Amplify Desmos Math

Geometry
GEO

Units 1–2 sampler



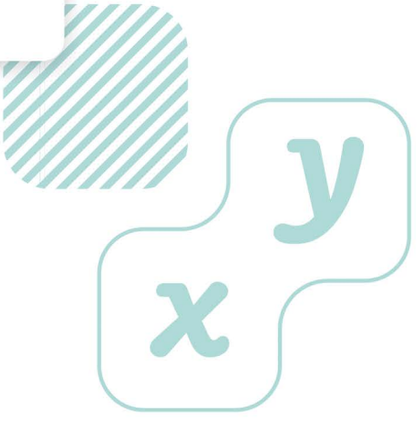
INSIDE YOU'LL FIND

- Program overview information
- Teacher Edition and Student Edition pages for Units 1–2

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.



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Dear Teacher,

Welcome to Amplify Desmos Math! We're so excited your school has chosen our program this year. Your hard work plays an essential role in your students' math experience, so we wanted to reach out to introduce ourselves.

We developed the program around the idea that a structured approach to problem-based instruction builds on students' curiosity to develop lasting grade-level understanding for each and every member of your classroom. Each lesson offers opportunities for you to build on students' understanding, connect their ideas, develop their skill fluency, and empower them to ask questions, explore, and make discoveries. This year, our mission is for your students to learn math — and to love learning math!

Here's what you can expect:

- **Interactive lessons** that blend paper-based and digital learning, including:
  - » Lessons that drive classroom discussions so students can work toward a shared understanding and sense of community.
  - » Responsive Feedback™ that interprets students' responses in context and encourages perseverance and revision.
  - » Easy-to-follow lesson plans tested in classrooms across the country, with clear teaching suggestions, strategies, and Math Language Routines.
- **Lesson practice** to support fluency and help students review previous topics.
- **Recommended differentiation moves** that meet the needs of diverse learners.
- **Formative and summative assessments** along with lesson-level checks for understanding.

We hope your students enjoy using technology to explore math, working with classmates to solve problems, and learning new and interesting concepts. We also hope you love experiencing it with them!

—The Amplify Desmos Math team



Unit 1 Constructions and Rigid Transformations

Students use a variety of tools to construct geometric figures. Students then extend what they learned about rigid transformations in Grade 8 by constructing the transformations precisely.

*This course is in development, so the goals and number of lessons per sub-unit may change slightly.

Pre-Unit Check (Optional) 1L

Sub-Unit 1 Constructions 2

1.01	Circles and Segments | An Introduction to Constructions 3A	HSG.CO.A.1, HSG.CO.D.12, MP7
1.02	Compass Constructions | Constructing Patterns 7A	HSG.CO.D.12, HSG.CO.D.13, MP6, MP7
1.03	Constructing Digitally | Geometric Constructions 15A	HSG.CO.D.12, HSG.CO.D.13, MP3, MP6
1.04	Right in the Middle | Constructing Perpendicular Bisectors 19A	HSG.CO.A.1, HSG.CO.D.12, MP1, MP6, MP8
1.05	Lines of Construction | Constructing Perpendicular and Parallel Lines 23A	HSG.CO.A.1, HSG.CO.D.12, MP3, MP6
1.06	Square Up | Constructing Squares 27A	HSG.CO.D.12, HSG.CO.D.13, MP3, MP6
1.07	Community Constructions | Using Perpendicular Bisectors 31A	HSG.CO.D.12, HSG.MG.A.2 ★, HSG.MG.A.3 ★, MP3, MP4

Practice Day 1 ..... 35A — HSG.CO.A.1, HSG.CO.D.12, HSG.CO.D.13, MP5

Sub-Unit Quiz ..... 36B — HSG.CO.A.1, HSG.CO.D.12, HSG.CO.D.13, MP5, MP6

Sub-Unit 2 Rigid Transformations 37–38

1.08	Going off the Grid, Part 1 | Introducing Sequences of Transformations 39A	HSG.CO.A.2, HSG.CO.A.5, MP6, MP7
1.09	Time to Reflect | Defining Reflections 43A	HSG.CO.A.2, HSG.CO.A.4, HSG.CO.A.5, MP5, MP6
1.10	Translation Information | Defining Translations 51A	HSG.CO.A.2, HSG.CO.A.4, HSG.CO.A.5, MP5, MP6
1.11	Rotation Devices | Defining Rotations 59A	HSG.CO.A.2, HSG.CO.A.4, HSG.CO.A.5, MP5, MP6
1.12	Going Off the Grid, Part 2 | Sequences of Transformations 69A	HSG.CO.A.2, HSG.CO.A.5, MP5, MP6, MP7
1.13	Tiles and Patterns | Geometric Art Using Constructions and Transformations 73A	HSG.CO.A.5, HSG.CO.D.12, MP7

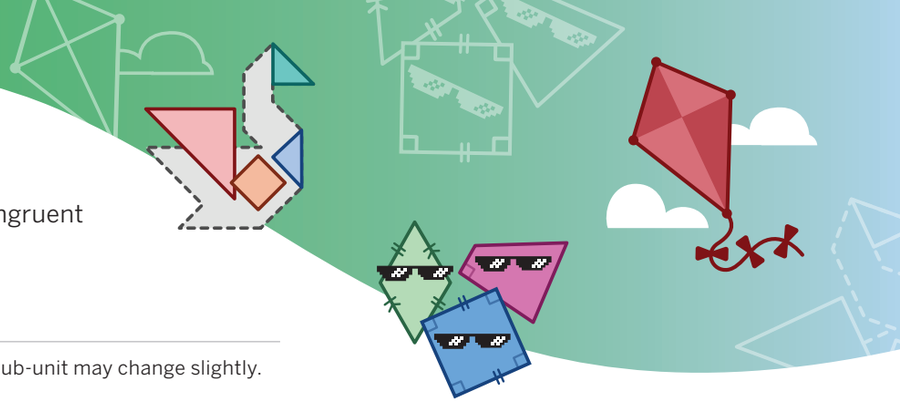
Practice Day 2 ..... 77A — HSG.CO.A.1, HSG.CO.A.2, HSG.CO.A.5, HSG.CO.D.12, MP5, MP6

End-of-Unit Assessment 78B

Unit 2 included in this sampler.

Unit 2 Congruence

Students use rigid transformations to explore and prove congruent relationships in angles, triangles, and quadrilaterals.



*This course is in development, so the goals and number of lessons per sub-unit may change slightly.

Pre-Unit Check (Optional) 79M

Sub-Unit 1 All About Angles 80

2.01	Is It? Or Is It Not? | Definitions and Their Consequences 81A	**HSG.CO.A.1,** MP3, MP6
2.02	Always True? | Making Arguments About Angle Relationships 85A	**HSG.CO.A.1, HSG.CO.A.2, HSG.CO.B.6, HSG.CO.C.9,** MP3, MP5, MP7
2.03	Angles and Transversals | Using Transformations to Prove Angle Relationships 93A	**HSG.CO.A.2, HSG.CO.C.9,** MP2, MP3
2.04	Doing a 180 | Triangle Angle Sum 97A	**HSG.CO.C.10,** MP3, MP6

Sub-Unit 2 Triangles: Congruence, Criteria, and Conjectures 101–102

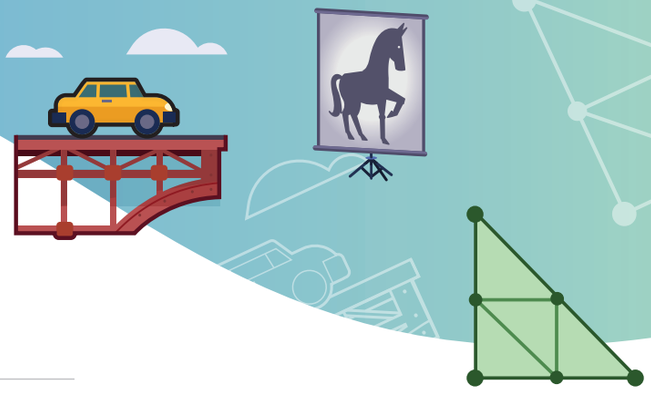
2.05	Stack 'Em Up | Defining Congruence 103A	**HSG.CO.A.2, HSG.CO.B.6,** MP1, MP3, MP6
2.06	Parts and Places | Congruence and Corresponding Parts 107A	**HSG.CO.B.7,** MP1, MP5, MP7
2.07	Corresponding Components | Congruence Statements 111A	**HSG.CO.A.2, HSG.CO.A.5, HSG.CO.B.7,** MP1, MP3, MP6
2.08	Less Is More | Exploring Requirements for Triangle Congruence 119A	**HSG.CO.B.7, HSG.CO.B.8,** MP3, MP8
2.09	Congruence Conditions | Criteria for Triangle Congruence 127A	**HSG.CO.B.8,** MP1, MP3, MP5
2.10	Resuming Construction | Using Triangle Congruence to Copy an Angle 131A	**HSG.CO.D.12,** MP1, MP3, MP5
2.11	Proving Points | Proving the Properties of Points on Perpendicular Bisectors 135A	**HSG.CO.C.9,** MP1, MP3, MP6, MP7
	Practice Day 1 ..... 143A	**HSG.CO.B.7, HSG.CO.B.8, HSG.CO.C.9, HSG.CO.C.10,** MP2, MP3, MP6
	Sub-Unit Quiz 144A	**HSG.CO.B.7, HSG.CO.B.8, HSG.CO.C.9,** MP3, MP6, MP7

Sub-Unit 3 Proving Theorems About Quadrilaterals 145–146

2.12	The Quad Squad (Optional) | Defining Quadrilaterals 147A	**Building Toward: HSG.CO.A.1, HSG.CO.C.11,** MP1, MP3, MP6
2.13	Go With the Flow | Proofs of Parallelogram Properties 151A	**HSG.CO.C.11,** MP3, MP6, MP7
2.14	A-Door-Able Parallelograms | Diagonals of Parallelograms 155A	**HSG.CO.B.6, HSG.CO.C.11,** MP3, MP5, MP8
2.15	Let's Go Build a Kite | Constructing an Angle Bisector 163A	**HSG.CO.D.12,** MP3, MP5, MP7
	Practice Day 2 ..... 167A	**HSG.CO.B.7, HSG.CO.B.8, HSG.CO.C.9, HSG.CO.C.11, HSG.CO.D.12,** MP3, MP5, MP6
	End-of-Unit Assessment 168B	

Unit 3 Similarity

Students build on their middle school understanding of dilations to prove when two figures are similar. Students investigate whether two figures are similar and use similarity to determine side lengths and angle measurements.



*This course is in development, so the goals and number of lessons per sub-unit may change slightly.

Pre-Unit Check (Optional) 169M

Sub-Unit 1 Dilations 170

3.01	Shadowy Figures, Part 1 | Dilations and Their Scale Factors 171A	**HSG.SRT.A.1, HSG.SRT.A.1.B,** MP1, MP2, MP4
3.02	Dilation Situations | Properties of Dilations 175A	**HSG.CO.A.2, HSG.SRT.A.1, HSG.SRT.A.1.B,** MP5, MP7
3.03	Lines of Thinking | Dilations of Lines and Their Angles 183A	**HSG.SRT.A.1, HSG.SRT.A.1.A,** MP3, MP8
3.04	Triangle Truths, Part 1 | Dividing Triangle Sides Proportionally 187A	**HSG.CO.C.10, HSG.SRT.B.4,** MP3, MP5, MP7
3.05	Shadowy Figures, Part 2 | Modeling With Dilations 191A	**HSN.Q.A.1 ★, HSN.Q.A.3 ★, HSG.MG.A.3 ★,** MP4, MP5, MP6
	Practice Day 1 ..... 199A	**HSG.SRT.A.1, HSG.SRT.A.1.A, HSG.SRT.A.1.B, HSG.CO.A.2, HSG.CO.C.10, HSG.SRT.B.4, HSN.Q.A.1 ★,** MP4, MP5, MP7
	Sub-Unit Quiz 200A	**HSG.SRT.A.1, HSG.SRT.A.1.A, HSG.SRT.A.1.B, HSG.CO.C.10, HSG.SRT.B.4,** MP3, MP7,

Sub-Unit 2 Similar Figures 201–202

3.06	Beyond a Shadow of a Doubt | Connecting Similarity and Transformations 203A	**HSG.SRT.A.2,** MP1
3.07	Revisiting Corresponding Components | Reasoning About Similarity 207A	**HSG.SRT.A.2,** MP2, MP3, MP8
3.08	More Than Pairs | Always Similar Shapes 211A	**HSG.C.A.1, HSG.SRT.A.2,** MP1, MP3, MP7
3.09	Similarity Conditions | Criteria for Triangle Similarity 219A	**HSG.SRT.A.3, HSG.SRT.B.5,** MP3, MP7, MP8
3.10	Triangle Truths, Part 2 | Parallel Lines and Triangles 223A	**HSG.SRT.B.4, HSG.SRT.B.5,** MP1, MP6
3.11	Similarity Search (Optional) | Investigations of Similar Triangles 227A	**HSG.SRT.B.5,** MP3, MP7
	Practice Day 2 ..... 235A	**HSG.SRT.A.1, HSG.SRT.A.2, HSG.SRT.B.4, HSG.SRT.B.5, HSG.C.A.1,** MP3
	End-of-Unit Assessment 238A	

Unit 4 Similarity in Right Triangles



Students explore relationships between the sides and angles of right triangles to build an understanding of ratios, leading to the naming of sine, cosine, and tangent as trigonometric ratios. They use similarity to estimate angles and sides using trigonometric ratios before they use the calculator to support reasoning quantitatively. Students also explore ways to solve design problems and model the world using right triangles.

*This course is in development, so the goals and number of lessons per sub-unit may change slightly.

Pre-Unit Check (Optional) 239M

Sub-Unit 1 Similarity in Right Triangles 240

4.01	Splitting Right | Similarity in Right Triangles 241A	**HSG.SRT.B.5,** MP1
4.02	The Proof Is in the Puzzling | Proving the Pythagorean Theorem 245A	**HSG.SRT.B.4, HSG.SRT.B.5,** MP2
4.03	Half-Time (Optional) | Exploring Relationships in Special Right Triangles 253A	**HSG.SRT.B.5,** MP1, MP6, MP8
4.04	Cheesy Challenges (Optional) | Applications of Special Right Triangles 261A	**HSG.SRT.B.5,** MP1, MP2, MP6

Sub-Unit 2 Trigonometric Ratios 265–266

4.05	Thwack Attack | Exploring Right Triangle Relationships 267A	**HSG.SRT.C.6,** MP1, MP7, MP8
4.06	Triangle Zapper | Using Right Triangle Relationships 271A	**HSG.SRT.C.6,** MP2, MP3, MP6
4.07	Careful Calculations | Solving for Lengths of Triangle Sides 275A	**HSG.SRT.C.8 ★, HSN.Q.A.3 ★,** MP5, MP6, MP8
4.08	Take It as a Complement | Sine and Cosine of Complementary Angles in Right Triangles 283A	**HSG.SRT.B.5, HSG.SRT.C.7,** MP1, MP2, MP8
4.09	Angle Inspector | Solving for Unknown Angles 291A	**HSG.SRT.C.6, HSG.SRT.C.8 ★, HSN.Q.A.1 ★, HSN.Q.A.3 ★,** MP2, MP4
4.10	Something is Non-Right (Optional) | Trigonometry in Non-Right Triangles 301A	**HSG.SRT.D.11(+), HSG.GMD.B.6,** MP1, MP2, MP3, MP8

Practice Day 1 ..... 305A — **HSG.SRT.B.5, HSG.SRT.C.6, HSG.SRT.C.7,** MP3, MP6

Sub-Unit Quiz ..... 306A — **HSG.SRT.B.5, HSG.SRT.C.6, HSG.SRT.C.7,** MP3, MP6

Sub-Unit 3 Modeling With Trigonometry 307–308

4.11	Slides, Swings, and Modeling Things | Modeling With Trigonometry 309A	**HSG.SRT.C.8 ★, HSG.MG.A.3 ★, HSN.Q.A.3 ★,** MP4, MP5
4.12	Encore, Encore! Part 1 | Creating a Model 313A	**HSG.SRT.C.8 ★, HSG.MG.A.3 ★, HSN.Q.A.2 ★,** MP4
4.13	Encore, Encore! Part 2 | Improving a Model 321A	**HSG.SRT.C.8 ★, HSG.MG.A.3 ★, HSN.Q.A.2 ★,** MP4

Practice Day 2 ..... 327A — **HSG.SRT.B.5, HSG.SRT.C.8 ★, HSG.SRT.C.6, HSG.SRT.C.7,** MP2, MP6

End-of-Unit Assessment 330B

Unit 5 Coordinate Geometry

Students use the coordinate plane to explore and extend their understanding of concepts such as transformations, parallel and perpendicular lines, circles, parabolas, and polygons.

Standards: HSA.SSE.A.1.A, HSA.SSE.A.2, HSG.CO.A.2, HSG.CO.A.3, HSG.CO.A.5, HSG.CO.C.10, HSG.GPE.A.1, HSG.GPE.A.2, HSG.GPE.B.4, HSG.GPE.B.5, HSG.GPE.B.6, HSG.GPE.B.7, HSG.MG.A.3

*This course is in development, so the goals and number of lessons per sub-unit may change slightly.

Pre-Unit Check (Optional)

Sub-Unit 1 Transformations on the Plane (7 lessons)

- Use function language to describe rules for symmetry and transformations.
- Prove the slope criteria for parallel and perpendicular lines.

Practice Day 1

Sub-Unit Quiz

Sub-Unit 2 Distances on the Plane (5 lessons)

- Derive the equations of circles and parabolas using the Pythagorean theorem.
- Describe and interpret the distance between two points on the coordinate plane.

Practice Day 2 (Optional)

Sub-Unit 3 Properties on the Plane (5 lessons)

- Determine the perimeters and areas of figures on the coordinate plane.
- Prove relationships about distances and angles on the coordinate plane, in and out of context.

Practice Day 3

End-of-Unit Assessment

Unit 6 Circles

Students deepen their understanding of circles, exploring angle relationships, proportions of a circle (arcs and sectors), and inscribing, circumscribing, and modeling with circles.

Standards: HSG.C.A.2, HSG.C.A.3, HSG.C.B.5, HSG.CO.A.1, HSG.GMD.A.1, HSG.GPE.B.7, HSG.MG.A.2

*This course is in development, so the goals and number of lessons per sub-unit may change slightly.

Pre-Unit Check (Optional)

Sub-Unit 1 Lines, Angles, and Circles (8 lessons)

- Generate and explore patterns in the relationships among the radiuses, arcs, and angles in a circle.
- Explain and use the relationship between radians and degrees.
- Describe angle properties created by chords within circles and tangent lines to circles.

Practice Day 1

Sub-Unit Quiz

Sub-Unit 2 Circles and Polygons (4 lessons)

- Prove properties of angles for a quadrilateral inscribed in a circle.
- Construct the inscribed and circumscribed circles of a triangle.
- Create and analyze mathematical models using the properties of circles

Practice Day 2

End-of-Unit Assessment

Unit 7 Solid Geometry

Students extend their geometric thinking into three-dimensional space to explore a variety of solids (prisms, cylinders, pyramids, and cones). They also discuss why volume formulas are true for different solids and develop mathematical models of three-dimensional figures.

Standards: HSG.GMD.A.1, HSG.GMD.A.3, HSG.GMD.B.4, HSG.GMD.B.5, HSG.MG.A.1, HSG.MG.A.2, HSG.MG.A.3, HSN.Q.A.1

*This course is in development, so the goals and number of lessons per sub-unit may change slightly.

Pre-Unit Check (Optional)

Sub-Unit 1 Introduction to Three-Dimensional Figures (5 Lessons + 1 Optional Lesson)

- Identify the three-dimensional solid created by rotating a two-dimensional figure around an axis.
- Visualize and draw cross-sections of three-dimensional figures.
- Calculate volumes of right and oblique prisms and cylinders, and figures composed of prisms and cylinders.
- Justify volume formulas for prisms and cylinders.
- Analyze solids using their two-dimensional cross-sections.

Practice Day 1

Sub-Unit Quiz

Sub-Unit 2 Cones, Pyramids, and Modeling (7 lessons)

- Show that the volume for any pyramid or cone is one-third the volume of a prism or cylinder with equal height and base.
- Calculate the volume of solids.
- Represent objects with three-dimensional geometric figures.
- Know that when figures are dilated by a scale factor of k, their areas are multiplied by k^2 and their volumes are multiplied by k^3.
- Use scaling, surface area, volume, and density concepts to model a context and solve design problems.

Practice Day 2

End-of-Unit Assessment

Unit 8 Conditional Probability

Students use conditional probability and independence to model and explore real situations in everyday life.

Standards: HSS.CP.A.1, HSS.CP.A.2, HSS.CP.A.3, HSS.CP.A.4, HSS.CP.A.5, HSS.CP.B.6, HSS.CP.B.7(+), HSS.CP.B.8(+), HSS.ID.B.5

*This course is in development, so the goals and number of lessons per sub-unit may change slightly.

Pre-Unit Check (Optional)

Sub-Unit 1 Probability and Chance (6 lessons)

- Estimate or calculate probabilities using data from a simulation or experiment.
- Use Venn diagrams and two-way tables to represent events and calculate probabilities.
- Apply the addition rule and interpret the answer.

Practice Day 1

Sub-Unit Quiz

Sub-Unit 2 Independence and Conditional Probability

(5 Lessons + 1 Optional Lesson)

- Distinguish between independent and dependent events.
- Apply concepts of conditional probability to justify whether events are independent or dependent.
- Apply probability concepts to investigate claims and mathematical models.

Practice Day 2

End-of-Unit Assessment

A curiosity-driven program that builds students' lifelong math proficiency

As we developed Amplify Desmos Math, we asked ourselves: how can we support teachers in creating a collaborative classroom of learners excited about math?

With that question in mind, we built the program around four core tenets:

A structured approach to problem-based learning

The program thoughtfully combines conceptual understanding, procedural fluency, and application. Each lesson is designed to tell a story by posing problems that invite a variety of approaches before guiding students to synthesize their understanding of the learning goals.

The Teacher Edition provides guidance for teachers to anticipate and monitor strategies students may use, select and sequence students' ideas, and orchestrate productive discussions to help students make connections between their own ideas and those of their classmates.[1]

Proficiency Progression

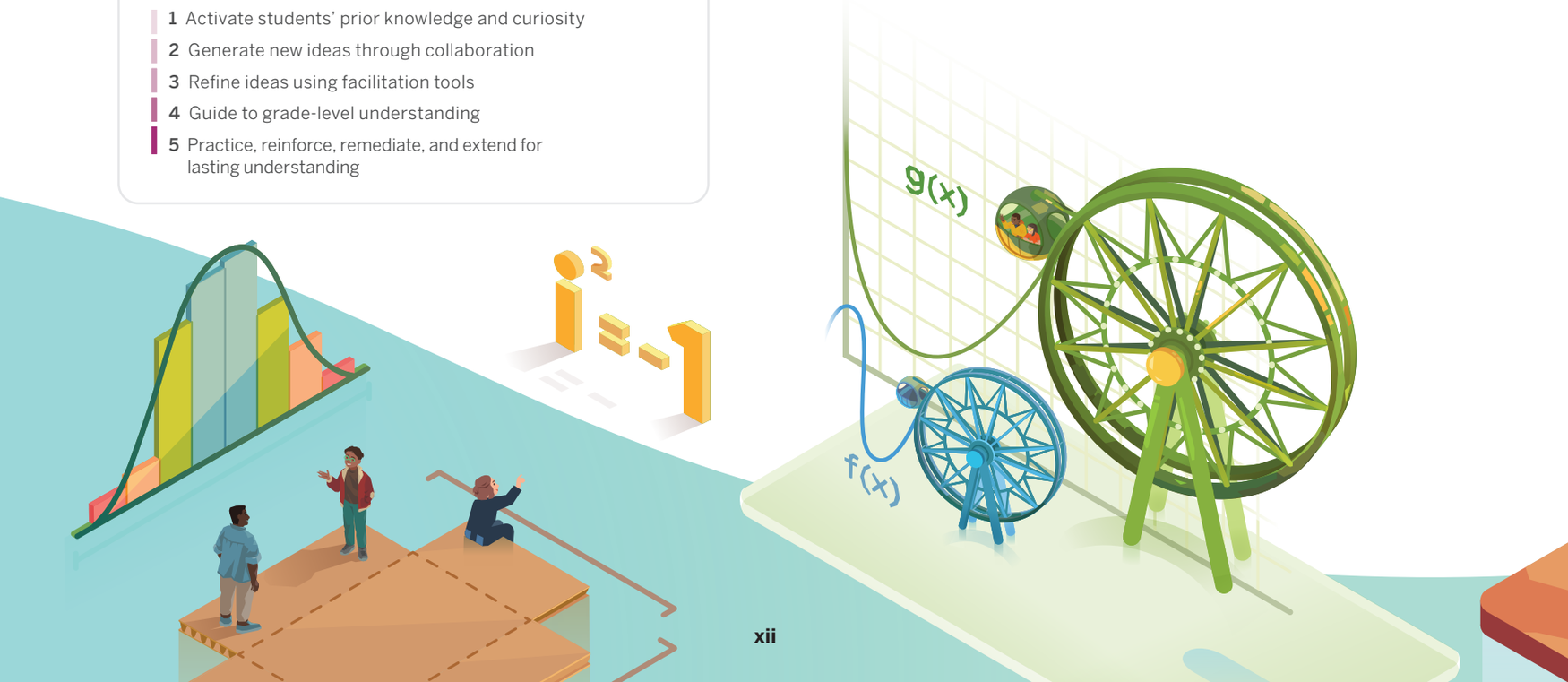
Lessons are designed around what we call the Proficiency Progression, a model that systematically builds on students' curiosity to develop lasting grade-level understanding.

1. Activate students' prior knowledge and curiosity
2. Generate new ideas through collaboration
3. Refine ideas using facilitation tools
4. Guide to grade-level understanding
5. Practice, reinforce, remediate, and extend for lasting understanding

Access to grade-level math for every student, every day

Tasks in each lesson are thoughtfully sequenced so that all students can engage with the math each day without any roadblocks. Every lesson includes suggestions for accessibility and differentiation to support, strengthen, and stretch students' understanding.

We also provide additional resources that integrate seamlessly with core instruction, including a suite of assessments, tailored practice resources that adjust to students' learning, and other intervention solutions. Cohesive differentiation and intervention resources support and challenge students on their path toward deeper understanding of the learning goals, ensuring all students a path toward a deeper understanding of the learning goals of the course.





Student thinking is valuable and can be made evident

Students take an active role in developing their own ideas first and then synthesize them as a class. To guide the learning process, students see each other's thinking, engage in conversations, and connect to each other through the understanding that they can use math to make sense of the world. This fuels classroom conversations and a shared understanding of math.

Rather than evaluating ideas as simply right or wrong, Responsive Feedback™ shows students what their ideas mean in context and offers opportunities for students to learn from each other's responses.[2] This feedback encourages students to explore different strategies and make sense of a variety of responses, so that students' ideas drive the learning process.

Math that motivates students

Picture a classroom where students are so eagerly engaged in a lesson, they wish it would not end. It is buzzing with the sounds of natural curiosity. There is an audible groan from students when their screens are paused. This is what an Amplify Desmos Math classroom looks like and sounds like. This is math that motivates.

Our curriculum supports social classrooms, invites mathematical creativity, and evokes wonder, empowering students to see themselves and their classmates as having brilliant mathematical ideas.

[1] Smith, M.S., & Stein, M.K. (2018). *5 practices for orchestrating productive mathematics discussions* (2nd ed.). SAGE Publications.

[2] Chase, C., Chin, D.B., Oppezzo, M., Schwartz, D.L. (2009). Teachable agents and the protégé effect: Increasing the effort towards learning. *Journal of Science Education and Technology 18*, 334–352. https://doi.org/10.1007/s10956-009-9180-4.



Scope and Sequence

Algebra 1, Geometry, Algebra 2

*Geometry and Algebra 2 are in development, so the number of days per unit may change slightly.

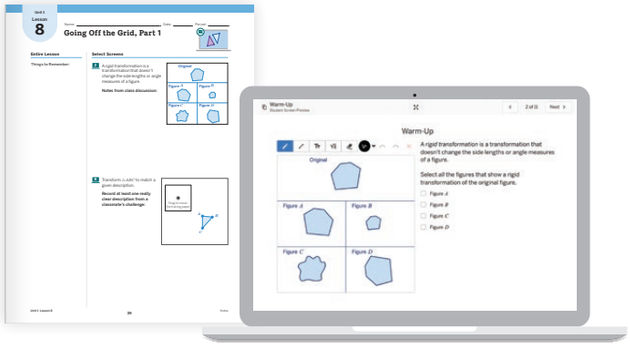
+ Optional: Pre-Unit Checks and optional lessons are considered optional days. Optional days are not included in the total day count.

	Unit 1	Unit 2	Unit 3	Unit 4
Algebra 1 144 days +23 optional days	**Patterns and Sequences** 8 instructional days 1 assessment day + 2 optional days 9 days total	**Solving Linear Equations and Inequalities** 16 instructional days 2 assessment days + 3 optional days 18 days total	**Describing Data** 18 instructional days 2 assessment days + 4 optional days 20 days total	**Functions** 19 instructional days 2 assessment days + 3 optional days 21 days total
Geometry 132 days +15 optional days	**Constructions and Rigid Transformations** 15 instructional days 2 assessment day + 2 optional days 17 days total	**Congruence** 16 instructional days 2 assessment days + 2 optional days 18 days total	**Similarity** 12 instructional days 2 assessment days + 2 optional days 14 days total	**Similarity in Right Triangles** 14 instructional days 2 assessment days + 2 optional days 16 days total
Algebra 2 141 days +13 optional days	**Transformations of Functions** 14 instructional days 2 assessment day + 2 optional days 16 days total	**Quadratic Functions and Complex Numbers** 13 instructional days 2 assessment days + 1 optional day 15 days total	**Polynomial Functions** 16 instructional days 2 assessment days + 1 optional day 18 days total	**Rational Functions** 11 instructional days 2 assessment days + 1 optional day 13 days total

Unit 5	Unit 6	Unit 7	Unit 8
Systems of Linear Equations and Inequalities	Exponents and Exponential Functions	Quadratic Functions	Quadratic Equations
13 instructional days	16 instructional days	19 instructional days	20 instructional days
2 assessment days	2 assessment days	2 assessment days	2 assessment days
+ 2 optional days	+ 4 optional days	+ 2 optional days	+ 3 optional days
15 days total	18 days total	21 days total	22 days total
Coordinate Geometry	Circles	Solid Geometry	Conditional Probability
19 instructional days	12 instructional days	15 instructional days	13 instructional days
2 assessment days	2 assessment days	2 assessment days	2 assessment days
+ 2 optional days	+ 2 optional day	+ 1 optional day	+ 1 optional day
21 days total	15 days total	17 days total	15 days total
Radical Functions and Equations	Exponential and Logarithmic Functions	Trigonometric Functions	Statistical Inferences
14 instructional days	16 instructional days	19 instructional days	20 instructional days
2 assessment days	2 assessment days	2 assessment days	2 assessment days
+ 1 optional day	+ 3 optional day	+ 2 optional days	+ 2 optional day
16 days total	18 days total	21 days total	22 days total

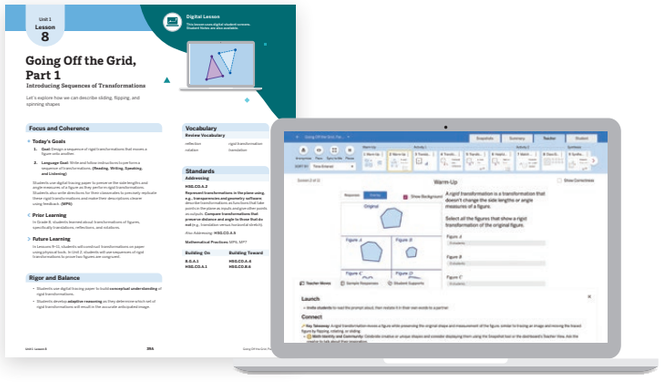
Program Resources

For Students



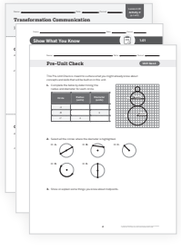
- Student Edition, including Student Notes sheets
- Digital access to interactive digital student screens, practice, and assessments
- Responsive Feedback™
- Collaboration tools like Challenge Creators and Polygraphs

For Teachers



- Teacher Edition pages (PDF)
- Digital access to planning and instruction resources
- Teacher Presentation Screens
- Facilitation and progress-monitoring tools
- Assessment and reporting suite

Additional Resources



Assessment and Lesson Resources

- Unit Assessments
- Show What You Know Lesson Assessments
- Rubrics and Teacher Answer Keys
- Activity Sheets and Cards



Intervention Resources

- Prior grade Mini-Lessons
- Prior grade Fluency Practice

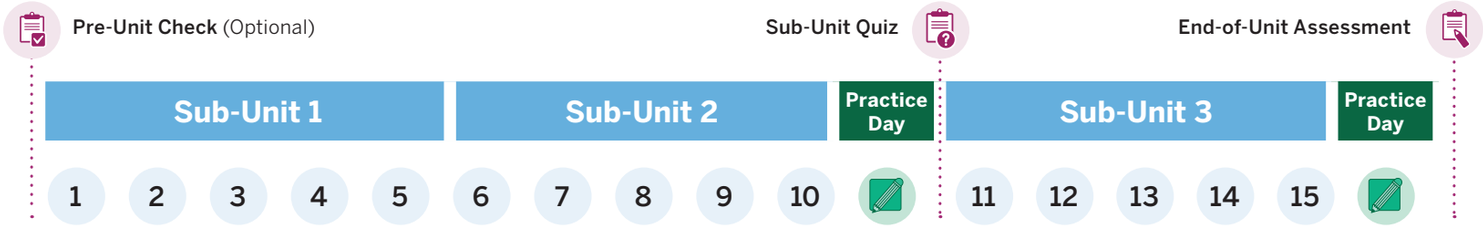
Program Architecture

Course

UNIT 1	UNIT 2	UNIT 3	UNIT 4	UNIT 5	UNIT 6	UNIT 7	UNIT 8
17 days	18 days	14 days	16 days	21 days	15 days	17 days	15 days

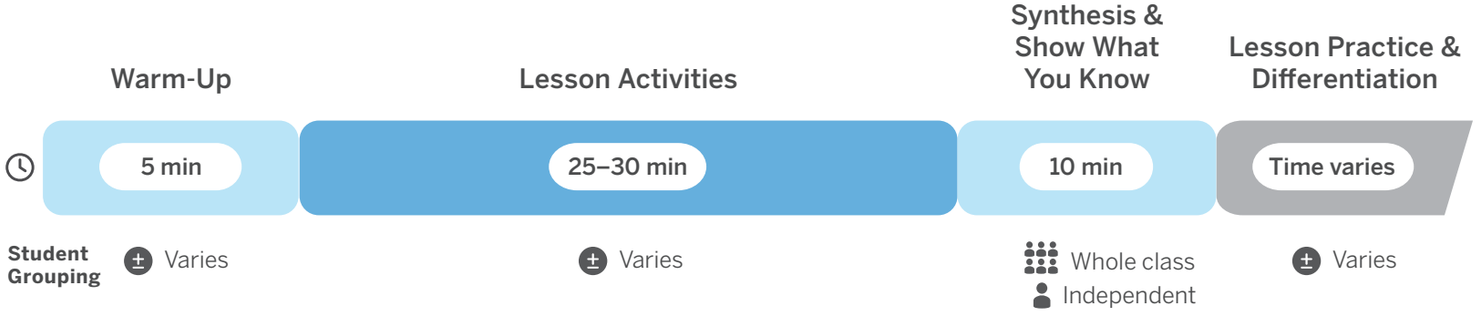
Note: The number of lessons varies from unit to unit. See Scope and Sequence pages for the full program scope.

Unit



Note: The number of sub-units, quizzes, and lessons within each unit varies. This depiction shows the general structure of a unit. See the course Table of Contents for more details.

Lesson

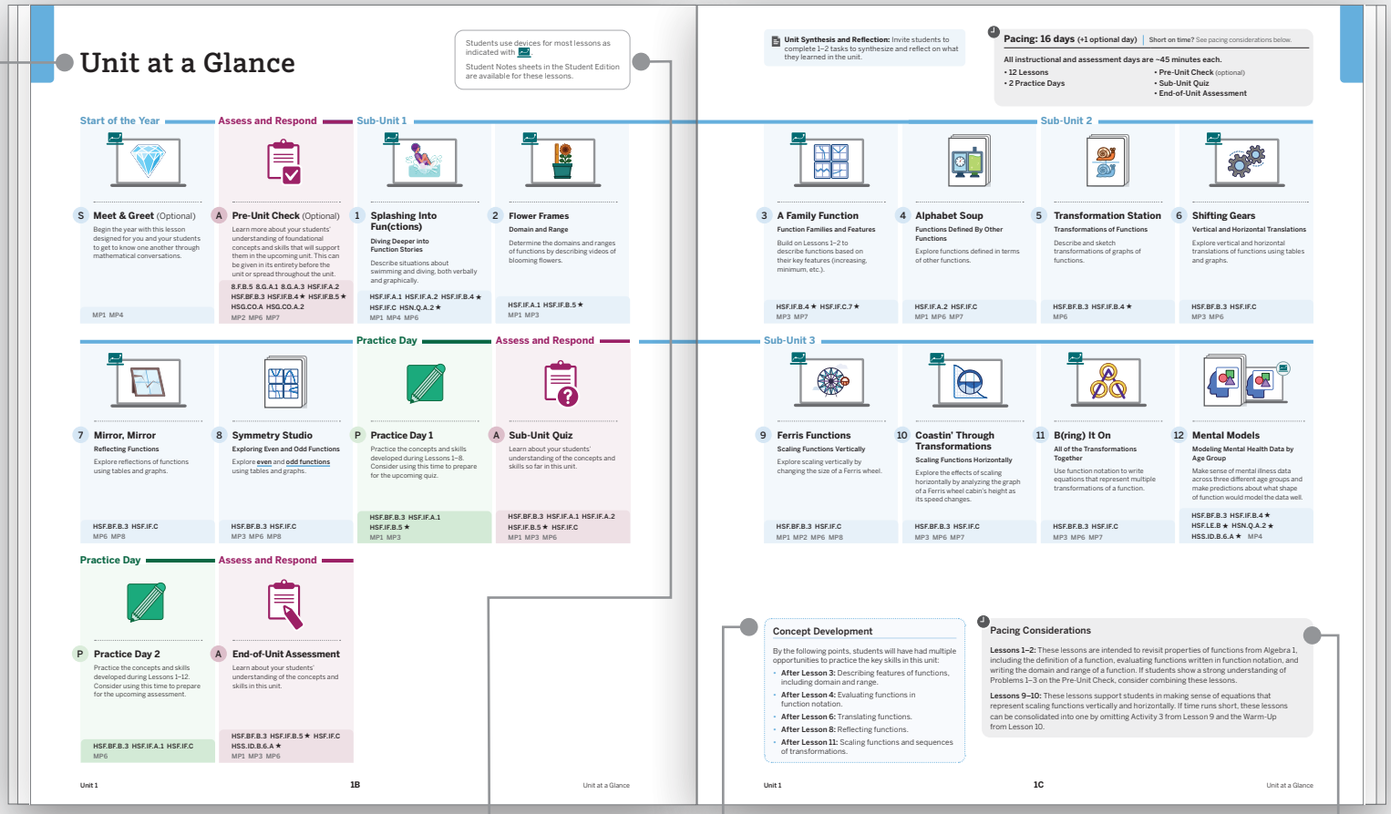


Navigating This Program

Once you have logged into the digital program, you can launch a guided tour with helpful navigation tips.

Unit & Sub-Unit Resources

Each unit includes a range of resources designed to support teachers in thinking through the progression of mathematics that students will engage with over the course of the unit. These resources can support teachers in their unit planning, as well as choices they make in response to students' thinking, strengths, and needs that arise over the course of the unit.



Unit at a Glance

Students use devices for most lessons as indicated with .

Student Notes sheets in the Student Edition are available for these lessons.

Start of the Year

S Meet & Greet (Optional)
Begin the year with this lesson designed for you and your students to get to know one another through mathematical conversations.
MP1 MP4

Assess and Respond

A Pre-Unit Check (Optional)
Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.
8.F.B.5 8.G.A.1 8.G.A.3 HSF.IF.A.2 HSF.BF.B.3 HSF.IF.B.4 ★ HSF.IF.B.5 ★ HSG.CO.A HSG.CO.A.2
MP2 MP6 MP7

Sub-Unit 1

1 Splashing Into Fun(ctions)
Diving Deeper into Function Stories
Describe situations about swimming and diving, both verbally and graphically.
HSF.IF.A.1 HSF.IF.A.2 HSF.IF.B.4 ★ HSF.IF.C HSN.Q.A.2 ★
MP1 MP4 MP6

2 Flower Frames
Domain and Range
Determine the domains and ranges of functions by describing videos of blooming flowers.
HSF.IF.A.1 HSF.IF.B.5 ★
MP1 MP3

Practice Day

7 Mirror, Mirror
Reflecting Functions
Explore reflections of functions using tables and graphs.
HSF.BF.B.3 HSF.IF.C
MP6 MP8

8 Symmetry Studio
Exploring Even and Odd Functions
Explore even and odd functions using tables and graphs.
HSF.BF.B.3 HSF.IF.C
MP3 MP6 MP8

P Practice Day 1
Practice the concepts and skills developed during Lessons 1–8. Consider using this time to prepare for the upcoming quiz.
HSF.BF.B.3 HSF.IF.A.1 HSF.IF.B.5 ★
MP1 MP3

Assess and Respond

A Sub-Unit Quiz
Learn about your students' understanding of the concepts and skills so far in this unit.
HSF.BF.B.3 HSF.IF.A.1 HSF.IF.A.2 HSF.IF.B.5 ★ HSF.IF.C
MP1 MP3 MP6

Practice Day

P Practice Day 2
Practice the concepts and skills developed during Lessons 1–12. Consider using this time to prepare for the upcoming assessment.
HSF.BF.B.3 HSF.IF.A.1 HSF.IF.C
MP6

Assess and Respond

A End-of-Unit Assessment
Learn about your students' understanding of the concepts and skills in this unit.
HSF.BF.B.3 HSF.IF.B.5 ★ HSF.IFC HSS.ID.B.6.A ★
MP1 MP3 MP6

Unit 1 1B Unit at a Glance

Unit Synthesis and Reflection: Invite students to complete 1–2 tasks to synthesize and reflect on what they learned in the unit.

Pacing: 16 days (+1 optional day) | Short on time? See pacing considerations below.

All instructional and assessment days are ~45 minutes each.
- 12 Lessons
- 2 Practice Days
- Pre-Unit Check (optional)
- Sub-Unit Quiz
- End-of-Unit Assessment

Sub-Unit 2

3 A Family Function
Function Families and Features
Build on Lessons 1–2 to describe functions based on their key features (increasing, minimum, etc.).
HSF.IF.B.4 ★ HSF.IF.C.7 ★
MP3 MP7

4 Alphabet Soup
Functions Defined By Other Functions
Explore functions defined in terms of other functions.
HSF.IF.A.2 HSF.IFC
MP1 MP6 MP7

5 Transformation Station
Transformations of Functions
Describe and sketch transformations of graphs of functions.
HSF.BF.B.3 HSF.IF.B.4 ★
MP6

6 Shifting Gears
Vertical and Horizontal Translations
Explore vertical and horizontal translations of functions using tables and graphs.
HSF.BF.B.3 HSF.IFC
MP3 MP6

Sub-Unit 3

9 Ferris Functions
Scaling Functions Vertically
Explore scaling vertically by changing the size of a Ferris wheel.
HSF.BF.B.3 HSF.IFC
MP1 MP2 MP6 MP8

10 Coastin' Through Transformations
Scaling Functions Horizontally
Explore the effects of scaling horizontally by analyzing the graph of a Ferris wheel cabin's height as its speed changes.
HSF.BF.B.3 HSF.IFC
MP3 MP6 MP7

11 B(ring) It On
All of the Transformations Together
Use function notation to write equations that represent multiple transformations of a function.
HSF.BF.B.3 HSF.IFC
MP3 MP6 MP7

12 Mental Models
Modeling Mental Health Data by Age Group
Make sense of mental illness data across three different age groups and make predictions about what shape of function would model the data well.
HSF.BF.B.3 HSF.IF.B.4 ★ HSF.LE.B ★ HSN.Q.A.2 ★ HSS.ID.B.6.A ★ MP4

Concept Development

By the following points, students will have had multiple opportunities to practice the key skills in this unit:
- **After Lesson 3:** Describing features of functions, including domain and range.
- **After Lesson 4:** Evaluating functions in function notation.
- **After Lesson 6:** Translating functions.
- **After Lesson 8:** Reflecting functions.
- **After Lesson 11:** Scaling functions and sequences of transformations.

Pacing Considerations

Lessons 1–2: These lessons are intended to revisit properties of functions from Algebra 1, including the definition of a function, evaluating functions written in function notation, and writing the domain and range of a function. If students show a strong understanding of Problems 1–3 on the Pre-Unit Check, consider combining these lessons.

Lessons 9–10: These lessons support students in making sense of equations that represent scaling functions vertically and horizontally. If time runs short, these lessons can be consolidated into one by omitting Activity 3 from Lesson 9 and the Warm-Up from Lesson 10.

Unit 1 1C Unit at a Glance

Every unit has a **Unit at a Glance** page that shows teachers everything they need to know to get started planning out their upcoming unit.

Student Notes sheets support student recall and retention when engaging in digital lessons.

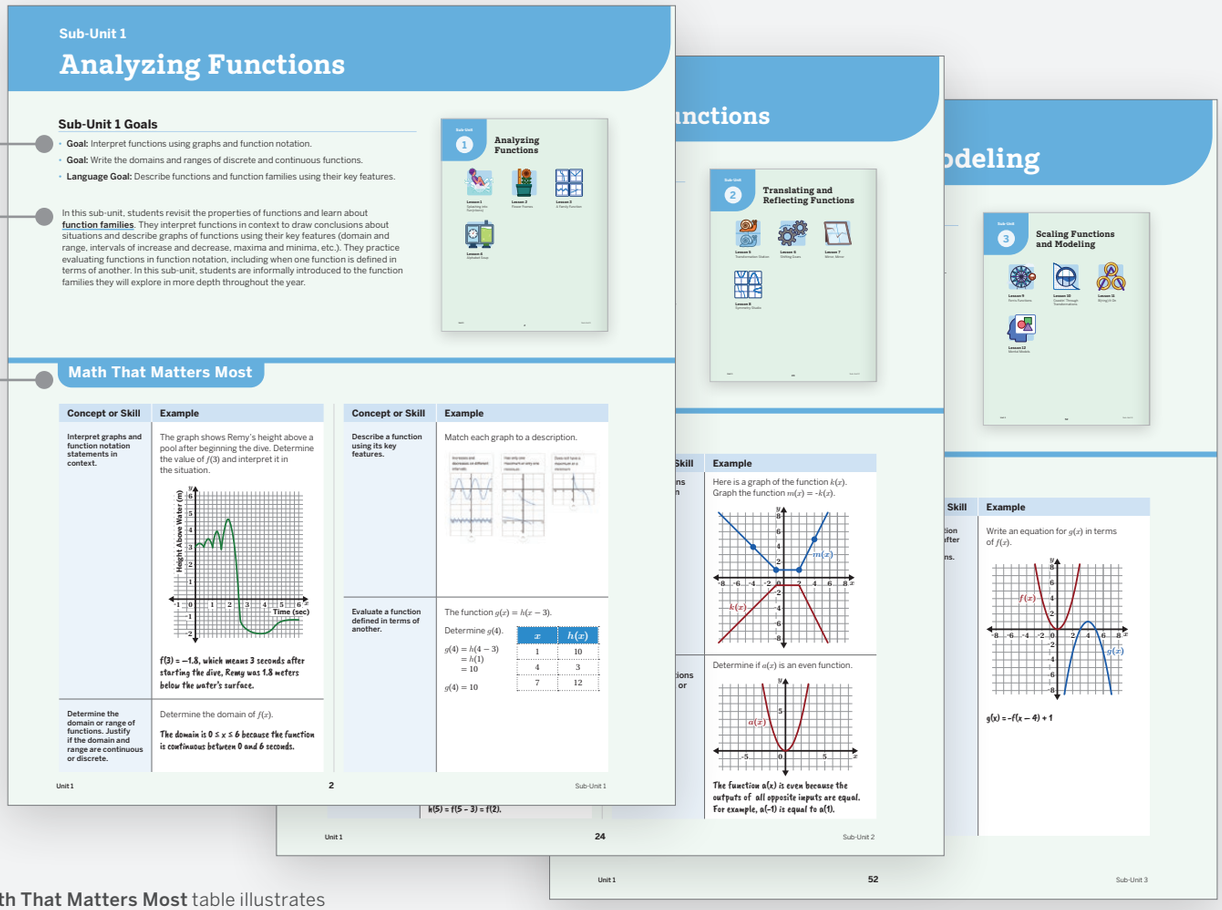
This overview of **concept development** supports teachers in making decisions about potential intervention points during the unit. It indicates at which point students have had multiple opportunities to focus on each key concept or skill. It does not necessarily mean there are no further opportunities to strengthen and develop these skills.

Teachers are provided with thoughtful **pacing considerations** for how they can adjust the pacing of the unit as needed without compromising unit learning goals.

Unit Overview pages

Teachers will find a comprehensive set of resources for each unit, including an overview of the math of the unit, a visual summary of the Unit at a Glance, materials, a preview of each of the unit assessments, and guidance about differentiation, supporting math identity and community, accessibility, language development, technology, and connections to future learning. Each Unit Overview also includes a professional development activity and a formative Pre-Unit Check that teachers can use to learn more about students' understanding of foundational concepts and skills that will support them in the upcoming unit.

The **Sub-Unit Overview** page summarizes the **key goals** and **vocabulary** for this part of the unit.



The **Math That Matters Most** table illustrates for teachers the most important progressions of strategies, skills, and language that develop over the course of the sub-unit.

Sub-Unit Overview pages

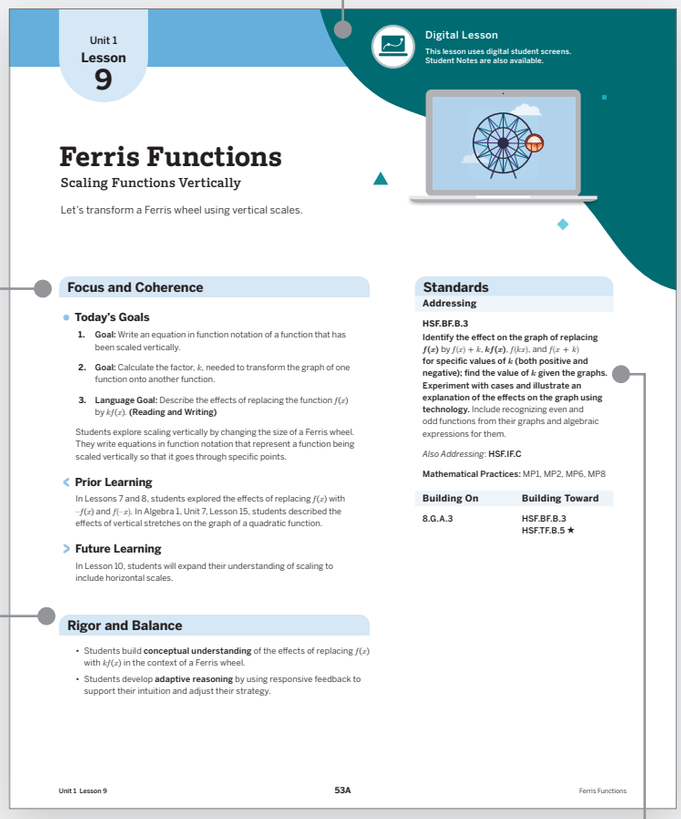
The lessons within each unit are grouped into sub-units that address a related group of concepts. All lessons in each sub-unit include lesson practice and Show What You Know formative assessments, and targeted sub-units include a practice day and assessment.

Navigating This Program

Lesson Supports

Throughout this Teacher Edition, lesson guidance for teachers is organized clearly and consistently so that they have all of the information they need at their fingertips.

Lesson modality shows teachers how they should plan to have students engage in the lesson.



In the **Focus and Coherence** section, teachers will find the goals and language goals for the lesson. There is also information on prior learning that has built to the math in this lesson, as well as future learning that this lesson is helping build to.

The **Rigor and Balance** section explains how students develop conceptual understanding, procedural fluency, application, strategic competence, adaptive reasoning, and productive disposition in this lesson.

Where applicable, the **Standards** section will list all standards addressed in this lesson, including standards the lesson builds on and standards the lesson builds toward. The **bolded** words indicate which parts of each standard described are covered in the lesson.

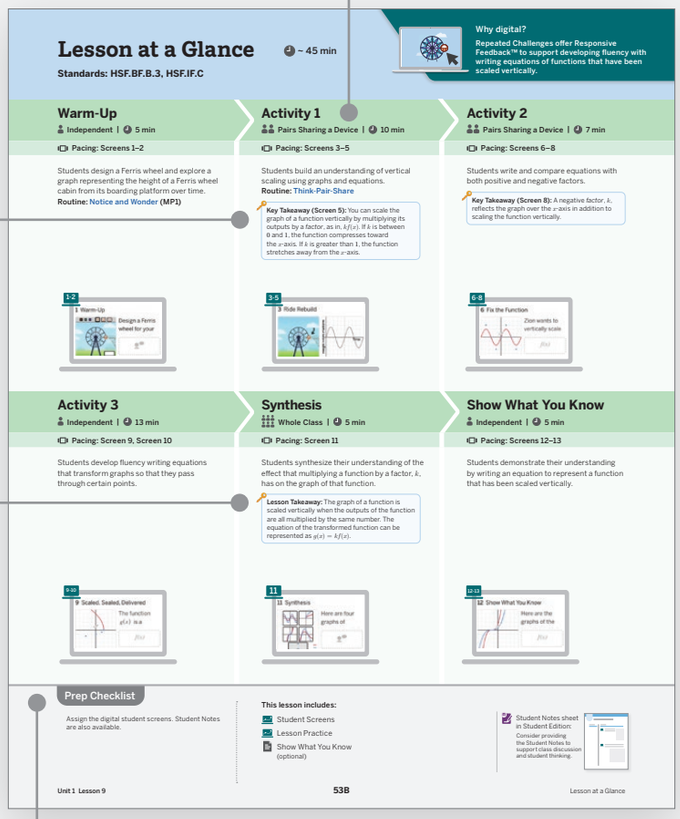
Lesson Overview

This introductory page orients teachers to the topic, standards, and key learning goals of the lesson, including any new vocabulary terms that will be introduced.

The **Key Takeaways** and **Lesson Takeaway** summarize the most important ideas in the lesson.

The **time frame** and suggested **student grouping** is listed for each part of the lesson.

The screen icon is used to show which **Presentation Screens** or **Digital Student Screens** align to each instructional moment.



Lesson at a Glance ~ 45 min

Standards: HSF.BF.B.3, HSF.IF.C

Why digital?
Repeated Challenges offer Responsive Feedback™ to support developing fluency with writing equations of functions that have been scaled vertically.

Warm-Up — Independent | 5 min — Pacing: Screens 1–2
Students design a Ferris wheel and explore a graph representing the height of a Ferris wheel cabin from its boarding platform over time.
Routine: Notice and Wonder (MP1)

Activity 1 — Pairs Sharing a Device | 10 min — Pacing: Screens 3–5
Students build an understanding of vertical scaling using graphs and equations.
Routine: Think-Pair-Share
Key Takeaway (Screen 5): You can scale the graph of a function vertically by multiplying its outputs by a factor, as in, $kf(x)$. If k is between 0 and 1, the function compresses toward the x-axis. If k is greater than 1, the function stretches away from the x-axis.

Activity 2 — Pairs Sharing a Device | 7 min — Pacing: Screens 6–8
Students write and compare equations with both positive and negative factors.
Key Takeaway (Screen 8): A negative factor, k, reflects the graph over the x-axis in addition to scaling the function vertically.

Activity 3 — Independent | 13 min — Pacing: Screen 9, Screen 10
Students develop fluency writing equations that transform graphs so that they pass through certain points.

Synthesis — Whole Class | 5 min — Pacing: Screen 11
Students synthesize their understanding of the effect that multiplying a function by a factor, k, has on the graph of that function.
Lesson Takeaway: The graph of a function is scaled vertically when the outputs of the function are all multiplied by the same number. The equation of the transformed function can be represented as $g(x) = kf(x)$.

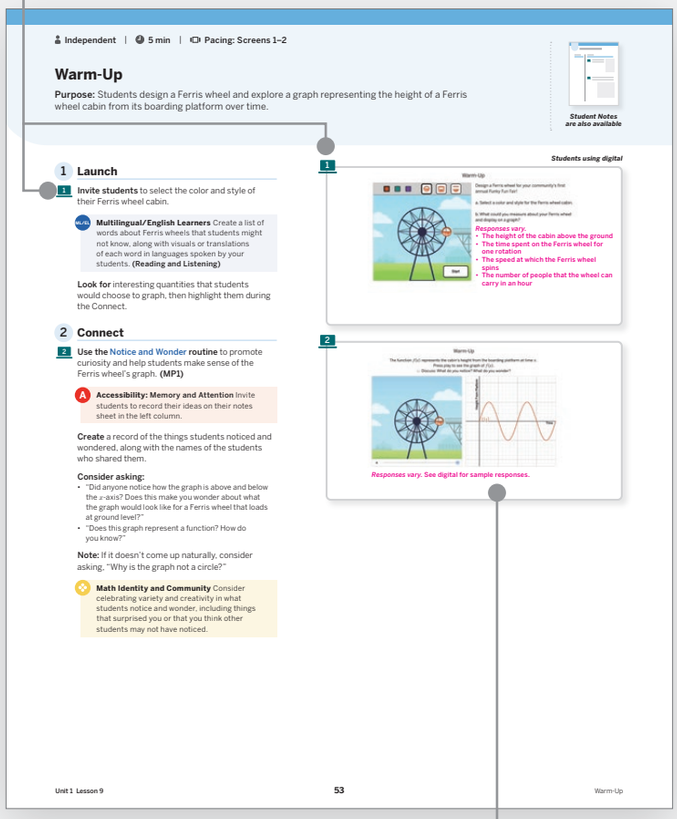
Show What You Know — Independent | 5 min — Pacing: Screens 12–13
Students demonstrate their understanding by writing an equation to represent a function that has been scaled vertically.

Prep Checklist
Assign the digital student screens. Student Notes are also available.

This lesson includes:
- Student Screens
- Lesson Practice
- Show What You Know (optional)

Student Notes sheet in Student Edition: Consider providing the Student Notes to support class discussion and student thinking.

Unit 1 Lesson 9 — 53B — Lesson at a Glance



Independent | 5 min | Pacing: Screens 1–2

Warm-Up

Purpose: Students design a Ferris wheel and explore a graph representing the height of a Ferris wheel cabin from its boarding platform over time.

Student Notes are also available

1 Launch

Invite students to select the color and style of their Ferris wheel cabin.

Multilingual/English Learners Create a list of words about Ferris wheels that students might not know, along with visuals or translations of each word in languages spoken by your students. **(Reading and Listening)**

Look for interesting quantities that students would choose to graph, then highlight them during the Connect.

2 Connect

Use the Notice and Wonder routine to promote curiosity and help students make sense of the Ferris wheel's graph. **(MP1)**

Accessibility: Memory and Attention Invite students to record their ideas on their notes sheet in the left column.

Create a record of the things students noticed and wondered, along with the names of the students who shared them.

Consider asking:
- "Did anyone notice how the graph is above and below the x-axis? Does this make you wonder about what the graph would look like for a Ferris wheel that loads at ground level?"
- "Does this graph represent a function? How do you know?"

Note: If it doesn't come up naturally, consider asking, "Why is the graph not a circle?"

Math Identity and Community Consider celebrating variety and creativity in what students notice and wonder, including things that surprised you or that you think other students may not have noticed.

Students using digital

Responses vary.
- The height of the cabin above the ground
- The time spent on the Ferris wheel for one rotation
- The speed at which the Ferris wheel spins
- The number of people that the wheel can carry in an hour

Responses vary. See digital for sample responses.

Unit 1 Lesson 9 — 53 — Warm-Up

The **Prep Checklist** displays all materials students will need for the lesson.

Sample student responses to the Warm-Up prompt are provided to help teachers prepare to facilitate the conversation.

Lesson at a Glance

The Lesson at a Glance page describes the purpose of the Warm-Up, Activities, Synthesis, and Show What You Know. Teachers will find suggested timing for each part of the lesson, as well as guidance on whether students should work individually, in pairs, in small groups, or with the whole class.

The page also lists which pages, Presentation Screens, or Digital Student Screens can be used with each part of the lesson, as well as any hands-on materials that may be needed.

Warm-Up

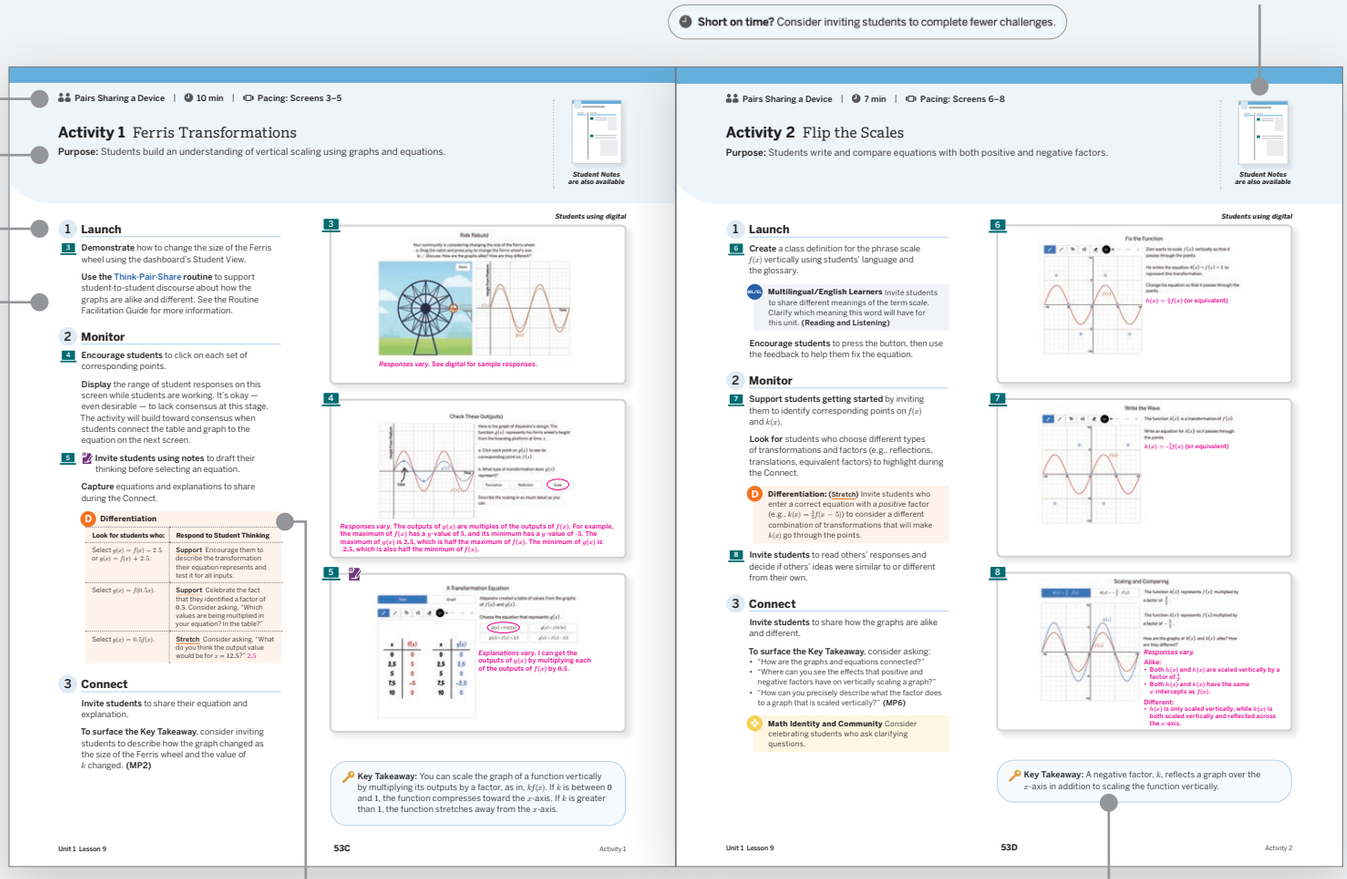
Every Amplify Desmos Math lesson begins with a Warm-Up to help draw students into the lesson. It might elicit information from their personal experience or intuition, remind them of a context they have seen before, invite them to think about the previous lesson, or preview a calculation that will appear in the current lesson.

Navigating This Program

The **Purpose** of each activity is highlighted here, as well as suggestions for the **student grouping**, **time frame**, and **screen pacing** for the activity.

Teachers are provided with thoughtful **Short on time?** suggestions for how they can adjust their facilitation of the lesson as needed without compromising lesson goals.

Each lesson notes the corresponding **Teacher Presentation Screens** or related **Student Edition** pages also available to support the lesson.



In the **Launch**, **Monitor**, **Connect** guidance, teachers will find ways to help students get started, suggested facilitation moves, and discussion questions.

The guidance for every lesson includes a **D Differentiation** table to support teachers in meeting the needs of all students during the activity. This table can help teachers anticipate the ways students may approach the activity, and provides prompts that they can use during the lesson to **Support**, **Strengthen**, and **Stretch** individual students in their thinking.

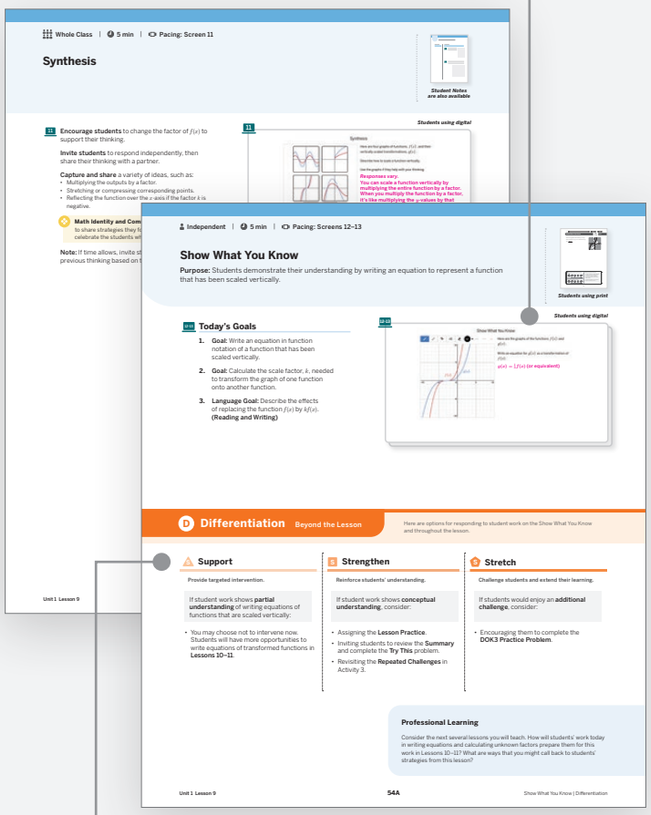
The **Key Takeaway** is called out to highlight the learning goal of the activity and provide teachers with an example of how to frame the big idea of the activity for students.

Activities

Each lesson includes one, two, or three activities. These activities are the heart of each lesson. Students notice, wonder, explore, calculate, predict, measure, explain their thinking, settle disputes, create challenges for their classmates, and more.

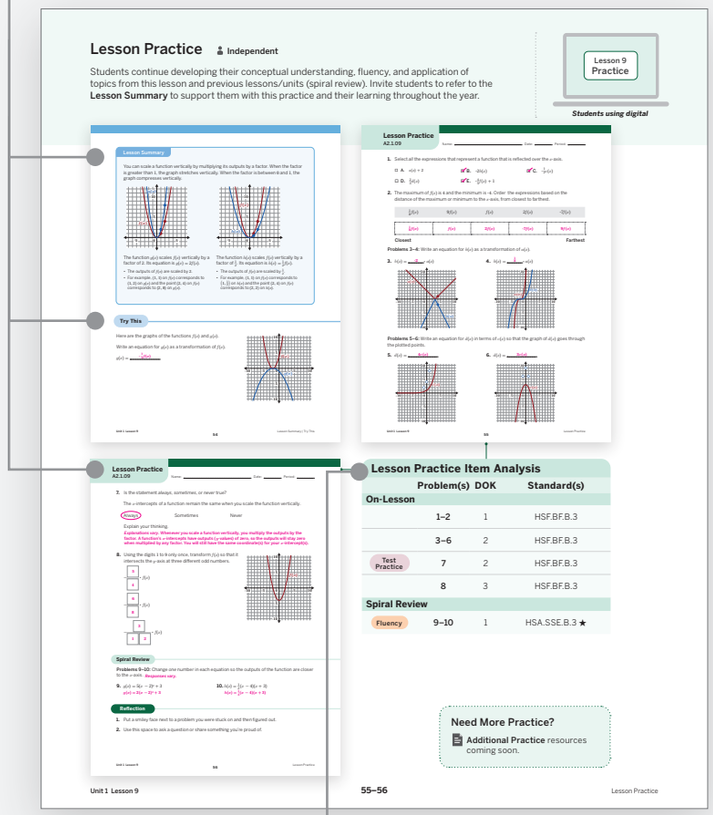
Guidance is provided to help teachers Launch, Monitor, and Connect students' thinking over the course of each activity. Teachers will also find suggestions for pacing, facilitation moves, discussion questions, examples of early student thinking, ideas for students who may enjoy a challenge, ways to increase access to the language and content, as well as opportunities to build and develop the math community in your classroom. The suggestions for facilitation provide comprehensive support for teachers as they prepare for and teach lessons.

Lessons conclude with the **Show What You Know**, which presents an opportunity for students to reflect on the main learning goals of the lesson. This is a great way for both students and teachers to access a formative check for understanding.



Each Show What You Know is accompanied by a table with suggestions in three categories: students who need **support**, would benefit from more practice to **strengthen** their understanding, or are interested in a **stretch** to deepen or extend their thinking.

Each **Lesson Practice** includes a **Summary** and **Try This** problem. Students can highlight big ideas of the lesson in the summary or share it with a caregiver or classmate. They can also engage with the Try This to further apply the lesson content.



A **Lesson Practice Item Analysis** table breaks down the problems by type, Depth of Knowledge (DOK), and corresponding standards.

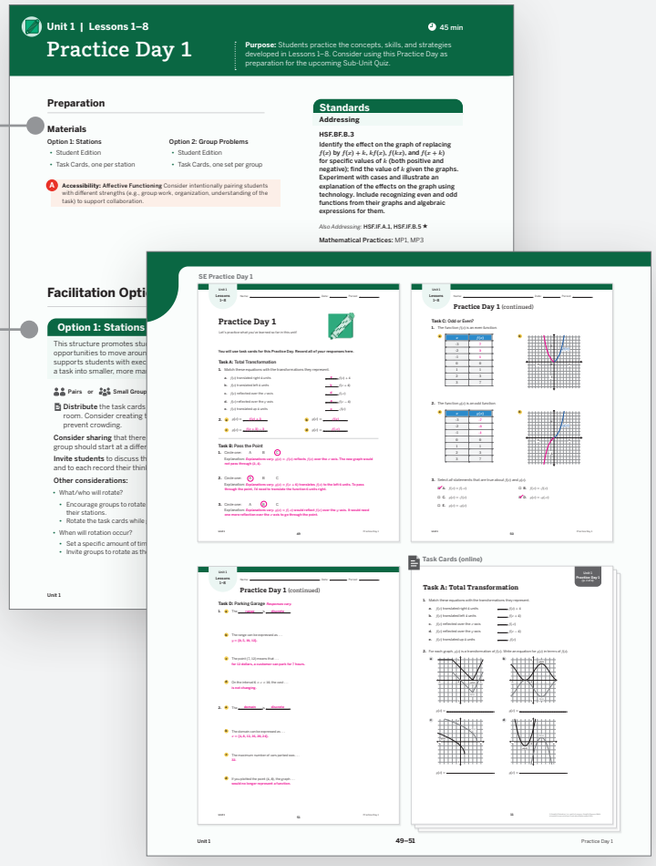
Synthesis

The Synthesis is an opportunity for students to put the key ideas from the lesson into their own words. There is typically an open-ended prompt followed by a discussion for students to consolidate and refine their ideas about the learning goals. If time allows, it is also an opportunity for students to revise their responses after the discussion.

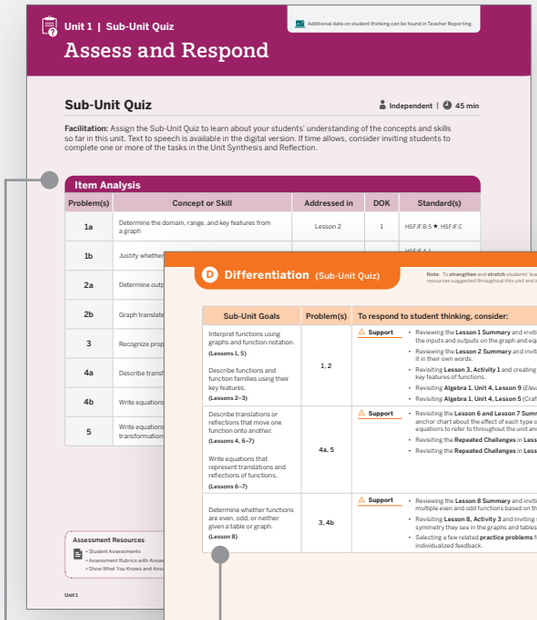
Lesson Practice

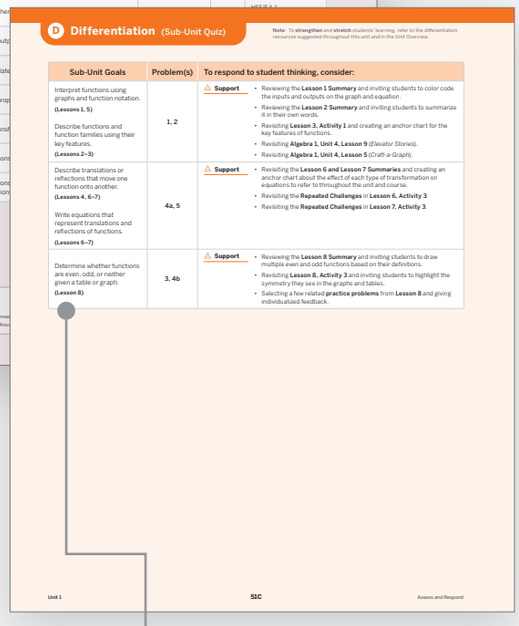
Daily practice problems for each lesson are included both online and in the Student Edition, including Fluency, Test Practice, and Spiral Review.

Practice Days are always **print-based** and may involve **materials** like Problem Cards.



The Teacher Guide typically provides **two different approaches for facilitating** Practice Day activities.





An **Item Analysis** is provided to show what concepts and skills are assessed in each problem.

A **Differentiation** table suggests specific materials from the unit to **Support**, **Strengthen**, or **Stretch** students' understanding based on their responses to different assessment problems.

Practice Day

Practice Days are included before each Sub-Unit Quiz and End-of-Unit Assessment. These lessons provide an opportunity for students to consolidate and apply their knowledge and skills from the preceding lessons. Practice Days often incorporate student movement and collaboration.

Assess and Respond

Each unit typically includes one or two Sub-Unit Quizzes. Quizzes are designed for students to show what they know and can do based on what they have learned so far in the unit. Each unit includes Assess and Respond guidance for the Pre-Unit Check, Sub-Unit Quizzes, and End-of-Unit Assessment.

Facilitating Lesson Activities

Launch, Monitor, Connect

Amplify Desmos Math is designed with a structured approach to problem-based learning that systematically builds on students' curiosity. Students are first invited to explore problems that create an intellectual need for new mathematical ideas. Then the teacher builds on strategies used by students and connects their ideas to the learning goals of the lesson. This approach is an interpretation of Smith and Stein's 5 Practices for Orchestrating Productive Mathematics Discussions.

1 Launch



The Launch is a short, whole-class conversation that creates a need or excitement, provides clarity, or helps students connect to their prior knowledge or personal experience, which ensures that everyone has access to the upcoming work.

Considerations for Launching

- Try to keep it short. Set students up to get started with a clear and catchy invitation to the math.
- Wait to model a specific way to solve. Leave space for a variety of different student approaches.
- Pair up. Encourage small groups of students to talk through their thinking as they work.

2 Monitor



As students work individually, in pairs, or in groups, teachers explore students' thinking, ask questions, and provide support to help move the conversations closer to the intended math learning goal.

Considerations for Monitoring

- Look for students' strategies and check in with students as they work.
- Ask questions to learn what students are thinking. The suggested differentiation moves can be used to support, strengthen, and stretch their ideas.
- Use the Differentiation table to help select and sequence student ideas to highlight during the Connect discussion.

3 Connect



Teachers connect students' ideas to the key learning goals of the lesson, facilitating class discussions that help students synthesize and solidify the big ideas.

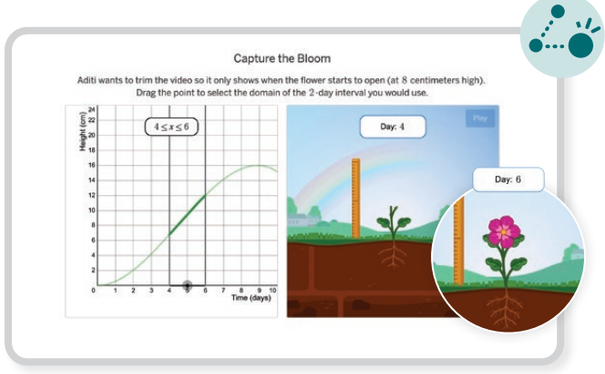
Considerations for Connecting

- Save a few minutes at the end of each lesson activity to bring students back together to discuss.
- It is okay to start the class discussion before all students have finished the activity.
- Students will be able to contribute to the discussion and learn from their classmates based on their in-progress work.
- Center discussions on students' ideas by displaying one or more of their responses and connecting the responses to the Key Takeaway of the activity.

Smith, M.S., & Stein, M.K. (2018). *5 practices for orchestrating productive mathematics discussions* (2nd ed.). SAGE Publications.

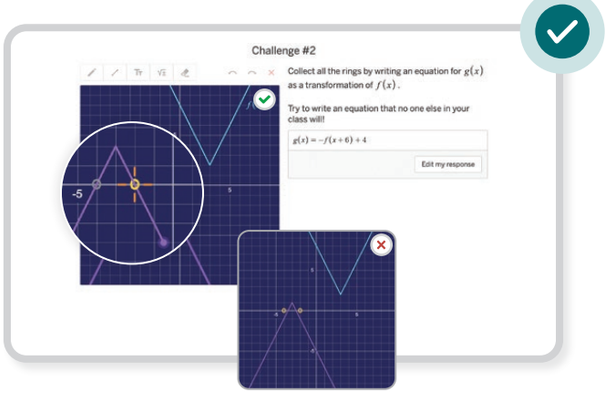
The Power of Digital

Lessons include visual and dynamic interactions that pique students' interest and invite all students to engage in the mathematics. The embedded interactions and animations allow students to test predictions, get feedback, share ideas, and connect representations.



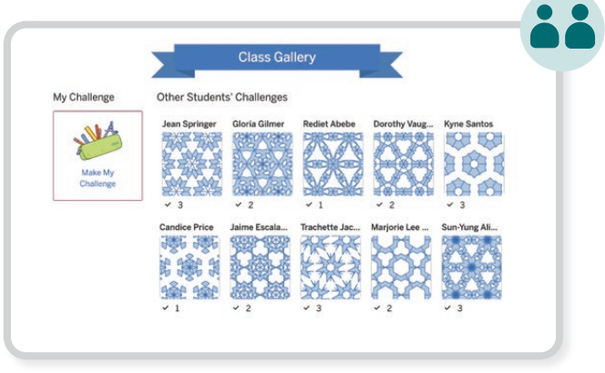
Delightful, engaging interactions

The digital interactions included in lesson activities are designed to elicit students' thinking in a way that feels fun and inviting. As students play and explore math concepts, teachers can highlight the ideas that students share, connect those ideas to other students' ideas, and build on their thinking through productive class discussion.



Responsive Feedback™

In Amplify Desmos Math, students are invited to try their thinking out — even if that thinking is still in the "rough draft" phase. As students interact with the digital screens, they see visuals and simulations that respond faithfully to their inputs. This meaningful feedback allows students to experience the joy of causing the animation to react to their mathematical ideas. As a result, students may notice interesting things about both correct and incorrect answers more readily.

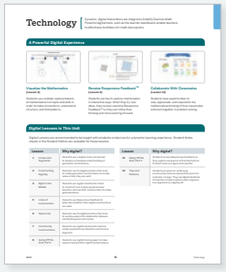


Social, collaborative experiences

Digital tools allow students to interact with each other's ideas in a variety of ways. Students can use the "Share With Class" button to exchange ideas with each other directly on an activity screen. Activities like Challenge Creator and Polygraph offer fun ways for students to play with the math together. Whether working solo or in tandem on devices, students are never alone as they work through activities.

Digital Lesson Recommendations

Check out the Technology page at the beginning of every unit to learn which lessons are recommended to be taught with students using devices for a dynamic learning experience. Lessons with digital screens have been carefully selected to support student sense-making with exploration and Responsive Feedback. An aligned notetaking page is available.

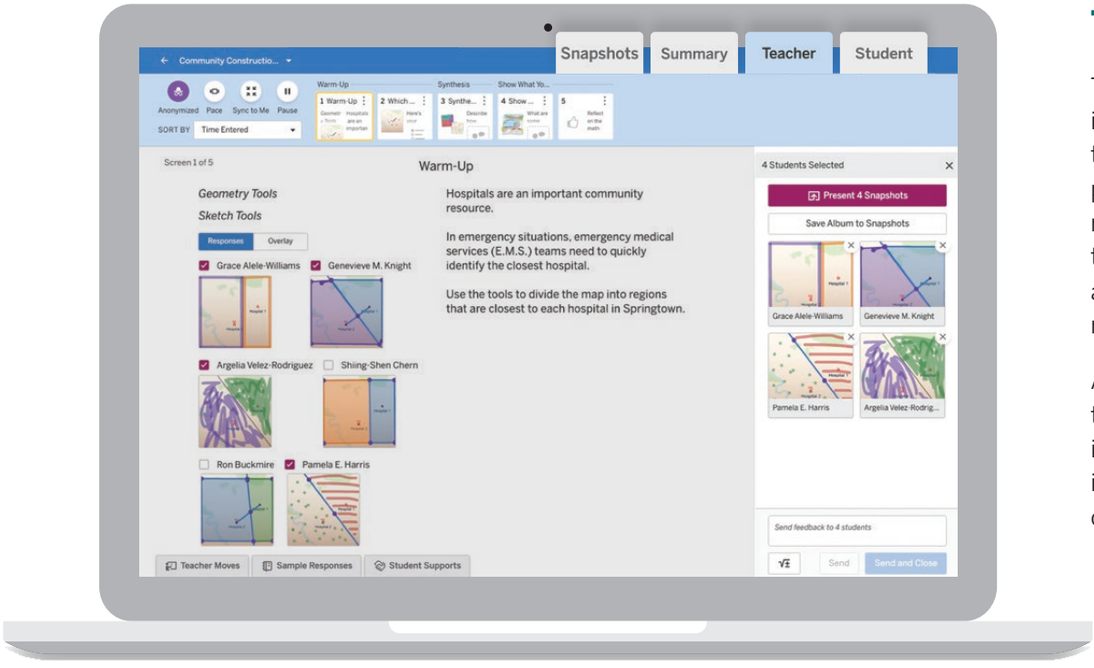


Teacher Presentation Screens

Print lessons also benefit from the power of digital. Teachers display Presentation Screens that include important animations and interactions to help guide and facilitate students' learning.

Digital Facilitation Tools

Amplify Desmos Math includes a suite of digital facilitation tools for lessons that are recommended to have students using devices. These tools foster collaborative classrooms and help teachers share their students' thinking.



Teacher Dashboard

The dashboard gives the teacher insight into students' thinking in real time. Teachers can zoom in on a particular student or view all students' responses at once. This can help to identify students that may need additional support and those who are ready for extensions.

After reviewing students' thinking, teachers can select and display specific ideas or the distribution of responses to invite students into productive, student-centered discussions.

Snapshot View

Select and sequence students' work to connect mathematical ideas. Teachers can even add their own questions to prompt further thinking.

Summary View

Monitor students' progress or accuracy for a set of screens. Click into any box to see a specific student's work on that screen.

Teacher View

Answer questions such as:

- How did all students answer this question?
- What answers were most common?

Student View

Preview screens for students. For example, the teacher might work through a screen with the class or talk through upcoming screens before students work on their own.

A powerful conversation toolkit

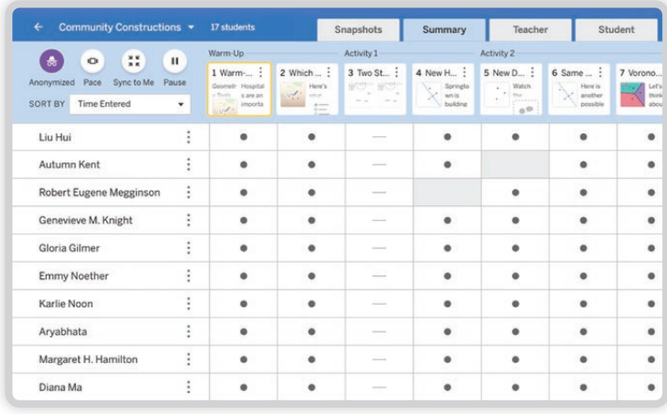
Pacing, pausing, and sync tools can support teachers as they encourage mathematical discourse and collaborative thinking in their classes.

Anonymize swaps out students' names to help them feel more comfortable sharing their ideas. Students' names are replaced by the names of famous mathematicians, with a special emphasis toward mathematicians with diverse backgrounds.

Pace allows the teacher to make a certain number of screens available for students to work on.

Sync to Me enables the teacher to bring all students to the same screen.

Pause stops students from working so that the teacher can gather everyone's attention for discussion.



Math Identity and Community

Amplify Desmos Math lessons are structured to elicit all students' powerful math ideas. The Math Identity and Community feature supports teachers in helping students build confidence in their own mathematical thinking, develop skills to work with and learn from others when doing math, and learn how math is an interwoven part of their broader community.

Math Identity is how students see themselves in relation to learning about and engaging with math.

Math Community is both the supportive learning environment of the math classroom as well as how math is a part of students' larger communities.

Each classroom is a unique combination of students, teachers, and school cultures. Teachers will find **embedded suggestions** within each lesson that they can use to provide opportunities for all students to mathematize their world. This is especially important for students who may not see themselves as belonging in math or may not see math as belonging in their lives.

Teachers can use the suggested prompts to broaden students' ideas about what it means to be good at math, highlight the value of each student's contributions, and celebrate math class as a place for coming together to think in flexible, creative, and interesting ways. These habits of mind can help students engage with math joyfully and successfully both in and outside of math class.

Embedded Suggestions

Here are some examples of the **Math Identity and Community** supports embedded in each lesson:

Math Identity and Community

Celebrate the variety of ways that students describe the same situation, highlighting how multiple perspectives help us have a broader understanding.

Math Identity and Community

Consider naming strategies after students in your class who share them. Encourage students to use each other's strategies by name throughout the lesson.

Math Identity and Community

Celebrate the fact that there are many possible strategies for connecting these different representations to one another. Consider asking the class why having more than one strategy might be useful.

Math Identity and Community

Remind students that confusion is a part of learning and thank students who ask questions and/or share their reasoning for supporting the class's collective learning.

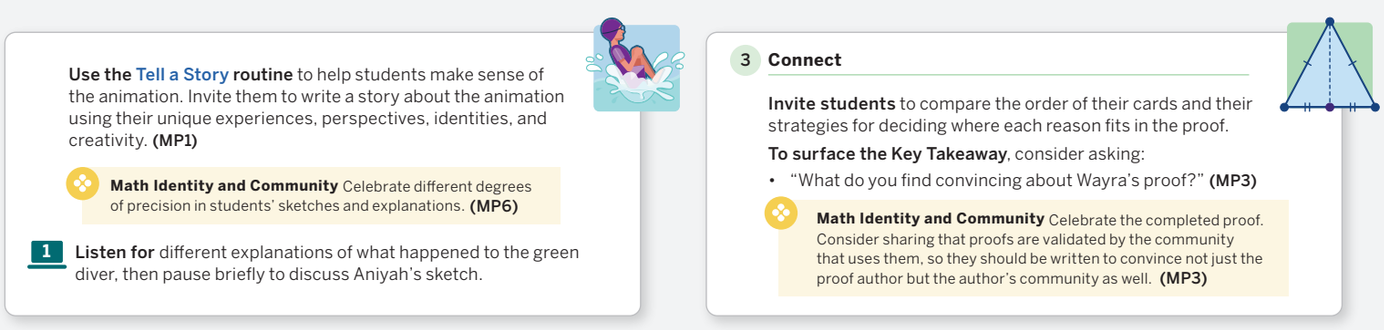
Facilitating Meaningful Conversations

Amplify Desmos Math is designed to support social and collaborative classrooms, where students make sense of mathematics by sharing their thinking in paired, small-group, and whole-class conversations.

Classroom conversations are an opportunity for students to connect and share their experiences, strengths, and knowledge for the benefit of the entire class. Students are encouraged to share their early thinking, consider the perspectives of others, and appreciate the value of different approaches and changing one's mind. Lessons celebrate math as a source of joy and as a tool for analyzing and making sense of the world. Teachers will find structures and tips to support them in facilitating meaningful conversations throughout each lesson.

Centering Personal Connections

Teachers are encouraged to consider the experiences of the students and communities represented in their classes and what connections they might have to various math topics and contexts. The Teacher Edition offers support for building conversations around the creativity and unique experiences that students bring to the classroom.

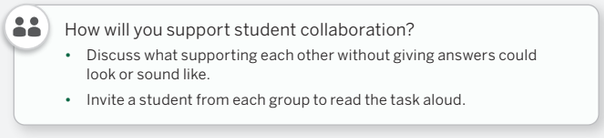


In the Algebra 2 lesson **Splashing Into Fun(ctions)**, students are invited to use the Tell a Story Routine to describe what is happening in an animation of a diver. This is a chance for students to use their own experiences to make sense of the mathematics.

In the Geometry lesson **Proving Points**, students are invited to order the arguments in a proof. The teacher's edition provides prompts to help students identify the characteristics of a proof that make it strong and convincing.

Supporting Student Collaboration

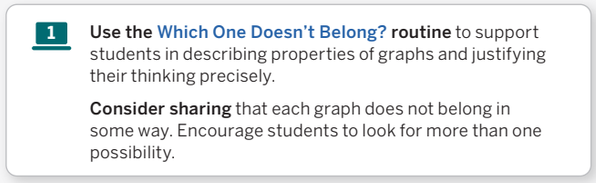
Different kinds of **groupings** are suggested for different phases of a lesson, including **whole-class** discussions and **pair** work. The conversations that these groupings encourage are opportunities for students to share their thinking and listen to each others' perspectives.



The Teacher Edition provides suggestions for intentional groupings to support access and language development and help students support their partners without simply sharing answers. Digital activities usually suggest that pairs of students share a single device. This 2:1 structure allows students to talk through tasks together in ways that inform each other's thinking.

Celebrating Different Perspectives

Many lesson activities invite students to consider a variety of different responses, including early or unfinished thinking.



In the Algebra 2 lesson **Symmetry Studio**, students use the **Which One Doesn't Belong?** routine to explore the properties of graphs and justify their thinking precisely.

Accessibility

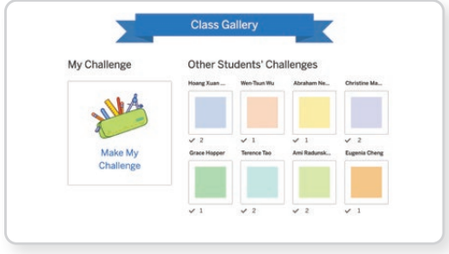
Amplify Desmos Math is built to support all students in accessing and participating in meaningful and challenging learning. This support is incorporated into the curriculum structure, lesson-level guidance, and digital tools.

Universal Design for Learning

Each lesson incorporates opportunities for engagement, representation, action, and expression based on the guidelines of Universal Design for Learning (UDL). UDL is a research-based framework designed to ensure meaningful learning experiences for all students.[1]

Multiple Means of Engagement

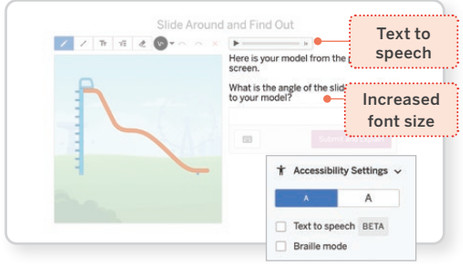
Individuals are each motivated in different ways, at different times, and in different contexts. Lessons are designed to welcome interests, support sustaining effort and persistence, and develop emotional capacity.



Sustaining Effort and Persistence: Students are invited to build their own challenge for other students to solve, which provides opportunities for choice and autonomy, as well as joy and play.

Multiple Means of Representation

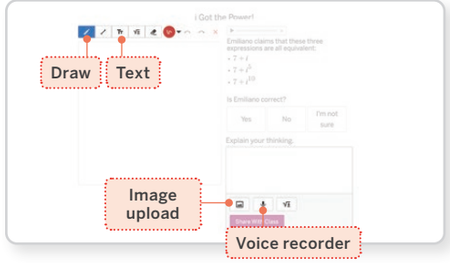
Learners make sense of information differently. Amplify Desmos Math includes options for presenting information in multiple ways to support comprehension and understanding of language.



Present Information in Multiple Ways: Students and teachers can use digital tools such as text to speech, enlarged font, and dynamic visuals.

Multiple Means of Action and Expression

Learners differ in how they navigate learning environments and express what they know. Amplify Desmos Math ensures that materials and interactions are accessible, support multiple means of students' expression and communication, and scaffold executive functioning.



Support Multiple Means of Student Expression and Communication: Students can communicate their ideas in multiple ways, including sketching, uploading photos, or responding with audio.

Lesson Facilitation Supports

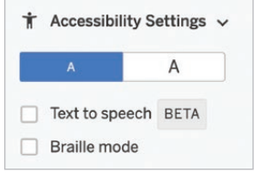
Every lesson includes at least one specific suggestion the teacher can use to increase access to the lesson without reducing the mathematical demand of the tasks. These suggestions address the following areas:

- Conceptual Processing
- Visual-Spatial Processing
- Executive Functioning
- Memory and Attention
- Fine Motor Skills
- Affective Functioning

A **Accessibility: Memory and Attention** To support keeping track of information, invite students to record the dimensions they try and whether or not they created a scaled copy.

Accessibility Tools

Students have the ability to control accessibility tools so that each learning experience is customized to their individual needs. In many instances, these tools can be turned on or off at any point of instruction.



Text to speech: Reads text instructions to students in multiple languages.

Enlarged font: Increases the size of all text on screen.

Braille mode: Includes narration of digital interactions.

Language selection: Toggles between languages.

[1] https://udlguidelines.cast.org/

Differentiation

Differentiation in Amplify Desmos Math enables teachers to ensure all students have access to grade-level math content. This support is organized for teachers into three categories:

Support

Provide targeted intervention for students by using these resources.

Strengthen

Reinforce students' understanding of the concepts assessed by using these resources.

Stretch

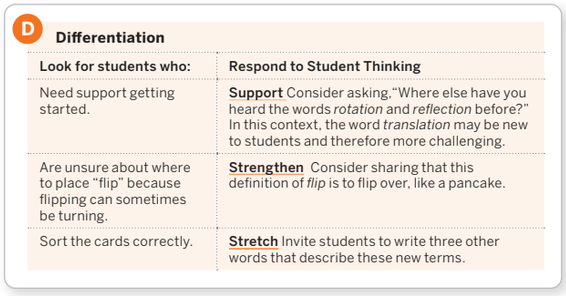
Challenge students and extend their learning with these resources.

Differentiation: In-Lesson Teacher Moves

Within every lesson activity, teachers can use the **Differentiation** suggestions to provide **in-the-moment instructional support** to learners as they engage in the work of the lesson.

Teachers are provided with clear student actions and understandings to look for, each matched with immediately usable suggestions for how to respond to the student thinking illustrated in each row of the table.

In addition to using these suggestions in the moment as teachers monitor student work, teachers can review the Differentiation table in advance to help them anticipate how students are likely to approach the activity.

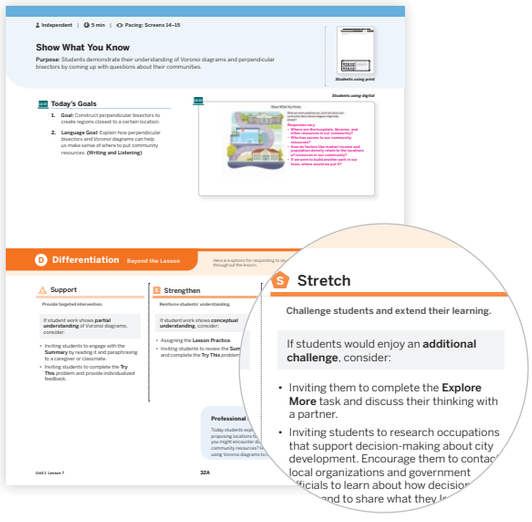


Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Where else have you heard the words *rotation* and *reflection* before?" In this context, the word *translation* may be new to students and therefore more challenging.
Are unsure about where to place "flip" because flipping can sometimes be turning.	**Strengthen** Consider sharing that this definition of *flip* is to flip over, like a pancake.
Sort the cards correctly.	**Stretch** Invite students to write three other words that describe these new terms.

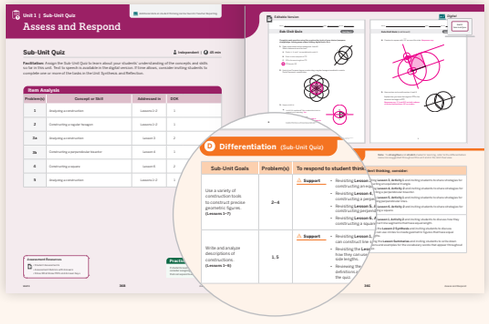
Differentiation: Beyond the Lesson

At the end of each lesson, guidance for how to support students' understanding of the learning goals is broken down for teachers into three categories: facilitation tips or resources that can be used to review, reinforce, or extend the key ideas. The guidance shows teachers which specific resources within the curriculum can be used to provide opportunities for each student to develop their understanding, including creative ways to stretch their thinking.



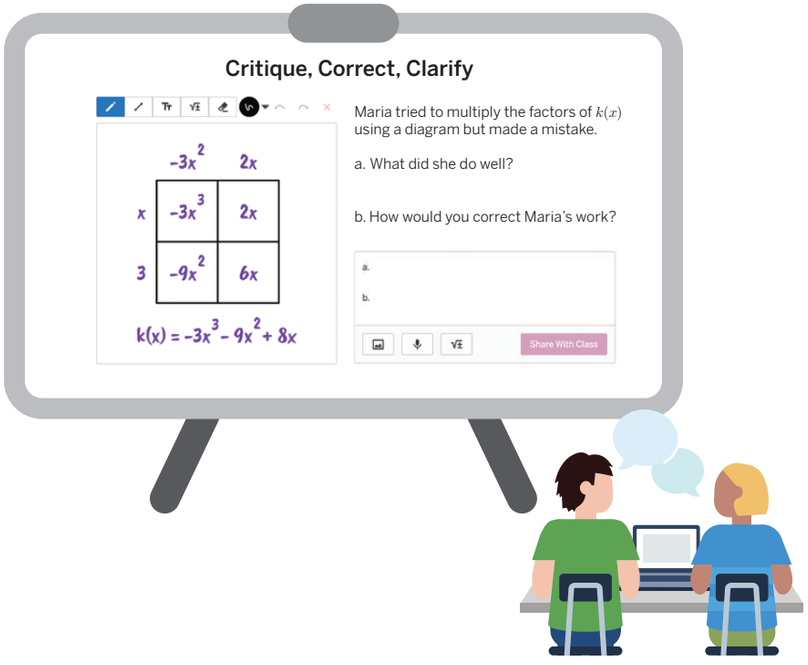
Differentiation: Assess and Respond

At each assessment point in a unit, teachers have the opportunity to respond to student understanding. For each assessment item, teachers are provided with clear suggestions for how to support, strengthen, or stretch student understanding of the corresponding concept based on their assessment responses.



Instructional Routines

Instructional routines that are used repeatedly create efficiencies for teachers so that they can attend to student thinking and communicate what is important in their classroom. Instead of focusing on the directions of an activity, students can focus on making sense of and communicating about the math.



Here are some examples:

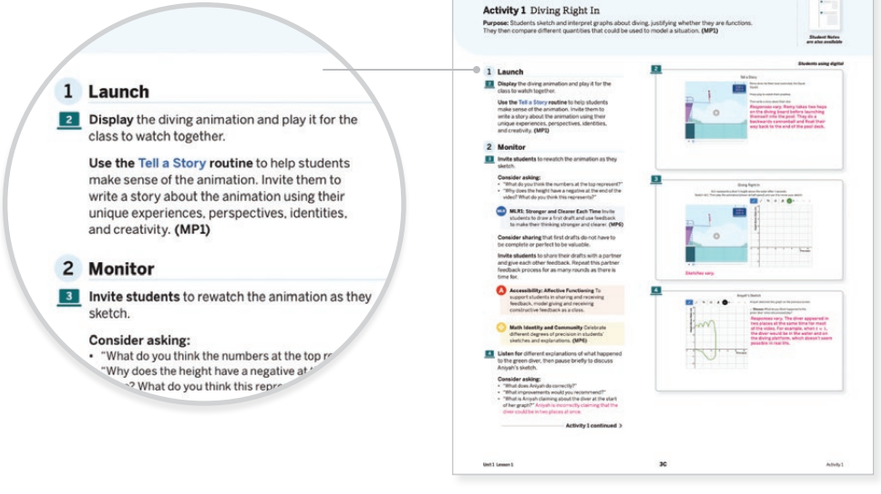
- The **Critque, Correct, and Clarify** routine is intended to help students communicate conceptual errors and ambiguities in language, as well as to model the process they might go through as they refine their own work. Students do more than just error analysis; they consider both the author's mathematical thinking as well as how they communicate their ideas.
- The **Stronger and Clearer Each Time** routine communicates the importance of feedback and creates an opportunity for students to learn from each other as they construct and refine their viable arguments.

Each instructional routine included in an Amplify Desmos Math lesson creates opportunities for conversations and supports meaningful discussion. Implementing these routines can be a practical tool for establishing a classroom learning community that values students' thinking.

Instructional Routines Embedded in the Curriculum

Instructional routines can be found throughout each lesson in the Teacher Edition. Here is a list of the instructional routines used in the Amplify Desmos Math curriculum:

- **MLR1:** Stronger and Clearer Each Time
- **MLR2:** Collect and Display
- **MLR3:** Critique, Correct, Clarify
- **MLR5:** Co-Craft Questions
- **MLR6:** Three Reads
- **MLR7:** Compare and Connect
- Decide and Defend
- Notice and Wonder
- Number Talk
- Tell a Story
- Think-Pair-Share
- Which One Doesn't Belong?



Math Language Development

At Amplify Desmos Math, we acknowledge that there is a strong interconnection between mathematical content, practices, and language. We believe that developing mathematical language is critical for all learners, while recognizing and supporting the unique needs of Multilingual/English Learners.

Our approach to math language development focuses on when, how, and why students are using language to make sense of and share their mathematical ideas. Every lesson in Amplify Desmos Math includes opportunities for all students to develop mathematical language as they experience the content, while providing intentional support for Multilingual/English Learners. We purposefully progress language throughout the units by cultivating students' language and supporting students in making their arguments and explanations stronger, clearer, and more precise as they progress from lesson to lesson.

Here are four ways we support students with developing mathematical language:

Vocabulary

Units and lessons build connections between students' language and the new vocabulary for that unit. This honors the language assets that students bring into their learning.

Some **new vocabulary** is marked as *optional*; these are terms students may encounter in other settings, but are not essential for them to access the lesson. Vocabulary is *review* if has been introduced and defined in a prior unit or course. The Unit Overview also lists **contextual vocabulary**, which are context-specific non-mathematical terms critical for students to understand the lesson.

To surface the Key Takeaway, consider asking:

- "How can you use these terms to describe dilations?"
- "How can you use dilations to model shadow puppets?" **(MP4)**

Create a class definition for the term **dilation** using students' language and the glossary.

Invite students using notes to annotate the shadow puppet image and capture the class definition of dilation.

Language Goals

Language goals attend to the mathematics students are learning and are written through the lens of one or more of four language modalities: **Reading**, **Writing**, **Speaking**, and **Listening**. They describe how students will grow their ability to analyze, compare, critique, describe, discuss, explain, interpret, and justify. Language goals appear at both the lesson and sub-unit level.

- **Today's Goals**
  1. **Goal:** Identify the key features of graphs from different *function families*.
  2. **Language Goal:** Describe graphs of functions using terms such as *positive, negative, maximum, minimum, increasing, and decreasing*. **(Reading, Writing, Speaking, and Listening)**
  3. **Language Goal:** Compare and contrast *function families* using their key features. **(Reading, Writing, Speaking, and Listening)**

Math Language Routines (MLR)

MLR **Math Language Routines**[1] are used within lessons to do one or more of the following: highlight student-developed language and ideas, cultivate conversation, support mathematical sense-making, and promote meta-cognition. Tips for facilitating MLRs are included when they would be helpful within lessons.

MLR **MLR7: Compare and Connect** Invite students to analyze each strategy, compare them, and then connect to their strategy. Consider asking:

- "Who used a strategy similar to Omari's? Ivory's?"
- "Where is the constant difference in each strategy?"
- "Why did different strategies lead to the same outcome?"

Multilingual/English Learners Support

Supports for ML/EL **Multilingual/English Learners** are called out at intentional points within each lesson. These suggested supports are specific, targeted actions that are beneficial for Multilingual/English Learners. They often describe a modification to increase access to the task or support with contextual or mathematical language development that can often be supportive of all learners. Multilingual/English Learner supports may also be attached to MLRs.

ML/EL **Multilingual/English Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

[1] Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). Principles for the Design of Mathematics Curricula: Promoting Language and Content Development. Retrieved from Stanford University, UL/SCALE website: http://ell.stanford.edu/content/mathematics-resources-additional-resources

Program Assessments

A variety of performance data in Amplify Desmos Math provides evidence of student learning while helping students bolster their skills and understanding.

Throughout lessons, units, and the entire program, you will find summative and formative assessments meant to provide insights into students' conceptual understandings. Student learning is never a surprise at the end of a unit — with Amplify Desmos Math, understanding is made continually visible.

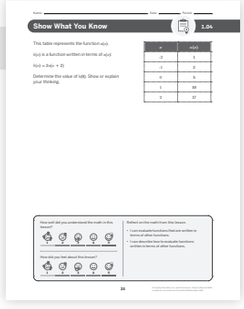
Amplify Desmos Math Reporting offers insightful and actionable data to help teachers understand their students' strengths, create grades,and modify instruction.

Lesson-Level Assessments

Amplify Desmos Math lessons are centered around sense-making and in-the-moment feedback. Daily moments of assessment provide insights into students' understandings of concepts and skills.

Show What You Know

Each lesson has a daily formative assessment focused on one of the **key concepts** in the lesson. Show What You Know moments are opportunities for students to show their teacher what they understand and what they are still learning.

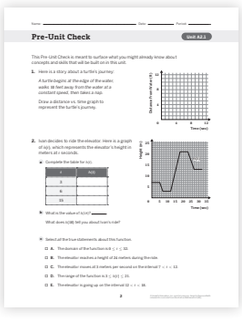


Unit-Level Assessments

Our **embedded unit assessments** offer key insights into students' conceptual understanding of math. These assessments provide regular, actionable information about how students are thinking about and processing math, with both **auto-scoring** and **in-depth rubrics** that help teachers anticipate and respond to students' learning needs.

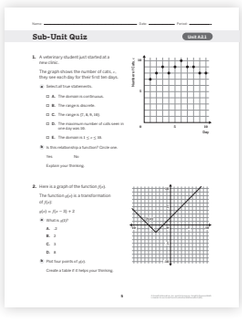
Pre-Unit Check (Optional)

Each unit begins with a check to determine students' **proficiency** with **prerequisite skills** that are helpful for success in the upcoming unit. This check serves as an affirmation of the knowledge and skills with which students come into the unit.



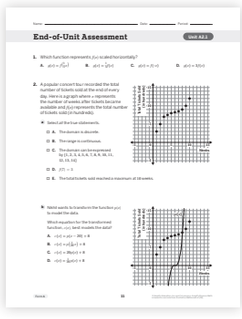
Sub-Unit Quizzes

In Sub-Unit Quizzes, students are assessed on a subset of **conceptual understandings and procedural fluencies** from the unit, with rubrics that help illuminate students' current understanding and provides guidance for responding to student thinking.



End-of-Unit Assessment

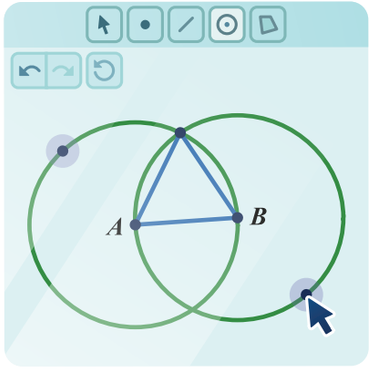
Students engage with rigorous grade-level mathematics through a variety of formats and tasks in the End-of-Unit Assessment. A combination of auto-scored and rubric-scored items provide deep conceptual insight.



The Geometry Tool

Many lessons in this course utilize the state-of-the-art digital Geometry Tool.

What is the Geometry Tool?



The Geometry Tool can be used to digitally construct mathematical objects, such as points, lines, segments, arcs, and circles. This tool supports students in constructing increasingly complex objects, such as perpendicular bisectors or Voronoi diagrams.

It can also be used to visualize and perform transformations such as translations, dilations, reflections, and rotations.

The Geometry Tool is one of many ways to explore geometric concepts, in addition to physical tools such as compasses and straightedges. Throughout the curriculum, students are invited to strategically consider the tools they use to support their exploration of geometric concepts and relationships.

Why Digital?

The Geometry Tool provides opportunities for:

Exploration

The Geometry Tool fosters curiosity and exploration. It supports students in testing their conjectures in real time, which can lead to surprise and delight. Students can construct precise mathematical objects, easily undoing steps and trying again as they further their understanding of what stays the same and what changes in geometric relationships.

Assessing Correctness

It can be difficult to tell at a glance whether a student has constructed an object with mathematical rigor.

A key feature of the Geometry Tool is its ability to assess whether an object has been constructed with sound mathematical logic — even if done in an unexpected way. This assessment is in part determined by whether a change in initial conditions results in the expected change in the constructed object.

Student Choice

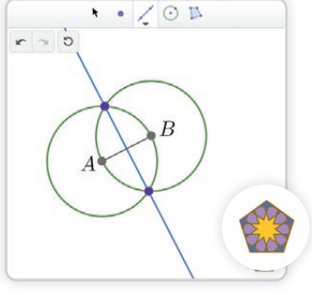
The Geometry Tool is several tools in one: It can be used to digitally construct line segments, circles, polygons, parallel and perpendicular lines, among other objects. Students thus have the ability to strategically choose sub-tools to help them solve problems.

Geometry Tool in Action

In this example, students are asked to construct a perpendicular bisector of $\overline{AB}$.

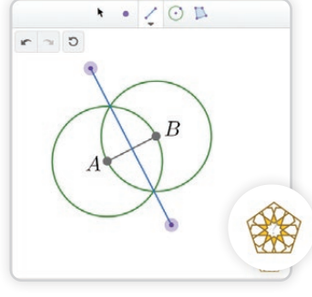
Correct

The student used the Geometry Tools to precisely construct a perpendicular bisector, receiving the full "Dramatic Star" in the bottom-right corner of their screen.



Partially or Indeterminately Correct

In a subtle but important difference, this construction has all the necessary elements, but the location of the perpendicular bisector was approximated. An unfilled star acknowledges the merits of this construction, with a small "?" to flag that this response may merit teacher inspection.



Student Notes

Providing students with the opportunity to take notes while engaging with digital lessons is a powerful way to enhance their learning experience. Thoughtful note-taking helps bridge the gap between digital exploration and lasting understanding, allowing students to structure their thinking, reflect on key concepts, and create a resource they can revisit.

Our approach to note-taking encourages students to decide what's important and how to record it meaningfully. By summarizing key ideas, choosing relevant examples, and annotating concepts, students deepen understanding, improve retention, and foster a sense of ownership over their learning. This practice also builds a classroom culture where their thinking is valued, flexible, and worth revisiting.

Components of a Student Notes sheet:

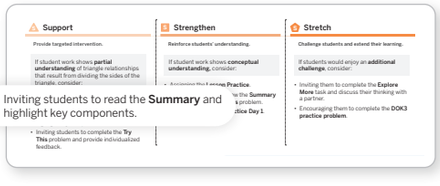
1. **Select Screens:** strategically identified pause points for teachers to invite students to respond to an additional prompt or to capture discussion notes that support their digital work
   - Teacher Edition connection: guidance on when and how to direct students to these screens.

   > **Invite students using notes** to record some important considerations from peer-to-peer discussion.

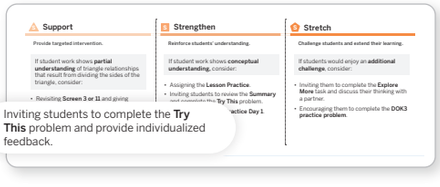
2. **Left Column:** encourages students to synthesize what they want to remember. Adapted from Cornell-style notes.
   - Teacher Edition connection: guidance on opportunities to scaffold this support for students

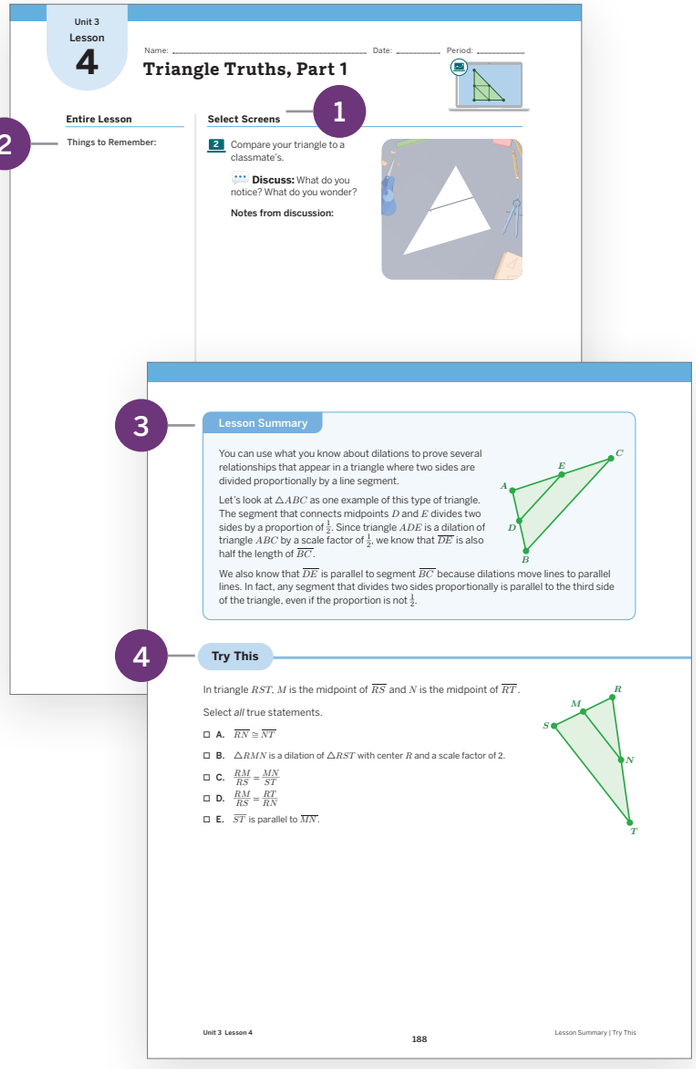
   > **Invite students using notes** to record ideas from the discussion in the left column while they unpack the proof and determine whether it is convincing.

3. **Lesson Summary:** captures core ideas for students to revisit and reflect upon after the lesson.
   - Teacher Edition connection: differentiation tips to prompt students to annotate or paraphrase the summary to a caregiver or classmate



4. **Try This:** an additional problem that allows students to practice applying the most important ideas of the lesson.
   - Teacher Edition connection: differentiation tips to prompt students to engage with this extra practice





Mathematical Modeling

Why Mathematical Modeling Matters

Living in our society requires navigating many quantitative situations, which are rarely as tidy as the typical math textbook problems. In today's world, technology handles most math computations, which puts a premium on the work of translating between real-world scenarios and mathematical models, and making decisions from the insights they offer.

*"Whatever trajectory a student's life takes after high school, being able to think flexibly, to make realistic assumptions about what may happen when the future is uncertain, and to understand how conclusions are reached based on prior knowledge are very valuable life tools."**

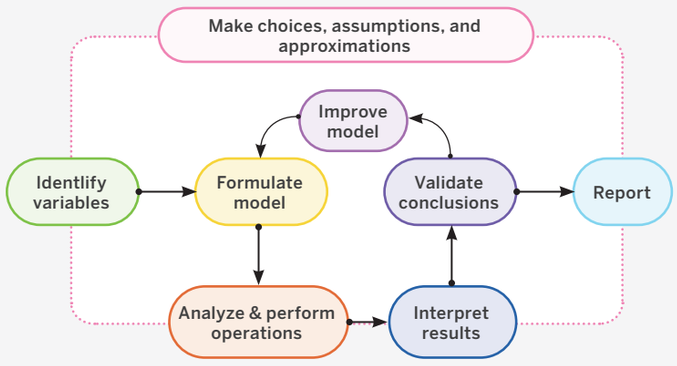
Students as Mathematical Modelers

Mathematical Modeling is a process by which learners use mathematics to represent, understand, and make predictions about real-world situations. Modeling is often cyclical, non-linear, and messy.

The diagram below shows one representation of the process, where each bubble represents an "action" students perform as they engage in the modeling process.

- **Make choices, assumptions, and approximations.**
- **Identify variables** and quantities that represent essential features.
- **Formulate a model** using geometric, graphical, tabular, algebraic, or statistical representations of variable relationships.
- **Analyze and perform operations** using the model to draw conclusions.
- **Interpret the results** in terms of the original situation.
- **Validate conclusions** by comparing them to the situation.
- Revisit steps to **improve the model** if needed.
- **Report** on conclusions and the reasoning behind them.

Mathematical Modeling Process



Mathematical Modeling in Amplify Desmos Math

Full Intent of Modeling Process

Each high school course includes lessons where students engage in all or most of the modeling actions. In the Teacher Edition, these lessons are marked with a modeling icon.

Modeling This lesson attends to the full intent of the modeling process.

Modeling Actions in Lessons

Many lessons have components where students engage in one or more modeling actions, identified with a modeling icon and description in the Teacher Edition.

Modeling Invite students to *identify multiple variables* they could measure about this situation.

Supplemental Tasks *(coming soon)*

Each high school course will include supplemental, full-cycle modeling tasks that are not tied to specific units.

*GAIMME (2019). Guidelines for Assessment and Instruction in Mathematical Modeling Education (2nd ed.). COMAP and SIAM. Retrieved from https://www.siam.org/media/wwjd5o2k/gaimme-2nd-ed-final-online-viewing-color.pdf

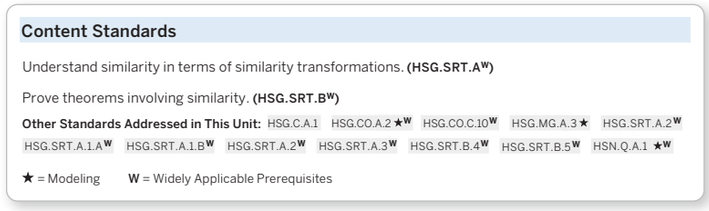
Foundations for the Future

Amplify Desmos Math is thoughtfully designed to prepare students for success in high school and beyond. Learning topics are carefully curated and pedagogical tenets are strategically designed to address the skills most important for post-secondary success, leaving students well-equipped for their academic, professional, and personal futures.

Widely Applicable Prerequisites

Widely Applicable Prerequisites are the "knowledge and skills that are widely applicable as prerequisites for postsecondary education." The Common Core State Standards identify a specific subset of standards as being widely applicable to be college and career ready (CCSSM, pp. 57, 84).

Throughout Amplify Desmos Math, teachers and students spend the majority of their time on these standard strands, knowledge, and skills. These standards are specifically outlined in the Publisher's Criteria for the CCSS in Mathematics (Table 1, p. 8). They are also marked with a "**W**" in this course within each unit opener.



College and Career Readiness

The Amplify Desmos Math curriculum is designed to prepare students not only for success in college-level mathematics but also for the demands of a rapidly changing world. The problem-based learning in this curriculum ensures that students are not simply "finding the answer" but also analyzing strategies, testing ideas, constructing arguments, and participating in mathematical conversations. These are precisely the skills required in both postsecondary education and the modern workplace, where answers are rarely provided upfront and individuals must make sense of complex situations through collaboration, reasoning, and adaptability.

Assessment Preparation

Amplify Desmos Math prepares students for success on standardized assessments like the SAT and ACT by addressing the full breadth of content included on these assessments. Through exposure to similar problem types within lesson practice sets, students gain familiarity with standardized assessment formats. This ample time devoted to deep engagement with content standards and mathematical practices, all while encouraging students to creatively problem solve, equips learners to tackle the most challenging and novel of assessment questions.

Empowered Problem Solvers

By engaging with rich problems, pattern analysis, and collaborative thinking, students build curiosity, creativity, critical thinking, resilience, and clear communication. These skills empower students to navigate complex ideas, make thoughtful decisions, and contribute meaningfully to their communities — preparing them not only for future studies and careers but also for the challenges and opportunities of an ever-changing world.

Amplify Desmos Math

Geometry

Teacher Edition

UNIT 1

Constructions and Rigid Transformations

Students use a variety of tools to construct geometric figures. Students then extend what they learned about rigid transformations in Grade 8 by constructing the transformations precisely.

Essential Questions

- How can we construct precise geometric figures using circles and lines?
- How can we describe transforming geometric figures using reflections, rotations, and translations?
- How can we use tools to construct geometric relationships?

Sub-Unit Goals

Sub-Unit 1	• Use a variety of construction tools (e.g., compass, straightedge, and dynamic geometry software) to construct precise geometric figures. • Write and analyze descriptions of constructions.
Sub-Unit 2	• Perform and describe a sequence of rigid transformations that moves a figure onto another. • Describe the relationships between an image and a pre-image when a figure is transformed.

The Math of the Unit

Focus Telling a story that narrows and deepens with the grade

Content Standards

Experiment with transformations in the plane. **(HSG.CO.A [W])**

Make geometric constructions. **(HSG.CO.D.12)**

Other Standards Addressed in This Unit: HSG.CO.A.1 [W] HSG.CO.A.2 [W] HSG.CO.A.4 [W] HSG.CO.A.5 [W] HSG.CO.D.13 HSG.MG.A.2 ★ HSG.MG.A.3 ★

★ = Modeling **W** = Widely Applicable Prerequisites

Standards for Mathematical Practice

- Students use perpendicular bisectors to construct Voronoi diagrams that help them make decisions about where additional resources in a community should be placed. **(MP4)**
- Students make decisions about which tools to use when constructing and transforming geometric figures. **(MP5)**
- Students communicate precisely by giving instructions to construct geometric figures with specific properties and to specify sequences of transformations that move one figure onto another. **(MP6)**

Also Addressed: **MP1, MP3, MP7, MP8**

Coherence Telling a story within and across grades

In This Unit

Students construct geometric figures using a variety of tools. They then use their construction strategies to perform rigid transformations.

Prior Learning

- **Grade 7,** Unit 3: Draw and construct geometric figures **(7.G.A)**
- **Grade 8,** Unit 1: Perform rigid transformations **(8.G.A)**

Future Learning

- **Geometry,** Unit 2: Prove congruence through rigid transformations **(HSG.CO.B, HSG.CO.C)**
- **Geometry,** Unit 3: Prove similarity through rigid transformations and dilations **(HSG.SRT.A)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lesson 5**, students develop conceptual understanding of how more complex geometric objects can be constructed by using known relationships.

Strategic Competence

In **Lessons 9–11,** students justify the properties of reflected, translated, and rotated images using constructions.

Procedural Fluency

In **Lesson 2**, students repeatedly use a compass and straightedge to construct increasingly sophisticated designs.

Adaptive Reasoning

In **Lesson 3**, students write and revise instructions for reproducing geometric constructions with precision.

Application

In **Lesson 7**, students apply their understanding of constructing perpendicular bisectors to justify the locations of community resources.

Productive Disposition

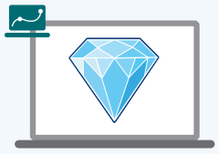
In **Lesson 13**, students relate the mathematics they do in class to the creation of geometric art.

Unit at a Glance

Students use devices for most lessons as indicated with .

Student Notes sheets in the Student Edition are available for these lessons.

Start of the Year



Meet & Greet

Begin the year with this lesson designed for you and your students to get to know each other.

Assess and Respond

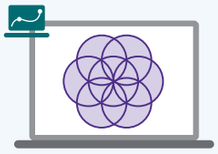


A Pre-Unit Check (Optional)

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

4.G.A 4.G.A.1 7.G.B.4 8.G.A 8.G.A.1 8.G.A.2 8.G.A.3 MP6 MP7

Sub-Unit 1



1 Circles and Segments

An Introduction to Constructions

Explore a simple digital tool that creates circles and line segments to engage with basic geometric constructions.

HSG.CO.A.1 HSG.CO.D.12 MP7



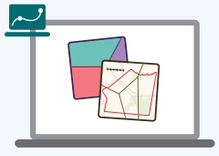
2 Compass Constructions

Constructing Patterns

Use a compass and straightedge to create precise, sophisticated designs.

HSG.CO.D.12 HSG.CO.D.13 MP6 MP7

Sub-Unit 2



7 Community Constructions

Using Perpendicular Bisectors

Use digital tools to create Voronoi diagrams to divide maps and analyze the locations of community resources, such as hospitals and libraries.

HSG.CO.D.12 HSG.MG.A.2 ★ HSG.MG.A.3 ★ MP3 MP4

Practice Day



P Practice Day 1

Practice the concepts and skills developed during Lessons 1–7. Consider using this time to prepare for the upcoming quiz.

HSG.CO.A.1 HSG.CO.D.12 HSG.CO.D.13 MP5

Assess and Respond

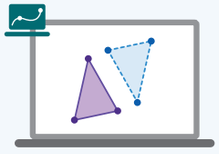


A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

HSG.CO.A.1 HSG.CO.D.12 HSG.CO.D.13 MP5 MP6

Sub-Unit 2



8 Going Off the Grid, Part 1

Introducing Sequences of Transformations

Perform and describe rigid transformations using digital tracing paper.

HSG.CO.A.2 HSG.CO.A.5 MP6 MP7

Practice Day



13 Tiles and Patterns

Geometric Art Using Constructions and Transformations

Use underlying constructions and geometry tools to create unique geometric patterns.

HSG.CO.A.5 HSG.CO.D.12 MP7



P Practice Day 2

Practice the concepts and skills developed during Lessons 1–13. Consider using this time to prepare for the upcoming assessment.

HSG.CO.A.1 HSG.CO.A.2 HSG.CO.A.5 HSG.CO.D.12 MP5 MP6

Assess and Respond



A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit.

HSG.CO.A.1 HSG.CO.A.2 HSG.CO.A.4 HSG.CO.A.5 HSG.CO.D.12 HSG.CO.D.13 MP5 MP6

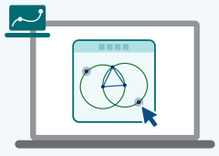
Unit Synthesis and Reflection: Invite students to complete 1–2 tasks to synthesize and reflect on what they learned in the unit.

Pacing: 17 days (+1 optional day) | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 13 Lessons
- 2 Practice Days
- **Pre-Unit Check** (optional)
- **Sub-Unit Quiz**
- **End-of-Unit Assessment**

Getting to Know Each Other and Unit Synthesis and Reflection activities are available digitally.

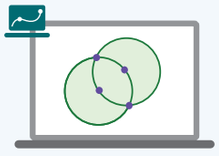


3 Constructing Digitally

Geometric Constructions

Explore the digital construction tools that will be used throughout the course.

HSG.CO.D.12 HSG.CO.D.13
MP3 MP6

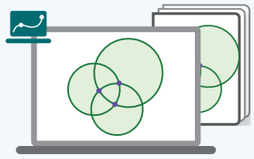


4 Right in the Middle

Constructing Perpendicular Bisectors

Explore equal distances by constructing **perpendicular bisectors**.

HSG.CO.A.1 HSG.CO.D.12
MP1 MP6 MP8

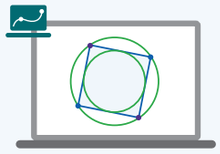


5 Lines of Construction

Constructing Perpendicular and Parallel Lines

Construct **perpendicular lines** with digital tools and **parallel lines** with physical tools.

HSG.CO.A.1 HSG.CO.D.12
MP3 MP6

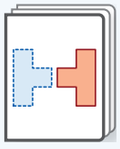


6 Square Up

Constructing Squares

Construct rectangles and squares, including squares that are **inscribed** in circles.

HSG.CO.D.12 HSG.CO.D.13
MP3 MP6



9 Time to Reflect

Defining Reflections

Develop construction techniques and an understanding of reflections.

HSG.CO.A.2 HSG.CO.A.4
HSG.CO.A.5
MP5 MP6



10 Translation Information

Defining Translations

Use tools to precisely translate points and figures.

HSG.CO.A.2 HSG.CO.A.4
HSG.CO.A.5 MP5 MP6

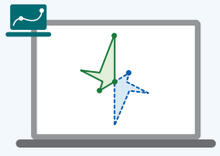


11 Rotation Devices

Defining Rotations

Precisely describe and perform rotations by indicating the center of rotation and the amount of rotation.

HSG.CO.A.2 HSG.CO.A.4
HSG.CO.A.5 MP5 MP6



12 Going Off the Grid, Part 2

Sequences of Transformations

Perform sequences of transformations that will move a figure onto another.

HSG.CO.A.2 HSG.CO.A.5
MP5 MP6 MP7

Concept Development

By the following points, students will have had multiple opportunities to practice the key skills in this unit:

- **After Lesson 3:** Use circles to represent line segments of equal length and the intersection of circles to create more complex geometric figures.
- **After Lesson 5:** Construct perpendicular bisectors to both bisect a segment and create right angles.
- **After Lesson 7:** Use a variety of geometry tools to precisely create and recreate geometric figures.
- **After Lesson 12:** Describe and perform geometric transformations using a variety of tools (compass and straightedge, tracing paper, folding paper, and dynamic geometry software).

Pacing Considerations

Lesson 7: This lesson gives students an opportunity to apply what they have learned about constructions in this unit to make sense of the placement of community resources. If this lesson is omitted, be sure to spend some time discussing how constructions can be useful in making decisions in different communities.

Lesson 8: This lesson is intended to revisit strategies for visualizing and performing rigid transformations with tracing paper from Grade 8. If students show a strong understanding on Problems 6–9 in the Pre-Unit Check, consider omitting this lesson.

Lesson 13: This lesson gives students an opportunity to apply what they have learned about constructions and transformations in this unit to make sense of geometric patterns found in Islamic architecture. If this lesson is omitted, be sure to discuss how constructions and transformations can be found in art and design around the world.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Having all items from the geometry toolkit available to students is recommended to build skills in selecting and using tools strategically. Specific tools recommended for each lesson are listed here.

Lesson 2

- **Copy:** Show What You Know *(one per student)*
- **Geometry toolkit:** compasses, straightedges

Lesson 5

- **Copy:** Activity 2 Sheet
- **Geometry toolkit:** compasses, straightedges

Lesson 7

- **Materials:** computers/devices *(for research)*

Practice Day 1

- **Cut:** Task Cards
- **Geometry toolkit:** compasses, straightedges

Lesson 8

- **Materials:** tracing paper *(as needed)*

Lesson 9

- **Copy:** Show What You Know *(one per student)*
- **Geometry toolkit:** compasses, straightedges, tracing paper

Lesson 10

- **Copy:** Show What You Know *(one per student)*
- **Cut:** Activity 2 Cards *(one set per pair)*
- **Geometry toolkit:** compasses, straightedges, tracing paper

Lesson 11

- **Copy:** Show What You Know *(one per student)*
- **Materials:** coloring tools or highlighters *(optional)*
- **Geometry toolkit:** compasses, straightedges, tracing paper

Lesson 12

- **Materials:** tracing paper *(as needed)*

Lesson 13

- **Materials:** computers/devices *(for research)*

Practice Day 2

- **Cut:** Task Cards
- **Geometry toolkit:** compasses, straightedges

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment, Form A
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards
- Synthesis and Reflection Tasks (optional)

Digital
PDF
Editable Version

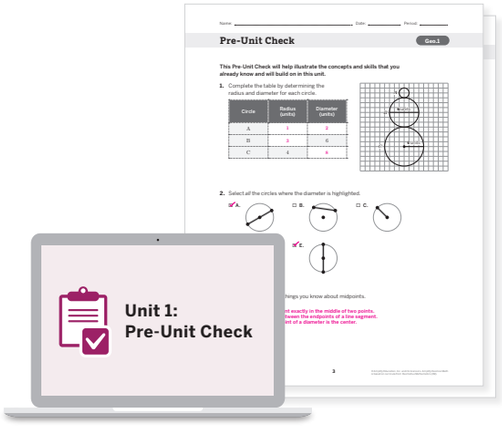
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check (Optional)

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

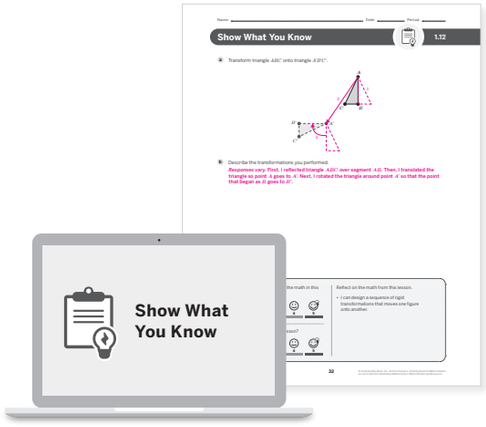
Standards assessed: 4.G.A, 4.G.A.1, 7.G.B.4, 8.G.A, 8.G.A.1, 8.G.A.2, 8.G.A.3



Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

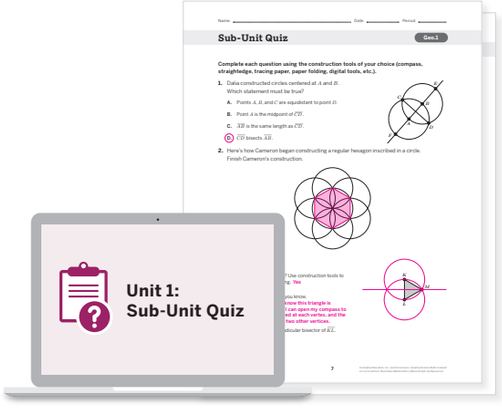
Standards assessed on Lesson 12 Show What You Know: HSG.CO.A.2, HSG.CO.A.5



Sub-Unit Quiz

Assign the **Sub-Unit Quiz** to learn about your students' understanding of the concepts and skills so far in this unit.

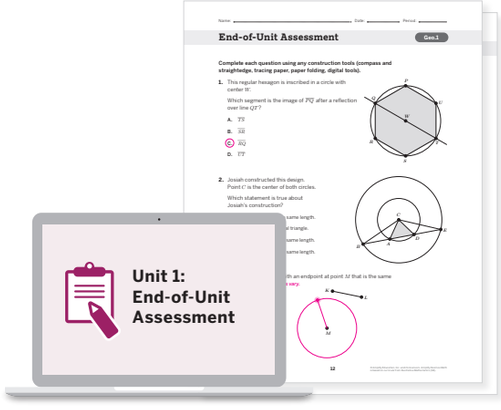
Standards assessed: HSG.CO.A.1, HSG.CO.D.12, HSG.CO.D.13



End-of-Unit Assessment

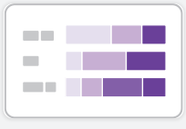
Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit.

Standards assessed: HSG.CO.A.1, HSG.CO.A.2, HSG.CO.A.4, HSG.CO.A.5, HSG.CO.D.12, HSG.CO.D.13



Reporting

Teacher and Administrator reporting is available for all digital assessments to help teachers understand their students' strengths, create grades, and modify instruction.



Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

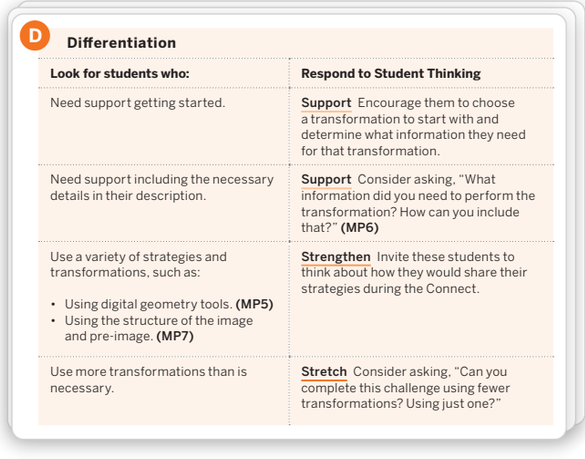
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–7) • Use a variety of construction tools (e.g., compass, straightedge, and dynamic geometry software) to construct precise geometric figures. • Write and analyze descriptions of constructions.	**Previous Lesson** • Algebra 1, Unit 5, Lesson 5 (*Lizard Lines*) **Student Notes** • Lessons 1, 3–7 **Teacher Moves** • Consider keeping a display of construction techniques for students to reference throughout the unit.	**Repeated Challenges** • Lesson 4 (*Right in the Middle*) **Challenge Creator** • Lesson 1 (*Circles and Segments*) • Lesson 3 (*Constructing Digitally*) **Practice Days** • Practice Day 1: Tasks A–D **Practice** • Lessons 1–7	**Explore More** • Lesson 4 (*Right in the Middle*) • Lesson 5 (*Lines of Construction*) • Lesson 6 (*Square Up*) • Lesson 7 (*Community Constructions*) **DOK3 Practice** • Lessons 1–7 **Teacher Moves** • Invite students to continue their research into how community resources are distributed using construction techniques.
Sub-Unit 2 (Lessons 8–13) • Perform and describe a sequence of rigid transformations that moves a figure onto another. • Describe the relationships between an image and a pre-image when a figure is transformed.	**Mini-Lesson** • Grade 8, Unit 1, (*Determining Coordinates After a Rotation*) **Student Notes** • Lessons 8, 12–13 **Previous Lessons** • Grade 8, Unit 1, Lesson 1 (*Transformers*) • Grade 8, Unit 1, Lesson 2 (*Spinning, Flipping, Sliding*) • Grade 8, Unit 1, Lesson 3 (*Transformation Targets*) • Grade 8, Unit 1, Lesson 7, Activity 2 (*No Bending, No Stretching*)	**Challenge Creator** • Lesson 8 (*Going Off the Grid, Part 1*) • Lesson 13 (*Tiles and Patterns*) **Practice Days** • Practice Day 2: Tasks A–D **Practice** • Lessons 8–13	**Explore More** • Lesson 11 (*Rotation Devices*) • Lesson 12 (*Going Off the Grid, Part 2*) **DOK3 Practice** • Lessons 8–13 **Teacher Moves** • Invite students to continue their research into how transformations can be used to create geometric patterns and art.

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 12**, where students engage with a series of challenges that utilize different sequences of transformations.



Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to choose a transformation to start with and determine what information they need for that transformation.
Need support including the necessary details in their description.	**Support** Consider asking, "What information did you need to perform the transformation? How can you include that?" **(MP6)**
Use a variety of strategies and transformations, such as: • Using digital geometry tools. **(MP5)** • Using the structure of the image and pre-image. **(MP7)**	**Strengthen** Invite these students to think about how they would share their strategies during the Connect.
Use more transformations than is necessary.	**Stretch** Consider asking, "Can you complete this challenge using fewer transformations? Using just one?"

Accessibility

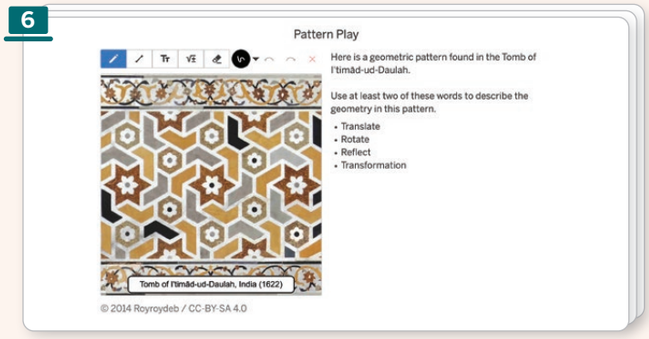
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Engagement.

Spotlight on Welcoming Interests & Identities . . .

Lesson 13: This lesson highlights the value and relevance of math among geometric patterns from Islamic cultures, constructions, and transformations through meaningful activities.



Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Fine Motor Skills . . .

Here are two suggestions in this unit for supporting students with their fine motor skills as they use different tools to construct geometric figures.

Lesson 2, Warm-Up

Accessibility: Fine Motor Skills To support students with using compasses, consider pairing students so they can take turns using the tool and giving real-time feedback on their partner's grip, the angle of the pencil, the pencil pressure, and the surface under their paper.

Lesson 9, Activity 1

Accessibility: Fine Motor Skills Consider varying the task demands by providing copies of the triangle for tracing or pre-filled in tracing paper.

Math Identity and Community

Every lesson includes suggestions to help develop and strengthen students' math identity and community.

Here are some suggestions in this unit for celebrating creativity and the broad range of student responses and perspectives.

Lesson 1, Activity 1

Math Identity and Community Invite students to use their creativity to explore different types of shapes and designs.

Lesson 6, Warm-Up

Math Identity and Community Celebrate the creative ways students engage with the tools when constructing a rectangle and the questions they develop in the process.

Lesson 7, Activity 3

Math Identity and Community Consider celebrating variety and creativity in what students notice and wonder, including things that surprise you or other students may not have noticed.

Lesson 13, Activity 2

Math Identity and Community Invite students to share patterns that they found delightful, beautiful, inspiring, or that they enjoyed recreating. Consider creating a class display of the images that students have created.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson
construction	1
equilateral triangle **regular polygon** *compass* (optional) *straightedge* (optional)	2
equidistant **midpoint** **perpendicular bisector**	4
parallel lines **perpendicular lines**	5
inscribe	6
Voronoi diagram	7
image **line of reflection** **pre-image**	9
prime notation	10

Review Vocabulary
center of rotation
circle
line segment
reflection
rigid transformation
rotation
translation
vertex

Contextual Vocabulary	Lesson
community resources population density	7

These are some of the non-mathematical words used in this unit, which may or may not be familiar to students. Consider presenting these words using contexts, visuals, and/or gestures to provide all students with greater access to these words.

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	2–3, 5, 8, 11–12
MLR2: Collect and Display	2, 4, 8, 11–12
MLR7: Compare and Connect	2, 4, 6–7, 12

Sub-Unit Language Goals

- Write and analyze descriptions of constructions.
- Describe the relationships between an image and a pre-image when a figure is transformed.

Multilingual / English Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

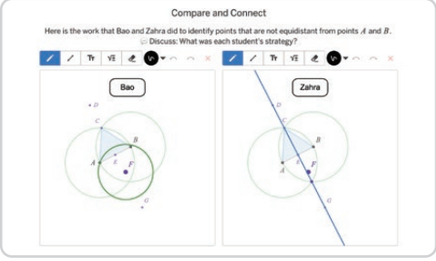
Here is one example from **Lesson 8**, where students visualize and engage with the vocabulary of rigid transformations.

ML/EL **Multilingual/English Learners** nvite students to research how to say *rigid transformation* in other languages. Create an anchor chart with the terms in multiple languages, along with examples. **(Reading and Listening)**

Technology

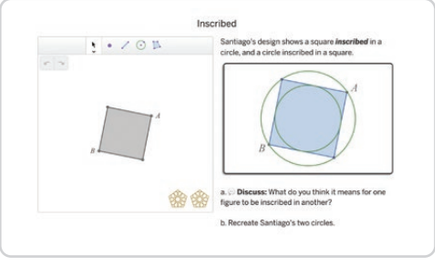
Dynamic, digital interactions are integral to Amplify Desmos Math. Powerful digital tools, such as the teacher dashboard, enable teachers to effectively facilitate rich math discussions.

A Powerful Digital Experience



Visualize the Mathematics
(Lesson 4)

Students use multiple representations of mathematical concepts and skills in order to make connections, understand structure, and find patterns.



Receive Responsive Feedback™
(Lesson 6)

Students are free to explore mathematics in interactive ways. When they try new ideas, they receive real-time Responsive Feedback™ so they can refine their thinking and move learning forward.



Collaborate With Classmates
(Lesson 13)

Students have opportunities to view, appreciate, and respond to the mathematical thinking of their classmates and work together in problem solving.

Digital Lessons in This Unit

Digital Lessons are recommended to be taught with students on devices for a dynamic learning experience. Student Notes sheets in the Student Edition are available for these lessons.

Lesson		Why digital?
1	Circles and Segments	Students use a digital circle and line tool to develop conceptual understanding of geometric constructions.
3	Constructing Digitally	Students use the digital construction tools to create geometric constructions and make sense of why they are valid.
4	Right in the Middle	Students use digital construction tools to construct and analyze perpendicular bisectors and use their constructions to make generalizations.
5	Lines of Construction	Students use Responsive Feedback to determine whether their digital constructions are valid.
6	Square Up	Students use the digital construction tools to quickly explore the relationship between inscribed squares and circles.
7	Community Constructions	Students use digital construction tools to create perpendicular bisectors and Voronoi diagrams.
8	Going Off the Grid, Part 1	Students use digital tracing paper to make sense of and perform rigid transformations.

Lesson		Why digital?
12	Going Off the Grid, Part 2	Students receive Responsive Feedback as they explore a sequence of transformations that will move one figure onto another.
13	Tiles and Patterns	Students are given an underlying construction and use intersection points to replicate a design. They use digital feedback to help them make revisions when angles or line segments are slightly off.

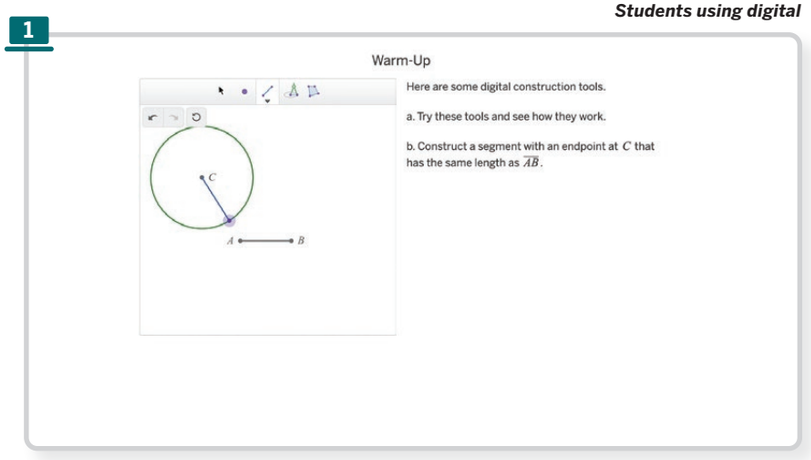
Professional Learning

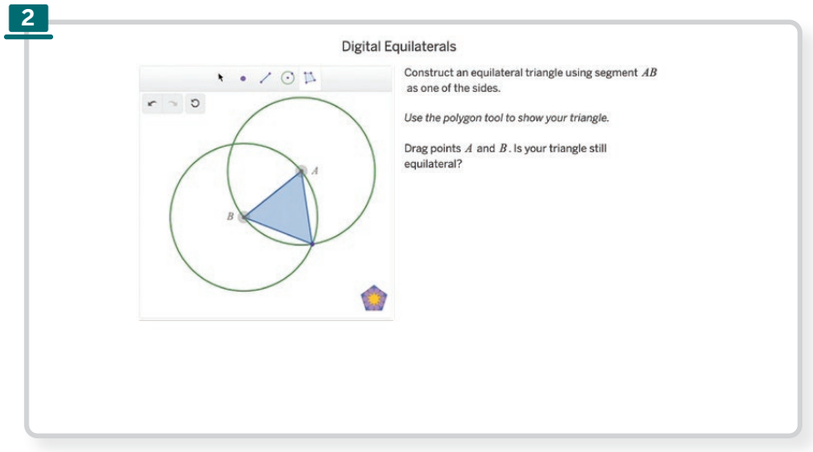
Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Digital Tools

Lesson 3, Screens 1–4

Students using digital





Context

In this activity, students develop procedural fluency using circles to construct segments with common lengths as they are introduced to digital geometry tools.

Try This

Put on your student hat and complete Screens 1–4.

Points to Ponder

- What different strategies and digital tools might students use to construct an equilateral triangle?
- What do students need to understand about precision in construction instructions to support their engagement with Aaliyah's instructions and different resulting constructions?
- How does constructing with digital tools build from their understanding of physical constructions?
- How will constructing circles support their understanding of equal distances in the future?

Every lesson includes an opportunity for teachers to reflect on their instruction. These include reflections about routines and activity structures, digital tools, mathematical concepts or language development, student thinking and strategies, math identity and community, and facilitating meaningful conversations.

Spotlight on Lesson 5

Professional Learning

Reflect on how students analyzed student work today. How did they talk about mistakes? Were they able to notice the correct parts, incorrect parts, or next steps of each student's construction? How can you support students in valuing learning from correct, incorrect, and unfinished thinking?

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed in their math journeys.

Use Rigid Transformations to Show Two Triangles Are Congruent

In this unit, students specified a sequence of rigid transformations that will move a figure onto another. In Unit 2, students will use rigid transformations to decide if two figures are congruent. They will use this definition of congruence in terms of rigid motions to explain the criteria for triangle congruence (ASA, SAS, and SSS).

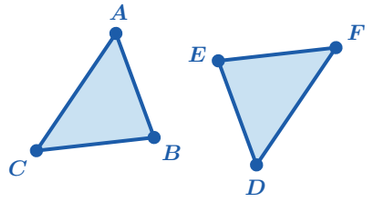
Standard Addressed: HSG.CO.B

Example:

Triangle ABC can be translated so that point A moves to point D.

Then the triangle can be rotated until point B is on top of point E.

Since there is a sequence of rigid transformations that moves triangle ABC to triangle DEF, the two triangles are congruent.



