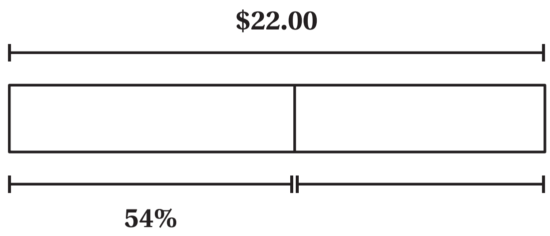
10 Solve as many challenges as you have time for.

Problem	Representation	Answer
The price of a space T-shirt is $22. 16% of every sale goes to material cost. Calculate the material cost.		
The price of a striped button-up shirt is $30. 9% of every sale goes to clothing company profit. Calculate the clothing company profit.		
The price of a striped long-sleeve T-shirt is $48. 8% of every sale goes to transport cost. Calculate the transport cost.		
The price of a blue pair of shoes is $65. 5% of every sale goes to factory profit. Calculate the factory profit.		

11 Synthesis

Describe a strategy for calculating a percentage of a number.

Use the example if it helps to explain your thinking.



14 Summary 3.12

When solving percentage problems related to money, you can:

- Determine the value of 1% and multiply that by the percentage you're looking for.
- Determine how many cents per dollar a given percentage represents.

Let's say a pair of pants costs $\$42$. If the factory makes a *profit* of 14% on the price of a pair of pants, how many dollars of profit does the factory make from each sale?

Strategy 1

	Cost (dollars)	Percentage (%)	
	42	100	
÷100	$\frac{42}{100}$	1	÷100
×14	$\frac{42}{100} \cdot 14$	14	×14

$$\frac{42}{100} \cdot 14 = 5.88$$

The factory makes $\$5.88$ of profit from each sale.

Strategy 2

- 14% profit means 14 cents of each dollar is profit.

  $\frac{14}{100} = 0.14$
- The price of the pants is $\$42$.
- $\frac{14}{100} \cdot 42 = 5.88$

The factory makes $\$5.88$ of profit from each sale.

Practice 3.12

Name: ______ Date: ______ Period: ______

Problems 1–4: Evaluate each expression.

1. 50% of 70

2. 10% of 70

3. 1% of 70

4. 2% of 70

5. A store is having a 30% off sale. The original price for a pair of headphones is $\$150$. How much would a customer save with this sale?

6. Order the following expressions from *least* to *greatest* value.

55% of 180	300% of 26	12% of 700

Least **Greatest**

7. To find 40% of 75, Jamal calculates $\frac{2}{5} \cdot 75$. Does his calculation give the correct value for 40% of 75? Explain your thinking.

8. Emika has a monthly budget for her cell phone bill. Last month, she spent 120% of her budget, and the bill was $\$60$. What is Emika's monthly budget?

9. Kyrie spent 75 minutes practicing the piano over the weekend. Yasmine practiced the violin for 152% as much time as Kyrie practiced the piano. How long did Yasmine practice?

Practice
3.12

Name: ________ Date: ________ Period: ________

10. Select *all* the expressions that could be used to calculate 45% of 60.

☐ **A.** $\frac{100}{45} \cdot 60$

☐ **B.** $\frac{60}{45} \cdot 100$

☐ **C.** $\frac{45}{100} \cdot 60$

☐ **D.** $\frac{100}{60} \cdot 45$

☐ **E.** $\frac{0.45}{100} \cdot 60$

☐ **F.** $\frac{60}{100} \cdot 45$

11. Fill in each blank using the numbers 0 to 9 only once, so that the expression on the left is greater than the expression on the right.

Left	Right
☐☐ **% of 50**	**50% of** ☐☐

Spiral Review

12. Two stores sell identical sandwich rolls in different-sized packages. Store A sells a six-pack for $\$5.28$. Store B sells a four-pack for $\$3.40$. Which store offers the better price per roll?

A. Store A

B. Store B

C. They are the same

Show or explain your thinking.

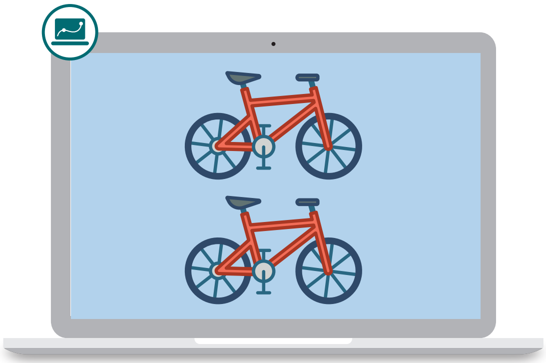
13. Neel buys 7 dog treats for $\$10.92$. What is the cost per dog treat?

14. Victor spent $\$345.96$ on 9 tickets to a basketball game. How much did he pay per ticket?

Unit 3
Lesson
13

Name: ______ Date: ______ Period: ______

Fraction Relationships | Model the World | 6.RP.3, 6.RP.3.c, SMP.3, SMP.8



More Bicycle Goals

Let's calculate unknown percentages.

Warm-Up

Evaluate each expression mentally. Try to think of more than one strategy.

1. $\frac{1}{3}$ of $\frac{1}{4}$

2. $\frac{1}{3} \cdot \frac{1}{4}$

3. $\frac{2}{3} \cdot \frac{1}{4}$

4. $\frac{2}{3} \cdot \frac{5}{4}$

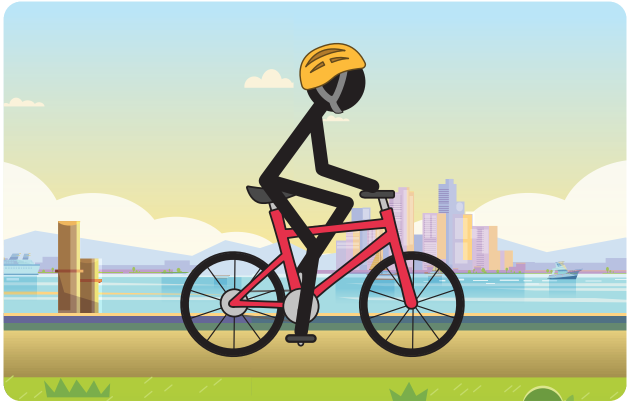
Name: ________________ Date: ________ Period: ________

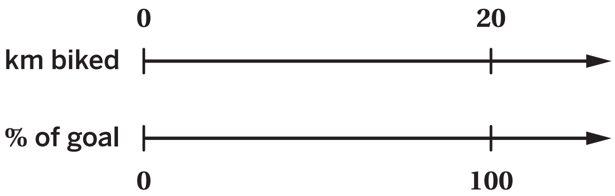
Chasing Goals

5 Alejandro's goal for Monday was to ride 20 kilometers.

His app says he rode 40% of his goal.

How far did he ride?

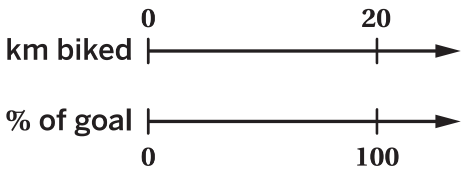




6 Alejandro's goal for Tuesday was to ride 20 kilometers.

His app says he rode 10 kilometers.

What percent of his goal did he ride?



7 On Wednesday, Alejandro and Basheera rode 17 kilometers.

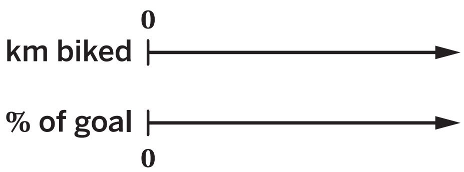
- Alejandro's goal was 20 kilometers.
- Basheera's goal was 50 kilometers.

Who rode a greater percent of their goal? Circle one.

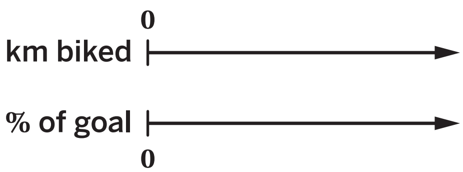
Alejandro   Basheera   Same percent

Explain your thinking.

Alejandro



Basheera



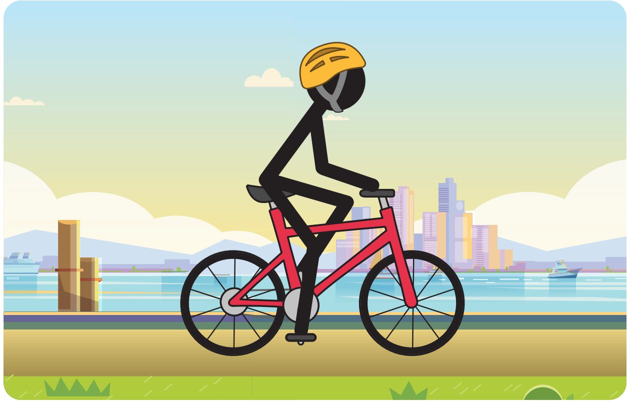
Name: ________ Date: ________ Period: ________

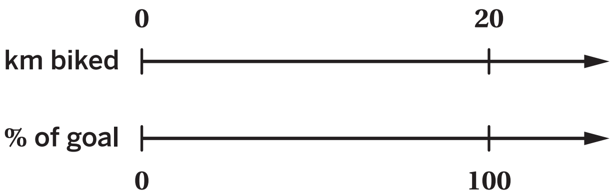
Chasing Goals (continued)

8 On Wednesday, Alejandro and Basheera rode 17 kilometers.

Alejandro's goal was 20 kilometers.

What percent of his goal did he ride?





9 Here is how Alejandro calculated 17 out of 20 as a percentage.

Explain how you could use Alejandro's strategy to calculate 13 out of 20 as a percentage.

	Distance (km)	Percent of Goal	
$\times \frac{1}{20}$	20	100	$\times \frac{1}{20}$
$\times 17$	1	$\frac{100}{20}$	$\times 17$
	17	$\frac{100}{20} \cdot 17$	

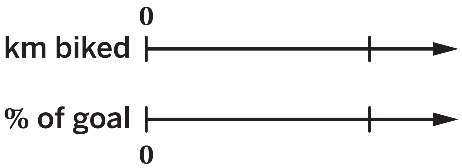
Name: ____________ Date: ________ Period: ________

Reaching Goals

10 Alejandro and Basheera set a new goal for Saturday: 30 kilometers.

They rode 36 kilometers.

What percent of their goal did they ride? Use the double number line to help explain your thinking.



11 Here are the expressions Alejandro and Basheera used to calculate 36 out of 30 as a percentage.

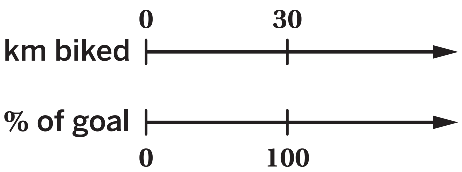
Alejandro: $\frac{100}{30} \cdot 36$

Basheera: $\frac{36}{30} \cdot 100$

Whose expression is correct? Circle one.

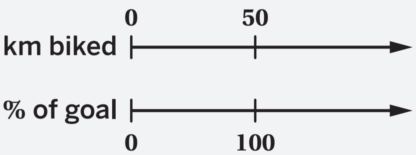
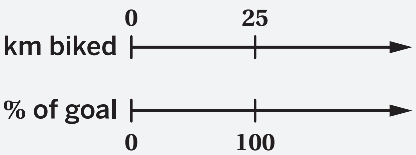
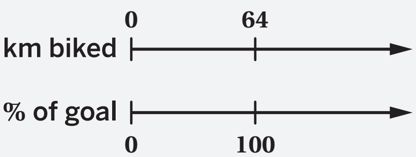
Alejandro   Basheera

Both   Neither



Explain your thinking.

12 On Sunday, Alejandro, Basheera, and Callen rode 40 kilometers. They each had different goals, as shown in the table. Calculate what percent of their goal each person rode.

Representation	Distance Traveled (km)	Goal (km)	Percent of Goal
Alejandro 	40	50	
Basheera 	40	25	
Callen 	40	64	

Name: ________ Date: ________ Period: ________

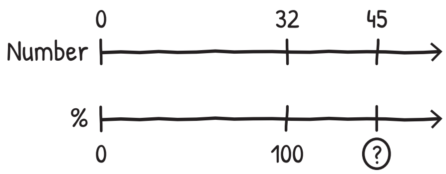
Reaching Goals (continued)

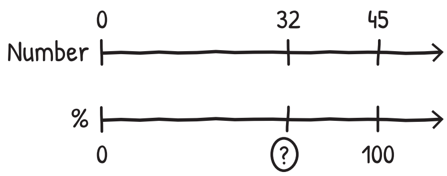
13 Match each question with a double number line and an expression. One choice will have no match.

a $\frac{32}{100} \cdot 45$

b $\frac{32}{45} \cdot 100$

c $\frac{45}{32} \cdot 100$

d


e


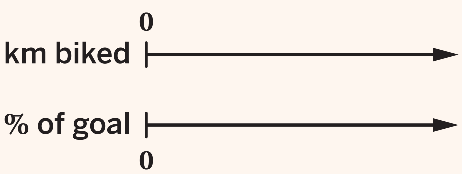
What is 32 out of 45 as a percentage?	What is 45 out of 32 as a percentage?

You're invited to explore more.

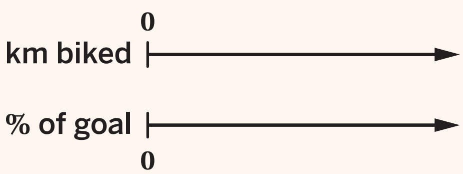
14 Basheera had a goal of riding 30 kilometers and rode 51 kilometers.
Callen had a goal of 20 kilometers and rode 41 kilometers.

Determine who rode a greater percent of their goal, in as many different ways as you can.

Basheera



Callen

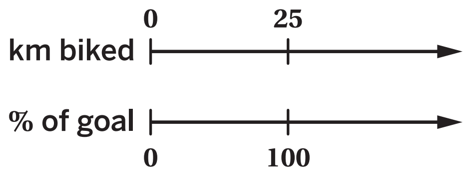


Explain your thinking.

15 Synthesis

Here is what Basheera wrote to solve a bicycle challenge.

$\frac{31}{25} \cdot 100 = 124$



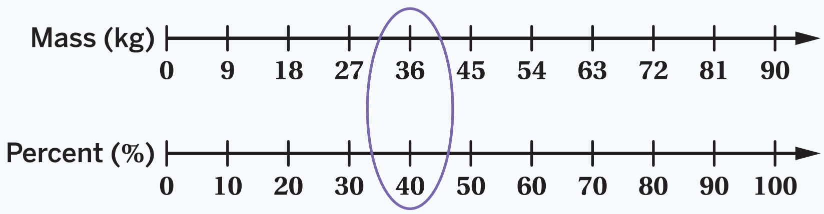
Discuss: What do 31, 25, and 124 represent in this scenario?

18 Summary 3.13

You can use ratios to compare percentages.

For example, you could compare the weights of an adult giant panda and a giant panda cub. To do this, you can determine the cub's weight as a percent of the adult's weight using several strategies.

Double Number Line



Ratio Tables

	Mass (kg)	Percent (%)	
	90	100	
$\times \frac{1}{90}$	1	$\frac{1}{90} \times 100$	$\times \frac{1}{90}$
$\times 36$	36	$\frac{36}{90} \times 100$	$\times 36$

- Determine the unit rate (what percent matches 1 kilogram).
- Use the unit rate to determine what percent 36 kilograms is of 90 kilograms.

Expressions

$36 \div 90 \cdot 100 = \frac{36}{90} \cdot 100 = 40$

Evaluate $\frac{p}{w} \cdot 100$ to determine what percent one value, p, is of the other value, w.

Practice
3.13

Name: .... Date: .... Period: ....

1. Select *all* the expressions that represent what percent 19 is of 20.

- ☐ A. $\frac{19}{20} \cdot 100$
- ☐ B. $\frac{19}{20} \div 100$
- ☐ C. $\frac{20}{19} \cdot 100$
- ☐ D. $19 \cdot \frac{100}{20}$
- ☐ E. $\frac{19}{100} \cdot 20$

2. At a hardware store, a tool set normally costs $\$80$. During a sale this week, the tool set costs $\$12$ less than normal. What percent of the original price can a customer save? Show or explain your thinking.

Problems 3–5: A 6th grade class did a weekend fitness challenge. Each student set a goal for 75 minutes of exercise.

3. Luca exercised for 54 minutes. What percent of the goal did she complete?

4. Brianna completed 64% of her goal. How many minutes did she exercise for?

5. Amari exercised for 78 minutes. What percent of the goal did Amari complete?

Practice
3.13

Name: .... Date: .... Period: ....

Problems 6–8: A middle school is collecting canned food to donate to a local food bank. Their goal is to collect 1,500 pounds of food in one week. Here is a table showing the total amount of food collected each day.

6. What percent of their goal did they complete with Monday's canned food donations?

7. What percent of their goal did they complete with all of the canned food donations between Monday and Thursday?

Day of the week	Amount of food collected (lbs)
Monday	345
Tuesday	225
Wednesday	135
Thursday	630
Friday	

8. What percent of their goal do they still need to collect on Friday?

Spiral Review

Problems 9–12: Determine each product.

9. $0.72 \cdot 15$

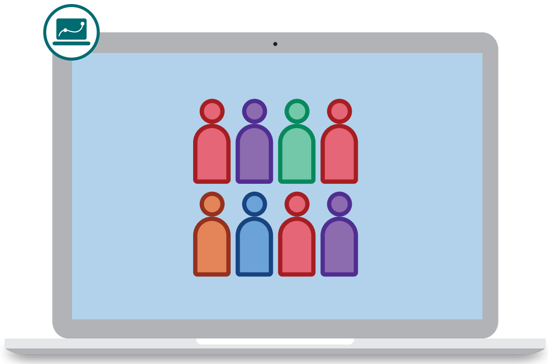
10. $0.72 \cdot 1.5$

11. $0.72 \cdot 0.15$

12. $72 \cdot 0.15$

Name: ____________ Date: ____________ Period: ____________

Model the World 6.RP.3, 6.RP.3.c, SMP.1, SMP.6



A Country as a Village

Let's explore different countries.

Warm-Up

1 Here are some facts about Indonesia from the year 2020.

What do you notice? What do you wonder?

- 2 out of 25 people own a car.
- 71 million people are under 15 years old.
- 54% have access to the internet.
- 39 out of 100 people speak Javanese.

Activity
1

Name: .... Date: .... Period: ....

Exploring Indonesia

2 The population of Indonesia was about 272 million people in 2020.

How many people in Indonesia have each of these characteristics?

Population of Indonesia: 272 million

2 out of 25 people own a car.

71 million people are under 15 years old.

54% have access to the internet.

39 out of 100 people speak Javanese.

Characteristic	Number of People (millions)
Speak Javanese	
Own a car	
Have internet access	
Under 15 years old	

3 It's hard to picture 272 million people.

Imagine Indonesia was a village with just 100 people.

How many people would have each of these characteristics?

Characteristic	Number of People in Village
Speak Javanese	
Own a car	
Have internet access	
Under 15 years old	

Activity 2

Name: ________ Date: ________ Period: ________

A Country as a Village

4 You and your partner will need the Activity 2 Sheet for this activity.

Pick a country. Use the information from 2020 on the Activity 2 Sheet to make a poster describing what this country would look like as a 100-person village.

Brazil China India Nigeria United States

Be sure to include these items in your poster:

- ☐ Your names and the name of the country you selected.
- ☐ An answer to the question: *If this country were a village of* 100 *people, how many of them would have each of these characteristics?*
- ☐ Your thinking and calculations for each characteristic.
- ☐ *At least two* other characteristics of this country you are interested in knowing about.

5 Synthesis

Discuss: How is working with percentages like working with a village of 100 people?

8 Summary 3.14

Working with real-world data and information can be interesting, but it presents challenges, like working with very large numbers or information presented in different forms.

Ratios, rates, and percentages can help you make sense of real-world situations and compare very large numbers.

Some of the benefits of ratios, rates, and percentages are:

- They allow you to compare quantities that are on different scales because they describe things in terms of multiplying and dividing instead of adding and subtracting.
- They bring everything to the same scale, most commonly with a reference point of either 1 or 100, which makes comparing numbers more straightforward.

For example, percentages can help us compare different-sized groups of people around the world to see what the distribution of people really looks like.

Name: .... Date: .... Period: ....

Problems 1–3: The sale price of every item in a store is 85% of its original price. Complete the table to show the prices of each item.

	Item	Original Price ($)	Sale Price ($)
1.	Backpack	30.00	
2.	Soccer Ball		15.30
3.	Jacket		21.08

Problems 4–6: Last Sunday, an amusement park had 1,575 visitors.

4. 56% of the visitors were adults. Calculate the number of adults that visited the park.

5. 16% of the visitors were teenagers. Calculate the number of teenagers that visited the park.

6. 28% of the visitors were children ages 12 and under. Calculate the number of children ages 12 and under that visited the park.

7. A veterinarian examined 45 animals on Wednesday. Of the animals she examined, 20% were cats. How many cats did the veterinarian examine on Wednesday?

A. 18 **B.** 9

C. 5 **D.** 14

Name: ______ Date: ______ Period: ______

Spiral Review

Problems 8–10: Fill in each blank to complete the sentence.

8. 5% of 70 is ______.

9. 25% of ______ is 6.

10. 12% of 700 is ______.

Problems 11–14: Use the conversion rate that makes the most sense to determine the approximate value of each missing quantity. Show your thinking.

1 foot = 12 inches	1 yard = 3 feet
1 meter = 100 centimeters	1 yard = 36 inches

11. A boat is 126 feet long. How long is it in yards?

12. An artist is stretching a canvas that measures 7 feet by 3 feet. What is the area of the canvas in square inches?

13. There is a crack in the sidewalk outside my apartment that is 0.5 meters long. How long is that in centimeters?

14. Tyron is building a dog house for his great dane puppy. He wants it to be 90 inches long and 144 inches wide. What are those measurements in yards?

Name: ________ Date: ________ Period: ________

Generalizing With Multiple Representations | Fraction Relationships | Model the World

6.RP.2, 6.RP.3, 6.RP.3.b, 6.RP.3.c, 6.RP.3.d, SMP.2, SMP.6

Practice Day 2

Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Running Teams

Shortest **Longest**

Task B: Lap Predictions

1. Solution: ________ laps per minute

2. Solution: ________ minutes per lap

3. Solution: ________ laps

4. Solution: ________ minutes

Number of Laps	Number of Minutes
4	6

Name: .... Date: .... Period: ....

Practice Day 2 (continued)

Task C: Lap Goals

1. Solution: .... laps

2. Solution: .... %

3. Solution: .... laps

4. Solution: .... laps

Task D: What's Missing?

1. Solution: ....

2. Solution: ....

3. Solution: ....

4. Solution: ....

5. Solution: ....

6. Solution: ....

You're invited to explore more.

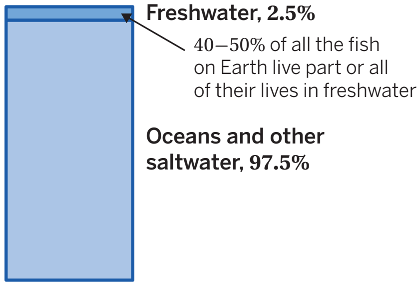
.... minutes

Notes:

Career Connection

71% of Earth is covered with water, yet only a small fraction of that is freshwater.

Only about 2.5% of Earth's total water is freshwater. Despite this small percentage, about 40–50% of all of the species of fish on Earth live in freshwater for all or part of their lives. Many species of freshwater fish are endangered – due to pollution, overfishing, invasive species, and more.



In 2024, due to successful conservation efforts, the U.S. Fish and Wildlife Service announced that several species of freshwater fish – the Roanoke logperch, lake sturgeon, and the Apache trout – are no longer endangered.

Freshwater conservation ecologists collect data to study the fish and plants that live in freshwater regions. They develop strategies to conserve and restore these populations. They might use percentages to predict the number of fish in a certain freshwater region each year.

Meet Laura Máiz-Tomé

Laura Máiz-Tomé is a freshwater conservation ecologist who collaborates with other scientists to help preserve freshwater ecosystems around the world. She has led research to predict the extinction risk of freshwater species. Laura Máiz-Tomé is particularly interested in species conservation related to expanding protected areas for species that live in or are dependent upon wetlands.

Are you interested in studying ecology? What can you do to learn more?

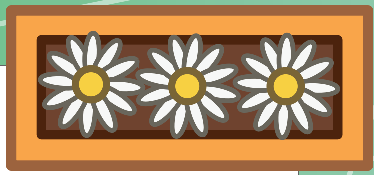
Math in the World

Earth's total water is estimated at about 370 quintillion gallons. (That's 370 with 18 zeros after it!) About how many gallons of Earth's water is freshwater?

Math Mindset

How can you use diagrams or reasoning to explain to a friend how to calculate 2.5% of 370?

Unit 4



Dividing Fractions

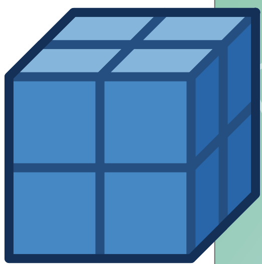
Big Ideas in This Unit

CC2 Fraction Relationships

Generalizing With Multiple Representations

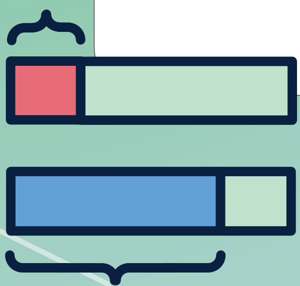
CC3 Nets and Surface Area

Questions for Investigation



- What are two ways to think about dividing by a fraction?
- How are division and multiplication related to each other?
- In what situations would there be fraction-sized groups or a number of groups that is a fraction?

