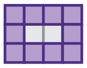
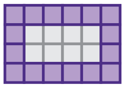
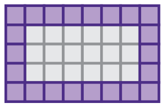
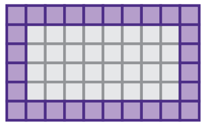
Equations and Inequalities

Accelerated 7
Unit 3

9 Synthesis

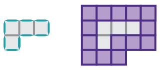
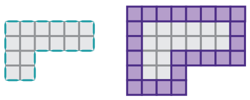
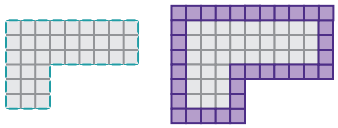
Here is a new pattern.

Stage 1	Stage 2	Stage 3	Stage 4
			
Border Tiles: 10	Border Tiles: 16	Border Tiles: 22	Border Tiles: 28

Describe how you can determine the number of border tiles at any stage.

Summary

Analyzing shape patterns can help you understand number patterns. Here is a design made with border toothpicks and border tiles.

Stage 1	Stage 2	Stage 3
		

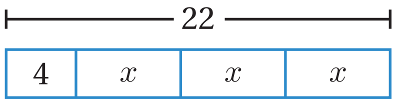
In Stage 1, there are 10 toothpicks. In Stage 2, there are 20 while in Stage 3, there are 30. The number of toothpicks increases by 10 each time. The table shows that the number of tiles is always 4 more than the number of toothpicks.

Stage	Border Toothpicks	Border Tiles
1	10	14
2	20	24
3	30	34

We can extend these rules to make predictions about any stage of the pattern. For example, in Stage 5 there will be 50 toothpicks and the number of border tiles will be 4 greater, 54.

10 Synthesis

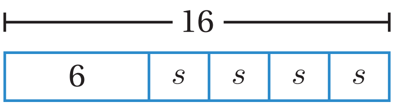
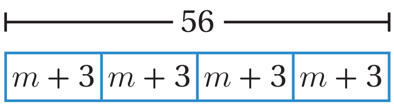
Write a story that this diagram could represent.



Summary

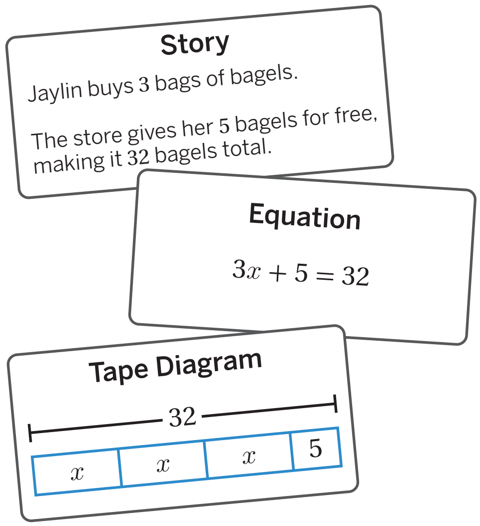
Tape diagrams and expressions can be used to represent stories.

Here are two scenarios and the tape diagrams that represent them.

Scenario A	Scenario B
The local farmer's market is selling fresh fruit and jam. Alma buys 4 pounds of strawberries for s dollars each and 1 jar of raspberry jam for \$6. The total bill is \$16.  $s = 2.5$ Each pound of strawberries costs \$2.50.	4 friends buy lunch from a food truck. The food truck charges a set price of m dollars for a meal and \$3 for a beverage. The total bill is \$56.  $m = 11$ Each meal costs \$11.00.

11 Synthesis

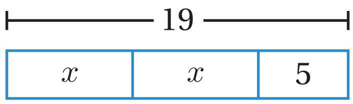
Here is a new situation. Explain how the number 9 is important in each representation.



Summary

We can use tape diagrams and equations to reason about unknown quantities and express how quantities in a scenario are related to each other.

Here is a story with the equation and tape diagram that represent it.

Story	Equation	Tape Diagram
Two students go to the movie theater. They purchase two tickets and a \$5 popcorn to share. In total, they spend \$19.	$2x + 5 = 19$	

In the equation and tape diagram, x represents the unknown price of a movie ticket, 2 represents the number of tickets that were purchased, 5 represents the \$5 spent on popcorn, and 19 represents the total amount spent.