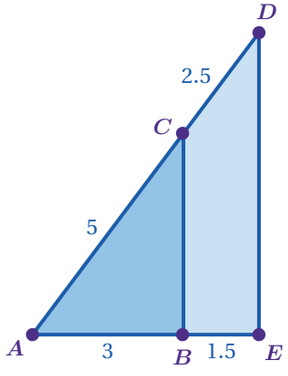
Prove Similarity Through Rigid Transformations and Dilations

In this unit, students specified a sequence of rigid transformations that will carry a given figure onto another. In Unit 3, students will extend the idea to a sequence of non-rigid transformations, to include dilations and to explain the properties of similar figures. They will use congruence and similarity to solve problems and prove relationships.

Standards Addressed: HSG.SRT.A, HSG.SRT.B

Example:

Point D is a dilation of point C using a center of dilation at point A and a scale factor of 1.5. Point E is a dilation of point B using the same center and scale factor. Because a sequence of rigid transformations and dilations can be used to move triangle ABC onto triangle ADE, they are similar.



Additional data on student thinking can be found in Teacher Reporting.

Assess and Respond

Pre-Unit Check (Optional)

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. Text to speech is available in the digital version. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or Skill	Preparation for	DOK	Standard(s)
1	Determine the measurement of the radius and diameter of a circle	Lesson 1	1	7.G.B.4
2	Identify the diameter of a circle	Lesson 1	1	7.G.B.4
3	Describe properties of midpoints	Lesson 4	2	4.G.A
4	Draw parallel and perpendicular lines using a grid	Lesson 5	2	4.G.A.1, 8.G.A
5	Identify parallel and perpendicular segments using a grid	Lesson 5	2	8.G.A
6	Identify corresponding points after a reflection	Lesson 8	1	8.G.A.1
7	Match types of rigid transformations	Lesson 8	2	8.G.A.2 MP7
8	Describe a rotation using a grid	Lesson 8	2	8.G.A.2, 8.G.A.3 MP6
9	Describe a sequence of rigid transformations using a grid	Lesson 12	2	8.G.A.2, 8.G.A.3 MP6

Assessment Resources

- Student Assessments
- Show What You Knows and Answers

Editable Version

Digital

Unit 1
Pre-Unit Check

Name: Date: Period:

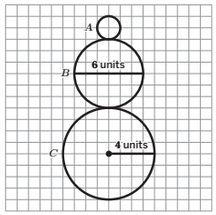
Pre-Unit Check

Unit Geo.1

This Pre-Unit Check is meant to surface what you might already know about concepts and skills that will be built on in this unit.

1. Complete the table by determining the radius and diameter for each circle.

Circle	Radius (units)	Diameter (units)
A	1	2
B	3	6
C	4	8



2. Select *all* the circles where the diameter is highlighted.

☑ A. 

☐ B. 

☐ C. 

☐ D. 

☑ E. 

3. Show or explain some things you know about midpoints.

Responses vary.
- A midpoint is the point exactly in the middle of two points.
- The point halfway between the endpoints of a line segment.
- In a circle, the midpoint of a diameter is the center.

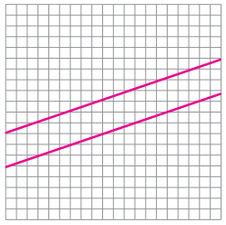
© Amplify Education, Inc. and its licensors. Amplify Desmos Math is based on curricula from Illustrative Mathematics (IM).

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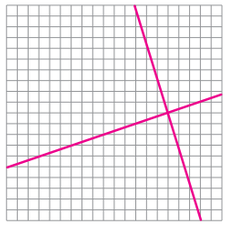
Pre-Unit Check (continued)

Unit Geo.1

4. a Draw two *parallel* lines. *Responses vary.*

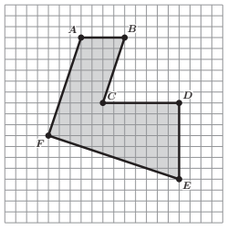


b Draw two *perpendicular* lines. *Responses vary.*



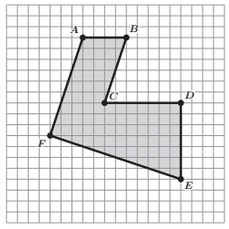
5. Here is figure $ABCDEF$.

a Identify two sides that are *parallel*.



Responses vary.
- Sides AF and BC
- Sides AB and CD

b Identify two sides that are *perpendicular*.



Responses vary.
- Sides AF and FE
- Sides CD and DE

Name: Date: Period:

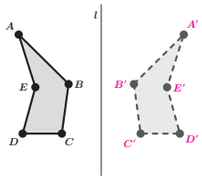
Pre-Unit Check (continued)

Unit Geo.1

6. Figure $A'B'C'D'E'$ is the image of figure $ABCDE$ after a reflection over line l.

Label each corresponding point on the image.

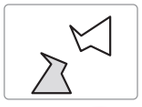
A' B' C' D' E'



7. For each picture, circle the transformation that best describes how one figure moves onto the other with a single transformation.

a 

Rotation Reflection
Translation

b 

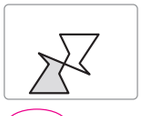
Rotation Reflection
Translation

c 

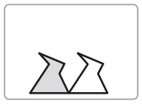
Rotation Reflection
Translation

d 

Rotation Reflection
Translation

e 

Rotation Reflection
Translation

f 

Rotation Reflection
Translation

Name: Date: Period:

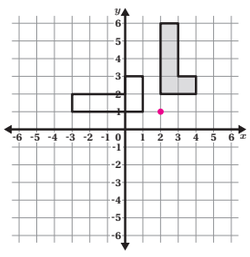
Pre-Unit Check (continued)

Unit Geo.1

8. a Plot the center of rotation that moves the shaded figure onto the unshaded figure.

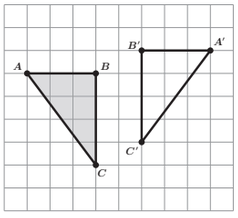
b Describe the rotation.

Responses vary. Rotate the shaded figure around (2, 1) until it exactly lines up with the unshaded figure.



9. Describe a sequence of transformations that moves triangle ABC onto triangle $A'B'C'$.

Responses vary. Reflect triangle ABC over line BC. Then translate the triangle up 1 unit and right 2 units.



D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Prior Knowledge	Problem(s)	To respond to student thinking, consider:
Identify the radius and diameter of a circle **(Preparation for Lessons 1–3)**	1–2	**Support** • Spending extra time on the design challenges on **Screens 3–6** of **Lesson 1** and creating a class anchor chart for the different parts of a circle (in particular the radius, center, and diameter). • Revisiting **Grade 7, Unit 3, Lesson 2, Activity 1** (*Is It a Circle?*) and inviting students to share strategies for identifying parts of a circle.
Identify properties of a midpoint **(Preparation for Lessons 4–7)**	3	**Support** • Drawing a segment and its midpoint and inviting students to discuss the properties they notice and questions they may have about the midpoint. • Giving students a grid and inviting students to draw segments and identify midpoints of those segments. Invite students to discuss how they determined the midpoint and what challenges they had. • Emphasizing these skills during **Lesson 4, Activity 1**.
Draw parallel and perpendicular lines **(Preparation for Lessons 5–6)**	4–5	**Support** • Revisiting **Grade 8, Unit 3, Lesson 8, Activity 1** (*Translations*) and inviting students to share what they notice and wonder about parallel lines on a grid. • Emphasizing these skills during **Lesson 5**.
Identify and describe rigid transformations **(Preparation for Lessons 8–12)**	6–9	**Support** • Spending more time on **Screens 3–5** of **Lesson 8**, discussing how the tracing paper represents rigid transformations, and creating a class anchor chart for each transformation that can be added to throughout the unit. • Using the ***Determining Coordinates After a Rotation*** **Mini-Lesson** from Grade 8, Unit 1.

Sub-Unit 1

Constructions

Sub-Unit 1 Goals

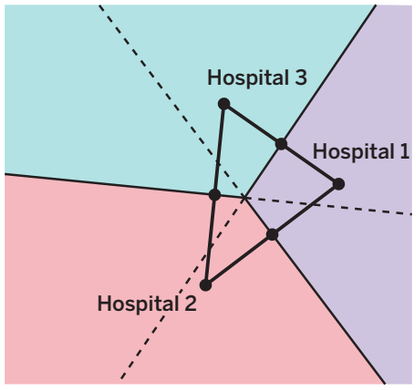
- **Goal:** Use a variety of construction tools (e.g., compass, straightedge, and dynamic geometry software) to construct precise geometric figures.
- **Language Goal:** Write and analyze descriptions of constructions.

In this sub-unit, students explore **constructions** with digital and physical tools, considering strategic uses of circles to measure equal distances. They apply their construction knowledge and skills to construct more complex figures. Students also write directions for their classmates to precisely replicate constructions and make their descriptions clearer using feedback.



Math That Matters Most

Concept or Skill	Example
A compass (or circle) is a helpful tool to create segments that have equal lengths.	Write instructions to construct an equilateral triangle using segment AB as one of the sides. When you drag points A and B, is your triangle still equilateral? *I used line segment AB as the length of the radius for two circles centered at points A and B. I connected one of the intersection points to points A and B. When I dragged the points around, it made all three segments longer, but they all stayed the same length since they were dependent on the length of AB. This means my triangle stayed equilateral.*
Intersection points of circles can be used to construct perpendicular bisectors.	Line m is a perpendicular bisector of $\overline{JK}$. Zahra says perpendicular bisectors pass through the segment's midpoint. Is this statement always, sometimes, or never true? *Always true. Every point on the perpendicular bisector is an equal distance from points J and K. Because the midpoint is the same distance from points J and K, the midpoint has to be on the perpendicular bisector.*

Concept or Skill	Example
Constructing perpendicular bisectors can be used to create regions closest to a certain location.	Divide the map into three regions, where each region shows all the locations closest to each hospital. *This Voronoi diagram shows regions that are closest to each hospital.* 

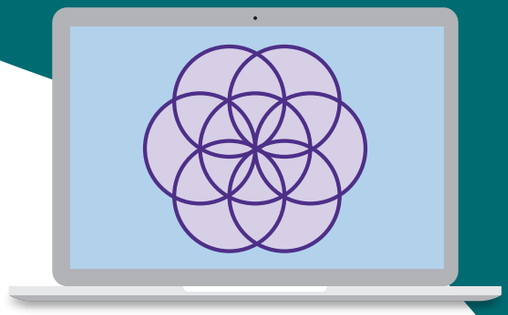
Unit 1

Lesson

1

Digital Lesson

This lesson uses digital student screens.
Student Notes are also available.



Circles and Segments

An Introduction to Constructions

Let's construct designs using circles and line segments.

Focus and Coherence

Today's Goals

1. **Goal:** Use a circle to construct different line segments of the same length.
2. **Language Goal:** Explain how circles help to construct figures with equal side lengths. **(Reading, Writing, Speaking, and Listening)**

Students are introduced to basic geometric **constructions** without explicit instructions or complex shapes. They explore with a simple digital tool that creates *circles* and *line segments* from intersections so they can focus on the reasoning behind the constructions instead of the tools. Students also use these tools to create their own designs and replicate others.

Prior Learning

In Grade 7, students drew geometric shapes with given conditions and tools (freehand, with a ruler and protractor, and with technology).

Future Learning

In Lesson 2, students will construct geometric objects on paper using a compass and straightedge. Throughout this unit, students will be introduced to different digital and paper-based tools to use for constructions.

Rigor and Balance

- Students use digital circle and line tools to develop their **conceptual understanding** of geometric constructions.

Vocabulary

New Vocabulary

construction

Review Vocabulary

circle *line segment*

Standards

Addressing

HSG.CO.D.12
Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, **dynamic geometric software**, etc.). **Copying a segment**; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

Also Addressing: **HSG.CO.A.1**

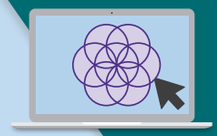
Mathematical Practice: MP7

Building On	Building Toward
HSG.CO.A.1	HSG.CO.D.12 HSG.CO.D.13

Lesson at a Glance

~ 45 min

Standards: HSG.CO.D.12, HSG.CO.A.1



Why digital?

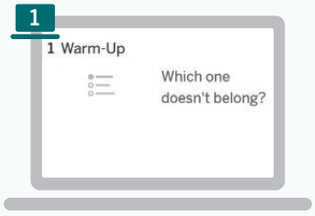
Students use a digital circle and line tool to develop conceptual understanding of geometric constructions.

Warm-Up

Independent | 5 min

Pacing: Screen 1

Students use the **Which One Doesn't Belong?** routine to see how *circles* can justify equal lengths.



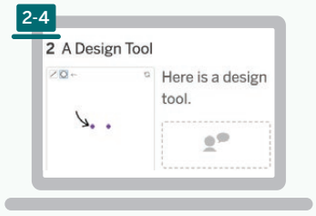
Activity 1

Pairs Sharing a Device | 10 min

Pacing: Screens 2–4

Students explore how to use circles and *line segments* by creating and matching precise designs.

Key Takeaway (Screen 4): Using *circles* of the same size to create intersection points can help us make precise designs.



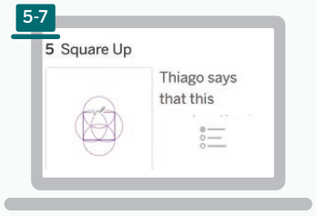
Activity 2

Pairs Sharing a Device | 10 min

Pacing: Screens 5–7

Students create and analyze **constructions** to make sense of how to use circles to create line segments with equal lengths.

Key Takeaway (Screen 7): Circles can be used to help make *line segments* with equal lengths. It is helpful to use intersection points of circles that are the same size to make precise **constructions**.

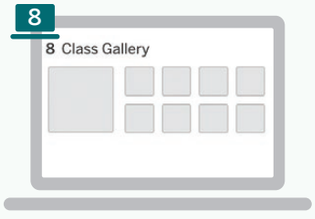


Activity 3

Independent | 10 min

Pacing: Screen 8

Students make their own geometric constructions and challenge their classmates to replicate their constructions using circles and line segments.



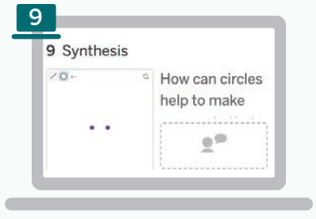
Synthesis

Whole Class | 5 min

Pacing: Screen 9

Students synthesize their understanding of using circles and line segments in geometric constructions.

Lesson Takeaway (Screen 4): A **construction** is a geometric drawing made by using a specific set of tools. Having a tool to create a *circle* is helpful to construct *line segments* of equal length.

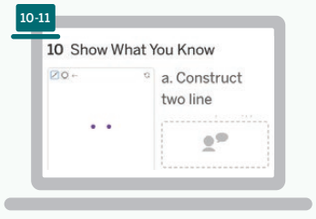


Show What You Know

Independent | 5 min

Pacing: Screens 10–11

Students demonstrate their understanding of using circles to copy a line segment by constructing two line segments with the same length.



Prep Checklist

Assign the digital student screens. Student Notes are also available.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)

Student Notes sheet in Student Edition:
Consider providing the Student Notes to support class discussion and student thinking.



Independent | 5 min | Pacing: Screen 1

Warm-Up

Purpose: Students use the **Which One Doesn't Belong?** routine to see how *circle*s can justify equal lengths.



Student Notes are also available

1 Launch

1 **Use the Which One Doesn't Belong? routine** to support students in looking for structure in each shape and understanding how the *circles* help to justify their thinking. **(MP7)**

Consider sharing that each shape does not belong in some way. Encourage students to look for more than one possibility.

Math Identity and Community Invite students to share how hearing the way different people think is valuable to everyone.

ML/EL **Multilingual/English Learners** Provide sentence frames (e.g., "_____doesn't belong because _____." or " _____is the only one that has/doesn't have _____."). **(Writing and Speaking)**

2 Connect

Discuss why *each* shape doesn't belong, asking students to justify their choice.

Capture and share student responses that mention circles as a way to justify equal lengths.

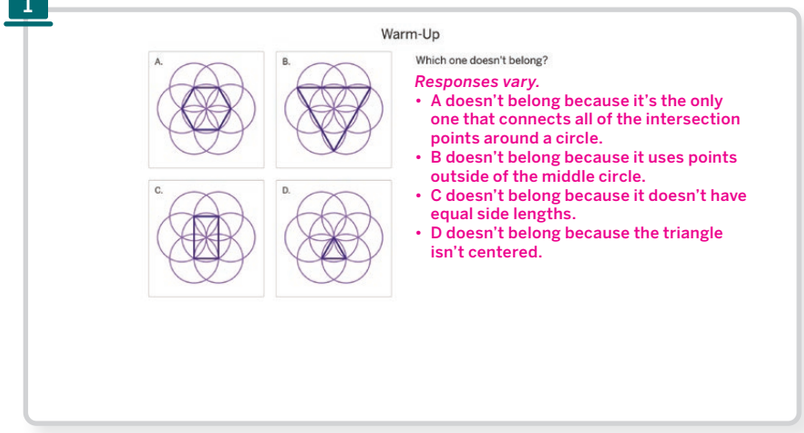
Consider asking:

- "How do the circles help us decide which shape doesn't belong?"
- "Would your answer change if the circles weren't visible in each image? Why?"

Note: If it doesn't come up naturally, consider asking, "How might circles be useful for comparing lengths of segments?"

Students using digital

1



Pairs Sharing a Device | 10 min | Pacing: Screens 2–4

Activity 1 Design Matcher

Purpose: Students explore how to use circles and *line segment*s by creating and matching precise designs.



Student Notes are also available

1 Launch

2 **Math Identity and Community** Invite students to use their creativity to explore different types of shapes and designs.

ML/EL **Multilingual/English Learners** Invite students to talk through their thinking with a partner before writing it down. **(Writing and Speaking)**

2 Monitor

Capture and share delightful designs, particularly if they have many segments of equal length.

Consider pausing to share and discuss student responses about how the digital design tool works.

Note: To create a circle, students must click on the points. If students need support getting started, consider demonstrating how more points appear when two intersecting circles are constructed.

3-4 **To support students getting started**, consider asking, "What shapes do you see in the design you're trying to match?"

Select and sequence rectangles, including those that used different intersection points from the design.

3 Connect

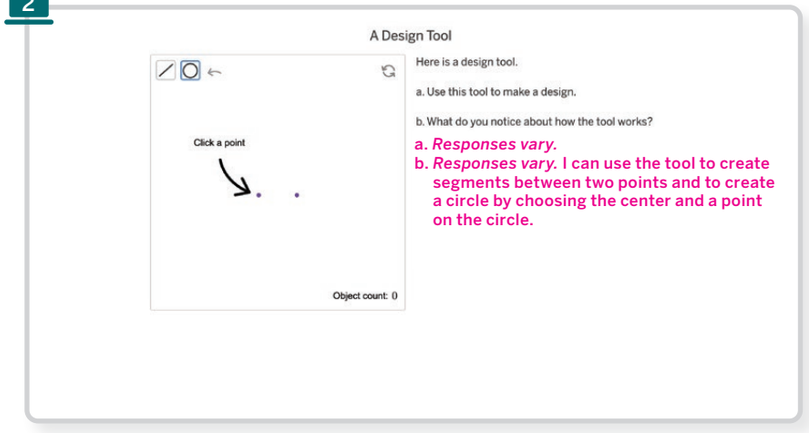
Share and discuss the captured rectangles. Invite students to share how specific rectangles were created using the circle and *line segments*.

A **Accessibility: Affective Functioning** To support students in sharing feedback as a whole class, model giving constructive feedback on one of the rectangles.

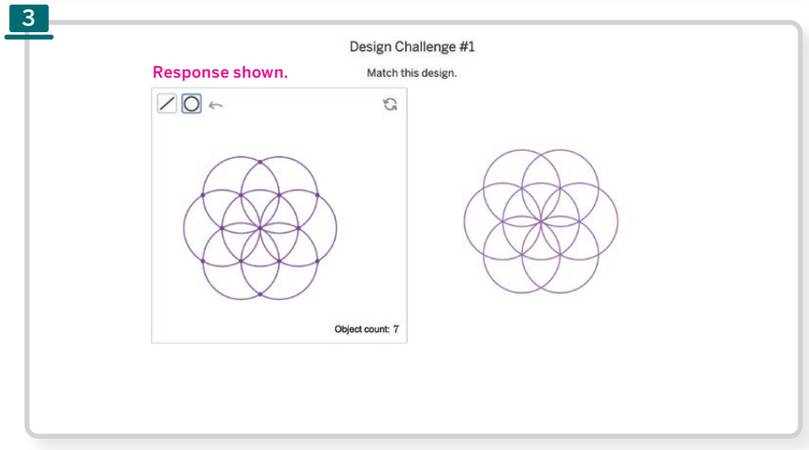
Consider asking, "How do the circles help us know that some of these rectangles are the same and some are different?"

Students using digital

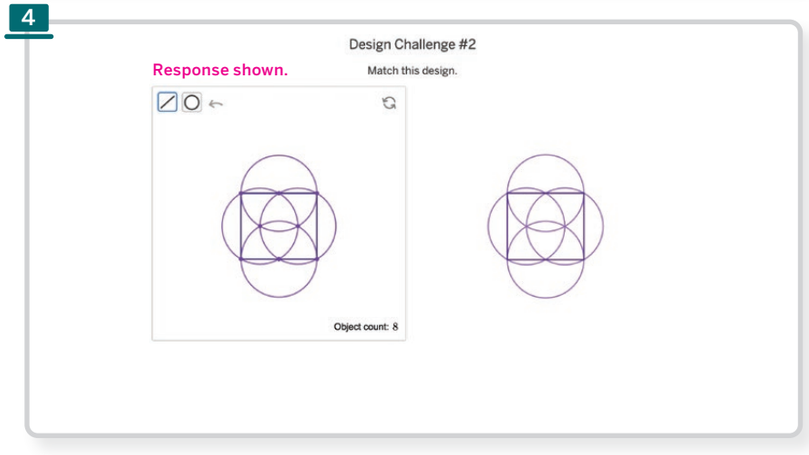
2



3



4



Key Takeaway: Using *circles* of the same size to create intersection points can help us make precise designs.

Pairs Sharing a Device | 10 min | Pacing: Screens 5–7



Student Notes are also available

Activity 2 Under Construction

Purpose: Students create and analyze **constructions** to make sense of how to use circles to create line segments with equal lengths.

Short on time? Consider creating a whole-class discussion for Screen 7 by asking each pair to contribute only one response to either part a or b.

1 Launch

5 **Invite students** to share with a partner what is required for a shape to be a square.

Create a class definition for the term **construction** using students' language and the glossary. To spark conversation, consider asking:

- "How have we been making our drawings in class today?"
- "What other tools could we use to create constructions?"

Invite students using notes to document the class definition of the term *construction*.

2 Monitor

6 **To encourage playfulness**, invite students to try to create an equilateral triangle that no one else in class will create.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How can you use circles to create line segments of the same length?"
Need support coming up with a second equilateral triangle.	**Support** If students use radius lengths as the side length, consider asking, "How can you combine radiuses to make longer equal segments?"
Create two congruent triangles with different orientations.	**Strengthen** Consider asking, "Can you create a triangle that is a different size than the first one?"

Capture different equilateral triangles, including congruent triangles that have different orientations.

Invite students using notes to answer, "How did you know the side lengths were equal in your construction?"

7 **Look for** responses that note line segments have a set length, while lines continue without end.

3 Connect

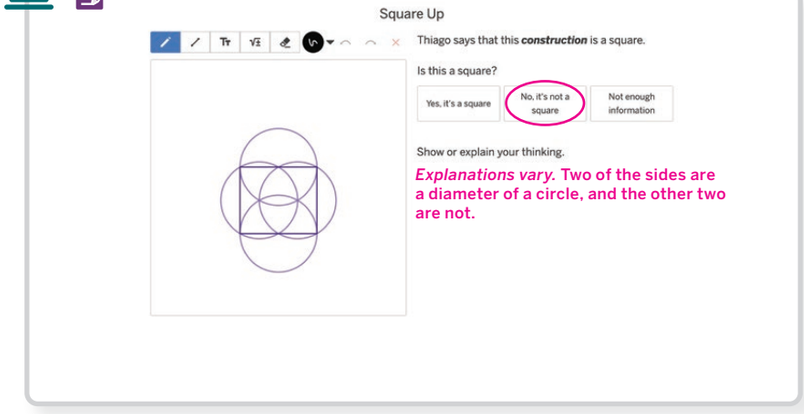
6 **Share and discuss** captured equilateral triangles.

7 **To surface the Key Takeaway**, consider asking:

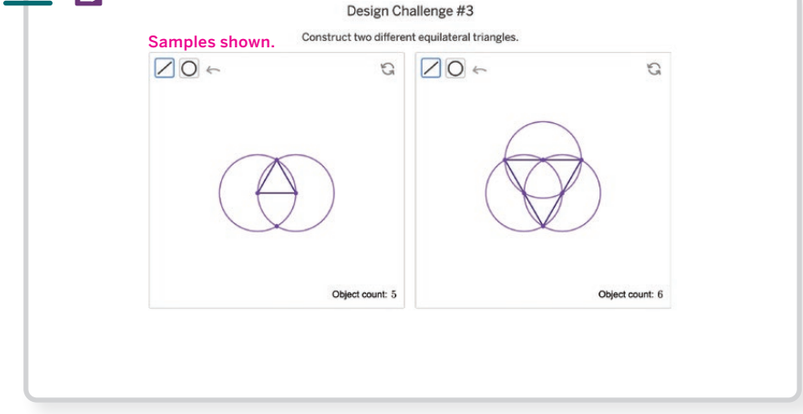
- "How did you create line segments of equal length?"
- "Which points from overlapping circles were helpful in constructing equilateral triangles?"

Students using digital

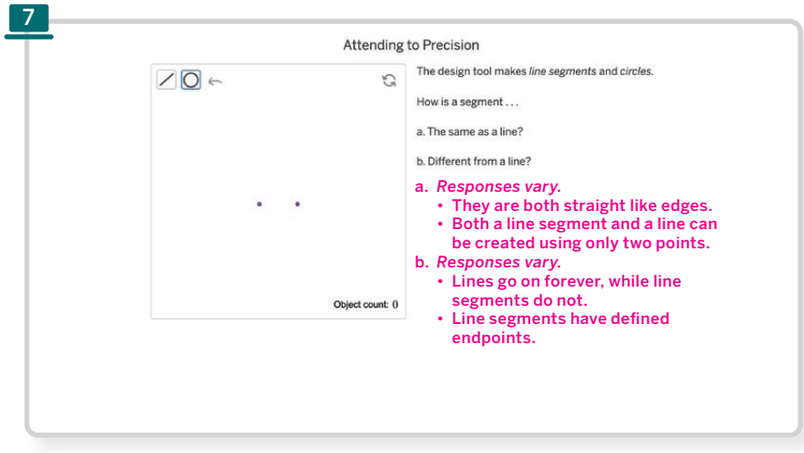
5



6



7



Key Takeaway: Circles can be used to help make *line segments* with equal lengths. It is helpful to use intersection points of circles that are the same size to make precise **constructions**.

Independent | 10 min | Pacing: Screen 8

Activity 3 Challenge Creator

Purpose: Students make their own geometric constructions and challenge their classmates to replicate their constructions using circles and line segments.



Student Notes are also available

1 Launch

8 **Consider sharing** that students will create their own construction and then replicate their classmates' designs.

To encourage creativity, invite students to make a challenge they think will be different from the ones their classmates create.

2 Monitor

Listen for the mathematical language students use in discussing strategies, such as *radius*, *diameter*, *intersection points*, or *equal length*.

Capture several students' designs to share during the Connect.

3 Connect

Math Identity and Community Invite students to celebrate a classmate's construction that they found particularly creative, interesting, or challenging and share what they liked about it.

Share the captured designs. Invite students to share what properties about the designs were easy to replicate and which required more thinking.

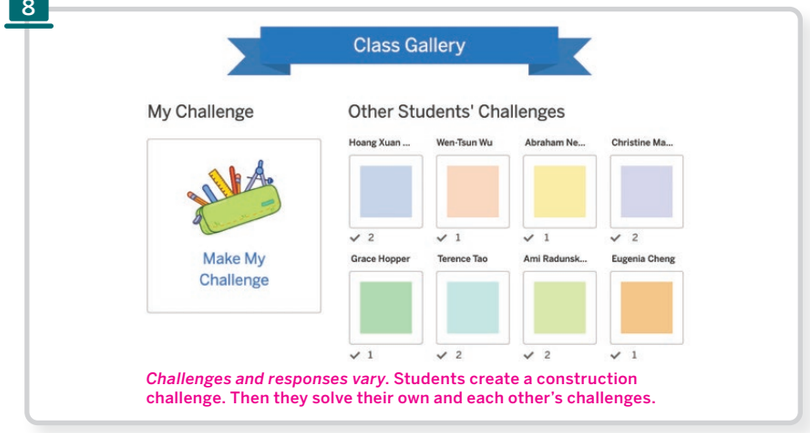
Invite students to share a variety of strategies for replicating their classmates' constructions.

Consider asking:

- "How did you pick the challenges you wanted to solve?"
- "What made some challenges more complex than others?"
- "Was there a strategy you heard that surprised you?"

Students using digital

8



Challenges and responses vary. **Students create a construction challenge. Then they solve their own and each other's challenges.**

Whole Class | 5 min | Pacing: Screen 9

Synthesis



Student Notes are also available

9 **Invite students** to respond independently, and then share their thinking with a partner.

ML/EL **Multilingual/English Learners** Provide a word bank to support students in expressing their ideas, including but not limited to: *circles*, *segments*, *radius*, *radiuses*, *diameter*, *constructions*, and *intersections*.

Capture and share a variety of ideas, such as:

- Using the radius of circles of the same size to identify equal distances.
- Overlapping circles with their centers on either endpoint of the same radius. This creates intersection points to continue creating equal-sized segments.

Consider asking:

- "In your constructions today, what tools did you use?"
- "What other tools might be helpful to create constructions?"
- "Why might circles be more precise for creating line segments of the same length compared to using a ruler?"

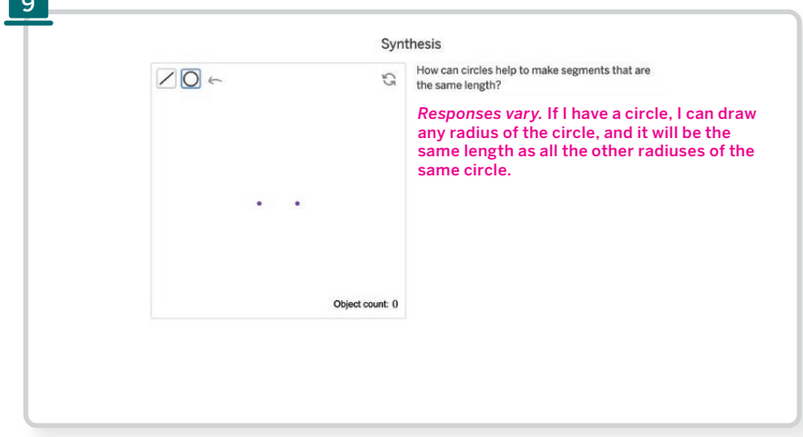
Math Identity and Community Invite students to share strategies they found helpful and give credit to the students who shared them.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.

Invite students using notes to document ideas from the class discussion in the left column.

Students using digital

9

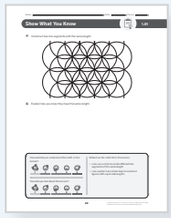


Lesson Takeaway: A **construction** is a geometric drawing made by using a specific set of tools. Having a tool to create a *circle* is helpful to construct *line segments* of equal length.

Independent | 5 min | Pacing: Screens 10–11

Show What You Know

Purpose: Students demonstrate their understanding of using circles to copy a line segment by constructing two line segments with the same length.

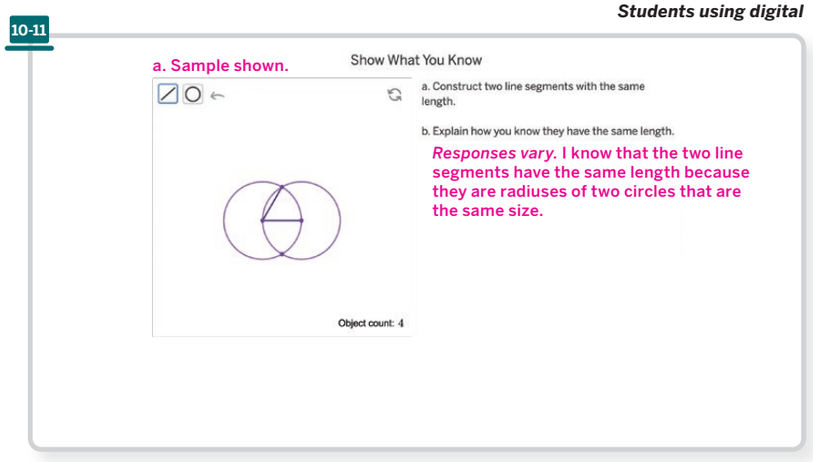


Students using print

10-11 Today's Goals

1. **Goal:** Use a circle to construct different line segments of the same length.
2. **Language Goal:** Explain how circles help to construct figures with equal side lengths. **(Reading, Writing, Speaking, and Listening)**

Students using digital



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

S Support

Provide targeted intervention.

If student work shows **partial understanding** of using circles to make precise constructions:

Students will have more opportunities to make geometric constructions using a variety of tools in **Lessons 2–3**. You may choose not to intervene now.

S Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice**.
- Inviting students to review the **Summary** and complete the **Try This** problem.
- Revisiting the **Challenge Creator** in **Activity 3** and inviting students to solve additional challenges and review responses to the challenge they created.

S Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Encouraging them to complete the **DOK3 practice problem**.

Professional Learning

Reflect on the strategies students used today. What opportunities were there to share specific students' strategies with the class and celebrate them? What facilitation techniques (e.g., using the Snapshot tool or naming strategies after the students) can help make this a part of your classroom culture? How might you continue to reference student strategies from today in the future?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

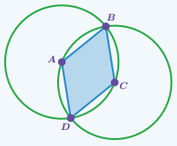
Lesson 1
Practice

Students using digital

Lesson Summary

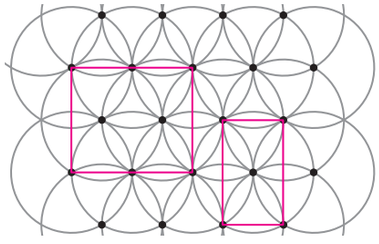
A **construction** is a geometric drawing made using precise tools. Because *circles* have a known radius, you can use them to construct *line segments* of equal length. When circles overlap with their centers on either end of the same radius, you can construct line segments with the same length.

For example, you know that each side length of quadrilateral $ABCD$ is the same because each side is a radius of one of two circles that are the same size.



Try This

a. Construct two different rectangles. *Responses vary.*



b. Explain how you know that opposite sides of your rectangles have the same length.

Responses vary. Since all of the circles are the same size, that means that every radius has the same length and every diameter has the same length. I can also use symmetry to see that the opposite sides of each of my rectangles have the same length.

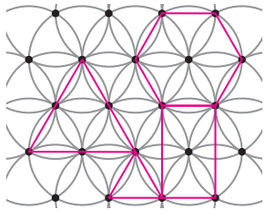
Lesson Practice
Geo.1.01

Name: _____ Date: _____ Period: _____

Problems 1–4: Use line segments and intersection points to construct each polygon.

Note: Each circle is the same size. *Responses vary.*

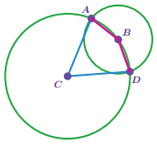
1. Equilateral triangle
2. Equilateral triangle with different side lengths from Problem 1
3. Hexagon
4. Rectangle



5. Points C and B are the centers of these circles. Highlight two segments that you know are the same length.

Responses vary.

- $\overline{AB}$ and $\overline{DB}$
- $\overline{AC}$ and $\overline{DC}$



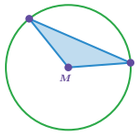
6. A triangle is drawn in a circle. Point M is the center of the circle. Which type of triangle is shown?

A. Equilateral: three sides are the same length

B. Right: one 90° angle

C. Scalene: no sides are the same length

D. Isosceles: two sides are the same length



Lesson Practice
Geo.1.01

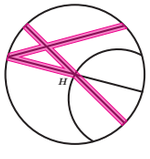
Name: _____ Date: _____ Period: _____

7. Describe how you could construct a line segment that's exactly 4 times the length of this line segment using only circles and line segments.

Responses vary. I could center a circle at point B with radius $\overline{AB}$, then extend the radius to create a diameter of the circle. Then I could center a circle at one of the endpoints with a radius the size of the diameter I just created. The diameter of the new circle will be 4 times the length of $\overline{AB}$.

Spiral Review

Problems 8–10: Here is a circle with center H.



8. Highlight a line segment that is a radius.

Explain how you know it's a radius.

Responses vary. The radius is a line segment that connects the center of the circle and a point on the circle.

9. Highlight a line segment that is a diameter.

Explain how you know it's a diameter.

Responses vary. The diameter is a line segment that passes through the center of the circle and has endpoints on the circle.

10. Highlight a line segment that is neither a radius nor a diameter.

How do you know it's neither?

Responses vary. The segment near the top of the circle is not a diameter because it doesn't go from one side of the circle to the other through the center, and it's not a radius because it does not have an endpoint at the center.

Reflection

1. Put a smiley face next to a problem you were stuck on and then figured out.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1–4	2	HSG.CO.D.12
	5	1	HSG.CO.A.1
Test Practice	6	2	HSG.CO.A.1
	7	3	HSG.CO.D.12
Spiral Review			
Fluency	8–10	1	5.G.B.4

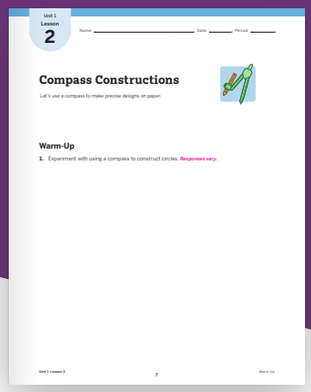
Need More Practice?

Additional Practice resources coming soon.

Unit 1

Lesson 2

This lesson uses print student pages and digital Teacher Presentation Screens.



Compass Constructions

Constructing Patterns

Let's use a compass to make precise designs on paper.

Focus and Coherence

Today's Goals

1. **Goal:** Use a compass and straightedge to construct *equilateral triangles* and *regular* hexagons.
2. **Language Goal:** Describe processes for performing compass and straightedge constructions. **(Reading, Writing, Speaking, and Listening)**

In this lesson, students use a *compass* and *straightedge* to create precise, sophisticated designs on paper using underlying constructions of intersecting circles with the same radius. **(MP6)**

Prior Learning

In Lesson 1, students constructed designs with circles and segments using a digital construction tool. This lesson builds on the learning from Lesson 1 by inviting students to construct circles and regular polygons on paper using a compass and straightedge.

Future Learning

In Lesson 3, students will deepen their investigation of constructions by using the digital construction tool to construct equilateral triangles and other figures. In Lesson 4, students will use the tools to construct perpendicular bisectors, and in Lesson 5, students will construct parallel and perpendicular lines.

Rigor and Balance

- Students develop **adaptive reasoning** as they construct designs using a compass and straightedge, building on their ideas, reasoning, and conclusions about construction tools from Lesson 1.
- Students develop **procedural fluency** as they repeatedly use a compass and straightedge to construct increasingly sophisticated designs.

Vocabulary

New Vocabulary

equilateral triangle

regular polygon

compass (optional)

straightedge (optional)

Standards

Addressing

HSG.CO.D.12
Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

HSG.CO.D.13
Construct an equilateral triangle, a square, and **a regular hexagon** inscribed in a circle.

Mathematical Practices: MP6, MP7

Building On	Building Toward
7.G.A.2	HSG.CO.D.12 HSG.CO.D.13

Lesson at a Glance

~ 45 min

Standards: HSG.CO.D.12, HSG.CO.D.13

Warm-Up

Pairs | 5 min

Students experiment with constructing circles using a compass.





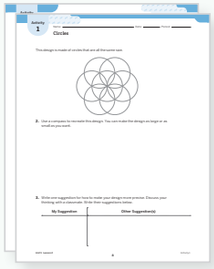
Presentation Screen 1

Activity 1

Small Groups | 15 min

Students make sense of the structure of circles to precisely construct **regular polygons**. MLR7: Compare and Connect; MLR1: Stronger and Clearer **(MP6, MP7)**

Key Takeaway (Screen 3): A **regular polygon** has sides that are the same length, such as an **equilateral triangle**. Circle constructions often involve constructing radiuses of the same length to create these types of figures.



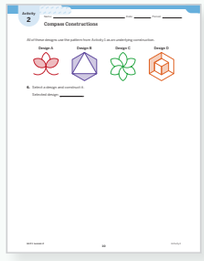


Presentation Screens 2–3

Activity 2

Pairs | 15 min

Students make sense of the underlying geometric structures of given designs, and use a compass and straightedge to replicate a pattern and explain their process. **(MP7)**





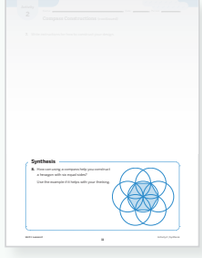
Presentation Screen 4

Synthesis

Whole Class | 5 min

Students synthesize their understanding of using compass constructions to create a regular hexagon. MLR2: Collect and Display

Lesson Takeaway: You can use compasses and straightedges to construct segments, circles, and **regular polygons** with precision.





Presentation Screen 5

Show What You Know

Independent | 5 min

Students demonstrate their understanding by constructing an **equilateral triangle** using a compass and straightedge.





Presentation Screen 6

Prep Checklist

Invite students to use their Student Edition. Display the Presentation Screens to help facilitate the lesson.

Consider making the geometry toolkit available to support students in selecting and using tools strategically.

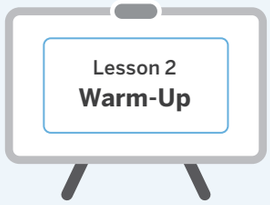
This lesson includes:

- Presentation Screens
- Student Edition
- Show What You Know
- geometry toolkit: compasses, straightedges

Pairs | 5 min

Warm-Up

Purpose: Students experiment with constructing circles using a compass.



Presentation Screen 1

1 Launch

Display the screen and invite students to think about tools they can use to make constructions.

Consider asking, "How might you use each tool to construct a circle?"

Distribute *compasses* to each student and ask, "How do you think this tool works?"

To encourage playfulness, invite students to explore the compass tool to make sense of how it works.

Math Identity and Community Acknowledge the playfulness of using a compass for the first time. Consider sharing that play can involve trying, struggling, failing, adjusting, and trying again.

Accessibility: Fine Motor Skills To support students with using compasses, consider pairing students so they can take turns using the tool and giving real-time feedback on their partner's grip, the angle of the pencil, the pencil pressure, and the surface under their paper.

2 Connect

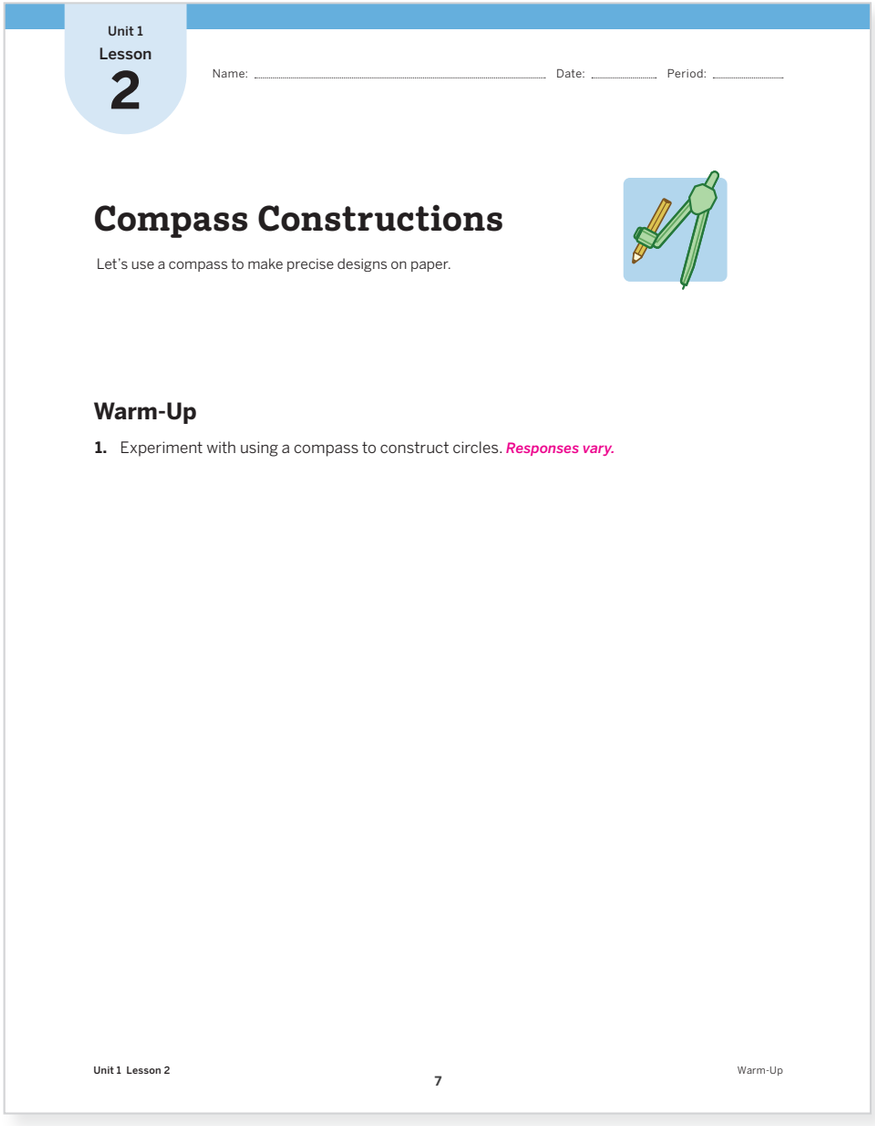
Capture and share examples of student constructions.

Invite students to identify how they made their circles.

To support making connections, consider asking:

- "How did you experiment with the compass to figure out how to use it?"
- "What tips might you give to another student about how to use a compass?"
- "How did you construct circles of different sizes?"

Note: If it doesn't come up naturally, consider modeling helpful ways to hold the compass tool in order to construct a circle.



Unit 1 Lesson 2

Name: _____ Date: _____ Period: _____

Compass Constructions

Let's use a compass to make precise designs on paper.

Warm-Up

1. Experiment with using a compass to construct circles. *Responses vary.*

Unit 1 Lesson 2 7 Warm-Up

Small Groups | 15 min

Activity 1 Circles

Purpose: Students make sense of the structure of circles to precisely construct regular polygons. **(MP6, MP7)**

Short on time? Consider shortening the discussion in the Launch of this activity.

Lesson 2
Activity 1

Presentation Screens
2–3

1 Launch

2 **Display** the side-by-side animations of circle constructions by pushing play on each video.

Note: Students saw this pattern in the previous lesson.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each construction technique, and then connect to the circles they constructed in the Warm-Up.

Consider asking, "How might you use a compass to make this pattern?"

2 Monitor

3 **Invite students** to attempt to construct the pattern on their own, using methods beyond tracing.

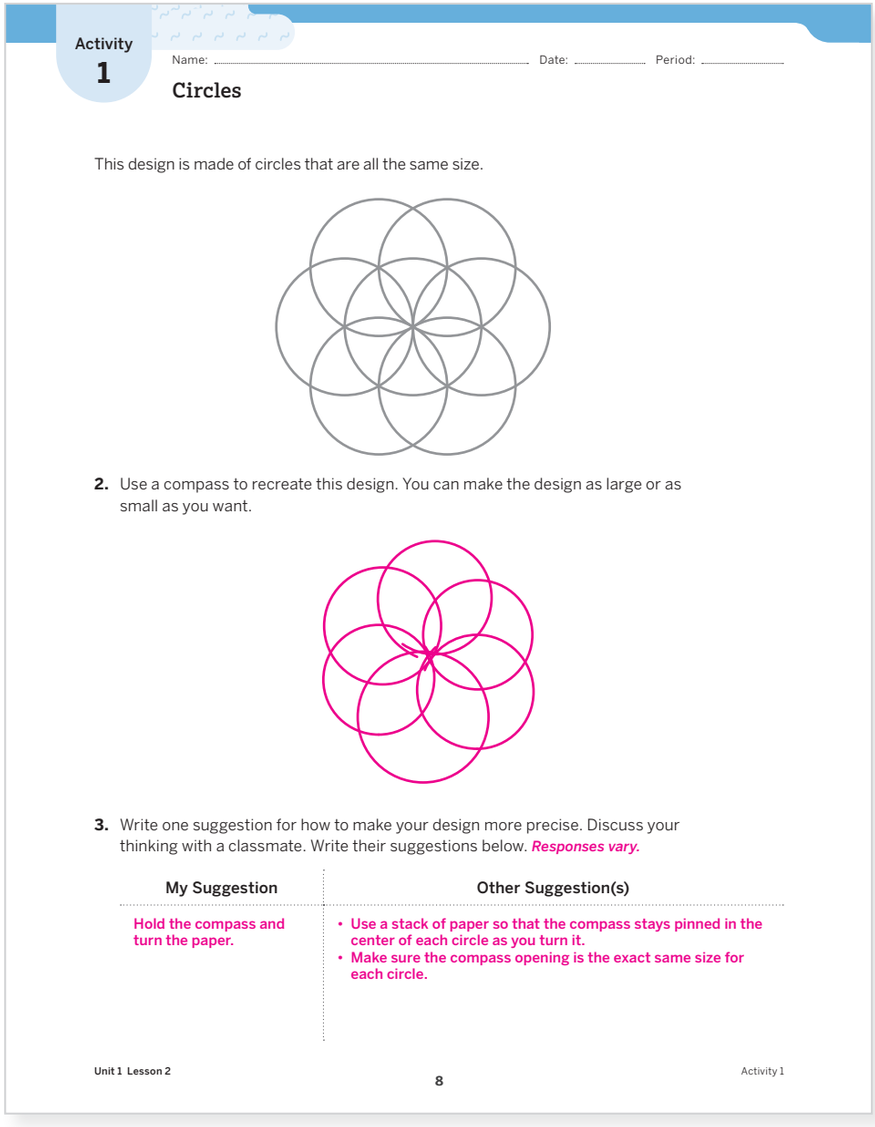
MLR **MLR1: Stronger and Clearer Each Time** Invite students to construct a first draft of the design and use feedback to make their construction stronger and clearer. **(MP6)**

Select and sequence student constructions to share during the Connect.

D **Differentiation** *(Problems 2–5)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Show the animation on the right again to support them making sense of constructions.
Select three points that are not equidistant for a triangle. Select six points that do not construct a regular hexagon. *(Problem 5)*	**Strengthen** Invite them to find additional feedback from a different classmate. Consider asking, "How do you know when you've constructed the figure you want?"

Activity 1 continued >



Activity 1

Name: Date: Period:

Circles

This design is made of circles that are all the same size.

2. Use a compass to recreate this design. You can make the design as large or as small as you want.

3. Write one suggestion for how to make your design more precise. Discuss your thinking with a classmate. Write their suggestions below. *Responses vary.*

My Suggestion	Other Suggestion(s)
Hold the compass and turn the paper.	• Use a stack of paper so that the compass stays pinned in the center of each circle as you turn it. • Make sure the compass opening is the exact same size for each circle.

Unit 1 Lesson 2 8 Activity 1

Activity 1 Circles (continued)

Purpose: Students make sense of the structure of circles to precisely construct regular polygons. **(MP6, MP7)**

Short on time? Consider shortening the discussion in the Launch of this activity.

Lesson 2
Activity 1

Presentation Screens 2–3

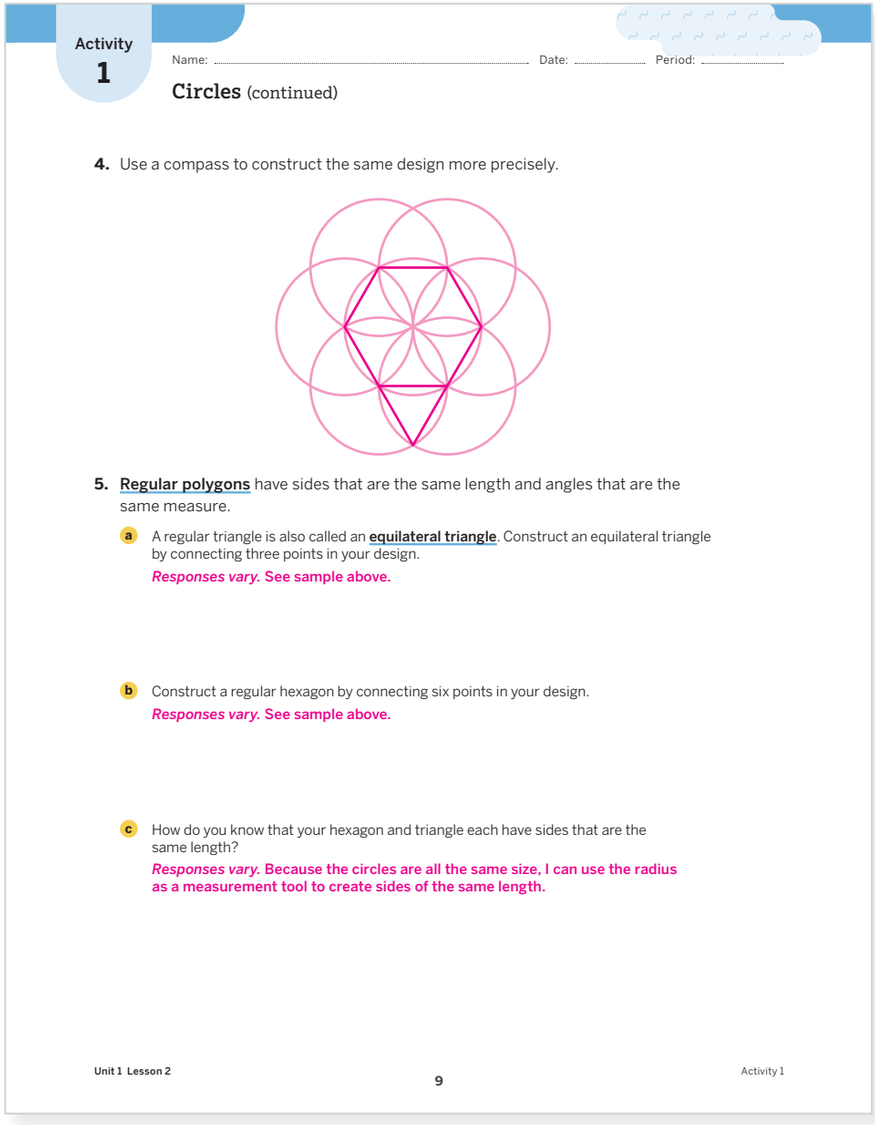
3 Connect

Display student constructions to discuss strategies and ideas about geometric shapes.

Create a class definition of the terms **regular polygon** and **equilateral triangle** using students' language and the glossary.

To surface the Key Takeaway, consider asking:

- "How would you define a *regular polygon* in your own words? An *equilateral triangle*?"
- "Why did we need to construct our circles with the same radius?" **(MP7)**



Activity 1

Name: Date: Period:

Circles (continued)

4. Use a compass to construct the same design more precisely.

5. Regular polygons have sides that are the same length and angles that are the same measure.

a. A regular triangle is also called an **equilateral triangle**. Construct an equilateral triangle by connecting three points in your design.

Responses vary. **See sample above.**

b. Construct a regular hexagon by connecting six points in your design.

Responses vary. **See sample above.**

c. How do you know that your hexagon and triangle each have sides that are the same length?

Responses vary. **Because the circles are all the same size, I can use the radius as a measurement tool to create sides of the same length.**

Unit 1 Lesson 2 9 Activity 1

Key Takeaway: A **regular polygon** has sides that are the same length, such as an **equilateral triangle**. Circle constructions often involve constructing radiuses of the same length to create these types of figures.

Pairs | 15 min

Activity 2 Compass Constructions

Purpose: Students make sense of the underlying geometric structures of given designs, and use a compass and straightedge to replicate a pattern and explain their process. **(MP7)**

Lesson 2
Activity 2

Presentation Screen 4

1 Launch

4 **Display** the designs and the discussion prompt and consider asking:

- "What part of the intersecting circles shows up in each pattern?"
- "What strategies can you use to construct these patterns?"

Distribute *straightedges* to pairs of students and invite them to discuss how this tool can be used to make these sophisticated patterns.

2 Monitor

A **Accessibility: Visual-Spatial Processing**
To support students with interpreting visual representations, eliminate extraneous visual stimuli by having students focus on a smaller number of possible designs.

Capture and record strategies to share during the Connect, including:

- Constructing the initial circle design to help them see different components of each design.
- Applying feedback tips from their partner from earlier in the lesson.
- Using a straightedge to construct additional segments.

D **Differentiation** *(Problems 6–7)*

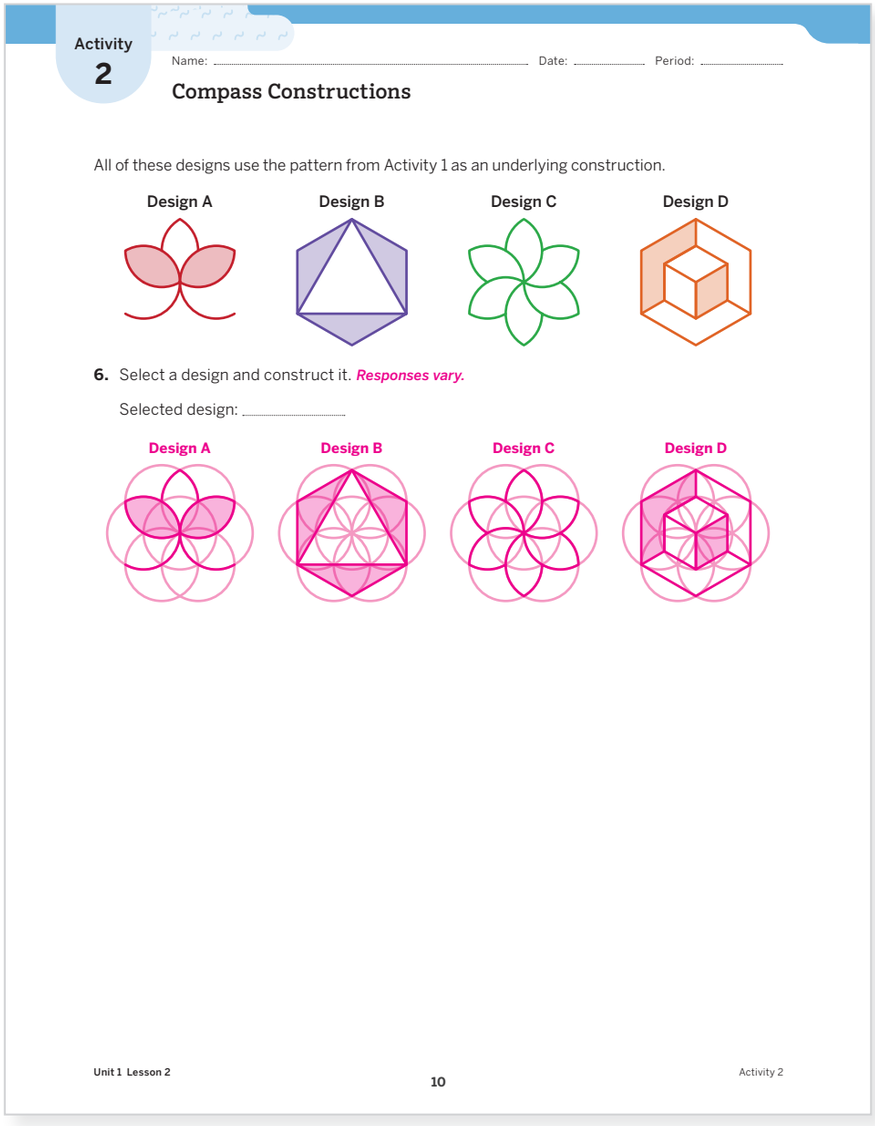
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to start with Design A or C and sketch the remaining circle parts that were erased in the design. Consider asking, "What steps could you take to create this new pattern?"
Would like an additional challenge.	**Stretch** Invite them to determine additional designs that can be made from the initial circle pattern.

3 Connect

Display examples of student constructions and invite students to share strategies for recreating these designs.

To support making connections, consider asking:

- "How did you use regular polygons to create each of these designs?"
- "How did you use peer feedback to help you in this process?"



Activity 2

Name: _____ Date: _____ Period: _____

Compass Constructions

All of these designs use the pattern from Activity 1 as an underlying construction.

Design A Design B Design C Design D

6. Select a design and construct it. *Responses vary.*

Selected design: _____

Design A Design B Design C Design D

Unit 1 Lesson 2 10 Activity 2

Whole Class | 5 min

Synthesis

Lesson 2
Synthesis

Presentation Screen
5

5 **Invite students** to respond independently, and then share their thinking with a partner.

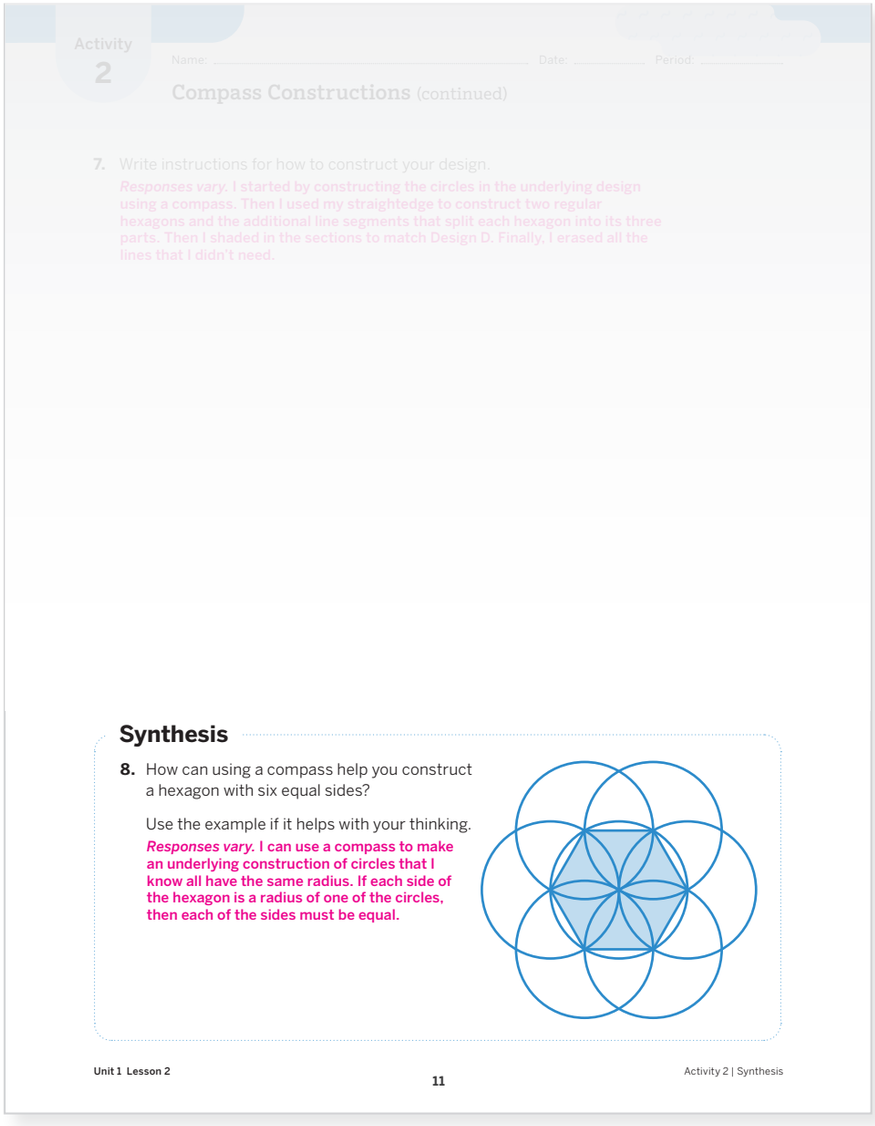
Capture and share a variety of ideas, such as:

- Compasses can be used to construct circles with the same radius, to measure lengths, and to compare segment lengths.
- A compass construction with intersecting circles of the same radius can reveal an underlying construction, which helps you create regular hexagons and other regular polygons.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *regular polygon* using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Math Identity and Community Consider naming construction strategies after the students who use them and using those names throughout the rest of this lesson and unit.

Note: If time allows, invite students to share with a partner what makes sense about each construction strategy and what questions they still have.



Activity 2

Name: Date: Period:

Compass Constructions (continued)

7. Write instructions for how to construct your design.

Responses vary. I started by constructing the circles in the underlying design using a compass. Then I used my straightedge to construct two regular hexagons and the additional line segments that split each hexagon into its three parts. Then I shaded in the sections to match Design D. Finally, I erased all the lines that I didn't need.

Synthesis

8. How can using a compass help you construct a hexagon with six equal sides?

Use the example if it helps with your thinking.

Responses vary. I can use a compass to make an underlying construction of circles that I know all have the same radius. If each side of the hexagon is a radius of one of the circles, then each of the sides must be equal.

Lesson Takeaway: You can use compasses and straightedges to construct segments, circles, and **regular polygons** with precision.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by constructing an equilateral triangle using a compass and straightedge.

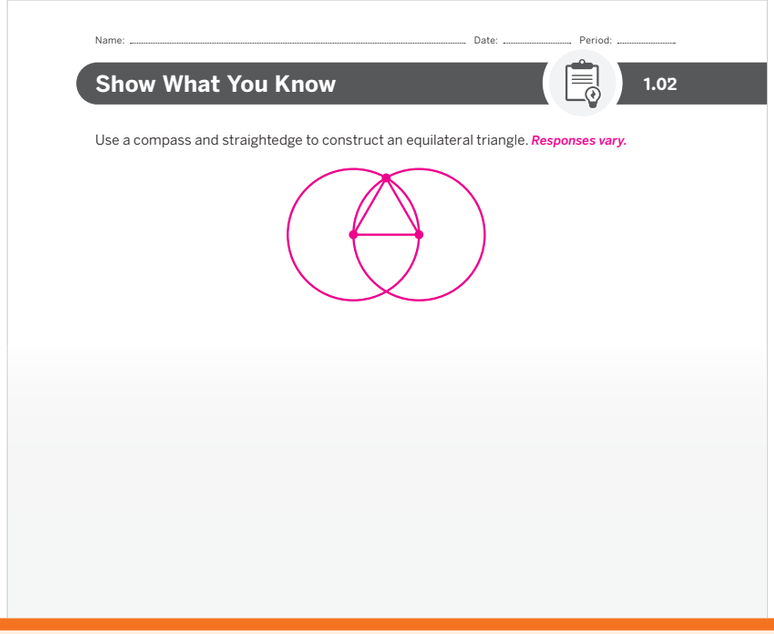


Presentation Screen 6

Today's Goals

1. **Goal:** Use a compass and straightedge to construct *equilateral triangles* and *regular* hexagons.
2. **Language Goal:** Describe processes for performing compass and straightedge constructions. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)


Name: _____ Date: _____ Period: _____

Show What You Know 1.02

Use a compass and straightedge to construct an equilateral triangle. ***Responses vary.***

Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of compass constructions, consider:

- Inviting students to construct a radius of their circle using a straightedge and to use this segment to envision the more complex designs. Consider asking, "Other than the center of your circle, where else could you place your compass using this same radius?"

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice**.
- Inviting students to review the **Summary** and complete the **Try This** problem.
- Encouraging students to apply additional strategies from peers to gain more facility with the compass.
- Inviting students to practice making additional constructions.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting them to explore additional designs that can be constructed from the initial circle pattern.

Professional Learning

Take a moment to reflect on how students used a compass and straightedge to perform constructions so far. How does their experience with compass and straightedge constructions compare with their experience using the digital construction tool from Lesson 1? How can you continue to support students in using both tools?

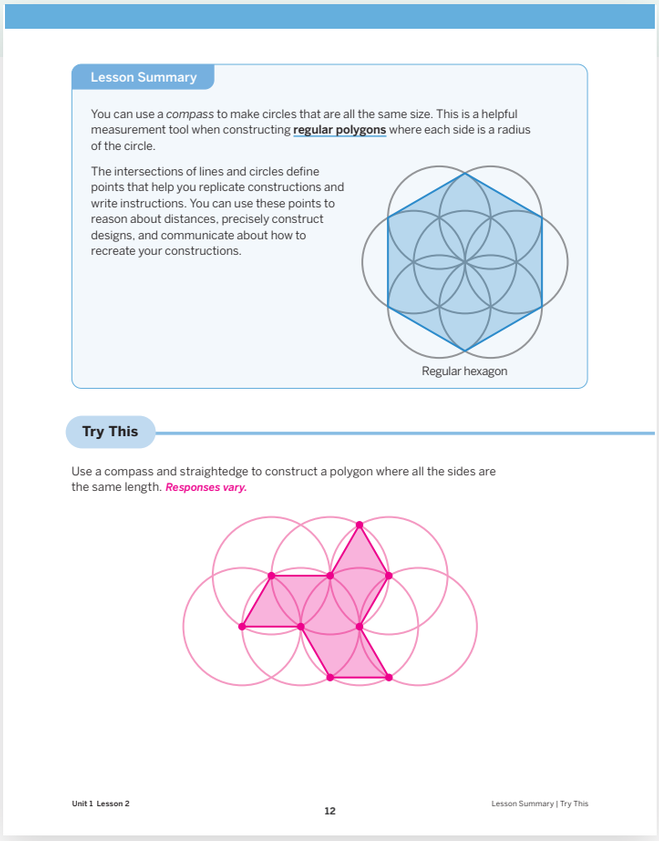
Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 2
Practice

Students using digital



Lesson Summary

You can use a *compass* to make circles that are all the same size. This is a helpful measurement tool when constructing **regular polygons** where each side is a radius of the circle.

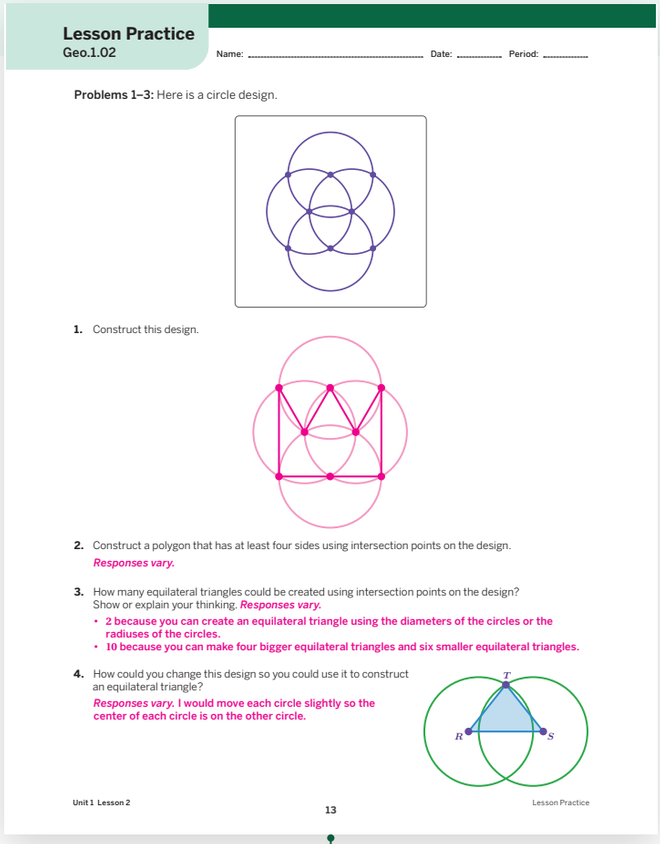
The intersections of lines and circles define points that help you replicate constructions and write instructions. You can use these points to reason about distances, precisely construct designs, and communicate about how to recreate your constructions.

Regular hexagon

Try This

Use a compass and straightedge to construct a polygon where all the sides are the same length. *Responses vary.*

Unit 1 Lesson 2 12 Lesson Summary | Try This



Lesson Practice
Geo.1.02

Name: _____ Date: _____ Period: _____

Problems 1–3: Here is a circle design.

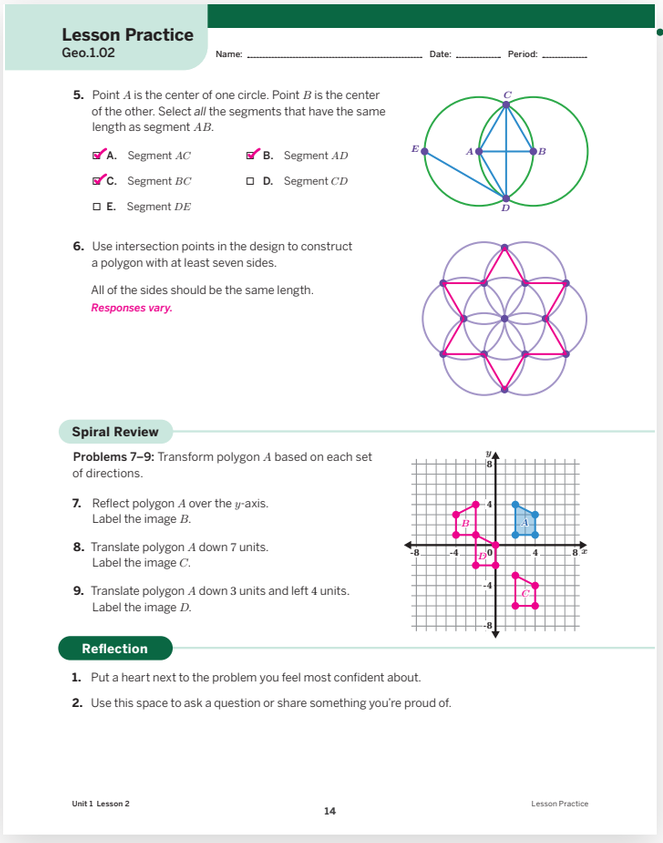
1. Construct this design.

2. Construct a polygon that has at least four sides using intersection points on the design.
Responses vary.

3. How many equilateral triangles could be created using intersection points on the design? Show or explain your thinking. *Responses vary.*
   - 2 because you can create an equilateral triangle using the diameters of the circles or the radiuses of the circles.
   - 10 because you can make four bigger equilateral triangles and six smaller equilateral triangles.

4. How could you change this design so you could use it to construct an equilateral triangle?
Responses vary. I would move each circle slightly so the center of each circle is on the other circle.

Unit 1 Lesson 2 13 Lesson Practice



Lesson Practice
Geo.1.02

Name: _____ Date: _____ Period: _____

5. Point A is the center of one circle. Point B is the center of the other. Select *all* the segments that have the same length as segment AB.

- ☑ A. Segment AC
- ☑ B. Segment AD
- ☑ C. Segment BC
- ☐ D. Segment CD
- ☐ E. Segment DE

6. Use intersection points in the design to construct a polygon with at least seven sides.

All of the sides should be the same length.
Responses vary.

Spiral Review

Problems 7–9: Transform polygon A based on each set of directions.

7. Reflect polygon A over the y-axis. Label the image B.
8. Translate polygon A down 7 units. Label the image C.
9. Translate polygon A down 3 units and left 4 units. Label the image D.

Reflection

1. Put a heart next to the problem you feel most confident about.
2. Use this space to ask a question or share something you're proud of.

Unit 1 Lesson 2 14 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1–2	2	HSG.CO.D.12
Test Practice	3–4	2	HSG.CO.D.13
	5	2	HSG.CO.D.12
	6	3	HSG.CO.D.12
Spiral Review			
Fluency	7–9	1	8.G.A.3

Need More Practice?

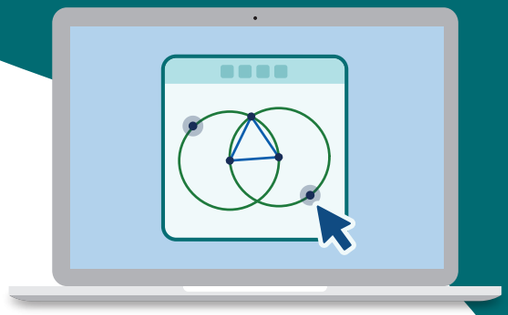
Additional Practice resources coming soon.

Unit 1

Lesson 3

Digital Lesson

This lesson uses digital student screens. Student Notes are also available.



Constructing Digitally

Geometric Constructions

Let's explore how to communicate to each other about constructions.

Focus and Coherence

Today's Goals

1. **Goal:** Use digital construction tools to construct geometric objects.
2. **Language Goal:** Write and follow precise instructions to create geometric objects. **(Reading and Writing)**

This lesson introduces students to the digital construction tools they will use to make constructions throughout this course. Students analyze the precision of language in written instructions so they can create precise instructions for their own constructions. **(MP6)**

Prior Learning

In Lesson 1, students used a digital line and circle tool to create geometric figures. In Lesson 2, they continued to reason about how circles can help to make sense of equal lengths.

Future Learning

In Lessons 4–7, students will continue to write and follow precise instructions for constructions. Throughout Unit 1, they will use the digital geometry tools to construct and perform transformations.

Rigor and Balance

- Students develop **procedural fluency** using circles to create segments with common lengths as they learn to use digital geometry tools.
- Students develop **adaptive reasoning** as they write and revise instructions for reproducing geometric constructions with precision.

Standards

Addressing

HSG.CO.D.12
Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, **dynamic geometric software**, etc.). **Copying a segment**; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

Also Addressing: **HSG.CO.D.13**

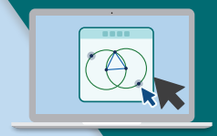
Mathematical Practices: MP3, MP6

Building On	Building Toward
HSG.CO.D.12	**HSG.CO.D.12**

Lesson at a Glance

~ 45 min

Standards: HSG.CO.D.12, HSG.CO.D.13



Why digital?

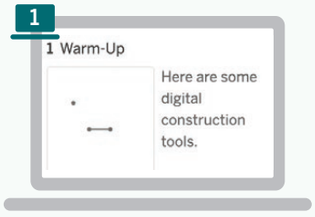
Students use the digital construction tools to make geometric constructions and justify why they are valid.

Warm-Up

Independent | 10 min

Pacing: Screen 1

Students copy a segment to explore and make sense of the digital construction tools.



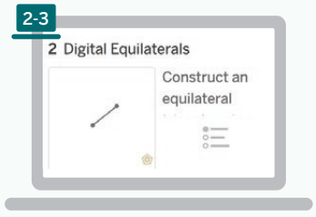
Activity 1

Pairs Sharing a Device | 10 min

Pacing: Screen 2, Screen 3

Students digitally construct an equilateral triangle to explore how dynamic points can help them to make sense of why a construction is valid.

Key Takeaway (Screen 3): Digital tools are helpful for constructing precise geometric figures, such as equilateral triangles. Draggable points can be used to determine if a construction works for any orientation or length.



Activity 2

Pairs Sharing a Device | 15 min

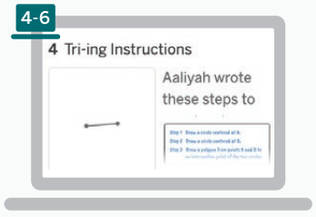
Pacing: Screens 4–5, Screen 6

Students practice writing and interpreting precise instructions by creating and revising steps to recreate a shape.

MLR1: Stronger and Clearer Each Time (MP3, MP6)

Key Takeaway (Screen 6): When writing instructions for constructions, it's important to be precise.

A few ways to attend to precision are:
- Defining centers and radius lengths for circles.
- Creating shapes using constructed intersection points.
- Labeling intersection points to have clear ways to talk about those points.



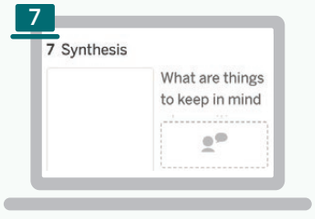
Synthesis

Whole Class | 5 min

Pacing: Screen 7

Students synthesize strategies for writing precise instructions when creating geometric constructions.

Lesson Takeaway: Digital tools can help construct geometric objects with precision. It's important to use specific language when writing instructions so that constructions can be made with precision.



Show What You Know

Independent | 5 min

Pacing: Screens 8–9

Students demonstrate their understanding of writing precise instructions by revising an imprecise step.



Prep Checklist

Assign the digital student screens. Student Notes are also available.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)

Student Notes sheet in Student Edition:
Consider providing the Student Notes to support class discussion and student thinking.



Independent | 10 min | Pacing: Screen 1

Warm-Up

Purpose: Students copy a segment to explore and make sense of the digital construction tools.



Student Notes are also available

1 Launch

1 **To encourage playfulness**, invite students to explore all of the construction tools before working explicitly to copy the segment. Consider asking, "What tools did you notice? What are you excited about using in your constructions today?"

To support making connections, consider asking, "How might the construction tools we used in the previous lesson be helpful here?"

Note: This may be the first time students have seen segment notation. Consider sharing that $\overline{AB}$ means *the line segment from points A to B*.

A **Accessibility: Fine Motor Skills** Consider strategically pairing students with different strengths to support the use of the digital construction tools.

2 Connect

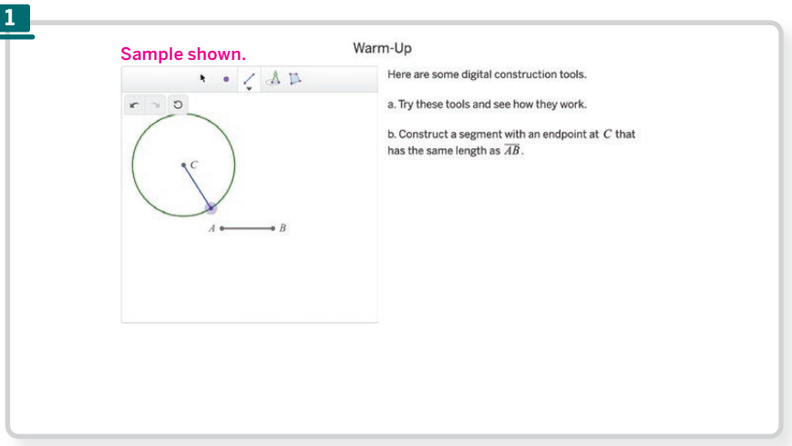
Math Identity and Community Celebrate the variety and creativity in the different approaches to copying a segment.

Capture and share student constructions. Consider asking:

- "Which new tools did you try to use? Can you describe what they do?"
- "Which of the digital construction tools relate to a compass? To a straightedge?"
- "How many different possible segments could be constructed to meet this criteria?"

Note: If they do not come up naturally, consider sharing some of the other features of the construction tools to support students throughout the lesson, such as changing the color, deleting, hiding, or labeling their constructions.

1



Pairs Sharing a Device | 10 min | Pacing: Screen 2, Screen 3

Activity 1 Digital Equilaterals

Purpose: Students digitally construct an equilateral triangle to explore how dynamic points can help them to make sense of why a construction is valid.



Student Notes are also available

1 Launch

2 **To encourage playfulness**, invite students to experiment first and revise their work for precision and clarity after.

Demonstrate how to use the polygon tool and the draggable points.

- Add any polygon using segment AB as one of the sides.
- Drag points A or B and consider asking, "What changes about the polygon as we drag the points? What stays the same?"

2 Monitor

Select and sequence constructions of triangles that stay equilateral when the points are dragged, and at least one that does not, if possible.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Before they use the Digital Construction Tools, encourage students to draw their plan on paper.
Have constructed circles using points close to points A and B, but are not exact.	**Support** Consider asking, "When you drag point A or point B, what happens to the triangle?"
Use points A and B as the centers of circles with segment AB as the radius.	**Stretch** Consider asking, "How many unique equilateral triangles can be constructed using segment AB as a side?"

Pause briefly and share the captured constructions. Invite students to determine which triangles are equilateral.

3 **Capture** student ideas to share during the Connect.

3 Connect

Share the student advantages and disadvantages of constructing with digital tools.

To surface the Key Takeaway, consider asking:

- "How is constructing digitally similar to or different from constructing on paper?"
- "What is important for us to remember when constructing digitally?"

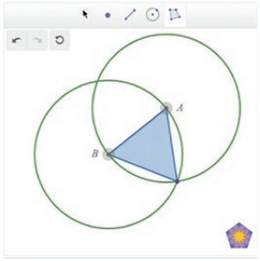
Invite students using notes to record some important considerations from the class discussion.

Students using digital

2

Sample shown.

Digital Equilaterals



Construct an equilateral triangle using segment AB as one of the sides.

Use the polygon tool to show your triangle.

Drag points A and B. Is your triangle still equilateral?

Responses vary.

- Yes. I used segment AB as the length of the radius for two circles centered at points A and B. I connected one of the intersection points to points A and B. When I dragged the points around, it made all three segments longer, but they all stayed the same length since they were dependent on the length of segment AB. This means my triangle stayed equilateral.
- No. When I moved points A and B, my third point did not move when I dragged it, so the triangle was not equilateral anymore.

3

Physical vs. Digital



We have made constructions on paper and digitally.

What are some advantages and disadvantages of constructing with digital tools?

Responses vary.

Advantages:

- When making constructions with the digital tool, I can be very precise.
- It's also nice to be able to drag points around to confirm the construction.

Disadvantages:

- It's harder to sketch what I think I should try to do with digital tools.
- It's a little harder to label points since I have to click through a few menus.
- I'm still learning how to best use all of the tools. Sometimes I click on points that I don't mean to click on.

Key Takeaway: Digital tools are helpful for constructing precise geometric figures, such as equilateral triangles. Draggable points can be used to determine if a construction works for any orientation or length.

Pairs Sharing a Device | 15 min | Screens 4–5, Screen 6

Activity 2 Construction Instructions

Purpose: Students practice writing and interpreting precise instructions by creating and revising steps to recreate a shape. **(MP3, MP6)**

Short on time? Consider assigning students to solve each other's challenges in pairs.



Student Notes are also available

1 Launch

4 **Invite students** to read the prompt aloud, then restate it in their own words.

Invite students using notes to draft their thinking before they discuss.

Display the overlay using the dashboard's Teacher View and consider asking, "How could the instructions be revised so everyone could create the same triangle?"

Record student examples of precise construction language, such as:

- "Draw a circle centered at point A, with radius $\overline{AB}$."
- "Draw a circle centered at point B, that goes through point A."

2 Monitor

5 **Look for** and celebrate the different shapes students construct, encouraging exploration of the construction tools and creativity.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to create a familiar shape (like a triangle or rectangle) for their construction.
Create a shape without using any circles that can't be recreated precisely.	**Support** Consider asking, "How could circles be utilized to reason about distances in your shape?"
Make a construction without labeled points.	**Support** Consider asking, "How could labels on points help clarify your steps?"
Need encouragement to transition to writing instructions for replicating their shape.	**Strengthen** Encourage them to start writing their instructions. Consider asking, "When writing instructions for your shape, what should you keep in mind?"

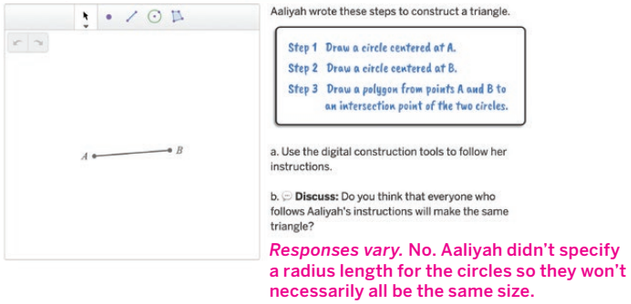
ML/EL **Multilingual/English Learners** Invite students to read each instruction aloud and paraphrase what it means. Clarify any unfamiliar words. **(Reading)**

Activity 2 continued >

Students using digital

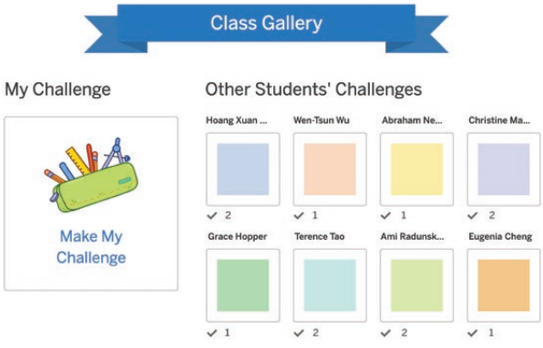
4

a. Sample shown.



Responses vary. No. Aaliyah didn't specify a radius length for the circles so they won't necessarily all be the same size.

5



Challenges and responses vary. Students create a construction challenge. Then they solve their own and each other's challenges.

Pairs Sharing a Device | 15 min | Pacing: Screens 4–5, Screen 6

Activity 2 Construction Instructions (continued)

Purpose: Students practice writing and interpreting precise instructions by creating and revising steps to recreate a shape. **(MP3, MP6)**

Short on time? Consider assigning students to solve each other's challenges in pairs.



Student Notes are also available

2 Monitor

5 **Capture** student instructions that result in precise reproductions and instructions that result in different shapes.

Pause and invite students to view how others followed their instructions.

Share the captured student instructions. Consider asking:

- "Which set(s) of instructions led to more students reproducing the shape exactly?"
- "What are some qualities of those instructions?"

6 MLR **MLR1: Stronger and Clearer Each Time**
Invite students to review the first draft of their instructions and use feedback from how other students followed their instructions to make their draft stronger and clearer. For students who wrote precise instructions the first time, consider having them list the qualities of their instructions that made them precise. **(MP6)**

D **Differentiation:** (Support) Consider asking, "Is there anything in someone else's instructions that you can use to make your instructions stronger and clearer?"

3 Connect

To surface the Key Takeaway, ask students to share revisions they made to their instructions so they are stronger and clearer. Record and display them for the class.

Note: If a Key Takeaway is not surfaced, consider asking:

- "For instructions about circles, what does each step mention?"
- "What is challenging about steps that deal with new points? What do each of these examples do to help make it easier to describe?"

Students using digital

6

Under Construction

How would you revise your instructions after seeing the designs your classmates constructed?

Responses vary. I need to be more precise in naming points. There were some designs that used a different intersection point than I had intended. As a result, some of their designs looked like mine but were in a different orientation.

Key Takeaway: When writing instructions for constructions, it's important to be precise.

A few ways to attend to precision are:

- Defining centers and radius lengths for circles.
- Creating shapes using constructed intersection points.
- Labeling intersection points to have clear ways to talk about those points.

Whole Class | 5 min | Pacing: Screen 7

Synthesis



Student Notes are also available

7 **Invite students** to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

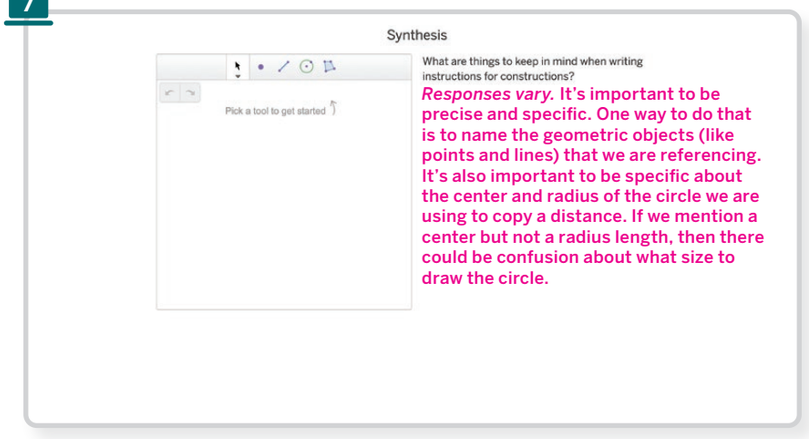
- Instructions are hard to follow without precise language, like labeling points and line segments or indicating radius lengths.
- Precise instructions can help us reproduce shapes and objects more accurately.
- Digital geometry tools are helpful for creating things with precision.

Math Identity and Community Invite students to share a classmate's construction instructions that they found particularly helpful or clear and what they appreciated about those instructions.

Invite students using notes to record ideas from the class discussion in the left column.

Students using digital

7



Lesson Takeaway: Digital tools can help construct geometric objects with precision. It's important to use specific language when writing instructions so that constructions can be made with precision.

Independent | 5 min | Pacing: Screens 8–9

Show What You Know

Purpose: Students demonstrate their understanding of writing precise instructions by revising an imprecise step.

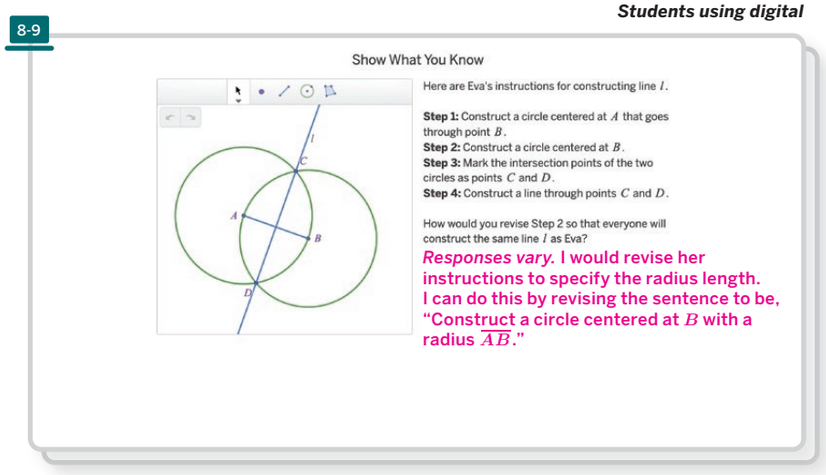


Students using print

8-9 Today's Goals

1. **Goal:** Use digital construction tools to construct geometric objects.
2. **Language Goal:** Write and follow precise instructions to create geometric objects. **(Reading and Writing)**

Students using digital



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of the Digital Tool, consider:

- Inviting students to engage with the **Summary** by reading it and paraphrasing to a caregiver or classmate.
- Selecting a few related **practice problems**, such as Problems 1–3, and giving individualized feedback.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding** with imprecisions, consider:

- Assigning the **Lesson Practice**.
- Inviting students to read the **Summary** and complete the **Try This** problem.
- Revisiting the **Challenge Creator** in Activity 2 and inviting students to solve additional problems.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting students to write their own problem that is similar to the **Try This** problem.
- Encouraging them to complete the **DOK3 practice problem**.

Professional Learning

Reflect on students' use of the construction tools throughout this lesson. How did these tools support students with constructing or participating in the lesson? What support might students need to use these tools effectively in the future?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the Lesson Summary to support them with this practice and their learning throughout the year.

Lesson 3
Practice

Students using digital

Lesson Summary

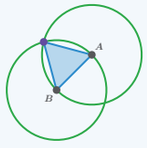
Digital tools are helpful in making precise constructions. For circles, the radius lengths will always be exactly what you define (and your compass won't slip like it can on paper).

You can define a center and a point on a circle to construct line segments of exactly the same length.

When using digital tools to construct, you can also drag points to see what relationships hold true about the constructions.

Using precise language can help you describe digital constructions. It's helpful to:

- Clearly define the center and radius length of a circle.
- Use points on constructed objects, like circles or line segments.
- Label intersection points to have clear ways to describe constructed points.



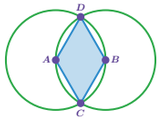
Try This

Here are instructions for constructing this polygon.

Step 1: Construct two circles.

Step 2: Label their intersection points C and D.

Step 3: Connect all the points to make a rhombus.



Select a step and revise it to make these instructions more precise.

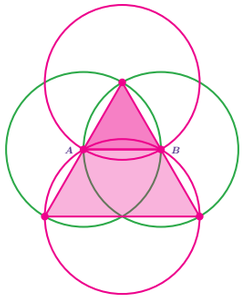
Responses vary. Step 1: Construct two circles, one centered at point A and one centered at point B, each with radius $\overline{AB}$.

Lesson Practice
Geo.1.03

Name: _____ Date: _____ Period: _____

Problems 1–3: Here is a diagram. *Responses vary.*

1. Construct an equilateral triangle using points A and B as vertices.
2. Construct two more circles centered at the intersections of the original circles. All of the circles should be the same size.
3. Use the intersections to construct an equilateral triangle that is larger than your first one.



4. Here are instructions for constructing an equilateral triangle.

   Which step should be revised? Explain your thinking.

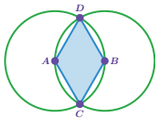
   Step 3. *Explanations vary.* Construct a circle with center B and a radius $\overline{AB}$.

Step 1 Construct a circle with center A.
Step 2 Mark a point on the circle and label it B.
Step 3 Construct a circle with center B.
Step 4 Mark a point at one of the intersection points of the circles and label it C.
Step 5 Construct segments AB, BC, and CA.

5. Roberto and Anh started with points A and B, and created quadrilateral $ACBD$.

   What steps could they write so that someone else can construct the exact same quadrilateral?

   Responses vary. Construct a circle with center A and radius $\overline{AB}$. Construct a circle with center B and radius $\overline{BA}$. Mark the intersection points of the two circles and label them C and D. Construct polygon $ACBD$.



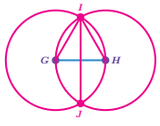
Lesson Practice
Geo.1.03

Name: _____ Date: _____ Period: _____

Problems 6–7: Here is segment GH.

6. Create a design that takes *exactly* 5 steps to complete.

   Responses vary.



7. List your instructions. *Responses vary.*

   Step 1: Construct a circle centered at point G with radius $\overline{GH}$.

   Step 2: Construct a circle centered at point H with radius $\overline{GH}$.

   Step 3: Label the top intersection point I and the bottom intersection point J.

   Step 4: Draw line segments from points G and H to point I.

   Step 5: Draw a line segment between points I and J.

Spiral Review

8. What information is helpful to include when writing precise directions for constructing a circle?

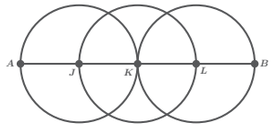
   Responses vary. Draw a center point. Place the center of the compass on the center point. Create a radius with the compass and create the circle.

9. These three circles are centered at points J, K, and L.

   The length of segment LB is 3 centimeters.

   What is the length of segment AL?

   $AL = 3(3) = 9$ centimeters



Reflection

1. Circle a problem you want to talk to a classmate about.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-lesson			
	1, 3–4	2	HSG.CO.D.13
Fluency	2	1	HSG.CO.D.12
	5	2	HSG.CO.D.12
	6–7	3	HSG.CO.D.12
Spiral Review			
	8	1	HSG.CO.D.12
Test Practice	9	2	HSG.CO.D.12

Need More Practice?

Additional Practice resources coming soon.

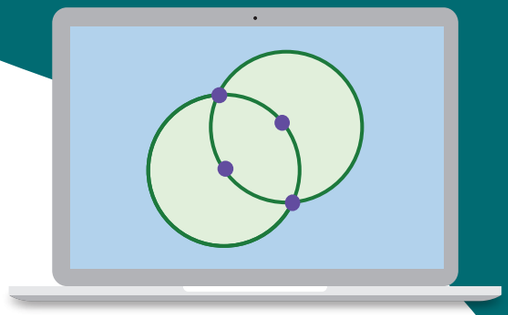
Unit 1

Lesson

4

Digital Lesson

This lesson uses digital student screens. Student Notes are also available.



Right in the Middle

Constructing Perpendicular Bisectors

Let's explore equal distances through constructions.

Focus and Coherence

Today's Goals

1. **Goal:** Construct the *perpendicular bisector* of a line segment.
2. **Goal:** Explore how points that are *equidistant* from a line segment's endpoints will be on the *perpendicular bisector* of that segment.
3. **Language Goal:** Describe that points that are *equidistant* from a line segment's endpoints will be on the *perpendicular bisector* of that segment. **(Writing and Speaking)**

Students explore equal distances between draggable points and the endpoints of a given line segment. They build upon this knowledge to construct **perpendicular bisectors** using digital tools. Students complete repeated challenges to construct a variety of perpendicular bisectors of different line segments and figures. **(MP8)**

Prior Learning

In Lesson 2, students reasoned about how circles can help them justify equal lengths. In Lesson 3, students explored the digital construction tools to construct precisely.

Future Learning

In Lesson 5, students will extend their understanding of constructing perpendicular bisectors to construct parallel and perpendicular lines. In Lesson 6, students will build upon this knowledge to construct squares.

Rigor and Balance

- Students develop **conceptual understanding** of equal distances by constructing points that are equidistant from a segment's endpoints and noticing that they fall on the perpendicular bisector.
- Students develop **strategic competence** as they compare strategies for constructing perpendicular bisectors in a variety of figures.

Vocabulary

New Vocabulary

equidistant

midpoint

perpendicular bisector

Standards

Addressing

HSG.CO.D.12

Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, **dynamic geometric software,** etc.). **Copying a segment;** copying an angle; **bisecting a segment;** bisecting an angle; **constructing perpendicular lines, including the perpendicular bisector of a line segment;** and constructing a line parallel to a given line through a point not on the line.

Also Addressing: **HSG.CO.A.1**

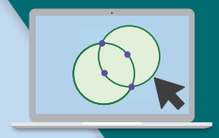
Mathematical Practices: MP1, MP6, MP8

Building On	Building Toward
7.G.A.2	HSG.CO.D.12 HSG.CO.D.13

Lesson at a Glance

~ 45 min

Standards: HSG.CO.D.12, HSG.CO.A.1



Why digital?

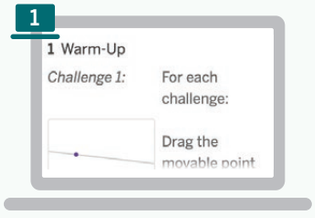
Students use digital construction tools to construct and analyze perpendicular bisectors and use their constructions to make generalizations.

Warm-Up

Independent | 5 min

Pacing: Screen 1

Students engage with a variety of challenges to place points an equal distance from the endpoints of a line segment. **(MP8)**

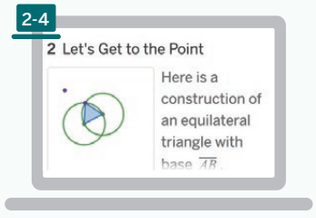


Activity 1

Pairs Sharing a Device | 10 min

Pacing: Screens 2–4

Students determine whether points are **equidistant** to the base of an equilateral triangle using their own strategies and MLR7: Compare and Connect. **(MP6)**



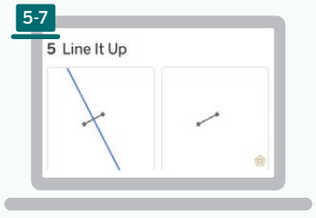
Activity 2

Pairs Sharing a Device | 12 min

Pacing: Screen 5, Screens 6–7

Students construct **perpendicular bisectors** and consider strategies for using multiple tools in their constructions. **(MP1)**

Key Takeaway (Screen 7): A **perpendicular bisector** is a line that passes through the **midpoint** of a line segment. Any point on the perpendicular bisector is **equidistant** from the endpoints of the segment it passes through.

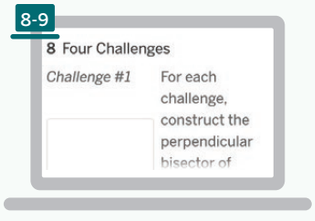


Activity 3

Pairs Sharing a Device | 8 min

Pacing: Screens 8–9

Students complete a series of perpendicular bisector construction challenges. **(MP8)**



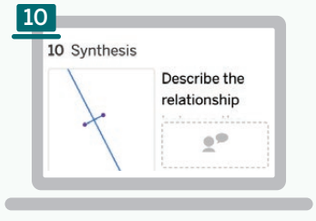
Synthesis

Whole Class | 5 min

Pacing: Screen 10

Students synthesize their understanding of perpendicular bisectors by describing how they relate to the endpoints of a line segment. MLR2: Collect and Display.

Lesson Takeaway: Points that are **equidistant** from a line segment's endpoints will be on the **perpendicular bisector** of that segment. To construct a perpendicular bisector, begin by constructing two equal-sized circles centered at the endpoints and connecting the points where two circles intersect with a line.

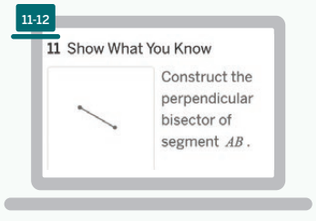


Show What You Know

Independent | 5 min

Pacing: Screens 11–12

Students demonstrate their understanding by constructing a perpendicular bisector of a line segment.



Prep Checklist

Assign the digital student screens. Student Notes are also available.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)

Student Notes sheet in Student Edition:
Consider providing the Student Notes to support class discussion and student thinking.



Independent | 5 min | Pacing: Screen 1

Warm-Up

Purpose: Students engage with a variety of challenges to place points an equal distance from the endpoints of a line segment. **(MP8)**

Short on time? Consider inviting students to complete fewer challenges.



Student Notes are also available

1 Launch

1 **Invite students** to solve as many challenges as they have time for. Share that the goal is to explore the figures, not to complete all three challenges.

A **Accessibility: Memory and Attention** Invite students who would benefit from having more time to think deeply to focus on the first challenge.

Math Identity and Community Celebrate students who use the structure of the figure as they progress through the challenges. **(MP8)**

2 Connect

Display the overlay of responses from Challenge 3 using the dashboard's Teacher View.

A **Accessibility: Conceptual Processing** Invite students to share how each figure is different from the previous one to help apply concepts to new situations.

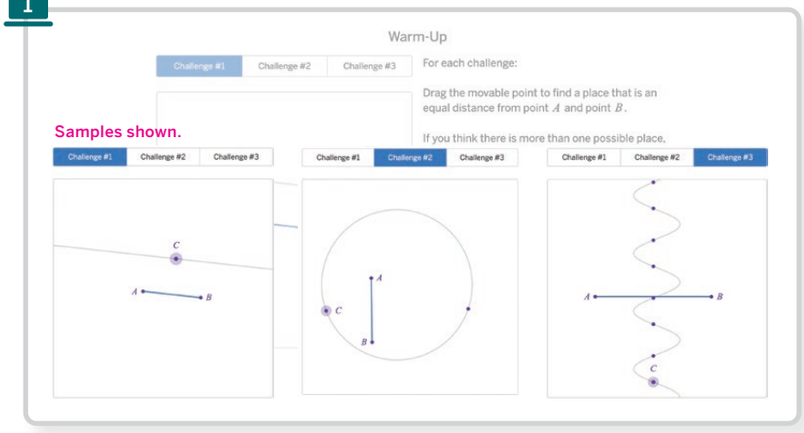
Invite students to share strategies they found helpful while solving these challenges. Consider asking:

- "Which challenge(s) did you find most challenging?"
- "Which challenge(s) had multiple solutions?"
- "What do you notice about the multiple solutions?"

Invite students to discuss the feedback from the interaction. Consider asking, "How did the feedback help you make your decision?"

Students using digital

1



Pairs Sharing a Device | 10 min | Pacing: Screens 2–4

Activity 1 Plotting Points

Purpose: Students determine whether points are **equidistant** to the base of an equilateral triangle using their own strategies and MLR7: Compare and Connect. **(MP6)**



Student Notes are also available

1 Launch

2 **Create** a definition for the term **equidistant** using students' language and the glossary.

Demonstrate how to drag a point using the dashboard's Student View. Encourage students to plot several points before moving on.

To support students getting started, consider asking, "How can we use the properties of a circle to identify equidistant points?"

2 Monitor

3 **Look for** a variety of strategies students use to identify which point is *not* equidistant.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to preview the next screen and consider asking, "Which construction tools might you use to try one of these strategies?"
Use multiple circles centered at each point to determine which do not go through points A and B.	**Strengthen** Celebrate these students one-on-one or publicly. Consider asking, "Why does your strategy work?"
Constructed a perpendicular bisector using the intersections of the circles.	**Strengthen** Consider asking, "Is there another construction that could be used to determine equidistant points?"

3 Connect

4 MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their strategy. Consider asking:

- "How could you use each construction to determine whether each point is equidistant from points A and B?" **(MP6)**
- "How are these two constructions connected? How can you see equal distances from points A and B in both representations?"

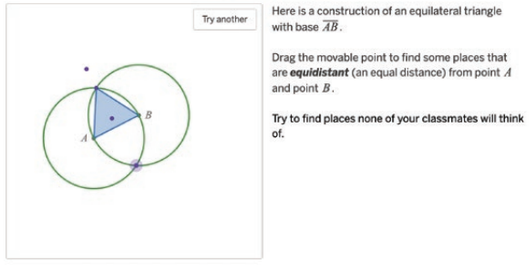
Invite students using notes to document some important considerations from the class discussion.

Math Identity and Community Consider naming strategies after the students who use them and using those names throughout the lesson and unit.

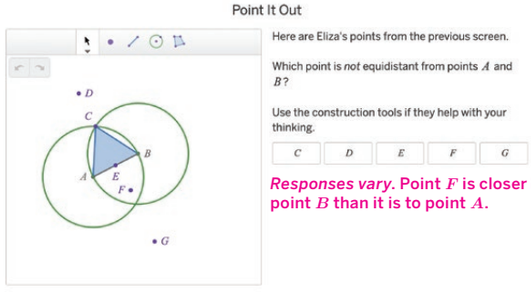
Students using digital

2

Sample shown.

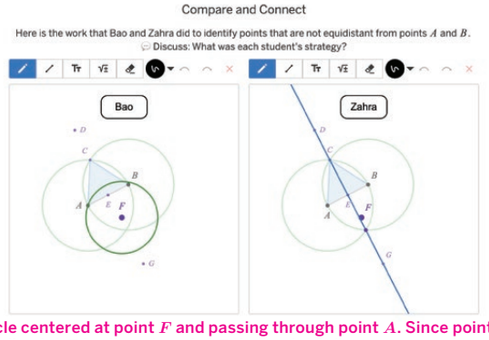


3



Responses vary. Point F is closer to point B than it is to point A.

4



Responses vary.

- Bao used a circle centered at point F and passing through point A. Since point B is inside the circle, he can tell point B is closer to point F than point A is.
- Zahra used the points where the circles intersect from the construction of the equilateral triangle to draw a line through all the points that are the same distance from points A and B.

Pairs Sharing a Device | 12 min | Pacing: Screen 5, Screens 6–7

Activity 2 Lining It Up

Purpose: Students construct **perpendicular bisectors** and consider strategies for using multiple tools in their constructions. **(MP1)**



Student Notes are also available

1 Launch

5 **To connect to prior knowledge**, consider asking, "What does it mean to be equidistant from points A and B? What does it mean to be perpendicular?" Invite students to discuss with their partner before sharing as a whole class.

ML/EL **Multilingual/English Learner** Use gestures to illustrate the meaning of the terms *equidistant* and *perpendicular*. **(Reading and Listening)**

2 Monitor

Look for student strategies for completing this construction, particularly those that try to mirror Charlie's or Zahra's constructions.

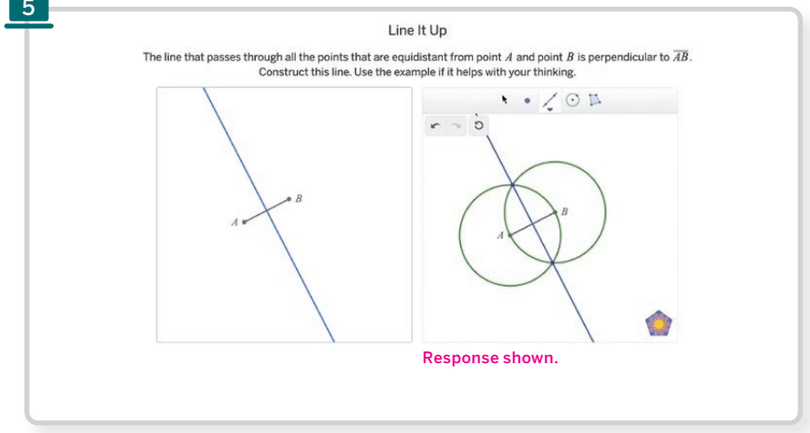
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to look back at Zahra's work from the previous screen to identify a first step.
Use two draggable points and a line to approximate the perpendicular line.	**Support** Consider asking, "How can you use circles to find points that are equidistant from a specific point?"
Construct two circles centered at the endpoints, but are unsure about how to move forward.	**Support** Consider asking, "Where can you see points that are equidistant from both endpoints? Can you use those points to form a line?"
Construct a segment between the intersections of the circles.	**Strengthen** Consider asking, "Does your construction capture all of the points that are equidistant from points A and B? What construction tool might be needed?"
Correctly construct a perpendicular bisector using two circles.	**Stretch** Consider asking, "How does constructing circles help ensure distances that are equidistant from points A and B?"

Pause to invite students to share how they know each point on their line is equidistant from both endpoints. Consider using the Student View to display their thinking as they share.

Students using digital

5



Response shown.

Activity 2 continued >

Pairs Sharing a Device | 12 min | Pacing: Screen 5, Screens 6–7

Activity 2 Lining It Up (continued)

Purpose: Students construct **perpendicular bisectors** and consider strategies for using multiple tools in their constructions. **(MP1)**



Student Notes are also available

2 Monitor

6 **Display** and read Zahra's claim aloud. Consider sharing with students, "This is the term for the construction you just made."

Create a class definition for the terms **midpoint** and **perpendicular bisector** using students' language and the glossary.

Invite students using notes to document the class definition of the terms *midpoint* and *perpendicular bisector* in the left column.

Invite students to continue to the next screen after they've read others' responses and compared them with their own.

3 Connect

7 **To encourage playfulness**, encourage students to engage with Zahra's construction. Consider asking:

- "Why will Zahra's strategy on the left not construct a perpendicular bisector?" **(MP1)**
- "Can someone describe steps they think Zahra took to construct the image on the right?" **(MP1)**

Note: Some students may struggle to distinguish the difference between the line segment constructed between the draggable point and point A on $\overline{AB}$. Show students that to use the circle by length tool, a segment needs to be selected and the goal here is to select the segment with the draggable length.

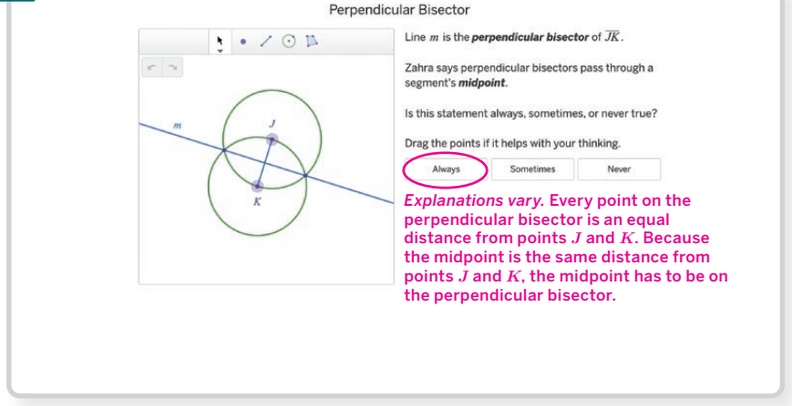
Display the construction tools and consider following along with the student descriptions. Consider asking, "How do circles (or compasses) help us precisely construct perpendicular bisectors?" We can construct two circles of the same size using endpoints of a segment as their centers. The intersection points of these circles are equidistant points from the endpoints of the segment.

To surface the Key Takeaway, consider asking, "What are properties that we can use to describe a perpendicular bisector?"

Invite students using notes to document some important considerations from the class discussion.

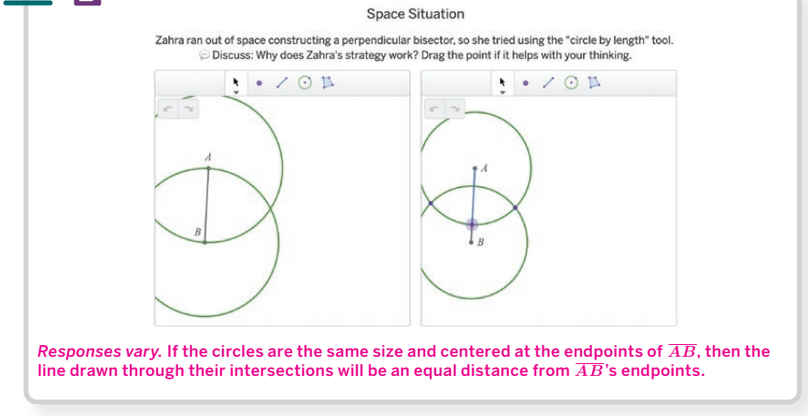
Students using digital

6



Explanations vary. Every point on the perpendicular bisector is an equal distance from points J and K. Because the midpoint is the same distance from points J and K, the midpoint has to be on the perpendicular bisector.

7



Responses vary. If the circles are the same size and centered at the endpoints of $\overline{AB}$, then the line drawn through their intersections will be an equal distance from $\overline{AB}$'s endpoints.

Key Takeaway: A **perpendicular bisector** is a line that passes through the **midpoint** of a line segment. Any point on the perpendicular bisector is **equidistant** from the endpoints of the segment it passes through.

Pairs Sharing a Device | 8 min | Pacing: Screens 8–9

Activity 3 Center Line

Purpose: Students complete a series of perpendicular bisector construction challenges. **(MP8)**

Short on time? Consider inviting students to complete fewer challenges.



Student Notes are also available

1 Launch

8 **Invite students** to set a goal for how many of the four challenges they want to solve. Share that it's more important to make sense of the challenges than it is to work quickly.

2 Monitor

Listen for strategies students use in discussing how to solve the challenges, such as using circles to represent equal distances and how the circle by length tool can be used when the intersection points of circles can't be seen.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to guess and use the feedback to think of a strategy.
Plot the line from point C to $\overline{AB}$ and wiggle it to get a star. (*Challenge 3*)	**Support** Consider asking, "What construction tools can we use to divide the triangle more precisely?"
Construct the circles, but are unsure how to construct the perpendicular bisector without the intersection. (*Challenge 2*)	**Strengthen** Celebrate their progress. Consider asking, "How might we use Zahra's work on Screen 7 to identify useful tools?"
Recognize the perpendicular bisector of $\overline{AB}$ is also the perpendicular bisector of $\overline{CD}$. (*Challenge 4*)	**Stretch** Consider asking, "What do you think is unique about the properties of this figure?" Invite students to construct a new figure with similar properties.

D **Differentiation: Explore More (Stretch)** Invite students who would like to further explore perpendicular bisectors to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

8 **Invite students** to share strategies they found helpful or what they learned from the mistakes they made while solving these challenges.

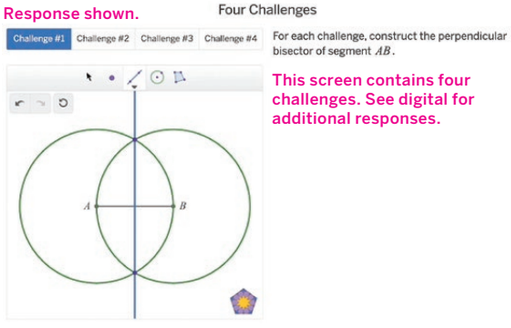
Consider asking:

- "What was a strategy that worked in every challenge?" **(MP8)**
- "What was a strategy that worked for some challenges, but had to be adjusted for others?"

Students using digital

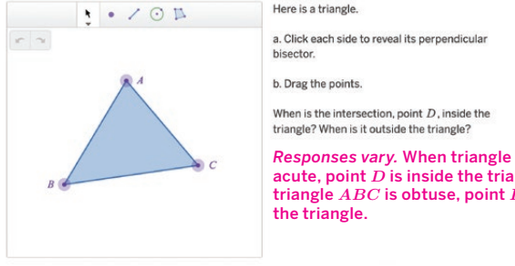
8

Response shown.



This screen contains four challenges. See digital for additional responses.

9



Responses vary. When triangle ABC is acute, point D is inside the triangle. When triangle ABC is obtuse, point D is outside the triangle.

Whole Class | 5 min | Pacing: Screen 10

Synthesis



Student Notes are also available

10 **Invite students** to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- A perpendicular bisector has to go through the midpoint of $\overline{AB}$.
- Points on the perpendicular bisector have to be equidistant from both points A and B.
- Circles can help us measure equal distances because the radiuses of a circle are always the same length.
- If I draw a circle with the center on the perpendicular bisector, both endpoints of segment AB will be on the circle.

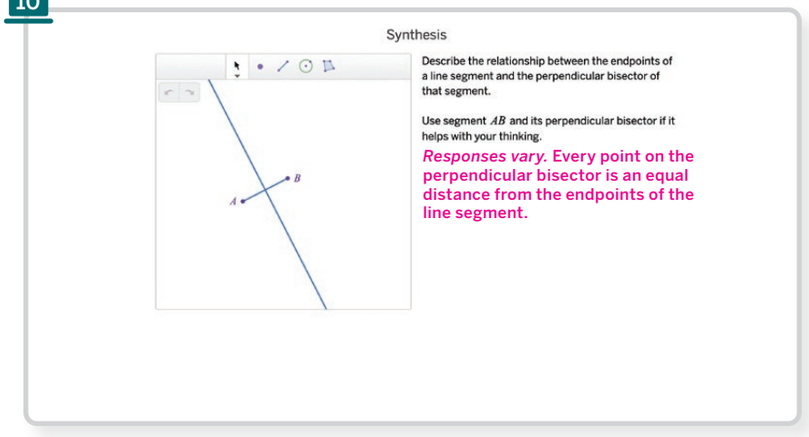
MLR **MLR2: Collect and Display** Use students' language to formalize the definition of *perpendicular bisectors* and the strategy for constructing them. Display any new language, examples, or diagrams for students to reference later.

Math Identity and Community Invite students to share strategies they found helpful and give credit to the students who shared them.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.

Students using digital

10



Lesson Takeaway: Points that are **equidistant** from a line segment's endpoints will be on the **perpendicular bisector** of that segment. To construct a perpendicular bisector, begin by constructing two equal-sized circles centered at the endpoints and connecting the points where two circles intersect with a line.

Independent | 5 min | Pacing: Screens 11–12

Show What You Know

Purpose: Students demonstrate their understanding by constructing a perpendicular bisector of a line segment.

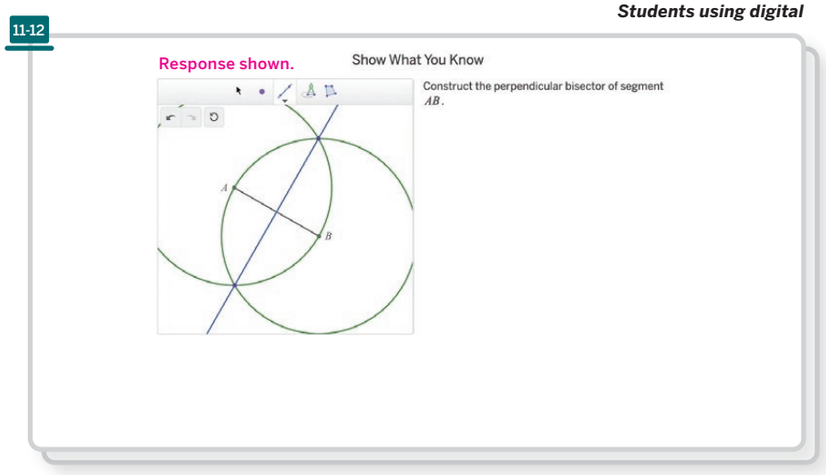


Students using print

11-12 Today's Goals

1. **Goal:** Construct the *perpendicular bisector* of a line segment.
2. **Goal:** Explore how points that are *equidistant* from a line segment's endpoints will be on the *perpendicular bisector* of that segment.
3. **Language Goal:** Describe that points that are *equidistant* from a line segment's endpoints will be on the *perpendicular bisector* of that segment. **(Writing and Speaking)**

Students using digital



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of perpendicular bisectors, consider:

- Inviting students to engage with the **Summary** by reading it and highlighting key components.
- Inviting students to do the **Try This** problem and provide individualized feedback.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice**.
- Inviting students to review the **Summary** and do the **Try This** problem.
- Revisiting **Screen 8**. Invite students to engage with the challenges they haven't yet completed.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting them to complete the **Explore More** task and discuss their thinking with a partner.
- Inviting students to write their own problem that is similar to the **Try This** problem.

Professional Learning

In this lesson, students were introduced to strategies for constructing and analyzing perpendicular bisectors. How will these strategies support their constructions in the upcoming lessons? How did the interpretive feedback in this lesson support student learning?

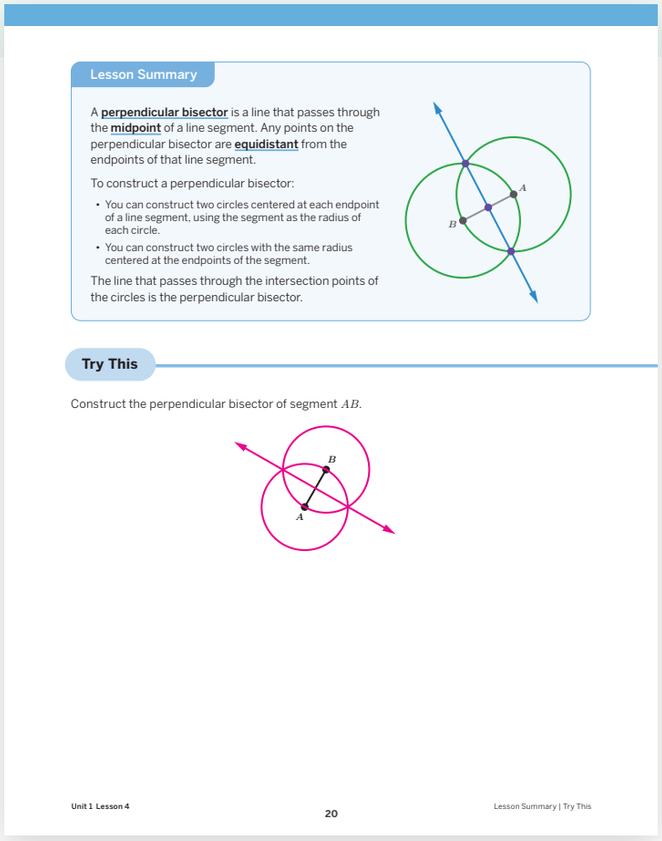
Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 4 Practice

Students using digital



Lesson Summary

A **perpendicular bisector** is a line that passes through the **midpoint** of a line segment. Any points on the perpendicular bisector are **equidistant** from the endpoints of that line segment.

To construct a perpendicular bisector:

- You can construct two circles centered at each endpoint of a line segment, using the segment as the radius of each circle.
- You can construct two circles with the same radius centered at the endpoints of the segment.

The line that passes through the intersection points of the circles is the perpendicular bisector.

Try This

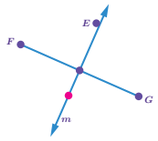
Construct the perpendicular bisector of segment AB.

Lesson Practice
Geo.1.04

Name: _____ Date: _____ Period: _____

Problems 1–2: Line m is the perpendicular bisector of $\overline{FG}$.

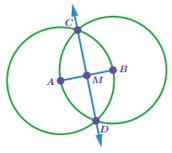
1. Plot a point that is the same distance from F as it is from G.
   Responses vary.



2. Circle the phrase that best completes the statement:
   Point E is _____.

   (Closer to point F) Closer to point G The same distance from point F as it is from point G

3. In the diagram, point A is the center of one circle and point B is the center of another. Select *all* the true statements.

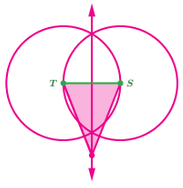


   - [x] **A.** Line CD is perpendicular to segment AB.
   - [x] **B.** Point M is the midpoint of segment AB.
   - [ ] **C.** The length of segment AB is equal to the length of segment CD.
   - [ ] **D.** Segment AM is perpendicular to segment BM.
   - [x] **E.** The length of segment CB is equal to the length of segment DB.

Problems 4–5: Here is $\overline{TS}$.

4. Construct the perpendicular bisector of $\overline{TS}$.
   Work varies.

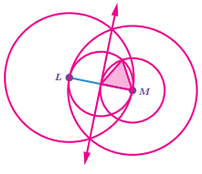
5. Using $\overline{TS}$ as a side, construct a triangle with *only* two sides of the same length.
   Responses vary.



Lesson Practice
Geo.1.04

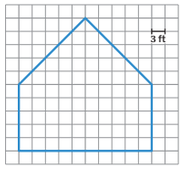
Name: _____ Date: _____ Period: _____

6. Construct an equilateral triangle with a side that is half the length of $\overline{LM}$. *Responses vary.*



Spiral Review

Problems 7–9: Here is a scale drawing of the front of a shed.



7. What is the height of the shed, in feet?
   30 feet

8. What is the width of the shed's base at its widest point, in feet?
   30 feet

9. Find the total area, in square feet.
   675 square feet

Reflection

1. Put a star next to the problem you understood best.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-lesson			
	1–2	1	HSG.CO.A.1
Test Practice	3	2	HSG.CO.D.12
	4–5	2	HSG.CO.D.12
	6	3	HSG.CO.D.12
Spiral Review			
	7–8	1	7.G.A.1
Fluency	9	1	7.G.A.1

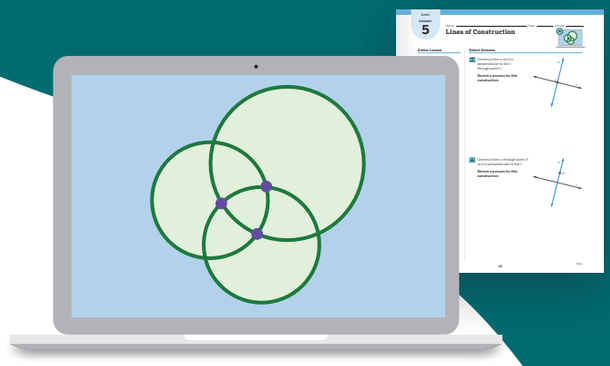
Need More Practice?

Additional Practice resources coming soon.

Unit 1

Lesson 5

This lesson uses print student pages and an assignable Digital Companion.



Lines of Construction

Constructing Perpendicular and Parallel Lines

Let's develop strategies to construct perpendicular and parallel lines.

Focus and Coherence

Today's Goals

1. **Goal:** Construct *perpendicular* and *parallel lines* that pass through given points.
2. **Language Goal:** Write precise instructions for completing a construction. **(Reading and Writing)**

Students construct **perpendicular lines** using digital tools. They construct a perpendicular line through a point on a line, extend that understanding to a point off the line, and then construct a **parallel line** using physical tools. Students write instructions for other students to replicate their construction and revise their instructions based on feedback. **(MP3)**

Prior Learning

In Grade 8, students learned about the relationship between interior angles of parallel lines cut by a transversal. In Lesson 4, students constructed a perpendicular bisector, which serves as a foundational construction for building parallel and perpendicular lines.

Future Learning

In Lesson 6, students will extend their understanding of constructing parallel and perpendicular lines to construct squares.

Rigor and Balance

- Students develop **conceptual understanding** of how more complex geometric objects can be constructed by using known relationships.
- Students develop **procedural fluency** with the construction tools as they use the perpendicular bisector construction to create parallel and perpendicular lines.

Vocabulary

New Vocabulary

parallel lines

perpendicular lines

Standards

Addressing

HSG.CO.D.12

Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, **dynamic geometric software**, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; **constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line**.

Also Addressing: **HSG.CO.A.1**

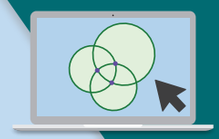
Mathematical Practices: MP3, MP6

Building On	Building Toward
8.G.A.5	HSG.CO.D.12
HSG.CO.D.12	HSG.CO.D.13

Lesson at a Glance

~ 45 min

Standards: HSG.CO.D.12, HSG.CO.A.1



Why digital?

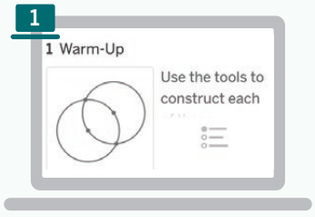
Students use Responsive Feedback to determine whether their digital constructions are valid.

Warm-Up

Pairs Sharing a Device | 5 min

Pacing: Screen 1

Students construct geometric objects and discuss strategies for constructing segments of equal length and **perpendicular lines**.



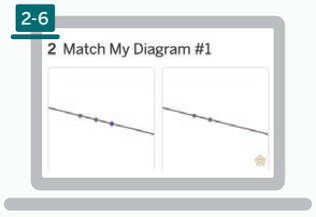
Activity 1

Pairs Sharing a Device | 15 min

Pacing: Screens 2–4, Screens 5–6

Students apply their understanding of perpendicular bisectors and equal distance to construct perpendicular lines through a point not on the line.

Key Takeaway (Screen 6): Geometric figures can be constructed by reasoning about properties of lines, circles, and segments. One example is to find two points on a line that are equidistant from a given point using a circle. Connecting these two points creates a line segment. The perpendicular bisector of this created line segment will also go through the given point, creating a **perpendicular line**.



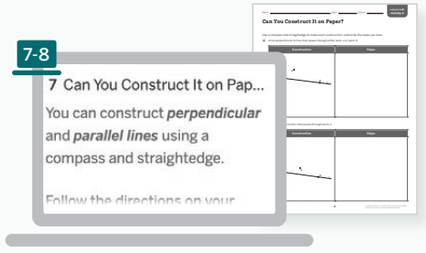
Activity 2

Independent | 15 min

Pacing: Screens 7–8

Students construct perpendicular and **parallel lines** using physical tools, and use **MLR1: Stronger and Clearer Each Time** to write precise steps for their constructions. **(MP6)**

Key Takeaway (Screen 8): One strategy for constructing **parallel lines** is to use the perpendicular bisector construction twice on the same line. This creates two parallel lines because they are perpendicular to the same line.



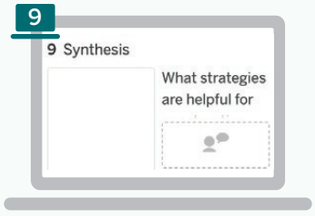
Synthesis

Whole Class | 5 min

Pacing: Screen 9

Students synthesize their understanding of constructing parallel and perpendicular lines.

Lesson Takeaway: The perpendicular bisector is helpful when constructing **perpendicular lines** (lines that form right angles). To construct **parallel lines**, it's helpful to think about their properties and how they can be used in constructions.

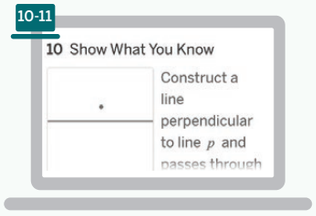


Show What You Know

Independent | 5 min

Pacing: Screens 10–11

Students demonstrate their understanding by constructing a line perpendicular through a point not on the given line.



Prep Checklist

Assign the digital student screens. Student Notes are also available.

Consider making the geometry toolkit available to support students in selecting and using tools strategically.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)
- Activity 2 Sheet (one per student)
  - geometry toolkit: compasses, straightedges
- Student Notes sheet in Student Edition:
  Consider providing the Student Notes to support class discussion and student thinking.



Pairs Sharing a Device | 5 min | Pacing: Screen 1

Warm-Up

Purpose: Students construct geometric objects and discuss strategies for constructing segments of equal length and **perpendicular lines**.



Student Notes are also available

1 Launch

1 **To encourage creativity**, challenge students to construct segments and figures they think may be different from their classmates.

ML/EL **Multilingual/English Learners** Give students time to think independently before sharing their ideas verbally. **(Speaking)**

Listen for students who are using a circle's fixed radius to justify why segments are the same length.

A **Accessibility: Visual-Spatial Processing** Invite pairs to think about the key features of the circles and line segments and to discuss them aloud before creating the objects to support distinguishing between visual representations.

Capture different examples of perpendicular bisectors, including those that construct additional points and circles.

Note: To provide students an easier way to discuss the construction, consider sharing, "You can refer to this circle based on their center, such as circle A."

2 Connect

Math Identity and Community Celebrate creative responses from students who added additional circles to create the geometric objects. Display their constructions and ask them to talk about their inspiration.

Display the captured perpendicular bisectors. Consider asking:

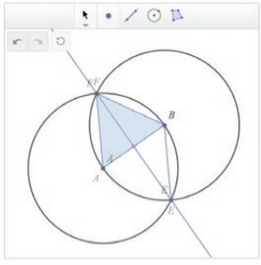
- "How do we know that all of these are perpendicular bisectors?"
- "How do we know that certain line segments in our diagram are the same length?"
- "Could we have created a perpendicular bisector that didn't use the identified points on the circles?"

Note: This lesson uses the construction tools to build on students' prior knowledge of using perpendicular bisectors to construct **perpendicular lines**.

Students using digital

1

Sample shown.



Responses vary.

- A radius is a line segment that connects the center of a circle to a point on the circle, so segment AB is a radius of circle A.
- An equilateral triangle has three equal sides. Segments AE, EB, and AB are all radiuses of circles A and B, so they are equal. That means triangle AEB is an equilateral triangle.

Pairs Sharing a Device | 15 min | Pacing: Screens 2–4, Screens 5–6

Activity 1 Construction Challenges

Purpose: Students apply their understanding of perpendicular bisectors and equal distance to construct perpendicular lines through a point not on the line.



Student Notes are also available

1 Launch

2 **Consider sharing** that students will complete several challenges. Encourage students to work until the star in the bottom corner animates, which shows that they have completed the challenge successfully.

Look for students who begin by dragging points to approximate equal lengths. Consider asking, "How can you construct equal distances using the tools?"

2 Monitor

3 **To support making connections**, encourage students to think about construction strategies they used in previous lessons and how those connect to creating a perpendicular line.

Note: It's okay if students are unsure about how to create a perpendicular line. Students will see strategies for approaching these constructions on later screens.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How can you use the perpendicular bisector construction from last class to support your thinking here?"
Construct a line and drag it to approximate a perpendicular line using dynamic star feedback.	**Support** Invite them to use the other construction tools to justify they have created a perpendicular line.
Construct points that are not at precise intersections of circles.	**Strengthen** Consider asking, "How can you adjust your construction so that the relationships between the objects stay the same even if points are dragged?"
Use the line segment tool instead of a line tool for the perpendicular line.	**Strengthen** Consider reminding them the task is to create a perpendicular line, not a line segment.

4 **Pause** to invite students to discuss how their strategy was similar to or different from Neo's.

Math Identity and Community Consider inviting students to share what they learned from making construction attempts.

Invite students using notes to record a process for constructing a perpendicular line through a point on a given line.

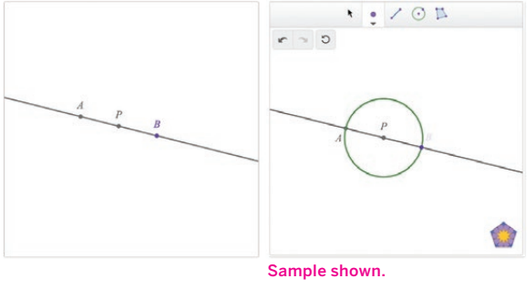
Activity 1 continued >

Students using digital

2

Match My Diagram #1

Construct point B so that point P is the midpoint of segment AB.

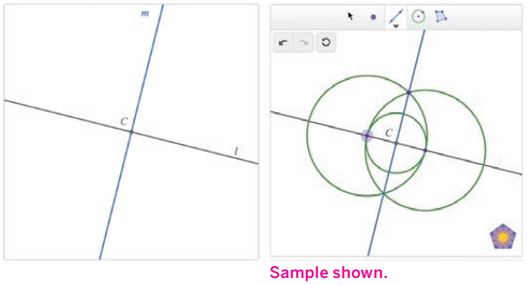


Sample shown.

3

Match My Diagram #2

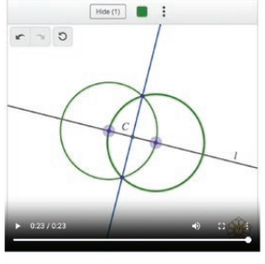
Construct line m so it is perpendicular to line l through point C.



Sample shown.

4

Neo's Construction



Watch Neo's strategy for the previous problem.

Discuss: How can Neo improve the construction so that the perpendicular line always goes through point C?

Responses vary. Neo can draw a circle centered at point C. Then Neo can use the points where the circle intersects line l as the centers of two circles, and use the distance between the two intersection points as the radius length. Neo can then draw the perpendicular line through the intersection points of the circles.

Pairs Sharing a Device | 15 min | Pacing: Screens 2–4, Screens 5–6

Activity 1 Construction Challenges (continued)

Purpose: Students apply their understanding of perpendicular bisectors and equal distance to construct perpendicular lines through a point not on the line.



Student Notes are also available

2 Monitor

5 **Look for** students who start their construction by using a circle centered at point C that intersects line l twice.

A **Accessibility: Conceptual Processing** Invite students to share how this construction is similar to and different from the previous one to help them apply concepts to new situations.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Why was a circle centered at the point a helpful first step in the previous construction?"
Construct a circle centered at point C that only intersects with line l at one point.	**Support** Invite students to drag their point along line l. Consider asking, "What happened? How can this help you make sure the line is exactly perpendicular?"
Complete the construction and receive the animated star feedback.	**Stretch** Consider asking, "Are there other ways we might construct this perpendicular line?"

Capture constructions that start with a circle centered at point C that intersects with line l twice.

3 Connect

Share the captured constructions and ask, "Are these perpendicular lines the same as the diagram we're trying to match? How do you know?" Invite students to discuss with a partner before discussing as a class.

6 **Display** Raven's strategy. Invite students to read other students' responses. Consider asking, "Does it matter what size Raven's circle is for this strategy?"

To surface the Key Takeaway, ask, "How is Raven's strategy similar to the start of the construction of a perpendicular line through a point on a line?" Both strategies start by constructing a circle centered at the point we want the perpendicular line to go through.

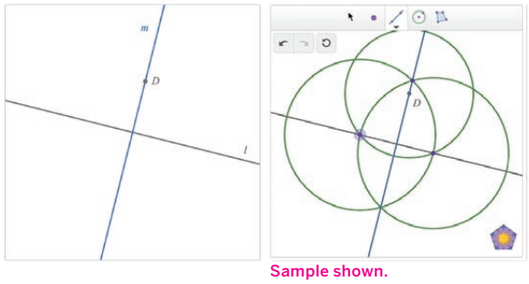
Invite students using notes to record a process for constructing a perpendicular line through a point not on a given line.

Students using digital

5

Match My Diagram #3

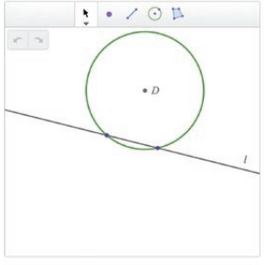
Construct line m through point D so it is perpendicular to line l.



Sample shown.

6

Raven's Approach



Here is Raven's first step for the previous problem.

Why do you think this is the first step in her

Responses vary. Raven constructed a circle centered at point D that intersects line l at two points, which makes a distance that can be used as a radius length. This allowed her to create two circles with the same-sized radius centered at the intersection points. The line through the intersection points of the two circles will be the line perpendicular to line l through point D.

Key Takeaway: Geometric figures can be constructed by reasoning about properties of lines, circles, and segments. One example is to find two points on a line that are equidistant from a given point using a circle. Connecting these two points creates a line segment. The perpendicular bisector of this created line segment will also go through the given point, creating a **perpendicular line**.

Independent | 15 min | Pacing: Screens 7–8

Activity 2 Can You Construct It on Paper?

Purpose: Students construct perpendicular and **parallel lines** using physical tools, and use **MLR1: Stronger and Clearer Each Time** to write precise steps for their constructions. **(MP6)**

Short on time? Consider omitting the perpendicular line construction on paper.



Student Notes are also available

1 Launch

7 **To connect to prior knowledge**, consider asking, "What are ways you have previously shown two lines are **parallel**?"

Note: Make sure all students have access to Activity 2.

Invite students to sketch a plan to make sense of the constructions before beginning with physical tools.

Math Identity and Community Celebrate and encourage students to be creative in their approaches and make sure students are clear in their instructions because of the multiple methods.

2 Monitor

To support students getting started with writing precise instructions, encourage them to refer back to their notes from Lesson 3 about the qualities of precise instructions.

MLR **MLR1: Stronger and Clearer Each Time** Invite students to write a first draft and use feedback to make their thinking stronger and clearer. **(MP6)**

Capture examples of precise instructions and consider asking:

- "Can you use point labels and centers of circles to better explain your thinking?"
- "Is there anything in someone else's explanation that you can use to make your writing stronger and clearer?"

8 D **Differentiation: Explore More (Stretch)** Invite students who would like to further explore constructing parallel and perpendicular lines to complete this optional task. Encourage them to discuss their thinking with a partner.

Activity 2 continued >

Students using digital

7

Can You Construct It on Paper?

You can construct ***perpendicular*** and ***parallel lines*** using a compass and straightedge.

Follow the directions on your worksheet.

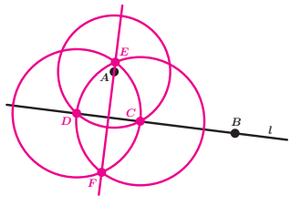
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Lesson 1.05 Activity 2

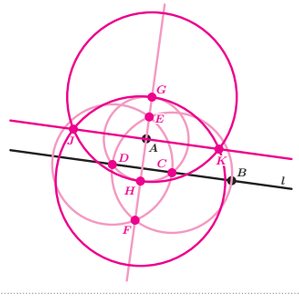
Can You Construct It on Paper?

Use a compass and straightedge to make each construction, and write the steps you took.

a A line perpendicular to line l that passes through *either* point A or point B. ***Responses vary.***

Construction	Steps
	• Construct a circle centered at point A that intersects line l at two points. • Label where circle A intersects line l as points C and D. • Construct a circle with center C that goes through point D. • Construct a circle with center D that goes through point C. • Label where the circles intersect as point E and point F. • Construct a line through points E and F.

b The line parallel to line l that passes through point A. ***Responses vary.***

Construction	Steps
	• Construct a perpendicular line through point A, as described above. • Label where circle A intersects the perpendicular line as points G and H. • Construct a circle with center G that goes through point H. • Construct a circle with center H that goes through point G. • Label where circles G and H intersect at points J and K. • Construct a line that passes through points J and K. Line JK is parallel to line l.

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Independent | 15 min | Pacing: Screens 7–8

Activity 2 Can You Construct It on Paper? (continued)

Purpose: Students construct perpendicular and **parallel lines** using physical tools, and use **MLR1: Stronger and Clearer Each Time** to write precise steps for their constructions. **(MP6)**

Short on time? Consider omitting the perpendicular line construction on paper.



Student Notes are also available

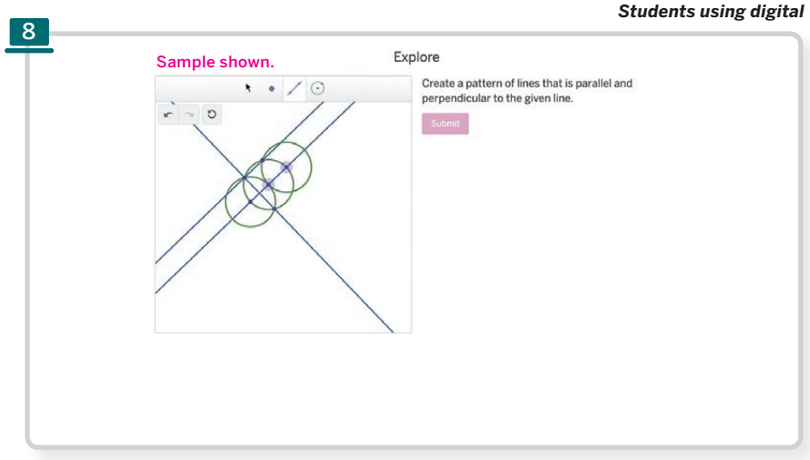
3 Connect

Share the captured instructions and invite students to discuss the qualities of the precise instructions they see, referring back to those listed earlier in class.

To surface the Key Takeaway, consider asking:

- "What construction tools were helpful for constructing parallel lines?"
- "How do we know lines are parallel based on the way they were constructed?"

Students using digital



Key Takeaway: One strategy for constructing **parallel lines** is to use the perpendicular bisector construction twice on the same line. This creates two parallel lines because they are perpendicular to the same line.

Whole Class | 5 min | Pacing: Screen 9

Synthesis



Student Notes are also available

9 **Invite students** to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Perpendicular bisectors are helpful for bisecting line segments.
- The perpendicular bisector creates lines that form right angles.
- A circle can be used to create a line segment where both endpoints are equidistant from a specific point.
- We can combine construction strategies to create objects that have specific properties. For example, using the perpendicular bisector construction twice to create two lines that are perpendicular to the same line.

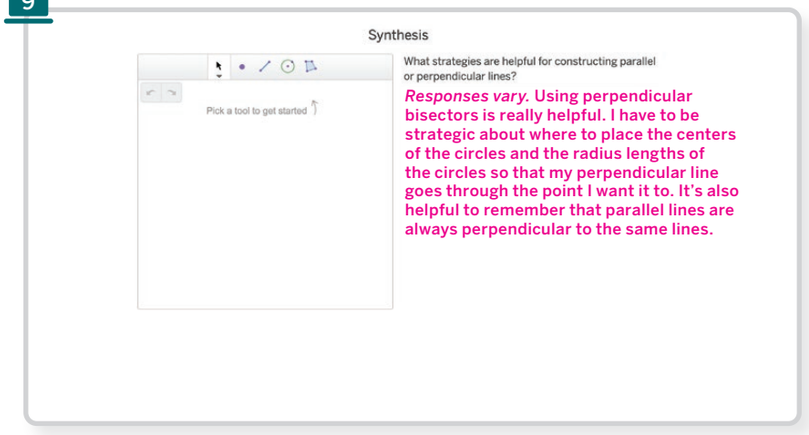
Math Identity and Community If time allows, consider sharing students' patterns from the Explore More task that used parallel and perpendicular lines.

Invite students using notes to document ideas from the class discussion in the left column.

Note: If time allows, invite students to share with a partner what makes sense about each construction strategy and what questions they still have.

Students using digital

9



Lesson Takeaway: The perpendicular bisector is helpful when constructing **perpendicular lines** (lines that form right angles). To construct **parallel lines**, it's helpful to think about their properties and how they can be used in constructions.

Independent | 5 min | Pacing: Screens 10–11

Show What You Know

Purpose: Students demonstrate their understanding by constructing a line perpendicular through a point not on the given line.

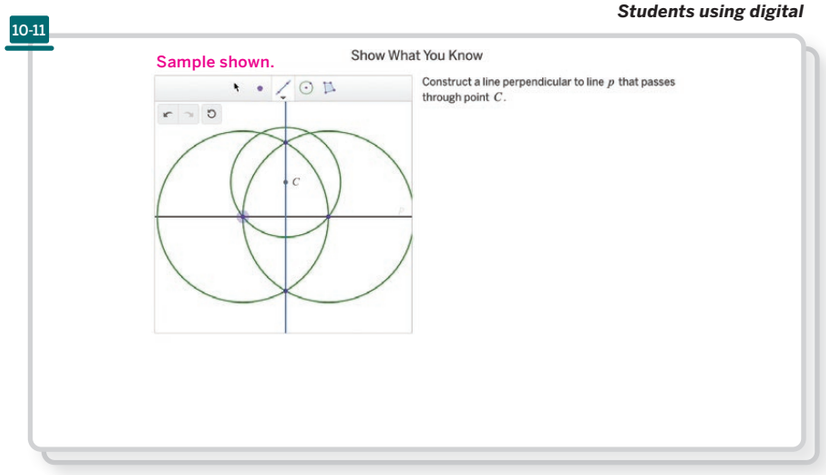


Students using print

10-11 Today's Goals

1. **Goal:** Construct *perpendicular* and *parallel lines* that pass through given points.
2. **Language Goal:** Write precise instructions for completing a construction. **(Reading and Writing)**

Students using digital



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of constructing perpendicular lines through a point not on the line:

You may choose not to intervene now. Students will have more opportunities to practice in **Lesson 6**.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding** with minor errors, consider:

- Assigning the **Lesson Practice**.
- Invite students to read the **Summary** and complete the **Try This** problem.
- Emphasizing this skill in the **Lesson 6 Warm-Up** before showing them the new construction tools options for perpendicular lines and parallel lines.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting them to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

Reflect on how students analyzed student work today. How did they talk about mistakes? Were they able to notice the correct parts, incorrect parts, or next steps of each student's construction? How can you support students in valuing learning from correct, incorrect, and unfinished thinking?

