Proportional Relationships

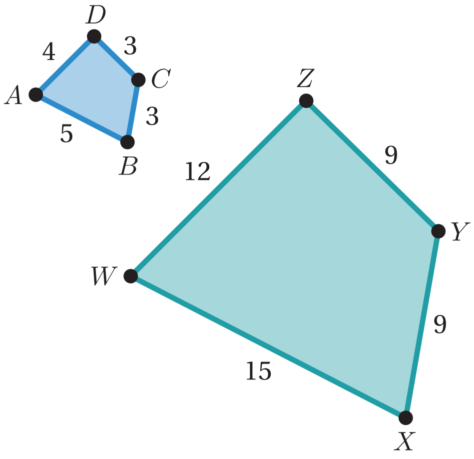
Accelerated 6

Unit 5

Synthesis

Equivalent ratios can help make matching paint colors.

Where do you see equivalent ratios in scale drawings?



Summary

Two ratios are equivalent if you can multiply each of the numbers in the first ratio by the same factor to obtain the numbers in the second ratio.

A table of ratios can be used to determine a number by which the values for one quantity are each multiplied to obtain the values for the other quantity. These ratios can then be said to be **equivalent ratios**. Notice that 1 white cup of paint mixed with 4 green cups of paint creates the same color as 2 white cups of paint mixed with 8 green cups of paint because the ratios are equivalent.

	White	Green	
	1	4	
×2	2	8	×2

Synthesis

1. Select *all* the relationships that you think are proportional.

A. A person's height in feet and that person's height in inches.

B. The number of cookies baked and the minutes in the oven.

C. The grams of flour needed to make different amounts of bread.

D. A person's time and total distance as they run a marathon.

E. The gallons of gasoline purchased and the total cost.

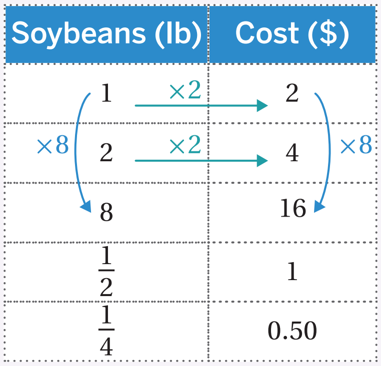
2. What determines whether a relationship is proportional?

Summary

In a **proportional relationship**, the values for one quantity are each multiplied by the same number to obtain the values for the other quantity.

In a table, this multiplication can happen across columns or between rows.

This table shows the cost for varying amounts of soybeans.



Soybeans (lb)	Cost ($)
1	2
2	4
8	16
$\frac{1}{2}$	1
$\frac{1}{4}$	0.50

Synthesis

Here are the instructions for cooking instant rice in the microwave.

Ingredients

Rice (cups)	1	2	3
Water (cups)	$1\frac{1}{2}$	3	$4\frac{1}{2}$

Cook Time

Rice (cups)	1	2	3
Time (minutes)	7	11	15

Use the tables shown if they help with your thinking.

1. How could you use a table to determine if a relationship is proportional?

2. If a relationship is proportional, how could you use a table to determine the constant of proportionality?

Summary

In a proportional relationship, the values for one quantity are multiplied by the same number to obtain the values for the other quantity. This number is called the **constant of proportionality**.

Feet	1	2	6
Inches	12	24	72

For example, feet and inches are in a proportional relationship.

From a table, a constant of proportionality (12) can be determined by dividing the number of inches by the number of feet in that column.

- The number of feet can be multiplied by 12 to determine the number of inches.
- The number of inches can be divided by 12 to determine the number of feet.

Synthesis

Examine the information about the three robot parts you worked on today.

Hat

Robot width (in.), w	Hat height (in.), h
3	6
1	2

The constant of proportionality is 2.

An equation for this relationship is $h = 2w$.

Shoes

Robot width (in.), w	Shoe height (in.), s
3	1
1	$\frac{1}{3}$

The constant of proportionality is $\frac{1}{3}$.

An equation for this relationship is $s = \frac{1}{3}w$.

