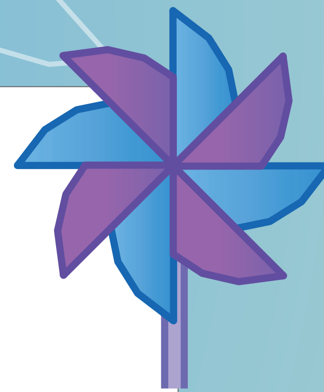
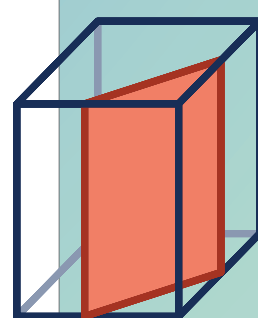
Unit 7



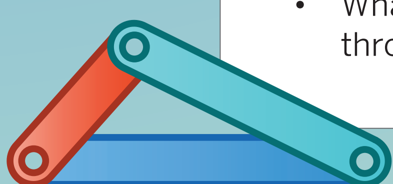
Angles, Triangles, and Prisms

There is always more than meets the eye. Angles and shapes can appear one way, but how can we know without a doubt? How do we know if what we perceive is also reality? Is there something special about a pair of angles? Are two triangles actually the same? Could I slice an object multiple ways? In this unit, you will explore special angle relationships, unique triangles, and cross sections of solids to answer questions like these.



Essential Questions

- What strategies are helpful for determining unknown angle measures?
- How many unique polygons are possible with different sets of side lengths and angle measurements?
- What different shapes can you make by slicing through a three-dimensional figure?



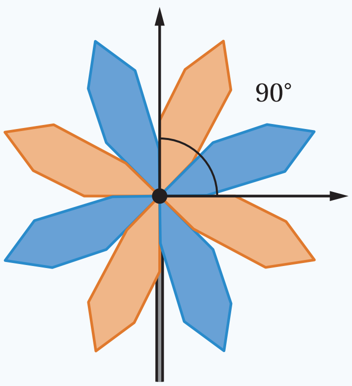
Summary | Lesson 1

One strategy for determining the measures of unknown angles is to compare them to angle measures you know.

Here are three familiar angles and how they can be used to determine the measure of an angle in a pinwheel made of identical shapes.

Right Angle (90°)

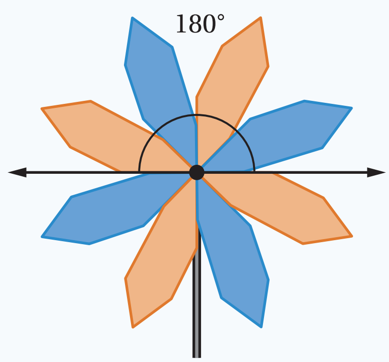
A *right angle* is half of a straight angle. It measures 90°.



$45^\circ + 45^\circ = 90^\circ$

Straight Angle (180°)

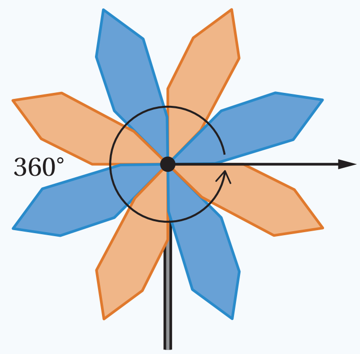
A *straight angle* forms a straight line. It measures 180°.



$\frac{180^\circ}{4} = 45^\circ$

Circle (360°)

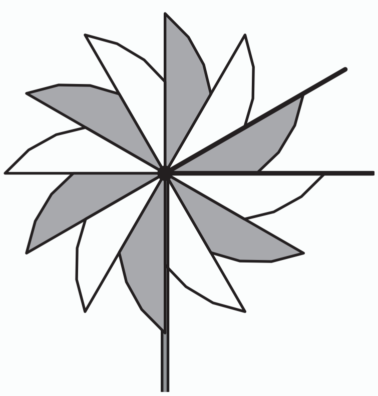
The measure around a circle is 360°.



$\frac{360^\circ}{8} = 45^\circ$

Try This

Here is Kadeem's pinwheel design.



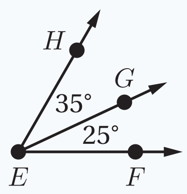
a What angle did he use?

b Explain your strategy for calculating Kadeem's angle.

Summary | Lesson 2

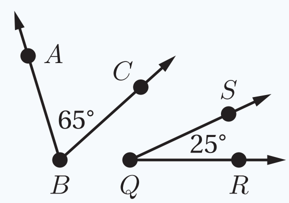
Here are three angle relationships that can help you determine missing angle measures.

Adjacent angles share a side and a vertex.



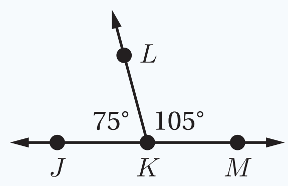
$\angle HEG$ and $\angle FEG$ are adjacent angles.

