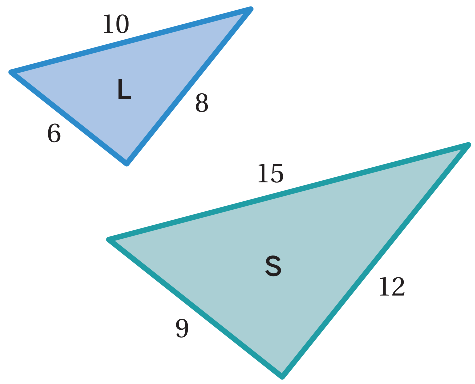
# Scale Drawings, Dilations, and Similarity

# Accelerated 7 Unit 2

## Synthesis

How can you use lengths to tell whether a figure is a scaled copy of another figure? Use Figures L and S to help you with your explanation.




*All measurements are in units.*

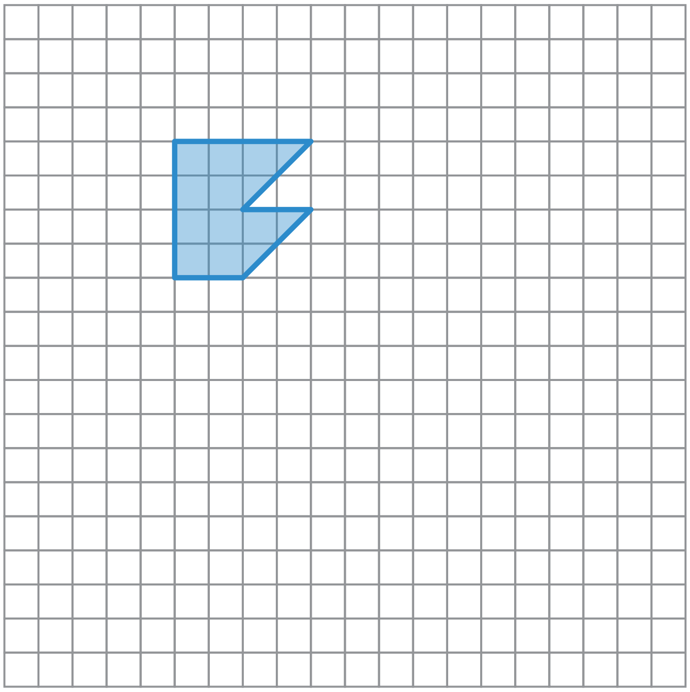
## Summary

The **scale factor** relates the lengths of one scaled copy to another. A figure can be considered a scaled copy of another figure if it appears to be the same shape and all side lengths are related by the same scale factor. If they cannot be related by the same scale factor, then the figures cannot be scaled copies.

## Synthesis

Describe how to draw a *scaled copy*. Include a few important things to remember.

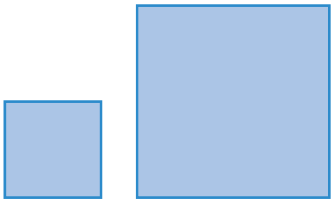
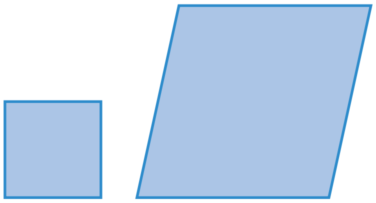
Use the figure shown if it helps with your thinking.



## Summary

Creating a *scaled copy* means making sure all the side lengths are multiplied by the same scale factor.

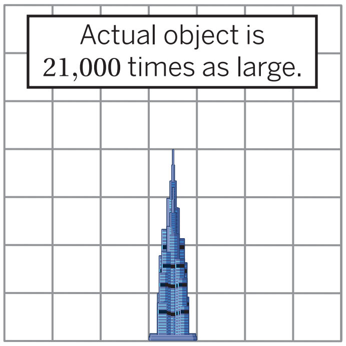
A scaled copy may have different side lengths than the original, but the angle measures will stay the same.

| Scaled copies | Not scaled copies |
|---|---|
|  |  |

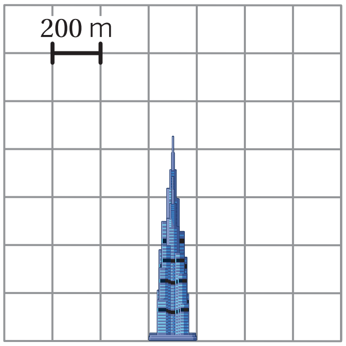
## Synthesis

How are *scale factor* and *scale* alike? How are they different?