Arms

Robot height (in.), r	Arm height (in.), a
9	5
1	$\frac{5}{9}$

The constant of proportionality is $\frac{5}{9}$.

An equation for this relationship is $a = \frac{5}{9}r$.

Describe a strategy for writing an equation from a table of a proportional relationship. Use the tables if they help you with your thinking.

Summary

Proportional relationships can be represented using the equation $y = kx$, where k is the constant of proportionality. For example, the table shows the proportional relationship between the cost and the number of pounds of soybeans at a certain store.

The cost of the soybeans is proportional to the weight, in pounds, with a constant of proportionality of 2.

If c represents the cost and w represents the weight, in pounds, of soybeans, then the proportional relationship can be represented with the equation $c = 2w$.

Weight (lb), w	Cost ($), c
$\frac{1}{2}$	1.00
1	2.00
2	4.00
w	$2w$

Synthesis

Some of the proportional relationships that you examined are represented below. Use these examples if they help you with your thinking.

Situation	Constants of proportionality	Equations
There are 100 centimeters, y, in every meter, x.	$100, \frac{1}{100}$	$y = 100x$, $x = \frac{1}{100}y$
It took Jayden 5 minutes, t, to fill a cooler with 8 gallons of water, w, at a steady rate.	$\frac{5}{8}, \frac{8}{5}$	$t = \frac{5}{8}w$, $w = \frac{8}{5}t$

1. In a situation, what is the relationship between the two constants of proportionality?

2. In a situation, what is the relationship between the two equations?

Summary

When two quantities x and y are in a proportional relationship, the equation $y = kx$ can be written to represent the proportional relationship. The number k is the *constant of proportionality*.

The equation $x = \frac{1}{k}y$ can also be written to represent a proportional relationship between x and y. In this equation, the number $\frac{1}{k}$ is the *constant of proportionality*. Both equations, $y = kx$ and $x = \frac{1}{\mathrm{k}}y$ represent a proportional relationship between x and y in two different ways.

If one pound of soybeans costs $\$2.00$, then . . .

- The cost, c, is proportional to the weight, w. The constant of proportionality is 2 showing that one pound of soybeans costs $\$2.00$, so the equation $c = 2w$ represents this situation.
- The weight, w, is proportional to the cost, c. The constant of proportionality is $\frac{1}{2}$ showing $\$1.00$ is the cost of a $\frac{1}{2}$ pound of soybeans, so the equation $w = \frac{1}{2}c$ represents this situation.

Notice that $\frac{1}{2}$ is the reciprocal of 2. **Reciprocals** are two numbers whose product is 1.

Synthesis

1. Write two equations: one that represents a proportional relationship and one that does not.

Proportional relationship	
Not a proportional relationship	

2. Describe how you know whether an equation represents a proportional relationship.

Summary

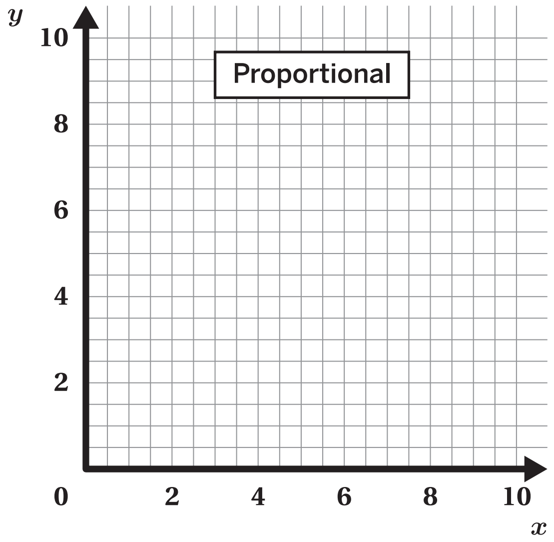
When two quantities x and y are in a relationship, the structure of an equation can indicate whether the relationship is proportional.

