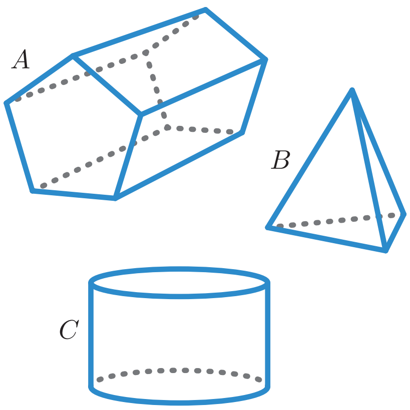
Volume and Surface Area

Accelerated 7
Unit 7

9 Synthesis

Describe why different cuts of a solid create different possible cross sections. Use the solids if they help with your thinking.

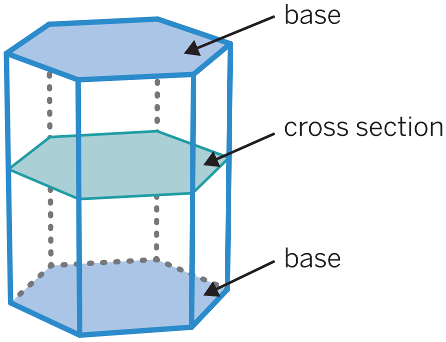
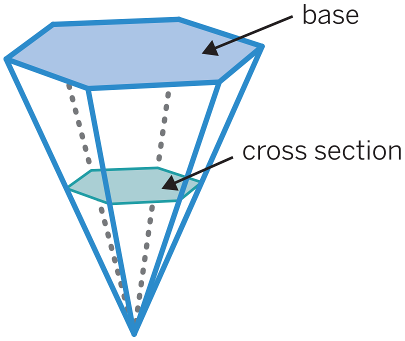


Summary

A **cross section** is the shape you see when you cut through a three-dimensional figure.

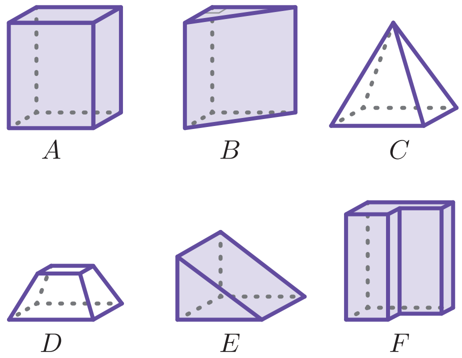
For example, if you cut a hexagonal *prism* parallel to the *base*, the cross section is a hexagon that is the same size as the base. If you make a vertical cut instead, the cross section is a rectangle that is as tall as the prism.

If you cut a hexagonal *pyramid* parallel to the base, the cross section is a hexagon that is smaller than the base. If you make a vertical cut instead, the cross section is a triangle that is taller than it is wide.

Prism	Pyramid
Hexagonal Prism  	**Hexagonal Pyramid** 

11 Synthesis

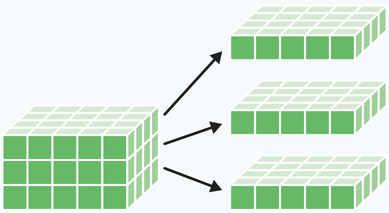
Describe how to determine the volume of a prism. Draw if it helps to show your thinking.



Summary

Any cross section of a prism that is parallel to the base will be identical to the base. This means you can slice a prism into layers to help you calculate its **volume**.

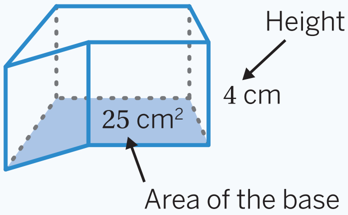
For example, if you have a rectangular prism that is 3 units tall and has a base that is 4 units by 5 units, you can imagine the prism as 3 layers of $4 \cdot 5$ cubic units.



That means the volume of this rectangular prism is $(4 \cdot 5) \cdot 3$ cubic units.

In general, you can calculate the volume of any prism by multiplying the area of its base by its height.

In other words, the volume of a prism is $V = B \cdot h$, where h is its height and B is the area of its base.