Complementary angles have measures that add up to $90°$.



$\angle ABC$ and $\angle RQS$ are complementary angles.

Supplementary angles have measures that add up to $180°$.



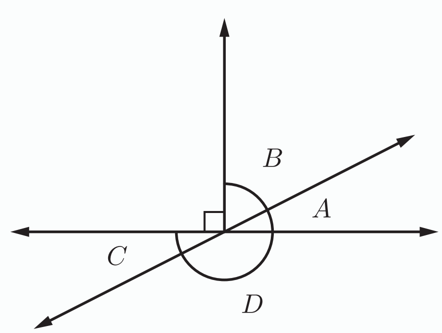
$\angle JKL$ and $\angle MKL$ are supplementary angles.

Try This

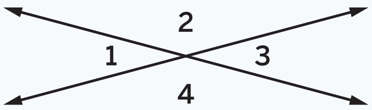
The measure of angle $A = 25°$.

A and B are complementary angles.

What is the measure of angle B?



Summary | Lesson 3



When two lines cross, the angles that are opposite each other have the same measure. These angles are called **vertical angles**.

$\angle 1$ and $\angle 3$ are a pair of vertical angles. Another pair is $\angle 2$ and $\angle 4$.

Using vertical angles can help you determine unknown angle measures.

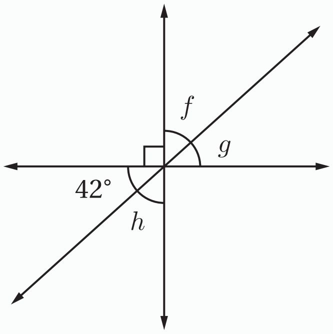
For example, if the measure of Angle 1 is $30°$, then:

- The measure of Angle 3 is $30°$ because $\angle 1$ and $\angle 3$ are vertical angles.
- The measure of Angle 2 is $150°$ because $\angle 1$ and $\angle 2$ are supplementary angles.
- The measure of Angle 4 is $150°$ because $\angle 2$ and $\angle 4$ are vertical angles.

Try This

Determine the values of f, g, and h.

Angle	Measure (degrees)
f	
g	
h	

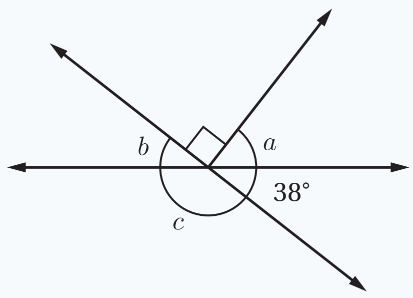


Summary | Lesson 4

Angle relationships can help you write equations to determine unknown angle measures.

Here are several equations that represent the angle relationships in this diagram.

Equation	Relationship
$a + 38 = 90$	Complementary angles
$c + 38 = 180$	Supplementary angles
$b + c = 180$	Supplementary angles
$b = 38$	Vertical angles
$a + b + c + 90 + 38 = 360$	Circle

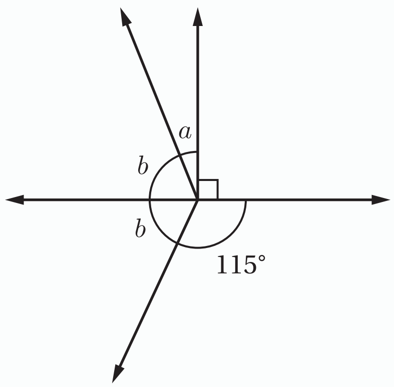


You can solve these equations to determine the values of a, b, and c.

In this diagram, $a = 52°$, $b = 38°$, and $c = 142°$.

Try This

Here is an angle diagram.



a Write *at least* one equation based on this diagram.

b Determine the values of a and b.

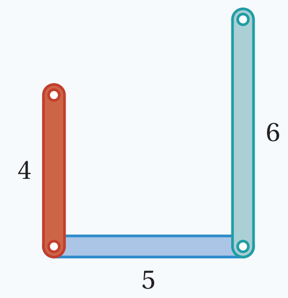
Angle	Measure (degrees)
a	
b	

Summary | Lesson 5

Three line segments do not always make a triangle.

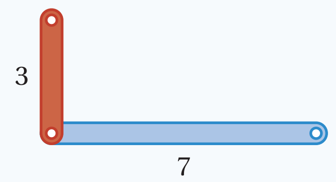
In order for three line segments to form a triangle, the sum of the two shorter segments' lengths must be greater than the third segment's length.

For example, $4 + 5 > 6$, so these three line segments would make a triangle.



You can use this relationship to determine what possible lengths would create a triangle.

For example, if two side lengths of a triangle are 3 units and 7 units, then the third side must be greater than 4 and less than 10 units.