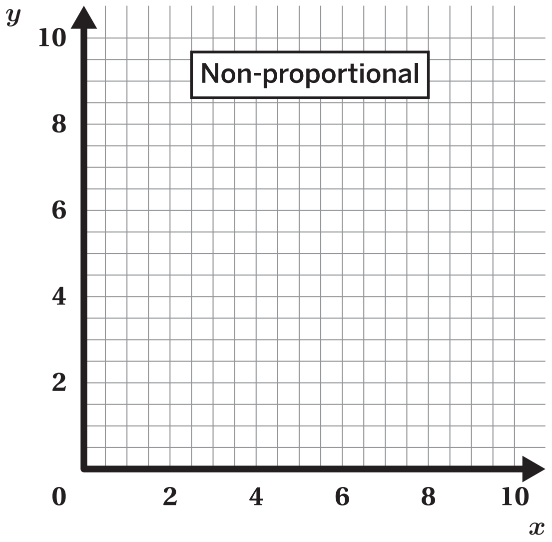
Sometimes, the same relationship can be written as different equations. For example, $y = \frac{x}{3}$ and $y = \frac{1}{3}x$ both represent the same relationship. Rewriting an equation in another form can help make it more recognizable as a proportional relationship.

Equations such as $y = 3x + 1$ and $y = x^2$ are not proportional relationships. If it is unclear whether an equation can be rewritten in the form of $y = kx$, setting up a table of values for the equation can be useful.

Synthesis

Make one graph that represents a proportional relationship and another that does not.





Discuss How can you determine whether a relationship is proportional using a graph?

Summary

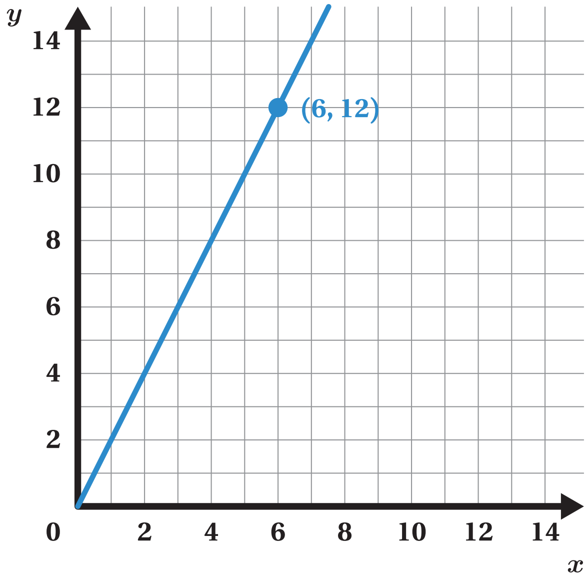
On a **coordinate plane**, graphs of proportional relationships are lines which pass through the **origin**, $(0, 0)$.

For some graphs where it is unclear if the points form a line, you can test if the ratios of the coordinates are equivalent.

Synthesis

What are two different ways you can find a constant of proportionality using a graph?

Use the graph if it helps you with your thinking.



Summary

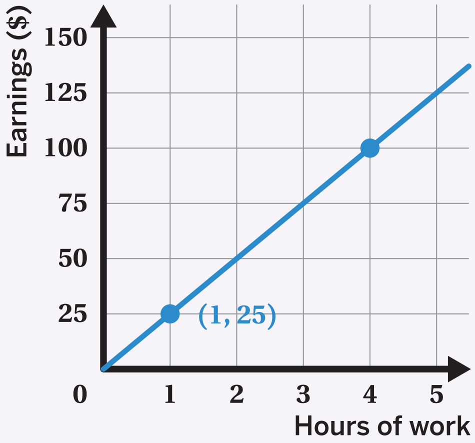
Each point on a graph of a proportional relationship tells a story using the quantities represented by x and y. The constant of proportionality is found on the graph of a proportional relationship by:

- Finding the value of y when x is equal to 1.
- Finding the ratio of $\frac{y}{x}$ for a given ordered pair.

Note that the relationship must be proportional for the constant of proportionality to exist and to be determined using these strategies.