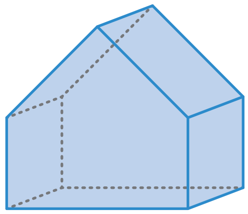


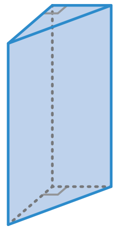
For example, this prism has a volume of 100 cubic centimeters because $25 \cdot 4 = 100$.

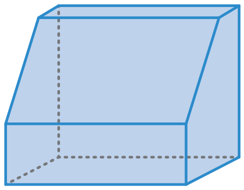
10 Synthesis

Here are several prisms you've seen in this lesson.

Describe a general strategy for determining the volume of any prism.

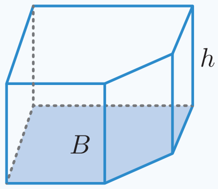


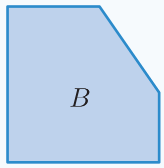




Summary

To calculate the volume of a prism, you can multiply the area of the base, B, by the height, h.

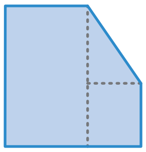
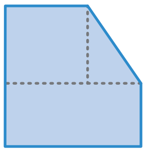
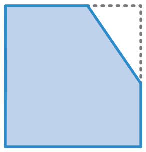




Sometimes the shape of a prism's base is a more complex *polygon*.

There are many strategies for calculating the area of a complex shape, including breaking it into rectangles and triangles, or surrounding the shape in a rectangle and subtracting the missing piece.

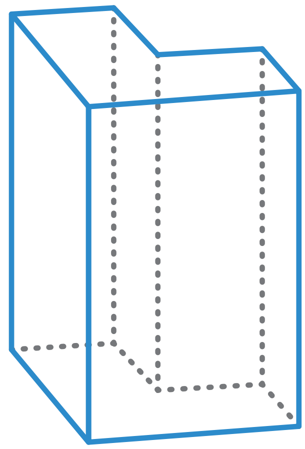
Here are three first steps you might take in calculating the area of this prism's base:

Example 1	Example 2	Example 3
		

Synthesis

10. Describe your favorite method for calculating the surface area of a prism.

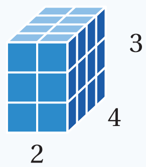
Use this prism if it helps you with your explanation.

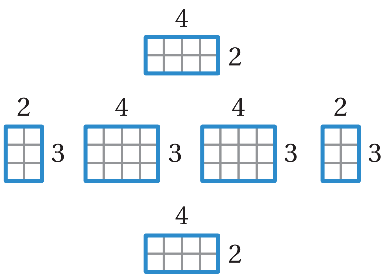
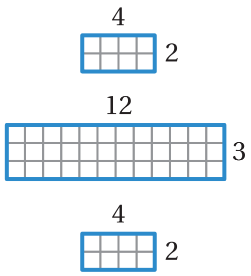


Summary

The **surface area** of a three-dimensional shape is the number of square units that cover all the faces of the polyhedron, without any gaps or overlaps.

There are multiple ways to calculate the surface area.

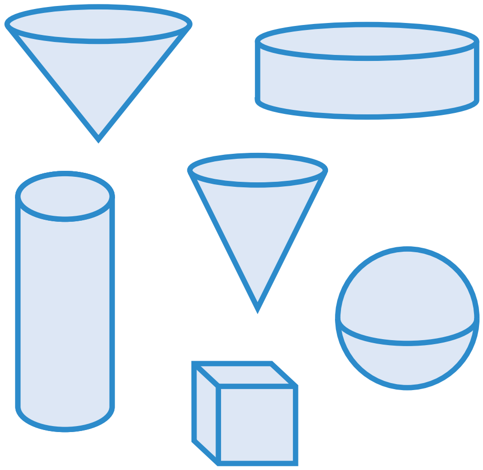


Strategy 1	Strategy 2
Calculate the area of each face separately and then add all of the areas.  $8 + 6 + 12 + 12 + 6 + 8 = 52$ Surface area: 52 square units	Break the prism into its two identical bases and unfold the sides into one long long rectangle. Add the three areas.  $8 + 36 + 8 = 52$ Surface area: 52 square units

Using either strategy to calculate the surface area or using equivalent calculations will result in the same total surface area, which in this case is 52 square units.

