Rigid Transformations and Congruence

Accelerated 7 Unit 1

Synthesis

Select one choice and describe what it does to a shape in your own words.

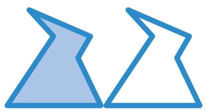
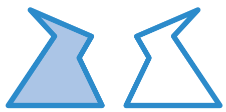
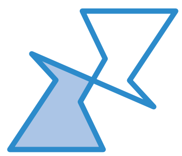
A. Translation

B. Reflection

C. Rotation

Summary

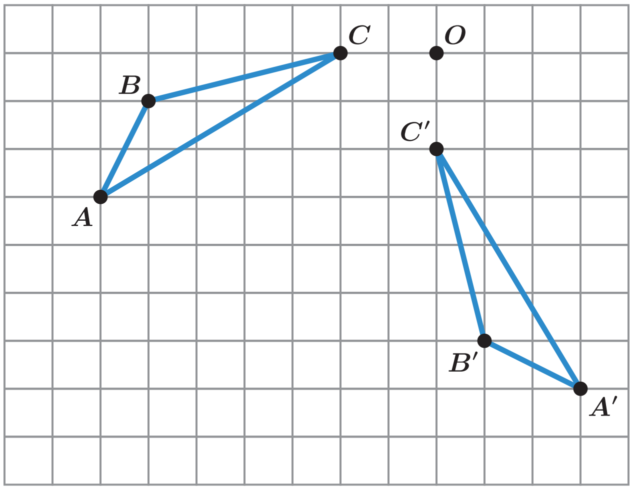
The terms **translation**, **reflection**, and **rotation** can be used to describe how a figure moves.

Translation	Reflection	Rotation
		
Slides a figure	Flips a figure	Spins a figure around a point

Synthesis

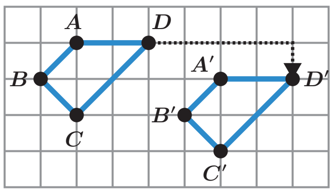
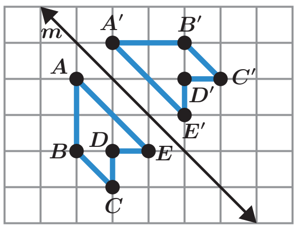
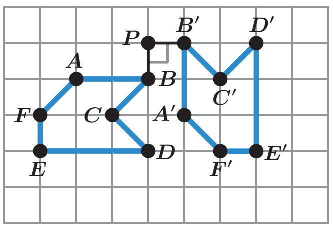
What details should you include when precisely describing a transformation?

Use this example if it helps with your thinking.



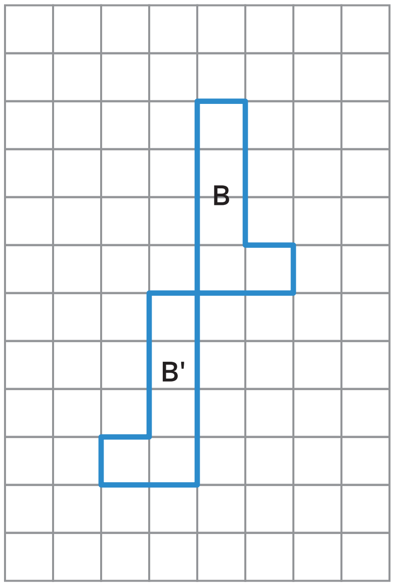
Summary

Translations, reflections, and rotations are all examples of **transformations**. The term **image** describes the new figure created by moving the original figure, the **pre-image**. Parts of the image **correspond** to parts of the pre-image.

Translation	Reflection	Rotation
Translation 4 units right and 1 unit down. • The grid shows that each point moved to the right 4 units and down 1 unit.	Reflection across line m. • The grid shows that the distance from each vertex to line m in the pre-image is maintained in the reflected image.	Rotation 90° **counterclockwise** (or 270° **clockwise**) around point P. • The grid shows that the distance between the center of rotation and each vertex of the pre-image is preserved in the rotated image. • Each of these distances forms a 90° angle with point P in the counterclockwise direction.
		

