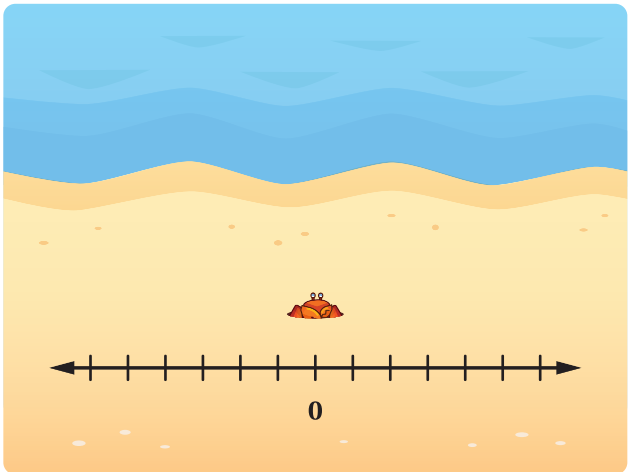
Positive and Negative Numbers

Accelerated 6
Unit 7

10 Synthesis

List at least two things you know about positive and negative numbers on a number line.

Draw on the image if it helps to show your thinking.



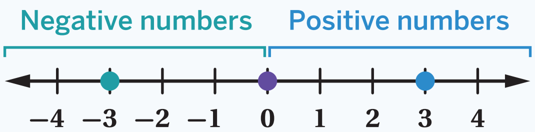
Summary

Positive numbers are numbers that are greater than 0. **Negative numbers** are numbers that are less than 0. Zero is neither positive nor negative.

You can extend a number line to the right of 0 to show *positive numbers*, and you can extend a number line to the left of 0 to show *negative numbers*.

The number 3 is 3 units to the right of 0 on the number line.

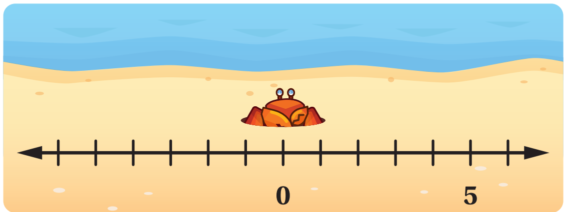
The number -3 is 3 units to the left of 0 on the number line.



11 Synthesis

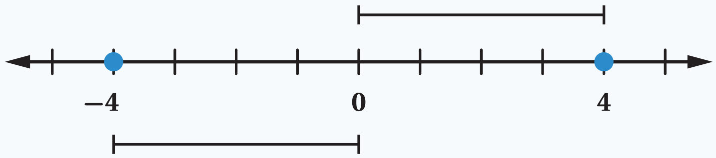
Describe where -3.2 is on a number line.

Draw on the image if it helps to show your thinking.



Summary

Two numbers are **opposites** if they are the same distance from 0 on different sides of the number line. For example, -4 and 4 are opposites because they are both 4 units away from 0.



Every number has an *opposite*, including fractions and decimals. 0 is its own opposite.

All positive and negative whole numbers and 0 are a group of numbers called *integers*. All positive and negative numbers that can be written as fractions, including whole numbers, are called *rational numbers*.

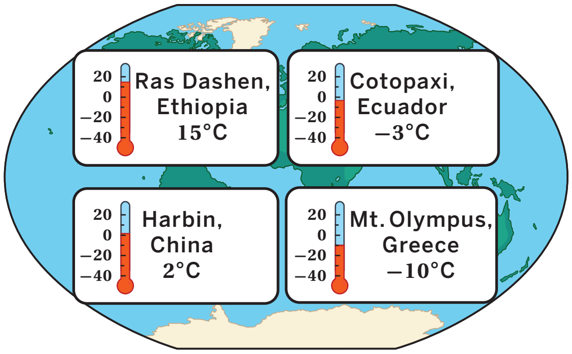
2 and −2 are both *integers* and *rational numbers*.

8.3, -8.3, $\frac{3}{2}$ and $-\frac{3}{2}$ are *rational numbers*, but *not integers*.

