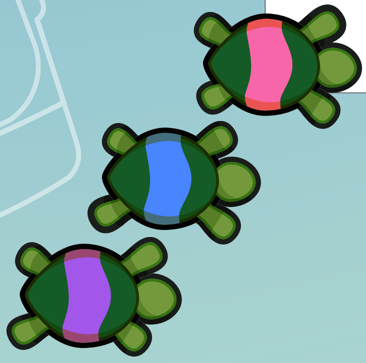
Unit 3

Proportional and Linear Relationships

The slope of a line tells us about its steepness, but did you know it can reveal even more? A simple line on a graph holds *endless* information. A line can tell us what's faster: a tortoise or a hare. It can help us sort coins in a piggy bank. It can even help us determine how many cups a water cooler can fill. Let's explore proportional and linear relationships, and learn all about what a line can represent!

Essential Questions

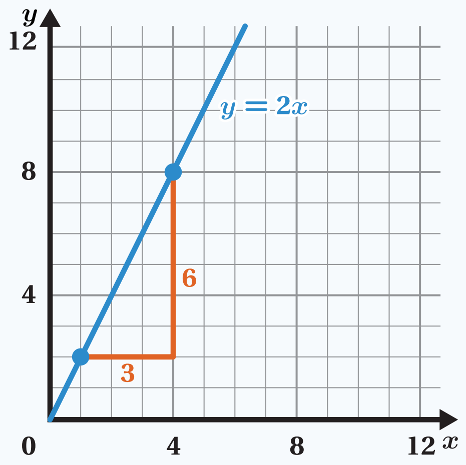
- What can proportional relationships and slope teach you about linear relationships?
- How do equations, tables, and graphs of linear relationships connect to one another?
- What does it mean for an ordered pair to be a solution to a linear equation?



Summary | Lesson 1

A line that passes through the origin, $(0, 0)$, represents a *proportional relationship*. The *slope* of the line represents a *unit rate* for this relationship.

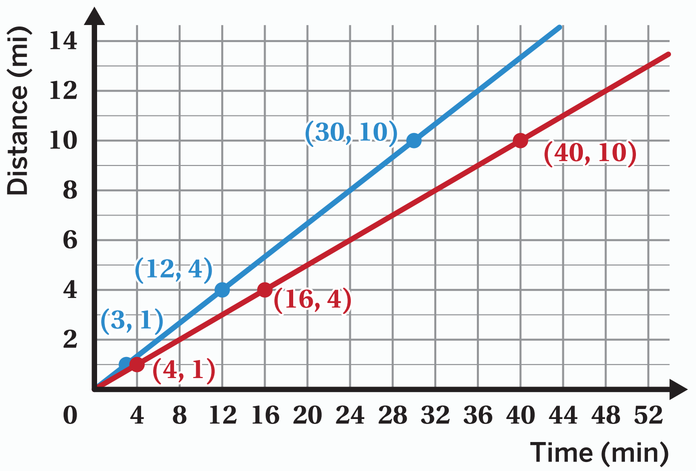
Here is a graph of the equation $y = 2x$. The slope of the line is $\frac{6}{3}$, or 2.



Try This

This graph shows the distance that Jasmine and Sothy travel on a long bike ride.

Jasmine rides 4 miles every 16 minutes, and Sothy rides 4 miles every 12 minutes.



a Do these lines represent proportional relationships? Explain your thinking.

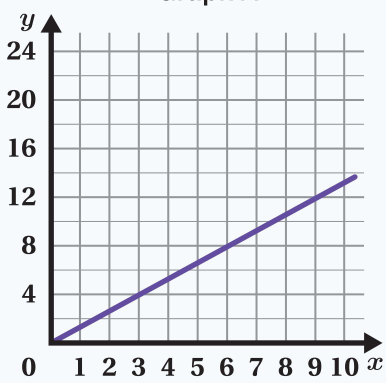
b What is the speed of each rider as a unit rate?

Summary | Lesson 2

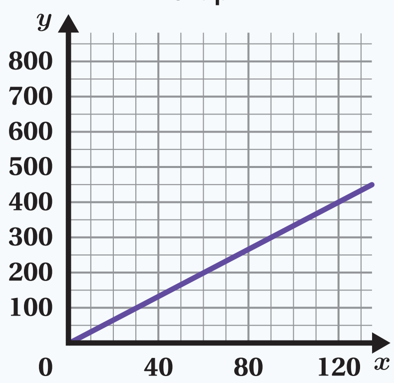
We can represent a proportional relationship using the equation $y = mx$, where m represents both a unit rate and the slope of the line.

When we represent these relationships on axes with different scales, we can use slope to compare these graphs. For example, you can compare Graphs A, B, and C using their slopes.