Synthesis

9. Here are two equations and their tape diagrams.

$3x + 15 = 90$	$3(x + 15) = 90$
90: x | x | x | 15	90: $x + 15$ | $x + 15$ | $x + 15$

Describe how the tape diagrams are alike and different.

Summary

We can use a tape diagram to help us make sense of a scenario. Two different tape diagrams and equations can often represent the same scenario.

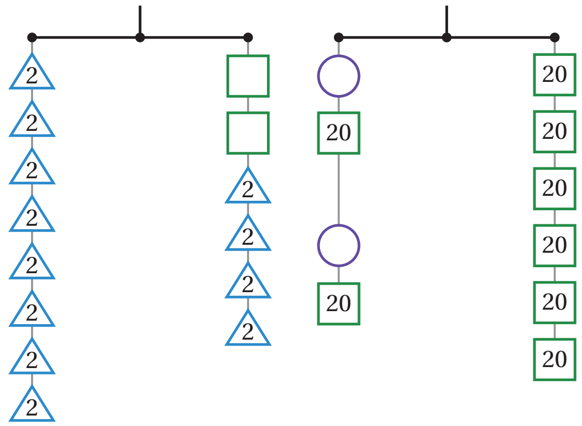
For example, Elena is training for a race. She trained 3 days this week for a total of 21 miles. On each training day, she ran several miles and biked 5 miles. If x represents the number of miles Elena ran, then this scenario can be modeled using two equations and tape diagrams.

$3x + 15 = 21$	$3(x + 5) = 21$
21: x | x | x | 5 | 5 | 5	21: $x + 5$ | $x + 5$ | $x + 5$

We can use either of the tape diagrams and equations to determine that Elena ran 2 miles on each training day.

8 Synthesis

Describe strategies for making a balanced hanger with fewer objects. Use the diagrams if they help with your thinking.



Summary

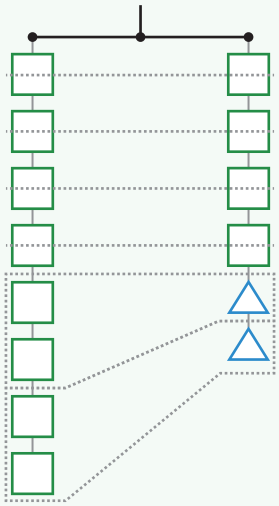
Hangers are balanced when the weight on both sides is the same. To determine unknown values, the same weight should be removed from both sides to maintain balance.

Different strategies can be used to determine unknown values. Here are some examples.

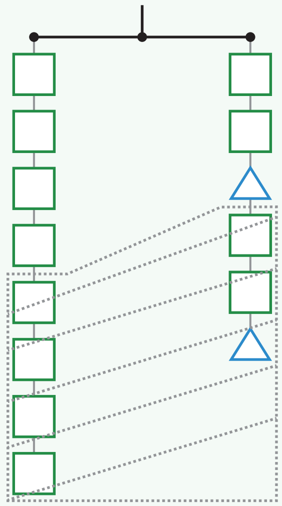
In Hanger A, 4 squares can be crossed off on each side while still keeping the sides balanced. When 4 squares to the left and 2 triangles to the right are balanced, this means they have the same weight. We can see that 1 triangle has the same weight as 2 squares.

In Hanger B, each cluster of 1 triangle and 2 squares on the right has the same weight as 4 squares on the left and can be removed. With fewer objects, we can see that 2 more squares can be removed from each side of the hanger, leaving the weight of 1 triangle equal to the weight of 2 squares.

Hanger A

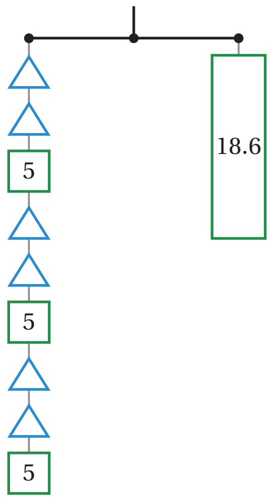


Hanger B



11 Synthesis

Describe how solving an equation is like solving for the weight of an object on a balanced hanger. Use the diagram if it helps with your thinking.