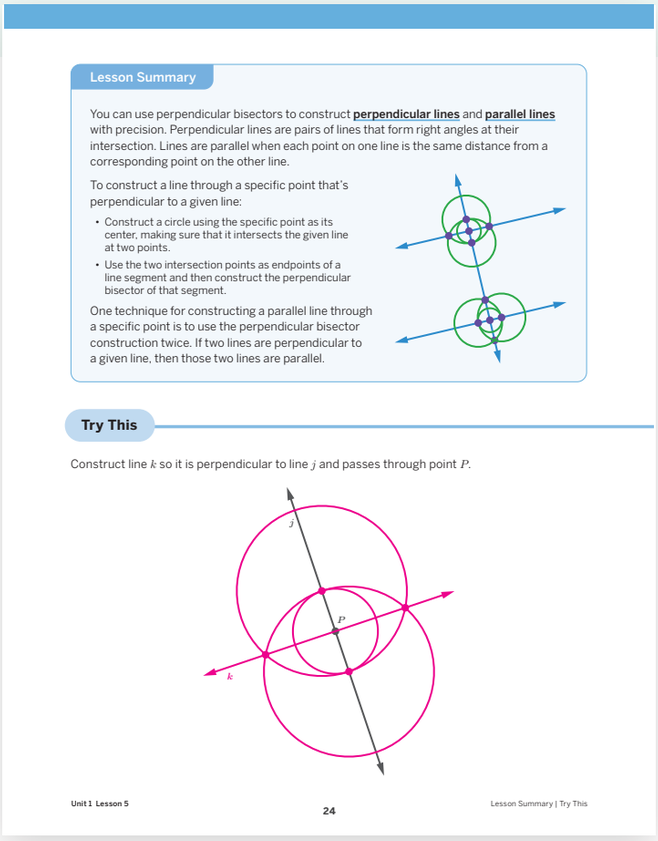
Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 5 Practice

Students using digital



Lesson Summary

You can use perpendicular bisectors to construct **perpendicular lines** and **parallel lines** with precision. Perpendicular lines are pairs of lines that form right angles at their intersection. Lines are parallel when each point on one line is the same distance from a corresponding point on the other line.

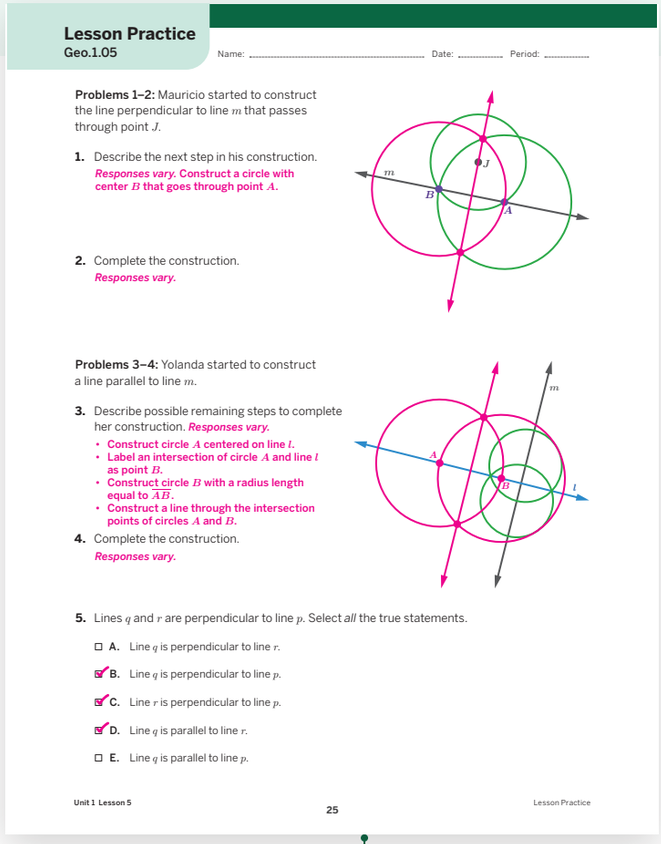
To construct a line through a specific point that's perpendicular to a given line:

- Construct a circle using the specific point as its center, making sure that it intersects the given line at two points.
- Use the two intersection points as endpoints of a line segment and then construct the perpendicular bisector of that segment.

One technique for constructing a parallel line through a specific point is to use the perpendicular bisector construction twice. If two lines are perpendicular to a given line, then those two lines are parallel.

Try This

Construct line k so it is perpendicular to line j and passes through point P.



Lesson Practice
Geo.1.05

Name: _____ Date: _____ Period: _____

Problems 1–2: Mauricio started to construct the line perpendicular to line m that passes through point J.

1. Describe the next step in his construction.
   Responses vary. Construct a circle with center B that goes through point A.

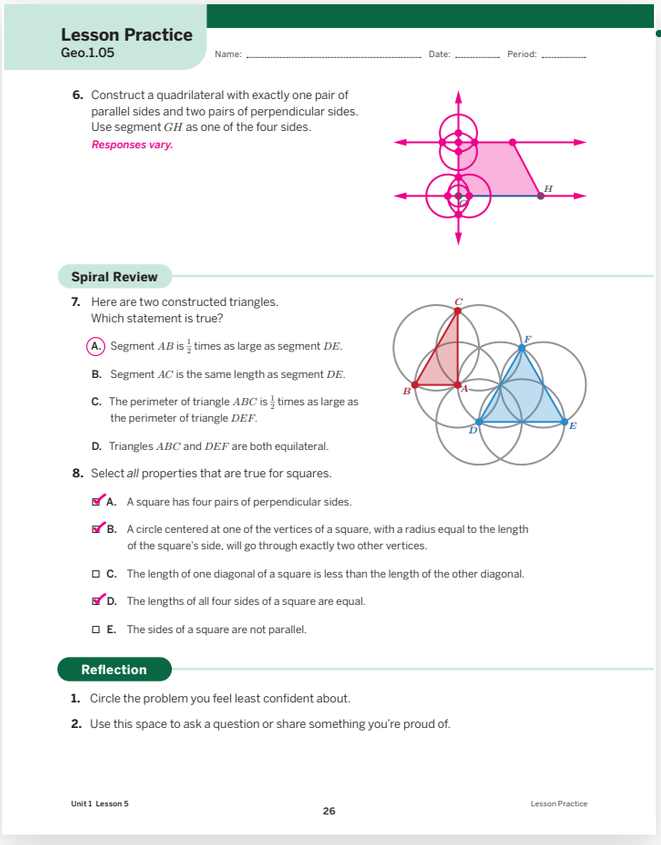
2. Complete the construction.
   Responses vary.

Problems 3–4: Yolanda started to construct a line parallel to line m.

3. Describe possible remaining steps to complete her construction. *Responses vary.*
   - Construct circle A centered on line l.
   - Label an intersection of circle A and line l as point B.
   - Construct circle B with a radius length equal to $\overline{AB}$.
   - Construct a line through the intersection points of circles A and B.

4. Complete the construction.
   Responses vary.

5. Lines q and r are perpendicular to line p. Select *all* the true statements.
   - ☐ A. Line q is perpendicular to line r.
   - ☑ B. Line q is perpendicular to line p.
   - ☑ C. Line r is perpendicular to line p.
   - ☑ D. Line q is parallel to line r.
   - ☐ E. Line q is parallel to line p.



Lesson Practice
Geo.1.05

Name: _____ Date: _____ Period: _____

6. Construct a quadrilateral with exactly one pair of parallel sides and two pairs of perpendicular sides. Use segment GH as one of the four sides.
   Responses vary.

Spiral Review

7. Here are two constructed triangles. Which statement is true?
   - (A.) Segment AB is $\frac{1}{2}$ times as large as segment DE.
   - B. Segment AC is the same length as segment DE.
   - C. The perimeter of triangle ABC is $\frac{1}{2}$ times as large as the perimeter of triangle DEF.
   - D. Triangles ABC and DEF are both equilateral.

8. Select *all* properties that are true for squares.
   - ☑ A. A square has four pairs of perpendicular sides.
   - ☑ B. A circle centered at one of the vertices of a square, with a radius equal to the length of the square's side, will go through exactly two other vertices.
   - ☐ C. The length of one diagonal of a square is less than the length of the other diagonal.
   - ☑ D. The lengths of all four sides of a square are equal.
   - ☐ E. The sides of a square are not parallel.

Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
Fluency	1	1	HSG.CO.D.12
	2–4	2	HSG.CO.D.12
Test Practice	5	1	HSG.CO.A.1
	6	3	HSG.CO.D.12
Spiral Review			
	7	1	HSG.CO.D.12*
	8	2	5.G.B.4

*This problem builds toward the standard shown.

Need More Practice?

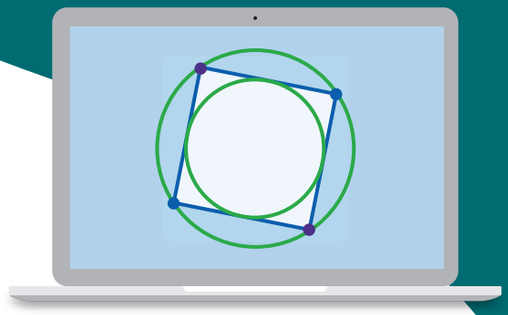
Additional Practice resources coming soon.

Unit 1

Lesson 6

Digital Lesson

This lesson uses digital student screens. Student Notes are also available.



Square Up

Constructing Squares

Let's make squares using circles.

Focus and Coherence

Today's Goals

1. **Goal:** Construct a square *inscribed* in a circle, and a circle inscribed in a square.
2. **Language Goal:** Describe how construction techniques, like determining a midpoint or a perpendicular line, are used to construct a square. **(Writing and Speaking)**

Students explore construction techniques and tools, particularly when to construct and use perpendicular lines, parallel lines, and midpoints. They start by constructing rectangles and squares, and progress to analyzing constructions for clarity and functionality. Students then consider **inscribed** figures and how their construction knowledge and skills can be applied to construct more complex figures. **(MP3)**

Prior Learning

In Lessons 4 and 5, students developed construction strategies for perpendicular and parallel lines.

Future Learning

In Sub-Unit 2, students will use perpendicular lines to perform rigid transformations.

Rigor and Balance

- Students **apply** methods used to construct perpendicular lines, parallel lines, and midpoints to construct squares.
- Students develop **adaptive reasoning** as they reflect on the relationships between inscribed squares and circles and the construction methods available to depict those relationships.

Vocabulary

New Vocabulary

inscribe

Review Vocabulary

vertex

Standards

Addressing

HSG.CO.D.13
Construct an equilateral triangle, **a square**, and a regular hexagon **inscribed in a circle.**

Also Addressing: **HSG.CO.D.12**

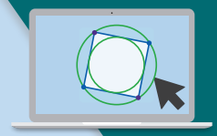
Mathematical Practices: MP3, MP6

Building On	Building Toward
HSG.CO.A.1 HSG.CO.D.12	HSG.CO.D.13

Lesson at a Glance

~ 45 min

Standards: HSG.CO.D.13, HSG.CO.D.12



Why digital?

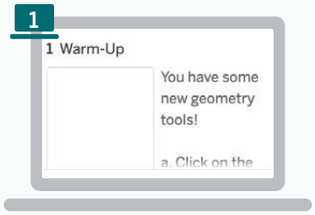
Students use the digital construction tools to quickly explore the relationship between inscribed squares and circles.

Warm-Up

Independent | 5 min

Pacing: Screen 1

Students explore using the parallel and perpendicular line tools to construct a rectangle. **(MP1)**



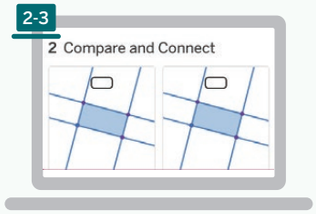
Activity 1

Pairs Sharing a Device | 10 min

Pacing: Screen 2, Screen 3

Students analyze and critique constructions of rectangles and squares to precisely determine the properties of these figures.
MLR7: Compare and Connect **(MP3, MP6)**

Key Takeaway (Screen 3): Parallel lines and perpendicular lines are helpful for constructing squares and rectangles. To construct precisely, it's important that the necessary properties always hold true.



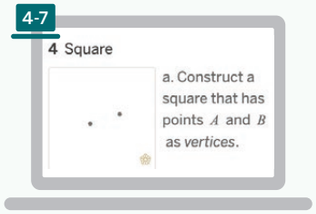
Activity 2

Pairs Sharing a Device | 20 min

Pacing: Screen 4, Screens 5–7

Students construct squares, including a square **inscribed** in a circle, and a circle inscribed in a square.
Routine: Think-Pair-Share

Key Takeaway (Screen 7): Midpoints can be helpful in constructing very specific types of squares that are **inscribed** in circles or by circles.



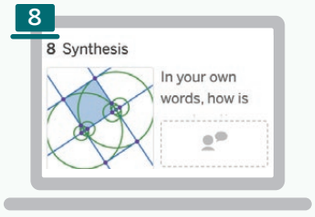
Synthesis

Whole Class | 5 min

Pacing: Screen 8

Students synthesize their understanding of construction strategies for squares.

Lesson Takeaway: Digital construction tools can help you construct geometric figures with precision. You can construct more complex figures by building on your knowledge of simpler constructions and using the construction tools intentionally.

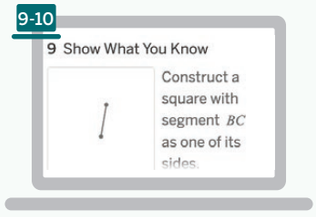


Show What You Know

Independent | 5 min

Pacing: Screens 9–10

Students demonstrate their understanding by constructing a square with a given line segment as one of the sides.



Prep Checklist

Assign the digital student screens. Student Notes are also available.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)

Student Notes sheet in Student Edition:
Consider providing the Student Notes to support class discussion and student thinking.



Independent | 5 min | Pacing: Screen 1

Warm-Up

Purpose: Students explore using the parallel and perpendicular line tools to construct a rectangle. **(MP1)**



Student Notes are also available

1 Launch

1 **To generate excitement**, invite students to explore the dropdown menus and identify new tools before constructing a rectangle. Consider asking, "What new tools did you notice? What are you excited about exploring through your constructions today?"

Note: The parallel and perpendicular line tools support students in creating different rectangles without having to use repetitive procedures to construct lines. Consider recommending these tools to focus students' learning on the reasoning and relationship of rectangles instead of the procedures of making them.

To connect to prior knowledge, consider asking, "What do you remember about rectangles that will help you construct the figure?"

ML/EL **Multilingual/English Learners** Use gestures or an anchor chart to review the meaning of the terms *midpoint*, *perpendicular lines*, and *parallel lines*. **(Reading and Listening)**

2 Connect

Math Identity and Community Celebrate the creative ways students engage with the tools when constructing a rectangle and the questions they develop in the process.

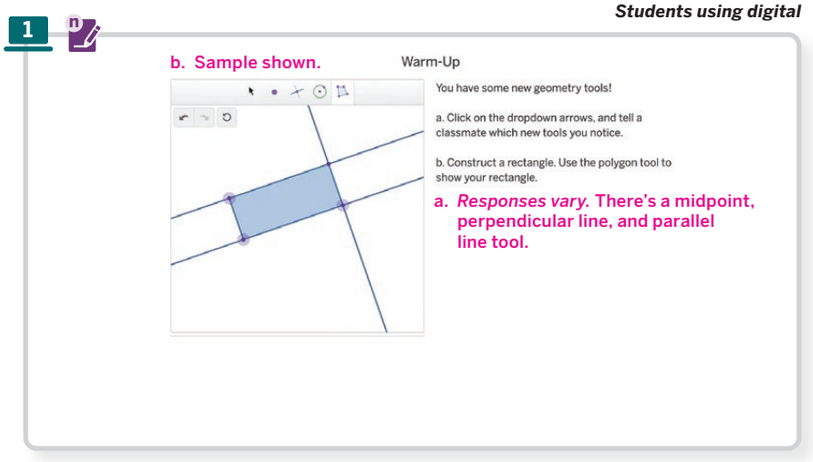
Capture and share student constructions. Consider asking:

- "Which new tools did you try? Can you describe what they do?"
- "How do you know you constructed a rectangle? Is it still a rectangle when you move the draggable points?"
- "How can we use our understanding of perpendicular bisectors to justify if these are right angles?"

Invite students using notes to record information about the new tools.

Note: If it doesn't come up naturally, consider sharing information about the midpoint, parallel line, and perpendicular line tools in the dropdowns to support additional constructions throughout the lesson.

Students using digital



Pairs Sharing a Device | 10 min | Pacing: Screen 2, Screen 3

Activity 1 Square or Not?

Purpose: Students analyze and critique constructions of rectangles and squares to precisely determine the properties of these figures. **(MP3, MP6)**



Student Notes are also available

1 Launch

2 **Display** Naoki's and Jada's constructions.

MLR **MLR7: Compare and Connect** Invite students to analyze each construction, then find ways in which the constructions are different (compare) and alike (connect). Invite students to explore by moving the draggable points. Consider asking, "What did each student do to construct a rectangle?"

Invite students to continue to the next screen after they've discussed with a partner.

2 Monitor

3 **Look for** and celebrate the different ways students think about Sam's construction.

Math Identity and Community Consider inviting students to share what they learned from constructions that did not work.

3 Connect

Display the student responses about Sam's construction and invite students to explain their thinking.

Note: It's okay if students are unsure of the steps to construct a square at this stage. They will use the construction tools to construct a square, receive Responsive Feedback, and share strategies in the next activity.

To surface the Key Takeaway, consider asking:

- "Why might someone think Sam's construction is a square?"
- "How can we test Sam's construction?"
- "What changes could we make so that Sam's construction always works?"
- "What tools might Sam have used when constructing his square?"

Note: If these ideas don't come up naturally, consider sharing the midpoint tool functions in two ways:

- If you click first on a point, then it expects a second point and will identify the midpoint between those two points.
- If you click first on a line segment, then it automatically identifies the midpoint of that line segment.

Students using digital

2

Compare and Connect

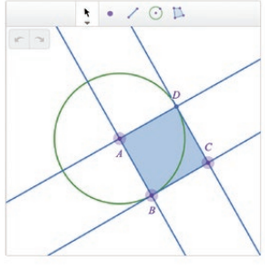
a. Drag the points to see how Naoki's and Jada's constructions work.
b. Discuss: Which constructions are rectangles? How do you know?



b. *Responses vary.* Naoki's construction is a rectangle. When I drag the points, the opposite sides stay parallel and the adjacent sides stay perpendicular to each other. Jada's construction is not a rectangle because when I drag the points, the angles are no longer all right angles.

3

Sam's Construction



a. Drag each movable point to see how Sam's construction works.

b. Did Sam construct a square?

Yes | No | I'm not sure

Explanations vary. When points A and B are dragged, the construction remains a square. When point C is dragged, the sides are no longer all the same length, and the angles are no longer all right angles. It looks like Sam created a line that looked parallel to segment AD instead of constructing one.

Key Takeaway: Parallel lines and perpendicular lines are helpful for constructing squares and rectangles. To construct precisely, it's important that the necessary properties always hold true.

Pairs Sharing a Device | 20 min | Pacing: Screen 4, Screens 5–7

Activity 2 Circles and Squares

Purpose: Students construct squares, including a square **inscribed** in a circle, and a circle inscribed in a square.

Short on time? Consider omitting Screen 5 or holding a whole-class discussion about how to hide objects.



Student Notes are also available

1 Launch

4 **To support making connections**, consider asking, "What is helpful to remember as you construct a square?"

Note: To use the perpendicular line tool, students will first need to construct a line segment and then select it.

Capture and celebrate the different ways that students construct a square and explore the construction tools.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What do we remember about squares?"
Use a strategy similar to Sam's: constructing two lines perpendicular to $\overline{AB}$, and then using the polygon tool for their square.	**Strengthen** Recognize their strategic use of tools. Consider asking, "Are there any tools you haven't used that might help ensure that your square construction always works?"
Identify one square with $\overline{AB}$ as a side length.	**Strengthen** Invite students to compare their construction with a classmate's. Consider asking, "Can you construct any other squares with points A and B as vertices?"
Identify multiple squares with $\overline{AB}$ as a side length.	**Stretch** Celebrate their creative thinking. Consider asking, "How might you construct a square where points A and B are not on a side length?"

Pause briefly to discuss the different squares that can be constructed with points A and B as *vertices*.

Note: If it doesn't come up naturally, consider sharing that a square can be made with points A and B on a diagonal. Consider asking, "What steps could we take to construct this square?"

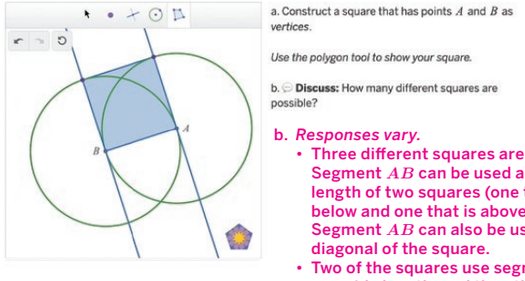
2 Monitor

5 **Display** Dylan's construction and consider asking, "Is this similar to any of the constructions we made? What is Dylan constructing with these circles?"

Students using digital

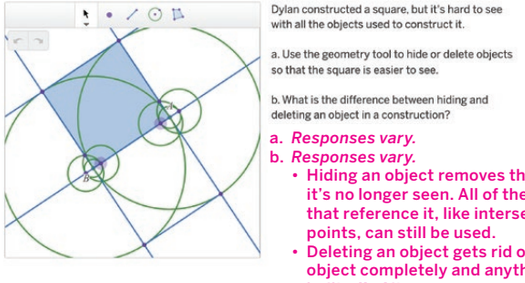
4

a. Sample shown.



b. *Responses vary.*
- Three different squares are possible. Segment AB can be used as the side length of two squares (one that is below and one that is above the line). Segment AB can also be used as the diagonal of the square.
- Two of the squares use segment AB as a side length, and the other square uses segment AB as a diagonal.

5



a. *Responses vary.*
b. *Responses vary.*
- Hiding an object removes the object so it's no longer seen. All of the objects that reference it, like intersection points, can still be used.
- Deleting an object gets rid of the object completely and anything else built off of it.

Activity 2 continued >

Pairs Sharing a Device | 20 min | Pacing: Screen 4, Screens 5–7

Activity 2 Circles and Squares (continued)

Purpose: Students construct squares, including a square inscribed in a circle, and a circle inscribed in a square.

Short on time? Consider omitting Screen 5 or holding a whole-class discussion about how to hide objects.



Student Notes are also available

2 Monitor

6 **Use the Think-Pair-Share routine** to help students shape their ideas about **inscribed** figures. See the Routine Facilitation Guide for more information.

ML/EL **Multilingual/English Learners** Invite students to share strategies in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

Listen to partner discussions for student ideas about inscribed figures and highlight them in the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to start by focusing on one of the two circles. Consider asking, "What do you know about your chosen circle?"
Construct the circle inscribed in the square with a draggable point on the side.	**Support** Invite them to drag the point. Consider asking: • "What happens when you move this point? Why?" • "What other tools might help identify a point on that line segment for your circle to touch?"
Guess the center of the square and construct a circle using that point to inscribe the square.	**Support** Invite them to drag the point. Consider asking, "What happens when you move this point? Why?"
Construct the two circles using a variety of construction tools.	**Strengthen** Consider asking, "How might you use the hide feature so your image would look like Santiago's?"

7 D **Differentiation: Explore More (Stretch)** Invite students who would like to further explore inscribed figures to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

6 **Display** student constructions and invite students to share the steps they used to construct the two circles.

To surface the Key Takeaway, consider inviting students to share how they would define *inscribed*. Consider asking, "What tools were helpful to construct inscribed figures?"

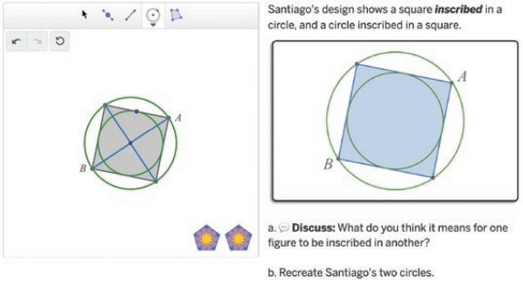
Invite students using notes to document some important considerations from the class discussion.

Students using digital

6

b. Sample shown.

Inscribed

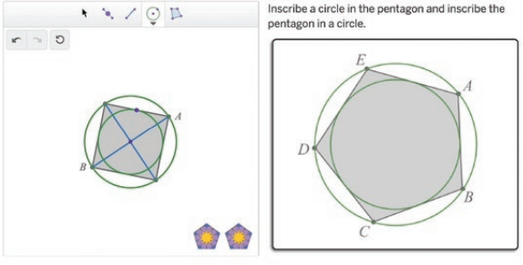


a. *Responses vary.* A figure is inscribed when it fits exactly inside of another figure. For the square, all of the vertices are on the circle. A circle is inscribed in a polygon when the circle touches, but does not cross, each of the polygon's sides.

7

Response shown.

Explore



Key Takeaway: Midpoints can be helpful in constructing very specific types of squares that are **inscribed** in circles or by circles.

Whole Class | 5 min | Pacing: Screen 8

Synthesis



Student Notes are also available

8 **Invite students** to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- When sketching a square, the lengths may not be exactly the same and the angles might not be exactly right angles. It just has to be close.
- When constructing a square, the process can be much more precise.
  - » Circles can be used to make sure the lengths of a square are all the same.
  - » Perpendicular and parallel lines can be used to construct precise right angles.
  - » Midpoints can be used to construct perpendicular bisectors.

Math Identity and Community If time allows, consider capturing and sharing student constructions that explore the pentagon challenge.

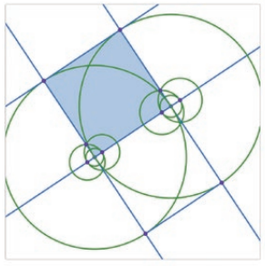
Note: If time allows, invite students to revise their previous thinking based on the discussion.

Invite students using notes to document ideas from the class discussion in the left column.

Students using digital

8

Synthesis



In your own words, how is constructing a square different from sketching one?

Responses vary. When constructing a square, I can use different construction techniques to be precise, like constructing perpendicular and parallel lines, and I can justify that I have created a square. When sketching a square, I can get the square to look like the side lengths are the same and each angle is a right angle, but I wouldn't know for sure how precise it is.

Lesson Takeaway: Digital construction tools can help you construct geometric figures with precision. You can construct more complex figures by building on your knowledge of simpler constructions and using the construction tools intentionally.

Independent | 5 min | Pacing: Screens 9–10

Show What You Know

Purpose: Students demonstrate their understanding by constructing a square with a given line segment as one of the sides.

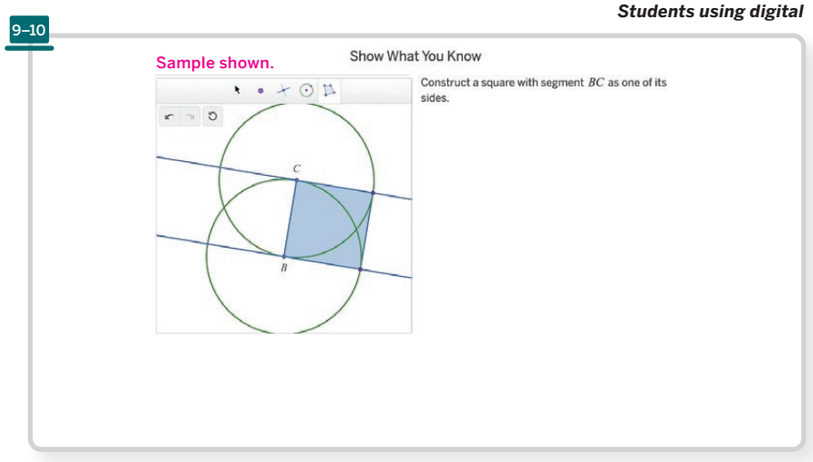


Students using print

9–10 Today's Goals

1. **Goal:** Construct a square *inscribed* in a circle, and a circle inscribed in a square.
2. **Language Goal:** Describe how construction techniques, like determining a midpoint or a perpendicular line, are used to construct a square. **(Writing and Speaking)**

Students using digital



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

S Support

Provide targeted intervention.

If student work shows **partial understanding** of constructing squares, consider:

- Revisiting **Geometry, Unit 1, Lesson 5** *(Perpendicular and Parallel Lines)*.
- Inviting students to engage with the **Summary** by reading it and writing it in their own words.

S Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding** with minor errors, consider:

- Assigning the **Lesson Practice**.
- Invite students to read the **Summary** and do the **Try This** problem.
- Revisiting this skill in **Practice Day 1**.

S Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting them to complete the **Explore More** task and discuss their thinking with a partner.
- Invite students to construct an argument about the best method for constructing a square.

Professional Learning

Reflect on the **Compare and Connect** routine used in this lesson. Students used this routine to consider the differences in construction techniques on Screen 2. In what ways did this routine support students in describing Naoki's and Jada's constructions? How did your facilitation of the routine support students in noticing the similarities and differences in the students' work? How might your facilitation change in a future lesson that uses this routine?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

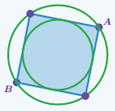
Lesson 6
Practice

Students using digital

Lesson Summary

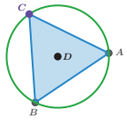
Parallel lines, perpendicular lines, and midpoints are helpful when constructing a square. Midpoints are especially helpful when constructing a square **inscribed** in a circle.

For a square (or any polygon) to be inscribed in a circle, every *vertex* of the square must be on the circle. For a circle to be inscribed in a square (or any polygon), the circle must fit inside the square and a point on the circle must be on each side of the square. When you construct either a square inscribed in a circle or a circle inscribed in a square, it's helpful to determine midpoints and perpendicular lines to identify the center point of the circle.



Try This

Triangle ABC is inscribed in a circle whose center is point D.



Show or explain how to construct a circle inscribed in $\triangle ABC$.

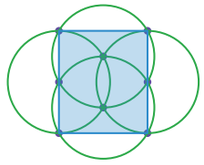
Responses vary. **Construct the midpoint of segment AC and connect it to point D with a line segment. Set that line segment as the radius of a circle that is also centered at point D.**

Lesson Practice
Geo.1.06

Name: _____ Date: _____ Period: _____

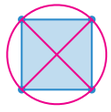
1. Is this construction a square? Explain your thinking.

No. Explanations vary. **The vertical sides are diameters of the circles, but the horizontal sides are not diameters and are a shorter length.**

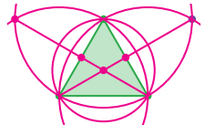


Problems 2–3: Here are regular polygons. Construct a circle so each polygon is inscribed in the circle.

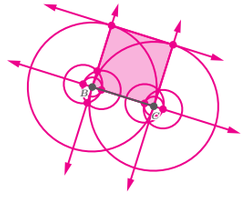
2.



3.



4. Construct a square using segment BC as a side. ***Responses vary.***



5. Construct a circle with the square from the previous problem inscribed in the circle.

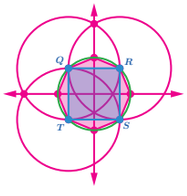
Responses vary.

Lesson Practice
Geo.1.06

Name: _____ Date: _____ Period: _____

6. Square $QRST$ is inscribed in a circle.

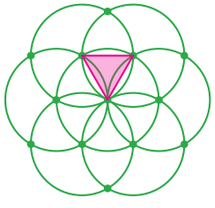
Construct a regular octagon that is inscribed in the circle.



Spiral Review

7. Construct an equilateral triangle using only the intersections of the circles. Explain how you know the triangle is equilateral.

Responses vary. **Each of the sides of my triangle is a radius of one of the circles. Since the circles are the same size, the side lengths are equal.**



8. Explain how to construct a line segment that is half the length of segment AB.



Responses vary. **Construct two circles, one centered at point A and one centered at point B, both with radius $\overline{AB}$. Construct the perpendicular bisector of segment AB to determine the midpoint. The line segment from point A or B to the midpoint would be half the length of segment AB.**

Reflection

1. Put a star next to a problem you could explain to a classmate.

2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1	1	HSG.CO.D.13*
	2–3, 5	2	HSG.CO.D.13
Test Practice	4	2	HSG.CO.D.13
	6	3	HSG.CO.D.13
Spiral Review			
Fluency	7	1	HSG.CO.D.13
	8	1	HSG.CO.D.12

*This problem builds toward the standard shown.

Need More Practice?

Additional Practice resources coming soon.

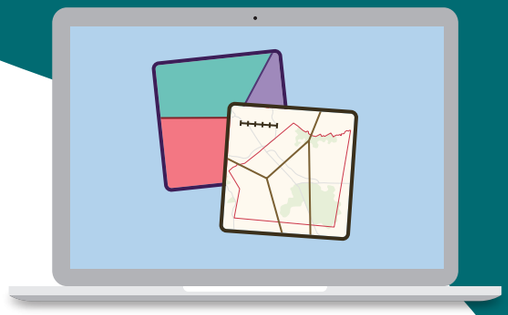
Unit 1

Lesson 7

Modeling This lesson attends to the full intent of the modeling process.

Digital Lesson

This lesson uses digital student screens. Student Notes are also available.



Community Constructions

Using Perpendicular Bisectors

Let's use constructions to help us make decisions.

Focus and Coherence

Today's Goals

1. **Goal:** Construct perpendicular bisectors to create regions closest to a certain location.
2. **Language Goal:** Explain how perpendicular bisectors and *Voronoi diagrams* can help us make sense of where to put community resources. **(Writing and Listening)**

Students use what they have learned about constructing perpendicular bisectors to analyze the locations of community resources, such as hospitals and libraries. They use digital construction tools to create Voronoi diagrams (made using perpendicular bisectors) to divide maps into regions that are closest to each resource. Students then consider factors for deciding the location of a new library and provide justifications for the location they choose. **(MP3)**

Prior Learning

In Lesson 4, students constructed perpendicular bisectors, and in Lesson 5, students developed strategies for constructing perpendicular and parallel lines.

Future Learning

In Sub-Unit 2, students will use perpendicular lines to perform rigid transformations (translations, reflections, and rotations).

Rigor and Balance

- Students **apply** their understanding of constructing perpendicular bisectors to justify the locations of community resources.
- Students have opportunities to develop a **productive disposition** as they see how constructions can help determine where community resources should be located.

Vocabulary

New Vocabulary

Voronoi diagram

Standards

Addressing

HSG.MG.A.3 ★

Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Also Addressing: **HSG.CO.D.12, HSG.MG.A.2 ★**

Mathematical Practices: MP3, MP4

Building On	Building Toward
HSG.CO.D.12	HSG.MG.A.3 ★

Lesson at a Glance

~ 45 min

Standards: HSG.MG.A.3 ★, HSG.CO.D.12, HSG.MG.A.2 ★



Why digital?

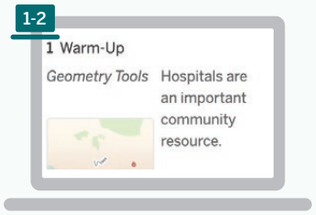
Students use digital construction tools to create perpendicular bisectors and Voronoi diagrams.

Warm-Up

Independent | 5 min

Pacing: Screens 1–2

Students divide a map into two regions and discuss which hospital is closest.

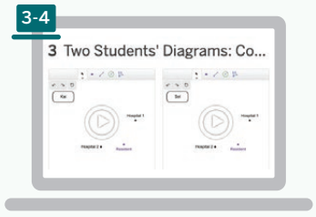


Activity 1

Pairs Sharing a Device | 10 min

Pacing: Screen 3, Screen 4

Students explore constructing diagrams to make arguments for which hospital a person should go to. MLR7: Compare and Connect (MP3)



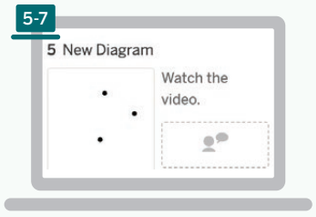
Activity 2

Pairs Sharing a Device | 10 min

Pacing: Screen 5, Screens 6–7

Students further their understanding by creating another **Voronoi diagram** and strengthening their arguments.

Key Takeaway (Screen 7): A **Voronoi diagram** uses perpendicular bisectors to construct regions that are closest to given points. These diagrams can be useful for thinking about where to place community resources.

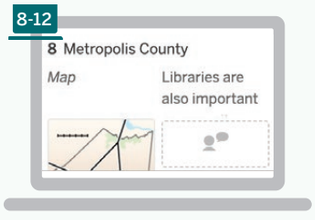


Activity 3

Pairs Sharing a Device | 10 min

Pacing: Screen 8, Screens 9–12

Students use information about a county and Voronoi diagrams to propose where to place a new library. **(MP4)**



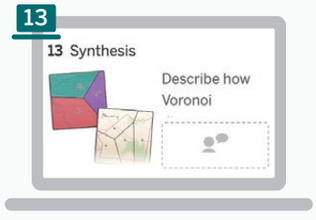
Synthesis

Whole Class | 5 min

Pacing: Screen 13

Students synthesize strategies for using constructions of perpendicular bisectors and Voronoi diagrams to help make real-world decisions

Lesson Takeaway: **Voronoi diagrams** and perpendicular bisectors split a given area into regions. This can be useful for considering the placement of community resources.



Show What You Know

Independent | 5 min

Pacing: Screen 14

Students demonstrate their understanding of Voronoi diagrams and perpendicular bisectors by coming up with questions about their communities.



Prep Checklist

Assign the digital student screens. Students Notes are also available.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)
- computers/devices (for research)

Student Notes sheet in Student Edition:
Consider providing the Student Notes to support class discussion and student thinking.



Independent | 5 min | Pacing: Screens 1–2

Warm-Up

Purpose: Students divide a map into two regions and discuss which hospital is closest.

Short on time? Consider omitting the discussion in the Launch.



Student Notes are also available

1 Launch

1 **To support making connections**, consider polling the class and asking:

- "Have you or anyone you know ever had to take a ride in an ambulance?"
- **Modeling** "What *variables might you identify* when deciding which hospital to go to?"

Display the map and read the prompt aloud.

ML/EL **Multilingual/English Learners** Invite students to read the problem aloud and paraphrase what it means. Encourage them to share with a partner what they think they are being asked to do. **(Reading)**

Invite students to utilize any of the tools, including the construction tools or sketch tools.

Note: Digging into topics like community resources might invite students to think about the resources, or lack thereof, in their own communities. Consider framing this lesson with a stance of curiosity (e.g., "I wonder what is happening here. I wonder what else I could study with this mathematics.").

2 Connect

2 **Invite students** to share by polling the class. It's okay — even desirable — to lack consensus at this stage. The lesson will build toward consensus when students see and make sense of strategies for dividing regions.

Display several student diagrams, including those that use perpendicular bisectors, those that divide the map in half using a line, those that use circular regions, or those with other strategies.

Modeling Invite students to explain how they made choices when dividing the map. Consider asking:

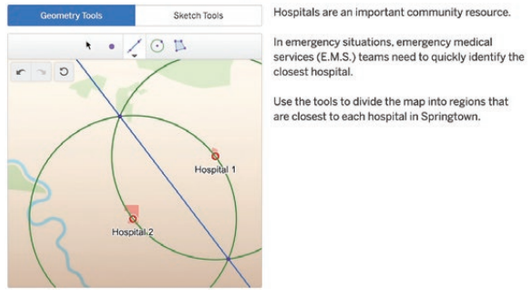
- "What different tools were used to divide the map into regions?"
- "What strategies made it clear which hospital the emergency medical service team should take the resident of Springtown to?"

Math Identity and Community Celebrate different degrees of precision in students' sketches and explanations.

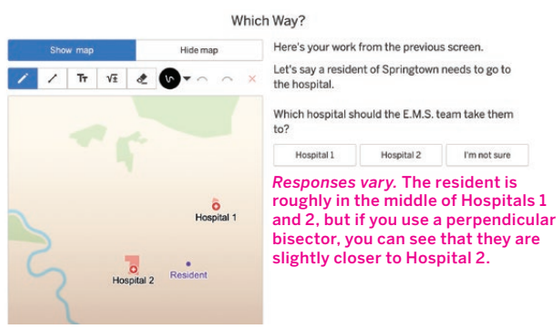
Students using digital

1

Sample shown.



2



Responses vary. The resident is roughly in the middle of Hospitals 1 and 2, but if you use a perpendicular bisector, you can see that they are slightly closer to Hospital 2.

Pairs Sharing a Device | 10 min | Pacing: Screen 3, Screen 4

Activity 1 Mapping Matters

Purpose: Students explore constructing diagrams to make arguments for which hospital a person should go to. **(MP3)**



Student Notes are also available

1 Launch

3 **Display** the two videos and consider sharing, "We will see two student strategies for splitting the map."

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their strategy from the Warm-Up. Consider asking:

- What did each student do to divide the map into regions?
- How does each strategy compare to your own?
- How did each student attend to precision in dividing the map into regions?
- What might you say to each student to help them be more precise? **(MP3, MP6)**

2 Monitor

4 **Select and sequence** a variety of student constructions and strategies for dividing the map.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How might we use Sol's work to divide the map for Hospital 1 and Hospital 3? Hospital 2 and Hospital 3?"
Estimate the lines between each hospital to divide the map.	**Support** Consider asking, "What construction tools can we use to divide the map more precisely?"
Construct the perpendicular bisectors, but are unsure how to use the lines to define regions.	**Strengthen** Celebrate their progress. Consider asking, "How do midpoints help us focus on which lines to consider?"

Modeling This screen utilizes the digital geometry tools to support students as they *formulate a model* for this situation.

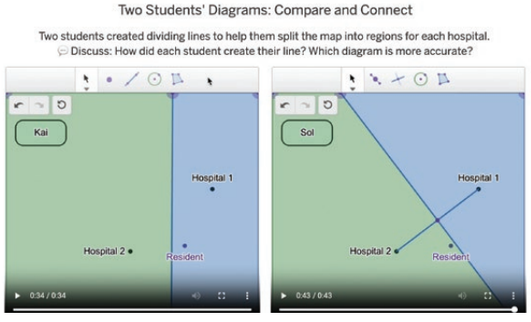
3 Connect

Display student constructions using the Snapshot tool. Encourage students to ask questions about each other's constructions.

Invite students using notes to show or explain their process for dividing the map into regions.

Students using digital

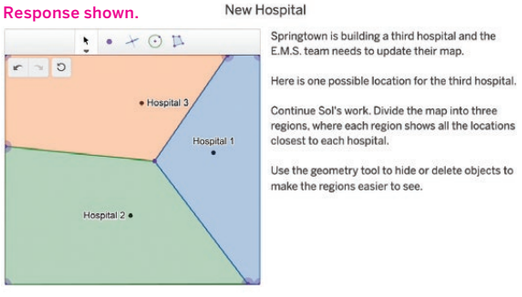
3



Responses vary.
- I think Kai drew a vertical line between the two hospitals. I think that Sol constructed a perpendicular bisector so that every point on the line is an equal distance from both hospitals.
- I think Sol's diagram is more accurate because it uses a perpendicular bisector to divide the map into two sections — all the locations that are closer to Hospital 1 and all the locations that are closer to Hospital 2.

4

Response shown.



Pairs Sharing a Device | 10 min | Pacing: Screen 5, Screens 6–7

Activity 2 City Construction

Purpose: Students further their understanding by creating another **Voronoi diagram** and strengthening their arguments.

Short on time? Consider omitting or limiting the amount of time students spend on Screen 6, as this is designed as fluency practice for student constructions.



Student Notes are also available

1 Launch

5 **Play** the video.

Create a class definition for the term **Voronoi diagram** using students' language and the glossary.

Invite students using notes to document the class definition of the term *Voronoi diagram* in the left column.

2 Monitor

6 **Look for** students' progress on completing another Voronoi diagram. (Consider referring to the Differentiation table on Screen 4 as well.)

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How might we use Sol's work to divide the map into regions for a different hospital?"
Divide the map using perpendicular bisectors and organize their diagram by hiding and coloring components.	**Stretch** Consider asking, "How might your diagram change if we added more hospitals?"

7 **Encourage students** to explore more complex Voronoi diagrams.

Note: If many students need support engaging with this screen, consider briefly pausing the class to demonstrate the interaction.

3 Connect

7 **Display** and celebrate a variety of student diagrams.

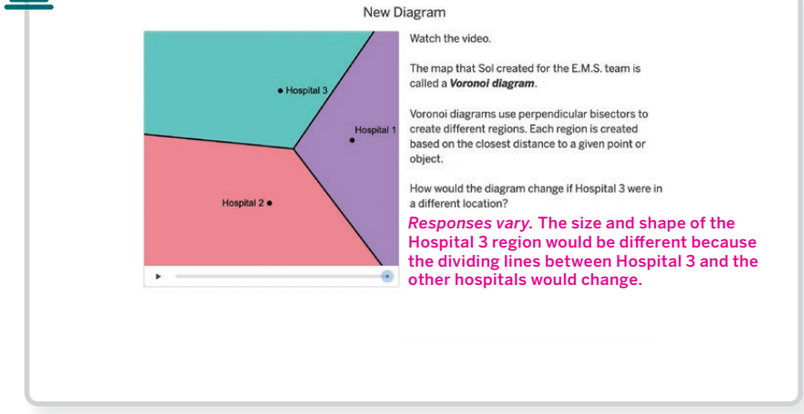
Math Identity and Community Consider celebrating students who persisted through struggle. Invite them to share what helped them persist.

To surface the Key Takeaway, consider asking:

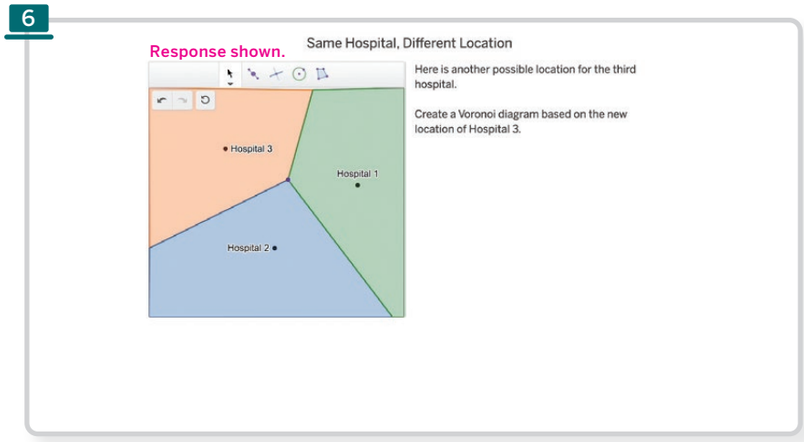
- "How can Voronoi diagrams help with community decision-making?"
- "Why might Springtown consider different locations for the new hospital?"
- "Who might be interested in this information?"
- "How did the construction of the regions change as more hospitals were added?" **(MP4)**

Students using digital

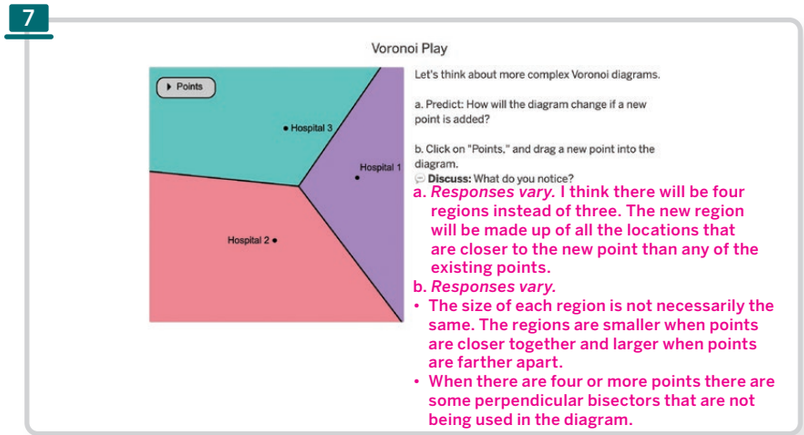
5



6



7



Key Takeaway: A **Voronoi diagram** uses perpendicular bisectors to construct regions that are closest to given points. These diagrams can be useful for thinking about where to place community resources.

Pairs Sharing a Device | 10 min | Pacing: Screen 8, Screens 9–12

Activity 3 Library Location

Purpose: Students use information about a county and Voronoi diagrams to propose where to place a new library. **(MP4)**



Student Notes are also available

1 Launch

8 **To support making connections**, consider asking, "Do you ever wish there was another library in your community?"

Note: To encourage students to think broadly about libraries, consider sharing, "In addition to access to books, libraries often provide things such as educational programs, computers and wifi, cultural events, programs for seniors and children, language classes, and job support."

Use the Notice and Wonder routine to promote curiosity and help students make sense of the multiple map views and current locations of libraries. See the Routine Facilitation Guide for more information. **(MP1)**

Math Identity and Community Consider celebrating variety and creativity in what students notice and wonder, including things that surprise you or other students may not have noticed.

Note: If it doesn't come up naturally, consider asking:

- "How would you describe the current library locations in terms of green spaces? Median income? Population density?" **(MP3, MP4)**
- "How is this like our community? How is it different?"
- "Where would libraries be in your ideal community?"

9 **Invite students** to continue to the rest of the activity using the Pace tool. Consider sharing, "There are no correct answers. All thinking is valuable and will be worthwhile to bring to our class discussion."

2 Monitor

Listen for partner discussions as they order the criteria to highlight in the Connect.

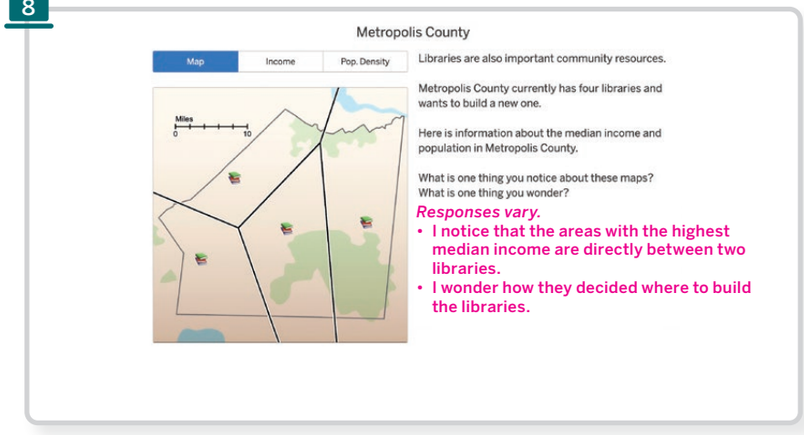
ML/EL **Multilingual/English Learners** As students share in partner discussions, provide sentence frames such as:

- I agree/disagree with you because . . .
- I think that makes sense because . . .

Activity 3 continued >

Students using digital

8



9



Activity 3 Library Location (continued)

Purpose: Students use information about a county and Voronoi diagrams to propose where to place a new library. **(MP4)**



Student Notes are also available

2 Monitor

10 **Modeling** Listen for different ways that students *validate their conclusions* about where to place the new library. Invite them to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Consider population density: placing their library in highly dense or rural areas.	Strengthen Invite students to compare their rationale with a classmate's and consider asking: • "How are you reasoning similarly? Differently?" • "How might someone use the other factors to make a different argument?"
Consider income: placing their library in highly affluent or low-income areas.	
Consider green spaces: placing their library near or far away from green areas.	

Note: If pairs struggle to agree on the criteria they most want to use to propose their location, encourage them to share both explanations to capture their discussion.

11-12 **D Differentiation: Explore More (Stretch)** Invite students who would like to explore using Voronoi diagrams for a topic they're interested in to complete this optional task. Encourage them to discuss their thinking with a partner or consider making space for them to share with the whole class another time.

3 Connect

Display the overlay of student points and invite the class to choose ones to discuss.

10 **Modeling** Invite students to report their conclusions about where to place the library. Consider asking:

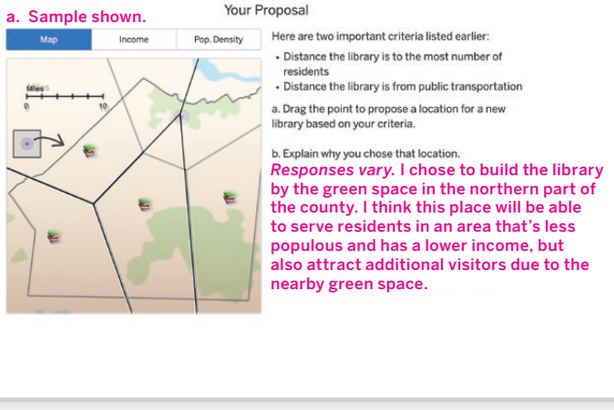
- "How can Voronoi diagrams and perpendicular bisectors help you make decisions about the location you proposed for a new library?" **(MP3, MP4)**
- "What are important things to keep in mind when choosing locations for community resources?"

Math Identity and Community Consider discussing why it's okay that students might come to different conclusions here, emphasizing that in math class, there is not always a correct answer.

Invite students using notes to document some important considerations from the class discussion.

Students using digital

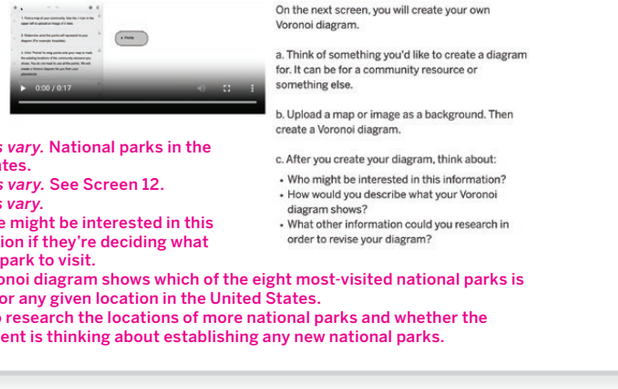
10



a. Sample shown.

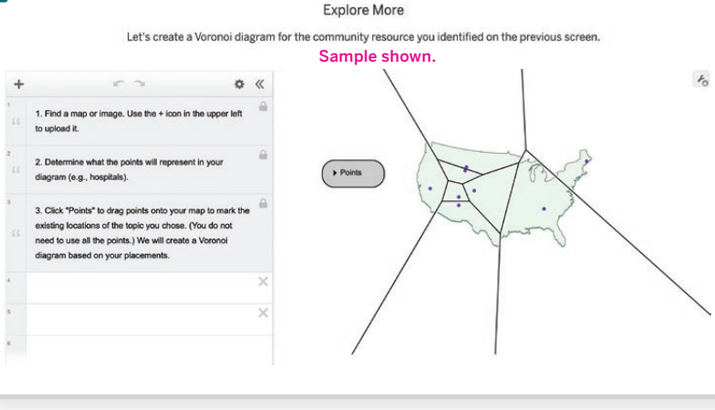
Responses vary. I chose to build the library by the green space in the northern part of the county. I think this place will be able to serve residents in an area that's less populous and has a lower income, but also attract additional visitors due to the nearby green space.

11



a. *Responses vary.* National parks in the United States.
b. *Responses vary.* See Screen 12.
c. *Responses vary.*
- Someone might be interested in this information if they're deciding what national park to visit.
- This Voronoi diagram shows which of the eight most-visited national parks is closest for any given location in the United States.
- I'd like to research the locations of more national parks and whether the government is thinking about establishing any new national parks.

12



Sample shown.

Whole Class | 5 min | Pacing: Screen 13

Synthesis



Student Notes are also available

13 **Invite students** to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

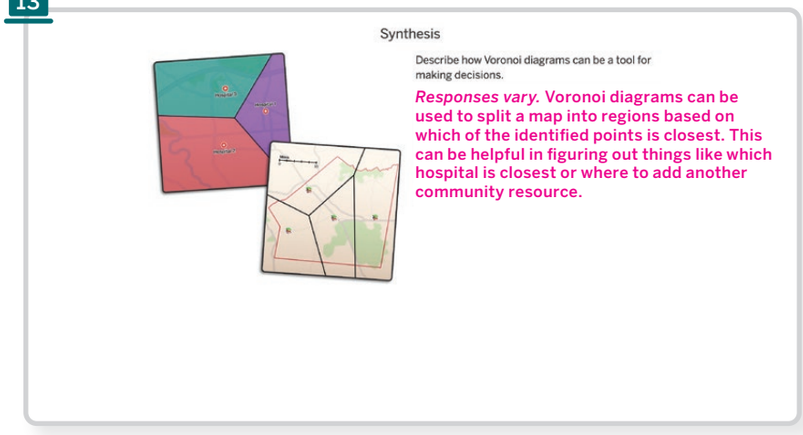
- Voronoi diagrams divide a map into regions to depict closeness to each point.
- Voronoi diagrams help show where community resources might be most helpful because they show all of the locations closest to each resource.

Note: If time allows, consider asking:

- "What would you like to share with your communities about what you learned today?"
- "What other topics are you interested in knowing more about, that can be analyzed by constructing Voronoi diagrams?"

Students using digital

13



Lesson Takeaway: Voronoi diagrams and perpendicular bisectors split a given area into regions. This can be useful for considering the placement of community resources.

Independent | 5 min | Pacing: Screens 14–15

Show What You Know

Purpose: Students demonstrate their understanding of Voronoi diagrams and perpendicular bisectors by coming up with questions about their communities.



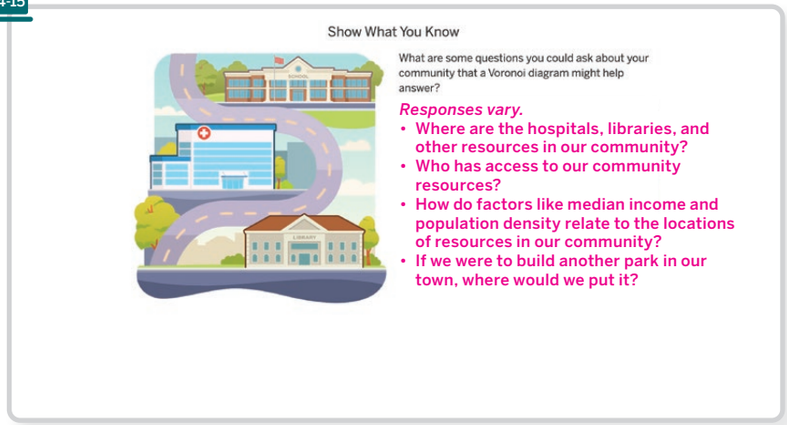
Students using print

14-15 Today's Goals

1. **Goal:** Construct perpendicular bisectors to create regions closest to a certain location.
2. **Language Goal:** Explain how perpendicular bisectors and *Voronoi diagrams* can help us make sense of where to put community resources. **(Writing and Listening)**

Students using digital

14-15



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of Voronoi diagrams, consider:

- Inviting students to engage with the **Summary** by reading it and paraphrasing to a caregiver or classmate.
- Inviting students to complete the **Try This** problem and provide individualized feedback.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice**.
- Inviting students to review the **Summary** and complete the **Try This** problem.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting them to complete the **Explore More** task and discuss their thinking with a partner.
- Inviting students to research occupations that support decision-making about city development. Encourage them to contact local organizations and government officials to learn about how decisions are made and to share what they learn.

Professional Learning

Today students explored Voronoi diagrams as a mathematical lens for proposing locations for community resources. What are some challenges you might encounter during a discussion about hospitals, libraries, and other community resources? How can you prepare to keep the discussion focused on using Voronoi diagrams to make sense of mapping?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 7 Practice

Students using digital

Lesson Summary

You can use constructions to solve real-world problems. **Voronoi diagrams** are constructed with perpendicular bisectors and divide an area into regions where all the locations inside a region are closest to an identified point.

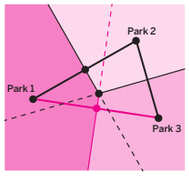
Voronoi diagrams are tools that help us make decisions in community planning. They provide evidence to support where specific services might be located and how these locations might impact a community.



Try This

Here is the start of a Voronoi diagram based on the locations of three parks.

a. Complete this Voronoi diagram.



b. What factors could be helpful to consider when choosing a location for an additional park?

Responses vary. It might be helpful to consider the locations of the other three parks, public transportation options to get to the new locations, the population density, and which populations the park could serve.

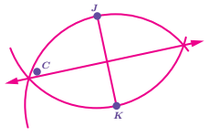
Lesson Practice
Geo.1.07

Name: _____ Date: _____ Period: _____

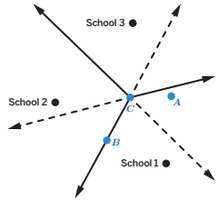
1. How can you use geometry tools to determine if point C is closer to point J or point K?

   Use the diagram if it helps to show your thinking.

   Responses vary. Construct the perpendicular bisector of segment JK. Point C is in the region that is closer to point J.



2. This is a Voronoi diagram based on three different schools. Select *all* the statements that are true.



- ☑ A. School 1 is the closest school to Location A.
- ☐ B. School 3 is the farthest school from Location A.
- ☐ C. Location B is equidistant to Schools 2 and 3.
- ☐ D. Location B is closer to School 1 than to School 2.
- ☑ E. Location C is equidistant to Schools 2 and 3.

Problems 3–4: This diagram represents the homes of three students who need to meet after school for a project.

3. Plot a point where you think the students located at point A and point B should meet. Explain your thinking.

   Responses vary. They could meet at the location that is the same distance from each of their homes. I found that by constructing the midpoint of segment AB.



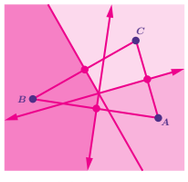
4. Plot a point at a location that is an equal distance from all three students' homes. Explain your thinking.

   Responses vary. I constructed perpendicular bisectors for segments AB, BC, and CA. The location that is an equal distance from all three students' homes is the intersection point of the perpendicular bisectors.

Lesson Practice
Geo.1.07

Name: _____ Date: _____ Period: _____

5. Create a Voronoi diagram based on these three points.



6. What are ways Voronoi diagrams could be used in your community to help make decisions?

   Responses vary. We could use a Voronoi diagram to figure out the best location for a park based on the location of other parks in our neighborhood. We could create a diagram that shows the areas that are closest to currently existing parks and then try to place a park in a region that would shorten the distance to any given park.

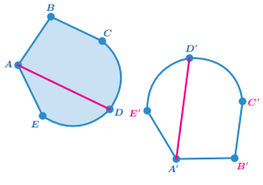
Spiral Review

Problems 7–8: The unshaded figure is a transformation of figure $ABCDE$.

7. Label the points corresponding to B, C, and E as B', C', and E'.

8. Construct segment AD and segment $A'D'$. What is the relationship between the lengths of these two line segments?

   Responses vary. They are the same length.



Reflection

1. Circle the problem you enjoyed doing the most.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1	2	HSG.CO.D.12
	2–5	2	HSG.MG.A.3 ★
	6	3	HSG.MG.A.3 ★
Spiral Review			
Fluency	7	1	8.G.A.1
Test Practice	8	1	8.G.A.1.A

Need More Practice?

Additional Practice resources coming soon.

45 min

Practice Day 1

Purpose: Students practice the concepts, skills, and strategies developed in Lessons 1–7. Consider using this Practice Day as preparation for the upcoming Sub-Unit Quiz.

Preparation

Materials:

Option 1: Partner Practice

- Student Edition
- geometry toolkit: compasses, straightedges
- Task Cards, one per pair

Option 2: Solve and Swap

- Student Edition
- geometry toolkit: compasses, straightedges
- Task Cards, one per student

Math Identity and Community Invite students to celebrate a classmate's thinking that was strong, clear, or creative.

Consider asking these questions to check for understanding:

- Task A: "How would you describe to someone else how you used tools to justify whether or not triangle ACE was equilateral?"
- Task C: "How do you know that line m bisects the segment you chose? Does line m bisect other segments?"

Standards

Addressing

HSG.CO.D.12
Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; **bisecting a segment;** bisecting an angle; **constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.**

Also Addressing: **HSG.CO.A.1, HSG.CO.D.13**

Mathematical Practice: MP5

Facilitation Options

Option 1: Partner Practice

This structure supports student collaboration and allows pairs to work at their own pace.

Pairs

Distribute one set of cards to each pair.

Invite students to solve each problem in their Student Edition.

Encourage students to discuss their reasoning with their partner as they work on each problem.

Other considerations:

- How will students get feedback?
  - Post the answers and invite students to check them.
  - Invite students to check in with you when they're ready for feedback.
- How will you support student collaboration?
  - Discuss what supporting each other without giving answers could look or sound like.
  - Encourage pairs to read each problem out loud and solve it before moving on to the next problem.

Option 2: Solve and Swap

This structure supports student collaboration, provides opportunities for movement, and positions students as experts as they discuss problems and support one another.

Revolving Pairs

Distribute one card to each student.

Invite students to solve the problem in their Student Edition.

Encourage students to move around the classroom, find a partner, and solve the problem on their partner's card in their Student Edition. Invite them to compare answers, discuss their reasoning, and work together to revise any mistakes before finding a new partner.

Consider sharing that there are four problems. If students encounter a problem they've already solved, they can compare strategies and solutions.

Other considerations:

- How will you support student collaboration?
  - Discuss what supporting each other without giving answers could look or sound like.
  - Consider encouraging students to discuss a check-in question prior to discussing the math problems.

SE Practice Day 1

Unit 1
Lessons 1–7

Name: _____ Date: _____ Period: _____

Practice Day 1

Let's practice what you've learned so far in this unit!



You will use task cards for this Practice Day. Record all of your responses here.

Task A: Equilateral Triangle *Responses vary.*

1. Statement 1: Point F is the midpoint of $\overline{AE}$.

   Statement 2: $\overline{EF}$ is the same length as $\overline{ED}$.

   Statement 3: $\overline{EB}$ is the perpendicular bisector of $\overline{AC}$.

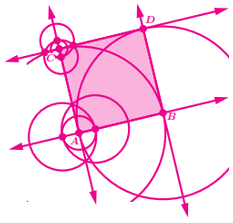
2. Circle one: (Yes) No Not enough information

   Explain your thinking. The endpoints of each side are on a circle with radius $\overline{FD}$.

3. Equilateral triangles: $\triangle EFD$, $\triangle AFB$, $\triangle BDC$, $\triangle BFD$

Task B: Square It *Responses vary.*

1.



2. Two parallel sides:
   Responses vary. $\overline{AB}$ and $\overline{CD}$

3. Two perpendicular sides:
   Responses vary. $\overline{AB}$ and $\overline{AC}$

Unit 1
Lessons 1–7

Name: _____ Date: _____ Period: _____

Practice Day 1 (continued)

Task C: Statements and Lines *Responses vary.*

1. Lines or segments:
   Line m is perpendicular to line KL.

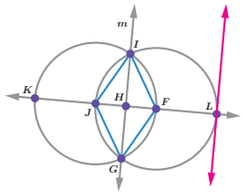
2. (a) Triangle JGF is equilateral.

   (b) Points I, F, and G are equidistant from point J.

   (c) Line m bisects segment JF.

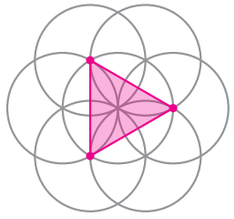
3. Statement:
   Points K, I, and F are equidistant to point J.

4.



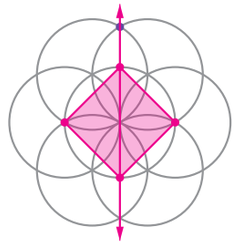
Task D: Inscribing *Responses vary.*

1. (a)



   (b) Description:
   I know this triangle is equilateral because I can open my compass to make a circle centered at each vertex and the circle intersects the two other vertices.

2. (a)



   (b) Description:
   I know this is a square because I can draw perpendicular diagonals with endpoints that are all on the same circle, so I know they are the same length.

Task Cards (online)

Task Cards

Unit 1
Practice Day 1
(p. 1 of 2)

Partner Practice Directions: Make and distribute one copy per pair of students.
Solve and Swap Directions: Make and distribute one copy per four students.

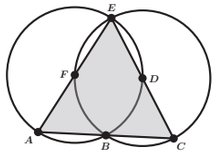
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Task A: Equilateral Triangle

Points F and D are the centers of the circles.

1. Write three true statements about the diagram.
2. Is triangle ACE an equilateral triangle?

   Explain your thinking.
3. Identify as many equilateral triangles as you can using the points in the diagram.



Task B: Square It

1. Construct a square inscribed in a circle. Label the vertices of the square $ABCD$.
2. Name two parallel sides in the square.
3. Name two perpendicular sides in the square.

Task Cards

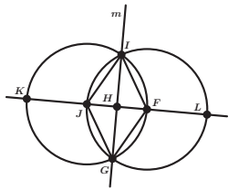
Unit 1
Practice Day 1
(p. 2 of 2)

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Task C: Statements and Lines

Points J and F are the centers of the circles.

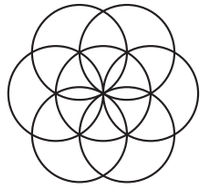
1. Name two perpendicular lines or segments.
2. Fill in the blanks to make each statement true.
   (a) Triangle _____ is equilateral.
   (b) Points _____, _____, and _____ are equidistant from point J.
   (c) Line m bisects segment _____.
3. Write your own true statement about this diagram.
4. Construct a line through point L that is parallel to line m.



Task D: Inscribing

Use this starting design to construct different shapes inscribed in a circle.

1. (a) Construct a triangle inscribed in a circle.
   (b) Describe how you know you constructed an equilateral triangle.
2. (a) Construct a square inscribed in a circle.
   (b) Describe how you know you constructed a square.



Assess and Respond

Additional data on student thinking can be found in Teacher Reporting.

Sub-Unit Quiz

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. Text to speech is available in the digital version. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or Skill	Addressed in	DOK	Standard(s)
1	Analyzing a construction	Lessons 1–2	1	HSG.CO.A.1, HSG.CO.D.12
2	Constructing a regular hexagon	Lessons 1–2	1	HSG.CO.D.13
3a	Analyzing a construction	Lesson 3	2	HSG.CO.D.12 MP5, MP6
3b	Constructing a perpendicular bisector	Lesson 4	1	HSG.CO.D.12 MP5
4	Constructing a square	Lesson 6	2	HSG.CO.D.12 MP5
5	Analyzing a construction	Lessons 1–2	1	HSG.CO.A.1

Assessment Resources

- Student Assessments
- Assessment Rubrics with Answers
- Show What Know PDFs and Answer Keys

Practice

If students need further review or practice, consider assigning the practice problems that correspond to each lesson.

Editable Version

Digital

Unit 1
Sub-Unit Quiz

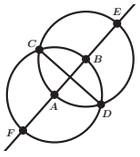
Name: Date: Period:

Sub-Unit Quiz

Unit Geo.1

Complete each question using the construction tools of your choice (compass, straightedge, tracing paper, paper folding, digital tools, etc.).

1. Dalia constructed circles centered at A and B. Which statement must be true?



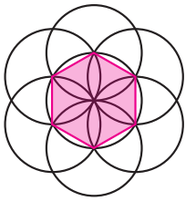
A. Points A, B, and C are equidistant to point D.

B. Point A is the midpoint of $\overline{CD}$.

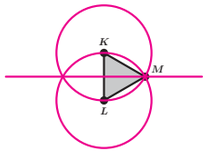
C. $\overline{AB}$ is the same length as $\overline{CD}$.

D. $\overline{CD}$ bisects $\overline{AB}$.

2. Here's how Cameron began constructing a regular hexagon inscribed in a circle. Finish Cameron's construction.



3. Here is $\triangle KLM$.



a. Is $\triangle KLM$ equilateral? Use construction tools to support your reasoning. Yes

Show or explain how you know.
Explanations vary. I know this triangle is equilateral because I can open my compass to make a circle centered at each vertex, and the circle intersects the two other vertices.

b. Construct the perpendicular bisector of $\overline{KL}$.

6

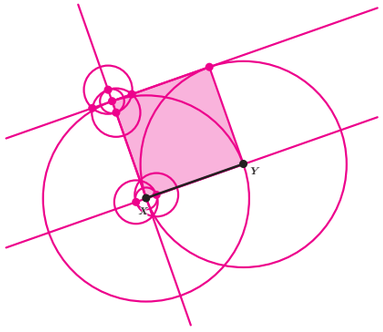
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Name: Date: Period:

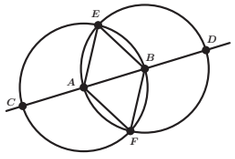
Sub-Unit Quiz (continued)

Uni

4. Construct a square with $\overline{XY}$ as one of its sides. *Responses vary.*



5. Here are two circles with centers A and B.



Explain how you know the length of $\overline{EA}$ is the same as the length of $\overline{BD}$.
Responses vary. $\overline{EA}$ and $\overline{BD}$ are both radiuses of circles that each have $\overline{AB}$ as a radius.

7

D Differentiation (Sub-Unit Quiz)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:
Use a variety of construction tools to construct precise geometric figures. **(Lessons 1–7)**	**2–4**	**Support** • Revisiting **Lesson 3, Activity 1** and inviting students to share strategies for constructing an equilateral triangle. • Revisiting **Lesson 4, Activity 2** and inviting students to share strategies for constructing a perpendicular bisector. • Revisiting **Lesson 5, Activity 1** and inviting students to share strategies for constructing perpendicular lines. • Revisiting **Lesson 6, Activity 2** and inviting students to share strategies for constructing a square.
Write and analyze descriptions of constructions. **(Lessons 1–6)**	**1, 5**	**Support** • Revisiting **Lesson 1, Activity 2** and inviting students to discuss how they can construct line segments that have equal length. • Revisiting the **Lesson 2 Synthesis** and inviting students to discuss how they can use circles to create geometric figures that have equal side lengths. • Reviewing the **Lesson Summaries** and inviting students to write down definitions and examples for the vocabulary words that appear throughout the quiz.

Rigid Transformations

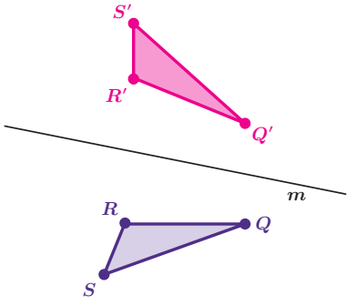
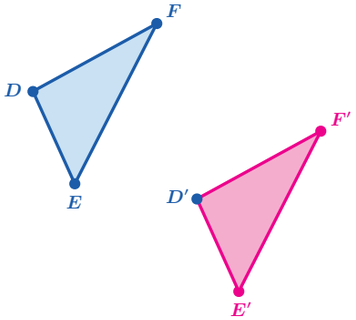
Sub-Unit 2 Goals

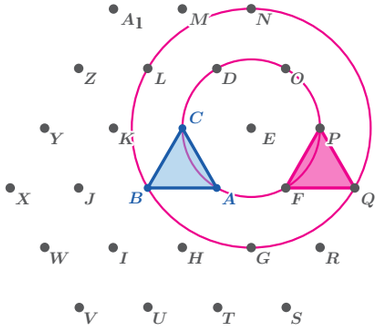
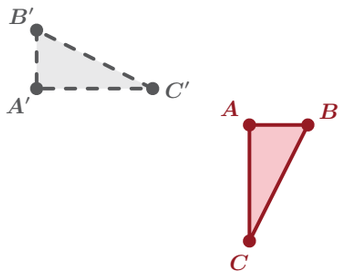
- **Goal:** Perform and describe a sequence of rigid transformations that moves a figure onto another.
- **Language Goal:** Describe the relationships between an image and a pre-image when a figure is transformed.

In this sub-unit, students build on their knowledge of rigid transformations from Grade 8 by constructing them using a variety of tools and strategies. They explore performing transformations off of a grid and precisely constructing them using a variety of construction tools, like tracing paper, compass and straightedge, and dynamic geometry software. They specify and perform sequences of transformations to move one figure onto another.



Math That Matters Most

Concept or Skill	Example
Perpendicular bisectors are useful to reflect geometric figures over a line of reflection.	Reflect $\triangle QRS$ over line m. Label the vertices Q', R', and S'. 
A translation is defined by a starting and ending point to move every point in a figure the same distance and in the same direction.	Translate $\triangle DEF$ so that point D goes to point D'. Label the vertices in the image using prime notation. 

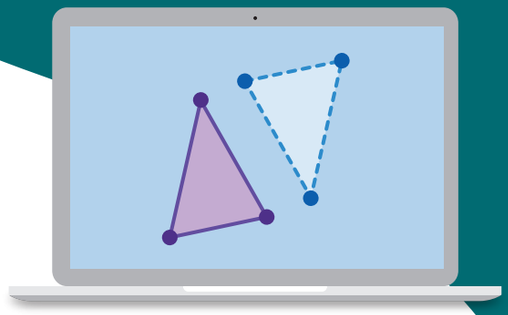
Concept or Skill	Example
A rotation moves all the points on a figure around a center of rotation where the original points and their images are equidistant from the center.	Construct the image of $\triangle ABC$, rotated around any point except A, B, or C. 
A sequence of rigid transformations transforms a figure while preserving the original shape and measurement of the figure.	Transform triangle ABC onto triangle $A'B'C'$. 

Unit 1

Lesson 8

Digital Lesson

This lesson uses digital student screens. Student Notes are also available.



Going Off the Grid, Part 1

Introducing Sequences of Transformations

Let's explore how we can describe sliding, flipping, and spinning shapes

Focus and Coherence

Today's Goals

1. **Goal:** Design a sequence of rigid transformations that moves a figure onto another.
2. **Language Goal:** Write and follow instructions to perform a sequence of transformations. **(Reading, Writing, Speaking, and Listening)**

Students use digital tracing paper to preserve the side lengths and angle measures of a figure as they perform *rigid transformations*. Students also write directions for their classmates to precisely replicate these rigid transformations and make their descriptions clearer using feedback. **(MP6)**

Prior Learning

In Grade 8, students learned about transformations of figures, specifically translations, reflections, and rotations.

Future Learning

In Lessons 9–11, students will construct transformations on paper using physical tools. In Unit 2, students will use sequences of rigid transformations to prove two figures are congruent.

Rigor and Balance

- Students use digital tracing paper to build **conceptual understanding** of rigid transformations.
- Students develop **adaptive reasoning** as they determine which set of rigid transformations will result in the accurate anticipated image.

Vocabulary

Review Vocabulary

reflection	*rigid transformation*
rotation	*translation*

Standards

Addressing

HSG.CO.A.2

Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. **Compare transformations that preserve distance and angle to those that do not** (e.g., translation versus horizontal stretch).

Also Addressing: **HSG.CO.A.5**

Mathematical Practices: MP6, MP7

Building On	Building Toward
8.G.A.1	**HSG.CO.A.4**
HSG.CO.A.1	**HSG.CO.B.6**

Lesson at a Glance

~ 45 min

Standards: HSG.CO.A.2, HSG.CO.A.5



Why digital?

Students use digital tracing paper to make sense of and perform rigid transformations.

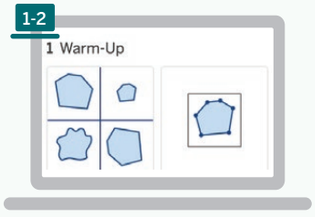
Warm-Up

Pairs Sharing a Device | 10 min

Pacing: Screens 1–2

Students create a unique figure that goes through a series of transformations and consider the features of *rigid transformations*, through examples and non-examples. **(MP7)**

Key Takeaway (Screen 2): A *rigid transformation* moves a figure while preserving the original shape and measurement of the figure, similar to tracing an image and moving the traced figure by flipping, rotating, or sliding.

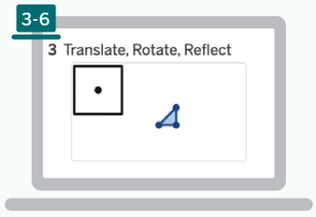


Activity 1

Pairs Sharing a Device | 10 min

Pacing: Screen 3, Screens 4–6

Students explore digital tracing paper as a construction tool for performing rigid transformations and consider language for precise descriptions of transformations. **(MP6)**



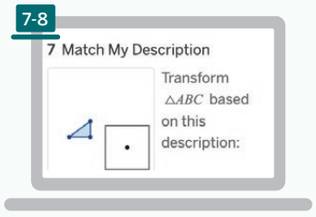
Activity 2

Pairs Sharing a Device | 15 min

Pacing: Screen 7, Screen 8

Students practice writing and interpreting descriptions of transformations to recreate figures using the digital tracing paper.
MLR1: Stronger and Clearer Each Time

Key Takeaway (Screen 8): When writing descriptions for transformations, it's important to be precise. A few ways to attend to precision are:
- Centering *rotations* on a vertex of the shape.
- Describing *reflections* as vertical, horizontal, or otherwise.
- Specifying the direction and length for *translations*.



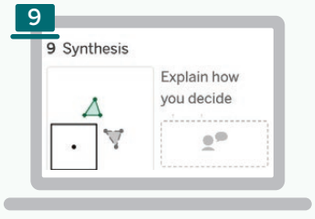
Synthesis

Whole Class | 5 min

Pacing: Screen 9

Students synthesize their understanding of *translations*, *rotations*, and *reflections*, and explain how to decide when each should be used. MLR2: Collect and Display

Lesson Takeaway: Digital tracing paper can help us precisely perform *rigid transformations* of geometric objects.



Show What You Know

Independent | 5 min

Pacing: Screens 10–11

Students demonstrate their understanding by using the digital tracing paper to perform and describe the steps for a transformation.



Prep Checklist

Assign the digital student screens. Student Notes are also available.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)
- tracing paper (as needed)

Student Notes sheet in Student Edition:
Consider providing the Student Notes to support class discussion and student thinking.



Pairs Sharing a Device | 10 min | Pacing: Screens 1–2

Warm-Up

Purpose: Students create a unique figure that goes through a series of transformations and consider the features of *rigid transformations*, through examples and non-examples. **(MP7)**



Student Notes are also available

1 Launch

1 **To encourage playfulness**, invite students to move the points until they have a shape they really like.

Consider asking, "How would you describe what is happening to your figure in each of the transformations?"

A **Accessibility: Visual-Spatial Processing** Invite students to select one point at a time to move and describe the result to support interpreting visual representations.

2 **Invite students**, to read the prompt aloud, then restate it in their own words to a partner.

2 Connect

Math Identity and Community Celebrate creative or unique shapes and consider displaying them using the Snapshot tool or the dashboard's Teacher View. Ask the creator to talk about their inspiration.

Display the distribution of responses using the dashboard's Teacher View, calling attention to any conflict or consensus.

To surface the Key Takeaway, consider asking, "How would you describe a rigid transformation in your own words?" **(MP7)**

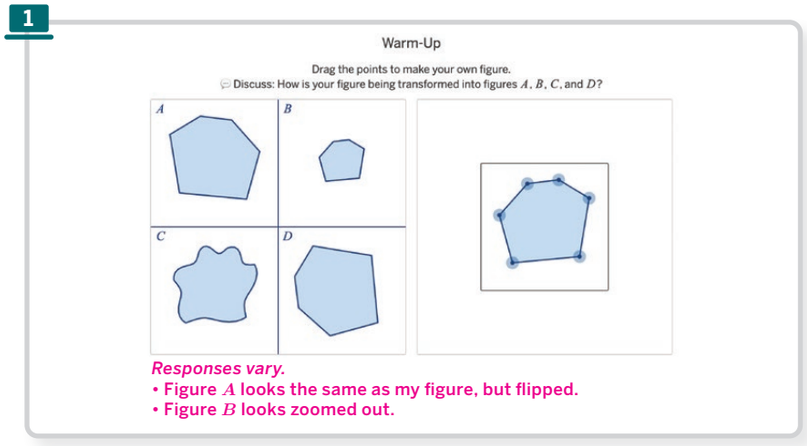
Capture a class definition of the term *rigid transformation* using students' language and the glossary. Consider asking:

- "How do the transformations in figures A and D represent rigid transformations? How can we describe what is happening in figures B and C?"
- "What is another way a rigid transformation could look?"

ML/EL **Multilingual/English Learners** Invite students to research how to say *rigid transformation* in other languages. Create an anchor chart with the terms in multiple languages, along with examples. **(Reading and Listening)**

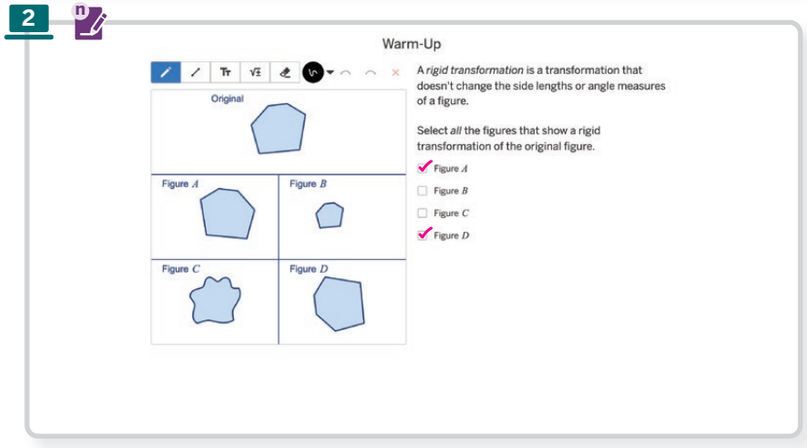
Invite students using notes to document some important points from the class discussion.

1



Responses vary.
- Figure A looks the same as my figure, but flipped.
- Figure B looks zoomed out.

2



Key Takeaway: A *rigid transformation* moves a figure while preserving the original shape and measurement of the figure, similar to tracing an image and moving the traced figure by flipping, rotating, or sliding.

Pairs Sharing a Device | 10 min | Pacing: Screen 3, Screens 4–6

Activity 1 Tracing and Placing

Purpose: Students explore digital tracing paper as a construction tool for performing rigid transformations and consider language for precise descriptions of transformations. **(MP6)**



Student Notes are also available

1 Launch

3 **To encourage playfulness**, invite students to explore the digital tracing paper before engaging with the questions.

To connect to prior knowledge invite students to think independently about *translations*, *rotations*, and *reflections* before discussing their ideas with a partner.

Demonstrate the digital tracing paper and consider asking, "Does the digital tracing paper perform a rigid transformation? How do you know?"

A **Accessibility: Visual-Spatial Processing** Make either tracing paper or a printed copy of the screen available to support students in deciding which transformation to specify throughout this activity.

Note: The digital tracing paper has been designed to fit each figure when the center is placed over a vertex. Consider sharing that students can use a vertex for the center of their tracing paper, which will be the center of rotation.

2 Monitor

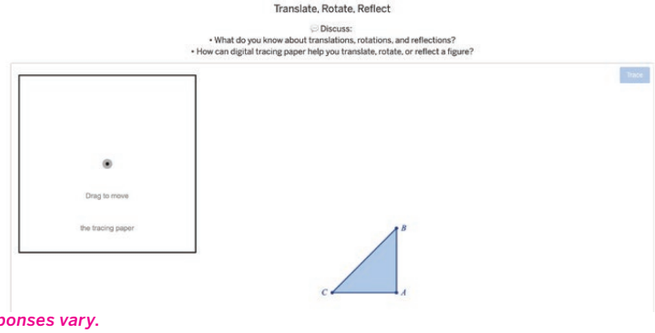
4-5 **Listen for** a variety of strategies students use as they experiment with the digital tracing paper.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to play with the digital tracing paper and use the feedback to think of a strategy.
Think the transformation cannot be completed after making an attempt.	**Support** Consider asking, "What do you know about each transformation? Is there an additional transformation that could help you complete the challenge?"
Center their tracing paper not on a vertex of the triangle.	**Strengthen** Consider asking, "How would you describe where you placed your digital tracing paper to someone else attempting this challenge?" Encourage them to think of a strategic place to align the tracing paper.
Complete the transformation.	**Stretch** Consider asking, "Are there other ways we could perform the transformation using digital tracing paper?"

Students using digital

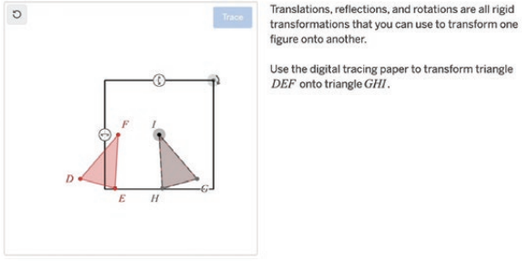
3



Responses vary.
- I know translations, rotations, and reflections are different ways I can move something around.
- After tracing, you can slide to translate, spin to rotate, and flip to reflect.

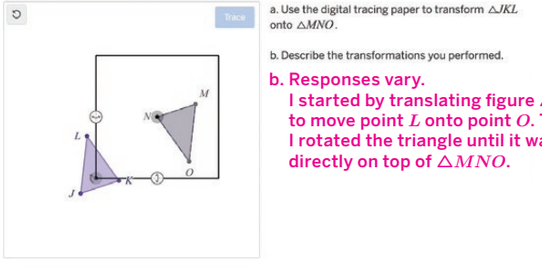
4

Response shown.



5

Response shown.



b. Responses vary.
I started by translating figure $\triangle JKL$ to move point L onto point O. Then I rotated the triangle until it was directly on top of $\triangle MNO$.

Activity 1 continued >

Pairs Sharing a Device | 10 min | Pacing: Screen 3, Screens 4–6

Activity 1 Tracing and Placing (continued)

Purpose: Students explore digital tracing paper as a construction tool for performing rigid transformations and consider language for precise descriptions of transformations. **(MP6)**



Student Notes are also available

2 Monitor

6 **Look for** similarities and differences in students' ordering of the descriptions.

ML/EL **Multilingual/English Learners** Invite students to read each description aloud and paraphrase what it means, clarifying any unfamiliar words. Consider inviting them to try to complete the transformation with what they understand. **(Reading)**

A **Accessibility: Visual-Spatial Processing** Invite students to reflect on the descriptions and attempt the transformations before ordering them by helpfulness.

3 Connect

Display the distribution of responses using the dashboard's Teacher View, calling attention to any conflict or consensus.

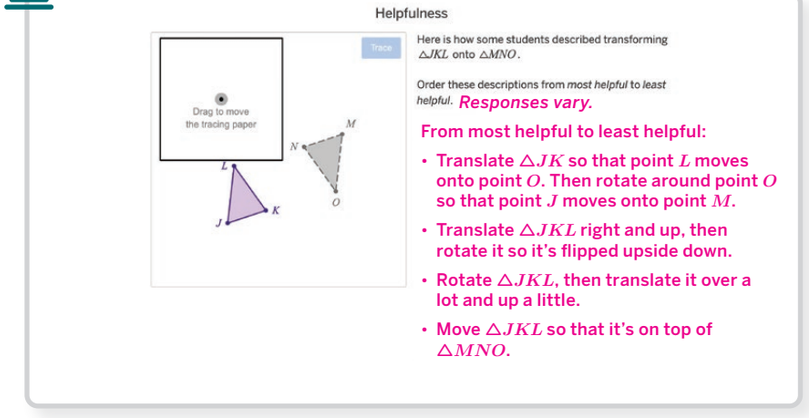
To support making connections, discuss the helpfulness of the descriptions. Consider asking:

- "How are each of these descriptions alike?"
- "Which description did you find least helpful? Why?"
- "Which description did you find most helpful? Why?"

Math Identity and Community Consider inviting students to share what they learned from descriptions that resulted in different transformations.

Students using digital

6



Pairs Sharing a Device | 15 min | Pacing: Screen 7, Screen 8

Activity 2 Trace and Transform

Purpose: Students practice writing and interpreting descriptions of transformations to recreate figures using the digital tracing paper. **(MP6)**

Short on time? Consider inviting students to complete fewer of their classmates' challenges.



Student Notes are also available

1 Launch

7 **Invite students** to read the description aloud, then restate it in their own words before working explicitly to complete the transformation.

Capture and display the range of student responses on this screen. It's okay — even desirable — to lack consensus at this stage. The activity will build toward consensus when students practice writing precise descriptions for transformations in the Challenge Creator.

Discuss the differences in transformations and consider asking, "What is important to consider when writing transformation instructions for someone else to follow?"

8 **Consider sharing** that students will create their own challenge where they will write descriptions of their transformations for others to match.

2 Monitor

Look for and celebrate the different transformations students construct, encouraging exploration and creativity as you observe.

D **Differentiation: (Strengthen)** For students who need encouragement transitioning to writing instructions for replicating their transformation, consider asking, "When writing instructions for your transformation, what should you keep in mind?"

ML/EL **Multilingual/English Learners** Use gestures to illustrate the meaning of the terms in each instruction. Clarify any unfamiliar words. **(Reading)**

Invite students using notes to document at least one clear transformation description.

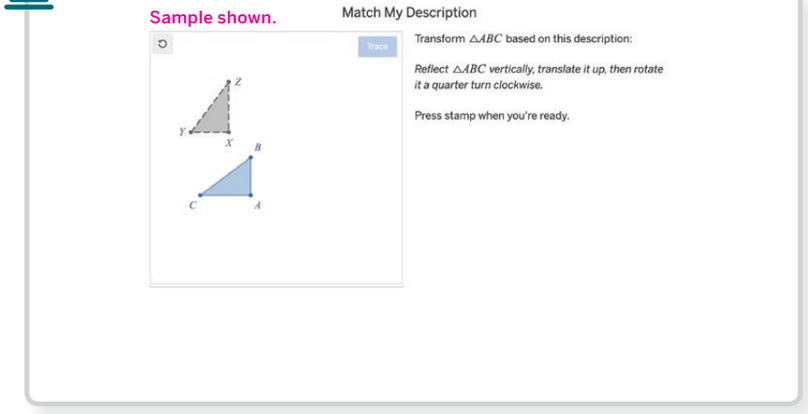
3 Connect

MLR **MLR1: Stronger and Clearer Each Time** Invite students to revisit the description they wrote and use feedback from how other students completed their challenge to make their descriptions stronger and clearer. For students who found pride in their descriptions the first time, consider having them list the qualities of their descriptions that felt precise. **(MP6)**

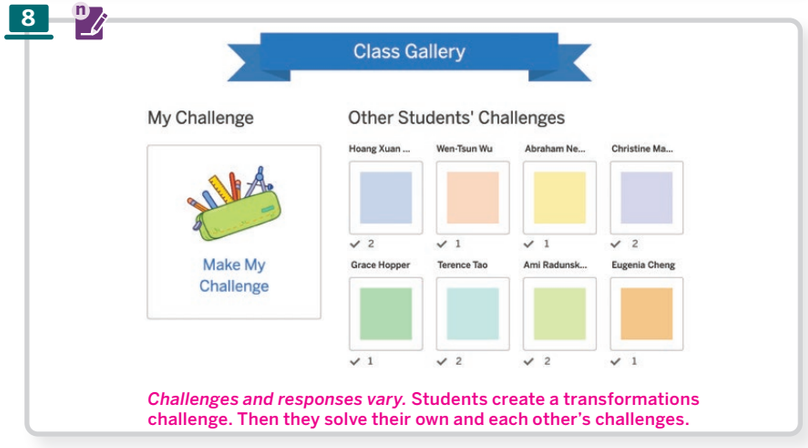
To surface the Key Takeaway, ask students to share revisions they made to make their descriptions stronger and clearer. Record and display them for the class.

Students using digital

7



8



Challenges and responses vary. **Students create a transformations challenge. Then they solve their own and each other's challenges.**

Key Takeaway: When writing descriptions for transformations, it's important to be precise. A few ways to attend to precision are:

- Centering *rotations* on a vertex of the shape.
- Describing *reflections* as vertical, horizontal, or otherwise.
- Specifying the direction and length for *translations*.

Whole Class | 5 min | Pacing: Screen 9

Synthesis



Student Notes are also available

9 **Invite students** to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Reflections are used to flip the figure vertically or horizontally.
- Rotations are used to spin the figure around a point.
- Translations are used to slide a figure a given distance in a given direction.

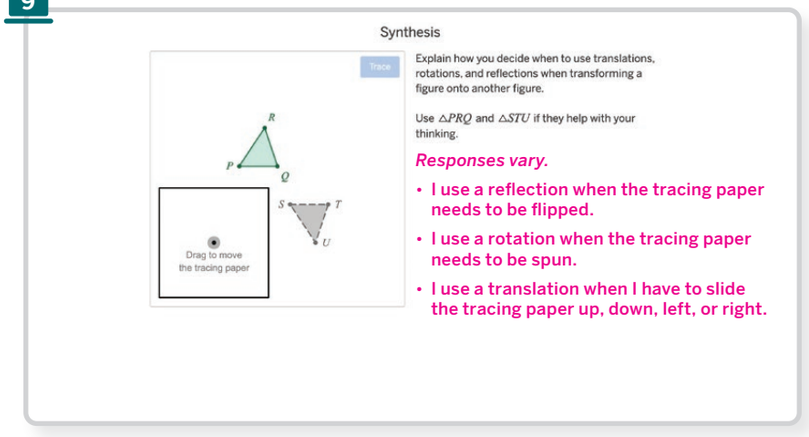
MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *rigid transformation* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to share with a partner what makes sense about each transformation and what questions they still have.

Invite students using notes to document ideas from the class discussion in the left column.

Students using digital

9

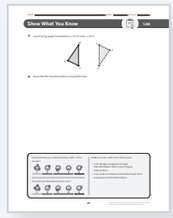


Lesson Takeaway: Digital tracing paper can help us precisely perform *rigid transformations* of geometric objects.

Independent | 5 min | Pacing: Screens 10–11

Show What You Know

Purpose: Students demonstrate their understanding by using the digital tracing paper to perform and describe the steps for a transformation.

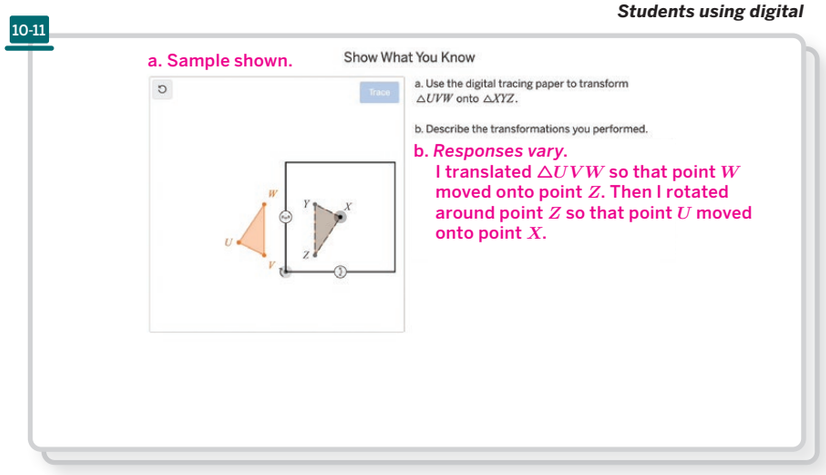


Students using print

10-11 Today's Goals

1. **Goal:** Design a sequence of rigid transformations that moves a figure onto another.
2. **Language Goal:** Write and follow instructions to perform a sequence of transformations. **(Reading, Writing, Speaking, and Listening)**

Students using digital



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

S Support

Provide targeted intervention.

If student work shows **partial understanding** of using digital tracing paper to perform rigid transformations, consider:

- Revisiting **Screen 5** and giving students individualized feedback.
- Inviting students to engage with the **Summary** by reading it and highlighting key components.
- Inviting students to do the **Try This** problem and provide individualized feedback.

S Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding** with minor errors, consider:

- Assigning the **Lesson Practice**.
- Inviting students to review the **Summary** and complete the **Try This** problem.
- Revisiting the **Challenge Creator** in **Activity 2** and inviting students to complete additional challenges.

S Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Encouraging them to complete the **DOK3 practice problem**.
- Inviting students to explore transformations using constructions with the Digital Tool.

Professional Learning

Reflect on students' use of the digital tracing paper in Activities 1 and 2. How did the digital tracing paper support students with performing *rigid transformations* and making sense of this new vocabulary? What support might students need in the future to effectively construct transformations using tracing paper or other tools?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 8
Practice

Students using digital

Lesson Summary

You can use tracing paper to perform *rigid transformations*, which don't change the side lengths or angle measures of a figure. With tracing paper, you can perform:

- *Translations*, which slide a figure up, down, left, or right.
- *Reflections*, which flip a figure.
- *Rotations*, which spin a figure.

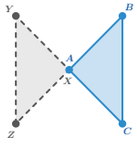
Using tracing paper can help you make sense of how to use rigid transformations to move one figure on top of another. Sometimes you only need one transformation, but other times you may need to use multiple transformations to perfectly align one figure to another.

Try This

Describe how to transform $\triangle ABC$ onto $\triangle XYZ$.

Use tracing paper if it helps with your thinking.

Responses vary. Reflect $\triangle ABC$ over a vertical line through point A.

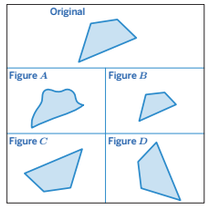


Lesson Practice
Geo.1.08

Name: _____ Date: _____ Period: _____

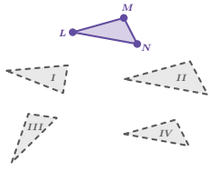
1. Select *all* the figures that show a rigid transformation of the original figure.

- ☐ **A.** Figure A
- ☐ **B.** Figure B
- ☑ **C.** Figure C
- ☑ **D.** Figure D

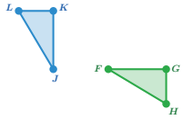


2. Which triangle could be the result of a single translation of $\triangle LMN$?

- **A.** I
- **B.** II
- **C.** III
- **D.** IV



Problems 3–4: Here are two triangles.



3. Describe a sequence of rigid transformations that transforms $\triangle FGH$ onto $\triangle JKL$.

Responses vary. Translate triangle FGH so that point F moves onto point J. Rotate around point J so that point G is on top of point K. Reflect so that point H moves onto point L.

4. Describe a *different* sequence that transforms $\triangle FGH$ onto $\triangle JKL$.

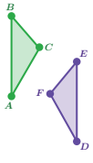
Responses vary. Rotate $\triangle FGH$ around point F until segment FG is parallel to segment KJ. Reflect over the perpendicular bisector of $\overline{GK}$.

Lesson Practice
Geo.1.08

Name: _____ Date: _____ Period: _____

5. Select *all* the sequence(s) of transformations that could transform $\triangle ABC$ onto $\triangle DEF$.

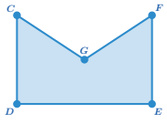
- ☑ **A.** A rotation and a reflection
- ☐ **B.** A translation
- ☑ **C.** A reflection and a translation
- ☐ **D.** A rotation and a translation
- ☐ **E.** A reflection



6. This polygon was made using only a triangle and its reflection.

Name one of the triangles.

$\triangle CDE$ or $\triangle FED$

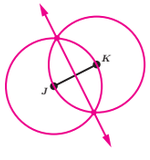


Spiral Review

7. Construct the perpendicular bisector of $\overline{JK}$.

8. Which point is closer to the perpendicular bisector you just constructed? Circle one.

Point J Point K They're the same distance.



Reflection

1. Put a question mark next to a response you'd like to compare with a classmate's.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-lesson			
	1–2	1	HSG.CO.A.2
	3–4	2	HSG.CO.A.5
Test Practice	5	2	HSG.CO.A.5
	6	3	HSG.CO.A.5
Spiral Review			
Fluency	7	1	HSG.CO.D.12
	8	1	HSG.CO.D.12

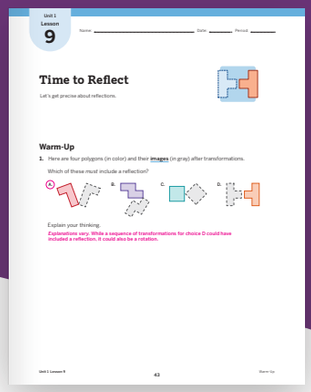
Need More Practice?

Additional Practice resources coming soon.

Unit 1

Lesson 9

This lesson uses print student pages and digital Teacher Presentation Screens.



Time to Reflect

Defining Reflections

Let's get precise about reflections.

Focus and Coherence

Today's Goals

1. **Goal:** Reflect figures precisely using a variety of tools.
2. **Language Goal:** Describe what happens to a *pre-image* when it is reflected over a given line. **(Speaking and Writing)**

Students move from digital tracing paper to physical tracing in order to develop their understanding of a reflection as a rigid transformation. They analyze ways that reflections can go wrong in order to better articulate the properties of reflections. Students develop construction techniques for reflecting a point over a line using a compass and straightedge.

Prior Learning

In Lesson 8, students used digital tracing paper to perform and describe sequences of transformations for triangles.

Future Learning

In Lessons 10 and 11, students will perform and describe translations and rotations. In Unit 2, students will use these rigid transformations as the foundation for geometric proof.

Rigor and Balance

- Students **apply** and extend their understanding of constructions as they use them to perform precise transformations.
- Students develop **strategic competence** as they justify the properties of reflected images using constructions.

Vocabulary

New Vocabulary

image

line of reflection

pre-image

Standards

Addressing

HSG.CO.A.5

Given a geometric figure and a rotation, **reflection**, or translation, **draw the transformed figure using, e.g., graph paper, tracing paper**, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

HSG.CO.A.4

Develop definitions of rotations, **reflections**, and translations **in terms of** angles, circles, **perpendicular lines**, parallel lines, **and line segments**.

Also Addressing: **HSG.CO.A.2**

Mathematical Practices: MP5, MP6

Building On	Building Toward
8.G.A.3	HSG.CO.A.5 HSG.CO.B.6

Lesson at a Glance

~ 45 min

Standards: HSG.CO.A.5, HSG.CO.A.4, HSG.CO.A.2

Warm-Up

Independent | 5 min

Students identify which of several **images** must have been reflected from its original polygon.



Lesson 9 Warm-Up

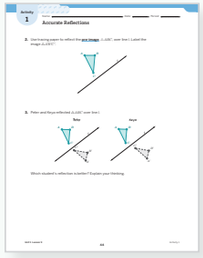
Presentation Screen 1

Activity 1

Pairs | 8 min

Students reflect a triangle and analyze two other reflections to discuss strategies and properties of reflections.

Key Takeaway: Points on a reflected **image** and the corresponding points on the **pre-image** are the same distance away from the line they are reflected over.



Lesson 9 Activity 1

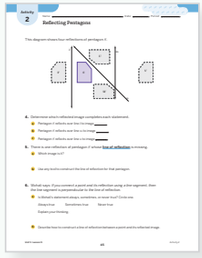
Presentation Screens 2–3

Activity 2

Pairs | 10 min

Students identify relationships between **lines of reflection**, images, and **pre-images**.

Key Takeaway: The **line of reflection** is the perpendicular bisector of the segment joining a point and its reflection. We can use vertices on the image and pre-image to construct the line of reflection.



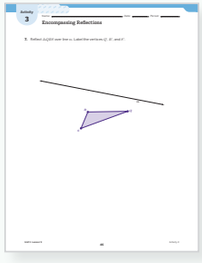
Lesson 9 Activity 2

Presentation Screens 4–5

Activity 3

Pairs | 12 min

Students use tools and the properties of reflections to precisely reflect a triangle and a point. **(MP5)**



Lesson 9 Activity 3

Presentation Screen 6

Synthesis

Whole Class | 5 min

Students synthesize their understanding about the relationship between an image, pre-image, and line of reflection.

Lesson Takeaway: Reflected points on an **image** are the same distance away from the **line of reflection** as the original points on the **pre-image**. To make sense of a *reflection*, you can use tracing paper, folding, or a compass and straightedge to construct a perpendicular bisector and circles to precisely reflect a point over a line.



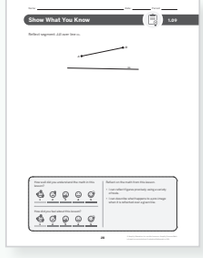
Lesson 9 Synthesis

Presentation Screen 7

Show What You Know

Independent | 5 min

Students demonstrate their understanding by reflecting a line segment.



Lesson 9 Show What You Know

Presentation Screen 8

Prep Checklist

Invite students to use their Student Edition. Display the Presentation Screens to help facilitate the lesson.

Consider making the geometry toolkit available to support students in selecting and using tools strategically.

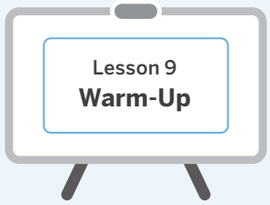
This lesson includes:

- Presentation Screens
- Student Edition
- Show What You Know
- geometry toolkit: compasses, straightedges, tracing paper
- coloring tools or highlighters (optional)

Independent | 5 min

Warm-Up

Purpose: Students identify which of several **images** must have been reflected from its original polygon.



Presentation Screen 1

1 Launch

Display the four polygons. To encourage creativity, invite students to list possible transformations for each **image**.

Distribute tracing paper, a compass, and a straightedge to each student. Invite students to use any of these tools if it helps with their thinking.

Note: It's okay if students are unsure about how to use these tools. They will have a chance to experiment and learn strategies throughout the lesson.

Accessibility: Visual-Spatial Processing Invite students to use coloring tools or highlighters to identify a portion of an image and pre-image that relate to one another. Then invite them to describe which transformations could have been performed before moving onto the next polygon.

2 Connect

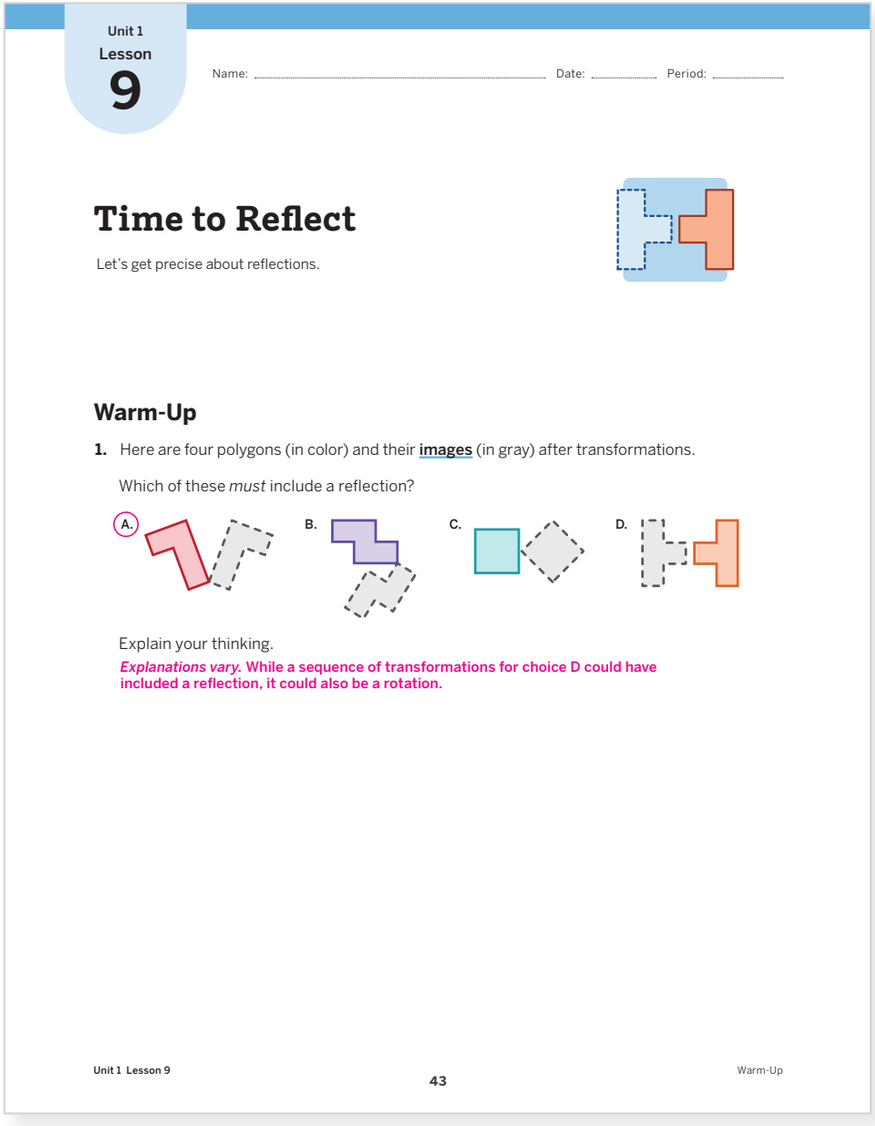
Invite students to share which choice must include a reflection by polling the class. If there is agreement among the class, consider inviting them to share why someone might think it is one of the other choices.

Discuss students' reasoning and consider using the Sketch tool to capture their ideas.

Create a class definition for the term *image* using students' language and the glossary.

Consider asking:

- "What is a relationship you notice between a polygon and its reflected image?"
- "What is a relationship you notice between a polygon and its reflected image?"



Unit 1 Lesson 9

Name: Date: Period:

Time to Reflect

Let's get precise about reflections.

Warm-Up

1. Here are four polygons (in color) and their **images** (in gray) after transformations.

Which of these *must* include a reflection?

A. B. C. D.

Explain your thinking.

Explanations vary. While a sequence of transformations for choice D could have included a reflection, it could also be a rotation.

Unit 1 Lesson 9 43 Warm-Up

Pairs | 8 min

Activity 1 Accurate Reflections

Purpose: Students reflect a triangle and analyze two other reflections to discuss strategies and properties of reflections.

Lesson 9
Activity 1

Presentation Screens
2–3

1 Launch

Invite students to discuss strategies for transferring the triangle from tracing paper to regular paper with a partner. Consider creating a class definition for image and **pre-image** to reference throughout the lesson.

A **Accessibility: Fine Motor Skills** Consider varying the task demands by providing copies of the triangle for tracing or pre-filled in tracing paper.

2 Monitor

Look for student strategies such as folding the paper over line l, annotating distances to show how far to move the tracing paper, or using a compass and straightedge to support constructing a transformation.

D **Differentiation** *(Problems 2)*

Look for students who:	Respond to Student Thinking
Perform a rotation with their tracing paper instead of a reflection.	**Support** Invite them to fold their paper over the line of reflection and notice if the image and pre-image align with one another.
Fold their paper to help make sense of or check their reflection.	**Strengthen** Celebrate their strategy. Consider asking, "How can you use your other tools to check your reflection?"
Draw the line of reflection on their tracing paper.	**Stretch** Highlight their problem solving skills and understanding of the problem. Consider asking, "How would you explain to a classmate why tracing the line is important?"

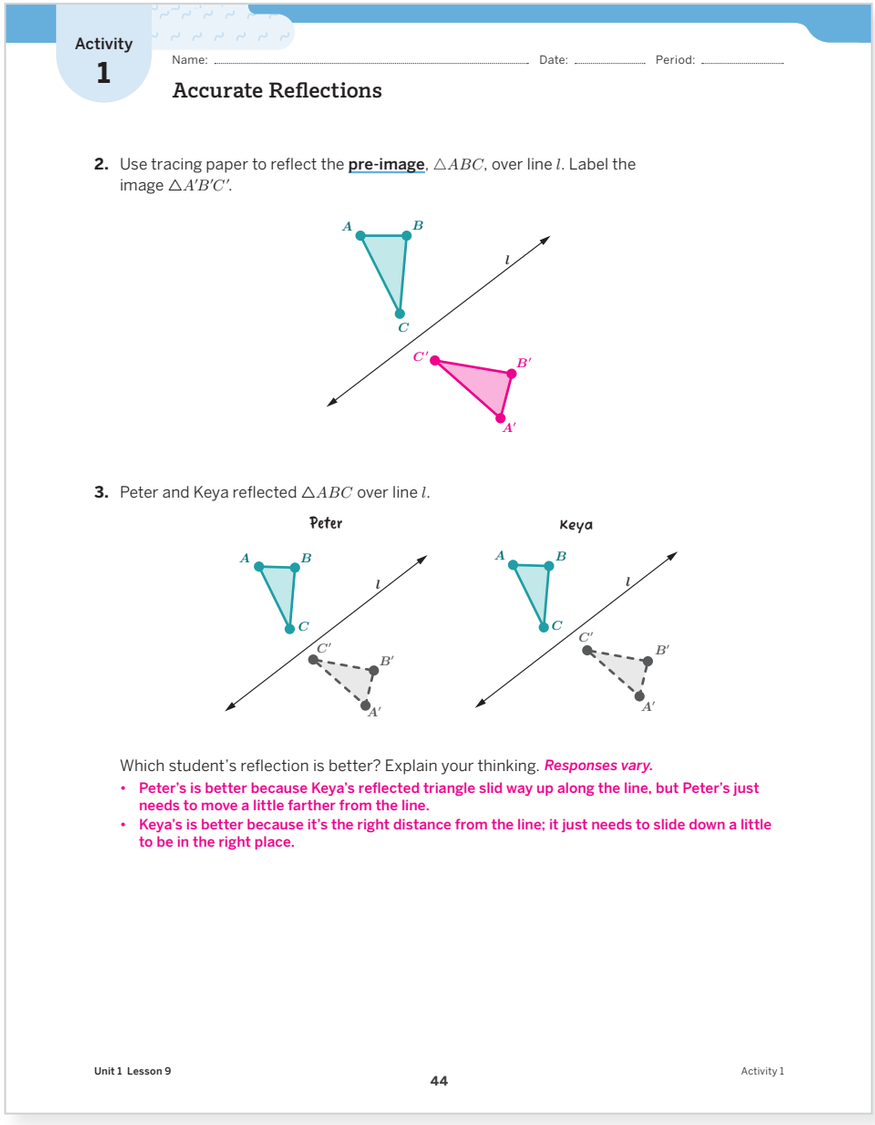
Note: It's okay if students' reflections vary in their precision. Use this activity as an opportunity to discuss properties that they would want to be more precise with later in the lesson.

Encourage students to use their tools to verify or support their thinking about which reflection is better.

3 Connect

Invite students to share which displayed reflection was more precise. Invite them to share how they attended to precision in their reflections and how their strategies were similar to the displayed reflections.

To surface the Key Takeaway, consider asking, "How do points on the image and pre-image relate to the line they are reflected over?"



Activity 1

Name: _____ Date: _____ Period: _____

Accurate Reflections

2. Use tracing paper to reflect the **pre-image**, $\triangle ABC$, over line l. Label the image $\triangle A'B'C'$.

3. Peter and Keya reflected $\triangle ABC$ over line l.

Peter

Keya

Which student's reflection is better? Explain your thinking. *Responses vary.*

- Peter's is better because Keya's reflected triangle slid way up along the line, but Peter's just needs to move a little farther from the line.
- Keya's is better because it's the right distance from the line; it just needs to slide down a little to be in the right place.

Unit 1 Lesson 9 44 Activity 1

Key Takeaway: Points on a reflected **image** and the corresponding points on the **pre-image** are the same distance away from the line they are reflected over.

Pairs | 10 min

Activity 2 Reflecting Pentagons

Purpose: Students identify relationships between **lines of reflection**, images, and **pre-images**.

Short on time? Consider displaying a completed diagram to discuss Problem 6 instead of having students construct the line of reflection.

Lesson 9
Activity 2

Presentation Screens 4–5

1 Launch

Encourage students to use multiple tools to help them with their thinking about each reflection.

Pause briefly and invite students to share how they determined which line creates each reflection. Consider playing the animation for each as students share.

Create a class definition for the term **line of reflection** using students' language and the glossary.

2 Monitor

Capture strategies students use to construct a line of reflection.

D Differentiation *(Problems 5)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What is a relationship between points on an image and pre-image? How can you use that to help make the line?"
Sketch a line of reflection.	**Support** Invite them to use their tools to construct a more precise line of reflection.
Construct midpoints to make the line precisely.	**Stretch** Consider asking, "What are some ways that you can use the image and pre-image to verify that you've constructed the line of reflection?"
Construct a perpendicular bisector as the precise line of reflection. **(MP6)**	

3 Connect

Display student strategies for constructing a line of reflection. Invite students to share their thinking.

Consider asking, "Why is Wohali's statement always true?" Consider using the geometry tools to make the student responses visible to the class.

To surface the Key Takeaway, consider connecting the vertices of pentagons S and W to show that the line of reflection goes through the midpoint of each segment. Consider asking, "What do you notice about the line of reflection and each segment?"

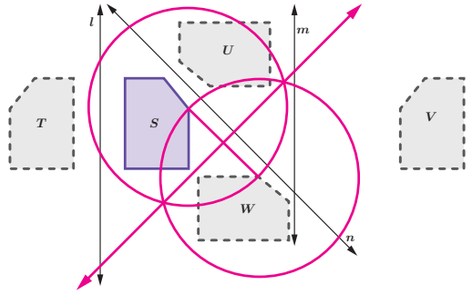
Math Identity and Community Invite students to celebrate the variety of mathematical thinking about how to determine and construct a line of reflection.

Activity 2

Name: _____ Date: _____ Period: _____

Reflecting Pentagons

This diagram shows four reflections of pentagon S.



4. Determine which reflected image completes each statement.

a. Pentagon S reflects over line l to image ___T___.

b. Pentagon S reflects over line m to image ___V___.

c. Pentagon S reflects over line n to image ___U___.

5. There is one reflection of pentagon S whose **line of reflection** is missing.

a. Which image is it?
Pentagon W

b. Use any tool to construct the line of reflection for that pentagon.
***Work varies.* Sample shown on diagram.**

6. Wohali says: *If you connect a point and its reflection using a line segment, then the line segment is perpendicular to the line of reflection.*

a. Is Wohali's statement *always*, *sometimes*, or *never* true? Circle one.

(Always true) Sometimes true Never true

Explain your thinking. ***Explanations vary.* This is like when we constructed the perpendicular bisector by folding the endpoints of a segment onto each other with tracing paper. Reflecting is like folding, so you'll always get the perpendicular bisector.**

b. Describe how to construct a line of reflection between a point and its reflected image.
***Responses vary.* If you construct a perpendicular bisector between a point and its reflection, the perpendicular bisector is the line of reflection.**

Unit 1 Lesson 9 45 Activity 2

Key Takeaway: The **line of reflection** is the perpendicular bisector of the segment joining a point and its reflection. We can use vertices on the image and pre-image to construct the line of reflection.

Pairs | 12 min

Activity 3 Encompassing Reflections

Purpose: Students use tools and the properties of reflections to precisely reflect a triangle and a point. **(MP5)**

Short on time? Consider having students only reflect 1–2 vertices in Problem 7.

Lesson 9
Activity 3

Presentation Screen 6

1 Launch

6 **Invite students** to strategize about how they can put together what they know about reflected points and constructions.

Consider asking, "What constructions might be helpful to complete this reflection?"

ML/EL **Multilingual/English Learners** Consider creating an anchor chart about reflections and strategies for performing them using constructions. Add student ideas and language to the chart throughout the lesson.

2 Monitor

Select and sequence strategies and tools students use to reflect the triangle.

D **Differentiation** *(Problems 7)*

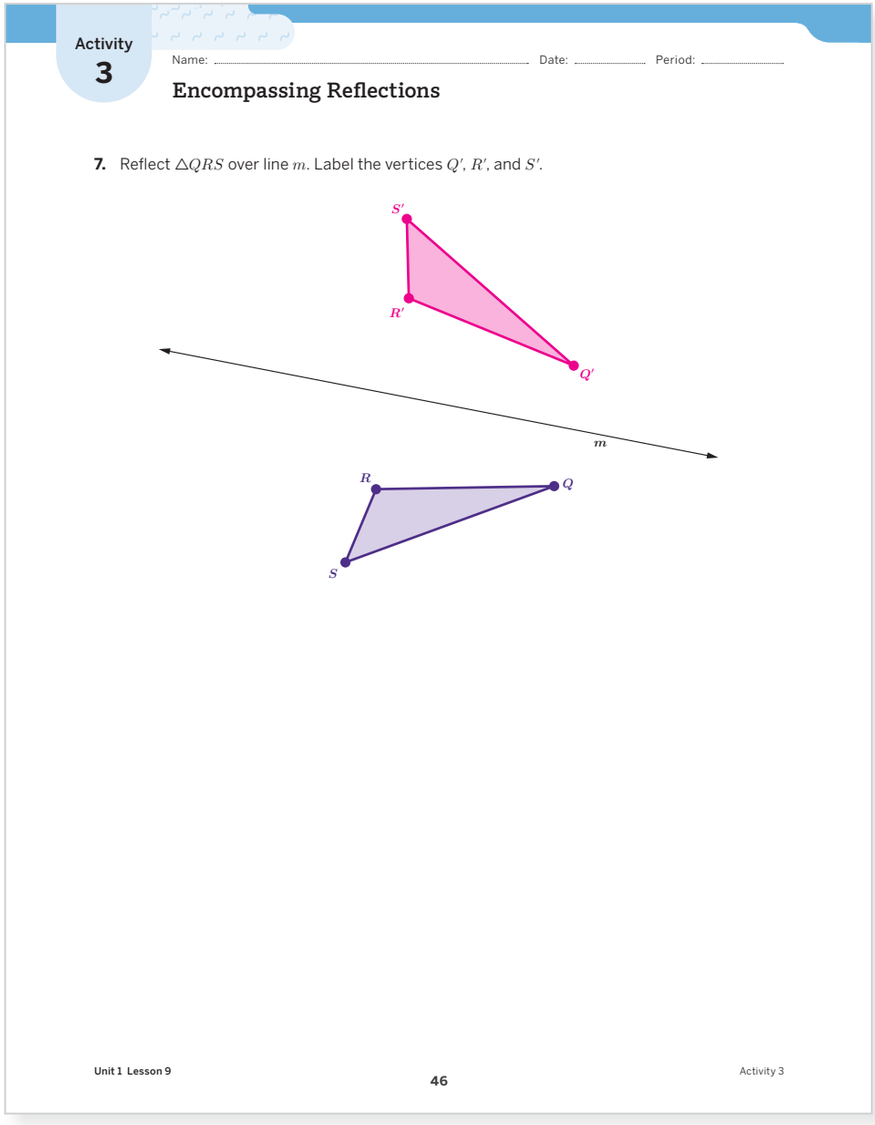
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How could drawing a circle centered at one of the vertices help you?"
Measure equal distances from the line, but not using line m as the perpendicular bisector of the points.	**Support** Celebrate their strategy for finding equal distances. Consider asking, "How can you use a circle to determine the perpendicular bisector?"
Use tracing paper to reflect the triangle's points.	**Strengthen** Consider asking, "How can you use a construction to validate your reflection?"
Use a variety of tools and constructions to reflect $\triangle QRS$. **(MP5)**	**Stretch** Invite students to explain the advantages and disadvantages of certain strategies.

Pause to display strategies that students used to perform the reflection. Consider sharing a selection of strategies and asking what we can learn from strategies that did not work.

Math Identity and Community Normalize learning from mistakes by encouraging students to add to a list of common errors made while attempting a construction.

Note: It's okay if students do not finish a precise reflection completely. They will have time to revise after they reflect a point in the next problem.

Activity 3 continued >



Activity 3

Name: _____ Date: _____ Period: _____

Encompassing Reflections

7. Reflect $\triangle QRS$ over line m. Label the vertices Q', R', and S'.

Unit 1 Lesson 9 46 Activity 3

Pairs | 12 min

Activity 3 Encompassing Reflections (continued)

Purpose: Students use tools and the properties of reflections to precisely reflect a triangle and a point. **(MP5)**

Short on time? Consider having students only reflect 1–2 vertices in Problem 7.

Lesson 9
Activity 3

Presentation Screen
6

1 Monitor

Select and sequence constructions that students used to precisely reflect point P.

D Differentiation *(Problem 8)*

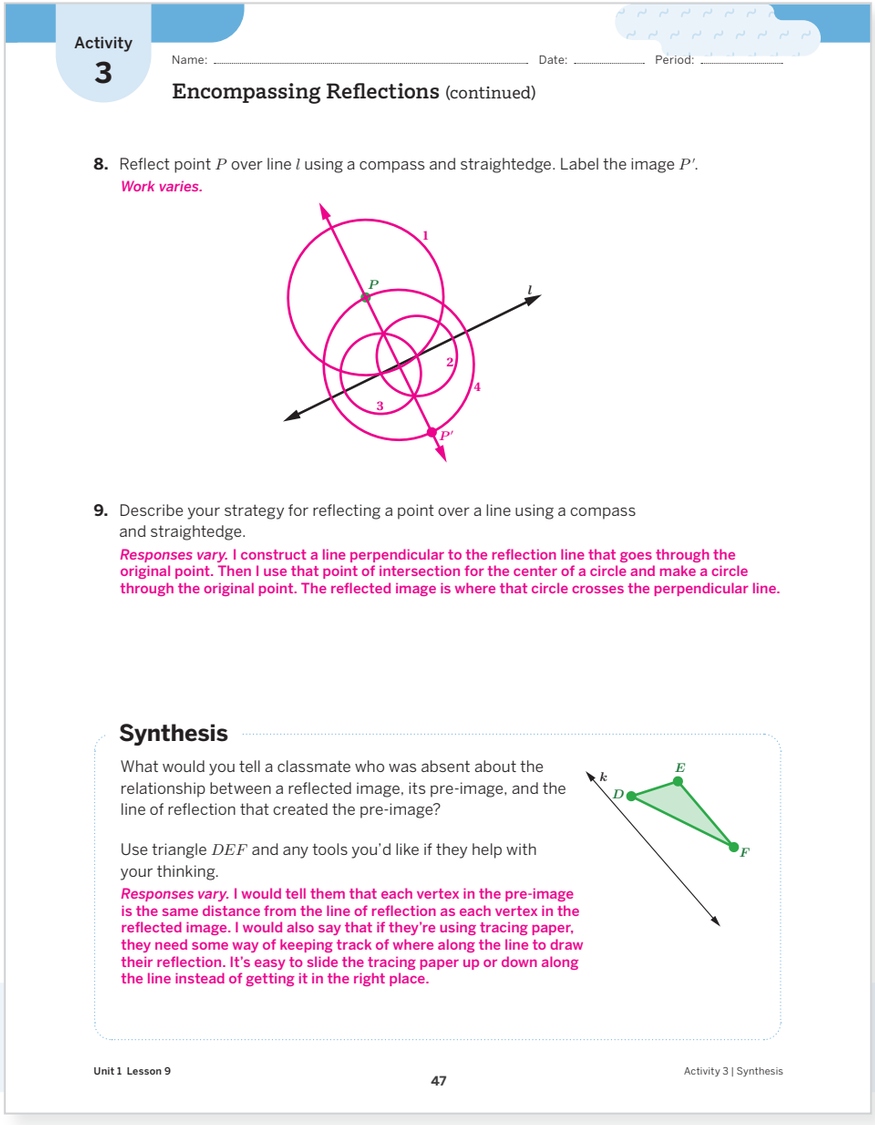
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Where is a strategic place to center a circle to start your construction?"
Construct a circle centered at P to create a perpendicular bisector.	**Strengthen** Consider asking, "How will you use a perpendicular bisector to justify that you have precisely reflected the point?"
Precisely reflected point P over line l.	**Stretch** Encourage students to go back to the reflection of the triangle and revise or justify their work using constructions.

Encourage students to discuss with their partner the strategies they used to reflect point P.

2 Connect

Invite students to share their strategies for reflecting a point. Consider asking:

- "Which tools would you rather use for doing a precise reflection: digital tracing paper, physical tracing paper, or compass and straightedge? Why?"
- "How does your chosen tool ensure the properties of a precise reflection (such as distance from the line of reflection)?" **(MP5)**



Activity 3

Name: _____ Date: _____ Period: _____

Encompassing Reflections (continued)

8. Reflect point P over line l using a compass and straightedge. Label the image P'.

Work varies.

9. Describe your strategy for reflecting a point over a line using a compass and straightedge.

Responses vary. I construct a line perpendicular to the reflection line that goes through the original point. Then I use that point of intersection for the center of a circle and make a circle through the original point. The reflected image is where that circle crosses the perpendicular line.

Synthesis

What would you tell a classmate who was absent about the relationship between a reflected image, its pre-image, and the line of reflection that created the pre-image?

Use triangle DEF and any tools you'd like if they help with your thinking.

Responses vary. I would tell them that each vertex in the pre-image is the same distance from the line of reflection as each vertex in the reflected image. I would also say that if they're using tracing paper, they need some way of keeping track of where along the line to draw their reflection. It's easy to slide the tracing paper up or down along the line instead of getting it in the right place.

Unit 1 Lesson 9 47 Activity 3 | Synthesis

Synthesis

Whole Class | 5 min

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- The line of reflection is the perpendicular bisector of the segment that connects two vertices in the image and pre-image.
- Tracing paper can be used to visualize a reflection by drawing a line along the line of reflection to help orient the image.

Create an anchor chart that outlines the important properties of reflections.

Lesson Takeaway: Reflected points on an **image** are the same distance away from the **line of reflection** as the original points on the **pre-image**. To make sense of a *reflection*, you can use tracing paper, folding, or a compass and straightedge to construct a perpendicular bisector and circles to precisely reflect a point over a line.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by reflecting a line segment.

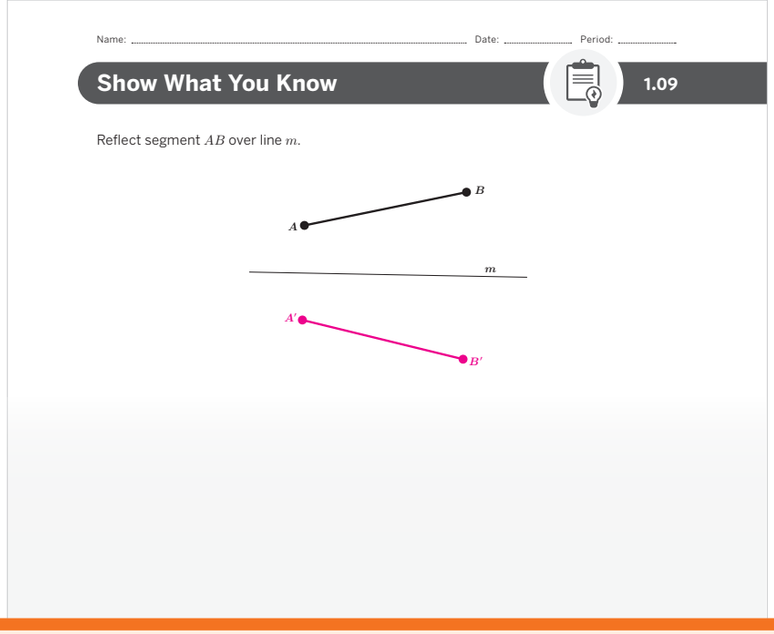


Presentation Screen 8

Today's Goals

1. **Goal:** Reflect figures precisely using a variety of tools.
2. **Language Goal:** Describe what happens to a *pre-image* when it is reflected over a given line. **(Speaking and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 1.09

Reflect segment AB over line m.

Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of reflecting a line segment, consider:

- Revisiting reflecting a point over a line in **Problem 8** and having them describe their strategy.
- Invite students to do the **Try This** problem and provide feedback on their reflection.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding** with minor errors, consider:

- Assigning the **Lesson Practice**.
- Invite students to review the **Summary** and complete the **Try This** problem.
- Inviting students to use another strategy (like tracing paper or compass and straightedge) to check their construction.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting students to complete the **DOK3 practice problem**.
- Inviting students to explore transforming images using digital construction tools.

Professional Learning

Take a moment to reflect on this lesson, where students used many strategies to perform reflections. How will this work support performing translation and rotations precisely?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 9
Practice

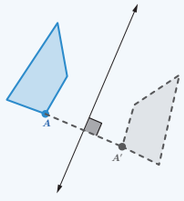
Students using digital

Lesson Summary

You can use tracing paper to explore the relationship between a **line of reflection** and the reflected image. Some properties of reflections are:

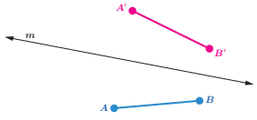
- Reflections are rigid transformations, so the reflected **image** will be the same size and shape as the **pre-image**.
- If you connect a point and its reflected image using a line segment, then the line segment is perpendicular to the line of reflection.
- A line of reflection bisects the line segment between a point and its reflected image.

You can use these properties to construct precise reflections. For example, a perpendicular line construction can help you determine the line that the reflected point must lie on. A circle can help you represent the correct distance from the line of reflection.



Try This

Reflect segment $\overline{AB}$ over line m.

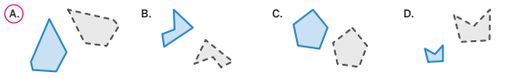


Lesson Practice
Geo.1.09

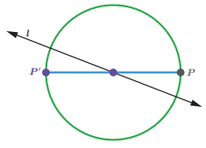
Name: _____ Date: _____ Period: _____

1. Which of the transformed images *must* include a reflection?

(A.) B. C. D.



Problems 2–3: Hailey tried to reflect point P over line l.



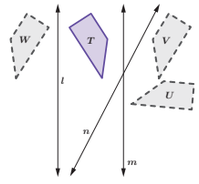
2. What is one thing Hailey did correctly?

Responses vary. She put point P' the same distance from line l as point P is.

3. How would you improve Hailey's reflection?

Responses vary. Construct a line perpendicular to the line of reflection l before reflecting point P.

Problems 4–6: Determine which reflected image completes each statement.

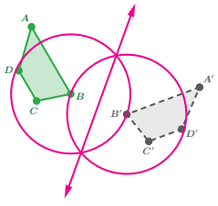


4. Trapezoid T reflects over line l to image W.

5. Trapezoid T reflects over line m to image V.

6. Trapezoid T reflects over line n to image U.

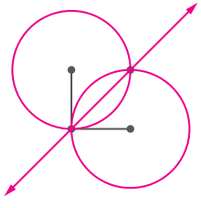
7. Construct the line of reflection.



Lesson Practice
Geo.1.09

Name: _____ Date: _____ Period: _____

8. Construct a line of reflection that transforms this figure onto itself. *Responses vary.*



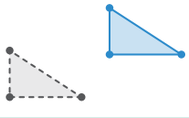
Spiral Review

9. Select *all* the statements that are always true about an image after a sequence of rigid transformations.

- ☑ A. The angle measures of the image and pre-image are the same.
- ☐ B. The orientation of the image and the pre-image are the same.
- ☑ C. The side lengths of the image and the pre-image are the same.
- ☐ D. The side lengths of the image are shorter than in the pre-image.
- ☐ E. The side lengths of the image are longer than in the pre-image.

10. Which type of single transformation is shown?

(A.) Translation B. Reflection
C. Rotation D. Dilation



Reflection

1. Star the problem you spent the most time on.

2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1	1	HSG.CO.A.5
	2–3, 7	2	HSG.CO.A.5
Test Practice	**4–6**	2	HSG.CO.A.5
	8	3	HSG.CO.A.5
Spiral Review			
Fluency	**9**	1	8.G.A.1
	10	1	HSG.CO.A.5

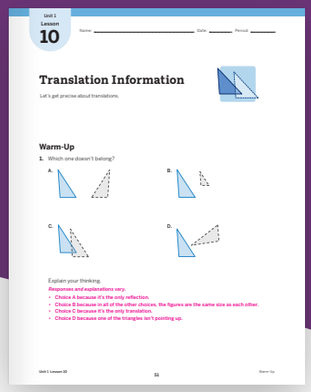
Need More Practice?

Additional Practice resources coming soon.

Unit 1

Lesson 10

This lesson uses print student pages and digital Teacher Presentation Screens.



Translation Information

Defining Translations

Let's get precise about translations.

Focus and Coherence

Today's Goals

1. **Goal:** Translate figures precisely using a variety of tools.
2. **Language Goal:** Describe translations and reflections precisely. **(Speaking and Listening)**

Students begin by using **prime notation** to describe translations and their properties. They use tools to precisely translate points and figures both to a specific point and with a given distance and direction. Students finish the lesson by describing translations and reflections to each other to precisely reproduce them. **(MP5, MP6)**

Prior Learning

In Lesson 9, students used a variety of tools to perform reflections.

Future Learning

In Lesson 11, students will perform and describe rotations. In Lesson 12, students will perform sequences of transformations digitally. In Unit 2, students will use these rigid transformations as the foundation for geometric proof.

Rigor and Balance

- Students **apply** and extend their understanding of constructions as they use them to perform precise transformations.
- Students develop **strategic competence** as they justify the properties of translated images using constructions.

Vocabulary

New Vocabulary

prime notation

Standards

Addressing

HSG.CO.A.5

Given a geometric figure and a rotation, **reflection**, **or translation, draw the transformed figure using, e.g., graph paper, tracing paper**, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

HSG.CO.A.4

Develop definitions of rotations, **reflections**, and **translations in terms** of angles, circles, **perpendicular lines**, parallel lines, **and line segments**.

Also Addressing: **HSG.CO.A.2**

Mathematical Practices: MP5, MP6

Building On	Building Toward
8.G.A.3	HSG.CO.A.5
	HSG.CO.B.6

Lesson at a Glance

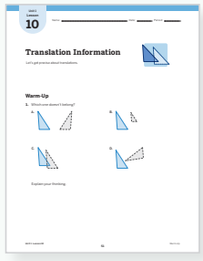
~ 45 min

Standards: HSG.CO.A.5, HSG.CO.A.4, HSG.CO.A.2

Warm-Up

Independent | 5 min

Students analyze four transformations to discuss the properties of rigid transformations, specifically translations.
Routine: Which One Doesn't Belong?





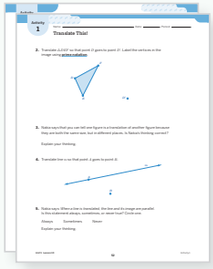
Presentation Screen 1

Activity 1

Pairs | 20 min

Students perform and analyze rigid transformations to explore the properties of translations.

Key Takeaway: A translation is a rigid transformation that moves every point on a geometric figure a given distance in a given direction. When performing translations with tracing paper, we need to make sure to not rotate the tracing paper.



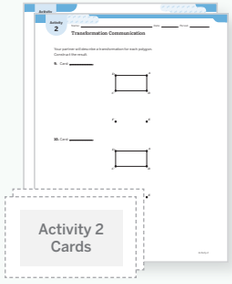


Presentation Screen 2

Activity 2

Pairs | 10 min

Students perform and describe transformations precisely. **(MP6)**





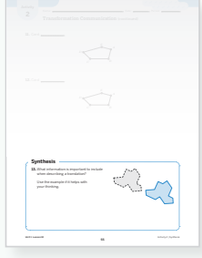
Presentation Screen 3

Synthesis

Whole Class | 5 min

Students synthesize their understanding of how to precisely describe a translation.

Lesson Takeaway: A translation is a rigid transformation that moves each point on the pre-image to another point on the image that is in the same direction and distance away. It is important to be precise when describing transformations and often helpful to use **prime notation** to show which points ended up in what locations.



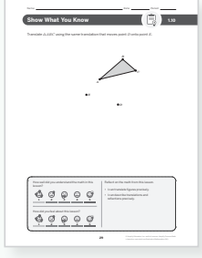


Presentation Screen 4

Show What You Know

Independent | 5 min

Students demonstrate their understanding of translating a triangle.





Presentation Screen 5

Prep Checklist

Invite students to use their Student Edition. Display the Presentation Screens to help facilitate the lesson.

Consider making the geometry toolkit available to support students in selecting and using tools strategically.

This lesson includes:

- Presentation Screens
- Student Edition
- Show What You Know
- Activity 2 Cards (one set per pair)
- geometry toolkit: compasses, straightedges, tracing paper

Independent | 5 min

Warm-Up

Purpose: Students analyze four transformations to discuss the properties of rigid transformations, specifically translations.



Presentation Screen 1

1 Launch

Display the four transformations. To encourage creativity, invite students to think of a reason that no one else in the class will consider.

Use the Which One Doesn't Belong? routine to support students in comparing and contrasting the properties of rigid transformations.

Consider sharing that each transformation does not belong in some way. Encourage students to look for more than one possibility.

ML/EL **Multilingual/English Learners** Invite students to talk through their thinking with a partner before writing it down. **(Writing and Speaking)**

2 Connect

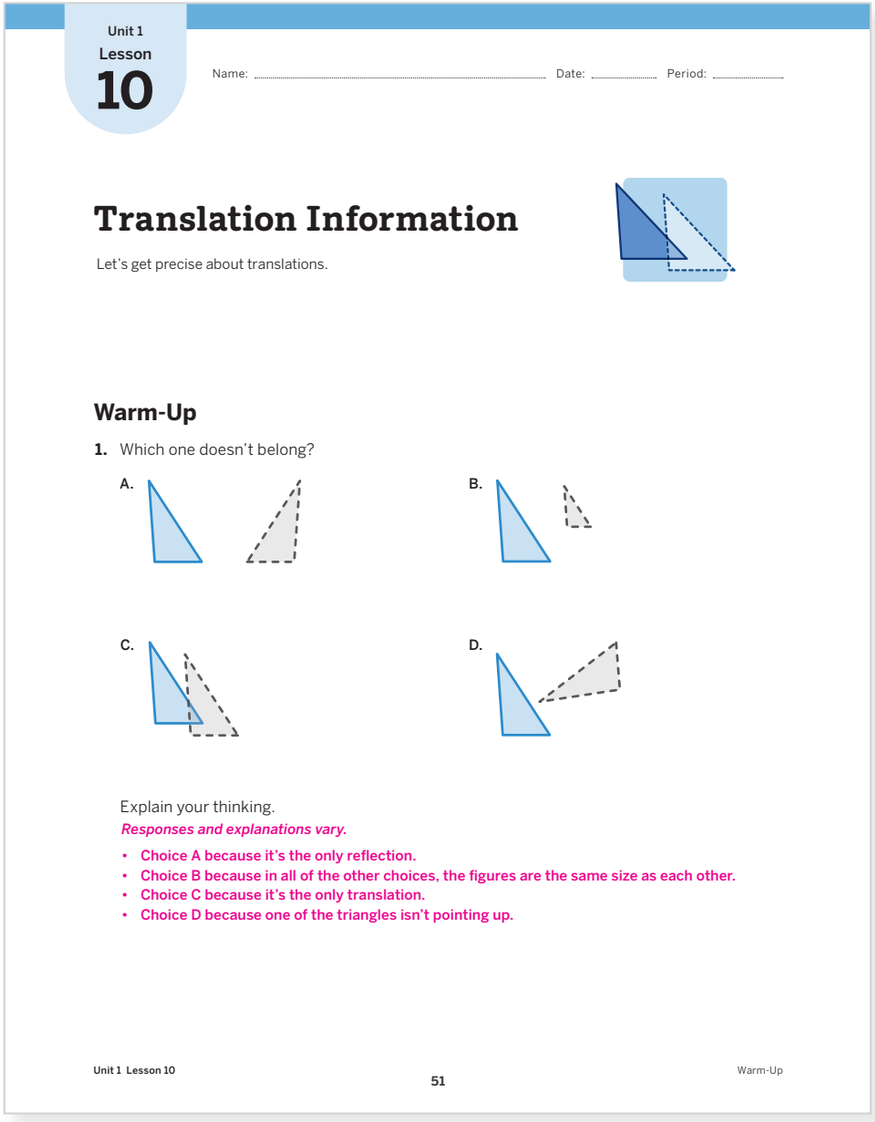
Invite students to share at least one reason why each transformation doesn't belong.

Create a record of why each transformation doesn't belong, along with the name of the student who shared the reasoning.

Consider asking:

- "How are the transformations similar to each other?"
- "Are there any of these transformations that had to use a reflection? A rotation? A translation?"

Note: If it doesn't come up naturally, consider asking, "Are there any of these that could not be done with a single transformation?"



Unit 1
Lesson
10

Name: _____ Date: _____ Period: _____

Translation Information

Let's get precise about translations.

Warm-Up

1. Which one doesn't belong?

A.

B.

C.

D.

Explain your thinking.

Responses and explanations vary.

- Choice A because it's the only reflection.
- Choice B because in all of the other choices, the figures are the same size as each other.
- Choice C because it's the only translation.
- Choice D because one of the triangles isn't pointing up.

Unit 1 Lesson 10 51 Warm-Up

Pairs | 20 min

Activity 1 Translate This!

Purpose: Students perform and analyze rigid transformations to explore the properties of translations.

Short on time? Consider omitting the discussion of reflections in Problems 6 and 7.

Lesson 10
Activity 1

Presentation Screen 2

1 Launch

To support making connections, consider asking, "What tools did you use to perform reflections that would be helpful for translations?" Consider discussing and defining **prime notation** using student language and the glossary.

Distribute a compass, straightedge, tracing paper, or other geometry tools to each student. Invite students to discuss with a partner before choosing what tools to use.

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to translate one point at a time.

2 Monitor

Look for students who use a variety of tools to translate the triangle. **(MP5)**

D **Differentiation** *(Problems 2–3)*

Look for students who:	Respond to Student Thinking
Draw the line segment from point D to point D, but need support to determine the distance and direction of the other two points.	**Support** Consider asking, "What can you use to construct a line segment of the same length? How can you construct something that would move the points in the same direction?"
Agree with Nakia's statement. *(Problem 3)*	**Support** Consider asking, "What other transformations result in a figure being the same size, but in different places?"
Use tracing paper to translate the triangle. *(Problem 2)*	**Strengthen** Consider asking, "How can you verify that you did not rotate your tracing paper at all when you translated?"
Use a compass and straightedge to translate the triangle. *(Problem 2)*	**Strengthen** Consider asking, "What construction strategies did you use to translate the triangle?"

ML/EL **Multilingual/English Learners** Invite students to highlight key phrases such as *translation*, *same size*, and *different places* to help them reach a conclusion about Nakia's claim. **(Reading)**

Pause briefly to display and discuss different strategies for precisely translating a triangle. If it doesn't come up, make sure to highlight how to use a compass to measure equal distances and ways to measure the same direction (e.g., using parallel lines or tracing paper).

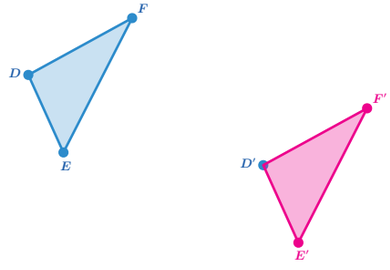
Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Translate This!

2. Translate $\triangle DEF$ so that point D goes to point D'. Label the vertices in the image using **prime notation**.



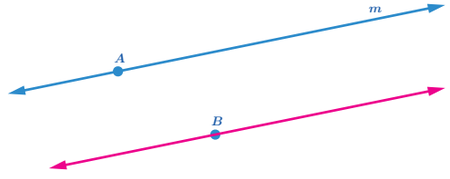
3. Nakia says that you can tell one figure is a translation of another figure because they are both the same size, but in different places. Is Nakia's thinking correct?

No

Explain your thinking.

Explanations vary. I disagree because the figure might be upside down or pointing in a different direction and still be the same size and in a new place, which is not just a translation.

4. Translate line m so that point A goes to point B.



5. Nakia says: *When a line is translated, the line and its image are parallel.* Is this statement *always*, *sometimes*, or *never* true? Circle one.

(Always) Sometimes Never

Explain your thinking.

Explanations vary. We don't turn the tracing paper when translating, so the new line has to be parallel.

Unit 1 Lesson 10 52 Activity 1

Pairs | 20 min

Activity 1 Translate This! (continued)

Purpose: Students perform and analyze rigid transformations to explore the properties of translations.

Short on time? Consider omitting the discussion of reflections on Problems 6 and 7.