Explore: Egyptian Fractions
Can all fractions be written as the sum of distinct unit fractions?



Watch Your Knowledge Grow

This is the math you'll explore in this unit. Rate your understanding to see how your knowledge grows!

Not yet | Almost | I got it!

I can . . .	Before	After
Decide if quotients of division situations are greater than 1, less than 1, or equal to 1.		
Draw diagrams that represent division situations involving whole numbers.		
Decide whether a division question is asking "How many groups?" or "How many in each group?"		
Use diagrams to represent and solve division problems asking "How many groups?" when the number of groups is a fraction.		
Use diagrams to represent and solve division problems asking "How many in 1 group?" when a whole number is divided by a fraction.		
Interpret a division problem as a question asking "How many in 1 group?"		
Interpret a division problem as a question asking "How many groups?"		
Use common denominators to divide fractions.		
Explain strategies for using common denominators to determine the quotient of any two fractions.		

I can . . .	Before	After
Identify and write equivalent expressions using division and multiplication with fractions.		
Explain why dividing by a fraction is the same as multiplying by its reciprocal.		
Solve problems involving division of fractions by fractions in context and explain my solution method.		
Write my own problem to represent a division expression.		
Divide fractions to solve problems about comparing lengths.		
Interpret a division expression as a question asking "How many times as long?"		
Use division and multiplication to solve problems about areas of polygons with lengths that are fractions.		
Calculate the volume of a rectangular prism with lengths that are fractions.		
Use fraction division and multiplication to solve problems about area and volume in context.		

Sub-Unit

1

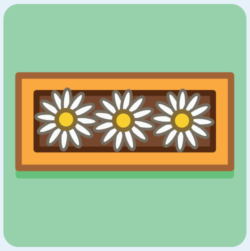
Introduction to Dividing Fractions

Explore
Egyptian Fractions

Lesson 1
Cookie Cutter

Lesson 2
Division Meanings

Lesson 3
Flour Planner



Lesson 4
Flower Planters

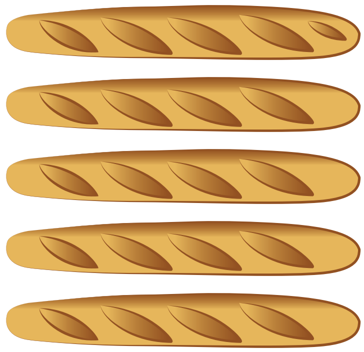
Name: .... Date: .... Period: ....

Fraction Relationships | Building Toward 6.NS.1, SMP.1, SMP.3, SMP.7

Explore: Egyptian Fractions

Can all fractions be written as the sum of distinct unit fractions?

Warm-up



1 Imagine this situation:

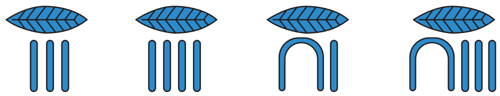
A scribe in ancient Egypt has to distribute 5 loaves of bread equally among 8 workers.

What is the least number of cuts the scribe can make so that each worker receives an equal share? Explain your thinking.

Name: .... Date: .... Period: ....

Sum of Distinct Unit Fractions

2 Here is how ancient Egyptians wrote fractions using *hieroglyphs*.



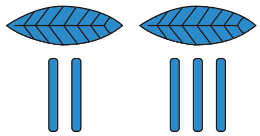
$\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{11}$ $\frac{1}{13}$

How do you think they wrote $\frac{1}{5}$ and $\frac{1}{10}$ using hieroglyphs?

a $\frac{1}{5}$

b $\frac{1}{10}$

3 Ancient Egyptian preferred to express $\frac{5}{6}$ as the sum of $\frac{1}{2}$ and $\frac{1}{3}$ instead of five $\frac{1}{6}$s. Why do you think that might be?



4 Some people define *Egyptian fractions* as the sum of two or more distinct *unit fractions*.

Work with your partner to express as many fractions as possible in the form of Egyptian fractions.

a $\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$

b $\frac{3}{4}$

c $\frac{2}{5}$

d $\frac{3}{5}$

e $\frac{4}{5}$

f $\frac{3}{8}$

g $\frac{5}{8}$

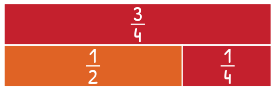
h $\frac{7}{8}$

Name: ________ Date: ________ Period: ________

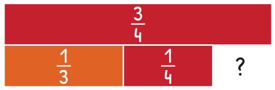
Sum of Distinct Unit Fractions (continued)

5 There are several ways to express a fraction as the sum of distinct unit fractions.

Sora $\frac{3}{4} = \frac{1}{2} + \frac{1}{4}$



Eva $\frac{3}{4} = \frac{1}{3} + \frac{1}{4} + \frac{1}{?}$



Luca $\frac{3}{4} = \frac{1}{2} + \frac{1}{5} + \frac{1}{?}$

a Here is how Sora, Eva, and Luca expressed $\frac{3}{4}$ using unit fractions.

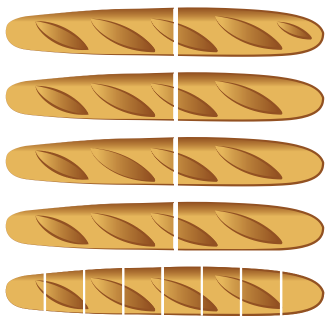
Determine the unknown fractions in Eva's and Luca's representations.

Eva: ________ Luca: ________

b **Discuss:** How many different ways can a fraction be represented as an Egyptian fraction?

6 Describe a strategy to represent any fraction in the form of Egyptian fractions using the least number of unit fractions.

7 **a** How can Egyptian fractions help us determine the fewest number of cuts in the problem from the Warm-Up?



b **Discuss:** Why might Egyptians prefer unit fractions for this kind of division?

Activity

Name: .... Date: .... Period: ....

Building Math habits of Mind

Discuss:

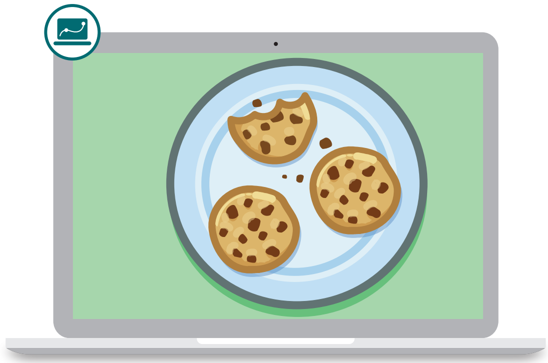
- Which of these habits of mind did you strengthen during this activity?
- How did you use the one(s) you selected?

I can slow down and first make sense of a challenging problem before trying to solve it. Not yet — Almost — I got it!	I can represent real-world problems using equations and inequalities and interpret their solutions within the context of the problem. Not yet — Almost — I got it!
I can justify my thinking and ask questions to help me understand the thinking of others. Not yet — Almost — I got it!	I can apply the math that I know to solve real-world problems, make assumptions and revise my thinking as needed. Not yet — Almost — I got it!
I can select an appropriate tool to help me solve problems. Not yet — Almost — I got it!	I can communicate my thinking and solutions clearly to others. Not yet — Almost — I got it!
I can look for structure or patterns to help me solve problems. Not yet — Almost — I got it!	I can look for repeated calculations and other repeated steps to make generalizations. Not yet — Almost — I got it!

Unit 4
Lesson
1

Name: .... Date: .... Period: ....

Fraction Relationships Building Toward 6.NS.1, SMP.1, SMP.3, SMP.7



Cookie Cutter

Let's estimate quotients.

Warm-Up

1 Write a story that could be represented by the expression $12 \div 3$.

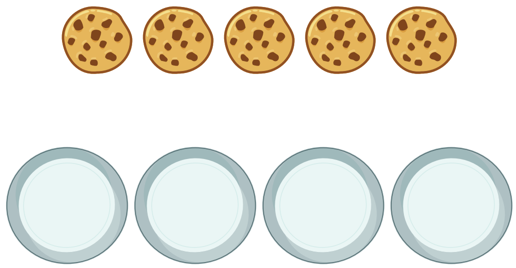
Draw a sketch if it helps you illustrate your story.

Activity 1

Name: .... Date: .... Period: ....

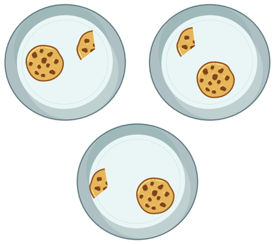
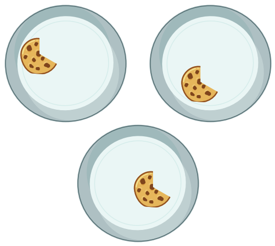
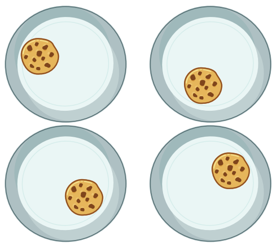
Sharing Cookies

2 The quotient of $5 \div 4$ can be represented by the number of cookies each person gets if there are 5 cookies shared equally on 4 plates.



Discuss: How do you know that each plate will have more than 1 cookie?

3 **a** Take a look at these three different situations about cookies and plates.

Cookies	Plates	Model	Expression
4	3		$4 \div 3$
2	3		$2 \div 3$
4	4		$4 \div 4$

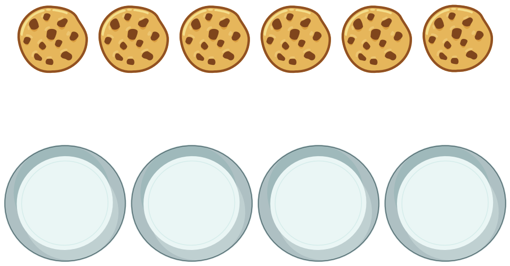
b How can you tell from each expression whether there will be more than 1, less than 1, or exactly 1 cookie per plate?

Activity
2

Name: .... Date: .... Period: ....

Crumbly Cookie Quotients

4 Kiandra wants to share 6 cookies on 4 plates.

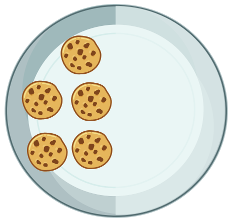


She wrote $6 \div 4 = 1\frac{1}{2}$.

What does $1\frac{1}{2}$ mean in this situation? Circle one.

Cookies Plates Plates per cookie Cookies per plate

5 There are 5 cookies for every $\frac{1}{2}$ of a plate.

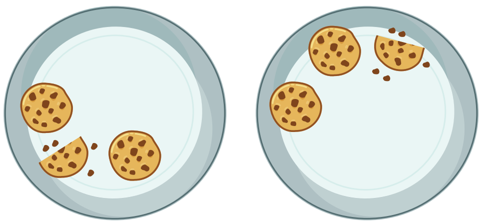


How many cookies would be on 1 whole plate?

Draw a sketch if it helps with your thinking.

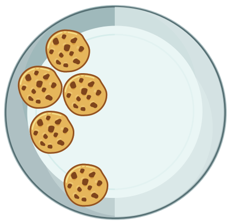
6 Here are two situations.

There are 5 cookies for every 2 plates. How many cookies would be on 1 plate?



$5 \div 2$

There are 5 cookies for every $\frac{1}{2}$ of a plate. How many cookies would be on 1 plate?



$5 \div \frac{1}{2}$

How are these situations alike? How are they different?

Activity 3

Name: ______ Date: ______ Period: ______

Quotient Card Sort

7 Sort these expressions by their quotient.

$9 \div 10000$	$\frac{1}{2} \div 2$	$\frac{1}{3} \div \frac{1}{3}$	$10000 \div 9$
$19 \div 19$	$10 \div 9$	$3 \div \frac{1}{4}$	

Less Than 1	Equal to 1	Greater Than 1

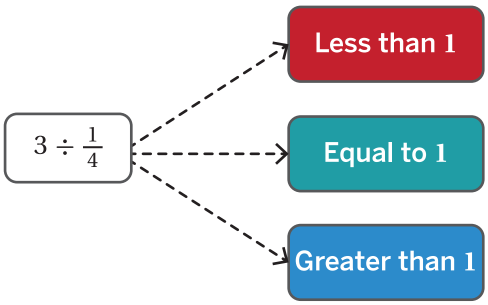
8 Where did you sort $3 \div \frac{1}{4}$? Select one.

A. Less than 1

B. Equal to 1

C. Greater than 1

Explain your reasoning.

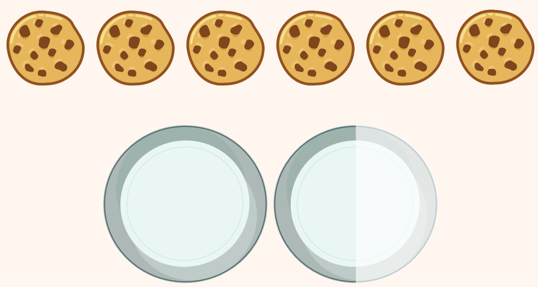


You're invited to explore more.

9 Kiandra wrote: $6 \div 1\frac{1}{2} = ?$

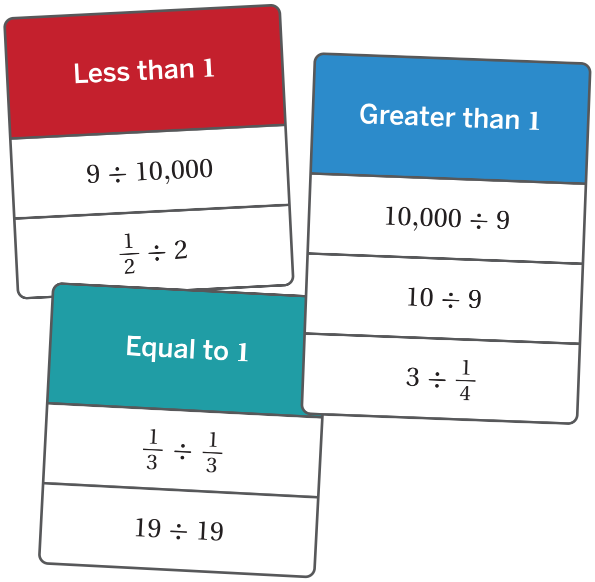
Then she drew 6 cookies and $1\frac{1}{2}$ plates.

How many cookies will be on 1 whole plate?



10 Synthesis

Discuss How can you tell whether a quotient will be less than 1, equal to 1, or greater than 1?



13 Summary 4.01

You can use the parts of an expression to estimate the size of its *quotient*. Take a look at these examples:

$$4 \div 1\frac{1}{2}$$

The quotient is greater than 1.

This expression represents 4 cookies separated onto $1\frac{1}{2}$ plates. Here, the *dividend* is greater than the *divisor*. This means each plate would have *more than* 1 cookie.

$$4 \div 4$$

The quotient is equal to 1.

This expression represents 4 cookies separated onto 4 plates. Here, the dividend is equal to the divisor. This means each plate would have *exactly* 1 cookie.

$$4 \div 4\frac{1}{2}$$

The quotient is less than 1.

This expression represents 4 cookies separated onto $4\frac{1}{2}$ plates. Here, the dividend is less than the divisor. This means each plate would have *less than* 1 cookie.

Practice 4.01

Name: .... Date: .... Period: ....

Problems 1–6: Without calculating, decide whether the value of each quotient is *greater than* 1, *less than* 1, or *equal to* 1.

1. $40 \div 40000$

2. $40 \div \frac{1}{2}$

3. $40 \div 39$

4. $40123 \div 40$

5. $\frac{1}{4} \div 40$

6. $\frac{1}{4} \div \frac{1}{4}$

7. Choose *one* of the expressions from Problems 1–6 and estimate its value. Explain your thinking.

Problems 8–10: Fill in each blank with a number that makes the sentence true.

8. The value of .... $\div$ 32 is less than 1.

9. The value of .... $\div$ 32 is equal to 1.

10. The value of .... $\div$ 32 is greater than 1.

Name: ________ Date: ________ Period: ________

Spiral Review

11. A class has $\$140$ to spend on tickets to a science museum. The teacher writes $140 \div 10 = 14$. Alma says that tickets are $\$14$ each. Mohamed says that 14 students are going on the trip. Who is correct? Circle one.

Alma Mohamed Both Neither

Explain your thinking.

Problems 12–14: Determine what fraction of each rectangle is shaded.

12. 

____

13. 

____

14. 

____

15. Determine each product.

a. $\frac{2}{3} \cdot 4$

b. $\frac{2}{3} \cdot \frac{1}{4}$

c. $\frac{2}{3} \cdot \frac{9}{10}$

Name: .... Date: .... Period: ....

Fraction Relationships 6.NS.1, SMP.7

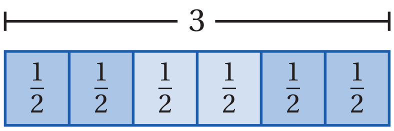
Division Meanings

Let's connect multiplication, division, and tape diagrams.

Warm-Up

1. Which one doesn't belong? Explain your thinking. **ELD.PI.6.11.Em, Ex, Br**

A.



B. $3 \div \frac{1}{2} = ?$

C. $3 \div 6 = \frac{1}{2}$

D. $6 \cdot \frac{1}{2} = 3$

Activity 1

Name: ________ Date: ________ Period: ________

Connecting Tape Diagrams

2. You will use tape diagram cards to complete Rounds 1–4.

Round 1: Write a situation to represent your card. Solve the problem in this situation. Include the units for your answer. **ELD.PI.6.10.Em, Ex, Br**

Round 2: Find a person whose card has the *same number in each group* as yours.

My Card

Multiplication equation:

________ groups of ________ is ________.

____

________ **'s Card**

Multiplication equation:

________ groups of ________ is ________.

____

Name: ________ Date: ________ Period: ________

Connecting Tape Diagrams (continued)

Round 3: Find a person whose card has the *same number of groups* as yours.

My Card

Tape diagram:

Division equation:

________'s Card

Tape diagram:

Division equation:

Round 4: Find a person whose card has the *same tape diagram total* as yours.

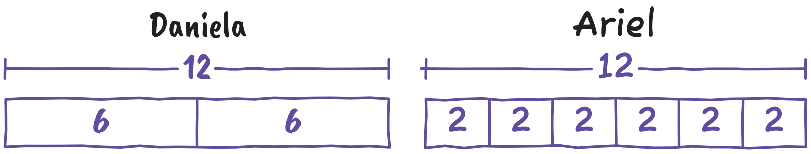
ELD.PI.6.10.Em, Ex, Br

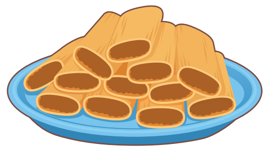
________'s Tape Diagram	My Division Equation	Explain how your classmate's tape diagram represents your division equation.

Name: .... Date: .... Period: ....

Tamales and Tape Diagrams

3. Nikolai put 12 tamales into equal-sized bags. Daniela and Ariel drew tape diagrams to represent the situation.





Discuss: What does each tape diagram tell us about the number of tamales and bags? **ELD.PI.6.3.Em, Ex, Br, ELD.PI.6.6.Em, Ex, Br,**

4. What division equations could the diagrams represent?

5. Draw a tape diagram to represent the equation $15 \div 3 = ?$

Activity 3

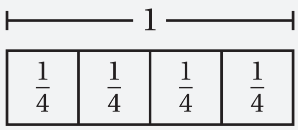
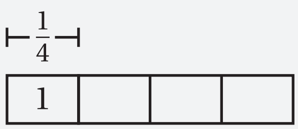
Name: ______ Date: ______ Period: ______

Situations, Diagrams, and Equations

Pair up with a classmate. Decide who will be Partner A and who will be Partner B.

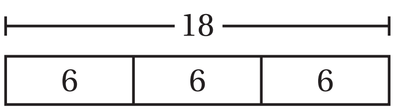
- Partner A will complete the odd numbered problems.
- Partner B will complete the even numbered problems.

When you're done, share and compare your responses. **ELD.PI.6.6.Em, Ex, Br,**

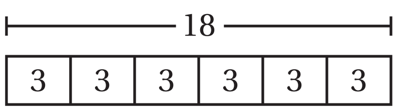
	Situation	Tape Diagram	Multiplication Equation	Division Equation
6.	I gave away 8 cookies. Each person got 2 cookies. How many people did I give cookies to?		$2 \bullet ? = 8$	
7.	I split 8 cookies equally between 2 people. How many cookies did each person get?		$? \bullet 2 = 8$	
8.	I split 12 flowers equally between 4 planters. How many flowers are in each planter?			$12 \div 4 = ?$
9.	I planted 12 flowers. Each planter holds 4 flowers. How many planters did I fill?			$12 \div 4 = ?$
10.	I need 1 cup of flour. I have a $\frac{1}{4}$-cup scoop. How many scoops do I need?			
11.	I used $\frac{1}{4}$ of a scoop to measure 1 cup of flour. How many cups does 1 whole scoop hold?			

12. Write a situation to represent each of these tape diagrams. **ELD.PI.6.6.Em, Ex, Br, ELD.PI.6.10.Em, Ex, Br**

Tape Diagram A



Tape Diagram B



Synthesis

13. Draw *two* different tape diagrams that each represent $30 \div 6$.

Summary 4.02

You can interpret division expressions in two ways, both of which involve thinking about equal-sized groups. One way answers "How many groups?" and the other answers "How many in each group?"

Let's use the division expression $35 \div 7$, where 35 represents the number of bagels. The quotient, 5, could answer two different questions: "How many boxes?" or "How many in each box?" Here's what the situation would be for each question.

How many boxes?	How many in each box?
35 bagels are placed in boxes so that there are 7 bagels in each box.	35 bagels are divided equally among 7 boxes.
Quotient: 5 boxes	**Quotient:** 5 bagels in each box

Practice 4.02

Name: ______________ Date: ________ Period: ________

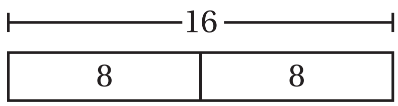
Problems 1–3: Match each situation to the diagram that best represents it.

1. I earn \$8 an hour for mowing the lawn. How much will I earn if I work for 2 hours? ________

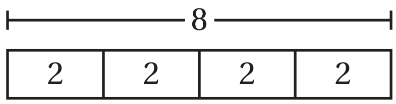
2. I divided 2 cakes equally between 8 people. How much cake does each person get? ________

3. I folded 8 socks into pairs. How many pairs do I have? ________

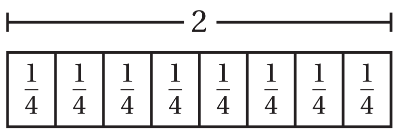
a



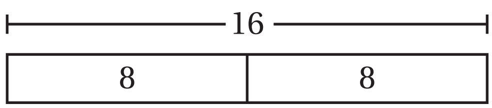
b



c



4. Three students wrote equations for this diagram.



Yolanda: $16 \div 8 = 2$

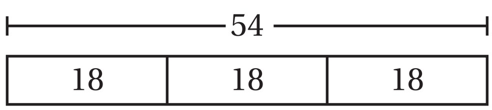
Riku: $16 \div 2 = 8$

Mariana: $8 \bullet 2 = 16$

Explain why they are each correct.

- Yolanda:
- Riku:
- Mariana:

5. Write *three* equations that could represent this tape diagram.



Practice
4.02

Name: ______ Date: ______ Period: ______

Spiral Review

Problems 6–8: Fill in each blank with a number that makes the sentence true.

6. The value of ______ $\div 6.1$ is greater than 1.

7. The value of ______ $\div 6.1$ is equal to 1.

8. The value of ______ $\div 6.1$ is less than 1.

Problems 9–11: Calculate each percentage.

9. 25% of 400

10. 75% of 200

11. 5% of $9{,}000$

Problems 12–13: A group of 12 students from the Science Club is on a field trip.

12. The group represents 20% of the total number of students in the club. What is the total number of students in the Science Club?

A. 20

B. 32

C. 60

D. 240

13. 25% of the students in the Science Club are also the members of the Art Club. How many students are in both clubs?

Unit 4
Lesson
3

Name: ______________ Date: ______________ Period: ______________

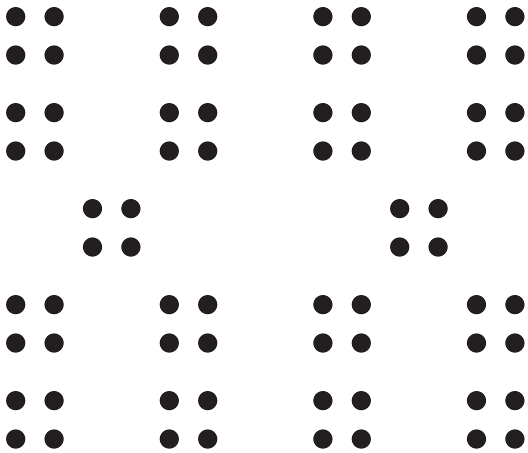
Fraction Relationships 6.NS.1, SMP.2

Flour Planner

Let’s think about fractions by using drawings and diagrams to ask, “How many groups?”

Warm-Up

1 a How many dots are in this image?



b Explain or show how you saw them.