Try This

Tasia wants to build two triangles.

a Will side lengths of 4, 8, and 10 units form a triangle?

Explain your thinking.

b Another triangle has two side lengths that are 7 and 13 units.

What is one possible length for the third side?

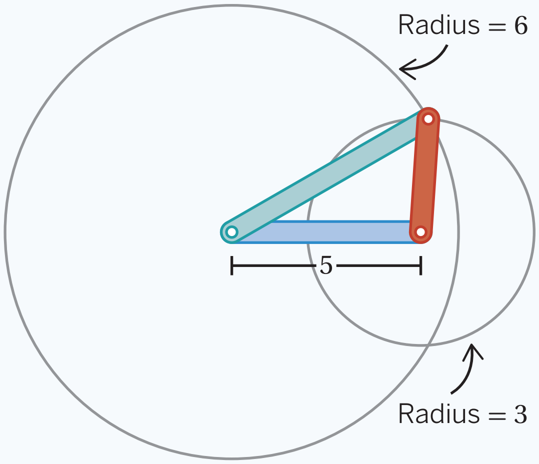
Explain your thinking.

Summary | Lesson 6

You can use circles to draw triangles.

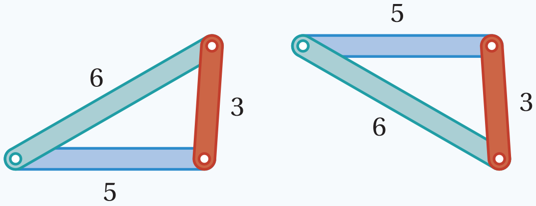
Here are the steps to draw a triangle with sides that are 5, 3, and 6 units long:

- Draw a line segment that is 5 units long.
- Draw a circle with a radius of 3 units centered at one end point.
- Draw a circle with a radius of 6 units centered at the other end point.
- Draw line segments connecting the end points of the 5 unit segment to a point where the two circles intersect.



All the triangles whose sides are 5, 3, and 6 units long will be **identical copies** because they have the same shape and size.

In fact, it is only possible to create one unique triangle if you know its three side lengths (unless you can't make a triangle at all).



Try This

How many non-identical triangles can you make using these lengths?

Show or explain your thinking.

a 4.5, 8, and 10 units

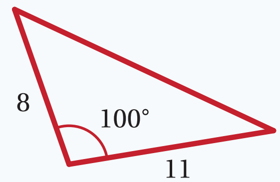
b 9, 11, and 21 units

Summary | Lesson 7

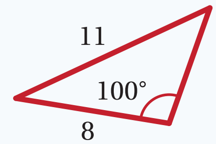
If three side lengths make a triangle, all triangles with those side lengths are identical copies. But what about a combination of side lengths and angles? Some combinations of measurements can make more than one unique triangle.

Here are three triangles: A, B, and C.

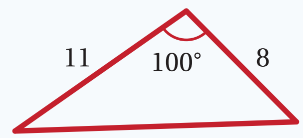
Triangle A



Triangle B



Triangle C



Each triangle has three of the same measurements: one side that is 8 units long, one side that is 11 units long, and one $100°$ angle.

Only triangles A and C are *identical copies* because the $100°$ angle is in between the side lengths of 8 and 11 units.

This means that there is more than one unique triangle that can be made with these measurements.

In general, knowing the order or placement of the sides and angles can help you determine whether two triangles are *identical copies* and how many unique triangles there are.

Try This

Mariam and Jamir are both drawing triangles that have a 5-centimeter side, a $60°$ angle, and a $45°$ angle.

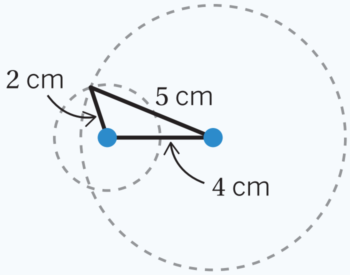
a Will Mariam's and Jamir's triangles be identical?

Show or explain your thinking.

b What information would Jamir need about Mariam's triangle so he can be sure he is creating an identical triangle?

Summary | Lesson 8

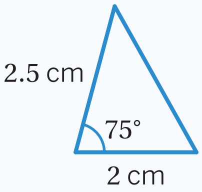
You can use a protractor, ruler, and compass to draw triangles with given measurements.



For example, when given three side lengths, using a ruler and a compass to draw circles with radiuses that match the given lengths can help you draw a triangle.

There is only one unique triangle that can be drawn when given three side lengths. When given two side lengths and an angle measure, multiple non-identical triangles can be drawn.

For example, here is one strategy for drawing a triangle that has a 75° angle, a side length of 2 centimeters, and a side length of 2.5 centimeters.

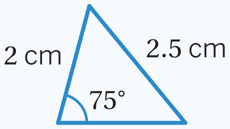


Step 1: Draw a 2-centimeter line segment.

Step 2: Draw a 75° angle using a protractor.