Lesson 10
Activity 1

Presentation Screen
2

2 Monitor

Invite students to construct the translated line and to analyze Nakia's statement in Problem 4. Consider asking:

- "If a new point was drawn on line 'm' and its image was drawn on the translated line, what would the distance between the two points be?" It would be the length of segment AB.
- "How can we use translations to define *parallel lines*?" Parallel lines are lines where each point on one line is the same distance from each corresponding point on the line's image.

Look for strategies for making sense of the reflections and translations, specifically for students who use a variety of tools to make sense of their constructions. **(MP5)**

D Differentiation *(Problems 6–8)*

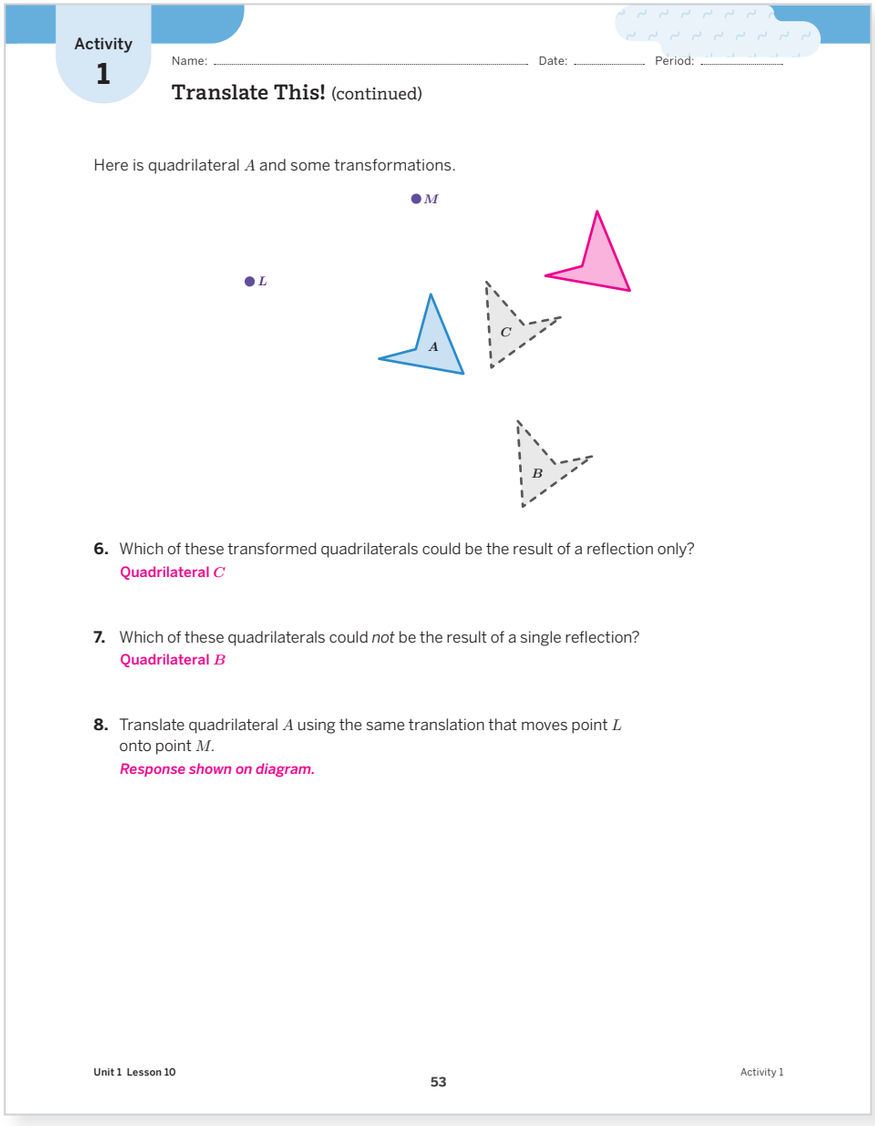
Look for students who:	Respond to Student Thinking
Need support with making sense of the reflections. *(Problems 6–7)*	**Support** Encourage students to fold their papers or to use tracing paper to see the reflections.
Need support getting started with the translation. *(Problem 8)*	**Support** Encourage them to use tracing paper to estimate a location for the translation. Then consider asking, "How can you use a compass to be more precise in your translation?"
Use a variety of tools to perform their translation precisely. *(Problem 8)*	**Strengthen** Consider asking, "How do each of your tools help you verify that you have translated each point the same distance and direction?"

3 Connect

Invite students to share their strategies for translating quadrilateral A. Encourage students to explain how they know each point on their image is the same direction and distance away from each corresponding point on the pre-image.

To surface the Key Takeaway, consider asking:

- "What are the two things we need to attend to when performing a translation?" Each point on the image needs to be the same distance away from the pre-image and in the same direction.
- "What are tools that we can use to be precise when performing translations?"



Activity 1

Name: _____ Date: _____ Period: _____

Translate This! (continued)

Here is quadrilateral A and some transformations.

6. Which of these transformed quadrilaterals could be the result of a reflection only?
   Quadrilateral C

7. Which of these quadrilaterals could *not* be the result of a single reflection?
   Quadrilateral B

8. Translate quadrilateral A using the same translation that moves point L onto point M.
   Response shown on diagram.

Unit 1 Lesson 10 53 Activity 1

Key Takeaway: A translation is a rigid transformation that moves every point on a geometric figure a given distance in a given direction. When performing translations with tracing paper, we need to make sure to not rotate the tracing paper.

Pairs | 10 min

Activity 2 Transformation Communication

Purpose: Students perform and describe transformations precisely. **(MP6)**

Short on time? Consider reducing the number of cards students complete.

Lesson 10
Activity 2

Presentation Screen
3

1 Launch

Display the instructions and review them as a class. Invite students to complete two cards from each set.

To generate excitement, consider sharing that students will take turns trying to construct an image by only following their classmate's instructions.

Distribute the Activity 2 Cards to each pair of students.

ML/EL **Multilingual/English Learners** Consider pairing students with partners who speak the same primary language and inviting them to describe the transformations in their primary language. **(Speaking and Listening)**

2 Monitor

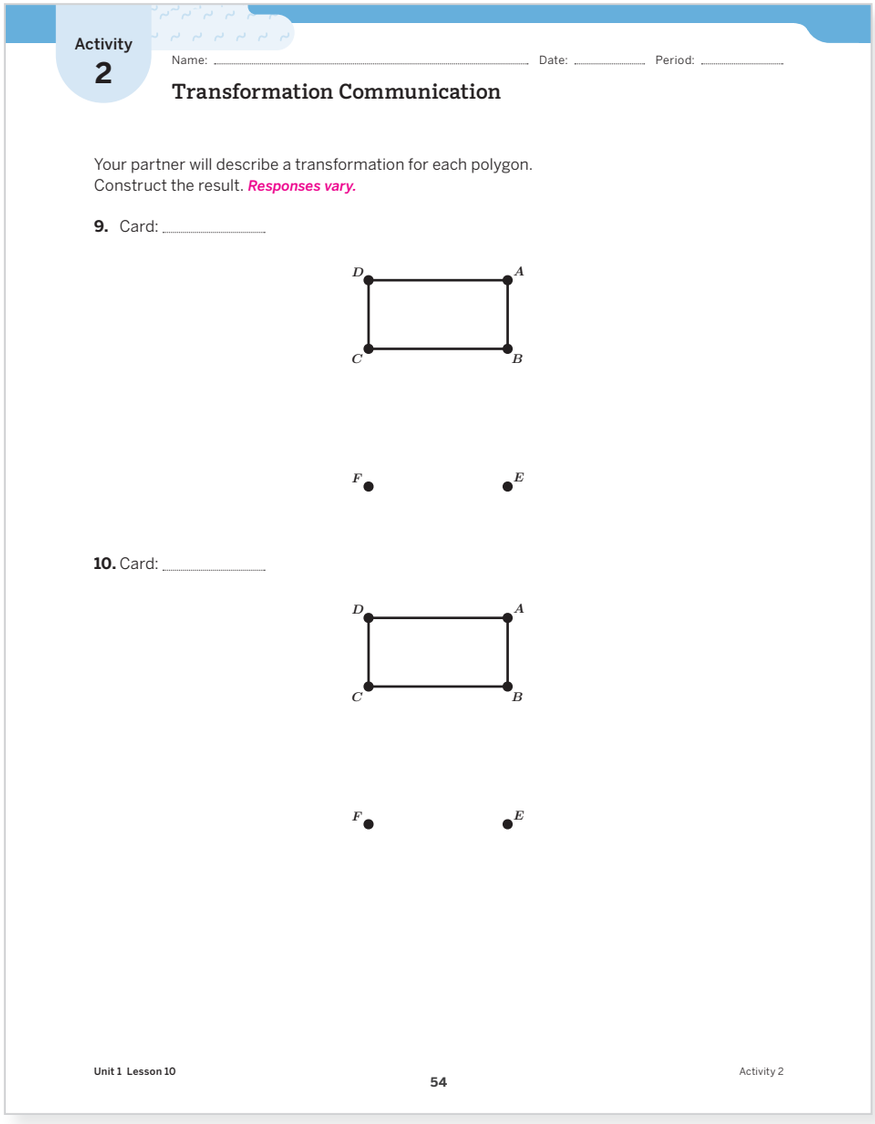
Listen for partner directions that include clear lines of reflection, distances and directions for translations, and clear labels for the transformed points.

Look for a variety of student strategies for constructing the rigid transformations.

D **Differentiation** *(Problems 9–10)*

Look for students who:	Respond to Student Thinking
Need support getting started with giving descriptions.	**Support** Encourage them to determine lines of reflection or distances and directions for translations and describe those first.
Need support understanding or following directions.	**Support** Encourage the direction giver to restate their description and be responsive to their partner's sketch.
Give the same descriptions for Cards 1 and 4.	**Support** Consider asking, "What is different about the labeling on the two cards?"
Describe Card 2 as a translation.	**Support** Consider asking, "What point would land on point F if the rectangle was translated down?"
Use tools to precisely translate or reflect rectangle $ABCD$.	**Stretch** Celebrate their strategies. Invite them to see if they can use another tool or strategy to construct the transformation.

Activity 2 continued ›



Activity 2

Name: _____ Date: _____ Period: _____

Transformation Communication

Your partner will describe a transformation for each polygon. Construct the result. ***Responses vary.***

9. Card: _____

10. Card: _____

Unit 1 Lesson 10 54 Activity 2

Pairs | 10 min

Activity 2 Transformation Communication (continued)

Purpose: Students perform and describe transformations precisely. **(MP6)**

Short on time? Consider reducing the number of cards students complete.

Lesson 10
Activity 2

Presentation Screen
3

2 Monitor

D Differentiation *(Problems 11–12)*

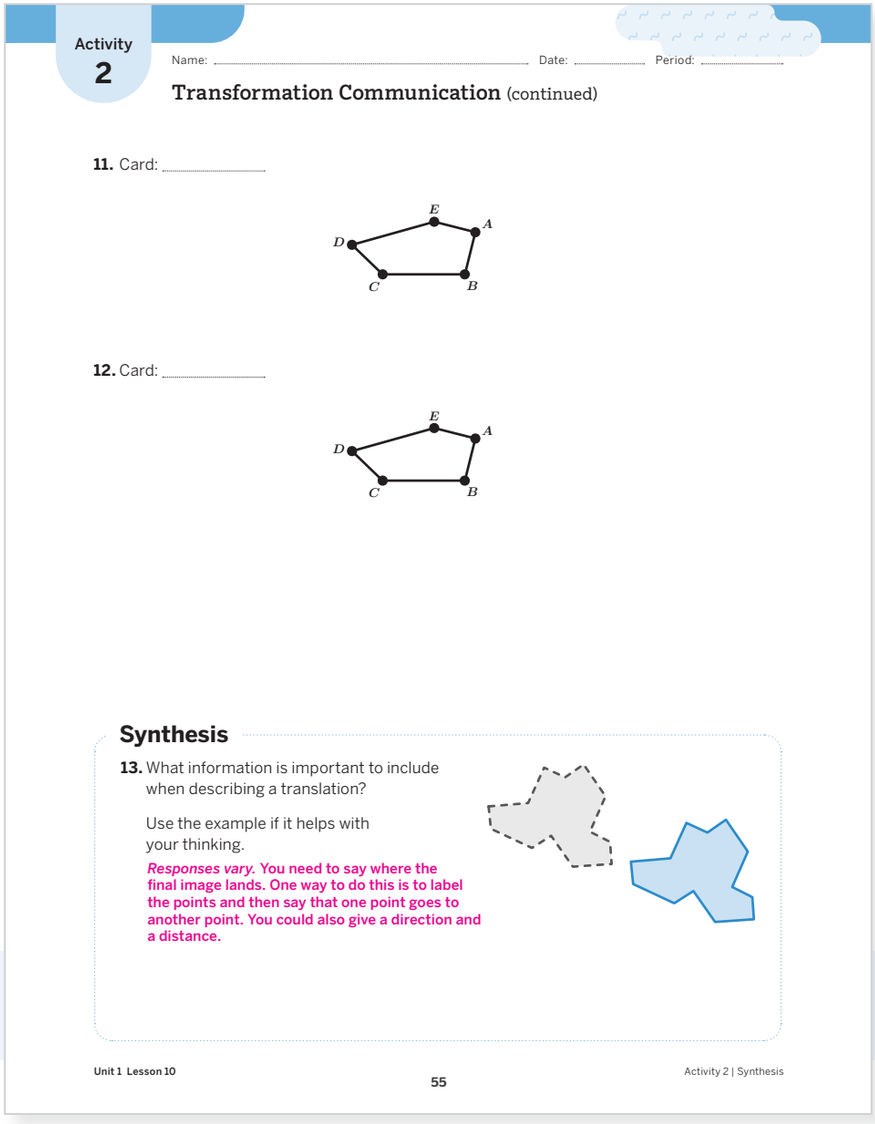
Look for students who:	Respond to Student Thinking
Use tracing paper to perform the transformations.	**Stretch** Consider asking, "How can you use a compass and straightedge to verify your transformations?"
Use precise labeling for their transformed images.	**Stretch** Consider asking, "Why is it important to label the images after you have performed a transformation?"

Math Identity and Community Encourage students to celebrate clear instructions from a partner or describe how a partner supported their understanding of transformations.

3 Connect

Consider asking:

- "Why is using precise language important for Cards 1 and 4?"
- "What sets of instructions were helpful to precisely perform the transformations?"
- "How can you quickly tell which transformations were translations or reflections?"



Activity 2

Name: _____ Date: _____ Period: _____

Transformation Communication (continued)

11. Card: _____

12. Card: _____

Synthesis

13. What information is important to include when describing a translation?

Use the example if it helps with your thinking.

Responses vary. You need to say where the final image lands. One way to do this is to label the points and then say that one point goes to another point. You could also give a direction and a distance.

Unit 1 Lesson 10 55 Activity 2 | Synthesis

Synthesis Whole Class | 5 min

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- The direction and distance of a translation.
- The starting point and endpoint of a translation (like point A goes to point A).
- It's often helpful to label the points on the image and pre-image using prime notation to show the relationship.

Create an anchor chart that outlines the important properties of translations.

Lesson Takeaway: A translation is a rigid transformation that moves each point on the pre-image to another point on the image that is in the same direction and distance away. It is important to be precise when describing transformations and often helpful to use **prime notation** to show which points ended up in what locations.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding of translating a triangle.

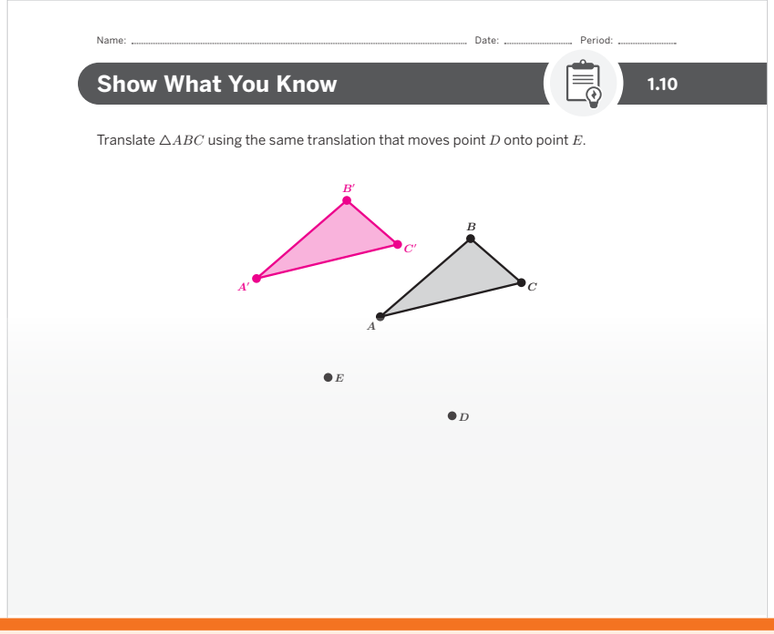


Presentation Screen 5

Today's Goals

1. **Goal:** Translate figures precisely using a variety of tools.
2. **Language Goal:** Describe translations and reflections precisely. (**Speaking and Listening**)

Show What You Know (Assessment Resources)


Name: Date: Period:

Show What You Know 1.10

Translate $\triangle ABC$ using the same translation that moves point D onto point E.

Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of translating a triangle, consider:

- Inviting students to revisit their strategies from **Problems 9–12**.
- Inviting students to complete the **Try This** problem and providing individualized feedback on their translation.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding** with minor errors, consider:

- Assigning the **Lesson Practice**.
- Inviting students to review the **Summary** and complete the **Try This** problem.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Encouraging them to complete the **DOK3 Practice Problem**.
- Inviting students to explore translating images using digital construction tools.

Professional Learning

In what ways did students' work in Activity 1 surprise you today? How did students think about *translations* in unexpected, delightful, or new ways?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 10
Practice

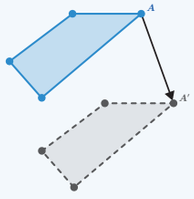
Students using digital

Lesson Summary

A translation is a type of rigid transformation that is defined by two points, a starting point and an endpoint. When you translate a starting point, A, to an endpoint, A', all other points in the translated figure move the same distance and in the same direction.

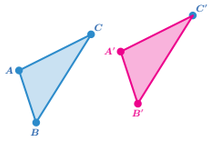
Translations move a figure without spinning or rotating. The translated figure is the same shape and size as the original figure but in a different location. When you translate a line, the image is a line parallel to the original.

When translating a figure, it's helpful to label translated points with **prime notation**, like A' or B', which are read out loud as A *prime* or B *prime*. This notation helps to show how the image relates to the pre-image.



Try This

Translate $\triangle ABC$ so that point C goes to point C'.



Unit 1 Lesson 10 56 Lesson Summary | Try This

Lesson Practice
Geo.1.10

Name: _____ Date: _____ Period: _____

1. Which one of these transformations cannot represent a *single* translation?

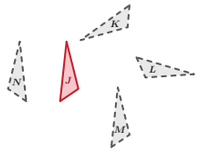
A. 

B. 

C. 

D. 

Problems 2–5: Here are four transformations of triangle J. Which of these images is the result of:



2. A single *translation* only?
Triangle M

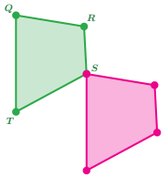
3. A single *reflection* only?
Triangle N

4. Both a single translation and a single reflection?
Triangle K

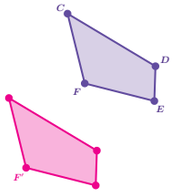
5. A single transformation that is not a translation or reflection?
Triangle L

Problems 6–7: Translate each quadrilateral so that:

6. Point Q goes to point S.



7. Point F goes to point F'.

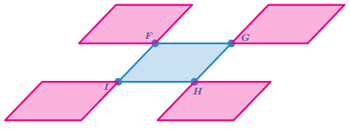


Unit 1 Lesson 10 57 Lesson Practice

Lesson Practice
Geo.1.10

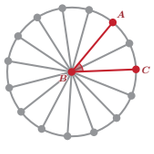
Name: _____ Date: _____ Period: _____

8. Construct translations of parallelogram $FGHI$ to create a final design that looks the same with a rotation but not a reflection. ***Responses vary.***

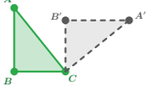


Spiral Review

9. A circle is divided into 15 equal pieces. What is the measure of angle ABC in degrees?
48°



10. Triangle ABC was rotated 270°. If angle $ABC = 90°$, what is the measure of angle $A'B'C'$?
90°



Reflection

1. Put a heart next to the problem you're most proud of.

2. Use this space to ask a question or share something you're proud of.

Unit 1 Lesson 10 58 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
Test Practice	1	1	HSG.CO.A.5
	2–5	2	HSG.CO.A.5
	6–7	2	HSG.CO.A.5
	8	3	HSG.CO.A.5
Spiral Review			
Fluency	9	1	4.MD.C.5.B
	10	1	8.G.A.1

Need More Practice?

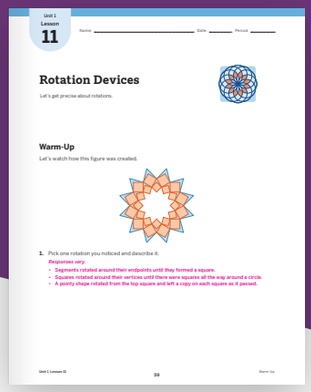
Additional Practice resources coming soon.

Unit 1

Lesson

11

This lesson uses print student pages and digital Teacher Presentation Screens.



Rotation Devices

Defining Rotations

Let's get precise about rotations.

Focus and Coherence

Today's Goals

1. **Goal:** Rotate figures precisely using a variety of tools.
2. **Language Goal:** Describe rotations using a center of rotation and the relationships between points. **(Writing and Speaking)**

Students perform rotations using predetermined points and give each other feedback to write precise directions. They then perform rotations and reflections and compare their transformations to see if they result in the same image. Throughout, students indicate *centers of rotation* and precisely describe the amount of rotation using prime notation.

Prior Learning

In Lessons 9 and 10, students used a variety of tools to perform reflections and translations.

Future Learning

In Lesson 12, students will perform sequences of transformations digitally. In Unit 2, students will use these translations, rotations, and reflections as the foundation for geometric proof.

Rigor and Balance

- Students **apply** and extend their understanding of constructions as they use them to perform precise transformations.
- Students develop **strategic competence** as they justify the properties of rotated images using constructions.

Vocabulary

Review Vocabulary

center of rotation

Standards

Addressing

HSG.CO.A.5

Given a geometric figure and a rotation, **reflection**, or translation, **draw the transformed figure using, e.g., graph paper, tracing paper**, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

HSG.CO.A.4

Develop definitions of rotations, **reflections**, and translations **in terms of angles**, **circles**, **perpendicular lines**, parallel lines, **and line segments**.

Also Addressing: **HSG.CO.A.2**

Mathematical Practices: MP5, MP6

Building On	Building Toward
8.G.A.3	HSG.CO.A.5
	HSG.CO.B.6

Lesson at a Glance

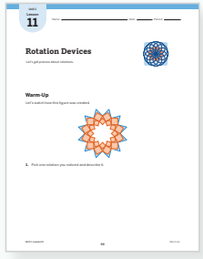
~ 45 min

Standards: HSG.CO.A.5, HSG.CO.A.4, HSG.CO.A.2

Warm-Up

Independent | 5 min

Students analyze an animation of rotated images to discuss how different *centers of rotation* impact an image.



Lesson 11
Warm-Up

Presentation Screen 1

Activity 1

Pairs | 15 min

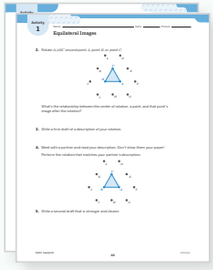
Students perform, describe, and revise rotations of a triangle to explore how to precisely describe a rotation.

MLR1: Stronger and Clearer Each Time (MP6)

Key Takeaway: When describing a rotation, two things must be identified:

- A *center of rotation*.
- How much each point is rotated around the center.

A helpful way to describe how much a point is rotated is to describe the point on the image it is supposed to rotate onto (e.g., "Rotate point A so that it goes to point B.").



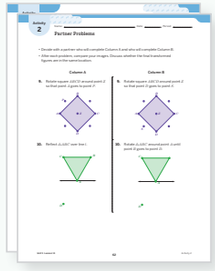
Lesson 11
Activity 1

Presentation Screens 2–3

Activity 2

Pairs | 15 min

Students perform rotations and reflections and compare their transformations to see if the points end up in the same location.



Lesson 11
Activity 2

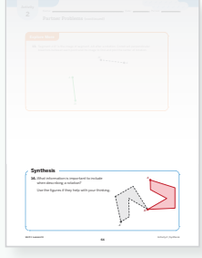
Presentation Screen 4

Synthesis

Whole Class | 5 min

Students synthesize the information needed to describe a rotation.

Lesson Takeaway: A rotation is a rigid transformation that moves each point on the pre-image around a *center of rotation* over a specified distance to a point on the image. Each corresponding point on the image and pre-image will be the same distance from the center of rotation.



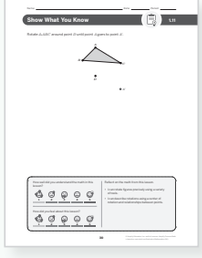
Lesson 11
Synthesis

Presentation Screen 5

Show What You Know

Independent | 5 min

Students demonstrate their understanding by rotating a triangle.



Lesson 11
Show What You Know

Presentation Screen 6

Prep Checklist

Invite students to use their Student Edition. Display the Presentation Screens to help facilitate the lesson.

Consider making the geometry toolkit available to support students in selecting and using tools strategically.

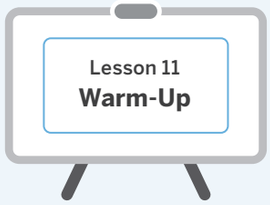
This lesson includes:

- Presentation Screens
- Student Edition
- Show What You Know
- geometry toolkit: compasses, straightedges, tracing paper
- coloring tools or highlighters (optional)

Independent | 5 min

Warm-Up

Purpose: Students analyze an animation of rotated images to discuss how different *centers of rotation* impact an image.



Presentation Screen 1

1 Launch

Play the animation. To encourage creativity, invite students to try to pick a rotation that no one else in the class will describe.

Note: If needed, consider playing the animation again after students have picked their rotation to support their descriptions.

ML/EL **Multilingual/English Learners** Invite students to talk through their thinking with a partner before writing it down. **(Writing and Speaking)**

Invite students to share their descriptions with a partner. Listen for students who discuss that the *center of rotation* is on the vertices of the square, is in the center of the shape, or is on the endpoints of the segments.

2 Connect

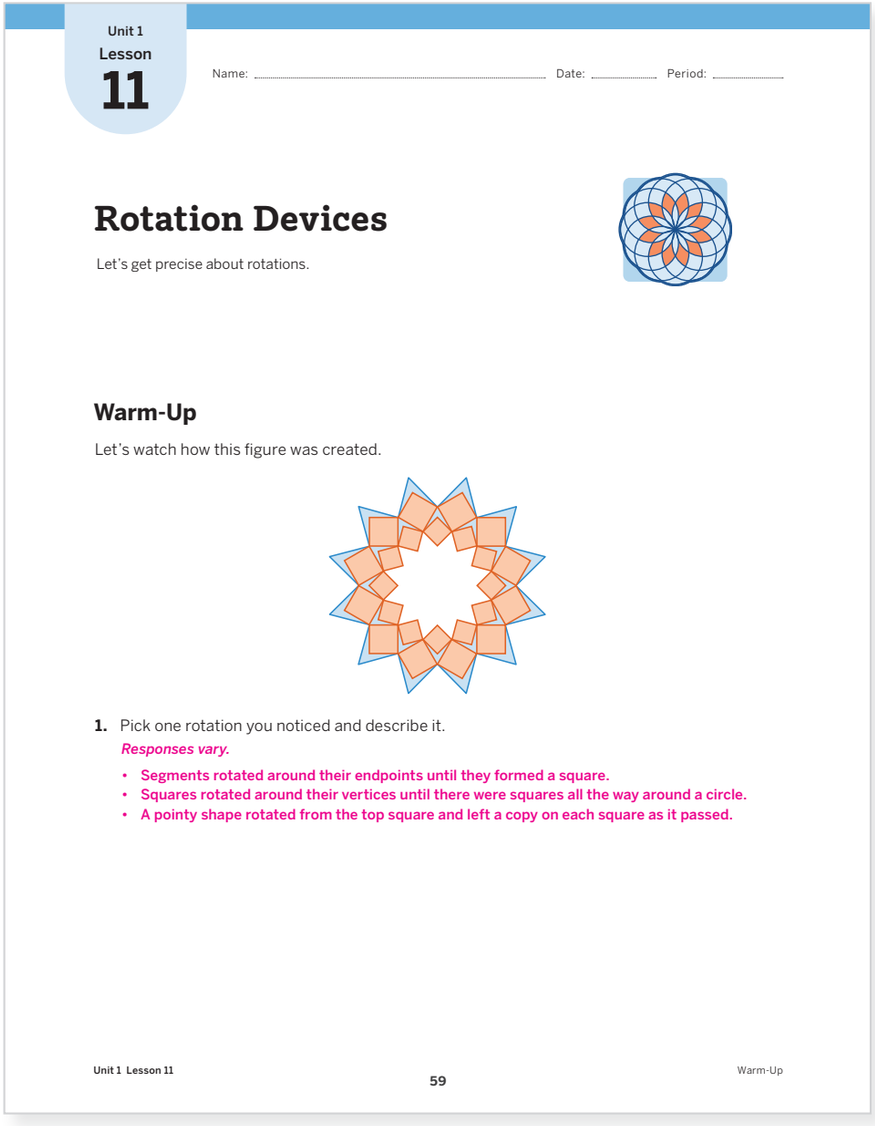
Invite students to share the rotations they noticed.

Consider asking:

- "What do each of these rotations (segment, square, and pointy quadrilateral) have in common?" They each rotated around a point.
- "How much did each line segment rotate to create an individual square at the beginning?" Each had to do a quarter-turn or rotate until the two segments formed a right angle.

MLR **MLR2: Collect and Display** (optional) Consider using this routine to create a class definition for the *center of rotation* using students' language. Display any new language, examples, or diagrams for students to reference later.

Math Identity and Community Encourage students to celebrate a classmate whose explanation of a rotation helped them see the figure in a new way.



Unit 1 Lesson 11

Name: Date: Period:

Rotation Devices

Let's get precise about rotations.

Warm-Up

Let's watch how this figure was created.

1. Pick one rotation you noticed and describe it.

Responses vary.

- Segments rotated around their endpoints until they formed a square.
- Squares rotated around their vertices until there were squares all the way around a circle.
- A pointy shape rotated from the top square and left a copy on each square as it passed.

Unit 1 Lesson 11 59 Warm-Up

Pairs | 15 min

Activity 1 Equilateral Images

Purpose: Students perform, describe, and revise rotations of a triangle to explore how to precisely describe a rotation. **(MP6)**

Lesson 11
Activity 1

Presentation Screens
2–3

1 Launch

2 **Display** the triangle and points. To surface connections, consider asking, "How will the points be helpful when describing transformations?"

2 Monitor

Invite students to discuss the relationship between the center of rotation, a point, and that point's image after the rotation. Consider asking, "How can a compass support us in representing a rotation?"

MLR **MLR1: Stronger and Clearer Each Time** Invite students to write a first draft and use feedback to make their thinking stronger and clearer. **(MP6)**

Consider sharing that first drafts don't have to be complete or perfect to be valuable.

Invite students to share their drafts with a partner and give each other feedback. Repeat this partner feedback process for as many rounds as there is time for.

Encourage students to use the feedback they got from their partners to write a second draft that is stronger, with more or better evidence, or clearer (with more precision, organization, etc.).

Listen for student explanations that include a clear center of rotation and directions for rotating one point to another.

D **Differentiation** *(Problems 3–5)*

Look for students who:	Respond to Student Thinking
Need describing the amount to rotate.	**Support** Invite them to identify a point and its image after the rotation. Consider asking, "How can you use those two points to describe the amount of rotation?"
Describe a rotation using an angle measure (like rotate triangle ABC by 60°).	**Strengthen** Celebrate their thinking of determining an angle measure. Consider asking, "How could you describe your transformation if you didn't know the angle measure?"

Pause to have students share their second draft. Consider asking, "What was a helpful piece of feedback you want to remember when describing rotations?"

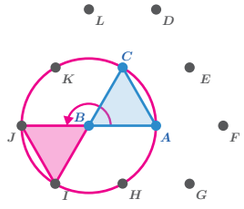
Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Equilateral Images

2. Rotate $\triangle ABC$ around point A, point B, or point C. ***Responses vary.***



What's the relationship between the *center of rotation*, a point, and that point's image after the rotation?

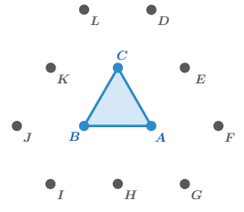
Responses vary. **Each point and its image are the same distance from the center of rotation.**

3. Write a first draft of a description of your rotation.

Responses vary. **I rotated around a center of rotation at point B.**

4. Meet with a partner and read your description. Don't show them your paper!

Perform the rotation that matches your partner's description. ***Responses vary.***



5. Write a second draft that is stronger and clearer.

Responses vary. **I rotated triangle ABC around point B so that point C goes to point I.**

Unit 1 Lesson 11 60 Activity 1

Pairs | 15 min

Activity 1 Equilateral Images (continued)

Purpose: Students perform, describe, and revise rotations of a triangle to explore how to precisely describe a rotation. **(MP6)**

Lesson 11
Activity 1

Presentation Screens 2–3

2 Monitor

Invite students to construct the image of triangle ABC. To support students getting started, consider asking, "What tools are helpful to visualize a center of rotation, and what points could be on the image of the rotation?"

A **Accessibility: Visual-Spatial Processing** Invite students to use tracing paper and different colors to represent different vertices being rotated to increase access to the task.

Capture the strategies and tools students use to perform a rotation, as well as descriptions that are precise with the center of rotation and amount to rotate.

D **Differentiation** *(Problems 6–7)*

Look for students who:	Respond to Student Thinking
Need support constructing a rotated image. *(Problem 6)*	**Support** Consider asking, "What tools could help you visualize rotating around a center?"
Need support describing their rotation. *(Problem 7)*	**Support** Consider asking, "What are the two properties you need to include in a description of a rotation?"
Construct circles centered at the center of rotation that go through each vertex of the triangle. *(Problem 6)* **(MP5)**	**Strengthen** Celebrate their thinking. Consider asking, "How do you know that the image after a rotation will be on one of the circles?"
Describe the rotation precisely with a center of rotation and amount to rotate. *(Problem 7)* **(MP6)**	**Stretch** Invite them to write a sentence frame for describing rotations and to share during the Connect.

3 Connect

Invite students to share their rotation descriptions. Display the presentation screen and follow along with their descriptions.

To surface the Key Takeaway, consider asking:

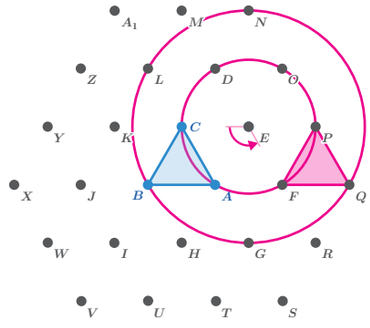
- "What are the two properties that must be determined to precisely describe a rotation?"
- "How do compasses (or circles) help us make sense of a rotation?" The center of rotation is the center of the circle. When I construct a circle with a vertex on the circle, then I know the vertex of the rotated image will also be on the circle.

Activity 1

Name: _____ Date: _____ Period: _____

Equilateral Images (continued)

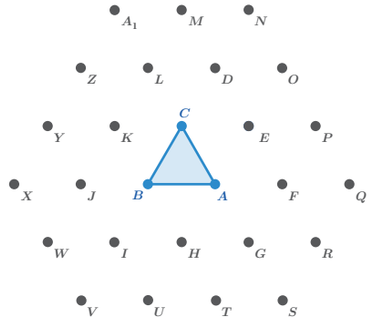
6. Rotate $\triangle ABC$ around any point *except* points A, B, or C. Make sure every vertex in the image is on a labeled point. ***Responses vary.***



7. Describe your rotation.
Responses vary. **I rotated $\triangle ABC$ around point E so that point C goes to point F.**

8. Meet with a partner and read your description. Don't show them your paper!

Perform the rotation that matches your partner's description. ***Responses vary.***



Unit 1 Lesson 11 61 Activity 1

Key Takeaway: When describing a rotation, two things must be identified:

- A *center of rotation*.
- How much each point is rotated around the center.

A helpful way to describe how much a point is rotated is to describe the point on the image it is supposed to rotate onto (e.g., "Rotate point A so that it goes to point B.").

Pairs | 15 min

Activity 2 Partner Problems

Purpose: Students perform rotations and reflections and compare their transformations to see if the points end up in the same location.

Short on time? Consider omitting Problem 11.

Lesson 11
Activity 2

Presentation Screen 4

1 Launch

Display the instructions on the screen. Remind students that they should compare their images to their partner's and discuss any differences they see.

Encourage students to use a variety of tools and strategies to construct each transformation.

ML/EL **Multilingual/English Learners** Consider modeling or inviting a student to model strategies for annotating the text to make sense of the different properties of rotations and reflections. **(Reading)**

2 Monitor

Look for students labeling points on the image using prime notation and listen for discussions about what is similar or different in the final image.

D **Differentiation** *(Problems 9–10)*

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 9)*	**Support** Encourage them to use their tools (i.e., tracing paper, compass, or straightedge) to make sense of the rotation.
Construct a circle centered at point Z to make sense of possible locations for the image of square $ABCD$. *(Problem 9)*	**Strengthen** Consider asking, "How does a circle help you to determine the image points?"
Use prime notation to label the transformed points on the image. *(Problems 9–10)*	**Strengthen** Consider asking, "How does prime notation help you to resolve any differences between you and your partner?"
Transform the triangle correctly, but think they have ended up in the same location as their partner. *(Problem 10)*	

Pause briefly to discuss strategies for rotations and how students are labeling their images. Invite students to share how they resolved any differences between their transformation and their partners.

Math Identity and Community Identify a student whose brilliance has not been recognized recently and celebrate their thinking publicly.

Activity 2 continued >

Activity 2

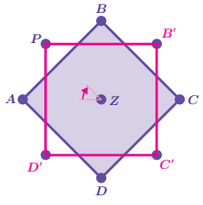
Name: _____ Date: _____ Period: _____

Partner Problems

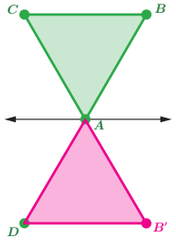
- Decide with a partner who will complete Column A and who will complete Column B.
- After each problem, compare your images. Discuss whether the final transformed figures are in the same location.

Column A

9. Rotate square $ABCD$ around point Z so that point A goes to point P.

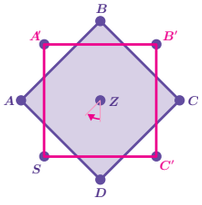


10. Reflect $\triangle ABC$ over line l.

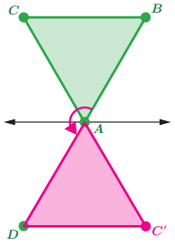


Column B

9. Rotate square $ABCD$ around point Z so that point D goes to point S.



10. Rotate $\triangle ABC$ around point A until point B goes to point D.



Unit 1 Lesson 11 62 Activity 2

Pairs | 15 min

Activity 2 Partner Problems (continued)

Purpose: Students perform rotations and reflections and compare their transformations to see if the points end up in the same location.

Short on time? Consider omitting Problem 11.

Lesson 11
Activity 2

Presentation Screen
4

2 Monitor

Listen for students who conjecture about the relationship between reflections and rotations.

D Differentiation *(Problems 11–12)*

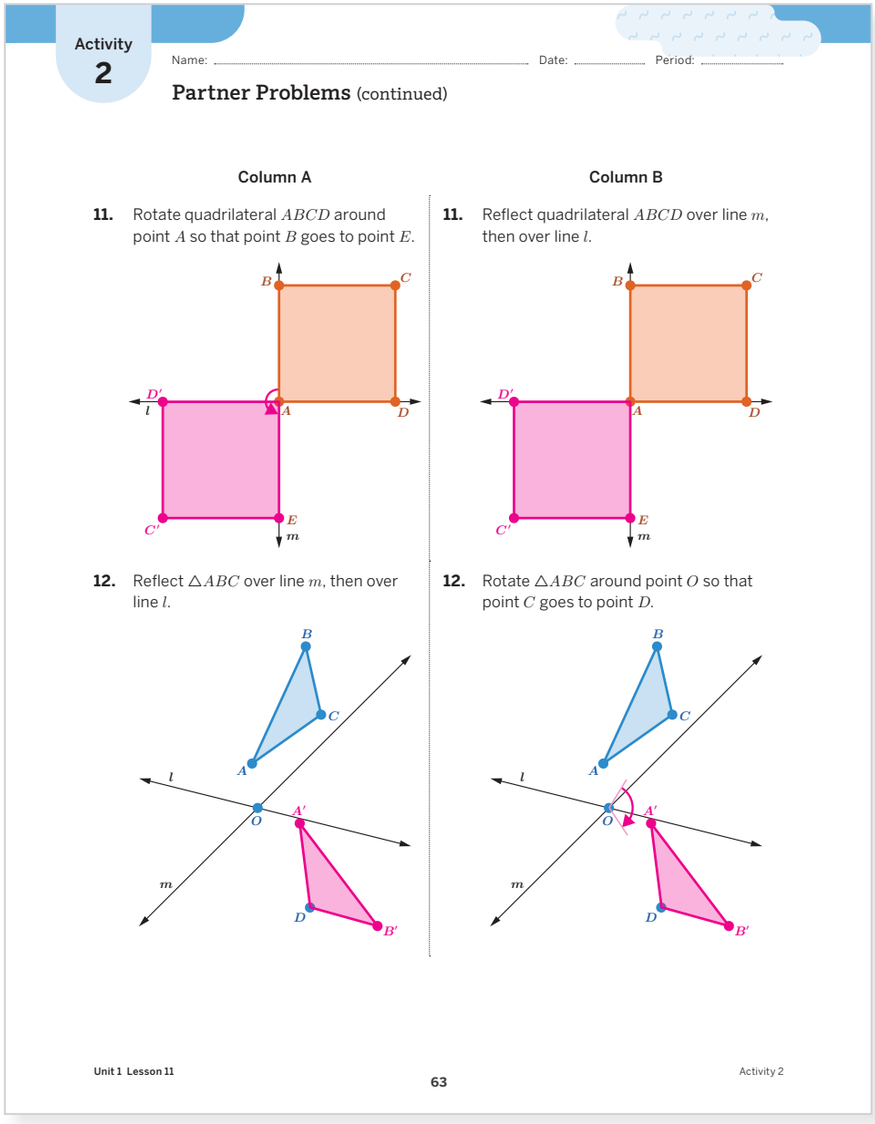
Look for students who:	Respond to Student Thinking
Need support performing the sequence of transformations.	**Support** Encourage them to perform one transformation at a time.
End up with different images from their partner.	**Support** Encourage them to describe each transformation one at a time and critique their partner's reasoning.
Make the conjecture that two reflections can be the same as one rotation.	**Stretch** Celebrate their thinking. Encourage them to become more specific in their conjecture by asking, "Where would the center of rotation be for your rotation so that it matches the two reflections?"

3 Connect

Invite students to share what they noticed between each of the columns. Consider asking:

- "What is the relationship between each point on the image, its corresponding point on the pre-image, and the center of rotation (i.e., between points A, A', and O in Problem 12)?" The points on the pre-image and image are the same distance away from the center of rotation because they lie on the same circle that is centered at O.
- "Why did you and your partner end up with the same exact image in some of the problems?" Two reflections can sometimes end up with the image being in the same location as one rotation. A reflection and a rotation don't always produce the same image.

Activity 2 continued >



Activity 2

Name: _____ Date: _____ Period: _____

Partner Problems (continued)

Column A

11. Rotate quadrilateral $ABCD$ around point A so that point B goes to point E.

12. Reflect $\triangle ABC$ over line m, then over line l.

Column B

11. Reflect quadrilateral $ABCD$ over line m, then over line l.

12. Rotate $\triangle ABC$ around point O so that point C goes to point D.

Unit 1 Lesson 11 63 Activity 2

Pairs | 15 min

Activity 2 Partner Problems (continued)

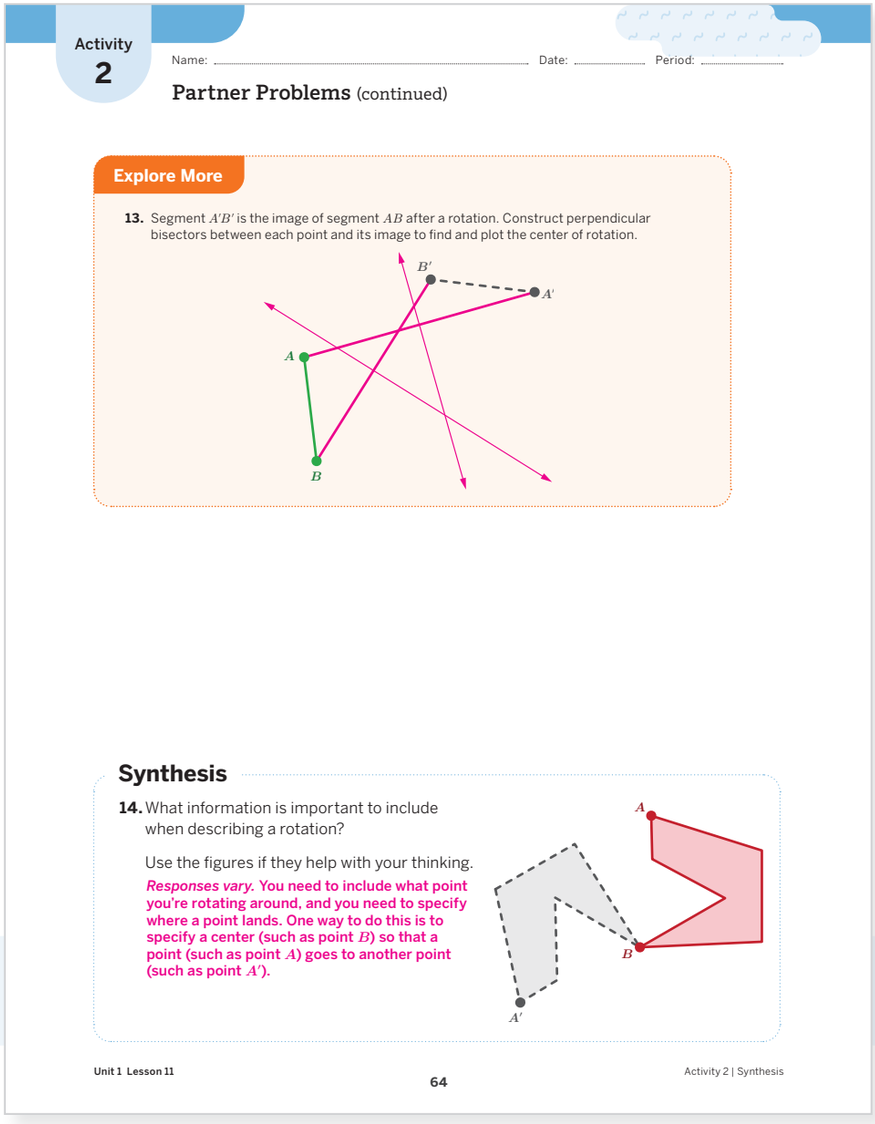
Purpose: Students perform rotations and reflections and compare their transformations to see if the points end up in the same location.

Short on time? Consider omitting Problem 11.

Lesson 11
Activity 2

Presentation Screen 4

D **Differentiation: Explore More (Stretch)** Invite students who would like to further explore the relationship between rotations and reflections to complete this optional task. Encourage them to discuss their thinking with a partner.



Activity 2

Name: _____ Date: _____ Period: _____

Partner Problems (continued)

Explore More

13. Segment $A'B'$ is the image of segment AB after a rotation. Construct perpendicular bisectors between each point and its image to find and plot the center of rotation.

Synthesis

14. What information is important to include when describing a rotation?

Use the figures if they help with your thinking.

Responses vary. You need to include what point you're rotating around, and you need to specify where a point lands. One way to do this is to specify a center (such as point B) so that a point (such as point A) goes to another point (such as point A').

Unit 1 Lesson 11 64 Activity 2 | Synthesis

Synthesis

Whole Class | 5 min

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- The center of rotation is important to define. We can construct rotations by centering circles at the center of rotation so that they go through important points on the pre-image and image.
- One way to communicate how much an image rotates is to describe the path a point will take from the pre-image to the image. In the example, we could say, "so that point A goes to point A."

Create an anchor chart that outlines the important properties of rotations.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.

Lesson Takeaway: A rotation is a rigid transformation that moves each point on the pre-image around a *center of rotation* over a specified distance to a point on the image. Each corresponding point on the image and pre-image will be the same distance from the center of rotation.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by rotating a triangle.

Lesson 11
Show What You Know

Presentation Screen
6

Today's Goals

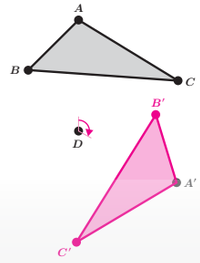
1. **Goal:** Rotate figures precisely using a variety of tools.
2. **Language Goal:** Describe rotations using a center of rotation and the relationships between points. **(Writing and Speaking)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.11

Rotate $\triangle ABC$ around point D until point A goes to point A'.



Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of how to rotate a triangle around a point, consider:

- Inviting students to complete the **Try This** problem and providing feedback on their rotation.
- Inviting students to revisit their strategies and try their partner's problems from **Problems 9–12**.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding** with minor errors, consider:

- Assigning the **Lesson Practice**.
- Inviting students to review the **Summary** and complete the **Try This** problem.
- Inviting students to revise their work using a compass. Encourage them to make adjustments based on the distance from the center of rotation.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting students to determine precisely when two reflections would be the same as a rotation.
- Inviting them to complete the **Explore More** task and discuss their thinking with a partner.
- Inviting students to explore transforming images using digital construction tools.

Professional Learning

Take a moment to reflect on this lesson. How did the **Stronger and Clearer Each Time** routine support students in describing rotations precisely? How might you facilitate it differently in the future?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 11
Practice

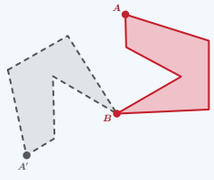
Students using digital

Lesson Summary

A rotation is a rigid transformation that moves all of the points in a figure around a given point called the *center of rotation*. The pre-image points and their images are always the same distance from the center of rotation.

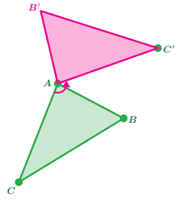
When a figure is rotated, the pre-image and the image have the same size and shape.

We can describe this rotation by saying "rotate the figure around point B so that point A goes to point A'."



Try This

Rotate $\triangle ABC$ around point A until point C goes to point C'.

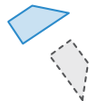


Unit 1 Lesson 11 65 Lesson Summary | Try This

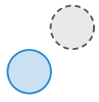
Lesson Practice
Geo.1.11

Name: _____ Date: _____ Period: _____

1. Which of these transformations could *not* be a single rotation?

A. 

B. 

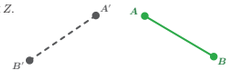
C. 

D. 

Problems 2–3: Rudra tried to rotate $\overline{AB}$ around point Z.

2. What is one thing Rudra did well?

Responses vary. His approach kept the lengths of $\overline{AB}$ and $\overline{A'B'}$ the same.

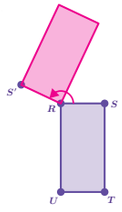


3. How could Rudra improve his rotation?

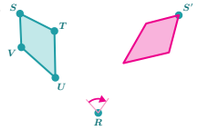
Responses vary. It looks like $\overline{AB'}$ is the result of a reflection. He should change the label of A' to B' and B' to A'.

Problems 4–5: Rotate each quadrilateral around point R so that point S goes to S'.

4.



5.



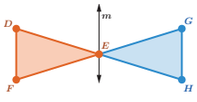
Unit 1 Lesson 11 67 Lesson Practice

Lesson Practice
Geo.1.11

Name: _____ Date: _____ Period: _____

6. Describe two different single transformations that move one of these triangles onto the other.

- Reflect $\triangle DEF$ over line m.
- Rotate $\triangle DEF$ until point F goes to point G.

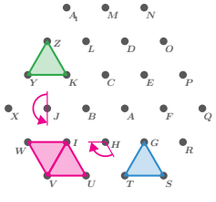


Problems 7–8: Use the grid.

7. Rotate each triangle *exactly once* to create the outline of a parallelogram. *Responses vary.*

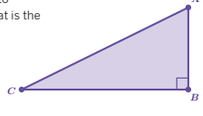
8. Fill in the blanks to describe your rotations.
Responses vary.
Rotate $\triangle YZK$ around point ___J___ until point Z goes to point ___V___.

Rotate $\triangle GST$ around point ___H___ until point T goes to point ___I___.



Spiral Review

Problems 9–10: Here is right triangle ABC. Describe how to use only rotations or only reflections to create an image that is the outline of a rectangle when combined with triangle ABC.



9. Description using only rotations:

Responses vary. Rotate around the midpoint of $\overline{AC}$ so that point A goes to point C.

10. Description using only reflections:

Responses vary. Reflect over the perpendicular bisector of $\overline{BC}$, then reflect over the perpendicular bisector of $\overline{AB}$.

Reflection

1. Put a star next to a problem you want to understand better.
2. Use this space to ask a question or share something you're proud of.

Unit 1 Lesson 11 68 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
Test Practice	1	2	HSG.CO.A.5
	2–3	2	HSG.CO.A.4
Fluency	4–5	2	HSG.CO.A.5
	6	2	HSG.CO.A.5
	7–8	3	HSG.CO.A.5
Spiral Review			
	9–10	2	8.G.A.2

Need More Practice?

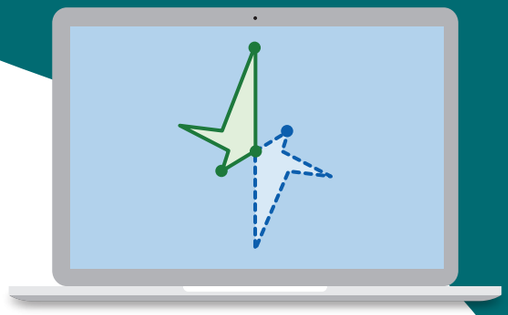
Additional Practice resources coming soon.

Unit 1

Lesson 12

Digital Lesson

This lesson uses digital student screens. Student Notes are also available.



Going Off the Grid, Part 2

Sequences of Transformations

Let's use geometry tools to perform sequences of rigid transformations that move one figure onto another.

Focus and Coherence

Today's Goals

1. **Goal:** Use geometry tools to perform a sequence of rigid transformations that moves one figure onto another.
2. **Language Goal:** Describe a sequence of rigid transformations that moves one figure onto another using mathematically precise language. **(Speaking and Listening)**

Students apply their understanding of translations, reflections, and rotations to specify and perform sequences of transformations that will move a figure onto another. Students use a variety of geometry tools to identify lines of reflection and centers of rotation precisely. When students decide what type of transformations to perform, they are considering the structures of pairs of congruent figures. **(MP5, MP6, MP7)**

Prior Learning

In Lesson 4, students constructed perpendicular bisectors. In Lesson 8, students used tracing paper to transform a figure onto another. In Lessons 9–11, students constructed transformations using physical tools on paper.

Future Learning

In Unit 2, students will use sequences of rigid transformations to prove two figures are congruent.

Rigor and Balance

- Students develop **conceptual understanding** of how to combine translations, reflections, and rotations to move one figure onto another.
- Students **apply** their understanding of perpendicular bisectors to construct lines of reflection.
- Students develop **strategic competence** as they decide which sequence of transformations will most efficiently move one figure onto another.

Standards

Addressing

HSG.CO.A.5

Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or **geometry software. Specify a sequence of transformations that will carry a given figure onto another**.

Also Addressing: **HSG.CO.A.2**

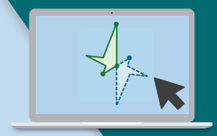
Mathematical Practices: MP5, MP6, MP7

Building On	Building Toward
HSG.CO.A.1	HSG.CO.A.2
HSG.CO.A.4	HSG.CO.B.6

Lesson at a Glance

~ 45 min

Standards: HSG.CO.A.5, HSG.CO.A.2



Why digital?

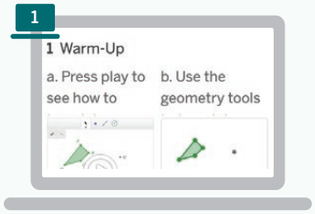
Students receive Responsive Feedback as they explore a sequence of transformations that will move one figure onto another.

Warm-Up

Pairs Sharing a Device | 5 min

Pacing: Screen 1

Students use the digital geometry tools to translate a polygon.

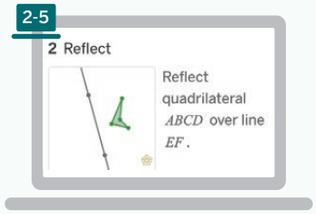


Activity 1

Pairs Sharing a Device | 10 min

Pacing: Screens 2–5

Students translate, reflect, and rotate polygons, and use the structure of polygons to transform one figure onto another. **(MP7)**



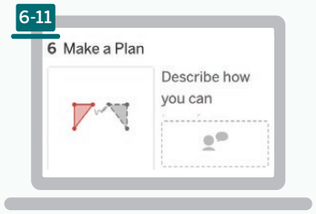
Activity 2

Pairs Sharing a Device | 20 min

Pacing: Screens 6–7, Screens 8–11

Students continue exploring digital geometry tools to perform and describe transformations. MLR7: Compare and Connect, MLR1: Stronger and Clearer Each Time **(MP5, MP6)**

Key Takeaway (Screen 11): Digital tools can help us visualize and revise transformations. We need to include precise details when specifying transformations, including the angle of rotation, center of rotation, and line of reflection. **(MP6, MP7)**



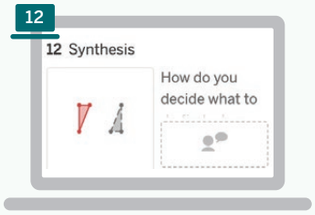
Synthesis

Whole Class | 5 min

Pacing: Screen 12

Students synthesize their understanding by describing strategies to transform one figure onto another using a sequence of transformations. MLR2: Collect and Display

Lesson Takeaway: There are often many ways to specify a sequence of transformations to move one figure onto another. Before you start, it's helpful to consider the structure of the image and pre-image, and make a plan.

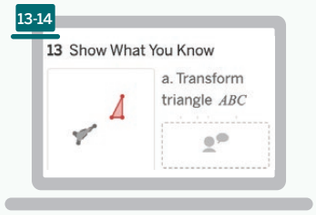


Show What You Know

Independent | 5 min

Pacing: Screens 13–14

Students demonstrate their understanding by performing and then describing a sequence of transformations to move one figure onto another.



Prep Checklist

Assign the digital student screens. Student Notes are also available.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)
- tracing paper (as needed)

Student Notes sheet
Student Edition:
Consider providing the Student Notes to support class discussion and student thinking.



Pairs Sharing a Device | 5 min | Pacing: Screen 1

Warm-Up

Purpose: Students use the digital geometry tools to translate a polygon.



Student Notes are also available

1 Launch

1 **To generate excitement**, consider sharing that additional digital geometry tools are available to explore. Invite students to view the video and share what they observed with a partner.

A **Accessibility: Visual-Spatial Processing** Invite students to pause the video several times and write down what is happening at each step to support interpreting visual representations.

To encourage playfulness, challenge students to experiment with the digital geometry tools so they can see how to perform different transformations.

Note: Students may benefit from a reminder that they can reset their work by clicking on the reset icon. Students will receive an animated star for performing the translation correctly.

2 Connect

Display the quadrilateral and invite students to share the steps they took to perform the translation. Consider asking:

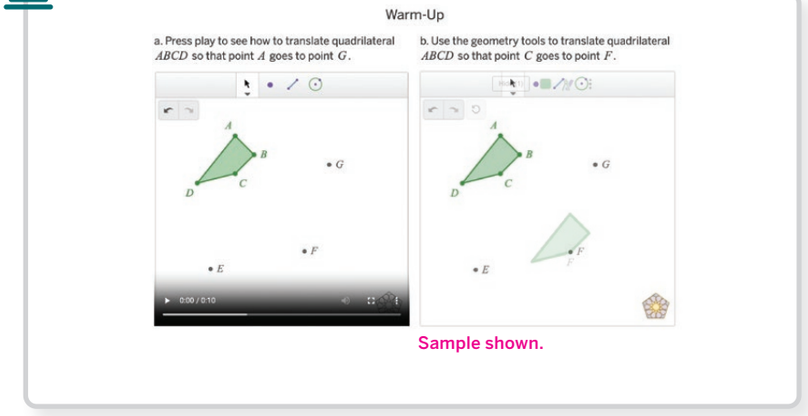
- "How is translating a quadrilateral digitally like translating a quadrilateral using tracing paper? How is it different?"
- "What is helpful to remember when performing transformations using the digital geometry tools?"

Math Identity and Community Invite students to share what they think we can learn from both correct and incorrect constructions.

Note: Students who use a draggable point to represent their translation will receive a partial star. Consider asking, "How can you define your translation precisely so it goes exactly from point C to point F?"

Students using digital

1



Sample shown.

Pairs Sharing a Device | 10 min | Pacing: Screens 2–5

Activity 1 Digital Transformations

Purpose: Students translate, reflect, and rotate polygons, and use the structure of polygons to transform one figure onto another. **(MP7)**

Short on time? Consider inviting students to complete fewer challenges on Screen 4.



Student Notes are also available

1 Launch

2 **To encourage playfulness**, invite students to explore all of the new digital geometry tools before they explicitly reflect the quadrilateral. Consider asking, "What information is needed to perform a reflection?"

A **Accessibility: Visual-Spatial Processing** Make either tracing paper or a printed copy of the screen available to support students in deciding which transformation to specify throughout this activity.

2 Monitor

3-4 **Listen for** students who are describing strategies for using the digital geometry tools to perform transformations.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to predict the location of the image for each transformation.
Start with incorrect answers for the description of the transformation.	**Support** Consider asking, "How might the structure of the pre-image and image help you decide which transformations to use?" **(MP7)**
Reflect the image using previous digital geometry tools.	**Strengthen** Celebrate their resourcefulness. Consider asking "How can you perform the transformation using the new transformation tools?" **(MP5)**
Guess and check the angle of rotation for rotations.	**Strengthen** Consider having students use the points on the screen to form an angle.

Listen for and celebrate the different ways that students discuss each transformation and explain what is happening with the construction of the rotation.

ML/EL **Multilingual/English Learners** Review an anchor chart that displays the properties of translations, rotations, and reflections to support reading comprehension. **(Reading)**

Note: If time allows, consider pacing to Screen 2 and inviting students to construct a line and then reflect the figure over it. Encourage them to move the line to see how the image changes. Consider asking, "How could you construct a line so you would know the reflection would end up on the image?"

Activity 1 continued >

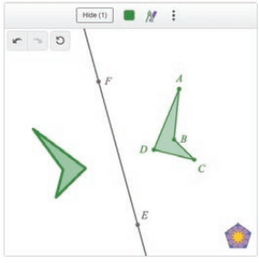
Students using digital

2

Sample shown.

Reflect

Reflect quadrilateral $ABCD$ over line EF.

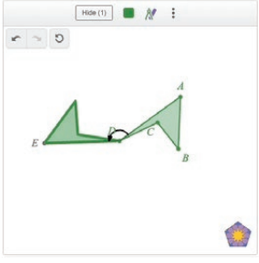


3

Sample shown.

Rotate

Rotate quadrilateral $ABCD$ around point D so that point A goes to point E.



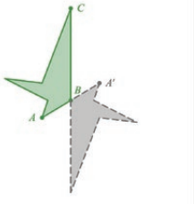
4

Transformation Descriptions

Which description will transform the solid polygon onto the dashed polygon?

- Translate so that point A goes to point A'.
- Reflect over segment BC.
- Reflect over segment AB.
- Rotate around point B so that point A goes to point A'.

Try It



Pairs Sharing a Device | 10 min | Pacing: Screens 2–5

Activity 1 Digital Transformations (continued)

Purpose: Students translate, reflect, and rotate polygons, and use the structure of polygons to transform one figure onto another. **(MP7)**

Short on time? Consider inviting students to complete fewer challenges on Screen 4.



Student Notes are also available

2 Monitor

5 **Invite students** to solve as many of the five challenges as they have time for. Share that it's more important to make sense of the transformations and the geometry tools than it is to work quickly.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to refer back to the previous screens to determine how to perform each transformation.
Struggle to identify the center of rotation.	**Support** Consider inviting students to use tracing paper to help them identify the center of rotation.
Use more transformations than needed. **Note:** Each challenge can be completed with a single transformation.	**Strengthen** Celebrate their persistence. Consider asking, "How might the structure of the pre-image and image help you decide which transformations to use?" **(MP7)**
	Stretch Celebrate the structure they noticed. Consider asking, "Can you complete this challenge using fewer transformations? Using just one?"

A **Accessibility: Conceptual Processing** Invite students to share how each problem is different from the previous one to help them apply concepts to new situations.

Capture different ways that students solved each challenge, particularly those that solved it using one transformation and those that solved it using multiple transformations.

3 Connect

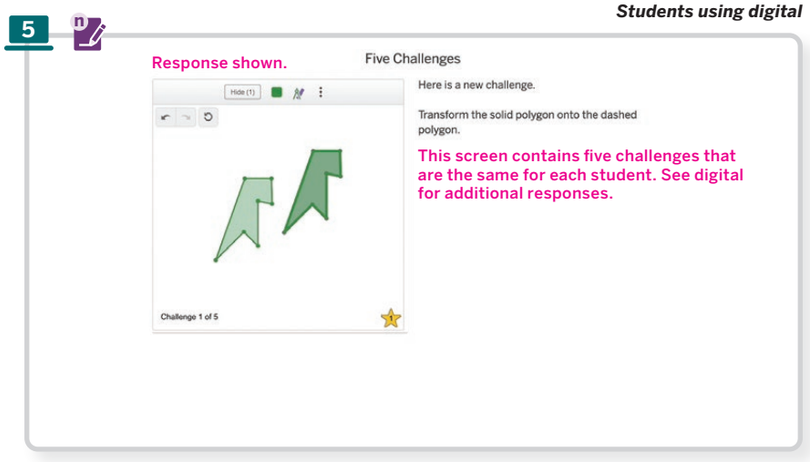
Display and celebrate a variety of student transformations. Invite students to share the strategies they found helpful or what they learned from the mistakes they made. Consider asking:

- "What types of transformations do we have access to in the geometry tools?"
- "What details are important to include when describing or performing a transformation?"

Invite students using notes to document some important considerations from the class discussion.

Math Identity and Community Consider asking the class why examining more than one way to transform one figure to another might be useful.

Students using digital



Pairs Sharing a Device | 20 min | Pacing: Screens 6–7, Screens 8–11

Activity 2 Sequences of Transformations

Purpose: Students continue exploring digital geometry tools to perform and describe transformations. **(MP5, MP6)**

Short on time? Consider inviting students to complete either the challenge on Screen 8 or Screen 9 and share with a partner.



Student Notes are also available

1 Launch

6 **To support making connections**, consider asking, "How is this transformation like those in the previous challenges? How is it different?"

2 Monitor

To support students getting started, invite them to make a plan using tracing paper or the sketch tool.

7 **Pause briefly** to discuss the strategies students used to transform the triangle on the previous screen.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their strategy on the previous screen. Consider asking:

- "How are Lan's and Jamar's strategies alike? How are they different?"
- "Why can different strategies lead to the same outcome?"
- "How might construction tools be helpful when transforming figures?" **(MP5)**

8 **Invite students** to continue to Screens 8–11 using the Pace tool. Look for a variety of strategies students might use to complete this screen.

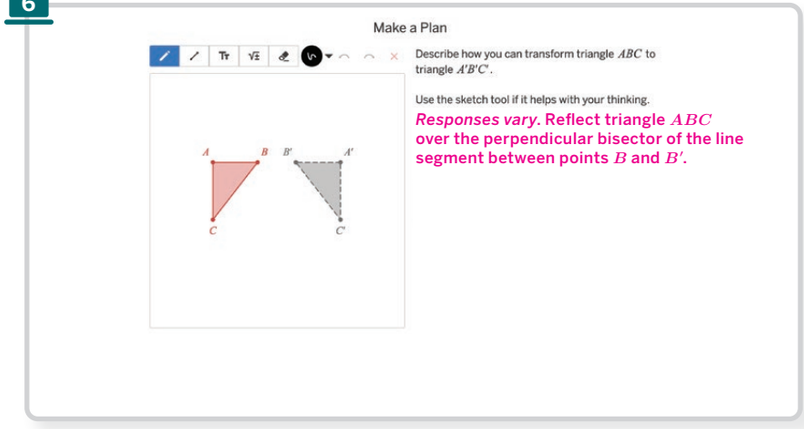
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to revisit Lan's and Jamar's strategies and consider asking, "What is a first step to try either of their strategies?"
Construct a line of reflection and use the draggable points until it matches with the image.	**Strengthen** Consider asking, "How can you construct the line so you know the image will appear every time?"
Construct a perpendicular bisector to use as the line of reflection and receive the animated star feedback.	**Stretch** Consider asking, "Are there other ways we might construct this transformation?"

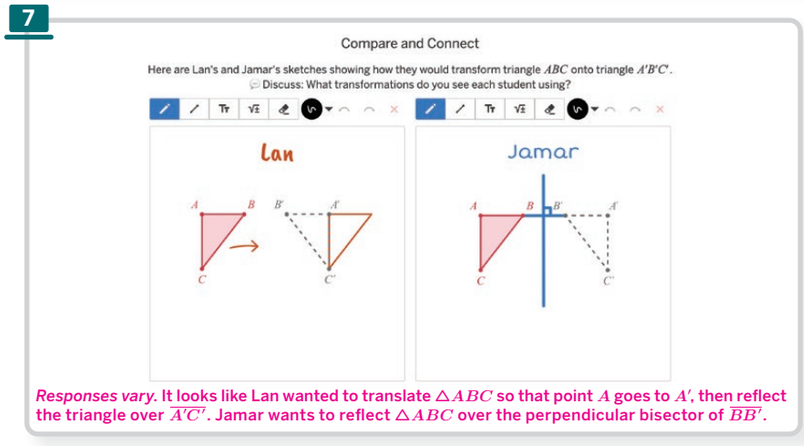
Activity 2 continued >

Students using digital

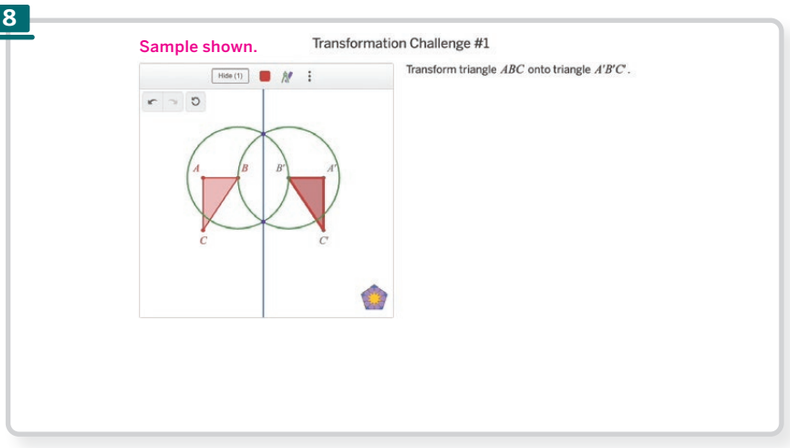
6



7



8



Pairs Sharing a Device | 20 min | Pacing: Screens 6–7, Screens 8–11

Activity 2 Sequences of Transformations (continued)

Purpose: Students continue exploring digital geometry tools to perform and describe transformations. **(MP5, MP6)**



Student Notes are also available

Short on time? Consider inviting students to complete the challenge on Screen 8 or Screen 9 and share with a partner.

2 Monitor

9-10 **Listen for** student responses that use different sequences of transformations to complete each challenge.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to choose a transformation to start with and determine what information they need for that transformation.
Need support including the necessary details in their description.	**Support** Consider asking, "What information did you need to perform the transformation? How can you include that?" **(MP6)**
Use a variety of strategies, such as: • Using digital geometry tools. **(MP5)** • Using the structure of the image and pre-image. **(MP7)**	**Strengthen** Invite these students to think about how they would share their strategies during the Connect.
Use more transformations than is necessary.	**Stretch** Consider asking, "Can you complete this challenge using fewer transformations? Using just one?"

A Accessibility: Conceptual Processing Invite students to revisit the descriptions of transformations on Screen 4 to help them apply concepts to new situations.

11 **D Differentiation: Explore More (Stretch)** Invite students who would like to further explore sequences of transformations to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

10 **MLR MLR1: Stronger and Clearer Each Time** Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. **(MP6)**

To surface the Key Takeaway, consider asking, "What details are helpful to make these descriptions clearer and more precise?"

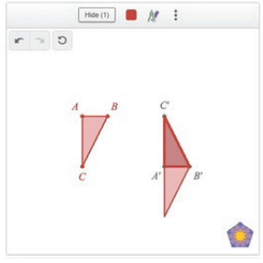
Math Identity and Community Celebrate different degrees of precision in students' descriptions. Consider creating class descriptions from students' responses.

Invite students using notes to capture some helpful details when describing transformations.

Students using digital

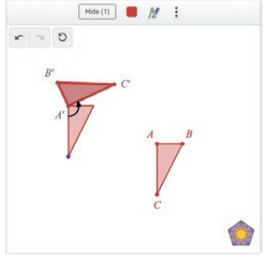
9

a. Sample shown.



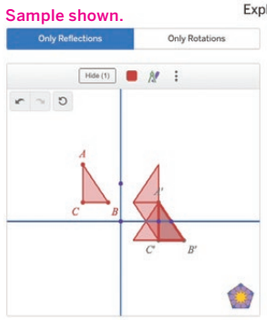
10

a. Sample shown.



11

Sample shown.



Key Takeaway: Digital tools can help us visualize and revise transformations. We need to include precise details when specifying transformations, including the angle of rotation, center of rotation, and line of reflection. **(MP6, MP7)**

Whole Class | 5 min | Pacing: Screen 12

Synthesis



Student Notes are also available

12 **Invite students** to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Looking at the orientation of the figure.
- Using your hand or tracing paper to check if you can rotate the figure around.
- Seeing if the figure moved from one place to another.
- Considering whether a construction might help to establish a line of reflection.
- Looking at where points from one figure have been transformed to in another figure (like A and A').

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies for performing transformations. Display for students to reference and add to for the rest of the unit.

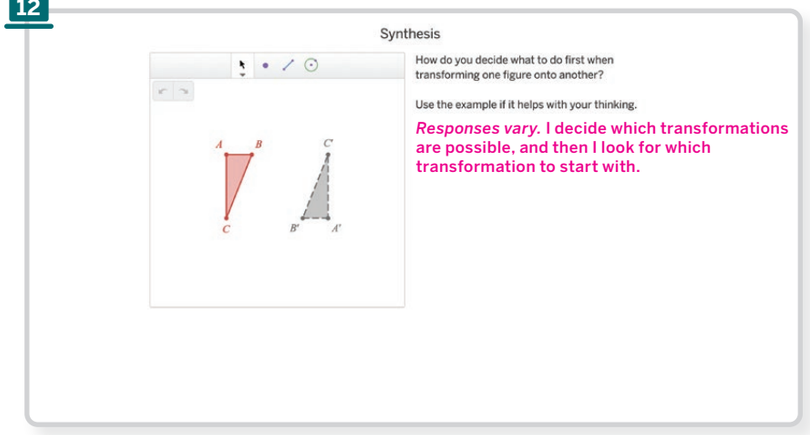
Math Identity and Community Invite students to share strategies they found helpful and give credit to the students who shared them.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.

Invite students using notes to document ideas from the class discussion in the left column.

Students using digital

12

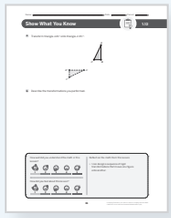


Lesson Takeaway: There are often many ways to specify a sequence of transformations to move one figure onto another. Before you start, it's helpful to consider the structure of the image and pre-image, and make a plan.

Independent | 5 min | Pacing: Screens 13–14

Show What You Know

Purpose: Students demonstrate their understanding by performing and then describing a sequence of transformations to move one figure onto another.

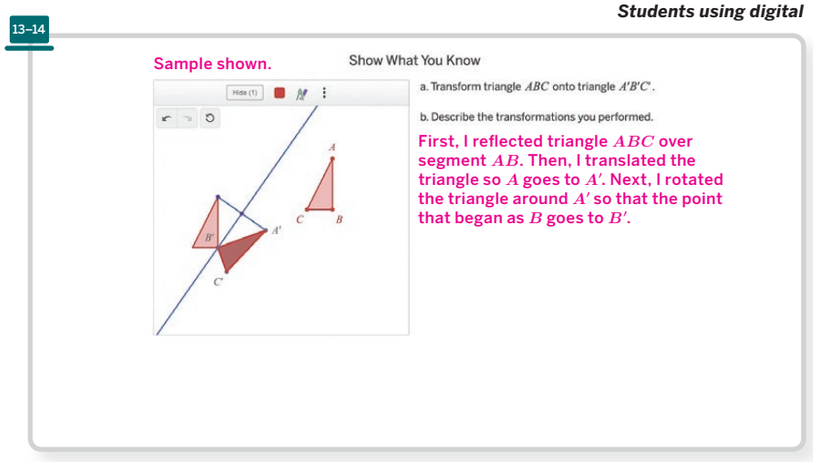


Students using print

13-14 Today's Goals

1. **Goal:** Use geometry tools to perform a sequence of rigid transformations that moves one figure onto another.
2. **Language Goal:** Describe a sequence of rigid transformations that moves one figure onto another using mathematically precise language. **(Speaking and Listening)**

Students using digital



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of determining which transformations to use to move one figure to another, consider:

- Revisiting **Grade 8**, **Unit 1**, **Lesson 4** (*Transformation Targets*).
- Inviting students to engage with the **Lesson Summary** by reading it and rewriting it in their own words.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice**.
- Inviting students to review the **Summary** and complete the **Try This** problem.
- Emphasizing this topic during **Lesson 13** and **Practice Day 2**.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting students to use the digital geometry tools to create their own challenges and then try to solve them.
- Encouraging them to complete the **DOK3 practice problem**.

Professional Learning

In what ways are students taking risks when trying out strategies for solving transformation challenges? What questions might encourage students to continue using creativity as they strengthen their understanding?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the Lesson Summary to support them with this practice and their learning throughout the year.

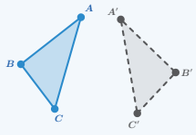
Lesson 12
Practice

Students using digital

Lesson Summary

There are many strategies to transform one figure onto another.

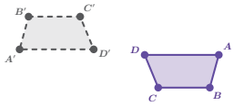
Here are three ways you could transform triangle ABC onto triangle $A'B'C'$.



1. Translate triangle ABC so that point B goes to B'. Rotate the image around B' until the point that was originally C moves onto C', and then reflect over segment $B'C'$.
2. Reflect triangle ABC over the perpendicular bisector of segment AA'.
3. Rotate triangle ABC around the midpoint between points A and A' until segment AB aligns with segment $A'B'$. Then reflect the image over segment $A'B'$.

Try This

Describe how to transform figure $ABCD$ onto figure $A'B'C'D'$.



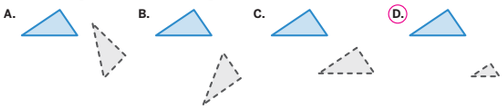
Use tracing paper if it helps with your thinking.

Responses vary. **First, translate figure $ABCD$ so that point D goes to point D'. Then rotate the new figure around point D' so that the point that began as A goes to A'.**

Lesson Practice
Geo.1.12

Name: _____ Date: _____ Period: _____

1. Which set of figures does *not* show a rigid transformation?



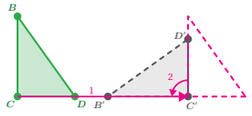
2. Select *all* the transformations that could move point A onto point B.



- ☐ A. Reflect point A over $\overline{AB}$.
- ☑ B. Reflect point A over the perpendicular bisector of $\overline{AB}$.
- ☑ C. Translate point A so that it moves onto point B.
- ☐ D. Rotate point A around point B.
- ☑ E. Rotate point A around the midpoint of $\overline{AB}$.

Problems 3–4: Transform the solid triangle onto the dashed triangle. Then describe the transformations you performed. ***Responses vary.***

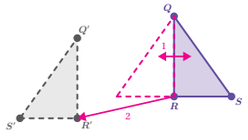
3.



Description:

Translate triangle BCD so point C moves onto C'. Then rotate the triangle around point C' so the translated $\overline{BC}$ lies on the same line as $\overline{B'C'}$.

4.



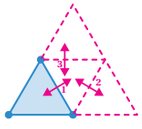
Description:

Reflect triangle QRS over segment QR. Then translate the triangle so that point R moves onto point R'.

Lesson Practice
Geo.1.12

Name: _____ Date: _____ Period: _____

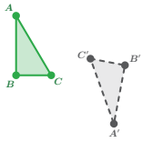
5. Here is an equilateral triangle. Create a larger triangle using only reflections. Show or explain your thinking. ***Responses vary.***



Spiral Review

6. Describe a strategy for determining the center of rotation for the transformation that would move triangle ABC onto triangle $A'B'C'$.

Responses vary. **I could construct the segments AA' and BB'. The center of rotation would be the point where these segments intersect.**



7. A triangle has one angle that measures 55°. What are possible values for the other two angles?

Responses vary. **55° and 70°, or any two angles whose sum is 125°.**

Reflection

1. Put a star next to a problem you're still wondering about.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1	1	HSG.CO.A.5
Test Practice	**2**	2	HSG.CO.A.5
	3–4	2	HSG.CO.A.5
	5	3	HSG.CO.A.5
Spiral Review			
	6	1	HSG.CO.A.4
Fluency	**7**	1	8.G.A.5

Need More Practice?

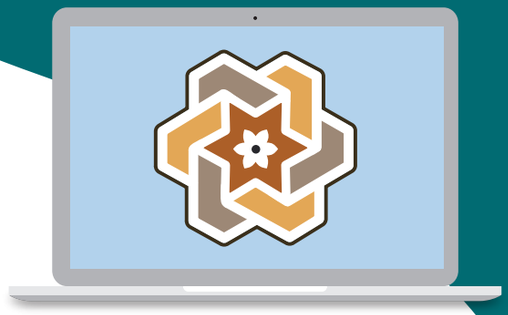
Additional Practice resources coming soon.

Unit 1

Lesson 13

Digital Lesson

This lesson uses digital student screens. Student Notes are also available.



Tiles and Patterns

Geometric Art Using Constructions and Transformations

Let's use transformations to describe and create patterns.

Focus and Coherence

Today's Goals

1. **Goal:** Use an underlying construction to create polygons that can be transformed into geometric patterns.
2. **Language Goal:** Describe connections between geometric patterns from Islamic cultures, constructions, and transformations. **(Reading, Writing, and Speaking)**

Students look at geometric patterns found in works by Muslim architects and research art that is used to recreate these types of patterns. They focus on one geometric pattern and use an underlying construction and digital geometry tools to recreate the design. Students also create their own geometric patterns for their classmates to recreate. **(MP7)**

Prior Learning

In Sub-Unit 1, students used circles and line segments (or a compass and straightedge) to perform precise constructions. In Sub-Unit 2, students transformed geometric figures using reflections, rotations, and translations.

Future Learning

In Unit 2, students will use transformations as the basis for proof of congruence.

Rigor and Balance

- Students **apply** their understanding of constructions and transformations to recreate patterns found in architecture.
- Students have opportunities to develop a **productive disposition** as they relate the mathematics they do in class to the creation of geometric art.

Standards

Addressing

HSG.CO.A.5

Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or **geometry software. Specify a sequence of transformations that will carry a given figure onto another.**

Also Addressing: **HSG.CO.D.12**

Mathematical Practice: MP7

Building On	Building Toward
HSG.CO.D.12	HSG.CO.B.6

Lesson at a Glance

~ 45 min

Standards: HSG.CO.A.5, HSG.CO.D.12



Why digital?

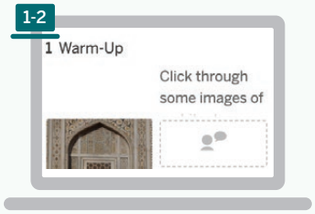
Students are given an underlying construction and use intersection points to replicate a design. They use digital feedback to help them make revisions when angles or line segments are slightly off.

Warm-Up

Pairs Sharing a Device | 10 min

Pacing: Screens 1–2

Students look at and research geometric patterns from Islamic cultures to discuss their characteristics. **Routine:** Notice and Wonder



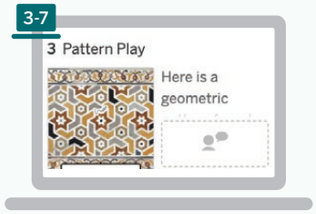
Activity 1

Pairs Sharing a Device | 12 min

Pacing: Screens 3–5, Screens 6–7

Students use an underlying construction and transformations to recreate a specific geometric pattern. **(MP7)**

Key Takeaway (Screen 7): Constructing a grid of intersecting circles and lines can help us be specific when creating (or recreating) designs. Transformations can help us replicate these designs to create intricate geometric patterns, such as those seen in works by Muslim architects.

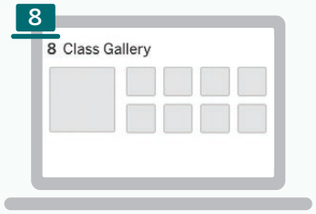


Activity 2

Pairs Sharing a Device | 13 min

Pacing: Screen 8

Students use an underlying construction and transformations to create a challenge for their classmates to recreate.



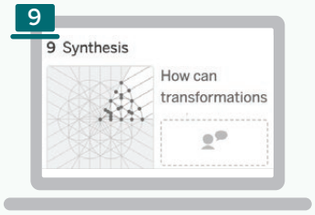
Synthesis

Whole Class | 5 min

Pacing: Screen 9

Students synthesize strategies for using transformations and constructions to create patterns.

Lesson Takeaway: Patterns, such as geometric patterns from Islamic cultures, can be created by precisely constructing objects and then describing how to transform them using translations, rotations, and reflections.



Show What You Know

Independent | 5 min

Pacing: Screens 10–11

Students demonstrate their understanding by describing how transformations can be used to recreate a pattern.



Prep Checklist

Assign the digital student screens. Student Notes are also available.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)
- computers/devices (for research)

Student Notes sheet in Student Edition:
Consider providing the Student Notes to support class discussion and student thinking.



Pairs Sharing a Device | 10 min | Pacing: Screens 1–2

Warm-Up

Purpose: Students look at and research geometric patterns from Islamic cultures to discuss their characteristics.



Student Notes are also available

1 Launch

1 **Use the Notice and Wonder routine** to promote curiosity and help students make sense of the ways constructions and transformations appear in geometric patterns from Islamic cultures. **(MP1)**

To support making connections, consider asking:

- "What do these different pieces of architecture have in common?"
- "What tools do you think architects used to create these patterns?"
- "What questions do you have?"

Create a record of what students noticed and wondered, along with the names of the students who shared them.

A **Accessibility: Visual-Spatial Processing** Consider providing students with enlarged copies of the images to support them in noticing specific details between the patterns.

2 **Invite students** to research answers to their and their classmates' questions about Samira Mian or art based on geometric patterns found in works by Muslim architects.

To support students getting started, consider inviting them to type keywords or phrases from the screen that they want to learn more about into a search engine.

Invite students using notes to record something interesting from their research.

2 Connect

Invite students to share something interesting from their research or from what their partner shared.

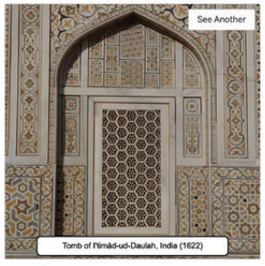
Math Identity and Community Celebrate students who recognize others' personal experiences in their research or share their own experiences with geometric patterns during the class discussion.

Note: If it doesn't come up in student research, consider sharing that these patterns are ornamental and were often used to beautify buildings, gardens, books, and fabrics in many different places. These geometric patterns are found in everyday items and in the grandest of secular and religious buildings of current and prior centuries.

Students using digital

1

Warm-Up



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Click through some images of architecture from different countries.

They contain examples of geometric patterns from Islamic cultures.

What do you notice? What do you wonder?

Responses vary.

- I notice that many of these images show art that uses repeating geometric figures.
- I notice that these images are from different places around the world and different time periods, but the patterns are similar.
- I wonder where this style of art originated and why they chose these shapes and styles. Do they represent different things? Do they have cultural significance?

2

Geometric Patterns From Islamic Cultures

Art | Artist



Samira Mian (born 1975)

You can use a compass and straightedge to create geometric patterns.

Spend some time researching and learning more about how artists like Samira Mian recreate geometric patterns found in works by Muslim architects.

As you explore, consider discussing:

- What is a pattern that stands out to you? Why?
- What are some characteristics of these patterns?

Responses vary. These geometric patterns can be found over a wide geographic region, in places like Turkey, Morocco, and Iran. While they do appear in sacred spaces such as mosques and tombs, they also appear in secular buildings and books. Polygons, overlapping figures, and symmetry are all common characteristics of these patterns.

Pairs Sharing a Device | 12 min | Pacing: Screens 3–5, Screens 6–7

Activity 1 Pattern Play

Purpose: Students use an underlying construction and transformations to recreate a specific geometric pattern. **(MP7)**



Student Notes are also available

1 Launch

3 **Display** the geometric pattern. To surface connections, invite students to describe different pieces of the pattern with different transformations.

ML/EL **Multilingual/English Learners** Invite students to sketch their thinking before expressing their ideas in words. **(Writing)**

4 **Play** the animation and give students some individual think time before discussing with a partner.

Note: Consider researching the pronunciation of "Tomb of I'timād-ud-Daulah" and "Samira Mian" prior to teaching this lesson to support students in speaking about the people and places being studied.

2 Monitor

5 **Invite students** to sketch different pieces of the pattern and see what appears on the pattern.

A **Accessibility: Visual-Spatial Processing** Invite students to use their tools (compass, straightedge, or tracing paper) to support sketching the distance from edges or vertices of the hexagon.

Note: Students may be confused by the lack of exact alignment due to the white grout of the tile. Invite students to consider what materials are used to build tiled structures to make sense of the pattern.

Capture student sketches that use intersection points to reconstruct pieces of the pattern.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Sketch the shapes with straightedges first. **(MP7)**	Strengthen Celebrate their strategies. Consider asking, "How can you use similar strategies to construct the curved shapes?"
Sketch the curved parts of the pattern.	Strengthen Celebrate how they leaned into the challenge. Consider asking, "What would be helpful in the underlying construction to recreate the curved portions?"

Pause briefly to display sketches with varying levels of completion.

Activity 1 continued >

Students using digital

3

Pattern Play



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Here is a geometric pattern found in the Tomb of I'timād-ud-Daulah.

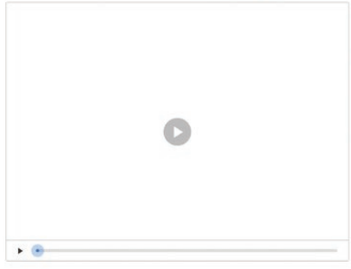
Use at least two of these words to describe the geometry in this pattern.

- Translate
- Rotate
- Reflect
- Transformation

Responses vary. This pattern has six-pointed stars and two different kinds of hexagons. We could translate each of the six-pointed stars onto each other without rotating or reflecting. Around each six-pointed star are hexagons with sides of different lengths. Those could be constructed by rotating one of them around the six-pointed star one-sixth of a turn. There are also regular hexagons that can be translated onto each other.

4

Underlying Construction



Here is an animation of the underlying construction that Samira Mian used to recreate the pattern from the Tomb of I'timād-ud-Daulah.

Discuss: How is this related to constructions you've done? How is it different?

Responses vary.

- It's made of intersecting circles and lines, just like constructions I've made.
- It's more complicated and fully symmetrical compared to many constructions I've made.

5

Use the underlying construction to outline the pattern.



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Sample shown.

Activity 1 Pattern Play (continued)

Purpose: Students use an underlying construction and transformations to recreate a specific geometric pattern. **(MP7)**



Student Notes are also available

2 Monitor

Math Identity and Community Consider celebrating students who persisted through struggle to sketch their pattern. Invite them to share what helped them persist.

6 **Encourage students**, to use their sketches shown from the previous screen to help them complete the portion of the design. Invite students to share how sketching first helped them recreate the polygon.

7 **Invite students** to play the animation. Invite them to discuss with their partner to see what transformations could be used to recreate the pattern before responding.

Look for responses that discuss symmetry and color.

D Differentiation

Look for students who:	Respond to Student Thinking
Mention rotations, but don't discuss the color of the pattern or the amount of rotation.	**Support** Consider asking, "Would a rotation also work with the colors of the tiles?"
Mention reflections because of symmetry, but without reference to color.	**Strengthen** Consider asking, "Which pieces of this are symmetric in terms of tile color and which are not?"
Discuss translations of the entire rotated design to tile the pattern. **(MP7)**	**Strengthen** Consider asking, "Does your design account for the entire geometric pattern? What parts do you still need to consider?"

3 Connect

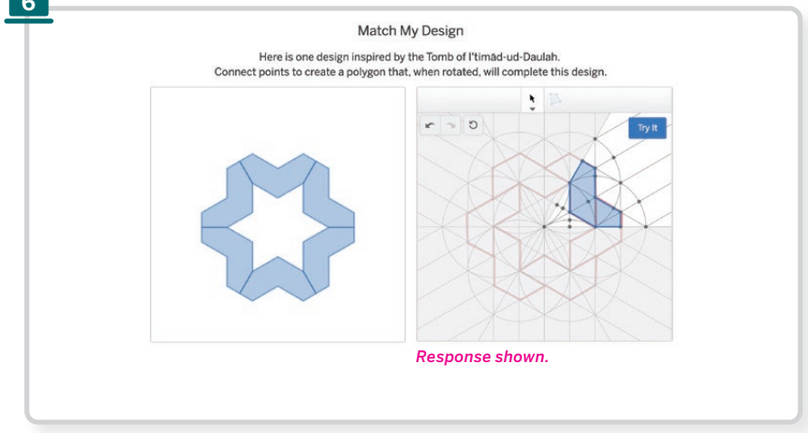
Invite students to share ways they could use transformations to recreate the geometric pattern. Consider asking, "Which of these transformations could work to make the pattern, but don't match the colors?"

Invite students using notes to record notes from the class discussion about ways transformations appear in the geometric pattern.

To surface the Key Takeaway, consider asking, "How can constructions and transformations help us recreate geometric patterns?"

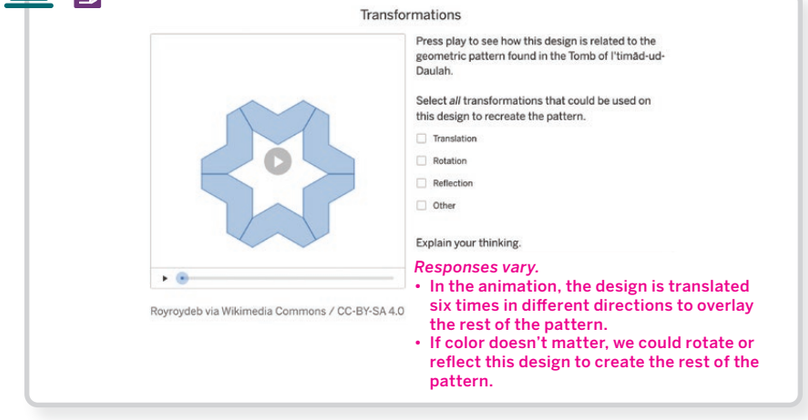
Students using digital

6



Response shown.

7



Responses vary.

- In the animation, the design is translated six times in different directions to overlay the rest of the pattern.
- If color doesn't matter, we could rotate or reflect this design to create the rest of the pattern.

Key Takeaway: Constructing a grid of intersecting circles and lines can help us be specific when creating (or recreating) designs. Transformations can help us replicate these designs to create intricate geometric patterns, such as those seen in works by Muslim architects.

Pairs Sharing a Device | 13 min | Pacing: Screen 8

Activity 2 Challenge Creator

Purpose: Students use an underlying construction and transformations to create a challenge for their classmates to recreate.

Short on time? Consider having students solve each other's challenges in pairs.



Student Notes are also available

1 Launch

8 **To encourage playfulness**, invite students to create a design that they would use to beautify buildings, books, or other everyday objects they use.

A **Accessibility: Visual-Spatial Processing** Eliminate extraneous visual stimuli by creating a simpler version of the construction to support interpreting visual representations.

2 Monitor

Look for and celebrate the different designs students construct, including those that create patterns that are more common, like stars or hexagons, and those who create more irregular figures, like polygons whose edges intersect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to create any polygon, see what the rotation looks like, and then revise based on the image.
Need support recreating their own challenge.	**Support** Consider asking, "What about your construction is challenging to recreate? How can you adjust your challenge to make it more clear?"

Capture student challenges that are creative, that are challenging to recreate, that many students chose to recreate, or that hold special importance.

3 Connect

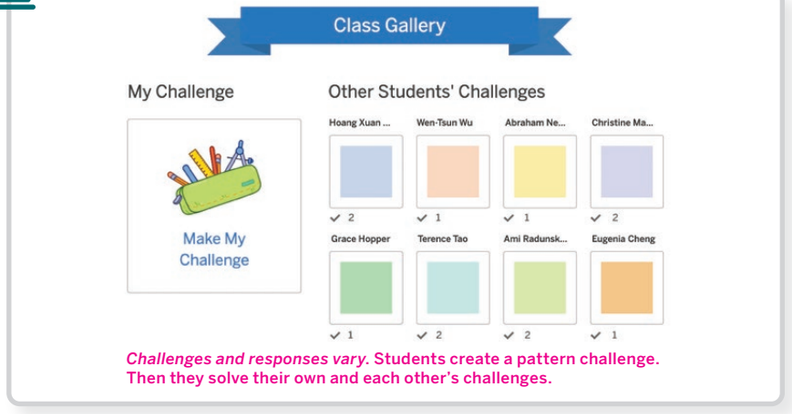
Display the captured patterns. Consider asking:

- "How do you know that each copy of the design will be the same after it's transformed?" Each design was created using the same constructed grid. Then each construction was created using rigid transformations, so all the copies will be the same.
- "Why might it be important to be able to recreate patterns like these precisely?" Symbols and patterns often have special significance or meaning. It's important to be able to replicate them precisely.

Math Identity and Community Invite students to share patterns that they found delightful, beautiful, inspiring, or that they enjoyed recreating. Consider creating a class display of the images that students have created.

Students using digital

8



Challenges and responses vary. Students create a pattern challenge. Then they solve their own and each other's challenges.

Whole Class | 5 min | Pacing: Screen 9

Synthesis



Student Notes are also available

9 **Invite students** to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

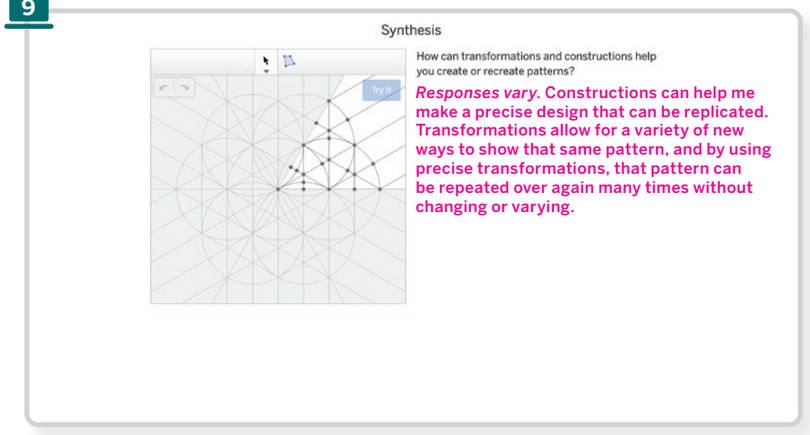
- Constructions can help to create an underlying grid to precisely replicate a pattern.
- Transformations allow us to replicate a design over and over again without changing it.
- If a pattern is symmetrical, we can construct a small piece of it and then use rotations and reflections to create the full design.

Math Identity and Community Invite students to share how recreating geometric patterns gave them insight into ways that constructions and transformations are used by others.

Invite students using notes to reflect on ways they used constructions and transformations to create geometric patterns.

Students using digital

9



Lesson Takeaway: Patterns, such as geometric patterns from Islamic cultures, can be created by precisely constructing objects and then describing how to transform them using translations, rotations, and reflections.

Independent | 5 min | Pacing: Screens 10–11

Show What You Know

Purpose: Students demonstrate their understanding by describing how transformations can be used to recreate a pattern.

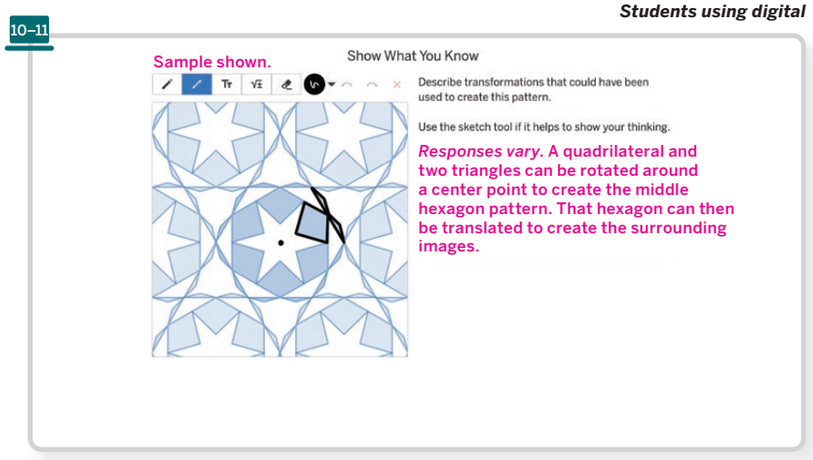


Students using print

10–11 Today's Goals

1. **Goal:** Use an underlying construction to create polygons that can be transformed into geometric patterns.
2. **Language Goal:** Describe connections between geometric patterns from Islamic cultures, constructions, and transformations. **(Reading, Writing, and Speaking)**

Students using digital



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of using transformations to create geometric patterns, consider:

- Inviting students to rewatch the animation on **Screen 7** and continue to read their classmates' responses.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice**.
- Invite students to review the **Summary** and complete the **Try This** problem.
- Revisiting the **Challenge Creator** in **Activity 2** and inviting students to complete additional challenges.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting students to create their own underlying construction to construct a geometric pattern.

Professional Learning

Take a moment to reflect on how students applied what they've learned about construction and transformation to recreating geometric patterns found in works by Muslim architects. In what ways were students able to make connections between math ideas and their or others' communities in this lesson? How might you support students in making connections between the math they learn and their world?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 13
Practice

Students using digital

Lesson Summary

Artists and mathematicians throughout history have developed strategies for making geometric patterns. Here is an example of a geometric pattern from the Tomb of I'timād-ud-Daulah in India.

There are many strategies to construct and recreate designs. You can use circles, intersection points, and line segments to precisely construct figures. You can transform those figures using translations, rotations, and reflections. By combining constructions and transformations, you can create complex designs made up of beautiful shapes and patterns.



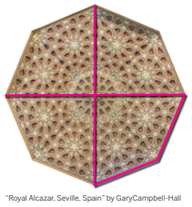
© 2014 Royroydeb / CC-BY-SA 4.0



Try This

Describe transformations that could be used to recreate this pattern from the Royal Alcazar in Spain. Draw on the image if it helps to show your thinking.

Responses vary. **If you construct one quarter of this pattern, you could then reflect that quarter over a vertical line to create half of the pattern, and then reflect that over a horizontal line to create the full pattern.**

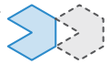


"Royal Alcazar, Seville, Spain" by GaryCampbell-Hall via Flickr

Lesson Practice
Geo.1.13

Name: _____ Date: _____ Period: _____

1. Which transformation cannot be achieved with a translation?

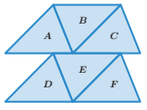
A. 

B. 

C. 

D. 

Problems 2–3: These triangles are rigid transformations of triangle A.



2. Select *all* the triangles that could be a translation of triangle A.

☐ B ☑ C ☑ D ☐ E ☑ F

3. Select *all* the triangles that could be a rotation of triangle A.

☑ B ☐ C ☐ D ☑ E ☐ F

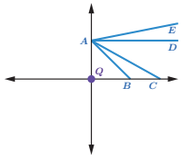
4. Which segment, when rotated around point Q into each quadrant, will form a square?

(A.) Segment AB

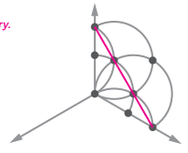
B. Segment AC

C. Segment AD

D. Segment AE



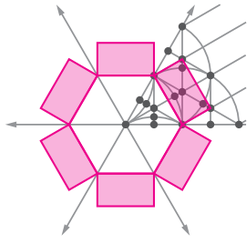
5. Use the intersection points to construct a line segment that will form an equilateral triangle when rotated. ***Responses vary.***



Lesson Practice
Geo.1.13

Name: _____ Date: _____ Period: _____

6. Connect points to create a polygon that, when rotated, will make a design that also includes at least one translation and one reflection. Use tracing paper to replicate your polygon to complete the design. ***Responses vary.***

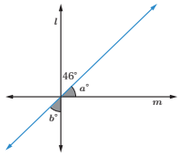


Spiral Review

Problems 7–8: Line l is perpendicular to line m. Determine the values of a and b.

7. $a =$ 44

8. $b =$ 46



Reflection

1. Put a heart next to the problem you found most interesting.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
Fluency	1	1	HSG.CO.A.5
	2–3, 6	2	HSG.CO.A.5
Test Practice	4	2	HSG.CO.A.5
	5	3	HSG.CO.A.5
Spiral Review			
	7–8	1	7.G.B.5

Need More Practice?

Additional Practice resources coming soon.

45 min

Practice Day 2

Purpose: Students practice the concepts, skills, and strategies developed in Lessons 1–13. Consider using this Practice Day as preparation for the upcoming End-of-Unit Assessment.

Preparation

Materials:

Option 1: Stations

- Student Edition
- Digital Companion
- Task Cards, one per station

Option 2: Partner Practice

- Student Edition
- Digital Companion
- Task Cards, one per pair

A **Accessibility: Executive Functioning** To support initiating complex tasks, consider making a plan with students about which tool they are going to use for each task. Invite students to describe a possible first step for each construction using paper-based tools or the digital companion.

Consider asking these questions to check for understanding:

- Task B: "What construction strategies were helpful to construct the line that reflected $\triangle ABC$ onto $\triangle DEF$?"
- Task C: "Why do you need transformations other than translations to transform $\triangle ABC$ onto $\triangle AFE$? Are there other triangles you could transform $\triangle ABC$ onto that would only require translations?"

Standards

Addressing

HSG.CO.A.5
Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Also Addressing: **HSG.CO.A.1, HSG.CO.A.2, HSG.CO.D.12**

Mathematical Practices: MP5, MP6

Facilitation Options

Option 1: Stations

This structure promotes student collaboration, provides opportunities to move around the classroom, and supports students with executive functioning by chunking a task into smaller, more manageable parts.

Small Groups

Distribute the tasks at stations around the room and assign the Digital Companion Screens. Consider creating two stations for each task to prevent crowding.

Consider sharing that there are four tasks and that each group should start at a different station.

Invite students to discuss their reasoning with their group and to each record their thinking in their Student Edition.

Other considerations:

- What/who will rotate?
  - Encourage groups to rotate while task cards stay at their stations.
  - Rotate the task cards while groups stay at their stations.
- When will rotation occur?
  - Set a specific amount of time to work at each station.
  - Invite groups to rotate as they feel ready.

Option 2: Partner Practice

This structure supports student collaboration and allows pairs to work at their own pace.

Pairs

Distribute one set of cards to each pair and assign the Digital Companion Screens.

Invite students to solve each problem in their Student Edition.

Encourage students to discuss their reasoning with their partner as they work on each problem.

Other considerations:

- How will students get feedback?
  - Post the answers and invite students to check them.
  - Invite students to check in with you when they're ready for feedback.
- How will you support student collaboration?
  - Discuss what supporting each other without giving answers could look or sound like.
  - Encourage pairs to read each problem out loud and solve it before moving on to the next problem.

SE Practice Day 2

Unit 1
Lessons 1–13

Name: ..... Date: ..... Period: .....

Practice Day 2

Let's practice what you've learned so far in this unit!

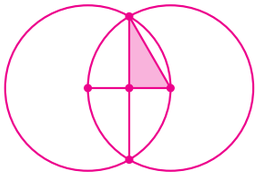


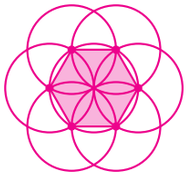
You will use task cards and the digital companion. Record your responses here.

Task A: My Favorite Tools *Responses vary.*

1. Choose one to construct below and one to construct using the digital companion.

	Constructed below	Digital companion
a Right triangle	☐	☐
b Regular hexagon	☐	☐





2. For geometric constructions, what is one advantage of using:

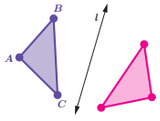
a Digital tools?
Digital tools make it easier to be precise.

b Paper-based tools?
Paper-based tools make it easy to sketch anywhere.

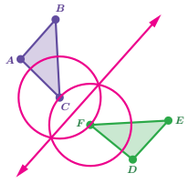
Task B: Reflected Triangles

1. Explanation: *Responses vary.* *When I fold the paper on line l, $\triangle DEF$ doesn't line up with $\triangle ABC$.*

2.



3.



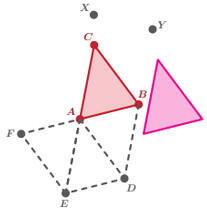
Unit 1
Lessons 1–13

Name: ..... Date: ..... Period: .....

Practice Day 2 (continued)

Task C: Translated Triangles

1.



2. Description: *Responses vary.*
Rotate $\triangle ABC$ around point A so that point B goes to point F.

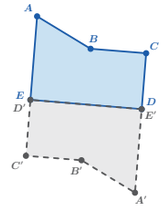
3. Circle one: Yes (No)

Explanation: *Responses vary.*
The triangles are oriented differently, so I have to reflect or rotate $\triangle ABC$ onto $\triangle ADB$.

Task D: Two Transformations *Responses vary.*

1. Sequence of rigid transformations #1:
Rotate pentagon $ABCDE$ around point D one half turn. Then translate the pentagon using the translation that moves point D to point E.

Sequence of rigid transformations #2:
Reflect pentagon $ABCDE$ over line DE, then reflect the pentagon over line BB'.



2. Circle one: (Yes) No

Explanations vary. *I agree with him because every segment in pentagon $ABCDE$ will have to transform the same way as $\overline{CD}$ for the pentagon to be the same shape after the sequence of transformations.*

Task Cards (online)

Unit 1
Practice Day 2
(p. 1 of 2)

Task Cards

Stations Directions: Make and distribute one copy per set of stations.
Partner Practice Directions: Make and distribute one copy per pair of students.

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Task A: My Favorite Tools

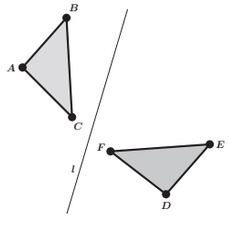
1. Choose one to construct on paper and one to construct using the digital companion.
   a. Right triangle
   b. Regular hexagon
2. For geometric constructions, what is one advantage of using:
   a. Digital tools?
   b. Paper-based tools?

Task B: Reflected Triangles

1. Explain how you know $\triangle DEF$ is *not* a reflected image of $\triangle ABC$ over line l.

Problems 2–3: Complete each construction.

2. Reflect $\triangle ABC$ over line l . . .
   a. Using the digital companion.
   b. Using paper-based tools.
3. Construct a line to reflect $\triangle ABC$ onto $\triangle DEF$ . . .
   a. Using the digital companion.
   b. Using paper-based tools.



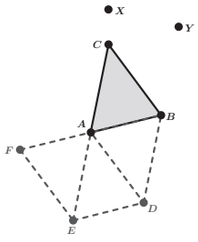
Unit 1
Practice Day 2
(p. 2 of 2)

Task Cards

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Task C: Translated Triangles

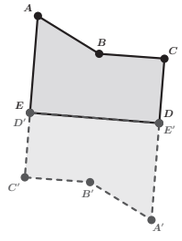
1. Translate $\triangle ABC$ using the translation that moves point X to point Y . . .
   a. Using the digital companion.
   b. Using paper-based tools.
2. Describe a sequence of rigid transformations that moves $\triangle ABC$ to $\triangle AFE$.
3. Is it possible to use only translations to transform $\triangle ABC$ onto $\triangle AFE$? Explain your reasoning.



Task D: Two Transformations

Here is a pentagon and its image after a sequence of transformations.

1. Describe *two different* sequences of rigid transformations that transform pentagon $ABCDE$ to pentagon $A'B'C'D'E'$.
2. Tyrone says: *If we transform pentagon $ABCDE$ by moving $\overline{CD}$ onto $\overline{C'D'}$, then the pentagon will be the same shape after the translation.* Do you agree? Explain your reasoning.



Assess and Respond

Additional data on student thinking can be found in Teacher Reporting.

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. Text to speech is available in the digital version. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or Skill	Addressed in	DOK	Standard(s)
1	Reflecting a segment over a line	Lesson 9	1	HSG.CO.A.5
2	Analyzing a construction	Lessons 1–3	2	HSG.CO.A.1
3	Constructing the copy of a segment	Lesson 3	2	HSG.CO.D.12 MP5
4a	Performing a translation of a triangle	Lesson 10	2	HSG.CO.A.5 MP5
4b	Performing a rotation of a triangle	Lesson 11	2	HSG.CO.A.5 MP5
5	Constructing a line of reflection	Lessons 4, 9	2	HSG.CO.A.2, HSG.CO.A.4 MP5
6	Describing a sequence of rigid transformations	Lesson 12	2	HSG.CO.A.4, HSG.CO.A.5 MP6
7	Constructing a design	Lessons 1–6	2	HSG.CO.D.12, HSG.CO.D.13 MP5

Assessment Resources

- Student Assessments
- Assessment Rubrics with Answers
- Show What Know PDFs and Answer Keys

Practice

If students need further review or practice, consider assigning the practice problems that correspond to each lesson.

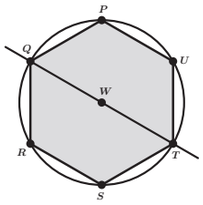
Name: Date: Period:

End-of-Unit Assessment

Geo.1

Complete each question using any construction tools (compass and straightedge, tracing paper, paper folding, digital tools).

1. This regular hexagon is inscribed in a circle with center W.



Which segment is the image of $\overline{PQ}$ after a reflection over line QT?

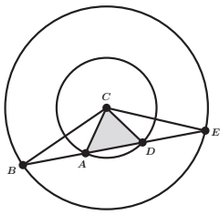
A. $\overline{TS}$

B. $\overline{SR}$

C. $\overline{RQ}$

D. $\overline{UT}$

2. Josiah constructed this design. Point C is the center of both circles.



Which statement is true about Josiah's construction?

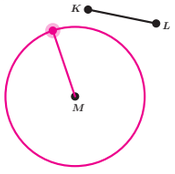
A. $\overline{AB}$ and $\overline{AD}$ have the same length.

B. $\triangle ACD$ is an equilateral triangle.

C. $\overline{AC}$ and $\overline{CD}$ have the same length.

D. $\overline{CB}$ and $\overline{AD}$ have the same length.

3. Construct a segment with an endpoint at point M that is the same length as $\overline{KL}$. *Responses vary.*



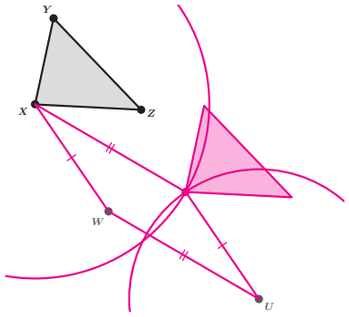
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Name: Date: Period:

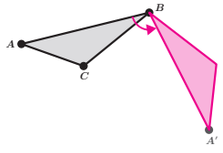
End-of-Unit Assessment (continued)

Geo.1

4. a Translate $\triangle XYZ$ using the translation that takes point W to point U.



b Rotate $\triangle ABC$ around point B so that point A goes to A'.



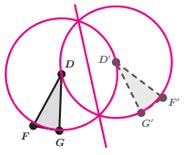
Name: Date: Period:

End-of-Unit Assessment (continued)

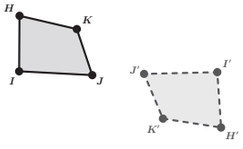
Geo.1

5. Here is triangle DFG and its image after a reflection.

Construct the line of reflection.



6. Here is a quadrilateral and its image after a sequence of transformations.



Describe two different sequences of rigid transformations that transform quadrilateral $HIJK$ onto quadrilateral $H'I'J'K'$. *Responses vary.*

a First sequence:
Rotate quadrilateral $HIJK$ around the midpoint of segment JJ' until point I goes to point I'.

b Second sequence:
Reflect quadrilateral $HIJK$ over the perpendicular bisector of segment JJ' and then reflect over the line $I'J'$.

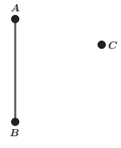
Name: Date: Period:

End-of-Unit Assessment (continued)

Geo.1

7. Here is segment AB. Create a design that includes one construction from each list: *Constructions vary.*

List A	List B
☐ A perpendicular bisector of $\overline{AB}$	☐ A line parallel to $\overline{AB}$ through point C
☐ An equilateral triangle that uses $\overline{AB}$ as one of its sides	☐ A line perpendicular to $\overline{AB}$ through point C
☐ A segment twice the length of $\overline{AB}$	☐ A square with $\overline{AB}$ as one of its sides



Reflection

Select a question to answer.

☐ What is something you're proud of from this unit?

☐ Write what you know about a topic from this unit that you weren't asked about today.

☐ Describe or show one strategy you found helpful in this unit. Name any students who helped you with this strategy.

☐ What else would you like your teacher to know?

Editable Version, Form B

Digital

Unit 1
End-of-Unit
Assessment

Coming Soon

Coming Soon

Coming Soon

Coming Soon

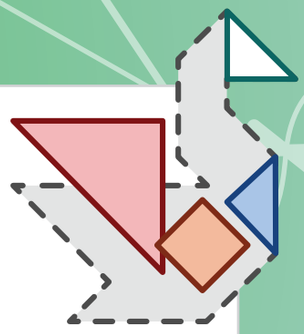
D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:
Use a variety of construction tools to construct precise geometric figures. **(Lessons 1–7)**	**3, 7**	**Support** • Revisiting **Lesson 3, Warm-Up** and inviting students to share strategies for using construction tools to copy a segment. • Revisiting **Lesson 4, Activity 2** and inviting students to share strategies for constructing a perpendicular bisector. • Revisiting **Lesson 5, Activity 2** and inviting students to share strategies for constructing perpendicular and parallel lines using physical tools. • Revisiting **Lesson 6, Activity 2** and inviting students to share strategies for constructing a square. • Inviting students to use the Student Notes to write down important constructions to reference. Students will use many of these constructions in later units to make sense of relationships and prove things about geometric relationships.
Write and analyze descriptions of constructions. **(Lessons 1–6)**	**2**	**Support** • Revisiting **Lesson 1, Activity 2** and inviting students to discuss how they can use circles to construct line segments that have equal length. • Inviting students to use a compass to make sense of each statement and to discuss how construction tools can help them verify the validity of statements.
Perform and describe a sequence of rigid transformations that moves a figure onto another. **(Lessons 8–13)**	**1, 4–6**	**Support** • Revisiting **Lesson 9, Activity 2** and inviting students to share strategies for visualizing the image of a reflection and constructing lines of reflection. • Revisiting **Lesson 10, Activity 2** and inviting students to reflect on the descriptions their partner gave and how they used tools to precisely transform their polygons. • Revisiting **Lesson 11, Activity 2** and inviting students to reflect on how they used tools to rotate the geometric figures. • Revisiting **Lesson 12, Activity 2** and inviting students to review the descriptions that they and their classmates wrote for the transformation challenges.

UNIT 2

Congruence



Students use rigid transformations to explore and prove congruent relationships in angles, triangles, and quadrilaterals.

Essential Questions

- How can I use rigid transformations to prove relationships in angles, triangles, and quadrilaterals?
- How can I prove a conjecture?

Sub-Unit Goals

Sub-Unit 1

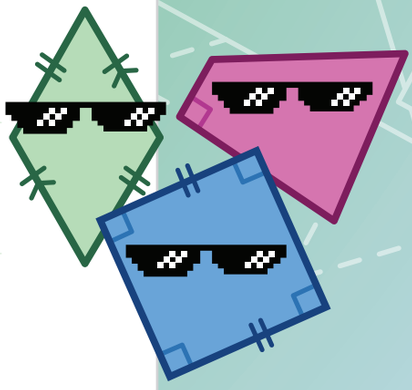
- Use rigid transformations to prove angle relationships.
- Describe a strategy using rigid transformations that proves a relationship is always true.

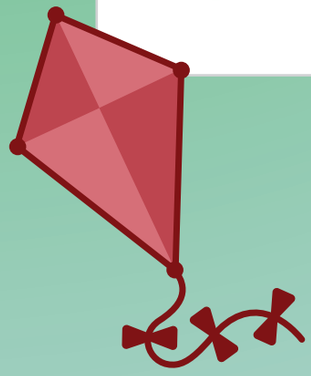
Sub-Unit 2

- Define congruence using rigid transformations.
- Justify how much information is needed to decide that two triangles are congruent.

Sub-Unit 3

- Make sense of constructions using properties and definitions of geometric figures.
- Prove theorems about quadrilaterals.





The Math of the Unit

Focus Telling a story that narrows and deepens with the grade

Content Standards

Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **(HSG.CO.B.6)**

Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.* **(HSG.CO.C.10[W])**

Other Standards Addressed in This Unit: HSG.CO.A.1[W] HSG.CO.A.2 HSG.CO.A.5[W] HSG.CO.B.7[W] HSG.CO.B.8[W] HSG.CO.C.9[W] HSG.CO.C.11 HSG.CO.D.12

W = Widely Applicable Prerequisites

Standards for Mathematical Practice

- Students construct arguments and critique the arguments of others as they prove theorems about lines and angles, triangles, and quadrilaterals. **(MP3)**
- Students attend to precision as they write statements to communicate which parts correspond in congruent figures. **(MP6)**
- Students choose which tools will best support them in making an argument that proves congruence in terms of rigid motion. **(MP5)**

Also Addressed: **MP1, MP2, MP7, MP8**

Coherence Telling a story within and across grades

In This Unit

Students use rigid transformations to explore relationships in and prove theorems about lines and angles, triangles, and quadrilaterals.

< Prior Learning

- **Grade 7**, Unit 7: Use facts about supplementary, complementary, and vertical angles to solve problems **(7.G.B.5)**
- **Grade 7**, Unit 7: Draw geometric shapes with given conditions **(7.G.A.2)**
- **Grade 8**, Unit 1: Understand that a figure is congruent to another if a rotation, reflection, or translation can map one to another **(8.G.A.2)**
- **Grade 8**, Unit 1: Use arguments to establish geometric facts **(8.G.A.5)**
- **Geometry**, Unit 1: Know definitions of geometric objects **(HSG.CO.A.1)**

> Future Learning

- **Geometry**, Unit 4: Use congruence and similarity to prove relationships in geometric figures **(HSG.SRT.B.5)**
- **Geometry**, Unit 5: Use the coordinate plane to prove geometric theorems **(HSG.GPE.B.5)**
- **Algebra 2**, Unit 3: Prove polynomial identities **(HSA.APR.C.4)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lesson 5,** students develop conceptual understanding of congruence as they use rigid transformations to stack figures.

Strategic Competence

In **Lessons 11 and 13–14,** students develop strategic competence as they use a variety of structures for writing a convincing argument to prove a conjecture.

Procedural Fluency

In **Lesson 9,** students develop procedural fluency as they identify which types of triangle congruence criteria are represented in different diagrams.

Adaptive Reasoning

In **Lesson 6,** students develop adaptive reasoning as they justify if two figures are congruent based on whether they have corresponding pairs of congruent parts.

Application

In **Lesson 10,** students apply triangle congruence criteria to help them construct a copy of an angle.

Productive Disposition

In **Lesson 1,** students have opportunities to develop a productive disposition as they consider that what is true for them may not be true for others.

Unit at a Glance

Students use devices for most lessons as indicated with .

Student Notes sheets in the Student Edition are also available for these lessons.

Assess and Respond



A Pre-Unit Check (Optional)

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

7.G.B.5 8.G.A.1 8.G.A.1.B 8.G.A.2 8.G.A.5 MP2 MP3 MP7

Sub-Unit 1



1 Is It? Or Is It Not?

Definitions and Their Consequences

Write precise definitions of real-life objects, and consider the consequences of those definitions.

HSG.CO.A.1
MP3 MP6

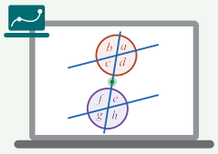


2 Always True

Making Arguments About Angle Relationships

Use rigid transformations to make arguments about angles and to generalize about relationships.

HSG.CO.A.1 HSG.CO.A.2 HSG.CO.B.6 HSG.CO.C.9
MP3 MP5 MP7



3 Angles and Transversals

Using Transformations to Prove Angle Relationships

Explore different relationships that are created when a transversal crosses parallel lines.

HSG.CO.A.2 HSG.CO.C.9
MP2 MP3

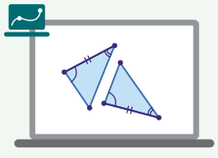


8 Less Is More

Exploring Requirements for Triangle Congruence

Investigate the minimum amount of information needed to prove that two triangles are congruent without having to confirm the congruence of all six corresponding parts.

HSG.CO.B.7 HSG.CO.B.8
MP3 MP8

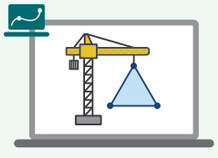


9 Congruence Conditions

Criteria for Triangle Congruence

Make sense of triangle congruence in terms of rigid transformations.

HSG.CO.B.8
MP1 MP3 MP5

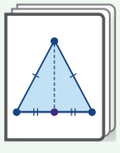


10 Resuming Construction

Using Triangle Congruence to Copy an Angle

Apply triangle congruence to construct copies of segments, angles, and triangles.

HSG.CO.D.12
MP1 MP3 MP5



11 Proving Points

Proving the Properties of Points on Perpendicular Bisectors

Apply an understanding of congruent triangles to prove that points on the perpendicular bisector of a line segment are exactly equidistant from the segment's endpoints.

HSG.CO.C.9
MP1 MP3 MP6 MP7

Practice Day

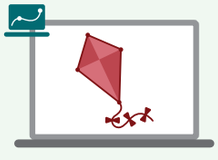


14 A-Door-Able Parallelograms

Diagonals of Parallelograms

Learn about and prove the properties of parallelograms and rectangles, specifically the properties that include diagonals.

HSG.CO.B.6 HSG.CO.C.11
MP3 MP5 MP8



15 Let's Go Build a Kite

Bisecting an Angle

Apply an understanding of transformations, constructions, and congruence to construct an angle bisector.

HSG.CO.D.12
MP3 MP5 MP7



P Practice Day 2

Practice the concepts and skills developed during Lessons 1–15. Consider using this time to prepare for the upcoming assessment.

HSG.CO.B.7 HSG.CO.B.8 HSG.CO.C.9 HSG.CO.C.11 HSG.CO.D.12
MP3 MP5 MP6

Assess and Respond



A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit.

HSG.CO.B.6 HSG.CO.B.7 HSG.CO.B.8 HSG.CO.C.11 HSG.CO.D.12
MP3 MP6

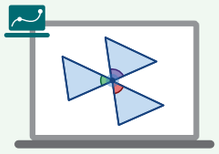
Unit Synthesis and Reflection: Invite students to complete 1–2 tasks to synthesize and reflect on what they learned in the unit.

Pacing: 18 days (+2 optional days) | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- **14 Lessons** (+1 optional lesson)
- **2 Practice Days**
- **Pre-Unit Check** (optional)
- **Sub-Unit Quiz**
- **End-of-Unit Assessment**

Sub-Unit 2

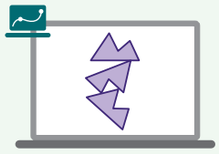


4 Doing a 180

Triangle Angle Sum

Explore different strategies for proving that the measures of interior angles of a triangle sum to 180 degrees.

HSG.CO.C.10
MP3 MP6

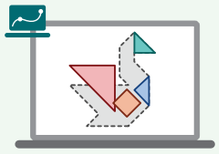


5 Stack 'Em Up

Defining Congruence

Define **congruence** by identifying a sequence of rigid transformations that will stack two figures perfectly.

HSG.CO.A.2 HSG.CO.B.6
MP1 MP3 MP6

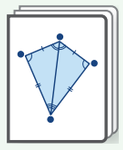


6 Parts and Places

Congruence and Corresponding Parts

Use the structure of tangram puzzles to understand that figures made with congruent parts are only congruent if the parts are **corresponding**.

HSG.CO.B.7
MP1 MP5 MP7



7 Corresponding Components

Congruence Statements

Use rigid transformations to analyze and make sense of corresponding parts of congruent figures.

HSG.CO.A.2 HSG.CO.A.5
HSG.CO.B.7 MP1 MP3 MP6

Practice Day



P Practice Day 1

Practice the concepts and skills developed during Lessons 1–11. Consider using this time to prepare for the upcoming quiz.

HSG.CO.B.7 HSG.CO.B.8
HSG.CO.C.9 HSG.CO.C.10
MP2 MP3 MP6

Assess and Respond

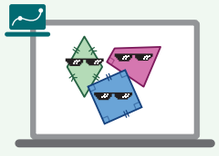


A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

HSG.CO.B.7 HSG.CO.B.8
HSG.CO.C.9
MP3 MP6 MP7

Sub-Unit 3



12 The Quad Squad (Optional)

Defining Quadrilaterals

Explore various quadrilaterals and analyze the common properties between some quadrilaterals based on their names and definitions.

HSG.CO.A.1 HSG.CO.C.11
MP1 MP3 MP6



13 Go With the Flow

Proofs of Parallelogram Properties

Use the structure of flow diagrams to prove **theorems** about parallelograms, specifically that their opposite sides and angles are congruent.

HSG.CO.C.11
MP3 MP6 MP7

Concept Development

By the following points, students will have had multiple opportunities to practice the key skills in this unit:

- **After Lesson 5:** Use rigid transformations to decide if two figures are congruent.
- **After Lesson 9:** Determine if there is enough information to decide if two triangles are congruent by SSS, SAS, or ASA.
- **After Lesson 14:** Write a convincing argument to prove a conjecture.

Pacing Considerations

Lesson 1: This lesson gives students an opportunity to consider the role that definitions play in geometry and proving conjectures. If time runs short, consider omitting Activity 2.

Lessons 2–3: These lessons support students in using rigid transformations to prove theorems about lines and angles from relationships they explored in Grade 8, including vertical angles being congruent and alternate interior and corresponding angles being congruent. If time runs short, these lessons can be combined together by omitting the Warm-Ups of both lessons and Activity 1 of Lesson 2.

Lesson 12 (Optional): This lesson addresses standards from a prior course and supports students in connecting to prior knowledge about the properties of specific quadrilaterals. If time runs short, consider omitting this lesson or combining it with Lesson 13.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Having all items from the geometry toolkit available to students is recommended to build skills in selecting and using tools strategically. Specific tools recommended for each lesson are listed here.

Lesson 2

Copy: Show What You Know *(one per student)*

Materials: tracing paper, straightedges *(optional)*, coloring tools *(optional)*

Lesson 5

Materials: tracing paper *(optional)*

Lesson 6

Materials: computers/devices *(for research, optional)*, tracing paper *(optional)*

Lesson 7

Copy: Show What You Know *(one per student)*

Materials: tracing paper, coloring tools *(as needed)* , straightedges *(as needed)*

Lesson 8

Copy: Show What You Know *(one per student)*

Materials: AngLegs® manipulatives or AngLegs® Sheet, tracing paper, coloring tools *(optional)*

Cut: Line Segment Sheet *(one per pair)*

Lesson 9

Materials: tracing paper *(optional)*

Lesson 10

Geometry toolkit: compasses, straightedges

Materials: tracing paper *(optional)*

Lesson 11

Copy: Show What You Know *(one per student)*

Materials: coloring tools *(optional)*

Cut: Activity 1 Cards PDF *(one set per pair)*

Practice Day 1

Copy: Task Cards

Materials: coloring tools *(optional)*, tracing paper *(optional)*

Lesson 13

Cut: Activity 2 Cards *(one set per pair)*

Materials: blank paper

Lesson 14

Copy: Activities 1 & 2 Sheet *(1–2 per pair,* Show What You Know *(one per student))*

Materials: scissors, tracing paper, straightedges *(as needed)*, whiteboards and dry erase markers *(optional)*

Lesson 15

Geometry toolkit: compasses, straightedges *(optional)*

Practice Day 2

Geometry toolkit: compasses, straightedges

Cut: Task Cards, one set per group

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment, Form A
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards
- Synthesis and Reflection Tasks (optional)

Digital

PDF

Editable Version

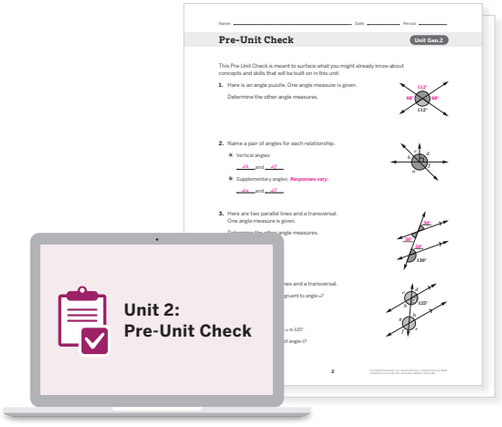
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check (Optional)

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

Standards assessed: 7.G.B.5, 8.G.A.1, 8.G.A.1.B, 8.G.A.2, 8.G.A.5



Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

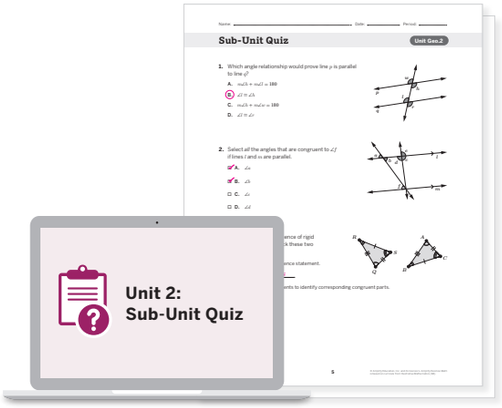
Standard assessed on Lesson 9 Show What You Know: HSG.CO.B.8



Sub-Unit Quiz

Assign the **Sub-Unit Quiz** to learn about your students' understanding of the concepts and skills so far in this unit.

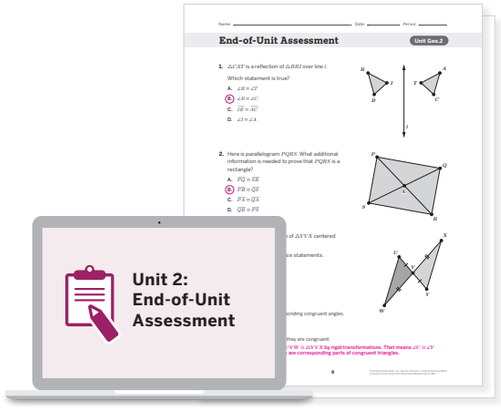
Standards assessed: HSG.CO.B.7, HSG.CO.B.8, HSG.CO.C.9



End-of-Unit Assessment

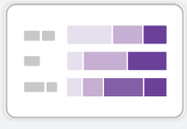
Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit.

Standards assessed: HSG.CO.B.6, HSG.CO.B.7, HSG.CO.B.8, HSG.CO.C.11, HSG.CO.D.12



Reporting

Teacher and Administrator reporting is available for all digital assessments to help teachers understand their students' strengths, create grades, and modify instruction.



Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

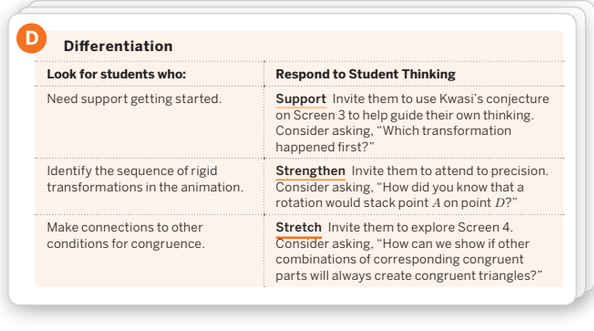
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–4) • Use rigid transformations to prove angle relationships. • Describe a strategy using rigid transformations that proves a relationship is always true.	**Previous Lessons** • Lesson 2, Activity 2 (*Always True?*) **Teacher Moves** • Invite students to read the Summary of Lesson 3 (*Angles and Transversals*) and write it in their own words or paraphrase it to a caregiver or classmate.	**Practice Days** • Practice Day 1: Task A **Practice** • Lessons 1–4	**Explore More** • Lesson 3 (*Angles and Transversals*) **DOK3 Practice** • Lessons 2–4 **Teacher Moves** • Invite students to revisit Lesson 2, Activity 1 and think of additional transformations of figures to make parallel lines.
Sub-Unit 2 (Lessons 5–11) • Define congruence using rigid transformations. • Justify how much information is needed to decide that two triangles are congruent.	**Previous Lessons** • Lesson 7, Screen 3 (*Corresponding Components*) • Lesson 7, Activity 1, Problem 4 (*Corresponding Components*) **Teacher Moves** • Invite students to revisit Lesson 7, Screen 3 (*Corresponding Components*) and explore the interaction again with a partner.	**Repeated Challenges** • Lesson 5 (*Stack 'Em Up*) **Practice Days** • Practice Day 1: Tasks B–D **Practice** • Lessons 5–11	**Explore More** • Lesson 5 (*Stack 'Em Up*) • Lesson 6 (*Parts and Places*) • Lesson 10 (*Resuming Construction*) **DOK3 Practice** • Lessons 5–11
Sub-Unit 3 (Lessons 12–15) • Make sense of constructions using properties and definitions of geometric figures. • Prove theorems about quadrilaterals.	**Previous Lessons** • Lesson 13, Activity 2 (*Go With The Flow*) • Lesson 14, Activity 1 (*A-Door-Able Parallelograms*) • Lesson 15, Activity 3 (*Let's Go Build a Kite*) **Teacher Moves** • Invite students to revisit the Summary for Lesson 10 (*Resuming Construction*) and paraphrase it for a caregiver.	**Practice Days** • Practice Day 2: Tasks A–D **Practice** • Lessons 5–11	**Explore More** • Lesson 12 (*The Quad Squad*) • Lesson 15 (*Let's Go Build a Kite*) **DOK3 Practice** • Lessons 12–15 **Teacher Moves** • Invite students to revisit Lesson 15, Activity 3 (*Let's Go Build a Kite*) and try a different method to construct an angle bisector.

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 9**, where students determine the minimum criteria necessary to prove congruent triangles.



Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite them to use Kwasi's conjecture on Screen 3 to help guide their own thinking. Consider asking, "Which transformation happened first?"
Identify the sequence of rigid transformations in the animation.	**Strengthen** Invite them to attend to precision. Consider asking, "How did you know that a rotation would stack point A on point D?"
Make connections to other conditions for congruence.	**Stretch** Invite them to explore Screen 4. Consider asking, "How can we show if other combinations of corresponding congruent parts will always create congruent triangles?"

Accessibility

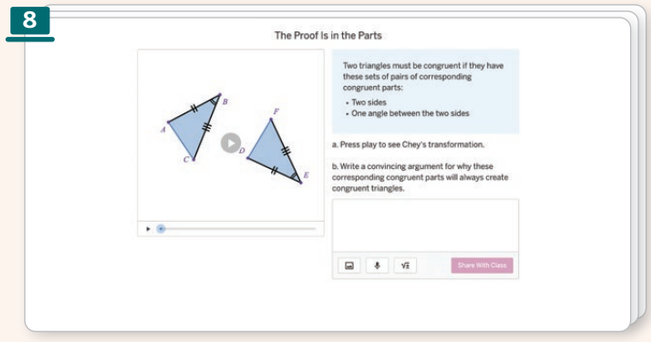
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Representation.

Spotlight on Language and Symbols . . .

Lesson 9: Students determine the minimum criteria that is needed to prove two triangles are congruent by making connections between the text and animation.



Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Affective Functioning . . .

Here are two suggestions in this unit for supporting students with sharing and receiving feedback.

Lesson 1, Activity 2

Accessibility: Affective Functioning To support students in sharing and receiving feedback, model giving and receiving constructive feedback as a class.

Lesson 11, Activity 2

Accessibility: Affective Functioning To support students in sharing and receiving feedback, use a structured process so students know what the feedback will look like.

Math Identity and Community

Every lesson includes suggestions to help develop and strengthen students' math identity and community.

Here are some suggestions in this unit for developing math community as the class acknowledges that proofs are created for and validated in community

Lesson 1, Synthesis

Math Identity and Community
Consider taking several minutes to name your class as a community. Encourage students to share that as a class, all of us are smarter than any one of us, and we need each other to be better listeners.

Lesson 4, Activity 2

Math Identity and Community
Acknowledge that whether an argument is convincing is determined by the community it is created in and for.

Lesson 2, Activity 1

Math Identity and Community
Consider acknowledging the importance of deciding on a definition as a community before using the term or building an argument.

Lesson 11, Activity 1

Math Identity and Community
Consider highlighting the value of collaboration by asking if any students revised the order of their cards by working with a partner.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson
conjecture	2
corresponding angles **prove**	3
congruent	5
corresponding	6
converse (optional) **theorem**	11
angle bisector	15

Review Vocabulary
congruent
equidistant
kite
parallelogram
perpendicular bisector
quadrilateral
rectangle
reflection
rhombus
rigid transformations
rotation
square
straight angle
translation
vertical angles

Contextual Vocabulary	Lesson
tangram	6

This is a non-mathematical word used in this unit, which may or may not be familiar to students. Consider presenting it using contexts, visuals, and/or gestures to provide all students with greater access.

Math Language Routines and Language Goals

Math Language Routine	Lessons
MLR1: Stronger and Clearer Each Time	1, 4, 11, 14
MLR2: Collect and Display	1–2, 4, 6–7, 11, 13
MLR3: Critique, Correct, Clarify	8, 13, 15
MLR6: Three Reads	11
MLR7: Compare and Connect	3, 5, 6, 15

Sub-Unit Language Goals

- Describe a strategy using rigid transformations that proves a relationship is always true.
- Justify how much information is needed to decide that two triangles are congruent.
- Prove theorems about quadrilaterals.

Multilingual/English Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

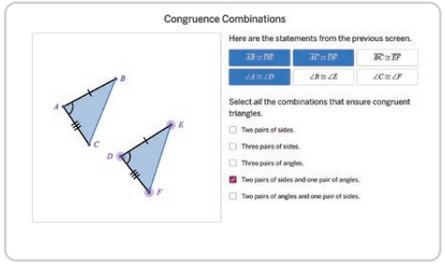
Here is one example from **Lesson 13**, where students make sense of the terms used to describe specific parts of parallelograms.

ML/EL **Multilingual/English Learners** Encourage students to make sense of each term that's shown. Consider asking, "What words do you think of when you hear the word *opposite? Adjacent?*" **(Speaking and Listening)**

Technology

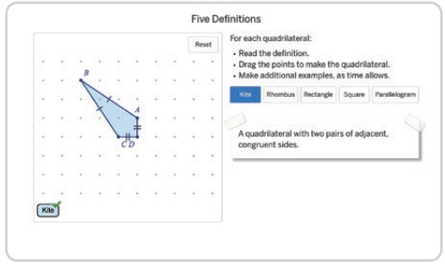
Dynamic, digital interactions are integral to Amplify Desmos Math. Powerful digital tools, such as the teacher dashboard, enable teachers to effectively facilitate rich math discussions.

A Powerful Digital Experience



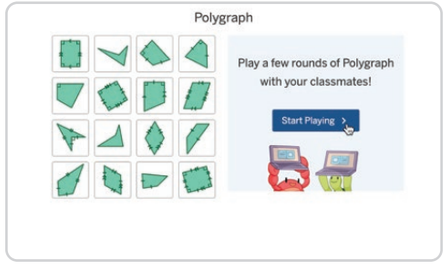
Visualize the Mathematics
(Lesson 9)

Students use multiple representations of mathematical concepts and skills in order to make connections, understand structure, and find patterns.



Receive Responsive Feedback™
(Lesson 12)

Students are free to explore mathematics in interactive ways. When they try new ideas, they receive Responsive Feedback™ so they can refine their thinking and move learning forward.



Collaborate With Classmates
(Lesson 12)

Students have opportunities to view, appreciate, and respond to the mathematical thinking of their classmates and work together in problem solving.

Digital Lessons in This Unit

Most lessons utilize student devices so students can have a dynamic learning experience. Student devices are needed for these lessons. Student Notes sheets in the Student Edition are available for these lessons.

Lesson		Why digital?
1	Is It? Or Is It Not?	Students are invited to edit the rough drafts of their definitions to make them stronger and clearer, while also using multi-select options to clarify their definitions.
3	Angles and Transversals	Students use digital geometry tools to perform rigid transformations to prove angle relationships.
4	Doing a 180	Students use digital interactions to support their exploration of strategies used to prove a conjecture is always true.
5	Stack 'Em Up	Students use digital geometry tools to visualize two figures stacking on top of each other using rigid transformations to determine if they are congruent.
6	Parts and Places	Students use digital manipulatives to drag figures to create congruent shapes and prove congruence using corresponding congruent parts.
9	Congruence Conditions	Students use digital interactions to develop conceptual understanding of which triangle congruence conditions will always create pairs of congruent triangles.

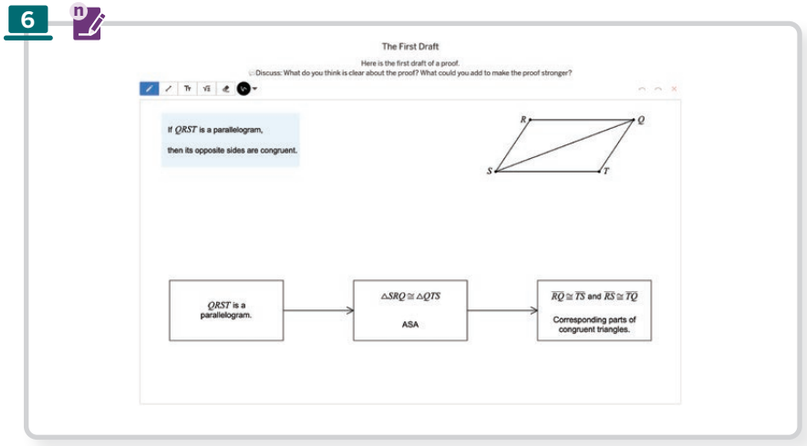
Lesson		Why digital?
10	Resuming Construction	Students use the digital geometry tools to make sense of and perform constructions.
12	The Quad Squad	The digital version of Polygraph coordinates pairings and the question exchange to support students in playfully developing language to describe quadrilaterals by their properties.
13	Go With the Flow	Students use an interactive flow diagram to make sense of how to prove conjectures about parallelogram properties.
15	Let's Go Build a Kite	Students use digital construction tools to develop strategies for constructing an angle bisector.

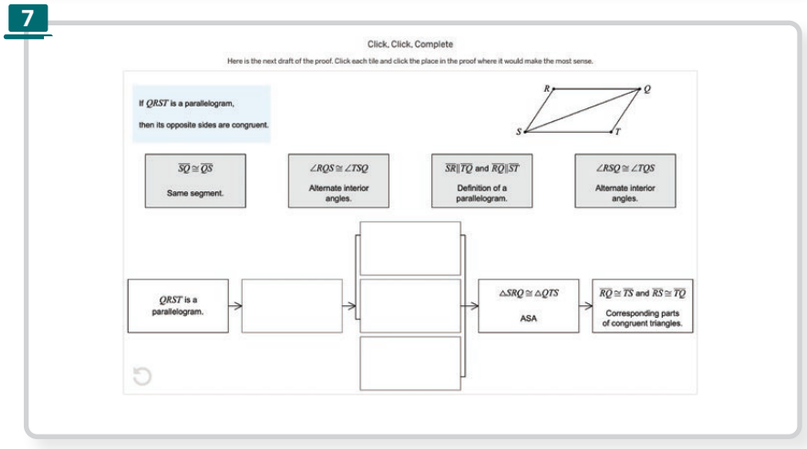
Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Strategic Competence

Lesson 13, Screens 6–7





Context

In this activity, students make a convincing argument to prove opposite sides are congruent in all parallelograms using a flow diagram. Throughout the unit, students develop their strategic competence for making a convincing argument using a variety of structures.

Try This

Put on your student hat and complete Screens 6 and 7.

Points to Ponder

- What do students need to understand about making a convincing argument to support their strategic competence?
- Some students may prefer to organize their thinking in a table. How might you support students in recognizing the value of having more than one way to structure an argument?
- How does anchoring convincing arguments in rigid transformations support students in proving conjectures?

Every lesson includes an opportunity for teachers to reflect on their instruction. These include reflections about routines and activity structures, digital tools, mathematical concepts or language development, student thinking and strategies, math identity and community, and facilitating meaningful conversations.

Spotlight on Lesson 4

Professional Learning

Take a moment to reflect on the strategies students used in this lesson. In what ways are students taking risks when trying out new strategies for proving conjectures? What opportunities were there to celebrate specific students' strategies for proving that the interior angles of a triangle always sum to 180°? How did students validate one another's thinking throughout the lesson?

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed in their math journeys.

Use Congruence and Similarity to Prove Relationships in Geometric Figures

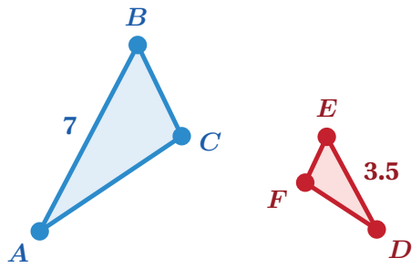
In this unit, students use rigid transformations (reflections, rotations, and translations) to prove that two figures are congruent. **In Unit 3**, students will use dilations to prove that two figures are similar.

Standard Addressed: HSG.SRT.B.5

Example:

Prove that $\triangle ABC \sim \triangle DEF$ by describing a sequence of transformations that moves $\triangle ABC$ onto $\triangle DEF$.

Reflect $\triangle ABC$ over the perpendicular bisector of $\overline{AD}$. Then dilate this image with a center at point D and a scale factor of $\frac{1}{2}$.



Use the Coordinate Plane to Prove Geometric Theorems

In this unit, students prove theorems by using rigid transformations on the Euclidean plane. **In Unit 5,** students will prove theorems on the coordinate plane.

