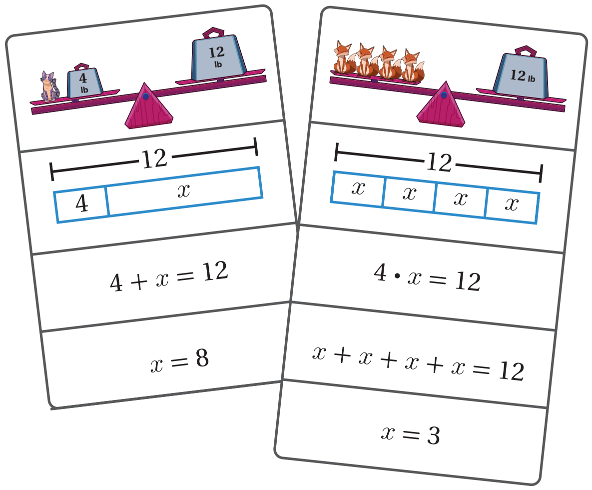
Expressions and Equations

Accelerated 6
Unit 4

10 Synthesis

How can you tell if an equation and a tape diagram match?

I can tell if an equation and a tape diagram match . . .

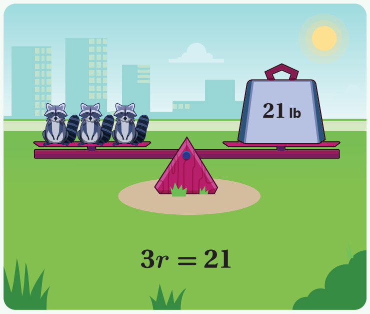


Summary

You can use seesaws and tape diagrams to represent *equations* and help determine unknown values.

We often use a letter, such as x or a, as a placeholder for an unknown number in tape diagrams and equations. This letter is called a **variable**.

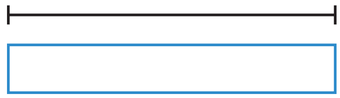
For example, if 3 equal-weight racoons weigh a total of 21 pounds, you can represent the weight of each raccoon with r and write the equation $3r = 21$.



Synthesis

8. Kwasi rides the subway 20 stops to get to work. After x stops, he has 5 stops left.

How can you tell which equation represents a situation? Use the example if it helps with your thinking.



$x + 5 = 20$

$5x = 20$

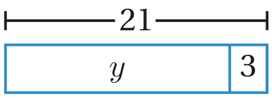
Summary

A tape diagram can help us visualize an equation and determine its solution. The **solution to an equation** is a value of a variable that makes the equation true.

When we work with an equation that represents a situation, it is important to determine what the variable represents when we determine the solution.

Here is an example.

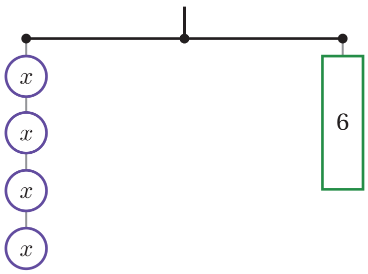
Emmanuel needed $21 to buy a gift. He had $3 and borrowed the rest from his parents.

Equation	Tape Diagram	Solution to the Equation	Solution's Meaning
$3 + y = 21$		$y = 18$	Emmanuel borrowed $18 from his parents.

10 Synthesis

How can a balanced hanger help determine the solution to an equation?

Use the hanger and equation if that helps you with your thinking.



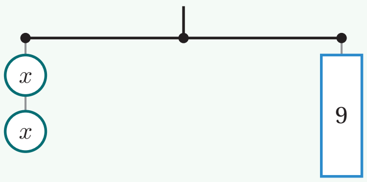
$4x = 6$

Summary

Hangers are a helpful way to represent equations because they demonstrate equality. A hanger is balanced when the weight on both sides is equal.

Here is an example.

This hanger represents the equation $2x = 9$, or $x + x = 9$. The solution to this equation is the value of x that will keep the hanger balanced. In this example, the solution is 4.5 because $4.5 + 4.5 = 9$ or $2(4.5) = 9$.



14 Synthesis

Describe a strategy for solving an equation.