Step 3: Measure 2.5 centimeters along the other ray of the angle and connect the triangle.

Here is another non-identical triangle that has the same three measurements. The 75° angle can be positioned in different places in relation to the 2- and 2.5-centimeter sides.



Try This

Draw a triangle using each set of measurements. Explain your steps.

a One 2-centimeter side, one 2.5-centimeter side, and one 75° angle

Drawing **My Steps**

b One 4-centimeter side, one 40° angle, and one 90° angle

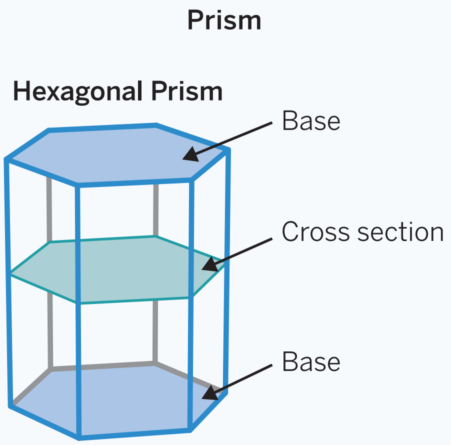
Drawing **My Steps**

Summary | Lesson 9

A **cross section** is the shape you see when you cut through a three-dimensional figure.

For example, if you cut a hexagonal *prism* parallel to the *base*, the cross section is a hexagon that is the same size as the base. If you make a vertical cut instead, the cross section is a rectangle that is as tall as the prism.

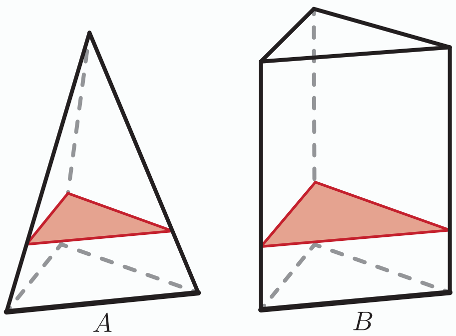
If you cut a hexagonal *pyramid* parallel to the base, the cross section is a hexagon that is smaller than the base. If you make a vertical cut instead, the cross section is a triangle that is taller than it is wide.



Try This

Vihaan has a triangular pyramid and a triangular prism.

He cut both the pyramid and the prism parallel to their bases.



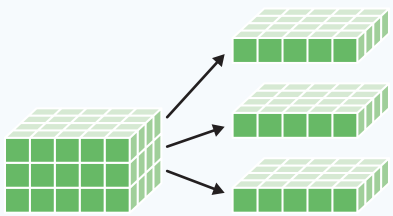
a How are the cross-sections similar?

b How are the cross-sections different?

Summary | Lesson 10

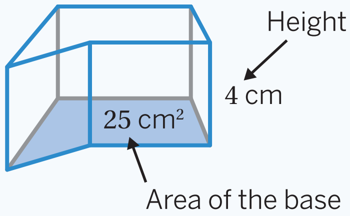
Any cross section of a prism that is parallel to the base will be identical to the base. This means you can slice a prism into layers to help you calculate its **volume**.

For example, if you have a rectangular prism that is 3 units tall and has a base that is 4 units by 5 units, you can imagine the prism as 3 layers of $4 \cdot 5$ cubic units.



That means the volume of this rectangular prism is $(4 \cdot 5) \cdot 3$ cubic units.

In general, you can calculate the volume of any prism by multiplying the area of its base by its height.

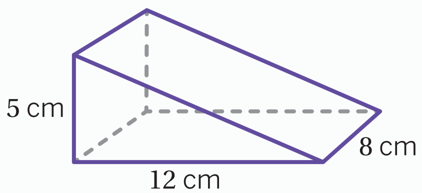


In other words, the volume of a prism is $V = B \cdot h$, where h is its height and B is the area of its base.

For example, this prism has a volume of 100 cubic centimeters because $25 \cdot 4 = 100$.

Try This

Here is a prism.

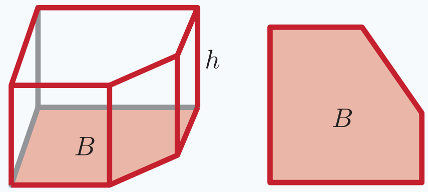


a Shade in a base of this prism.

b Calculate the volume of the prism.

Show your thinking.

Summary | Lesson 11



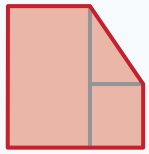
To calculate the volume of a prism, you can multiply the area of the base, B, by the height, h.

Sometimes the shape of a prism's base is a more complex *polygon*.

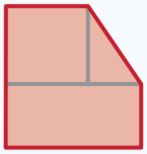
There are many strategies for calculating the area of a complex shape, including breaking it into rectangles and triangles, or surrounding the shape in a rectangle and subtracting the missing piece.