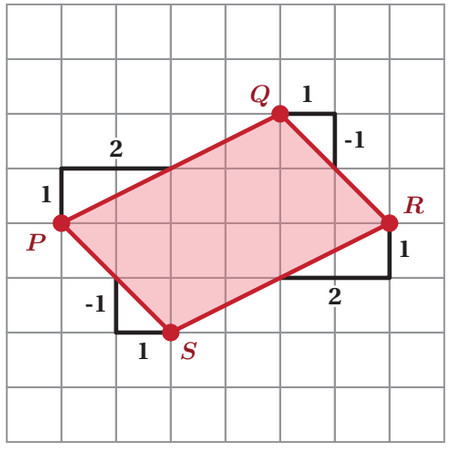
Standard Addressed: HSG.GPE.B.5

Example:

Prove that $PQRS$ is a parallelogram.

$\overline{PQ}$ and $\overline{SR}$ each have a slope of $\frac{1}{2}$.
$\overline{QR}$ and $\overline{PS}$ each have a slope of -1.

Both pairs of opposite sides are parallel, so by definition, $PQRS$ is a parallelogram.



Prove Polynomial Identities

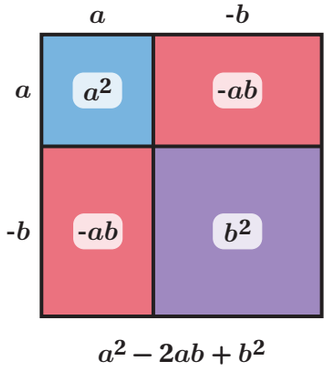
In this unit, students prove geometric theorems about lines, angles, triangles, and quadrilaterals. **In Algebra 2,** students will prove polynomial identities.

Standard Addressed: HSA.APR.C.4

Example:

Prove that $a^2 - 2ab + b^2 = (a - b)^2$.

By rewriting $(a - b)^2$ as $(a - b)(a - b)$, you can multiply the factors to reveal its equivalent identity.



Assess and Respond

Additional data on student thinking can be found in Teacher Reporting.

Pre-Unit Check (Optional)

Independent | ~45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. Text to speech is available in the digital version. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or Skill	Preparation for	DOK	Standard(s)
1	Determine missing angle measures from vertical and supplementary angle relationships	Lesson 2	1	8.G.A.1, 8.G.A.1.B MP2
2	Identify vertical and supplementary angles	Lesson 2	1	7.G.B.5
3	Determine missing angle measures from parallel line relationships	Lessons 2–4	1	8.G.A.5 MP2
4	Identify congruent angles from parallel lines cut by a transversal	Lesson 3	1	8.G.A.5 MP2
5	Use the triangle angle sum to determine missing angle measures	Lesson 3	1	8.G.A.5
6	Apply the triangle angle sum to possible angle measures of triangles	Lesson 4	2	8.G.A.5 MP3
7	Identify rigid transformations to show congruent relationships	Lesson 5	1	8.G.A.2
8	Justify if two figures are congruent using rigid transformations	Lesson 5	2	8.G.A.2 MP7
9	Identify congruent triangles from given angle measures and side lengths	Lessons 8–10	2	8.G.A.2

Assessment Resources

- Student Assessments
- Show What You Know PDFs and Answer Keys

Editable Version

Digital

Unit 2
Pre-Unit Check

Name: ..... Date: ..... Period: .....

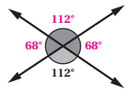
Pre-Unit Check

Unit Geo.2

This Pre-Unit Check is meant to surface what you might already know about concepts and skills that will be built on in this unit.

1. Here is an angle puzzle. One angle measure is given.

Determine the other angle measures.



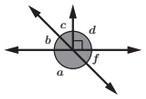
2. Name a pair of angles for each relationship.

a Vertical angles:

$\angle b$ and $\angle f$

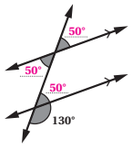
b Supplementary angles: ***Responses vary.***

$\angle a$ and $\angle f$



3. Here are two parallel lines and a transversal. One angle measure is given.

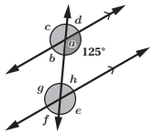
Determine the other angle measures.



4. Here are two parallel lines and a transversal.

a Which angles are congruent to angle a?

Angles c, g, and e



b The measure of angle a is 125°.

What is the measure of angle b?

55°

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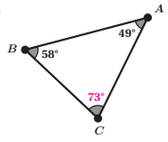
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Pre-Unit Check (continued)

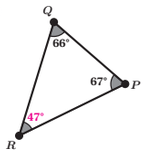
Uni

5. Determine the measure of each missing angle.

a



b



6. Is it possible to have a triangle with two right angles? Circle one.

Yes (No)

Show or explain your thinking.

Explanations vary. **The angles in a triangle sum to 180°, and triangles have three angles. If two of the angles are each 90°, then the third angle must measure 0°, which is impossible.**

7. Select a sequence of rigid transformations that could show the figures are congruent.

a



A. A translation
B. A rotation and a translation
(C.) A reflection and a translation
D. None. They're not congruent.

b



A. A translation
(B.) A rotation and a translation
C. A reflection and a translation
D. None. They're not congruent.

c



A. A translation
B. A rotation and a translation
C. A reflection and a translation
(D.) None. They're not congruent.

D Differentiation (Pre-Unit Check)

Note: To **Strengthen** and **Stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

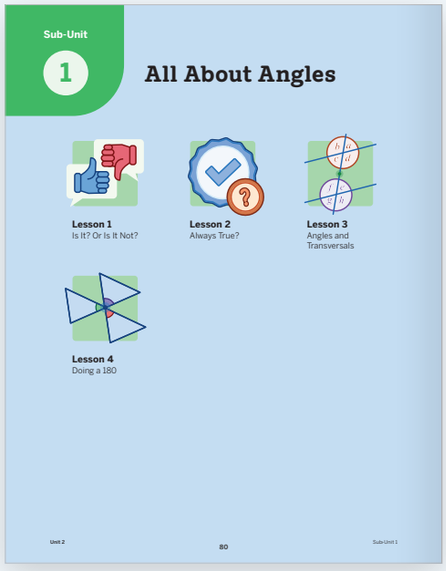
Prior Knowledge	Problem(s)	To respond to student thinking, consider:
Vertical angle and supplementary angle relationships **(Preparation for Lesson 2)**	**1–2**	**Support** • Revisiting **Grade 8, Unit 1, Lesson 10, Activity 1** (*Transforming Angles*). • Inviting students to share prior knowledge about angle relationships.
Parallel lines and angles **(Preparation for Lessons 2–4)**	**3–4**	**Support** • Revisiting **Grade 8, Unit 1, Lesson 10, Activity 1** (*Transforming Angles*). • Pausing to highlight angle relationships in **Lesson 3** after students make convincing arguments to prove conjectures about these relationships. • Creating and referring to a class anchor chart describing these relationships throughout the unit.
Triangle angle sum **(Preparation for Lesson 4)**	**5–6**	**Support** • Revisiting **Grade 8, Unit 1, Lesson 11** (*Tearing It Up*). • Inviting students to summarize the triangle angle sum theorem at the end of **Lesson 4.**
Congruence of figures shown through a series of rigid transformations **(Preparation for Lessons 2–5)**	**7–8**	**Support** • Revisiting **Grade 8, Unit 1, Lesson 2** (*Spinning, Flipping, Sliding*). • Capturing student ideas that surface prior knowledge about transformations at the start of **Lesson 2**. • Capturing, displaying, and discussing student work that highlights translations, rotations, and reflections as examples of rigid transformations.

All About Angles

Sub-Unit 1 Goals

- **Goal:** Use rigid transformations to prove angle relationships.
- **Language Goal:** Describe a strategy using rigid transformations that proves a relationship is always true.

In this sub-unit, students use translations, rotations, and reflections to **prove** theorems about lines and angles. They write definitions and consider the implications of those definitions within their class community. Students perform rigid transformations to prove whether vertical angles are always congruent and under what conditions alternate interior and **corresponding angles** are congruent. They also prove that the interior angles of a triangle sum to 180 degrees.



Math That Matters Most

Concept or Skill	Example
Use rigid transformations to prove vertical angles are congruent.	Here's a conjecture: *Vertical angles are congruent*. How might you convince someone that this conjecture is always true? Rigid transformations keep angles and distances the same. I can rotate any angle 180° around its vertex onto its vertical angle. This strategy works even if I draw the vertical angles a different way.
Use rigid transformations to prove that when a transversal crosses parallel lines, corresponding angles are congruent.	Here's a conjecture: *When a transversal crosses parallel lines, corresponding angles are congruent*. How might you convince someone that this conjecture is always true? Rigid transformations keep angles and distances the same. If I translate one line onto its parallel line, the angles in corresponding positions stack perfectly, which means they're congruent.

Concept or Skill	Example
Use rigid transformations to prove that when a transversal crosses parallel lines, alternate interior angles are congruent.	Here's a conjecture: *When a transversal crosses parallel lines, alternate interior angles are congruent.* How might you convince someone that this conjecture is always true? If I rotate one line crossed by a transversal around a center point that isn't the point of intersection, I can create alternate interior angles. Rigid transformations keep angles and distances the same, so these angles will always be congruent no matter what.
Prove that the interior angles of a triangle sum to 180 degrees.	Here's a conjecture: *The interior angles of a triangle sum to* 180°. How might you prove that this conjecture is always true? If I construct a line parallel to one side of a triangle through the vertex across from it, I can use alternate interior angles being congruent and rigid transformations to show that all three angles within a triangle must add to 180°.

Unit 2

Lesson 1

Digital Lesson

This lesson uses digital student screens. Student Notes are also available.



Is It? Or Is It Not?

Definitions and Their Consequences

Let's think about definitions and their consequences.

Focus and Coherence

Today's Goals

1. **Goal:** Write a definition and consider its implications, revising the definition as necessary.
2. **Language Goal:** Analyze a definition by considering examples and non-examples. **(Reading and Writing)**

Students are introduced to a situation that requires them to write precise definitions of real-life objects and consider the consequences of those definitions. They use examples and non-examples to analyze and critique definitions to make them stronger and clearer. Students learn that what is true for them may not also be true for everyone else in their community. **(MP6)**

Prior Learning

In Grade 8, Unit 1, students defined congruence using examples and non-examples. In Unit 1 of this course, students learned definitions of several geometric terms, including *parallel* and *perpendicular lines*, as well as *perpendicular bisectors*.

Future Learning

In Lesson 2, students use the definition of *vertical angles* to prove that they are always congruent. Throughout the unit, students use definitions to construct arguments and critique the arguments of others.

Rigor and Balance

- Students develop **adaptive reasoning** as they reflect on the definitions they write, using examples and non-examples to make them stronger and clearer.
- Students have opportunities to develop a **productive disposition** as they consider that what is true for them may not be true for others.

Standards

Addressing

HSG.CO.A.1
Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, **based on the undefined notions of point, line, distance along a line, and distance around a circular arc.**

Mathematical Practices: MP3, MP6

Building Toward

HSG.CO.C.9
HSG.CO.C.10

Lesson at a Glance

~ 45 min

Standard: HSG.CO.A.1



Why digital?

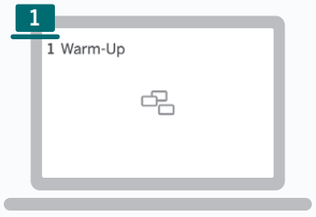
Students are invited to edit the rough drafts of their definitions to make them stronger and clearer, while also using multi-select options to clarify their definitions.

Warm-Up

Pairs Sharing a Device | 5 min

Pacing: Screen 1

Students complete a card sort of several food images and decide: Is it a sandwich?

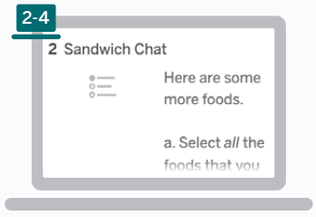


Activity 1

Pairs Sharing a Device | 10 min

Pacing: Screens 2–3, Screen 4

Students sort through examples and non-examples to help them write a definition of *sandwich*. MLR1: Stronger and Clearer



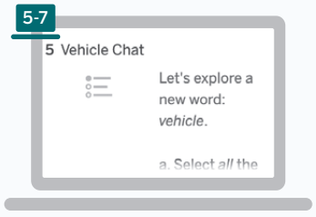
Activity 2

Pairs Sharing a Device | 8 min

Pacing: Screens 5–7

Students explore how to define a new word, *vehicle*, using examples and non-examples to develop a precise and clear definition.
MLR1: Stronger and Clearer (MP6)

Key Takeaway (Screen 7): Definitions are often as unique as the people who create them. Knowing how one person defines a term can help us understand one another better.
See the Activity page for full Key Takeaway.

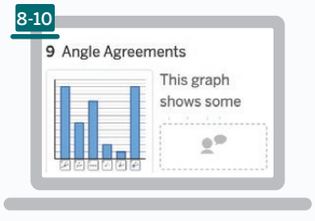


Activity 3

Pairs Sharing a Device | 12 min

Pacing: Screens 8–9, Screen 10

Students apply their thinking to understand when it can be helpful to have shared definitions and then create a class definition of *angle*.
MLR2: Collect and Display



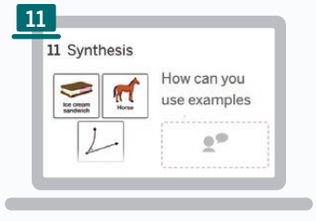
Synthesis

Whole Class | 5 min

Pacing: Screen 11

Students synthesize their understanding of how examples and non-examples can help make a definition more precise.
MLR2: Collect and Display

Lesson Takeaway: Definitions help us communicate with one another and are often made to help us live in community and speak clearly with others.
See the Synthesis page for full Lesson Takeaway.

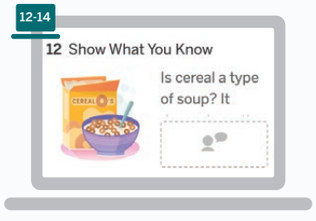


Show What You Know

Independent | 5 min

Pacing: Screens 12–13

Students demonstrate their understanding by writing definitions of *soup* that would make cereal an example or a non-example.



Prep Checklist

Assign the digital student screens.
Student Notes are also available.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)

Student Notes sheet in Student Edition:
Consider providing the Student Notes to support class discussion and student thinking.



Pairs Sharing a Device | 5 min | Pacing: Screen 1

Warm-Up

Purpose: Students complete a card sort to consider several images and decide: Is it a sandwich?



Student Notes are also available

1 Launch

1 **Invite students** to think about each item and decide: Is it a sandwich?

ML/EL **Multilingual/English Learners** Encourage students to discuss how to group the images in their primary language, clarifying any unknown words. **(Speaking and Reading)**

Encourage students to share their reasoning with a partner about how they each sorted the cards. It is okay — even desirable — for students to sort their cards differently and not come to a consensus on this screen.

Note: If time allows, invite pairs to compare their groupings, justify their thinking, and make revisions based on their conversation.

2 Connect

Consider asking:

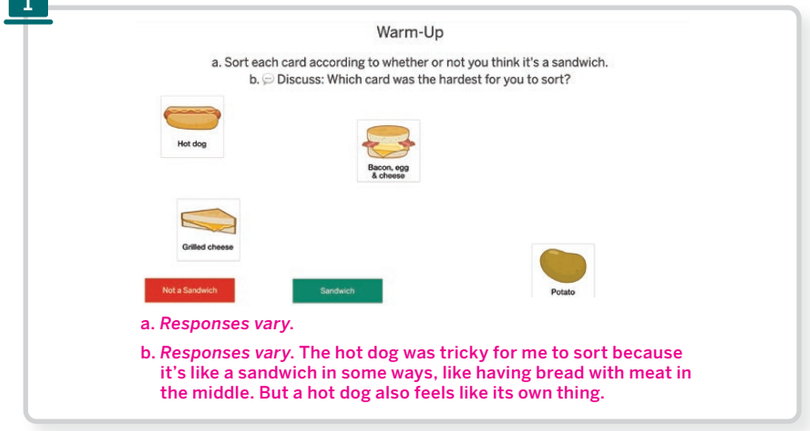
- "How did you decide to group each item?"
- "Why do you think a [hot dog] is a sandwich but a potato isn't?"
- "What is it about a ___ that made it difficult to decide whether it was a sandwich or not?"

Math Identity and Community Consider highlighting the value of changing one's mind by asking if any students revised their groups after discussing their thinking with a partner.

Consider sharing that in this lesson, students will wrestle with what counts as a thing and what doesn't, finding ways to navigate those tricky decisions.

Students using digital

1



a. *Responses vary.*

b. *Responses vary.* The hot dog was tricky for me to sort because it's like a sandwich in some ways, like having bread with meat in the middle. But a hot dog also feels like its own thing.

Pairs Sharing a Device | 10 min | Pacing: Screens 2–3, Screen 4

Activity 1 Sandwich Chat

Purpose: Students sort through examples and non-examples to help them write a definition of *sandwich*.



Student Notes are also available

1 Launch

2 ML/EL **Multilingual/English Learners** Read the label for each item aloud. Identify any unknown items and encourage others to describe them (e.g., A calzone is a type of pizza that is folded in half before cooking). **(Reading)**

Invite students to select all the items they think are sandwiches, then discuss with a partner.

Display the dashboard's Teacher View to show which items the class defined as sandwiches.

2 Monitor

3 **Invite students using notes** to write their definition of *sandwich*. Consider sharing that first drafts do not have to be perfect to be valuable.

Pause briefly and invite several students to share their definitions. Consider asking, "How are these definitions alike? How are they different?"

Invite students to go to Screen 4 using the Pace tool.

3 Connect

4 MLR **MLR1: Stronger and Clearer Each Time** Encourage students to use the case of a taco to write a second draft that is stronger, (with more or better evidence), and clearer (with more precision, organization, etc.) than their first draft.

Invite students to share their definition with a partner and give each other feedback. Encourage students to use other cases to support them in revising their definition, such as lasagna, quesadilla, or torta. **(MP3)**

Math Identity and Community Consider using other cases that are culturally relevant to your students.

Consider asking:

- "How do examples and non-examples help us define *sandwich* more clearly?"
- "If you think sandwiches must have two slices of bread, how does that change what a sandwich is?"
- "Is it okay if we have different definitions? When might it not be okay?"

Consider sharing that in 2006, a sandwich shop in Massachusetts had an agreement with a mall that no other shop in that mall could sell sandwiches. The mall rented to a new shop that was going to sell burritos. The sandwich shop claimed that burritos were a sandwich, preventing the new shop from opening.

Students using digital

2

Sandwich Chat



Here are some more foods.

a. Select *all* the foods that you think are sandwiches. *Responses vary.*

b. **Discuss:** What properties make each food item a sandwich or non-sandwich?

Responses vary.

- Gyro: This is a sandwich because it has meat, cheese, veggies, and a form of bread, pita.
- Peanut butter & jelly: This is a sandwich because it has two distinct pieces of bread and a filling.
- Ice cream sandwich: This has "sandwich" in the name, but I think it's not a sandwich because it's sweet, not savory.
- Avocado toast: This isn't a sandwich because the "filling" is mostly exposed rather than enclosed.
- Bagel with cream cheese: This was a close call for me, but I think it's a sandwich because it has bread and cheese.
- Calzone: This is not a sandwich because the filling is totally enclosed by the crust. A sandwich has to let the filling breathe!

3

What's a Sandwich?



Here are the sandwiches and non-sandwiches according to you.

Write a definition of *sandwich* that is true for you.

Responses vary.

- A food that has a savory filling and some form of bread.
- A food with meat and one or two pieces of bread.
- A savory food that is surrounded by bread on two or more sides.

4

Let's Taco 'Bout It



Here is a definition of a sandwich:

A savory food that is surrounded by bread on two or more sides.

a. According to the definition, is a taco a sandwich?

Yes | No | I'm not sure

b. Use this taco (or other foods) to make the definition stronger and clearer.

a. *Responses vary.*

b. *Responses vary.*

- A savory food that is about 25% to 75%surrounded by bread.
- An edible container made from a flour or wheat base, with an edible filling inside.

Pairs Sharing a Device | 8 min | Pacing: Screens 5–7

Activity 2 Vehicle Chat

Purpose: Students explore how to define a new word, *vehicle*, using examples and non-examples to develop a precise and clear definition. **(MP6)**

Short on time? Consider omitting Screen 7.



Student Notes are also available

1 Launch

5 ML/EL **Multilingual/English Learners** Read the label for each item. Identify any unknown items and encourage others to describe them (e.g., A horse is an animal that is used for riding, racing, and carrying and pulling loads.). **(Reading)**

Invite students to share which objects they said where vehicles and to name which properties they thought were important in deciding. Consider asking, "Is it okay if someone names a different property of a vehicle than you? Why or why not?"

2 Monitor

6 **Invite students using notes** to write the first draft of their definition of *vehicle*.

D **Differentiation: (Support)** Look for students who need support getting started in writing a definition of *vehicle*. Consider asking, "What's one property that you think all vehicles must have? How does that help you write your definition?"

ML/EL **Multilingual/English Learners** Invite students to write in their primary language and then translate their second draft into English second draft. **(Writing)**

7 A **Accessibility: Affective Functioning** To support students in sharing and receiving feedback, model giving and receiving constructive feedback as a class.

3 Connect

MLR **MLR1: Stronger and Clearer Each Time** Encourage students to use the example of an escalator and other cases (e.g., zip line or sled) to write a second draft of their definition that is stronger and clearer than their first draft.

Encourage students to share their definition with a partner and give each other feedback by asking questions like: **(MP3)**

- "Do you think all vehicles have the property of _____? How can you include that in your definition?"
- "What do you mean when you say_____?"

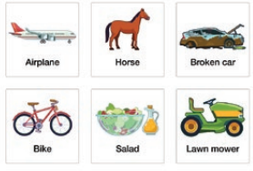
To surface the Key Takeaway, consider asking:

- "What is the purpose of a definition?"
- "Who gets to decide if a definition is precise or clear enough, or right?" **(MP6)**
- "Why might it be helpful to know if your definition is different from someone else's?"

Students using digital

5

Vehicle Chat



Let's explore a new word: *vehicle*.

a. Select *all* the items that you think are vehicles. *Responses vary.*

b. **Discuss:** What properties make each item a vehicle or non-vehicle?

Responses vary.
- Airplane: This is a vehicle because it transports a lot of people.
- Horse: A horse transports people, but I think it's not a vehicle because it's a living thing.
- Broken car: A broken car is a vehicle because it used to be able to transport people.
- Bike: A bike is a vehicle even though it's not motorized because it gets a person from here to there.
- Salad: Salad is definitely a vehicle for dressing.
- Lawn mower: A lawn mower is not a vehicle because it's not used for transporting people.

6

What's a Vehicle?



Here are the vehicles and non-vehicles according to you.

Write a definition of *vehicle* that is true for you.

Responses vary.
- A thing that transports or carries something.
- A motorized device that people can use to get from one place to another.
- A non-living thing that takes people from here to there.

7

Escalating Your Definition



Here is a definition of a vehicle:

> A motorized device that people can use to get from one place to another.

a. According to the definition, is an escalator a vehicle?

Yes | No | I'm not sure

b. Use this escalator (or other items) to make the definition stronger and clearer.

a. *Responses vary.*

b. *Responses vary.*
- A machine that moves and carries people or things from one place to another.
- A thing that is a carrier for another thing, person, or group of people.

Key Takeaway: Definitions are often as unique as the people that create them. One person's definition of *vehicle* can be different from another's, and that is okay. Knowing how someone defines a term can help us understand one another better.

Pairs Sharing a Device | 12 min | Pacing: Screens 8–9, Screen 10

Activity 3 Angle Chat

Purpose: Students apply their thinking to understand when it can be helpful to have shared definitions and create a class definition of *angle*.



Student Notes are also available

1 Launch

8 **Encourage students** to select the images they think are angles, then compare their selections with a partner, justify their thinking, and make revisions based on their conversation.

Math Identity and Community Encourage students to celebrate a classmate who used reasoning to support their thinking, or how a classmate's argument about an image changed their mind. **(MP3)**

2 Monitor

9 **Pause** briefly to discuss the class distribution of angle selections. Consider asking:

- "How is thinking about the properties of angles similar to thinking about properties about vehicles and sandwiches? How is it different?"
- "Based on the choices we made, what are some properties of angles we should include in our class definition?"

Capture and share several ideas about what should be included in a class definition of *angle*, highlighting those that name specific properties.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas and language of what should be included in the definition of *angle*. Add any examples and non-examples from the bar graph.

Consider asking after drafting the class definition, "Is there anything we can make stronger or clearer about our definition so it is more precise?" **(MP6)**

3 Connect

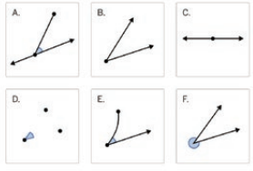
Consider asking:

- "Why do you think definitions are important in Geometry class? Outside of Geometry class?"
- "When in Geometry class do you think it's okay to have different definitions? When might we need to have the same definition?"
- "Why might it be important for everyone in our classroom community to have the same definition of *angle*? In the larger mathematics community?"
- "Who gets to decide what the definition of an angle is?" *Responses vary.* We do! We create definitions to use in our community and to support us in understanding one another. In our class, we will decide together on definitions so we can all communicate with each other precisely.

Students using digital

8

Angle Chat



Now let's think about another word: *angle*.

a. Select *all* the images that you think show an angle. *Responses vary.*

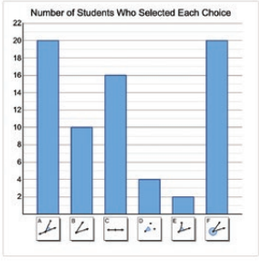
b. **Discuss:** What properties make each image an angle or non-angle?

Responses vary.

- A is an angle because the blue arc shows space between two straight lines.
- B is an angle because two rays meet at a point.
- C is not an angle because it's just a line with a point on it.
- D is not an angle because it doesn't have any lines, segments, or rays.
- E is not an angle because one of the black segments isn't straight.
- F is an angle because the blue arc shows the amount of turn from one ray to another.

9

Angle Agreements



This graph shows some students' selections from the previous screen.

What should be included in a class definition of *angle*? What should not be included?

Responses vary.

- A class definition of "angle" should mention that rays, segments, and straight lines must be used. The definition shouldn't allow for any curved lines like the one in Choice E.
- The definition can say that little arcs can be used to show the angle better, but it's still an angle even if there is no arc.

10

When Should We Agree?



Kris and Tasia have some disagreements about sandwiches and angles.

Discuss: When can it be helpful to agree on the definition for a word? When can it be okay to disagree?

Responses vary.

- I think it's usually OK if we have different definitions for the same word. If I think salad is a vehicle and you don't, life will go on! Maybe we'll need a short conversation to clarify what we mean, and that's OK too.
- I can see how in math class, it's helpful to agree on definitions for important math terms. There might be other situations where we should agree on words like "sandwich" and "vehicle." If my friend asks, "Hey, want me to get a sandwich for you at the store?" I don't want them to get me a bike.

Whole Class | 5 min | Pacing: Screen 11

Synthesis



Student Notes are also available

11 **Invite students** to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Examples and non-examples help us think about properties that are important to include in a definition.
- Items that are examples or non-examples for me may be different from those that are examples or non-examples for others, and that's okay.
- Using a specific case and deciding if something has properties that should be included in our definition helps make our definitions stronger and clearer.

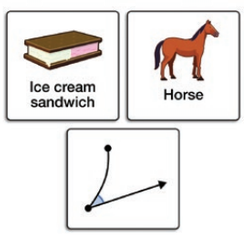
MLR **MLR2: Collect and Display** Invite students to revisit their ideas and language of what should be included in the definition of *angle*. Consider asking how examples and non-examples helped shape their definition.

Math Identity and Community Consider taking several minutes to name your class as a community. Encourage students to share that as a class, all of us are smarter than any one of us, and we need each other to be better listeners.

Students using digital

11

Synthesis



How can you use examples and non-examples to help make a definition more precise?

Responses vary. Examples and non-examples can help me realize my definition is too inclusive or not inclusive enough. For instance, I don't think an ice cream sandwich counts as a sandwich, but my initial definition could include ice cream sandwiches. That's why I added the word "savory" to my definition so it excluded sweet things.

Lesson Takeaway: Definitions help us communicate with one another and are often made to help us live in community and speak clearly with others. It is often okay to have different definitions from another person's. There are also times when having a shared definition will help the community communicate clearly and precisely with one another. **(MP6)**

Independent | 5 min | Pacing: Screens 12–13

Show What You Know

Purpose: Students demonstrate their understanding by writing definitions of *soup* that would make cereal an example or a non-example.

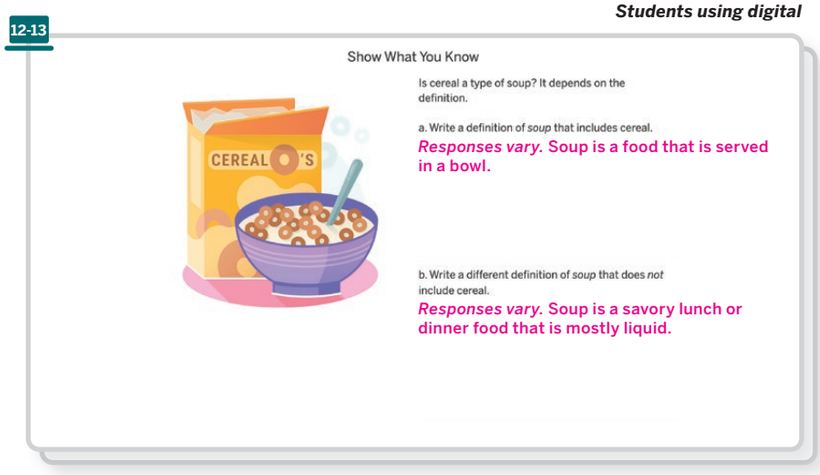


Students using print

12-13 Today's Goals

1. **Goal:** Write a definition and consider its implications, revising the definition as necessary.
2. **Language Goal:** Analyze a definition by considering examples and non-examples. **(Reading and Writing)**

Students using digital



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

S Support

Provide targeted intervention.

If student work shows **partial understanding** of writing definitions by considering examples and non-examples, consider:

- Inviting students to read the **Summary** and highlight key components.

S Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice**.
- Revisiting **Screens 3 and 6** and inviting students to consider how they wrote definitions for *sandwich* and *vehicle* on these screens.
- Inviting students to review the **Summary** and complete the **Try This** problem.

S Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting them to write their own problem that is similar to the **Try This** problem.

Professional Learning

Take a moment to reflect about definitions and the role they play in Geometry and in general conversation. How did writing definitions about sandwiches and vehicles support students in thinking about the role definitions play in Geometry? How did students discuss when it is okay to disagree about definitions and when it is not okay? How can you support students in writing precise definitions to support them as they construct arguments and critique the arguments of others in this unit? **(MP3, MP6)**

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 1 Practice

Students using digital

Lesson Summary

Definitions help us communicate with one another. They are often created to help us understand one another as we live in community. We can use examples and non-examples to help us think about what to include in a definition.

Definitions are often as unique as the people that create them. For example, one person's definition of a vehicle can be very different from another person's, and that is okay. When members of the same community use different definitions, we can begin by asking how each person defines a term so that we can better understand each other.

There are also times when a shared definition can help a community speak clearly and precisely with one another. Creating a shared definition can have consequences, and a community that has a shared definition can make sense of those consequences together.

Try This

a. Select *all* the activities that you think are sports. ***Responses vary.***

☐ Poker ☐ Chess ☐ Fishing

☐ Dominoes ☐ Bowling

b. Write a definition of *sport* that is true for you and the choices you made above.
Responses vary. **A sport is an activity that involves competition and requires physical effort.**

Unit 2 Lesson 1 82 Lesson Summary | Try This

Lesson Practice
Geo.2.01

Name: _____ Date: _____ Period: _____

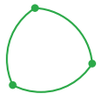
1. Select *all* the items that you think are cake. ***Responses vary.***

☑ A. 
☑ B. 
☑ C. 
☑ D. 
☐ E. 
☐ F. 

2. Write a definition of *cake* that is true for you and the choices you made in the previous problem.
Responses vary. **A sweet baked food topped with icing or frosting.**

Problems 3–5: Raine's definition of a triangle is a figure that has three angles.

3. Select *all* the figures that fit Raine's definition. ***Responses vary.***

☑ A. 
☑ B. 
☐ C. 
☑ D. 
☐ E. 
☑ F. 

Unit 2 Lesson 1 83 Lesson Practice

Lesson Practice
Geo.2.01

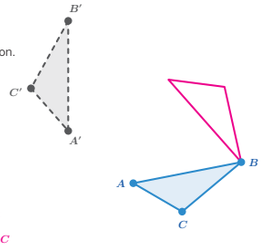
Name: _____ Date: _____ Period: _____

4. Arnav has a different definition of a triangle, which includes only choices *B*, *D*, *E*, and *F*. What is a possible definition that Arnav is using?
Responses vary. **A figure that has three segments.**

5. What is a possible definition of a triangle that includes only choices *B* and *F*?
Responses vary. **A polygon that has exactly three interior angles and three segments.**

Spiral Review

Problems 6–8: Here is a diagram.



6. Select the rigid transformation that moves $\triangle ABC$ onto $\triangle A'B'C'$ in a single transformation.

A. Rotation

B. Translation

C. Reflection

7. Describe a strategy for completing the transformation that moves $\triangle ABC$ onto $\triangle A'B'C'$.
Responses vary. **I could construct the segment AA'. Then I could construct the perpendicular bisector of this segment. Last, I'd reflect $\triangle ABC$ over that line.**

8. Sketch a new triangle that could be the result of a rotation of $\triangle ABC$, followed by a reflection.
Responses vary. **Sample shown.**

Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

Unit 2 Lesson 1 84 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-lesson			
	1	1	HSG.CO.A.4*
	2	2	HSG.CO.A.4*
	3–4	2	HSG.CO.A.1*
	5	2	HSG.CO.A.1
Spiral Review			
Test Practice	6	1	HSG.CO.A.4
	7	2	HSG.CO.A.5
Fluency	8	2	HSG.CO.A.5

*This problem builds toward the standard shown.

Need More Practice?

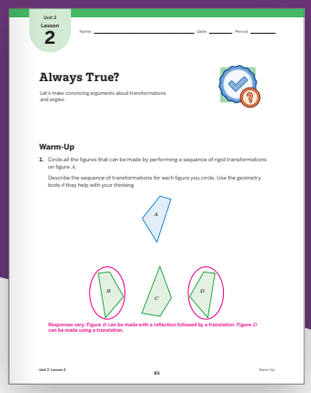
Additional Practice resources coming soon.

Unit 2

Lesson

2

This lesson uses print Student Edition pages and digital Teacher Presentation Screens.


Unit 2 Lesson 2

Always True?

Let's make convincing arguments about transformations and angles.

Warm-Up

1. Circle all the figures that can be made by performing a sequence of rigid transformations on figure A.

Describe the sequence of transformations for each figure you circle. Use the geometry tools if they help with your thinking.

Responses vary. Figure B can be made with a reflection followed by a translation. Figure D can be made using a translation.

Unit 2 Lesson 2 85 Warm-Up

Always True?

Making Arguments About Angle Relationships

Let's make convincing arguments about transformations and angles.

Focus and Coherence

Today's Goals

1. **Goal:** Use rigid transformations to explore and investigate *conjectures*.
2. **Goal:** Prove vertical angles are congruent using rigid transformations.
3. **Language Goal:** Describe a rigid transformation that will meet specific conditions. **(Reading, Writing, Speaking, and Listening)**

Students use rigid transformations to make arguments about angles and to generalize about relationships. Students use transformation strategies and tools of their choice to support their arguments and help prove their **conjectures**.

Prior Learning

In Grade 8, students learned that a rigid transformation is a transformation that doesn't change the side lengths or angle measures of a figure. They were also introduced to translations, reflections, and rotations, which they used to verify that *vertical angles* are *congruent*. In Geometry Unit 1, Lessons 8–12, students described sequences of rigid transformations with precise language and used geometry tools to perform the transformations.

Future Learning

In Lesson 3, students will apply rigid transformations to explore and prove conjectures about angle relationships created by parallel lines. In Lesson 4, students will use a variety of strategies to prove angle relationships in triangles. In Lesson 5, students will use transformations to prove the congruence in polygons.

Rigor and Balance

- Students **apply** rigid transformations as a way to prove angle relationships.
- Students develop **strategic competence** as they investigate different transformation strategies in order to show a variety of relationships.

Vocabulary

New Vocabulary

conjecture

converse (optional)

Review Vocabulary

congruent

vertical angles

Standards

Addressing

HSG.CO.C.9

Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

HSG.CO.B.6

Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

Also Addressing: **HSG.CO.A.1, HSG.CO.A.2**

Mathematical Practices: MP3, MP5, MP7

Building On	Building Toward
8.G.A.1	**HSG.CO.A.1**
8.G.A.2	**HSG.CO.C.9**

Lesson at a Glance

~ 45 min

Standards: HSG.CO.C.9, HSG.CO.B.6, HSG.CO.A.1, HSG.CO.A.2

Warm-Up

Pairs | 5 min

Students use geometry tools and rigid transformations strategically to verify *congruent* figures. **(MP5)**





Presentation Screen 1

Activity 1

Pairs | 15 min

Students make use of structure as they perform rigid transformations to create geometric objects. **(MP7)**

Key Takeaway: Translations, reflections, and rotations are all examples of rigid transformations; they preserve distances and angle measures. Because of this definition, they can be helpful in proving geometric relationships.
See the Activity page for full Key Takeaway.





Presentation Screens 2–6

Activity 2

Pairs | 15 min

Students use geometry tools strategically to prove *vertical angle* relationships. **(MP5)**

Key Takeaway: Rigid transformations can be used to prove that *vertical angles* are always congruent. A $180°$ rotation of one angle will move it onto the other in the vertical angle pair, no matter the size of the angles. This proves the **conjecture** is always true.





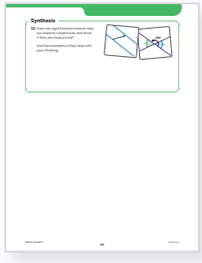
Presentation Screen 7

Synthesis

Whole Class | 5 min

Students synthesize strategies that use rigid transformations to prove geometric relationships. MLR2: Collect and Display

Lesson Takeaway: Rigid transformations are tools that can help make convincing arguments as we explore and prove geometric relationships. Because rigid transformations preserve distances and angle measures, they can be used to prove *congruence*.
See the Synthesis page for full Lesson Takeaway.



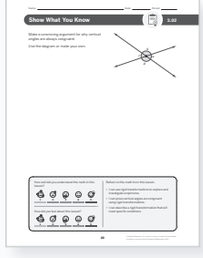


Presentation Screen 8

Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing a convincing argument about why vertical angles are always congruent. **(MP3)**





Presentation Screen 9

Prep Checklist

Invite students to use their Student Edition. Display the Presentation Screens to help facilitate the lesson.

This lesson includes:

- Presentation Screens
- Student Edition
- Show What You Know
- tracing paper
- coloring tools (optional)
- straightedges (optional)

Pairs | 5 min

Warm-Up

Purpose: Students use geometry tools and rigid transformations strategically to verify *congruent* figures. **(MP5)**



Presentation Screen 1

1 Launch

Display the figures. Consider asking, "What tools might help you determine which figures can be made with a rigid transformation?"

Encourage students to strategically use tools, such as tracing paper, coloring tools, or straightedges, to support their thinking. **(MP5)**

To connect to prior knowledge, consider asking:
- "What are examples of rigid transformations?"
- "What stays the same during rigid transformations? What changes?"

To encourage creativity, consider inviting students to try a strategy that is different from their partner's strategy.

Look for a variety of strategies used, such as:
- Using tracing paper and/or paper folding to perform a translation, reflection, or rotation.
- Using a straightedge to connect corresponding points in the pre-image and image.

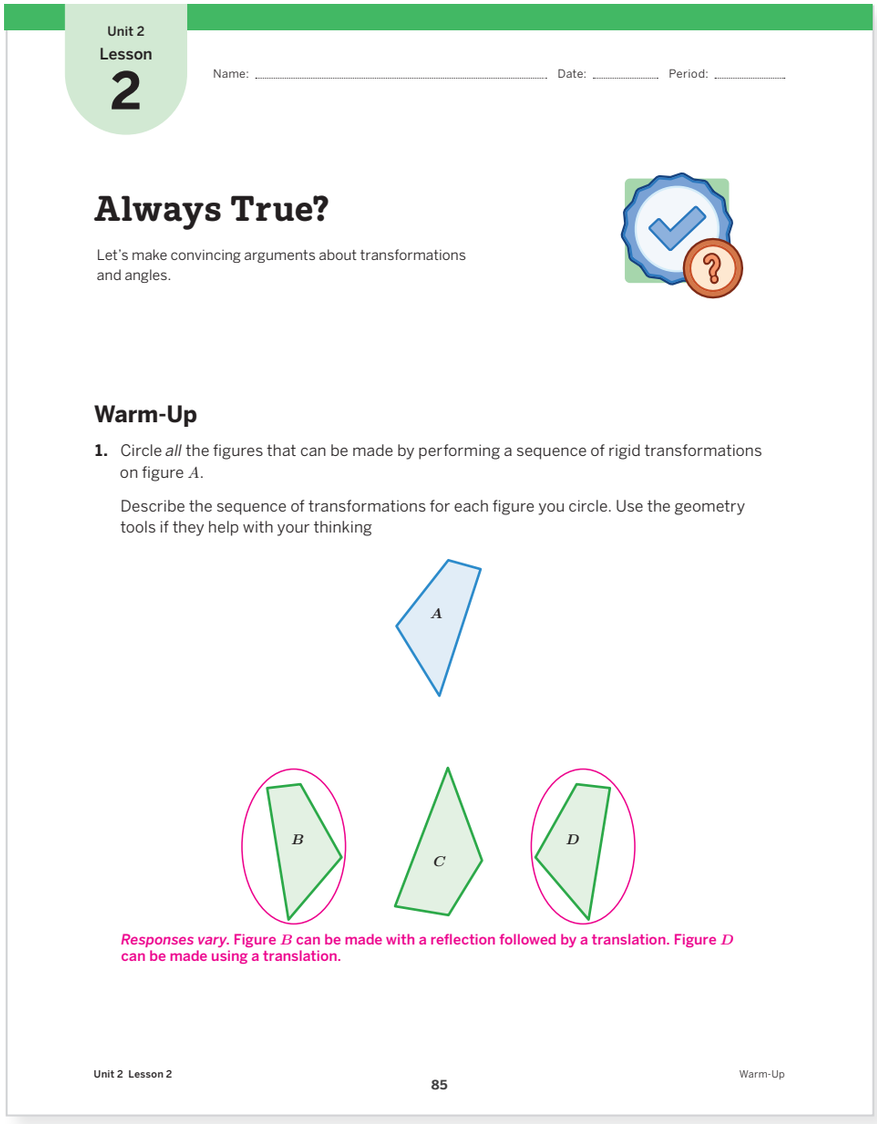
2 Connect

To support making connections, consider encouraging students to compare strategies with a partner.

Invite students to share the different strategies that used rigid transformations. Consider asking:
- "What rigid transformations did you use?"
- "How did you use different tools to perform a translation? A reflection? A rotation?" **(MP5)**
- "Why can different sequences of transformations make the same images?"
- "Why couldn't you make figure C using rigid transformations?" *Responses vary.* I can tell figure C can't be made because the angles and side lengths are different from figure A.

Demonstrate how to use the digital geometry tools to perform the rigid transformations that students describe.

A **Accessibility: Fine Motor Skills** To reduce anxiety about precision, share that the goal is to communicate ideas about congruence rather than to be precise when using geometry tools.



Unit 2 Lesson 2

Name: _____ Date: _____ Period: _____

Always True?

Let's make convincing arguments about transformations and angles.

Warm-Up

1. Circle *all* the figures that can be made by performing a sequence of rigid transformations on figure A.

   Describe the sequence of transformations for each figure you circle. Use the geometry tools if they help with your thinking

Responses vary. Figure B can be made with a reflection followed by a translation. Figure D can be made using a translation.

Unit 2 Lesson 2 85 Warm-Up

Pairs | 15 min

Activity 1 Applying Rigid Transformations

Purpose: Students make use of structure as they perform rigid transformations to create geometric objects. **(MP7)**

Short on time? Consider facilitating two of the three challenges in Problem 2 as a class.

Lesson 2
Activity 1

Presentation Screens
2–6

1 Launch

2 **Display** the examples of a ray, line, and angle. Consider asking, "How would you define a *ray*? A *line*? An *angle*?"

Math Identity and Community Consider acknowledging the importance of deciding on a definition as a community before using the term or building an argument.

3 **Display** Challenge 1. Invite students to discuss multiple ways of completing the challenge with their partner before deciding what to write down on their own.

2 Monitor

4-5 **Pause** briefly after some time has passed to invite students to find a new partner for Challenge 2. After students complete Challenge 2, repeat this step and invite them to find a new partner for Challenge 3.

Select and sequence transformation strategies to share during the Connect.

D Differentiation *(Problem 2)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to try any rigid transformation with a given object. Consider asking, "What tool can you use if you want to rotate a ray? What is created when you rotate the ray?"
Describe a rigid transformation to create their object.	**Strengthen** Invite them to use geometry tools to perform the transformation.

Activity 1 continued >

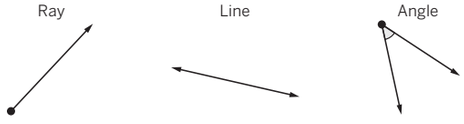
Activity 1

Name: _____ Date: _____ Period: _____

Applying Rigid Transformations

What happens when we apply rigid transformations to rays, lines, and angles?

Ray Line Angle



2. Your teacher will show a series of challenges. Work with a new partner for each challenge and describe a rigid transformation that completes the challenge.

	Partner's Name	Start With	Make	Rigid Transformation
Challenge 1		A ray	A line	*Responses vary.* • Rotate the ray 180° around its endpoint. • Reflect the ray over a perpendicular line.
Challenge 2		An angle	A congruent angle	*Responses vary.* • Reflect the angle over one of its rays. • Translate the angle anywhere.
Challenge 3		A line	A perpendicular line	*Responses vary.* • Rotate the line 90°. • Reflect the line over a line that makes a 45° angle with the original line.

3. Write your own challenge. Make sure it can be solved using rigid transformations.
Responses vary.

a **My challenge:** Starting with a ray, make a right angle.

b Complete someone else's challenge. Classmate's name: _____

Rigid transformation solution: *Responses vary.*

Unit 2 Lesson 2 86 Activity 1

Pairs | 15 min

Activity 1 Applying Rigid Transformations (continued)

Purpose: Students make use of structure as they perform rigid transformations to create geometric objects. **(MP7)**

Short on time? Consider facilitating two of the three challenges in Problem 2 as a class.

Lesson 2
Activity 1

Presentation Screens
2–6

3 Connect

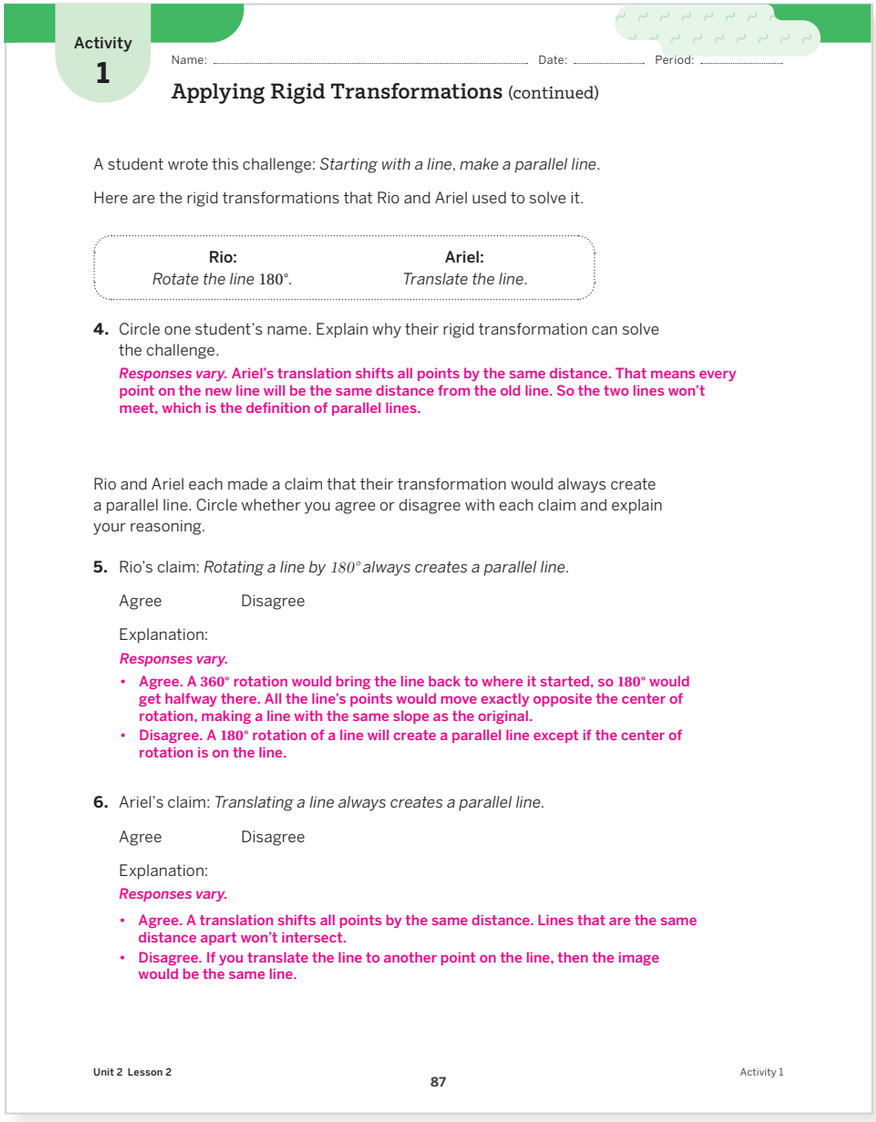
6

Display the two claims from Problems 5–6. Show different places to put the movable point and watch the transformation. Invite students to reflect on how the different definitions of *parallel* may lead people to different conclusions.

Invite students to choose one of the conjectures they disagree with and rewrite it so that it's always true.

To surface the Key Takeaway, consider asking: **(MP7)**

- "How is the structure of rigid transformations a tool for making sense of geometric relationships?"
- "How can rigid transformations prove that a relationship is always true?"



Activity 1

Name: ..... Date: ..... Period: .....

Applying Rigid Transformations (continued)

A student wrote this challenge: *Starting with a line, make a parallel line.*

Here are the rigid transformations that Rio and Ariel used to solve it.

Rio:	Ariel:
Rotate the line 180°.	*Translate the line.*

4. Circle one student's name. Explain why their rigid transformation can solve the challenge.

Responses vary. **Ariel's translation shifts all points by the same distance. That means every point on the new line will be the same distance from the old line. So the two lines won't meet, which is the definition of parallel lines.**

Rio and Ariel each made a claim that their transformation would always create a parallel line. Circle whether you agree or disagree with each claim and explain your reasoning.

5. Rio's claim: *Rotating a line by 180° always creates a parallel line.*

Agree Disagree

Explanation:

Responses vary.

- **Agree. A 360° rotation would bring the line back to where it started, so 180° would get halfway there. All the line's points would move exactly opposite the center of rotation, making a line with the same slope as the original.**
- **Disagree. A 180° rotation of a line will create a parallel line except if the center of rotation is on the line.**

6. Ariel's claim: *Translating a line always creates a parallel line.*

Agree Disagree

Explanation:

Responses vary.

- **Agree. A translation shifts all points by the same distance. Lines that are the same distance apart won't intersect.**
- **Disagree. If you translate the line to another point on the line, then the image would be the same line.**

Unit 2 Lesson 2 87 Activity 1

Key Takeaway: Translations, reflections, and rotations are all examples of rigid transformations; they preserve distances and angle measures. Because of this definition, they can be helpful in proving geometric relationships. For example, parallel lines can be created by performing a translation of a line to a point off the line. Parallel lines could also be created by rotating a line 180° with a center of rotation that is off the line.

Pairs | 15 min

Activity 2 Conjecture Venture

Purpose: Students use appropriate geometry tools strategically to prove *vertical angle* relationships. **(MP5)**

Lesson 2
Activity 2

Presentation Screen 7

1 Launch

Display Problem 7 of the Student Edition and invite students to make sense of the directions before completing the activity.

Consider sharing that a **conjecture** is another word for a reasonable guess that can be proved or disproved, and that students will explore a conjecture about *vertical angles*.

Invite students to use geometry tools that help support their thinking (e.g., tracing paper, straightedges, scratch paper, coloring tools). **(MP5)**

2 Monitor

Listen for the arguments that students use to prove their conjectures. Share them during the Connect.

D Differentiation *(Problems 8–9)*

Look for students who:	Respond to Student Thinking
Identify a rigid transformation using words. *(Problem 9)*	**Strengthen** Consider asking, "How do rigid transformations convince us that this conjecture is true?" **(MP3)**
Generalize about the relationship between vertical angles to understand that this conjecture is always true. *(Problem 8)*	**Stretch** Celebrate their generalization. Invite them to share during the Connect. **(MP7)**

3 Connect

Display the rigid transformation animation to help discuss vertical angles in Problem 11.

Play the animation. Invite students to make a convincing argument that explains why rigid transformations can be used to prove vertical angles are always congruent.

Math Identity and Community Invite students to share something that was convincing about a partner's argument.

To surface the Key Takeaway, consider asking:

- "How can rigid transformations help us understand the term *congruent*?"
- "What was something you noticed in the animation? What relationship was shown?"
- "What would convince us that vertical angles are always congruent?" **(MP3)**
- "How do you know this rigid transformation shows that vertical angles will always be *congruent*?"

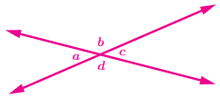
Activity 2

Name: _____ Date: _____ Period: _____

Conjecture Venture

A **conjecture** is a reasonable claim that we want to prove. Let's use rigid transformations to prove a conjecture about angles

7. Use construction tools to draw two intersecting lines. Label the angles you created.



8. Fill in the blanks to complete a conjecture. Use tracing paper if it helps with your thinking.

In my diagram, ∠ *b* is *congruent* to ∠ *d*.

Responses vary based on student constructions.

9. a. Describe a rigid transformation that proves your conjecture.

Responses vary. **A 180° rotation around the intersection point.**

b. Describe a different rigid transformation you could use or explain why that's not possible.

Responses vary. **I could also use a reflection.**

10. Compare papers with a partner.

Discuss:

- How are your diagrams for Problem 7 different? How are they alike?
- Will *your partner's* transformation in Problem 9 also prove *your* conjecture?

Responses vary.

- **We both have two pairs of congruent angles that are across from each other. The sizes of the pairs of angles are different though.**
- **Their transformation does work for my conjecture.**

When two lines intersect, the angles that are opposite each other are called vertical angles.

11. Here's another conjecture: *Vertical angles are congruent.* How might you convince someone that this conjecture is always true?

Responses vary. **Rigid transformations always preserve angle measures. So I can use a rigid transformation to move any angle onto its vertical angle. This strategy isn't specific to a particular set of angles. Even if I draw them in a different size, the reasoning stays the same.**

Unit 2 Lesson 2 88 Activity 2

Key Takeaway: Rigid transformations can be used to prove that *vertical angles* are always congruent. A 180° rotation of one angle will move it onto the other in the vertical angle pair, no matter the size of the angles. This proves the **conjecture** is always true.

Whole Class | 5 min

Synthesis

Lesson 2
Synthesis

Presentation Screen
8

8 **Invite students** to respond independently, and then share their thinking with a partner.

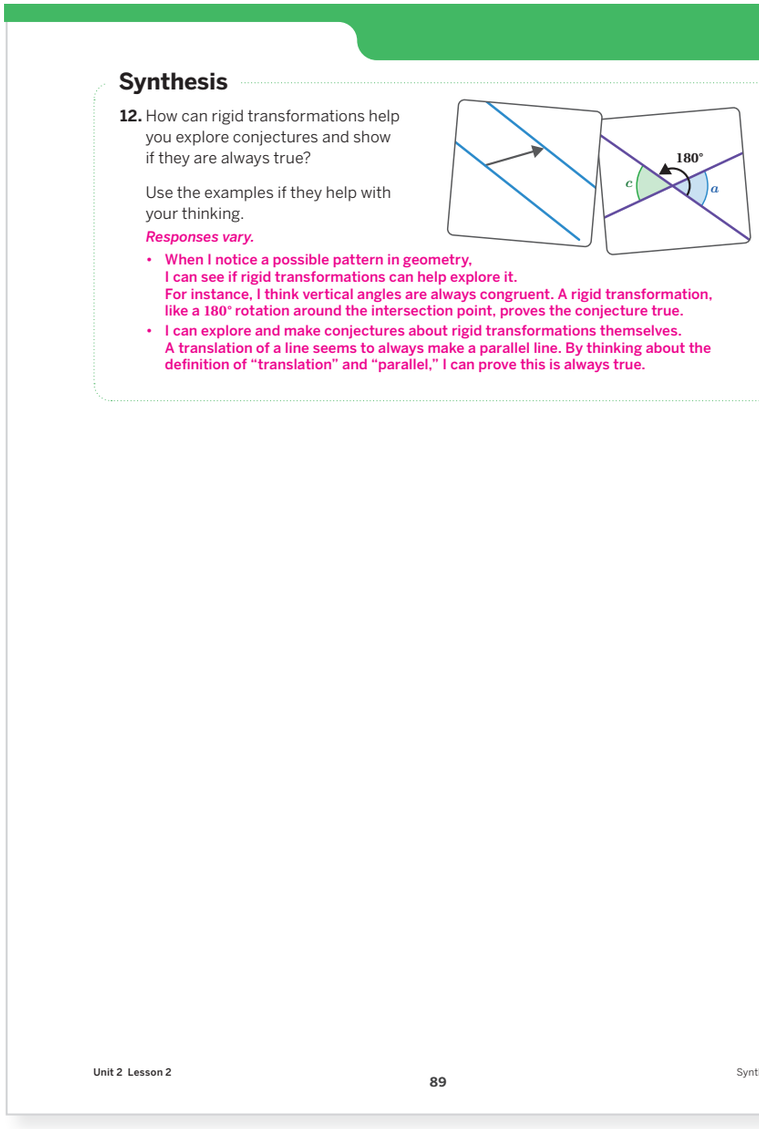
Capture and share a variety of ideas, such as:

- Patterns in geometry can be explored and proved using rigid transformations.
- Rigid transformations preserve distance and angle measure.
- Rigid transformations can help us prove conjectures or confirm things we think are true.
- Rigid transformations help us prove that vertical angles are always congruent.

Note: If time allows, invite students to revise their thinking on Problem 11 based on the discussion.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies for making convincing arguments with rigid transformations as students share during the Connect. Display the strategies for students to reference and add to for the rest of the unit.

Math Identity and Community Invite students to share strategies they found helpful and give credit to the students who shared them.



Synthesis

12. How can rigid transformations help you explore conjectures and show if they are always true?

Use the examples if they help with your thinking.

Responses vary.

- When I notice a possible pattern in geometry, I can see if rigid transformations can help explore it. For instance, I think vertical angles are always congruent. A rigid transformation, like a 180° rotation around the intersection point, proves the conjecture true.
- I can explore and make conjectures about rigid transformations themselves. A translation of a line seems to always make a parallel line. By thinking about the definition of "translation" and "parallel," I can prove this is always true.

Unit 2 Lesson 2 89 Synthesis

Lesson Takeaway: Rigid transformations are a tool that can help make convincing arguments as we explore and prove geometric relationships. Because rigid transformations preserve distances and angle measures, they can be used to prove congruence. For example, *vertical angles* are always *congruent* because one angle can be made from the other using a reflection. This is true no matter the size of the angle.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing a convincing argument about why vertical angles are always congruent. **(MP3)**

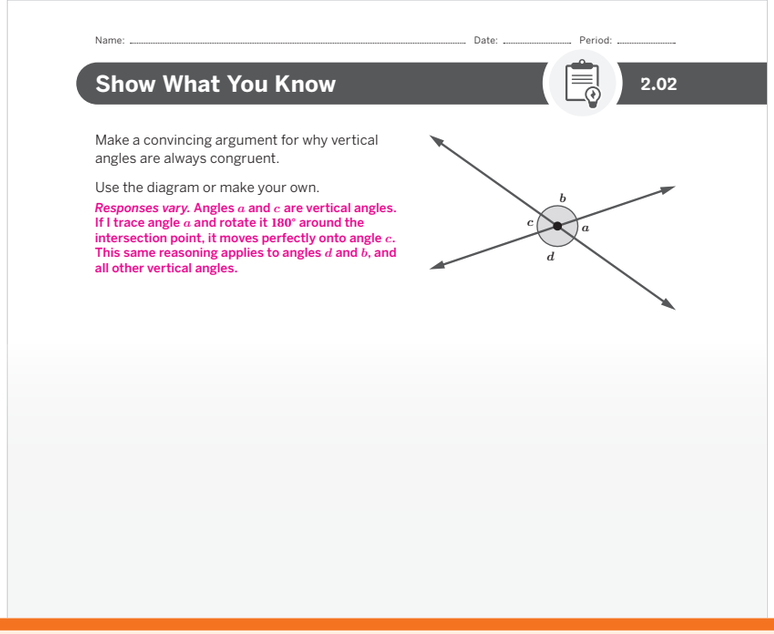


Presentation Screen 9

Today's Goals

1. **Goal:** Use rigid transformations to explore and investigate *conjectures*.
2. **Goal:** Prove vertical angles are congruent using rigid transformations.
3. **Language Goal:** Describe a rigid transformation that will meet specific conditions. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 2.02

Make a convincing argument for why vertical angles are always congruent.

Use the diagram or make your own.

Responses vary. Angles a and c are vertical angles. If I trace angle a and rotate it 180° around the intersection point, it moves perfectly onto angle c. This same reasoning applies to angles d and b, and all other vertical angles.

Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of using rigid transformations to prove vertical angles are always congruent, consider:

- Revisiting **Screen 6** and inviting students to explain how rigid transformations can be used to make a convincing argument.
- Giving individualized feedback on a few related **practice problems** (Problems 3–5).

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice**.
- Emphasizing rigid transformations as a strategy to prove congruence in **Lesson 4**.
- Inviting students to review the **Summary** and complete the **Try This** problem.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Encouraging them to complete the **DOK3 practice problem**.
- Inviting them to create their own challenge where they use rigid transformations on a geometric figure to construct a new geometric figure.

Professional Learning

Consider the next several lessons when answering these questions: How have students made sense of rigid transformations as a tool that can help convince them a conjecture is true? How will students' understanding of rigid transformations as a way of proving the congruence of vertical angles prepare them for proving angle relationships in Lessons 3–4? How might you call back to students' strategies from this lesson?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 2 **Practice**

Students using digital

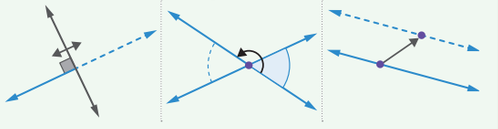
Lesson Summary

You can use rigid transformations and their definitions to explore geometric relationships.

- A translation moves any point a specific distance and direction.
- A reflection moves any point to a point on the opposite side of a line of reflection.
- A rotation moves any point around a center of rotation by a specific angle and direction.

Because rigid transformations keep all angles and distances the same, they can be helpful tools for proving **conjectures** about geometric relationships.

Reflection	Rotation	Translation
You can reflect a ray across a perpendicular line to make another line.	You can rotate an angle 180° around its vertex to prove that *vertical angles are congruent*.	You can translate a line to a point off of the line in order to create a parallel new line.



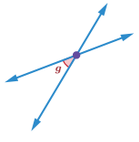
Try This

a If you applied a 180° rotation to a line, what could be the result? Select *all* that apply.

- ☑ **A.** A line that's parallel to the original line
- ☐ **B.** A line that's perpendicular to the original line
- ☑ **C.** Just the original line

b The diagram shows two lines and the angle g. Describe the result of rotating everything by 180° around the intersection point.

Responses vary. Each line rotates back onto itself. Angle g rotates onto its vertical angle, showing the two angles are congruent.

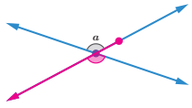


Lesson Practice
Geo.2.02

Name: _____ Date: _____ Period: _____

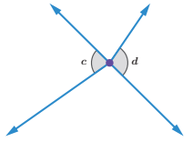
1. Construct one ray to make an angle that is congruent to angle a.

Responses vary.

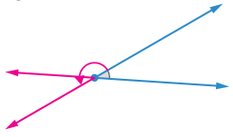


2. Which type of transformation would move $\angle c$ exactly onto $\angle d$?

- **A.** Reflection
- **B.** Rotation
- **C.** Translation
- **D.** None of these



Problems 3–5: Here is an angle.



3. Show or describe a rigid transformation (or sequence of transformations) that can create a vertical angle.

Responses vary. Rotate each of the angle's rays 180° around the vertex.

4. How does your transformation prove that the angle and its vertical angle are congruent?

Responses vary. I used a rotation. Rotations are rigid transformations, which means they preserve angle measures.

5. Is there an angle between 0° and 180° that *cannot* be transformed to make a vertical angle? Explain your thinking.

No. *Explanations vary.* My answer for Problem 4 isn't limited to a specific angle. Any angle between 0° and 180° can be rotated 180°, and that will make its vertical angle.

Lesson Practice
Geo.2.02

Name: _____ Date: _____ Period: _____

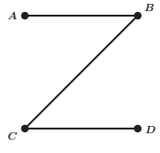
6. Which statement is false?

- **A.** A ray can be rotated around its endpoint to form an angle.
- **B.** A ray can be rotated 180° around its endpoint to form a line.
- **C.** A ray can be reflected to form an angle.
- **D.** A ray can be translated to form a line.

7. Three line segments form the letter Z. Rotate the figure counterclockwise around the midpoint of $\overline{BC}$ by 180°.

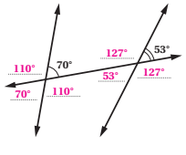
Explain why the result looks like the same letter Z. Use tracing paper if it helps with your thinking.

Responses vary. The 180° rotation takes point B to point C because they are exactly opposite each other through the center of rotation. The same reasoning applies to points A and D. That means the rotation takes $\overline{BC}$ to itself, and takes $\overline{AB}$ and $\overline{DC}$ to one another.



Spiral Review

8. This diagram shows three lines with some missing angle measures. Complete the diagram by writing in all of the missing angle measures.



Reflection

1. Put a star next to a problem you want to understand better.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-lesson			
	1	2	HSG.CO.A.1, HSG.CO.A.2
	2	1	HSG.CO.A.1, HSG.CO.A.2
	3–5	2	HSG.CO.A.2, HSG.CO.C.9
Test Practice	6	2	HSG.CO.A.1
	7	3	HSG.CO.A.4
Spiral Review			
Fluency	8	1	HSG.CO.C.9

Need More Practice?

Additional Practice resources coming soon.

Unit 2

Lesson 3

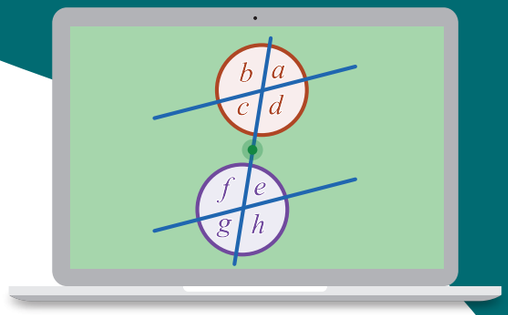
Angles and Transversals

Using Transformations to Prove Angle Relationships

Let's prove relationships along lines and angles.

Digital Lesson

This lesson uses digital student screens. Student Notes are also available.



Focus and Coherence

Today's Goals

1. **Goal:** *Prove* relationships along lines and angles using rigid transformations.
2. **Language Goal:** Describe the different angle relationships that are created when a transversal crosses parallel lines. **(Speaking and Listening)**

Students explore the relationships that are created when a transversal crosses parallel lines. They use rigid transformations to **prove** that some angle pairs are congruent and some are supplementary. They also briefly explore a converse: when alternate interior angles are congruent, the two lines crossed by a transversal must be parallel.

Prior Learning

In Grade 8, Unit 1, students used rigid transformations to verify congruent angle relationships created when a transversal crosses parallel lines. In Lesson 2, students used rigid transformations and tracing paper to prove that vertical angles are congruent.

Future Learning

In Lesson 4, students will apply angle relationships within parallel lines to prove that a triangle's interior angles will sum to $180°$. In Lesson 5, students will use rigid transformations to prove that figures are congruent when they stack perfectly on top of one another.

Rigor and Balance

- Students **apply** their knowledge of angle relationships created when a transversal crosses parallel lines to prove those relationships using rigid transformations.
- Students develop **strategic competence** as they experiment with using digital tools to perform rigid transformations to prove angle relationships.

Vocabulary

New Vocabulary

corresponding angles

prove

Standards

Addressing

HSG.CO.C.9

Prove theorems about lines and angles. Theorems include: vertical angles are congruent; **when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent;** points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

Also Addressing: **HSG.CO.A.2**

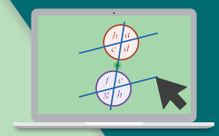
Mathematical Practices: MP2, MP3

Building On	Building Toward
8.G.A.1	**HSG.CO.C.9**
8.G.A.5	

Lesson at a Glance

~ 45 min

Standards: HSG.CO.C.9, HSG.CO.A.2



Why digital?

Students use digital geometry tools to perform rigid transformations to prove angle relationships.

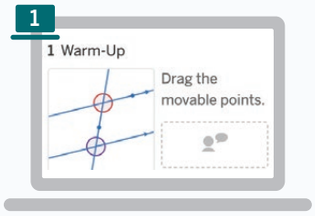
Warm-Up

Independent | 5 min

Pacing: Screen 1

Students reason about the relationships between different angles along lines that are crossed by a transversal. **(MP2)**

Key Takeaway: When parallel lines are crossed by a transversal, the resulting angles within the figure have congruent and supplementary relationships.



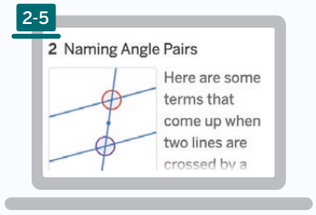
Activity 1

Pairs Sharing a Device | 15 min

Pacing: Screens 2–4, Screen 5

Students use rigid transformations to **prove** that congruent **corresponding angles** are created when parallel lines are crossed by a transversal. **(MP3)**

Key Takeaway (Screen 5): A translation can carry one angle of a corresponding angle pair onto another, **proving** they are always congruent.
See the Activity page for full Key Takeaway.



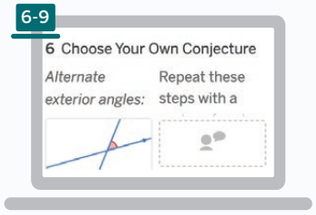
Activity 2

Pairs Sharing a Device | 15 min

Pacing: Screens 6–7, Screens 8–9

Students use rigid transformations to prove their conjectures about the angle relationships that are created when two parallel lines are crossed by a transversal.
MLR7: Compare and Connect

Key Takeaway (Screen 8): We can use rigid transformations to prove that specific angles are congruent when two parallel lines are crossed by a transversal, or to prove that two lines must be parallel when those angles are congruent.
See the Activity page for full Key Takeaway.



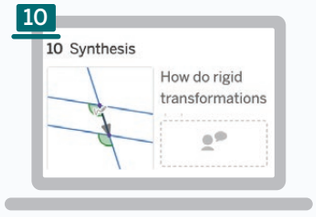
Synthesis

Whole Class | 5 min

Pacing: Screen 10

Students synthesize their understanding of how rigid transformations can prove relationships along lines and angles.

Lesson Takeaway: Rigid transformations can help **prove** conjectures because they preserve angle measures and distances. If a transformation carries one angle onto another, then you can prove the angles are congruent.
See the Synthesis page for full Lesson Takeaway.

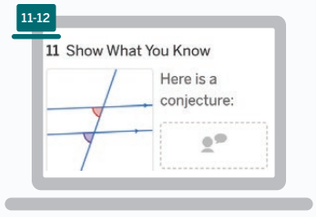


Show What You Know

Independent | 5 min

Pacing: Screens 11–12

Students demonstrate their understanding by performing a rigid transformation to prove that corresponding angles are congruent and explaining how they know.



Prep Checklist

Assign the digital student screens.
Student Notes are also available.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)

Student Notes sheet in Student Edition:
Consider providing the Student Notes to support class discussion and student thinking.



Independent | 5 min | Pacing: Screen 1

Warm-Up

Purpose: Students reason about the relationships between different angles along lines that are crossed by a transversal. **(MP2)**



Student Notes are also available

1 Launch

1 **To support making connections**, invite students to drag the points and make conjectures about the relationships they observe.

Invite students to first think independently about the relationships that are created by two lines crossed by a transversal before sharing with a partner. **(MP2)**

Encourage students to hypothesize about the angle measurements before clicking on the angles to confirm their conjecture.

2 Connect

Capture and share student observations about the figure.

Math Identity and Community Consider using the Snapshot tool to display and celebrate a variety of students' observations.

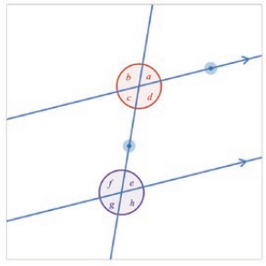
To surface the Key Takeaway, consider asking:

- "How did you predict certain angle relationships?"
- "What stayed the same when you dragged the lines to no longer make them parallel? What changed?" **(MP2)**
- "How would you describe some of the angle relationships you noticed?" *Responses vary.* I noticed that vertical angles were always congruent. I also noticed that when the lines were parallel, some angles were congruent and some angles added to $180°$.

Students using digital

1

Warm-Up



Drag the movable points.

What are some things you notice?

Click angles to see some measurements if that helps with your thinking.

Responses vary.

- I notice that every angle always has an adjacent angle, and together they add up to $180°$.
- I notice that when the lines are parallel, I'm seeing the same angle measures in several places. Angles a, c, e, g have the same measure. Same for angles b, d, f, h.
- I notice that when the lines are parallel, the four angles along the top line have the same angle measures in the same order as those along the bottom line.
- I notice that when the lines are not parallel, the four angles along the top line do not have the same measures as those along the bottom line.

Key Takeaway: When parallel lines are crossed by a transversal, the resulting angles within the figure have congruent and supplementary relationships.

Pairs Sharing a Device | 15 min | Pacing: Screens 2–4, Screen 5

Activity 1 Naming Angle Pairs

Purpose: Students use rigid transformations to **prove** that congruent **corresponding angles** are created when parallel lines are crossed by a transversal. **(MP3)**



Student Notes are also available

Short on time? Consider facilitating Screen 4 as a class.

1 Launch

2 **To encourage playfulness**, invite students to predict with their partner which angles relate to each term before clicking on the term buttons.

ML/EL **Multilingual/English Learners** Encourage students to make sense of each word in the terms. Consider asking, "What words do you think of when you hear the word *exterior*? *Interior*? *Alternate*? *Same side*?" **(Speaking and Listening)**

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to choose one angle term at a time.

Invite students using notes to write down descriptions of each angle term in their own words.

2 Monitor

3 **Pause briefly** to invite students to recall what a *conjecture* is from the previous lesson.

Consider asking, "What makes this statement a conjecture?"

Invite students to move to Screen 4 using the Pace tool.

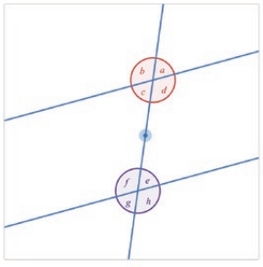
4 **To support students getting started**, encourage pairs to read the directions aloud together.

D **Differentiation: (Support)** Look for students who need support navigating the digital tools. Invite them to identify which figure is undergoing a transformation using the "Box Select" tool and to explore the rigid motions available. Consider asking, "What transformation do you see on the screen?"

Activity 1 continued >

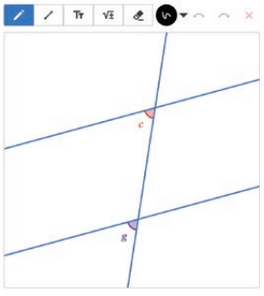
Students using digital

2



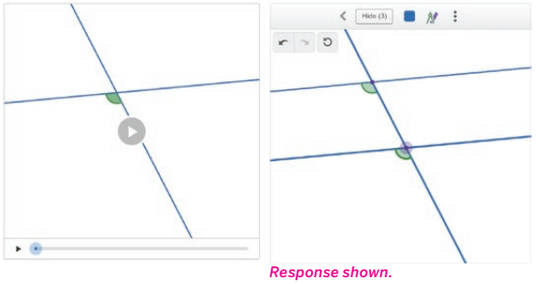
Responses vary.
- Exterior angles are on the "outside" of the lines. Interior angles are on the "inside," facing each other.
- Alternate angles are on opposite sides of the transversal. "Same side" means the angles are on the same side of the transversal.

3



Responses vary. Jasmine could create a diagram based on the first part of her conjecture (a transversal crossing parallel lines) and see if the result is congruent angles. She can make parallel lines using a translation of a line, or a 180° rotation.

4



Response shown.

Pairs Sharing a Device | 15 min | Pacing: Screens 2–4, Screen 5

Activity 1 Naming Angle Pairs (continued)

Purpose: Students use rigid transformations to **prove** that congruent **corresponding angles** are created when parallel lines are crossed by a transversal. **(MP3)**

Short on time? Consider facilitating Screen 4 as a class.



Student Notes are also available

2 Monitor

5 **Select and sequence** explanations to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Say that the translation shows the angles are the same.	**Support** Encourage them to think about what else they would need to write a convincing argument. Consider asking, "What would prove this conjecture is *always* true?" **(MP3)**
Recall that a translation is a rigid transformation that preserves angle measures and distances.	**Strengthen** Celebrate them for their thinking. Consider asking, "How does this idea prove that this conjecture is *always* true?"
Describe that translations carry one angle onto another, proving that corresponding angles are always congruent.	**Stretch** Invite them to consider other transformations that could also prove this conjecture or related conjectures.

3 Connect

Share the captured explanations and invite students to reflect on how using translations to verify is different from using them to **prove**. **(MP2)**

To surface the Key Takeaway, consider asking:

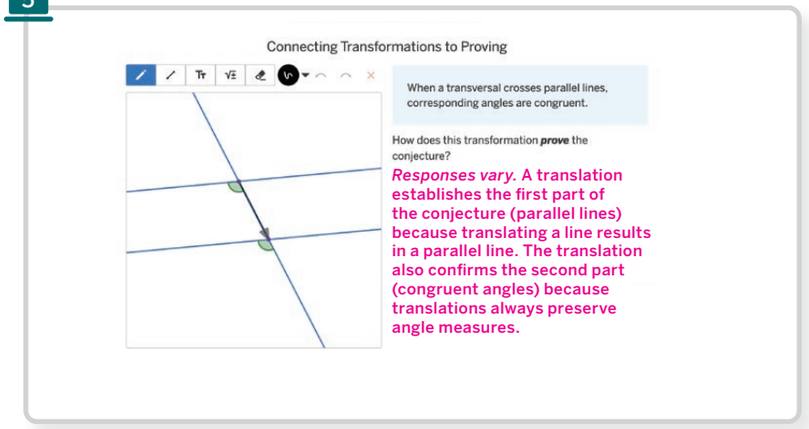
- "If we can use a translation to prove this conjecture, what else can we use to prove that conjectures are true?"
- "How are proving and verifying alike? How are they different?" *Responses vary.* To verify means you're making sure, and to prove means you're convincing someone that something is always true.
- "When you prove a conjecture, what does that mean about who you need to convince?" **(MP3)**

Math Identity and Community Consider sharing with students that proofs are validated by the community in which they are used. If members of that community are not convinced, then there's value in revising the argument to make it more convincing.

ML/EL Multilingual/English Learners Invite students to share what a new term reminds them of (e.g., "When I hear *prove*, I think . . ."). This may help surface multiple meanings and encourage students to connect to their prior knowledge. **(Reading and Listening)**

Students using digital

5



Lesson Takeaway: Translations carry all of the points of a line the same distance away, creating a pair of parallel lines. When those lines are crossed by a transversal, the pair of angles formed on the same side of the transversal are called **corresponding angles**. A translation can also carry one angle of a corresponding angle pair onto another, **proving** they are always congruent.

Pairs Sharing a Device | 15 min | Pacing: Screens 6–7, Screens 8–9

Activity 2 Choose Your Own Conjecture

Purpose: Students use rigid transformations to prove their conjectures about the angle relationships that are created when two parallel lines are crossed by a transversal.



Student Notes are also available

1 Launch

6 **To generate excitement**, consider sharing that students will explore new angle relationships that are both congruent and not congruent.

Invite pairs of students to choose an angle relationship they would like to explore, write a conjecture about the relationship, and then use the digital tools to prove the relationship.

2 Monitor

Invite students using notes to write their conjectures on their paper before using the digital tools to prove them.

Look for the different conjectures students write using the dashboard's Teacher View.

A **Accessibility: Executive Functioning** Invite students to work together on a challenge before trying one independently.

ML/EL **Multilingual/English Learners** Consider pairing students with partners who speak the same primary language, inviting them to share their thinking in their primary language. **(Writing, Speaking, and Listening)**

D **Differentiation:** (Stretch) Invite students who would like an additional challenge to prove all four of the angle relationships. Consider asking, "Is there more than one transformation you could use to prove these relationships?"

Math Identity and Community Look for students who write a conjecture similar to Kele's on Screen 7. If a student used a similar transformation to Kele, consider renaming Kele's conjecture after them and invite them to share their thinking.

7 **Pause briefly** to discuss Kele's thinking. Capture and share several students' reasoning about how the transformation helps prove the conjecture. Consider asking:

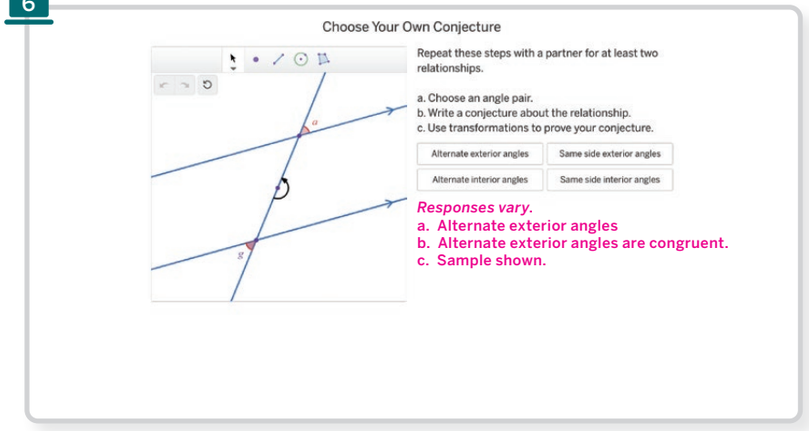
- "What type of transformation did Kele use to prove the conjecture?"
- "Are you convinced by Kele's transformation?"

Invite students to continue using the Pace tool.

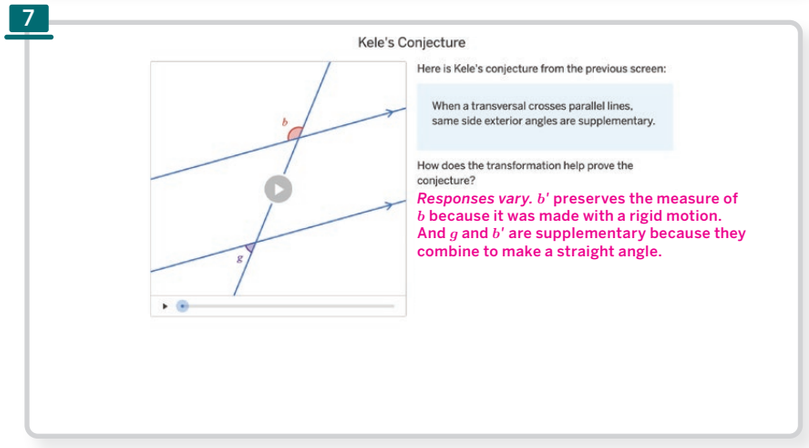
Activity 2 continued >

Students using digital

6



7



Pairs Sharing a Device | 15 min | Pacing: Screens 6–7, Screens 8–9

Activity 2 Choose Your Own Conjecture (continued)

Purpose: Students use rigid transformations to prove their conjectures about the angle relationships that are created when two parallel lines are crossed by a transversal.



Student Notes are also available

2 Monitor

8 **Listen for** students who notice that Nyanna's and Avi's conjectures are similar in structure, but are in a different order. Invite these students to share during the Connect.

A **Accessibility: Executive Functioning** To support multi-step activities, invite students to play the animation while focusing on only one conjecture at a time.

D **Differentiation: (Support)** To support students in getting started, consider asking, "What transformation is being performed? How can you tell?"

9 D **Differentiation: Explore More (Stretch)** Invite students who would like to further explore conjectures to complete this screen. Encourage them to discuss how the wording of a conjecture can affect whether it's true.

3 Connect

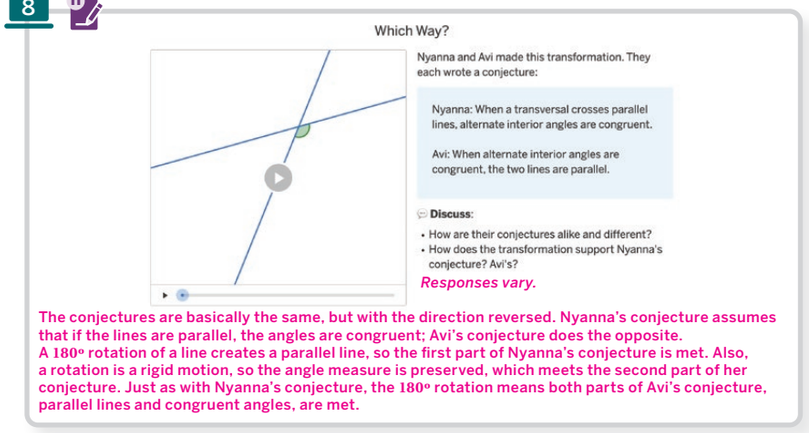
8 MLR **MLR7: Compare and Connect** Invite students to analyze each conjecture, then play the animation before discussing how each conjecture is connected to the animation.

To surface the Key Takeaway, consider asking:

- "How does what you see in the transformation connect to Nyanna's conjecture? Avi's?"
- "If alternate interior angles are congruent when created by a pair of lines crossed by a transversal, then must those lines always be parallel?"
- "Do you think that this type of conjecture, written in both directions, would be true for other angle relationships along parallel lines (e.g., *corresponding angles*)?"

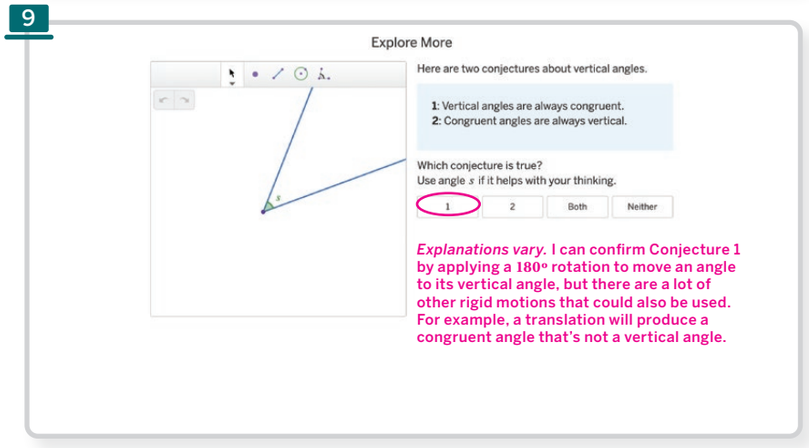
Students using digital

8



The conjectures are basically the same, but with the direction reversed. Nyanna's conjecture assumes that if the lines are parallel, the angles are congruent; Avi's conjecture does the opposite. A 180° rotation of a line creates a parallel line, so the first part of Nyanna's conjecture is met. Also, a rotation is a rigid motion, so the angle measure is preserved, which meets the second part of her conjecture. Just as with Nyanna's conjecture, the 180° rotation means both parts of Avi's conjecture, parallel lines and congruent angles, are met.

9



Key Takeaway: We can use rigid transformations to prove relationships within angles and lines because rigid transformations preserve angle measures and distances. Reflections, rotations, and translations can prove that specific angles are congruent when parallel lines are crossed by a transversal, or they can prove that two lines must be parallel when those specific angles are congruent.

Whole Class | 5 min | Pacing: Screen 10

Synthesis



Student Notes are also available

10 **Invite students** to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

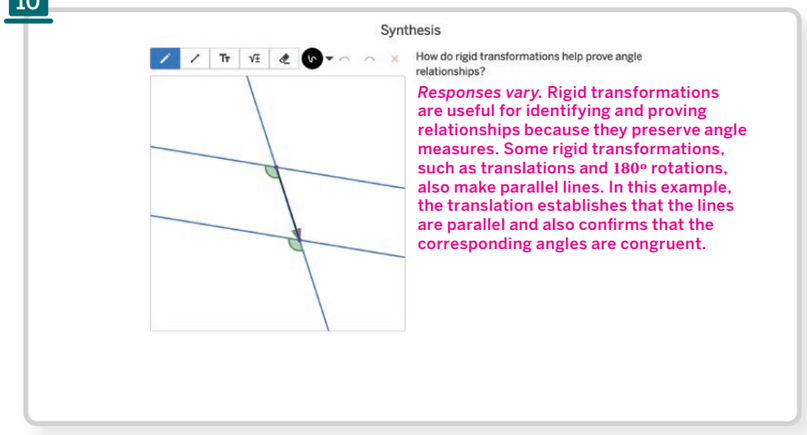
- Rigid transformations preserve angle measures and distances.
- Translations, reflections, and rotations are all rigid transformations that can be used to prove two angles are congruent.
- Determining a transformation that carries one angle onto the other is a helpful strategy.
- There is more than one transformation that can prove two angles are congruent.
- Proving means making a convincing argument that justifies whether or not a conjecture must always be true.

Math Identity and Community Invite students to share strategies they found helpful and give credit to the students who shared them.

Create an anchor chart to record the different angle relationships and the rigid transformations that can be used to prove them using students' language. Display any new language, examples, or diagrams for students to reference later.

Students using digital

10

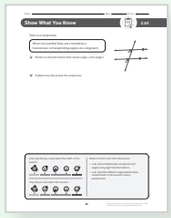


Lesson Takeaway: Rigid transformations can help **prove** conjectures because they preserve angle measures and distances. If there is a reflection, translation, or rotation that carries one angle onto another, then you can prove they are congruent.

Independent | 5 min | Pacing: Screens 11–12

Show What You Know

Purpose: Students demonstrate their understanding by performing a rigid transformation to prove that corresponding angles are congruent and explaining how they know.

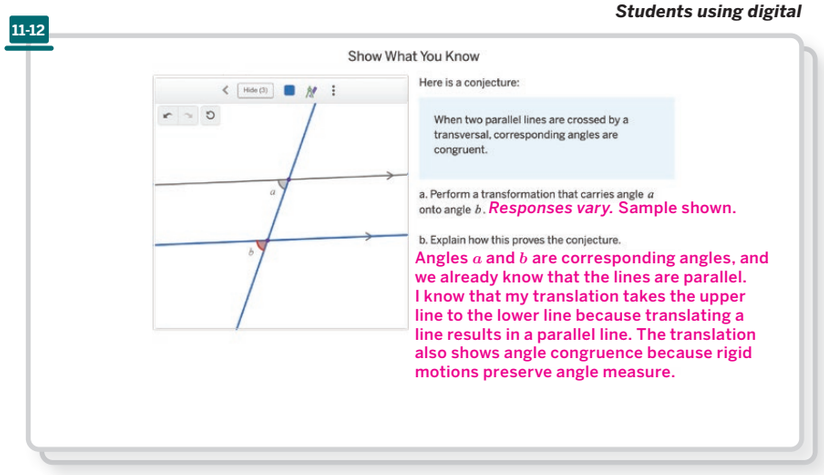


Students using print

11-12 Today's Goals

1. **Goal:** Prove relationships along lines and angles using rigid transformations.
2. **Language Goal:** Describe the different angle relationships that are created when a transversal crosses parallel lines. **(Speaking and Listening)**

Students using digital



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of performing rigid transformations to prove a conjecture, consider:

- Revisiting **Screen 6** as a class and inviting students to share strategies for proving conjectures for each angle relationship.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice**.
- Inviting students to review the **Summary** and complete the **Try This** problem.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting them to complete the **Explore More** task and discuss their thinking with a partner.
- Inviting them to complete any additional transformations on **Screen 6**.

Professional Learning

In this lesson, students used transformations to explore and prove conjectures about relationships within angles and lines. Rigid transformations are a powerful tool for proving conjectures because they preserve angle measure and distance. How did students make sense of what it means to prove in this lesson?

Lesson Practice

Independent

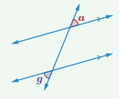
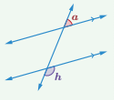
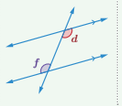
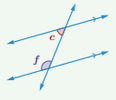
Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 3 Practice

Students using digital

Lesson Summary

There are often several different ways to explain why statements are true. Comparing the different ways can lead to new insights or more flexible understanding. Consider the different angles that form when two parallel lines are cut by a transversal.

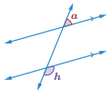
Alternate Exterior Angles	Same Side Exterior Angles	Alternate Interior Angles	Same Side Interior Angles
			

You can **prove** that there are relationships between these angles using rigid transformations, such as translations, reflections, and rotations. As an example for alternate exterior angles, by translating angle a to the vertex of angle g, you can prove that angle a is congruent to angle g because they become vertical angles.

Try This

Here is a conjecture.

When two parallel lines are crossed by a transversal, same side exterior angles are supplementary.



a. Describe a transformation that would show this relationship.

Responses vary. I can translate the top line onto the bottom line because they are parallel lines.

b. Explain how this proves the conjecture.

Responses vary. Angle a would be adjacent to angle h, and their angles would form a straight angle. This means they are supplementary.

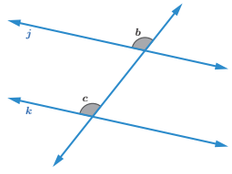
Unit 2 Lesson 3 94 Lesson Summary | Try This

Lesson Practice
Geo.2.03

Name: _____ Date: _____ Period: _____

1. Describe one more piece of information you would need to show that angle c and angle b are congruent.

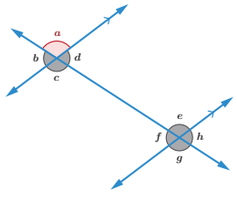
Responses vary. Line j is parallel to line k.



Problems 2–5: Two parallel lines are crossed by a transversal.

2. Select *all* the angles that are congruent to angle a.

- ☐ b
- ☐ c
- ☐ d
- ☑ e
- ☐ f
- ☑ g
- ☐ h



3. Identify a pair of vertical angles.

Responses vary. Angles f and h

4. Identify a pair of alternate interior angles.

Responses vary. Angles d and f

5. Identify a pair of corresponding angles.

Responses vary. Angles d and h

Unit 2 Lesson 3 95 Lesson Practice

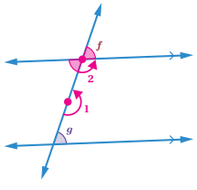
Lesson Practice
Geo.2.03

Name: _____ Date: _____ Period: _____

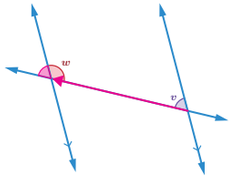
6. Show or describe a sequence of rigid transformations that carries angle g onto angle f.

Responses vary.

- Translate angle g up along the transversal so that the vertex of g goes to the vertex of f.
- Rotate angle g 180° as shown in the diagram, which results in a vertical angle with f. Then rotate that angle 180° around its vertex.



Problems 7–8: Two parallel lines are crossed by a transversal.



7. Write a conjecture about angles v and w.

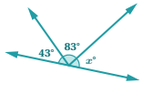
Responses vary. Same side interior angles sum to 180 degrees.

8. Describe a transformation that proves your conjecture. Explain your thinking.

Responses vary. Translate one of the parallel lines to the other. Since rigid transformations preserve angle measures, the two angles now share a vertex and form a straight angle, which measures 180°.

Spiral Review

Problems 9–10: Here are three angles that form a straight angle.



9. Write an equation that includes x.

Responses vary. $43 + 83 + x = 180$

10. Determine the value of x.

$x =$ 54°

Reflection

1. Put a star next to a problem where you revised your thinking.
2. Use this space to ask a question or share something you're proud of.

Unit 2 Lesson 3 96 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-lesson			
	1	2	HSG.CO.C.9*
Test Practice	2	2	HSG.CO.C.9
	3–5	1	HSG.CO.C.9
	6	2	HSG.CO.A.2, HSG.CO.A.5, HSG.CO.C.9
	7	3	HSG.CO.A.2, HSG.CO.A.5, HSG.CO.C.9
Spiral Review			
Fluency	9–10	1	7.G.B.5

*This problem builds toward the standard shown.

Need More Practice?

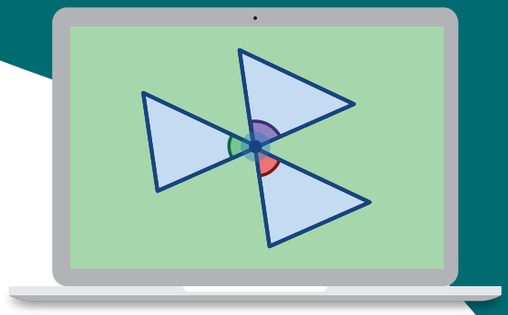
Additional Practice resources coming soon.

Unit 2

Lesson 4

Digital Lesson

This lesson uses digital student screens. Student Notes are also available.



Doing a 180

Triangle Angle Sum

Let's prove that the interior angles of a triangle sum to 180 degrees.

Focus and Coherence

Today's Goals

1. **Goal:** Prove that the measures of interior angles of a triangle sum to 180 degrees.
2. **Language Goal:** Justify a strategy that proves a relationship is always true. **(Reading, Writing, Speaking, and Listening)**

Students explore different strategies for proving that the measures of interior angles of a triangle sum to 180 degrees. They use the different angle relationships they have proved so far to explore a variety of ways to show that this conjecture is always true. Students look for opportunities to strengthen their arguments by making them more convincing and precise. **(MP3, MP6)**

Prior Learning

In Grade 8, Unit 1, students constructed arguments that explained why the interior angles of a triangle sum to 180°. In Lessons 2–3, students used rigid transformations to prove relationships in parallel lines and vertical angles.

Future Learning

In Lesson 5, students will use rigid transformations to prove and define congruence in figures. In Lesson 11, students will prove a conjecture by organizing their argument using a table or by writing a paragraph.

Rigor and Balance

- Students **apply** their understanding of angle relationships to prove a conjecture about the sum of interior angle measures in triangles.
- Students develop **strategic competence** as they explore different strategies to prove a conjecture is always true.
- Students develop **adaptive reasoning** as they reflect on how to make a convincing argument stronger and clearer.

Vocabulary

Review Vocabulary

straight angle

Standards

Addressing

HSG.CO.C.10

Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

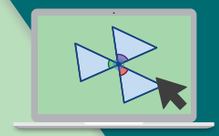
Mathematical Practice: MP3

Building On	Building Toward
7.G.B.5 8.G.A.5 HSG.CO.C.9	HSG.CO.C.10

Lesson at a Glance

~ 45 min

Standard: HSG.CO.C.10



Why digital?

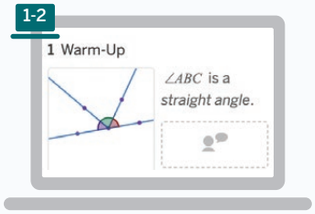
Students use digital interactions to support their exploration of strategies used to prove a conjecture is always true.

Warm-Up

Pairs Sharing a Device | 5 min

Pacing: Screens 1–2

Students explore geometric relationships in which the angles sum to $180°$. **Routine:** Decide and Defend



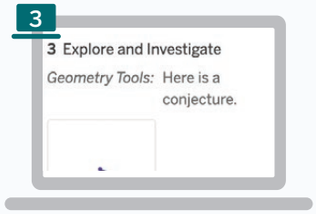
Activity 1

Pairs Sharing a Device | 10 min

Pacing: Screen 3

Students use different angle relationships that they've proven to explore a new conjecture: that the measures of the interior angles of a triangle always sum to $180°$.

Key Takeaway (Screen 3): You can prove conjectures in many different ways. One strategy for proving a conjecture is to start with other relationships you have already proven to be true. See the Activity page for full Key Takeaway.



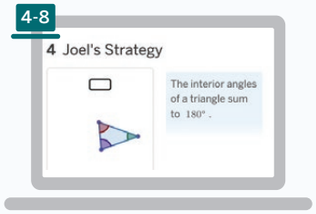
Activity 2

Pairs Sharing a Device | 20 min

Pacing: Screens 4–6, Screens 3–8

Students investigate different strategies for proving that the measures of the interior angles of a triangle always sum to $180°$.
MLR1: Stronger and Clearer Each Time

Key Takeaway (Screen 6): There is more than one way to prove that a conjecture is true. See the Activity page for full Key Takeaway.



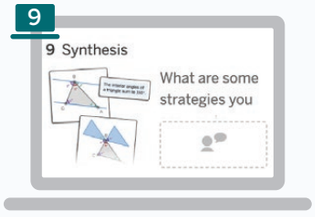
Synthesis

Whole Class | 5 min

Pacing: Screen 9

Students synthesize their strategies for proving a conjecture is *always* true.
MLR2: Collect and Display

Lesson Takeaway: Proving a conjecture means showing it is always true mathematically. Proving a conjecture is true for one case is not the same as proving that a conjecture is *always* true. See the Synthesis page for full Lesson Takeaway.



Show What You Know

Independent | 5 min

Pacing: Screens 10–11

Students demonstrate their understanding by explaining how to use a diagram to prove that the interior angles of a triangle always sum to $180°$.



Prep Checklist

Assign the digital student screens. Student Notes are also available.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)

Student Notes sheet in Student Edition:
Consider providing the Student Notes to support class discussion and student thinking.



Pairs Sharing a Device | 5 min | Pacing: Screens 1–2

Warm-Up

Purpose: Students explore geometric relationships in which the angles sum to $180°$.



Student Notes are also available

1 Launch

1 **To support making connections**, consider asking:
- "What is a *straight angle*?"
- "What is the measure of a straight angle, in degrees?"

Encourage students to drag points D and E to investigate the relationship between $\angle ABE$, $\angle EBD$, $\angle DBC$, and $\angle ABC$.

Look for students who create unique combinations of angles to highlight during the Connect, such as dragging point D to point E or point A.

ML/EL **Multilingual/English Learners** Invite students to share what a new term reminds them of (e.g., "When I hear *straight angle*, I think . . ."). This may help surface multiple meanings and encourage students to connect to to their prior knowledge. **(Reading and Listening)**

Capture several student explanations for why $\angle ABE$, $\angle EBD$, and $\angle DBC$ will always sum to $180°$, no matter their measure.

2 **Encourage students** to play with the diagram and make observations as they move the vertices and change the angle measures.

2 Connect

Share the captured explanations from Screen 1. Invite several students to share their explanation.

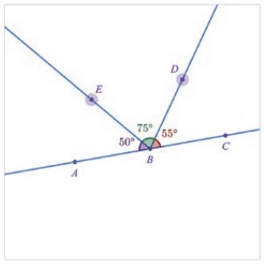
Use the Decide and Defend routine to support students in making arguments for whether or not they think the interior angles of a triangle always sum to $180°$. Consider asking:
- "How are these two diagrams alike? How are they different?"
- "How does each interaction help convince you that these relationships will always be true, no matter the angle measure?" **(MP3)**

Math Identity and Community Celebrate different degrees of precision in students' explanations. It is okay — even desirable — for students to lack precision at this point in the lesson. Students will have several opportunities to explore more ideas and revise their thinking throughout this lesson. **(MP6)**

Students using digital

1

Warm-Up



$\angle ABC$ is a *straight angle*.

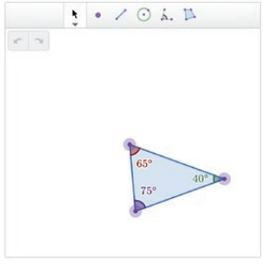
a. Drag points E and D to explore the diagram.

b. How can you convince someone that these angles always sum to $180°$?

Responses vary. The three angles will always add to $180°$ because together the angles form $\angle ABC$. $\angle ABC$ measures $180°$ because it's a straight angle. It's like if I had a 5-centimeter segment and split it into three smaller segments, the total length will still be 5 centimeters.

2

Try-Angles



Here are three more angles that sum to $180°$.

a. Drag the points to explore the diagram.

b. **Discuss**: Do you think a triangle's angles *always* sum to $180°$? Why or why not?

Use the geometry tools if they help with your thinking.

Responses vary.
- I'm noticing that as I move the points around, some angle measures get smaller as some get larger. The changes balance each other out so the total sum doesn't change.
- I do think a triangle's angles will always sum to $180°$ because I learned it in middle school. I remember tearing off the three vertices of a paper triangle and arranging them into a straight line.

Pairs Sharing a Device | 10 min | Pacing: Screen 3

Activity 1 Try-Angles

Purpose: Students use different angle relationships that they've proven to explore a new conjecture: that the measures of the interior angles of a triangle always sum to 180°.



Student Notes are also available

1 Launch

3 **Consider sharing** that today's lesson is focused on using the relationships they've already proven to prove a new conjecture. To connect to prior knowledge, consider asking, "What do we know about vertical angles? Alternate interior angles? Corresponding angles?"

ML/EL **Multilingual/English Learners** Invite students to read the conjecture aloud. Invite them to share what they think each word in the term *interior angles* reminds them of (e.g., "When I hear the word *interior*, I think..." and "When I hear the word *angles*, I think of..."). Then invite students to share what they think it means when the two words are used together. **(Reading and Listening)**

2 Monitor

Encourage students to explore different ways for proving this conjecture.

Math Identity and Community Normalize that not all explorations will be fruitful. Remind them that mathematics is not about the end result, but about thinking deeply.

D **Differentiation: (Support)** Encourage students to use the geometry tools or draggable triangles to create one of the angle relationships listed on the screen.

Look for a variety of strategies students explore. Students may create parallel lines and alternate interior angles, vertical angles by extending the segments into lines, or copies of the triangle to make a straight angle.

Capture several strategies for proving the conjecture.

Invite students using notes to record different relationships they explored for proving the conjecture.

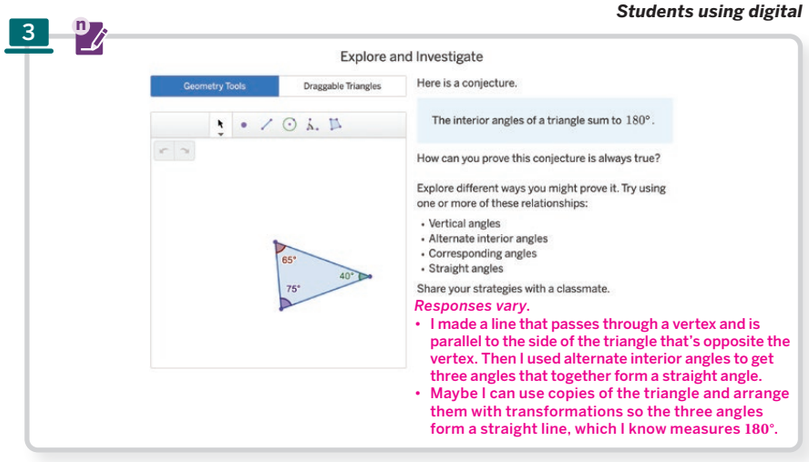
3 Connect

Share the captured strategies.

To surface the Key Takeaway, consider asking:

- "What were some strategies you explored to prove that the interior angles of a triangle always sum to 180°?"
- "How did _____'s strategy convince you? What questions do you still have about their strategy?" **(MP3)**
- "How did the relationships you've already proven help you explore a strategy for proving this new conjecture?"

Students using digital



Lesson Takeaway: You can prove conjectures in many different ways. One strategy for proving a conjecture is to start with other relationships you have already proven to be true. For example, to prove that the interior angles of a triangle always sum to 180°, you might explore rigid transformations, vertical angles, alternate interior angles, corresponding angles, *straight angles*, or other angles from parallel lines to construct a convincing argument.

Pairs Sharing a Device | 20 min | Pacing: Screens 4–6, Screens 3–8

Activity 2 Proving Strategies

Purpose: Students investigate different strategies for proving that the measures of the interior angles of a triangle always sum to 180°.

Short on time? Consider discussing Screen 5 as a whole class instead of having students write their responses.



Student Notes are also available

1 Launch

4 **Encourage students** to press "Show" to reveal Joel's strategy.

Discuss the questions on the screen as a whole class. Then consider asking:

- "What relationships did Joel use in his strategy?" Joel used parallel lines and alternate interior angles.
- "What are some claims in Joel's diagram that need to be verified? Which claims are you already convinced of?" I know from the previous lesson that when a transversal crosses parallel lines, the alternate interior angles are congruent. I need to verify that this relationship works for triangles with other angle measures.
- "What strategies could you use to verify them?"

2 Monitor

5 **Capture** several student explanations for why replacing the numbers with variables and making the points movable strengthens Joel's argument. Some students may share that:

- The variables help show that the relationship doesn't depend on the measure of the angles.
- The draggable points show that the relationship is still true for different triangles.

A **Accessibility: Conceptual Processing** Invite students to answer a more concrete question first to support abstract thinking, such as, "What changed between Screen 4 and Screen 5?"

Pause briefly to discuss how Joel's changes strengthened his argument.

Consider asking: (MP3)

- "Are you more convinced by Joel's argument on Screen 4 or Screen 5? Why?"
- "What does this diagram verify that the previous diagram did not?"
- "What's the difference between proving for one case and proving for all cases?

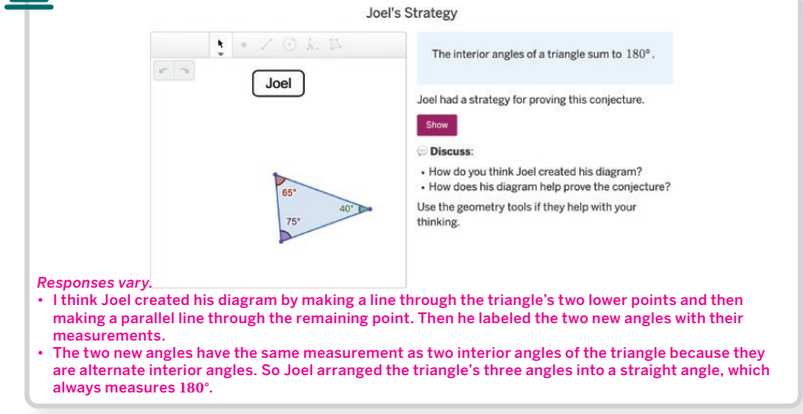
Math Identity and Community Acknowledge that whether an argument is convincing is determined by the community it is created in and for.

Invite students using notes to record some important considerations from the class discussion.

Activity 2 continued >

Students using digital

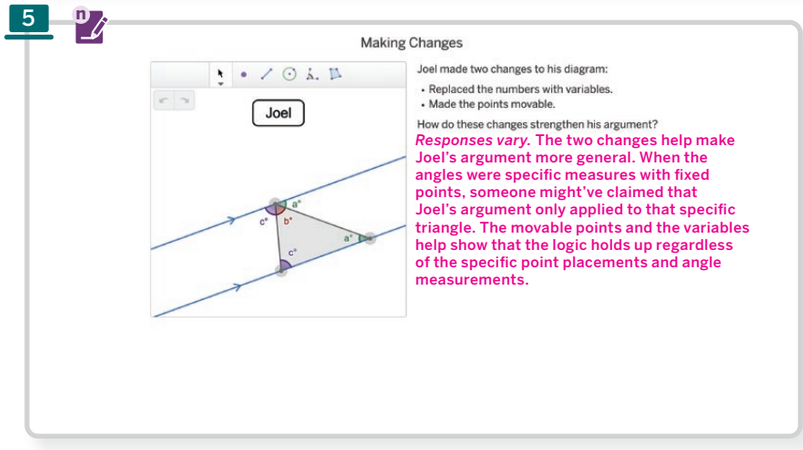
4



Responses vary.

- I think Joel created his diagram by making a line through the triangle's two lower points and then making a parallel line through the remaining point. Then he labeled the two new angles with their measurements.
- The two new angles have the same measurement as two interior angles of the triangle because they are alternate interior angles. So Joel arranged the triangle's three angles into a straight angle, which always measures 180°.

5



Responses vary. The two changes help make Joel's argument more general. When the angles were specific measures with fixed points, someone might've claimed that Joel's argument only applied to that specific triangle. The movable points and the variables help show that the logic holds up regardless of the specific point placements and angle measurements.

Pairs Sharing a Device | 20 min | Pacing: Screens 4–6, Screens 3–8

Activity 2 Proving Strategies (continued)

Purpose: Students investigate different strategies for proving that the measures of the interior angles of a triangle always sum to $180°$.

Short on time? Consider discussing Screen 5 as a whole class instead of having students write their responses.



Student Notes are also available

2 Monitor

6 **Encourage students** to spend several minutes making sense of Shanice's strategy on this screen. Consider asking, "What rigid transformations do you think Shanice explored?"

7 **Invite students** to return to Screen 3 to review their initial explorations for proving this relationship.

MLR **MLR1: Stronger and Clearer Each Time** Use this routine to support students in revising the first draft of their argument to make their argument stronger and clearer. Invite students to return their initial explorations on Screen 3. Encourage them to explore new ideas that would lead to a convincing argument that proves this conjecture. **(MP3, MP6)**

Consider sharing that first drafts do not have to be complete or perfect to be valuable.

Invite students to share their drafts with a partner and give each other feedback.

Encourage students to use the feedback they got from their partner to revise their strategy to make it stronger, with more or better evidence, or clearer (with more precision, organization, etc.). **(MP6)**

ML/EL **Multilingual/English Learners** To support students in giving feedback, consider displaying prompts such as:

- "How do you know that .....?"
- "I'm not convinced yet when you say....."
- "What relationship are you building your argument around?"

(Writing and Speaking)

8 D **Differentiation: Explore More (Stretch)** Invite students who would like to further explore a different strategy for proving that the interior angles of a triangle always sum to $180°$ to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

6 **To surface the Key Takeaway**, consider asking:

- "What did Shanice do?"
- "What are the claims that need to be verified?"
- "How does translating copies of the original triangle make a convincing argument for why interior angles of a triangle always sum to $180°$?"

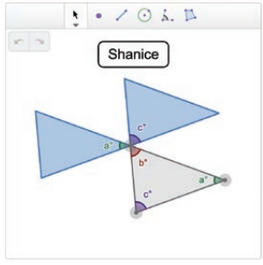
7 **To surface the Key Takeaway**, consider asking:

- "What did you explore in your strategy?"
- "What claims in your strategy need to be verified?"
- "How did using something you know help make a convincing argument?" **(MP3)**

Students using digital

6

Shanice's Strategy



The interior angles of a triangle sum to $180°$.

Shanice had a different strategy for proving this conjecture.

Discuss:

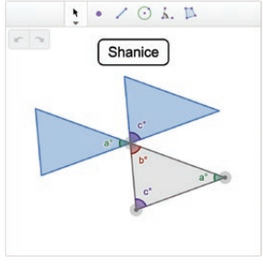
- How do you think Shanice created her diagram?
- What do you think Shanice's strategy is?

Responses vary.

- It looks like Shanice made two copies of the triangle using translations. The three triangles meet at a single point, and Shanice labeled the triangles' interior angles that have their vertices at that point.
- I think Shanice is going to use vertical angles to label more angles with $a°$, $b°$, or $c°$. This will let her show that specific combinations of those angles form a straight angle ($180°$) or a full circle ($360°$).

7

Prove It



The interior angles of a triangle sum to $180°$.

Here is Shanice's diagram again.

Continue with her strategy. Show or explain how to prove the conjecture.

Use the geometry tools if they help with your thinking.

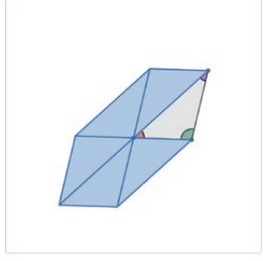
Explanations vary. I can start with the red angle, $b°$, and mark its vertical angle $b°$ too because vertical angles are congruent. The diagram now shows that three angles marked $a°$, $b°$, and $c°$ can make a straight angle, which means they sum to $180°$. Since $a°$, $b°$, and $c°$ are the three angle measures of the original triangle, this means that a triangle's angles always sum to $180°$.

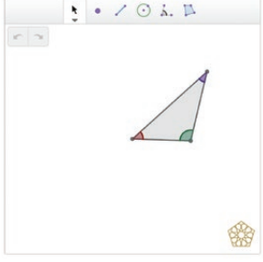
8

Explore More

Use the geometry tools to match the diagram.

Discuss: How does this diagram help prove that the interior angles of a triangle sum to $180°$?





Responses vary. The six triangles meet at a single point. The sum of all the angles at that point is $360°$ because it is a full circle. Those six angles are actually two identical sets of the triangle's three angles, so one set of the triangle's angles should sum to $\frac{360}{2} = 180°$.

Key Takeaway: There is more than one way to prove that a conjecture is true. One strategy that could help you begin proving a new conjecture is to use relationships that you have already proved to be true or that have been established as fact.

Whole Class | 5 min | Pacing: Screen 9

Synthesis



Student Notes are also available

9 **Invite students** to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Starting from a relationship you know.
- Creating vertical angles that are congruent.
- Creating parallel lines to use angle relationships, such as alternate interior angles.
- Making a copy of the original figure using rigid transformations.
- Replacing numbers with variables to see if the relationship is true for all values.
- Making points draggable in a diagram to see if the relationship holds true for all cases.

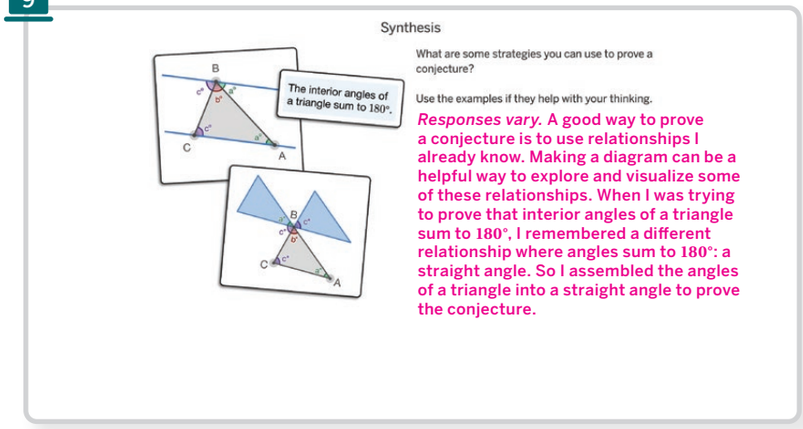
Math Identity and Community Consider inviting students to reflect on how their arguments became stronger and clearer over the course of the lesson. Invite students to give credit to the students who helped shape their ideas from helpful feedback.

Note: If time allows, invite students to revise their previous thinking based on the discussion.

MLR **MLR2: Collect and Display** Invite students to revisit their strategies for making convincing arguments with rigid transformations on the Collect and Display from a previous lesson. Encourage students to add additional strategies they learned in this lesson and sub-unit. Display the strategies for students to reference and add to for the rest of the unit.

Students using digital

9

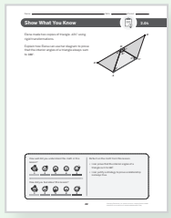


Lesson Takeaway: You can prove conjectures in many different ways. Some strategies include using relationships you already know are true, replacing numbers with variables to show that it works for all values, using rigid transformations to create congruent shapes, and making points draggable to see if it holds true for all cases. Proving a conjecture true for one case is not the same as proving that a conjecture is *always* true.

Independent | 5 min | Pacing: Screens 10–11

Show What You Know

Purpose: Students demonstrate their understanding by explaining how to use a diagram to prove that the interior angles of a triangle always sum to 180°.

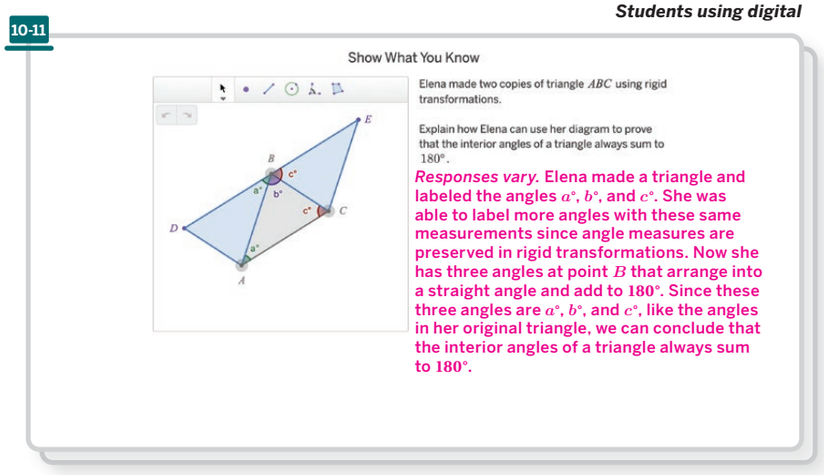


Students using print

10-11 Today's Goals

1. **Goal:** Prove that the measures of interior angles of a triangle sum to 180 degrees.
2. **Language Goal:** Justify a strategy that proves a relationship is always true. **(Reading, Writing, Speaking, and Listening)**

Students using digital



Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of writing a convincing argument:

- You may choose not to intervene now. Students will have more opportunities to write a convincing argument in **Lessons 5–15**.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding** with minor errors, consider:

- Assigning the **Lesson Practice**.
- Inviting students to review the **Summary** and complete the **Try This** problem.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting them to complete the **Explore More** task and discuss their thinking with a partner.
- Encouraging them to complete the **DOK3 practice problems**.

Professional Learning

Take a moment to reflect on the strategies students used in this lesson. In what ways are students taking risks when trying out new strategies for proving conjectures? What opportunities were there to celebrate specific students' strategies for proving that the interior angles of a triangle always sum to 180°? How did students validate one another's thinking throughout the lesson?

Lesson Practice

Independent

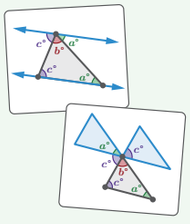
Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 4 Practice

Students using digital

Lesson Summary

Conjectures can be proven in many different ways. As you make a plan to prove a conjecture, it can be helpful to begin with a relationship or property that has already been proven true. For example, to prove that the interior angles of a triangle always sum to 180°, you might begin with what you know about vertical angles, alternate interior angles, *straight angles*, or other angle relationships.



Proving a conjecture is true for one case is not the same as proving that a conjecture is always true.

There are several strategies for proving a conjecture is always true.

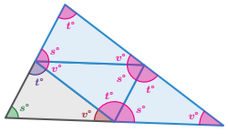
- You can explain why the reasoning that applies to one case can be generalized to all other possible cases.
- You can use draggable points to show that a relationship stays the same for all of the possible cases.
- You can also replace numbers with variables to show that a relationship stays the same for *all* cases.

Try This

A student is trying to prove this conjecture: *The interior angles of a triangle sum to* 180°.

The student starts by making a triangle, and then uses rigid transformations to make three copies of the triangle.

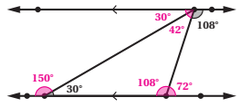
Label each angle in the copies with the appropriate variable.



Lesson Practice
Geo.2.04

Name: _____ Date: _____ Period: _____

1. Label all the angle measurements that are possible to know from the given information.



Problems 2–5: Here is a conjecture: *The interior angles of a triangle sum to* 180°. A student started to prove the conjecture by making a triangle and then a copy of the triangle.

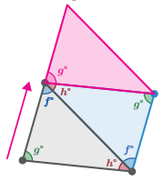
2. Which rigid transformation was used to make the copy of the triangle?

   A. Translation

   B. Reflection

   C. Rotation

Diagram A

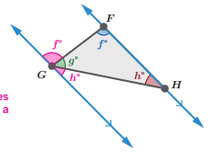


3. Show or describe what the student could do next to prove the conjecture.

   Responses vary. The student could translate the original triangle up as shown on the diagram.

4. Here is a new diagram that proves the same conjecture. Label the angles at point G with the appropriate variables.

Diagram B



5. Explain how to prove the conjecture is always true using one or both of the diagrams.

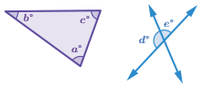
   Responses vary. In both diagrams, the three original angles of the triangle (measuring $f°$, $g°$, and $h°$) are arranged into a straight angle, which must measure 180°.

Lesson Practice
Geo.2.04

Name: _____ Date: _____ Period: _____

6. Determine the average of a, b, c, d, and e.

   72°



   Explain your thinking.

   Explanations vary. a, b, and c sum to 180° because they are angles in a triangle. d and e also sum to 180° because they form a straight angle. Therefore, $360 \div 5 = 72$.

Spiral Review

7. Select *all* the true statements.

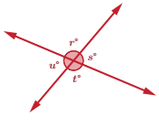
   ☑ A. $r + s = 180$

   ☐ B. $u + s = r + t$

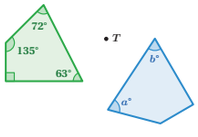
   ☐ C. $t + r = 180$

   ☑ D. $s = u$

   ☑ E. $u + t = r + s$



Problems 8–9: A polygon is rotated around point T. Its image is shown.



Determine the value of each variable.

8. $a =$ 72

9. $b =$ 63

Reflection

1. Put a heart next to the problem you're most proud of.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-lesson			
	1	2	HSG.CO.C.9*
	2	1	HSG.CO.A.5
	3	3	HSG.CO.C.10
	4	1	HSG.CO.C.10
	5	2	HSG.CO.C.10
	6	3	HSG.CO.C
Spiral Review			
Test Practice	7	2	HSG.CO.C.9*
Fluency	8–9	1	8.G.A.2

*This problem builds toward the standard shown.

Need More Practice?

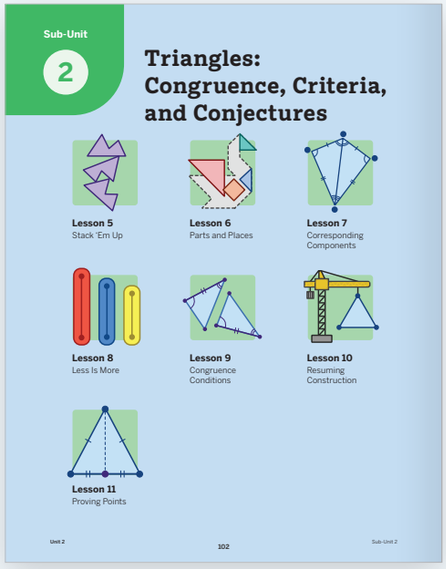
Additional Practice resources coming soon.

Triangles: Congruence, Criteria, and Conjectures

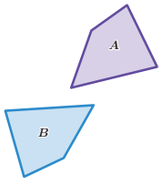
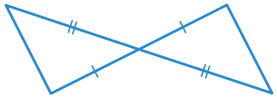
Sub-Unit 2 Goals

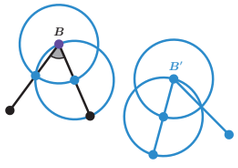
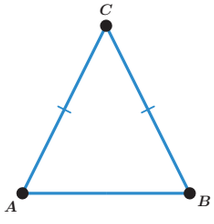
- **Goal:** Define congruence using rigid transformations.
- **Language Goal:** Justify how much information is needed to decide that two triangles are congruent.

In this sub-unit, students use the definition of congruence and rigid transformations to decide if figures are **congruent**. They make sense of how two figures can be congruent if there is a sequence of transformations that carries one figure onto the other, or if the **corresponding** sides and angles in the two figures are congruent. Students also determine the minimum information needed to prove two triangles congruent, and they use triangle congruence criteria to construct a copy of a segment and an angle.



Math That Matters Most

Concept or Skill	Example
Decide if two figures are congruent by describing a sequence of transformations that carries one figure onto the other.	Are polygon A and polygon B congruent? Explain your thinking. I can translate then rotate polygon B onto polygon A. Since there is a sequence of transformations that stacks them, the polygons are congruent. 
Identify and describe corresponding parts in congruent figures.	$\triangle ABC \cong \triangle DEF$. Identify all of the congruent sides and angles between the two triangles. $\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}, \overline{AC} \cong \overline{DF}$ $\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$
Determine if specific criteria will always prove congruent triangles (SSS, SAS, ASA).	Is there enough information to prove that these two triangles are congruent?  Vertical angles are always congruent, so these triangles are congruent by SAS.

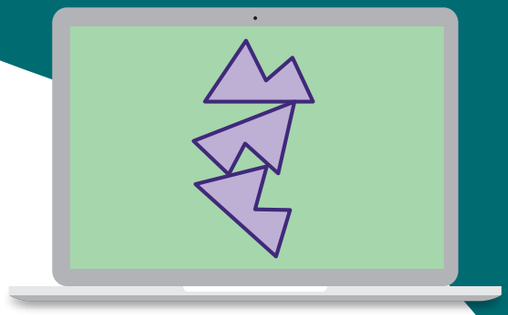
Concept or Skill	Example
Construct a copy of an angle.	Construct a copy of $\angle B$. 
Prove theorems about triangles.	Here's a conjecture: *In an isosceles triangle, the base angles are congruent.* Explain how you can prove this conjecture is true.  First I can construct point Q at the midpoint of $\overline{AB}$ and draw $\overline{CQ}$. Then I can prove that $\triangle ACQ \cong \triangle BCQ$ because of SSS. Corresponding parts in congruent triangles are congruent, so $\angle A \cong \angle B$.

Unit 2

Lesson 5

Digital Lesson

This lesson uses digital student screens. Student Notes are also available.



Stack 'Em Up

Defining Congruence

Let's use rigid transformations to define congruence.

Focus and Coherence

Today's Goals

1. **Goal:** Use rigid transformations to investigate congruence claims.
2. **Goal:** Use the definition of *congruence* in terms of rigid transformations to decide if two figures stack perfectly.
3. **Language Goal:** Justify whether two figures are congruent by describing a sequence of transformations that will stack the figures perfectly. **(Writing, Speaking, and Listening)**

Students define **congruence** by identifying a sequence of *rigid transformations* that will stack two figures perfectly. Students then use *rotations*, *reflections*, and *translations* to verify their conjectures about whether two figures are congruent. **(MP3)**

Prior Learning

In Unit 1, students were introduced to reflections, rotations, and translations as rigid transformations that preserve angle measure and distance. In Lessons 2–3, students used rigid transformations to prove congruence in angle relationships.

Future Learning

In Lesson 6, students will justify that figures are congruent if and only if their corresponding parts are congruent. In Lesson 8, students will consider what minimum number of corresponding congruent parts are needed in order to prove two triangles are congruent.

Rigor and Balance

- Students develop **conceptual understanding** of the definition of *congruence* as they use rigid transformations to stack figures.
- Students develop **strategic competence** as they compare different strategies that use sequences of rigid transformations to solve challenges.

Vocabulary

New Vocabulary

congruent

Review Vocabulary

reflection
rotation
rigid transformations
translation

Standards

Addressing

HSG.CO.B.6
Understand congruence in terms of rigid motions. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; **given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.**

Also Addressing: **HSG.CO.A.2**

Mathematical Practices: MP1, MP3, MP6

Building On	Building Toward
8.G.A.1 **8.G.A.2**	**HSG.CO.B.8**

Lesson at a Glance

~ 45 min

Standards: HSG.CO.B.6, HSG.CO.A.2



Why digital?

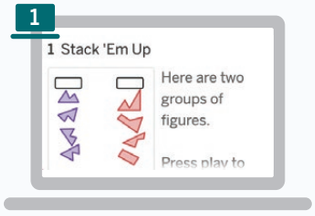
Students use digital geometry tools to visualize two figures stacking on top of each other and use rigid transformations to determine if they are congruent.

Warm-Up

Pairs Sharing a Device | 5 min

Pacing: Screen 1

Students use the **Notice and Wonder** routine to explore the concept of stacking figures perfectly.



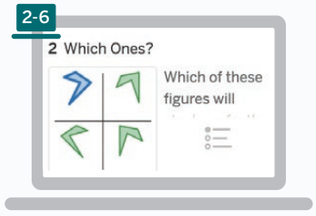
Activity 1

Pairs Sharing a Device | 15 min

Pacing: Screens 2–4, Screens 5–6

Students solve challenges by determining if sets of figures will stack perfectly through a sequence of *rigid transformations*.
MLR7: Compare and Connect **(MP1)**

Key Takeaway (Screen 6): *Rigid transformations* do not change the size or shape of a figure. If you can find a sequence of rigid transformations that makes two figures stack perfectly, then you know the figures are the same size and shape.



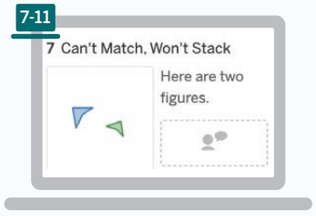
Activity 2

Pairs Sharing a Device | 15 min

Pacing: Screens 7–8, Screen 9, Screens 10–11

Students solidify their definition of **congruence** by completing additional challenges where they stack figures through a sequence of rigid transformations.

Key Takeaway (Screen 9): When figures can be stacked perfectly through a sequence of rigid transformations, they are called **congruent**. There is more than one way to transform figures to show their congruence.



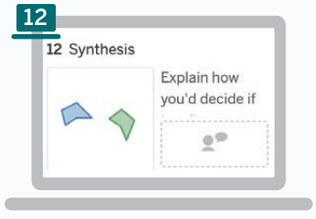
Synthesis

Whole Class | 5 min

Pacing: Screen 12

Students synthesize their understanding of how to determine if two figures are congruent by using rigid transformations.

Lesson Takeaway: Rigid transformations, such as *reflections*, *translations*, and *rotations*, keep the size and shape of a figure the same. If a sequence of rigid transformations stacks two figures perfectly, that means the figures are the same size and shape, or **congruent**.
See the Synthesis page for full Lesson Takeaway.

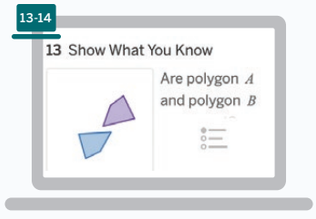


Show What You Know

Independent | 5 min

Pacing: Screens 13–14

Students demonstrate their understanding by determining if two figures are congruent and explaining their reasoning.



Prep Checklist

Assign the digital student screens. Student Notes are also available.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)
- tracing paper (optional)

Student Notes sheet in Student Edition:
Consider providing the Student Notes to support class discussion and student thinking.



Pairs Sharing a Device | 5 min | Pacing: Screen 1

Warm-Up

Purpose: Students use the Notice and Wonder routine to introduce the concept of stacking figures perfectly.



Student Notes are also available

1 Launch

1 **Play** the animation.

To support making connections, consider asking, "What does it mean to stack something?"

Use the Notice and Wonder routine to promote curiosity and help students make sense of the animation. **(MP1)**

ML/EL **Multilingual/English Learners** Provide sentence frames to help students explain their strategy (e.g., "I notice ....." or "I wonder ....."). **(Writing and Speaking)**

Listen for students who discuss how they could have predicted Group A would stack perfectly while Group B would not without using the animation. Consider asking these students to share during the Connect.

2 Connect

Create a record of things that students notice and wonder, along with the names of the students who shared their thoughts with the class.

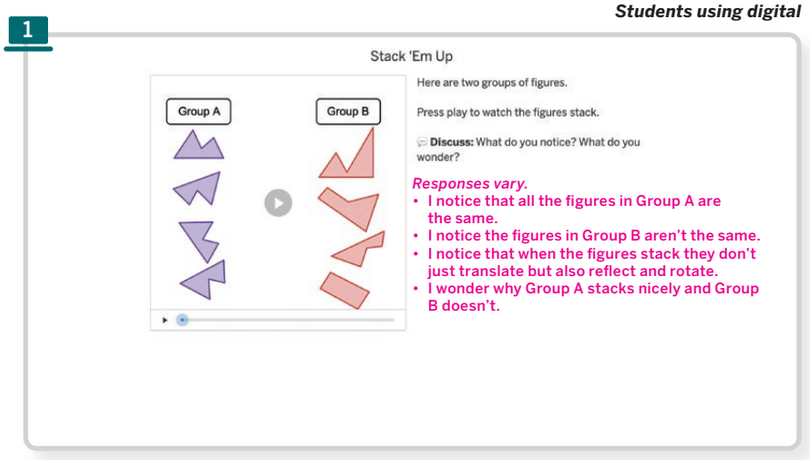
Math Identity and Community Consider celebrating variety and creativity in what students notice and wonder, including things that surprised you or that you think other students may not have noticed.

Consider asking:

- "Did anyone notice how the figures in Group A compare to the figures in Group B?"
- "How can we predict if the figures will stack perfectly before we stack them?"
- "What can make us more confident in our predictions?"

Note: Students will learn the term *congruent* on Screen 8 of this lesson. Until then, it is okay — even preferable — for students to use their own words when describing the animations they see on this screen and in Activity 1.

Students using digital



Pairs Sharing a Device | 15 min | Pacing: Screens 2–4, Screens 5–6

Activity 1 Stacking Challenges

Purpose: Students solve challenges by determining if sets of figures will stack perfectly through a sequence of *rigid transformations*. **(MP1)**



Student Notes are also available

1 Launch

2 **Display** the four figures.

Consider asking, "Can you predict which figures will stack perfectly on figure A? How do you know?" **(MP1)**

Demonstrate first how to select a figure to make a prediction. Then demonstrate how to drag the figure.

Invite students to revise their thinking after they have dragged the figure to confirm their prediction.

Consider asking:

- "How do you know if two figures are stacked perfectly?"
- "What is it about figure B and figure D that makes you think they will stack perfectly with figure A?"
- "What is it about figure C that makes you think it might *not* stack perfectly with figure A?"
- "What transformations did you use to stack figure B perfectly onto figure A? What about figure D?"

2 Monitor

3 **Encourage students** to decide if they think the figures will stack perfectly before they use the geometry tools to confirm their thinking.

Invite students to hide earlier images. This could help them focus on the current transformation.

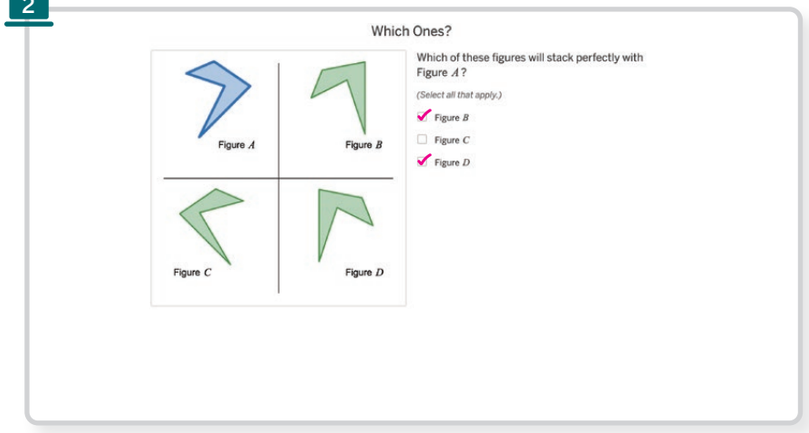
Look for the different ways students solve the challenge on this screen.

D Differentiation

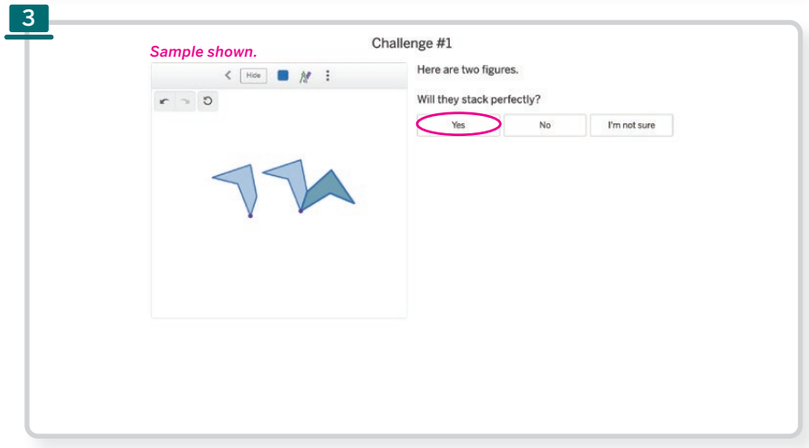
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to predict whether the figures will stack perfectly. Consider asking, "What transformations could show you how to stack that?"
Translate across a directed line segment between the two lower vertices.	**Strengthen** Celebrate the sequence of transformations they found. Consider asking, "Is there another sequence that would show how these figures stack perfectly?"
Start their proof strategy using a sequence of transformations like Miko's on Screen 4.	**Strengthen** Invite these students to share their approach during the discussion on the next screen. Consider asking, "What was the last step of your proof strategy?"

Students using digital

2



3



Activity 1 continued >

Activity 1 Stacking Challenges (continued)

Purpose: Students solve challenges by determining if sets of figures will stack perfectly through a sequence of *rigid transformations*. **(MP1)**



Student Notes are also available

2 Monitor

4 **Encourage students** to go back to Screen 3 to determine the next steps in Miko's proof strategy.

Pause briefly to discuss the different ways students could finish Miko's strategy. Some students might:

- Reflect over a side and then rotate.
- Create a line of symmetry to reflect over.

Consider asking, "Why might *rigid transformations* be useful for showing that figures stack perfectly?" Rigid transformations like reflections, rotations, and translations don't change a figure's size or shape.

Invite students to continue to Screen 5 using the Pace tool.

5 **Look for** the different ways students solve the challenge on this screen.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to decide first if the figures are congruent. Consider asking, "What is a transformation that could help you see if these figures stack perfectly?"
Solve this challenge by: • Translating along a directed line segment between two vertices, then reflecting. • Translating, rotating, and then reflecting.	**Strengthen** Celebrate their transformational proof strategy. Consider asking, "Is there another way to solve this challenge? Is there a way to solve this with fewer steps?"
Would like an additional challenge.	**Strengthen** Invite them to solve this challenge using only one transformation.

3 Connect

6 **MLR MLR7: Compare and Connect** Invite students to analyze and compare Dakota's and Sakari's strategies before connecting to their own strategy.

Consider asking:

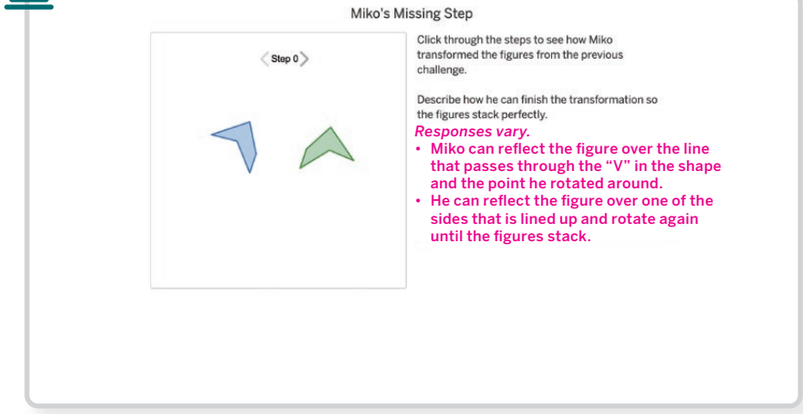
- "What transformations did each student perform?"
- "How are their strategies alike? How are they different?"
- "How does each strategy compare to your own?"

Math Identity and Community Consider asking, "Why might it be useful to have more than one strategy?"

Invite students using notes to record different sequences of transformations that solve this challenge.

Students using digital

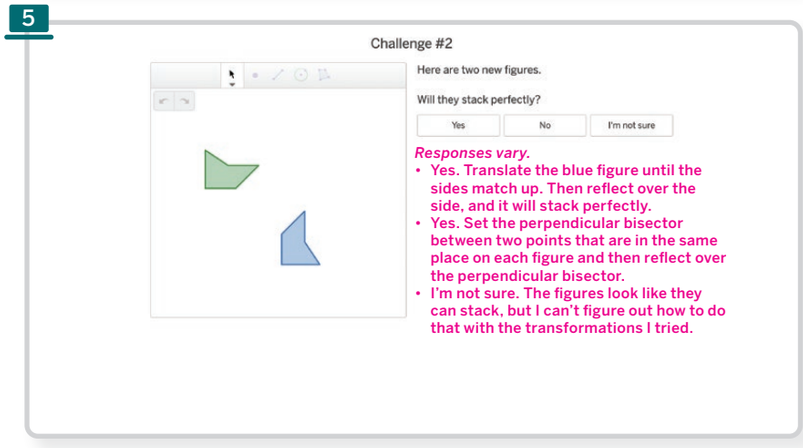
4



Responses vary.

- Miko can reflect the figure over the line that passes through the "V" in the shape and the point he rotated around.
- He can reflect the figure over one of the sides that is lined up and rotate again until the figures stack.

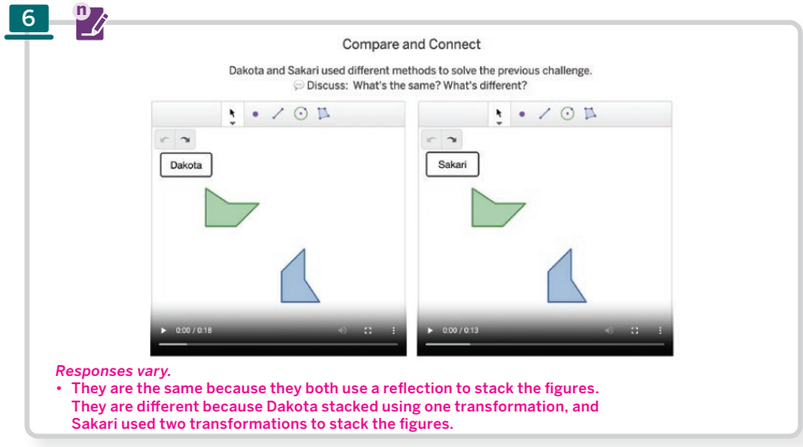
5



Responses vary.

- Yes. Translate the blue figure until the sides match up. Then reflect over the side, and it will stack perfectly.
- Yes. Set the perpendicular bisector between two points that are in the same place on each figure and then reflect over the perpendicular bisector.
- I'm not sure. The figures look like they can stack, but I can't figure out how to do that with the transformations I tried.

6



Responses vary.

- They are the same because they both use a reflection to stack the figures. They are different because Dakota stacked using one transformation, and Sakari used two transformations to stack the figures.

Key Takeaway: *Rigid transformations* do not change the size or shape of a figure. If you can find a sequence of rigid transformations that makes two figures stack perfectly, then you know the figures are the same size and shape.

Pairs Sharing a Device | 15 min | Pacing: Screens 7–8, Screen 9, Screens 10–11

Activity 2 Transformation Station

Purpose: Students solidify their definition of **congruence** by completing additional challenges where they stack figures through a sequence of rigid transformations.

Short on time? Consider inviting students to complete fewer Repeated Challenges on Screens 9–10.



Student Notes are also available

1 Launch

7 **Capture and share** ideas about why the figures won't stack perfectly, highlighting the ideas about how there is no sequence of rigid transformations that will make the figures the same size.

Consider asking, "Is there any sequence of rigid transformations that will make these stack perfectly? If yes, what would that mean?"

2 Monitor

8 **Encourage students** to decide if they think the figures are **congruent** before they use the geometry tools to check their thinking.

Pause briefly to discuss how the definition of *congruent figures* is rooted in rigid transformations. Consider asking:

- "Why do you think reflections, rotations, and translations are called rigid transformations?"
- "Why does finding a sequence of rigid transformations verify that two figures must be congruent?" **(MP3)**

Invite students to continue to Screen 9 using the Pace tool.

9 **Encourage students** to determine one successful sequence using as many transformations as they need, and then using fewer moves for their next attempts.

Capture strategies to share during the Connect.

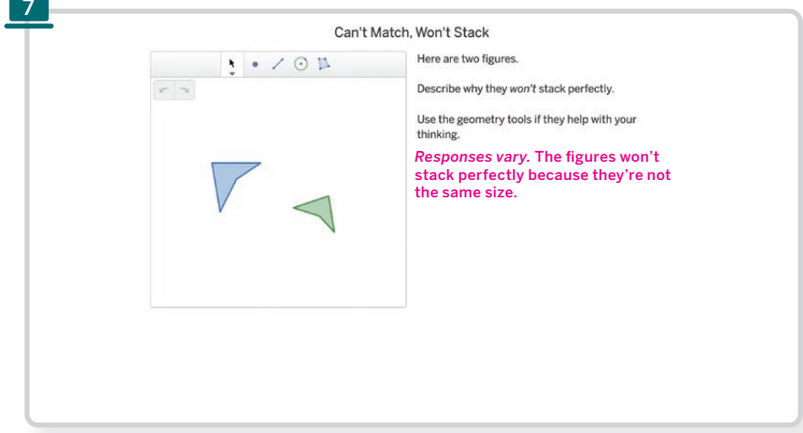
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to try any rigid transformation and reflect on how it works.
Determine one transformational proof, but need support getting started on a different one.	**Strengthen** Celebrate their successful proof. Invite them to choose a new proof strategy. Consider asking, "I see you used a reflection. Can you try to solve this with a rotation?"
Determine a sequence of transformations using many transformations.	**Strengthen** Celebrate their successful proof. Consider asking, "Could any of your sequences of transformations be done using fewer steps?" **(MP6)**

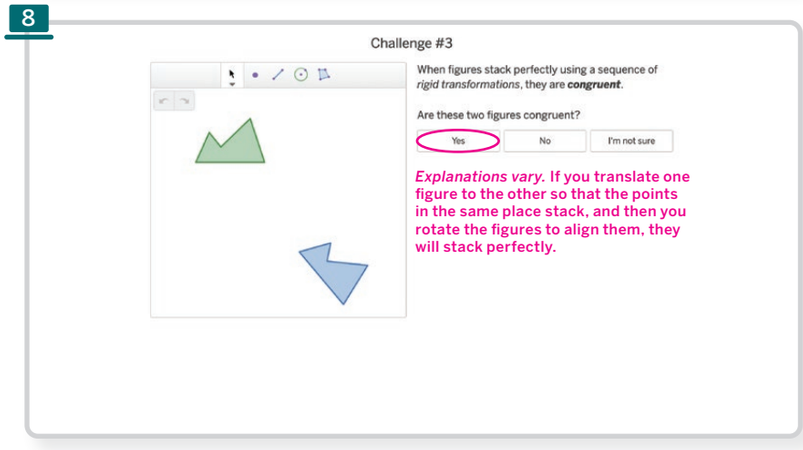
Invite students using notes to record their favorite sequence of transformations to solve this challenge.