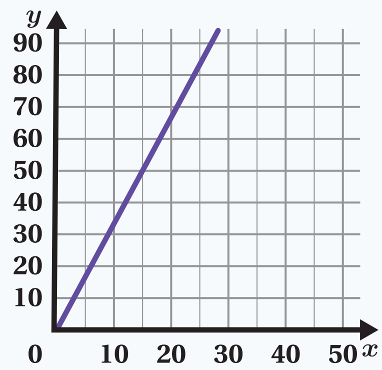
Graph A



Graph B



Graph C



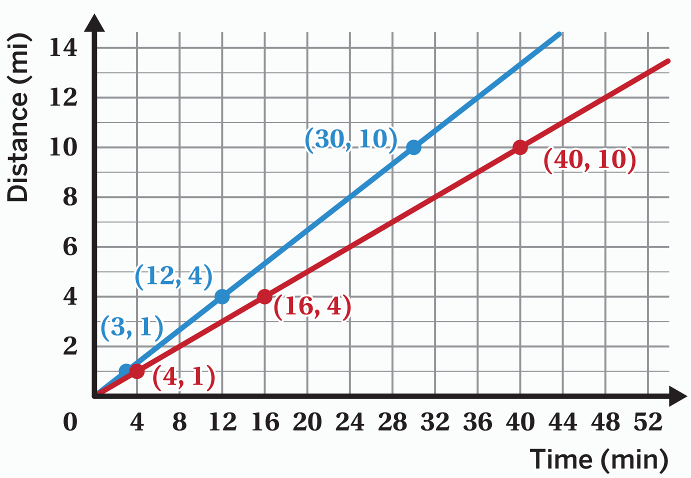
- Graph A slope: $\frac{200}{150} = \frac{4}{3}$
- Graph B slope: $\frac{100}{30} = \frac{10}{3}$
- Graph C slope: $\frac{50}{15} = \frac{10}{3}$

You can see that the slopes for Graph B and Graph C are equivalent. This means they have the same proportional relationship, even though the lines may look like they don't have the same steepness.

Try This

This graph shows the distance that Jasmine and Sothy travel on a long bike ride.

Jasmine rides 4 miles every 16 minutes, and Sothy rides 4 miles every 12 minutes.



a Determine the slope of each line.

b Write an equation to represent each line.

Summary | Lesson 3

Using different representations is helpful when comparing proportional relationships, such as equations, tables, and graphs.

You can represent proportional relationships with the equation $y = mx$, where m is the slope of a line and also represents a unit rate for the situation. You can identify the slope or unit rate using all of the different representations, or by using slope triangles within a graph.

Try This

As of 2024, the federal minimum wage is $7.25 per hour.

Create an equation, table, and graph to represent the relationship between hours worked and money earned for a person making minimum wage.

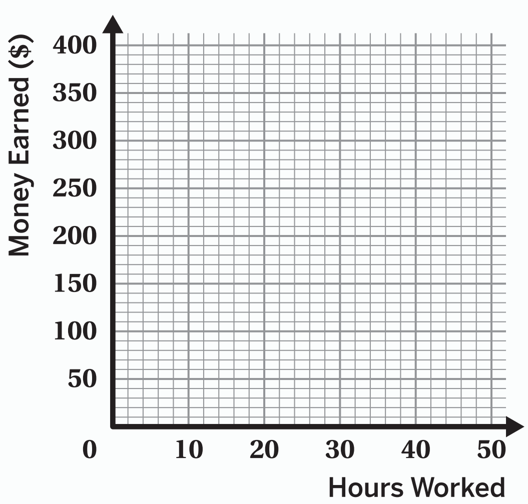
Table

Hours Worked	Money Earned ($)

Equation

____

Graph



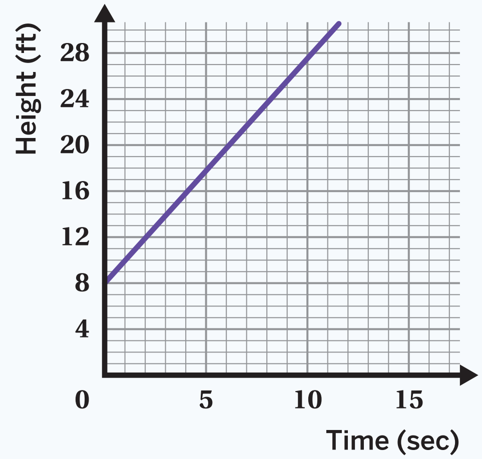
Summary | Lesson 4

Linear relationships are graphs that are lines. Some linear relationships are proportional relationships and some are not.

For example, this graph represents a flag's height, in feet, over time.

- The line starts at $(0, 8)$, which means the flag is at first 8 feet off the ground.
- The slope is 2, which represents the number of feet the flag rises each second.
- The equation $y = 8 + 2x$ represents the height of the flag y after x seconds.

This relationship is linear, but it's non-proportional because the line does not pass through the origin.



Try This

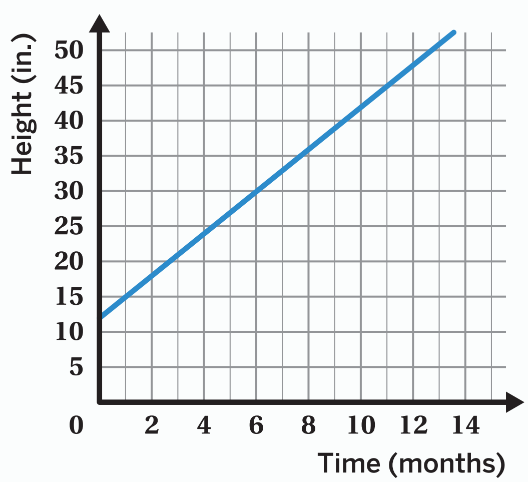
The graph shows the height, in inches, of a bamboo plant each month after it was planted.

The equation for this line is $y = 3x + 12$.

a What is the slope of this line? What does this value mean in context?

b How tall was the bamboo plant when it was planted?

c Is this a linear relationship? Explain your thinking.

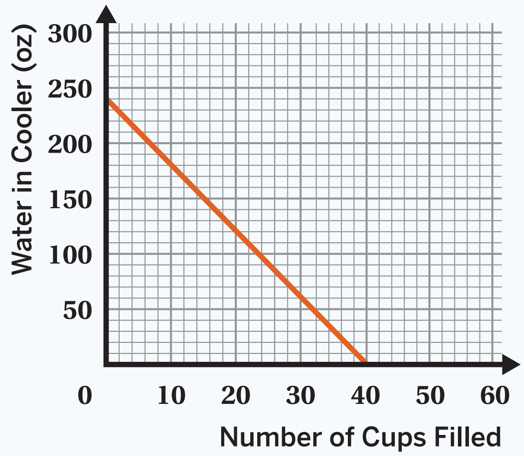


Summary | Lesson 5

When a *linear relationship* has a negative slope, this means that as the x-values increase, the y-values decrease at a constant rate.