11 Synthesis

When comparing two temperatures, how can you tell which is warmer and which is colder?

Use the examples if they help with your thinking.

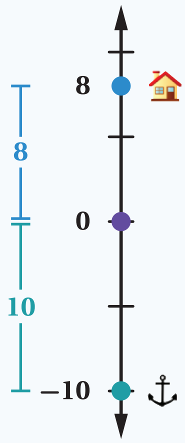


Summary

A vertical number line is a different way to represent positive and negative numbers. On a vertical number line, points above 0 are positive and points below 0 are negative.

When talking about elevation, 0 feet represents sea level. This means that a positive elevation is above sea level and a negative elevation is below sea level.

For example, an anchor that is 10 meters below sea level has an elevation of -10 meters. A house whose elevation is 8 meters is 8 meters above sea level. To show that the anchor has a lower elevation than the house, you can write $-10 < 8$.

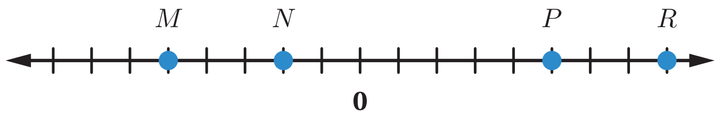


When talking about temperature, $0°C$ means the temperature is freezing. If the temperature in Mt. Olympus is $-10°C$, that means it has a temperature of $10°C$ below $0°C$, or below freezing.

If the temperature in Cotopaxi is $-3°C$, you can write $-10 < -3$, which means that it is colder in Mt. Olympus than it is in Cotopaxi.

6 Synthesis

Describe how to compare numbers on a number line. Use the example if it helps with your thinking.



Summary

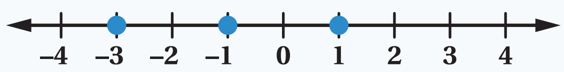
Number lines are helpful for comparing numbers with different **signs** (like -4 and 3) or numbers with the same *sign* (like -4 and -3). Numbers like 4 and $+4$ have the same sign, even though one has a $+$ symbol and one does not.

The order of numbers from least to greatest is the same order as they appear on the number line from left to right. This means that any positive number is greater than any negative number, and negative numbers farther from 0 are less than negative numbers that are closer to 0.

To compare numbers, we can use the symbols $>$ ("greater than") and $<$ ("less than").

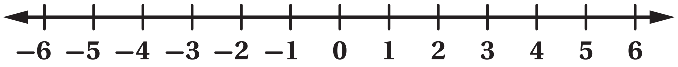
Let's say you want to compare -3 and -1. On a number line, -1 is to the right of -3. This means that -1 is greater than -3, or $-3 < -1$. This also makes sense because -1 is closer to 0 than -3 is.

A number line can also help you order numbers from least to greatest. 1 is greater than -1 and -3 because it is the farthest to the right on the number line.



13 Synthesis

Explain 2–3 things you know about absolute value. Use the number line if it helps to show your thinking.



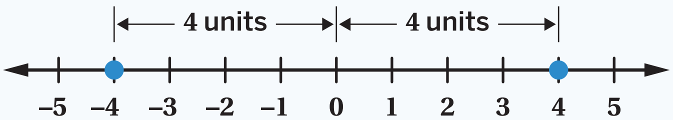
Summary

The **absolute value** of a number is a way to describe its distance from 0. For example:

The *absolute value* of -4 is 4, because it is 4 units away from 0. $\lvert -4 \rvert = 4$	The *absolute value* of 4 is also 4, because it is 4 units away from 0. $\lvert -4 \rvert = 4$

The distance from 0 to itself is 0, so $|0| = 0$.

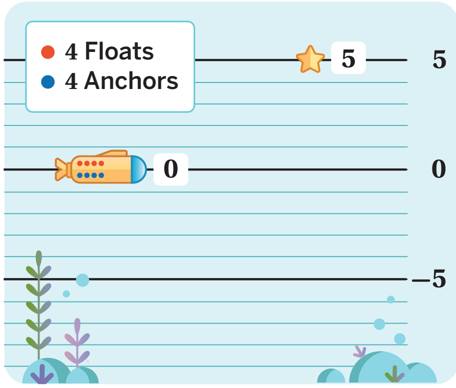
Opposites always have the same *absolute value* because they are the same distance from 0 on a number line, but in opposite directions.