Use the examples if they help with your thinking.

$3x = 18$

$3 + x = 15.6$

Summary

There are many strategies that can be used when solving equations, such as drawing models, using number sense to determine the value that makes an equation true, making a hanger balanced, or using inverse operations to isolate a variable.

Here are two examples that use inverse operations to solve an equation.

Equation	Explanation
$x + 1.5 = 3.25$	Original equation
$x + 1.5 - 1.5 = 3.25 - 1.5$	To isolate x, we subtract 1.5 from both sides.
$x = 1.75$	The solution to this equation is 1.75.

Equation	Explanation
$\frac{1}{2}y = 54$	Original equation
$\frac{1}{2}y \div \frac{1}{2} = 54 \div \frac{1}{2}$	To isolate y, we divide both sides by $\frac{1}{2}$.
$y = 108$	The solution to this equation is 108.

Synthesis

8. What do you think is important to remember when writing equations to represent solutions?

Takeshi has \$10 to spend on laundry. It costs \$2.50 to wash and dry each load. Takeshi can wash p loads of laundry. $2.50p = 10$

Summary

Writing an equation to match a situation is a helpful tool when trying to determine an unknown value. The equation can be solved using a variety of strategies such as tape diagrams, hanger diagrams, or inverse operations. We can check the solution to an equation by substituting the value of the variable to see if it makes the equation true. Once we have a solution to the equation, it's important to determine the meaning of the solution.

Here is an example.

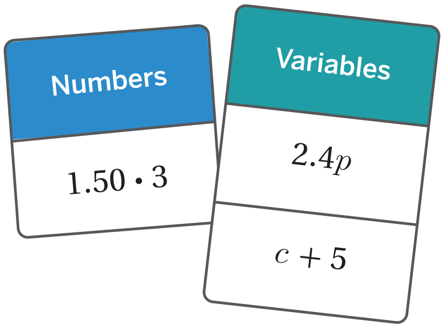
Situation	Equation	Solution	Solution Check	Solution's Meaning
Adah has \$42 to spend on music downloads. Each download costs \$7. She can buy x downloads.	$7x = 42$	$x = 6$	$7 \cdot 6 = 42$	She can buy 6 music downloads.

13 Synthesis

Here are two types of expressions: expressions with numbers and expressions with variables.

When might each kind of expression be useful?

Expressions with numbers are useful when . . .



Expressions with variables are useful when . . .

Summary

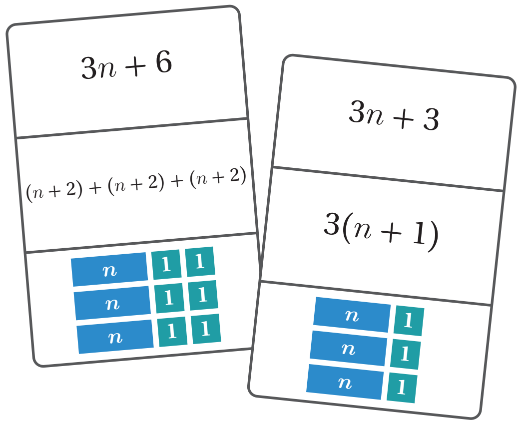
We can use an expression with a variable to represent situations with known and unknown values. Each part in the expression represents a different value in the situation. Here are a few examples.

- The cost of 1 pound of grapes is $\$2.25$. Let p represent pounds of grapes. The variable expression $\$2.25p$ can be used to calculate the total cost for any number of pounds of grapes. This expression only has one **term**, $2.25p$.
- A grocery store adds a $\$10$ fee to the cost of groceries for delivery. Let c represent the cost of groceries. The variable $c + 10$ can be used to calculate the total cost for any cost of groceries. This expression has two *terms*, c and 10.

13 Synthesis

How can you decide if two expressions are *equivalent*?

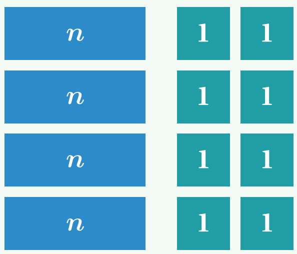
Use the examples if they help you explain your thinking.