Synthesis

Describe a sequence of transformations that will move the pre-image onto the image.



Summary

When more than one transformation is applied to a pre-image, that series of moves is called a **sequence of transformations**. There can be more than one sequence of transformations that moves a pre-image onto an image.

Synthesis

If you know the coordinates of a pre-image, how can you determine the coordinates of the image after . . .

a A reflection across the x-axis?

b A translation 2 units right and 3 units up?

Summary

Coordinates can be used to describe the position of points and determine patterns in the coordinates of transformed points.

Reflecting a point across the . . .

- x-axis makes the y-value of the point in the image the opposite of the y-value of the point in the pre-image. The x-coordinate remains the same.
- y-axis makes the x-value of the point in the image the opposite of the x-value of the point in the pre-image. The y-coordinate remains the same.

Translating a point . . .

- To the left or the right changes the value of the x-coordinate.
- Up or down changes the value of the y-coordinate.

Synthesis

How can knowing the coordinates of points help you rotate a point or figure?

Summary

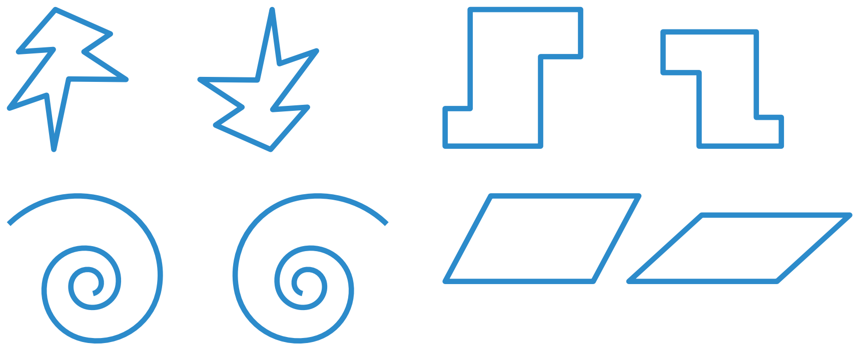
Coordinates can be used to describe points and determine patterns in the coordinates of transformed points. Rotating a point around the origin results in an image whose coordinates are related to the coordinates of the pre-image, as follows.

$90°$ counterclockwise or $270°$ clockwise	$90°$ clockwise or $270°$ counterclockwise	$180°$ in either direction
The x- and y-coordinates switch places. The x-coordinate of the image has the opposite sign of the y-coordinate of the pre-image. **Example:** **Pre-image:** $(-3, 2)$ **Image:** $(-2, -3)$	The x- and y-coordinates switch places. The y-coordinate of the image has the opposite sign of the x-coordinate of the pre-image. **Example:** **Pre-image:** $(-3, 2)$ **Image:** $(2, 3)$	The order of the x- and y-coordinates of the image stay in the same place as the pre-image, but have opposite signs. **Example:** **Pre-image:** $(-3, 2)$ **Image:** $(3, -2)$

Synthesis

How can you determine whether two figures are congruent?

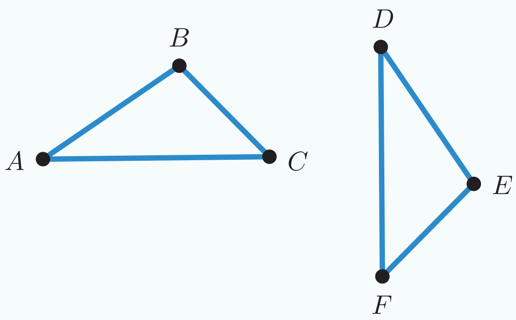
Use the images shown if they help your thinking.



Summary

Two figures are **congruent** if one figure moves onto the other figure by using a sequence of rigid transformations.

The congruence symbol ≅ can be used to show that two figures are congruent. For example, $\Delta ABC \cong \Delta DEF$ means that the two triangles are congruent. The statement is read as, "Triangle ABC is congruent to Triangle DEF." The vertices in a congruence statement are listed in the same order to show corresponding parts.



Synthesis

Select all of the statements that are enough to claim that two figures are congruent.