Let's say the equation $y = 240 - 6x$ represents the amount of water in a cooler, y, after x cups have been filled.

- The **vertical intercept**, also called the y-*intercept*, is $(0, 240)$. In this situation, the vertical intercept represents the starting amount of water in the cooler.
- The slope is -6. This means that the amount of water decreases by 6 ounces for each cup filled. Because the amount of water decreases each time, the slope is negative.
- The **horizontal intercept**, also called the x-*intercept*, is $(40, 0)$. In this situation, the horizontal intercept represents how many cups can be filled before the cooler runs out of water.

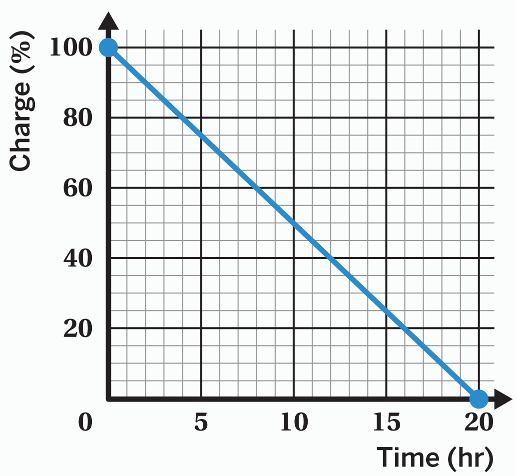


Try This

This graph shows a phone's charge over time.

a What is the vertical intercept of the line, and what does it represent?

b What is the horizontal intercept of the line, and what does it represent?



Summary | Lesson 6

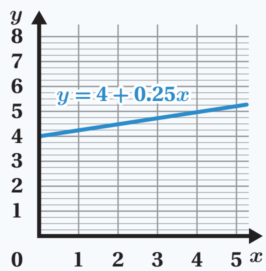
Equations of linear relationships, or linear equations, can be written in the form $y = mx + b$, where m represents the slope and b represents the vertical intercept.

For linear relationships with a *positive* slope, the y-values increase at a constant rate as the x-values increase. For linear relationships with a *negative* slope, the y-values decrease at a constant rate as the x-values increase.

Here are two examples.

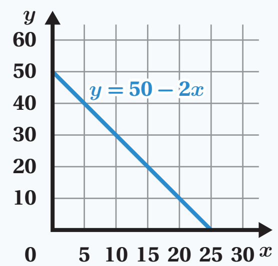
Positive slope: A medium-sized frozen yogurt costs \$4, plus \$0.25 per topping.

Let y represent the total cost of the frozen yogurt after adding x toppings.



Negative slope: A student loads an arcade game card with \$50. Every time she plays a game, \$2 is subtracted from the amount available on the game card.

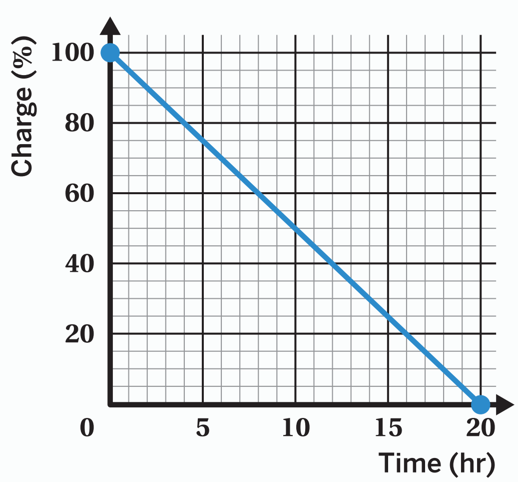
Let y represent the amount in dollars on the card after the student plays x games.



Try This

This graph shows a phone's charge over time.

Let y represent the percent charge after x hours.



a What is the slope of the line?

b What is the vertical intercept?

c Write an equation that represents this situation.

Summary | Lesson 7

Linear relationships can help us make predictions. We can use the graph or the equation of a linear relationship to determine the value of one variable when we're given the other variable.

Let's say we want to know how many stacked cups will have a height of 50 centimeters. We can look at a graph like the one in the Synthesis and determine the number of cups, x, that correspond to $y = 50$. In this situation, the slope of the line tells us how many centimeters the height of the stacked cups increases by every time we add a cup to the stack.

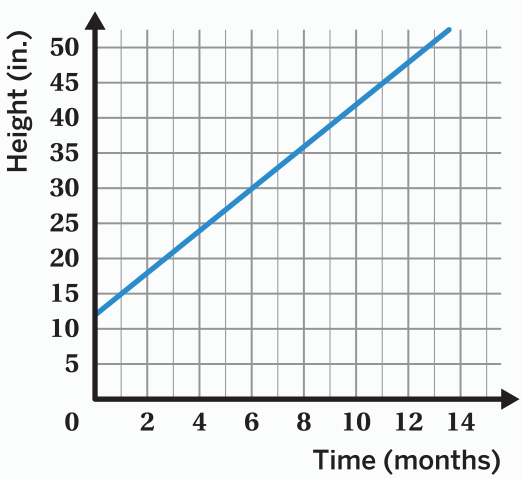
We might think of the y-intercept as the stack height when there are 0 cups, but this doesn't make sense in this situation. Instead, the y-value tells us the distance from the bottom of the first cup to its rim. Unlike a proportional relationship, the graph of this linear relationship doesn't pass through the origin.