Students using digital

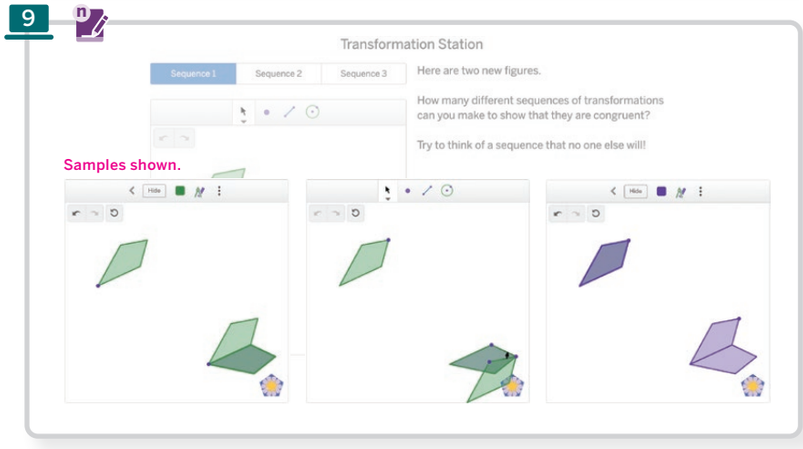
7



8



9



Activity 2 continued >

Pairs Sharing a Device | 15 min | Pacing: Screens 7–8, Screen 9, Screens 10–11

Activity 2 Transformation Station (continued)

Purpose: Students solidify their definition of **congruence** by completing additional challenges where they stack figures through a sequence of rigid transformations.

Short on time? Consider inviting students to complete fewer Repeated Challenges on Screens 9–10.



Student Notes are also available

2 Monitor

10 **Invite students** to solve as many of the three challenges as they have time for. Share that it is more important to make sense of the challenges than it is to work quickly.

Listen for strategies students use when they discuss how to determine a sequence of transformations that will stack one figure on top of the other perfectly.

Note: Use the dashboard's Teacher View to see the number of challenges each student completes correctly. The Responses View shows how each student completes each challenge; in this view, students who correctly solve the challenge will receive a star.

Encourage students who solve all three challenges to try and solve them again with as few transformation moves as possible. **(MP6)**

Math Identity and Community Consider pausing to celebrate students who persisted through struggle. Invite them to share strategies that worked for them.

11 D **Differentiation: Explore More (Stretch)** Invite students who would like to further explore how sequences of rigid transformations show congruence to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

9 **Share** the captured strategies, highlighting at least one strategy that uses a *reflection*, *rotation*, or *translation*. Consider also highlighting strategies that use sequences of transformations.

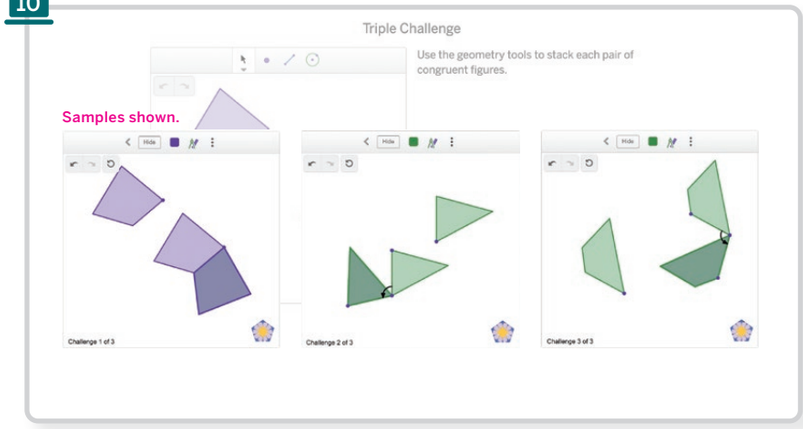
Invite students to share their sequences of transformations that stack the figures perfectly.

To surface the Key Takeaway, consider asking:

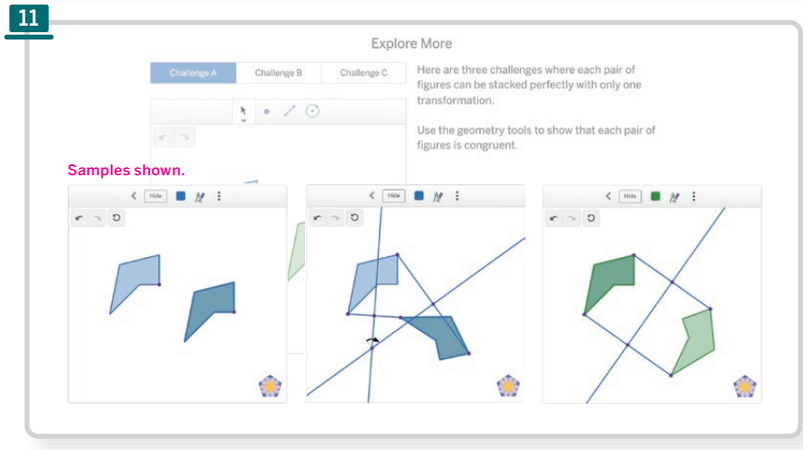
- "How did you decide what your first step would be in proving the figures are congruent?"
- "Does it matter how many rigid transformations you use to prove two figures are congruent?"
- "When might fewer transformations be useful?" **(MP6)**

Students using digital

10



11



Key Takeaway: When figures can be stacked perfectly through a sequence of rigid transformations, they are called **congruent**. There is more than one way to transform figures to show their congruence.

Whole Class | 5 min | Pacing: Screen 12

Synthesis



Student Notes are also available

12 **Invite students** to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

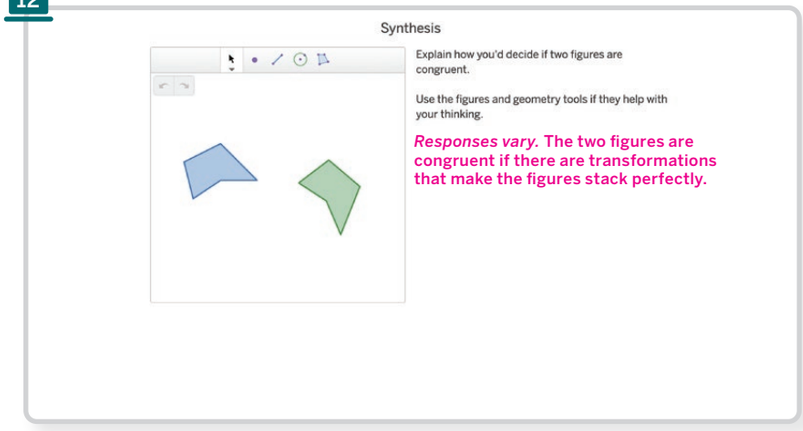
- Finding a sequence of rigid transformations that make the figures stack perfectly.
- Because reflections, rotations, and translations are rigid transformations, we can use them to decide if figures will stack.
- Rigid transformations don't change the size and shape of a figure, so we can use them to decide if figures are congruent.

Math Identity and Community Invite students to share strategies they found helpful and celebrate the students who shared them.

Note: If time allows, invite students to revise their previous thinking based on the discussion.

Students using digital

12

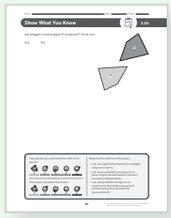


Lesson Takeaway: In Geometry, a main idea that we are often trying to prove is whether two figures or objects are exactly the same size and shape. *Rigid transformations*, such as *reflections*, *translations*, and *rotations*, keep the size and shape of a figure the same. If a sequence of rigid transformations stacks two figures perfectly, that means the figures are the same size and shape, or **congruent**.

Independent | 5 min | Pacing: Screens 13–14

Show What You Know

Purpose: Students demonstrate their understanding by determining if two given figures are congruent and explaining their reasoning.

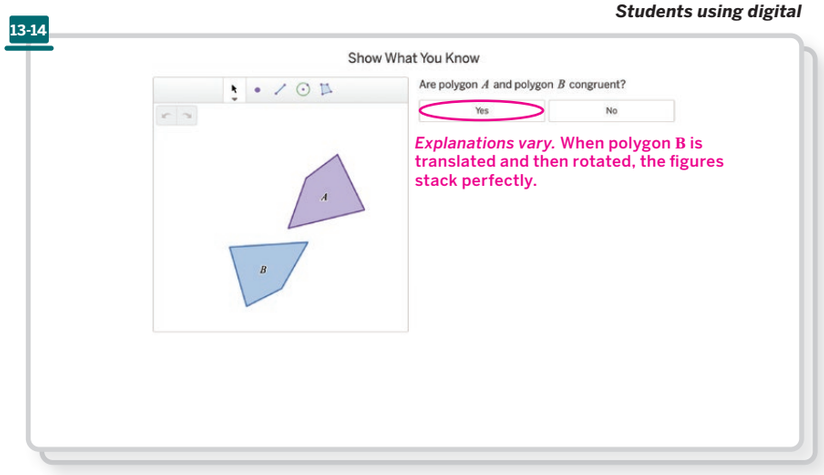


Students using print

13-14 Today's Goals

1. **Goal:** Use rigid transformations to investigate congruence claims.
2. **Goal:** Use the definition of *congruence* in terms of rigid transformations to decide if two figures stack perfectly.
3. **Language Goal:** Justify whether two figures are congruent by describing a sequence of transformations that will stack the figures perfectly. **(Writing, Speaking, and Listening)**

Students using digital



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

S Support

Provide targeted intervention.

If student work shows **partial understanding** of using rigid transformations to determine congruence between two figures, consider:

- Inviting students to read the **Summary** and paraphrase it to a caregiver or classmate.
- Using the ***Justifying Congruent Figures* Mini-Lesson** from Grade 8, Unit 1.

S Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding** with minor errors, consider:

- Assigning the **Lesson Practice**.
- Inviting students to review the **Summary** and complete the **Try This** problem.
- Revisiting the **Repeated Challenges** in Activity 2 and determining a different way to solve each challenge.

S Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting them to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

Take a moment to reflect on this lesson where students develop their understanding of how to use rigid transformations to prove that two figures are congruent. How did your class community come to accept proof by rigid transformation as a valid strategy? How does proof by rigid transformation support students in defining *congruence*? How will this approach continue to support students in determining triangle congruence in Lessons 8–9? Reflect on the ways that you experienced proving congruence as a math learner. How is this lesson's introduction to proving different from your experiences?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 5 Practice

Students using digital

Lesson Summary

You can use sequences of rigid transformations to decide if two figures are **congruent**. *Translations*, *rotations*, and *reflections* are all *rigid transformations*, which means they keep the size or shape of a figure the same. If two figures stack perfectly using rigid transformations, then they are congruent.

Let's look at two examples.

These figures are congruent because there is a sequence of transformations that will make them stack perfectly.

One way to stack the figures perfectly is to translate and then rotate the figure.

Congruent



These two figures are *not* congruent.

There is no sequence of rigid transformations that will stack the figures perfectly.

Not Congruent



Try This

Are these two figures congruent? Circle one.



(Yes) No I'm not sure

Explain your thinking.

Responses vary. **I used tracing paper to translate and rotate until both figures stacked perfectly. This means the figures are congruent.**

Lesson Practice
Geo.2.05

Name: _____ Date: _____ Period: _____

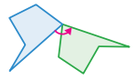
1. Select *all* transformations that carry point C onto point D.



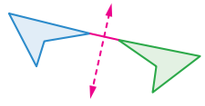
- ☐ **A.** Rotate 180° with point C as the center
- ☐ **B.** Rotate 180° with point D as the center
- ☑ **C.** Rotate 180° around the midpoint of segment CD
- ☑ **D.** Reflect over the perpendicular bisector of segment CD
- ☐ **E.** Reflect over segment CD

Problems 2–3: Show or explain *one* rigid transformation that can determine if the two figures are congruent. ***Responses vary.***

2.

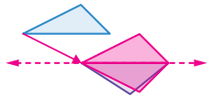


3.

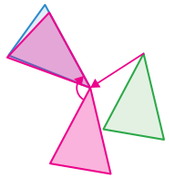


Problems 4–5: Are the two figures congruent? Circle your response and show or explain your sequence of rigid transformations. Use tracing paper if it helps with your thinking. ***Explanations vary.***

4. Congruent (Not congruent)



5. Congruent (Not congruent)

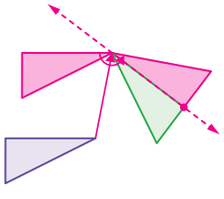


Lesson Practice
Geo.2.05

Name: _____ Date: _____ Period: _____

6. Use at least one reflection, rotation, and translation to determine if the figures are congruent. Circle your response and show or explain your thinking. ***Explanations vary.***

(Congruent) Not congruent



Spiral Review

Problems 7–8: Imagine figure B is an image of figure A.

7. Select *all* transformations that could have been used.

- ☑ **A.** Translation
- ☑ **B.** Rotation
- ☑ **C.** Reflection
- ☐ **D.** Dilation

8. Select *all* transformations that could not have been used.

- ☐ **A.** Translation
- ☐ **B.** Rotation
- ☐ **C.** Reflection
- ☑ **D.** Dilation

Reflection

1. Circle the problem that was the most challenging for you.

2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-lesson			
Test Practice	**1**	1	HSG.CO.A.2
	2–3	1	HSG.CO.A.2, HSG.CO.B.6
	4–5	2	HSG.CO.A.2, HSG.CO.B.6
	6	3	HSG.CO.A.2, HSG.CO.B.6
Spiral Review			
Fluency	**7–8**	1	8.G.A.2

Need More Practice?

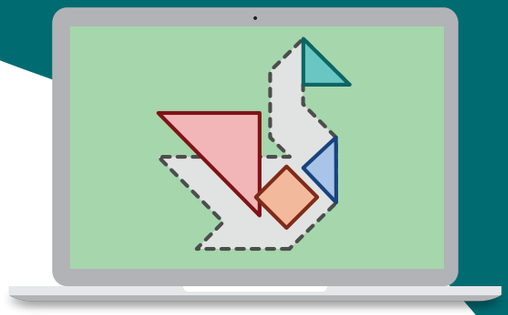
Additional Practice resources coming soon.

Unit 2

Lesson 6

Digital Lesson

This lesson uses digital student screens. Student Notes are also available.



Parts and Places

Congruence and Corresponding Parts

Let's make connections between corresponding parts and congruence.

Focus and Coherence

Today's Goal

1. **Language Goal:** Explain that two triangles are congruent if and only if all *corresponding* sides and angles are congruent. **(Writing and Speaking)**

Students use the structure of tangram puzzles to understand that figures made of congruent parts are only congruent if those parts are **corresponding**. They determine that if all corresponding parts of a pair of triangles are congruent, then the triangles themselves must be congruent, and this relationship holds true in both directions. Students explore congruence further by stacking triangles using rigid transformations. They also make sense of how notation can communicate congruence between two figures. **(MP1, MP7)**

Prior Learning

In Lesson 2, students used rigid transformations to prove that vertical angles are always congruent. In Lesson 5, students defined congruence as two figures stacking perfectly onto one another through a sequence of rigid transformations.

Future Learning

In Lesson 7, students will write and interpret congruence statements that communicate correspondence and congruence in triangle relationships. In Lessons 8–9, students will determine the minimum number of corresponding congruent parts needed to prove that two triangles are congruent.

Rigor and Balance

- Students develop **strategic competence** as they decide whether to use rigid transformations or corresponding congruent parts to prove two figures are congruent.
- Students develop **adaptive reasoning** as they justify if two figures are congruent based on whether they have corresponding pairs of congruent parts.

Vocabulary

New Vocabulary

corresponding

Standards

Addressing

HSG.CO.B.7

Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Mathematical Practices: MP1, MP5, MP7

Building On

8.G.A
8.G.A.2
HSG.CO.B.6

Lesson at a Glance

~ 45 min

Standard: HSG.CO.B.7



Why digital?

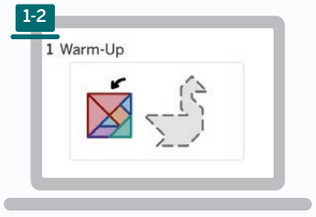
Students use digital manipulatives to drag figures to create congruent shapes and prove congruence using corresponding congruent parts.

Warm-Up

Independent | 5 min

Pacing: Screens 1–2

Students solve a tangram puzzle and explore how the same congruent parts can make many different figures. **(MP1)**



Activity 1

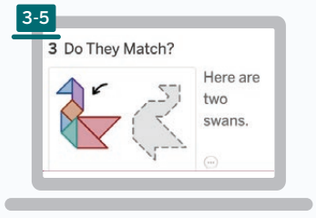
Pairs | 10 min

Pacing: Screens 3–4, Screen 5

Students use the structure of tangram puzzles to explore **corresponding** congruent parts in congruent figures.

MLR7: Compare and Connect **(MP7)**

Key Takeaway (Screen 5): Two figures are congruent when all of their **corresponding** parts are also congruent. This means that the parts are the same size and shape with the same placement and orientation in each figure.



Activity 2

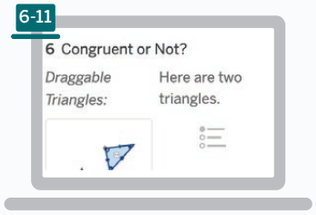
Pairs | 20 min

Pacing: Screens 6–7, 8–9, 10–11

Students analyze and apply different strategies to determine if two triangles are congruent using their corresponding parts.

MLR7: Compare and Connect

Key Takeaway (Screen 10): Using rigid transformations to stack figures can prove congruent triangles. Determining that all the corresponding parts of two triangles are congruent can also prove this relationship.



Synthesis

Whole Class | 5 min

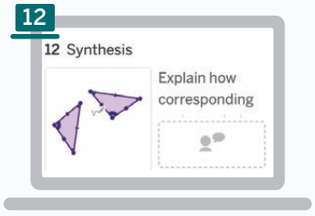
Pacing: Screen 12

Students synthesize their understanding of congruent figures. MLR2: Collect and Display

Lesson Takeaway: You can prove congruence by using **corresponding** parts of two figures:

- If all of the corresponding parts within two figures are congruent, then the figures are congruent.
- If two figures are congruent, then all of the parts within those two figures must be congruent and corresponding.

See the Synthesis page for full Lesson Takeaway.

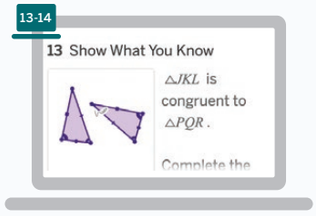


Show What You Know

Independent | 5 min

Pacing: Screens 13–14

Students demonstrate their understanding by identifying the corresponding congruent parts of congruent triangles.



Prep Checklist

Assign the digital student screens. Student Notes are also available.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)
- computers/devices (for research, optional)
- tracing paper (optional)

Student Notes sheet in Student Edition:

Consider providing the Student Notes to support class discussion and student thinking.



Independent | 5 min | Pacing: Screens 1–2

Warm-Up

Purpose: Students solve a tangram puzzle and explore how the same congruent parts can make many different figures. **(MP1)**

Short on time? Consider omitting Screen 2.



Student Notes are also available

1 Launch

1 **To encourage playfulness**, invite students to solve a tangram puzzle by making a swan. **(MP1)**

Demonstrate how to move the individual pieces by dragging the vertex to rotate the shape and clicking the reflection icon to reflect the shape.

A **Accessibility: Visual-Spatial Processing** Invite students to first select a shape by clicking on it. They can then use the arrows on the shape to reflect and rotate it to explore how the shape can fit into a specific space in the tangram puzzle

3 Connect

Invite students to compare their tangram images with a partner and share different strategies they used to solve the puzzle.

Consider asking:

- "How were your strategies alike? How were they different?"
- "Was it possible to build your swans with parts in different places?"

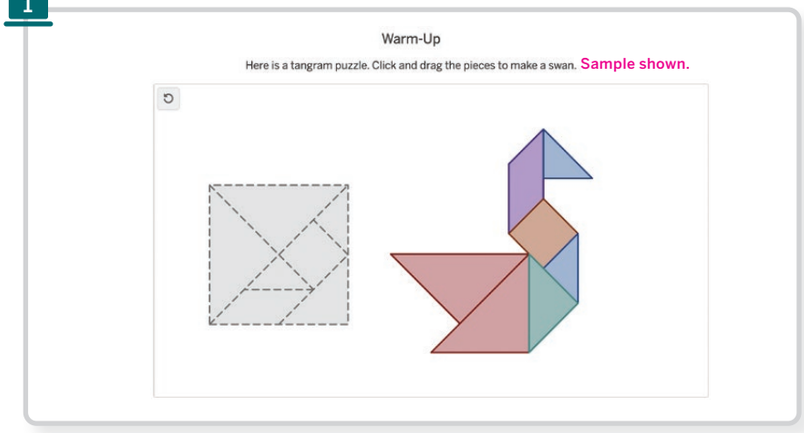
2 **Invite students** to view more examples of the tangram puzzle and spend a few minutes researching its history and uses. Consider asking, "What's one thing you learned about tangrams from your research?"

Consider sharing that a tangram is a puzzle that is typically made of seven specific geometric figures, called tans, that are used to explore a variety of concepts. The tans can be arranged to form many different shapes and used as a tool to prove conjectures.

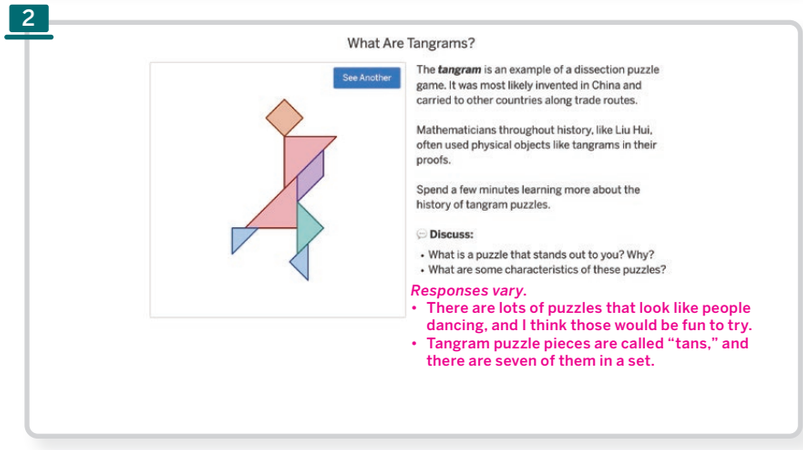
Math Identity and Community Consider sharing that proving in geometry can involve a variety of media including puzzles and physical objects. Invite students to think creatively about the variety of tools that can be used to construct arguments in this course. **(MP5)**

Students using digital

1



2



Pairs | 10 min | Pacing: Screens 3–4, Screen 5

Activity 1 Congruent Swans

Purpose: Students use the structure of tangram puzzles to explore **corresponding** parts in congruent figures. **(MP7)**



Student Notes are also available

1 Launch

3 **Invite students** to discuss the various strategies they might use to show the swans are congruent.

Demonstrate how to use the rotation and reflection toggles on a few puzzle pieces and explain what the symbols represent.

2 Monitor

3-4 **Capture** ideas that make use of structure with the congruence of **corresponding** parts to determine the congruence of the whole figure. **(MP7)**

4 MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their strategy.

Pause briefly to surface connections between Aki's and Mariam's thinking. Consider asking:
- "What do you think was helpful about each strategy?"
- "How does each strategy compare to your own?"

Invite students using notes to record some important considerations from the class discussion.

5 **To support making connections**, consider asking, "How could you use Mariam's strategy to identify the secret pair?"

Math Identity and Community Acknowledge the playfulness of identifying the congruent pair of swans. Consider sharing that play can involve perseverance — trying, struggling, failing, adjusting, and trying again. **(MP1)**

ML/EL **Multilingual/English Learners** Invite students to share what each word in *corresponding parts* reminds them of (e.g., "When I hear the word *corresponding*, I think…". "When I hear the word *parts*, I think…"). Consider asking, "What do these words mean together?" **(Listening and Speaking)**

Capture explanations to share during the Connect.

3 Connect

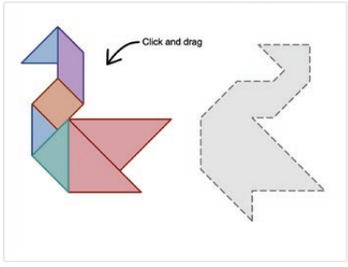
Share and discuss explanations that use corresponding parts to determine congruent figures.

To surface the Key Takeaway, consider asking:
- "How can you determine when two swans are congruent? How can you determine when they're *not*?"
- "What is the difference between stacking figures and deciding their corresponding parts are congruent?"

Students using digital

3

Do They Match?



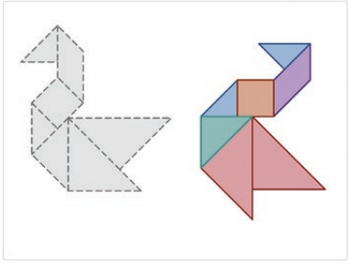
Here are two swans.

Discuss: Are these swans congruent? How do you know?

Responses vary.
- I can move all the pieces from the left to the swan shadow on the right, and the pieces all fit so they are congruent.
- I traced the swan with the tangrams using tracing paper and then rotated it to see that it stacks on the gray swan.

4

Swan Strategies



Here are the two tangram swans from the previous screen.

Aki says: *I used tracing paper to make an outline of one swan to see if it stacked perfectly on the other swan.*

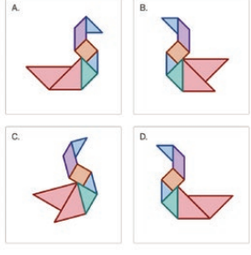
Mariam says: *I can tell the swans are congruent because the same tiles are in the same place on each swan.*

Discuss: How are their strategies alike? How are they different?

Responses vary.
- They are the same because there is only one way to use the pieces to assemble the swan, so looking at individual pieces and looking at the whole swan do the same thing.
- They are different because Mariam looked at the pieces of the puzzle to see if they fit, and Aki looked at the whole shape to see if they stack.

5

The Secret Pair



Two figures are congruent when all of their parts are congruent and ***corresponding***.

a. Select the secret pair of congruent swans.

b. Explain how you know they are congruent.

Responses vary. The swans are congruent because the tangram pieces are in the same place on each swan.

Lesson Takeaway: Two figures are congruent when all of their **corresponding** parts are also congruent. This means that the parts are the same size and shape with the same placement and orientation in each figure.

Pairs | 20 min | Pacing: Screens 6–7, Screens 8–9, Screens 10–11

Activity 2 Congruent or Not?

Purpose: Students analyze and apply different strategies to determine if two triangles are congruent using their corresponding parts.



Student Notes are also available

1 Launch

6 **Display** the triangle diagram and invite students to use both the manipulative and the sketch tool to investigate the figures.

Look for different strategies students apply when they determine congruence (e.g., using rigid transformations to stack figures, identifying and confirming corresponding congruent parts).

2 Monitor

7 **Listen for** specific language students use to distinguish the two strategies, and invite students to share these ideas.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their strategy.

Pause briefly to share student ideas.

To support making connections, consider asking:

- "What strategies can we use to determine if two triangles are congruent?"
- "How can rigid transformations help prove that corresponding parts of congruent triangles are also congruent?"

Invite students to continue to Screens 8–9 using the Pace tool.

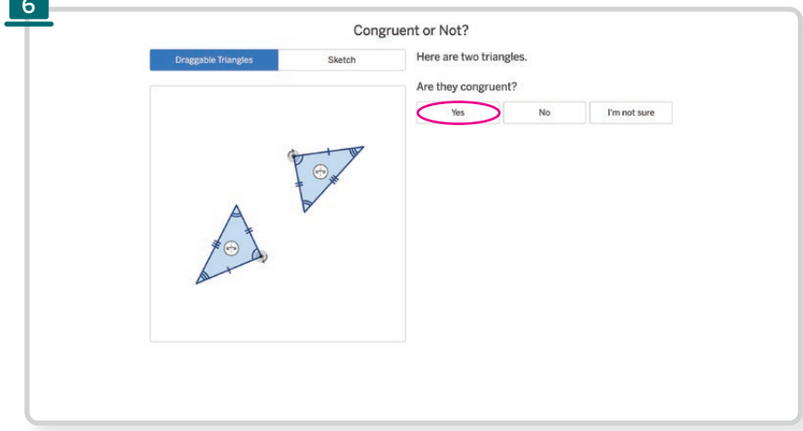
8 **Look for** language used by students to justify that these triangles are not congruent.

ML/EL **Multilingual/English Learners** Provide sentence frames to help students explain their strategy, such as, "I know the triangles are not congruent because _____, so I _____." or "First, I noticed that _____, which means that _____." **(Writing and Speaking)**

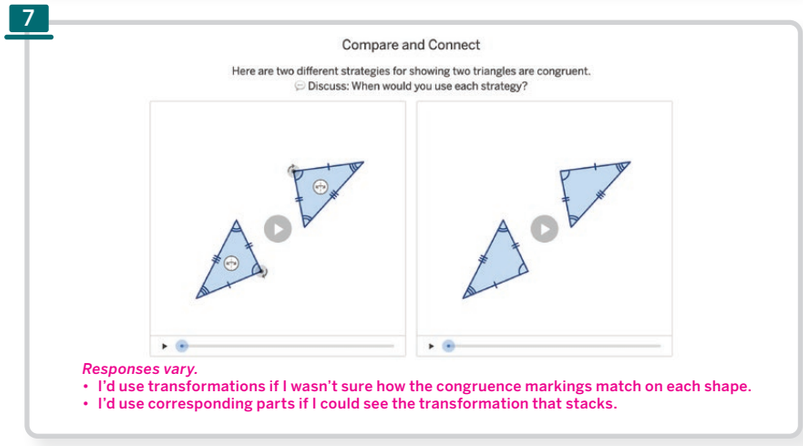
Activity 2 continued >

Students using digital

6



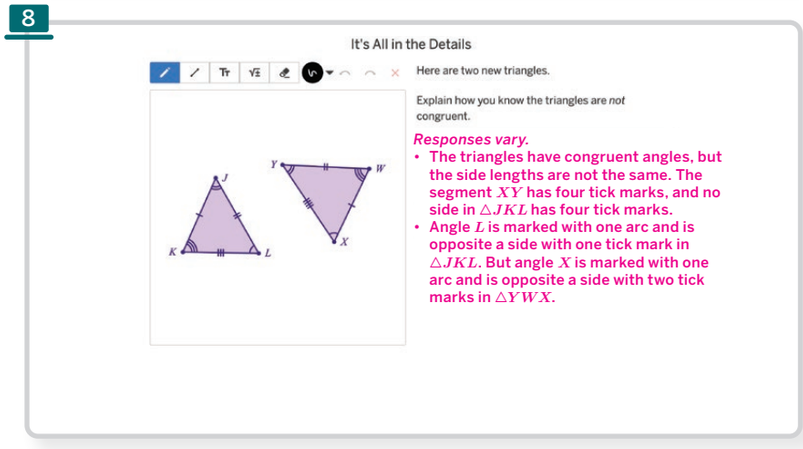
7



Responses vary.

- I'd use transformations if I wasn't sure how the congruence markings match on each shape.
- I'd use corresponding parts if I could see the transformation that stacks.

8



Responses vary.

- The triangles have congruent angles, but the side lengths are not the same. The segment XY has four tick marks, and no side in $\triangle JKL$ has four tick marks.
- Angle L is marked with one arc and is opposite a side with one tick mark in $\triangle JKL$. But angle X is marked with one arc and is opposite a side with two tick marks in $\triangle YWX$.

Pairs | 20 min | Pacing: Screens 6–7, Screens 8–9, Screens 10–11

Activity 2 Congruent or Not? (continued)

Purpose: Students analyze and apply different strategies to determine if two triangles are congruent using their corresponding parts.



Student Notes are also available

2 Monitor

9 **Pause** to demonstrate how to add the overline or angle symbol to the math input box.

Note: To locate the overline or angle symbol, select the keypad button, then "geometry," and then $\overline{AB}$ or $\angle$. Select the symbol before typing capital letters to format it correctly. Students will still receive feedback that they have correctly identified a congruent segment even if they write the segment in a transposed order. Students will learn about how orientation is related to order in the next lesson.

Invite students to continue using the Pace tool.

10 **Look for** ways students engage with this screen.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to try any combination of vertices they see on the screen and use the feedback to revise their thinking.
Identify the corresponding segment as DE without using the overline symbol.	**Strengthen** Invite them to attend to precision. Consider asking, "What symbols will identify DE as a line segment?" **(MP6)**
Enter the segment with vertices in transposed order (e.g., $\overline{ED}$ instead of $\overline{DE}$).	**Stretch** Celebrate that they correctly identified a corresponding segment. Consider asking, "If you stacked these triangles perfectly, would point D stack with point A or point E? How do you know?"

11 **D Differentiation: Explore More (Stretch)** Invite students who want to further explore corresponding parts to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share their identified segments and angles and display the transformations that accompany each congruence statement.

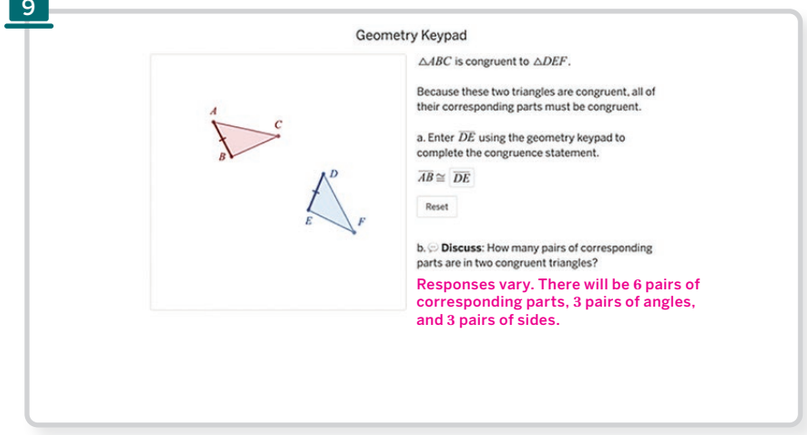
Invite students using notes to answer, "How did you identify the corresponding parts in the figures?"

To surface the Key Takeaway, consider asking:

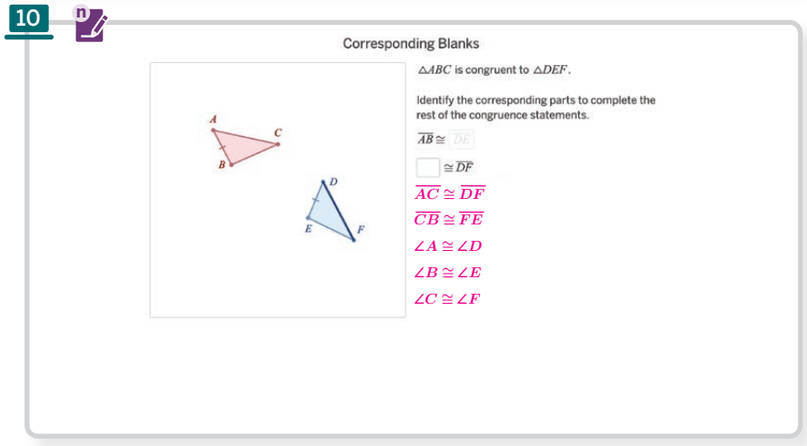
- "How did rigid transformations convince us of which corresponding parts were congruent?"
- "Why might it be helpful to have more than one strategy for determining congruence?"

Students using digital

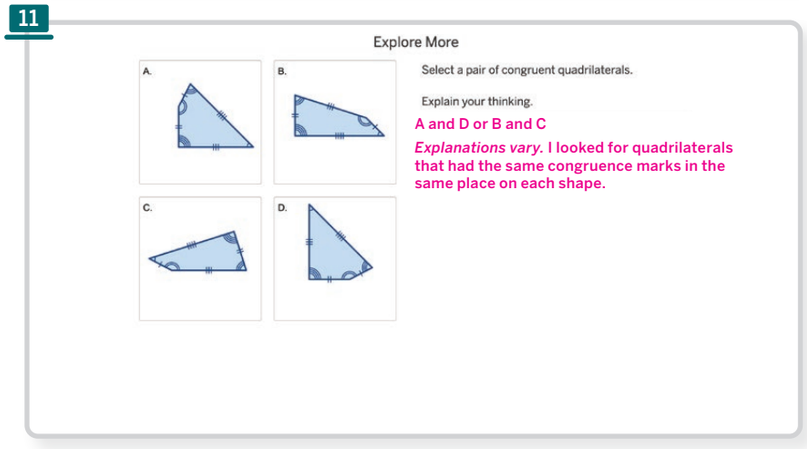
9



10



11



Key Takeaway: Using rigid transformations to stack figures can prove congruent triangles. Determining that all the corresponding parts of two triangles are congruent can also prove this relationship.

Whole Class | 5 min | Pacing: Screen 12

Synthesis



Student Notes are also available

12 **Invite students** to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- When congruence markings are the same in two figures, it means the parts are congruent.
- If all the congruent parts are in the same place and orientation in two figures, then the figures are congruent.
- If two figures are congruent, then all of the parts must be in the same place, in the same orientation, and must be congruent.
- Rigid transformations can help convince us of which parts are corresponding and congruent between two figures.
- Rigid transformations of whole figures is not the only way to determine if two figures are congruent.

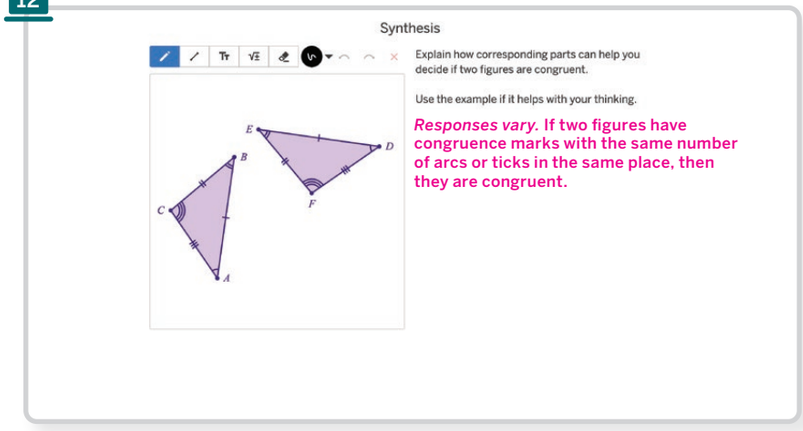
MLR **MLR2: Collect and Display** Use this routine to collect students' ideas and language about identifying corresponding parts to decide if two figures are congruent. Create an anchor chart to display throughout the unit that includes congruence markings, notation, and rigid transformations to support students' thinking.

Math Identity and Community Invite students to share strategies they found helpful and celebrate the students who shared them.

Note: If time allows, invite students to revise their previous thinking on Screen 8 based on the discussion.

Students using digital

12



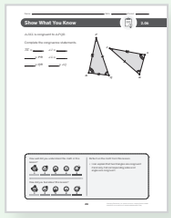
Lesson Takeaway: Using rigid transformations of whole figures is not the only strategy for determining if two figures are congruent. Another strategy for proving congruence is to determine whether the **corresponding** parts of two figures are congruent, and this relationship works in both directions.

- If all of the corresponding parts within two figures are congruent, then the figures are congruent.
- If two figures are congruent, then all of the parts within those two figures must be congruent and corresponding.

Independent | 5 min | Pacing: Screens 13–14

Show What You Know

Purpose: Students demonstrate their understanding by identifying corresponding congruent parts of congruent triangles.

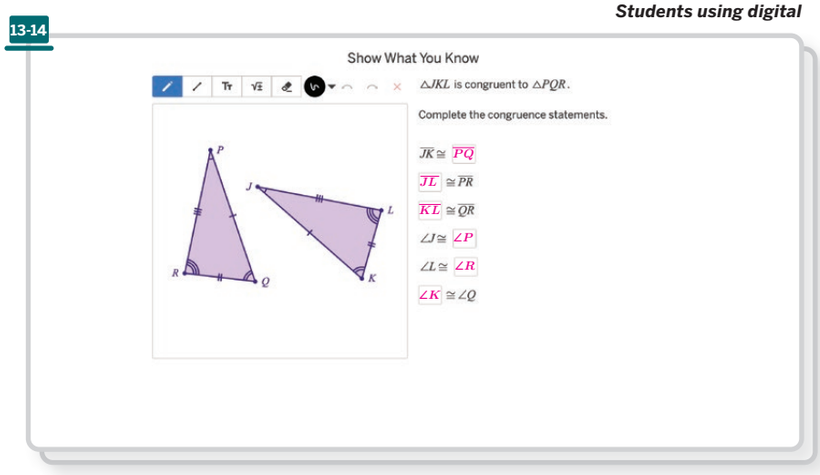


Students using print

13-14 Today's Goal

1. **Language Goal:** Explain that two triangles are congruent if and only if all *corresponding* sides and angles are congruent. **(Writing and Speaking)**

Students using digital



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of identifying corresponding congruent parts in congruent triangles, consider:

- Inviting students to complete the **Try This** problem and providing individualized feedback.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice**.
- Revisiting **Screens 9–10**. Invite students to try incorrect line segments and discuss with a partner how rigid transformations show that these parts are not corresponding.
- Inviting students to review the **Summary** and complete the **Try This** problem.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting them to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

Reflect on your facilitation of **MLR7: Compare and Connect** in this lesson. How did students use this routine to make connections between rigid transformations and corresponding parts? How did your facilitation of the routine enhance student engagement? How could you facilitate it differently in the future?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 6 Practice

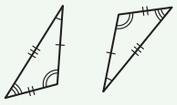
Students using digital

Lesson Summary

You can determine if two figures are congruent in two different ways:

- Determining a sequence of rigid transformations that stacks the figures perfectly.
- Verifying that all the **corresponding** parts in the figures are congruent.

Corresponding parts are in the same position and orientation in each figure. If *all* the pairs of congruent sides and angles are corresponding in each figure, then the figures are congruent.

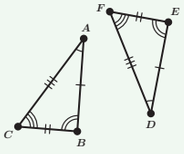


To show that congruent segments are corresponding, you can mark them with tick marks. To show that congruent angles are corresponding, you can mark them with arcs.

If two triangles are congruent, then they have 6 sets of corresponding congruent parts: 3 pairs of sides and 3 pairs of angles.

$\overline{AB} \cong \overline{DE}$ $\quad \overline{BC} \cong \overline{EF}$ $\quad \overline{AC} \cong \overline{DF}$

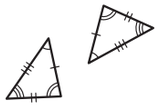
$\angle A \cong \angle D$ $\quad \angle B \cong \angle E$ $\quad \angle C \cong \angle F$



Try This

Explain how you know that these two triangles must be congruent.

Responses vary. The triangles are congruent because they have congruent corresponding parts.



Lesson Practice
Geo.2.06

Name: _____ Date: _____ Period: _____

Problems 1–2: Here is a pair of triangles.



1. Explain how you know that $\triangle EMP \cong \triangle ASD$.

   Explanations vary. All of the corresponding angles and corresponding segments for the two triangles are congruent.

2. Write the congruence statements for the corresponding parts in the triangles.

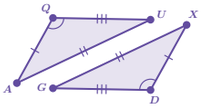
   $\angle P \cong \angle$ D $\quad \angle E \cong \angle$ A $\quad \angle M \cong \angle$ S

   $\overline{EM} \cong$ $\overline{AS}$ $\quad \overline{PE} \cong$ $\overline{DA}$ $\quad \overline{PM} \cong$ $\overline{DS}$

3. What two additional congruence statements are needed to make $\triangle QUA \cong \triangle DGX$ true?

   $\angle A \cong \angle X$

   $\angle U \cong \angle G$

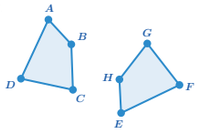


4. Quadrilateral $ABCD$ is congruent to quadrilateral $EHGF$. Select *all* the statements that must be true.

   ☐ A. $\angle C \cong \angle F$ ☐ B. $\overline{BC} \cong \overline{GF}$

   ☑ C. $\angle B \cong \angle H$ ☑ D. $\overline{EH} \cong \overline{AB}$

   ☑ E. $\angle A \cong \angle E$ ☐ F. $\overline{DA} \cong \overline{EF}$

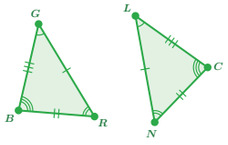


5. Here are two congruent triangles, $\triangle GRB \cong \triangle LNC$.

   Complete the following congruence statements:

   $\angle R \cong$ $\angle N$ $\quad \overline{BG} \cong$ $\overline{CL}$ $\quad \angle B \cong$ $\angle C$

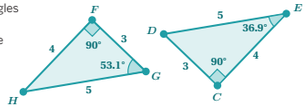
   $\overline{GR} \cong$ $\overline{LN}$ $\quad \angle G \cong$ $\angle L$ $\quad \overline{RB} \cong$ $\overline{NC}$



Lesson Practice
Geo.2.06

Name: _____ Date: _____ Period: _____

6. DeShawn thinks that these two triangles are congruent. Lucy does not think these triangles are congruent. Whose thinking is correct? Circle one.



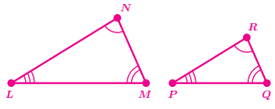
   (DeShawn's) Lucy's Neither

   Explain your reasoning.

   Explanations vary. Finding the missing angle measures in each triangle shows that all the corresponding sides and corresponding angles are congruent.

7. Draw two triangles, $\triangle LMN$ and $\triangle PQR$, that have 3 pairs of congruent angles.

   Responses vary.



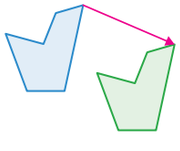
8. Do your triangles from Problem 7 have to be congruent?

   No

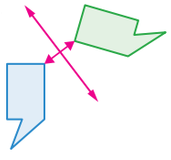
Spiral Review

Problems 9–10: Sketch one rigid transformation that shows if the two figures are congruent.

9.



10.



Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-lesson			
	1–3, 5–6	2	HSG.CO.B.7
Test Practice	4	2	HSG.CO.B.7
	7–8	3	HSG.CO.B.7
Spiral Review			
Fluency	9–10	1	HSG.CO.A.2, HSG.CO.B.6

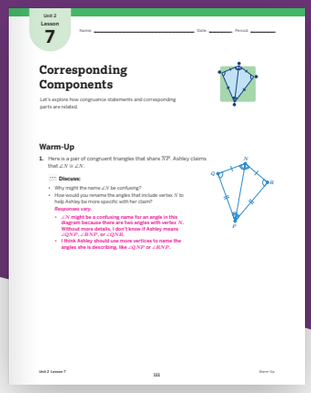
Need More Practice?

Additional Practice resources coming soon.

Unit 2

Lesson 7

This lesson uses print Student Edition pages and digital Teacher Presentation Screens.



Corresponding Components

Congruence Statements

Let's explore how congruence statements and corresponding parts are related.

Focus and Coherence

Today's Goals

1. **Goal:** Use congruence statements to identify and describe corresponding parts in congruent figures.
2. **Goal:** Draw a figure when given a sequence of transformations represented by a congruence statement.
3. **Language Goal:** Describe a sequence of transformations that stacks corresponding vertices in congruent figures. **(Reading, Writing, Speaking, and Listening)**

Students use rigid transformations to analyze and make sense of the corresponding parts of congruent figures. Then they apply their understanding of congruence to determine the corresponding parts of congruent triangles and to write congruence statements precisely. **(MP1, MP6)**

Prior Learning

In Lesson 5, students used rigid transformations to prove that two figures are congruent. In Lesson 6, students defined congruence by showing that two triangles are congruent if and only if all corresponding pairs of sides and angles are congruent.

Future Learning

In Lessons 8–9, students will use rigid transformations to determine the minimum number of corresponding congruent parts that prove two triangles are congruent.

Rigor and Balance

- Students **apply** their understanding of corresponding parts when they write and read congruence statements.
- Students develop **strategic competence** as they use rigid transformations to identify corresponding pairs of congruent parts in two triangles.

Standards

Addressing

HSG.CO.B.7

Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

HSG.CO.A.5

Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Also Addressing: **HSG.CO.A.2**

Mathematical Practices: MP1, MP3, MP6

Building On

8.G.A
8.G.A.2

Lesson at a Glance

~ 45 min

Standards: HSG.CO.B.7, HSG.CO.A.5, HSG.CO.A.2

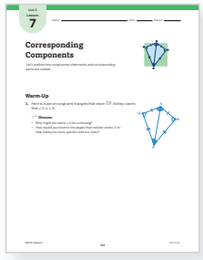
Warm-Up

Pairs | 5 min

Students use corresponding congruent parts between two congruent triangles to generate the need for precise labels in figures.

Key Takeaway: To determine corresponding parts of congruent figures, it is helpful to identify the parts as precisely as possible. **(MP6)**

See the Activity page for full Key Takeaway.





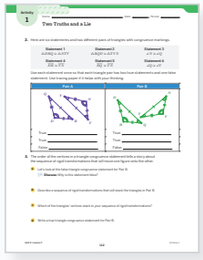
Presentation Screen 1

Activity 1

Pairs | 15 min

Students identify two truths and a lie to understand how congruence statements and the order of vertices show that one figure stacks perfectly onto another.

Key Takeaway: Congruence statements can show corresponding parts of congruent figures. The order of vertices in congruence statements tells a story about the sequence of rigid transformations that will stack figures perfectly.



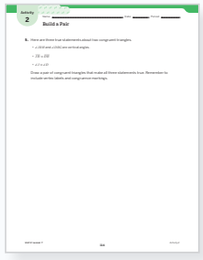


Presentation Screens 2–3

Activity 2

Pairs | 15 min

Students use congruence statements to draw precise diagrams of triangles with corresponding congruent parts. **(MP4)**





Presentation Screen 4

Synthesis

Whole Class | 5 min

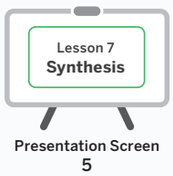
Students synthesize their understanding of corresponding congruent parts of triangles from congruence statements. MLR2: Collect and Display

Lesson Takeaway: There are multiple ways to identify corresponding parts:

- Using rigid transformations to stack the figures and their corresponding vertices.
- Reading congruence markings that show which parts correspond in each figure.
- Interpreting congruence statements.

See the Synthesis page for full Lesson Takeaway.



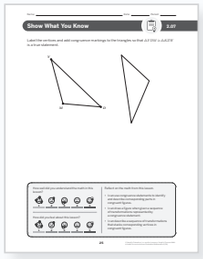


Presentation Screen 5

Show What You Know

Independent | 5 min

Students demonstrate their understanding by labeling a diagram with congruence markings to represent corresponding congruent parts in congruent triangles.





Presentation Screen 6

Prep Checklist

Invite students to use their Student Edition. Display the Presentation Screens to help facilitate the lesson.

This lesson includes:

- Presentation Screens
- Student Edition
- Show What You Know
- coloring tools (as needed)
- straightedges (as needed)
- tracing paper (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students use corresponding congruent parts between two congruent triangles to generate the need for precise labels in figures.



Presentation Screen 1

1 Launch

Display the diagram for the Warm-Up and invite students to think independently about the discussion prompt.

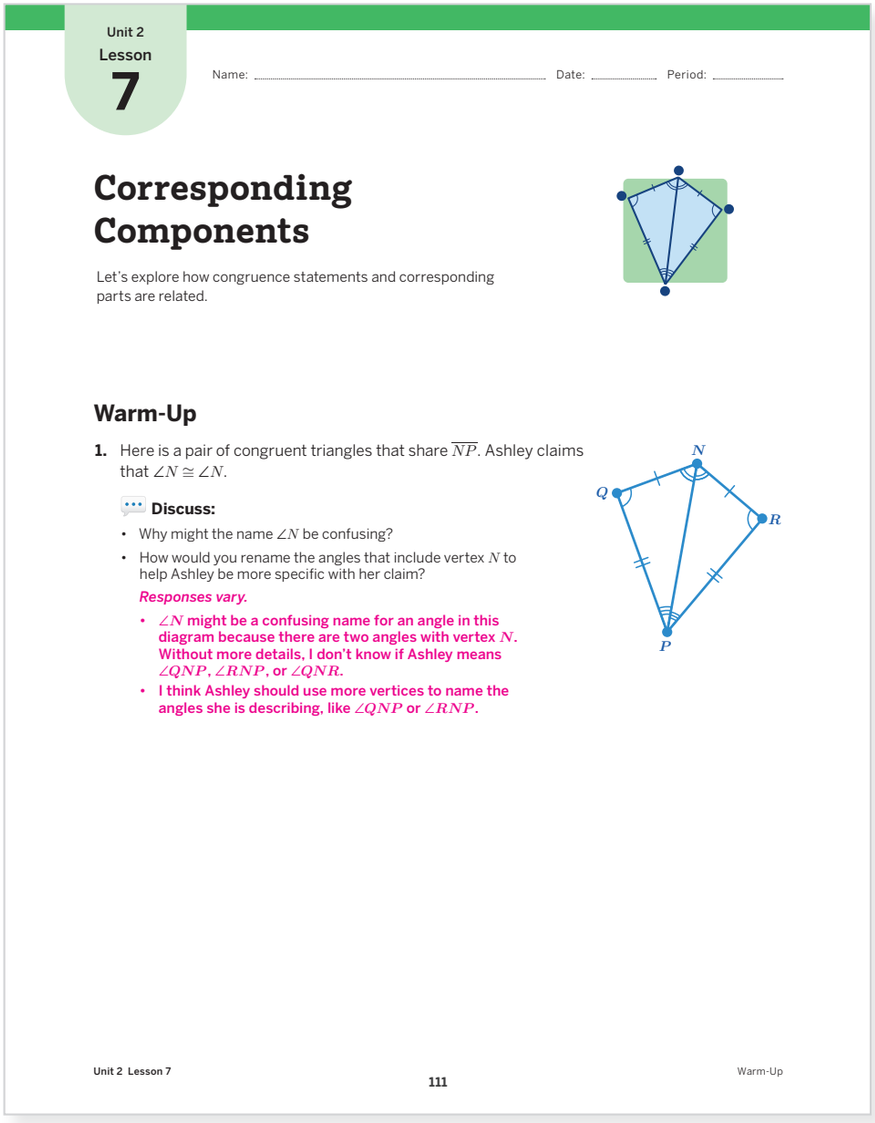
Use the Think-Pair-Share routine to support student-to-student discourse. See the Routine Facilitation Guide for more information.

2 Connect

Listen for ways students refer to the other angles in the diagram that include vertex N.

To surface the Key Takeaway, consider asking:

- "Is Ashley's claim incorrect because of how she described her angle?" *Responses vary.* No. Ashley's claim is not incorrect, but it could be more precise.
- "How can you identify the vertices of angle names?"
- "What is the difference between referring to the angle as $\angle QNP$ versus $\angle PNQ$? Why might someone use one label instead of the other?" *Responses vary.* These angle labels refer to the same angle but use a different orientation. Someone might use $\angle QNP$ when they also want to refer to $\angle RNP$, but they might use $\angle PNQ$ when referencing $\angle PNR$.



Unit 2 Lesson 7

Name: Date: Period:

Corresponding Components

Let's explore how congruence statements and corresponding parts are related.

Warm-Up

1. Here is a pair of congruent triangles that share $\overline{NP}$. Ashley claims that $\angle N \cong \angle N$.

   Discuss:

   - Why might the name $\angle N$ be confusing?
   - How would you rename the angles that include vertex N to help Ashley be more specific with her claim?

   Responses vary.

   - $\angle N$ might be a confusing name for an angle in this diagram because there are two angles with vertex N. Without more details, I don't know if Ashley means $\angle QNP$, $\angle RNP$, or $\angle QNR$.
   - I think Ashley should use more vertices to name the angles she is describing, like $\angle QNP$ or $\angle RNP$.

Unit 2 Lesson 7 111 Warm-Up

Key Takeaway: To identify corresponding parts of congruent figures, it is helpful to identify the parts as specifically and precisely as possible so it is clear what part of each figure you are describing. **(MP6)**

Pairs | 15 min

Activity 1 Two Truths and a Lie

Purpose: Students identify two truths and a lie to understand how congruence statements and the order of vertices tell how one figure stacks perfectly onto another.

Lesson 7
Activity 1

Presentation Screens 2–3

1 Launch

2 **Display** the pairs of triangles. Invite students to share how they are alike and different.

Encourage students to use tracing paper or coloring tools to make sense of the congruence statements.

2 Monitor

To support students getting started, consider using the sketch tool to demonstrate how a congruence statement represents a relationship shown in the diagram.

Select and sequence different statements students considered to be true or false.

D Differentiation *(Problems 2–4)*

Look for students who:	Respond to Student Thinking
Need support determining which vertices correspond in each triangle pair.	**Support** Encourage students to use coloring tools to help them visualize the corresponding parts.
Identify a congruence statement for the triangle pair that lists the vertices for the triangle in non-corresponding order. *(Problem 2)*	**Support** Invite them to use tracing paper and rigid transformations to confirm their thinking. Consider asking, "Does the order of the labels matter? Why or why not?"
Name $\overline{DR} \cong \overline{TN}$ as a true statement for Pair A. *(Problem 2)*	**Strengthen** Consider asking, "Imagine stacking point D onto point T. Will the triangles be stacked perfectly?"
Would like an additional challenge.	**Stretch** Invite them to describe a second sequence of transformations that stacks the triangles in Pair A.

ML/EL **Multilingual/English Learners** Provide a word bank to support students in expressing their ideas with words such as: *translate*, *rotate*, *reflect*, etc. **(Writing and Speaking)**

Activity 1

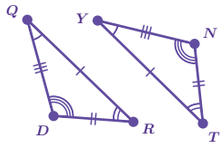
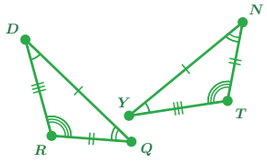
Name: _____ Date: _____ Period: _____

Two Truths and a Lie

2. Here are six statements and two different pairs of triangles with congruence markings.

Statement 1	Statement 2	Statement 3
$\triangle DRQ \cong \triangle NTY$	$\triangle RQD \cong \triangle TYN$	$\angle N \cong \angle Q$
Statement 4	**Statement 5**	**Statement 6**
$\overline{DR} \cong \overline{TN}$	$\overline{RQ} \cong \overline{TN}$	$\angle Q \cong \angle Y$

Use each statement once so that each triangle pair has two true statements and one false statement. Use tracing paper if it helps with your thinking.

Pair A	Pair B
	
True: 1. $\triangle DRQ \cong \triangle NTY$	True: 3. $\angle N \cong \angle Q$
True: 6. $\angle Q \cong \angle Y$	True: 5. $\overline{RQ} \cong \overline{TN}$
False: 4. $\overline{DR} \cong \overline{TN}$	False: 2. $\triangle RQD \cong \triangle TYN$

3. The order of the vertices in a triangle congruence statement tells a story about the sequence of rigid transformations that will move one figure onto the other.

a. Let's look at the false triangle congruence statement for Pair B.

Discuss: Why is this statement false?

Responses vary. If $\triangle RQD \cong \triangle TYN$ were true, then a transformation would align angle Q with angle Y and angle D with angle N, but those pairs of angles aren't congruent.

b. Describe a sequence of rigid transformations that will stack the triangles in Pair B.

Responses vary. Translate $\triangle DRQ$ until angle D' is on top of angle Y. Rotate $\triangle D'R'Q'$ around point D' until the two triangles stack.

c. Which of the triangles' vertices stack in your sequence of rigid transformations?

Vertices D and Y, vertices R and T, vertices Q and N (or equivalent)

d. Write a *true* triangle congruence statement for Pair B.

$\triangle RQD \cong \triangle TNY$ (or equivalent)

Unit 2 Lesson 7 112 Activity 1

Pairs | 15 min

Activity 1 Two Truths and a Lie (continued)

Purpose: Students identify two truths and a lie to understand how congruence statements and the order of vertices tell how one figure stacks perfectly onto another.

Lesson 7
Activity 1

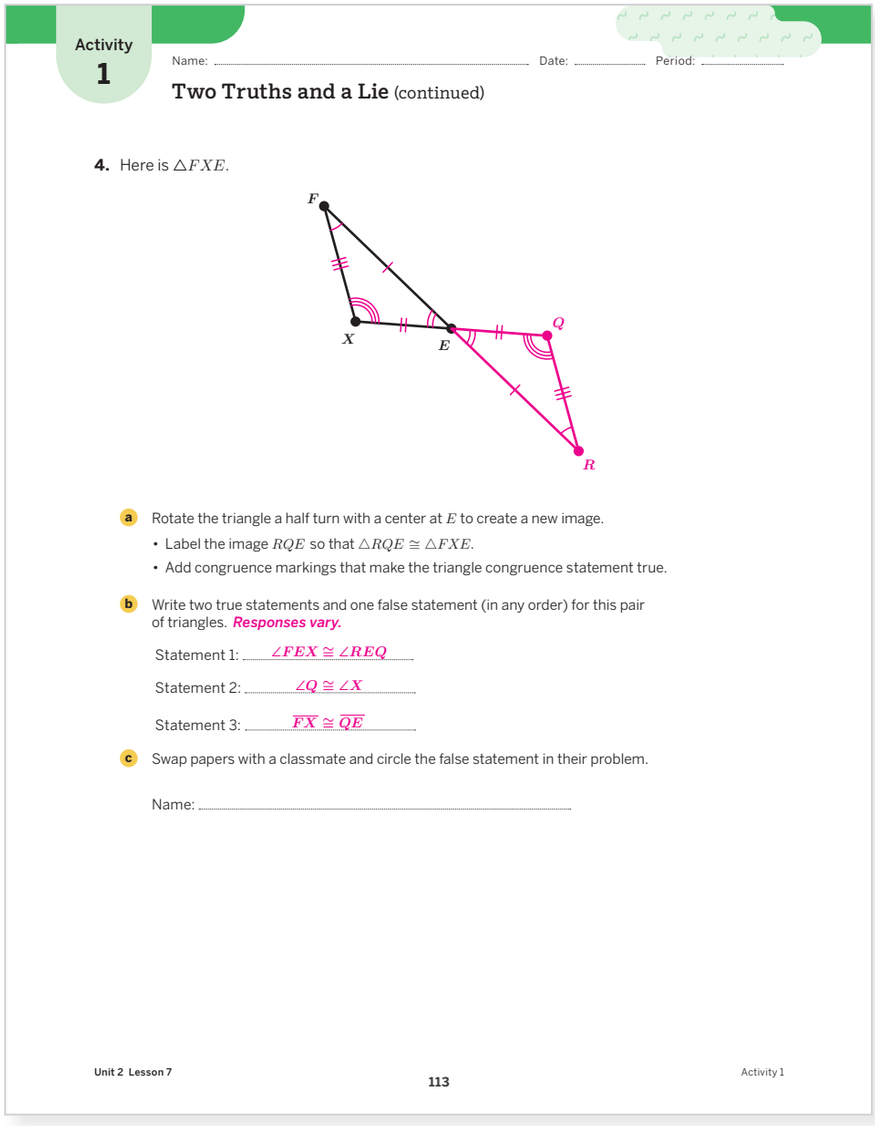
Presentation Screens 2–3

3 Connect

3 **Display** the Pair B triangles from Problem 2 and invite students to suggest false congruence statements to demonstrate in the interaction.

To surface the Key Takeaway, consider asking:

- "Which vertex from $\triangle RQD$ will stack perfectly onto vertex N in $\triangle TYN$? How do you know?"
- "How does each rigid transformation — translations, rotations, and reflections — stack congruent parts?"



Activity 1

Name: _____ Date: _____ Period: _____

Two Truths and a Lie (continued)

4. Here is $\triangle FXE$.

a. Rotate the triangle a half turn with a center at E to create a new image.
- Label the image RQE so that $\triangle RQE \cong \triangle FXE$.
- Add congruence markings that make the triangle congruence statement true.

b. Write two true statements and one false statement (in any order) for this pair of triangles. ***Responses vary.***

Statement 1: $\angle FEX \cong \angle REQ$

Statement 2: $\angle Q \cong \angle X$

Statement 3: $\overline{FX} \cong \overline{QE}$

c. Swap papers with a classmate and circle the false statement in their problem.

Name: _____

Unit 2 Lesson 7 113 Activity 1

Key Takeaway: Congruence statements can show corresponding parts of congruent figures. The order of vertices in congruence statements tells a story about the sequence of rigid transformations that will stack figures perfectly.

Pairs | 15 min

Activity 2 Build a Pair

Purpose: Students use congruence statements to draw precise diagrams of triangles with corresponding congruent parts. **(MP6)**

Short on time? Consider completing Problem 6 as a whole class instead of in pairs.

Lesson 7
Activity 2

Presentation Screen
4

1 Launch

Display the screen. Invite students to discuss with a partner what they think the triangles might look like based on the three congruence statements. **(MP1)**

Consider asking, "What must be the same in everyone's drawings? What can be different?"

A **Accessibility: Conceptual Processing**
To support abstract thinking, invite students to sketch one triangle on tracing paper and to fold or trace on the paper to imagine what the other triangle would look like.

2 Monitor

Pause briefly to encourage students to reflect on their drawings. Consider asking: **(MP3)**

- "How do you know your triangles are congruent?"
- "What sequence of rigid transformations would convince you that the triangles are congruent?"

Invite students to revise their drawings to make them more precise before continuing to Problem 6. **(MP6)**

Look for strategies to share during the Connect.

D **Differentiation** *(Problems 5–6)*

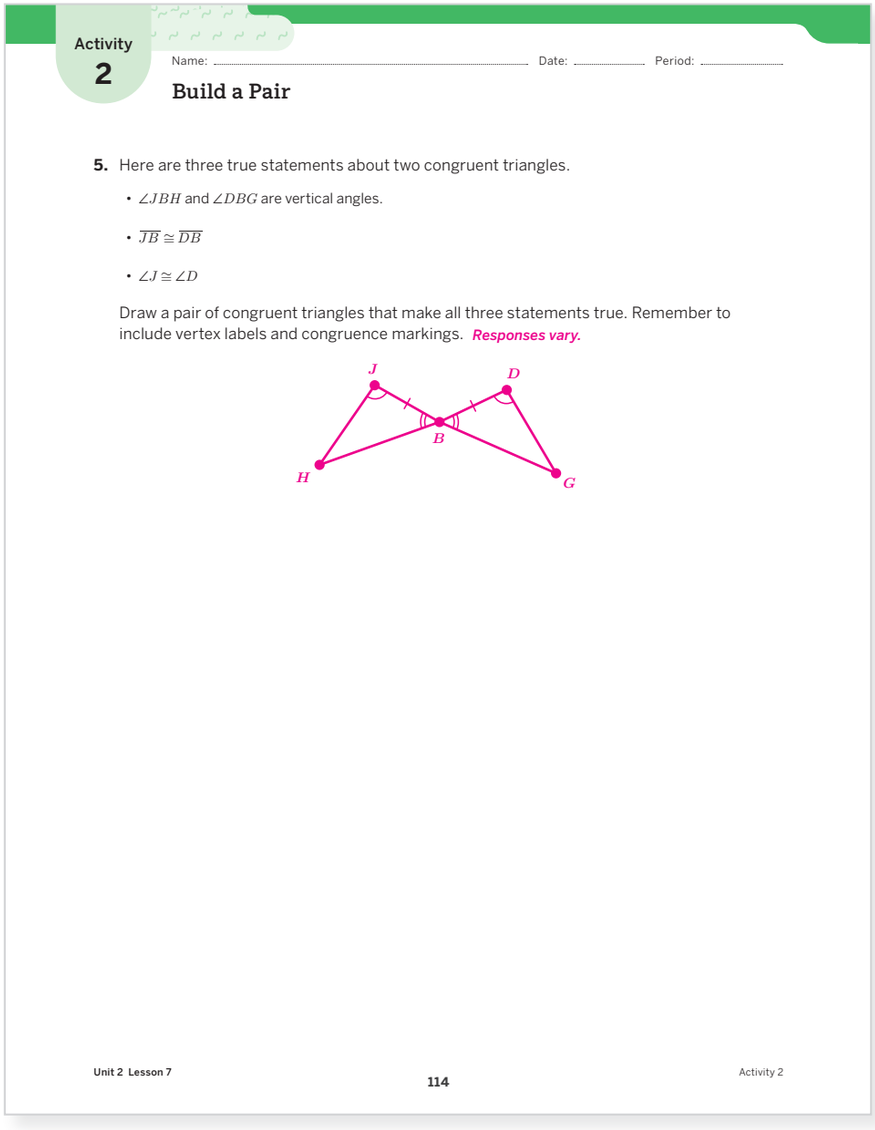
Look for students who:	Respond to Student Thinking
Sketch congruent vertical angles that are not in the corresponding order of vertices. (*Problem 5*)	**Strengthen** Encourage them to use coloring tools or tracing paper to identify corresponding parts.
Would like an additional challenge. (*Problem 6*)	**Stretch** Encourage them to continue iterating. Consider asking, "How might your figures change if you include one more statement?"

ML/EL **Multilingual/English Learners** Invite students to read each statement aloud and share what they think it means. Consider asking, "What does the $\ncong$ mean here? What does $m\angle C$ mean?" (**Reading**)

3 Connect

To support making connections, consider asking:

- "What strategies did you use to draw the diagrams precisely?" **(MP6)**
- "How do diagrams communicate which parts are corresponding and congruent? How do congruence statements communicate the same information?"



Activity 2

Name: _____ Date: _____ Period: _____

Build a Pair

5. Here are three true statements about two congruent triangles.

- $\angle JBH$ and $\angle DBG$ are vertical angles.
- $\overline{JB} \cong \overline{DB}$
- $\angle J \cong \angle D$

Draw a pair of congruent triangles that make all three statements true. Remember to include vertex labels and congruence markings. *Responses vary.*

Unit 2 Lesson 7 114 Activity 2

Whole Class | 5 min

Synthesis

Lesson 7
Synthesis

Presentation Screen
5

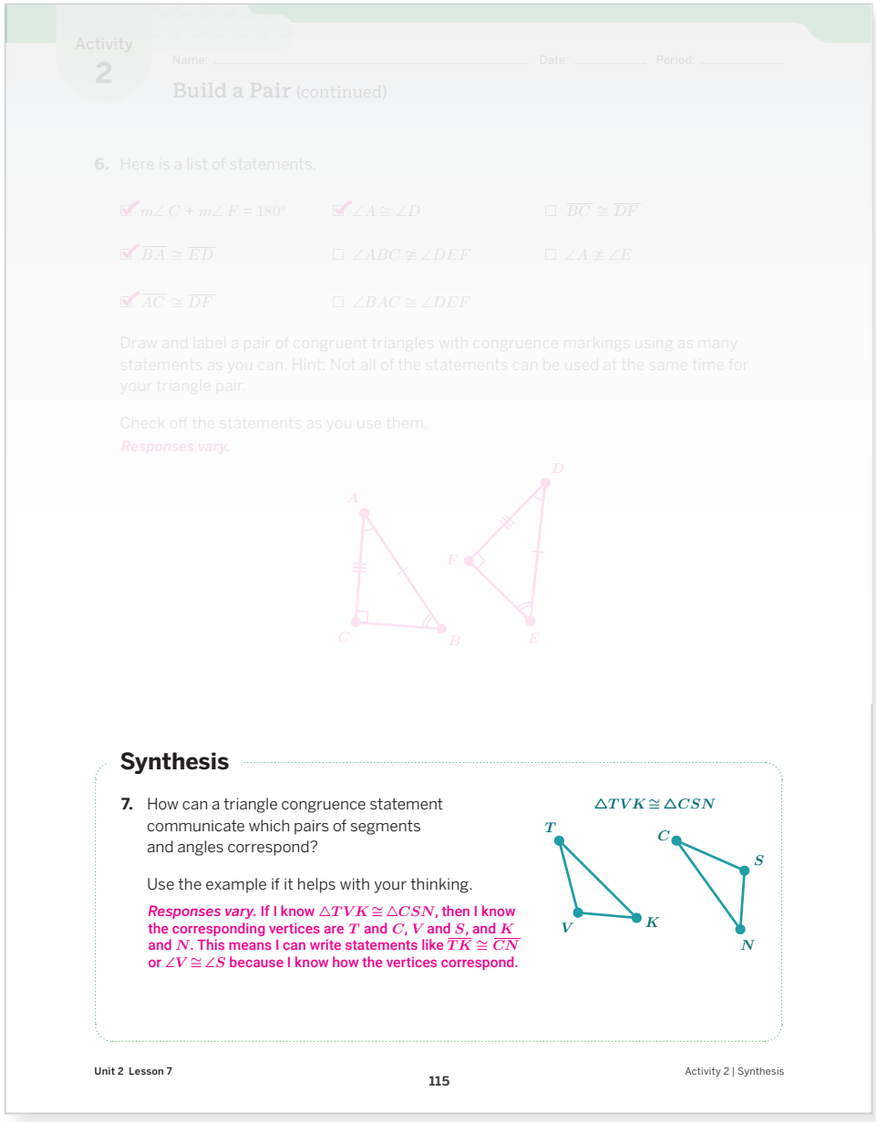
5 **Invite students** to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Congruence statements communicate which vertices correspond in congruent figures.
- The order of vertices in triangle congruence statements tells a story about the sequence of rigid transformations that will stack the triangles perfectly.
- Congruence markings show which parts are corresponding and congruent in two figures.
- Being precise when reading and writing congruence statements helps communicate clearly which parts correspond between two figures.

MLR **MLR2: Collect and Display** Use this routine to collect additional student ideas and language about identifying corresponding parts to decide if two figures are congruent. Revisit the anchor chart from the previous lesson that includes congruence markings, notation, and rigid transformations, to support students' thinking.

Math Identity and Community Consider acknowledging how students used their creativity in this lesson. Encourage students to celebrate a classmate and share how their classmate's creativity helped with their understanding.



Activity 2

Name: Date: Period:

Build a Pair (continued)

6. Here is a list of statements.

- ☑ $m\angle C + m\angle F = 180°$
- ☑ $\angle A \cong \angle D$
- ☐ $\overline{BC} \cong \overline{DF}$
- ☑ $\overline{BA} \cong \overline{ED}$
- ☐ $\angle ABC \ncong \angle DEF$
- ☐ $\angle A \ncong \angle E$
- ☑ $\overline{AC} \cong \overline{DF}$
- ☐ $\angle BAC \cong \angle DEF$

Draw and label a pair of congruent triangles with congruence markings using as many statements as you can. Hint: Not all of the statements can be used at the same time for your triangle pair.

Check off the statements as you use them.

Responses vary.

Synthesis

7. How can a triangle congruence statement communicate which pairs of segments and angles correspond?

Use the example if it helps with your thinking.

Responses vary. If I know $\triangle TVK \cong \triangle CSN$, then I know the corresponding vertices are T and C, V and S, and K and N. This means I can write statements like $\overline{TK} \cong \overline{CN}$ or $\angle V \cong \angle S$ because I know how the vertices correspond.

$\triangle TVK \cong \triangle CSN$

Unit 2 Lesson 7 115 Activity 2 | Synthesis

Lesson Takeaway: The congruent parts that are in the same place on a pair of congruent figures are called corresponding parts. There are multiple ways to identify corresponding parts in figures:

- Using rigid transformations to stack the figures and their corresponding vertices.
- Reading congruence markings that show which parts correspond in each figure.
- Interpreting congruence statements precisely to know which vertices stack with each other.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by labeling a diagram with congruence markings to represent corresponding congruent parts of congruent triangles.

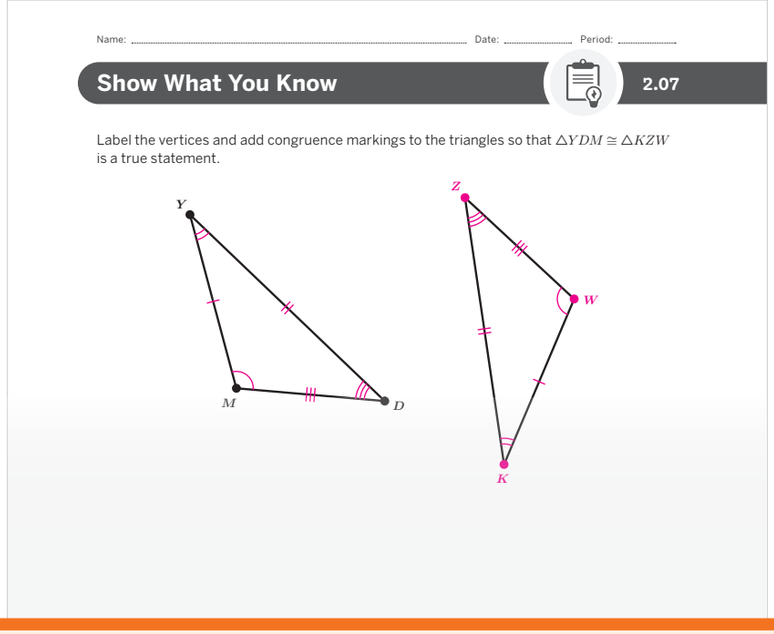


Presentation Screen 6

Today's Goals

1. **Goal:** Use congruence statements to identify and describe corresponding parts in congruent figures.
2. **Goal:** Draw a figure when given a sequence of transformations represented by a congruence statement.
3. **Language Goal:** Describe a sequence of transformations that stacks corresponding vertices in congruent figures. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 2.07

Label the vertices and add congruence markings to the triangles so that $\triangle YDM \cong \triangle KZW$ is a true statement.

Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of labeling corresponding congruent parts in congruent triangles, consider:

- Inviting students to complete the **Try This** problem and providing individualized feedback.
- Giving individualized feedback on a few related **practice problems**.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice**.
- Inviting students to review the **Summary** and complete the **Try This** problem.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting them to write their own problem that is similar to **Problem 6**.

Professional Learning

Take a moment to reflect on how students attended to precision in today's lesson. What evidence did students give that they were engaged in communicating their ideas precisely? **(MP6)**

Lesson Practice

Independent

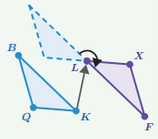
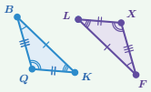
Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 7
Practice

Students using digital

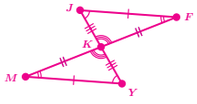
Lesson Summary

The congruent parts that are in the same position and orientation within congruent figures are called *corresponding parts*. You can identify them in multiple ways:

Transformations	Diagrams	Congruence Statements
		 $\triangle BQK \cong \triangle FXL$
The sequence of transformations shows a translation that stacks K on L and a rotation that stacks Q on X and B on F. Since $\overline{BQ}$ stacks perfectly on $\overline{FX}$, $\overline{BQ} \cong \overline{FX}$.	The markings on the pair of triangles show which parts are congruent and corresponding. Since $\angle Q$ and $\angle X$ are both marked with a double arc, $\angle Q \cong \angle X$.	The order in which vertices appear in triangle congruence statements shows which vertices correspond in two figures. Since K and L are both the third letter in the triangle congruence statement, $\angle K \cong \angle L$.

Try This

Draw and label two triangles so that $\triangle YMK \cong \triangle JFK$. Remember to include congruence markings. ***Responses vary.***



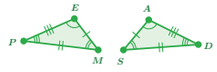
Unit 2 Lesson 7 116 Lesson Summary | Try This

Lesson Practice
Geo.2.07

Name: _____ Date: _____ Period: _____

Problems 1–2: Here are two triangles.

1. Which rigid transformation stacks one triangle on the other perfectly?
   Responses vary. Reflection



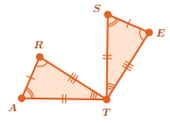
2. What are two possible triangle congruence statements for this pair? ***Responses vary.***

   △ *PEM* ≅ △ *DAS*

   △ *MEP* ≅ △ *SAD*

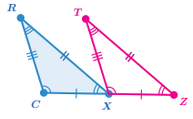
Problems 3–5: Use the triangle pair to determine whether the statements are true or false. Circle one.

3. $\overline{RT} \cong \overline{ST}$ True (False)
4. $\triangle ART \cong \triangle SET$ (True) False
5. $\angle ATR \cong \angle ETS$ (True) False



Problems 6–7: Here is $\triangle CXR$.

6. Translate the triangle until point C goes to point X and label the image so $\triangle CXR \cong \triangle XZT$.



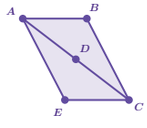
7. Add congruence markings that make the triangle congruence statement true.

Unit 2 Lesson 7 117 Lesson Practice

Lesson Practice
Geo.2.07

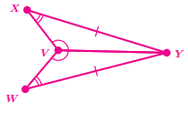
Name: _____ Date: _____ Period: _____

8. Triangle CEA is the image of triangle ABC after a 180° rotation around point D. Select *all* statements that must be true.



- ☐ A. $\triangle ABC \cong \triangle AEC$
- ☐ B. $\angle BAC \cong \angle EAC$
- ☑ C. $\triangle BAC \cong \triangle ECA$
- ☐ D. $\overline{AE} \cong \overline{AB}$
- ☑ E. $\angle EAC \cong \angle BCA$
- ☑ F. $\overline{EC} \cong \overline{BA}$

9. Draw a pair of congruent triangles that make all three statements true. Remember to include vertex labels and congruence markings. ***Responses vary.***
   - $\angle X \cong \angle W$
   - $\angle XVY \cong \angle WVY$
   - $\overline{XY} \cong \overline{WY}$

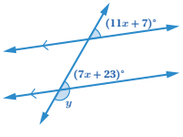


Spiral Review

Problems 10–11: Determine the value of each variable.

10. x = 4

11. y = 129



Reflection

1. Put a star next to the problem you spent the most time on.
2. Use this space to ask a question or share something you're proud of.

Unit 2 Lesson 7 118 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1	1	8.G.A.2
	2–5	2	HSG.CO.B.7
	6, 8	2	HSG.CO.A.5
	7	1	HSG.CO.B.7
	9	3	HSG.CO.A.5
Spiral Review			
Fluency / Test Practice	10–11	2	HSG.CO.C.9, HSA.REI.B.3

Need More Practice?

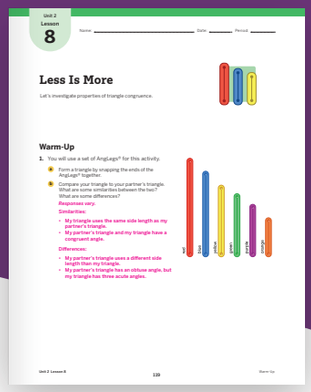
Additional Practice resources coming soon.

Unit 2

Lesson

8

This lesson uses print Student Edition pages and digital Teacher Presentation Screens.



Less Is More

Exploring Requirements for Triangle Congruence

Let's investigate properties of triangle congruence.

Focus and Coherence

Today's Goals

1. **Goal:** Explore the combinations of sides and angles that always, sometimes, or never create congruent triangles.
2. **Goal:** Generate and investigate congruence claims using rigid transformations.
3. **Language Goal:** Analyze pairs of corresponding congruent parts to determine if two triangles are congruent. **(Writing, Speaking, and Listening)**

Students investigate the minimum amount of information needed to prove that two triangles are congruent without having to confirm the congruence of all six corresponding parts. They use repeated reasoning to identify combinations of sides and angles that will always produce congruent triangles. **(MP8)**

Note: Students can use AngLegs®, an AngLegs® Sheet, or an AngLegs® Digital Manipulative to investigate properties of triangle congruence.

Prior Learning

In Lesson 5, students learned that two figures are congruent when they stack perfectly through a sequence of rigid transformations. In Lessons 6–7, students made sense of how two figures are congruent if and only if all of the corresponding sides and angles are also congruent.

Future Learning

In Lesson 9, students will solidify their understanding of ASA, SAS, and SSS congruence theorems, which always produce congruent triangles. They will investigate why two sides and an adjacent angle do not always create congruent triangles.

Rigor and Balance

- Students develop **conceptual understanding** by using different combinations of sides and angles to build congruent triangles.
- Students develop **adaptive reasoning** as they analyze different combinations of triangle parts that create congruent triangles.

Standards

Addressing

HSG.CO.B.7

Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

HSG.CO.B.8

Understand congruence in terms of rigid motions. Explain how the criteria for triangle congruence (ASA, SAS, and **SSS) follow from the definition of congruence in terms of rigid motions.**

Mathematical Practices: MP3, MP8

Building On	Building Toward
HSG.CO.B.7	**HSG.CO.B.8**
HSG.CO.B.6	**HSG.CO.C.10**

Lesson at a Glance

~ 45 min

Standards: HSG.CO.B.7, HSG.CO.B.8

Warm-Up

Pairs | 5 min

Students are introduced to the manipulatives they will use to build and compare triangles throughout the lesson.





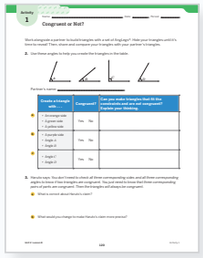
Presentation Screen 1

Activity 1

Pairs | 15 min

Students investigate the minimum amount of information needed to prove that two triangles are congruent. MLR3: Critique, Correct, Clarify

Key Takeaway: It is possible to prove two triangles are congruent without confirming that all three corresponding sides and all three corresponding angles are congruent. See the Activity page for full Key Takeaway.





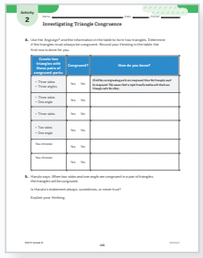
Presentation Screen 2

Activity 2

Pairs | 15 min

Students use repeated reasoning to generalize which combinations of angles and sides will always create congruent triangles.
Routine: Decide and Defend

Key Takeaway: Three pairs of corresponding congruent sides will *always* create congruent triangles. Two pairs of corresponding congruent sides and a pair of corresponding congruent angles will *sometimes* create congruent triangles.





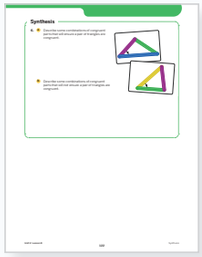
Presentation Screens 3–4

Synthesis

Whole Class | 5 min

Students synthesize their understanding of the combinations of sides and angles that will always create congruent triangles.

Lesson Takeaway: Combinations of sides and angles depend on their location to ensure congruence. Three pairs of congruent sides always produce congruent triangles.
See the Synthesis page for full Lesson Takeaway.





Presentation Screen 5

Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining and justifying if two different triangles can be created with three specific side lengths.





Presentation Screen 6

Prep Checklist

Invite students to use their Student Edition. Display the Presentation Screens to help facilitate the lesson.

This lesson includes:

- Presentation Screens
- Student Edition
- Show What You Know
- AngLegs®, AngLegs® Sheet, or AngLegs® Digital Manipulative
- tracing paper
- coloring tools (optional)

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to the manipulatives they will use to build and compare triangles throughout the lesson.



Presentation Screen 1

1 Launch

Display images of the AngLegs® manipulatives and invite students to make sense of the different pieces by size and color.

A **Accessibility: Visual-Spatial Processing**
To support students who are color-blind in differentiating the AngLegs®, consider reviewing the labels on Screen 1. Encourage students to make a connection between the colors of the AngLegs® and their relative sizes.

Note: Each pair of students need either one set of AngLegs®, a copy of the AngLegs® Sheet, or access to the AngLegs® Digital Manipulative.

To encourage playfulness, invite students to experiment with the manipulatives for several minutes, to create different shapes. Then, invite students to create a triangle that they think is unique or interesting.

Note: For students using AngLegs®, encourage them to snap together different sides and make sense of the rigidity created with different segments.

2 Connect

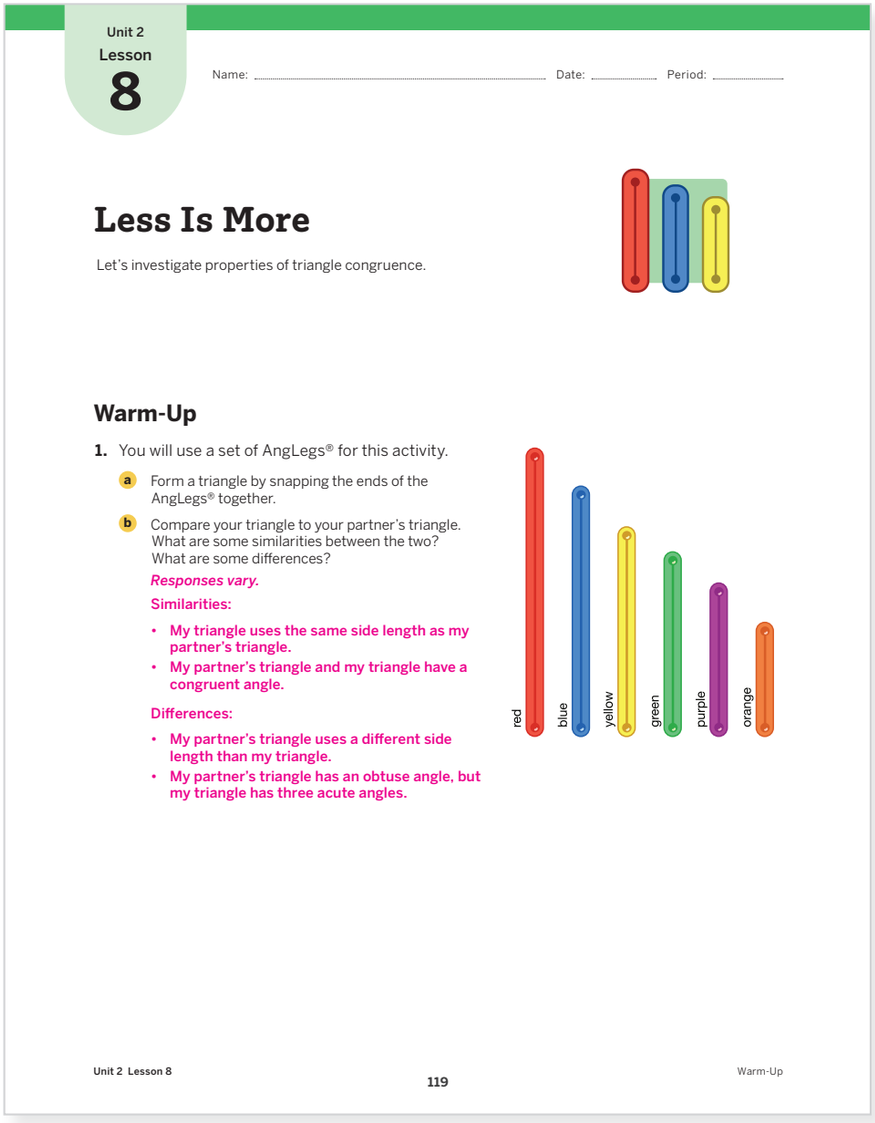
Invite several students to share examples of different triangles they made and how their triangles compared to their partners' triangles.

To support making connections, consider asking:

- "What is interesting to you about your triangle? What makes it unique?"
- "Did anyone create the exact same triangle? How do you know?

ML/EL **Multilingual/English Learners** Invite students to describe how one angle is positioned relative to the sides. Provide a word bank to support students in expressing their ideas with words such as: *adjacent*, *between*, *across from*, *next to*, etc. **(Writing and Speaking)**

Math Identity and Community Consider celebrating students who ask clarifying questions.



Unit 2 Lesson 8

Name: _____ Date: _____ Period: _____

Less Is More

Let's investigate properties of triangle congruence.

Warm-Up

1. You will use a set of AngLegs® for this activity.
   a. Form a triangle by snapping the ends of the AngLegs® together.
   b. Compare your triangle to your partner's triangle. What are some similarities between the two? What are some differences?

   Responses vary.

   Similarities:
   - My triangle uses the same side length as my partner's triangle.
   - My partner's triangle and my triangle have a congruent angle.

   Differences:
   - My partner's triangle uses a different side length than my triangle.
   - My partner's triangle has an obtuse angle, but my triangle has three acute angles.

Unit 2 Lesson 8 119 Warm-Up

Pairs | 15 min

Activity 1 Congruent or Not?

Purpose: Students investigate the minimum information needed to prove that two triangles are congruent.

Short on time? Consider omitting Problem 2c.

Lesson 8
Activity 1

Presentation Screen
2

1 Launch

Display the directions and invite students to create a triangle using a purple AngLeg®, angle B, and another AngLeg® of their choice.

Demonstrate how to create an angle that is congruent to angle B using segments. Encourage students to use tracing paper if it helps them with their thinking.

To generate excitement, tell students to keep their triangles hidden until it is time to reveal them.

To support making connections, consider asking:
- "Is it possible to create a different triangle with these parts? How do you know?"
- "If your triangle and your partner's triangle are the same, how can you show that they are congruent?" I can stack them using rigid transformations.

Consider sharing with students that they will go through the same "build, reveal, and compare" process using different combinations of parts.

2 Monitor

Select and sequence explanations to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Notice that the location of the angle creates different triangles.	**Strengthen** Invite them to share in the Connect. Encourage them to use terms like *adjacent* and *between* to discuss the angle's location.
Identify that three sides will always produce congruent triangles.	**Stretch** Invite them to share during the Connect. Consider asking, "Are there other combinations that will always produce congruent triangles?"

3 Connect

Share the selected student explanations.

MLR3: Critique, Correct, Clarify Use this routine to help students describe strengths, errors, and ambiguities in the claim. Invite students to share their responses from the activity. **(MP3)**

To surface the Key Takeaway, consider asking, "What would a counterexample to Haruto's claim look like?"

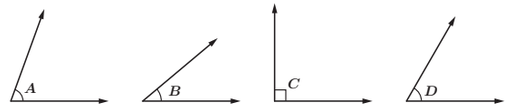
Activity 1

Name: _____ Date: _____ Period: _____

Congruent or Not?

Work alongside a partner to build triangles with a set of AngLegs®. Hide your triangles until it's time to reveal! Then, share and compare your triangles with your partner's triangles.

2. Use these angles to help you create the triangles in the table.



Partner's name: _____

	Create a triangle with . . .	Congruent?	Can you make triangles that fit the constraints and are *not* congruent? Explain your thinking.
a	• An orange side • A green side • A yellow side	(Yes) No	No. *Responses vary.* These triangles must be congruent because every triangle with these sides stacks perfectly on top of each other.
b	• A purple side • Angle A • Angle B	Yes (No)	Yes. *Responses vary.* I can make the triangles not congruent if I put the purple side between the two given angles or adjacent to the two angles.
c	• Angle C • Angle D	Yes (No)	Yes. *Responses vary.* I can make the triangles not congruent if I use different-sized AngLegs®.

3. Haruto says: *You don't need to check all three corresponding sides and all three corresponding angles to know if two triangles are congruent. You just need to know that three corresponding pairs of parts are congruent. Then the triangles will always be congruent.*

a. What is correct about Haruto's claim?
Responses vary. Haruto is right about not needing to check all three sides and all three angles in both triangles to know the triangles will be congruent.

b. What would you change to make Haruto's claim more precise?
Responses vary. Some combinations of three pairs of congruent parts will make the triangles congruent. For example, triangles with three congruent sides are always congruent. But the triangles with one congruent side and two congruent angles are not always congruent.

Unit 2 Lesson 8 120 Activity 1

Key Takeaway: Congruent triangles have six pairs of corresponding congruent parts. It is possible to prove two triangles congruent with fewer than all three corresponding sides and all three corresponding angles being congruent, but not every combination works.

Pairs | 15 min

Activity 2 Investigating Triangle Congruence

Purpose: Students use repeated reasoning to generalize which combinations of angles and sides will always create congruent triangles.

Short on time? Consider building the triangles for Problems 2e and 2f as a whole class.

Lesson 8
Activity 2

Presentation Screens
3–4

1 Launch

3

Display the two triangles shown. Consider asking:

- "What parts of these triangles are congruent? Do the congruent parts correspond?"
- "How can you prove that these two triangles are not congruent?" **(MP3)**

Invite students to use tracing paper or coloring tools to support their thinking as they identify the corresponding parts.

2 Monitor

4

Display the directions for Activity 2 and encourage students to make sense of the first example before exploring the rest of the table with their partner. Students should complete in consecutive order to support their discovery of congruence relationships.

Note: Students will learn the names for the triangle congruence relationships (e.g., ASA, SSS, SAS) in Lessons 9–10.

Listen for the different ways that students justify whether two triangles will always be congruent.

D Differentiation *(Problem 4)*

Look for students who:	Respond to Student Thinking
Are not sure if three pairs of congruent sides and a pair of congruent angles will always create congruent triangles. (*Problem 4*)	**Support** Encourage them to build triangles using their manipulatives. Consider asking, "Will these triangles always stack perfectly? How do you know?"
Think that two pairs of congruent sides and a pair of congruent angles will always create congruent triangles. (*Problem 4*)	**Strengthen** Encourage them to try and create triangles that are not congruent. Consider asking, "How does changing the location of the congruent angle change the triangle?"

3 Connect

Use the Decide and Defend routine to support students in critiquing Haruto's reasoning. **(MP3)**

To surface the Key Takeaway, consider asking:

- "What evidence supports your claim?"
- "How can we know if two sides and an angle will always create congruent triangles?" **(MP8)**
- "Why does it matter that the congruent parts are in corresponding positions?"

Activity 2

Name: _____ Date: _____ Period: _____

Investigating Triangle Congruence

4. Use the AngLegs® and the information in the table to form two triangles. Determine if the triangles must always be congruent. Record your thinking in the table; the first row is done for you.

Create two triangles with these pairs of congruent parts:	Congruent?	How do you know?
• Three sides • Three angles	(Yes) No	If all the corresponding parts are congruent, then the triangles must be congruent. This means that a rigid transformation will stack one triangle onto the other.
• Three sides • One angle	(Yes) No	*Responses vary.* If I have three congruent pairs of sides, all of the corresponding angles of the triangles are always congruent.
• Three sides	(Yes) No	*Responses vary.* When I make a triangle with three sides, I find there's no other triangle that can be made with the same three sides.
• Two sides • One angle	Yes (No)	*Responses vary.* First, I made a triangle with purple, orange, and green AngLegs®. Then, I made a different triangle by replacing the orange side with another green side while keeping the same angle.
You choose: • Three angles	Yes (No)	*Responses vary.* I can create similar triangles with three pairs of angles, but they don't need to be congruent triangles.
You choose: • Two sides • Two angles	(Yes) No	*Responses vary.* If I have two congruent pairs of angles, the third angle must also be congruent. Since two pairs of sides are also congruent, there's only one length the third side can be.

5. Haruto says: *When two sides and one angle are congruent in a pair of triangles, the triangles will be congruent.*

Is Haruto's statement *always*, *sometimes*, or *never* true? Sometimes true.

Explain your thinking.

Responses vary. What Haruto said is sometimes true because a pair of triangles are always congruent if the angle is between two adjacent sides. But if the angle is not between two adjacent sides, the triangles can't always be congruent.

Unit 2 Lesson 8 121 Activity 2

Key Takeaway: Three pairs of corresponding congruent sides will *always* create congruent triangles. Two pairs of corresponding congruent sides and a pair of corresponding congruent angles will *sometimes* create congruent triangles.

Whole Class | 5 min

Synthesis

Lesson 8
Synthesis

Presentation Screen
5

5 **Invite students** to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Congruence of all three sides and all three angles will always create congruent triangles.
- Three pairs of corresponding congruent sides will always create congruent triangles.
- If the combination has two sides and one angle, then the location of the angle matters.
- Two pairs of corresponding congruent sides and a pair of corresponding congruent angles will always create congruent triangles when the angle is located between the two sides, but not when the angle is adjacent to the two sides.
- Two pairs of congruent angles and one pair of congruent sides will always create congruent triangles if the side is located between or adjacent to the two angles.

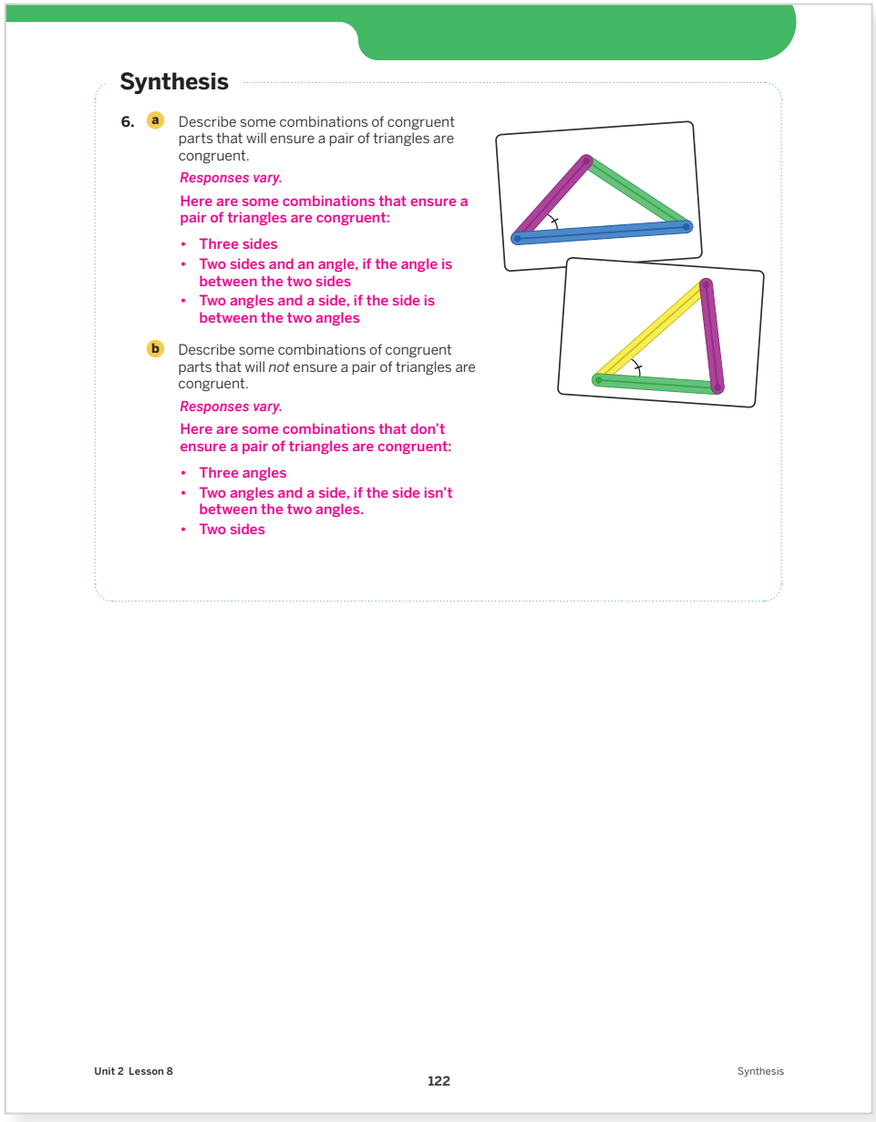
Consider asking:

- "How did the AngLegs® help you better understand what it means for a transformation to be rigid?"
- "How do rigid transformations help prove congruence?"

Math Identity and Community Remind students that confusion is a part of learning. Thank the students who asked questions for supporting the class's collective learning.

Create an anchor chart with the combinations of sides and angles that *always* create congruent triangles for students to reference and build on for the rest of the unit.

Note: If there is time, invite students to reflect on and revise their thinking in Problem 4.



Synthesis

6. a. Describe some combinations of congruent parts that will ensure a pair of triangles are congruent.

Responses vary.

Here are some combinations that ensure a pair of triangles are congruent:

- **Three sides**
- **Two sides and an angle, if the angle is between the two sides**
- **Two angles and a side, if the side is between the two angles**

b. Describe some combinations of congruent parts that will *not* ensure a pair of triangles are congruent.

Responses vary.

Here are some combinations that don't ensure a pair of triangles are congruent:

- **Three angles**
- **Two angles and a side, if the side isn't between the two angles.**
- **Two sides**

Lesson Takeaway: Congruent triangles have three pairs of corresponding, congruent sides and three pairs of corresponding congruent angles. But, it is possible to prove that the triangles are congruent with a smaller combination of sides and angles. Three pairs of congruent sides will always produce two congruent triangles, but combinations of sides and angles will depend on the location of angles relative to the sides.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining and justifying if a different triangle can be created by three specific side lengths.

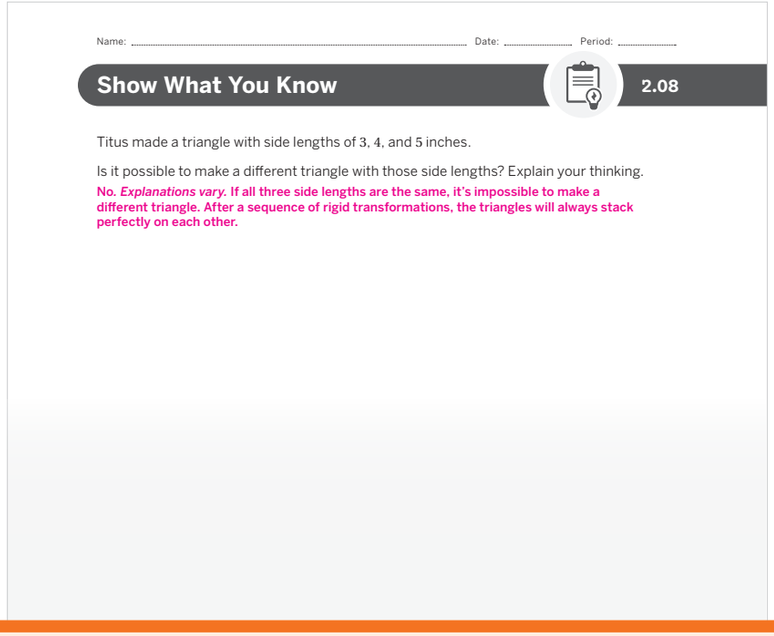


Presentation Screen 6

Today's Goals

1. **Goal:** Explore the combinations of sides and angles that always, sometimes, or never create congruent triangles.
2. **Goal:** Generate and investigate congruence claims using rigid transformations.
3. **Language Goal:** Analyze pairs of corresponding congruent parts to determine if two triangles are congruent. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 2.08

Titus made a triangle with side lengths of 3, 4, and 5 inches.

Is it possible to make a different triangle with those side lengths? Explain your thinking.

No. *Explanations vary.* If all three side lengths are the same, it's impossible to make a different triangle. After a sequence of rigid transformations, the triangles will always stack perfectly on each other.

Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of congruence, consider:

- Inviting students to complete the **Try This** problem and providing individualized feedback.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice**.
- Inviting students to review the **Summary** and complete the **Try** This problem.
- Emphasizing the triangle congruence theorems in **Lesson 9**.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting them to think of more combinations of sides and angles that will always create congruent triangles.
- Inviting them to apply their thinking from this lesson to quadrilaterals.

Professional Learning

In what ways are students taking risks when they try new strategies to determine the conditions for triangle congruence? What questions could encourage students to move beyond exploration and to start establishing convincing criteria in Lessons 9–10?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

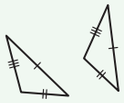
Lesson 8
Practice

Students using digital

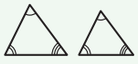
Lesson Summary

If you compare two triangles and every corresponding pair of sides and angles is congruent, then you know the two triangles are congruent. Sometimes, comparing just a few corresponding parts is enough to determine if two triangles are congruent.

Having three pairs of corresponding congruent sides means the triangles will always be congruent.



But there are exceptions to keep in mind! For example, having three pairs of corresponding congruent angles does not mean that the triangles will always be congruent.



Try This

Azul made two triangles with three pairs of corresponding congruent angles.

Are these triangles always congruent? Explain your thinking.

No. *Responses vary.* The triangles can have three pairs of congruent corresponding angles, but one of the triangles can be bigger than the other. This means they're not congruent.

Lesson Practice
Geo.2.08

Name: _____ Date: _____ Period: _____

Problems 1–2: Determine the number of sides and angles that . . .

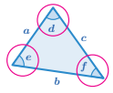
1. *Always* ensures a pair of congruent triangles. *Responses vary.*

3 side(s) and 0 angle(s)

2. *Sometimes* ensures a pair of congruent triangles. *Responses vary.*

1 side(s) and 2 angle(s)

3. Two congruent triangles have the same congruent corresponding parts. Circle a combination of sides and angles that would *not* ensure that the triangles are congruent. *Responses vary.*



4. What is the minimum number of pairs of congruent sides that ensures two triangles are congruent, without considering any angles?

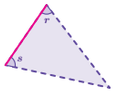
A. 1 **B.** 2 **C.** 3 **D.** Not possible

5. What is the minimum number of pairs of congruent angles that ensures two triangles are congruent, without considering any sides?

A. 1 **B.** 2 **C.** 3 **D.** Not possible

Problems 6–7: The congruent parts *r* and *s* are given for a pair of triangles. Ensure the triangles are congruent by marking . . .

6. One congruent side.



7. One congruent angle.



Lesson Practice
Geo.2.08

Name: _____ Date: _____ Period: _____

8. Congruent parts can also be used to determine if other types of polygons are congruent.

Can four congruent sides ensure that two quadrilaterals are congruent? Show or explain your thinking.

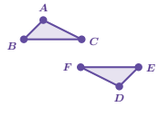
No. *Explanations vary.* Different quadrilaterals can be made depending on the order the sides are used, so there is no guarantee that SSS will make two of the same quadrilaterals.

Spiral Review

Problems 9–10: Triangle ABC is congruent to triangle DEF.

9. Determine a sequence of rigid transformations that carries triangle ABC onto triangle DEF.

Responses vary. First, translate point C onto F. Then rotate a half turn.



10. Determine a *different* sequence of rigid transformations that carries triangle ABC onto triangle DEF.

Responses vary. Start by translating point C onto E. Then reflect $\triangle ABC$ over segment BC. And then reflect $\triangle ABC$ over the perpendicular bisector of segment BC.

Reflection

1. Put a star next to a problem you're still feeling confused about.

2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-lesson			
	1–2	1	HSG.CO.B.8
	3, 6–7	2	HSG.CO.B.8
Test Practice	4–5	2	HSG.CO.B.8
	8	3	HSG.CO.B.8
Spiral Review			
Fluency	9–10	2	HSG.CO.A.5

Need More Practice?

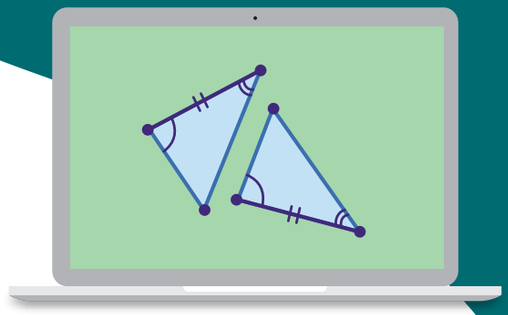
Additional Practice resources coming soon.

Unit 2

Lesson 9

Digital Lesson

This lesson uses digital student screens. Student Notes are also available.



Congruence Conditions

Criteria for Triangle Congruence

Let's get specific about criteria that proves triangle congruence.

Focus and Coherence

Today's Goals

1. **Goal:** Determine the minimum criteria needed to prove that two triangles are congruent (ASA, SAS, and SSS).
2. **Language Goal:** Identify when two corresponding sides and a corresponding angle can prove triangle congruence and when they cannot. **(Reading and Writing)**

Students make sense of triangle congruence in terms of rigid transformations. They investigate which congruence conditions are sufficient to prove two triangles are congruent. Students use rigid transformations to justify why specific triangle congruence criteria will always create congruent pairs of triangles. **(MP3)**

Prior Learning

In Lesson 7, students learned that two triangles are congruent if all six corresponding congruent parts are congruent. In Lesson 8, students used AngLeg® manipulatives to explore different combinations of sides and angles that always, sometimes, or never create congruent triangles.

Future Learning

In Lesson 10, students will apply the SSS triangle congruence conditions to construct a copy of an angle. In Lesson 11, students apply triangle congruence criteria to prove other relationships within triangles are always true.

Rigor and Balance

- Students develop **conceptual understanding** of why specific triangle congruence criteria always create pairs of congruent triangles.
- Students develop **procedural fluency** as they identify which types of triangle congruence criteria are represented in different diagrams.

Standards

Addressing

HSG.CO.B.8
Understand congruence in terms of rigid motions. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

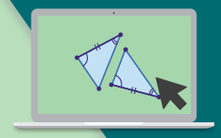
Mathematical Practices: MP1, MP3, MP5

Building On	Building Toward
8.G.A.1 **8.G.A.2** **HSG.CO.B.6** **HSG.CO.B.7**	**HSG.CO.C.10**

Lesson at a Glance

~ 45 min

Standard: HSG.CO.B.8



Why digital?

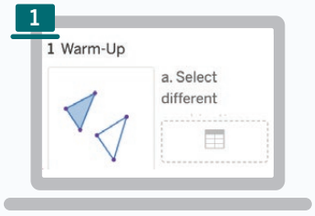
Students use digital interactions to develop conceptual understanding of which triangle congruence conditions will always create pairs of congruent triangles.

Warm-Up

Independent | 5 min

Pacing: Screen 1

Students create different combinations of congruent parts and make sense of what happens between a pair of triangles. **(MP1)**



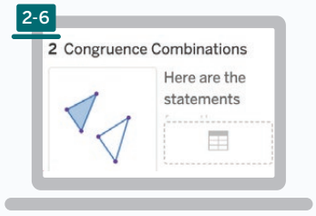
Activity 1

Pairs Sharing a Device | 20 min

Screens 2–3, Screens 4–5, Screen 6

Students use rigid transformations to construct arguments that justify why certain congruence criteria always result in congruent triangles. **(MP3)**

Key Takeaway (Screen 6): Congruent parts must correspond in a precise order between two triangles, and only some combinations ensure triangle congruence. ASA, SAS, and SSS congruence always create congruent triangles.
See the Activity page for full Key Takeaway.



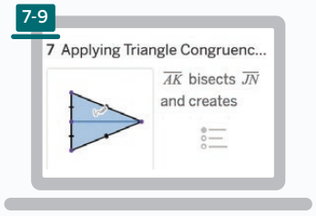
Activity 2

Pairs Sharing a Device | 10 min

Pacing: Screens 7–9

Students develop fluency to determine how triangle congruence criteria can be applied to different triangle pairs.

Key Takeaway (Screen 9): The congruence of two sides and an adjacent angle (side-side-angle) does not always provide conditions that ensure triangle congruence. As a result, SSA is not congruence criteria.



Synthesis

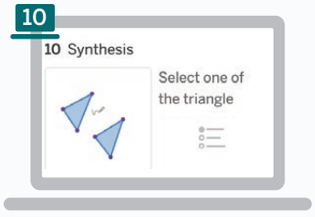
Whole Class | 5 min

Pacing: Screen 10

Students synthesize their understanding of which congruence criteria will always create congruent triangles.

Lesson Takeaway: Two triangles are congruent when they have the following pairs of corresponding congruent parts:
- Three sides (SSS)
- Two sides and an included angle (SAS)
- Two angles and an included side (ASA)

See the Synthesis page for full Lesson Takeaway.



Show What You Know

Independent | 5 min

Pacing: Screens 11–12

Students demonstrate their understanding by reasoning from a diagram which criteria can be used to prove two triangles are congruent.



Prep Checklist

Assign the digital student screens. Student Notes are also available.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)
- tracing paper (optional)

Student Notes sheet in Student Edition:
Consider providing the Student Notes to support class discussion and student thinking.



Independent | 5 min | Pacing: Screen 1

Warm-Up

Purpose: Students create different combinations of congruent parts and make sense of what happens between a pair of triangles. **(MP1)**



Student Notes are also available

1 Launch

1 **Use the Notice and Wonder routine** to promote curiosity and help students make sense of what is happening in the pair of triangles. **(MP1)**

Encourage students to drag the vertices of the triangles as they explore different combinations of congruence statements and make observations. Consider asking:

- "What happens when you select a congruence statement? What if you drag vertex E?"
- "Are the triangles congruent? How do you know?"
- "How can you tell if segments and angles are corresponding and congruent?"

Math Identity and Community Consider inviting students to slow down on this screen. Remind them that mathematics is not about speed, but about thinking deeply.

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to make sense of what happens with the triangles, one congruence statement at a time.

2 Connect

To support making connections, consider asking:

- "What did you notice? What did you wonder?"
- "What does it mean if both triangles are shaded in?"
- "How can you tell if your triangles are congruent?"
- "Were there any conditions that didn't make congruent triangles?"

Demonstrate a specific example using the dashboard's Student View. Select the congruence statements: $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\angle B \cong \angle E$. Drag point E until two different triangles are constructed. Consider asking, "Why doesn't this combination of congruent parts always create congruent triangles?"

Note: Students were introduced to conditions for triangle congruence in the previous lesson. By the end of this lesson, students will learn the abbreviated terms for specific criteria that prove triangle congruence (ASA, SAS, and SSS congruence). They will also learn that SSA congruence does not always create congruent triangles.

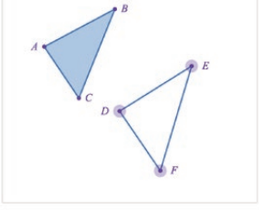
Students using digital

1

Warm-Up

a. Select different combinations of congruence statements. Drag the points.

$\overline{AB} \cong \overline{DE}$	$\overline{AC} \cong \overline{DF}$	$\overline{BC} \cong \overline{EF}$
$\angle A \cong \angle D$	$\angle B \cong \angle E$	$\angle C \cong \angle F$



b. What do you notice?

Responses vary.

- I notice the triangles are shaded when they're congruent.
- I notice $\triangle ABC$ and $\triangle DEF$ aren't congruent if I pick two pairs of congruent sides.
- I notice I can make different triangles even if three pairs of corresponding angles are congruent.

Pairs Sharing a Device | 20 min | Pacing: Screens 2–3, Screens 4–5, Screen 6

Activity 1 Which Combinations?

Purpose: Students use rigid transformations to construct arguments that justify why certain congruence criteria always result in congruent triangles. **(MP3)**



Student Notes are also available

1 Launch

2 **Invite students** to investigate which combinations of congruent parts will always create congruent triangles.

Consider sharing that students can refresh the screen to explore a different combination of congruence statements.

Look for responses that include three pairs of sides, two pairs of sides, and one pair of angles, or two pairs of angles and one pair of sides using the dashboard's Teacher View.

2 Monitor

3 ML/EL **Multilingual/English Learners** Invite students to focus on the word "between." Consider asking, "How can you tell that the pair of congruent sides is *between* the two congruent angles?" **(Reading)**

A **Accessibility: Visual-Spatial Processing** Invite students to pause the video several times and describe what is happening at each step to support interpreting visual representations.

Pause briefly to discuss how the rigid transformations prove that all three pairs of vertices will stack onto one another. Consider asking, "Is there a situation where Kwasi's conjecture will not be true?"

Encourage students to go back to Screen 2 and test their conjectures. **(MP3)**

Invite students using notes to record some important considerations from the class discussion.

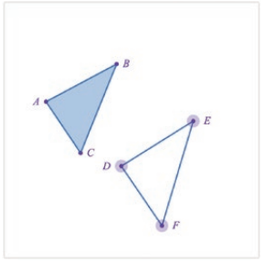
Invite students to continue to Screens 4 and 5 using the Pace tool.

4 **Listen for** strategies that students use to determine if Chey's conjecture is true. *Responses vary.* I noticed that this combination of congruence statements locks the triangle in a way that won't let me change the side lengths to create a different shape.

Students using digital

2

Congruence Combinations



Here are the statements from the previous screen.

$\overline{AB} \cong \overline{DE}$	$\overline{AC} \cong \overline{DF}$	$\overline{BC} \cong \overline{EF}$
$\angle A \cong \angle D$	$\angle B \cong \angle E$	$\angle C \cong \angle F$

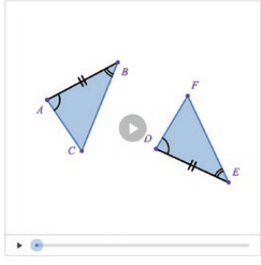
Select *all* the combinations that ensure congruent triangles.

- ☐ Two pairs of sides.
- ☑ Three pairs of sides.
- ☐ Three pairs of angles.
- ☑ Two pairs of sides and one pair of angles.
- ☑ Two pairs of angles and one pair of sides.

For "two pairs of sides and one pair of angles," students may or may not select this choice because congruence is ensured only if the angle is between the two sides.

3

Kwasi's Conjecture



Kwasi made a conjecture:

Two triangles must be congruent if they have these sets of pairs of corresponding congruent parts:
- Two angles
- One side between the two angles

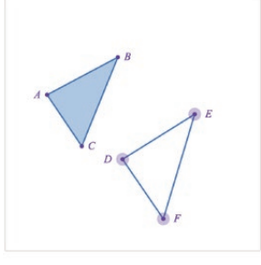
Kwasi says this will always be true because:
- Point A is translated to point D.
- Point B will move to point E because $\overline{AB} \cong \overline{DE}$.

Discuss: How do you know point C will always stack perfectly on point F?

Responses vary. If I extend the rays from $\angle A$ and $\angle B$, I know that they will only intersect in one place, point C. Because $\angle A = \angle D$ and $\angle B = \angle E$, then point F must also be in the same location on $\triangle DEF$. After reflection, point C will always stack perfectly on point F.

4

Chey's Conjecture



Chey made a new conjecture:

Two triangles must be congruent if they have these sets of pairs of corresponding congruent parts:
- Two sides
- One angle between the two sides

a. Select combinations of congruence statements. Drag the points to explore Chey's conjecture.

$\overline{AB} \cong \overline{DE}$	$\overline{AC} \cong \overline{DF}$	$\overline{BC} \cong \overline{EF}$
$\angle A \cong \angle D$	$\angle B \cong \angle E$	$\angle C \cong \angle F$

b. **Discuss:** Do you think this conjecture is *always* true? Why or why not?

a. *Selections vary.* $\overline{AB}$, $\overline{DE}$, $\overline{AC}$, $\overline{DF}$, $\angle A$, $\angle D$ (Any combination with two sides and an angle between the two sides.)

b. *Responses vary.* I think the conjecture is always true because when I drag the points I can't make a triangle that's different from $\triangle ABC$.

Pairs Sharing a Device | 20 min | Pacing: Screens 2–3, Screens 4–5, Screen 6

Activity 1 Which Combinations? (continued)

Purpose: Students use rigid transformations to construct arguments that justify why certain congruence criteria always result in congruent triangles. **(MP3)**



Student Notes are also available

2 Monitor

5 **Select and sequence** written arguments to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite them to use Kwasi's conjecture on Screen 3 to help guide their own thinking. Consider asking, "Which transformation happened first?"
Identify the sequence of rigid transformations in the animation.	**Strengthen** Invite them to attend to precision. Consider asking, "How did you know that a rotation would stack point A on point D?"
Make connections to other conditions for congruence.	**Stretch** Invite them to explore Screen 4. Consider asking, "How can we show if other combinations of corresponding congruent parts will always create congruent triangles?"

ML/EL **Multilingual/English Learners** Provide sentence frames to help students write their arguments (e.g., "I can prove the conjecture by _____. To do this, I will _____, and then I will _____.") (**Writing and Speaking**)

Pause briefly to discuss how rigid transformations prove the criteria for SAS congruence conditions.

To support making connections, consider asking:

- "A sequence of rigid transformations is one way to prove that triangles are congruent. What's another strategy?"
- "How did rigid transformations convince us that certain criteria will prove congruent triangles?" **(MP3)**

Invite students to move to Screen 6 using the Pace tool.

3 Connect

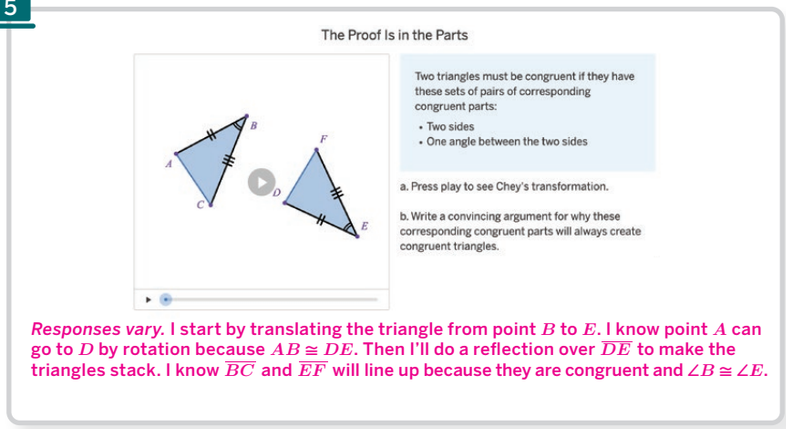
6 **Invite students** to make sense of other combinations of corresponding congruent parts that always create congruent triangles.

To surface the Key Takeaway, consider asking:

- "How can you tell if congruent parts are corresponding between two triangles?"
- "How does ASA congruence also prove that AAS congruence will always create two congruent triangles?" *Responses vary.* If I have ASA congruence, then I know the third pair of angles will also be congruent because the interior angles of a triangle add up to 180°. That means I have AAS congruence too.

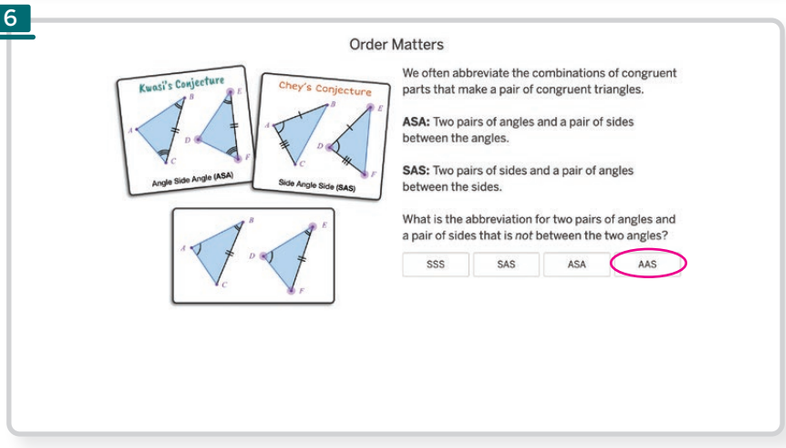
Students using digital

5



Responses vary. I start by translating the triangle from point B to E. I know point A can go to D by rotation because $AB \cong DE$. Then I'll do a reflection over $\overline{DE}$ to make the triangles stack. I know $\overline{BC}$ and $\overline{EF}$ will line up because they are congruent and $\angle B \cong \angle E$.

6



Key Takeaway: Rigid transformations prove how certain conditions for congruence always create congruent triangles, even without knowing all six corresponding parts are congruent between the two triangles. But the order of the corresponding congruent parts matters. Congruent parts must correspond in a precise order between two triangles, and only some combinations are sufficient. ASA congruence, SAS congruence, and SSS congruence conditions always create congruent triangles.

Pairs Sharing a Device | 10 min | Pacing: Screens 7–9

Activity 2 Applying Triangle Congruence Criteria

Purpose: Students develop fluency to determine how triangle congruence criteria can be applied to different triangle pairs.

Short on time? Consider facilitating Screen 8 as a whole class.



Student Notes are also available

1 Launch

7 **Invite students** to make sense of the diagram using the sketch tool.

To support making connections, consider asking:

- "How do you know that $\overline{JA}$ is congruent to $\overline{NA}$? $\overline{JK}$ is congruent to $\overline{NK}$?"
- "How do you know that $\overline{KA}$ is congruent to $\overline{KA}$?"
- "How can you determine which congruence conditions are met by a diagram?"

2 Monitor

8 **Encourage students** to share their reasoning with a partner and work together to reach an agreement about how to sort the cards.

A **Accessibility: Executive Functioning** Invite students to display and sort one card at a time by clicking on each image to enlarge the diagram.

Look for partners that do not identify $\triangle KJL \cong \triangle MNL$ as SAS criteria. Invite them to share their reasoning during the Connect on Screen 9.

9 **Invite students** to determine if the triangles are congruent using an appropriate tool of their choice. **(MP5)**

D **Differentiation: (Support)** Look for students who predict that the triangles are congruent. Invite them to use the tools to prove if all of the corresponding parts are congruent. Consider asking, "What could you use to confirm your thinking? The "circle by length" tool? The geometry tools?"

3 Connect

Math Identity and Community Consider highlighting the creative strategies that students use to show the triangles are not congruent.

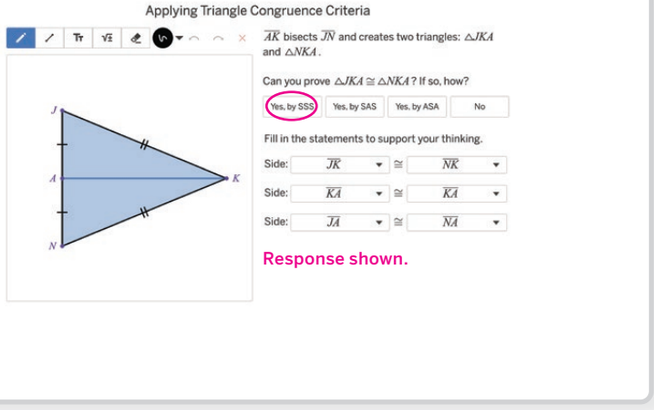
To surface the Key Takeaway, consider asking:

- "How does side-side-angle differ from the side-angle-side triangle congruence criteria?"
- "Why don't the congruence markings mean that the triangles will always be congruent?"
- "What's important to remember about a criteria with two sides and an angle that's *not* between them when you're thinking about triangle congruence?"

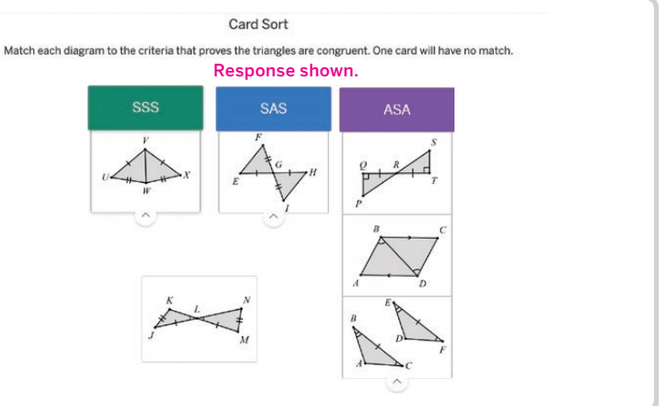
Invite students using notes to record some important considerations from the class discussion.

Students using digital

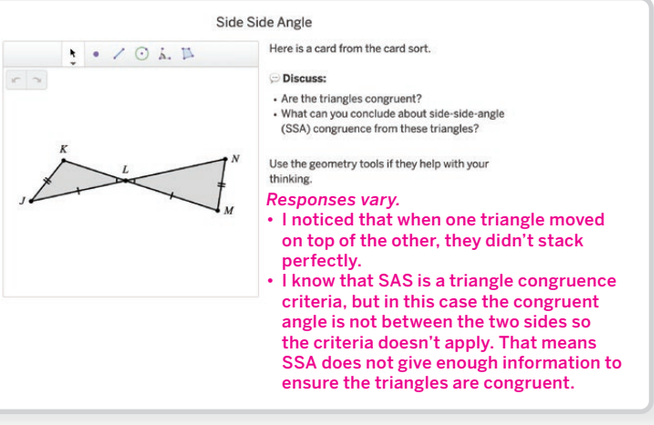
7



8



9



Key Takeaway: The congruence of two sides and an adjacent angle (side-side-angle) does not always provide conditions that ensure triangle congruence. As a result, SSA is not congruence criteria.

Whole Class | 5 min | Pacing: Screen 10

Synthesis



Student Notes are also available

10 **Invite students** to respond independently after discussing each question with their partner.

Encourage students to select a criteria that is different from their partner.

Capture and share a variety of ideas, such as:

- When there are certain congruence criteria between two triangles, there is always a sequence of rigid transformations that will carry one triangle onto the other.
- Side-side-side (SSS) congruence requires all three pairs of corresponding congruent sides to create congruent triangles.
- Side-angle-side (SAS) congruence requires two pairs of corresponding congruent sides and a corresponding congruent angle between them to create congruent triangles.
- Angle-side-angle (ASA) congruence requires two pairs of corresponding congruent angles and a corresponding congruent side between them to create congruent triangles.
- The congruent parts between the two triangles must correspond.
- Side-side-angle does not create sufficient conditions to prove a pair of triangles will always be congruent, even if the markings suggest it.

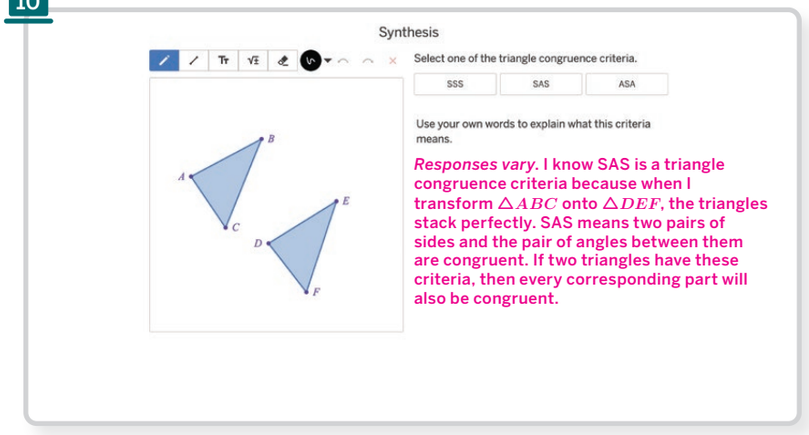
Invite students to share their ideas with another person before sharing them as a whole class.

Display the anchor chart from the previous lesson with abbreviated congruence criteria and the diagram with congruence markings for the combinations of sides and angles that *always* create congruent triangles (ASA, SAS, SSS). Display for students to reference and add to for the rest of the unit.

Math Identity and Community Invite students to celebrate a classmate's thinking that was strong, clear, or creative.

Students using digital

10



Lesson Takeaway: Two triangles are always congruent when they have the following pairs of corresponding congruent parts:

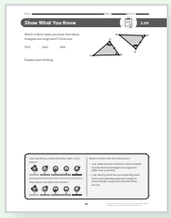
- Three sides (SSS)
- Two sides and an included angle (SAS)
- Two angles and an included side (ASA)

Using a sequence of rigid transformations to carry one triangle onto another is one way to prove that two triangles are congruent. Determining if there is a specific set of corresponding congruent parts within the pair of triangles is another strategy.

Independent | 5 min | Pacing: Screens 11–12

Show What You Know

Purpose: Students demonstrate their understanding by reasoning from a diagram which criteria can be used to prove two triangles are congruent.

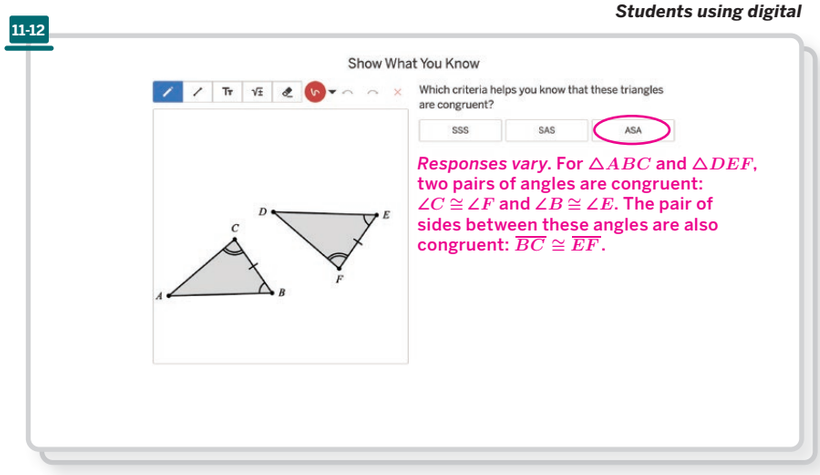


Students using print

11-12 Today's Goals

1. **Goal:** Determine the minimum criteria needed to prove that two triangles are congruent (ASA, SAS, and SSS).
2. **Language Goal:** Identify when two corresponding sides and a corresponding angle can prove triangle congruence and when they cannot. **(Reading and Writing)**

Students using digital



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of identifying conditions for congruence, consider:

- Inviting students to complete the **Try This** problem and providing individualized feedback.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice**
- Inviting students to review the **Summary** and complete the **Try This** problem.
- Emphasizing congruence criteria in **Lessons 10–11**.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting them to use the geometry tools to prove why the side-side-angle conditions do not ensure congruence.
- Encouraging them to complete the **DOK3 practice problem**.

Professional Learning

Take a moment to reflect on how students have proven congruence in different ways over the course of the unit so far. How do rigid transformations support students in making sense of congruence criteria as a new way of proving that triangles are congruent? How did identifying conditions for congruence shape the way that students can now prove triangle relationships? How are students expanding their available resources as a result of this lesson?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

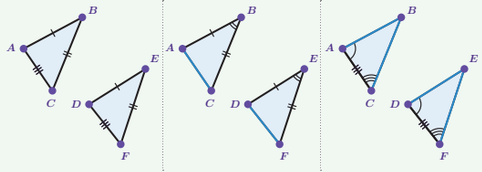
Lesson 9 Practice

Students using digital

Lesson Summary

You can use specific triangle congruence criteria to create triangles that are always congruent. Two triangles are congruent when they have these pairs of corresponding congruent parts:

SSS (side-side-side)	SAS (side-angle-side)	ASA (angle-side-angle)
Three sides	Two sides and the angle between them	Two angles and the side between them

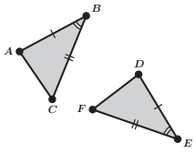


However, two corresponding congruent sides and an adjacent pair of corresponding angles (known as SSA or side-side-angle) do not always create congruent triangles.

Try This

Which criteria helps you know that these triangles are congruent? Circle one.

SSS (SAS) ASA



Explain your thinking.

Explanations vary. **For $\triangle ABC$ and $\triangle DEF$, two pairs of sides are congruent: $\overline{BC} \cong \overline{EF}$ and $\overline{AB} \cong \overline{DE}$. The pair of angles between these sides are also congruent: $\angle B \cong \angle E$.**

Unit 2 Lesson 9 128 Lesson Summary | Try This

Lesson Practice
Geo.2.09

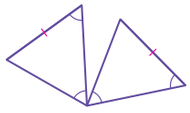
Name: _____ Date: _____ Period: _____

1. Which combination of congruent parts is *not* sufficient to know whether two triangles are congruent?

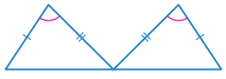
- **A.** Angle-angle-angle
- **B.** Side-side-side
- **C.** Angle-side-angle
- **D.** Side-angle-side

Problems 2–3: Mark one additional pair of congruent corresponding parts to match the given combination of congruent parts.

2. AAS ***Responses vary.***



3. SAS



Problems 4–6: Which criteria proves the triangles are congruent?

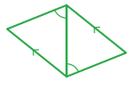
4. Circle one.

- **A.** SSS
- **B.** ASA
- **C.** SAS
- **D.** SAA



5. Circle one.

- **A.** SSS
- **B.** ASA
- **C.** SAS
- **D.** SAA



6. Circle one.

- **A.** SSS
- **B.** ASA
- **C.** SAS
- **D.** SAA

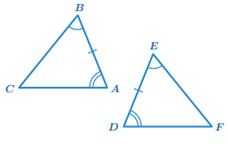


Unit 2 Lesson 9 129 Lesson Practice

Lesson Practice
Geo.2.09

Name: _____ Date: _____ Period: _____

7. Fill in the blanks to prove $\triangle ABC \cong \triangle DEF$.



- I can translate $\triangle ABC$ so that point A goes to point ___D___.
- If I rotate this image around point D, I know that point B will align with point E because ___$\overline{AB}$___ $\cong \overline{DE}$.
- I can reflect this image over $\overline{DE}$.
- I know that $\angle A$ will align with $\angle$ ___D___ and $\angle B$ will align with $\angle$ ___E___ because angle measure is preserved with rigid transformations.
- After the reflection, I know that point C will stack onto point ___F___.
- After proving all of the corresponding vertices align, I know that ___$\triangle ABC \cong \triangle DEF$___.

8. Here is a conjecture.

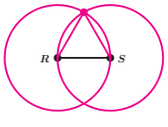
> Two triangles must be congruent if they have these pairs of corresponding congruent parts:
> - Two sides
> - One angle not between the two sides

When is this conjecture true? Circle one. Always (Sometimes) Never

Explain your thinking. ***Explanations vary.*** **If the included angle is 90° or greater, there is only one possible triangle that can be made, but if it is less than 90°, then two different triangles may be formed.**

Spiral Review

9. Construct an equilateral triangle using $\overline{RS}$ as a side. ***Responses vary.***



Reflection

1. Put a star next to a problem where you revised your thinking.
2. Use this space to ask a question or share something you're proud of.

Unit 2 Lesson 9 130 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-lesson			
Test Practice	1	1	HSG.CO.B.8
	2–7	2	HSG.CO.B.8
	8	3	HSG.CO.B.8
Spiral Review			
Fluency	9	1	HSG.CO.C.12

Need More Practice?

Additional Practice resources coming soon.

Unit 2

Lesson 10

Digital Lesson

This lesson uses digital student screens. Student Notes are also available.



Resuming Construction

Using Triangle Congruence to Copy an Angle

Let's copy angles!

Focus and Coherence

Today's Goals

1. **Goal:** Construct a copy of an angle using triangle congruence criteria.
2. **Language Goal:** Justify whether a construction will copy an angle using triangle congruence criteria. **(Reading and Writing)**

Students perform a variety of constructions that build toward one which they are encountering for the first time: copying an angle. They apply what they learned about triangle congruence criteria to justify their constructions. Students also make connections between triangle congruence criteria and the figures generated by the construction tool.

Prior Learning

In Sub-Unit 1 of Unit 1, students were introduced to geometric constructions. In Lesson 8, students investigated possible conditions that prove triangle congruence using AngLeg® manipulatives. In Lesson 9, students looked at specific sets of criteria that ensure triangle congruence (SSS, SAS, and ASA).

Future Learning

In Lesson 11, students will use constructions and congruent triangles to help them prove the properties of perpendicular bisectors. In Lesson 15, students will use the properties of kites to support their understanding of how to construct angle bisectors.

Rigor and Balance

- Students develop **conceptual understanding** by justifying why their constructions are valid copies.
- Students **apply** triangle congruence criteria to help them construct a copy of an angle.
- Students develop **strategic competence** as they use various tools to construct copies of angles.

Standards

Addressing

HSG.CO.D.12
Make geometric constructions. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding**, dynamic geometric software,** etc.**). Copying a segment; copying an angle;** bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

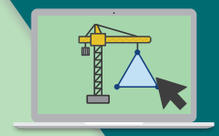
Mathematical Practices: MP1, MP3, MP5

Building On	Building Toward
HSG.CO.B.7 **HSG.CO.B.8**	**HSG.CO.D.13**

Lesson at a Glance

~ 45 min

Standard: HSG.CO.D.12



Why digital?

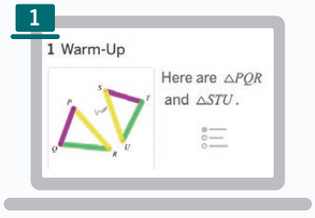
Students use the digital geometry tools to make sense of and perform constructions.

Warm-Up

Pairs Sharing a Device | 5 min

Pacing: Screen 1

Students revisit triangle congruence criteria using a diagram of AngLeg® manipulatives to elicit ideas about the corresponding congruent parts of congruent triangles.



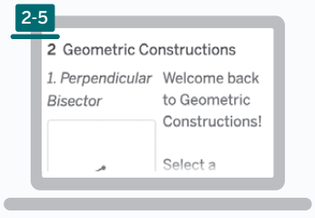
Activity 1

Pairs Sharing a Device | 15 min

Pacing: Screens 2–5

Students recall prior constructions and apply their learning as they construct a copy of a triangle.

Key Takeaway (Screen 5): You can construct a copy of a segment by using the "circle by length" tool (or a compass on paper). A copy of a triangle can be constructed by copying its three line segments.



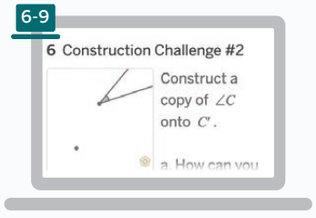
Activity 2

Pairs Sharing a Device | 15 min

Pacing: Screens 6–9

Students construct a copy of an angle and justify the validity of their copy using triangle congruence criteria. **(MP3)**

Key Takeaway (Screen 8): If you construct a copy of a triangle, the two triangles will be congruent. Congruent triangles have corresponding congruent parts, including congruent angles. This relationship is key to copying an angle using constructions.



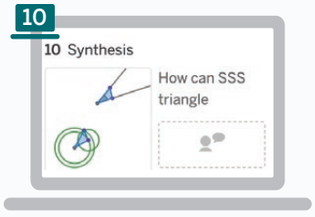
Synthesis

Whole Class | 5 min

Pacing: Screen 10

Students synthesize their ideas about using triangle congruence criteria to verify constructions that are copies of an angle.

Lesson Takeaway: To copy an angle, construct a triangle on the original angle. Then, use the "circle by length" tool to copy the sides of the triangle. By SSS congruence criteria, the triangle and its copy are congruent. All corresponding parts of congruent triangles are congruent, so a congruent angle was constructed.
See the Synthesis page for full Lesson Takeaway.

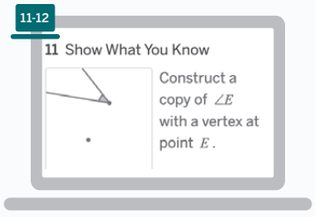


Show What You Know

Independent | 5 min

Pacing: Screens 11–12

Students demonstrate their understanding by constructing a copy of an angle.



Prep Checklist

Assign the digital student screens. Student Notes are also available.

Consider making the geometry toolkit available to support students in selecting and using tools strategically.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)
- geometry toolkit: compasses, straightedges
- tracing paper (optional)

Student Notes sheet in Student Edition:
Consider providing the Student Notes to support class discussion and student thinking.



Pairs Sharing a Device | 5 min | Pacing: Screen 1

Warm-Up

Purpose: Students revisit triangle congruence criteria using a diagram of AngLeg® manipulatives to elicit ideas about the corresponding congruent parts of congruent triangles.



Student Notes are also available

1 Launch

1 **To connect to prior knowledge**, consider asking:

- "What do you remember about AngLegs®? How can they help us decide if two triangles are congruent?"
- "How can you tell if any of the parts shown are corresponding parts?"

Invite students to discuss their thinking with a partner before selecting an answer.

A **Accessibility: Visual-Spatial Processing** To support students in identifying the three lengths of the AngLegs®, consider sharing that both diagrams have a purple, a yellow, and a green AngLegs®.

Look for explanations that make connections to SSS triangle congruence.

2 Connect

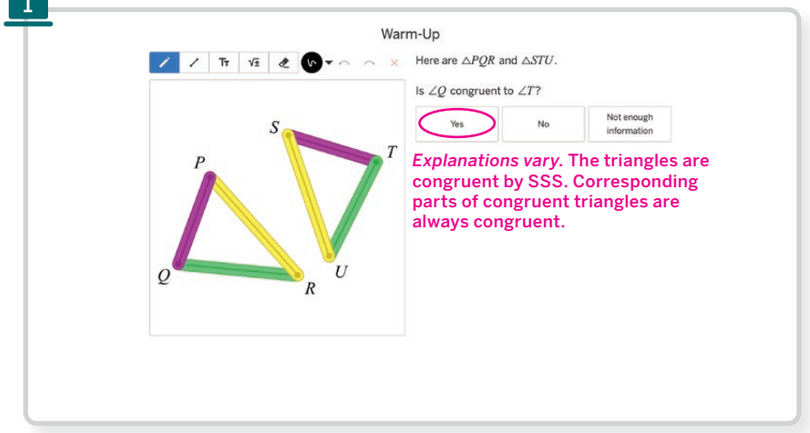
To support making connections, consider asking:

- "What was your strategy for deciding if $\angle Q \cong \angle T$?"
- "Why did we need to prove that these two triangles were congruent first?"
- "Which congruence criteria did you use to determine that these triangles are congruent?"
- "If we have proven that these two triangles are congruent, what else do we know about these triangles?"

Math Identity and Community Invite students to celebrate a classmate's thinking that was strong, clear, or creative, especially if their thinking made connections to previous lessons.

Students using digital

1



Explanations vary. The triangles are congruent by SSS. Corresponding parts of congruent triangles are always congruent.

Pairs Sharing a Device | 15 min | Pacing: Screens 2–5

Activity 1 Geometric Constructions

Purpose: Students recall prior constructions and apply their learning as they construct a copy of a triangle.



Student Notes are also available

1 Launch

2 **To connect to prior knowledge**, invite students to discuss with their partner what they remember about each construction.

Consider asking:

- "Which tools are available when we're doing constructions? Which tools aren't?" **(MP5)**
- "What is a perpendicular bisector?"

Invite students to complete the constructions one at a time with their partner, then continue to Screen 3.

Note: The geometry tools are intentionally disabled for this lesson because the lesson's focus is on using construction tools.

2 Monitor

Look for constructions that show copies of segments in different orientations. Consider sharing examples in the Connect using the Snapshot tool.

3 ML/EL **Multilingual/English Learners** Provide sentence frames to help students reflect on Julian's work (e.g., "Something Julian did well was _____." or "Something Julian could revise is _____.") **(Writing and Speaking)**

4 **Capture** different strategies for constructing a copy of $\triangle ABC$, particularly those that use the "circle by length" tool to copy segments.

Note: Students will need to use the "polygon" tool to show their triangle to receive a full star for a correct construction. A *possibly* correct construction will result in a partially filled-in star.

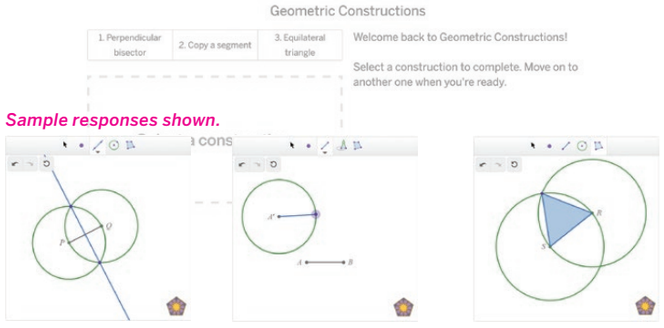
D **Differentiation:** (Support) Look for constructions that use a strategy like Julian's from Screen 3. Consider asking, "How do you know that the copied line segments will always be congruent to the corresponding segments of $\triangle ABC$?"

Math Identity and Community Consider pausing to celebrate students who persisted through struggle. Invite them to share strategies for persistence.

Activity 1 continued >

Students using digital

2



Sample responses shown.

3

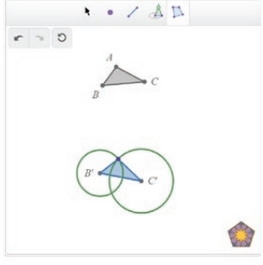


Responses vary.

- Something that Julian did well was make a new segment instead of a line or ray. He also knew that $\overline{A'B'}$ needed to look like $\overline{AB}$.
- Julian can be more precise by using the length of $\overline{AB}$ to make a circle with the "circle by length" tool. Then he can move the circle onto point A' to make his congruent segment.

4

Sample shown.



Pairs Sharing a Device | 15 min | Pacing: Screens 2–5

Activity 1 Geometric Constructions (continued)

Purpose: Students recall prior constructions and apply their learning as they construct a copy of a triangle.



Student Notes are also available

2 Monitor

5 **Listen for** interpretations of Anong's claim, particularly those saying the "circle by length" tool helps to create three congruent segments from $\triangle ABC$.

Invite students using notes to capture their ideas from the discussion before sharing with the class.

A **Accessibility: Affective Functioning** To support students in sharing their thinking, check in with students in advance to let them know you will ask for their thoughts.

3 Connect

To connect to prior knowledge, consider asking, "How is SAS congruence different from SSS congruence?"

Share the captured constructions from Screen 4.

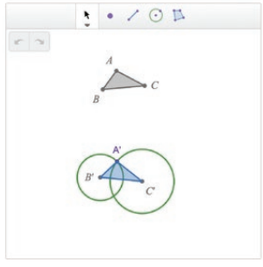
To surface the Key Takeaway, consider asking:

- "If you construct a copy of a figure, what is the relationship between the original figure and the new figure?" *Responses vary.* The original figure and the constructed figure are congruent.
- "How did you use the "circle by length" tool to copy segments from the original triangle?"
- "Why might you use the "circle by length" tool instead of the circle tool?" *Responses vary.* The "circle by length" tool lets me precisely copy a segment from the triangle and see all the possible congruent segments that can also be constructed.
- "What would the construction need if Anong used SAS congruence instead?" *Responses vary.* I need to copy an angle to do a construction using SAS. But I don't know how to do that yet.