A. The figures are both right triangles.

B. The figures are both isosceles right triangles.

C. You can trace one figure on tracing paper and move it perfectly on top of the other.

D. The figures are both rectangles that have areas of 8 square units.

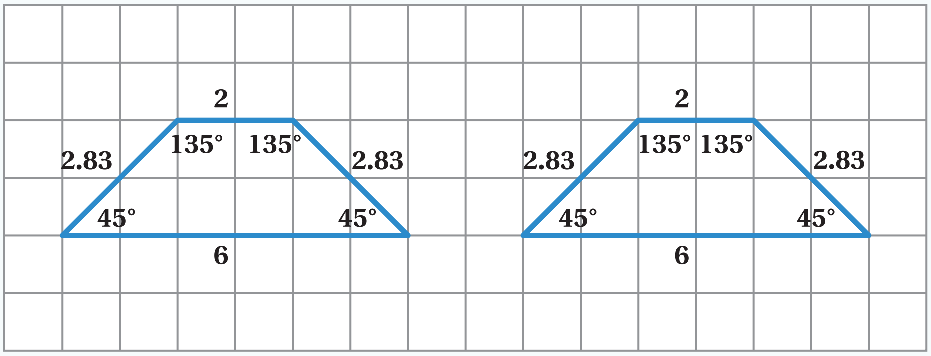
E. You can move one figure on top of the other by translating 8 units left.

Summary

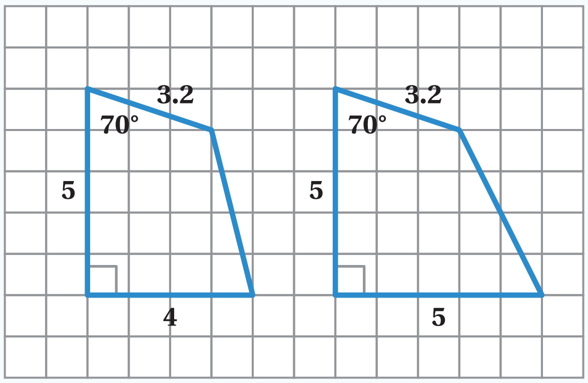
Two figures are congruent when a rigid transformation or sequence of rigid transformations moves one figure onto the other. Recall that translations, reflections, and rotations are rigid transformations. That means that in congruent figures, corresponding sides have the same length and corresponding angles have the same measure.

Two figures are not congruent if their corresponding side lengths or angle measures are not the same, or if they have different perimeters or areas. Even if some of the corresponding side lengths or angle measures are the same, the figures are not congruent unless *all* of the corresponding parts have the same measure.

Congruent



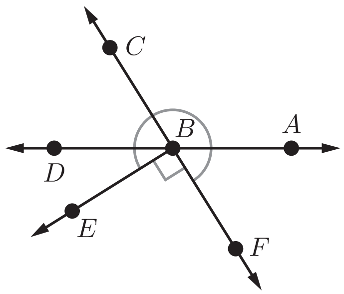
Not congruent



8 Synthesis

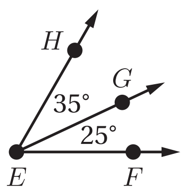
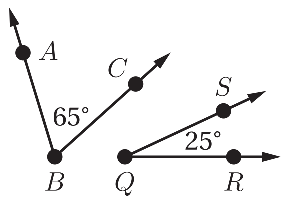
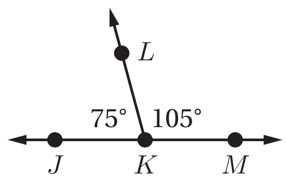
Here is a diagram. Describe or show as many angle relationships as you can.

Use the terms *complementary* and *supplementary* in your description.



Summary

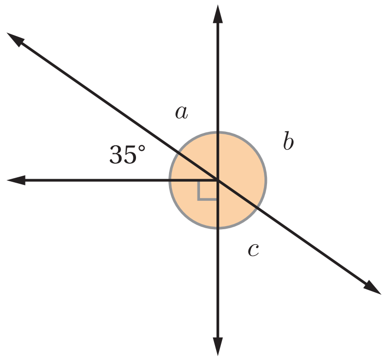
Here are three angle relationships that can help you determine missing angle measures.