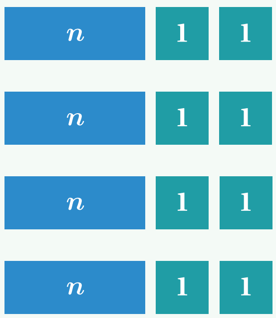
Summary

Equivalent expressions are expressions that are equal for every value of a variable, such as $4n + 8$ and $4(n + 2)$. Diagrams that represent these expressions can help us visually decide if the expressions are equivalent.

$4n + 8$



$4(n + 2)$

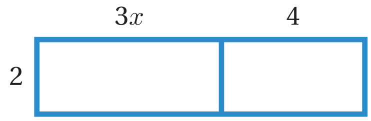


The diagrams for $4n + 8$ and $4(n + 2)$ both show 4 n-tiles and 8 one-tiles. Therefore, $4n + 8 = 4(n + 2)$ are equal for every value of the variable, n, and are equivalent expressions.

9 Synthesis

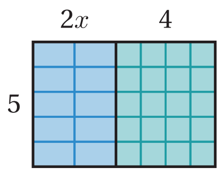
Describe how you can use the area of a rectangle to write two or more equivalent expressions.

Use the example if it helps to show your thinking.



Summary

You can use areas of rectangles (called *area models*) to write equivalent expressions. Two expressions that are equivalent are a product expression and a sum expression because they refer to the same area. No matter what value you substitute for a variable, the total area is the same.

Area Model	Product of Two Side Lengths $\ell \cdot w$	Sum of Two Areas $\ell \cdot w + \ell \cdot w$
	length width $5(2x + 4)$ $10x + 20$	$5 \cdot 2x + 5 \cdot 4$ $10x + 20$

Synthesis

18. Explain how you can show that two expressions are equivalent.

Use these expressions if they help with your thinking.

Equivalent Expressions

$3x + 4(x + 2)$
$7x + 8$

Not Equivalent

$7(x + 2)$

Summary

It can be helpful to have new words to describe more complicated expressions. For example, the expression $8x + 2$ has two terms, and the term $8x$ has a **coefficient** of 8. The expression $2(x + 1) + 3(2x)$ also has two terms, $2(x + 1)$ and $3(2x)$, but the terms are more complex.

To decide if two expressions are *equivalent*, you can draw models, substitute values, or rewrite the expressions. If the expressions are equivalent, you can use the distributive property and other operations to rewrite one expression to look like the other.

Here is an example: Determine whether $2(x + 1) + 3(2x)$ is equivalent to $8x + 2$.

$$\begin{aligned} 2(x + 1) + 3(2x) &= 2x + 2 + 6x \\ &= 2x + 6x + 2 \\ &= 8x + 2 \end{aligned}$$

$2(x + 1) + 3(2x)$ and $8x + 2$ are *equivalent expressions* because after using the distributive property and adding the like terms, the expressions are the same.

13 Synthesis

Without calculating, how can you tell whether expressions with exponents are equivalent?

$11 + 11 + 11 + 11 + 11$

$11^4 \cdot 11$

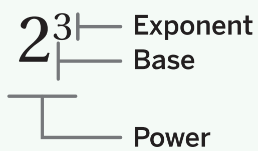
$11 \cdot 5$

$11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$

5^{11}

11^5

Summary



Exponents are used to represent repeated multiplication. In the expression 2^n, 2 is the **base**, and n is the **exponent**. If n is a positive whole number, it represents how many times 2 should be multiplied to determine the value of the expression.

Here are some examples.

$2^1 = 2$ $\qquad$ $2^3 = 2 \cdot 2 \cdot 2$

There are several different ways to say "2^3".

- "Two to the power of three"
- "Two raised to the power of three"
- "Two to the third power"
- "Two cubed"

Synthesis

12. What are some things to remember when determining the value of expressions with exponents?

Use these examples if they help with your thinking.