Here are three first steps you might take in calculating the area of this prism's base:

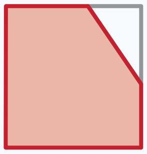
Example 1



Example 2



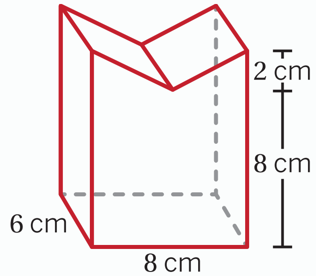
Example 3



Try This

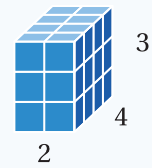
Calculate the volume of this prism.

Show your thinking.



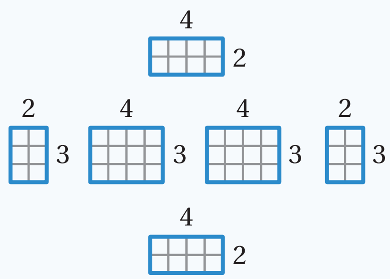
The **surface area** of a three-dimensional shape is the number of square units that cover all the faces of the polyhedron, without any gaps or overlaps.

Here are two strategies for calculating the surface area of a rectangular prism.



Strategy 1

Calculate the area of each face separately and then add all of the areas.

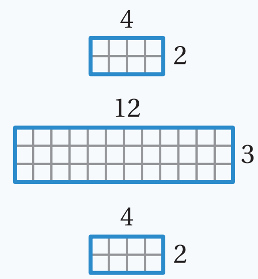


$8 + 6 + 12 + 12 + 6 + 8 = 52$

Surface area: 52 square units

Strategy 2

Break the prism into its two identical bases and unfold the sides into one long rectangle. Add the three areas.



$8 + 36 + 8 = 52$

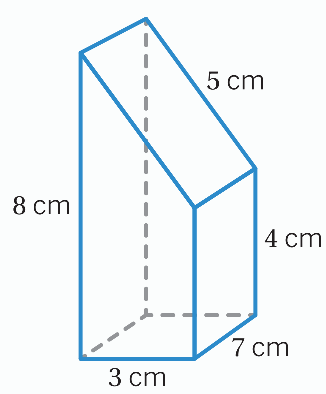
Surface area: 52 square units

Using either strategy to calculate the surface area or using equivalent calculations will result in the same total surface area.

Try This

Calculate the surface area of this prism.

Show your thinking.



Summary | Lesson 13

Knowing when to calculate volume and surface area can be helpful in answering questions about situations in context.

Questions related to volume:

- How much water can a container hold?
- How much material did it take to build a solid object?

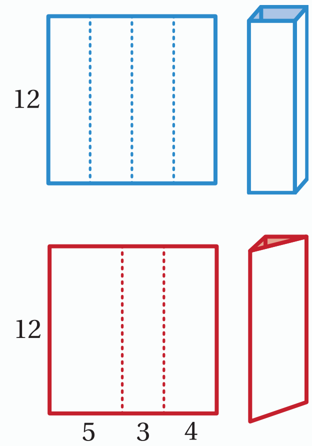
Questions related to surface area:

- How much fabric is needed to cover a surface?
- How much of an object needs to be painted?

One way to decide if a question is asking about volume or surface area is to think about the units of measure. Volume is measured in cubic units and surface area is measured in square units.

Try This

Zahra is folding origami paper to make pencil holders for her room. She folds two holders: a square prism and a right triangular prism.



a Which container holds more pencils?

Calculate the amount each container can hold to support your claim.

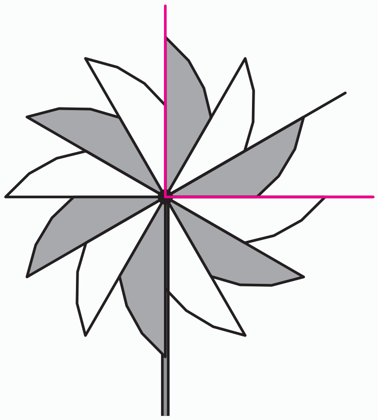
b Zahra added a bottom and a top to each container. Which container uses more paper?

Lesson 1

a $30°$

b *Explanations vary.*

- There are 12 pieces, and all the way around is $360°$, so one angle is $\frac{360}{12} = 30°$.
- There are 3 pieces in each right angle, and $30 \cdot 3 = 90°$.



Lesson 2

$65°$

Explanation: If A and B are complementary angles, they add up to $90°$. $90 - 25 = 65$

Lesson 3

Angle	Measure (degrees)
f	48
g	42
h	48

Lesson 4

a *Responses vary.* $a + 65 = 90$, $b + 115 = 180$, $a + b = 90$, $a + b + b + 90 + 115 = 360$ (or equivalent)

b

Angle	Measure (degrees)
a	25
b	65

Lesson 5

a Yes. *Explanations vary*. If the sum of the two shorter sides is longer than the third side, then it will make a triangle. $4 + 8 > 10$, so these three sides will form a triangle.

b *Responses and explanations vary*. A length of 8 units would create a triangle because if you connected it to the other side of 11 units, it would be long enough to connect to the 7-unit side as well. $7 + 8 > 11$

Explanation: Any length greater than 6 and less than 20 would create a triangle.