Adjacent Angles	Complementary Angles	Supplementary Angles
Adjacent angles share a side and a vertex.	**Complementary angles** have measures that add up to $90°$.	**Supplementary angles** have measures that add up to $180°$.
		
$\angle HEG$ and $\angle FEG$ are adjacent angles.	$\angle ABC$ and $\angle RQS$ are complementary angles.	$\angle JKL$ and $\angle MKL$ are supplementary angles.

12 Synthesis

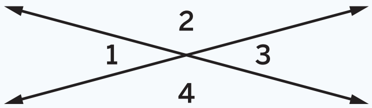
Describe what you know about vertical angles.

Use the example if it helps with your thinking.



Summary

When two lines cross, the angles that are opposite each other have the same measure. These angles are called **vertical angles**.



$\angle 1$ and $\angle 3$ are a pair of vertical angles. Another pair is $\angle 2$ and $\angle 4$.

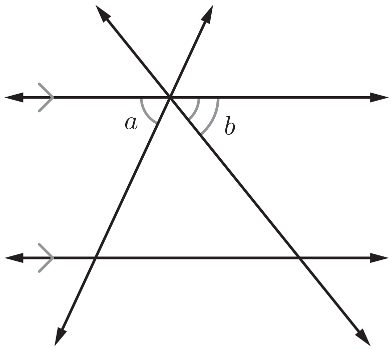
Using vertical angles can help determine missing angle measures.

For example, if the measure of $\angle 1$ is 30°, then:

- The measure of $\angle 3$ is 30° because $\angle 1$ and $\angle 3$ are *vertical angles*.
- The measure of $\angle 2$ is 150° because $\angle 1$ and $\angle 2$ are *supplementary angles*.
- The measure of $\angle 4$ is 150° because $\angle 2$ and $\angle 4$ are *vertical angles*.

Synthesis

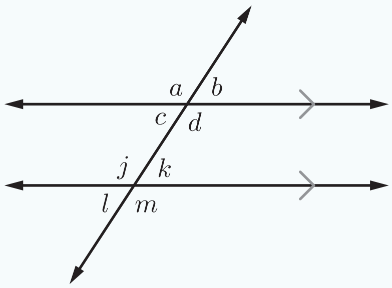
Consider the diagram shown.



1. Mark *all* the angles that are congruent to angle a using one arc.
2. Mark *all* the angles that are congruent to angle b using two arcs.
3. Explain how you know that each angle is congruent by describing the transformations you might use.

Summary

A rigid transformation moves lines to lines and parallel lines to parallel lines.



Vertical angles are opposite angles that share the same vertex.

- They are formed by a pair of intersecting lines.
- A rotation or reflection can be used to show that vertical angles are congruent.

Congruent angles can be written using the symbols $\angle$ and $\cong$. For example, $\angle a \cong \angle d$ is read as, "Angle a is congruent to angle d."

A **transversal** is a line that intersects parallel lines.

- *Interior angles* are formed when parallel lines are cut by a transversal. These angles lie inside the parallel lines.
- *Alternate interior angles* lie inside the parallel lines, but on opposite sides of the transversal.
- Alternate interior angles are congruent.

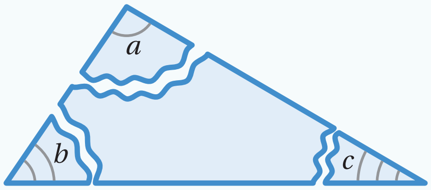
Vertical angles	Alternate interior angles
$\angle a \cong \angle d$	$\angle c \cong \angle k$
$\angle b \cong \angle c$	$\angle d \cong \angle j$
$\angle j \cong \angle m$	
$\angle k \cong \angle l$	

Synthesis

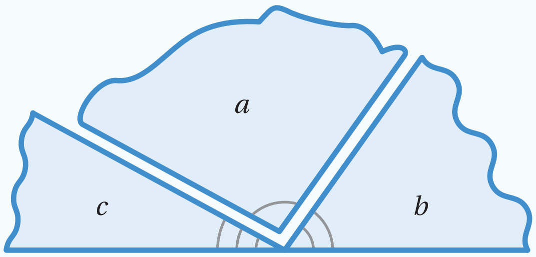
What is true about the sum of the measures of the three angles in a triangle? Explain your thinking.

