Exponents and Scientific Notation

Accelerated 7 Unit 8

12 Synthesis

When might it be helpful to write values or expressions using *exponents*?

Use the examples if they help with your thinking.

$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

$6 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 5$

$10 + 10 + 10 + 10 + 10$

Summary

Expressions with *exponents* are useful for representing repeated multiplication. In the expression 3^5, 5 is the exponent. When the exponent is a positive integer, it says how many times the number or expression is multiplied.

For example, $3^5 = \underbrace{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}_{\text{5 times}}$. Imagine writing 3^{100} using multiplication!

Here are a few more examples:

- $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^5$
- $5 \cdot 8 \cdot 5 \cdot 8 \cdot 5 \cdot 8 \cdot 5 = 5^4 \cdot 8^3$
- $10 \cdot 10 \cdot 10 + 10 \cdot 10 = 10^3 + 10^2$

10 Synthesis

Describe some strategies for writing equivalent expressions involving exponents.

Use the examples if they help with your thinking.

$3^4 \cdot 3^2 = 3^6$

$3^5 \cdot 4^5 = 12^5$

$(3^2)^4 = 3^8$

Summary

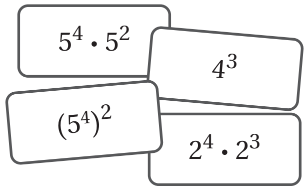
Expanding is one strategy for determining if expressions with exponents are equivalent.

Here are two **powers of ten** that are equivalent to 10^8. Each expression can be expanded to "$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$."

- $10^5 \cdot 10^3 = (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) = 10^8$
- $(10^2)^4 = (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) = 10^8$

Synthesis

10. What are some important things to remember when determining whether expressions with exponents are equivalent?



Summary

Rewriting powers can help you make sense of different bases with the same exponent. Here is one example.

If you expand $4^6 \cdot 3^6$, it is equal to $(4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4)\ (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)$. You can rearrange the factors to get $(4 \cdot 3)\ (4 \cdot 3)\ (4 \cdot 3)\ (4 \cdot 3)\ (4 \cdot 3)\ (4 \cdot 3)$, which is equal to 12^6. This means that $4^6 \cdot 3^6$ is equivalent to 12^6.

You may not need to expand an expression completely to determine whether it is equivalent to another expression. For example, $(12^4)^2$ is not equivalent to 12^6 because it is $(12^4) \cdot (12^4) = 12^8$.

8 Synthesis

Describe a strategy for rewriting an expression as a single power.

Use these expressions if they help with your thinking.

$\frac{6^7 \cdot 6^7}{(6^3)^2}$

$2^8 \cdot 3^8$

$\frac{12^8}{2^8}$

Summary

Rewriting expressions as a single power like 7^3 can help you make sense of more complex expressions, especially ones that involve division. Expanding is one strategy for rewriting expressions with exponents as single powers.

Here are two examples.

$$\begin{aligned}\frac{(3^3)^2}{3^4} &= \frac{(3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3 \cdot 3} \\ &= \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot 3 \cdot 3 \\ &= 1 \cdot 1 \cdot 1 \cdot 1 \cdot 3 \cdot 3 \\ &= 3^2\end{aligned}$$

$$\begin{aligned}\frac{9^2 \cdot 3^5}{3^3} &= \frac{(9 \cdot 9) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3} \\ &= \frac{(3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3} \\ &= \frac{3}{3} \cdot \frac{3}{3} \cdot \frac{3}{3} \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \\ &= 1 \cdot 1 \cdot 1 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \\ &= 3^6\end{aligned}$$

Knowing that $\frac{(3^3)^2}{3^4} = 3^2$ and $\frac{9^2 \cdot 3^5}{3^3} = 3^6$ can help you compare the two: $\frac{9^2 \cdot 3^5}{3^3}$ is greater than $\frac{(3^3)^2}{3^4}$ because 3^6 is greater than 3^2.

10 Synthesis

How could you use the table to convince someone that $6^0 = 1$ and $6^{-1} = \frac{1}{6}$?

Exponent Form	Value
6^3	216
6^2	36
6^1	6
6^0	
6^{-1}	

Summary

Positive, negative, and zero exponents are all related.

Positive exponents represent the number of times 1 is *multiplied* by the base. For example: $3^2 = 1 \cdot 3 \cdot 3$.

Negative exponents represent the number of times 1 is *divided* by the base. For example: $3^{-2} = \frac{1}{3^2} = \frac{1}{3 \cdot 3} = \frac{1}{9}$. Since $\frac{1}{3^2} = \left(\frac{1}{3}\right)^2$, we can also say that $3^{-2} = \left(\frac{1}{3}\right)^2$.