Activity 1

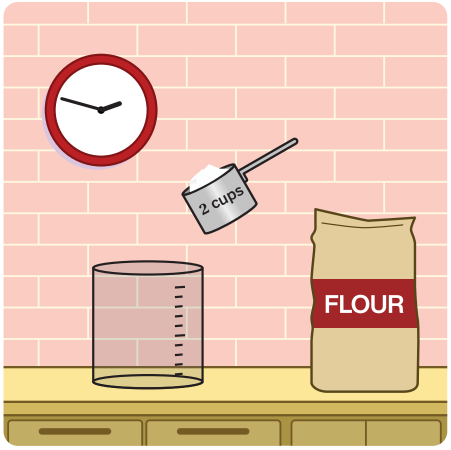
Name: ________ Date: ________ Period: ________

Fractional Scoops

2 Tres leches cake is a popular dessert in Mexico and Central America that's made with three kinds of milk.

Alexis needs 6 cups of flour to make tres leches cake, but there is only a 2-cup measuring scoop.

How many scoops does Alexis need?



3 Circle an equation that you could use to determine how many 2-cup scoops make 6 cups of flour.

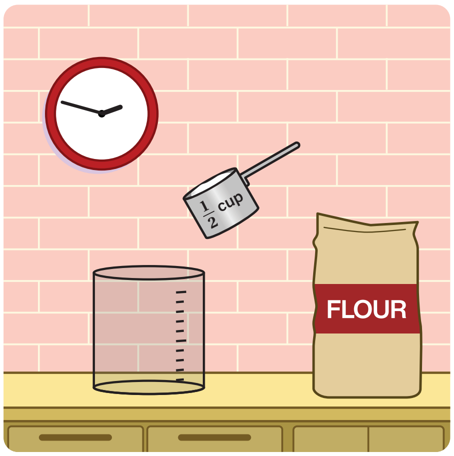
$6 \cdot ? = 2$ $\qquad$ $6 \div 2 = ?$ $\qquad$ $2 \div 6 = ?$ $\qquad$ $2 \cdot ? = 6$

Explain your thinking.

4 LaShawn also needs 6 cups of flour to make tres leches cake.

There is only a $\frac{1}{2}$-cup measuring scoop.

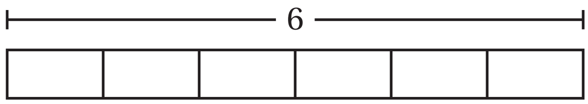
How many scoops does LaShawn need?



Name: .... Date: .... Period: ....

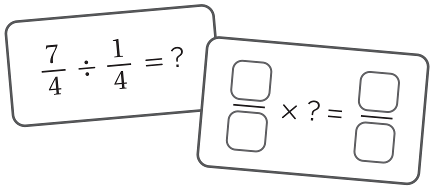
Fractional Scoops (continued)

5 **a** Let's watch how LaShawn determined the number of $\frac{1}{2}$-cup scoops needed for 6 cups of flour.



b Explain how this tape diagram helped LaShawn determine that 12 scoops are needed.

6 Alexis needs $\frac{7}{4}$ cups of flour to make tres leches cake. There is only a $\frac{1}{4}$-cup measuring scoop. Write a multiplication expression to determine how many scoops are needed?



Activity 2

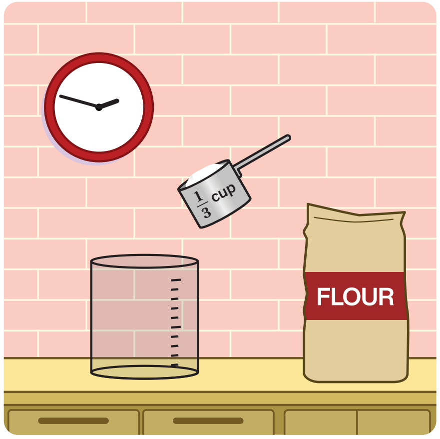
Name: ________ Date: ________ Period: ________

A Bigger Scoop

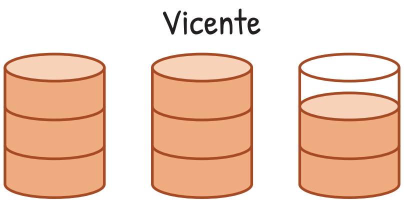
7 Sirnee is a sweet dish that is often made for Islamic celebratory feasts.

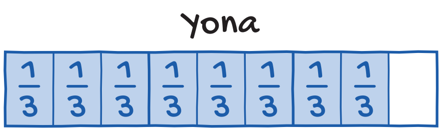
Hamza needs $2\frac{2}{3}$ cups of flour to make sirnee, but he only has a $\frac{1}{3}$-cup measuring scoop.

How many scoops does he need?



8 Vicente and Yona each sketched a diagram to determine how many $\frac{1}{3}$-cup scoops they need to measure $2\frac{2}{3}$ cups of flour.





Discuss: How could each diagram help us calculate the number of scoops needed?

Activity
2

Name: ________ Date: ________ Period: ________

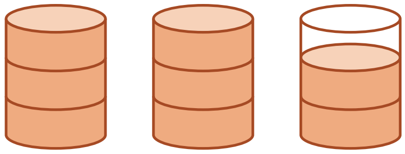
A Bigger Scoop (continued)

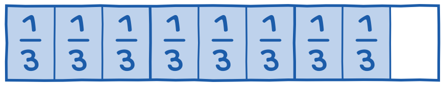
9 Hamza found a $\frac{2}{3}$-cup measuring scoop to use to make sirnee.

How many of these scoops would he need to measure $2\frac{2}{3}$ cups of flour?

Use the multiplication expression, cups diagram, and tape diagram if they help with your thinking.

$\frac{2}{3} \times ? = 2\frac{2}{3}$

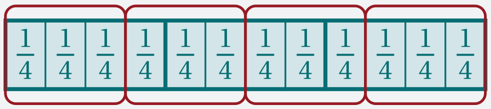




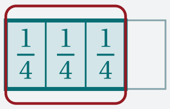
Explain your thinking.

10 Group together the choices that represent the same situation.

$3 \div \frac{3}{4} = ?$



$\frac{3}{4} \div 3 = ?$

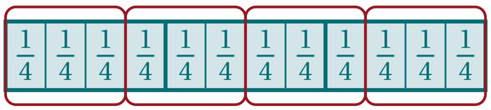
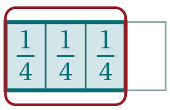


$\frac{3}{4} \cdot ? = 3$

4 scoops

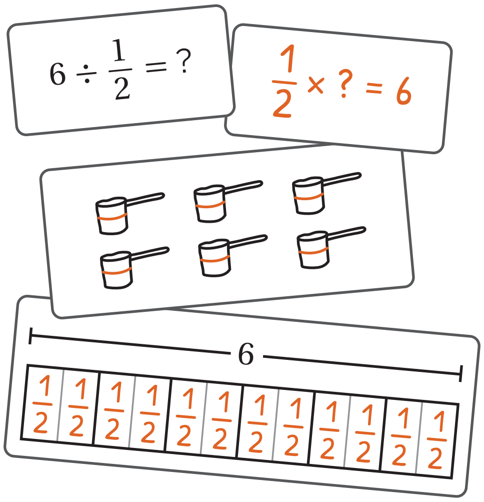
$3 \cdot ? = \frac{3}{4}$

$\frac{1}{4}$ scoops

Alexis needs 3 cups of flour, but there is a $\frac{3}{4}$-cup measuring scoop.	LaShawn needs $\frac{3}{4}$ cups of flour, but there is a 3-cup measuring scoop.
	

11 Synthesis

Discuss: How can you use an equation or a diagram to determine how many $\frac{1}{2}$-cup scoops you need to make 6 cups?



14 Summary 4.03

You can answer the question "How many groups?" using different representations that include both whole numbers and fractions.

Here's the problem "How many $\frac{2}{5}$s are in 2?" represented using a tape diagram, a multiplication equation, and a division equation.

Tape Diagram

2

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Multiplication Equation

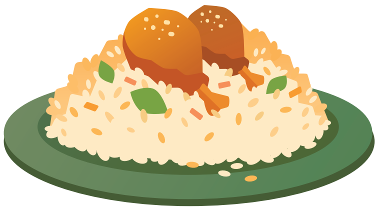
$$\frac{2}{5} \cdot ? = 2$$

Division Equation

$$2 \div \frac{2}{5} = ?$$

Because there are 5 groups of $\frac{2}{5}$ in 2, the value 5 makes both equations $\frac{2}{5} \cdot 5 = 2$ and $2 \div \frac{2}{5} = 5$ true.

Name: ______________________ Date: __________ Period: __________



Problems 1–4: Biryani is a rice dish from South Asia. Three students made Alisha's biryani recipe using different-sized scoops. If the recipe calls for 4 cups of rice, how many scoops of rice does each student need?

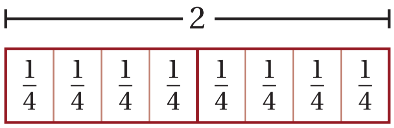
1. Alisha: 2-cup scoop

2. Lukas: $\frac{1}{2}$-cup scoop

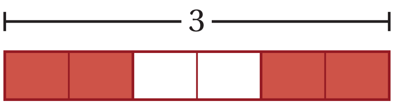
3. Emma: $\frac{1}{3}$-cup scoop

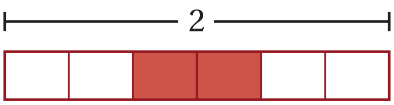
4. Explain how the equation $4 \div \frac{1}{3} = ?$ represents Emma's situation.

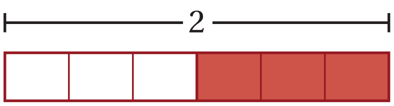
5. Lukas drew this diagram to represent the question "How many $\frac{1}{4}$s make 2?" Write a division equation to represent Lukas's diagram.

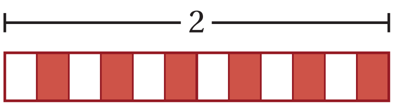


6. Allison has a 2-pound bag of cat food. She has 6 cats. Her cats eat $\frac{2}{3}$ pounds of cat food per day. Which model best represents how many days her 2-pound bag of food will last?

A.


B.


C.


D.


Practice
4.03

Name: ______ Date: ______ Period: ______

Spiral Review

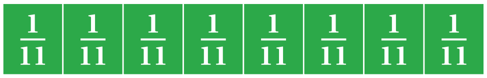
Problems 7–10: Match the tape diagrams with the multiplication expressions.

$8 \cdot \frac{1}{11}$ $\quad$ $11 \cdot \frac{1}{8}$ $\quad$ $\frac{2}{7} \cdot 4$ $\quad$ $\frac{4}{7} \cdot 2$

7.

______

8.

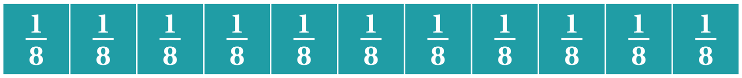


______

9.

______

10.



______

11. *When you multiply one number by another, the result will be larger than the first number.*

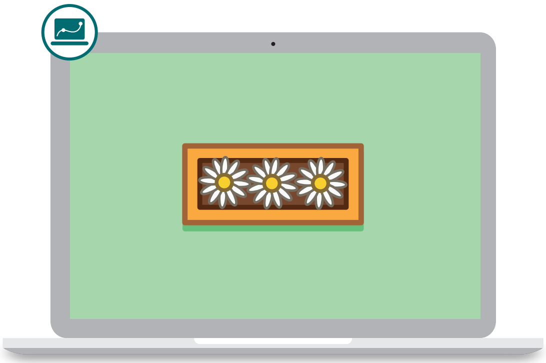
Is this statement *always*, *sometimes*, or *never* true? Circle one.

Always $\qquad$ Sometimes $\qquad$ Never

Explain your thinking.

Name: ______________ Date: ______ Period: ______

Fraction Relationships 6.NS.1, SMP.1, SMP.6, SMP.8



Flower Planters

Let's use flower planters to answer the question "How many in *one* group?"

Warm-Up

1 Order these expressions from *least* to *greatest* by the value of the quotient.

$12 \div 12$	$12 \div \frac{2}{3}$	$12 \div 1$
$12 \div 3$	$12 \div \frac{1}{4}$	

	Least
	Greatest

Activity 1

Name: .... Date: .... Period: ....

Plenty of Planters

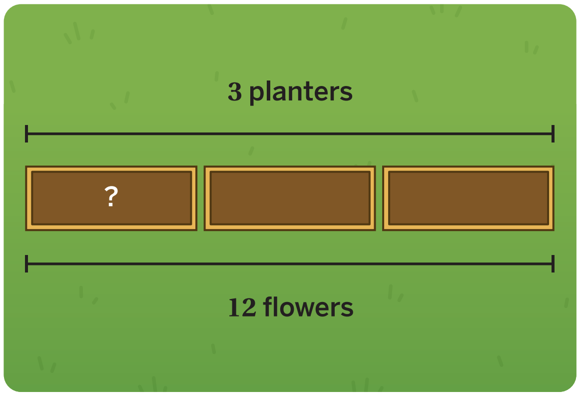
2 Write a story that could be represented by the expression $12 \div \frac{1}{3}$.

Draw a sketch if it helps you illustrate your story.

3 Brianna is planting flowers in her class garden.

12 flowers fill 3 small planters.

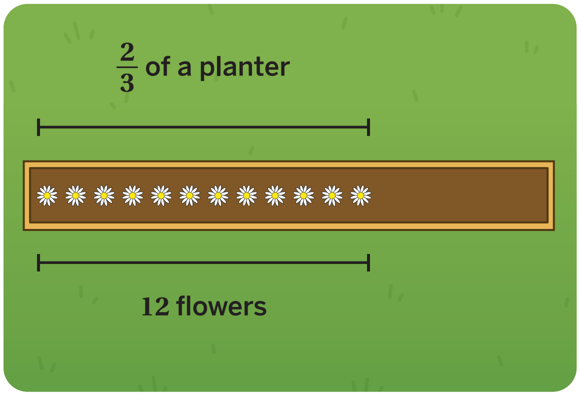
How many flowers fill 1 small planter?



4 Brianna also put flowers in a big planter.

12 flowers fill $\frac{2}{3}$ of a big planter.

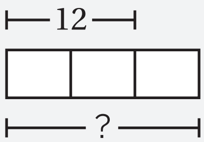
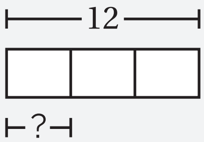
How many flowers fill 1 big planter?



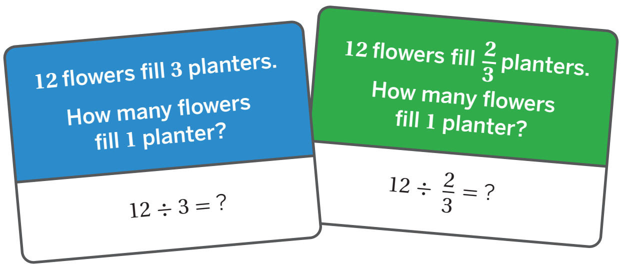
Name: ______ Date: ______ Period: ______

Plenty of Planters (continued)

5 Match each representation with a question.

	12 flowers fill 3 planters. How many flowers fill 1 planter?	12 flowers fill $\frac{2}{3}$ of a planter. How many flowers fill 1 planter?
$12 \div 3 = ?$		
$12 \div \frac{2}{3} = ?$		
$\frac{2}{3} \cdot ? = 12$		
$3 \cdot ? = 12$		
		
		

6 How are these expressions alike? How are they different?



Alike:

Different:

Activity

2

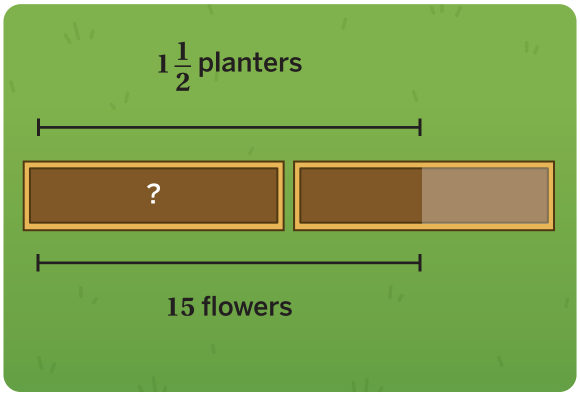
Name: ____ Date: ____ Period: ____

Practicing With Planters

7 Brianna has 15 flowers to put in her planters.

The flowers fill $1\frac{1}{2}$ planters.

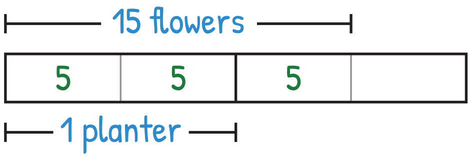
How many flowers fill 1 planter?



8 Here is a diagram Brianna made to calculate how many flowers fill 1 planter when 15 flowers fill $1\frac{1}{2}$ planters.

Explain how Brianna can use this diagram to help her answer the question.

$$15 \div 1\frac{1}{2} = ?$$

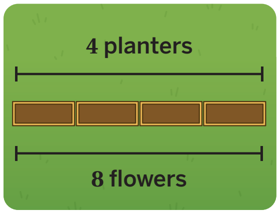
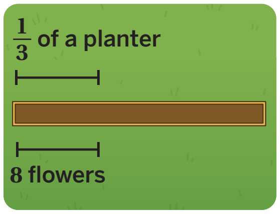
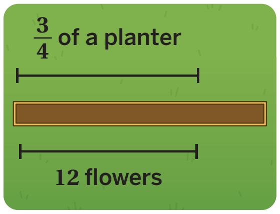
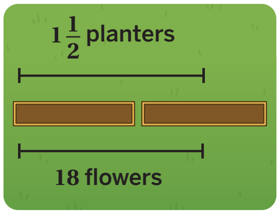
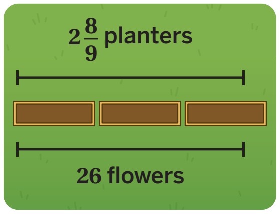


Activity 2

Name: ______ Date: ______ Period: ______

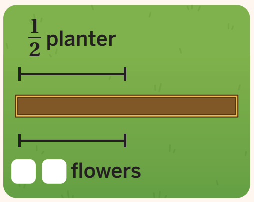
Practicing With Planters (continued)

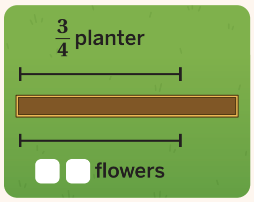
9 Solve as many challenges as you have time for.

	Situation	Diagram	How many flowers fill 1 planter?
a	8 flowers fill 4 planters.		
b	8 flowers fill $\frac{1}{3}$ of a planter.		
c	12 flowers fill $\frac{3}{4}$ of a planter.		
d	18 flowers fill $1\frac{1}{2}$ planters.		
e	26 flowers fill $2\frac{8}{9}$ of a planter.		

You're invited to explore more.

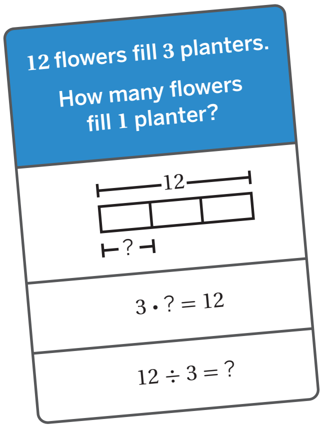
10 Fill in each blank using the numbers 0 to 9 only once, so that the same number of flowers fill each planter.

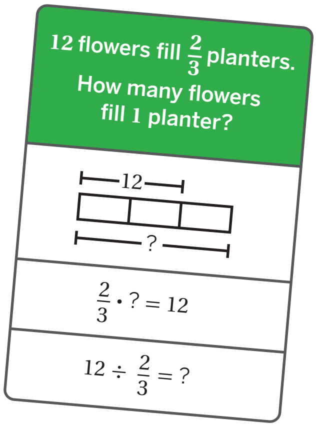




11 Synthesis

Discuss: How can a tape diagram represent a division problem?

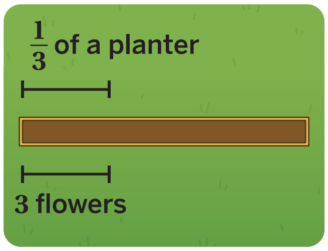
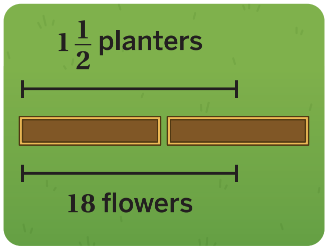




14 Summary 4.04

You can answer "How many are in *one* group?" by:

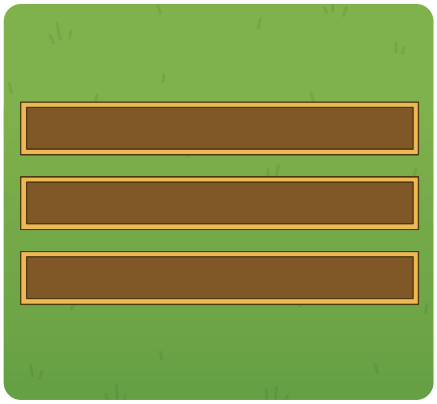
- Evaluating division and multiplication expressions.
- Using tape diagrams that represent division and multiplication expressions.

Situation	Diagram	Expressions	Number of Flowers in 1 Planter
3 flowers fill $\frac{1}{3}$ of a planter.		$3 \div \frac{1}{3} = ?$ or $\frac{1}{3} \cdot ? = 3$	9
18 flowers fill $1\frac{1}{2}$ planters.		$18 \div 1\frac{1}{2} = ?$ or $1\frac{1}{2} \cdot ? = 18$	12

Practice
4.04

Name: ______________________ Date: __________ Period: __________

Problems 1–4: Abena is planting vegetables in her backyard. Determine how many of each vegetable plant Abena can fit in 1 planter.



Use the diagrams if they help with your thinking.

1. Onion plants, if 10 onion plants fill $\frac{1}{2}$ of a planter.

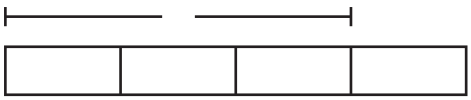
2. Asparagus crowns, if 8 asparagus crowns fill $\frac{2}{3}$ of a planter.

3. Potato plants, if 6 potato plants fill $\frac{3}{4}$ of a planter.

4. Abena wrote the expression $6 \div \frac{3}{4}$ to represent how many potato plants fill 1 planter. Describe a situation that represents the expression $8 \div \frac{4}{5}$.

Problems 5–6: Ashley picks 9 strawberries from her backyard, which fill $\frac{3}{4}$ of a cup.

5. Label the tape diagram to represent Ashley's situation.



6. Determine how many strawberries fill 1 cup. Use the tape diagram if it helps with your thinking.

7. George is painting a mural. He uses 3 gallons of paint for $\frac{3}{8}$ of the mural. How many gallons of paint would he need to paint the whole mural?

Name: .... Date: .... Period: ....

Spiral Review

Problems 8–9: Karima made 9 pairs of earrings in 6 hours.

8. How long will it take Karima to make 12 pairs of earrings?

9. How many pairs of earrings can Karima make in 10 hours?

Problems 10–12: Calculate each unknown number.

10. 5 is 50% of what number?

11. 300 is 10% of what number?

12. 18 is 150% of what number?

Problems 13–16: Determine half of each unit fraction.

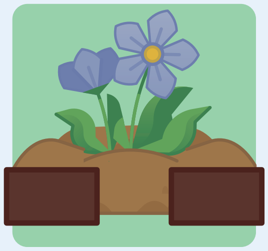
13. $\frac{1}{2}$ **14.** $\frac{1}{3}$ **15.** $\frac{1}{5}$ **16.** $\frac{1}{8}$

Sub-Unit

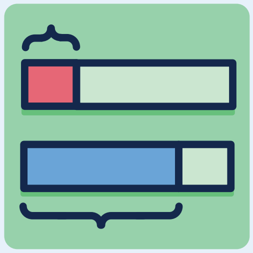
2

Dividing Fractions

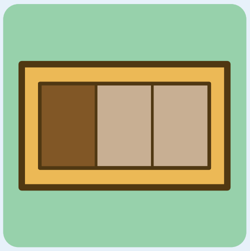
Lesson 5
Garden Bricks



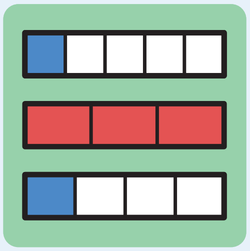
Lesson 6
Fill the Gap



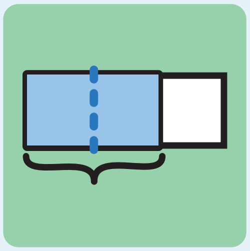
Lesson 7
Break It Down



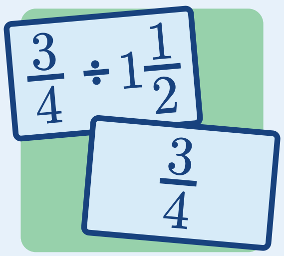
Lesson 8
Potting Soil



Lesson 9
Division Challenges



Lesson 10
Action Fractions



Lesson 11
Swap Meet

Name: ______________ Date: ________ Period: ________

Fraction Relationships 6.NS.1, SMP.2, SMP.3, SMP.6

Garden Bricks

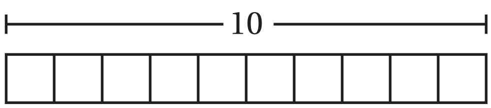
Let's use tape diagrams to think about, "How many groups?"