For example, this graph shows a proportional relationship between x, hours worked, and y, money earned in dollars. The constant of proportionality is 25 because $25 is earned for each hour of work.

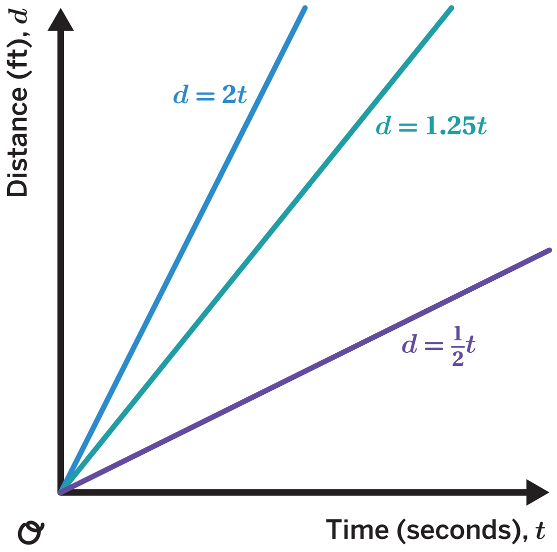
The ordered pair $(4, 100)$ shows that $100 is earned for 4 hours of work which is equivalent to earning $25 per hour.



Synthesis

Discuss **This graph shows the distance traveled over time by three different turtles.**

Discuss the following questions. Then select one question and record your response.



1. How can you tell from the graphs which turtle moved the fastest?

2. How can you tell from the equations which turtle moved the fastest?

Summary

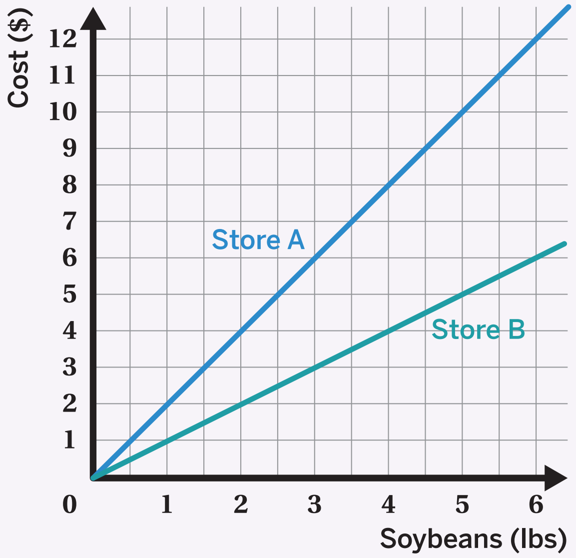
Graphs of proportional relationships can be compared when on the same coordinate plane. The steeper the line, the greater the constant of proportionality.

For example, this graph shows the cost of soybeans at two different stores.

On the graph, the line representing Store A is steeper than the line representing Store B, so it has a greater constant of proportionality.

In context, this means Store A charges more per pound than Store B because its line is steeper.

Store A charges \$2 per pound ($k = 2$), while Store B charges \$1 per pound ($k = 1$). The constant of proportionality of Store A is greater than that of Store B.



Synthesis

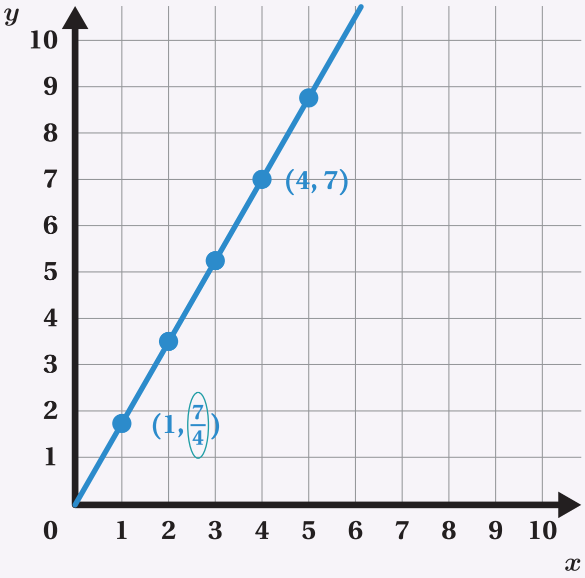
For each of the representations, describe how you can tell if a relationship is proportional.