Zero exponents represent 1 multiplied by the base 0 times. This means the value of any non-zero base with a 0 exponent is equal to 1. Here are some other examples of powers with 0 exponents.

- $3^0 = 1$
- $14^0 = 1$
- $\left(\frac{1}{5}\right)^0 = 1$
- $(-3)^0 = 1$

Synthesis

Describe how exponent rules can help you rewrite exponential expressions.

Summary

There are several rules about powers that can be helpful when rewriting or comparing expressions with exponents.

Types of Powers	Rule With Variables	Example
Multiplying Powers With the Same Base	$a^n \bullet a^m = a^{n+m}$	$6^2 \bullet 6^7 = 6^{2+7} = 6^9$
Dividing Powers With the Same Base	$\frac{a^n}{a^m} = a^{n-m}$	$\frac{3^{11}}{3^4} = 3^{11-4} = 3^7$
Powers of Powers	$(a^n)^m = a^{n \bullet m}$	$(1.7^5)^3 = 1.7^{(5 \bullet 3)} = 1.7^{15}$
Negative Exponents	$a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$	$14^{-2} = \frac{1}{14^2} = \left(\frac{1}{14}\right)^2$
Powers With Different Bases	$a^n \bullet b^n = (ab)^n$ $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$	$2^3 \bullet 5^3 = 10^3$ $\frac{2^3}{5^3} = \left(\frac{2}{5}\right)^3$
Zero Exponents	$a^0 = 1$	$\left(\frac{5}{9}\right)^0 = 1$

Note: The variables a and b are not equal to 0, and n and m are integers.

10 Synthesis

What are some strategies for writing a number as a combination of powers of 10? Use the examples if they help with your thinking.

- 2560
- $2 \cdot 10^3 + 5 \cdot 10^2 + 6 \cdot 10^1$
- $256 \cdot 10^1$
- $2.56 \cdot 10^3$
- $25 \cdot 10^2 + 6 \cdot 10^1$
- $25.6 \cdot 10^2$

Summary

Large numbers can be written as a combination of *powers of* 10 to make them less awkward to work with and to prevent having to count 0s.

The number 90,700,000 can be written many different ways using *powers of* 10.

For example:

- $90{,}700{,}000 = 9 \cdot 10^7 + 7 \cdot 10^5$
- $90{,}700{,}000 = 90 \cdot 10^6 + 7 \cdot 10^5$
- $90{,}700{,}000 = 9.07 \cdot 10^7$

9 Synthesis

What are some strategies for writing very small values as a number times a single power of 10? Use the examples if they help with your thinking.

0.00083

$8.3 \cdot 10^{-4}$

$83 \cdot 10^{-5}$

Summary

Like large numbers, small numbers can be rewritten using combinations of powers of 10. Numbers less than 1 will use negative powers of 10.

For example:

- $0.000000877 = 8 \cdot 10^{-7} + 7 \cdot 10^{-8} + 7 \cdot 10^{-9}$
- $0.00000000034 = 3 \cdot 10^{-10} + 4 \cdot 10^{-11}$
- $0.00000049 = 4 \cdot 10^{-7} + 9 \cdot 10^{-8}$

Writing large and small values as a number times a single power of 10 can also be helpful for comparing those values and for getting a sense of their scale.

For example:

- $42000000000 = 4.2 \cdot 10^{10}$
- $2500000000 = 25 \cdot 10^{8}$
- $0.00000000034 = 3.4 \cdot 10^{-10}$
- $0.00000049 = 49 \cdot 10^{-8}$

9 Synthesis

Describe a strategy for writing a number in scientific notation. Use the examples if they help with your thinking.

Not in Scientific Notation	In Scientific Notation
36,000,000	$3.6 \cdot 10^7$
6,700,000	
0.00024	
$417 \cdot 10^3$	

Summary

There are many ways to express a number using a power of 10. One specific way is called **scientific notation**, which can be helpful for comparing very large or very small numbers. When a number is written in scientific notation, the first part is a number greater than or equal to 1, but less than 10. The second part is an integer power of 10.

For example:

- 425,000,000 is $4.25 \cdot 10^8$ in scientific notation
- 0.0000000000783 is $7.83 \cdot 10^{-11}$ in scientific notation

Synthesis

11. Describe a strategy for multiplying or dividing numbers written in scientific notation. Use the examples if they help with your thinking.

$$(4 \cdot 10^{-2}) \cdot (5 \cdot 10^{8})$$

$$\frac{5 \cdot 10^{9}}{2 \cdot 10^{4}}$$

Summary

Multiplying numbers written in *scientific notation* is an extension of multiplying decimals.