Synthesis

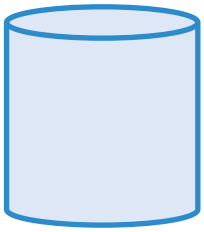
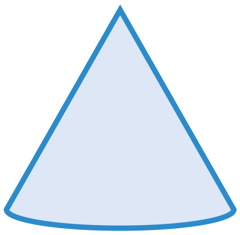
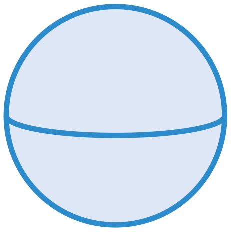
Describe one volume relationship you discovered during the lesson.



Summary

The *volume* of an object is the number of cubic units that fill its three-dimensional region without any gaps or overlaps.

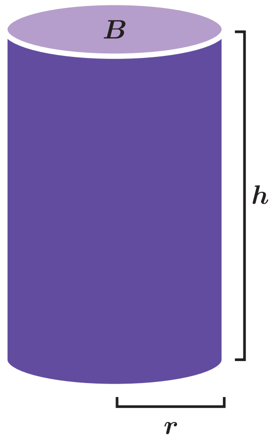
Circles have a **radius**, which is a line segment that connects the center of the circle with a point on the circle. The term *radius* can also refer to the length of this segment. Here are examples of circular solids.

Cylinder	Cone	Sphere
A **cylinder** is a three-dimensional figure that has two parallel circular bases connected by a curved surface.	A **cone** is a three-dimensional figure that tapers from one circular base to a point.	A **sphere** is a three-dimensional figure in which all cross-sections in every direction are circles. In spheres, a radius can extend from the center of the shape to any part of its exterior.
		

Synthesis

Here are two formulas for the volume of a cylinder.

$V = \pi r^2 \cdot h$ $\qquad$ $V = B \cdot h$



Describe how the formulas are related.

Summary

The volume of a cylinder can be determined using the radius, r, and height, h, with two previously introduced mathematical concepts.

- The volume of a rectangular prism is a result of multiplying the area of its base by its height.
- The base of the cylinder is a circle with radius r, so the base area is determined by the expression πr^2.

The base of a cylinder with radius r units has an area of πr^2 square units. If the height is h units, then the volume, V, in cubic units is $V = \pi r^2 \cdot h$.

Synthesis

Discuss

- Why do you think the relationship between height and volume is a linear function?
- Why do you think the relationship between radius and volume is a non-linear function?

Then select one of the questions and record your response.

Summary

Volume depends on the radius and height of the cylinder — if you know the radius and height, you can determine the cylinder's volume. The formula for the volume of a cylinder is $V = \pi r^2 h$. When the height h increases, the volume V also increases proportionally, showing that there is a linear relationship between the height and volume of a cylinder.

In general, when one quantity in a proportional relationship changes by a given factor, the other quantity changes by the same factor. Remember that proportional relationships are examples of linear relationships, which can also be thought of as functions.

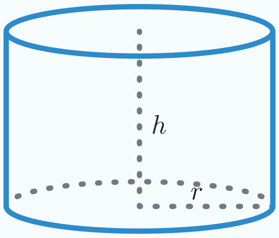
On the other hand, the relationship between the radius and the volume of a cylinder is non-linear because the ratio of the volume to the radius changes as the radius increases.

Synthesis

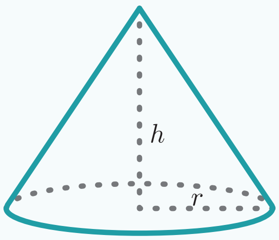
One way of writing the formula for volume of a cone is $V = \frac{1}{3}\pi r^2 \bullet h$. Where does each part of the formula come from?

Summary

Recall that the volume of a cylinder can be found by calculating $V = \pi r^2 h$. If a cone and a cylinder have the same base and the same height, then the volume of the cone is one-third the volume of the cylinder. If the radius and the height are known, the volume can be determined by using the formula for the volume of a cone: $V = \frac{1}{3}\pi r^2 h$.