Summary

The three angles in a triangle can be arranged to form a straight angle. A straight angle measures $180°$, so the sum of the measures of the *interior angles* of a triangle is $180°$.

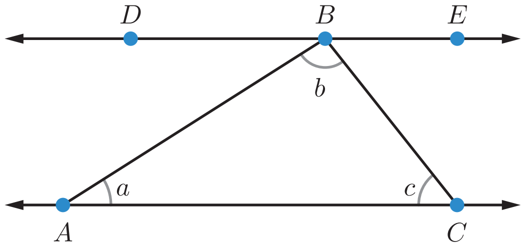


If the measures of two angles in a triangle are known, the third angle can be determined by subtracting the sum of the measures of the two known angles from $180°$.



Synthesis

Explain why the sum of the angle measures in any triangle is $180°$. Use the diagram if it helps your thinking.

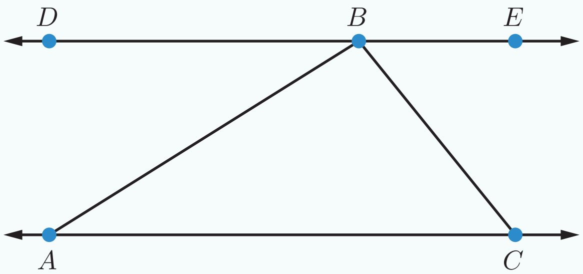


Summary

Rigid transformations, parallel lines, and angle relationships can be used to informally establish that the sum of the interior angles in *any* triangle is always $180°$.

Refer to parallel lines DE and AC.

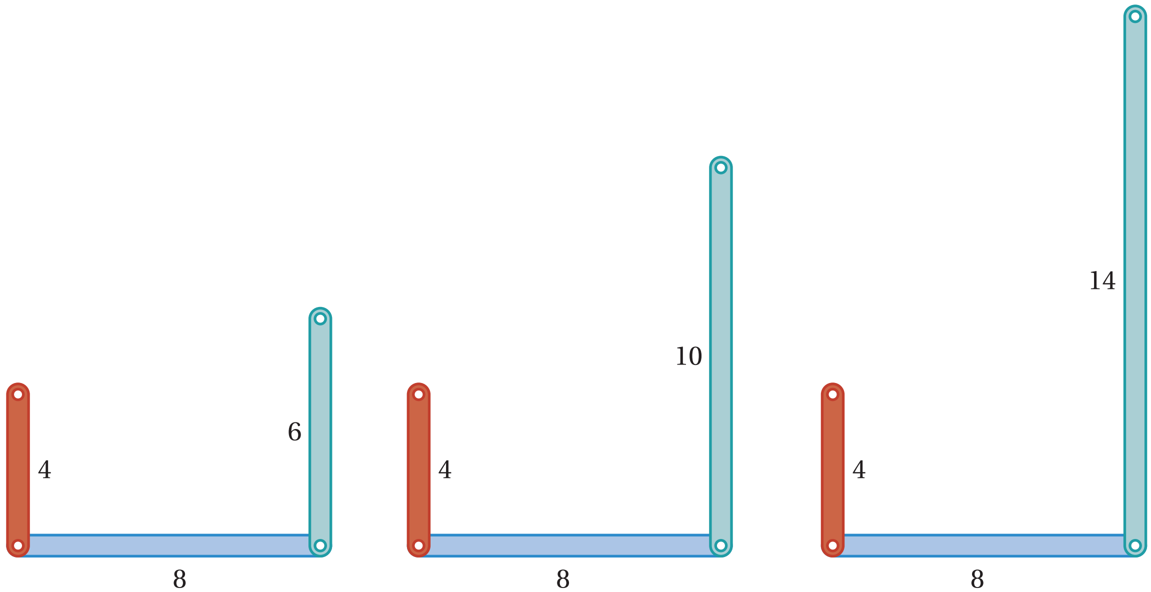
- Angles ABD and BAC are congruent and angles ACB and CBE are congruent because the angles in each angle pair are alternate interior angles.
- Angles ABD, ABC, and CBE form a straight angle, so their measures sum to $180°$.