Try This

This graph shows the height, in inches, of a bamboo plant each month after it was planted.

The equation of this line is $y = 3x + 12$.

How tall will the bamboo plant be after 12 months?



Summary | Lesson 8

A *translation* of a line that represents a proportional relationship creates a line that is parallel to the pre-image, but changes the location of the vertical intercept, also known as the y-intercept.

The equation $y = mx$ represents a line that passes through the origin. The equation $y = mx + b$ represents a vertical translation of line $y = mx$ by b units.

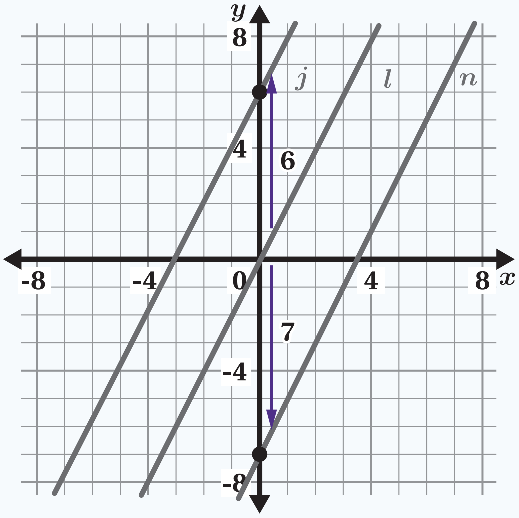
If $b > 0$, the line is translated up.

If $b < 0$, the line is translated down.

For example, the equation of line l is $y = 2x$.

If line l is translated 6 units up to produce line j, the equation of line j is $y = 2x + 6$.

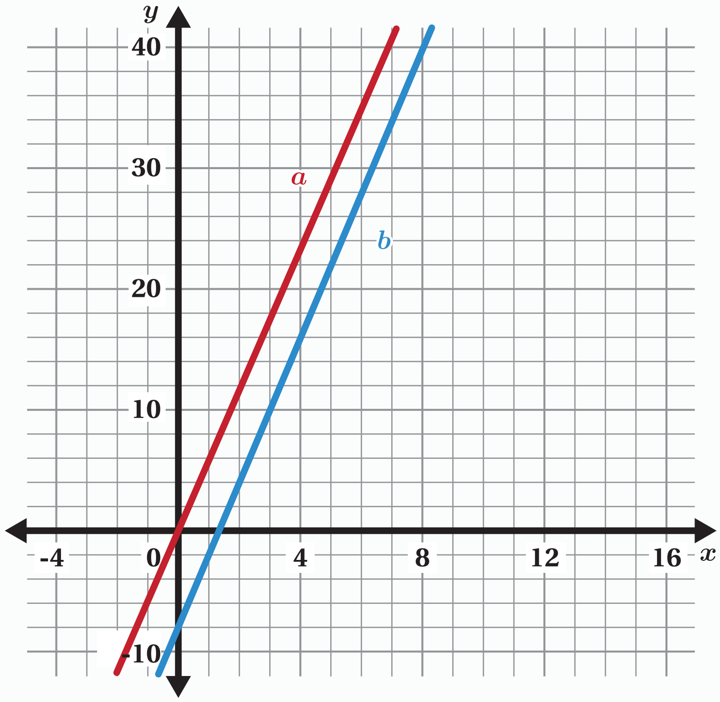
If line l is translated 7 units down to produce line n, the equation of line n is $y = 2x - 7$.



Try This

Here are the graphs of line a and line b. The equation for line a is $y = 6x$.

Write the equation for line b.



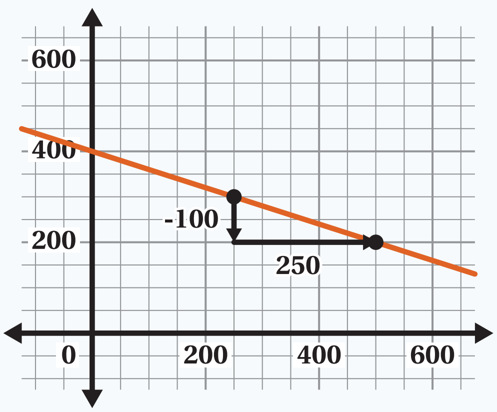
Summary | Lesson 9

You can determine the slope of a line using two points on that line. Lines with positive slopes increase in height from left to right, while lines with negative slopes decrease in height from left to right.

You can use slope triangles to calculate the vertical change and horizontal change between two points on a coordinate plane. You can also calculate the slope by listing the coordinates in a table and then determining the difference between the y-coordinates (the vertical change) and the difference between the x-coordinates (the horizontal change).

The slope is the ratio of the vertical change to the horizontal change.

Using Slope Triangles



Slope $= \frac{\text{vertical change}}{\text{horizontal change}}$

$= -\frac{100}{250}$

$= -\frac{2}{5}$

Using Coordinates in a Table

x	y
250	300
500	200

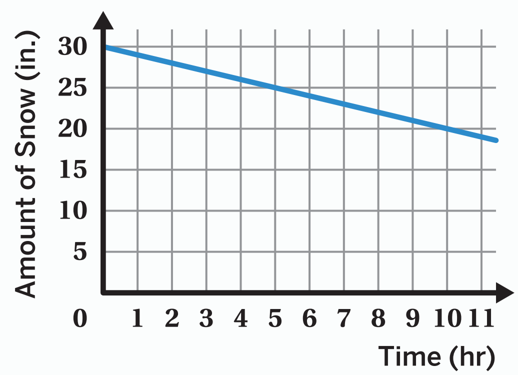
+250 (change in x), -100 (change in y)

$\frac{\text{change in } y}{\text{change in } x} = \frac{-100}{250} = -\frac{2}{5}$

Try This

This graph represents how much snow there is on the ground after it starts to melt in the sun.