Absolute values are helpful when you are interested in the size of a difference or measurement but its direction is not important. For example, measuring how far a dart is from a target.

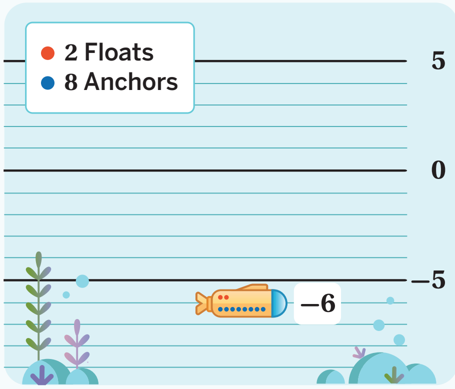
Synthesis

Describe a set of actions that would allow this submarine to collect the star at 5 units. Try to come up with something none of your classmates will.



Summary

Using models such as floats and anchors on a vertical number line can be useful when representing addition and subtraction of positive and negative numbers.



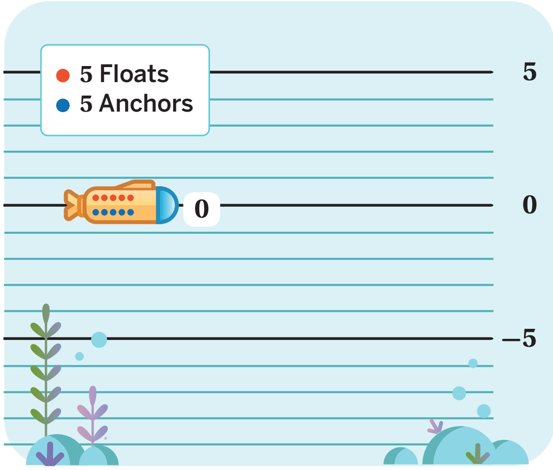
For example, consider a submarine whose position is at -6 units. The submarine will move from its position as 3 floats are added and 2 anchors are removed.

- Adding 3 floats can be expressed as $+3$ and represents moving the submarine up 3 units.
- Removing 2 anchors can be expressed as $-(-2)$ and represents moving the submarine up 2 units.

The submarine's new position would be -1 units. To move the submarine to 0 units from -1 units, 1 float can be added, represented by $+1$. -1 and 1 are *opposites* which add to 0.

Synthesis

Use the floats and anchors scenario to explain why it makes sense that $0 - (-5)$ is equivalent to $0 + 5$.

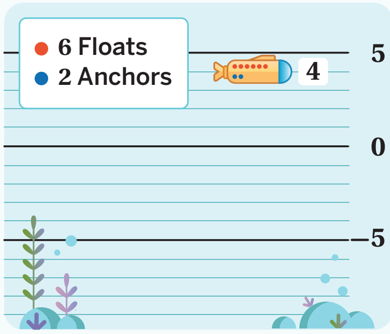


Summary

Different combinations of floats and anchors can arrive at the same result. Consider these examples:

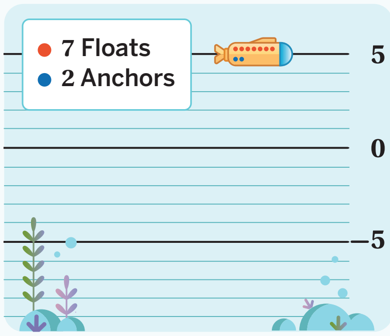
- If a submarine starts at 4 units, adding 2 floats or removing 2 anchors will both result in the submarine moving up to 6 units. Therefore, adding a positive number is the same as subtracting a negative number.