This means the sum of the angles in Triangle ABC is $180°$. The same reasoning could be applied to any triangle, so the sum of the interior angles of any triangle is $180°$.

11 Synthesis

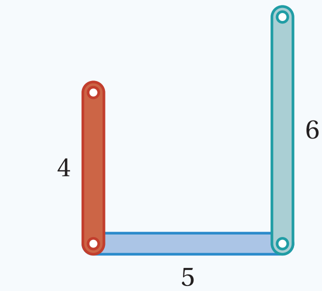
Explain how you can determine whether three line segments will form a triangle.



Summary

Three line segments do not always make a triangle.

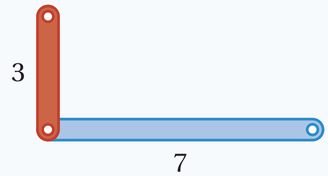
In order for three line segments to form a triangle, the sum of the two shorter segments' lengths must be greater than the third segment's length.



For example, $4 + 5 > 6$, so these three line segments would make a triangle.

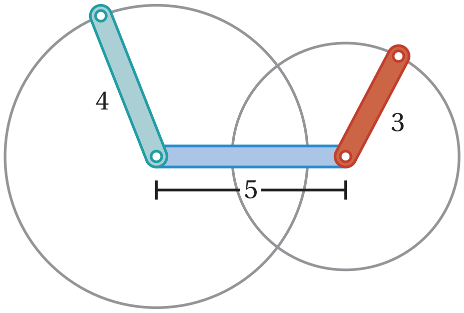
You can use this relationship to determine what possible lengths would create a triangle.

For example, if two side lengths of a triangle are 3 units and 7 units, then the third side must be greater than 4 and less than 10 units.



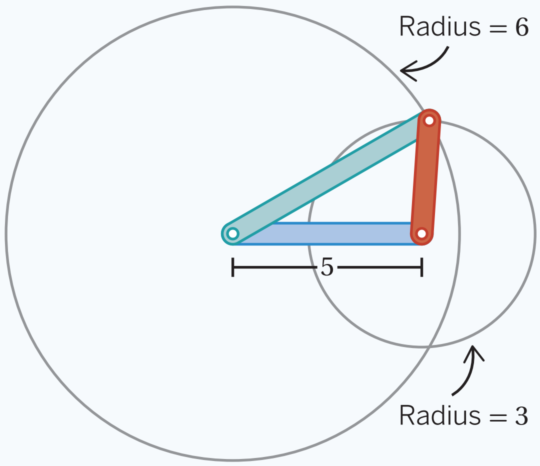
13 Synthesis

a Describe how to create a triangle given three side lengths.



b Explain why there will be only one possible triangle.

Summary



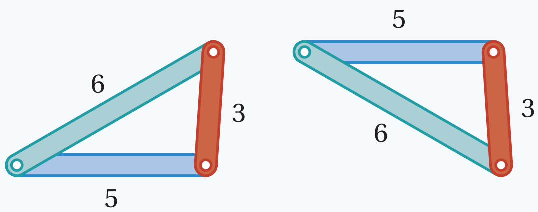
You can use circles to draw triangles.

You can draw a triangle with sides that are 5, 3, and 6 units long by drawing:

- A line segment that is 5 units long.
- A circle of *radius* 3 units centered at one end point.
- A circle of radius 6 units centered at the other end point.