What is the slope of the line?



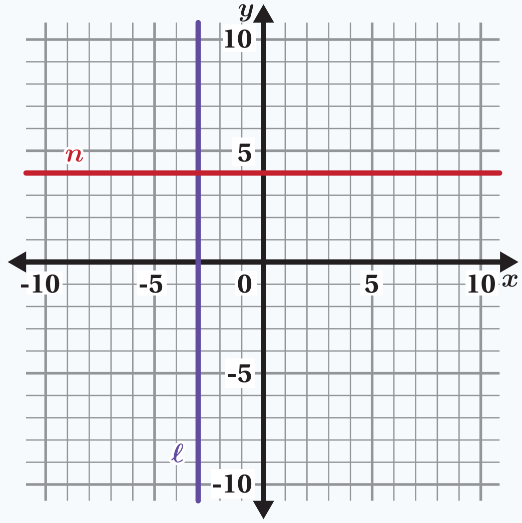
Summary | Lesson 10

On the coordinate plane:

- Horizontal lines represent situations where the y-value is constant and the x-values change. Horizontal lines have a slope of 0.
- Vertical lines represent situations where the x-value is constant and the y-values change. Vertical lines have an *undefined* slope.

For example, the equation $y = 4$ represents the horizontal line n because every point on the line has the same y-coordinate, 4.

The equation $x = \text{-}3$ represents the vertical line ℓ because every point on the line has the same x-coordinate, -3.

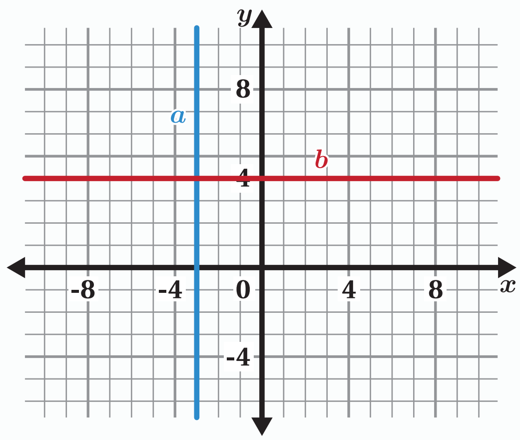


Try This

Write an equation for each line:

a Line a: ____________________

b Line b: ____________________



Here are two different strategies for writing the equation of a line using two given points.

Strategy Using a Table

First, calculate the slope using a table. Next, substitute the coordinates of one of the points into the equation $y = mx + b$ to determine the y-intercept. Then write the equation in the form $y = mx + b$.

x	y
1	8
3	2

+2 (x), −6 (y)

slope: $\frac{-6}{2} = -3$

$y = -3x + b$

Substitute (1, 8) in for x and y.

$8 = -3(1) + b$

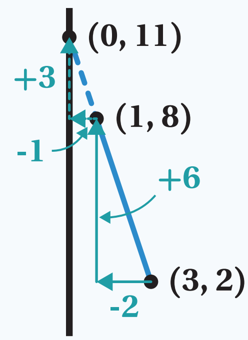
$8 = -3 + b$

$11 = b$

$y = -3x + 11$

Strategy Using Slope Triangles

Draw a line and use similar triangles to determine the slope and y-intercept of the line. Then write the equation in the form $y = mx + b$.



$8 + 3 = 11$

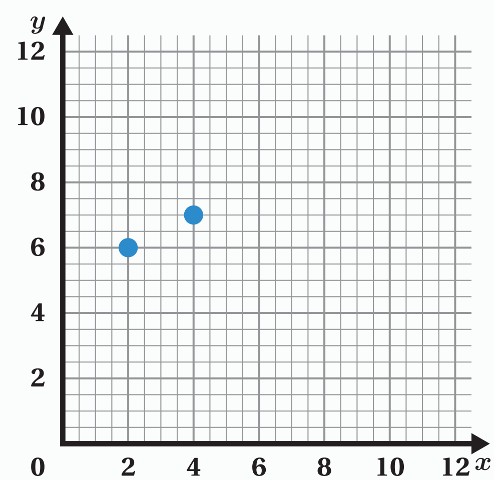
y-intercept: $(0, 11)$

Slope: -3

$y = -3x + 11$

Try This

Write an equation of a line that goes through points $(2, 6)$ and $(4, 7)$.



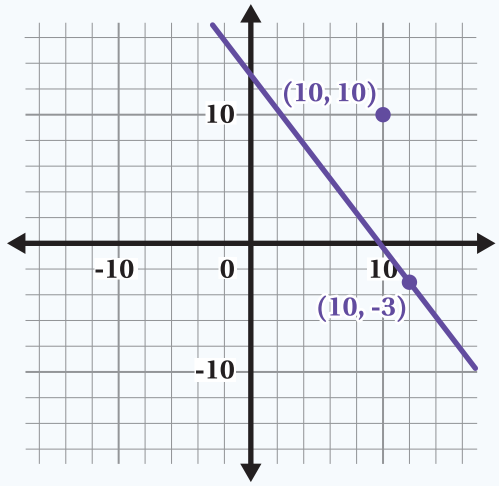
Summary | Lesson 12

A **solution** to an equation with two variables is a set of values that makes the equation true. Solutions are often written as an ordered pair, (x, y).

Every point that lies on a line is a solution to that equation. Points that do not lie on the line are *not* solutions to the equation.

Here is a graph of the linear equation $3x + 2y = 24$.