$3(2x + 5) = 18.6$

Summary

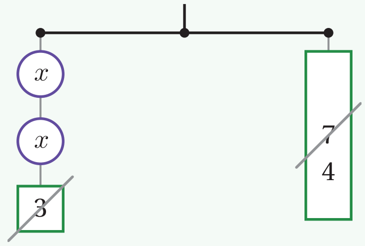
We can use a hanger diagram to represent an equation and help us understand how to find an unknown value in that equation. The steps for finding an unknown value in an equation can be written without using a hanger.

For example, the equation $2x + 3 = 7$ can be solved using these steps:

In the equation, subtract 3 from both sides.

$$2x + 3 = 7$$
$$2x + 3 - 3 = 7 - 3$$
$$2x = 4$$

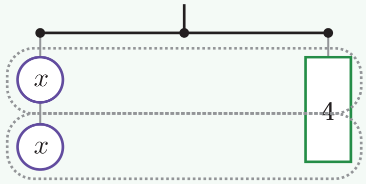
Remove 3 from both sides.



In the equation, divide both sides by 2.

$$2x = 4$$
$$2x \div 2 = 4 \div 2$$
$$x = 2$$

Divide into two equal groups.



Synthesis

12. **a** Write an equation that would belong in Group B.

b What advice would you give to help someone solve an equation like yours?

Group B
$2(x - 1) = -200$
$900 = -100(x - 3)$
$3(x + 4.5) = 36$

Summary

When solving an equation, the same operations should be applied to both sides of the equation at each step so that the equation remains true.

Here are two examples.

Equation 1	Equation 2
$3x - 6 = 9$ $3x - 6 + 6 = 9 + 6$ $3x = 15$ $3x \div 3 = 15 \div 3$ $x = 5$	$3(x - 6) = 9$ $3(x - 6) \div 3 = 9 \div 3$ $x - 6 = 3$ $x - 6 + 6 = 3 + 6$ $x = 9$

The *solution to an equation* is a value of a variable that makes the equation true. You can check your solution by substituting the value in for the variable and evaluating to determine whether the equation is true.

	Equation 1	Equation 2
Checking a Solution	$3(5) - 6 = 9$ $15 - 6 = 9$ $9 = 9$ ✅	$3(9 - 6) = 9$ $3(3) = 9$ $9 = 9$ ✅

Synthesis

10. **a** What are two possible first steps you could use when solving an equation like $6(x + 4) = 30$?

b What are some advantages to having different ways to solve an equation?

Summary

Equations of the form $p(x + r) = q$, where p is a **factored** term, can be solved by either expanding or dividing as the first step.

When **expanding** first, the distributive property allows us to multiply the factored term by each term on the inside of the parentheses.

When dividing first, both sides of the equation are divided by the factored term that is outside of the parentheses.

For example, here are two ways to solve the equation $3(x + 1) = 9$. The first steps are different, but the value of x, the *solution to the equation*, is the same.

Expanding First (using the distributive property)	Dividing First
$3(x + 1) = 9$ $3x + 3 = 9$ $3x + 3 - 3 = 9 - 3$ $3x = 6$ $3x \div 3 = 6 \div 3$ $x = 2$	$3(x + 1) = 9$ $3(x + 1) \div 3 = 9 \div 3$ $x + 1 = 3$ $x + 1 - 1 = 3 - 1$ $x = 2$

12 Synthesis

How can you determine whether two expressions are equivalent to each other? Use these examples if they help with your thinking.

$24 - 8x$

$-4(-6 + 2x)$

$-8(x - 24)$

$\frac{1}{2}(16x - 48)$

$8(x - 3)$

$-24 + 8x$

$8x - 24$

Summary

Expressions that give the same output for every input are called **equivalent expressions**. To determine whether expressions are equivalent, you can test several inputs to see if they produce the same output.

In this example, $3x + 6$ and $3(x + 2)$ are equivalent because they give the same output for every input. You can test other values and they will always give matching outputs. $6(x + 3)$ is not equivalent because it doesn't give the same output for every input (it is only the same when $x = -4$).

x	$3x + 6$	$6(x + 3)$	$3(x + 2)$
10	36	78	36
7	27	60	27
–4	–6	–6	–6

12 Synthesis

Describe how to write an equivalent expression using the fewest number of terms. Use this expression if it helps with your thinking.