$5 \cdot 3^2$
$(3 + 5)^2$
$(3 \cdot 5)^2$
$5^2 + 3^2$

Summary

There is a specific *order of operations* we use to evaluate expressions with more than one operation, like $5 \cdot 2^4$ or $(5 \cdot 2)^4$.

With Parentheses	Without Parentheses
Evaluate the operations in parentheses first:	Evaluate the term with the exponent first:
$(5 \cdot 2)^4$	$5 \cdot 2^4$
$(10)^4$	$5 \cdot (2 \cdot 2 \cdot 2 \cdot 2)$
$10 \cdot 10 \cdot 10 \cdot 10$	$5 \cdot 16$
10,000	80

14 Synthesis

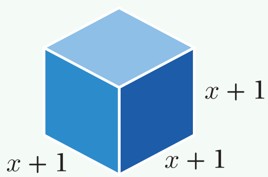
Describe how to evaluate $(x + 1)^3$ when $x = 3$.

Summary

To *evaluate* an expression is to determine its value for a specific value of a variable. When evaluating expressions using the order of operations, evaluate the operations in parentheses first; in expressions without parentheses, exponents should be evaluated first.

You can use area and volume models to evaluate expressions with exponents 2 and 3.

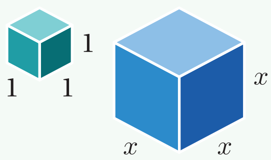
For example, when $x = 2$.



$(x + 1)^3$ is

$(2 + 1)^3 = 3^3$

$= 27$



$x^3 + 1$ is

$2^3 + 1 = 8 + 1$

$= 9$

When an expression has an exponent larger than 3, you substitute the value of the variable and perform the operations using the order of operations.

For example, when $x = 2$.

$(x + 1)^4$ is

$(2 + 1)^4 = 3^4$

$= 81$

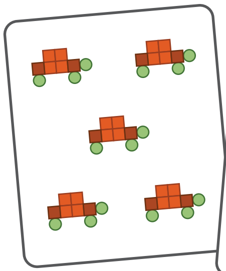
$x^5 + 1$ is

$2^5 + 1 = 32 + 1$

$= 33$

11 Synthesis

How are relationships between *independent* and *dependent variables* represented in tables and equations?



Number of Turtles, t	Total number of tiles, a
1	9
3	27
5	45

$a = 9t$

Summary

Tables and equations can be used to represent and describe a relationship between two variables or quantities.

- The **dependent variable** is the variable in a relationship that is the effect or result.
- The **independent variable** is the variable in a relationship that is the cause. It is used to calculate the value of the dependent variable.

For example, if a boat can travel 36 miles in 3 hours, the variables could be:

- The dependent variable is the distance traveled, d.
- The independent variable is the amount of time, t.

Table

t (hr)	d (mi)
3	36
1	12
2	24

The speed of the boat is 12 miles per hour.

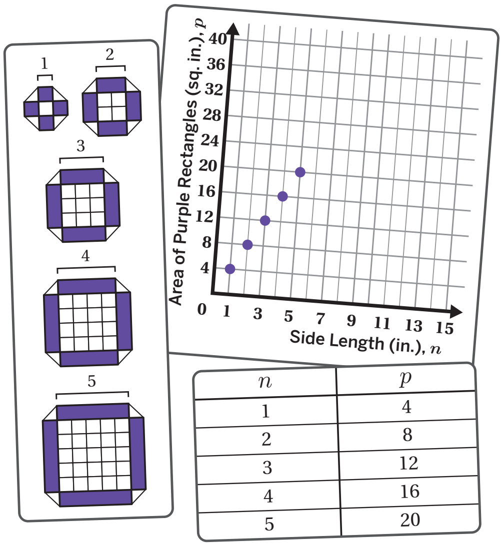
Equation

$d = 12t$

In 6 hours, the boat will travel $d = 12 \cdot 6 = 72$ miles.	The boat can travel 120 miles in $120 = 12t = 10$ hours.

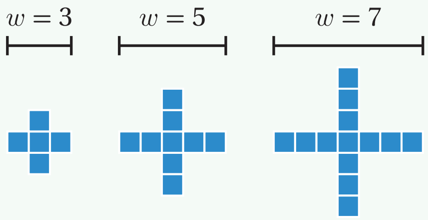
13 Synthesis

How can you tell that a table, a graph, and an image show the same relationship?