Adding floats	Removing anchors
$4 + 2 = 6$	$4 - (-2) = 6$



- If a submarine starts at 5 units, removing 3 floats or adding 3 anchors will both result in the submarine moving down to 2 units. Therefore, subtracting a number is the same as adding its opposite.

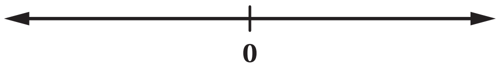
Removing floats	Adding anchors
$5 - 3 = 2$	$5 + (-3) = 2$



When adding two values that are opposites, the sum is always 0. These numbers are also called *additive inverses* of each other.

Synthesis

Describe how you can use a number line to determine the value of x in an equation like $3.1 + x = -2$. Use the number line if it helps you show your thinking.

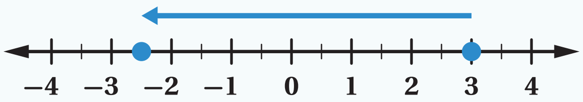


Summary

When adding positive and negative decimals and fractions, it may help to use a number line and to think of each equation as representing $start + change = end$.

For example, consider the equation $3 + (-5.5) = x$.

3 represents the starting location, -5.5 represents the change by moving 5.5 units to the left, and x represents the ending location, which is -2.5.



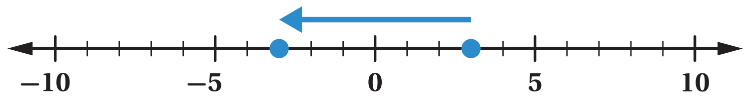
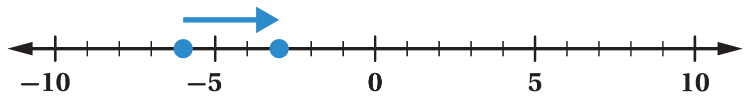
Synthesis

What happens to the value of a subtraction expression when you rearrange the order of the numbers? Use the examples if they help you with your explanation.

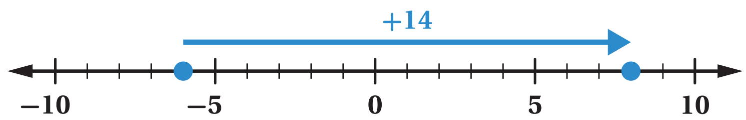
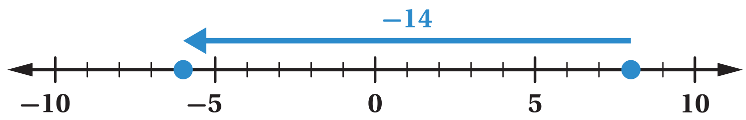
$7 - 4$ and $4 - 7$
$-2 - (-3.5)$ and $-3.5 - (-2)$

Summary

When determining the *sum*, the order of the values in an expression does not change the final result.

$3 + (-6) = -3$	
$-6 + 3 = -3$	

When determining the *difference*, the order of the values in an expression changes the final result. A positive or negative difference indicates whether the second value is greater than or less than the first value. The two values from the expression can be plotted on the number line. Beginning at the second value, draw an arrow to the first value. When the arrow moves right, the difference is positive. When the arrow moves left, the difference is negative.

$8 - (-6) = 14$	
$-6 - 8 = -14$	

Synthesis

Imagine subtracting a pair of numbers. Describe how you can tell whether the result will be positive or negative.

$\square - \square = ?$

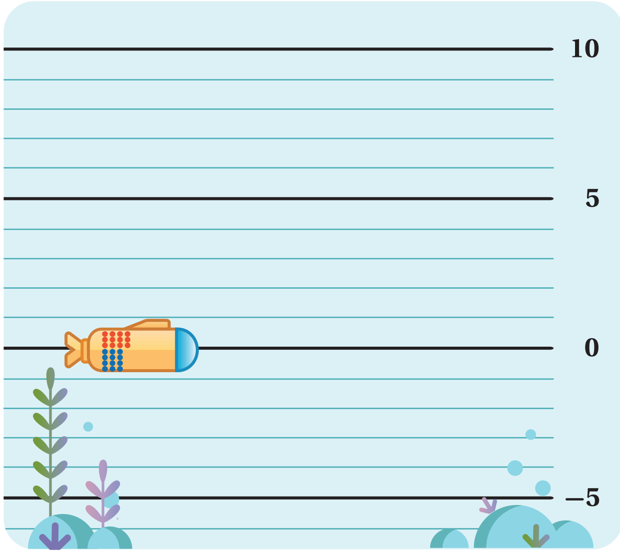
Summary

When adding and subtracting rational numbers, there are multiple paths to the same solution. Consider some of the following strategies for adding and subtracting signed numbers.