$5x - 2(6x - 4)$

Summary

Here are two expressions.

$-\frac{1}{3}(3x - 6)$ $\qquad$ $2x - 4x + 5$

The sum of these expressions can be written as:

$-\frac{1}{3}(3x - 6) + 2x - 4x + 5$

By adding and expanding terms, we can write the sum using the fewest number of *terms*:

$$-\frac{1}{3}(3x - 6) + 2x - 4x + 5$$
$$= -x + 2 - 2x + 5$$
$$= -3x + 7$$

Synthesis

5. There are different ways to solve the equation $2(-3 + 8x) = -10$.

a List two different first steps you could take to solve this equation.

b Which first step do you prefer? Explain your thinking.

Summary

An equation can be solved in many different ways, depending on how the terms are added or expanded.

Here is an equation and one way to solve it.

$-6(3x - 5) = 75$	Rewrite subtracting 5 as adding (-5).
$-6(3x + (-5)) = 75$	Distribute the -6 to the $3x$ and -5.
$-18x + 30 = 75$	Subtract 30 from both sides.
$-18x = 45$	Divide each side by -18.
$x = -2.5$	

Here is a different strategy for solving the same equation.

$-6(3x - 5) = 75$	Divide both sides of the equation by -6.
$3x - 5 = -12.5$	Add 5 to both sides.
$-3x = -7.5$	Divide each side by -3.
$x = -2.5$	

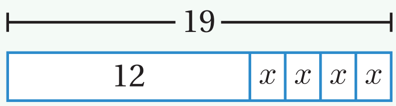
Synthesis

6. What do you think is important to remember when solving problems using visual representations and equations?

Summary

Visual representations and equations can represent and answer questions about situations.

For example, Zahra buys 4 pens and a binder for $\$12$. She pays a total of $\$19$. This tape diagram represents Zahra's situation.



Zahra can determine the price of each pen by writing and solving the equation $12 + 4x = 19$:

$$\begin{aligned} 12 + 4x &= 19 \\ 4x &= 7 \\ x &= 1.75 \end{aligned}$$

Zahra pays $\$1.75$ for each pen.

Synthesis

Dalia used a balancing move to begin to solve the equation $6x + 12 = 10x - 4$.

The result of Dalia's first step was $12 = 4x - 4$. Describe the first step Dalia made to solve the equation.

Summary

A balanced hanger diagram can be represented by an equation.

Hanger diagrams can be used to see that mathematically valid moves create *equivalent equations*, such as:

- Adding the same number to or subtracting the same number from each side of the equation.
- Multiplying or dividing the expressions on each side of an equation by the same non-zero number.
- Adding the same expression to or subtracting the same expression from each side of an equation.

Synthesis

Consider the equation $2x - 6 = 4 - 8x$. There are several ways to solve this equation. What are some different first steps that you could take to solve it? Explain your thinking.

Summary

A **solution** to an equation is a value that makes the equation true. Performing the same operation on each side of an equation results in equivalent equations. Therefore, the solution can be checked by substituting the value of the solution into the original equation. If the resulting equation is true, then the solution is correct.

For example, consider the equation $-x - 8 = 4x + 7$.

Because each operation shown was performed on both sides, all four of these equations are equivalent to each other.

The solution, $x = -3$, can be substituted into any of these equations to make a true equation.

$-x - 8 = 4x + 7$	Add x to each side.
$-8 = 5x + 7$	Subtract 7 from each side.
$-15 = 5x$	
$-3 = x$	Divide both sides by 5.

Synthesis

Solve this equation in two different ways: $5(2x-3)-10=15x-10$.

Summary

There are many ways to solve equations in one variable. Generally, you want to perform steps which will move you closer to an equation where the variable is isolated, such as an equation of the form *variable* $=$ *number*.

You can use these steps to solve an equation:

- Use the Distributive Property to remove any parentheses.
- Collect like terms on each side. (Recall that a **term** is an expression with constants or variables that are multiplied or divided. Like terms have the same variable.)
- Add or subtract expression(s) so that the variable term(s) are by themselves on only one side. Collect like terms, as needed.
- Multiply or divide by the coefficient of the variable, if needed, to obtain an equation of the form *variable* $=$ *number*.