**Scale Factor**




**Scale**




| Alike | Different |
|---|---|
| | |

## Summary

**Scale** tells how actual measurements are represented on the drawing. The line segment indicates what 1 unit on the grid represents in the actual object.

For example, consider a building that is placed on a grid with a scale of "1 unit to 10 meters." This scale represents the ratio 1 unit to 10 meters which means that every 1 grid unit represents 10 meters. If the building is 3 units tall on the grid, then this represents its actual height of 30 meters.

## Synthesis

Explain how you could use Karima's scale drawing to calculate the actual distance across the center court circle.

| | Distance across the center court circle |
|---|---|
| Scale drawing | 1.8 cm |
| Actual court | |

## Summary

**Scale drawings** are two-dimensional representations of actual objects or places. Floor plans and maps are examples of scale drawings.

On a scale drawing:

- Every part or section corresponds to a part or section in the actual object.
- Lengths on the drawing are enlarged or reduced by the same scale factor.
- A scale tells you how actual measurements are represented on the drawing. For example, if a map has a scale of "1 inch to 5 miles," then a 0.5-inch line segment on that map would represent an actual distance of 2.5 miles.

A scale drawing may not show every detail of the actual object. However, the features that are shown correspond to the actual object and follow the specified scale.

## Synthesis

What do you think will always be the same about scale drawings of the same object? What do you think can be different? Use the scale drawings created in Activity 2 to help with your thinking.

## Summary

When creating a scale drawing, keep these things in mind:

- When going from an original figure to a scale drawing, multiply each side length by the same scale factor.
- Draw the angles so they have the same measure in the original figure and in the scale drawing.

For example, to create a scale drawing of a room's floor plan that has the scale shown, multiply the actual lengths in the room (in feet) by $\frac{1}{4}$ to determine the corresponding lengths (in inches) for your drawing.

**Scale:**

1 inch on the drawing is equal to 4 feet in the room

## Synthesis

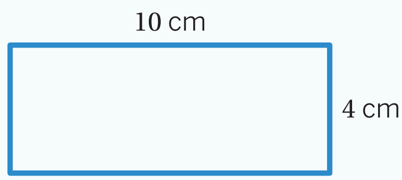
Suppose there are two scale drawings of the same building. Drawing A uses the scale 1 centimeter to 2 meters and Drawing B uses the scale 1 centimeter to 4 meters.

Which drawing is larger? Explain your thinking.

## Summary

The scales of drawings can be changed. As the scale decreases, the size of the scale drawing increases.

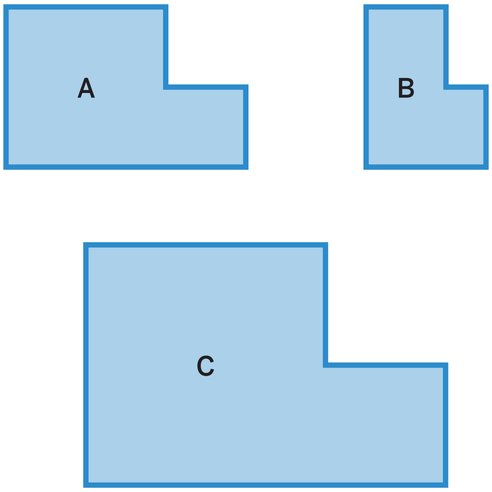
Suppose a scale drawing of a rectangular park with dimensions 360 meters by 900 meters was created using a scale of 1 centimeter to 90 meters. The dimensions of the scale drawing are 4 centimeters by 10 centimeters as shown.




Suppose you now want to create a new scale drawing of the park using a scale of 1 centimeter to 30 meters. The dimensions of the new scale drawing will be 12 centimeters by 30 centimeters. The size of the new scale drawing will be larger than the previous because each centimeter represents a shorter distance.

## Synthesis

Which figure could be a result of a dilation to Figure A? Explain your thinking.

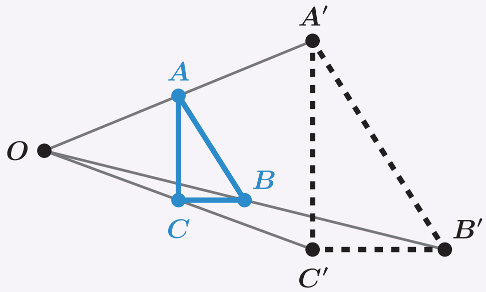



## Summary

A **dilation** is a type of transformation that creates scaled copies.

Triangle $ABC$ was dilated using a fixed point $O$ to create Triangle $A'B'C'$.