- To add two numbers with the same sign, determine the sum of the absolute value of each number, and then give the sum the same sign as the addends. For example, $-3 + (-4) = -(|-3| + |-4|) = -(3 + 4)$ or -7.
- To add two numbers with different signs, determine the difference of the absolute values, and then give the result the same sign as the number with the greater absolute value. For example, $3 + (-4) = -(|-4) - |3|) = -(4 - 3)$ or -1.
- When subtracting signed numbers, rewrite the expression as addition using the additive inverse of the second term. For example, $-3 - 4 = -3 + (-4)$ or -7.

Synthesis

Use the floats and anchors scenario to explain why it makes sense that $(-2)(-4)$ is positive.



Summary

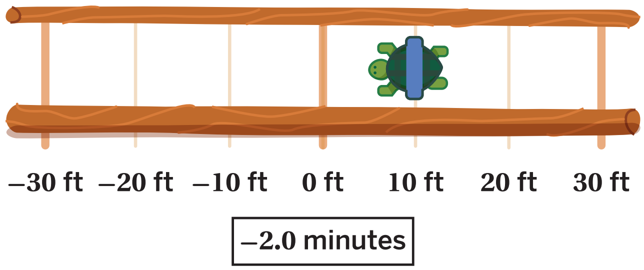
Models such as floats and anchors can also be used to make sense of multiplying positive and negative integers. Here are some examples.

Action	Representation	Submarine's direction	Final Value
Adding 2 groups of 3 floats	$2 \cdot 3$	Up	6
Removing 2 groups of 3 floats	$-2 \cdot 3$	Down	-6
Adding 2 groups of 3 anchors	$2 \cdot (-3)$	Down	-6
Removing 2 groups of 3 anchors	$-2 \cdot (-3)$	Up	6

- The product of two integers with the same sign, negative • negative or positive • positive, is positive.
- The product of two integers with different signs, negative • positive or positive • negative, is negative.

Synthesis

Use the turtle scenario to explain why it makes sense that $(-5)(-2)$ is positive.

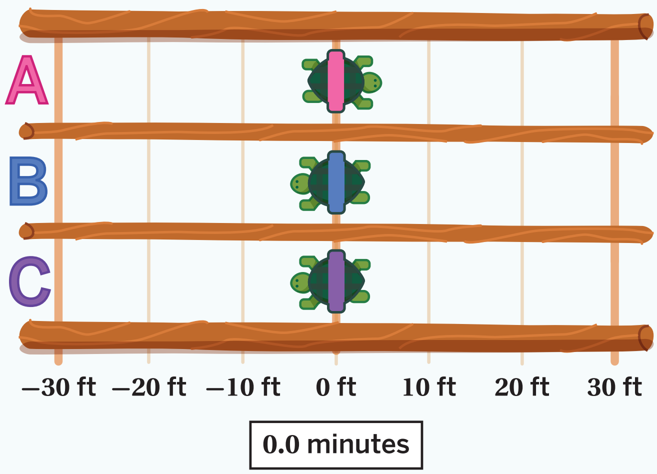


Summary

Multiplying rational numbers can help represent position, rate, and time. The position of an object is equal to the walking rate multiplied by the time or $rate \cdot time = position$.

Consider these three turtles. They are all together at 0 feet.