Students using digital

5

Anong's Construction



Anong constructed a copy of $\triangle ABC$ by making a circle with a radius of AB and a circle with a radius of AC.

Anong says: *I can prove that I copied the triangle successfully using SSS congruence.*

Discuss:

- What does Anong's statement mean?
- Do you think SAS could be used instead? Why or why not?

Responses vary.

- Anong copied all three segments of $\triangle ABC$ to make the triangle. SSS congruence means that if three corresponding sides are congruent, the triangles are congruent. So we know that Anong's triangle is a successful copy of $\triangle ABC$.
- It'd be hard to use SAS to copy the triangle because I need to copy an angle, and I don't know how to copy an angle using the construction tools yet.

Key Takeaway: You can construct a copy of a segment by using the "circle by length" tool (or a compass on paper). A copy of a triangle can be constructed by copying its three line segments.

Pairs Sharing a Device | 15 min | Pacing: Screens 6–9

Activity 2 Construction Challenges

Purpose: Students construct a copy of an angle and justify the validity of their copy using triangle congruence criteria. **(MP3)**

Short on time? Consider facilitating Screen 7 together as a class.



Student Notes are also available

1 Launch

6 **Invite students** to generate a few ideas with their partner before trying a strategy.

Consider asking, "Which available digital tools could you use?" **(MP5)**

2 Monitor

To support students getting started, consider asking:

- "How could you use the last construction to help you think of a new strategy?"
- "How could you use triangle congruence criteria to help you copy an angle?"

7 ML/EL **Multilingual/English Learners** Invite students to read each step of Caleb's strategy aloud and paraphrase what it means. Clarify any words they don't know. **(Reading)**

MLR **MLR6: Three Reads (optional)** Consider inviting students to read Caleb's strategy three times to help them make sense of the task. Encourage students to examine the strategy through a different lens each time.

- What is Caleb's strategy?
- Which construction steps does Caleb plan to use?
- What is the goal of the construction?

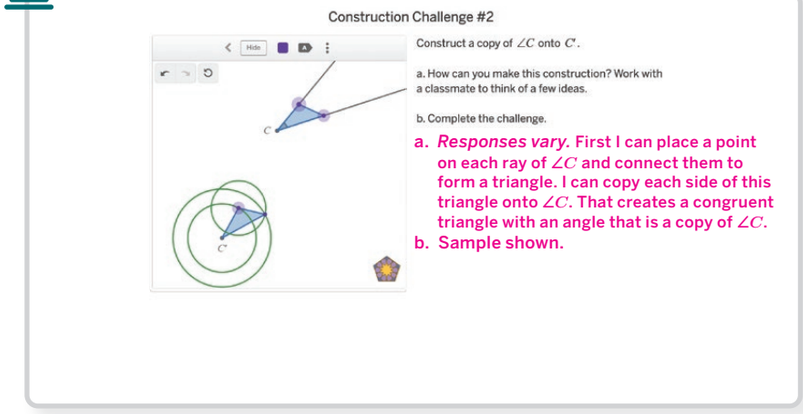
See the Routine Facilitation Guide for more information. **(MP1)**

D **Differentiation: (Support)** Look for students who notice that the "circle by length" tool is not accessible. Consider sharing that the "circle by length" tool requires a segment to determine a radius. Encourage students to construct a segment and see if the tool becomes available then.

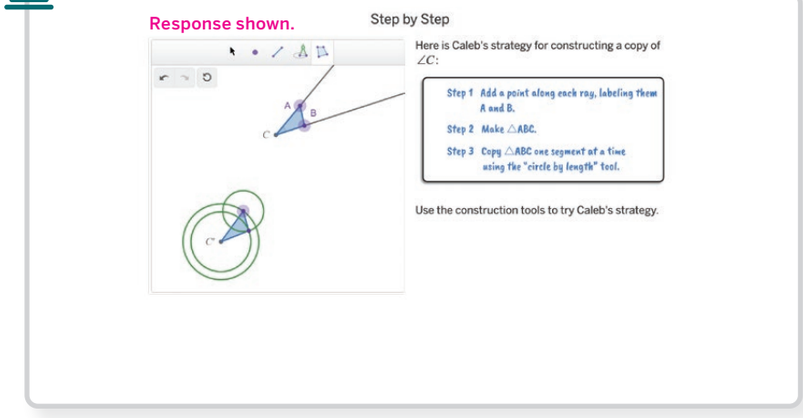
Activity 2 continued >

Students using digital

6



7



Pairs Sharing a Device | 15 min | Pacing: Screens 6–9

Activity 2 Construction Challenges (continued)

Purpose: Students construct a copy of an angle and justify the validity of their copy using triangle congruence criteria. **(MP3)**

Short on time? Consider facilitating Screen 7 together as a class.



Student Notes are also available

2 Monitor

8 **Look for** the ways that students make sense of Caleb's strategy.

D Differentiation

Look for students who:	Respond to Student Thinking
Use measurement of the angle to justify the congruence relationship.	Strengthen Celebrate their strategy. Consider asking, "What would convince you that these angles are congruent without measuring them?"
Justify that Caleb's construction of $\angle C'$ is congruent to $\angle C$ because the angles are corresponding congruent parts of two congruent triangles.	Stretch Consider sharing that Caleb's method involves constructing three circles. Consider asking, "Is there a way to copy an angle using only two circles? Go back to Screen 6 to try it out!"

9 **D Differentiation: Explore More. (Stretch)** Invite students who would like to further explore constructions and congruence of copied figures to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

8 **Display** Caleb's work and invite students to name the different construction steps.

Consider asking, "How did constructing $\triangle ABC$ help Caleb know he constructed a copy of $\angle C$?" *Responses vary.* By constructing a congruent triangle, Caleb could confirm that $\angle C'$ and $\angle C$ are also congruent.

To surface the Key Takeaway, consider asking:

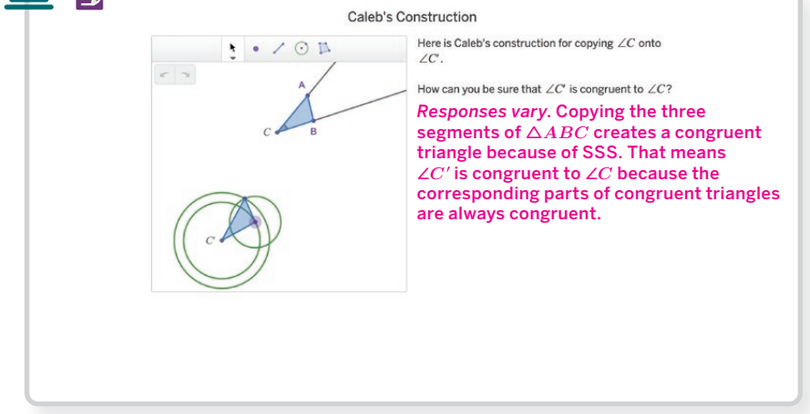
- "What is convincing about Caleb's strategy?" **(MP3)**
- "How do we know that this construction strategy will always create congruent angles?"
- "If two triangles are congruent, what do we know about their corresponding parts?"

Note: If time allows, consider asking, "What might look different about Caleb's strategy if he used a physical compass instead?"

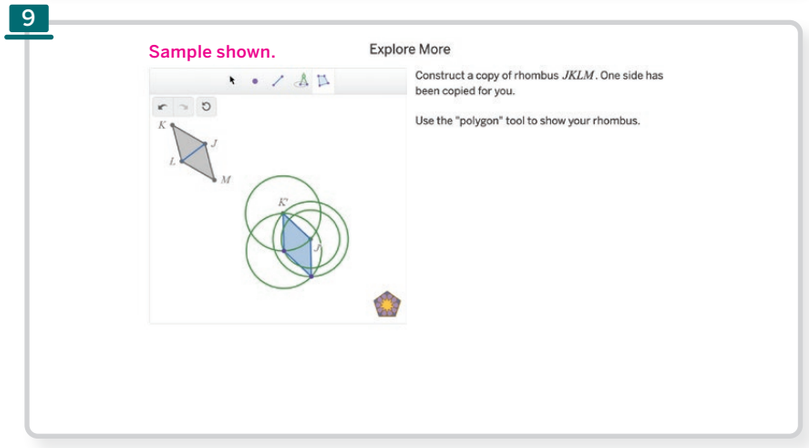
Invite students using notes to answer, "How does Caleb's strategy connect with what you did in the Warm-Up?"

Students using digital

8



9



Key Takeaway: If you construct a copy of a triangle, the two triangles will be congruent. Congruent triangles have corresponding congruent parts, including congruent angles. This relationship is key to copying an angle using constructions.

Whole Class | 5 min | Pacing: Screen 10

Synthesis



Student Notes are also available

10 **Invite students** to respond independently after discussing each question with their partner.

Capture and share a variety of ideas, such as:

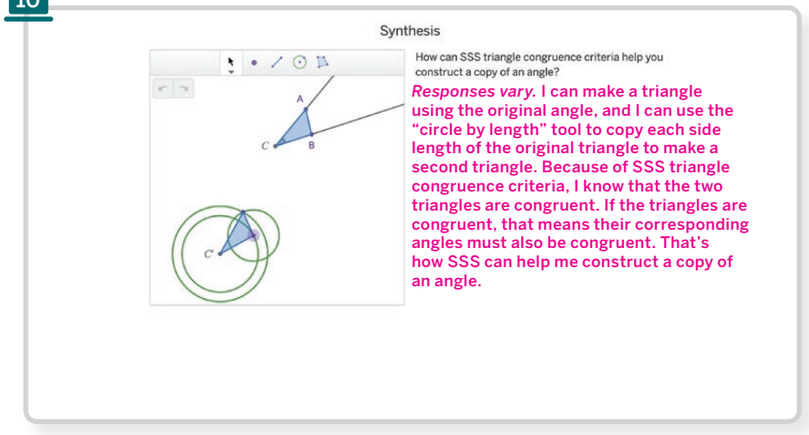
- SSS congruence criteria proves that two triangles are congruent.
- To construct a copy of an angle, we can construct a triangle containing that angle.
- We can use SSS congruence criteria to construct a copy of a triangle by constructing copies of each of the triangle's three segments.
- Corresponding angles of congruent triangles are always congruent.
- By constructing a copy of a triangle, we also copy the angles in the triangle.

Math Identity and Community Invite students to share strategies they found helpful and give credit to the students who shared them.

Note: If time allows, invite students to revise their thinking from Screen 8 based on the discussion.

Students using digital

10



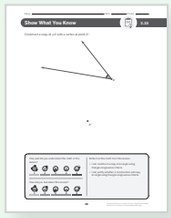
Lesson Takeaway: One strategy you could use to construct a copy of an angle is to construct a triangle using an additional point on each ray of the original angle. Then you could use the "circle by length" tool to copy the sides of the triangle. By SSS congruence criteria, the triangle and its constructed copy are congruent.

This means that the copy of the triangle must contain a copy of the original angle because all corresponding parts of congruent triangles are also congruent.

Independent | 5 min | Pacing: Screens 11–12

Show What You Know

Purpose: Students demonstrate their understanding by constructing a copy of an angle.

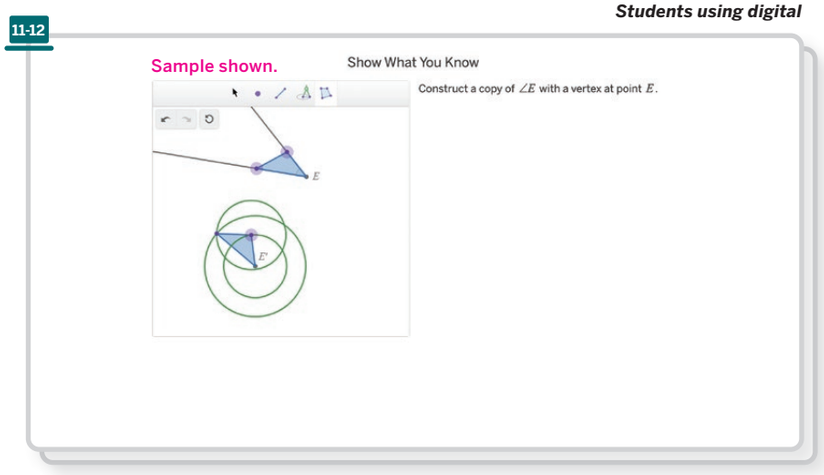


Students using print

11-12 Today's Goals

1. **Goal:** Construct a copy of an angle using triangle congruence criteria.
2. **Language Goal:** Justify whether a construction will copy an angle using triangle congruence criteria. **(Reading and Writing)**

Students using digital



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of using triangle congruence criteria to perform constructions, consider:

- Inviting students to complete the **Try This** problem and providing individualized feedback.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice**.
- Inviting students to review the **Summary** and complete the **Try This** problem.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting them to complete the **Explore More** task and discuss their thinking with a partner.
- Inviting them to create their own "angle copy construction" problem and discuss additional strategies with a partner.

Professional Learning

Take a moment to reflect on how students used digital tools to construct a copy of an angle in this lesson. How did the tools encourage students to work strategically to perform constructions? How did students develop their understanding of this tool as they were presented with different challenges?

Lesson Practice

Independent

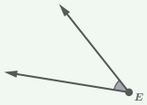
Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 10
Practice

Students using digital

Lesson Summary

When constructing a copy of an angle, it can be helpful first to construct a copy of a triangle. For example, you can add a point on each ray of $\angle E$. If you connect these points, then you have constructed a triangle.



The "circle by length" tool can be used to copy the side lengths of this triangle. The intersection of the new circles represents the vertices of the copied triangle. The constructed triangle is congruent to the original because of the SSS congruence criteria.

By constructing a congruent triangle, you have also constructed a copy of $\angle E$ because corresponding parts of congruent triangles are always congruent.

Try This

Describe how to construct a copy of $\angle J$ using a straightedge and a compass.

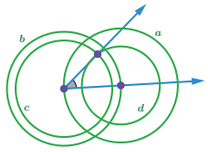


Responses vary. First, I'll make two more points, K and L, one on each ray. Then I can make a triangle that connects all three points, $\triangle JKL$. I'll use the compass to create a circle by length and copy each side length to make a congruent triangle using SSS congruence criteria. I know the angle is copied because every corresponding part of a congruent triangle is also congruent.

Lesson Practice
Geo.2.10

Name: _____ Date: _____ Period: _____

Problems 1–2: Here is an angle being copied.



1. Which circle is not necessary to copy the angle?

   (A.) Circle a B. Circle b

   C. Circle c D. Circle d

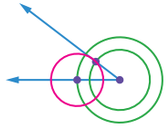
2. Which circle copies the segment opposite the angle?

   A. Circle a B. Circle b

   C. Circle c (D.) Circle d

Problems 3–4: Here is part of the construction to copy an angle.

3. Construct a circle to complete the first half of the construction. *Responses vary.*

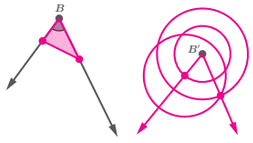


4. Which congruence criteria can be used to prove that the angle is copied perfectly?

   A. SAS B. AAS

   C. ASA (D.) SSS

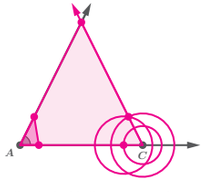
5. Copy $\angle B$ with the vertex at point B'. *Responses vary.*



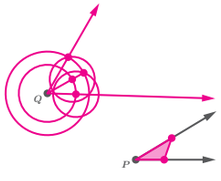
Lesson Practice
Geo.2.10

Name: _____ Date: _____ Period: _____

6. Copy $\angle A$ with the vertex at point C to construct an isosceles triangle ABC.

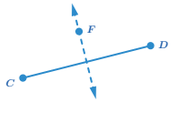


7. Construct an angle at point Q that is double the measure of $\angle P$. *Responses vary.*



Spiral Review

8. Point F is on the perpendicular bisector of $\overline{CD}$. Which statement must be true about point F?



   A. It is closer to point C than to point D.

   B. It is closer to point D than to point C.

   (C.) It is equal distance to points C and D.

   D. There is not enough information to know if it is closer to point C or point D.

Reflection

1. Put a star next to a problem you could explain to a classmate.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-lesson			
	1–3, 5	2	HSG.CO.D.12
Test Practice	4	2	HSG.CO.B.8
	6–7	3	HSG.CO.D.12
Spiral Review			
Fluency	8	1	HSG.CO.A.1

Need More Practice?

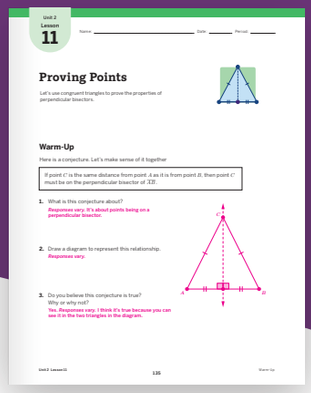
Additional Practice resources coming soon.

Unit 2

Lesson

11

This lesson uses print Student Edition pages and digital Teacher Presentation Screens.



Proving Points

Proving the Properties of Points on Perpendicular Bisectors

Let's use congruent triangles to prove the properties of perpendicular bisectors.

Focus and Coherence

Today's Goals

1. **Goal:** Prove the properties of points on perpendicular bisectors.
2. **Language Goal:** Analyze an argument about the properties of points on perpendicular bisectors to make it stronger and clearer. **(Reading, Writing, Speaking, and Listening)**

Students apply their understanding of congruent triangles to prove that the points on the *perpendicular bisector* of a line segment are exactly *equidistant* from the segment's endpoints. They follow a plan to prove the **theorem** by splitting it into a conjecture and its *converse*.

Prior Learning

In Unit 1, students constructed perpendicular bisectors of given segments and lines. In Lesson 9, students determined which specific congruence conditions always create congruent triangles. In Lesson 10, students used SSS congruence criteria to construct a copy of an angle.

Future Learning

In Lessons 13–14, students will prove the properties of parallelograms. In Lesson 15, students will use what they've learned about proving the properties of quadrilaterals and triangles to construct angle bisectors.

Rigor and Balance

- Students **apply** their understanding of congruent triangles to prove the specific properties of perpendicular bisectors.
- Students develop **strategic competence** as they practice making a plan to prove a conjecture.

Vocabulary

New Vocabulary

theorem

converse (optional)

Review Vocabulary

equidistant

perpendicular bisector

Standards

Addressing

HSG.CO.C.9

Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; **points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.**

Mathematical Practices: MP1, MP3, MP6, MP7

Building On

HSG.CO.B.8
HSG.CO.D.12

Lesson at a Glance

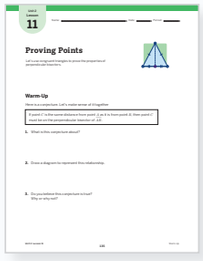
~ 45 min

Standard: HSG.CO.C.9

Warm-Up

Whole Class | 10 min

Students use MLR6: Three Reads to make sense of a conjecture that they will prove in this lesson. **(MP1)**





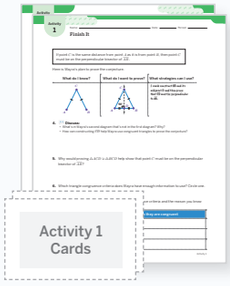
Presentation Screens 1–3

Activity 1

Pairs | 12 min

Students make sense of and finish a plan to prove that points that are *equidistant* from a segment's endpoints are on the *perpendicular bisector*.

Key Takeaway: Making a convincing argument that proves a conjecture is true includes justifying the things you know to be true mathematically. Writing a reason for each statement helps to make the argument more convincing for you and others in your community





Presentation Screen 4

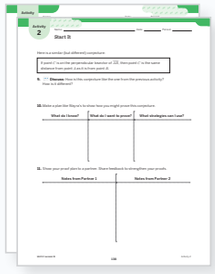
Activity 2

Pairs | 13 min

Students make a plan and prove the *converse* of the conjecture from Activity 1: The points on the perpendicular bisector of a segment are equidistant from its endpoints.
MLR1: Stronger and Clearer

Key Takeaway: Writing a convincing argument is about more than creating a sequence of logical statements and reasons. A convincing argument includes planning, drafting, feedback, and revision.

See the Activity page for full Key Takeaway.





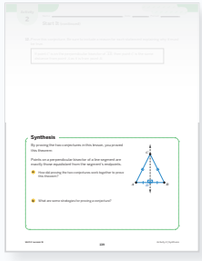
Presentation Screens 5–6

Synthesis

Whole Class | 5 min

Students synthesize strategies for strengthening their proofs of a conjecture.
MLR2: Collect and Display

Lesson Takeaway: Proving conjectures and **theorems** involves making convincing arguments. A convincing proof includes a reason that shows how you know each statement must be true. A table can help you stay organized and ensure that there's a reason for each statement.



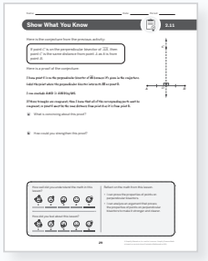


Presentation Screen 7

Show What You Know

Independent | 5 min

Students demonstrate their understanding by critiquing a proof of this conjecture: Points on a perpendicular bisector of a line segment are equidistant from the segment's endpoints.





Presentation Screen 8

Prep Checklist

Invite students to use their Student Edition. Display the Presentation Screens to help facilitate the lesson.

This lesson includes:

- Presentation Screens
- Student Edition
- Show What You Know
- Activity 1 Cards (one set per pair)
- coloring tools (optional)

Whole Class | 10 min

Warm-Up

Purpose: Students use **MLR6: Three Reads** to make sense of a conjecture they will prove in this lesson. **(MP1)**

Lesson 11
Warm-Up

Presentation Screens 1–3

1 Launch

MLR **MLR6: Three Reads** Invite students to read the text three times, each with a particular focus, to support students in making sense of the conjecture they will prove in Activity 1. **(MP1)**

1 **Display** the screen and invite a student to read the conjecture aloud before discussing it as a class.

ML/EL **Multilingual/English Learners** After this read aloud, invite students to paraphrase what the conjecture means. Clarify any words they don't know. **(Reading)**

To support making connections, consider asking:
- "What would it look like for point C to be the same distance from point A and point B in a diagram?"
- "If point C is on the perpendicular bisector of $\overline{AB}$, what other information do we know? What would that information look like in a diagram?" *Responses vary.* The perpendicular bisector makes a 90° angle with $\overline{AB}$ and goes through the midpoint.

2 **Display** the screen and invite a student to read the conjecture aloud again. Then invite students to sketch a diagram to represent the conjecture.

Encourage students to share their drawings with a partner. Consider asking:
- "What must be the same in all of our diagrams? What might be different?"
- "What parts of the diagram do we already know to be true? What parts are we trying to prove?"

Invite students to revise their diagram.

3 **Display** the screen and invite a student to read the conjecture for a third time. Then, invite students to discuss the questions with a partner.

3 Connect

Consider asking: (MP3)
- "What's a strategy for showing that this conjecture is always true?"
- "What is the value in proving something that we already know is true?"

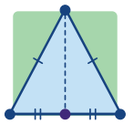
Math Identity and Community Invite students to share why reading a conjecture multiple times helped them make sense of the information.

Unit 2
Lesson
11

Name: ..... Date: ..... Period: .....

Proving Points

Let's use congruent triangles to prove the properties of perpendicular bisectors.



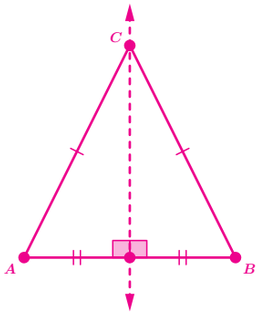
Warm-Up

Here is a conjecture. Let's make sense of it together

> If point C is the same distance from point A as it is from point B, then point C must be on the perpendicular bisector of $\overline{AB}$.

1. What is this conjecture about?
***Responses vary.* It's about points being on a perpendicular bisector.**

2. Draw a diagram to represent this relationship.
Responses vary.



3. Do you believe this conjecture is true? Why or why not?
Yes. *Responses vary.* I think it's true because you can see it in the two triangles in the diagram.

Unit 2 Lesson 11 135 Warm-Up

Pairs | 12 min

Activity 1 Finish It

Purpose: Students make sense of and finish a student's plan to prove that points that are *equidistant* from a segment's endpoints are on the *perpendicular bisector*.

Lesson 11
Activity 1

Presentation Screen 4

1 Launch

To connect to prior knowledge, consider asking, "What makes an argument convincing? When does an argument have enough proof?"

4 **Invite students** to review Wayra's plan to prove the conjecture. Then play the animation to support students in making connections to Wayra's strategy.

Consider sharing that there are many different paths Wayra could have chosen to prove the conjecture. Two possible paths are to prove:

- $\overline{CD}$ is perpendicular to $\overline{AB}$, and point D is the midpoint.
- There is a line that is a perpendicular bisector of $\overline{AB}$ and C is a point on that line.

Consider asking, "What are some other strategies Wayra could have used to prove this conjecture?"

2 Monitor

Listen for student discussions that identify how proving $\triangle ACD \cong \triangle BCD$ would help prove the conjecture. Highlight the student conversations that make sense of how proving triangle congruence can be a first step to proving corresponding congruent parts are in each triangle.

D Differentiation *(Problems 4–6)*

Look for students who:	Respond to Student Thinking
Need support when making sense of the congruence markings in Wayra's second diagram. *(Problem 4)*	**Support** Encourage them to make connections to the conjecture. Consider asking, "Why is $\overline{AC}$ congruent to $\overline{BC}$, according to this conjecture?".
Need support visualizing the two triangles in Wayra's diagram. *(Problem 5)*	**Support** Encourage them to use coloring tools to highlight each triangle.
Choose a different triangle congruence criteria than SSS. *(Problem 6)*	**Strengthen** Invite them to sketch the diagram and label any congruent parts they know at this point. Consider asking, "What parts are congruent between the two triangles?"
Identify congruent parts but write them without correspondence. *(Problem 6)*	**Strengthen** Invite them to use precision. Consider asking, "Which vertices stack when you reflect $\triangle ACD$ over $\overline{CD}$?" **(MP6)**

Activity 1 continued >

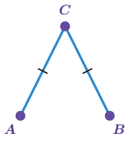
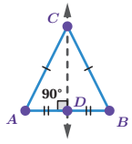
Activity 1

Name: _____ Date: _____ Period: _____

Finish It

If point C is the same distance from point A as it is from point B, then point C must be on the perpendicular bisector of $\overline{AB}$.

Here is Wayra's plan to prove the conjecture.

What do I know?	What do I want to prove?	What strategies can I use?
		I could construct $\overline{AB}$ and its midpoint D and then prove that $\overline{CD}$ must be perpendicular to $\overline{AB}$.

4. Discuss:

- What's in Wayra's second diagram that's not in the first diagram? Why?
- How can constructing $\overline{CD}$ help Wayra use congruent triangles to prove the conjecture?

Responses vary.

- Wayra's second diagram includes the perpendicular bisector and the 90° angles at the intersection of $\overline{AB}$ and $\overline{CD}$. It also includes markings showing that $\overline{AD} \cong \overline{BD}$.
- Constructions can help Wayra prove this conjecture because adding a midpoint on $\overline{AB}$ can help make two triangles that we can show are congruent.

5. Why would proving $\triangle ACD \cong \triangle BCD$ help show that point C must be on the perpendicular bisector of $\overline{AB}$?

Responses vary. Once we prove that $\triangle ACD \cong \triangle BCD$, then we know $\angle ADC \cong \angle BDC$ because they are corresponding parts of congruent triangles. Since $\angle ADC$ and $\angle BDC$ form a straight angle, we can conclude that they are both 90°, which proves that $\overline{CD}$ must be perpendicular to $\overline{AB}$.

6. Which triangle congruence criteria does Wayra have enough information to use? Circle one.

(SSS) SAS ASA AAS

List the congruent pairs for each part of the congruence criteria and the reason you know they are congruent. *Explanations vary.*

Congruent Pair	Reason they are congruent
$\overline{AC} \cong \overline{BC}$	This information is given in the conjecture.
$\overline{AD} \cong \overline{BD}$	Constructing point D at the midpoint of $\overline{AB}$ means that these segments must be congruent.
$\overline{CD} \cong \overline{CD}$	It's the same segment in both triangles.

Unit 2 Lesson 11 136 Activity 1

Pairs | 12 min

Activity 1 Finish It (continued)

Purpose: Students make sense of and finish a student's plan to prove that points that are *equidistant* from a segment's endpoints are on the *perpendicular bisector*.

Lesson 11
Activity 1

Presentation Screen 4

2 Monitor

Distribute the Activity 1 cards.

Invite students to make sense of each reason before deciding where it fits in the proof.

Note: The reasons are written in student-friendly language to support their understanding. In future lessons, the word "Given" will be used as the first reason for a proof.

Encourage students to work together to reach an agreement about the placement of each card and justify their thinking as they go.

A **Accessibility: Executive Function** To support students in carrying out multiple steps, invite students to focus on one card at a time.

Math Identity and Community Consider highlighting the value of collaboration by asking if any students revised the order of their cards by working with a partner.

Invite students to read through the proof from beginning to end to support them with Problem 8.

3 Connect

Invite students to compare the order of their cards and their strategies for deciding where each reason fits in the proof.

To surface the Key Takeaway, consider asking:

- "What do you find convincing about Wayra's proof?" *Responses vary.* Wayra constructed a point! I didn't know we could do that to prove a conjecture. **(MP3)**
- "Is there a way to make Wayra's proof stronger and clearer?" *Responses vary.* Wayra can make it clearer by saying the corresponding angles are congruent. **(MP6)**
- "How did the table help structure Wayra's argument?" **(MP7)**

Math Identity and Community Celebrate the completed proof. Consider sharing that proofs are validated by the community that uses them, so they should be written to convince not just the proof author but the author's community as well. **(MP3)**

Activity 1

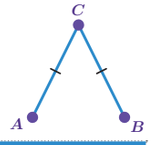
Name: Date: Period:

Finish It (continued)

You can make a proof more convincing by including a reason each statement must be true. A table can help organize reasons and statements.

7. Use a set of cards to finish the table and complete Wayra's proof.

If point C is the same distance from point A as it is from point B, then point C must be on the perpendicular bisector of $\overline{AB}$.



Statement	Reason that this must be true
$\overline{AC} \cong \overline{BC}$	This information is given in the conjecture.
$\overline{AD} \cong \overline{BD}$	Constructing point D at the midpoint of $\overline{AB}$ means that these segments must be congruent.
$\overline{CD} \cong \overline{CD}$	f. The same segment in both triangles.
$\triangle ACD \cong \triangle BCD$	a. Congruent by SSS.
$\angle ADC \cong \angle BDC$	b. These are corresponding angles in congruent triangles.
$m\angle ADC + m\angle BDC = 180°$	e. $\overline{AB}$ is a line segment, so these angles form a straight angle.
$m\angle ADC = 90°$ and $m\angle BDC = 90°$	d. These angles have the same measure and must sum to 180°.
$\overline{CD}$ is a perpendicular bisector of $\overline{AB}$.	c. Since $\angle ADC$ and $\angle BDC$ are each 90°, $\overline{CD}$ must be perpendicular to $\overline{AB}$.

8. What do you find convincing about Wayra's proof? What questions do you have?

Responses vary. Wayra's proof is convincing because it has a reason for each step. My question is: How do I know how much detail to use for each statement?

Unit 2 Lesson 11 137 Activity 1

Key Takeaway: Making a convincing argument that proves a conjecture is true includes justifying the things you know to be true mathematically. Writing a reason for each statement helps to make the argument more convincing for you and others in your community.

Pairs | 13 min

Activity 2 Start It

Purpose: Students make a plan and prove the *converse* of the conjecture from Activity 1: the points on a perpendicular bisector of a segment are equidistant from its endpoints.

Short on time? Consider completing only one round of partner feedback.

Lesson 11
Activity 2

Presentation Screens
5–6

1 Launch

Invite students to share how this conjecture is the same and different from the one proved in Activity 1.

Note: The conjecture in Activity 2 is the *converse* of the conjecture in Activity 1. Students do not need to know the term "converse" to engage with this conjecture, and it will not be assessed. You may choose to introduce the term to your students if it helps with their thinking.

5 **Display** the screen. Encourage students to sketch a diagram that represents this conjecture. Then invite students to share their sketches.

Consider asking, "What are some things that we know? What are we trying to prove?"

2 Monitor

6 **Display** the diagram. Encourage students to make a plan independently first. Then invite students to share their plan with a partner and give each other feedback. Have students complete this feedback process again with a different partner.

ML/EL **Multilingual/English Learners** To support students giving feedback, consider displaying prompts such as:
- "Are you planning to construct anything?"
- "Which triangle congruence criteria do you think you might be able to use?"
- "Are there any other conjectures that we've proven in class that you plan on using?"

(Writing and Speaking)

Math Identity and Community Acknowledge the vulnerability that comes with critiquing someone's argument. Consider sharing how giving feedback is like giving a gift; by critiquing an argument we can help make it stronger and clearer. We all want to be understood well, and feedback is one way we can support each other in communicating clearly. **(MP3)**

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

Start It

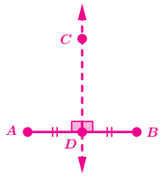
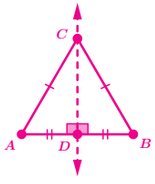
Here is a similar (but different) conjecture:

> If point C is on the perpendicular bisector of $\overline{AB}$, then point C is the same distance from point A as it is from point B.

9. **Discuss:** How is this conjecture like the one from the previous activity? How is it different?

Responses vary. This conjecture has the same words as the one in Activity 1, but the "if" and "then" parts are swapped.

10. Make a plan like Wayra's to show how you might prove this conjecture.

What do I know?	What do I want to prove?	What strategies can I use?
		*Responses vary.* • I know that $m\angle ADC = 90°$ and $m\angle BDC = 90°$ because $\overline{DC}$ is given to be perpendicular. • I could use SAS to show that $\triangle ACD \cong \triangle BCD$.

11. Show your proof plan to a partner. Share feedback to strengthen your proofs.

Notes from Partner 1	Notes from Partner 2
Responses vary.	*Responses vary.*

Unit 2 Lesson 11 138 Activity 2

Pairs | 13 min

Activity 2 Start It (continued)

Purpose: Students make a plan and prove the *converse* of the conjecture from Activity 1: the points on a perpendicular bisector of a segment are equidistant from its endpoints.

Short on time? Consider completing only one round of partner feedback.

Lesson 11
Activity 2

Presentation Screens
5–6

2 Monitor

Invite students to write a first draft of their own proof.

Encourage students to write their argument using any structure that makes sense to them (e.g., using a table, writing a paragraph, or making a flowchart). **(MP7)**

D Differentiation *(Problems 10, 12)*

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 10)*	**Support** Encourage them to review Wayra's plan from Activity 1. Consider asking, "How do you think Wayra made a plan to prove that conjecture?"
Make an argument by writing statements without any reasons. *(Problem 12)*	**Strengthen** Invite them to go back to Problem 7. Consider asking, "How did the reasons in Wayra's proof help explain their thinking? How can you add reasons to your statements to help explain your thinking?"
Make a convincing argument that has some additional statements and reasons. *(Problem 12)*	**Strengthen** Celebrate their convincing argument. You may choose to acknowledge that their proof is convincing and sufficient. If you do choose to respond, consider asking, "Are there any steps that you can remove without weakening your argument?" **(MP6)**

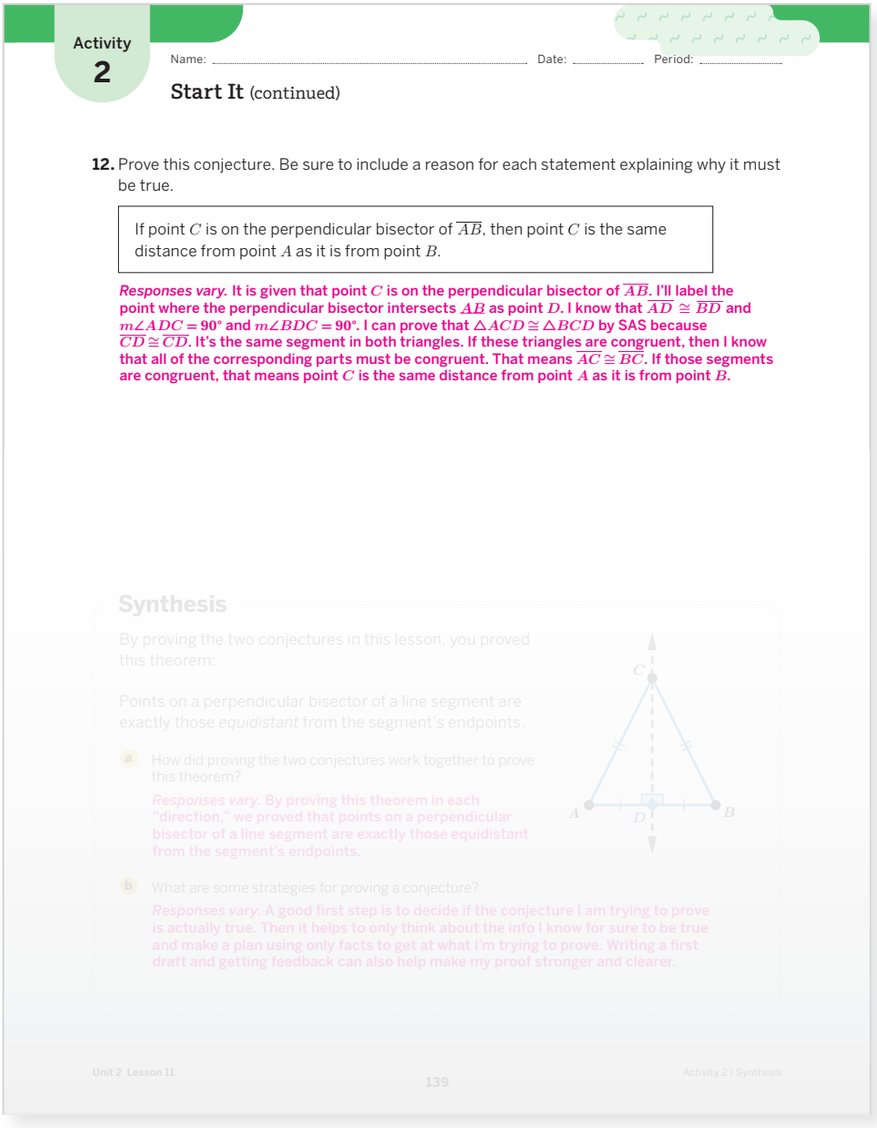
3 Connect

MLR MLR1: Stronger and Clearer Each Time Invite students to share the drafts of their proofs with a partner and give each other feedback. Encourage students to share feedback with 1–2 different partners, and then use the feedback to write a clearer second draft. See the Routine Facilitation Guide for more information. **(MP6)**

A Accessibility: Affective Functioning To support students in sharing and receiving feedback, use a structured process so students know what the feedback will look like.

Consider asking, "Are you convinced by your argument? Is your community convinced?"

Math Identity and Community Remind students that a proof is validated by the communities they are used in. Encourage students to use feedback from a partner to see if something could make their proof more convincing. **(MP3)**



Activity 2

Name: _____ Date: _____ Period: _____

Start It (continued)

12. Prove this conjecture. Be sure to include a reason for each statement explaining why it must be true.

If point C is on the perpendicular bisector of $\overline{AB}$, then point C is the same distance from point A as it is from point B.

Responses vary. It is given that point C is on the perpendicular bisector of $\overline{AB}$. I'll label the point where the perpendicular bisector intersects $\overline{AB}$ as point D. I know that $\overline{AD} \cong \overline{BD}$ and $m\angle ADC = 90°$ and $m\angle BDC = 90°$. I can prove that $\triangle ACD \cong \triangle BCD$ by SAS because $\overline{CD} \cong \overline{CD}$. It's the same segment in both triangles. If these triangles are congruent, then I know that all of the corresponding parts must be congruent. That means $\overline{AC} \cong \overline{BC}$. If those segments are congruent, that means point C is the same distance from point A as it is from point B.

Synthesis

By proving the two conjectures in this lesson, you proved this theorem:

Points on a perpendicular bisector of a line segment are exactly those *equidistant* from the segment's endpoints.

a. How did proving the two conjectures work together to prove this theorem?

Responses vary. By proving this theorem in each "direction," we proved that points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

b. What are some strategies for proving a conjecture?

Responses vary. A good first step is to decide if the conjecture I am trying to prove is actually true. Then it helps to only think about the info I know for sure to be true and make a plan using only facts to get at what I'm trying to prove. Writing a first draft and getting feedback can also help make my proof stronger and clearer.

Unit 2 Lesson 11 139 Activity 2 | Synthesis

Key Takeaway: Writing a convincing proof is about more than creating a sequence of logical statements and reasons. Writing proofs includes planning, drafting, feedback, and revision. It also includes looking for and identifying relationships that you've already shown to be true, such as congruent triangles.

Whole Class | 5 min

Synthesis

Lesson 11
Synthesis

Presentation Screen
7

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Proving that the points on a perpendicular bisector of a line segment are *exactly* those equidistant from the segment's endpoints requires proving conjectures written in each direction.
- Making a plan and sketching a diagram.
- Identifying any relationships you already know are true.
- Identifying what you are trying to prove.
- Having a reason for each statement that explains why you know something is true.
- Writing a first draft and using feedback from a partner to make the proof stronger and clearer.

Consider asking:

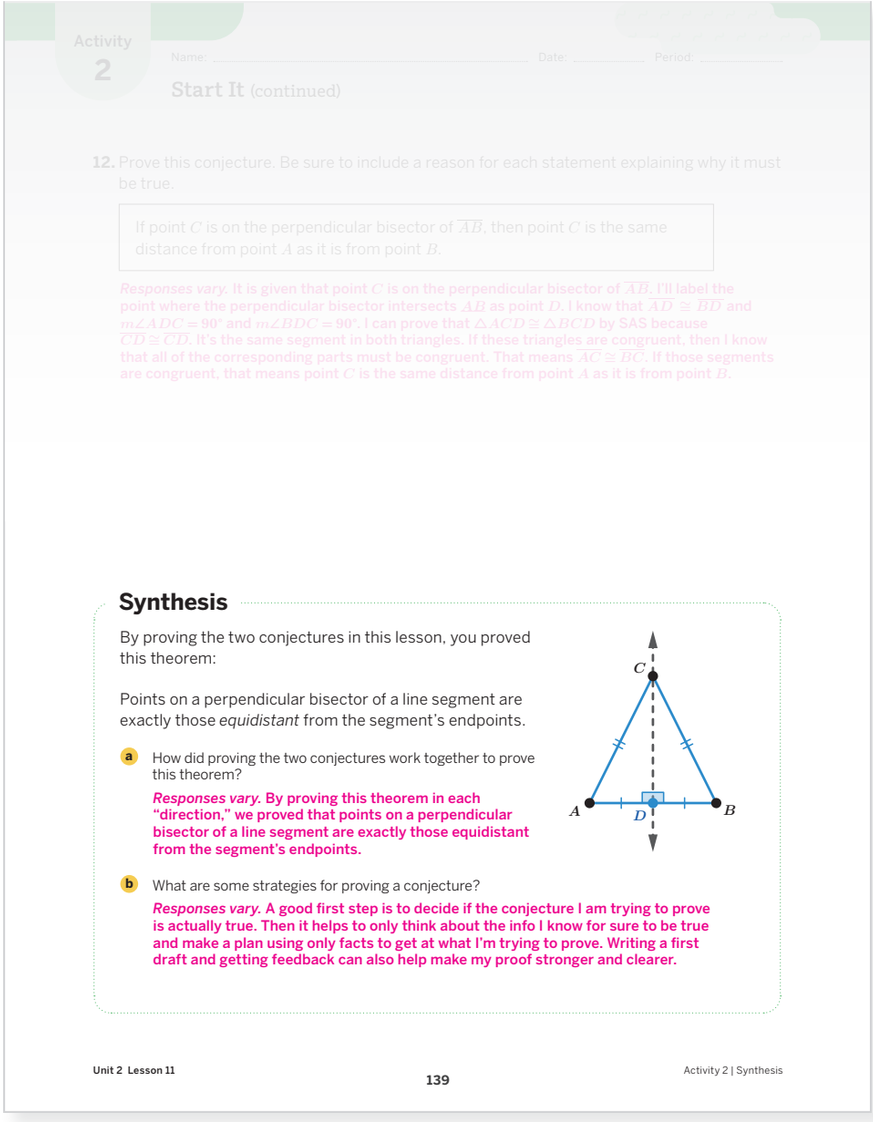
- "What is a theorem? How is it different from a conjecture?" A theorem is a conjecture that's proven to be true. A conjecture is a reasonable claim of something I'm trying to prove or disprove.
- "What is something fun about writing a proof? What is something challenging?"
- "How did the table help you organize your thinking?" **(MP7)**
- "What is the value in proving something that we already know is true?"

Consider sharing that to prove the points on the perpendicular bisector of a line segment are *exactly* those equidistant from the segment's endpoints, we need to analyze both directions of this theorem.

Encourage students to reflect on the different ways they've learned how to organize a proof so far, such as using a table in this lesson or writing paragraphs like they did in Lesson 4. Students will encounter even more ways to organize their proofs later in this unit. **(MP7)**

MLR **MLR2: Collect and Display** Invite students to revisit their strategies for making convincing arguments. Encourage students to add additional strategies they learned in this lesson and sub-unit. Display the strategies for students to reference and add to for the rest of the unit.

Math Identity and Community Acknowledge the vulnerability that comes with critiquing someone else's argument. Remind students that critiquing an argument is different from critiquing the argument's author. Receiving feedback about how a proof can be strengthened or made clearer will support us in developing precision as mathematicians. **(MP6)**



Activity 2

Name: Date: Period:

Start It (continued)

12. Prove this conjecture. Be sure to include a reason for each statement explaining why it must be true.

If point C is on the perpendicular bisector of $\overline{AB}$, then point C is the same distance from point A as it is from point B.

Responses vary. It is given that point C is on the perpendicular bisector of $\overline{AB}$. I'll label the point where the perpendicular bisector intersects $\overline{AB}$ as point D. I know that $\overline{AD} \cong \overline{BD}$ and $m\angle ADC = 90°$ and $m\angle BDC = 90°$. I can prove that $\triangle ACD \cong \triangle BCD$ by SAS because $\overline{CD} \cong \overline{CD}$. It's the same segment in both triangles. If these triangles are congruent, then I know that all of the corresponding parts must be congruent. That means $\overline{AC} \cong \overline{BC}$. If those segments are congruent, that means point C is the same distance from point A as it is from point B.

Synthesis

By proving the two conjectures in this lesson, you proved this theorem:

Points on a perpendicular bisector of a line segment are exactly those *equidistant* from the segment's endpoints.

a. How did proving the two conjectures work together to prove this theorem?

Responses vary. By proving this theorem in each "direction," we proved that points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

b. What are some strategies for proving a conjecture?

Responses vary. A good first step is to decide if the conjecture I am trying to prove is actually true. Then it helps to only think about the info I know for sure to be true and make a plan using only facts to get at what I'm trying to prove. Writing a first draft and getting feedback can also help make my proof stronger and clearer.

Unit 2 Lesson 11 139 Activity 2 | Synthesis

Lesson Takeaway: Proving conjectures and **theorems** to be true involves making convincing arguments. A convincing proof includes a reason that shows how you know each statement must be true. A table can help to stay organized and ensure that there's a reason for each statement.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by critiquing a student's proof that points on a perpendicular bisector of a line segment are equidistant from the segment's endpoints.

Lesson 11
Show What You Know

Presentation Screen
8

Today's Goals

1. **Goal:** Prove the properties of points on perpendicular bisectors.
2. **Language Goal:** Analyze an argument about the properties of perpendicular bisectors to make it clearer and more precise. **(Reading, Writing, Speaking, and Listening)**

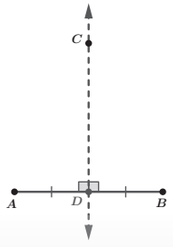
Show What You Know (Assessment Resources)

Name: ..... Date: ..... Period: .....

Show What You Know 2.11

Here is the conjecture from the previous activity:

If point C is on the perpendicular bisector of $\overline{AB}$, then point C is the same distance from point A as it is from point B.



Here is a proof of the conjecture:

I know point C is on the perpendicular bisector of $\overline{AB}$ because it's given in the conjecture.

Label the point where the perpendicular bisector intersects $\overline{AB}$ as point D.

I can conclude $\triangle ACD \cong \triangle BCD$ by SAS.

If these triangles are congruent, then I know that all of the corresponding parts must be congruent, so point C must be the same distance from point A as it is from point B.

a. What is convincing about this proof?
Responses vary. There's a reason for each statement.

b. How could you strengthen this proof?
Responses vary.
- The second sentence mentions SAS but the pairs of corresponding sides and angles are not listed, and there isn't a clear reason for how they know each part is congruent.
- It will be stronger if the proof includes a statement and reason for how congruent triangles are connected to the conclusion that point C must be the same distance from point A as it is from point B.

Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of making sense of an argument, consider:

- Inviting students to read the **Summary** and highlight key components.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice**.
- Inviting students to review the **Summary** and complete the **Try This** problem.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting them to prove a similar conjecture, e.g., "If $\triangle ABC$ is an equilateral triangle, then the angle bisector of $\angle ABC$ is also the perpendicular bisector of $\overline{AC}$."

Professional Learning

Reflect on the routines used in this lesson. Students used the **Three Reads** routine to help make sense of a conjecture, and they used **MLR1: Stronger and Clearer** to support them when writing their first proof. What about these routines felt helpful? How did these routines support students in seeing proof writing as an involved process that requires deep thinking, planning, drafting, feedback, and revision? How might you call back to these routines in the future?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 11
Practice

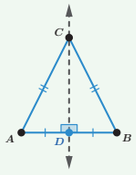
Students using digital

Lesson Summary

The *perpendicular bisector* of a line segment is *exactly* those points that are the same distance from both endpoints of the line segment.

This **theorem** can be split into two conjectures:

1. If a point is on the perpendicular bisector of a segment, then it must be the same distance from both endpoints of the segment.
2. If a point is the same distance from both endpoints of a segment, then it must be on the perpendicular bisector of the segment.

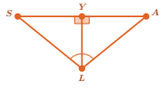


Convincing arguments include a reason explaining how you know that each statement must be true. A table can help you stay organized and make sure there is a reason for each statement.

One first step when writing a convincing argument is to decide if you believe the conjecture is true. This can help you focus on the information that must be true, and you can make a plan that only uses facts to prove the conjecture. Writing a first draft and getting feedback from a partner can help make your proof stronger and clearer.

Try This

Fill in the missing reasons to prove this conjecture:



If $\overline{YL}$ is the perpendicular bisector of $\overline{SA}$, then $\overline{LS} \cong \overline{LA}$.

Responses vary.

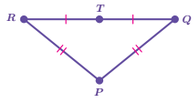
	Statement	Reason
	$\overline{YL}$ is the perpendicular bisector of $\overline{SA}$.	Given in the conjecture.
	$\overline{SY} \cong \overline{AY}$, $m\angle SYL = 90°$, and $m\angle AYL = 90°$.	Definition of a perpendicular bisector.
a	$\angle SYL \cong \angle AYL$	All right angles are congruent.
	$\overline{YL} \cong \overline{YL}$	The same segment in both triangles.
b	$\triangle YSL \cong \triangle YAL$	Congruent by SAS.
c	$\overline{LS} \cong \overline{LA}$	Corresponding sides in congruent triangles.

Lesson Practice
Geo.2.11

Name: _____ Date: _____ Period: _____

Problems 1–2: Mark the diagram with congruency markings to match the statement.

1. Point P is the same distance from points Q and R.
2. Point T is the midpoint of $\overline{QR}$.

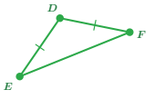


Problems 3–4: Does point D have to be on the perpendicular bisector of $\overline{EF}$? Circle one.

3. (Yes) No

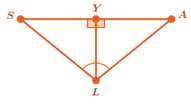


4. (Yes) No



Problems 5–7: Fill in the missing reasons to complete this proof. *Responses vary.*

If $\angle SYL$ and $\angle AYL$ are right angles and $\angle SLY \cong \angle ALY$, then $\overline{YL}$ must be the perpendicular bisector of $\overline{SA}$.



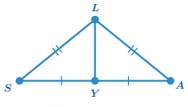
	Statement	Reason
	$\angle SYL$ and $\angle AYL$ are right angles and $\angle SLY \cong \angle ALY$.	Given in the conjecture.
5.	$\angle SYL \cong \angle AYL$	All right angles are congruent.
	$\overline{YL} \cong \overline{YL}$	The same segment in both triangles.
6.	$\triangle YSL \cong \triangle YAL$	Congruent by SAS.
7.	$\overline{SY} \cong \overline{AY}$	These are corresponding sides in congruent triangles.
	$\overline{YL}$ must be the perpendicular bisector of $\overline{SA}$.	$\overline{YL}$ is perpendicular to $\overline{SA}$, and since $\overline{SY} \cong \overline{AY}$ we know $\overline{YL}$ bisects $\overline{SA}$.

Lesson Practice
Geo.2.11

Name: _____ Date: _____ Period: _____

8. Prove this conjecture:

If $\overline{SY} \cong \overline{AY}$ and $\overline{SL} \cong \overline{AL}$, then $\overline{YL}$ must be the perpendicular bisector of $\overline{SA}$.



Responses vary. I know that $\overline{SY} \cong \overline{AY}$ and $\overline{SL} \cong \overline{AL}$ because it's given in the conjecture, and $\overline{YL} \cong \overline{YL}$ because it is the same segment in each triangle. This means that $\triangle SYL \cong \triangle AYL$ by SSS. If these triangles are congruent, then $\angle LYS \cong \angle LYA$ because they are corresponding angles of congruent triangles. These angles are each 90° because they are congruent and form a straight angle, meaning that $\overline{YL}$ is perpendicular to $\overline{AS}$. So I can conclude that $\overline{YL}$ must be the perpendicular bisector of $\overline{SA}$.

Spiral Review

9. Which property must a square always have that a rhombus might not have?

A. Parallel sides
B. Four congruent sides
C. Two pairs of supplementary angles
(D.) All congruent angles

10. Which property must a rectangle always have that a parallelogram might not have?

A. Parallel sides
B. Four congruent sides
C. Two pairs of supplementary angles
(D.) All congruent angles

Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-lesson			
	1–2	1	HSG.CO.C.9*
	3–4	2	HSG.CO.C.9*
Test Practice	5–7	2	HSG.CO.C.9
	8	3	HSG.CO.C.9
Spiral Review			
Fluency	9–10	1	5.G.B.4

*This problem builds toward the standard shown.

Need More Practice?

Additional Practice resources coming soon.

~ 45 min

Practice Day 1

Purpose: Students practice the concepts, skills, and strategies developed in Lessons 1–11. Consider using this Practice Day as preparation for the upcoming Sub-Unit Quiz.

Preparation

Materials

Option 1: Partner Practice

- Student Edition
- Task Cards, one set per pair
- coloring tools (optional)
- tracing paper (optional)

Option 2: Solve and Swap

- Student Edition
- Task Cards, one card per student
- coloring tools (optional)
- tracing paper (optional)

A **Accessibility: Conceptual Understanding** Invite students to use tracing paper or coloring tools to support them with each task.

Consider asking these questions to check for understanding:

- Task C: "What strategies are you using to draw the figures precisely?"
- Task D: "Does your argument convince you? Does it convince others?"

Standards

Addressing

HSG.CO.C.10
Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; **base angles of isosceles triangles are congruent**; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

Also Addressing: **HSG.CO.C.9, HSG.CO.B.8, HSG.CO.B.7**

Mathematical Practices: MP2, MP3, MP6

Facilitation Options

Option 1: Partner Practice

This structure supports student collaboration and allows pairs to work at their own pace.

Pairs

Distribute one set of cards to each pair.

Invite students to solve each problem in their Student Edition.

Encourage students to discuss their reasoning with their partner as they work on each problem.

Other considerations:

- How will students get feedback?
  - Post the answers and invite students to check them.
  - Invite students to check in with you when they're ready for feedback.
- How will you support student collaboration?
  - Discuss what supporting each other without giving away answers could look or sound like.
  - Encourage pairs to read each problem out loud and solve it before moving on to the next problem.

Option 2: Solve and Swap

This structure supports student collaboration, provides opportunities for movement, and positions students as experts as they discuss problems and support one another.

Revolving Pairs

Distribute one card to each student.

Invite students to solve the problem in their Student Edition.

Encourage students to move around the classroom, find a partner, and solve the problem on their partner's card in their Student Edition. Invite students to compare answers, discuss their reasoning, and work together to revise any mistakes before finding a new partner.

Consider sharing that there are four problems. If students encounter a problem they've already solved, they can compare strategies and solutions with their partner.

Other considerations:

- How will you support student collaboration?
  - Discuss what supporting each other without giving answers could look or sound like.
  - Encourage students to discuss a check-in question prior to discussing the math problems.

SE Practice Day 1

Unit 2
Lessons 1–11

Name: Date: Period:

Practice Day 1

Let's practice what you've learned so far in this unit!



You will use task cards for this Practice Day. Record all of your responses here.

Task A: Angle Properties

1. a. $\angle e$ and $\angle c$
   b. *Responses vary.* $\angle g$ and $\angle h$
   c. $\angle b$ and $\angle d$

2. Explanation:
   Responses vary. $\angle a$ must be supplementary to $\angle e$ and $180 - 66 = 114$.

3. $m\angle a = 66°$ $m\angle b =$ 124° $m\angle c =$ 114° $m\angle d =$ 124°
   $m\angle e =$ 114° $m\angle f =$ 56° $m\angle g =$ 58° $m\angle h = 122°$

Task B: Triangle Congruence Criteria

1.	2.	3.	4.
SSS	SSS	SSS	SSS
ASA	ASA	ASA	ASA
SAS	SAS	SAS	SAS
Not congruent	Not congruent	Not congruent	Not congruent

5. Pair: $\overline{RF} \cong \overline{MD}$
6. Pair: $\angle CYQ \cong \angle LYQ$
7. Pair: $\overline{WF} \cong \overline{KG}$

Unit 2
Lessons 1–11

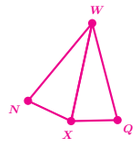
Name: Date: Period:

Practice Day 1 (continued)

Task C: Build a Pair

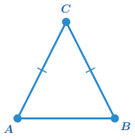
- ☑ A. $\angle N \cong \angle Q$
- ☑ B. $\overline{WN} \cong \overline{WQ}$
- ☐ C. $\overline{NX} \cong \overline{WQ}$
- ☐ D. $\angle NWX \cong \angle QXW$
- ☑ E. $\overline{WX} \ncong \overline{QX}$
- ☐ F. $m\angle NXQ + m\angle QXW = 180°$
- ☑ G. $\overline{XN} \cong \overline{XQ}$
- ☐ H. $\overline{NW} \cong \overline{WX}$

Responses vary.



Task D: Prove It

In an isosceles triangle, the base angles are congruent.



Response:

Responses vary.

Construct point Q at the midpoint of segment $\overline{AB}$, then construct $\overline{CQ}$.
$\overline{AQ} \cong \overline{BQ}$ because of the definition of a bisector.
$\overline{CA} \cong \overline{CB}$ because it is given by the definition of an isosceles triangle.
$\overline{CQ} \cong \overline{CQ}$ because it is the same segment in both triangles.
$\triangle ACQ \cong \triangle BCQ$ by SSS triangle congruence. $\angle A \cong \angle B$ because they are corresponding parts of congruent triangles.

Task Cards (online)

Unit 2
Practice Day 1
(p. 1 of 2)

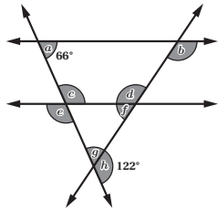
Task Cards

Partner Practice Directions: Make and distribute one copy per pair of students.
Solve and Swap Directions: Make and distribute one copy per four students and give each student one card.

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Task A: Angle Properties

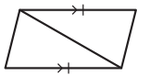
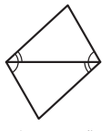
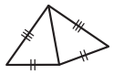
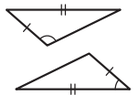
Here are two parallel lines and two transversals.



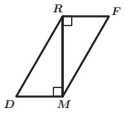
1. Name a pair of:
   a. Vertical angles.
   b. Supplementary angles.
   c. Congruent alternate interior angles.
2. Explain why the measure of $\angle e$ must be 114°.
3. Determine the measures of the labeled angles.

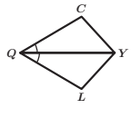
Task B: Triangle Congruence Criteria

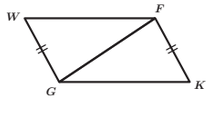
For each pair of triangles, circle the criteria that proves the triangles are congruent.

1. 
2. 
3. 
4. 

Identify one additional pair of congruent corresponding parts so that the congruence criteria applies.

5. SAS


6. ASA


7. SSS


Unit 2
Practice Day 1
(p. 2 of 2)

Task Cards

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Task C: Build a Pair

Here is a list of statements.

A. $\angle N \cong \angle Q$ B. $\overline{WN} \cong \overline{WQ}$ C. $\overline{NX} \cong \overline{WQ}$
D. $\angle NWX \cong \angle QXW$ E. $\overline{WX} \ncong \overline{QX}$ F. $m\angle NXQ + m\angle QXW = 180°$
G. $\overline{XN} \cong \overline{XQ}$ H. $\overline{NW} \cong \overline{WX}$

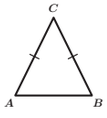
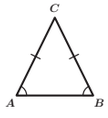
Draw and label a pair of congruent triangles with congruence markings using as many statements as you can.

Hint: Not all of the statements can be used at the same time for a triangle pair.

Task D: Prove It

Here is a plan to prove this conjecture.

In an isosceles triangle, the base angles are congruent.

What do I know?	What do I want to prove?	What strategies can I use?
		I could construct point Q at the midpoint of AB and draw CQ. Then I can use congruent triangles to prove that ∠A ≅ ∠B.

Write a convincing argument that proves this conjecture.

Assess and Respond

Additional data on student thinking can be found in Teacher Reporting.

Sub-Unit Quiz

Independent | ~45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. Text to speech is available in the digital version. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or Skill	Addressed in	DOK	Standard(s)
1	Prove parallel lines using angle relationships	Lesson 3	1	HSG.CO.C.9
2	Identify congruent angle relationships using rigid transformations	Lessons 2–3	1	HSG.CO.C.9
3a	Complete a congruence statement from a sequence of rigid transformations	Lesson 7	2	HSG.CO.B.7 MP6
3b	Identify corresponding parts of congruent triangles	Lessons 6–7	2	HSG.CO.B.7 MP6
4	Use triangle congruence theorems and the definition of congruence in terms of rigid transformations to prove if two triangles are congruent	Lesson 9	2	HSG.CO.B.8 MP3
5	Use corresponding parts of congruent triangles to draw triangles from possible criteria	Lessons 6–7	3	HSG.CO.B.7 MP7

Assessment Resources

- Student Assessments
- Assessment Rubrics with Answers
- Show What You Know PDFs and Answer Keys

Practice

If students need further review or practice, consider assigning the practice problems that correspond to each lesson.

Editable Version

Digital

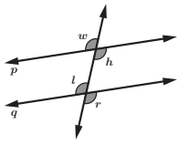
Unit 2
Sub-Unit Quiz

Name: Date: Period:

Sub-Unit Quiz

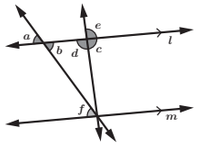
Unit Geo.2

1. Which angle relationship would prove line p is parallel to line q?



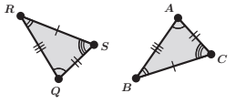
- A. $m\angle h + m\angle l = 180$
- B. $\angle l \cong \angle h$
- C. $m\angle h + m\angle w = 180$
- D. $\angle l \cong \angle r$

2. Select *all* the angles that are congruent to $\angle f$ if lines l and m are parallel.



- ☑ A. $\angle a$
- ☑ B. $\angle b$
- ☐ C. $\angle c$
- ☐ D. $\angle d$
- ☐ E. $\angle e$

3. Sora performed a sequence of rigid transformations to stack these two trianglesperfectly.



a. Complete the congruence statement.

$\triangle ABC \cong \triangle$ *QRS*

b. Complete the statements to identify corresponding congruent parts.

$\angle A \cong \angle Q$

∠C $\cong \angle S$

∠B $\cong \angle R$

$\overline{AB} \cong$ *$\overline{QR}$*

$\overline{BC}$ $\cong \overline{RS}$

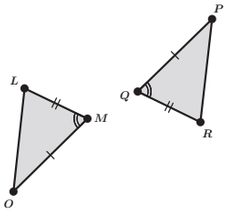
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Name: Date: Period:

Sub-Unit Quiz (continued)

Uni

4. Are these two triangles congruent?



Yes No Not enough information

Write a convincing argument for your choice.

Responses vary.

- The triangles must be congruent because they have two corresponding pairs of congruent sides and a pair of corresponding congruent angles that are adjacent to those sides. By SAS, these triangles are congruent.
- I can use rigid transformations to perfectly stack $\triangle LMO$ onto $\triangle RQP$. I can translate $\triangle LMO$ so that point M goes to point Q, and then rotate $\triangle LMO$ a half turn (180°) so that point L stacks onto point R and point O stacks onto point P.

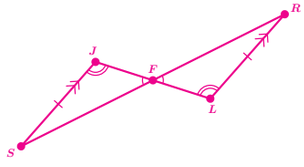
5. Here are some statements about two triangles.

- ☑ $\angle JFS$ and $\angle RFL$ are vertical angles.
- ☑ $\overline{JS} \cong \overline{LR}$
- ☐ $\angle SJF \cong \angle LRF$
- ☐ $\angle SJF \ncong \angle RLF$
- ☐ $\overline{JS}$ is *not* parallel with $\overline{LR}$
- ☑ $\angle SJF$ and $\angle LRF$ are *not* in the same triangle.
- ☑ $\angle SJF$ and $\angle RLF$ are alternate interior angles.

Draw and label one pair of congruent triangles with congruence markings, using *at least three* of these statements. Not all of the statements can be used in the same triangle pair.

Check off each statement as you use it.

Responses vary.



D Differentiation (Sub-Unit Quiz)

Note: To **Strengthen** and **Stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:
Use rigid transformations to prove angle relationships. Describe a strategy using rigid transformations that proves a relationship is always true. **(Lessons 1–4)**	1–2	**Support** • Inviting students to read the **Summary** of each lesson in this Sub-Unit and write it in their own words or paraphrase it to a caregiver or classmate. • Revisiting **Lesson 2, Activity 1** (*Always True?*) and inviting students to paraphrase how transformations always create parallel lines. • Revisiting **Lesson 3, Activity 2** (*Angles and Transversals*) and inviting students to use transformation tools to illustrate conjectures about alternate interior angles and parallel lines.
Define congruence using rigid transformations. **(Lesson 5–7)**	3, 5	**Support** • Revisiting the **Pre-Unit Check, Screens 10–11** and inviting students to reflect on what strategies are used to identify corresponding parts. • Revisiting the **Lesson 7 Summary** (*Corresponding Components*) and annotating key information. • Revisiting a few related **practice problems** in **Lesson 7** and giving individualized feedback.
Justify how much information is needed to decide that two triangles are congruent. **(Lessons 8–11)**	4	**Support** • Revisiting **Lesson 9, Activity 2** (*Congruence Conditions*) and inviting students to make sense of which diagrams represented which congruence conditions. • Inviting students who need support with writing convincing arguments to complete the **Try This** problem from **Lesson 9** and offering individualized feedback.

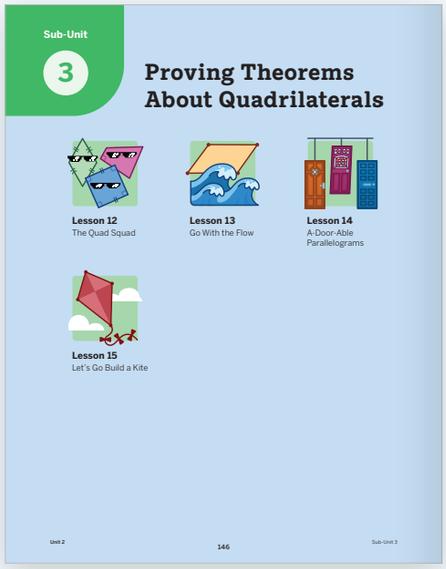
Sub-Unit 3

Proving Theorems About Quadrilaterals

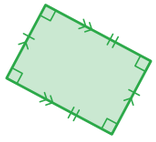
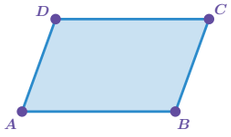
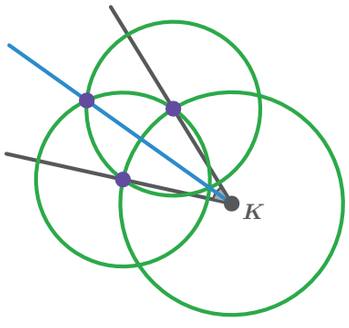
Sub-Unit 3 Goals

- **Goal:** Make sense of constructions using properties and definitions of geometric figures.
- **Language Goal:** Prove theorems about quadrilaterals.

In this sub-unit, students apply their understanding of proving angles and triangles congruent from Sub-Units 1 and 2 to support them in proving theorems about quadrilaterals. They make sense of the definitions of specific quadrilaterals and the implications of those definitions. They use strategic reasoning as they construct diagonals to create congruent triangles in parallelograms, and they apply triangle congruence criteria to prove properties of parallelograms. Students also use properties of quadrilaterals to support them in developing strategies for constructing the **angle bisector** of a given angle.



Math That Matters Most

Concept or Skill	Example
Analyze a quadrilateral by its properties.	Based on the markings in the diagram, what names could apply to this quadrilateral?  Since opposite sides are parallel and congruent, this figure is called a parallelogram. It also has four equal angles, so it's a rectangle too.
Prove theorems about parallelograms.	Prove that the adjacent angles in parallelogram $ABCD$ are supplementary.  Opposite sides are parallel in a parallelogram, so $\overline{DC} \parallel \overline{AB}$. $\angle A \cong \angle C$ because they are alternate interior angles. $\angle C$ and $\angle D$ form a straight angle, so they are supplementary. This means $\angle A$ and $\angle D$ are also supplementary.
Construct the angle bisector of a given angle.	

Unit 2

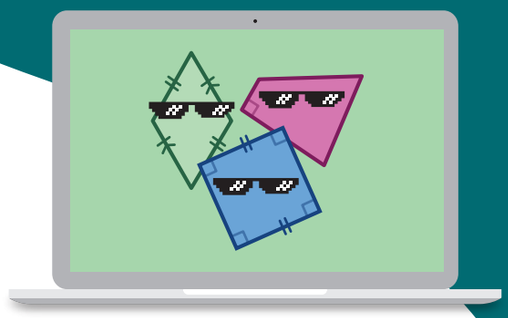
Lesson 12

OPTIONAL

This lesson is optional as it addresses standards from a prior course, supporting students in surfacing prior knowledge

Digital Lesson

This lesson uses digital student screens. Student Notes are also available.



The Quad Squad

Defining Quadrilaterals

Let's identify quadrilaterals by using their definitions.

Focus and Coherence

Today's Goals

1. **Goal:** Create quadrilaterals according to their properties, names, and definitions.
2. **Language Goal:** Analyze quadrilaterals according to their properties, names, and definitions. **(Reading, Writing, Speaking, and Listening)**

Students explore various *quadrilaterals* and analyze the common properties between some quadrilaterals based on their names and definitions. They play a collaborative game of Polygraph to surface language for describing quadrilaterals by their properties. Students are introduced to the definitions for specific quadrilaterals and attend to precision as they use those definitions to make figures based on their properties. **(MP6)**

Prior Learning

In Grade 3, students learned that shapes with different names (such as rhombuses, rectangles, and squares) can share attributes with and be part of a larger category (such as parallelograms or quadrilaterals). In Grade 6, Unit 1, students identified parallelograms and calculated their area. In Grade 7, Unit 7, students constructed polygons with specific features.

Future Learning

In Lessons 13–15, students will prove conjectures about parallelograms and kites.

Rigor and Balance

- Students develop **adaptive reasoning** as they consider how definitions can support them in analyzing the properties of different quadrilaterals.
- Students have opportunities to develop a **productive disposition** as they play a collaborative game to surface language for describing quadrilaterals by their properties.

Vocabulary

Review Vocabulary

kite	*parallelogram*
quadrilateral	*rectangle*
rhombus	*square*

Standards

Building Toward

HSG.CO.A.1

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Also Building Toward: **HSG.CO.C.11**

Mathematical Practices: MP1, MP3, MP6

Building On

3.G.A.1
5.G.B.3
6.G.A.1
7.G.A.6

Lesson at a Glance

~ 45 min

Standards: Building Toward HSG.CO.A.1, HSG.CO.C.11



Why digital?

The digital version of Polygraph coordinates pairings and the question exchange to support students in playfully developing language to describe quadrilaterals by their properties.

Warm-Up

Pairs Sharing a Device | 5 min

Pacing: Screen 1

Students use the Which One Doesn't Belong? routine to surface language for identifying properties of *quadrilaterals*.



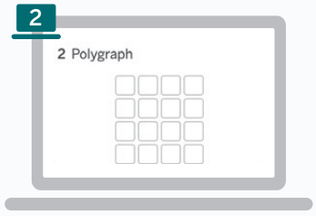
Activity 1

Pairs Sharing a Device | 15 min

Pacing: Screen 2

Students play a collaborative game to surface language for describing quadrilaterals by their properties.

Key Takeaway (Screen 2): Polygons with exactly four sides are called *quadrilaterals*. One way to identify a quadrilateral is by analyzing its properties.



Activity 2

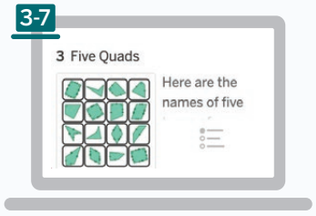
Pairs Sharing a Device | 10 min

Pacing: Screens 3–4, Screens 5–6, Screen 7

Students explore the definitions of specific quadrilaterals and attend to precision as they use those definitions to make figures.

Routine: Notice and Wonder **(MP6)**

Key Takeaway (Screen 6): Names and definitions of quadrilaterals can help us identify common properties in figures with the same name. A figure can have more properties than what is mentioned in its definitions.



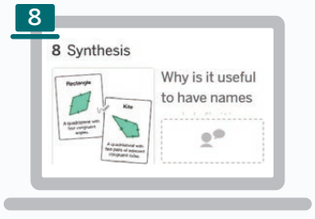
Synthesis

Whole Class | 5 min

Pacing: Screen 8

Students demonstrate their understanding by using definitions of quadrilaterals to determine if it's possible to make a *rhombus* that is *not a kite*.

Lesson Takeaway: *Quadrilaterals* are four-sided polygons. Some quadrilaterals have specific properties that help to identify them by name and definition.
See the Synthesis page for full Lesson Takeaway.

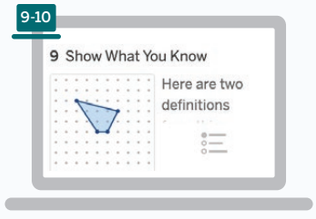


Show What You Know

Independent | 5 min

Pacing: Screens 9–10

Students demonstrate their understanding by using definitions of quadrilaterals to determine if it's possible to make a *rhombus* that is *not a kite*.



Prep Checklist

Assign the digital student screens.
Student Notes are also available.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)

Student Notes sheet in Student Edition:
Consider providing the Student Notes to support class discussion and student thinking.



Pairs Sharing a Device | 5 min | Pacing: Screen 1

Warm-Up

Purpose: Students use the **Which One Doesn't Belong?** routine to surface language for identifying properties of *quadrilaterals*.



Student Notes are also available

1 Launch

1 **Use the Which One Doesn't Belong?** routine to support students in noticing and precisely describing the properties of quadrilaterals. **(MP3, MP6)**

Consider sharing that each quadrilateral does not belong in some way, and we can assume that the diagrams are drawn to scale. Encourage students to look for more than one possibility.

2 Connect

Invite students to share at least one reason why *each* figure doesn't belong.

Create a record of why each figure doesn't belong. Include the name of the student who shared each reasoning.

Math Identity and Community Celebrate the mathematical brilliance of students attending to precision. (e.g., "I saw _____ attending to precision in their explanation by describing the opposite sides as congruent."). **(MP6)**

Consider asking, "You can call each of these figures a *quadrilateral*. What do you think that means?"

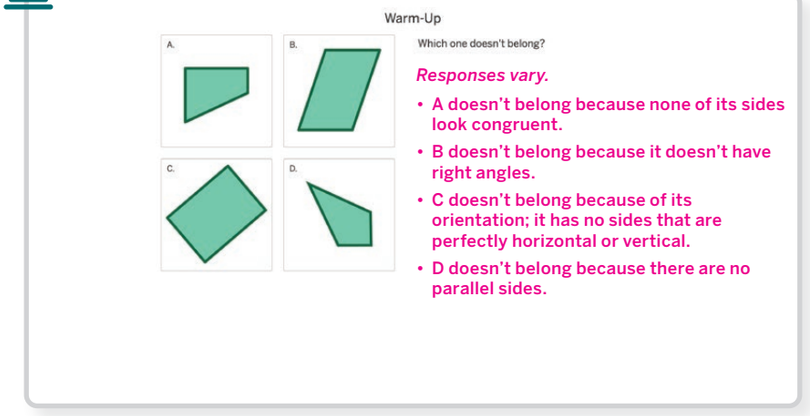
ML/EL **Multilingual/English Learners** Invite students to share what each part of the term *quadrilateral* reminds them of (e.g., "When I hear the word *quad*, I think of..." and "When I hear the word *lateral*, I think of..."). Then invite students to share what they think it means when the two words are used together. **(Listening and Speaking)**

Consider asking:

- "How is quadrilateral B similar to the other quadrilaterals? How is it different?"
- "What is another way to explain why quadrilateral C doesn't belong?"
- "Is there another reason why quadrilateral D doesn't belong?"

Students using digital

1



Pairs Sharing a Device | 15 min | Pacing: Screen 2

Activity 1 Polygraph

Purpose: Students play a collaborative game to surface language for describing quadrilaterals by their properties.

Short on time? Consider playing fewer rounds of Polygraph.



Student Notes are also available

1 Launch

2 **Invite students** to work on their own devices. Students can start by playing a practice round against the computer. Then the platform will assign them with a random partner for each round of the game.

2 Monitor

Encourage students to try and guess the card using as few questions as possible.

Note: Students will explore the definitions of these figures later in this lesson.

Capture interesting or effective questions about topics, such as: congruent and non-congruent sides, angles, diagonals, and other properties.

Pause briefly to discuss common questions and terms that students are using.

Invite students using notes to record any words or phrases they would like to try using.

Invite students to continue playing Polygraph and referencing the anchor chart if necessary.

A **Accessibility: Executive Functioning** To support students carrying out multiple steps, encourage them to focus on one image at a time when they are determining which images to eliminate.

ML/EL **Multilingual/English Learners** Consider pairing students of different English language proficiency levels, so students can work together to interpret and write questions. **(Speaking and Listening)**

3 Connect

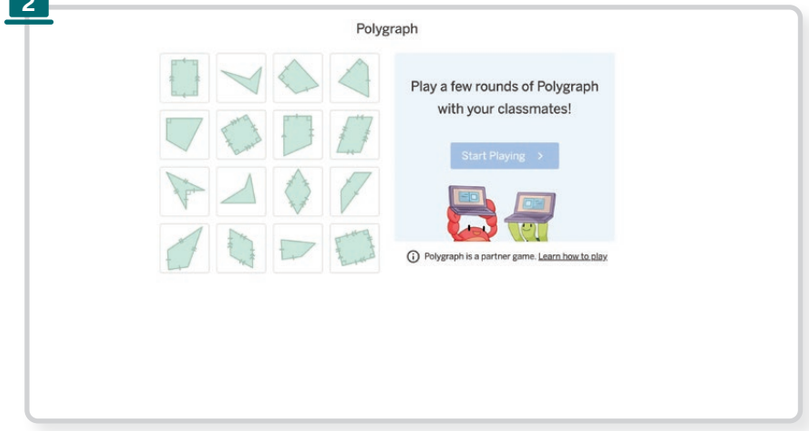
To encourage playfulness, consider asking, "What were some fun or interesting questions that were asked?"

To support making connections, consider asking, "What words or phrases did you find particularly helpful?"

Math Identity and Community Celebrate the variety of language used during this activity and acknowledge that there are many ways we can communicate about geometric figures.

Students using digital

2



Key Takeaway: Polygons with exactly four sides are called *quadrilaterals*. One way to identify a quadrilateral is by analyzing its properties.

Pairs Sharing a Device | 10 min | Pacing: Screens 3–4, Screens 5–6, Screen 7

Activity 2 Five Quads

Purpose: Students explore the definitions of specific quadrilaterals and attend to precision as they use those definitions to make figures. **(MP6)**



Student Notes are also available

1 Launch

3 **Use the Notice and Wonder routine** to promote curiosity and help students make sense of the properties of each specific quadrilateral. **(MP1)**

Encourage students to click on each quadrilateral term and identify any shared properties among the highlighted figures.

Consider asking:

- "Is there any figure you are surprised to see highlighted with a specific name?"
- "Based on the highlighted figures, what must be true of all kites? Rhombuses? Rectangles? Parallelograms? Squares?"

Invite students using notes to record the definition of each quadrilateral.

Math Identity and Community Consider celebrating the variety and creativity in what students notice and wonder. Include some things that surprised you or that you think other students may not have noticed.

ML/EL **Multilingual/English Learners** Invite students to research how to say the new names in other languages to increase access to the task. Create an anchor chart with the names in multiple languages. **(Reading and Listening)**

2 Monitor

4 **Encourage students** to read the definition aloud with a partner before dragging any points. Then invite students to press "Analyze" and use the feedback to help them with their thinking.

ML/EL **Multilingual/English Learners** Invite students to turn on the "Text to Speech" feature to support their reading of the definitions. This feature is available in the Accessibility Settings area in the student sidebar (press the hamburger in the top-left corner of an activity). **(Reading and Listening)**

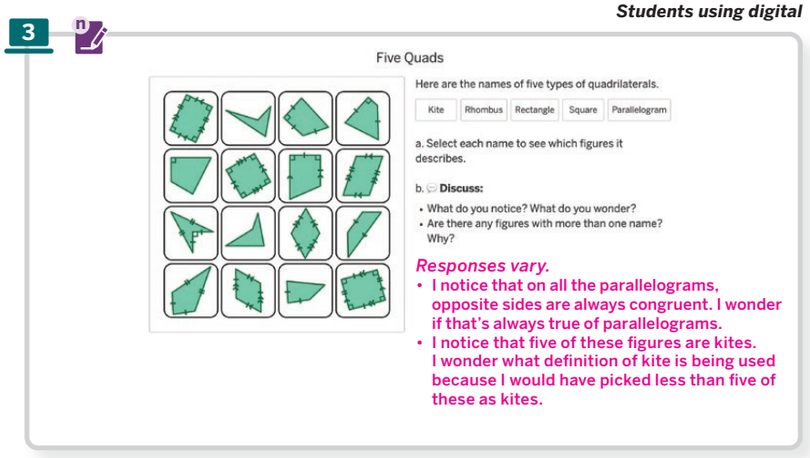
Pause briefly to discuss what students noticed. Consider asking: "What properties did the scanner check for each quadrilateral?"

Note: Students may see a "?" when it is ambiguous whether the quadrilateral fits the definition. If there is time, invite students to discuss whether they think the definition has been met.

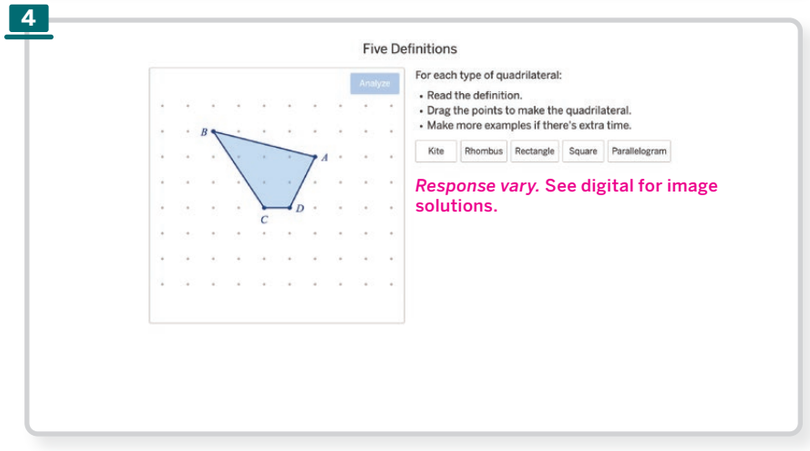
Invite students to continue to Screen 5 using the Pace tool.

Students using digital

3



4



Activity 2 continued >

Pairs Sharing a Device | 10 min | Pacing: Screens 3–4, Screens 5–6, Screen 7

Activity 2 Five Quads (continued)

Purpose: Students explore the definitions of specific quadrilaterals and attend to precision as they use those definitions to make figures. **(MP6)**



Student Notes are also available

2 Monitor

5 **Encourage students** to think with a partner about what each option could look like before dragging the points.

Invite students to notice which names are highlighted as they drag the points, and which are not.

Note: When students create a specific quadrilateral precisely, the names that are represented by the quadrilateral will be highlighted.

Encourage students to use the feedback to support their thinking.

D **Differentiation:** (Support) Some students may think they can create a rectangle that's not a parallelogram, or a rhombus that's not a kite. Invite these students to return to Screen 4 and read the definitions of the relevant quadrilaterals. Consider asking, "Why is it impossible to create a rectangle that's not a parallelogram?"

6 **To encourage creativity,** invite students to spend several minutes inventing a new quadrilateral. Encourage students to try inventing a quadrilateral that's unique (or one that they've never seen before) and give it a fun name.

Capture several student-invented quadrilaterals and their definitions to share during the Connect.

7 D **Differentiation: Explore More.** (Stretch) Invite students who would like to explore the names and definitions of quadrilaterals further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

6 **Share** the captured student-invented quadrilaterals and definitions. Invite several students to share their thinking.

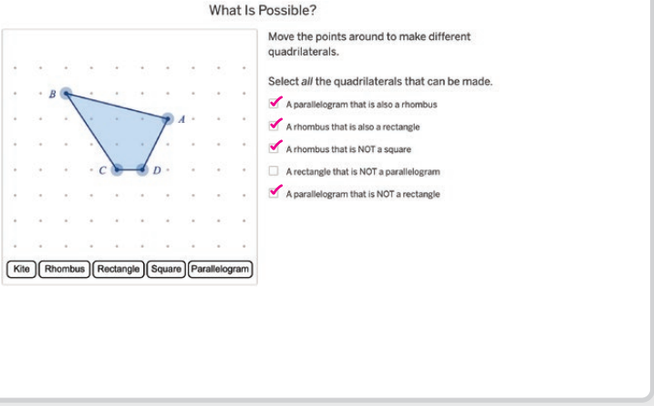
Math Identity and Community Celebrate the creatively invented quadrilaterals. Ask the students to talk about what inspired their creation.

To surface the Key Takeaway, consider asking:

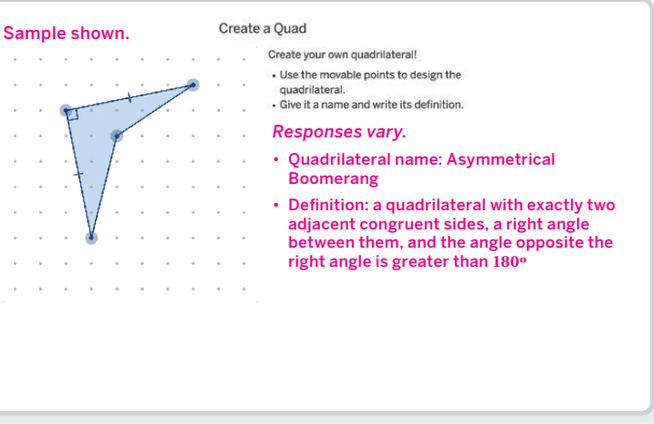
- "How does the name of a quadrilateral help communicate what it might look like? What about the definition?" **(MP6)**
- "Do definitions have to list every property that every shape with the same name has in common? Why or why not?" No; definitions often don't list all common properties of a shape. For example, a rhombus is a name that describes all quadrilaterals with four congruent sides. All rhombuses also have opposite sides that are parallel.

Students using digital

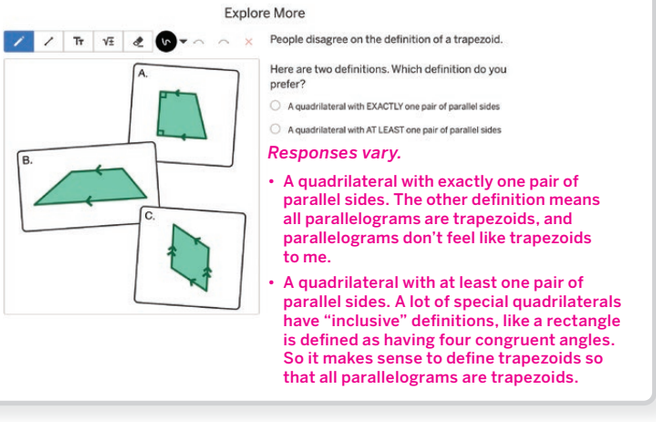
5



6



7



Key Takeaway: Names and definitions of quadrilaterals can help us identify common properties in figures with the same name. A figure can have more properties than what is mentioned in its definitions.

Whole Class | 5 min | Pacing: Screen 8

Synthesis



Student Notes are also available

8 **Invite students** to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

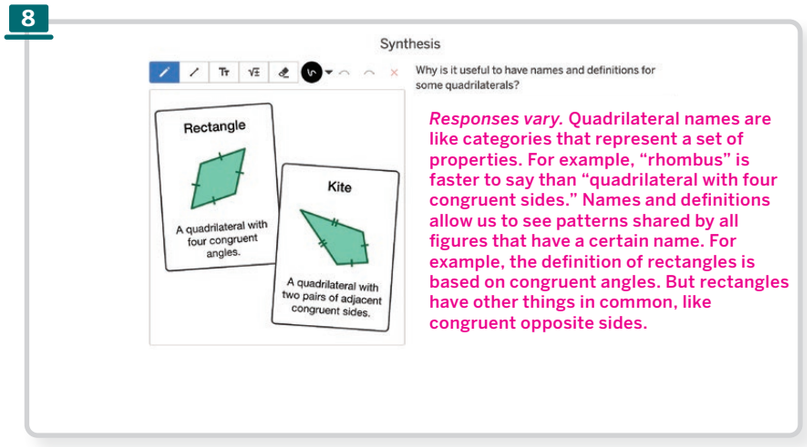
- Quadrilateral names are like categories that represent a set of properties.
- A quadrilateral's name can help us determine what properties we expect it to have.
- Names are also faster ways of communicating about a specific set of properties (e.g., "rhombus" is a faster way of saying "quadrilateral with four congruent sides").
- Definitions help us identify the key property that defines a quadrilateral, even though quadrilaterals with the same name can have more properties than what is mentioned in its definition.
- Names and definitions help us communicate with precision and clarity when we're talking about the same figure or category of figures. **(MP6)**
- Some quadrilaterals have overlapping properties but not overlapping definitions.

Create an anchor chart to support students in identifying the quadrilaterals defined in this lesson by their properties using students' language. Include examples and diagrams and display it for students to reference throughout the rest of the unit.

Math Identity and Community Invite students to share strategies they found helpful and give credit to the students who shared them.

Note: If time allows, invite students to revise their previous thinking based on the discussion.

8



Lesson Takeaway: *Quadrilaterals* are four-sided polygons. Some quadrilaterals have specific properties that help to identify them by name and definition.

- A *kite* is a quadrilateral with two pairs of adjacent congruent sides.
- A *rhombus* is a quadrilateral with four congruent sides.
- A *rectangle* is a quadrilateral with four congruent angles.
- A *square* is a quadrilateral with four congruent angles and sides.
- A *parallelogram* is a quadrilateral with opposite sides that are parallel.

Independent | 5 min | Pacing: Screens 9–10

Show What You Know

Purpose: Students demonstrate their understanding by using definitions of quadrilaterals to decide whether it is possible to make a *rhombus* that is *not* a *kite*.

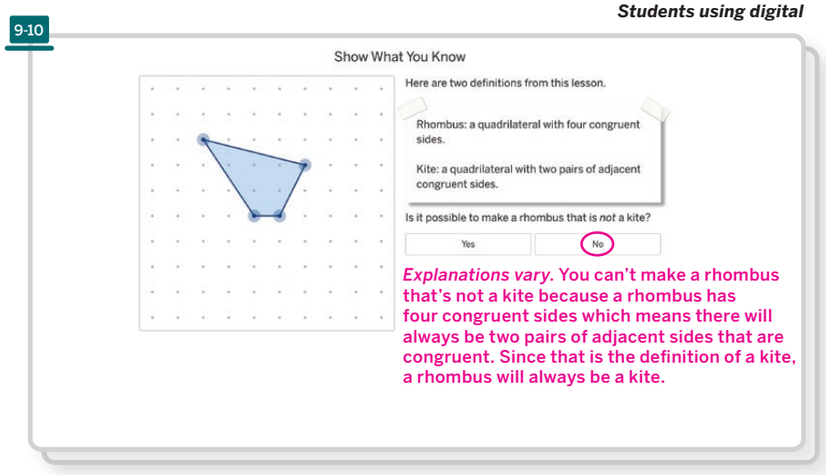


Students using print

9-10 Today's Goals

1. **Goal:** Create quadrilaterals according to their properties, names, and definitions.
2. **Language Goal:** Analyze quadrilaterals according to their properties, names, and definitions. **(Reading, Writing, Speaking, and Listening)**

Students using digital



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

S Support

Provide targeted intervention.

If student work shows **partial understanding** of identifying quadrilaterals by its properties, consider:

- Revisiting **Activity 1, Screen 4** as a class and inviting students to share strategies for identifying quadrilaterals by their definitions.
- Inviting students to read the **Summary** and highlight key components.

S Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice.**
- Inviting students to review the **Summary** and complete the **Try This** problem.
- Revisiting **Activity 1** and inviting students to play some additional rounds of Polygraph.

S Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting them to complete the **Explore More** task and discuss their thinking with a partner.
- Encouraging them to complete the **DOK3 practice problems**.

Professional Learning

In this lesson, students learned how to identify quadrilaterals by playing a collaborative game of Polygraph. How did playing a game support students in experiencing math as delightful? When in the lesson did you see evidence of students experiencing math as joy? What were some ways that you celebrated math as joy in today's lesson?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson # **Practice**

Students using digital

Lesson Summary

Quadrilaterals can look very different from each other, and some quadrilaterals are defined by specific properties. Here are the names and definitions of five quadrilaterals.

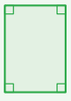
Kite
A quadrilateral with two pairs of adjacent congruent sides.



Rhombus
A quadrilateral with four congruent sides.



Rectangle
A quadrilateral with four congruent angles.



Square
A quadrilateral with four congruent angles and sides.



Parallelogram
A quadrilateral with opposite sides that are parallel.



These defining properties can sometimes overlap. For example, a rhombus is also a kite because it has two pairs of adjacent sides. A square is both a rhombus and a rectangle because it has four congruent sides and four congruent angles. The definitions of quadrilaterals often imply that there are additional properties. For example, a rectangle has four congruent angles, which means that each of its angles must be a right angle.

Try This

Select *all* the names that apply to this quadrilateral.

- ☐ **A.** Rectangle
- ☐ **B.** Square
- ☑ **C.** Parallelogram
- ☑ **D.** Kite
- ☑ **E.** Rhombus



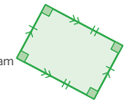
Unit 2 Lesson 12 148 Lesson Summary | Try This

Lesson Practice
Geo.2.12

Name: _____ Date: _____ Period: _____

Problems 1–2: Select *all* the names that apply to the quadrilateral.

1.
- ☑ **A.** Rectangle
- ☐ **B.** Square
- ☑ **C.** Parallelogram
- ☐ **D.** Kite
- ☐ **E.** Rhombus

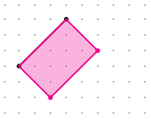


2.
- ☐ **A.** Rectangle
- ☐ **B.** Square
- ☑ **C.** Parallelogram
- ☑ **D.** Kite
- ☑ **E.** Rhombus

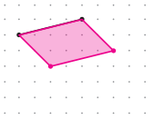


Problems 3–5: Draw the remaining three sides to complete each quadrilateral.

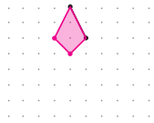
3. Rectangle



4. Parallelogram



5. Kite



6. Which of these quadrilaterals are possible to make? Select *all* that apply.

- ☑ **A.** A parallelogram that is also a rhombus
- ☑ **B.** A rectangle that is a kite
- ☑ **C.** A rhombus that is *not* a rectangle
- ☐ **D.** A square that is *not* a rhombus
- ☐ **E.** All of the above

7. Is it possible to construct a figure that can be described by *exactly* four of these quadrilateral names? Quadrilateral names: rectangle, square, parallelogram, kite, rhombus
Explain your thinking.

No. *Explanations vary.* If a quadrilateral is a square, all the other names will apply. So I can't make a square. But if my constructed figure is a rectangle, parallelogram, kite, and rhombus, then it has to also be a square. So it's not possible to construct a figure with exactly four of the quadrilateral names.

Unit 2 Lesson 12 149 Lesson Practice

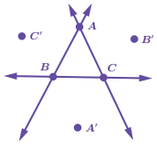
Lesson Practice
Geo.2.12

Name: _____ Date: _____ Period: _____

8. In this diagram:
- $\overline{AB} \cong \overline{AC}$.
- Points A', B', and C', are all reflections over the opposite side of triangle ABC.

How many kites can you make using any of these points as vertices?

Three kites



Spiral Review

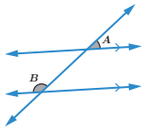
Problems 9–10: Two parallel lines are crossed by a transversal.

9. Determine a possible pair of values for angle A and angle B.

Responses vary.

Angle A: 40°

Angle B: 140°



10. Could angle A and angle B both be obtuse angles?
Explain your thinking.

No. *Explanations vary.* Angles A and B must add up to 180°. If both are obtuse angles, then they will add up to more than 180°.

Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

Unit 2 Lesson 12 150 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-lesson			
	1	1	5.G.B.3
Test Practice	**2**	1	5.G.B.3
	3–5	2	5.G.B.3
	6–7	3	HSG.CO.C.11*
	8	2	HSG.CO.C.11*
Spiral Review			
Fluency	**9**	1	8.G.A.5
	10	1	8.G.A.5

*This problem builds toward the standard shown.

Need More Practice?

Additional Practice resources coming soon.

Unit 2

Lesson 13

Digital Lesson

This lesson uses digital student screens. Student Notes are also available.



Go With the Flow

Proving Theorems About Parallelograms

Let's prove theorems about the properties of parallelograms.

Focus and Coherence

Today's Goals

1. **Goal:** Prove that parallelograms have opposite sides and angles that are congruent.
2. **Goal:** Use congruent triangles to prove theorems about parallelograms.
3. **Language Goal:** Analyze a proof to make it stronger and clearer. **(Reading, Speaking, and Listening)**

Students use the structure of flow diagrams to prove theorems about the properties of parallelograms, specifically that parallelograms have opposite sides and angles congruent. Students construct congruent triangles to help prove that parts of parallelograms are congruent. They also analyze first drafts of proofs to make them stronger and clearer. **(MP6, MP7)**

Prior Learning

In Lesson 6, students proved that corresponding parts of congruent figures are congruent. In Lesson 9, students determined the minimum criteria for proving triangle congruence. In Lesson 12, students revisited the definitions of specific quadrilaterals, including parallelograms.

Future Learning

In Lesson 14, students will prove additional theorems about the diagonals of parallelograms, such as all rectangles having congruent diagonals and the diagonals of parallelograms bisecting each other.

Rigor and Balance

- Students **apply** their understanding of corresponding parts in congruent triangles always being congruent in order to prove the properties of parallelograms.
- Students develop **adaptive reasoning** as they reflect on drafts of proofs to make them stronger and clearer.

Standards

Addressing

HSG.CO.C.11
Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

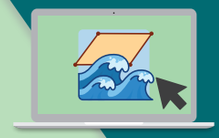
Mathematical Practices: MP3, MP6, MP7

Building On	Building Toward
HSG.CO.B.7	**HSG.CO.C.11**

Lesson at a Glance

~ 45 min

Standard: HSG.CO.C.11



Why digital?

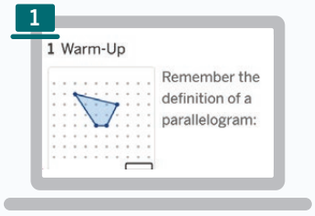
Students use an interactive flow diagram to make sense of how to prove conjectures about parallelogram properties.

Warm-Up

Pairs Sharing a Device | 5 min

Pacing: Screen 1

Students use the definition of a parallelogram to construct a variety of figures.



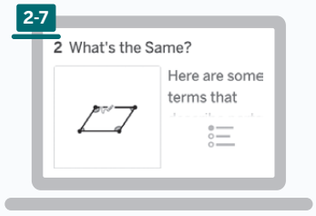
Activity 1

Pairs Sharing a Device | 20 min

Pacing: Screens 2–6, Screen 7

Students apply strategies for proving triangle congruence to prove opposite sides of a parallelogram are congruent. MLR3: Critique, Correct, Clarify

Key Takeaway (Screen 6): Strong arguments often come in an order that makes them more convincing. This can mean you first prove triangle congruence within a figure before proving the properties of a parallelogram.



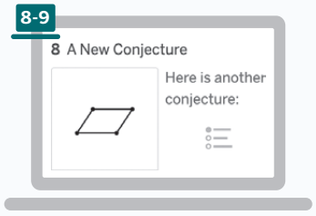
Activity 2

Pairs Sharing a Device | 10 min

Pacing: Screens 8–9

Students prove that the opposite angles of a parallelogram are congruent.

Key Takeaway (Screen 9): Opposite angles in parallelograms are congruent. To prove this conjecture, you can make congruent triangles by constructing a diagonal and using corresponding pairs of congruent parts.



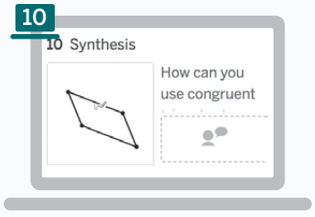
Synthesis

Whole Class | 5 min

Pacing: Screen 10

Students synthesize their understanding of how congruent triangles can help to prove the properties of parallelograms.
MLR2: Collect and Display

Lesson Takeaway: Constructing diagonals in parallelograms creates congruent triangles. By creating congruent triangles, you can use their corresponding congruent parts to prove properties of parallelograms. Flow diagrams are one way to organize a proof.

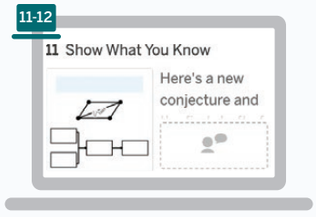


Show What You Know

Independent | 5 min

Pacing: Screens 11–12

Students demonstrate their understanding by critiquing the first draft of a proof about a property of parallelograms.



Prep Checklist

Assign the digital student screens. Student Notes are also available.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)
- Activity 2 Cards (one set per pair)
- blank paper

Student Notes sheet in Student Edition:
Consider providing the Student Notes to support class discussion and student thinking.



Pairs Sharing a Device | 5 min | Pacing: Screen 1

Warm-Up

Purpose: Students use the definition of a parallelogram to construct a variety of figures.



Student Notes are also available

1 Launch

1 **To connect to prior knowledge**, consider asking, "According to the definition, what properties does a parallelogram have?"

To support students getting started, invite them to press "Analyze" and make sense of the feedback.

Encourage students to make as many different parallelograms as they can.

2 Connect

Capture and share unique parallelograms that students create.

Math Identity and Community Celebrate the variety and creativity of parallelograms made by the class.

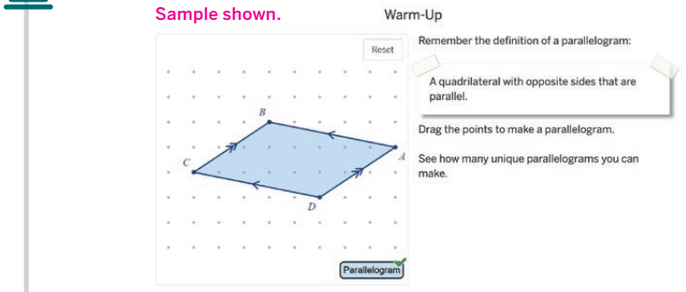
Consider asking:

- "What do all of our parallelograms have in common? How do you know?"
- "How did you use the grid to make parallel sides?"
- "Were any of our parallelograms unique in any way?"
- "Do any of our parallelograms have other names they could be called by?"

Students using digital

1

Sample shown.



Pairs Sharing a Device | 20 min | Pacing: Screens 2–6, Screen 7

Activity 1 Opposite Sides Congruent

Purpose: Students apply strategies for proving triangle congruence to prove opposite sides of a parallelogram are congruent.



Student Notes are also available

1 Launch

2 **To encourage playfulness,** invite students to predict with their partner which parts will be highlighted before they click on each term.

ML/EL **Multilingual/English Learners** Encourage students to make sense of each term that's shown. Consider asking, "What words do you think of when you hear the word *opposite*? *Adjacent*?" **(Speaking and Listening)**

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to choose and make sense of one term at a time.

Invite students using notes to describe each term in their own words in the left column.

Capture and share several conjectures that students create. Consider asking, "How are these conjectures connected to the terms?"

2 Monitor

3 **Look for** a variety of ways students explore this screen.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to drag the vertices to make sense of the conjecture.
Drag the vertices onto one another to create an overlapping segment.	**Strengthen** Celebrate their strategy. Consider asking, "Will this idea show that all parallelograms have opposite sides that are congruent?"
Reference a variety of parallelograms to help inform their thinking.	**Stretch** Celebrate their reasoning. Invite them to share during the Connect why they felt it was valuable to reference multiple parallelograms.

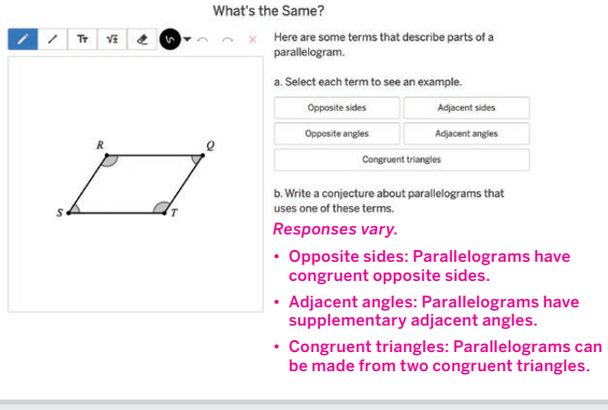
4 **Look for** the different ways that students diagram the parallelograms on this screen.

D **Differentiation:** (Support) Some students might only mark one set of opposite sides as congruent. Consider asking, "If you prove that one set of opposite sides is congruent, have you proved the entire conjecture? Why or why not?"

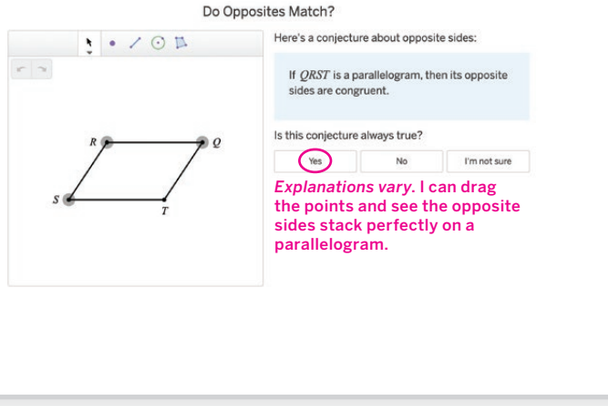
Activity 1 continued >

Students using digital

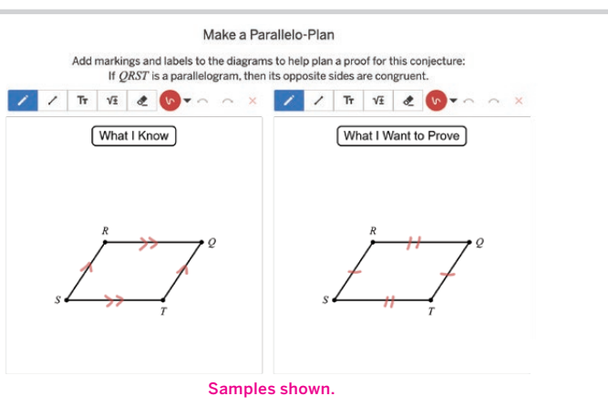
2



3



4



Pairs Sharing a Device | 20 min | Pacing: Screens 2–6, Screen 7

Activity 1 Opposite Sides Congruent (continued)

Purpose: Students apply strategies for proving triangle congruence to prove opposite sides of a parallelogram are congruent.



Student Notes are also available

2 Monitor

5 **Look for** several ways that students create congruent triangles in the parallelogram using the dashboard's Teacher View. Students may draw 1–2 diagonals and identify different pairs of triangles.

Note: The parallelogram on Screen 6 has a diagonal from point Q to point S. For students who drew a different diagonal, encourage them to make sense of the proof using the diagonal they drew. Consider asking, "How does the way the diagonal is drawn change what needs to be proved?"

6 **Pause** to facilitate a class discussion.

Consider sharing that this proof is organized in a flow diagram. Encourage students to make sense of the diagram's structure. Consider asking: **(MP7)**

- "What do the arrows mean? Why are the tiles arranged in this way?"
- "What information is included inside each tile?"

A **Accessibility: Executive Functioning** To support multi-step thinking, invite students to make sense of the tiles one at a time and analyze how each tile connects to the next one. Consider asking, "How does knowing that $QRST$ is a parallelogram help you prove these two triangles are congruent by ASA?"

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in the first draft of the proof. As a class:

- Discuss what is *clear* about the proof.
- Discuss what is *incorrect* or *unclear*.
- Discuss what could be added to strengthen the proof.

Invite students to continue to the next screen using the Pace tool.

7 **Demonstrate** how to click on each tile to place it in the flow diagram and how to undo selections.

Invite students to make arguments and justify their reasoning for the placement of each tile. **(MP3)**

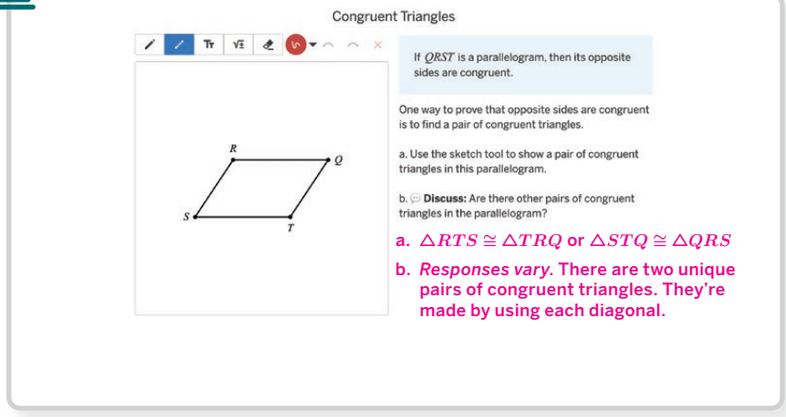
3 Connect

To surface the Key Takeaway, consider asking:

- "How did you decide where to place each tile?"
- "Does the order of the tiles in the center column matter? Why or why not?"
- "How was the flow diagram helpful for organizing the proof of this conjecture?" **(MP7)**

Students using digital

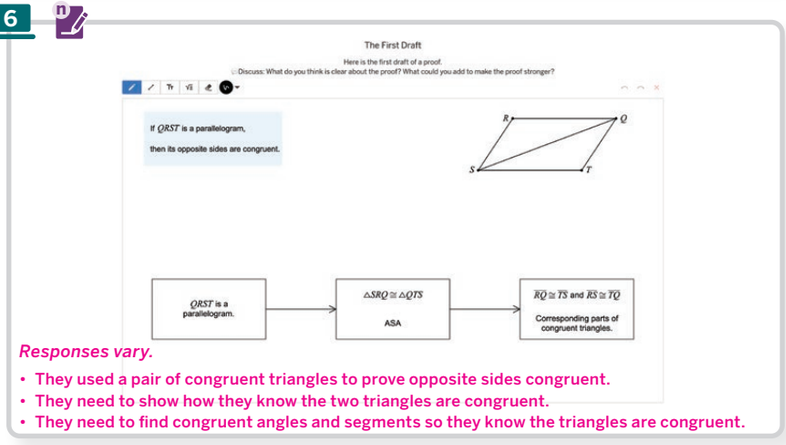
5



a. $\triangle RTS \cong \triangle TRQ$ or $\triangle STQ \cong \triangle QRS$

b. *Responses vary.* There are two unique pairs of congruent triangles. They're made by using each diagonal.

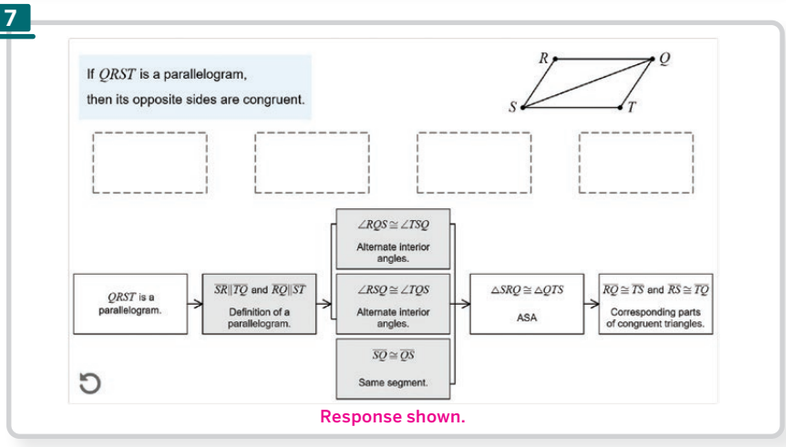
6



Responses vary.

- They used a pair of congruent triangles to prove opposite sides congruent.
- They need to show how they know the two triangles are congruent.
- They need to find congruent angles and segments so they know the triangles are congruent.

7



Response shown.

Key Takeaway: Strong arguments often come in an order that makes them more convincing. This can mean you first prove triangle congruence within a figure before proving the properties of a parallelogram.

Pairs Sharing a Device | 10 min | Pacing: Screens 8–9

Activity 2 Opposite Angles Congruent

Purpose: Students prove that the opposite angles of a parallelogram are congruent.



Student Notes are also available

Short on time? Consider completing the flow diagram as a class.

1 Launch

8 **Consider sharing** that in this activity students will look at a new conjecture: The opposite angles of a parallelogram are congruent.

Encourage students to decide whether they think the conjecture is true before exploring the diagram.

Consider asking, "How is this conjecture similar to the last conjecture? How is it different?"

2 Monitor

9 **Distribute** the Activity 2 cards.

Invite students to create their own flow diagram using the cards. Encourage them to lay the cards on blank paper and draw arrows to connect the ideas communicated by each tile.

D **Differentiation: (Support)** To support students in getting started, consider inviting them to identify which card they think is the beginning of the proof and which one is the end.

Invite students using notes to make a plan to prove this new conjecture.

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to order the cards to create a convincing argument.

Note: If time allows, invite student pairs to compare how they grouped the cards in their flow diagram. Encourage students to justify their thinking and make revisions based on their conversation.

3 Connect

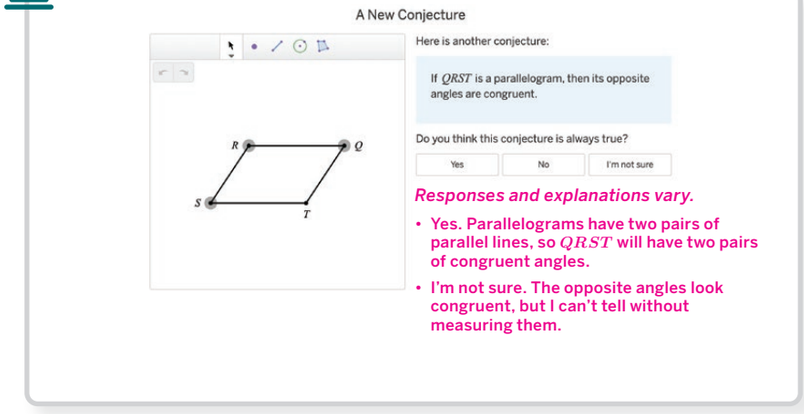
To surface the Key Takeaway, consider asking:

- "What was your plan for proving this conjecture?"
- "How did you decide what order the cards should go in?"
- "Are you convinced by this proof?" **(MP3)**
- "Is there anything that would make this proof stronger or clearer?" **(MP6)**

Math Identity and Community Consider highlighting the value of collaboration by asking if any students revised their groups through working with a partner.

Students using digital

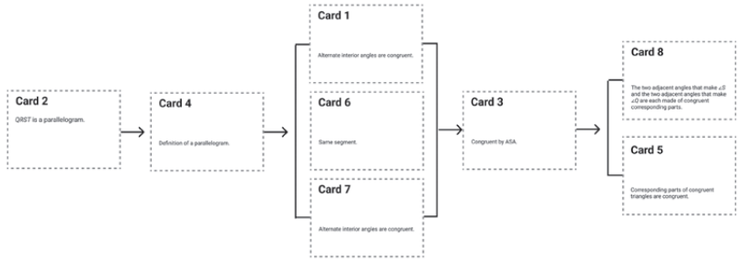
8



Responses and explanations vary.

- Yes. Parallelograms have two pairs of parallel lines, so $QRST$ will have two pairs of congruent angles.
- I'm not sure. The opposite angles look congruent, but I can't tell without measuring them.

9



Response shown.

Key Takeaway: Opposite angles in parallelograms are congruent. To prove this conjecture, you can make congruent triangles by constructing a diagonal and using corresponding pairs of congruent parts.

Whole Class | 5 min | Pacing: Screen 10

Synthesis



Student Notes are also available

10 **Invite students** to respond independently after discussing each question with their partner.

Capture and share a variety of ideas, such as:

- Using what you know about a parallelogram from its definition.
- Constructing diagonals within parallelograms creates congruent triangles.
- Using the definition of a parallelogram to prove that two triangles are congruent by ASA or another triangle congruence criteria.
- If I can prove that the triangles are congruent, then I can prove their parts are congruent.
- Corresponding parts of congruent triangles are congruent.

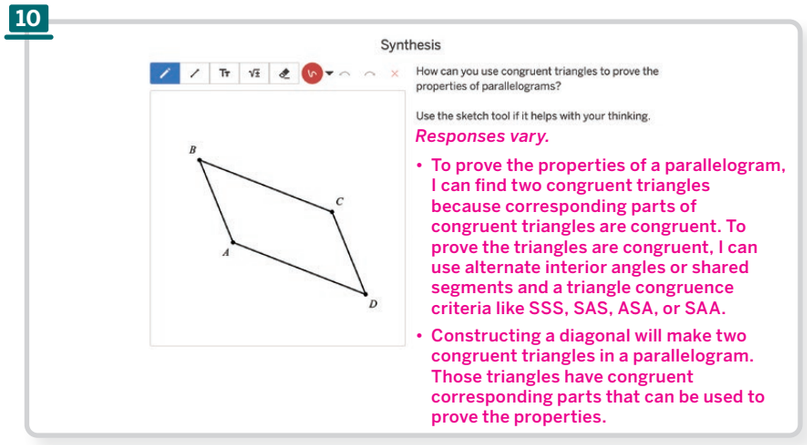
Create an anchor chart that displays the properties of parallelograms proven in this lesson using students' language, diagrams, and examples to reference for the rest of the unit.

MLR **MLR2: Collect and Display** Use this routine to revisit students' strategies for making convincing arguments. Encourage them to add or edit their ideas as they have changed throughout the unit. Display the strategies for students to reference and add to for the rest of the unit.

Math Identity and Community Invite students to share strategies they found helpful and give credit to the students who shared them

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.

10

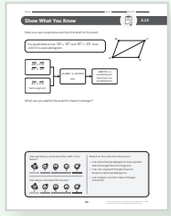


Lesson Takeaway: Constructing diagonals in parallelograms creates congruent triangles. By creating congruent triangles, you can use their corresponding congruent parts to prove properties of parallelograms. Flow diagrams are one way to organize a proof.

Independent | 5 min | Pacing: Screens 11–12

Show What You Know

Purpose: Students demonstrate their understanding by critiquing the first draft of a proof about a property of parallelograms.

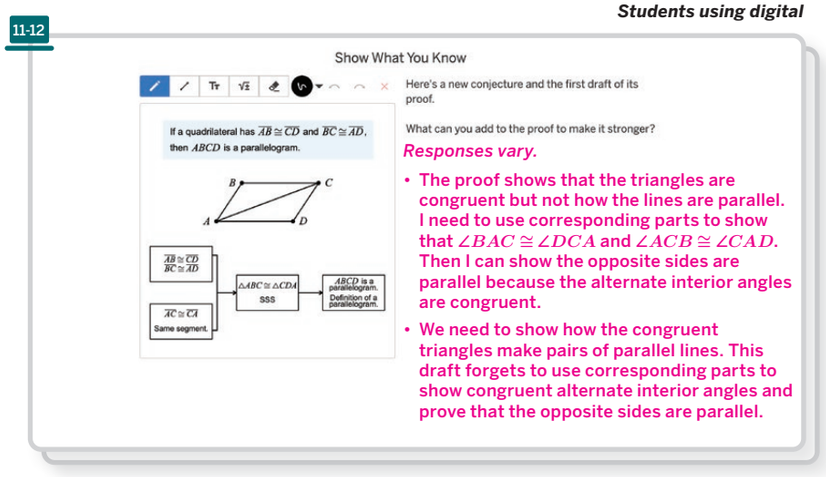


Students using print

11-12 Today's Goals

1. **Goal:** Prove that parallelograms have opposite sides and angles that are congruent.
2. **Goal:** Use congruent triangles to prove theorems about parallelograms.
3. **Language Goal:** Analyze a proof to make it stronger and clearer. **(Reading, Speaking, and Listening)**

Students using digital



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of how to strengthen a proof, consider:

- Revisiting **Screens 6–7** as a class and inviting students to share strategies for strengthening a proof.
- Inviting students to read the **Summary** and write it in their own words.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice**.
- Inviting students to review the **Summary** and complete the **Try This problem**.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting them to prove another theorem about parallelograms, e.g., "If one angle is right in a parallelogram, then the parallelogram is also a rectangle."
- Encouraging them to complete the **DOK3 practice problem.**

Professional Learning

Take a moment to reflect on the different formats students have seen to prove arguments in this unit so far. In Lesson 9, students wrote arguments in paragraphs. In Lesson 11, students used a table to organize the statements and reasons needed to write a convincing argument. In this lesson, students used a flow diagram to visualize parts of a proof. Which formats did your students find helpful when writing their arguments? Why is it helpful for students to explore different types of formats for writing a convincing argument?

In Lesson 14, students will have the opportunity to prove more conjectures about parallelograms. How can you support students in choosing an organizational structure for proofs that makes sense to them?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 13
Practice

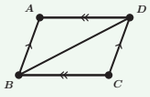
Students using digital

Lesson Summary

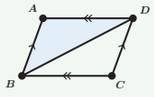
Constructing diagonals can be a helpful strategy for proving the properties of parallelograms.

When you construct a diagonal in a parallelogram, you create congruent triangles. You can use the triangles' corresponding pairs of congruent parts to prove the properties of parallelograms.

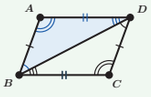
Here's one way you can use diagonals to prove that parallelograms have congruent opposite sides and congruent opposite angles.



Construct the diagonal $\overline{BD}$.



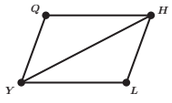
Prove $\triangle ABD \cong \triangle CDB$.



Look for corresponding congruent parts.

Try This

Here is parallelogram $YQHL$ with diagonal $\overline{YH}$.



Select *all* the true statements.

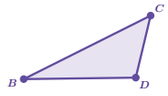
- ☐ A. $\overline{QY} \cong \overline{LY}$
- ☑ B. $\angle QYH \cong \angle LHY$
- ☑ C. $\overline{QH} \cong \overline{LY}$
- ☐ D. $\angle YHL \cong \angle YHQ$
- ☑ E. $\overline{YQ} \cong \overline{HL}$
- ☑ F. $\angle YQH \cong \angle HLY$

Unit 2 Lesson 13 152 Lesson Summary | Try This

Lesson Practice
Geo.2.13

Name: _____ Date: _____ Period: _____

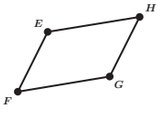
Problems 1–2: Here is $\triangle BCD$.

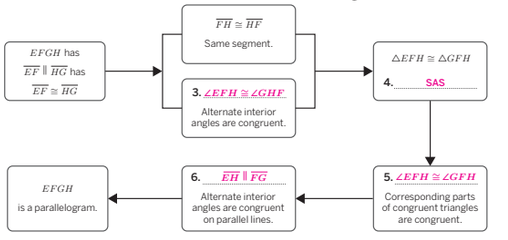


1. If $\triangle BCD$ was rotated around the midpoint of $\overline{BC}$, it would create a:
   - A. Kite
   - B. Rectangle
   - (C.) Parallelogram
   - D. Rhombus
2. If $\triangle BCD$ was reflected over $\overline{BC}$, it would create a:
   - (A.) Kite
   - B. Rectangle
   - C. Parallelogram
   - D. Rhombus

Problems 3–6: Use quadrilateral $EFGH$ to fill in the missing information and prove the conjecture:

If $EFGH$ is a quadrilateral with $\overline{EF} \parallel \overline{GH}$ and $\overline{EF} \cong \overline{GH}$, then it is a parallelogram.



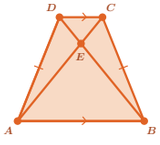


Unit 2 Lesson 13 153 Lesson Practice

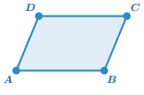
Lesson Practice
Geo.2.13

Name: _____ Date: _____ Period: _____

7. Select *all* the true statements based on quadrilateral $ABCD$.
   - ☐ A. Segment EB is congruent to segment AD.
   - ☑ B. Segment DB is congruent to segment CA.
   - ☐ C. Angle CBE is congruent to angle ABE.
   - ☑ D. Angle CEB is congruent to angle DEA.
   - ☑ E. Triangle AED is congruent to triangle BEC.



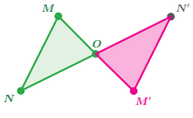
8. Prove that the adjacent angles in a parallelogram are supplementary. Use parallelogram $ABCD$ if it helps with your thinking.



Responses vary. Opposite sides are parallel in a parallelogram, so $\overline{DC} \parallel \overline{AB}$. $\angle A \cong \angle C$ because they are alternate interior angles. $\angle C$ and $\angle D$ are supplementary. This means $\angle A$ and $\angle D$ are also supplementary.

Spiral Review

Problems 9–10: Here is triangle MNO and point N'.



9. Rotate triangle MNO around point O until point N aligns with point N'.
10. How are segments MN and $M'N'$ related to each other?

Responses vary.
- Segments MN and $M'N'$ are parallel to each other.
- Segments MN and $M'N'$ are congruent to each other.

Reflection

1. Circle the problem you feel most confident about.
2. Use this space to ask a question or share something you're proud of.

Unit 2 Lesson 13 154 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-lesson			
	1	1	HSG.CO.C.11*
	2	1	HSG.CO.A.5
	3–6	2	HSG.CO.C.11
Test Practice	7	2	HSG.CO.B.7
	8	3	HSG.CO.C.11
Spiral Review			
Fluency	9	1	HSG.CO.A.5
	10	1	HSG.CO.B.6

*This problem builds toward the standard shown.

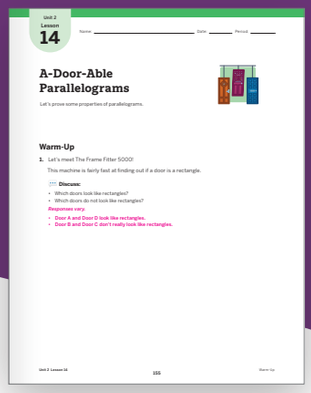
Need More Practice?

Additional Practice resources coming soon.

Unit 2

Lesson 14

This lesson uses print Student Edition pages and digital Teacher Presentation Screens.



A-Door-Able Parallelograms

Diagonals of Parallelograms

Let's prove some properties of parallelograms.

Focus and Coherence

Today's Goals

1. **Goal:** Construct diagonals to create pairs of congruent triangles in parallelograms.
2. **Language Goal:** Prove properties of parallelograms using congruent triangle pairs. **(Writing and Reading)**

Students learn about the properties of parallelograms and rectangles by making sense of The Frame Fitter 5000, a machine that tests to see if doors are rectangles. They make conjectures about properties of parallelograms, specifically the properties that include diagonals. Students make plans to develop a convincing argument and prove these conjectures. **(MP3)**

Prior Learning

In Lessons 8–9, students learned about the criteria necessary to prove congruent triangles. In Lesson 12, students explored the names, definitions, and properties of different quadrilaterals.

Future Learning

In Lesson 15, students will make sense of and prove the properties of a kite as they learn how to construct an angle bisector. In Unit 5, students will explore and prove properties of quadrilaterals using the coordinate plane.

Rigor and Balance

- Students **apply** their understanding of congruent triangles to prove conjectures about parallelograms.
- Students develop **strategic competence** as they devise different plans and consider different formats for proving properties of parallelograms.

Standards

Addressing

HSG.CO.C.11
Prove geometric theorems. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, **the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.**

Also Addressing: **HSG.CO.B.6**

Mathematical Practices: MP3, MP5, MP8

Building On

HSG.CO.B.8

Lesson at a Glance

~ 45 min

Standards: HSG.CO.C.11, HSG.CO.B.6

Warm-Up

Pairs | 5 min

Students make sense of The Frame Fitter 5000, which is a machine that tests to see if doors are rectangles.





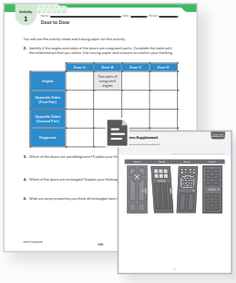
Presentation Screen 1

Activity 1

Pairs | 10 min

Students generalize the properties of rectangles by using tools to repeatedly analyze different parallelograms. **(MP8)**

Key Takeaway: Identifying congruent triangles in parallelograms is a helpful strategy for proving that all rectangles have congruent diagonals.





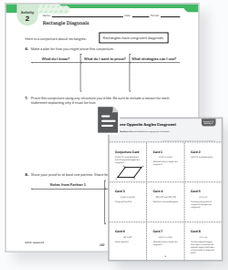
Presentation Screen 2

Activity 2

Pairs | 10 min

Students create plans to prove the conjectures about diagonals in rectangles. MLR1: Stronger and Clearer Each Time **(MP3)**

Key Takeaway: Constructing diagonals to represent congruent triangles in parallelograms is a helpful strategy to prove that all rectangles have congruent diagonals.





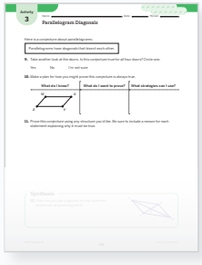
Presentation Screens 3–4

Activity 3

Pairs | 10 min

Students prove that if a quadrilateral is a parallelogram, then its diagonals bisect each other. **(MP3)**

Key Takeaway: Constructing diagonals can be used to create congruent triangles that prove diagonals of a parallelogram bisect each other.





Presentation Screen 5

Synthesis

Whole Class | 5 min

Students synthesize their understanding of how diagonals and congruent triangles can help prove the properties of parallelograms.

Lesson Takeaway: Constructing diagonals can reveal congruent triangles within a parallelogram. The triangles' corresponding congruent parts can prove the properties of parallelograms. See the Synthesis page for full Lesson Takeaway.



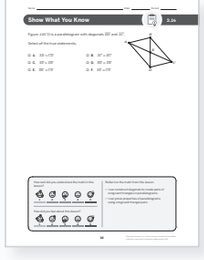


Presentation Screen 6

Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying congruent statements related to the properties of parallelograms they have proved. **(MP2)**





Presentation Screen 7

Prep Checklist

Invite students to use their Student Edition. Display the Presentation Screens to help facilitate the lesson.

This lesson includes:

- Presentation Screens
- Student Edition
- Show What You Know
- Activities 1 & 2 Sheet (1–2 per pair)
- scissors, tracing paper
- straightedges (as needed)
- whiteboards and dry erase markers (optional)

Pairs | 5 min

Warm-Up

Purpose: Students make sense of The Frame Fitter 5000, which is a machine that tests to see if doors are rectangles.



Presentation Screen 1

1 Launch

Display the Frame Fitter 5000 and the four suspended doors.

To connect to prior knowledge, consider asking:
- "How would you describe the shape of each door?"
- "What do you think The Frame Fitter 5000 will do to test the doors?" **(MP1)**
- "Why would it be helpful to know if a door is a rectangle?"

To encourage playfulness, invite students to act out what might happen if they tried to close each door into a door frame.

Distribute the Activity Sheet and invite students to make sense of these figures. Consider sharing that this resource may be helpful throughout the lesson.

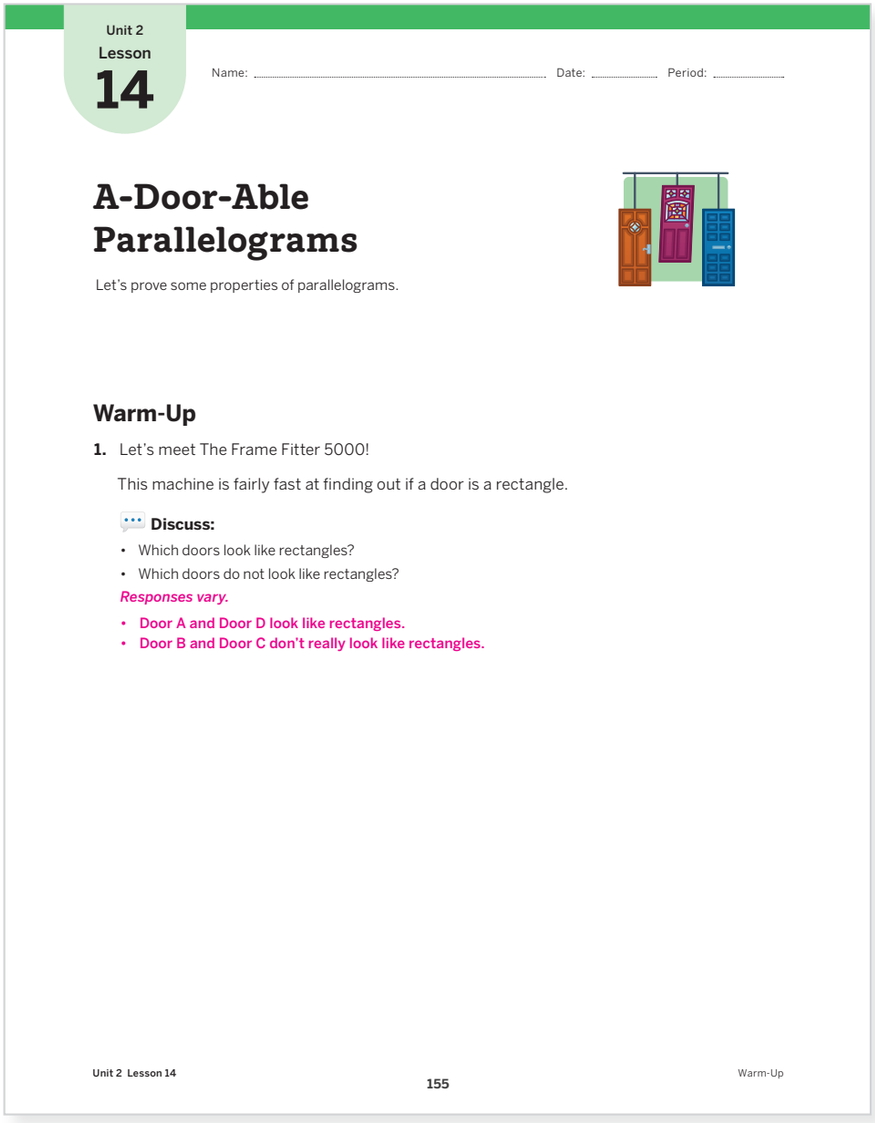
A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to make sense of one door at a time.

3 Connect

Invite students to justify which doors look like rectangles.

Play the animation by clicking on each door to observe what happens when they are dropped into the frame.

To support making connections, consider asking:
- "What does it mean if The Frame Fitter 5000 says a door is a rectangle? What does that tell you about the door?" **(MP1)**
- "What properties do Door A and Door D have in common that Door B and Door C do not?"
- "Why didn't Door B or Door C fit into the frame?"
- "What properties could The Frame Fitter 5000 test for to decide if a door is a rectangle?" *Responses vary.* The Frame Fitter 5000 can check if all the angles are congruent or if they are all right angles.



Unit 2 Lesson 14

Name: _____ Date: _____ Period: _____

A-Door-Able Parallelograms

Let's prove some properties of parallelograms.

Warm-Up

1. Let's meet The Frame Fitter 5000!

This machine is fairly fast at finding out if a door is a rectangle.

Discuss:
- Which doors look like rectangles?
- Which doors do not look like rectangles?

Responses vary.
- Door A and Door D look like rectangles.
- Door B and Door C don't really look like rectangles.

Unit 2 Lesson 14 155 Warm-Up

Pairs | 10 min

Activity 1 Door to Door

Purpose: Students generalize the properties of rectangles by using tools to repeatedly analyze different parallelograms. **(MP8)**

Short on time? Consider inviting student pairs to divide the doors so that each student focuses on two doors.

Lesson 14
Activity 1

Presentation Screen 2

1 Launch

Display the four doors and invite students to examine them closely before sharing their ideas aloud.

Distribute tracing paper, scissors, and straightedges, if needed, to support students in their exploration.

Note: Consider making additional copies of the Activity Sheet available and ensuring that the tracing paper is large enough to trace the doors.

2 Monitor

Look for a variety of conjectures that surfaced in Problem 5, specifically ones that identify congruent diagonals.

D Differentiation *(Problems 3, 5)*

Look for students who:	Respond to Student Thinking
Conclude that Door A and Door D do not have congruent diagonals. *(Problem 3)*	**Support** Invite them to attend to precision. Consider asking, "How might you use other tools to confirm your findings?" **(MP5)**
Determine that Door A and Door D are rectangles because their angles are all right angles or because their opposite sides are congruent. *(Problem 3)*	**Strengthen** Celebrate their thinking. Encourage them to reflect on additional relationships. Consider asking, "Are there any other properties that you can use to identify Door A and Door D as rectangles?"
Write a conjecture that claims all rectangles have congruent diagonals. *(Problem 5)*	**Stretch** Consider asking, "Does your conjecture hold true for squares? Rhombuses?"

3 Connect

Invite students to look for patterns in the table that are related to the congruent parts of each door. **(MP8)**

Consider asking:

- "What was different about Doors A and D compared to B and C?"
- "What quadrilateral name would you use to describe the doors that are not rectangles?"
- "For the doors that are rectangles, what properties do they share?"

Activity 1

Name: _____ Date: _____ Period: _____

Door to Door

You will use the activity sheet and tracing paper for this activity.

2. Identify if the angles and sides of the doors are congruent parts. Complete the table with the relationships that you notice. Use tracing paper and scissors to confirm your thinking.

Responses vary.

	Door A	Door B	Door C	Door D
Angles	All congruent	Two pairs of congruent angles	Two pairs of congruent angles	All congruent
Opposite Sides (First Pair)	Congruent	Congruent	Congruent	Congruent
Opposite Sides (Second Pair)	Congruent	Congruent	Congruent	Congruent
Diagonals	Congruent	Not congruent	Not congruent	Congruent

3. Which of the doors are parallelograms? Explain your thinking.

Responses vary. All four of the doors are parallelograms because they have two pairs of congruent angles and two pairs of congruent sides.

4. Which of the doors are rectangles? Explain your thinking.

Responses vary. A and D because they have four congruent angles.

5. What are some properties you think all rectangles have in common?

Responses vary.

- Rectangles have congruent diagonals.
- Rectangles are made up of two congruent right triangles.
- Rectangles have opposite sides that are congruent.

Unit 2 Lesson 14 156 Activity 1

Key Takeaway: Identifying congruent triangles in parallelograms is a helpful strategy for proving that all rectangles have congruent diagonals.

Pairs | 10 min

Activity 2 Rectangle Diagonals

Purpose: Students create a plan and prove conjectures about diagonals in rectangles. **(MP3)**

Lesson 14
Activity 2

Presentation Screens
3–4

1 Launch

3 **Display** the rectangle and consider asking:
- "What do we know about rectangles from their definition?"
- "Since we know rectangles are parallelograms, what else do we know about rectangles?"

2 Monitor

To support students getting started, consider asking, "How can we use what we know about rectangles to prove that their diagonals are always congruent?"

4 **Display** the screen and encourage students to think about how the animation can help them create a plan.

Look for a variety of plans students make.

D **Differentiation** *(Problems 6–7)*

Look for students who:	Respond to Student Thinking
Need support applying their plan. *(Problem 6)*	**Support** Invite them to apply structures that have supported their thinking in this unit so far. Consider asking, "What structure would help you organize your argument in an order that makes sense?"
Think that they can use congruent triangles. *(Problem 6)*	**Strengthen** Celebrate their thinking. Consider asking, "How can you prove triangle congruence?" **(MP3)**
Use SAS triangle congruence criteria to prove congruent triangles. *(Problem 7)*	**Stretch** Invite them to reflect on the strength of their proof. Consider asking, "Does this convince you that all rectangles have congruent diagonals?"

MLR **MLR1: Stronger and Clearer Each Time** Invite students to write a first draft of their proof and use feedback to make their argument stronger and clearer. **(MP3)**

3 Connect

Play the animation and invite students to share their strategies for using congruent triangles to prove the conjecture.

Math Identity and Community Consider inviting students to share their partner's strategies or the helpful feedback they received from their partner.

To surface the Key Takeaway, consider asking, "How did separating the triangles help your thinking?"

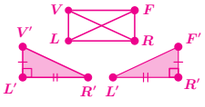
Activity 2

Name: _____ Date: _____ Period: _____

Rectangle Diagonals

Here is a conjecture about rectangles: Rectangles have congruent diagonals.

6. Make a plan for how you might prove this conjecture. *Responses vary.*

What do I know?	What do I want to prove?	What strategies can I use?
		*Responses vary.* I can draw the diagonals to create two congruent triangles. Then I can separate the triangles to look at the relationship.

7. Prove this conjecture using any structure you'd like. Be sure to include a reason for each statement explaining why it must be true.

Statement	Reason that this must be true
$VFRL$ is a rectangle	This information is given in the conjecture
$\angle V$, $\angle L$, $\angle F$, and $\angle R$ are congruent	Definition of rectangle
$\overline{VL} \cong \overline{FR}$	Parallelograms have congruent opposite sides
$\overline{LR} \cong \overline{RL}$	They are the same segment
$\triangle VRL \cong \triangle FLR$	SAS
$\overline{VR} \cong \overline{FL}$	Corresponding parts

8. Show your proof to at *least* one partner. Share feedback to strengthen your proofs.

Notes from Partner 1	Notes from Partner 2
Responses vary.	*Responses vary.*

Unit 2 Lesson 14 157 Activity 2

Key Takeaway: Constructing diagonals to represent congruent triangles in parallelograms is a helpful strategy to prove that all rectangles have congruent diagonals.

Pairs | 10 min

Activity 3 Parallelogram Diagonals

Purpose: Students prove that if a quadrilateral is a parallelogram, then its diagonals bisect each other. **(MP3)**

Lesson 14
Activity 3

Presentation Screen 5

1 Launch

Display the doors from the Activity Sheet and invite student pairs to discuss the conjecture.

To connect to prior knowledge, consider asking, "What does it mean when diagonals bisect each other? How would that look on a diagram?"

Play the animation to reveal the diagonals.

2 Monitor

Encourage students to prove this conjecture is *always* true and not just true for these four parallelograms.

Select and sequence proofs to share during the Connect.

D Differentiation *(Problems 10–11)*

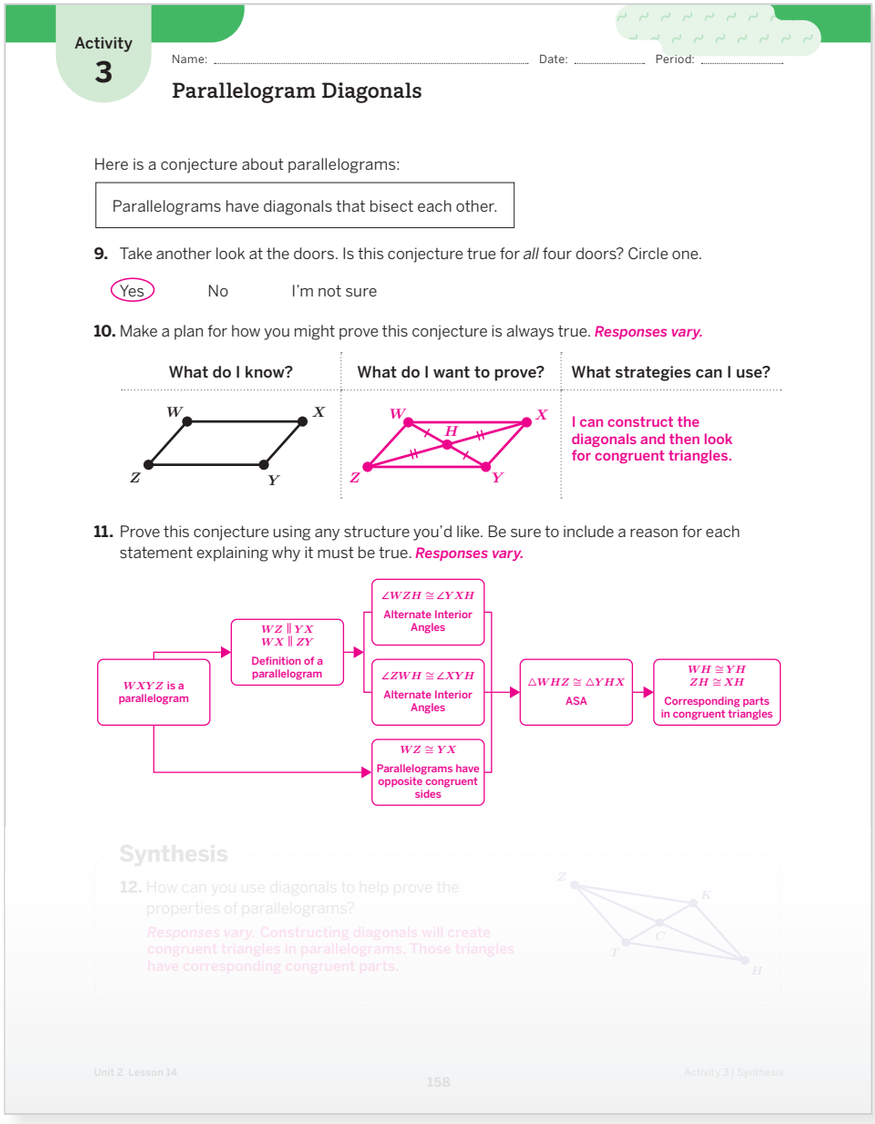
Look for students who:	Respond to Student Thinking
Are unsure of how to organize their argument to prove the conjecture. *(Problem 10)*	**Support** Invite them to use other tools such as whiteboards and dry erase markers to support flexible thinking.
Need support identifying corresponding parts of congruent triangles in their proof. *(Problem 11)*	**Strengthen** Invite them to sketch the diagram to see two triangles in the same orientation. Consider asking, "Which vertices would stack onto each other?"
Construct two diagonals in their plan. *(Problem 10)*	**Strengthen** Celebrate their strategy. Invite them to consider additional congruent relationships that could be applied. Consider asking, "Have you created any new figures? What else have you learned about proving?"

3 Connect

Share the proofs. Invite students to reflect on the strategies that were the most helpful.

To surface the Key Takeaway, consider asking, "What would convince our community that these diagonals bisect each other?" **(MP3)**

Math Identity and Community Consider highlighting the value of having multiple ways to prove a conjecture.



Activity 3

Name: _____ Date: _____ Period: _____

Parallelogram Diagonals

Here is a conjecture about parallelograms:

Parallelograms have diagonals that bisect each other.

9. Take another look at the doors. Is this conjecture true for *all* four doors? Circle one.

Yes No I'm not sure

10. Make a plan for how you might prove this conjecture is always true. ***Responses vary.***

What do I know?	What do I want to prove?	What strategies can I use?
W X Z Y	W H X Z Y	I can construct the diagonals and then look for congruent triangles.

11. Prove this conjecture using any structure you'd like. Be sure to include a reason for each statement explaining why it must be true. ***Responses vary.***

$WXYZ$ is a parallelogram

$WZ \parallel YX$, $WX \parallel ZY$ — Definition of a parallelogram

$\angle WZH \cong \angle YXH$ — Alternate Interior Angles

$\angle ZWH \cong \angle XYH$ — Alternate Interior Angles

$WZ \cong YX$ — Parallelograms have opposite congruent sides

$\triangle WHZ \cong \triangle YHX$ — ASA

$WH \cong YH$, $ZH \cong XH$ — Corresponding parts in congruent triangles

Synthesis

12. How can you use diagonals to help prove the properties of parallelograms?

Responses vary. Constructing diagonals will create congruent triangles in parallelograms. Those triangles have corresponding congruent parts.

Unit 2 Lesson 14 158 Activity 3 | Synthesis

Key Takeaway: Constructing diagonals can be used to create congruent triangles that prove diagonals of a parallelogram bisect each other.

Whole Class | 5 min

Synthesis

Lesson 14
Synthesis

Presentation Screen
6

6 **Invite students** to respond independently, and then share their thinking with a partner.