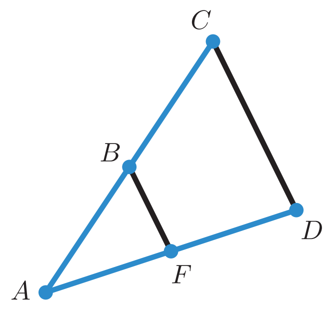
The triangles are scaled copies of one another.



## Synthesis

Triangle $ABF$ was dilated with a scale factor of 2.

Precisely explain the steps for dilating $AF$ using the same center and a scale factor of 6.

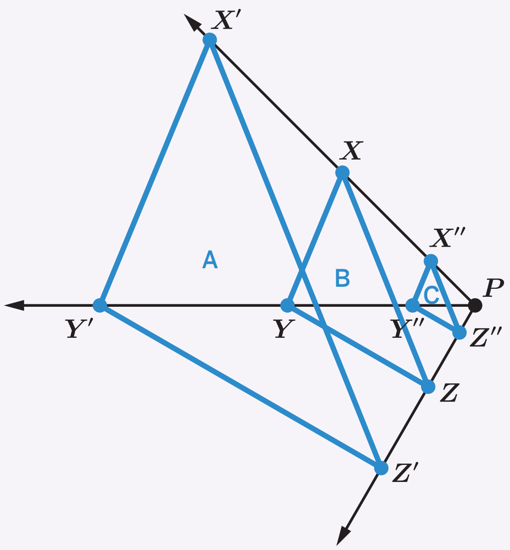



## Summary

A *dilation* is a transformation which is defined by a fixed point $P$, called the **center of dilation**, and a **scale factor**, $k$.

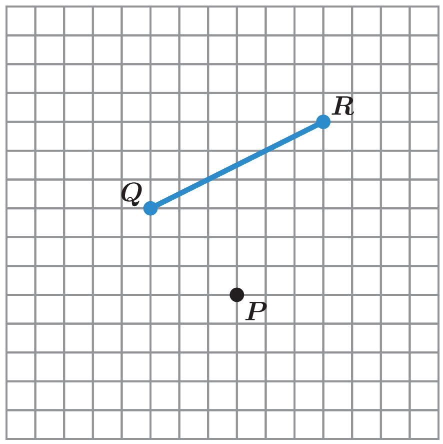
In the figure shown, the dilations move Triangle $XYZ$ to Triangles $X'Y'Z'$ and $X''Y''Z''$, such that the distances of the vertices from $P$ change by the scale factor. The *scale factor* is determined by finding the ratio of the distance between the center of dilation and the image, and the distance between the center of dilation and the pre-image.

- Triangle $XYZ$ is dilated using point $P$ as the center of dilation and a scale factor of 2 to produce Triangle $X'Y'Z'$.
- Triangle $XYZ$ is dilated using point $P$ as the center of dilation and a scale factor of $\frac{1}{3}$ to produce Triangle $X''Y''Z''$.



## Synthesis

Describe how to dilate segment $QR$ with center $P$ and a scale factor of $\frac{1}{3}$.




## Summary

Square grids can be useful for showing dilations, especially when the center of dilation and the point(s) being dilated are marked at the intersections of grid squares.

Instead of using a ruler to measure the distance between the points, counting grid units can help determine the location of the image.

Compared to the pre-image, a dilation with:

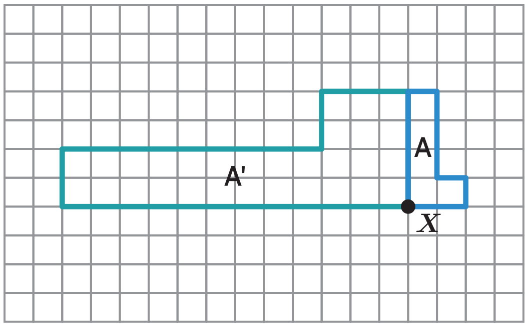
- A scale factor between $0$ and $1$ produces a smaller image.
- A scale factor greater than $1$ produces a larger image.
- A scale factor equal to $1$ produces a congruent image.

## Synthesis

Camila says the following sequence of transformations will move the pre-image onto the image:

- Rotate Figure A $90^\circ$ counterclockwise around point $X$.
- Dilate using $X$ as the center of dilation.

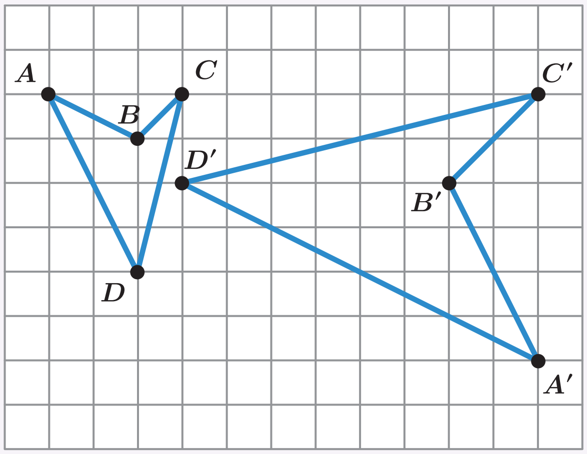
Do you agree or disagree with Camila? Explain your thinking.