Warm-Up

Question	Expression
How many groups of $2\frac{1}{2}$ are in 10?	$10 \div 2\frac{1}{2}$

1. **Discuss:** How do you know that the expression represents the question?
   ELD.PI.6.1.Em, Ex, Br

2. Use the tape diagram to answer the question.



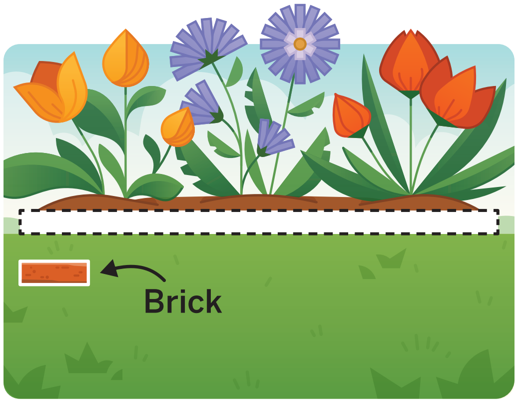
Activity 1

Name: .... Date: .... Period: ....

How Many Bricks?

Deja and Emma are upgrading their class gardens by placing bricks along the front of each garden.

3. The first garden is 4 feet long. Deja is using small bricks, which are $\frac{1}{3}$ of a foot long. How many small bricks does Deja need? Draw a tape diagram to show your thinking.



4. The second garden is also 4 feet long. Emma is using large bricks, which are $\frac{2}{3}$ of a foot long. How many large bricks does Emma need? Draw a tape diagram to show your thinking.

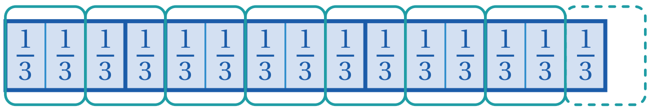
5. The third garden is 5 feet long. Here is how each student predicts the number of large bricks. **ELD.PI.6.6.Em, Ex, Br**

Deja

$5 \div \frac{2}{3}$

"I need less than 5 bricks because I am dividing 5 by $\frac{2}{3}$, the result must be less than 5"

Emma



"I need more than 7 bricks because there are 7 groups of $\frac{2}{3}$ and some left over"

Who do you agree with?

Explain your thinking.

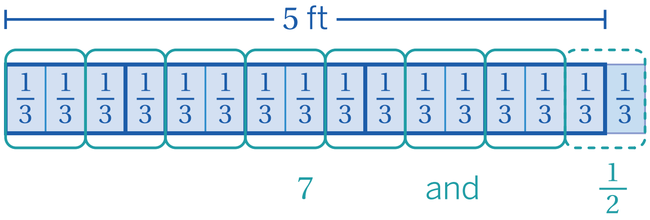
Name: .... Date: .... Period: ....

How Many Bricks? (continued)

6. Emma calculated the number of large bricks needed to be $7\frac{1}{2}$.

ELD.PI.6.1.Em, Ex, Br, ELD.PI.6.6.Em, Ex, Br

Discuss: How did Emma determine the number of bricks as $7\frac{1}{2}$?

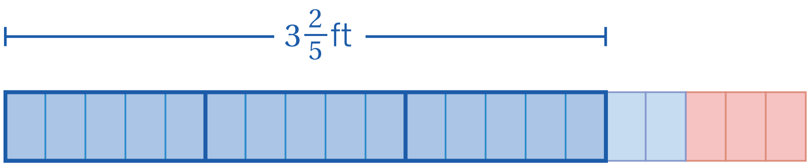


7. Deja wants to cover a 3 feet long garden using $\frac{4}{5}$ ft bricks. How many large bricks does Deja need? Draw a tape diagram to show your thinking.

8. Emma wrote $3\frac{2}{5} \div \frac{4}{5}$ to help answer a different question about bricks and gardens.

a Explain what $3\frac{2}{5}$ and $\frac{4}{5}$ mean in this situation.

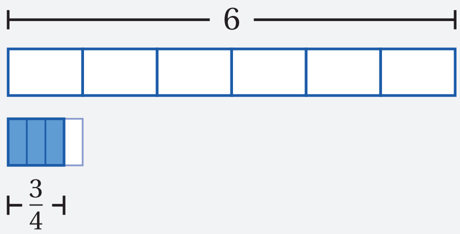
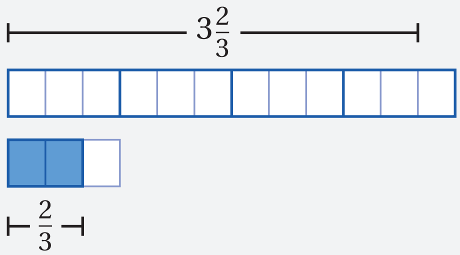
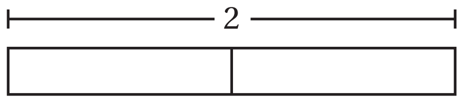
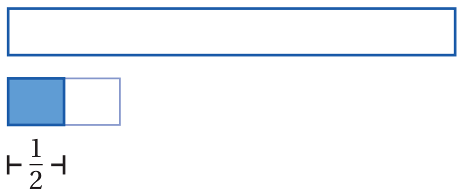
b Complete Emma's tape diagram to determine the value of $3\frac{2}{5} \div \frac{4}{5}$



Name: ______ Date: ______ Period: ______

What's Missing?

9. Complete each row in the table.

	Expression	Tape Diagram	Quotient
a	$6 \div \frac{3}{4}$		
b			
c	$2 \div \frac{3}{5}$		
d			7

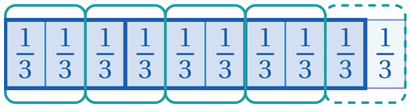
You're invited to explore more.

9. **a** Write a division expression.
b On a separate piece of paper, draw a tape diagram that represents your expression.
c Trade tape diagrams with a partner. Determine their division expression and calculate its quotient.

Synthesis

10. **Discuss:** How can you use the tape diagram to help determine the value of $3 \div \frac{2}{3}$?

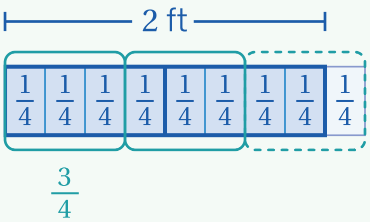
ELD.PI.6.1.Em, Ex, Br



Summary 4.05

You can use division to determine how many groups fit into a whole. For example, the expression $2 \div \frac{3}{4}$ can represent how many $\frac{3}{4}$-foot-long bricks fit along a 2-foot garden wall. You can use tape diagrams or reasoning about equal groups to determine how many groups (bricks) fit into the whole (along the garden wall).

Tape Diagram



$$2 \div \frac{3}{4} = 2\frac{2}{3}$$

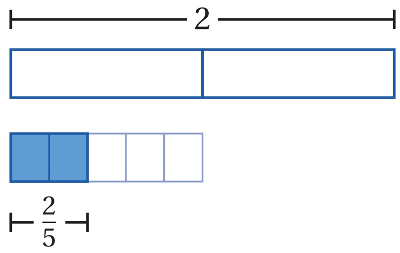
Reasoning About Equal Groups

- To calculate how many lengths of $\frac{3}{4}$ fit into 2, it would help to determine how many $\frac{1}{4}$s there are in 2 wholes.
- I can rewrite 2 as $\frac{8}{4}$.
- There are two groups of $\frac{3}{4}$ in 2 with $\frac{2}{4}$ left over.
- The leftover $\frac{2}{4}$ is the $\frac{2}{3}$ of the $\frac{3}{4}$. (It has 2 of the 3 parts needed to complete the last group of $\frac{3}{4}$.)
- That means there are 2 and $\frac{2}{3}$, $2\frac{2}{3}$, groups of $\frac{3}{4}$ in 2.

Practice 4.05

Name: ________ Date: ________ Period: ________

1. How many $\frac{2}{5}$s are in 2? Use the diagram if it helps with your thinking.



Problems 2–3: Think about how many $\frac{1}{4}$s are in 3.

2. Draw a tape diagram to represent the situation.

3. Determine how many $\frac{1}{4}$s are in 3.

Problems 4–5: Think about the expression $3\frac{2}{5} \div \frac{4}{5}$.

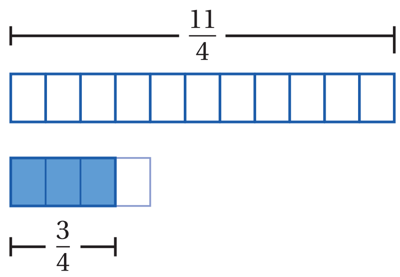
4. Draw a tape diagram to represent this expression.

5. Calculate the quotient.

Problems 6–7: Here is a tape diagram.

6. What expression does this tape diagram represent?

________ $\div$ ________



7. Calculate the quotient for this expression.

Practice
4.05

Name: ______ Date: ______ Period: ______

Problems 8–9: Think about the expression $6\frac{1}{2} \div \frac{3}{4}$.

8. Draw a tape diagram to represent this expression.

9. Calculate the quotient.

10. Kayleen buys one 3-pound bag of rice. Her family eats about $\frac{3}{4}$ of a pound every week. How many weeks does one bag last? Use a tape diagram if it helps you with your thinking.

Spiral Review

11. Complete the table.

Fraction	Decimal	Percent
$\frac{1}{4}$	0.25	25%
	0.1	
$\frac{1}{5}$		
		140%

Problems 12–14: Determine the unknown values.

12. $\frac{2}{5} \times$ ______ $= \frac{8}{5}$

13. $\frac{3}{7} \times$ ______ $= 1\frac{2}{7}$

14. $\frac{6}{5} \times$ ______ $= 4\frac{4}{5}$

Unit 4
Lesson
6

Name: ________ Date: ________ Period: ________

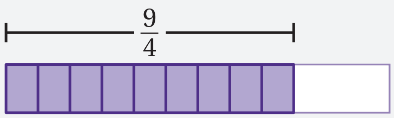
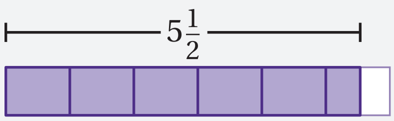
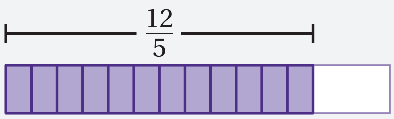
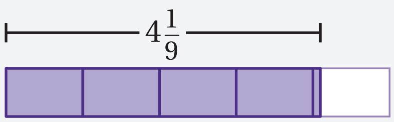
Fraction Relationships 6.NS.1, SMP.1, SMP.3, SMP.7

Fill the Gap

Let's use garden bricks to determine whether the number of groups is greater or less than 1.

Warm-Up

1-2 Complete the table.

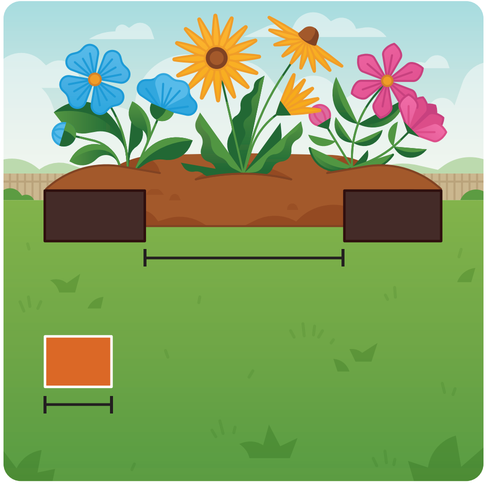
Tape Diagram	Fraction	Mixed Number
	$\frac{9}{4}$	
		$5\frac{1}{2}$
	$\frac{12}{5}$	
		$4\frac{1}{9}$

Name: Date: Period:

More or Less Than One Group

3 Deja is filling a gap along the front of this garden.

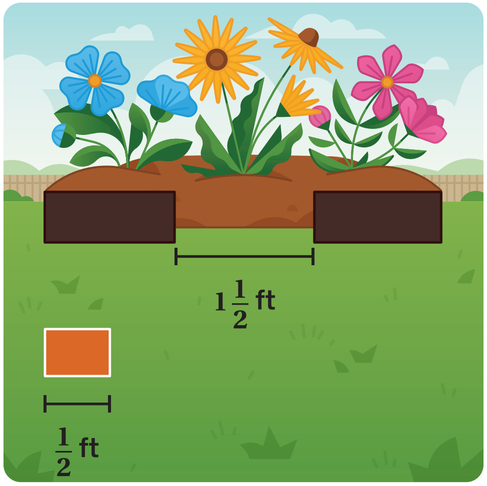
About how many bricks does she need?



4 The gap in Deja's garden is $1\frac{1}{2}$ feet long. Each brick is $\frac{1}{2}$ of a foot long.

a **Discuss:** Do you think Deja will need more than 1 brick or less than 1 brick to fill the gap?

b How many bricks does Deja need to fill the gap?



5 Deja and Emma each wrote an expression to represent the number of bricks needed to fill the gap.

Deja wrote $1\frac{1}{2} \div \frac{1}{2}$. Emma wrote $\frac{1}{2} \div 1\frac{1}{2}$. Whose expression is correct? Circle one.

Deja's Emma's Both Neither

Explain your thinking.

Name: ______ Date: ______ Period: ______

More or Less Than One Group (continued)

6 Deja evaluated her expression $1\frac{1}{2} \div \frac{1}{2}$.

Her correct work is shown.

Discuss: What is Deja's strategy?

$$1\frac{1}{2} \div \frac{1}{2}$$

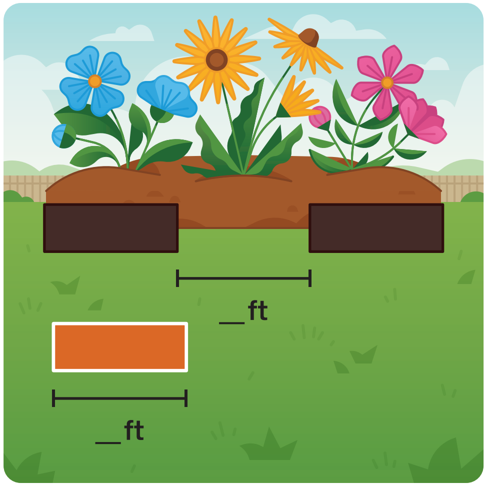
$$\frac{3}{2} \div \frac{1}{2}$$

$$3 \div 1$$

$$3$$

7 Here is Emma's expression: $\frac{1}{2} \div 1\frac{1}{2}$.

a. Draw a sketch to represent this expression in the garden situation.



b. The value of $\frac{1}{2} \div 1\frac{1}{2}$ is:

Less than 1 | Greater than 1 | Equal to 1

c. Determine the value of $\frac{1}{2} \div 1\frac{1}{2}$. Show or explain your thinking.

8 Sort these expressions by the value of their quotient.

$2\frac{1}{4} \div \frac{3}{4}$	$\frac{1}{4} \div \frac{3}{8}$	$\frac{3}{8} \div \frac{1}{4}$	$1 \div \frac{1}{4}$
$\frac{5}{4} \div 1\frac{1}{4}$	$\frac{3}{8} \div \frac{3}{8}$	$1 \div 4$	

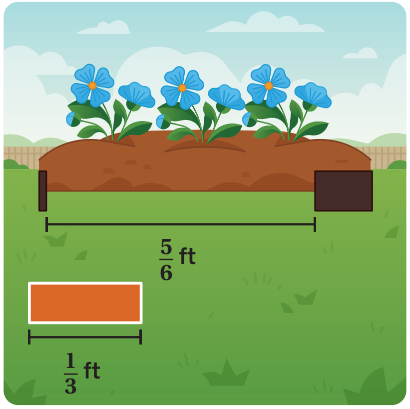
Less than 1	Greater than 1	Equal to 1

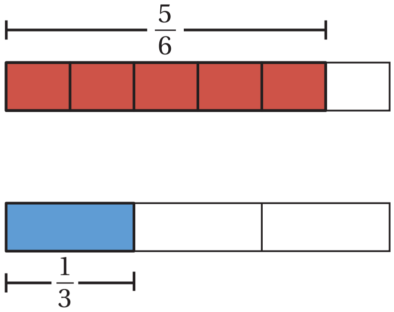
Name: .... Date: .... Period: ....

Equal-Sized Pieces

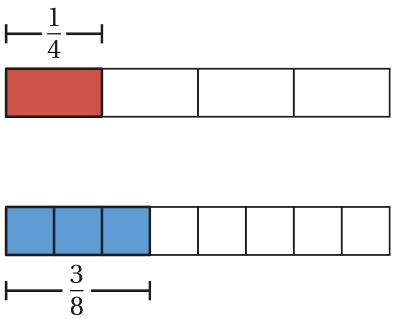
9 Here is a new expression: $\frac{5}{6} \div \frac{1}{3}$.

Use the garden or tape diagram to determine its value.

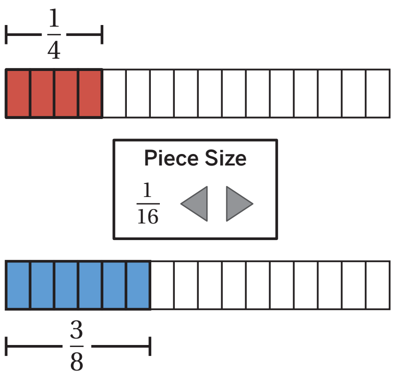




10 What is $\frac{1}{4} \div \frac{3}{8}$?



11 Deja broke $\frac{1}{4}$ and $\frac{3}{8}$ into $\frac{1}{16}$-sized pieces.



a. **Discuss:** How does Deja's strategy show that $\frac{1}{4} \div \frac{3}{8} = \frac{4}{6}$?

b. Let's determine other helpful ways to break up $\frac{1}{4}$ and $\frac{3}{8}$.

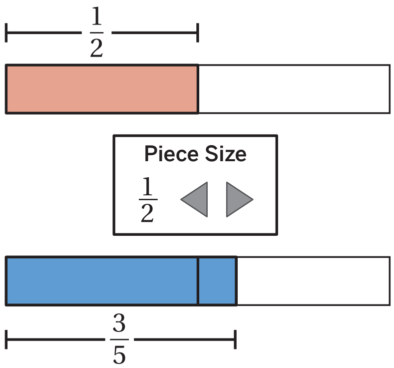
Activity 3

Name: .... Date: .... Period: ....

The Return of Common Denominators

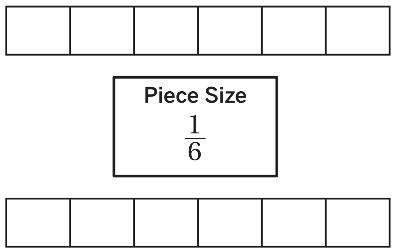
12 **a** Let's look at how we can break these fractions into equal pieces and set up a *common denominator*.

b Draw on the diagram to break the fractions into equal pieces. Then calculate $\frac{1}{2} \div \frac{3}{5}$.



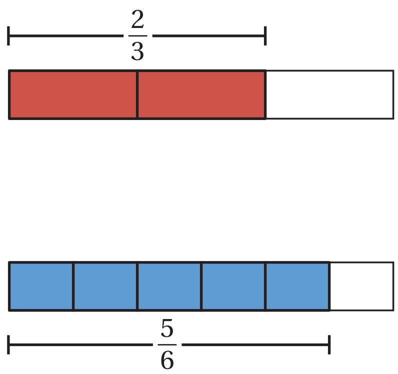
13 Calculate $\frac{2}{3} \div \frac{1}{2}$.

Use the diagram if it helps you with your thinking.



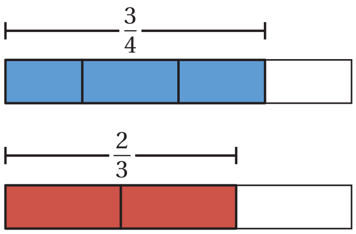
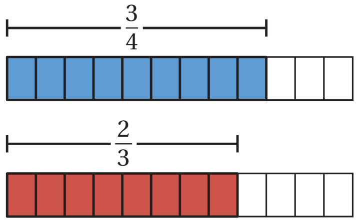
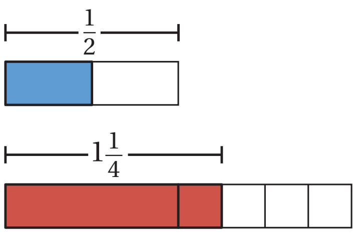
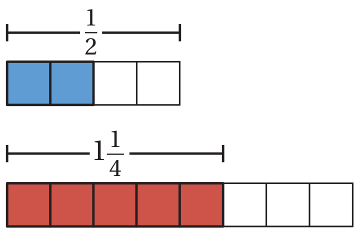
14 Synthesis

Explain how you can show that $\frac{2}{3} \div \frac{5}{6} = \frac{4}{5}$. Use the tape diagrams if they help with your thinking.



17 Summary 4.06

Creating equal-sized pieces, or using a **common denominator**, is a helpful strategy for calculating quotients involving fractions and determining when there is more or less than 1 group.

	Expression and Tape Diagram	Expression and Tape Diagram Using a Common Denominator
More Than 1 Group	$\frac{3}{4} \div \frac{2}{3}$ 	$\frac{9}{12} \div \frac{8}{12} = \frac{9}{8}$  A common denominator of 4 and 3 is 12.
Less than 1 Group	$\frac{1}{2} \div 1\frac{1}{4}$ 	$\frac{2}{4} \div \frac{5}{4} = \frac{2}{5}$  A common denominator of 2 and 4 is 4.

Practice
4.06

Name: .... Date: .... Period: ....

1. Select *all* the expressions whose value is greater than 1.

☐ **A.** $\frac{2}{3} \div 5$ ☐ **B.** $5 \div \frac{2}{3}$ ☐ **C.** $\frac{5}{3} \div 4$ ☐ **D.** $\frac{1}{3} \div \frac{4}{5}$ ☐ **E.** $\frac{4}{5} \div \frac{1}{3}$

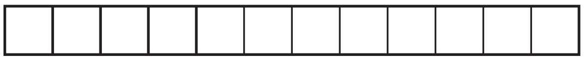
2. Afia uses a $\frac{1}{2}$-cup scoop for flour. How many scoops does Afia need for each amount of flour? Draw a diagram if it helps with your thinking.

Flour (cups)	Number of Scoops
1	
$\frac{1}{4}$	
$\frac{3}{4}$	

Problems 3–4: Here is a diagram.

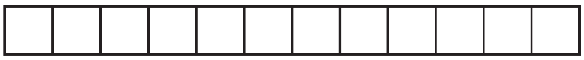
3. Determine if the value of $1\frac{1}{2} \div \frac{2}{3}$ is:

Less than 1 Greater than 1



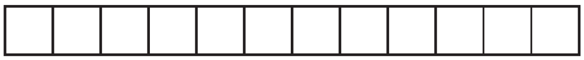
4. Calculate the value of the expression in Problem 3.

Problems 5–6: Here is a diagram.



5. Determine if the value of $\frac{4}{3} \div \frac{3}{2}$ is:

Less than 1 Greater than 1



6. Calculate the value of the expression in Problem 5.

Name: ______________ Date: ______ Period: ______

Spiral Review

Problems 7–9: Determine the missing value that creates a pair of equivalent fractions. Draw diagrams if it helps with your thinking.

7. $\frac{2}{3} = \frac{\square}{9}$

8. $2\frac{1}{2} = \frac{\square}{8}$

9. $\frac{4}{\square} = \frac{10}{25}$

Problems 10–11: A school's Latino Student Union has a budget of $\$240$ for the year.

10. The club wants to spend 40% of their budget on snacks. How much money will they spend on snacks?

11. The club spent $\$36$ on decorations for Día de los Muertos. What percent of their budget is that?

12. Mini scones cost $\$3.00$ per dozen.

- Andre says, "I have $\$2.00$, so I can afford 8 mini scones."
- Elena says, "I want to get 16 mini scones, so I will pay $\$4.00$."

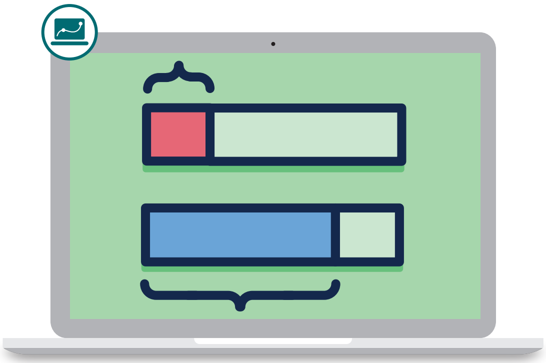
Do you agree with one, both, or neither of them?

Explain your thinking.

Unit 4
Lesson
7

Name: ______ Date: ______ Period: ______

Fraction Relationships 6.NS.1, SMP.3, SMP.7, SMP.8



Break It Down

Let's divide fractions by rewriting with common denominators.

Warm-Up

1 Calculate the following:

a $12 \div 3$

b $\frac{12}{5} \div \frac{3}{5}$

How are these problems alike? How are they different?