- Turtle A walks to the right at a rate of 6 feet per minute. 3 minutes ago, Turtle A was at -18 feet because $6 \cdot (-3) = -18$.
- Turtle B walks to the left at a rate of 5 feet per minute. 6 minutes ago, Turtle B was at 30 feet because $(-5)(-6) = 30$.
- Turtle C walks to the left at a rate of 3 feet per minute. In 4 minutes, Turtle C will be at -12 feet because $-3 \cdot 4 = -12$.



Synthesis

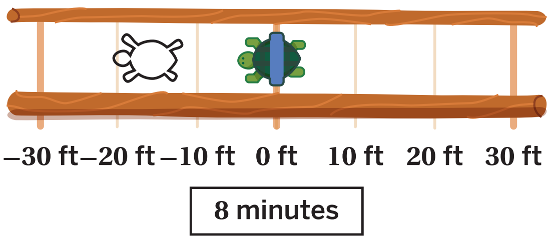
Explain why these expressions have the same value.

$-\frac{6}{3}$
$\frac{6}{-3}$

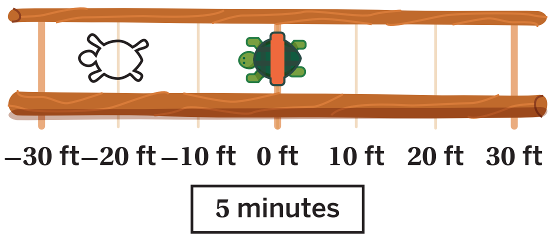
Summary

Dividing rational numbers can help represent position, rate, and time.

Time	Rate
The *time* that an object travels is equal to the position of the object divided by the rate at which it is traveling. $\frac{\text{position}}{\text{rate}} = \text{time}$	The *rate* at which an object travels is equal to the position of the object divided by the time. $\frac{\text{position}}{\text{time}} = \text{rate}$



Turtle A walks -2 feet per minute. It begins at 0 feet and will end at -16 feet. Turtle A will walk for 8 minutes because $\frac{-16}{-2} = 8$.



Turtle B walks -20 feet in 5 minutes. Turtle B's walking rate is -4 feet per minute because $\frac{-20}{5} = -4$.

Synthesis

Describe strategies you can use to decide if a statement is always, sometimes, or never true. Use the examples in the box if they help you with your explanation.

$\frac{-x}{x}$ is positive

$\frac{y}{2}$ is less than y

Summary

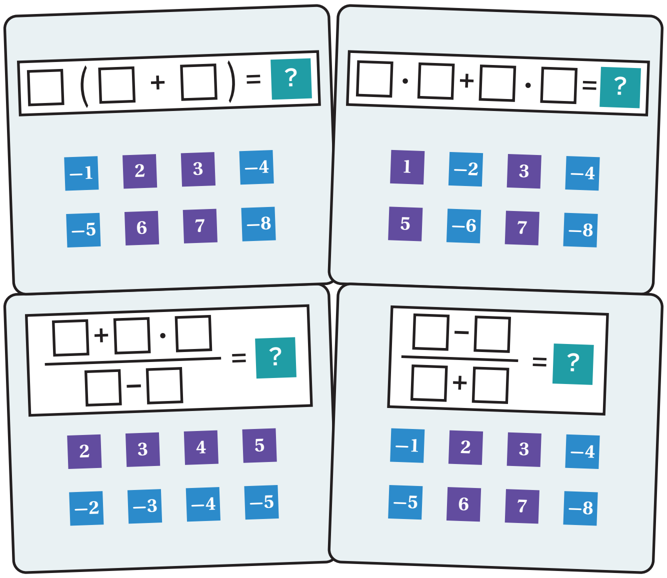
Reasoning about the value of variable expressions requires some flexible thinking. Using number lines and testing with example values are two strategies that can help to make sense of these types of expressions.

Be careful — some things that are familiar from using positive numbers are not the same when working with negative numbers!

For example, when x is a positive number, $10 - x$ will be less than 10. But if x is a negative number, $10 - x$ will be greater than 10.

Synthesis

Describe something you learned today that might help someone else as they complete puzzles like these.