Some of these steps can be performed in a different order if it makes the process more efficient. Just remember . . .

- Always maintain equality by using balancing moves when moving terms across the equal sign.
- You can always check your solution by substituting the value into the original equation and evaluating to see whether the resulting equation is true.

Synthesis

1. Write an equation that you would consider challenging to solve.

2. What makes your equation challenging to solve?

3. What are some strategies that you know for solving equations like this?

Summary

Sometimes you are asked to solve equations that contain more complicated expressions than just a variable term and a constant. (Recall that a **constant term** is a value that does not change, meaning it is not a variable.)

For example, consider the equation $3(4-2x)+6=4-2x$.

$3(4-2x)+6=4-2x$	
$12-6x+6=4-2x$	Use the Distributive Property.
$18-6x=4-2x$	Combine like terms on each side.
$18=4+4x$	Add $6x$ to each side.
$14=4x$	Subtract 4 from each side.
$\frac{14}{4}=x$	Divide each side by the coefficient.

So, $x=\frac{7}{2}$

Synthesis

Discuss How do you know whether an equation will be true for all values of x, one value of x, or no values of x? Write an equation that is true for:

a No values of x

b All values of x

c One value of x

Summary

It is possible for some equations to have one solution, no solutions, or **infinitely many solutions**.

Here are some examples.

One solution:

$$3x + 8 = 6 + 2 - 3x$$
$$3x + 8 = 8 - 3x$$
$$6x + 8 = 8$$
$$6x = 0$$
$$x = 0$$

This equation is only true when $x = 0$.

A linear equation has *one solution* when the coefficients are different on each side.

No solutions:

$$3(x + 4) = 3x + 7$$
$$3x + 12 = 3x + 7$$
$$12 = 7$$

This equation is never true for any value of x.

A linear equation has *no solutions* when the coefficients are the same on each side, but the constants are not.

Infinitely many solutions:

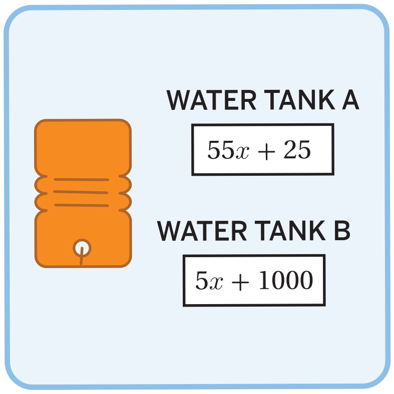
$$10 - 3x = 8 - 3x + 2$$
$$10 - 3x = 10 - 3x$$
$$10 = 10$$

This equation is always true for any value of x.

A linear equation has *infinitely many solutions* when the coefficients and constants are the same on each side.

Synthesis

The image shows expressions that represent the amount of water, in liters, in two water tanks. Let x represent the number of seconds that pass. How could you determine when the tanks will have the same amount of water?



Summary

Two expressions can be written to represent relationships between two different variables for the same real-world scenario. These expressions can be set equal to each other to form one equation — in one variable — which can be solved.

For example, imagine two hikers walking toward each other on a flat trail. Consider when the hikers will meet each other. This is when they are at the same location on the trail at the same time.

- To determine when this time occurs, an expression can be used to represent the location of each hiker.

  Hiker 1 location: Expression 1 **Hiker 2 location:** Expression 2

- These two expressions can be set equal to each other to form one equation that can be solved.

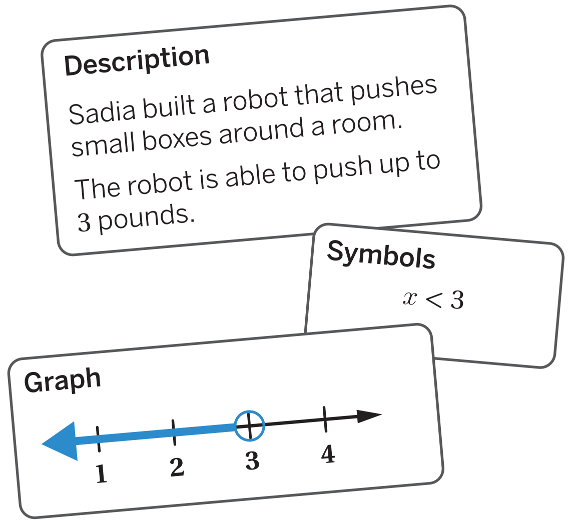
  Expression 1 = Expression 2

12 Synthesis

Circle one representation and explain how it shows that Sadia's robot can push a 2-pound box.