Description	Table of values
Graph	Equation

Summary

The constant of proportionality, k, can be found using various representations.

Graph



Equation

$$y = \boxed{\frac{7}{4}}x$$

Table

x	y
0	0
1	$\boxed{\frac{7}{4}}$
2	$\frac{7}{2}$
3	$\frac{21}{4}$
4	7

The value of k can be found by looking for the corresponding y-value paired with an x-value of 1. If an ordered pair (x, y) is known, then $k = \frac{y}{x}$. An equation of a proportional relationship is of the form $y = kx$, and the coefficient of x is the constant of proportionality.

Synthesis

1. Which representation did you find the most useful for answering the question?

2. What assumptions did you make to help you answer the question?

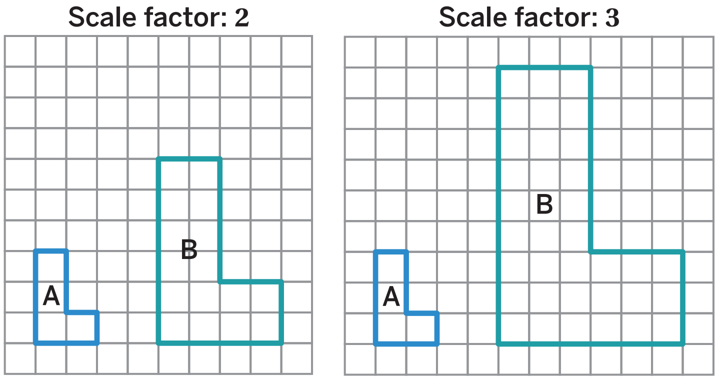
Summary

Understanding proportional relationships is useful when modeling the amount of water used while taking a shower or taking a bath. Choosing how to represent your model is also important because it affects how the information will be interpreted by others.

This is important work! By creating this model, you were able to turn a problem without a clear answer into information that someone else can use.

Synthesis

Explain why the relationship between the side length of a scaled copy and its perimeter is proportional. Use Figures A and B as examples if they help to support your thinking.



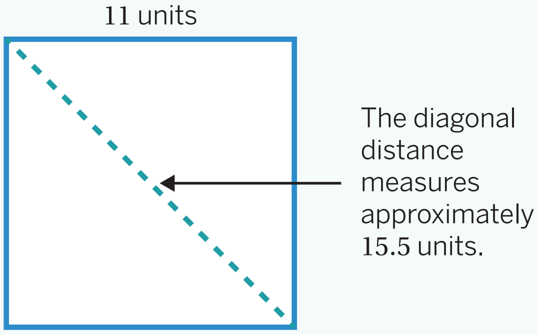
Summary

Plotting points on a coordinate plane or calculating the quotients in a relationship in a table can help determine whether two quantities are proportional. Some examples of proportional relationships include:

- The side lengths of a square and its perimeter.
- The diagonal length of a square and its perimeter.
- The side lengths of a scaled copy and its perimeter.

Because the relationship between the diagonal length and the perimeter of a square is proportional, the diagonal length or perimeter can be determined if one of the measurements is known.

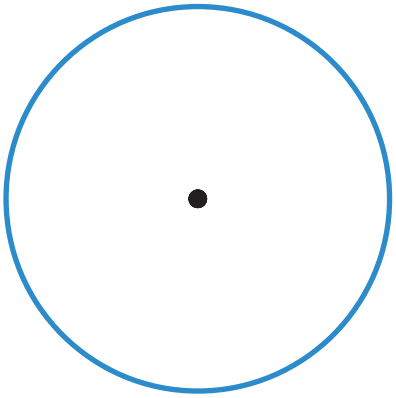
For example, consider a square whose perimeter is 44 units. Because the diagonal length of a square can be approximated by dividing the perimeter by the constant of proportionality, approximately 2.83, the length of the diagonal of a square with a perimeter of 44 units would measure $44 \div 2.83 \approx 15.5$ units.