Lesson 6

a One triangle. *Explanations vary*. You can make exactly one triangle with these three side lengths because the sum of the two shortest sides is greater than the third side: $4.5 + 8 > 10$. Any other triangle you make using these three side lengths would be an identical copy because it's the same shape and size as the original and would fit right on top of the original triangle if flipped or turned.

b Zero triangles. *Explanations vary*. The sum of the two shortest sides is less than the longest side, so these sides will not form a triangle. $9 + 11 < 21$

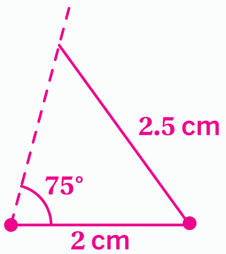
Lesson 7

a Maybe. *Explanations vary*. The triangles could be identical, but Mariam and Jamir could have put the side in different places, which would create different triangles.

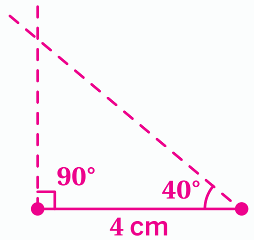
b Jamir would need to know if the 5-centimeter side is between the two angles, adjacent to only the $60°$ angle, or adjacent to only the $45°$ angle.

Lesson 8

a

Drawing	My Steps
 (or equivalent)	*Responses vary*. Step 1: Draw a 2-centimeter line. Step 2: Draw a $75°$ angle at one of the endpoints of the 2-centimeter line. Step 3: Draw a 2.5-centimeter line from the other endpoint of the 2-centimeter line.

b

Drawing	My Steps
 (or equivalent)	*Responses vary*. Step 1: Draw a 4-centimeter line. Step 2: Draw a $90°$ angle and a $40°$ angle. Step 3: Extend the angles until the lines intersect.

Lesson 9

a *Responses vary*.

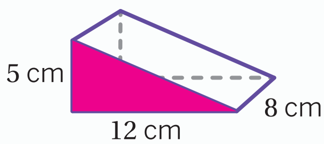
- They are both triangles.
- They are both the same shape as the base.
- They are both scaled copies of the base.

b *Responses vary*.

- The cross-section of the prism is the same size as its base.
- The cross-section of the pyramid is smaller than its base.
- The cross-section of the pyramid looks like it's smaller than the cross-section of the prism.

Lesson 10

a *Responses vary*.

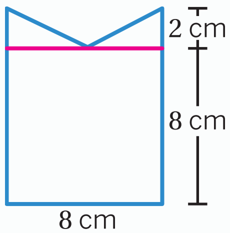


b 240 cubic centimeters

Explanation: One strategy is to multiply the area of the base by the height of the prism. $V = \frac{1}{2}(5 \cdot 12) \cdot 8 = 240$

Lesson 11

432 cubic centimeters. *Work varies*. Find the area of the base and multiply it by the height of the prism. Since the base is a complex shape, it can be broken into two identical triangles and a square. To find the area of the base, add the area of the square, $8 \cdot 8 = 64$, to the areas of the 2 triangles, $\frac{1}{2}(4 \cdot 2) + \frac{1}{2}(4 \cdot 2) = 8$. The area of the base is $64 + 8 = 72$ square centimeters and $V = 72 \cdot 6 = 432$ cubic centimeters.



Lesson 12

176 square centimeters. *Work varies*. Calculate the area of each face and add them together. There are 6 faces of this prism. $SA = 18 + 18 + 28 + 35 + 56 + 21 = 176$

Lesson 13

a. The square prism. *Explanations vary*. The base of the square prism pencil holder is 3-by-3 units, so the area of its base is 9 square units. The area of the base of the right triangular prism pencil holder is $\frac{1}{2} \cdot 4 \cdot 3 = 6$ square units. Since both pencil holders are the same height, we can compare the areas of the bases to determine that the square prism pencil holder has a larger volume and can hold more pencils.

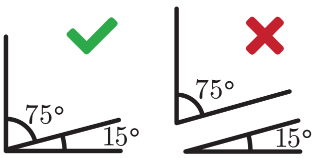
b. The square prism. *Explanations vary*. The amount of paper used for the side faces of each prism are the same since they are different ways of folding the same-sized piece of paper. This means that the only difference is the areas of the bases. We already know the area of the base of the square prism is larger than the area of the base of the right triangular prism, so it must also use more paper.