Description Symbols Graph

Explain your thinking.

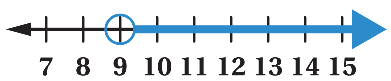


Summary

You can use variables, verbal descriptions, symbols, and number lines to represent inequalities related to real-world situations.

To represent an inequality on a number line, you can shade part of the number line to indicate that every point covered by the shaded region is a solution. This means every rational number in the shaded region is a solution. Then draw an arrow on one end of the number line to show the possible solutions continue on forever in that direction.

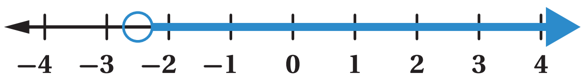
Here is an example.

Situation	Solution's Description	Inequality	Number Line
A two-year-old sleeps more than 9 hours a day.	Any value greater than 9.	$x > 9$	

13 Synthesis

What does it mean for a number to be a *solution to an inequality*?

$x > -2.5$



Summary

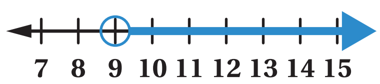
You drew and labeled number line diagrams to represent solutions to an inequality. A **solution to an inequality** is any value that makes the inequality true.

For example, for the inequality $c < 10$ you could say:

- 5 is a solution because $5 < 10$ is a true statement, and
- 12 is not a solution because $12 < 10$ is not a true statement.

Inequalities with a variable like $c < 10$ have an infinite number of solutions — also called the solution set. We use inequality statements with variables and symbols $<$ or $>$ to represent all the solutions without having to list them.

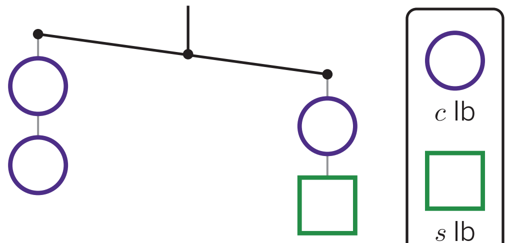
Here is an example.

Inequality	Description	Possible Solutions	Number Line
$x > 9$	Any value greater than 9.	$9.75, 10, 11.3, 82$	

9 Synthesis

Describe how an inequality is like an unbalanced hanger.

Use the diagram if it helps with your thinking.

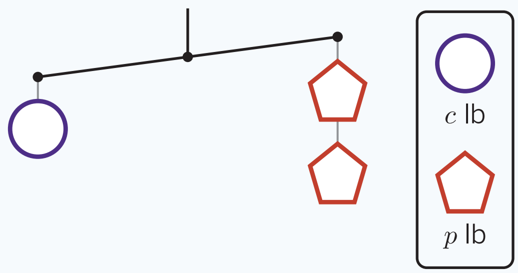


$2c < c + s$

Summary

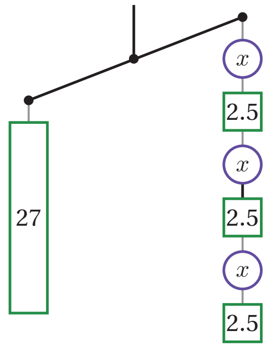
A hanger is balanced when the total weights on both sides are equal, so inequalities will be represented by unbalanced hangers. Inequalities may include a *variable* as a placeholder for an unknown value.

For example, this hanger shows that the weight of the circle, c, is heavier than the weight of two pentagons, $2p$. This relationship can be represented by the inequality $c > 2p$ because these symbols mean that the circle has a greater weight than 2 pentagons.

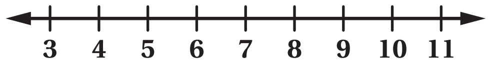


12 Synthesis

Describe a process you can use to determine the solutions to an inequality.
Use the hanger if it helps show your thinking.