Summary

Properties of operations, order of operations, and multiplication and division of integers can be used as strategies to solve integer expression puzzles. For example, making the following inequality true may involve making a decision about whether the sum inside the parentheses needs to be positive or negative depending on the sign of the value outside of the parentheses.

Make the inequality true. $\_\_\_\_ (\_\_\_\_ + \_\_\_\_) > 0$

Consider that:

positive • positive = positive → Makes the inequality true

negative • negative = positive → Makes the inequality true

positive • negative = negative → Makes the inequality false

negative • positive = negative → Makes the inequality false

To maximize the value of an expression involving fractions, you must use order of operations for the numerator, consider the smallest possible value for the denominator, and apply your knowledge of multiplication and division of integers.

Synthesis

What new questions do you have about the changing climate after exploring this lesson?

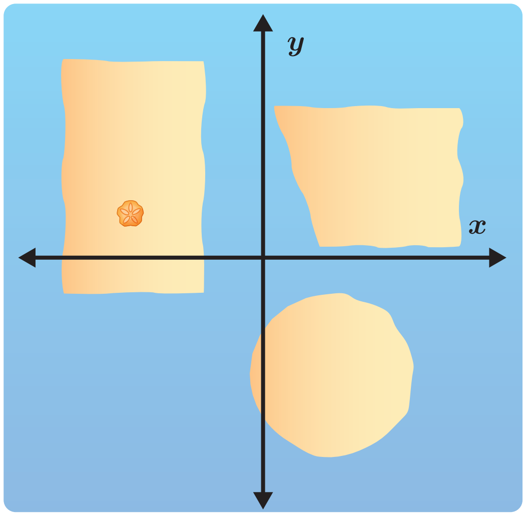
Summary

Adding and subtracting rational values helps to solve problems involving real-world situations. Consider these important steps.

1. Reason about which operation is most important to represent the real-world situation.
2. Predict whether the sums or differences will be positive or negative prior to completing any computations.
3. Apply strategies for addition and subtraction to simplify expressions involving both operations, noting that there are multiple paths to the same solution.

11 Synthesis

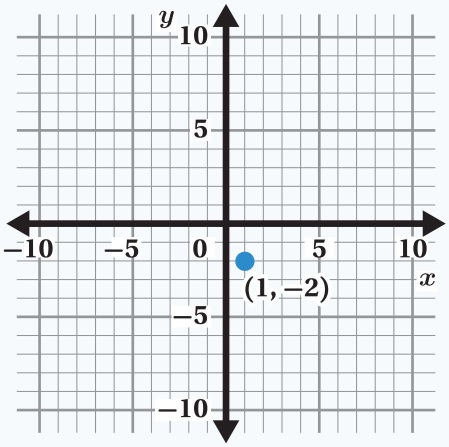
Explain what you know about the coordinates of this sand dollar.



Summary

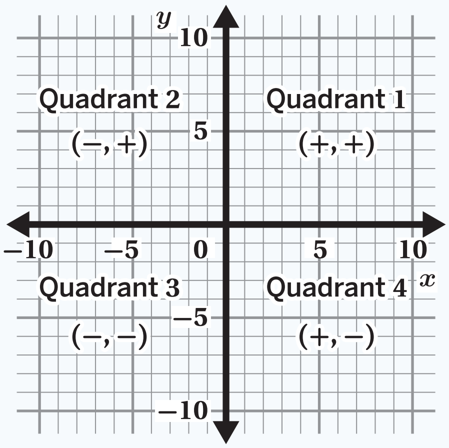
Just like a number line, the *coordinate plane* can include positive and negative numbers along the x- and y-axes. The x and y axes cross at the *origin*, or the point $(0, 0)$.

Coordinate pairs are written (x, y), where the x-value is the horizontal location (left and right) and the y-value is the vertical location (up and down). For example, the point $(1, -2)$ is 1 unit to the right and 2 units down from the origin.



The four regions of the *coordinate plane* are called **quadrants**. They are numbered 1–4 starting with the top right quadrant and going in a circle counter-clockwise.

The image shows each *quadrant*, along with the sign of the x- and y-values in that quadrant.