To multiply two numbers written in scientific notation:

- Multiply the first parts of each number.
- Multiply the powers of 10 using what you have learned about exponent properties.

For example: $(2 \cdot 10^{3}) \cdot (4 \cdot 10^{6}) = (2 \cdot 4) \cdot 10^{(3+6)} = 8 \cdot 10^{9}$

To divide two numbers written in scientific notation, it can be helpful to rewrite the expression as a fraction.

- Divide the first part in the numerator by the first part in the denominator.
- Divide the powers of 10 using what you have learned about exponent properties.

For example: $\frac{8 \cdot 10^{7}}{4 \cdot 10^{2}} = \frac{8}{4} \cdot 10^{(7-2)} = 2 \cdot 10^{5}$

Synthesis

5. Describe something you learned about scientific notation while making your poster.

Summary

Scientific notation and exponent rules can be helpful when solving real-world problems that include very large or very small numbers. For example, you can use the rules to calculate how many dollars worth of food are wasted in the United States each year or the total amount of student debt in the United States.

When solving a real-world problem, it is important to look at the information you know, determine what information is needed to solve the problem, and think about appropriate units of measurement. You can also use rounding to make some quantities simpler to work with.

10 Synthesis

What are some important things to remember when adding or subtracting numbers written in scientific notation?

Use the examples if they help with your thinking.

$4.6 \cdot 10^7 + 3.2 \cdot 10^6$

$1.57 \cdot 10^8 - 4 \cdot 10^6$

Summary

Scientific notation can be useful for adding or subtracting very large or very small numbers. Just like when you work with decimals, it is important to pay attention to place value when adding and subtracting numbers written in scientific notation.

For example: Let's add $3.4 \cdot 10^5 + 2.1 \cdot 10^6$.

It may appear that you can add the first parts: 3.4 and 2.1. However, these numbers *do not* have the same place value because they are multiplied by different powers of 10.

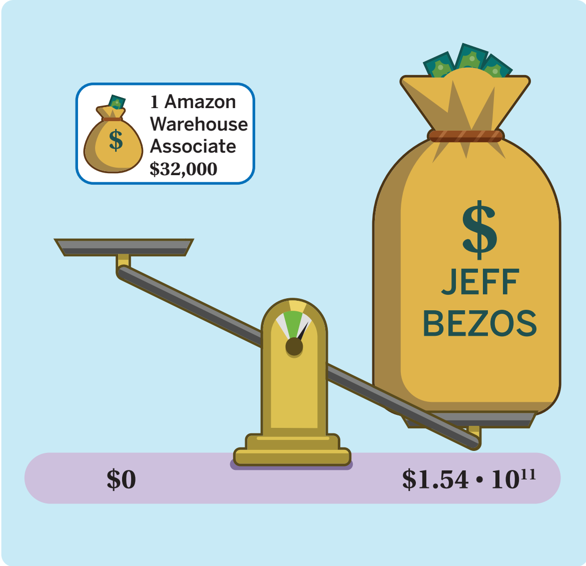
If you rewrite one number so that both numbers have the same power of 10, then you can add their first parts. In this case, let's rewrite $2.1 \cdot 10^6$ as $21 \cdot 10^5$.

$$\begin{aligned} 3.4 \cdot 10^5 + 2.1 \cdot 10^6 &= 3.4 \cdot 10^5 + 21 \cdot 10^5 \\ &= 24.4 \cdot 10^5 \\ &= 2.44 \cdot 10^6 \end{aligned}$$

Now that the power of 10 is the same, you can add 3.4 and 21. The sum is $24.4 \cdot 10^5$, or $2.44 \cdot 10^6$ when rewritten in scientific notation.

Synthesis

11. What are some important things to remember about adding, subtracting, multiplying, and dividing numbers written in scientific notation?



Summary

Scientific notation is a useful tool for adding, subtracting, multiplying, dividing, and comparing very small or very large numbers.

- You can rewrite the number $39,000,000,000,000$ as $3.9 \cdot 10^{13}$ and still convey just how large the number is.
- To add or subtract numbers written in scientific notation, it is useful to rewrite the numbers so they have the same power of 10.
- To multiply or divide numbers written in scientific notation, it is useful to multiply or divide the numbers that come before the powers of 10. Then you can use exponent rules to multiply or divide the powers of 10.
- If the product or quotient is not written in scientific notation, you can always rewrite it to be in that form.
- Sometimes it can be helpful to round numbers written in scientific notation when exact values are less important.

Some situations that involve very large and very small numbers include salaries of wealthy people, talking about large groups like the total number of workers at Amazon, or the sizes of microscopic objects like cells and bacteria.