Alike:

Different:

Activity
1

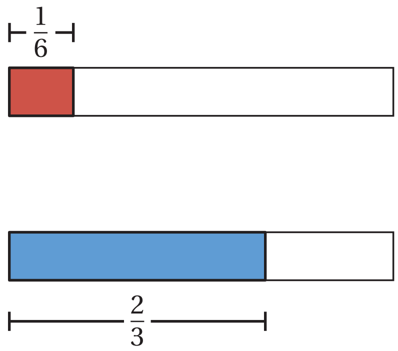
Name: ______ Date: ______ Period: ______

Common Denominators

2 The value of $\frac{1}{6} \div \frac{2}{3}$ is:

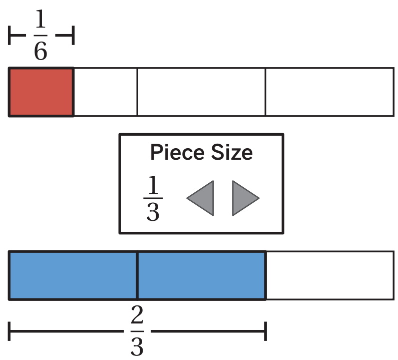
Less than 1 Greater than 1 Equal to 1

Explain your thinking.



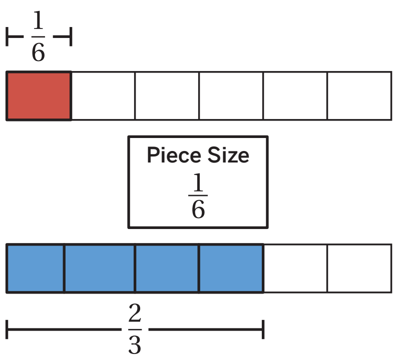
3 **a** Let's look at how we can break both of these fractions into equal pieces and make common denominators.

b Draw on the diagram to break the fractions into equal pieces. Then calculate $\frac{1}{6} \div \frac{2}{3}$.



4 Here's how Ahmed calculated $\frac{1}{6} \div \frac{2}{3}$.

Discuss: Why do you think Ahmed used $\frac{1}{6}$-sized pieces?



$\frac{1}{6} \div \frac{2}{3}$

$\frac{1}{6} \div \frac{4}{6}$

$1 \div 4$

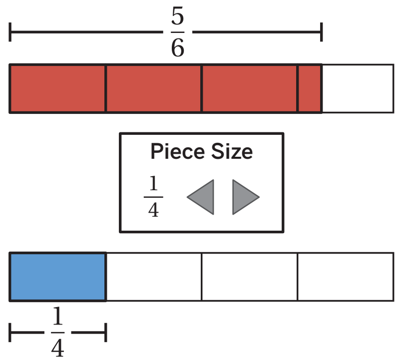
$\frac{1}{4}$

Name: ________ Date: ________ Period: ________

Common Denominators (continued)

5 **a** Let's look at how to make common denominators.

b Draw on the diagram to break the fractions into equal pieces. Then calculate $\frac{5}{6} \div \frac{1}{4}$.



6 Ahmed and Zoe calculated the previous problem without a diagram. Their calculations are both correct.

How are their strategies alike? How are they different?

Ahmed	Zoe
$\frac{5}{6} \div \frac{1}{4}$	$\frac{5}{6} \div \frac{1}{4}$
$\frac{10}{12} \div \frac{3}{12}$	$\frac{20}{24} \div \frac{6}{24}$
$10 \div 3$	$20 \div 6$
$\frac{10}{3}$	$\frac{20}{6}$

Alike:

Different:

7 Zoe says she can't use common denominators to calculate $2 \div \frac{3}{4}$ because 2 is a whole number. What advice would you give Zoe?

Zoe

$2 \div \frac{3}{4}$

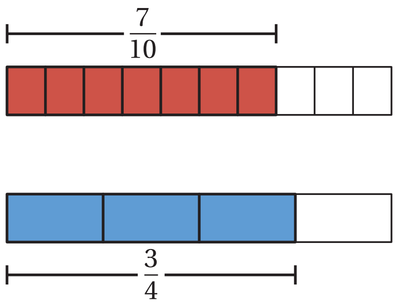
$\frac{?}{?} \quad \frac{?}{?}$

Activity
2

Name: .... Date: .... Period: ....

Dividing with Common Denominators

8 **a** Calculate $\frac{7}{10} \div \frac{3}{4}$



b **Discuss:** What was your strategy?

9 Solve as many challenges as you have time for.

a $\frac{4}{3} \div \frac{2}{3}$

b $\frac{1}{6} \div \frac{5}{6}$

c $\frac{3}{8} \div \frac{1}{4}$

d $2 \div \frac{1}{3}$

e $\frac{3}{10} \div \frac{2}{5}$

f $\frac{5}{6} \div \frac{3}{4}$

g $4 \div \frac{3}{4}$

h $\frac{11}{4} \div \frac{2}{3}$

i $2\frac{1}{2} \div \frac{2}{3}$

j $1\frac{4}{5} \div \frac{1}{2}$

10 Synthesis

Discuss: How can using a common denominator help you divide a fraction with another fraction?.

$\frac{4}{3} \div \frac{2}{3}$ $\frac{5}{2} \div \frac{4}{3}$

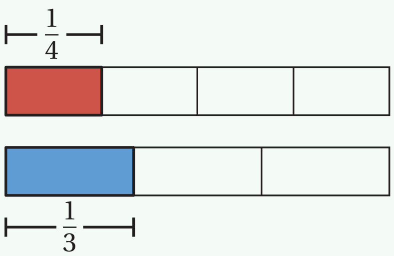
Use these examples if they help you explain your thinking.

13 Summary 4.07

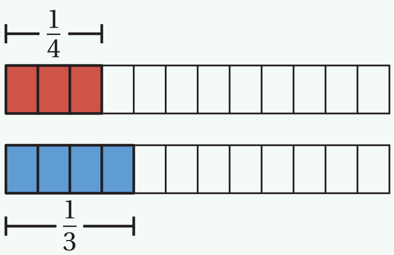
You can use common denominators to determine quotients involving fractions.

For example, in $\frac{1}{4} \div \frac{1}{3}$, you can use 12 as a common denominator of 4 and 3. Then you can rewrite the division expression as $\frac{3}{12} \div \frac{4}{12}$. This helps you determine that there are $\frac{3}{4}$ groups of $\frac{4}{12}$ in $\frac{3}{12}$.

Tape Diagram of Original Problem



Tape Diagram With Common Denominator



Equivalent Fractions With Common Denominator

$$\frac{1}{4} \div \frac{1}{3}$$

$$\frac{3}{12} \div \frac{4}{12}$$

$$3 \div 4$$

$$\frac{3}{4}$$

Practice
4.07

Name: ______________ Date: ________ Period: ________

1. Here is Irelle's work for calculating $\frac{2}{3} \div \frac{3}{4}$. Explain what you think Irelle did at each step.

Irelle

$\frac{2}{3} \div \frac{3}{4}$

Step 1: $\frac{8}{12} \div \frac{9}{12}$

Step 2: $\frac{8}{9}$

Problems 2–5: Calculate the value of each expression. Draw a diagram if it helps with your thinking.

2. $5 \div \frac{2}{3}$

3. $2\frac{1}{2} \div \frac{5}{8}$

4. $\frac{4}{3} \div \frac{5}{2}$

5. $\frac{10}{4} \div \frac{4}{5}$

6. Sahana's work for Problem 5 is incorrect. What advice would you give her?

Sahana

$\frac{10}{4} \div \frac{4}{5}$

$10 \div 5 = 2$ and $4 \div 4 = 1$

$\frac{2}{1} = 2$

7. **Test Practice** Kelly made $2\frac{1}{2}$ cups of slime. The shaded part of the rectangles show how many cups of slime she has.

Kelly is putting the slime into small containers. Each container holds $\frac{2}{3}$ of a cup of slime. What is the greatest number of containers Kelly can completely fill with slime?

A. 2 containers

B. 3 containers

C. 4 containers

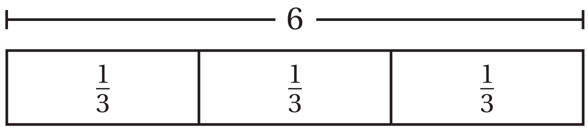
D. 5 containers

Name: ______________________ Date: __________ Period: __________

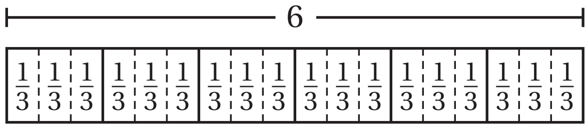
Spiral Review

8. Which of these tape diagrams represent the expression $6 \div \frac{1}{3}$?

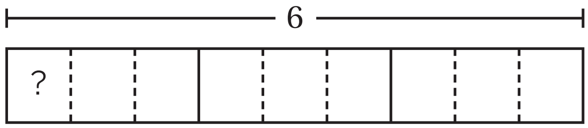
A.



B.



C.



9. Eliza and Isabella are running on a track. Isabella starts 10 meters ahead of Eliza. Eliza runs 120 meters in 24 seconds. Isabella runs 120 meters in 25 seconds. If they both continue running at this pace, how long will it take for Eliza to catch up to Isabella? Explain your thinking.

Problems 10–12: A rocking horse has a weight limit of 60 pounds.

10. What percent of the weight limit is 33 pounds?

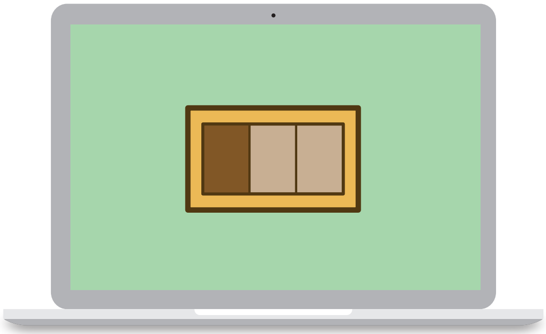
11. What weight is 95% of the weight limit?

12. What percent of the weight limit is 114 pounds?

Unit 4
Lesson
8

Name: .... Date: .... Period: ....

Fraction Relationships 6.NS.1, SMP.2, SMP.8

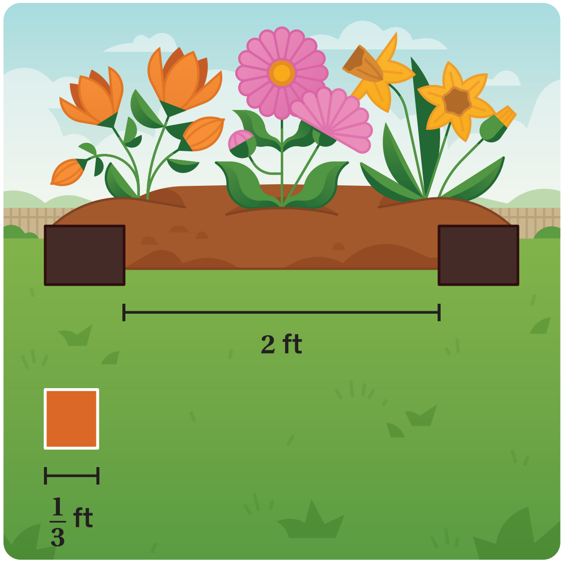


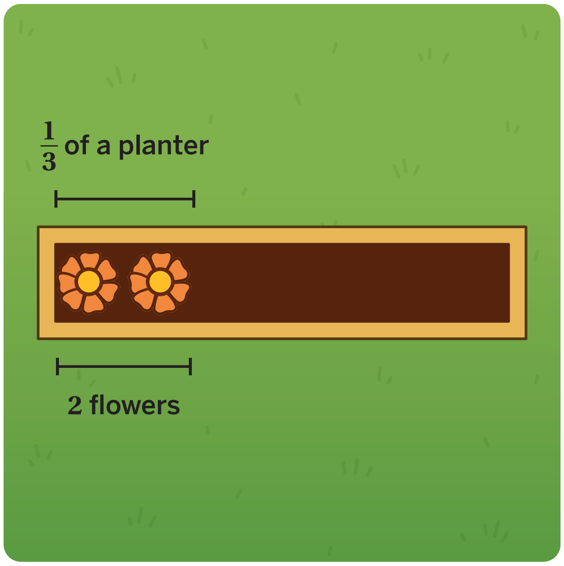
Potting Soil

Let's explore another strategy for dividing fractions.

Warm-Up

1 Habib says $2 \div \frac{1}{3}$ represents the brick situation. Inola says $2 \div \frac{1}{3}$ represents the flower situation.





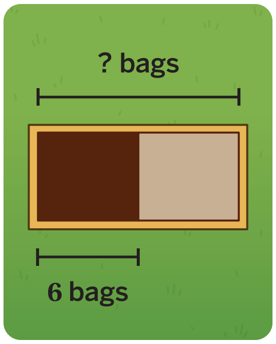
Discuss: Why are they both correct?

Activity 1

Name: .... Date: .... Period: ....

Digging Into Fraction Division

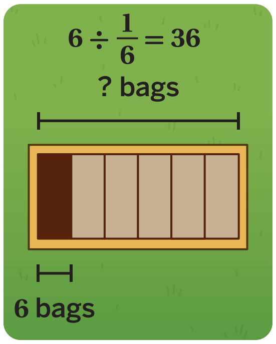
2 Habib and Inola are filling planters with potting soil so that their class can grow vegetables.

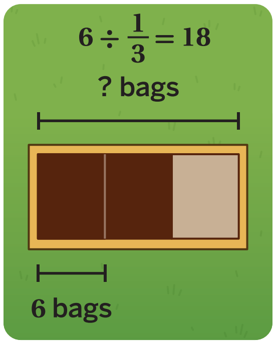


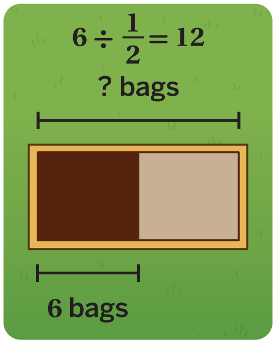
a Let's take a look at how many bags of soil fill $\frac{1}{2}$ of a planter.

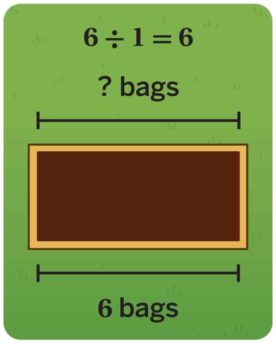
b How many bags does it take to fill 1 planter?

3 **a** Take a look at four different soil situations.









b What do you notice? What do you wonder?

I notice:

I wonder:

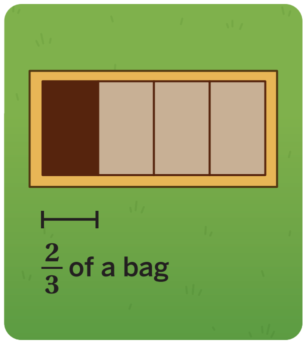
Activity 1

Name: ________ Date: ________ Period: ________

Digging Into Fraction Division (continued)

4 It takes $\frac{2}{3}$ of a bag of soil to fill $\frac{1}{4}$ of this planter.

How many bags does it take to fill 1 planter?



5 Habib wrote $\frac{2}{3} \div \frac{1}{4} = \frac{8}{3}$ to solve the previous problem.

What does each fraction mean in this situation?

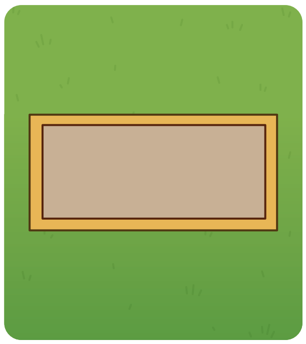
$\frac{2}{3}$ means . . .

$\frac{1}{4}$ means . . .

$\frac{8}{3}$ means . . .

6 Inola wrote $5\frac{1}{3} \div \frac{1}{2}$ to solve a new problem.

Draw or describe a situation about planters and potting soil that represents Inola's expression.

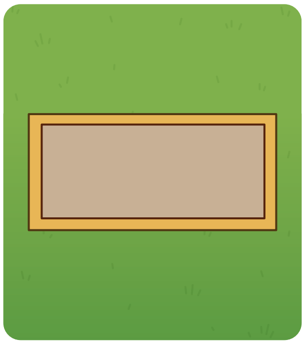


Name: ________ Date: ________ Period: ________

Different Operation, Same Value

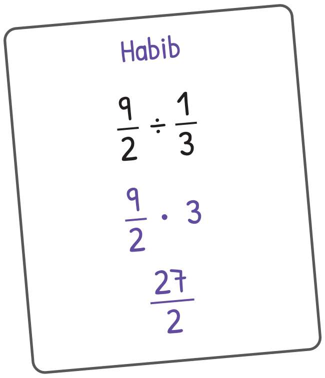
7 **Discuss:** How could you think about the expression $\frac{9}{2} \div \frac{1}{3}$ in terms of a planter?

Draw a diagram if it helps you with your thinking.



8 What is $\frac{9}{2} \div \frac{1}{3}$?

9 Habib says that $\frac{9}{2} \div \frac{1}{3}$ has the same value as $\frac{9}{2} \cdot 3$.



a **Discuss:** How would you show Habib's strategy using a tape diagram?

b Use Habib's strategy to calculate $\frac{2}{3} \div \frac{1}{7}$.

Name: .... Date: .... Period: ....

Different Operation, Same Value (continued)

10 a Calculate the value of each expression.

Expression	Value
$\frac{4}{3} \div \frac{1}{3}$	
$\frac{4}{3} \div \frac{1}{6}$	
$\frac{4}{3} \div \frac{1}{5}$	
$1\frac{2}{3} \div \frac{1}{4}$	

b Discuss your answers and strategies with a classmate.

11 Synthesis

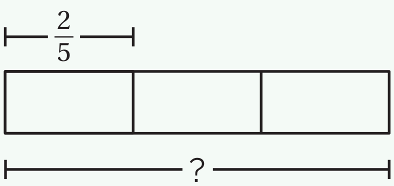
Discuss: What is a strategy for dividing a number by a unit fraction, such as $2\frac{1}{3} \div \frac{1}{5}$?

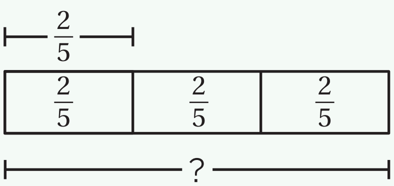
14 Summary 4.08

When you divide a number by a unit fraction $\frac{1}{b}$, it's generally the same as multiplying the number by b.

For example, think about the expression $\frac{2}{5} \div \frac{1}{3}$. In our planter and soil situation, this means it takes $\frac{2}{5}$ bags of soil to fill $\frac{1}{3}$ of a planter.

To fill the entire planter, you would need 3 times $\frac{2}{5}$ bags of soil, or $\frac{2}{5} \cdot 3$.



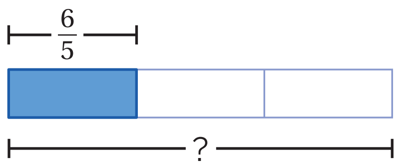


$$\begin{aligned} &\frac{2}{5} \div \frac{1}{3} \\ &= \frac{2}{5} \cdot 3 \\ &= \frac{6}{5} \\ &= 1\frac{1}{5} \end{aligned}$$

Practice
4.08

Name: .... Date: .... Period: ....

1. Calculate $\frac{6}{5} \div \frac{1}{3}$. Use the tape diagram if it helps with your thinking.



2. $\frac{2}{3}$ cups of apple chips fill $\frac{1}{4}$ of a jar. Write and evaluate an expression to determine how many cups fill 1 jar.

Problems 3–4: Determine whether each statement is *always*, *sometimes*, or *never* true. Circle your answer and explain your thinking.

3. Dividing the same numbers in a different order keeps the value the same, like $2 \div 3 = 3 \div 2$.

Always Sometimes Never

4. Dividing a number by $\frac{1}{3}$ produces the same value as multiplying the number by 3.

Always Sometimes Never

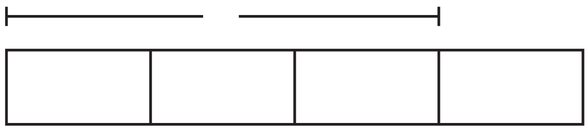
5. **Test Practice** $\frac{2}{5}$ of the student population walked to school on a given Friday. If 150 students walked to school that day, how many total students go to the school?

Practice
4.08

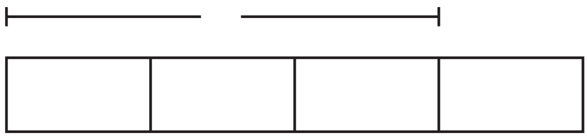
Name: ________________ Date: ________ Period: ________

Problems 6–7: Complete the tape diagram to represent and solve each problem.

6. Mai picked 1 cup of strawberries, which is enough for $\frac{3}{4}$ of a pan of strawberry oatmeal bars. How many cups does she need for a whole pan?



7. Prisha picked $1\frac{1}{2}$ cups of raspberries, which is enough for $\frac{3}{4}$ of a loaf of raspberry bread. How many cups does she need for a whole loaf?



Spiral Review

Problems 8–10: Determine each quotient.

8. $6 \div \frac{1}{3}$

9. $4 \div \frac{1}{9}$

10. $\frac{1}{10} \div 8$

Problems 11–13: Use the numbers 20, 5, and 4 to write a division expression with the given quotient.

11. Greater than 1

12. Less than 1

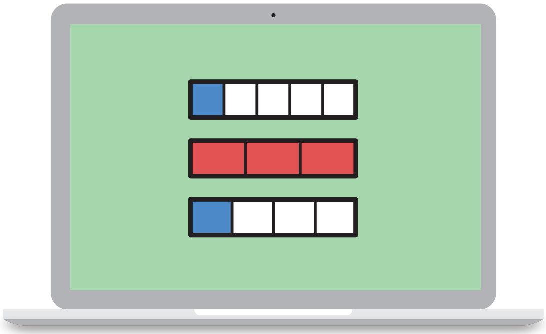
13. Close to 1, but not equal to 1

Name: .... Date: .... Period: ....

Fraction Relationships 6.NS.1, SMP.3

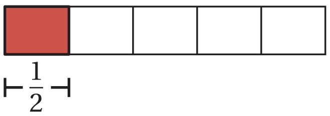
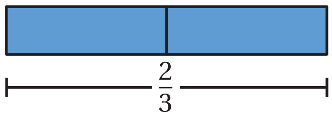
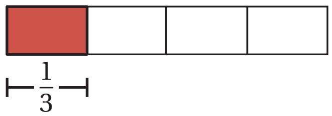
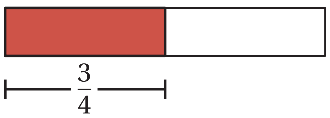
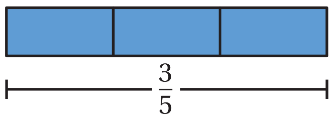
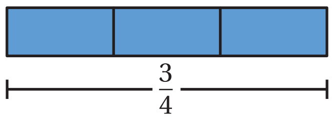
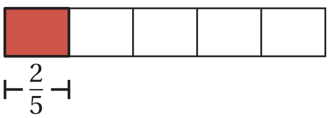
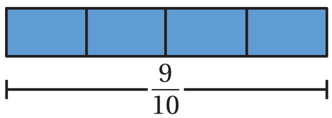
Division Challenges

Let's compare strategies for dividing fractions with and without tape diagrams.



Warm-Up

1 Solve as many challenges as you have time for. Try some problems of each type.

Multiplying	Dividing	Surprise Me!
 What is the value of $5 \cdot \frac{1}{2}$?	 What is the value of $\frac{2}{3} \div 2$?	 What is the value of $4 \cdot \frac{1}{3}$?
 What is the value of $2 \cdot \frac{3}{4}$?	 What is the value of $\frac{3}{5} \div 3$?	 What is the value of $\frac{3}{4} \div 3$?
 What is the value of $5 \cdot \frac{2}{5}$?	 What is the value of $\frac{9}{10} \div 4$?	 What is the value of $3 \cdot \frac{2}{5}$?

Activity 1

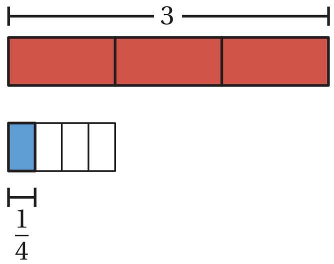
Name: .... Date: .... Period: ....

Two Strategies With Tape Diagrams

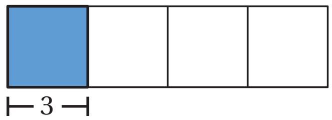
2 Jalen and Katie drew diagrams to calculate $3 \div \frac{1}{4}$.

a Take a look at each student's diagram. Discuss each strategy.

Jalen



Katie



b Calculate $3 \div \frac{1}{4}$.

3 Here is a new expression: $\frac{4}{3} \div \frac{1}{5}$.

Jalen says the quotient is $\frac{20}{3}$. Katie says the quotient is $\frac{4}{15}$.

Diagram 1

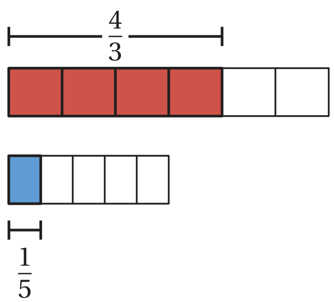
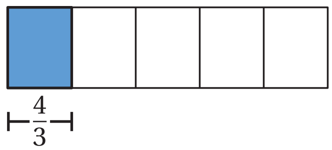


Diagram 2



a Whose quotient is correct? Circle one.

Jalen's   Katie's   Both   Neither

b Use one of the diagrams to help explain your thinking.