Grade 7 Unit 7 Glossary/7.º grado Unidad 7 Glosario

English | Español

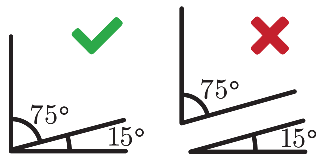
A

adjacent angles Angles that share a side and a vertex.



The image with the check mark shows a 75° angle and a 15° angle when they are adjacent.

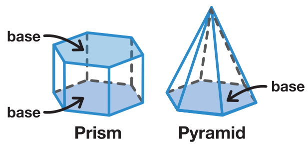
ángulos adyacentes Ángulos que comparten un lado y un vértice.



La imagen con la marca de verificación muestra un ángulo de 75° y un ángulo de 15° cuando son adyacentes.

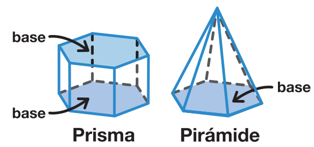
B

base (of a pyramid or prism) The face of a pyramid or prism that gives the solid its name. A prism has two identical bases that are parallel. A pyramid has one base.



For example, a hexagonal prism has 2 bases with six sides and a hexagonal pyramid has 1 base with six sides.

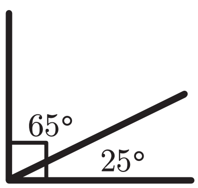
base (de una pirámide o un prisma) La cara de una pirámide o un prisma que da el nombre al cuerpo geométrico. Un prisma tiene dos bases idénticas que son paralelas. Una pirámide tiene una base.



Por ejemplo, un prisma hexagonal tiene 2 bases con seis lados y una pirámide hexagonal tiene 1 base con seis lados.

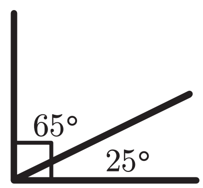
C

complementary angles Two angles whose measures add up to 90°.



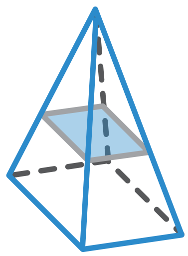
For example, a 65° angle and a 25° angle are complementary.

ángulos complementarios Dos ángulos cuyas medidas suman 90°.



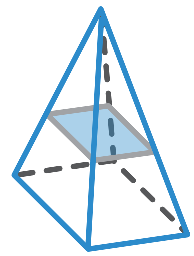
Por ejemplo, un ángulo de 65° y otro ángulo de 25° son complementarios.

cross section The shape you see when you slice through a 3-D figure.



For example, if you slice a rectangular pyramid parallel to the base, you get a smaller rectangle as the cross section.

sección transversal La figura que se ve al cortar una figura tridimensional.



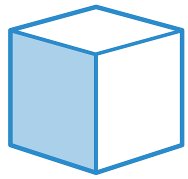
Por ejemplo, si se corta una pirámide rectangular de forma paralela a la base, se obtiene un rectángulo más pequeño como sección transversal.

English	Español

F

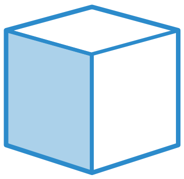
face Each flat side of a polyhedron.

A cube has six faces and they are all squares.



cara Cada lado plano de un poliedro.

Un cubo tiene seis caras y todas son cuadrados.



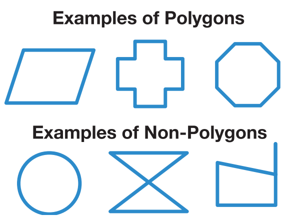
I

identical copy A copy of a figure that has the same shape and size as the original.

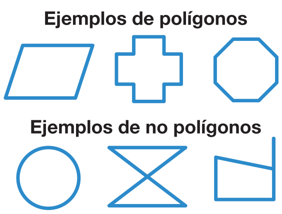
copia idéntica Una copia de una figura que tiene la misma forma y el mismo tamaño que la figura original.

P

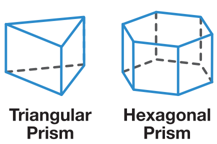
polygon A closed 2-D shape with straight sides that do not cross each other.



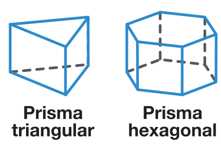
polígono Una figura bidimensionalcerrada con lados rectos que no se cruzan.



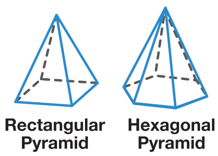
prism A solid with two bases that are identical copies. The base of a prism gives the solid its name.



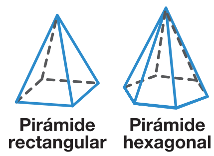
prisma Un cuerpo geométrico con dos bases que son copias idénticas. La base de un prisma da nombre al cuerpo geométrico.



pyramid A solid in which the base is a polygon and all of the other faces are triangles that meet at a single vertex. The base of a pyramid gives the solid its name.