## Summary

Two figures are scaled copies of each other if there is a sequence of reflections, rotations, translations, or dilations that moves one figure onto the other.

For example, here is one sequence of transformations that moves Polygon $ABCD$ onto Polygon $A'B'C'D'$.




**Step 1:** Dilate Polygon $ABCD$ using point $D$ as the center of dilation and a scale factor of 2.

**Step 2:** Translate the image so that point $D$ moves onto point $D'$.

**Step 3:** Rotate the new image $90^\circ$ clockwise around point $D'$.

**Step 4:** Reflect the new image across the horizontal line that contains points $D'$ and $B'$.

## Synthesis

How are coordinates useful when describing and drawing dilations?

## Summary

One important use of coordinates on a coordinate plane is to communicate geometric information precisely. Performing a dilation of a polygon requires three essential pieces of information:

1. The coordinates of the vertices.
2. The coordinates of the center of dilation.
3. The scale factor of the dilation.

With this information, any dilation of a figure can be precisely described and drawn.

To get the coordinates of the point $(x, y)$ after dilating it with center $(0, 0)$ and a scale factor of $k$, multiply both coordinates by $k$.

For example, a dilation using $(0, 0)$ as the center of dilation and a scale factor of $3$ changes the coordinates of point $(-2, 4)$ to $(-6, 12)$.

## Synthesis

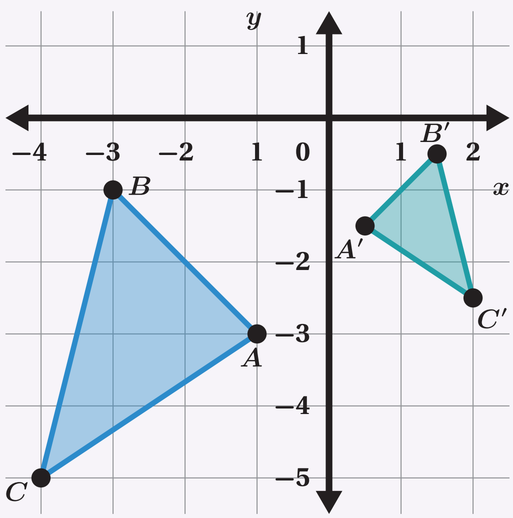
Determine whether each statement is *always true*, *sometimes true*, or *never true*. Place a check mark in the appropriate column.

| Statement | Always true | Sometimes true | Never true |
|---|---|---|---|
| **1.** If two figures are congruent, then they cannot be similar. | | | |
| **2.** If two figures are similar, then they are congruent. | | | |
| **3.** Two figures are similar if they can be moved onto one another through a sequence of transformations. | | | |
| **4.** If two figures have congruent corresponding angles and a common scale factor between corresponding sides, then the figures are similar. | | | |

## Summary

Figures are **similar** if one figure can be moved onto another by a sequence of dilations and rigid transformations. There may be many correct sequences of transformations, but you only need to describe one to show that two figures are similar.

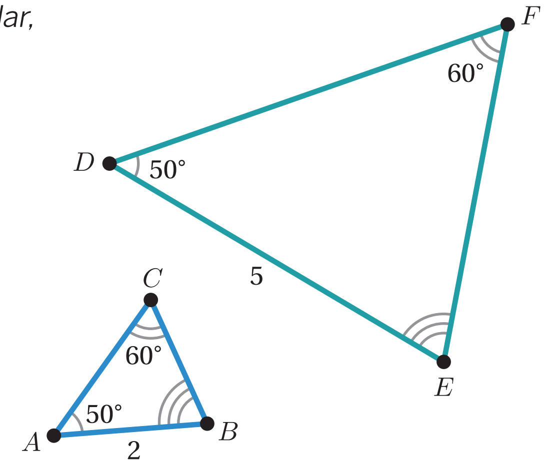
The symbol ~ indicates that two figures are similar. In the diagram, Triangle $ABC$ ~ Triangle $A'B'C'$. Here is one sequence of transformations that moves Triangle $ABC$ onto Triangle $A'B'C'$.




- Rotate Triangle $ABC$ $180°$.
- Dilate the image with center $(0, 0)$ and with a scale factor of $\frac{1}{2}$.
- Reflect the image across the $x$-axis.