- $(10, \text{-}3)$ is a solution to the equation $3x + 2y = 24$ because the point is on the graph of the line, and because $3(10) + 2(\text{-}3) = 24$.
- $(10, 10)$ is *not* a solution because the point is not on the line, and because $3(10) + 2(10) = 50$, not 24.
- Although we can't see it on the graph, $(\text{-}10, 27)$ is also a solution because $3(\text{-}10) + 2(27) = 24$.

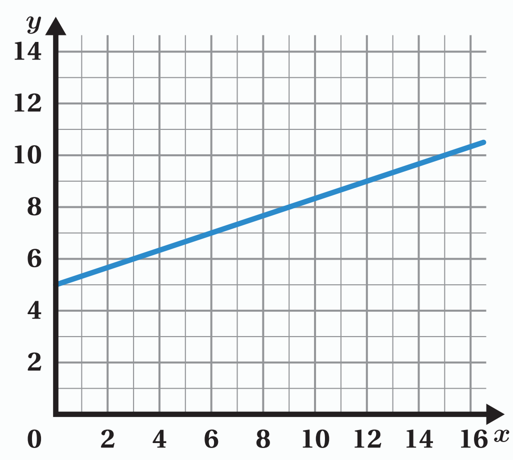


Try This

This graph shows the line $y = \frac{1}{3}x + 5$.

a Is $(10, 8)$ a solution to this equation? Explain your thinking.

b Is $(6, 7)$ a solution to this equation? Explain your thinking.



Summary | Lesson 13

The four representations of a linear relationship — table, graph, equation, and verbal description — are all useful when solving real-world problems.

Let's say a coach has a $120 budget to buy dinner for their team. Pizzas cost $20 and sandwiches cost $8. x represents the number of pizzas bought and y represents the number of sandwiches bought.

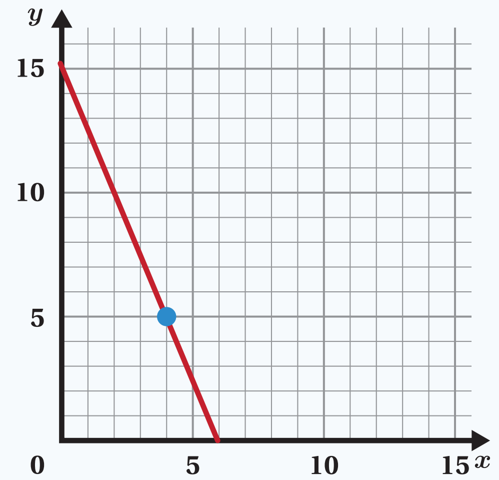
This situation can be modeled by the linear relationship $20x + 8y = 120$.

Here are two ways to show that 4 pizzas and 5 sandwiches is one solution to the equation:

- The values $x = 4$ and $y = 5$ make the equation true.

$$20(4) + 8(5) = 120$$
$$80 + 40 = 120$$
$$120 = 120$$

- The point $(4, 5)$ is on the graph of the linear relationship.



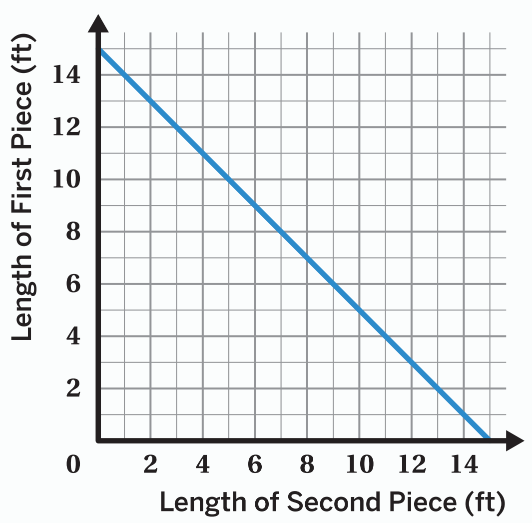
Try This

A 15-foot piece of ribbon is cut into two pieces.

This graph shows the possible lengths of each piece. The equation of the line is $y = -x + 15$.

a Is $(6, 9)$ a solution to this equation? Explain your thinking.

b Choose *one* solution to the equation and interpret what it means in this situation.



Lesson 1

a Yes. *Explanations vary.* Both lines pass through the origin, $(0, 0)$, so they both represent proportional relationships.

b Jasmine: $\frac{1}{4}$ or 0.25 miles per minute. Sothy: $\frac{1}{3}$ or about 0.33 miles per minute. [To find Jasmine's speed, divide 4 miles by 16 minutes, or $\frac{4}{16} = \frac{1}{4}$. To find Sothy's speed, divide 4 miles by 12 minutes, or $\frac{4}{12} = \frac{1}{3}$.]

Lesson 2

a Jasmine: $\frac{1}{4}$ or 0.25. Sothy: $\frac{1}{3}$ or about 0.33.

[To find the slope of Jasmine's line, divide 4 miles by 16 minutes, or $\frac{4}{16} = \frac{1}{4}$. To find the slope of Sothy's line, divide 4 miles by 12 minutes, or $\frac{4}{12} = \frac{1}{3}$.]

b Jasmine: $y = \frac{1}{4}x$

Sothy: $y = \frac{1}{3}x$

Lesson 3

Table

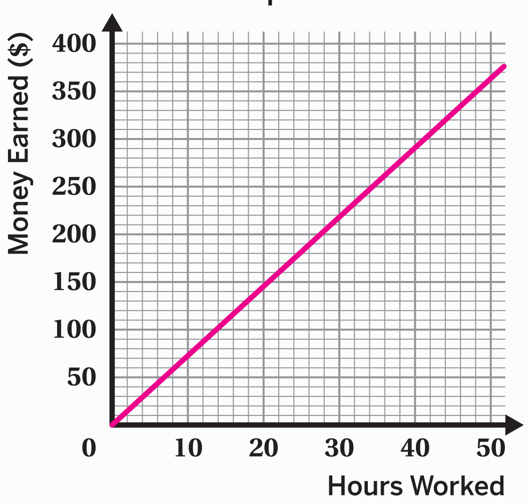
Responses vary.