Synthesis

Describe the relationship between the length of the radius, length of the diameter, and circumference of a circle.

Draw a sketch if it helps you to show your thinking.

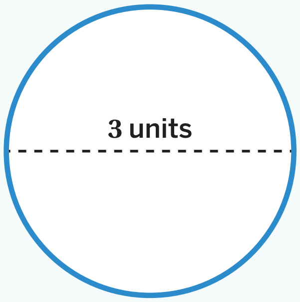


Summary

The distance around the circle is called the **circumference**. In the same way there is a proportional relationship between the length of a diagonal of a square and its perimeter, there is also a proportional relationship between the length of a diameter of a circle and its circumference.

For any diameter length, d, the circumference, C, can be calculated using the equation $C = \pi d$ where **pi** is the constant of proportionality which is often approximated as 3.14 or $\frac{22}{7}$.

For example, if the diameter of a circle is 3 units long, the circumference of the circle can be calculated approximately as $3 \cdot 3.14 = 9.42$. More accurately, $C = 3\pi$ units.

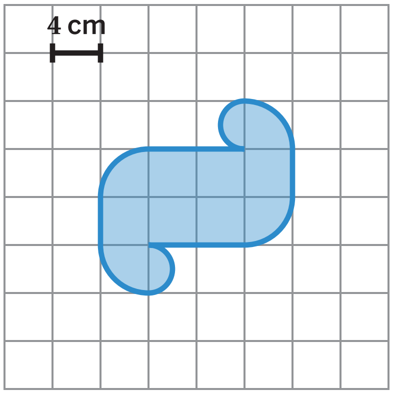


For any radius length r, the circumference C, can be calculated using the equation $C = 2\pi r$, where 2π, is the constant of proportionality.

Synthesis

Describe a strategy for determining the total perimeter of a shape that is made up of squares and parts of circles.

Use this example if it helps you show your thinking.



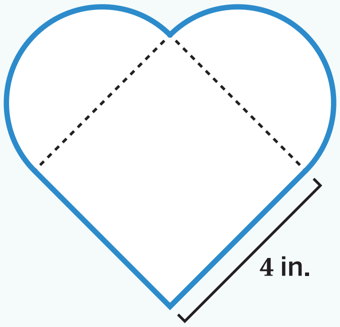
Summary

The symbol π is used to give an exact value for some circle measurements, such as circumference. Even using the π button on the calculator will technically still yield an approximation of the non-ending decimal value, so it is not as accurate as leaving a response in terms of π.

The circumference of a circle is found using the formula, $C = \pi d$. Because $d = 2r$, the formula $C = 2\pi r$ can also be used. These formulas also help to analyze and determine the perimeter of shapes involving circular pieces.

For example:

This heart shape consists of two semicircles and a square. Since the two semicircles have the same diameter, the sum of their circumferences is equivalent to the circumference of a full circle with the same diameter. To determine the exact perimeter of the figure, add the circumference of the circle to the two exterior sides of the square.



$P = \pi d + 2s$

$P = \pi \cdot 4 + 2 \cdot 4$

$P = 4\pi + 8$

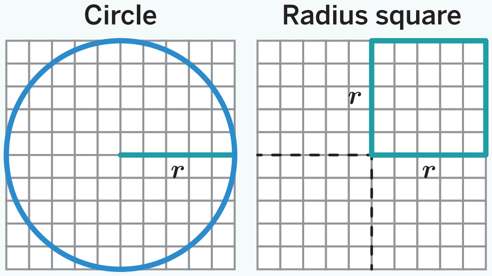
The exact perimeter would be $4\pi + 8$ inches.

Synthesis

1. Precious says you can estimate the area of a circle by calculating $3 \cdot r^2$. What do you think each part of her expression means?

2. Do you agree with Precious? Use your earlier work to help support your thinking.

Summary



The area of a circle can be measured using unit squares.