## Synthesis

Determine if the triangles are *similar, not similar,* or if there is *not enough information*. Explain your thinking.

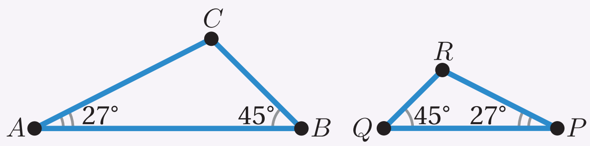



*The figures may not be drawn to scale.*

## Summary

For two triangles, one pair of corresponding congruent angles is not enough to determine whether the triangles are similar. However, two corresponding congruent angles is enough to show that two triangles are similar. These triangles actually share three pairs of corresponding congruent angles because the sum of the interior angles of any triangle is $180°$, so the measure of the unknown third angle must be the same value for both triangles.

For example, for Triangles $CAB$ and $RPQ$ the unknown angle for both triangles is $108°$. There are three corresponding congruent angles, so Triangle $CAB$ is similar to Triangle $RPQ$.

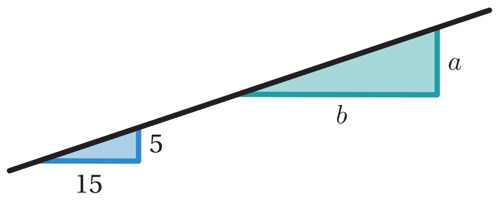



*The figures may not be drawn to scale.*

## Synthesis

**These two triangles are similar.**

Determine the value of $\frac{a}{b}$. Explain your thinking.




## Summary

The ratio of a pair of side lengths in one triangle is equal to the ratio of the corresponding side lengths in similar triangles. For a pair of similar triangles, a missing side length can be calculated by using the ratios of side lengths within a triangle or by using the scale factor between the triangles.

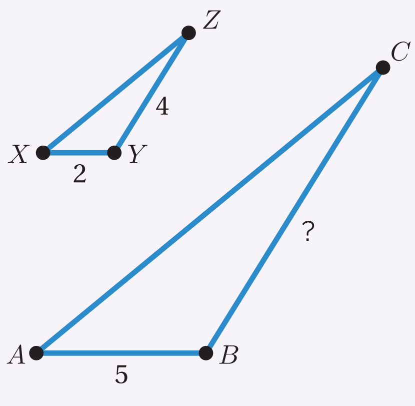
For example, consider similar Triangles $ABC$ and $XYZ$. Here are two methods that can be used to determine the length of side $BC$.

**Method 1: Use the scale factor *between* the triangles.**

The ratio of the corresponding side lengths $AB$ to $XY$ is $5:2$, so the scale factor is $2.5$. Multiply $YZ$ by the scale factor to determine $BC$; $4 \cdot 2.5 = 10$. The length of side $BC$ is $10$ units.

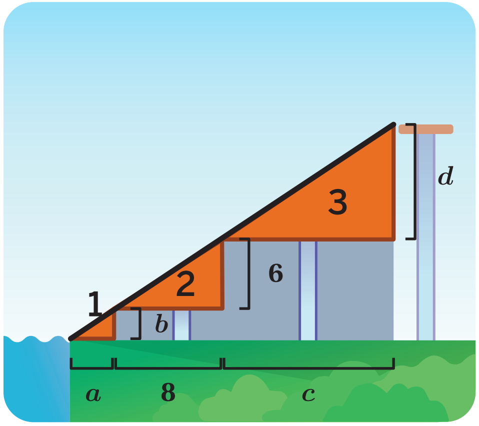
**Method 2: Use ratios of side lengths *within* one triangle.**

The ratio of $YZ$ to $XY$ is $4:2$, or $2$. Therefore, $BC$ is twice the length of $AB$; $5 \cdot 2 = 10$. The length of side $BC$ is $10$ units.



## Synthesis

The dimensions of Ramp 2 are shown.
What must be true about the dimensions of Ramps 1 and 3 for the slide to be smooth?




## Summary

The three ramps in the Synthesis will result in a smooth ride on the water slide because all three triangles are similar with corresponding side lengths that are in proportion.

In order for the longest side of each triangle to line up, the **slope** of each triangle must be the same. The *slope* is determined by the height-to-base ratio of each triangle.

Consider these three ramps on a grid.

The height-to-base ratio of Ramp 2 is $\frac{6}{8}$, or $\frac{3}{4}$.

That means the height-to-base ratio of Ramp 1 and Ramp 3 is also $\frac{3}{4}$, and the line formed has a slope of $\frac{3}{4}$.