Activity
2

Name: ______ Date: ______ Period: ______

Two Strategies Revisited

4 Here are four expressions.

a Order these expressions by value from *least* to *greatest*.

$\frac{4}{3} \div \frac{2}{5}$	$\frac{4}{3} \div 1$	$\frac{4}{3} \div \frac{1}{5}$	$\frac{4}{3} \div 2$

Least **Greatest**

b **Discuss:** How are these expressions alike? How are they different?

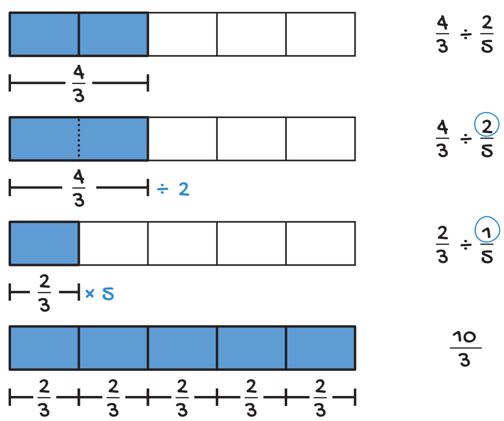
5 Here is an expression from the previous problem:

$\frac{4}{3} \div \frac{2}{5}$

Calculate its value.

6 Here is Katie's strategy for evaluating $\frac{4}{3} \div \frac{2}{5}$.

Describe Katie's strategy.

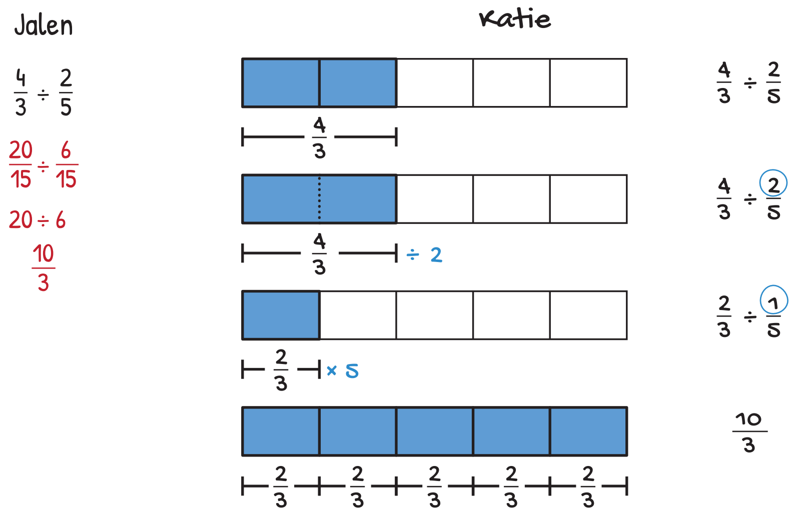


Name: ______ Date: ______ Period: ______

Two Strategies Revisited (continued)

7 Here is how Jalen and Katie calculated $\frac{4}{3} \div \frac{2}{5}$.

a Take a look at each of their strategies.



b Which strategy would you use to calculate $\frac{9}{10} \div \frac{3}{4}$? Circle one.

Jalen's Katie's My own

c If you chose Jalen's or Katie's strategy, what would your first step be for the strategy you chose? Otherwise, describe your own strategy.

Activity
3

Name: .... Date: .... Period: ....

Fraction Fluency

8 Here is an expression from the previous problem:

$\frac{9}{10} \div \frac{3}{4}$

Calculate its value.

9 Here is a new expression: $\frac{6}{5} \div \frac{2}{3}$.

The three answers below are *not* correct.

$\frac{12}{15}$ $\frac{6}{5}$ 3

Circle your favorite (wrong) answer and explain why it cannot be correct.

Activity 3

Name: ______ Date: ______ Period: ______

Fraction Fluency (continued)

10 Solve as many challenges as you have time for. Calculate each expression.

a $5 \div \frac{2}{3}$

b $\frac{1}{2} \div \frac{3}{4}$

c $\frac{6}{5} \div \frac{2}{3}$

d $\frac{1}{2} \div \frac{5}{3}$

e $\frac{3}{4} \div \frac{3}{5}$

f $\frac{9}{4} \div \frac{7}{10}$

g $\frac{5}{9} \div \frac{5}{3}$

h $\frac{1}{9} \div \frac{3}{5}$

i $\frac{9}{7} \div \frac{1}{7}$

j $\frac{6}{7} \div \frac{4}{7}$

k $2 \div \frac{8}{9}$

l $\frac{6}{5} \div \frac{2}{5}$

11 Synthesis

Describe a strategy for calculating the quotient of two fractions, such as $\frac{2}{5} \div \frac{3}{4}$.

Draw a diagram if it helps you with your thinking.

14 Summary 4.09

You don't have to use tape diagrams to determine the quotient of two fractions!

Here are two ways to calculate the quotient of the expression $\frac{9}{10} \div \frac{3}{4}$: by using common denominators and by simplifying numerators.

Common Denominators

- Rewrite the expression using common denominators.
  $\frac{18}{20} \div \frac{15}{20}$
- Then divide the numerator of the first fraction by the numerator of the second fraction.
  $18 \div 15 = \frac{18}{15}$ or $\frac{6}{5}$

Simplifying Numerators

- Divide the first fraction by the numerator of the divisor to create a unit fraction.
  $\frac{3}{10} \div \frac{1}{4}$
- To divide by the unit fraction, multiply the dividend by the denominator of the divisor.
  $\frac{3}{10} \cdot 4 = \frac{12}{10}$ or $\frac{6}{5}$

Practice
4.09

Name: .... Date: .... Period: ....

Problems 1–4: Use any strategy to calculate each quotient.

1. $10 \div \frac{1}{5}$

2. $10 \div \frac{3}{5}$

3. $3\frac{3}{4} \div \frac{3}{8}$

4. $\frac{1}{2} \div \frac{5}{3}$

5. How many groups of $\frac{3}{4}$ are in $4\frac{1}{2}$?

6. How many groups of $\frac{3}{4}$ are in $2\frac{2}{3}$?

7. Use the equation $2\frac{1}{2} \div \frac{1}{8} = 20$ to determine $2\frac{1}{2} \div \frac{5}{8}$. Explain your thinking.

Spiral Review

8. **Test Practice** Basheera has 90 songs on a playlist. She listened to 40% of the songs. How many songs did Basheera listen to?

Name: .... Date: .... Period: ....

9. One batch of trail mix uses 2 cups of cereal, $\frac{1}{4}$ cups of raisins, and $\frac{2}{3}$ cups of almonds. Complete the table to show how much of each ingredient you would need to make 3 or 4 batches of trail mix.

	Cereal (cups)	Raisins (cups)	Almonds (cups)
3 Batches			
4 Batches			

10. Two stores sell the same brand of juice. The first store sells 2 liters for $\$3.80$. The second store sells 1.5 liters for $\$2.70$. Which is the better deal? Explain your thinking.

Problems 11–12: Here are three expressions.

$56 \div 8$	$56 \div 8000000$	$56 \div 0.000008$

11. Without calculating, order the quotients from *least* to *greatest*.

Least **Greatest**

12. Explain how you ordered the three quotients.

Name: .... Date: .... Period: ....

Fraction Relationships 6.NS.1, SMP.1, SMP.3, SMP.8

Action Fractions

Let's rewrite fraction division as multiplication.

Warm-Up

1. Callen and Kiandra both evaluated $\frac{2}{3} \div \frac{3}{5}$. Here are their strategies.

Callen

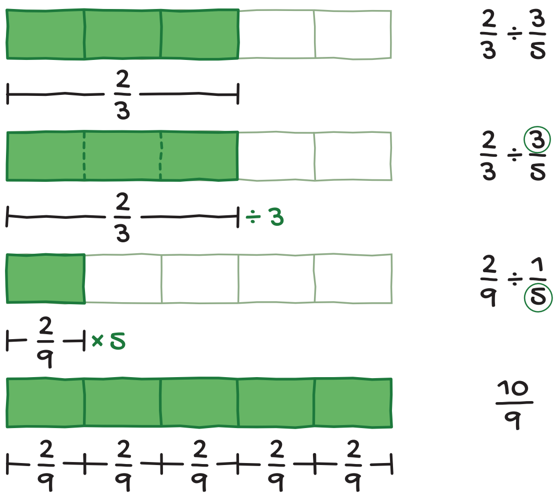
$\frac{2}{3} \div \frac{3}{5}$

$\frac{10}{15} \div \frac{9}{15}$

$10 \div 9$

$\frac{10}{9}$

Kiandra



What do you notice? What do you wonder?

ELD.PI.6.6.Em, Ex, Br

I notice:

I wonder:

Activity 1

Name: ________ Date: ________ Period: ________

Creating New Strategies

2. Polina also evaluated $\frac{2}{3} \div \frac{3}{5}$. She said: *My work is similar to Kiandra's*.

Take a look at Polina's work. How is her strategy similar to Kiandra's?

ELD.PI.6.6.Em, Ex, Br

Polina

$$\frac{2}{3} \div \frac{3}{5}$$
$$\frac{2}{9} \div \frac{1}{5}$$
$$\frac{2}{9} \cdot 5$$
$$\frac{10}{9}$$

3. Nia looked at Polina's work and said she could determine the answer in fewer steps.

ELD.PI.6.3.Em, Ex, Br, ELD.PI.6.6.Em, Ex, Br

Here is Nia's work.

Nia

$$\frac{2}{3} \div \frac{3}{5}$$
$$\frac{2}{3} \cdot \frac{5}{3}$$
$$\frac{10}{9}$$

a Is Nia's work correct? Explain your thinking.

b **Discuss:** Do you think you can use Nia's strategy on *all* division expressions?

Name: .... Date: .... Period: ....

Creating New Strategies (continued)

4. Nia multiplied by the **reciprocal**.

a Use Nia's strategy to evaluate $\frac{3}{5} \div \frac{7}{10}$.

b Then evaluate the same expression using a different strategy.

Nia's Strategy	A Different Strategy

5. **Discuss:** What are the advantages of Nia's strategy? What are the advantages of the other strategy you selected?

ELD.PI.6.3.Em, Ex, Br

Activity
2

Name: ________ Date: ________ Period: ________

Card Sort: Equivalent Expressions

6. You will use a set of cards for this activity.

Sort the cards into groups of equivalent expressions.

Equivalent to $\frac{5}{4}$	Equivalent to $\frac{9}{5}$	Equivalent to $\frac{5}{9}$

7. Choose *one* group from the card sort. Then explain how you and your partner made these matches. **ELD.PI.6.11.Em, Ex, Br**

Activity 3

Name: .... Date: .... Period: ....

Partner Problems

Decide with your partner who will complete Column A and who will complete Column B. The solutions in each row should be the same.

Compare your solutions and strategies, then discuss and resolve any differences.

	Column A	Column B
8.	$\frac{1}{2} \div \frac{2}{3}$	$\frac{5}{8} \div \frac{5}{6}$
9.	$\frac{5}{9} \div \frac{7}{4}$	$\frac{10}{7} \div \frac{9}{2}$
10.	$1\frac{2}{3} \div \frac{8}{15}$	$1\frac{1}{4} \div \frac{2}{5}$
11.	$\frac{9}{10} \div 1\frac{1}{5}$	$\frac{5}{6} \div 1\frac{1}{9}$

Synthesis

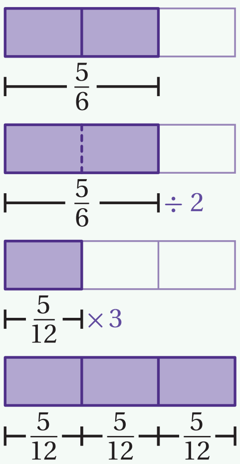
12. Show that $\frac{3}{4} \div \frac{2}{3}$ is equivalent to $\frac{3}{4} \cdot \frac{3}{2}$.

Summary 4.10

In general, when you divide a number by a unit fraction, $\frac{1}{b}$, it's the same as multiplying the number by b (which is the **reciprocal** of $\frac{1}{b}$).

Here are three strategies you can use when you divide a number by a fraction, $\frac{a}{b}$.

Tape Diagram



Simplifying Numerators

Divide by the numerator, a, then multiply the result by the denominator, b.

$$\frac{5}{6} \div \frac{2}{3}$$
$$= \frac{5}{12} \div \frac{1}{3}$$
$$= \frac{5}{12} \cdot 3$$
$$= \frac{15}{12}$$
$$= \frac{5}{4}$$

Multiplying by the Reciprocal

Multiply by the reciprocal of the fraction $\left(\frac{b}{a}\right)$.

$$\frac{5}{6} \div \frac{2}{3}$$
$$= \frac{5}{6} \cdot \frac{3}{2}$$
$$= \frac{15}{12}$$
$$= \frac{5}{4}$$

reciprocal The reciprocal of a fraction $\frac{a}{b}$ is $\frac{b}{a}$. The product of two fractions that are reciprocals of one another is 1.

Practice
4.10

Name: .... Date: .... Period: ....

1. Select *all* the statements that provide the correct steps for evaluating the expression $\frac{14}{15} \div \frac{7}{5}$.

   - ☐ A. Multiply $\frac{14}{15}$ by $\frac{1}{7}$, then multiply by 5.
   - ☐ B. Divide $\frac{14}{15}$ by 5, then multiply by $\frac{1}{7}$.
   - ☐ C. Multiply $\frac{14}{15}$ by 7, then multiply by $\frac{1}{5}$.
   - ☐ D. Divide $\frac{14}{15}$ by 7, then multiply by 5.

Problems 2–5: Determine the value of each expression. Show or explain your thinking.

2. $\frac{8}{9} \div 4$

3. $\frac{3}{4} \div \frac{1}{2}$

4. $\frac{9}{2} \div \frac{3}{8}$

5. $3\frac{1}{3} \div \frac{2}{9}$

6. Clare incorrectly said that $\frac{4}{3} \div \frac{5}{2}$ is $\frac{10}{3}$ because $\frac{4}{3} \cdot 5 = \frac{20}{3}$ and $\frac{20}{3} \div 2 = \frac{10}{3}$. Determine the correct quotient for $\frac{4}{3} \div \frac{5}{2}$. Then explain why Clare's quotient and reasoning are incorrect.

7. Determine the quotient of $3\frac{1}{5}$ divided by $\frac{8}{9}$.

Name: ______ Date: ______ Period: ______

8. Select *all* of the expressions that represent the same value as $n \div \frac{a}{b}$.

☐ **A.** $n \cdot \frac{a}{b}$

☐ **B.** $\frac{n \cdot b}{a}$

☐ **C.** $\frac{n}{b \cdot a}$

☐ **D.** $n \cdot \frac{b}{a}$

☐ **E.** $\frac{b}{a} \cdot n$

Spiral Review

9. Without calculating, determine how the expressions $98 \cdot 25$ and $(100 \cdot 25) - (2 \cdot 25)$ are related. Explain your thinking.

10. **Test Practice** Kiran and Nicolas are comparing the numbers $1{,}000$ and 10. Kiran says that $1{,}000$ is 100 times as large as 10. Nicolas says that 10 is $\frac{1}{100}$ times as large as $1{,}000$. Whose thinking is correct? Circle one.

Kiran's | Nicolas's | Both | Neither

Explain your thinking.

Unit 4

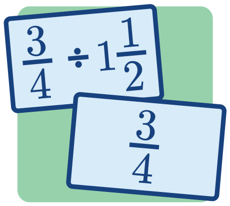
Lesson

11

Name: ______________________ Date: __________ Period: __________

Fraction Relationships 6.NS.1, SMP.1, SMP.2. SMP.7

Swap Meet



Let's solve division problems using different strategies.

Warm-Up

1. Write a division expression to represent each question. Then answer the question. **ELD.PI.6.6.Em, Ex, Br**

Question	Division Expression	Answer
How many groups of $2\frac{1}{3}$ are in 21?		
How many groups of 21 are in $2\frac{1}{3}$?		

Name: Date: Period:

Match and Solve

You will use a set of cards for this activity. **ELD.PI.6.6.Em, Ex, Br**

- Match each question to an expression and its answer.
- Write an expression and answer for the missing cards. Make sure to include a unit of measurement for each answer.

	Question	Expression	Answer
2.	Hailey lives $1\frac{1}{2}$ miles from school. Luis lives $\frac{3}{4}$ miles from school. How many more miles from school does Hailey live?		
3.	Darius is making a small garden that is $\frac{3}{4}$ feet long and $1\frac{1}{2}$ feet wide. What is the area of the garden?		
4.	A cookie recipe uses $1\frac{1}{2}$ cups of sugar per batch. Ana has $\frac{3}{4}$ cups of sugar. How many batches of cookies can she make?		
5.	Kanna biked $1\frac{1}{2}$ miles, which is $\frac{3}{4}$ of the distance between her home and school. What is the distance between her home and school?		

6. Choose *one* of the expressions from the table. Explain how you decided which question it represents. **ELD.PI.6.10.Em, Ex, Br**

Activity 2

Name: .... Date: .... Period: ....

Write, Trade, Solve!

A. $1\frac{1}{2} \div 3$	B. $\frac{5}{6} \cdot \frac{2}{3}$	C. $5 \div 1\frac{1}{4}$	D. $\frac{2}{5} \div \frac{9}{2}$	E. $4\frac{1}{2} \cdot \frac{1}{3}$
F. $\frac{3}{4} \div 1\frac{1}{2}$	G. $\frac{5}{6} \div \frac{2}{3}$	H. $5 \div \frac{3}{4}$	I. $\frac{9}{2} \div \frac{2}{5}$	J. $4\frac{1}{2} \div \frac{1}{3}$

7. Circle an expression from the table.

8. On a separate sheet, write a question that can be answered by the expression you chose. **ELD.PI.10.Em, Ex, Br, ELD.PI.6.12.Em, Ex, Br**

9. Calculate the value of your expression and answer your question. Show your thinking. (Don't write any of this on the sheet of paper!)

a. Expression: ....

b. Value of expression:

c. Answer to your question (include units):

Name: ______ Date: ______ Period: ______

Write, Trade, Solve! (continued)

Find a partner, then trade questions with them. ELD.PI.6.6.Em, Ex, Br

- Write an expression that can be used to represent their question.
- Calculate the value of the expression. Show your thinking.

Repeat with other partners.

	Partner's Name	Expression	Answer
10.			
11.			
12.			
13.			
14.			

Synthesis

15. Ricardo biked $1\frac{1}{4}$ miles, which is $\frac{3}{5}$ of the distance between his home and school. What is the distance between his home and school? **ELD.PI.6.11.Em, Ex, Br**

Circle the expression that represents this situation. Explain your thinking.

$1\frac{1}{4} \div \frac{3}{5}$ $\qquad\qquad$ $\frac{3}{5} \div 1\frac{1}{4}$

Summary 4.11

There are many real-life situations where you can use fraction division.

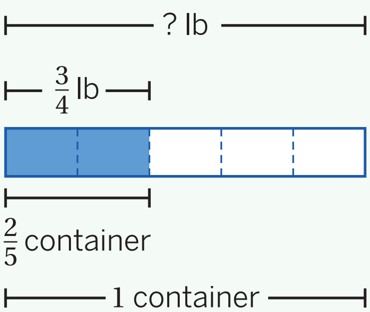
For example, let's say $\frac{3}{4}$ pounds of rice fills $\frac{2}{5}$ of a container.

There are two possible questions you can ask:

- How many pounds of rice fill 1 container?
- How many containers are filled by 1 pound of rice?

Here's how you can use different division expressions and tape diagrams to answer each question.

How many pounds fill 1 container?

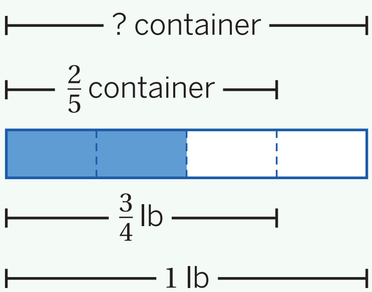


$1\frac{7}{8}$ pounds fill 1 container.

$\frac{3}{4} \div \frac{2}{5}$

$= \frac{15}{8}$ or $1\frac{7}{8}$

How many containers for 1 pound?



1 pound would fill $\frac{8}{15}$ of a container.

$\frac{2}{5} \div \frac{3}{4}$

$= \frac{8}{15}$

Practice 4.11

Name: ______________________ Date: __________ Period: __________

Problems 1–4: Use any strategy to calculate each quotient.

1. $2\frac{1}{2} \div \frac{5}{8}$

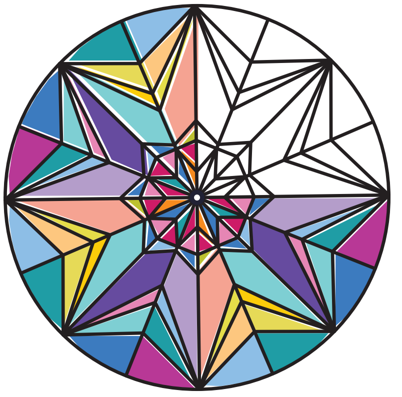
2. $\frac{4}{3} \div \frac{5}{2}$

3. $3\frac{1}{2} \div \frac{1}{3}$

4. $3 \div \frac{2}{3}$

5. Write a situation that could be represented by the expression $3 \div \frac{2}{3}$. Explain what the quotient means in your situation.

6. Elena has 5 tubes of blue glass paint to color her window. She used 4 tubes to paint $\frac{3}{4}$ of the window. Does she have enough blue paint to completely color her window? Show or explain your thinking.



7. Engineers designed superchargers to decrease the amount of time needed to charge the battery of an electric car. A supercharger can charge $\frac{3}{4}$ of a car's battery in 25 minutes. What fraction of the battery gets charged every 5 minutes? Show or explain your thinking.

Name: ____________ Date: ________ Period: ________

8. **Test Practice** A recipe requires $1\frac{1}{3}$ cups of flour for every batch of cookies. How many full batches of cookies can be made with $6\frac{3}{4}$ cups of flour? Show or explain your thinking.

Spiral Review

Problems 9–10: Ethan works as a server in a restaurant. He gets a 15% tip on the cost of every order.

9. What tip would he get if the order costs $\$50$?

10. Ethan got a $\$9$ tip. What was the cost of the order?

Problems 11–12: Karima made 12 pairs of earrings in 9 hours, working at a consistent rate.

11. How long did it take Karima to make 8 pairs of earrings?

12. How many pairs of earrings can Karima make in 15 hours?

Name: ______________________ Date: __________ Period: __________

Fraction Relationships   6.NS.1, SMP.2

Practice Day 1

Let's practice what you've learned so far in this unit!

Start with any of the scavenger hunt sheets.

- Record the sheet shape, solve the problem, and write your answer.
- Look for your answer at the top of another scavenger hunt sheet. Solve that problem.
- Repeat until you make it back to your starting sheet.

Sheet: ______________ **Answer** →	↙ **Sheet:** ______________ **Answer**
Sheet: ______________ **Answer** →	**Sheet:** ______________ **Answer** ↗

Name: ______ Date: ______ Period: ______

Practice Day 1 (continued)

Sheet: ______ **Answer** →	Sheet: ______ ↙ **Answer**
Sheet: ______ **Answer** →	Sheet: ______ ↙ **Answer**
Sheet: ______ **Answer** →	Sheet: ______ **Answer**

You're invited to explore more.

1. Describe a situation that could be represented by $7 \div \frac{3}{4}$.

2. Write two different division expressions that have the quotient $\frac{13}{7}$.

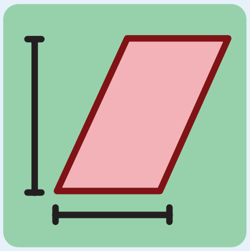
Notes:

Sub-Unit

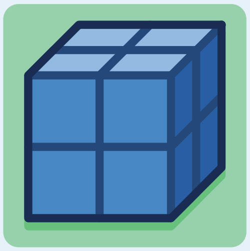
3

Area and Volume With Fractions

Lesson 12
Classroom Comparisons



Lesson 13
Puzzling Areas



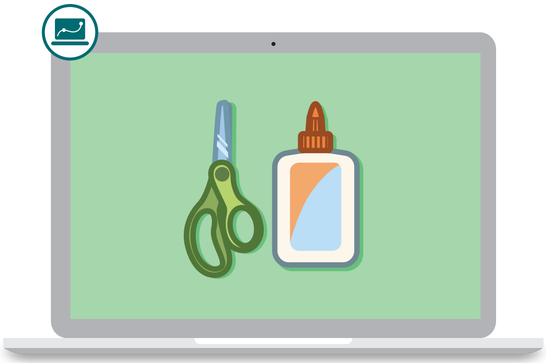
Lesson 14
Volume Challenges

Lesson 15
Planter Planner

Unit 4
Lesson
12

Name: ____________ Date: ________ Period: ________

Fraction Relationships 6.NS.1 SMP.1, SMP.3, SMP.6



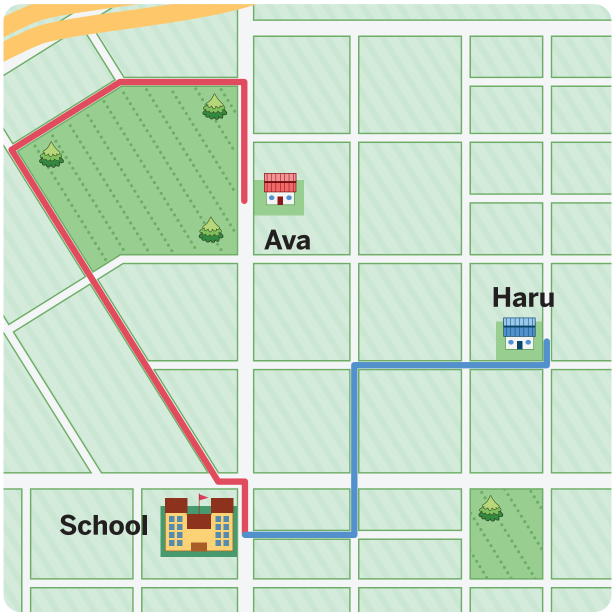
Classroom Comparisons

Let's compare the size of familiar objects by asking, "How many times as long?"