Volume of a cylinder:
$V = \pi r^2 h$



Volume of a cone:
$V = \frac{1}{3}\pi r^2 h$

Synthesis

Work with a partner. Have one partner pick a value for the radius and one partner pick a value for the volume of the cylinder. Write your values below. Then work together to determine the height. Make a sketch of your cylinder and label the dimensions.

Radius: .... centimeters Volume: .... cubic centimeters

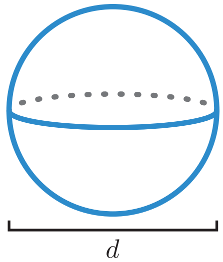
Summary

The volume of a cylinder depends on its radius and height. If the radius and height are known, the cylinder's volume can be determined using the formula $V = \pi r^2 h$. It is also true that, if the volume and one dimension (either radius or height) are known, the other dimension can be determined. This can be done by writing an equation and solving for the missing dimension.

The volume of a cone is also a function of the radius of the base and of the height. As with a cylinder, if the volume and one of the dimensions, either the radius or height, is known, the other dimension can be determined by writing and solving an equation using the formula for the volume of a cone, $V = \frac{1}{3}\pi r^2 h$.

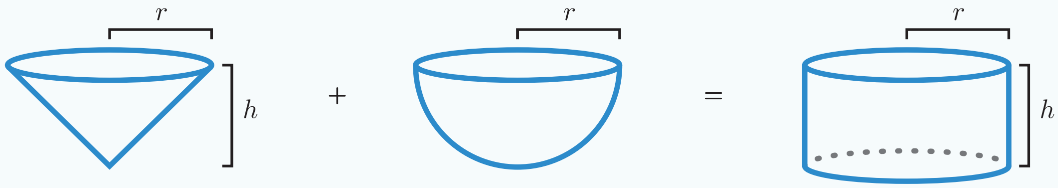
Synthesis

Imagine that all you know about a sphere is its diameter. Describe how to find the sphere's volume step by step.



Summary

The volume of a cylinder is equivalent to the sum of the volume of a cone with the same height and radius and the volume of a hemisphere with the same radius. In other words, if a cone and hemisphere were filled with water and that water was poured into the cylinder, then the cylinder would be completely filled.



Volume of cone ($\frac{1}{3}V$ of cylinder)

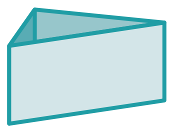
Volume of hemisphere ($\frac{2}{3}V$ of cylinder)

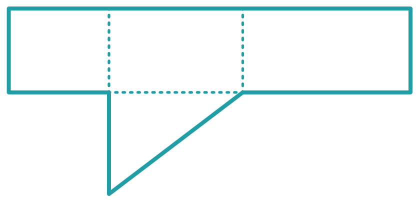
Volume of cylinder

In a previous lesson, it was determined that the volume of a cone is $\frac{1}{3}$ the volume of a cylinder with the same height and radius, and can be calculated using the formula $V = \frac{1}{3}\pi r^2 h$. In this lesson, it was shown that the volume of a hemisphere is $\frac{2}{3}$ of the volume of the cylinder with the same radius, and can be calculated using the formula $V = \frac{2}{3}\pi r^3$ (if the radius and height are equal). Since the volume of a sphere is twice the volume of a hemisphere, a formula for the volume of a sphere can be determined by doubling the volume of a hemisphere: $V = 2 \bullet \frac{2}{3}\pi r^3 = \frac{4}{3}\pi r^3$.

9 Synthesis

What is important to remember when calculating the surface area or volume of a prism? Use the example if it helps with your thinking.





Summary

Knowing when to calculate volume and surface area can be helpful in answering questions about situations in context.

Questions related to volume:

- How much water can a container hold?
- How much material did it take to build a solid object?

Questions related to surface area:

- How much fabric is needed to cover a surface?
- How much of an object needs to be painted?

One way to decide if a question is asking about volume or surface area is to think about the units of measure. Volume is measured in cubic units and surface area is measured in square units.