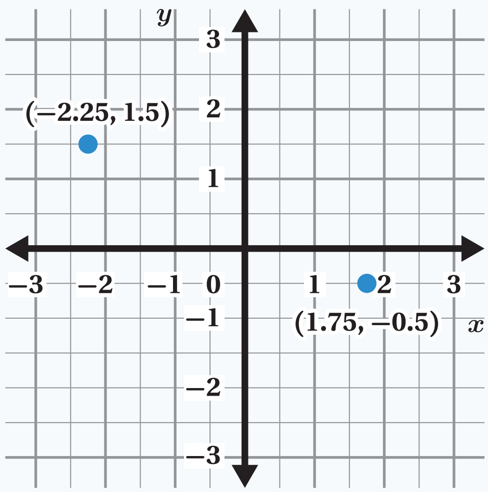
11 Synthesis

Look back over the challenges and select the one you are most proud of solving.

Write some advice for someone solving that challenge.

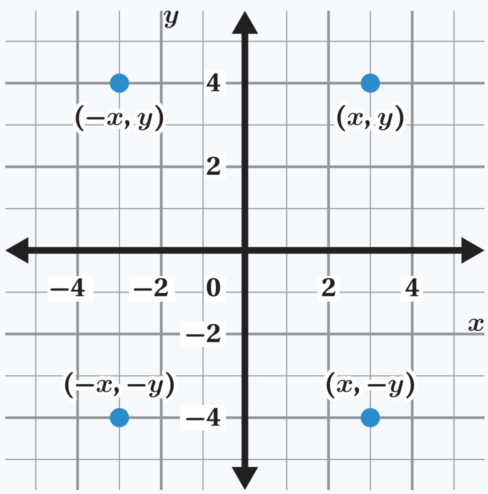
Summary

While the scale of 1 is often used, coordinate planes sometimes use different scales to show very big or very small numbers. This means that sometimes points are plotted in between tick marks. Consider the points $(1.75, -0.5)$ and $(-2.25, 1.5)$ and where they appear on the graph shown.



The points $(3, 4)$ and $(3, -4)$ have the same x-coordinate and the y-coordinates only differ by their sign. We can see on the graph that those points are a reflection, or a mirror, of each other across the x-axis.

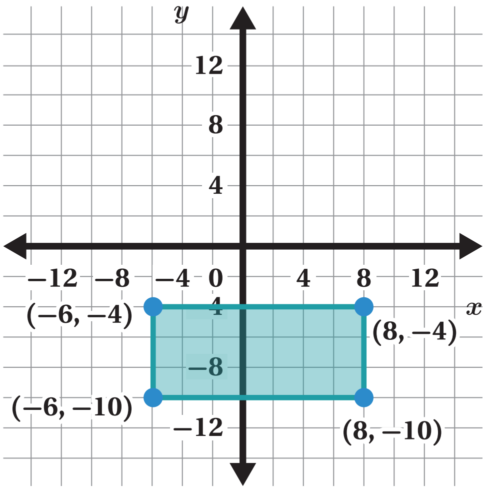
The points $(-3, -4)$ and $(3, -4)$ have the same y-coordinate and the x-coordinates only differ by their sign. We can see on the graph that those points are a reflection, or a mirror, of each other across the y-axis.



9 Synthesis

Describe how you can use the coordinates to calculate the side lengths of a rectangle. Use the table and graph if they help with your explanation.

Point	Coordinates
A	$(-6, -4)$
B	$(8, -4)$
C	$(8, -10)$
D	$(-6, -10)$



Summary

You can plot points on the coordinate plane to create polygons. When the vertices of a polygon are horizontally or vertically aligned in the graph you can count the number of units between them to determine the length of that side.

You can also calculate the side lengths using the coordinates of each vertex. Here are two calculation strategies:

- If the coordinates are in the same quadrant, like points A and B, find the length by subtracting the coordinates that are different.
- If the coordinates are in different quadrants, like points A and C, use the absolute value to determine the distance each point is from the axis between them.