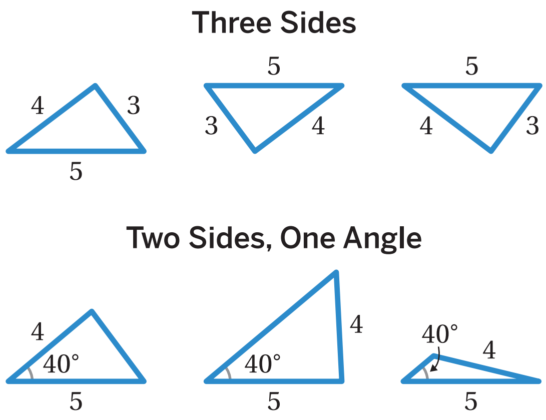
All the triangles whose sides are 5, 3, and 6 units long will be **identical copies** because they have the same shape and size.

In fact, it is only possible to create one unique triangle if you know its three side lengths (unless you can't make a triangle at all).



10 Synthesis

Explain why triangles with three of the same side lengths are *identical copies*, but triangles with the same two side lengths and one angle measure aren't always identical copies.



Summary

It is only possible to make one unique triangle if you know its three side lengths (unless you can't make a triangle at all).

How many unique triangles can be made if you know a combination of side lengths and angles?

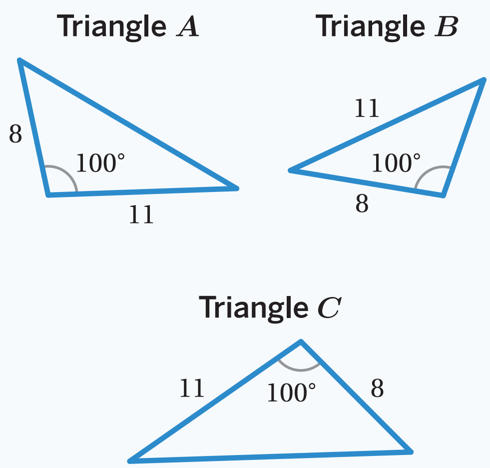
Here are three triangles: A, B, and C.

Each triangle has three of the same measurements: one side that is 8 units long, one side that is 11 units long, and one 100° angle.

Only triangles A and C are *identical copies* because the 100° angle is in between the side lengths of 8 and 11 units.

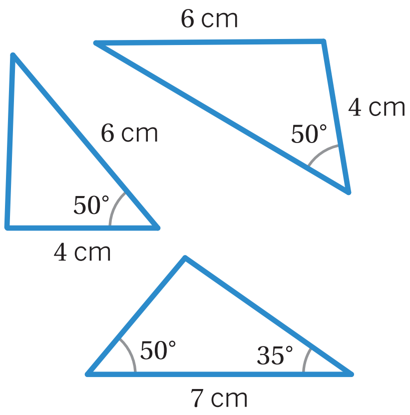
This means that there is more than one unique triangle that can be made with these measurements.

In general, knowing the order or placement of the sides and angles can help you determine whether two triangles are *identical copies* and how many unique triangles there are.



Synthesis

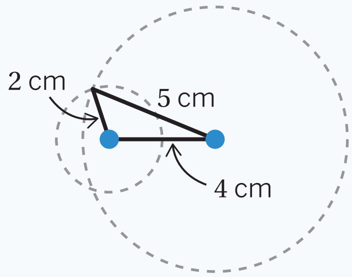
12. Describe how many non-identical triangles can be made with different combinations of measurements.



Figures not drawn to scale.

Summary

You can use a protractor, ruler, and compass to draw triangles with given measurements.

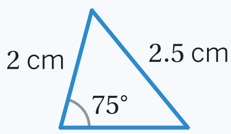


For example, when given three side lengths, use a ruler and a compass to draw radiuses, which can help you draw a triangle.

There is only one unique triangle that can be drawn when given three side lengths.

When given two side lengths and an angle measure, multiple non-identical triangles can be drawn.

For example, here is one strategy for drawing a triangle that has a 75° angle, a side length of 2 centimeters, and a side length of 2.5 centimeters.



Step 1: Draw a 2-centimeter line segment.

Step 2: Draw a 75° angle using a protractor.

Step 3: Measure 2.5 centimeters along the other side of the angle and connect the triangle.

Here are two other non-identical triangles that have the same measurements. The 75° angle can be positioned in different places in relation to the side lengths that are 2 and 2.5 centimeters.