$27 > 3(x + 2.5)$



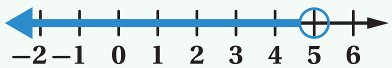
Summary

You can solve an inequality in similar ways that you solve an equation to determine the values of x that make the inequality true. These values are known as the **solutions to an inequality**. You can test values by substituting them into the inequality.

For example, consider the inequality $4x + 2 < 22$.

- To determine the value of x that balances the hanger, solve the equation $4x + 2 = 22$.
- When $x = 5$, the hanger is balanced. All values less than 5 will make the inequality true because $4x + 2$ needs to be less than 22.

The solution shown on the graph means that all values of x *less than* 5 will make the inequality true.



To check the solution, substitute any value less than 5 into the original inequality.

$$4(4) + 2 < 22$$

$$16 + 2 < 22$$

$$18 < 22$$ ✓

Synthesis

6. Tay has a $\$30$ gift card to Tea Time Cafe. They spend $\$2.50$ on a tasty beverage every school day. Tay wants to know how many beverages they can buy using the gift card. Explain how the inequality $30 - 2.50x \geq 0$ represents Tay's situation.

Summary

Inequalities can be used to solve real-world problems, like budgeting.

For example, Aditi has $\$5$ and sells homemade greeting cards for $\$1.50$ each. Her goal is to have $\$20$ total.

The solution to the equation $1.50x + 5 = 20$ represents the number of greeting cards, x, that she needs to sell in order to have exactly $\$20$. If she sells 10 greeting cards, she will have $\$20$ because $1.50(10) + 5 = 20$.

What if Aditi wants to have more than $\$20$? The inequality $1.50x + 5 > 20$ represents this situation.

The solution to the previous equation can help us make sense of the solution to the inequality $x > 10$. Aditi will need to sell *more than* 10 cards to have *more than* $\$20$ total.

Aditi

$1.50x + 5 = 20$

$1.50x + 5 - 5 = 20 - 5$

$1.50x = 15$

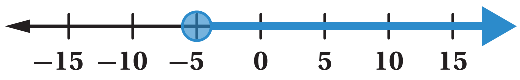
$x = 10$

10 cards makes $20

x > 10 represents making more than $20

10 Synthesis

Explain how you solve and graph the solutions to any inequality. Use the inequality $4 - 3x \leq 19$ if it helps you with your thinking.



Summary

When solving an inequality, it can help to start by solving a related equation. The solution to the equation tells you the *boundary point* of the graph of the *solutions to the inequality*. Once you determine the boundary point, you still need to decide whether the solutions include values greater or less than the boundary point. This can be done by testing a value to the right or left of the boundary point on the number line.

Here is the inequality $-3x + 6 < 18$ and steps for solving it.

Equation	Explanation
$-3x + 6 = 18$	Write the inequality as an equation.
$-3x + 6 - 6 = 18 - 6$	Subtract 6 on both sides.
$-3x \div (-3) = 12 \div (-3)$	Divide by -3 on both sides.
$x = -4$	This is the boundary point.

You can show the solution on a number line by drawing an open circle on -4, the boundary point. To determine if the solutions to the inequality are to the right or left of -4, choose a value, such as 0, to test in the original inequality.

In this case, 0 is to the right of -4 on the number line. $-3(0) + 6 < 18$ is true, which means all values to the right of -4 are solutions to this inequality. The solutions to the inequality are $x > -4$.

Synthesis

6. Sahana works at the pet store and gets paid $9.50 per hour. She needs to make at least $235 each week in order to pay her bills. Describe how to write an inequality that represents Sahana's situation.

Summary

Inequalities can be used to represent and solve real-world problems. When writing an inequality, it can be helpful to first decide what quantity the variable represents. After making that decision, write an inequality based on the relationships between the quantities in the situation.

As you solve the inequality, keep the meaning of each quantity in mind. This helps determine if the solutions to the inequality make sense in the context of the situation. When interpreting solutions, sometimes those with only whole number values (e.g., number of people, number of buses, items that can be purchased, etc.) make sense while other solutions can include decimal or fractional values (e.g., height of a roller-coaster rider, weight of a package, etc.).