Hours Worked	Money Earned ($)
10	72.50
20	145
30	217.50
40	290

Equation

$y = 7.25x$

Graph



Lesson 4

a 3. *Responses vary*. This means the bamboo plant grows 3 inches each month.

b 12 inches. [The graph starts at $(0, 12)$, which means that the plant was 12 inches tall when it was planted, at 0 months.]

c Yes. *Explanations vary.* The graph forms a straight line.

Lesson 5

a (0, 100). *Responses vary*. The phone's charge was 100% at 0 hours.

b (20, 0). *Responses vary*. The phone's charge was at 0% after 20 hours.

Lesson 6

a -5

b (0, 100)

c $y = \text{-}5x + 100$ (or equivalent)

Lesson 7

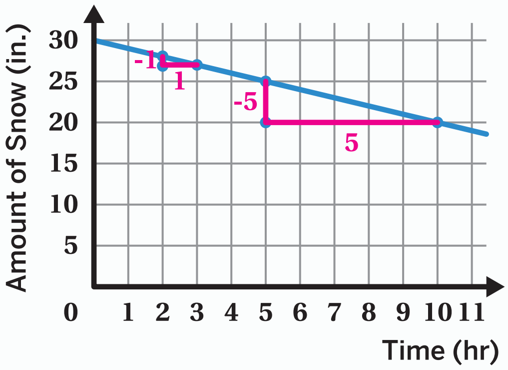
48 inches. [$y = 3(12) + 12 = 48$]

Lesson 8

$y = 6x - 8$

Lesson 9

-1. [Here is one strategy for determining the slope using slope triangles:



Slope: $\frac{-1}{1} = \text{-}1$ or $\frac{-5}{5} = \text{-}1$]

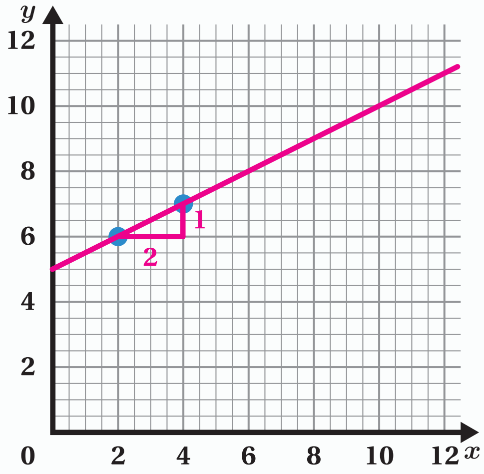
Lesson 10

a $x = \text{-}3$

b $y = 4$

Lesson 11

$y = \frac{1}{2}x + 5$. [Here is one strategy for determining the equation using a slope triangle:



The slope triangle shows that the change in y-values is 1 and the change in x-values is 2, which means the slope is $\frac{1}{2}$. The y-intercept is $(0, 5)$, so the equation is $y = \frac{1}{2}x + 5$.]

Lesson 12

- **a** No. *Explanations vary*. The point $(10, 8)$ is not on the line and $8 \neq \frac{1}{3}(10) + 5$.
- **b** Yes. *Explanations vary*. The point $(6, 7)$ is on the line and $7 = \frac{1}{3}(6) + 5$.

Lesson 13

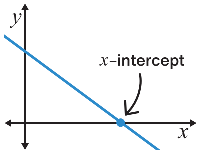
- **a** Yes. *Explanations vary*. The point $(6, 9)$ is on the line and $9 = -6 + 15$. This makes sense because if a 15-foot ribbon is cut so that one piece is 6 feet long, the other piece would be 9 feet long.
- **b** *Responses vary*. The point $(10, 5)$ means that if the first piece is 5 feet long the second piece is 10 feet long.

Grade 8 Unit 3 Glossary/8.º grado Unidad 3 Glosario

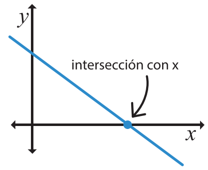
English | Español

H

horizontal intercept The point where the graph of a line crosses the horizontal axis or when $y = 0$. The horizontal intercept is sometimes called the x-intercept.

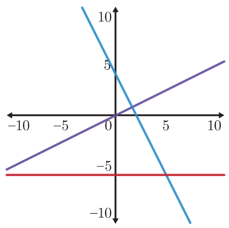


intersección horizontal El punto donde la gráfica de una recta se cruza con el eje horizontal o cuando $y = 0$. La intersección horizontal a veces se denomina intersección con el eje x.

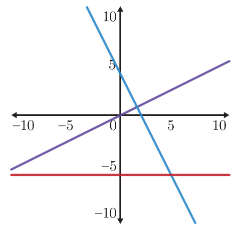


I

linear relationship A relationship is called linear when its graph is a line.



relación lineal Una relación se llama lineal cuando su gráfica es una recta.



P

proportional relationship A set of equivalent ratios. The values for one quantity are each multiplied by the same number to get the values for the other quantity.

Carpet (sq. ft.)	Cost (dollars)
10 ×1.5 →	15.00
20 ×1.5 →	30.00
50 ×1.5 →	75.00

For example, every cost in this table is equal to 1.5 times the number of square feet of carpet.

relación proporcional Un conjunto de razones equivalentes. Cada uno de los valores de una cantidad se multiplica por el mismo número para obtener los valores de la otra cantidad.