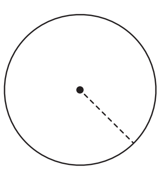


pirámide Un cuerpo geométrico en el que la base es un polígono y todas las demás caras son triángulos que confluyen en un único vértice. La base de una pirámide da nombre al cuerpo geométrico.

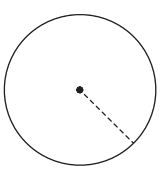


R

radius A line segment that connects the center of a circle with a point on the circle. Every radius of a circle is the same length.



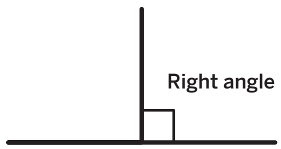
radio Un segmento de recta que conecta el centro de un círculo con un punto del círculo. Todos los radios de un círculo tienen la misma longitud.



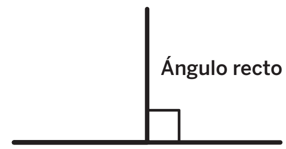
Grade 7 Unit 7 Glossary/7.º grado Unidad 7 Glosario

English / **Español**

right angle An angle whose measure is $90°$.

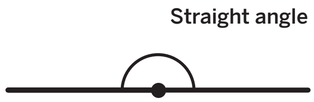


ángulo recto Un ángulo cuya medida es $90°$.

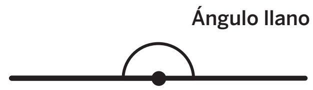


S

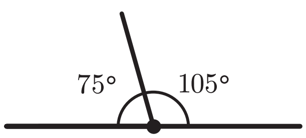
straight angle An angle that forms a straight line. A straight angle measures $180°$.



ángulo llano Un ángulo que forma una línea recta. Un ángulo llano mide $180°$.

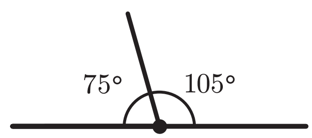


supplementary angles Two angles whose measures add up to $180°$.



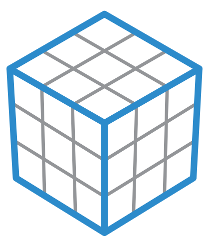
For example, a $75°$ angle and a $105°$ angle are supplementary.

ángulos suplementarios Dos ángulos cuyas medidas suman $180°$.



Por ejemplo, un ángulo de $75°$ y otro ángulo de $105°$ son suplementarios.

surface area The number of square units needed to cover all the faces of a polyhedron, without any gaps or overlaps.



For example, the six faces of this cube each have an area of 9 square centimeters, so the surface area of the cube is $6 \cdot 9$, or 54 square centimeters.

área de superficie La cantidad de unidades cuadradas que se necesitan para cubrir todas las caras de un poliedro, sin vacíos ni superposiciones.

Por ejemplo, cada una de las seis caras de este cubo tiene un área de 9 centímetros cuadrados, por lo tanto, el área de superficie del cubo mide $6 \cdot 9$ o 54 centímetros cuadrados.

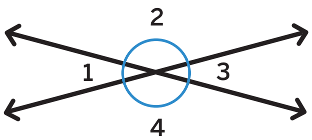
V

vertex A corner or a point where two or more lines or rays meet. An angle is named based on its vertex.

vértice Una esquina o un punto donde se encuentran dos o más rectas o semirrectas. Un ángulo recibe su nombre a partir de su vértice.

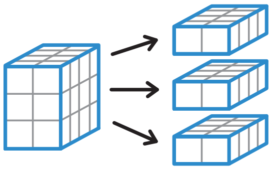
English

vertical angles Angles that are opposite each other when two lines cross. Vertical angles have the same measure.



For example, Angles 1 and 3 are a pair of vertical angles. Another pair of vertical angles is 2 and 4.

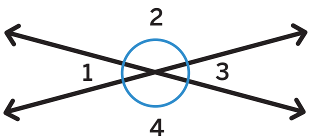
volume The number of cubic units that fill a 3-D region without any gaps or overlaps.



For example, the volume of this rectangular prism is 24 cubic units because it is made of 3 layers that are each 8 cubic units.

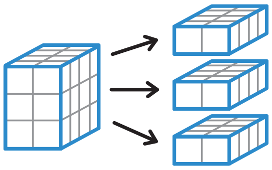
Español

ángulos verticales Ángulos que son opuestos cuando se cruzan dos rectas. Los ángulos verticales tienen la misma medida.



Por ejemplo, los ángulos 1 y 3 son un par de ángulos verticales. Otro par de ángulos verticales son 2 y 4.

volumen La cantidad de unidades cúbicas que llenan un espacio tridimensional, sin vacíos ni superposiciones.



Por ejemplo, el volumen de este prisma rectangular mide 24 unidades cúbicas porque se compone de 3 capas de 8 unidades cúbicas cada una.