Warm-Up

1 Here is how Ava and Haru walked to school on Monday.

What do you notice? What do you wonder?



I notice:

I wonder:

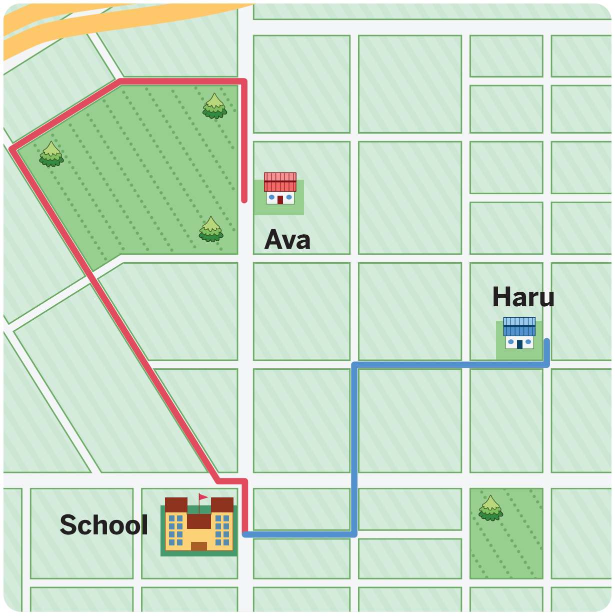
Activity 1

Name: ______ Date: ______ Period: ______

Comparing Distances

2 Ava walked farther than Haru.

About how many times as far do you think Ava walked?



3 Ava walked $1\frac{1}{4}$ miles. Haru walked $\frac{3}{4}$ of a mile.

How many times as far did Ava walk?

4 Select *all* the expressions that represent how many times as far Ava walked compared to Haru.

- ☐ A. $\frac{5}{4} - \frac{3}{4}$
- ☐ B. $1\frac{1}{4} \cdot \frac{3}{4}$
- ☐ C. $1\frac{1}{4} \div \frac{3}{4}$
- ☐ D. $\frac{3}{4} \div 1\frac{1}{4}$
- ☐ E. $\frac{5}{4} \div \frac{3}{4}$

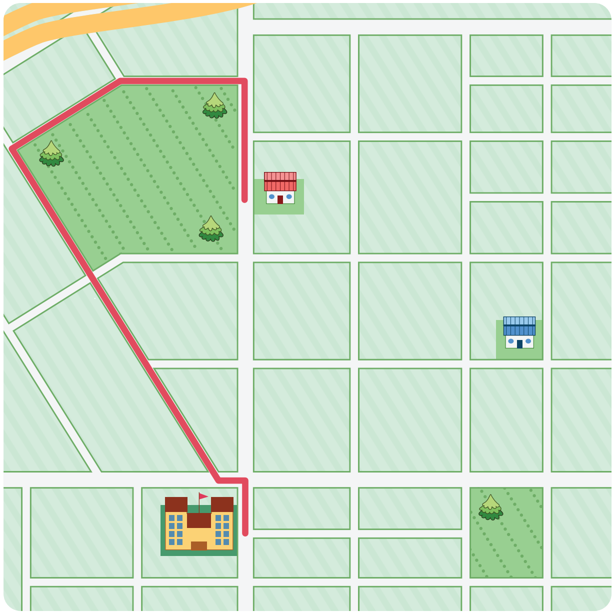
Activity 1

Name: ________ Date: ________ Period: ________

Comparing Distances (continued)

5 Ava decides to walk a different path to school on Tuesday.

Draw a path she could walk.

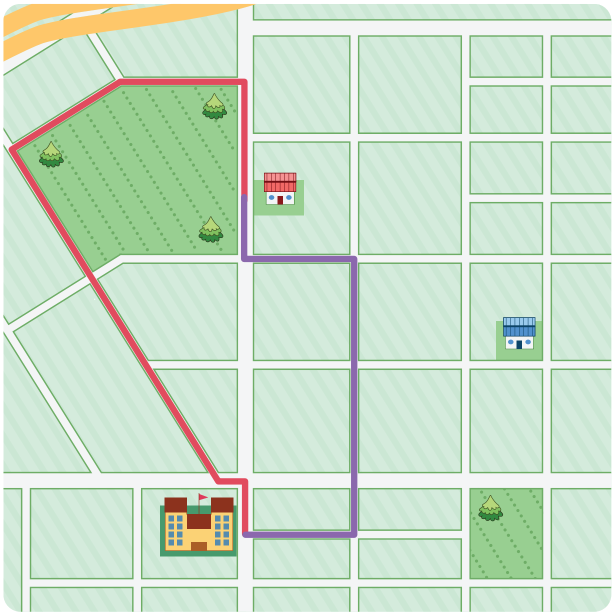


6 The map shows Ava's paths on Monday and Tuesday.

Monday: Ava walked $1\frac{1}{4}$ miles.

Tuesday: Ava walked $\frac{7}{8}$ of a mile.

How many times as far did Ava walk on Tuesday than on Monday?



Activity 2

Name: ________ Date: ________ Period: ________

Comparing Classroom Objects

7 Ava and Haru are comparing two objects in their classroom.

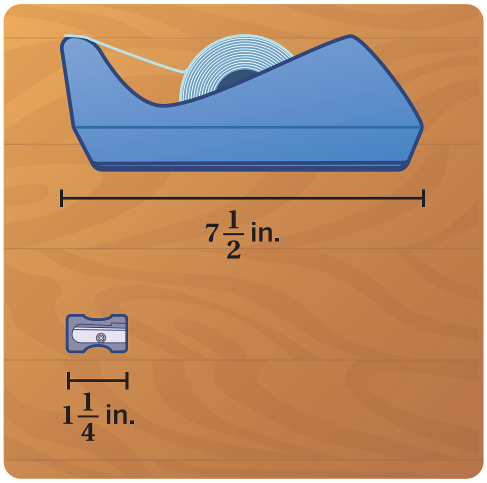
Ava says: *The tape dispenser is* 6 *times the length of the pencil sharpener.*

Haru says: *The pencil sharpener is* $\frac{1}{6}$ *of the length of the tape dispenser.*

Whose thinking is correct? Circle one.

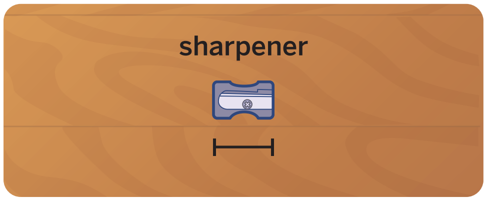
Ava's Haru's Both Neither

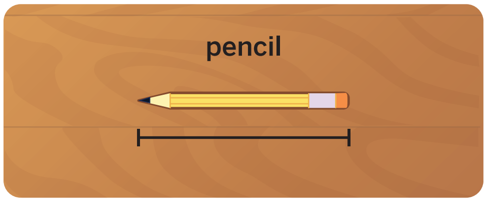
Explain your thinking.



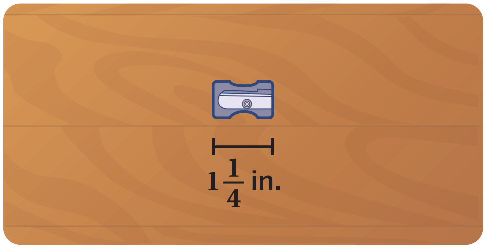
8 The sharpener is ________ times the length of the pencil.

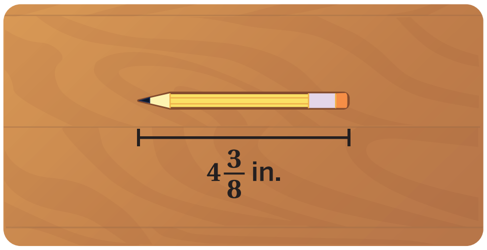
Estimate a value.





9 The sharpener is what fraction of the length of the pencil?





Name: ______ Date: ______ Period: ______

Comparing Classroom Objects (continued)

10 Here is a collection of classroom objects, along with their lengths (in inches).

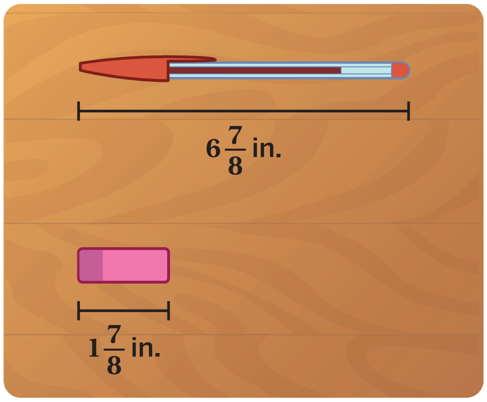
Scissors	$6\frac{1}{4}$	Eraser	$1\frac{7}{8}$
Marker	$5\frac{5}{8}$	Stapler	5
Red Pen	$6\frac{7}{8}$	Pencil Sharpener	$1\frac{1}{4}$
Tape Dispenser	$7\frac{1}{2}$	Glue Bottle	$3\frac{1}{8}$
Highlighter	$3\frac{3}{4}$	Calculator	$2\frac{1}{2}$
Large Pencil	$8\frac{1}{8}$	Small Pencil	$4\frac{3}{8}$

Choose pairs of objects to compare. Write an expression and a statement comparing their lengths. Solve as many challenges as you have time for!

	Objects	Expression	Statement
a	______ and ______		The ______ is ______ times the length of the ______.
b	______ and ______		The ______ is ______ times the length of the ______.
c	______ and ______		The ______ is ______ times the length of the ______.
d	______ and ______		The ______ is ______ times the length of the ______.

11 Synthesis

Describe a strategy for solving problems like this: *The pen is how many times the length of the eraser?*



14 Summary 4.12

You can use division to determine how many times as large one quantity is compared to another.

For example, let's say a song is $1\frac{1}{2}$ minutes long, and another song is $3\frac{3}{4}$ minutes long. You can compare the lengths of the two songs by answering either of these questions:

How many times longer is the second song than the first song?

$$3\frac{3}{4} \div 1\frac{1}{2} = ?$$
$$= \frac{15}{4} \div \frac{3}{2}$$
$$= \frac{15}{4} \cdot \frac{2}{3}$$
$$= \frac{30}{12} \text{ or } 2\frac{1}{2}$$

The second song is $2\frac{1}{2}$ times as long as the first song.

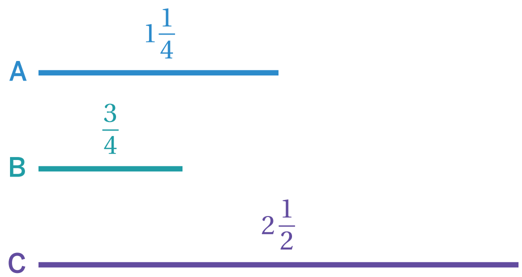
What fraction of the second song is the first song?

$$1\frac{1}{2} \div 3\frac{3}{4} = ?$$
$$= \frac{3}{2} \div \frac{15}{4}$$
$$= \frac{6}{4} \div \frac{15}{4}$$
$$= \frac{6}{15} \text{ or } \frac{2}{5}$$

The first song is $\frac{2}{5}$ as long as the second song.

Name: .... Date: .... Period: ....

1. Segment A is is $1\frac{1}{4}$ centimeters long. Segment B is $\frac{3}{4}$ centimeters long, and Segment C is $2\frac{1}{2}$ centimeters long.



Match each question with the expression that could be used to answer it.

Question		Expression
a. How much longer is Segment A than Segment B?	....	$\frac{3}{4} \div 1\frac{1}{4}$
b. Segment B is how many times the length of Segment A?	....	$1\frac{1}{4} \div 2\frac{1}{2}$
c. Segment A is how many times the length of Segment B?	....	$1\frac{1}{4} \div \frac{3}{4}$
d. What fraction of Segment A is Segment C?	....	$1\frac{1}{4} - \frac{3}{4}$
e. What fraction of Segment C is Segment A?	....	$2\frac{1}{2} \div 1\frac{1}{4}$

Problems 2–3: Deiondre set a daily goal to ride a bicycle $4\frac{1}{2}$ miles.

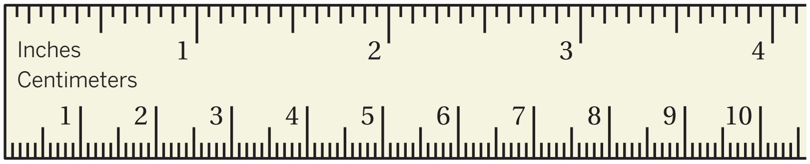
2. On Monday, Deiondre biked 6 miles. How many times the goal did Deiondre ride?

3. On Tuesday, Deiondre biked $1\frac{4}{5}$ miles. What fraction of the goal did Deiondre ride?

4. A security guard works $9\frac{1}{2}$-hour-long shifts. At one point during a shift, the guard looks at the clock and realizes it has been $3\frac{3}{4}$ hours since the shift started. Calculate exactly how much of the shift the guard has worked. Show or explain your thinking.

Name: .... Date: .... Period: ....

Problems 5–7: When taking measurements, engineers in the U.S. use the U.S. customary system, while engineers in Canada use the metric system. On an international project, it's important to convert measurements precisely. For example, one inch is the same length as $2\frac{27}{50}$ centimeters.



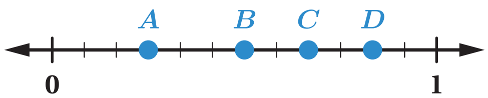
5. How many centimeters long is 3 inches? Show or explain your thinking.

6. What fraction of 1 inch is 1 centimeter? Show or explain your thinking.

7. What question can you answer about this situation by determining the value of $10 \div 2\frac{27}{50}$?

Spiral Review

8. Here are four points plotted on a number line. Which point best represents $66\frac{2}{3}\%$ of the distance between 0 and 1?



A. Point A

B. Point B

C. Point C

D. Point D

Problems 9–10: Determine each product.

9. $\frac{2}{3} \cdot \frac{9}{20}$

10. $1\frac{2}{5} \cdot \frac{3}{14}$

Problems 11–12: An airline has a luggage weight limit of 50 pounds.

11. What percent of the weight limit is 23 pounds?.

12. What weight is 94% of the weight limit?

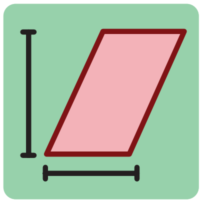
Name: ______ Date: ______ Period: ______

Fraction Relationships | Generalizing With Multiple Representations | Nets and Surface Area

6.EE.2.c, 6.NS.1, 6.G.1, SMP.1

Puzzling Areas

Let's explore the areas of rectangles and triangles with fractional side lengths.



Warm-Up

Evaluate each expression mentally.

1. $3 \cdot 4$

2. $\frac{1}{3} \cdot 4$

3. $\frac{1}{3} \cdot \frac{1}{4}$

4. $2 \cdot \frac{1}{3} \cdot \frac{1}{4}$

5. $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4}$

Name: ________ Date: ________ Period: ________

Areas

6. Use any strategy to determine the area of as many figures as you can. Use the workspace below if it helps with your thinking.

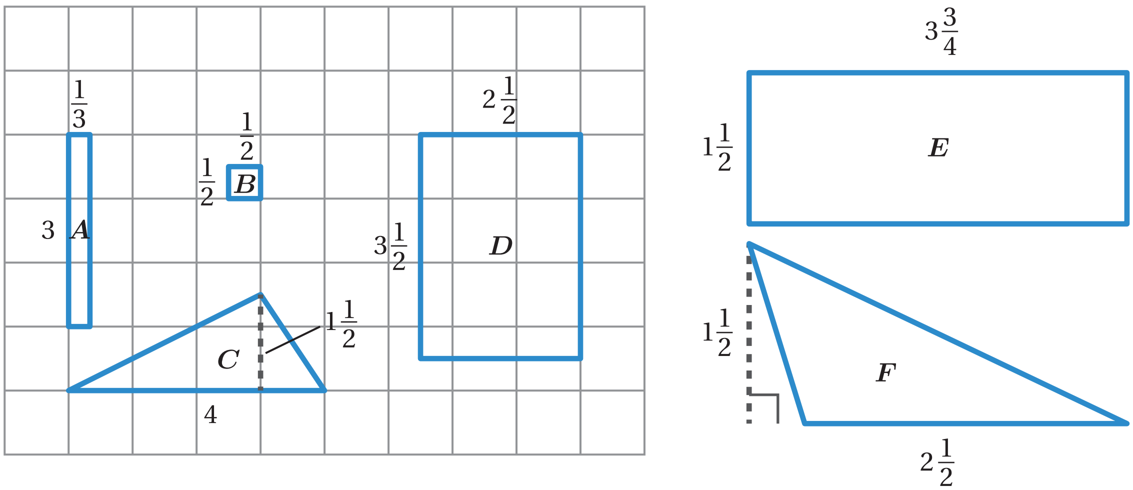


Figure	A	B	C	D	E	F
Area (sq. units)						

Workspace:

7. Compare your strategies for determining the area with a partner.

ELD.PI.6.3.Em, Ex, Br, ELD.PI.6.6.Em, Ex, Br

Discuss:

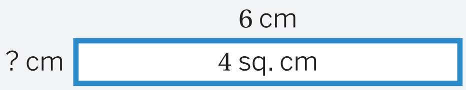
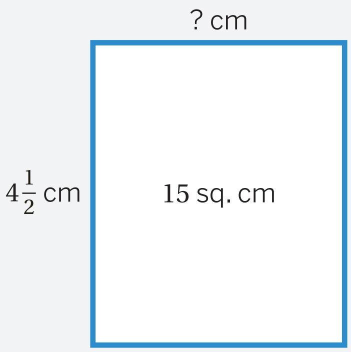
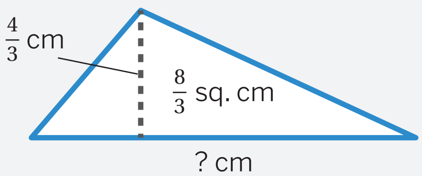
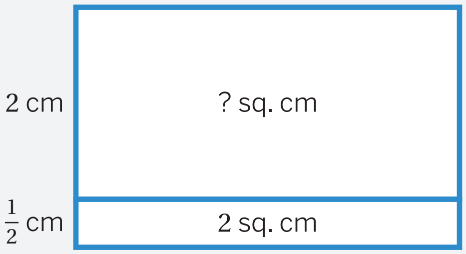
- How are your strategies the same?
- How are they different?
- Was there a difference in how you each approached the problems on a grid versus the ones not on a grid?

Activity 2

Name: ______________________ Date: __________ Period: __________

Level Up Area Puzzles

8. Use area formulas to determine the unknown side length or area.

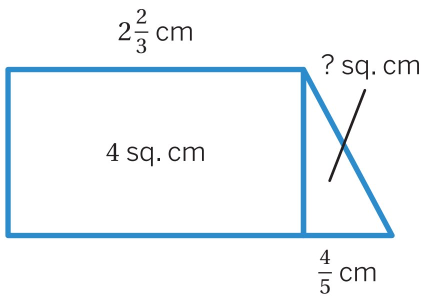
	Puzzle	Workspace
a		? = ______ centimeters
b		? = ______ centimeters
c		? = ______ centimeters
d		? = ______ square centimeters

Name: ________ Date: ________ Period: ________

Level Up Area Puzzles (continued)

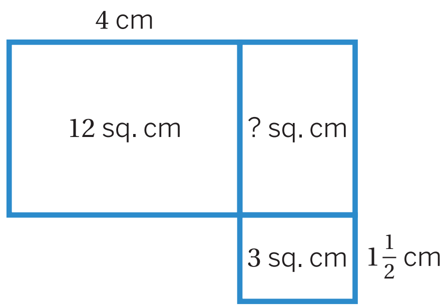
9. Solve as many puzzles as you have time for. You can work on them in any order.

Puzzle A



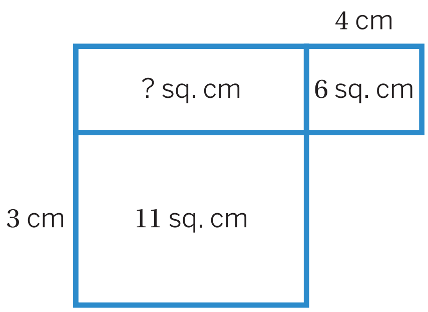
? = ________ square centimeters

Puzzle B



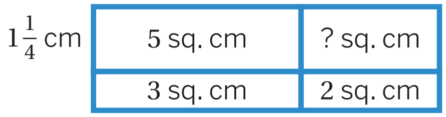
? = ________ square centimeters

Puzzle C



? = ________ square centimeters

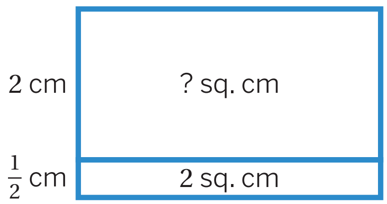
Puzzle D



? = ________ square centimeters

Synthesis

10. **a** When is multiplication helpful for solving problems about areas?

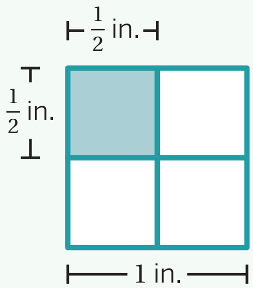


b When is division helpful for solving problems about areas?

Summary 4.13

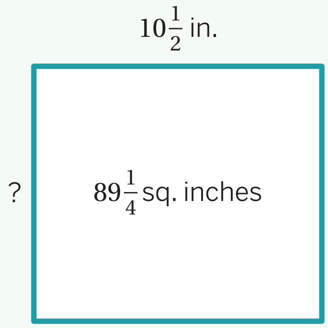
You can determine the area of a polygon that has fractional side lengths just like you would a polygon that has whole-number side lengths.

For example, you can calculate the area of the shaded square using the the formula $A = l \cdot w$. The area is equal to $\frac{1}{2} \cdot \frac{1}{2}$, or $\frac{1}{4}$ square inches.



You can also use area formulas to determine an unknown length. If you know the area and one side length of a rectangle, you can divide to determine the other side length.

For example, to determine the missing side length of this rectangle, you can calculate $89\frac{1}{4} \div 10\frac{1}{2} = 8\frac{1}{2}$. The missing side length is $8\frac{1}{2}$ inches.



Practice
4.13

Name: ____________ Date: ________ Period: ________

1. A rectangular lawn has an area of $7\frac{1}{3}$ square yards and a width of $2\frac{1}{5}$ yards. What is the length of the lawn, in yards?

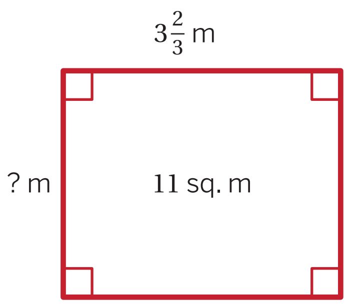
A. $9\frac{8}{15}$ **B.** $3\frac{1}{3}$ **C.** $\frac{3}{10}$ **D.** $5\frac{2}{15}$

2. A television screen has a length of $16\frac{1}{2}$ inches, a width of w inches, and an area of 462 square inches. Select *all* the equations that represent the relationship between the dimensions of the television.

- ☐ **A.** $w \cdot 462 = 16\frac{1}{2}$
- ☐ **B.** $16\frac{1}{2} \cdot w = 462$
- ☐ **C.** $462 \div 16\frac{1}{2} = w$
- ☐ **D.** $462 \div w = 16\frac{1}{2}$
- ☐ **E.** $16\frac{1}{2} \cdot 462 = w$

Problems 3–6: Determine the missing length or lengths in each figure.

3.



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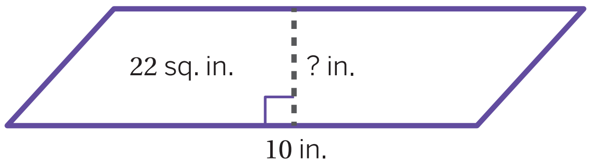
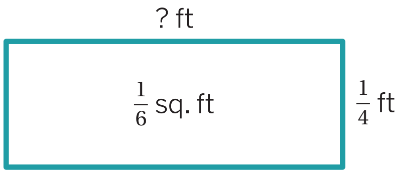
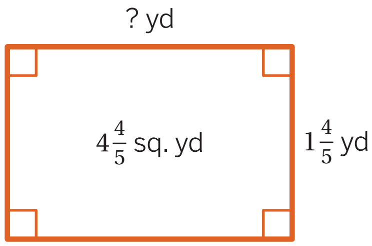


5.



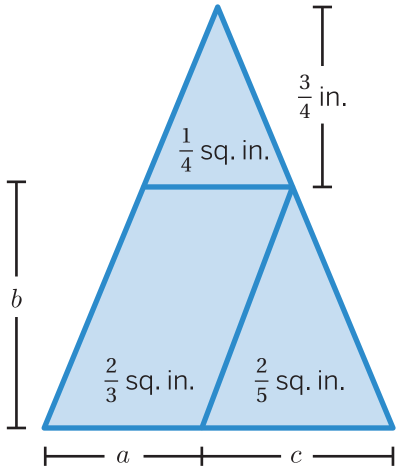
6.