Summary

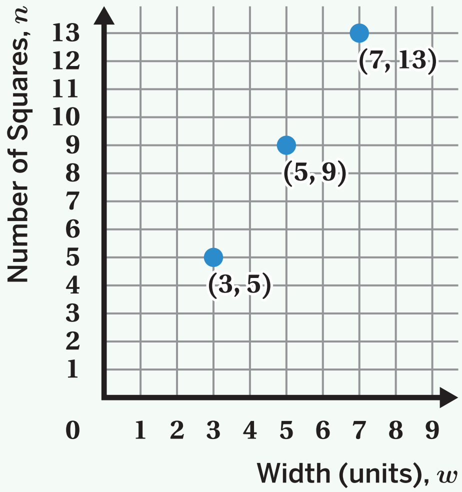
Like tables and equations, graphs are another way of representing the relationship between two quantities. For example, in this pattern, the *independent variable* is the width of the figure, w, and the *dependent variable* is the number of squares, n.



Here is the table and graph of this relationship.

Width of the Figure (units), w	Number of Squares, n
3	5
5	9
7	13

The numbers in each row of the table indicate an *ordered pair* on the coordinate plane. In the first row of the table, w is 3 and n is 5, which is represented by the point $(3, 5)$ on the graph.

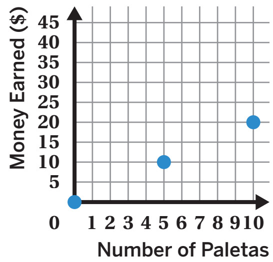


While representing a relationship using a graph, the common practice is to use the x-axis for the independent variable.

Synthesis

11. Explain how tables, equations, or graphs represent the same relationship.

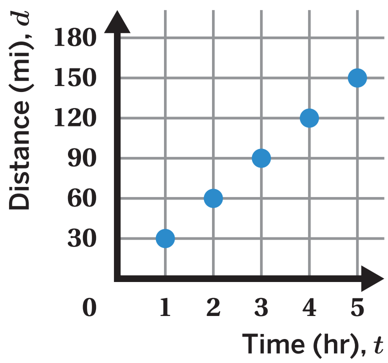
Use the example if it helps with your explanation.

<table>
<tr><th>Situation</th><th>Table</th><th>Graph</th><th>Equation</th></tr>
<tr><td>Amanda sells paletas for $2 each.</td><td><table><tr><th>p</th><th>m</th></tr><tr><td>0</td><td>0</td></tr><tr><td>5</td><td>10</td></tr><tr><td>10</td><td>20</td></tr></table></td><td>
</td><td>$m = 2p$</td></tr>
</table>

Summary

All three representations — tables, equations, and graphs — hold the same mathematical information described in a situation but display it in different ways.

For example, if a car travels 30 miles per hour at a constant speed, you can determine the distance the car traveled in 4 hours using a table, a graph, or an equation.

<table>
<tr><th>Table</th><th>Graph</th><th>Equation</th></tr>
<tr><td><table><tr><th>Time, t (hr)</th><th>Distance, d (mi)</th></tr><tr><td>1</td><td>30</td></tr><tr><td>2</td><td>60</td></tr><tr><td>4</td><td>120</td></tr></table></td><td>
</td><td>$d = 30t$
$d = 30(4)$
$d = 120$</td></tr>
</table>

In all three representations, we can see that the car traveled 120 miles in 4 hours.

Synthesis

8. How can making a graph and a table help us understand relationships in the world, such as subway fares?

Summary

We can use data in tables, graphs, and equations to help make decisions in real-world situations. When it comes to analyzing subway ticket fares, these representations can help us make informed decisions about what type of transportation ticket to purchase.

Using the graph, we can see that the regular fare ticket is the best choice if we ride 5 times or less and do not qualify for the reduced fare. If we extend each line on the graph, we'll be able to determine when the price of an unlimited 7-day pass will be lower than the regular fare.

We can use these tools to make sure we get the best subway ticket for our needs.