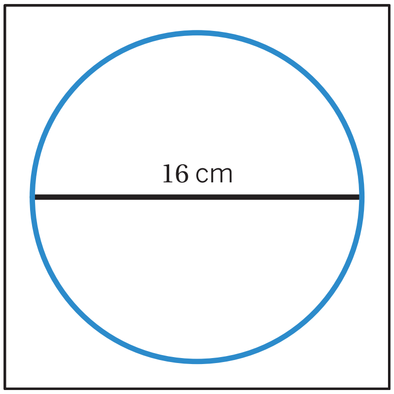
Decomposing radius squares and rearranging them helps to figure out an estimate for the area of any circle.

- The length of the radius of a circle, r, is the distance from the center of the circle to any point on the circle.
- Radius squares have side lengths that are the same length as the radius of the circle, r.
- The area of a radius square is equal to its side length squared, r^2.
- To estimate the area of a circle, determine how many radius squares fit inside the circle and multiply this by the area of one radius square.

It takes a little more than 3 radius squares to cover a circle, so we can estimate the area of a circle using the formula $3 \cdot r^2$.

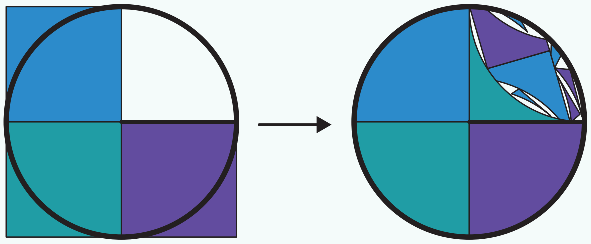
Synthesis

Describe a strategy to calculate the area of a circle if you know its diameter. Use the circle to help you with your explanation.



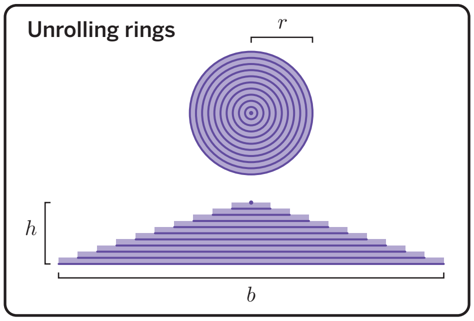
Summary

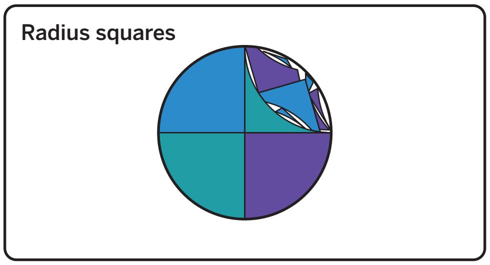
The area of a circle with a radius of length r is equal to the area of a little more than 3 radius squares, where each radius square has an area of r^2. To be exact, the area of a circle is equal to the area of π radius squares and can be expressed by the formula $A = \pi \cdot r^2$.



Synthesis

Choose one of these representations.





How does the representation you chose show that the area of a circle is $A = \pi r^2$?

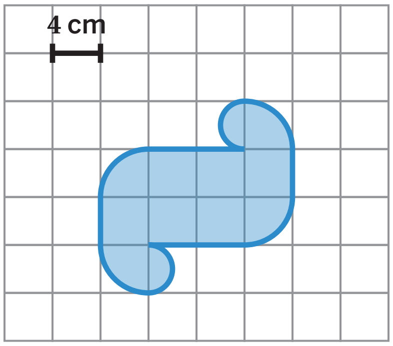
Summary

The relationship between the radius measurement and the area of a circle is not proportional.

By decomposing a circle and rearranging it into a triangle, the formula for the area of a circle can be related to the formula for the area of the triangle. This helps to make sense of each part of the formula for the area of a circle.

Synthesis

Describe a strategy for determining the area of a shape like the one shown here.



Summary

Determining the area of figures composed of circles and parts of circles allows for multiple ways of thinking and invites different strategies.

Solving complex problems like these, where the path to the solution is not obvious, is important to your growth as a mathematical thinker.