Name: ______________ Date: ______ Period: ______

7. Determine the missing lengths in this figure made up of a parallelogram and two triangles.

Unknown	Length (in.)
a	
b	
c	



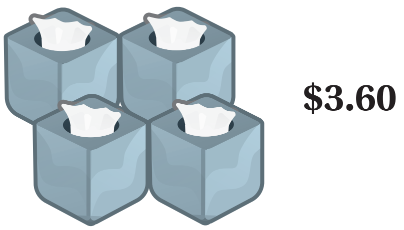
Spiral Review

Problems 8–9: A bookshelf is 42 inches long.

8. How many books will fit on the bookshelf if each book is $1\frac{1}{2}$ inches wide? Show your thinking.

9. A bookcase has five of these 42-inch-long bookshelves. How many total feet of shelf space does the bookcase have? Show your thinking.

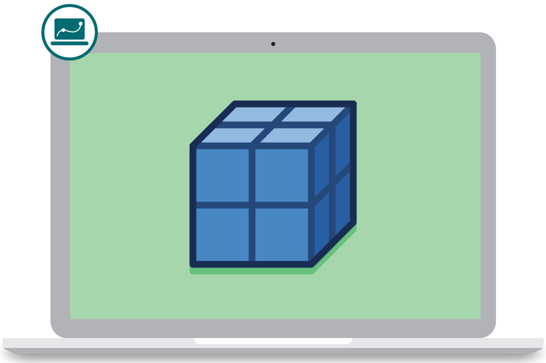
10. If 4 tissue boxes cost \$3.60, what is the cost of 7 tissue boxes?



Name: ________ Date: ________ Period: ________

Fraction Relationships | Generalizing With Multiple Representations | Nets and Surface Area

6.EE.2.c, 6.G.2, 6.NS.1, SMP.3



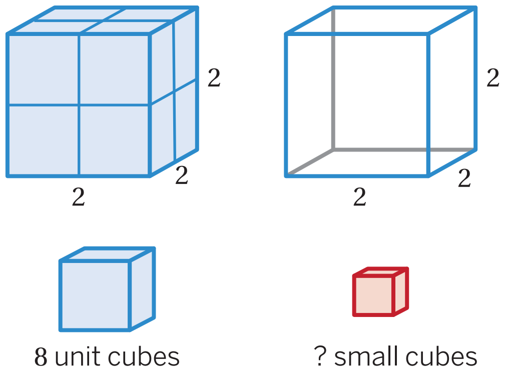
Volume Challenges

Let's explore the volume of prisms with fractional dimensions.

Warm-Up

1 A unit cube is a $1 \times 1 \times 1$ prism.

a **Discuss:** How do you know that it takes 8 unit cubes to fill a $2 \times 2 \times 2$ prism?



b A small cube measures $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ units. How many small cubes do you need to fill the $2 \times 2 \times 2$ prism?

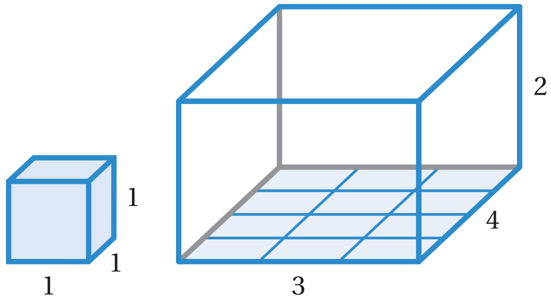
Activity 1

Name: ______ Date: ______ Period: ______

Volume Strategies

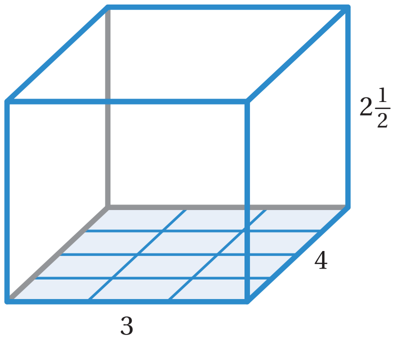
2 *Volume* is the number of unit cubes it takes to fill a container.

What is the volume of this rectangular prism?

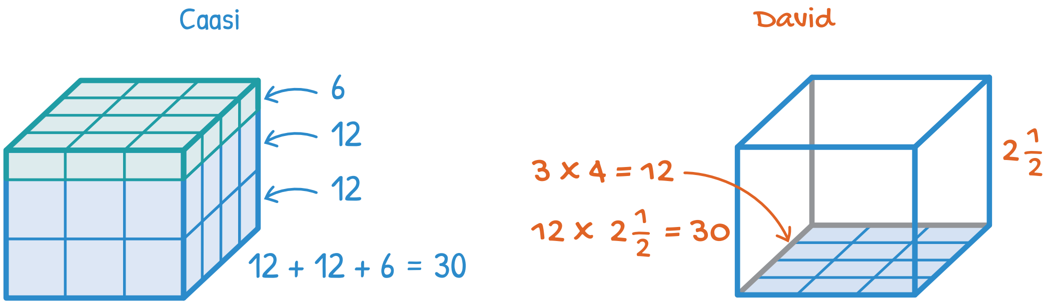


3 This prism has the same base as the one in the previous problem, but it's a $\frac{1}{2}$-unit taller.

What is the volume of this rectangular prism?



4 Here is the work that Caasi and David did to calculate the volume of the prism from the previous problem.



Discuss: What was each student's strategy?

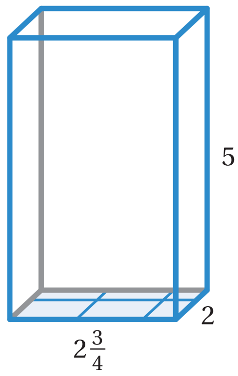
Activity
2

Name: ______ Date: ______ Period: ______

Calculating Volume

5 Here is another rectangular prism.

What is its volume?



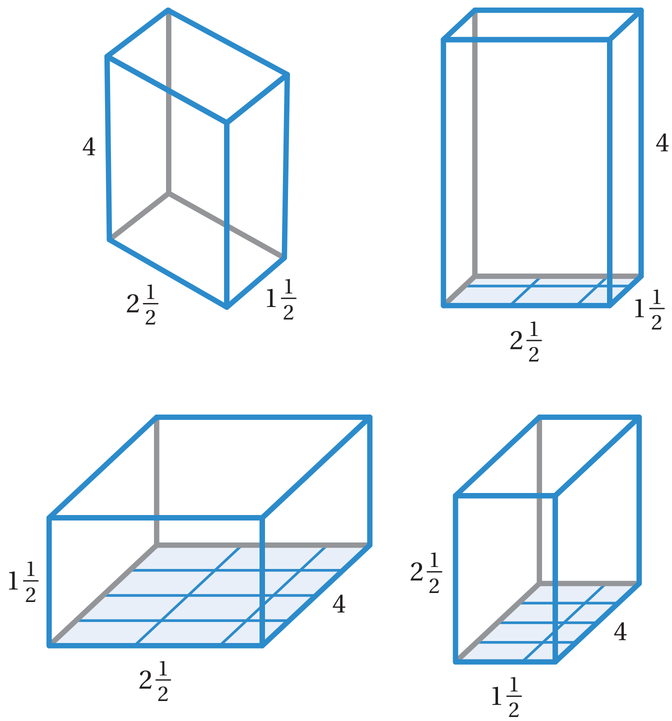
6 Caasi and David are calculating the volume of a new prism. Here are four different views of that prism.

Caasi thinks the area of the base is 10 square units. David thinks the area of the base is 6 square units.

Whose thinking is correct?

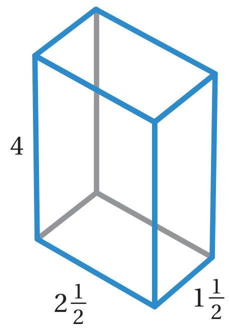
Caasi's David's Both Neither

Explain your thinking.



7 Here is the prism from the previous problem.

What is its volume?

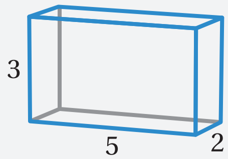
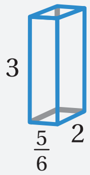
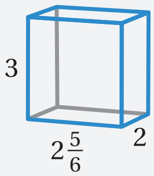
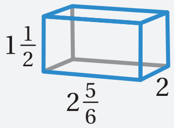


Activity 2

Name: ______ Date: ______ Period: ______

Calculating Volume (continued)

8 Calculate the volume of each prism.

Prism	Dimensions (units)	Volume (cubic units)
	$5 \times 2 \times 3$	
	$\frac{5}{6} \times 2 \times 3$	
	$2\frac{5}{6} \times 2 \times 3$	
	$2\frac{5}{6} \times 2 \times 1\frac{1}{2}$	

You're invited to explore more.

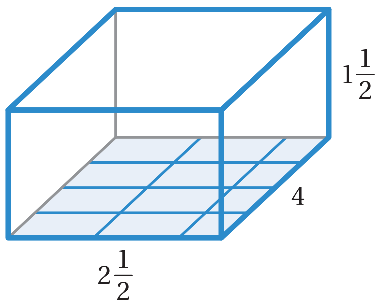
9 How many rectangular prisms can you make that have a volume of 12 cubic inches? List their dimensions. Only use dimensions measuring 24 inches or less.

The first one has been done for you.

Length (in.)	Width (in.)	Height (in.)	Volume (cu. in.)
2	2	3	12

10 Synthesis

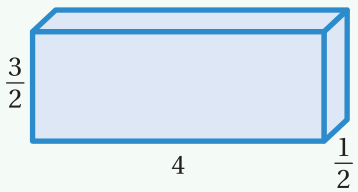
Describe a strategy for calculating the volume of a rectangular prism like this one.



13 Summary 4.14

You can determine the *volume* of a prism by multiplying its dimensions.

For example, here is a rectangular prism with side lengths measuring 4 units, $\frac{3}{2}$ units, and $\frac{1}{2}$ units.

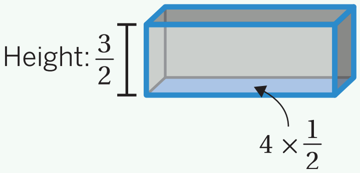


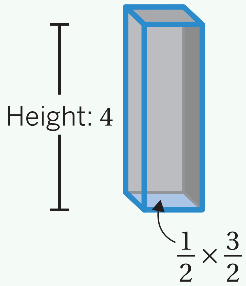
Volume

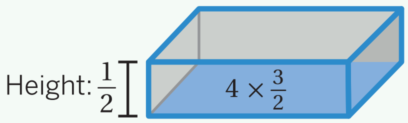
$4 \cdot \frac{3}{2} \cdot \frac{1}{2}$

$= 3$ cubic units

You can also calculate the volume of a prism as the product of its base area and the height. You can choose any of the rectangles as the base.







If you choose the 4-by-$\frac{1}{2}$ rectangle as the base, then the base area will be 2 square units.

The volume is $2 \cdot \frac{3}{2} = 3$ cubic units.

If you choose the $\frac{3}{2}$-by-$\frac{1}{2}$ rectangle as the base, then the base area will be $\frac{3}{4}$ square units.

The volume is $\frac{3}{4} \cdot 4 = 3$ cubic units.

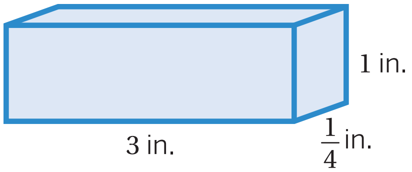
If you choose the 4-by-$\frac{3}{2}$ rectangle as the base, then the base area will be 6 square units.

The volume is $6 \cdot \frac{1}{2} = 3$ cubic units.

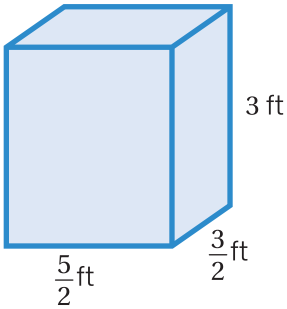
Name: .... Date: .... Period: ....

Problems 1–3: Determine the volume of these rectangular prisms.

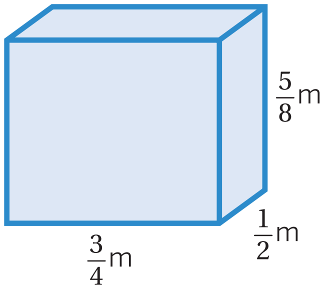
1.



2.



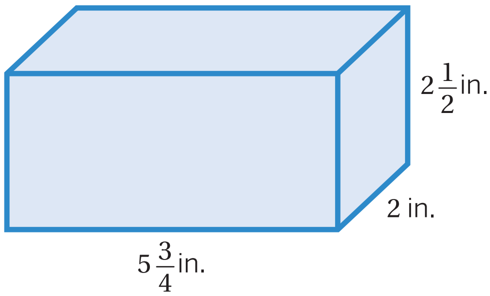
3.



Problems 4–5: Complete the table for each rectangular prism.

	Base Area (sq. in.)	Height (in.)	Volume (cu. in.)
4.	4	$1\frac{1}{3}$	
5.		$2\frac{2}{3}$	8

6. **Test Practice** How many small cubes with edge lengths measuring $\frac{1}{4}$ inches can be packed into this right rectangular prism?

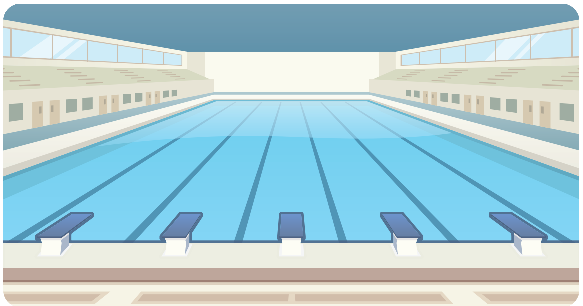


7. Diego says that he needs 108 cubes, each with an edge length of $\frac{1}{3}$ inches, to fill a rectangular prism measuring 3-by-1-by-$1\frac{1}{3}$ inches. Do you agree with Diego? Explain your thinking.

Practice 4.14

Name: .... Date: .... Period: ....

Problems 8–9: A swimming pool used in the Olympics is 25 meters wide, 50 meters long, and about $2\frac{1}{2}$ meters deep.



8. Determine the volume of this swimming pool. Show your thinking.

9. There are 1,000 liters of water in 1 cubic meter. Calculate the number of liters needed to fill this pool.

Spiral Review

10. Select *all* the fractions that are equivalent to $\frac{2}{3}$.

☐ A. $\frac{4}{6}$ ☐ B. $\frac{8}{15}$ ☐ C. $\frac{12}{13}$ ☐ D. $\frac{20}{30}$ ☐ E. $\frac{14}{21}$

11. A teacher wants to make a paste with flour and water. The table shows the ratio of the number of cups of flour to the number of cups of water they need. Complete the table to show the other equivalent ratios.

Flour (cups)	Water (cups)
1	$\frac{1}{2}$
4	
	3
$\frac{1}{2}$	

Problems 12–14: Evaluate each percentage problem.

12. 45% of 300 **13.** 1% of 800 **14.** 10% of 650

Unit 4
Lesson
15

Name: ________ Date: ________ Period: ________

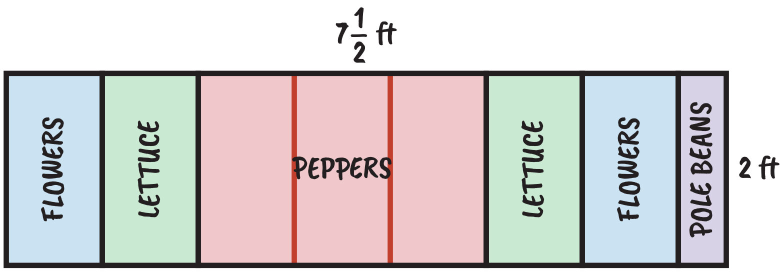
Fraction Relationships | Nets and Surface Area | 6.NS.1, 6.G.2, SMP.2, SMP.4

Planter Planner

Let's apply division of fractions to real-world scenarios.

Warm-Up

Jin sketched a plan for a planter.



1. What do you notice? What do you wonder?

I notice:

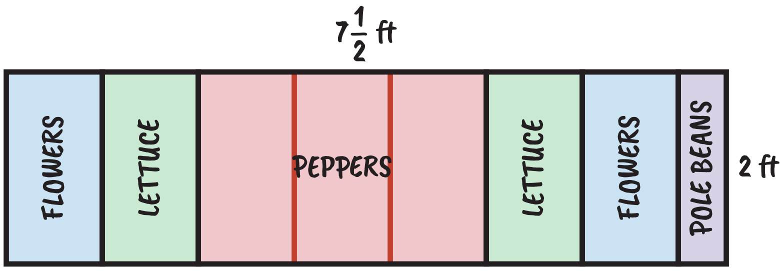
I wonder:

2. Jin says 8 lettuce plants can fit in each lettuce section of the planter. Do you agree? Why or why not? ELD.PI.6.11.Em, Ex, Br

Name: ________ Date: ________ Period: ________

Jin’s Planter

Here is Jin’s planter diagram again.



3. Use the Activities 1 & 2 Sheet to help Jin figure out how many of each plant can be grown.

a ________ flower plants

b ________ lettuce plants

c ________ pepper plants

d ________ pole bean plants

4. How many servings of pole beans can Jin grow in the planter?

5. Will Jin get more servings of food from the pepper plants or the pole bean plants? Explain your thinking. **ELD.PI.6.10.Em, Ex, Br**

6. Jin needs to fill the planter with soil.

Help Jin determine how many bags of soil are needed.

Activity 2

Name: .... Date: .... Period: ....

Build Your Own Planter

7. You are planning a planter for your school's greenhouse.

a Select *at least three* types of plants from the Activities 1 & 2 Sheet to grow in your planter.

b Select a planter to grow your plants in.

c Determine how many of each type of plant you can fit in your planter. Be sure each plant has enough space to grow! Show your work.

d Create a poster about your planter.

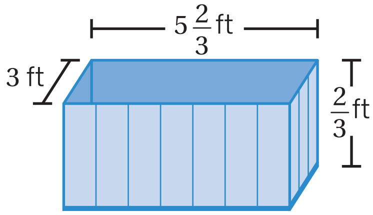
- ☐ Sketch your planter, showing the locations of each type of plant.
- ☐ List the types of plants in the planter.
- ☐ List your answer from part c.
- ☐ Calculate the amount of soil you will need to fill your planter. Show your work. (Each bag fills $\frac{3}{4}$ of a cubic foot.)
- ☐ Calculate how many servings of food and bunches of flowers you expect to grow.

You're invited to explore more.

8. Xavier has 20 cubic feet of soil. He wants to build a planter that is $2\frac{1}{2}$ feet wide and $1\frac{1}{4}$ feet deep. How many feet long should he make the planter so that he uses all of his soil? Explain your thinking.

Synthesis

9. **Discuss:** What are some important things to remember when calculating volume using dimensions that are fractions?
ELD.PI.6.1.Em, Ex, Br



Summary 4.15

Take a moment to review what you know about fractions and operations.

To multiply fractions . . .

- Multiply the denominators as a way to determine a common denominator and make same-sized parts.
- Then multiply the numerators to determine how many of those parts exist.

Example:

$$\frac{3}{8} \bullet \frac{5}{9}$$

$$= \frac{3 \bullet 5}{8 \bullet 9}$$

$$= \frac{15}{72}$$

$$= \frac{5}{24}$$

To divide by a fraction, $\frac{a}{b}$ . . .

Multiply the dividend by the reciprocal, $\frac{b}{a}$.

Example:

$$\frac{4}{7} \div \frac{5}{3}$$

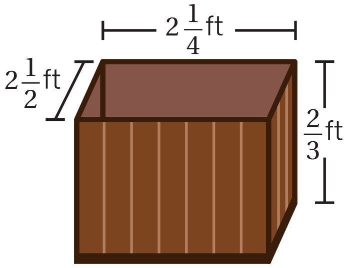
$$= \frac{4}{7} \bullet \frac{3}{5}$$

$$= \frac{12}{35}$$

Practice 4.15

Name: ______________________ Date: __________ Period: __________

Problems 1–3: Raven is building a new planter for her class garden.

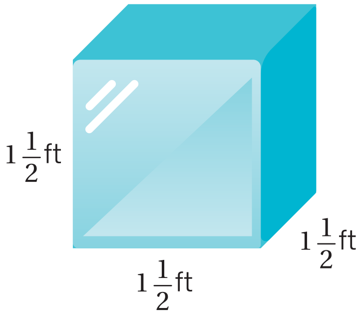


1. The base of her planter is $2\frac{1}{2}$ feet by $2\frac{1}{4}$ feet. What is its area?

2. If the planter is $\frac{2}{3}$ feet high, what volume of soil does she need to fill it?

3. Each bag holds $\frac{3}{4}$ cubic feet of soil. How many bags of soil does Raven need?

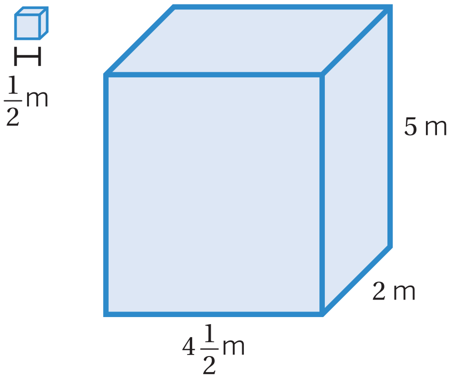
Problems 4–5: Before home freezers were introduced in the 1940s, some people had large blocks of ice delivered to their homes.



4. **Test Practice** Delivery wagons held $7\frac{1}{2}$ feet by 6 feet by 6 feet of ice. What volume of ice could a delivery wagon hold?

5. Each ice block was a $1\frac{1}{2}$-foot cube. How many ice blocks could fit in a delivery wagon? Explain your thinking.

6. A small cube has edge lengths measuring $\frac{1}{2}$ meter. How many small cubes could be packed into a rectangular prism that measures $4\frac{1}{2}$-by-2-by-5 meters?



Practice
4.15

Name: .... Date: .... Period: ....

Spiral Review

Problems 7–10: Determine each quotient.

7. $8 \div \frac{2}{9}$

8. $\frac{5}{6} \div 15$

9. $\frac{4}{7} \div \frac{8}{12}$

10. $\frac{23}{2} \div \frac{14}{5}$

Problems 11–14: If a smoothie recipe calls for 6 cups of ice, how many scoops of ice does each student need?

11. Alisha has a 2-cup scoop

12. Emma has a $1\frac{1}{2}$-cup scoop

13. Lukas has a $\frac{1}{5}$-cup scoop

14. Explain how the equation $6 \div \frac{1}{5} = ?$ represents Lukas's situation.

Name: ______________________ Date: ____________ Period: ____________

Fraction Relationships | Generalizing With Multiple Representations | Nets and Surface Area

6.EE.2.c, 6.NS.1, 6.G.2, SMP.8

Practice Day 2

Let’s practice what you’ve learned so far in this unit!

You will use problem cards for this Practice Day. Record all of your responses here.

Card 1

Circle one: A B C D

Card 2

Drawing:

Solution: ____________ scoops

Card 3

Quotient: ____________

Card 4

Quotient: ____________

Card 5

Quotient: ____________

Card 6

Quotient: ____________

Name: ________ Date: ________ Period: ________

Practice Day 2 (continued)

Card 7

Volume: ________ cubic inches

Card 8

Area: ________ square inches

Card 9

Volume: ________ cubic inches

Card 10

Missing length: ________ inches

Card 11

Rudra biked ________ times as far as Na'ilah.

Card 12

Zwena biked ________ times as far as Deven.

Notes:

Career Connection

How can math help predict the popularity of a movie?

Drazen Zigic/Shutterstoc.com.

Today, streaming is a popular choice for watching movies. Some movies are still shown in theaters. For theater-based movies, how well a movie performs during its opening weekend can help predict how well it might perform overall. Mathematician Ron Buckmire has used math to help predict the fraction of a movie's total earnings that comes after its opening weekend.

Suppose the fraction of total earnings that came from Movie A's opening weekend was $\frac{7}{25}$ and the fraction for Movie B was $\frac{7}{20}$. How can you compare these fractions?

Applied mathematicians use math to model and help solve problems that arise in society, science, business, healthcare, government, transportation, entertainment, and more. They might divide with fractions to compare different part-to-whole relationships.

Meet Ron Buckmire

Ron Buckmire is Dean of the School of Computer Science and Mathematics and Professor of Mathematics at Marist College in Poughkeepsie, New York. His research focuses on numerical analysis and applied mathematics, including mathematical modeling. In 2023, mathematicians from around the world honored Buckmire with a prestigious fellowship in recognition of his "exemplary research and outstanding service to the discipline of applied mathematics and computational science."

Are you interested in studying how mathematics can be used to solve problems in our world? What can you do to learn more?

Community Connection

Explain to an adult at home how you can use more than one strategy to divide $\frac{5}{8}$ by $\frac{5}{6}$.

Math Mindset

Without calculating, how do you know if the quotient of $\frac{2}{3} \div \frac{1}{4}$ is less than, equal to, or greater than $\frac{2}{3}$?