Alfombra (pies cuadrados)	Costo (dólares)
10 ×1.5 →	15.00
20 ×1.5 →	30.00
50 ×1.5 →	75.00

Por ejemplo, cada costo en la tabla es igual a 1.5 veces el número de pies cuadrados de alfombra.

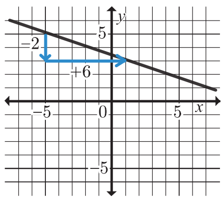
R

rate of change See *slope*.

tasa de cambio Véase *pendiente*.

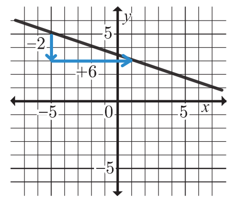
S

slope A number that describes the direction and steepness of a line. Slope represents the amount that y changes when x increases by 1. That's why the slope of a line is sometimes called a rate of change. To calculate the slope, divide the vertical distance between any two points on the line by the horizontal distance between those points.



The slope of this line is $-\frac{2}{6} = -\frac{1}{3}$.

pendiente Un número que describe la dirección e inclinación de una línea. La pendiente representa la cantidad en la que cambia y cuando x se incrementa en 1. Es por eso que la pendiente de una recta a veces se denomina tasa de cambio. Para calcular la pendiente, la distancia vertical entre dos puntos cualesquiera en la recta se divide entre la distancia horizontal entre dichos puntos.



La pendiente de esta recta es $-\frac{2}{6} = -\frac{1}{3}$.

English

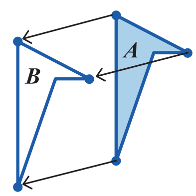
solution The value or set of values that makes the equation true. A solution to an equation with two variables is a pair of values that makes the equation true, often written as an ordered pair, (x, y).

The solution to the equation $x + 15 = 8$ is $x =$ -7 because (-7) $+ 15 = 8$. One solution to the equation $4x + 3y = 24$ is $(6, 0)$ because $4(6) + 3(0) = 24$.

T

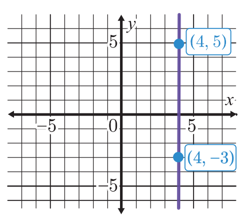
transformation An action or rule for moving or changing figures on a plane. Transformations include translations, reflections, rotations, and dilations.

translation A translation moves every point in a figure a given distance in a given direction.



U

undefined An expression or term that has no understandable value. Dividing any value by 0 creates an undefined expression because we cannot divide by 0.



The slope of a vertical line is undefined because there is 0 horizonal change between any two points on the line. This means any vertical change would be divided by 0, which creates an undefined value.

unit rate A rate that describes how one quantity changes when the other quantity changes by exactly 1 unit.

For example, if 12 people share 3 pizzas equally, then one unit rate is 4 people per pizza. Another unit rate in this situation is $\frac{1}{4}$ pizza per person.

Español

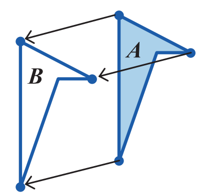
solución El valor o conjunto de valores que hacen que la ecuación sea verdadera. Una solución de una ecuación con dos variables es un par de valores que hacen que la ecuación sea verdadera, y a menudo se escribe como un par ordenado, (x, y).

La solución de la ecuación $x + 15 = 8$ es $x =$ -7 porque (-7) $+ 15 = 8$. Una solución de la ecuación $4x + 3y = 24$ es $(6, 0)$ porque $4(6) + 3(0) = 24$.

T

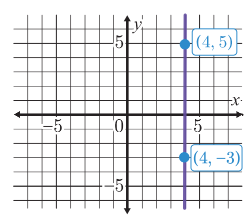
transformación Una acción o regla para mover o cambiar figuras en un plano. Las transformaciones incluyen traslaciones, reflexiones, rotaciones y dilataciones.

traslación Una traslación mueve cada punto de una figura una determinada distancia en una determinada dirección.



U

indefinido Una expresión o un término que no tiene ningún valor comprensible. La división de un valor (diferente de cero) entre 0 produce una expresión indefinida porque no es posible dividir por 0.



La pendiente de una recta vertical es indefinida porque hay un cambio horizontal de 0 entre dos puntos cualesquiera en la recta. Esto significa que cualquier cambio vertical se dividiría por 0, lo cual produce un valor indefinido.

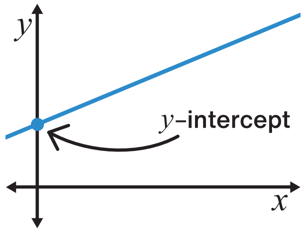
tasa unitaria Una tasa que describe cómo cambia una cantidad cuando la otra cantidad cambia en exactamente 1 unidad.

Por ejemplo, si 12 personas se reparten 3 pizzas en partes iguales, entonces una tasa unitaria es 4 personas por pizza. Otra tasa unitaria en esta situación es $\frac{1}{4}$ de pizza por persona.

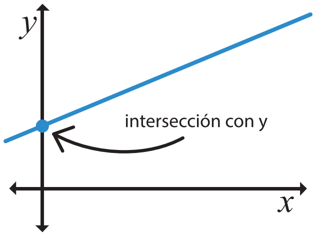
English | Español

V

vertical intercept The point where the graph of a line crosses the vertical axis or when $x = 0$. The vertical intercept is sometimes called the y-intercept.



intersección vertical El punto donde la gráfica de una recta se cruza con el eje vertical o cuando $x = 0$. La intersección vertical a veces se denomina intersección con el eje y.



X

x-intercept See *horizontal intercept*.

intersección con el eje x Véase *intersección horizontal*.

Y

y-intercept See *vertical intercept*.

intersección con el eje y Véase *intersección vertical*.