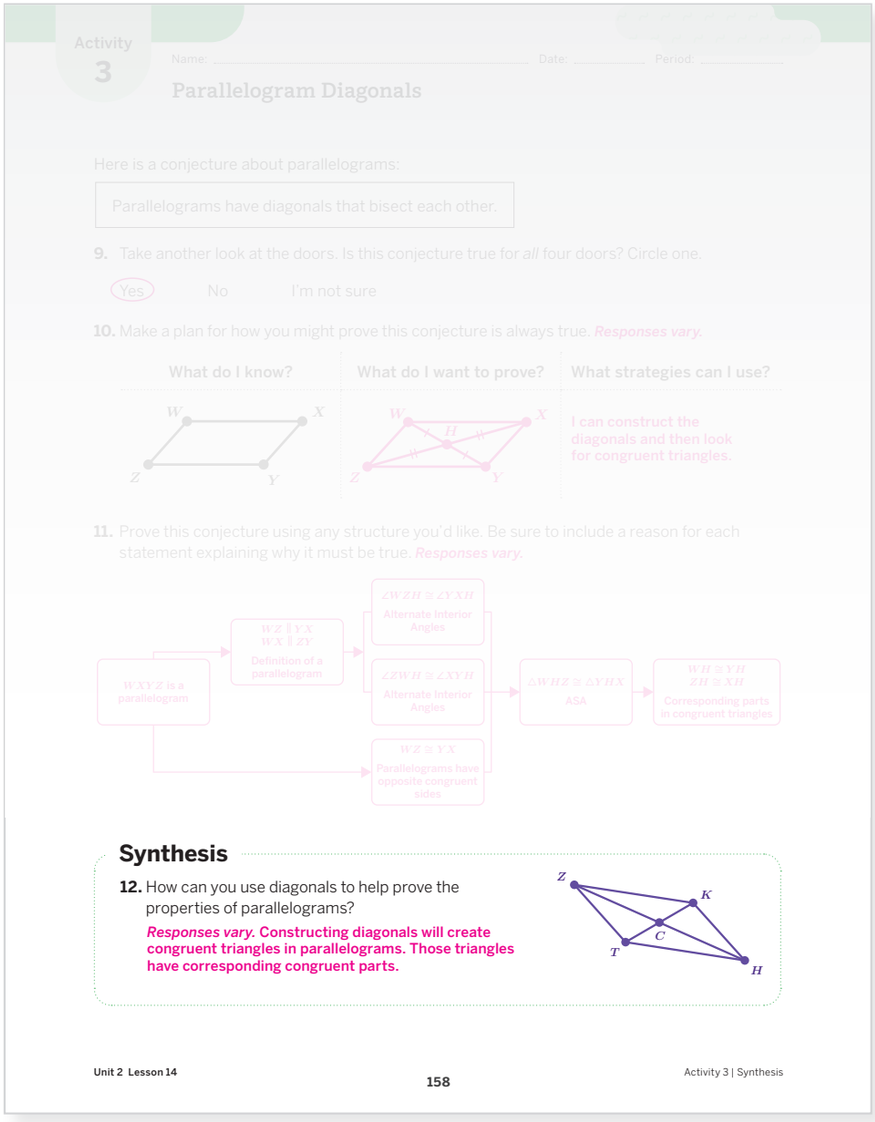
Capture and share a variety of ideas, such as:

- Constructing diagonals in parallelograms reveals congruent triangles.
- Proving triangles congruent using triangle congruence criteria helps prove the properties of parallelograms.
- Corresponding parts of congruent triangles must be congruent, which can prove properties that must always be true in parallelograms.

Create an anchor chart of parallelogram properties that were proved in this lesson. Display the chart for students to reference and add to for the rest of the unit.

Math Identity and Community Invite students to share strategies they found helpful and give credit to the students who shared them.

Note: If time allows, invite students to revise their previous thinking on Problems 6 or 10 based on the discussion.



Activity 3

Name: Date: Period:

Parallelogram Diagonals

Here is a conjecture about parallelograms:

Parallelograms have diagonals that bisect each other.

9. Take another look at the doors. Is this conjecture true for *all* four doors? Circle one.

Yes No I'm not sure

10. Make a plan for how you might prove this conjecture is always true. *Responses vary.*

What do I know?	What do I want to prove?	What strategies can I use?
		I can construct the diagonals and then look for congruent triangles.

11. Prove this conjecture using any structure you'd like. Be sure to include a reason for each statement explaining why it must be true. *Responses vary.*

Synthesis

12. How can you use diagonals to help prove the properties of parallelograms?

Responses vary. Constructing diagonals will create congruent triangles in parallelograms. Those triangles have corresponding congruent parts.

Unit 2 Lesson 14 158 Activity 3 | Synthesis

Lesson Takeaway: Constructing diagonals can be a helpful strategy to reveal congruent triangles within a parallelogram. If these triangles are proved congruent, then their corresponding parts must also be congruent. These corresponding congruent parts can help prove properties of parallelograms.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying properties of parallelograms they have proven to be true. **(MP2)**

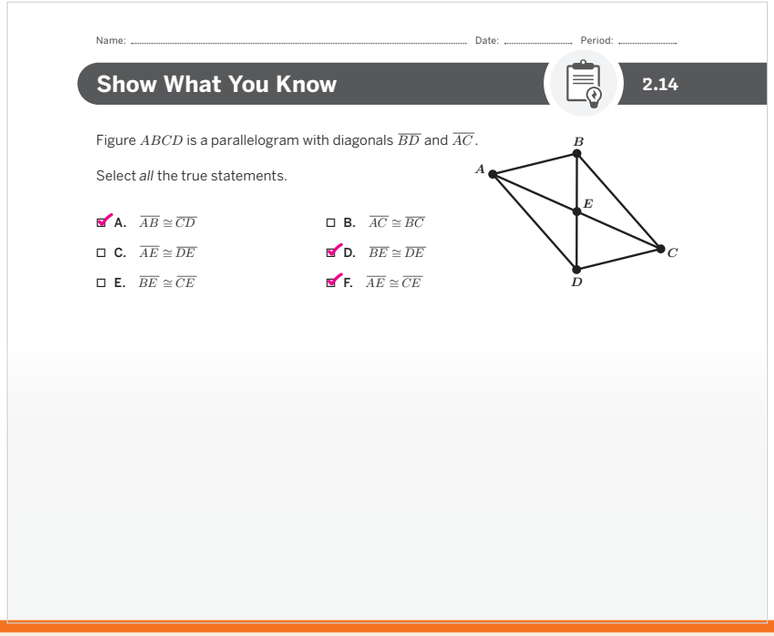


Presentation Screen 7

Today's Goals

1. **Goal:** Construct diagonals to create pairs of congruent triangles in parallelograms.
2. **Language Goal:** Prove properties of parallelograms using congruent triangle pairs. **(Writing and Reading)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 2.14

Figure $ABCD$ is a parallelogram with diagonals $\overline{BD}$ and $\overline{AC}$.

Select *all* the true statements.

- ☑ A. $\overline{AB} \cong \overline{CD}$
- ☐ B. $\overline{AC} \cong \overline{BC}$
- ☐ C. $\overline{AE} \cong \overline{DE}$
- ☑ D. $\overline{BE} \cong \overline{DE}$
- ☐ E. $\overline{BE} \cong \overline{CE}$
- ☑ F. $\overline{AE} \cong \overline{CE}$

Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of proving properties of parallelograms using diagonals, consider:

- Inviting students to read the **Summary** and highlight key components.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice**
- Inviting students to review the **Summary** and complete the **Try This** problem.
- Emphasizing this skill in **Practice Day 2, Cards B, C, and D.**

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Encouraging them to complete the **DOK3 practice problem**.
- Inviting them to write their own conjecture about parallelograms and discuss how to prove it with a partner.

Professional Learning

Take a moment to reflect on how students communicated their proof ideas on paper throughout the lesson. Which students' ideas were you able to highlight during the Connect in each activity? How did learning on paper affect how often and which type of student ideas were shared? What could you consider for future print lessons?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

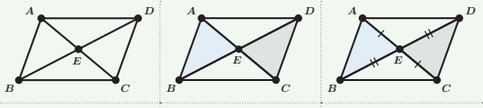
Lesson 14
Practice

Students using digital

Lesson Summary

When you construct one or two diagonals in a parallelogram, they make congruent triangles. Since congruent triangles have corresponding pairs of congruent parts, this step helps prove the properties of parallelograms, such as opposite sides and angles being congruent.

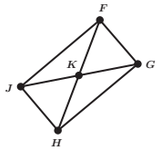
Here is one example of how constructing diagonals can be a helpful strategy for proving this property of parallelograms: *The diagonals of a parallelogram bisect each other.*



Construct the diagonals $\overline{AC}$ and $\overline{BD}$. Label the intersection point E.	Prove $\triangle AEB \cong \triangle CED$.	Look for corresponding congruent parts. $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$, so point E bisects both $\overline{AC}$ and $\overline{BD}$.

Try This

Figure $FGHJ$ is a rectangle with diagonals $\overline{JG}$ and $\overline{FH}$.



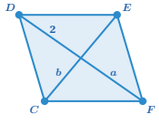
Select *all* the true statements.

- ☐ A. $\overline{JK} \cong \overline{FG}$
- ☑ B. $\overline{JK} \cong \overline{GK}$
- ☑ C. $\overline{JG} \cong \overline{FH}$
- ☐ D. $\overline{FH} \cong \overline{JF}$
- ☑ E. $\overline{JH} \cong \overline{FG}$
- ☐ F. $\overline{GH} \cong \overline{JH}$

Lesson Practice
Geo.2.14

Name: _____ Date: _____ Period: _____

Problems 1–2: Here is quadrilateral $CDEF$.



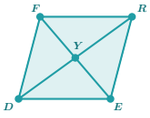
1. What is the value of a if figure $CDEF$ is a parallelogram?

   $a =$ 2

2. What is the value of b if figure $CDEF$ is a rectangle?

   $b =$ 2

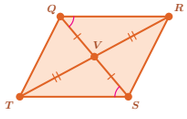
Problems 3–4: Name two different pairs of congruent triangles in parallelogram $FRED$. *Responses vary.*



3. $\triangle FED \cong \triangle EFR$

4. $\triangle DYE \cong \triangle RYF$

Problems 5–7: Here is quadrilateral $QRST$.



5. Which triangle congruence criteria can be used to prove $\triangle SVT \cong \triangle QVR$?

   A. SSS
   B. SAS
   C. ASA
   D. SAA

6. If $\triangle SVT \cong \triangle QVR$, mark a pair of angles that show $\overline{QR} \parallel \overline{ST}$.

   Responses vary.

7. Prove that $\overline{QT} \parallel \overline{SR}$.

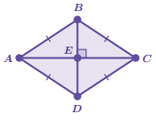
   Responses vary. Since $\overline{QV} \cong \overline{SV}$, $\overline{RV} \cong \overline{TV}$, and $\angle QVT \cong \angle SVR$ (vertical angles), then $\triangle QVT \cong \triangle SVR$ by SAS. $\angle QTV \cong \angle SRV$ because they are corresponding parts of congruent triangles. So $\overline{QT} \parallel \overline{SR}$ because alternate interior angles are congruent on parallel lines.

Lesson Practice
Geo.2.14

Name: _____ Date: _____ Period: _____

8. Prove that the diagonals of a rhombus bisect each other. Use the rhombus $ABCD$ if it helps with your thinking.

   Responses vary. A rhombus is a quadrilateral with four congruent sides and opposite sides parallel. $\angle BAE \cong \angle DCE$ and $\angle ABE \cong \angle CDE$ because they are both alternate interior angles. So $\triangle ABE \cong \triangle CDE$ by ASA. Congruent triangles have congruent corresponding parts, so $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$. This means that the diagonals bisect each other.



Spiral Review

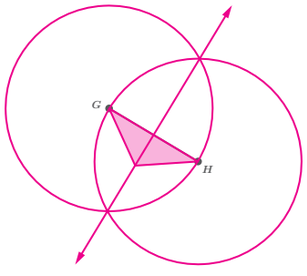
Problems 9–10: Here is $\overline{GH}$.

9. Plot a point that is equidistant to points G and H.

   Responses vary.

10. Construct an isosceles triangle that is *not* equilateral. Use $\overline{GH}$ as the base of your triangle.

    Responses vary.



Reflection

1. Circle the problem you feel most confident about.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-lesson			
Fluency	1–2	1	HSG.CO.C.11*
	3–4	2	HSG.CO.B.6
	5–6	2	HSG.CO.C.11
Test Practice	7	2	HSG.CO.C.11
	8	3	HSG.CO.C.11
Spiral Review			
	9	2	HSG.CO.A.1
	10	1	HSG.CO.D.13

*This problem builds toward the standard shown.

Need More Practice?

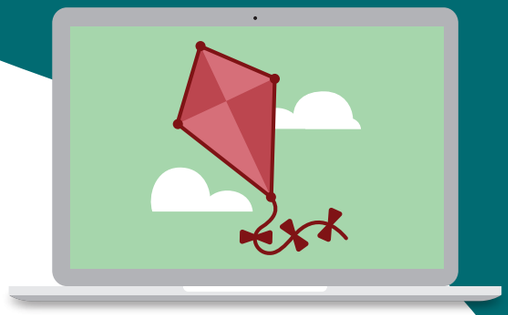
Additional Practice resources coming soon.

Unit 2

Lesson 15

Digital Lesson

This lesson uses digital student screens. Student Notes are also available.



Let's Go Build a Kite

Constructing an Angle Bisector

Let's develop strategies to construct an angle bisector.

Focus and Coherence

Today's Goals

1. **Goal:** Construct the *angle bisector* of a given angle.
2. **Language Goal:** Prove that a line is the *angle bisector* of a given angle using constructions, transformations, and triangle congruence theorems. **(Speaking and Listening)**

Students apply what they have learned about transformations, constructions, and congruence to construct an **angle bisector**. They make sense of the properties of a kite, such as one of the diagonals being an angle bisector, and they apply this information to surface strategies for construction. **(MP1)**

Prior Learning

In Unit 1, students were introduced to geometric constructions. They used the properties of circles to construct precise geometric figures. In Lesson 10, students constructed a copy of an angle. In Lesson 12, students were introduced to different types of quadrilaterals, such as kites and rhombuses, and their properties.

Future Learning

In Unit 5, students will prove the properties of quadrilaterals by using angle measurements and the coordinate plane.

Rigor and Balance

- Students develop **conceptual understanding** by making connections between angle bisectors and properties of quadrilaterals.
- Students **apply** their understanding of triangle congruence to prove the relationships between the diagonals and congruent parts of kites.
- Students develop **strategic competence** as they make sense of a variety of strategies for constructing an angle bisector.

Vocabulary

New Vocabulary

angle bisector

Standards

Addressing

HSG.CO.D.12

Make geometric constructions. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, **dynamic geometric software, etc.)**. Copying a segment; copying an angle; bisecting a segment; **bisecting an angle;** constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

Mathematical Practices: MP3, MP5, MP7

Building On

HSG.CO.B.8

Lesson at a Glance

~ 45 min

Standard: HSG.CO.D.12



Why digital?

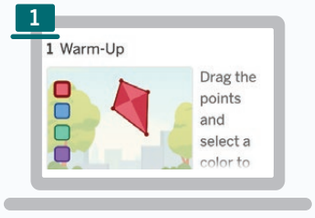
Students use digital construction tools to develop strategies for constructing an angle bisector.

Warm-Up

Independent | 5 min

Pacing: Screen 1

Students play with and customize different kite designs to make connections to kite properties.



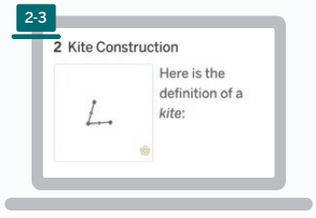
Activity 1

Pairs Sharing a Device | 5 min

Pacing: Screens 2–3

Students make use of structure to construct a kite from a definition and generalize properties of the quadrilateral. **(MP7)**

Key Takeaway (Screen 3): There are properties of kites that we can determine by looking at their diagonals, specifically the property that one of the diagonals bisects two of the angles.



Activity 2

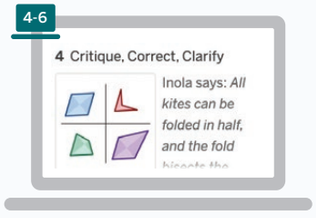
Pairs Sharing a Device | 10 min

Pacing: Screens 4–5, Screen 6

Students generalize about the properties of kites and their diagonals to surface possible strategies for constructing **angle bisectors.**

MLR3: Critique, Correct, and Clarify

Key Takeaway (Screen 5): A property of every kite is that it has at least one diagonal that bisects angles. Other quadrilaterals can also have diagonals that are **angle bisectors**.

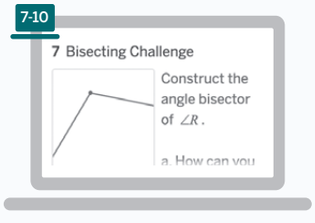


Activity 3 (Optional)

Pairs Sharing a Device | 15 min

Pacing: Screens 7–8, Screens 9–10, Screen 9

Students construct an angle bisector and use MLR7: Compare and Connect to surface strategies for the construction.



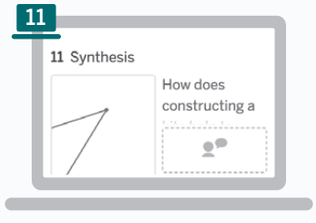
Synthesis

Whole Class | 5 min

Pacing: Screen 11

Students synthesize their understanding of strategies they use to construct angle bisectors.

Lesson Takeaway: There are many strategies for constructing an **angle bisector**. One strategy is to construct a kite using the given angle. Since all kites have a diagonal that is an angle bisector, constructing a kite and using its definition is one way to prove that you also constructed an angle bisector.

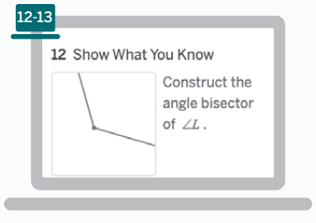


Show What You Know

Independent | 5 min

Pacing: Screens 12–13

Students demonstrate their understanding by constructing an angle bisector using digital construction tools.



Prep Checklist

Assign the digital student screens. Student Notes are also available.

Consider making the geometry toolkit available to support students in selecting and using tools strategically.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)
- geometry toolkit: compasses, straightedges (optional)

Student Notes sheet in Student Edition:

Consider providing the Student Notes to support class discussion and student thinking.



Independent | 5 min | Pacing: Screen 1

Warm-Up

Purpose: Students play with and customize different kite designs to make connections to kite properties.



Student Notes are also available

1 Launch

1 **To encourage playfulness,** invite students to share their experiences with flying kites. Consider asking, "What's a memory you have about flying a kite?"

Invite students to drag the points and create their own design. Encourage students to try designing a kite that is unique.

Math Identity and Community Celebrate creative or interesting kite designs. Invite the author to share their inspiration.

Encourage students to press "Fly It" when they have finished their design.

Connect

Capture and share different kite designs that students generated, especially those that have concave angles.

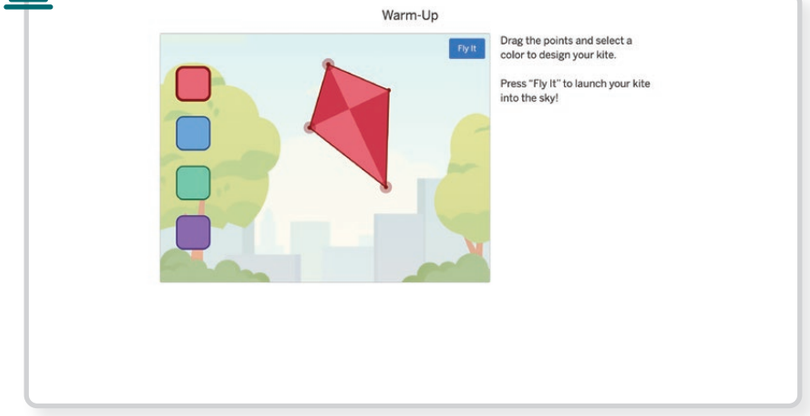
To support making connections, consider asking:

- "How were our designs alike? How were they different?"
- "When you drag the points, what relationships in the kite stay the same? What relationships change?" *Responses vary.* There are two pairs of triangles that always seem congruent. The side lengths of the triangles can change though.

Math Identity and Community Acknowledge that math can be both playful and interesting. Encourage students to lean on these connections when learning about geometric relationships.

Students using digital

1



Pairs Sharing a Device | 5 min | Pacing: Screens 2–3

Activity 1 Building Kites

Purpose: Students make use of structure to construct a kite from a definition and generalize properties of the quadrilateral. **(MP7)**

Short on time? Consider facilitating Screen 3 as a class.



Student Notes are also available

1 Launch

2 **To support making connections,** consider asking:
- "What figures are we starting with?"
- "What do we need to construct when making a kite?"

ML/EL **Multilingual/English Learners** Invite students to share different meanings of the term *kite*. Clarify which meaning of the term will be used for this lesson. Consider asking, "How does a kite as an object relate to a kite as a quadrilateral?" **(Reading and Listening)**

Note: Remind students to use the polygon tool to show their kite to receive a full star.

2 Monitor

3 **To support students getting started,** consider asking:
- "How can you use the digital tools to compare each kite's sides or angles?" **(MP5)**
- "How might a diagonal help you see the structure within a kite?" **(MP7)**
- "What transformations can you use to show the properties that every kite has?"

Capture properties that students identify. Highlight the properties that discuss relationships that include diagonals.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support with verifying the congruent sides of the kites.	**Support** Encourage them to use the "circle by length" tool and see what happens when they hover their cursor over a figure.
Notice that the diagonals create congruent adjacent angles.	**Stretch** Consider asking, "How might you describe the diagonals of kites as perpendicular bisectors?"

Connect

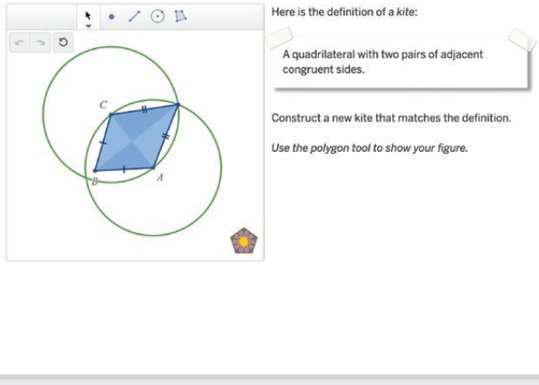
Share the captured properties identified by students.

To surface the Key Takeaway, consider asking:
- "What properties are true for all of these figures?"
- "How can we convince each other of the properties of all kites?" *Responses vary.* We can use rigid transformations to show congruent triangles because they have corresponding congruent segments. We can also use triangle congruence criteria. **(MP3)**

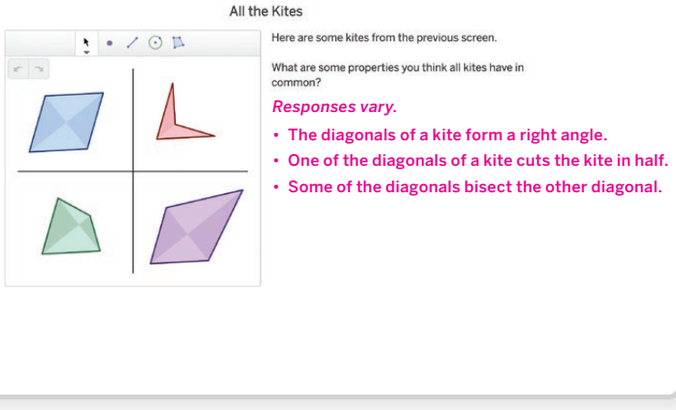
Students using digital

2

Sample shown.



3



Key Takeaway: There are properties of kites that we can determine by looking at their diagonals, specifically the property that one of the diagonals bisects two of the angles.

Pairs Sharing a Device | 10 min | Pacing: Screens 4–5, Screen 6

Activity 2 Kite Diagonals

Purpose: Students generalize about the properties of kites and their diagonals to surface possible strategies for constructing **angle bisectors**.

Short on time? Consider facilitating Screen 6 as a class.



Student Notes are also available

1 Launch

4 **To support making connections,** invite students to restate Inola's conjecture in their own words.

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Inola's statement. Invite students to share their responses from the activity. See the Routine Facilitation Guide for more information. **(MP3)**

2 Monitor

D **Differentiation: (Support)** Look for students who think that the conjecture does not need to be strengthened. Invite them to make sense of both the diagonals. Consider asking, "How can you use diagonals to support Inola's conjecture?" **(MP3)**

5 **Pause briefly** to discuss the term **angle bisector**. Consider asking:

- "What strategies can you use to prove $\overline{DF}$ is an angle bisector?"
- "How can you tell when a diagonal is not an angle bisector?"

ML/EL **Multilingual/English Learners** Invite students to demonstrate or explain how *angle bisector* is related to review terms like *angle*, *bisector*, and *adjacent angles*. **(Reading and Listening)**

Invite students using notes to summarize the strategies they used to prove $\overline{DF}$ is an angle bisector.

Invite students to continue to Screen 6 using the Pace tool.

6 **Look for** students who identify squares and rhombuses as quadrilaterals with diagonals that are angle bisectors. Invite these students to share during the Connect.

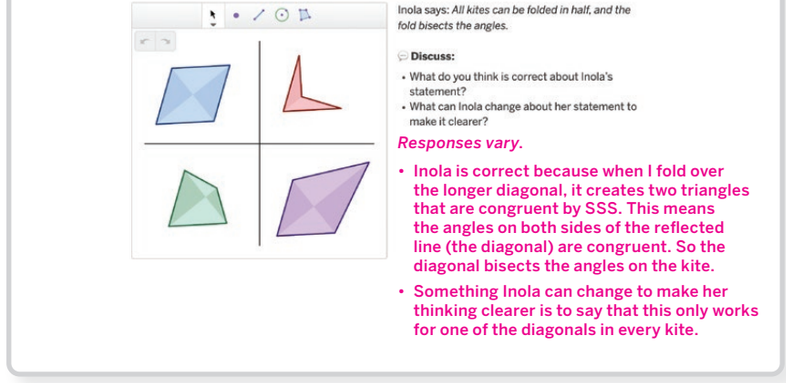
3 Connect

To surface the Key Takeaway, consider asking:

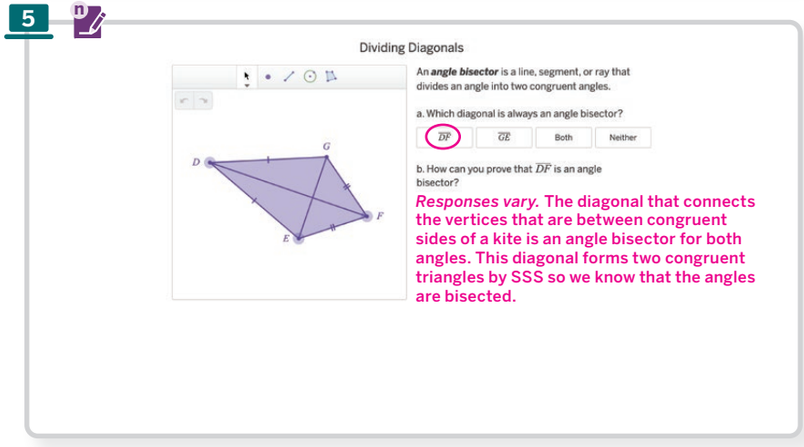
- "What is something true that you can say about all kites and angle bisectors?" *Responses vary.* One diagonal of all kites is an angle bisector.
- "What is a strategy for proving that a diagonal is an angle bisector?"
- "How do diagonals and angle bisectors help us make sense of other quadrilaterals, like squares and rhombuses?

Students using digital

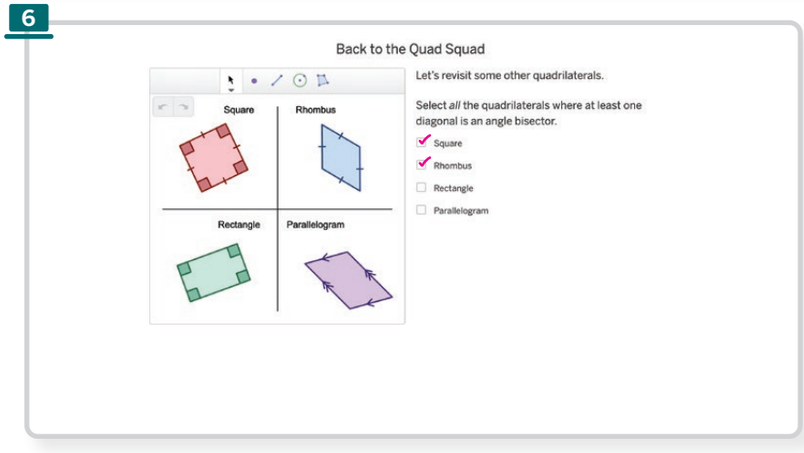
4



5



6



Key Takeaway: A property of every kite is that it has at least one diagonal that bisects angles. Other quadrilaterals can also have diagonals that are **angle bisectors**.

Pairs Sharing a Device | 15 min | Pacing: Screens 7–8, Screens 9–10, Screen 9

Activity 3 Bisecting Challenge (Optional)

Purpose: Students construct an angle bisector and use **MLR7: Compare and Connect** to surface strategies for the construction.



Student Notes are also available

1 Launch

7 **To connect to prior knowledge,** consider asking, "How can you create a strategy using the properties you previously learned about?"

Note: Consider sharing that the transformation tools are turned off for this activity.

2 Monitor

A **Accessibility: Conceptual Processing** Invite students to talk aloud about their construction strategies to support self-monitoring of understanding.

Select and sequence constructions to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Approximate an angle bisector by constructing a ray, line, or segment and moving it until a star appears.	**Support** Invite them to consider a strategy that will precisely construct an angle bisector. Consider asking, "What do we know about figures with two adjacent congruent angles?"
Construct a parallelogram with a diagonal that they think is the angle bisector.	**Strengthen** Consider asking, "How do you know the adjacent angles are congruent?"
Construct a kite using an initial circle and two additional circles of equal radius to determine the angle bisector.	**Stretch** Celebrate their strategy. Encourage them to think of other strategies for constructing an angle bisector. Consider asking, "What are other quadrilaterals that you can use to construct an angle bisector?"

8 **Pause briefly** to discuss Malik's and Tamiya's strategies.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy before connecting to their own strategy. See the Routine Facilitation Guide for more information.

Consider asking:
- "How are their strategies alike? Different?"
- "Why did both strategies use a quadrilateral to construct an angle bisector?"
- "How does each strategy compare to your own?"

Invite students to continue to Screen 9 using the Pace tool.

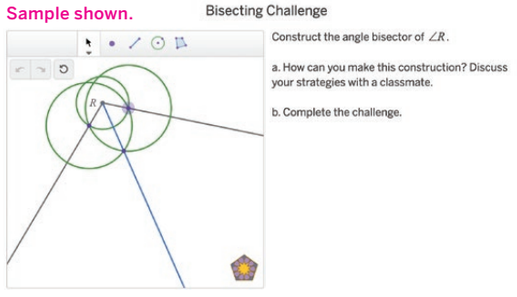
9 **Capture** explanations to share during the Connect.

Activity 3 continued >

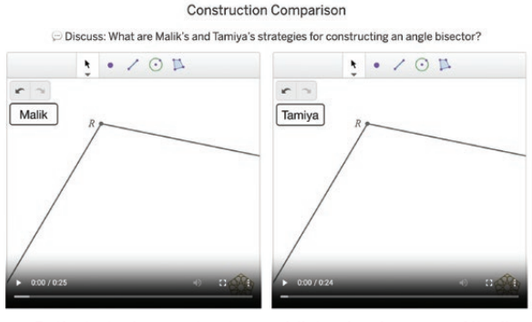
Students using digital

7

Sample shown.

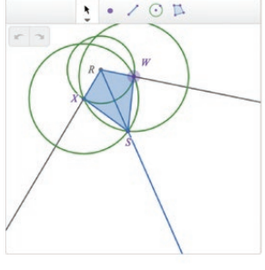


8



Responses vary. Both strategies construct a quadrilateral and use one of its diagonals to make the angle bisector. They construct the angle bisector by using a property they know about specific quadrilaterals: The diagonal bisects the angles.

9



Responses vary.

- Malik constructed a kite, and we know that the diagonal of a kite bisects the angles that are between two congruent sides. This means we know that $\angle WRS$ is congruent to $\angle XRS$.
- Malik constructed two congruent triangles ($\triangle WRS$ is congruent to $\triangle XRS$) that are congruent by SSS. Since we know that corresponding angles in congruent triangles are congruent, we know that $\angle WRS$ is congruent to $\angle XRS$.

Pairs Sharing a Device | 15 min | Pacing: Screens 7–8, Screens 9–10, Screen 9

Activity 3 Bisecting Challenge (continued)

Purpose: Students construct an angle bisector and use **MLR7: Compare and Connect** to surface strategies for the construction.



Student Notes are also available

2 Monitor

10 D **Differentiation: Explore More (Stretch)** Invite students who would like to further explore angle bisectors to complete this optional task. Encourage them to discuss their thinking with a partner.

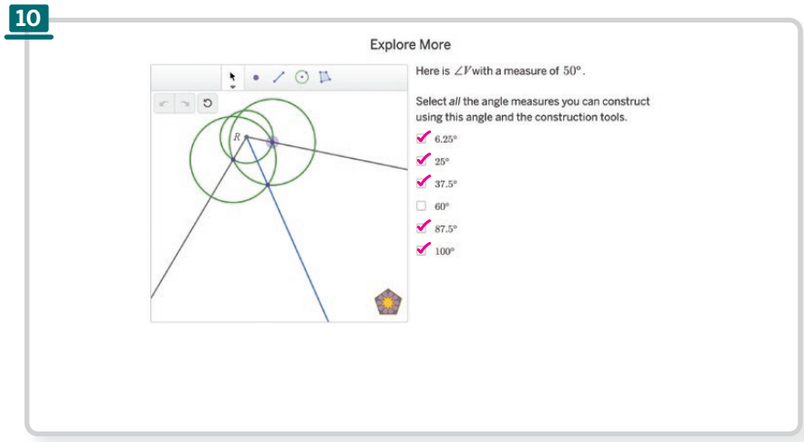
3 Connect

9 **Display** captured constructions and explanations and consider asking:

- "How would you describe Malik's proof strategy? Why did he choose to construct a kite?"
- "How did Malik's strategy convince you that he constructed an angle bisector?"

Invite students using notes to explain how they know $\angle WRS \cong \angle XRS$.

10



Synthesis

Whole Class | 5 min | Pacing: Screen 11

11 **Invite students** to respond independently, and then share their thinking with a partner.

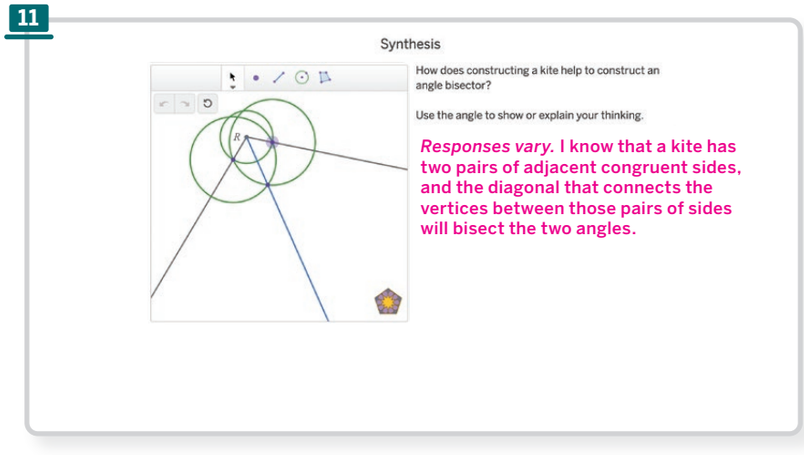
Capture and share a variety of ideas, such as:

- Constructing a kite as a strategy for constructing an angle bisector because of a property all kites have, that at least one diagonal is an angle bisector.
- Using the "circle by length" tool to measure where to construct a point can help to create adjacent congruent angles.

Note: Consider sharing that there is an additional "angle bisector" tool that is currently unavailable. Students will be able to explore this new construction capability in future lessons or in the construction tools.

Math Identity and Community Invite students to share strategies they found helpful and give credit to the students who shared them.

11

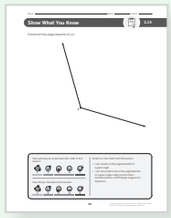


Lesson Takeaway: There are many strategies for constructing an **angle bisector**. One strategy is to construct a kite using the given angle. Since all kites have a diagonal that is an angle bisector, constructing a kite and using its definition is one way to prove that you also constructed an angle bisector.

Independent | 5 min | Pacing: Screens 12–13

Show What You Know

Purpose: Students demonstrate their understanding by constructing an angle bisector using digital construction tools.

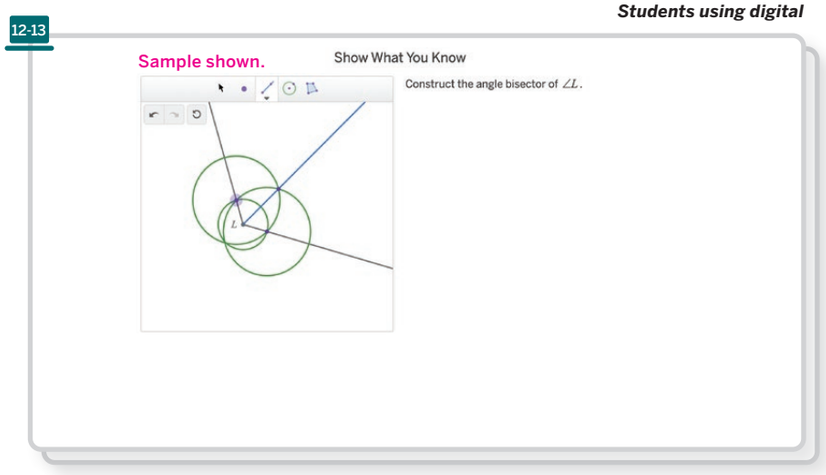


Students using print

12-13 Today's Goals

1. **Goal:** Construct the *angle bisector* of a given angle.
2. **Language Goal:** Prove that a line is the *angle bisector* of a given angle using constructions, transformations, and triangle congruence theorems. **(Speaking and Listening)**

Students using digital



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of how to construct an angle bisector, consider:

- Inviting students to complete the **Try This** problem and providing individualized feedback.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding,** consider:

- Assigning the **Lesson Practice**.
- Inviting students to review the **Summary** and complete the **Try This** problem.
- Revisiting this skill in **Practice Day 2**.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge,** consider:

- Inviting them to complete the **Explore More** task and discuss their thinking with a partner.
- Inviting them to construct their own angle challenge like the one in the **Explore More** task and trade with a partner to solve each other's challenge.

Professional Learning

Take a moment to reflect on the strategies students used to construct an angle bisector. How did students tap into their creativity today? How did you recognize and encourage students' creativity? In what ways did their methods surprise you or extend beyond the connections explored in this lesson?

Lesson Practice

Independent

Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

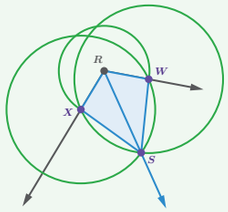
Lesson 15
Practice

Students using digital

Lesson Summary

An **angle bisector** is a line, segment, or ray that divides an angle into two adjacent congruent angles.

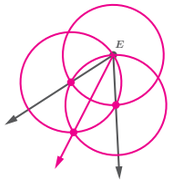
You can make sense of the construction of an angle bisector by using figures that form angle bisectors. For example, the diagonal that connects the vertices between two congruent sides of a kite will also bisect two of the kite's angles. If you can construct a kite, then you can use your understanding of a kite's properties to construct an angle bisector.



Other quadrilaterals, such as squares and rhombuses, also have diagonals that are angle bisectors. You can use figures that you already know how to construct to guide your thinking about figures that you haven't constructed yet.

Try This

Construct the angle bisector of $\angle E$. ***Responses vary.***



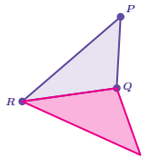
Lesson Practice
Geo.2.15

Name: _____ Date: _____ Period: _____

Problems 1–2: Here is $\triangle PQR$.

1. Reflect $\triangle PQR$ over $\overline{QR}$. Explain why the resulting quadrilateral is a kite.

   Responses vary. **When I reflect over one side of triangle PQR, the image and pre-image make two adjacent congruent sides. This is the definition of a kite.**

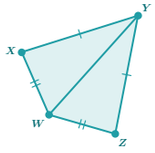


2. If you reflected over a different side of $\triangle PQR$, would you still form a kite? Explain your thinking.

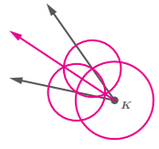
   Yes. *Explanations vary.* Because the reflection will always form two adjacent congruent sides, it doesn't matter which side I reflect over.

3. Kiandra claims that the diagonal $\overline{WY}$ bisects angles ZWX and ZYX. Is her claim correct? Explain your thinking.

   Yes. *Explanations vary.* Quadrilateral $WXYZ$ is a kite because of its two adjacent congruent sides. So the diagonal that connects the vertices between the congruent sides bisects the angle.



4. Construct the angle bisector of $\angle K$.



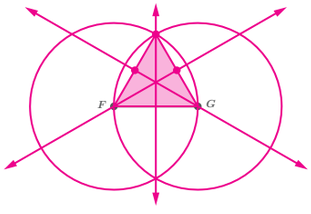
5. Is it possible to construct an angle bisector of an angle that is larger than $180°$? Explain your thinking.

   Yes. *Explanations vary.* Construct the angle bisector of the part of the angle that is less than $180°$ and just stretch the bisector line through the vertex.

Lesson Practice
Geo.2.15

Name: _____ Date: _____ Period: _____

6. Construct a triangle where the angle bisectors are the same lines as the perpendicular bisectors. Use $\overline{FG}$ as one side of the triangle.



Spiral Review

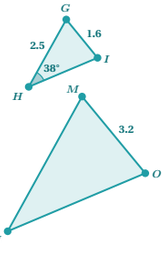
Problems 7–8: Triangles GHI and MNO are similar triangles.

7. What is the length of $\overline{MN}$?

   5

8. What is the measure of angle N?

   38°



Reflection

1. Circle a problem you want to talk to a classmate about.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK	Standard(s)
On-lesson			
	1–2	2	HSG.CO.C.12*
	3	2	HSG.CO.C.12
Test Practice	4	2	HSG.CO.C.12
	5–6	3	HSG.CO.C.12
Spiral Review			
Fluency	7–8	1	HSG.SRT.B.5

*This problem builds toward the standard shown.

Need More Practice?

Additional Practice resources coming soon.

~ 45 min

Practice Day 2

Purpose: Students practice the concepts, skills, and strategies developed in Lessons 1–15. Consider using this Practice Day as preparation for the upcoming End-of-Unit Assessment.

Preparation

Materials

Option 1: Group Problems

- Student Edition
- Task Cards, one set per group
- geometry toolkit: compasses, straightedges

Option 2: Level Up

- Student Edition
- Task Cards, one set per group
- geometry toolkit: compasses, straightedges

Math Identity and Community Invite students to use a variety of formats to help them organize their arguments for Task Cards C and D.

D **Differentiation: (Support)** To support students in using definitions, invite students to use the glossary, their student notes, or Lesson 12, Screen 4.

Consider asking these questions to check for understanding:

- Task A: "What tools will you use to construct the angle bisector?" **(MP5)**
- Task B: "How did you decide which conjecture matched each diagram?"

Standards

Addressing

HSG.CO.C.11

Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Also Addressing: **HSG.CO.B.7, HSG.CO.B.8, HSG.CO.C.9, HSG.CO.D.12**

Mathematical Practices: MP3, MP5, MP6

Facilitation Options

Option 1: Group Problems

This structure encourages student collaboration and focuses students' attention on one task at a time.

Small Groups

Distribute one set of task cards to each group.

Consider sharing that each group will engage with all four task cards, one at a time. If time allows, invite students to order the cards based on their level of complexity to help them prioritize which task cards to start with.

Invite students to discuss their reasoning with their group.

Other considerations:

- How will students get feedback?
  - Post the answers and invite groups to check them.
  - Invite groups to check in with you when they're ready for feedback.
- How will you support student collaboration?
  - Invite a student from each group to read the task aloud.

Option 2: Level Up

This structure supports student collaboration and focuses students' attention on one task at a time.

Small Groups

Consider sharing that there are four tasks that groups will complete one at a time.

Distribute Task A to each group.

Invite students to discuss their reasoning with their group.

Other considerations:

- How will students get feedback and transition to the next task?
  - Post the answers. Invite groups to check their answers and retrieve the next task card as they're ready.
  - Invite groups to check in with you when they're ready for feedback and the next task card.
- How will you support student collaboration?
  - Discuss what supporting each other without giving answers could look or sound like.

SE Practice Day 2

Unit 2
Lessons 1–15

Name: Date: Period:

Practice Day 2

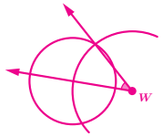
Let's practice what you've learned so far in this unit!



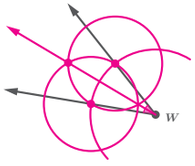
You will use task cards for this Practice Day. Record all of your responses here.

Task A: Construct It! *Responses vary.*

1.



2.



3. Explanation:

I know that if I construct three sides of a triangle to be congruent to three corresponding sides of another triangle, the corresponding angles must also be congruent.

4. Explanation:

I know ray $\overline{AF}$ will bisect angle A because $\triangle ADF \cong \triangle AEF$ by SSS, so $\angle DAF \cong \angle EAF$ since they are corresponding parts of congruent triangles.

Task B: Match It! *Responses vary.*

1.

Side-Angle-Side (SAS)	Angle-Side-Angle (ASA)	Side-Side-Side (SSS)
$\triangle PSR \cong \triangle QSR$ $\triangle HYJ \cong \triangle FYW$	$\triangle ABC \cong \triangle CDA$	$\triangle ZBA \cong \triangle ZIA$ $\triangle BZI \cong \triangle IAB$

2. (a) Figure $FJHW$ (b) Figure $POQR$

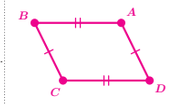
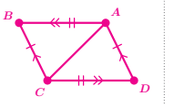
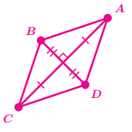
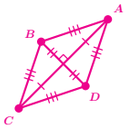
(c) Figure $ZIAB$ (d) Figure $ABCD$

Unit 2
Lessons 1–15

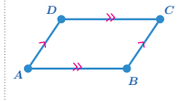
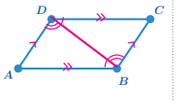
Name: Date: Period:

Practice Day 2 (continued)

Task C: Plan It! *Responses vary.*

	What do I know?	What do I want to prove?	What strategies could I use?
c			$\triangle ABC \cong \triangle CDA$ by SSS. This means that corresponding angles are congruent. I can use this to show that the opposite sides are parallel because alternate interior angles are congruent.
e			Fold quadrilateral $ABCD$ over $\overline{AC}$. Point B will land on point D, which shows $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{CD}$. Fold quadrilateral $ABCD$ over $\overline{BD}$, and point A will land on point C, which shows $\overline{CB} \cong \overline{AB}$ and $\overline{CD} \cong \overline{AD}$. Therefore all of the sides are congruent.

Task D: Prove It! *Responses vary.*

	What do I know?	What do I want to prove?	What strategies could I use?
1.			Construct $\overline{BD}$. $\angle CDB \cong \angle ABD$ and $\angle ADB \cong \angle CBD$ because they are alternate interior angles. These pairs of congruent corresponding angles must be congruent.

2. $\angle CDB \cong \angle ABD$ and $\angle ADB \cong \angle CBD$ because they are alternate interior angles of parallel lines. $\angle D$ is formed by combining $\angle CDB$ and $\angle ADB$. $\angle B$ is formed by combining $\angle ABD$ and $\angle CBD$. Since $\angle B$ and $\angle D$ are each formed by combining pairs of congruent angles, they must be congruent to each other.

Task Cards (online)

Unit 2
Practice Day 2
(p. 1 of 2)

Task Cards

Directions: Make and distribute one copy per group of students.

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Task A: Construct It!

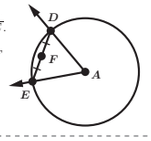
1. Construct a copy of angle W.

2. Construct an angle bisector of angle W.

3. Explain how SSS triangle congruence criteria helps you construct a congruent angle.

4. Here is a circle with center A. Point F is the midpoint of $\overline{DE}$.

Explain how you know ray AF will bisect angle A.



Task B: Match It!

Figure $POQR$
$\overline{OR} \perp \overline{PQ}$, $\overline{PS} \cong \overline{QS}$

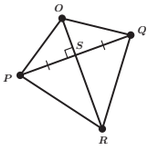


Figure $FJHW$
$\overline{JW} \perp \overline{FH}$, $\overline{FY} \cong \overline{HY}$, $\overline{JY} \cong \overline{WY}$

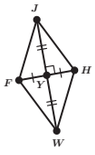


Figure $ZIAB$
$\overline{ZI} \cong \overline{IA} \cong \overline{AB} \cong \overline{BZ}$

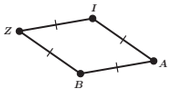
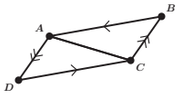


Figure $ABCD$
$\overline{DC} \parallel \overline{BA}$, $\overline{AD} \parallel \overline{CB}$



1. Write at least one congruence statement that proves two triangles are congruent based on the criteria given in the diagram. Be sure to write the vertices in corresponding order.

2. Match each conjecture with a diagram that could be used to prove it.

(a) A quadrilateral with perpendicular diagonals that bisect each other is equilateral.

(b) If one diagonal of a quadrilateral is the perpendicular bisector of the other, then two pairs of adjacent sides are congruent.

(c) Opposite angles in an equilateral quadrilateral are congruent.

(d) In a parallelogram, opposite sides are congruent.

Unit 2
Practice Day 2
(p. 2 of 2)

Task Cards

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Task C: Plan It!

1. Pick two conjectures and convince yourself they are true. Write the letter for the conjectures you select in the first column of the table.

Use the definitions of each quadrilateral to help with your thinking.

(a) All rectangles are parallelograms.

(b) If a parallelogram has (at least) one right angle, then it is a rectangle.

(c) If a quadrilateral has two pairs of opposite sides that are congruent, then it is a parallelogram.

(d) If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

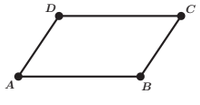
(e) If the diagonals of a quadrilateral bisect each other and they are perpendicular, then the quadrilateral is a rhombus.

2. Complete the table to make a plan to prove each conjecture you chose.

Task D: Prove It!

Here is a conjecture.

If quadrilateral $ABCD$ is a parallelogram, then $\angle B$ is congruent to $\angle D$.



1. On your own, complete a plan to prove the conjecture.

2. With a partner, discuss each other's plans.

- What do you notice about your partner's understanding of the problem?
- What revision would help them move forward?

3. Prove the conjecture.

Assess and Respond

Additional data on student thinking can be found in Teacher Reporting.

End-of-Unit Assessment

Independent | ~45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. Text to speech is available in the digital version. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or Skill	Addressed in	DOK	Standard(s)
1	Identify what corresponding parts of figures are congruent after a transformation	Lesson 7	2	HSG.CO.B.7
2	Identify properties of parallelograms	Lesson 14	2	HSG.CO.C.11
3	Identify corresponding parts of congruent triangles	Lesson 3	2	HSG.CO.B.7 MP6
4	Determine if there is sufficient criteria to prove two triangles are congruent and identify corresponding parts of congruent triangles	Lesson 9	2	HSG.CO.B.8
5	Prove that two triangles are congruent using rigid transformations	Lesson 5	2	HSG.CO.B.6 MP3
6a	Construct an angle bisector	Lesson 15	2	HSG.CO.D.12
6b	Construct a copy of an angle	Lesson 10	2	HSG.CO.D.12
7a–b	Analyze a proof about parallelograms	Lesson 9	2	HSG.CO.C.11 MP3
7c	Prove theorems about parallelograms	Lesson 13	3	HSG.CO.C.11 MP3

Assessment Resources

- Student Assessments (Form A)
- Assessment Rubrics with Answers
- Show What You Know PDFs and Answer Keys

Practice

If students need further review or practice, consider assigning the practice problems that correspond to each lesson.

Name: Date: Period:

End-of-Unit Assessment

Unit Geo.2

1. $\triangle CAT$ is a reflection of $\triangle BRI$ over line l.

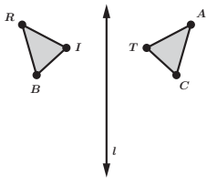
Which statement is true?

A. $\angle R \cong \angle T$

B. $\angle B \cong \angle C$

C. $\overline{IB} \cong \overline{AC}$

D. $\angle I \cong \angle A$



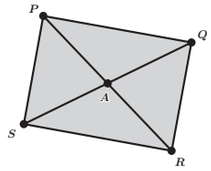
2. Here is parallelogram $PQRS$. What additional information is needed to prove that $PQRS$ is a rectangle?

A. $\overline{PQ} \cong \overline{SR}$

B. $\overline{PR} \cong \overline{QS}$

C. $\overline{PA} \cong \overline{QA}$

D. $\overline{QR} \cong \overline{PS}$



3. $\triangle UVW$ is a 180° rotation of $\triangle YVX$ centered at point V.

Complete the congruence statements.

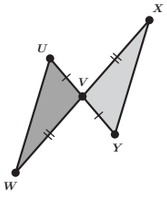
a. $\overline{UV} \cong$ $\overline{YV}$

b. $\overline{WV} \cong$ $\overline{XV}$

c. Name a pair of corresponding congruent angles.

Responses vary.

$\angle U \cong \angle Y$



Explain how you know they are congruent.

Explanations vary. $\triangle UVW \cong \triangle YVX$ by rigid transformations. That means $\angle U \cong \angle Y$ because these angles are corresponding parts of congruent triangles.

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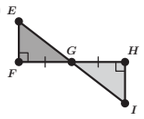
Name: Date: Period:

End-of-Unit Assessment (continued)

Unit Geo.2

4. Is there enough information to prove the following triangle pairs are congruent? If so, how?

a.



A. Yes, by SSS

B. Yes, by SAS

C. Yes, by ASA

D. No

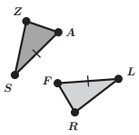
If yes, identify the congruent pairs from the criteria.

$\angle F \cong \angle H$

$\overline{FG} \cong \overline{HG}$

$\angle FGE \cong \angle HGI$

b.



A. Yes, by SSS

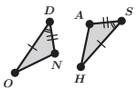
B. Yes, by SAS

C. Yes, by ASA

D. No

If yes, identify the congruent pairs from the criteria.

c.



A. Yes, by SSS

B. Yes, by SAS

C. Yes, by ASA

D. No

If yes, identify the congruent pairs from the criteria.

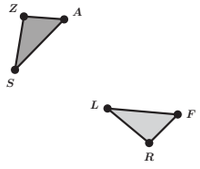
$\overline{OD} \cong \overline{HS}$

$\angle D \cong \angle S$

$\overline{DN} \cong \overline{SA}$

5. Describe a sequence of rigid transformations to prove whether $\triangle ZAS \cong \triangle RFL$.

Responses vary. Translate $\triangle ZAS$ so that point S goes to point L. Next, rotate this translated triangle with point L as the center so that point A goes to point F. Then reflect this new triangle over $\overline{FL}$. The triangles stack perfectly, so $\triangle ZAS \cong \triangle RFL$.



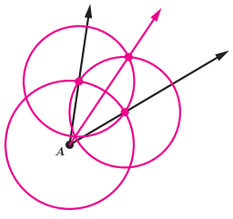
Name: Date: Period:

End-of-Unit Assessment (continued)

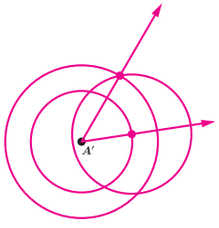
Unit Geo.2

6. Here is angle A.

a. Construct an angle bisector of $\angle A$.



b. Construct a copy of $\angle A$.



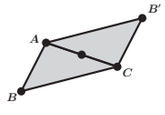
Name: Date: Period:

End-of-Unit Assessment (continued)

Unit Geo.2

7. Lola rotated $\triangle ABC$ 180° centered at the midpoint of $\overline{AC}$. Here is her proof that quadrilateral $ABCB'$ is a parallelogram. *Responses vary.*

I know this is a parallelogram because rotations make congruent figures. Corresponding parts of these triangles are congruent, and $\overline{CB}$ and $\overline{AB'}$ are parallel. If opposite sides are parallel, then ABCB' is a parallelogram.



a. Describe one convincing thing about this proof.

- It states that rotations make congruent figures, which means corresponding parts of these triangles are congruent.
- It identifies that $\overline{CB}$ and $\overline{AB'}$ are parallel.

b. Describe one way to strengthen this proof.

- Lola can justify how she knows that $\overline{CB}$ and $\overline{AB'}$ are parallel.
- She can add more information to prove this is a parallelogram. One pair of parallel sides is not enough.

c. Write your own proof that quadrilateral $ABCB'$ is a parallelogram.

$\triangle ABC \cong \triangle CB'A$ because rotations result in congruent figures. This means corresponding parts are congruent, so $\overline{CB} \cong \overline{AB'}$ and $\angle ACB \cong \angle CAB'$. $\angle ACB$ and $\angle CAB'$ are alternate interior angles, so if they are congruent, this means $\overline{CB}$ and $\overline{AB'}$ are parallel. If a quadrilateral has a pair of sides that are both congruent and parallel, then the figure must be a parallelogram.

Reflection

Select a question to answer.

- ☐ What is something you're proud of from this unit?
- ☐ Write what you know about a topic from this unit that you weren't asked about today.
- ☐ Describe or show one strategy you found helpful in this unit. Name any students who helped you with this strategy.
- ☐ What else would you like your teacher to know?

Editable Version, Form B

Digital

Unit X
End-of-Unit
Assessment

Coming Soon

Coming Soon

Coming Soon

Coming Soon

D Differentiation (End-of-Unit Assessment)

Note: To **Strengthen** and **Stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:
Use rigid transformations to prove angle relationships. Describe a strategy using rigid transformations that proves a relationship is always true. **(Lessons 1–4)**	3c	**Support** • Revisiting **Lesson 2, Activity 2** (*Always True?*) and inviting students to precisely describe the transformations taking place. • Inviting students to read the **Lesson 3 Summary** (*Angles and Transversals*) and write it in their own words or paraphrase it to a caregiver or a classmate.
Define congruence using rigid transformations.	1, 3a–b, 5	**Support** • Inviting them to read the **Lesson 6 Summary** (*Parts and Places*) and highlight key components in the diagram and table. • Revisiting **Lesson 7, Activity 1, Problem 4** (*Corresponding Components*) and inviting them to sketch a new triangle pair example with different vertex labels. • Revisiting **Lesson 7, Screen 3** (*Corresponding Components*) and inviting them to explore the interaction again with a partner. • Revisiting **Lesson 5, Screen 9** (*Stack 'Em Up*) and discussing each transformation challenge with a partner.
Justify how much information is needed to decide that two triangles are congruent. **(Lessons 5–11)**	4	**Support** • Inviting them to read the **Lesson 9 Summary** (*Congruence Conditions*) and highlight important information in each diagram. • Inviting them to complete the **Try This** problem in **Lesson 9** (*Congruence Conditions*) and providing individualized feedback.
Prove theorems about quadrilaterals.	2, 7	**Support** • Inviting students to read the **Lesson 14 Summary** (*A-Door-Able Parallelograms*) and discuss with a partner what they know to be true about rectangles. • Revisiting the **Lesson 12 Summary** (*The Quad Squad*) and reflecting on how to use the different properties to identify each quadrilateral.
Make sense of constructions using properties and definitions of geometric figures. **(Lessons 12–15)**	6	**Support** • Inviting them to complete the **Try This** problem for **Lesson 15** (*Let's Go Build a Kite*) and giving individualized feedback. • Revisiting **Lesson 15, Activity 3** and summarizing the different strategies used to perform an angle bisector construction. • Inviting them to complete the **Try This** problem for **Lesson 10** (*Resuming Construction*) and providing individualized feedback. • Revisiting the **Lesson 10 Summary** and paraphrasing it for a caregiver or classmate.

Student Edition pages

Amplify Desmos Math

Geometry

Student Edition

Unit 1

Constructions and Rigid Transformations

There are many ways to create objects in geometry. You can use drawing tools, such as a compass or straightedge, or use design software to create things digitally. In this unit, you'll explore that creation process by constructing figures using circles and lines and using transformations to move, flip, and rotate those figures.

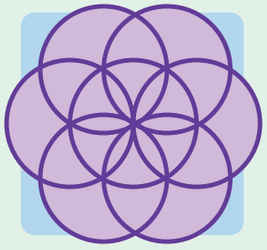
Essential Questions

- How can we construct precise geometric figures using circles and lines?
- How can we describe transforming geometric figures using reflections, rotations, and translations?
- How can we use tools to construct geometric relationships?

Sub-Unit

1

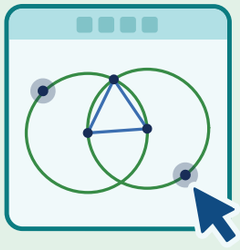
Constructions



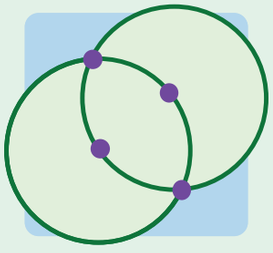
Lesson 1
Circles and Segments



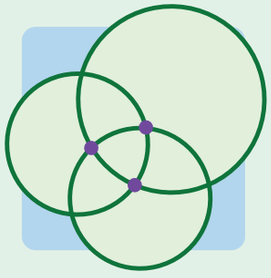
Lesson 2
Compass Constructions



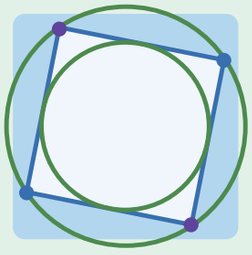
Lesson 3
Constructing Digitally



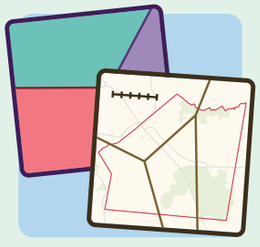
Lesson 4
Right in the Middle



Lesson 5
Lines of Construction



Lesson 6
Square Up

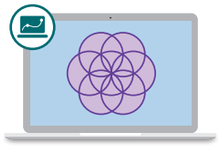


Lesson 7
Community Constructions

Unit 1
Lesson
1

Name: ..... Date: ..... Period: .....

Circles and Segments

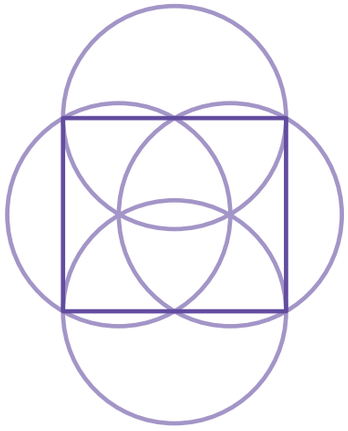


Entire Lesson

Things to Remember:

Select Screens

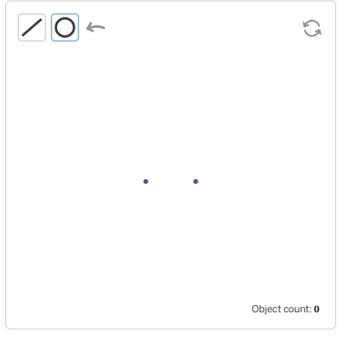
5 Thiago says that this **construction** is a square.



Class definition of *construction*:

6 Construct two different equilateral triangles.

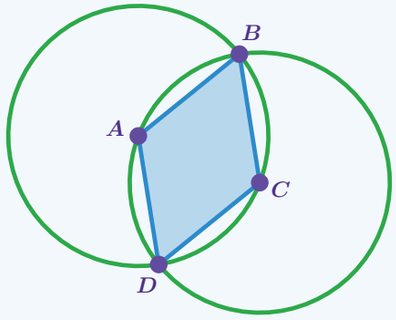
How did you know the side lengths were equal in your construction?



Lesson Summary

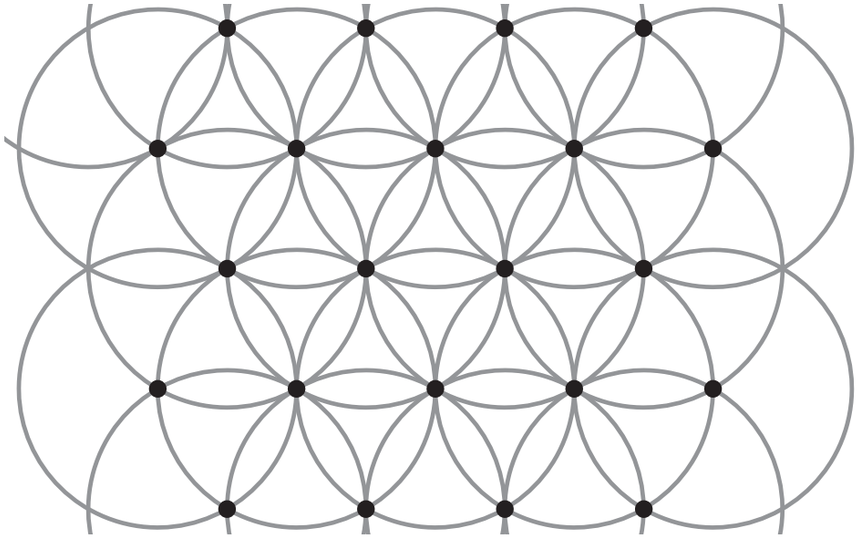
A **construction** is a geometric drawing made using precise tools. Because *circles* have a known radius, you can use them to construct *line segments* of equal length. When circles overlap with their centers on either end of the same radius, you can construct line segments with the same length.

For example, you know that each side length of quadrilateral $ABCD$ is the same because each side is a radius of one of two circles that are the same size.



Try This

a Construct two different rectangles.



b Explain how you know that opposite sides of your rectangles have the same length.

Lesson Practice
Geo.1.01

Name: ..... Date: ..... Period: .....

Problems 1–4: Use line segments and intersection points to construct each polygon.

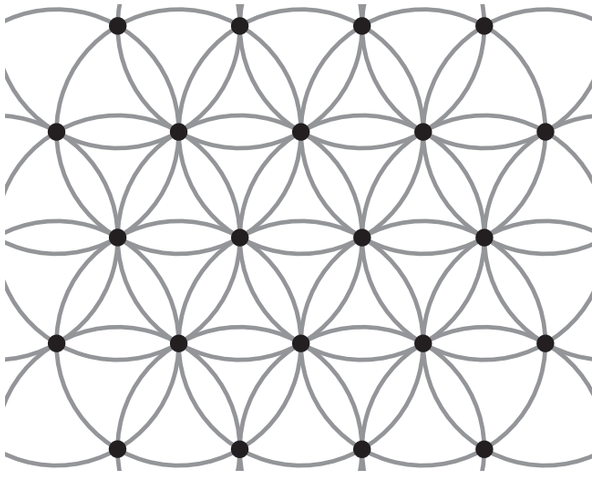
Note: Each circle is the same size.

1. Equilateral triangle

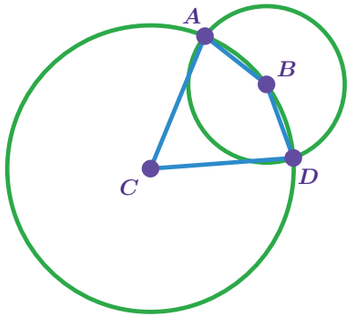
2. Equilateral triangle with different side lengths from Problem 1

3. Hexagon

4. Rectangle



5. Points C and B are the centers of these circles. Highlight two segments that you know are the same length.



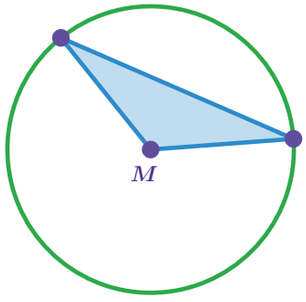
6. A triangle is drawn in a circle. Point M is the center of the circle. Which type of triangle is shown?

A. Equilateral: three sides are the same length

B. Right: one $90°$ angle

C. Scalene: no sides are the same length

D. Isosceles: two sides are the same length



Lesson Practice
Geo.1.01

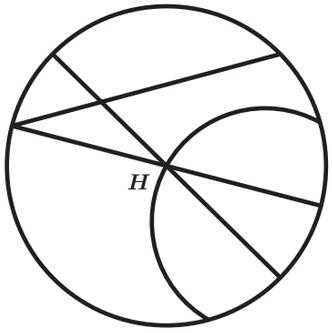
Name: _____ Date: _____ Period: _____

7. Describe how you could construct a line segment that's exactly 4 times the length of this line segment using only circles and line segments.



Spiral Review

Problems 8–10: Here is a circle with center H.



8. Highlight a line segment that is a radius.

Explain how you know it's a radius.

9. Highlight a line segment that is a diameter.

Explain how you know it's a diameter.

10. Highlight a line segment that is neither a radius nor a diameter.

How do you know it's neither?

Reflection

1. Put a smiley face next to a problem you were stuck on and then figured out.

2. Use this space to ask a question or share something you're proud of.

Unit 1
Lesson
2

Name: _____ Date: _____ Period: _____

Compass Constructions



Let's use a compass to make precise designs on paper.

Warm-Up

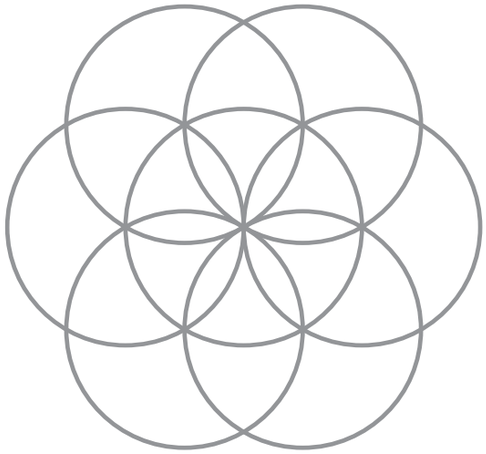
1. Experiment with using a compass to construct circles.

Activity
1

Name: ..... Date: ..... Period: .....

Circles

This design is made of circles that are all the same size.



2. Use a compass to recreate this design. You can make the design as large or as small as you want.

3. Write one suggestion for how to make your design more precise. Discuss your thinking with a classmate. Write their suggestions below.

My Suggestion	Other Suggestion(s)

Activity 1

Name: ..... Date: ..... Period: .....

Circles (continued)

4. Use a compass to construct the same design more precisely.

5. **Regular polygons** have sides that are the same length and angles that are the same measure.

a A regular triangle is also called an **equilateral triangle**. Construct an equilateral triangle by connecting three points in your design.

b Construct a regular hexagon by connecting six points in your design.

c How do you know that your hexagon and triangle each have sides that are the same length?

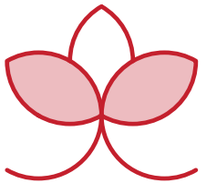
Activity
2

Name: ..... Date: ..... Period: .....

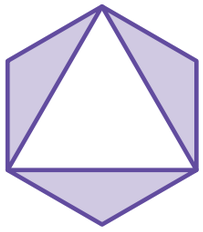
Compass Constructions

All of these designs use the pattern from Activity 1 as an underlying construction.

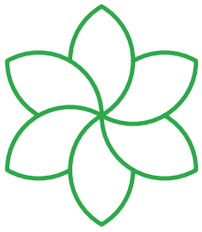
Design A



Design B



Design C



Design D



6. Select a design and construct it.

Selected design: .....

Activity 2

Name: Date: Period:

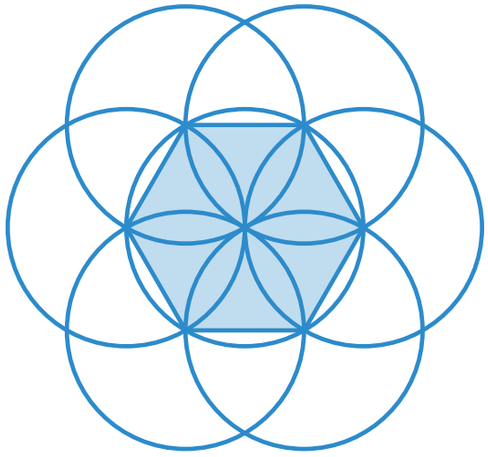
Compass Constructions (continued)

7. Write instructions for how to construct your design.

Synthesis

8. How can using a compass help you construct a hexagon with six equal sides?

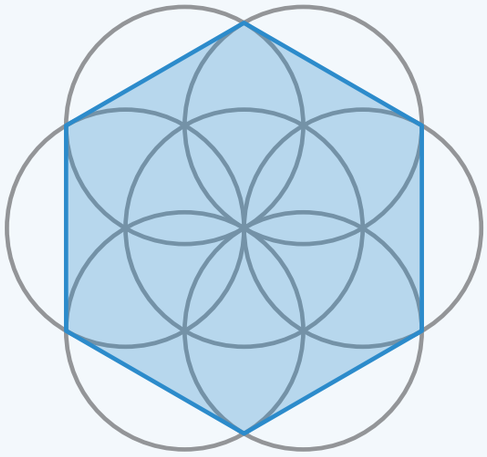
Use the example if it helps with your thinking.



Lesson Summary

You can use a *compass* to make circles that are all the same size. This is a helpful measurement tool when constructing **regular polygons** where each side is a radius of the circle.

The intersections of lines and circles define points that help you replicate constructions and write instructions. You can use these points to reason about distances, precisely construct designs, and communicate about how to recreate your constructions.



Regular hexagon

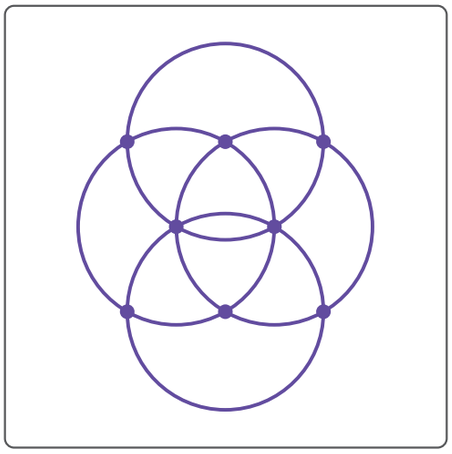
Try This

Use a compass and straightedge to construct a polygon where all the sides are the same length.

Lesson Practice
Geo.1.02

Name: _____ Date: _____ Period: _____

Problems 1–3: Here is a circle design.

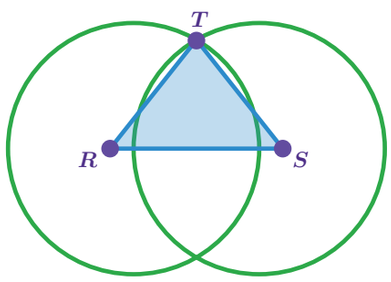


1. Construct this design.

2. Construct a polygon that has at least four sides using intersection points on the design.

3. How many equilateral triangles could be created using intersection points on the design? Show or explain your thinking.

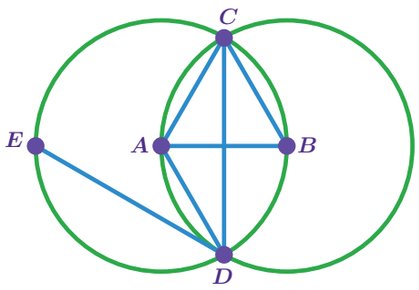
4. How could you change this design so you could use it to construct an equilateral triangle?



Lesson Practice
Geo.1.02

Name: _____ Date: _____ Period: _____

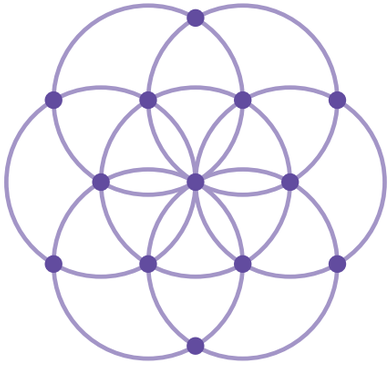
5. Point A is the center of one circle. Point B is the center of the other. Select *all* the segments that have the same length as segment AB.



- ☐ **A.** Segment AC
- ☐ **B.** Segment AD
- ☐ **C.** Segment BC
- ☐ **D.** Segment CD
- ☐ **E.** Segment DE

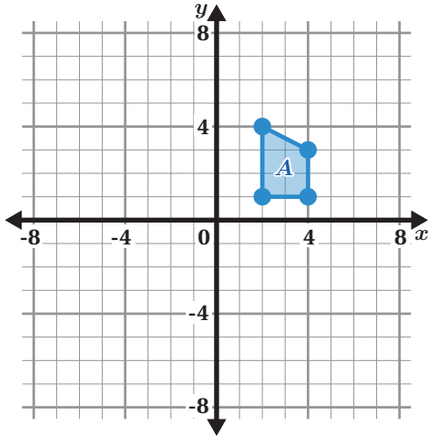
6. Use intersection points in the design to construct a polygon with at least seven sides.

All of the sides should be the same length.



Spiral Review

Problems 7–9: Transform polygon A based on each set of directions.



7. Reflect polygon A over the y-axis. Label the image B.

8. Translate polygon A down 7 units. Label the image C.

9. Translate polygon A down 3 units and left 4 units. Label the image D.

Reflection

1. Put a heart next to the problem you feel most confident about.

2. Use this space to ask a question or share something you're proud of.

Unit 1
Lesson
3

Name: ..... Date: ..... Period: .....

Constructing Digitally



Entire Lesson

Things to Remember:

Select Screens

3 What are some advantages and disadvantages of constructing with digital tools?

Notes from class discussion:



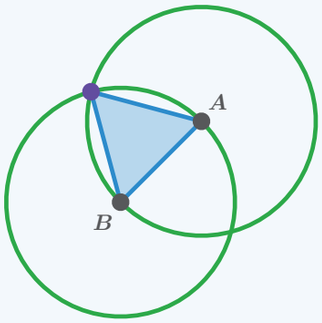
4 **Discuss:** Do you think that everyone who follows Aaliyah's instructions will make the same triangle?

> Step 1 Draw a circle centered at A.
> Step 2 Draw a circle centered at B.
> Step 3 Draw a polygon from points A and B to an intersection point of the two circles.

Draft your thinking before you discuss:

Lesson Summary

Digital tools are helpful in making precise constructions. For circles, the radius lengths will always be exactly what you define (and your compass won't slip like it can on paper).



You can define a center and a point on a circle to construct line segments of exactly the same length.

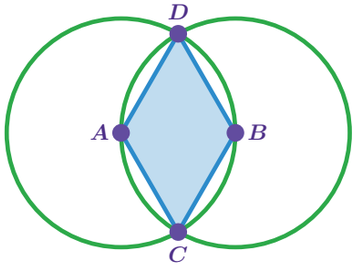
When using digital tools to construct, you can also drag points to see what relationships hold true about the constructions.

Using precise language can help you describe digital constructions. It's helpful to:

- Clearly define the center and radius length of a circle.
- Use points on constructed objects, like circles or line segments.
- Label intersection points to have clear ways to describe constructed points.

Try This

Here are instructions for constructing this polygon.



Step 1: Construct two circles.

Step 2: Label their intersection points C and D.

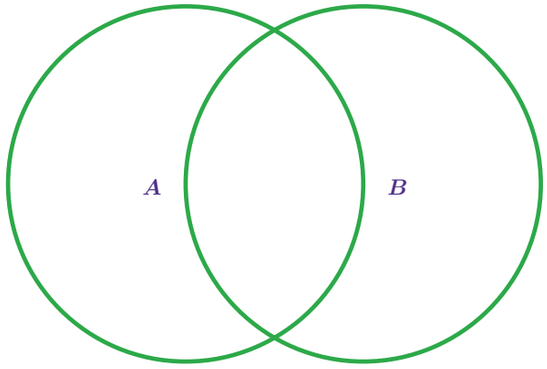
Step 3: Connect all the points to make a rhombus.

Select a step and revise it to make these instructions more precise.

Lesson Practice
Geo.1.03

Name: _____ Date: _____ Period: _____

Problems 1–3: Here is a diagram.



1. Construct an equilateral triangle using points A and B as vertices.

2. Construct two more circles centered at the intersections of the original circles. All of the circles should be the same size.

3. Use the intersections to construct an equilateral triangle that is larger than your first one.

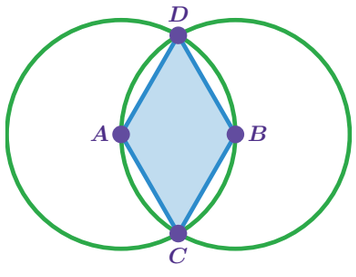
4. Here are instructions for constructing an equilateral triangle.

Which step should be revised? Explain your thinking.

Step 1 Construct a circle with center A.
Step 2 Mark a point on the circle and label it B.
Step 3 Construct a circle with center B.
Step 4 Mark a point at one of the intersection points of the circles and label it C.
Step 5 Construct segments AB, BC, and CA.

5. Roberto and Anh started with points A and B, and created quadrilateral $ACBD$.

What steps could they write so that someone else can construct the exact same quadrilateral?



Lesson Practice
Geo.1.03

Name: _____ Date: _____ Period: _____

Problems 6–7: Here is segment GH.

6. Create a design that takes *exactly* 5 steps to complete.



7. List your instructions.

Step 1:

Step 2:

Step 3:

Step 4:

Step 5:

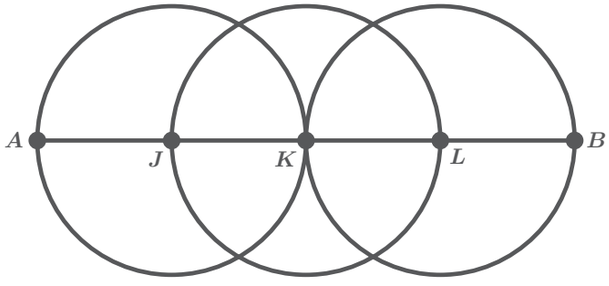
Spiral Review

8. What information is helpful to include when writing precise directions for constructing a circle?

9. These three circles are centered at points J, K, and L.

The length of segment LB is 3 centimeters.

What is the length of segment AL?



Reflection

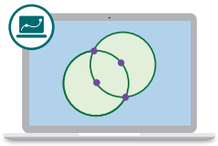
1. Circle a problem you want to talk to a classmate about.

2. Use this space to ask a question or share something you're proud of.

Unit 1
Lesson
4

Name: ..... Date: ..... Period: .....

Right in the Middle

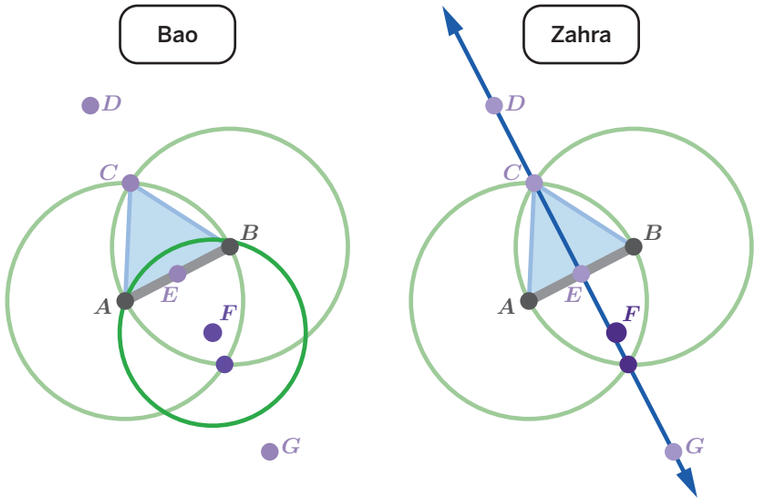


Entire Lesson

Things to Remember:

Select Screens

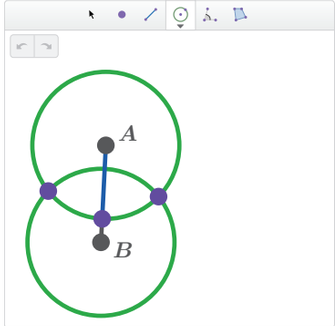
4 Here is the work that Bao and Zahra did to identify points that are not equidistant from points A and B.



Notes from class discussion:

7 Zahra ran out of space constructing a perpendicular bisector, so she used the "circle by length" tool instead.

Notes from class discussion:



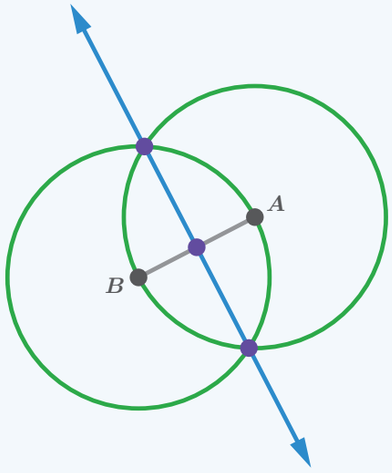
Lesson Summary

A **perpendicular bisector** is a line that passes through the **midpoint** of a line segment. Any points on the perpendicular bisector are **equidistant** from the endpoints of that line segment.

To construct a perpendicular bisector:

- You can construct two circles centered at each endpoint of a line segment, using the segment as the radius of each circle.
- You can construct two circles with the same radius centered at the endpoints of the segment.

The line that passes through the intersection points of the circles is the perpendicular bisector.



Try This

Construct the perpendicular bisector of segment AB.

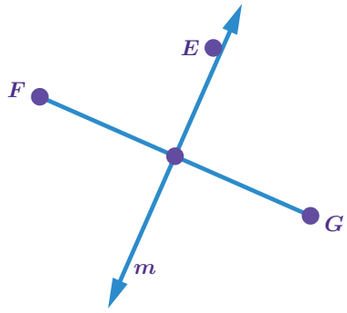


Lesson Practice
Geo.1.04

Name: _____ Date: _____ Period: _____

Problems 1–2: Line m is the perpendicular bisector of $\overline{FG}$.

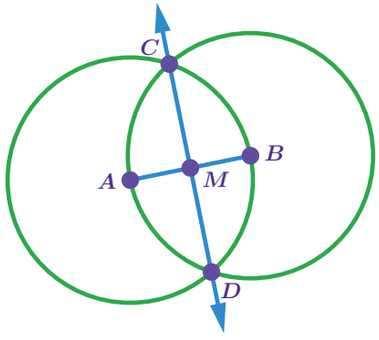
1. Plot a point that is the same distance from F as it is from G.



2. Circle the phrase that best completes the statement:
Point E is _____.

Closer to point F | Closer to point G | The same distance from point F as it is from point G

3. In the diagram, point A is the center of one circle and point B is the center of another. Select *all* the true statements.



- ☐ **A.** Line CD is perpendicular to segment AB.
- ☐ **B.** Point M is the midpoint of segment AB.
- ☐ **C.** The length of segment AB is equal to the length of segment CD.
- ☐ **D.** Segment AM is perpendicular to segment BM.
- ☐ **E.** The length of segment CB is equal to the length of segment DB.

Problems 4–5: Here is $\overline{TS}$.

4. Construct the perpendicular bisector of $\overline{TS}$.



5. Using $\overline{TS}$ as a side, construct a triangle with *only* two sides of the same length.

Lesson Practice
Geo.1.04

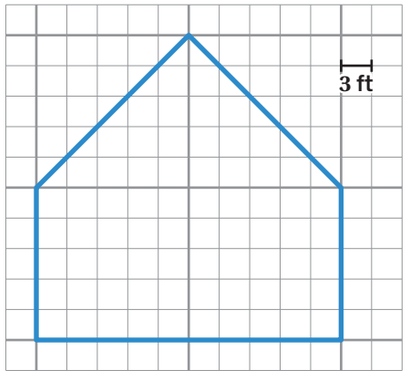
Name: _____ Date: _____ Period: _____

6. Construct an equilateral triangle with a side that is half the length of $\overline{LM}$.



Spiral Review

Problems 7–9: Here is a scale drawing of the front of a shed.



7. What is the height of the shed, in feet?

8. What is the width of the shed's base at its widest point, in feet?

9. Find the total area, in square feet.

Reflection

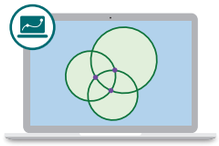
1. Put a star next to the problem you understood best.

2. Use this space to ask a question or share something you're proud of.

Unit 1
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5

Name: ..... Date: ..... Period: .....

Lines of Construction



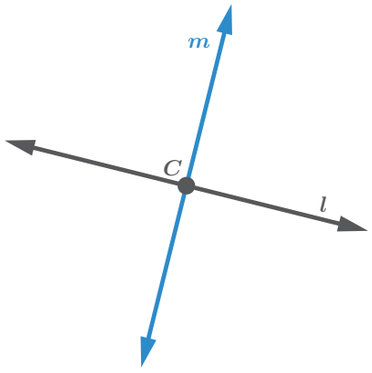
Entire Lesson

Things to Remember:

Select Screens

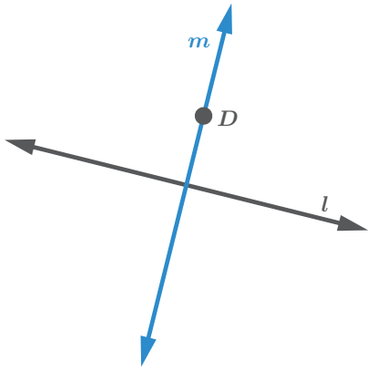
3-4 Construct line m so it is perpendicular to line l through point C.

Record a process for this construction:



5-6 Construct line m through point D so it is perpendicular to line l.

Record a process for this construction:



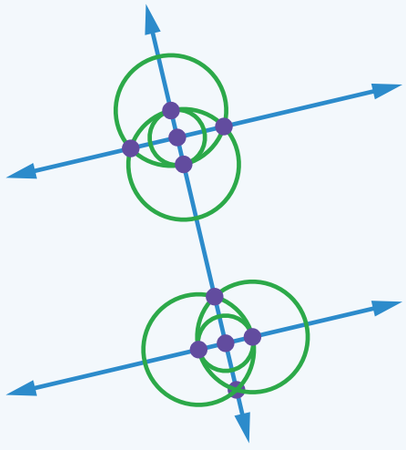
Lesson Summary

You can use perpendicular bisectors to construct **perpendicular lines** and **parallel lines** with precision. Perpendicular lines are pairs of lines that form right angles at their intersection. Lines are parallel when each point on one line is the same distance from a corresponding point on the other line.

To construct a line through a specific point that's perpendicular to a given line:

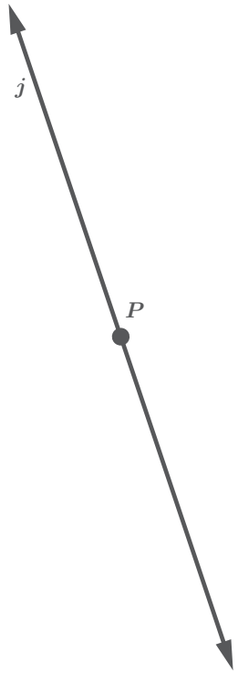
- Construct a circle using the specific point as its center, making sure that it intersects the given line at two points.
- Use the two intersection points as endpoints of a line segment and then construct the perpendicular bisector of that segment.

One technique for constructing a parallel line through a specific point is to use the perpendicular bisector construction twice. If two lines are perpendicular to a given line, then those two lines are parallel.



Try This

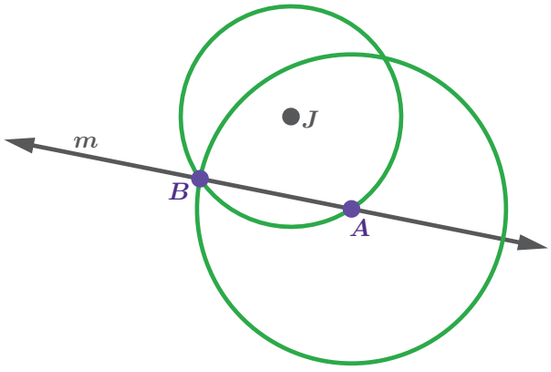
Construct line k so it is perpendicular to line j and passes through point P.



Lesson Practice
Geo.1.05

Name: _____ Date: _____ Period: _____

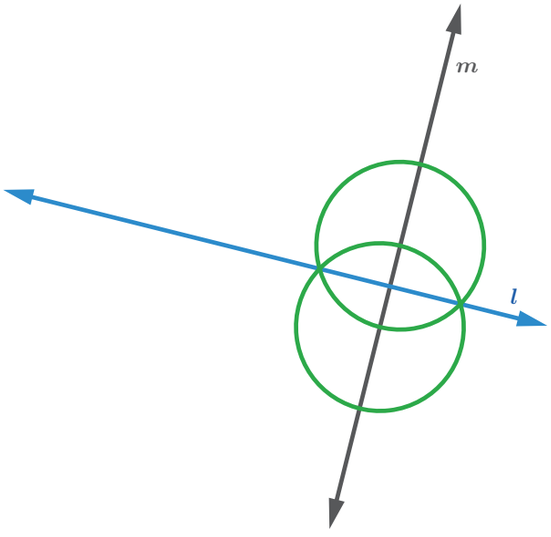
Problems 1–2: Mauricio started to construct the line perpendicular to line m that passes through point J.



1. Describe the next step in his construction.

2. Complete the construction.

Problems 3–4: Yolanda started to construct a line parallel to line m.



3. Describe possible remaining steps to complete her construction.

4. Complete the construction.

5. Lines q and r are perpendicular to line p. Select *all* the true statements.

- ☐ **A.** Line q is perpendicular to line r.
- ☐ **B.** Line q is perpendicular to line p.
- ☐ **C.** Line r is perpendicular to line p.
- ☐ **D.** Line q is parallel to line r.
- ☐ **E.** Line q is parallel to line p.

Lesson Practice
Geo.1.05

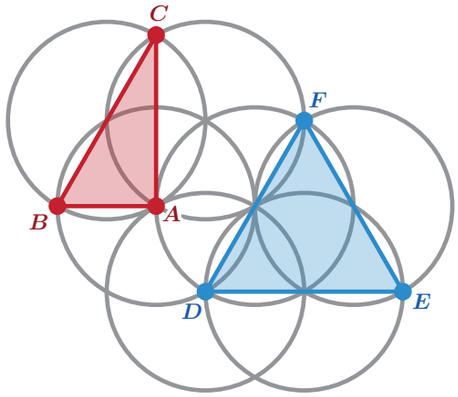
Name: _____ Date: _____ Period: _____

6. Construct a quadrilateral with exactly one pair of parallel sides and two pairs of perpendicular sides. Use segment GH as one of the four sides.



Spiral Review

7. Here are two constructed triangles. Which statement is true?



A. Segment AB is $\frac{1}{2}$ times as large as segment DE.

B. Segment AC is the same length as segment DE.

C. The perimeter of triangle ABC is $\frac{1}{2}$ times as large as the perimeter of triangle DEF.

D. Triangles ABC and DEF are both equilateral.

8. Select *all* properties that are true for squares.

- ☐ **A.** A square has four pairs of perpendicular sides.
- ☐ **B.** A circle centered at one of the vertices of a square, with a radius equal to the length of the square's side, will go through exactly two other vertices.
- ☐ **C.** The length of one diagonal of a square is less than the length of the other diagonal.
- ☐ **D.** The lengths of all four sides of a square are equal.
- ☐ **E.** The sides of a square are not parallel.

Reflection

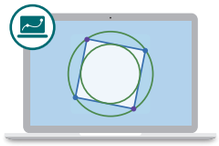
1. Circle the problem you feel least confident about.

2. Use this space to ask a question or share something you're proud of.

Unit 1
Lesson
6

Name: ..... Date: ..... Period: .....

Square Up



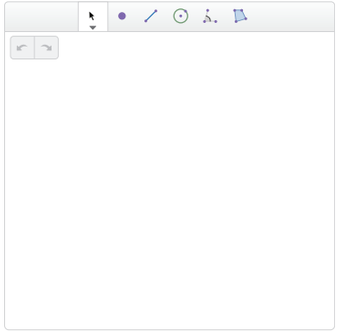
Entire Lesson

Things to Remember:

Select Screens

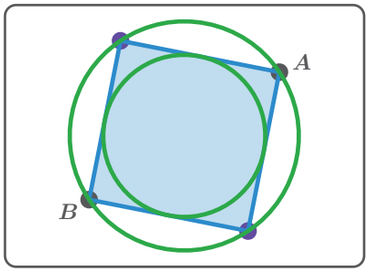
1 You have some new geometry tools!

Record information about the new tools:



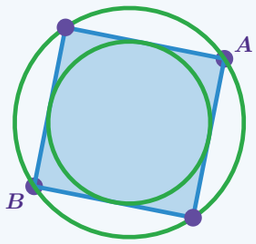
6 Santiago's design shows a square **inscribed** in a circle, and a circle inscribed in a square.

Notes from class discussion:



Lesson Summary

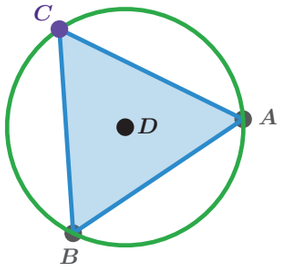
Parallel lines, perpendicular lines, and midpoints are helpful when constructing a square. Midpoints are especially helpful when constructing a square **inscribed** in a circle.



For a square (or any polygon) to be inscribed in a circle, every *vertex* of the square must be on the circle. For a circle to be inscribed in a square (or any polygon), the circle must fit inside the square and a point on the circle must be on each side of the square. When you construct either a square inscribed in a circle or a circle inscribed in a square, it's helpful to determine midpoints and perpendicular lines to identify the center point of the circle.

Try This

Triangle ABC is inscribed in a circle whose center is point D.

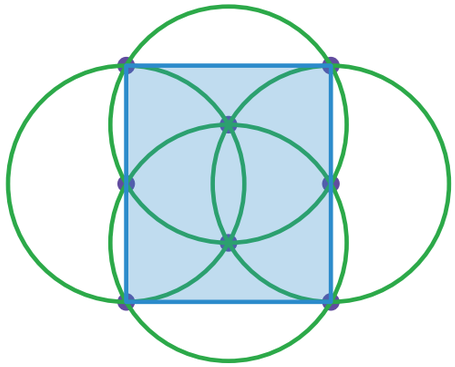


Show or explain how to construct a circle inscribed in $\triangle ABC$.

Lesson Practice
Geo.1.06

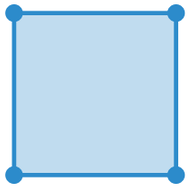
Name: ..... Date: ..... Period: .....

1. Is this construction a square? Explain your thinking.

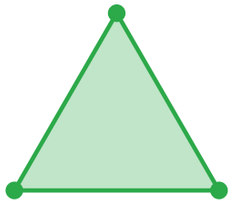


Problems 2–3: Here are regular polygons. Construct a circle so each polygon is inscribed in the circle.

2.



3.



4. Construct a square using segment BC as a side.



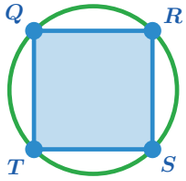
5. Construct a circle with the square from the previous problem inscribed in the circle.

Lesson Practice
Geo.1.06

Name: _____ Date: _____ Period: _____

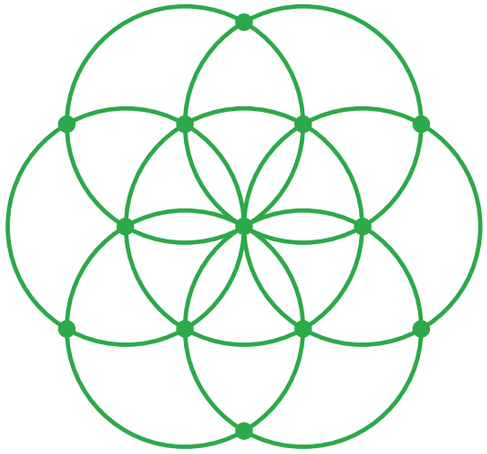
6. Square $QRST$ is inscribed in a circle.

Construct a regular octagon that is inscribed in the circle.



Spiral Review

7. Construct an equilateral triangle using only the intersections of the circles. Explain how you know the triangle is equilateral.



8. Explain how to construct a line segment that is half the length of segment AB.



Reflection

1. Put a star next to a problem you could explain to a classmate.

2. Use this space to ask a question or share something you're proud of.

Unit 1
Lesson
7

Name: ..... Date: ..... Period: .....

Community Constructions



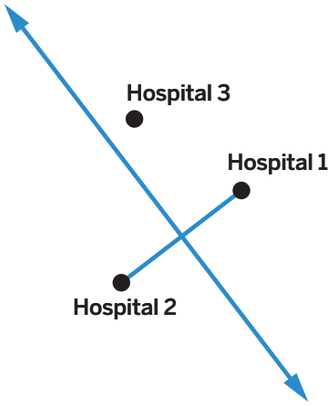
Entire Lesson

Things to Remember:

Select Screens

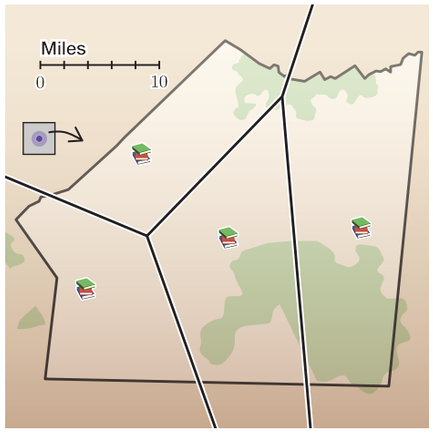
4 Springtown is building a third hospital and the E.M.S. team needs to update their map.

Show or explain your process for dividing the map into regions:



10 Propose a location for a new library based on your criteria.

Notes from class discussion:



Lesson Summary

You can use constructions to solve real-world problems. **Voronoi diagrams** are constructed with perpendicular bisectors and divide an area into regions where all the locations inside a region are closest to an identified point.

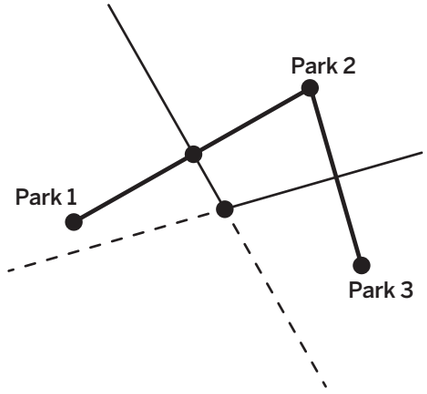
Voronoi diagrams are tools that help us make decisions in community planning. They provide evidence to support where specific services might be located and how these locations might impact a community.



Try This

Here is the start of a Voronoi diagram based on the locations of three parks.

a Complete this Voronoi diagram.



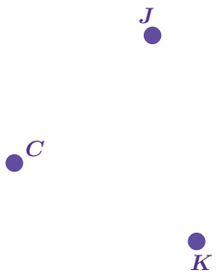
b What factors could be helpful to consider when choosing a location for an additional park?

Lesson Practice
Geo.1.07

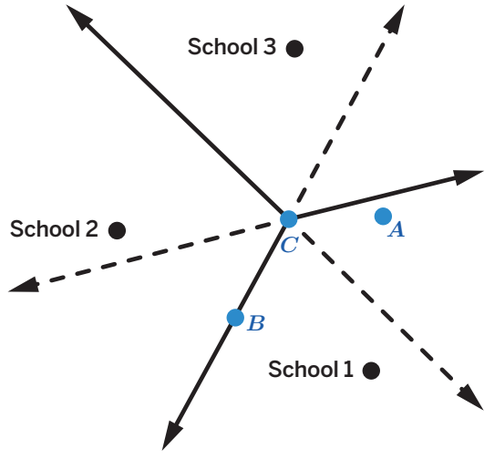
Name: _____ Date: _____ Period: _____

1. How can you use geometry tools to determine if point C is closer to point J or point K?

Use the diagram if it helps to show your thinking.



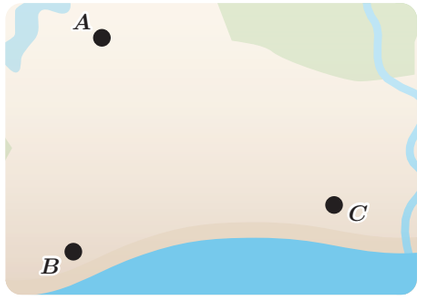
2. This is a Voronoi diagram based on three different schools. Select *all* the statements that are true.



- ☐ **A.** School 1 is the closest school to Location A.
- ☐ **B.** School 3 is the farthest school from Location A.
- ☐ **C.** Location B is equidistant to Schools 2 and 3.
- ☐ **D.** Location B is closer to School 1 than to School 2.
- ☐ **E.** Location C is equidistant to Schools 2 and 3.

Problems 3–4: This diagram represents the homes of three students who need to meet after school for a project.

3. Plot a point where you think the students located at point A and point B should meet. Explain your thinking.



4. Plot a point at a location that is an equal distance from all three students' homes. Explain your thinking.

Lesson Practice
Geo.1.07

Name: _____ Date: _____ Period: _____

5. Create a Voronoi diagram based on these three points.



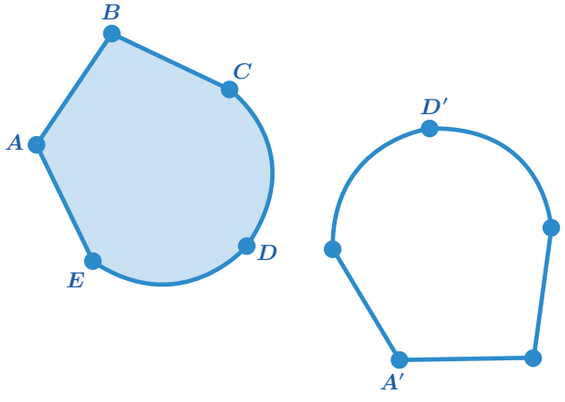
6. What are ways Voronoi diagrams could be used in your community to help make decisions?

Spiral Review

Problems 7–8: The unshaded figure is a transformation of figure $ABCDE$.

7. Label the points corresponding to B, C, and E as B', C', and E'.

8. Construct segment AD and segment $A'D'$. What is the relationship between the lengths of these two line segments?



Reflection

1. Circle the problem you enjoyed doing the most.

2. Use this space to ask a question or share something you're proud of.

Name: _____ Date: _____ Period: _____

Practice Day 1

Let's practice what you've learned so far in this unit!



You will use task cards for this Practice Day. Record all of your responses here.

Task A: Equilateral Triangle

1. Statement 1:

Statement 2:

Statement 3:

2. Circle one: Yes No Not enough information

Explain your thinking.

3. Equilateral triangles:

Task B: Square It

1.

2. Two parallel sides:

3. Two perpendicular sides:

Name: _____ Date: _____ Period: _____

Practice Day 1 (continued)

Task C: Statements and Lines

1. Lines or segments:

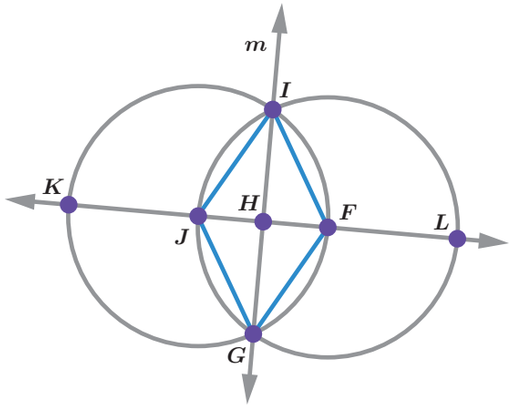
2. **a** Triangle _____ is equilateral.

b Points _____, _____, and _____ are equidistant from point J.

c Line m bisects segment _____.

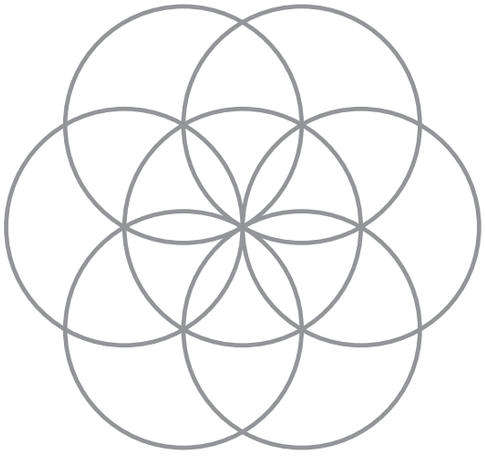
3. Statement:

4.



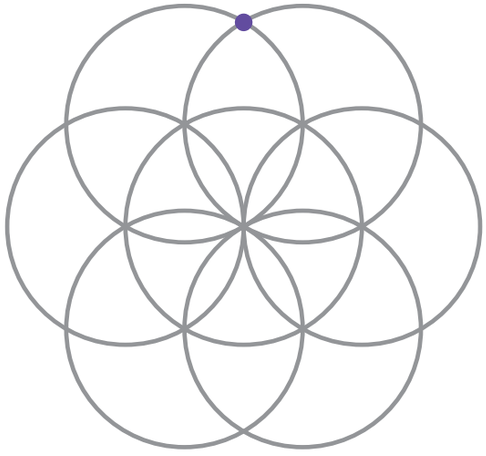
Task D: Inscribing

1. **a**



b Description:

2. **a**



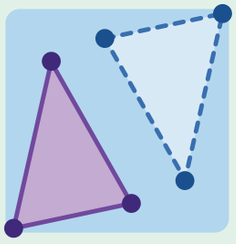
b Description:

Student Edition pages

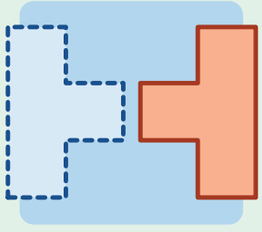
Sub-Unit

2

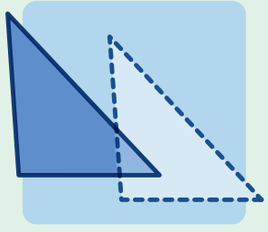
Rigid Transformations



Lesson 8
Going Off the Grid, Part 1



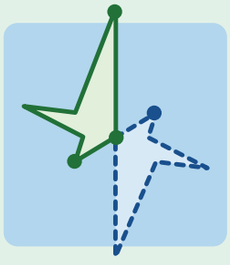
Lesson 9
Time to Reflect



Lesson 10
Translation Information



Lesson 11
Rotation Devices



Lesson 12
Going Off the Grid, Part 2

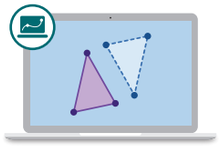


Lesson 13
Tiles and Patterns

Unit 1
Lesson
8

Name: ..... Date: ..... Period: .....

Going Off the Grid, Part 1



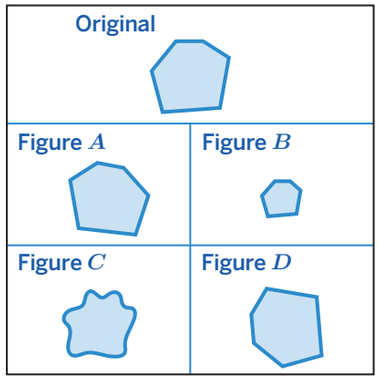
Entire Lesson

Things to Remember:

Select Screens

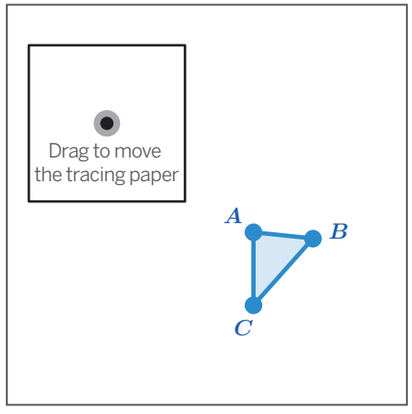
2 A *rigid transformation* is a transformation that doesn't change the side lengths or angle measures of a figure.

Notes from class discussion:



8 Transform $\triangle ABC$ to match a given description.

Record at least one really clear description from a classmate's challenge:



Lesson Summary

You can use tracing paper to perform *rigid transformations*, which don't change the side lengths or angle measures of a figure. With tracing paper, you can perform:

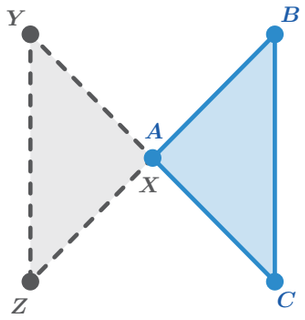
- *Translations*, which slide a figure up, down, left, or right.
- *Reflections*, which flip a figure.
- *Rotations*, which spin a figure.

Using tracing paper can help you make sense of how to use rigid transformations to move one figure on top of another. Sometimes you only need one transformation, but other times you may need to use multiple transformations to perfectly align one figure to another.

Try This

Describe how to transform $\triangle ABC$ onto $\triangle XYZ$.

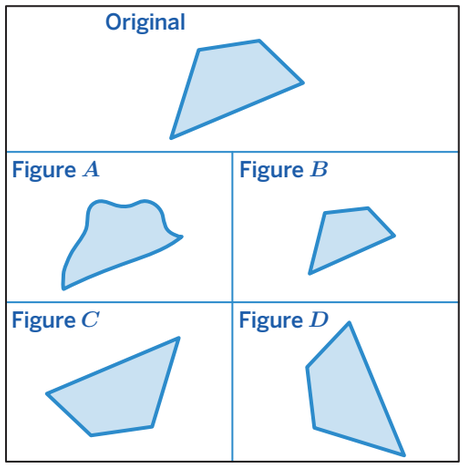
Use tracing paper if it helps with your thinking.



Lesson Practice
Geo.1.08

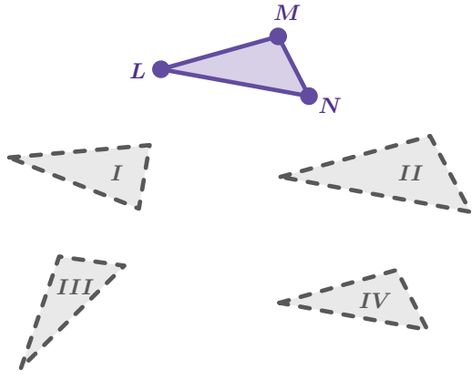
Name: _____ Date: _____ Period: _____

1. Select *all* the figures that show a rigid transformation of the original figure.



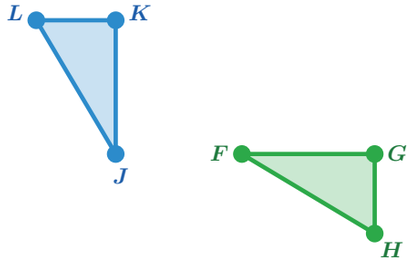
- ☐ **A.** Figure A
- ☐ **B.** Figure B
- ☐ **C.** Figure C
- ☐ **D.** Figure D

2. Which triangle could be the result of a single translation of $\triangle LMN$?



- **A.** I
- **B.** II
- **C.** III
- **D.** IV

Problems 3–4: Here are two triangles.



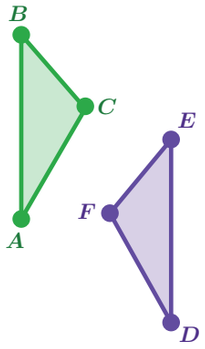
3. Describe a sequence of rigid transformations that transforms $\triangle FGH$ onto $\triangle JKL$.

4. Describe a *different* sequence that transforms $\triangle FGH$ onto $\triangle JKL$.

Lesson Practice
Geo.1.08

Name: _____ Date: _____ Period: _____

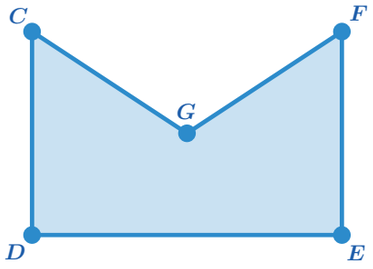
5. Select *all* the sequence(s) of transformations that could transform $\triangle ABC$ onto $\triangle DEF$.



- ☐ **A.** A rotation and a reflection
- ☐ **B.** A translation
- ☐ **C.** A reflection and a translation
- ☐ **D.** A rotation and a translation
- ☐ **E.** A reflection

6. This polygon was made using only a triangle and its reflection.

Name one of the triangles.



Spiral Review

7. Construct the perpendicular bisector of $\overline{JK}$.

8. Which point is closer to the perpendicular bisector you just constructed? Circle one.



Point J Point K They're the same distance.

Reflection

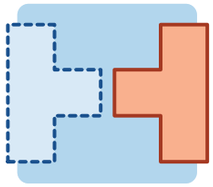
1. Put a question mark next to a response you'd like to compare with a classmate's.

2. Use this space to ask a question or share something you're proud of.

Unit 1
Lesson
9

Name: ..... Date: ..... Period: .....

Time to Reflect



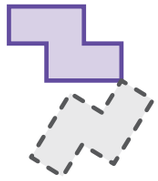
Let's get precise about reflections.

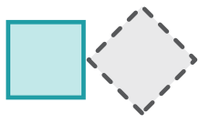
Warm-Up

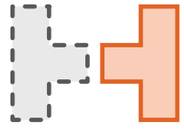
1. Here are four polygons (in color) and their **images** (in gray) after transformations.

Which of these *must* include a reflection?

A. 

B. 

C. 

D. 

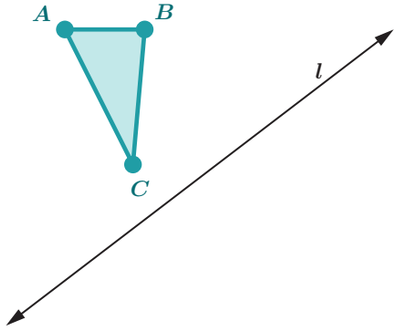
Explain your thinking.

Activity 1

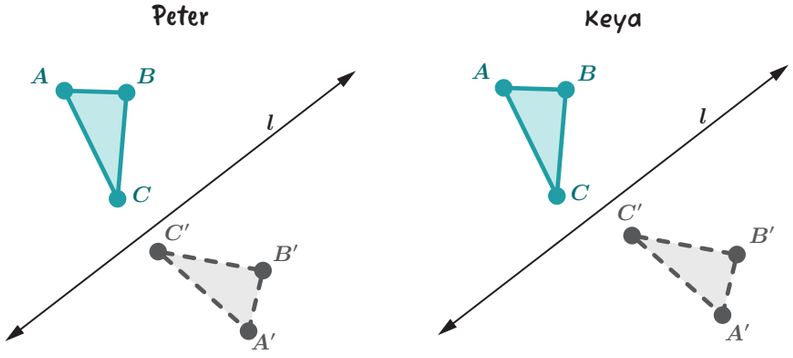
Name: ..... Date: ..... Period: .....

Accurate Reflections

2. Use tracing paper to reflect the **pre-image**, $\triangle ABC$, over line l. Label the image $\triangle A'B'C'$.



3. Peter and Keya reflected $\triangle ABC$ over line l.



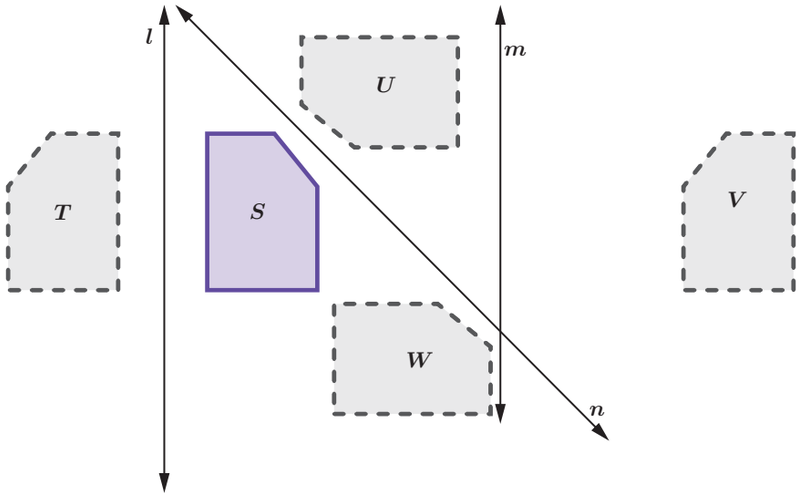
Which student's reflection is better? Explain your thinking.

Activity 2

Name: ..... Date: ..... Period: .....

Reflecting Pentagons

This diagram shows four reflections of pentagon S.



4. Determine which reflected image completes each statement.

a Pentagon S reflects over line l to image .....

b Pentagon S reflects over line m to image .....

c Pentagon S reflects over line n to image .....

5. There is one reflection of pentagon S whose **line of reflection** is missing.

a Which image is it?

b Use any tool to construct the line of reflection for that pentagon.

6. Wohali says: *If you connect a point and its reflection using a line segment, then the line segment is perpendicular to the line of reflection.*

a Is Wohali's statement *always*, *sometimes*, or *never* true? Circle one.

Always true Sometimes true Never true

Explain your thinking.

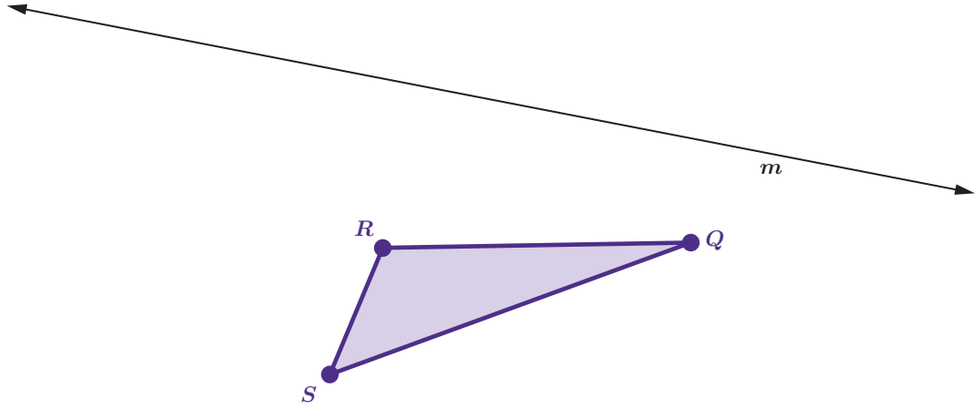
b Describe how to construct a line of reflection between a point and its reflected image.

Activity
3

Name: ..... Date: ..... Period: .....

Encompassing Reflections

7. Reflect $\triangle QRS$ over line m. Label the vertices Q', R', and S'.

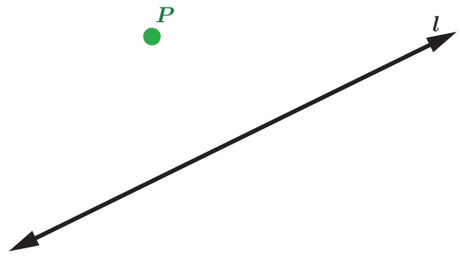


Activity 3

Name: Date: Period:

Encompassing Reflections (continued)

8. Reflect point P over line l using a compass and straightedge. Label the image P'.

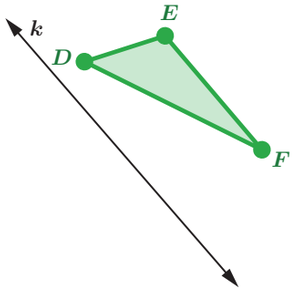


9. Describe your strategy for reflecting a point over a line using a compass and straightedge.

Synthesis

What would you tell a classmate who was absent about the relationship between a reflected image, its pre-image, and the line of reflection that created the pre-image?

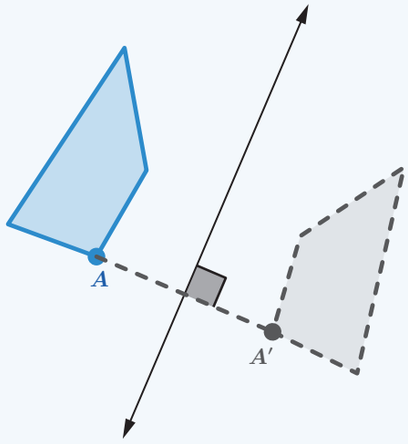
Use triangle DEF and any tools you'd like if they help with your thinking.



Lesson Summary

You can use tracing paper to explore the relationship between a **line of reflection** and the reflected image. Some properties of reflections are:

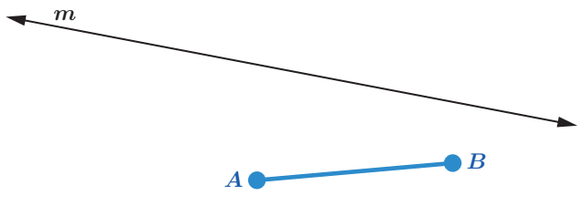
- Reflections are rigid transformations, so the reflected **image** will be the same size and shape as the **pre-image**.
- If you connect a point and its reflected image using a line segment, then the line segment is perpendicular to the line of reflection.
- A line of reflection bisects the line segment between a point and its reflected image.



You can use these properties to construct precise reflections. For example, a perpendicular line construction can help you determine the line that the reflected point must lie on. A circle can help you represent the correct distance from the line of reflection.

Try This

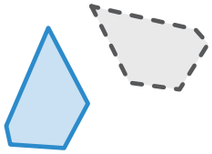
Reflect segment $\overline{AB}$ over line m.

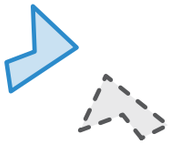


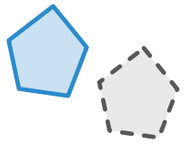
Lesson Practice
Geo.1.09

Name: _____ Date: _____ Period: _____

1. Which of the transformed images *must* include a reflection?

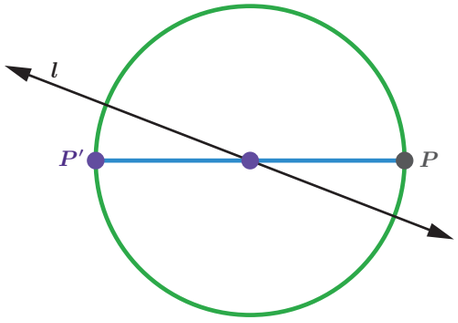
A. 

B. 

C. 

D. 

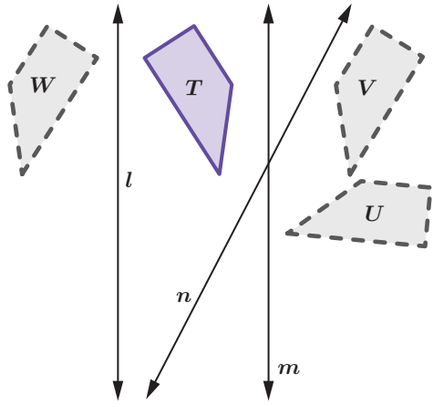
Problems 2–3: Hailey tried to reflect point P over line l.



2. What is one thing Hailey did correctly?

3. How would you improve Hailey's reflection?

Problems 4–6: Determine which reflected image completes each statement.

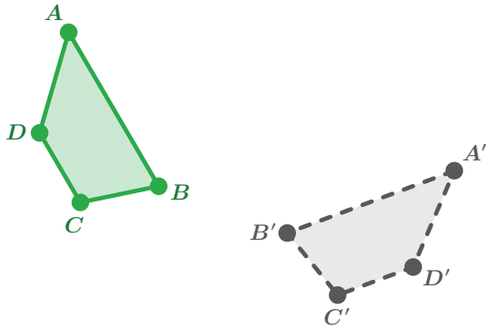


4. Trapezoid T reflects over line l to image _____.

5. Trapezoid T reflects over line m to image _____.

6. Trapezoid T reflects over line n to image _____.

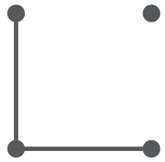
7. Construct the line of reflection.



Lesson Practice
Geo.1.09

Name: _____ Date: _____ Period: _____

8. Construct a line of reflection that transforms this figure onto itself.



Spiral Review

9. Select *all* the statements that are always true about an image after a sequence of rigid transformations.

- ☐ **A.** The angle measures of the image and pre-image are the same.
- ☐ **B.** The orientation of the image and the pre-image are the same.
- ☐ **C.** The side lengths of the image and the pre-image are the same.
- ☐ **D.** The side lengths of the image are shorter than in the pre-image.
- ☐ **E.** The side lengths of the image are longer than in the pre-image.

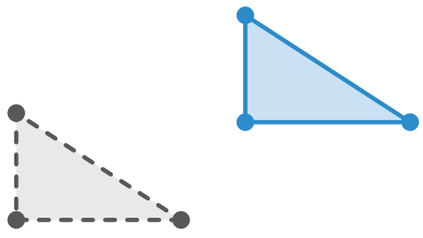
10. Which type of single transformation is shown?

A. Translation

B. Reflection

C. Rotation

D. Dilation



Reflection

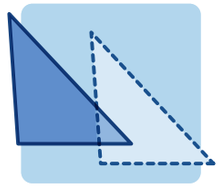
1. Star the problem you spent the most time on.

2. Use this space to ask a question or share something you're proud of.

Unit 1
Lesson
10

Name: ..... Date: ..... Period: .....

Translation Information



Let's get precise about translations.

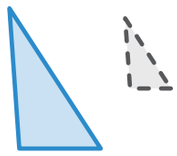
Warm-Up

1. Which one doesn't belong?

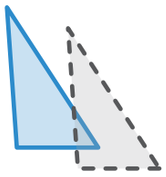
A.



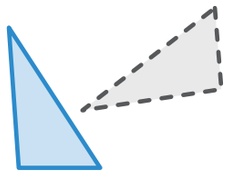
B.



C.



D.



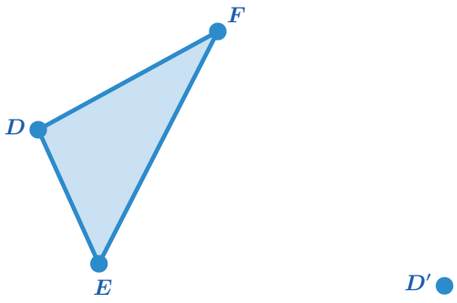
Explain your thinking.

Activity 1

Name: ..... Date: ..... Period: .....

Translate This!

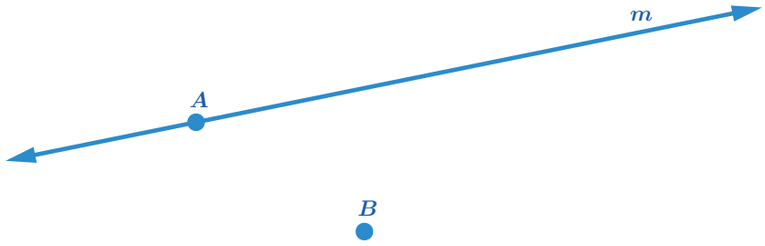
2. Translate $\triangle DEF$ so that point D goes to point D'. Label the vertices in the image using **prime notation**.



3. Nakia says that you can tell one figure is a translation of another figure because they are both the same size, but in different places. Is Nakia's thinking correct?

Explain your thinking.

4. Translate line m so that point A goes to point B.



5. Nakia says: *When a line is translated, the line and its image are parallel.* Is this statement *always*, *sometimes*, or *never* true? Circle one.

Always Sometimes Never

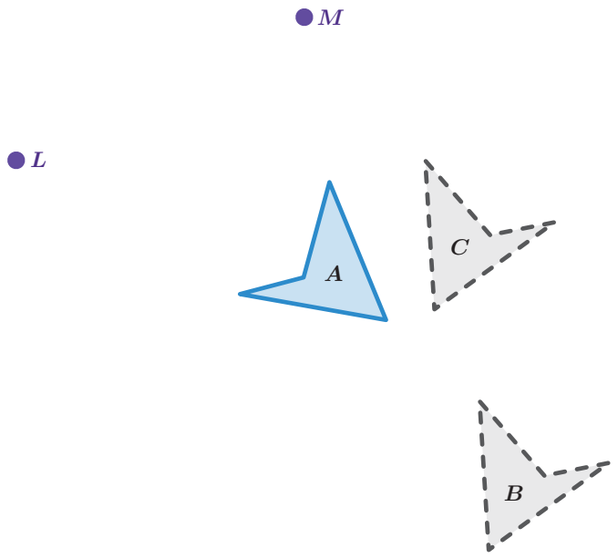
Explain your thinking.

Activity 1

Name: ..... Date: ..... Period: .....

Translate This! (continued)

Here is quadrilateral A and some transformations.



6. Which of these transformed quadrilaterals could be the result of a reflection only?

7. Which of these quadrilaterals could *not* be the result of a single reflection?

8. Translate quadrilateral A using the same translation that moves point L onto point M.

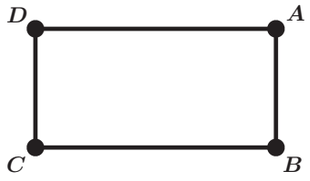
Activity
2

Name: ..... Date: ..... Period: .....

Transformation Communication

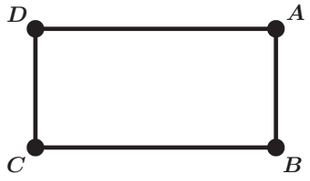
Your partner will describe a transformation for each polygon.
Construct the result.

9. Card: .....





10. Card: .....



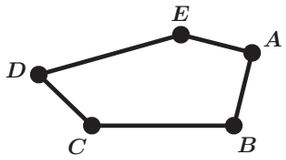


Activity 2

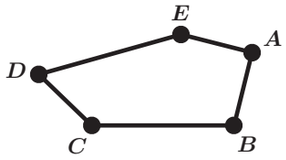
Name: _____ Date: _____ Period: _____

Transformation Communication (continued)

11. Card: _____



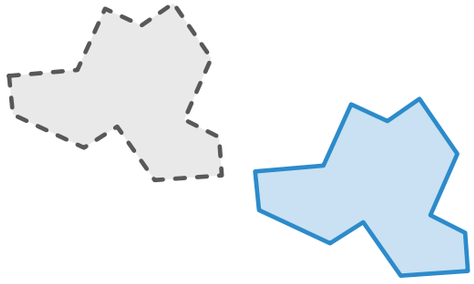
12. Card: _____



Synthesis

13. What information is important to include when describing a translation?

Use the example if it helps with your thinking.

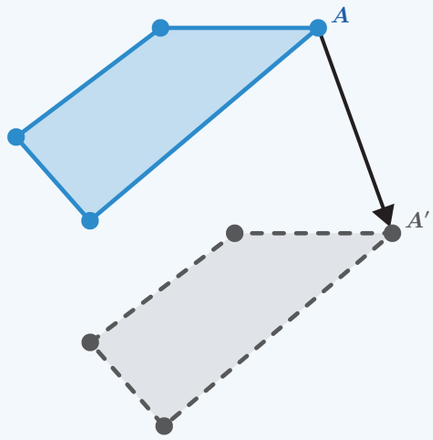


Lesson Summary

A translation is a type of rigid transformation that is defined by two points, a starting point and an endpoint. When you translate a starting point, A, to an endpoint, A', all other points in the translated figure move the same distance and in the same direction.

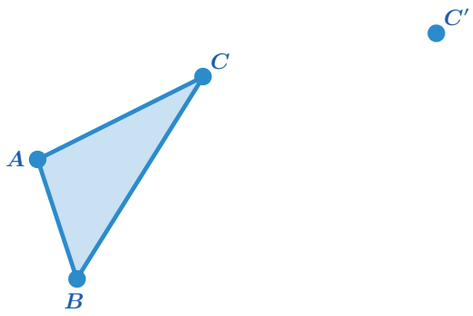
Translations move a figure without spinning or rotating. The translated figure is the same shape and size as the original figure but in a different location. When you translate a line, the image is a line parallel to the original.

When translating a figure, it's helpful to label translated points with **prime notation**, like A' or B', which are read out loud as A *prime* or B *prime*. This notation helps to show how the image relates to the pre-image.



Try This

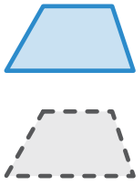
Translate $\triangle ABC$ so that point C goes to point C'.

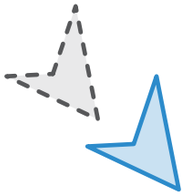


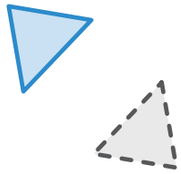
Lesson Practice
Geo.1.10

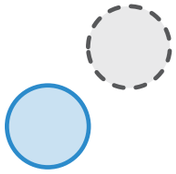
Name: ..... Date: ..... Period: .....

1. Which one of these transformations cannot represent a *single* translation?

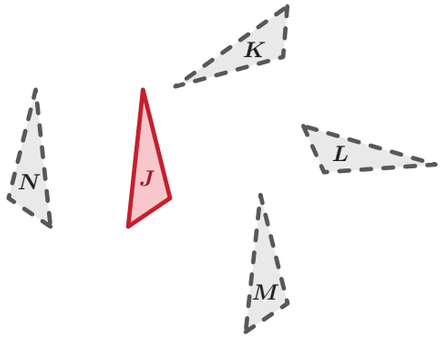
A. 

B. 

C. 

D. 

Problems 2–5: Here are four transformations of triangle J.
Which of these images is the result of:



2. A single *translation* only?

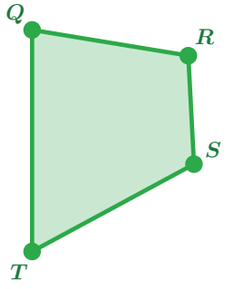
3. A single *reflection* only?

4. Both a single translation and a single reflection?

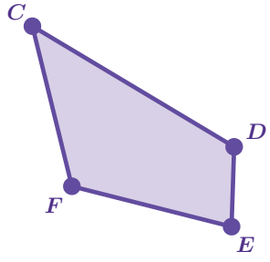
5. A single transformation that is not a translation or reflection?

Problems 6–7: Translate each quadrilateral so that:

6. Point Q goes to point S.



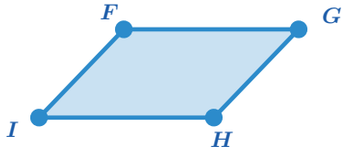
7. Point F goes to point F'.



Lesson Practice
Geo.1.10

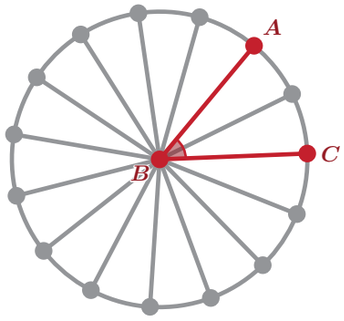
Name: _____ Date: _____ Period: _____

8. Construct translations of parallelogram $FGHI$ to create a final design that looks the same with a rotation but not a reflection.

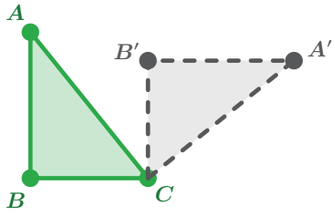


Spiral Review

9. A circle is divided into 15 equal pieces. What is the measure of angle ABC in degrees?



10. Triangle ABC was rotated $270°$. If angle $ABC = 90°$, what is the measure of angle $A'B'C$?



Reflection

1. Put a heart next to the problem you're most proud of.

2. Use this space to ask a question or share something you're proud of.

Unit 1
Lesson
11

Name: ..... Date: ..... Period: .....

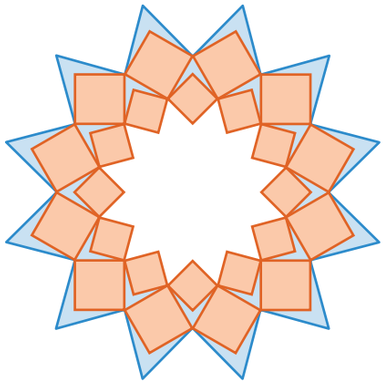
Rotation Devices



Let's get precise about rotations.

Warm-Up

Let's watch how this figure was created.



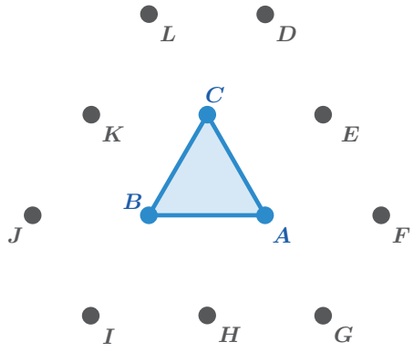
1. Pick one rotation you noticed and describe it.

Activity
1

Name: ..... Date: ..... Period: .....

Equilateral Images

2. Rotate $\triangle ABC$ around point A, point B, or point C.

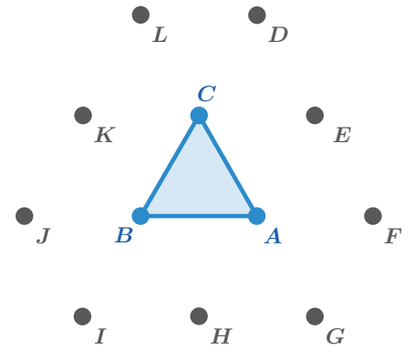


What's the relationship between the *center of rotation*, a point, and that point's image after the rotation?

3. Write a first draft of a description of your rotation.

4. Meet with a partner and read your description. Don't show them your paper!

Perform the rotation that matches your partner's description.



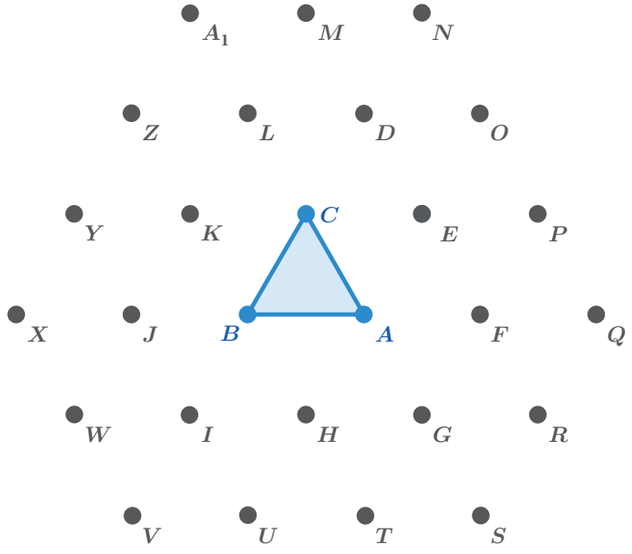
5. Write a second draft that is stronger and clearer.

Activity 1

Name: Date: Period:

Equilateral Images (continued)

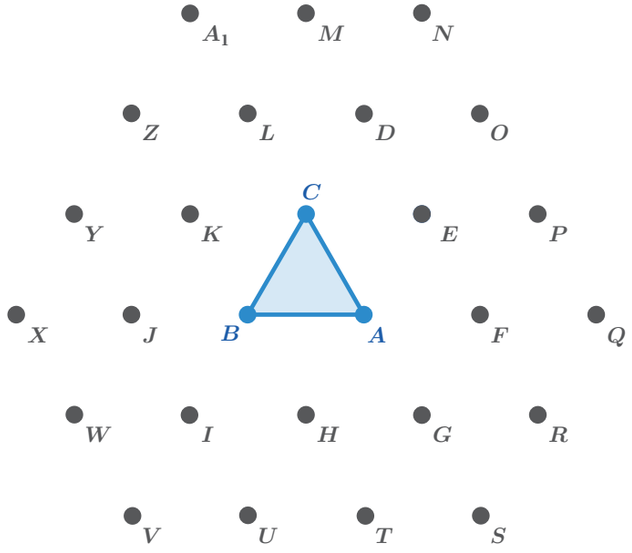
6. Rotate $\triangle ABC$ around any point *except* points A, B, or C. Make sure every vertex in the image is on a labeled point.



7. Describe your rotation.

8. Meet with a partner and read your description. Don't show them your paper!

Perform the rotation that matches your partner's description.



Activity 2

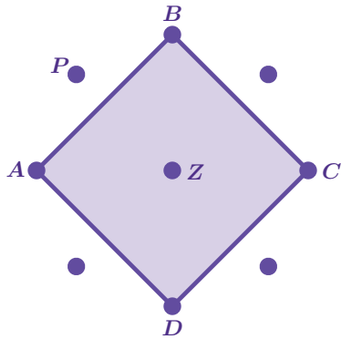
Name: ..... Date: ..... Period: .....

Partner Problems

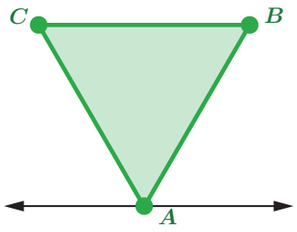
- Decide with a partner who will complete Column A and who will complete Column B.
- After each problem, compare your images. Discuss whether the final transformed figures are in the same location.

Column A

9. Rotate square $ABCD$ around point Z so that point A goes to point P.



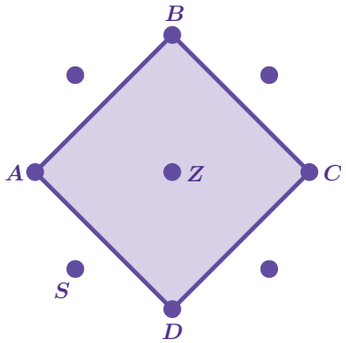
10. Reflect $\triangle ABC$ over line l.



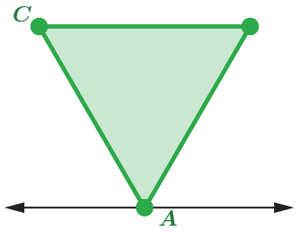


Column B

9. Rotate square $ABCD$ around point Z so that point D goes to point S.



10. Rotate $\triangle ABC$ around point A until point B goes to point D.





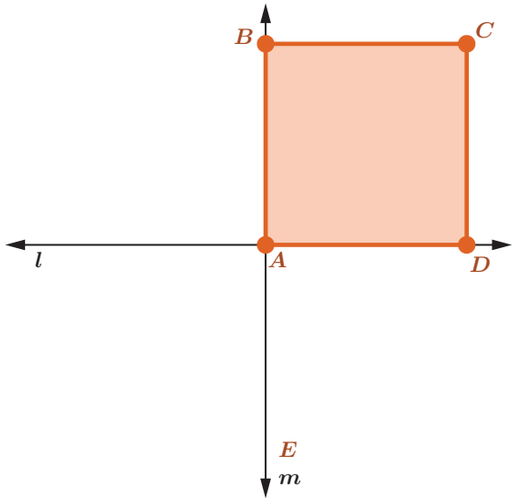
Activity 2

Name: ..... Date: ..... Period: .....

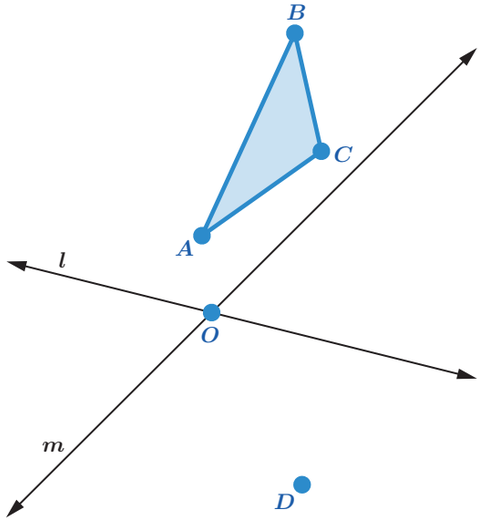
Partner Problems (continued)

Column A

11. Rotate quadrilateral $ABCD$ around point A so that point B goes to point E.

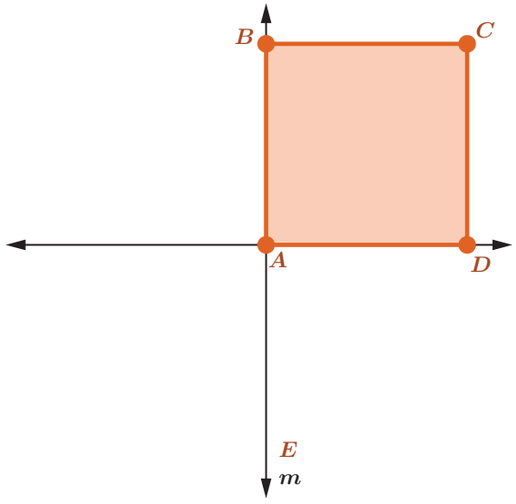


12. Reflect $\triangle ABC$ over line m, then over line l.

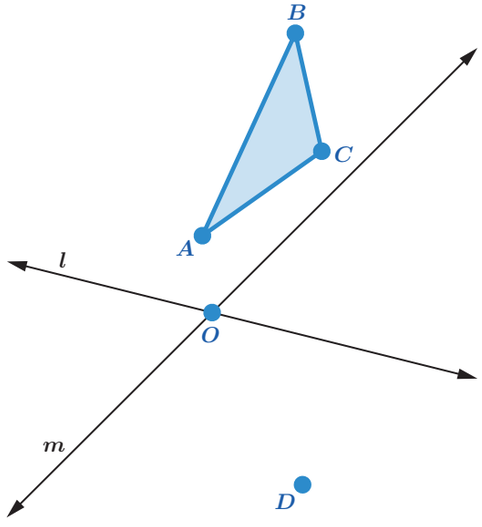


Column B

11. Reflect quadrilateral $ABCD$ over line m, then over line l.



12. Rotate $\triangle ABC$ around point O so that point C goes to point D.



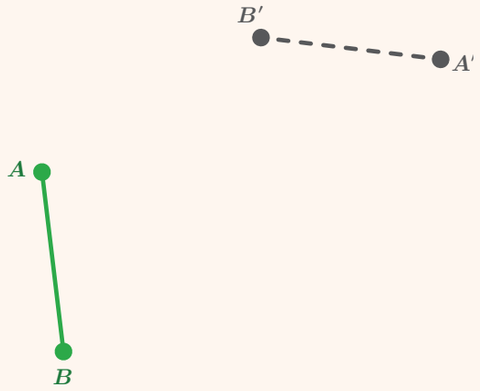
Activity 2

Name: ..... Date: ..... Period: .....

Partner Problems (continued)

Explore More

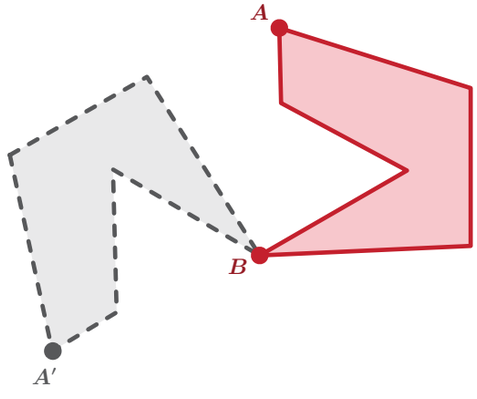
13. Segment $A'B'$ is the image of segment AB after a rotation. Construct perpendicular bisectors between each point and its image to find and plot the center of rotation.



Synthesis

14. What information is important to include when describing a rotation?

Use the figures if they help with your thinking.

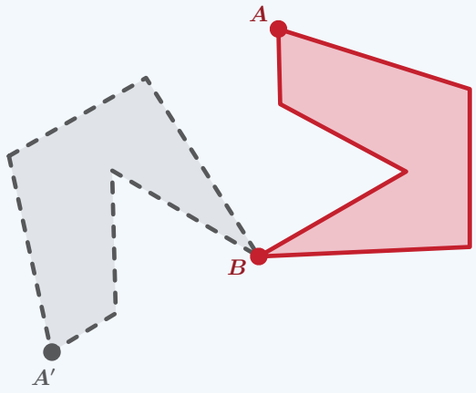


Lesson Summary

A rotation is a rigid transformation that moves all of the points in a figure around a given point called the *center of rotation*. The pre-image points and their images are always the same distance from the center of rotation.

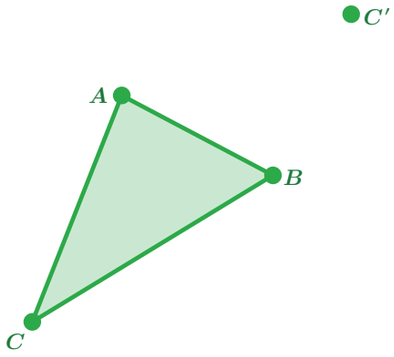
When a figure is rotated, the pre-image and the image have the same size and shape.

We can describe this rotation by saying "rotate the figure around point B so that point A goes to point A'."



Try This

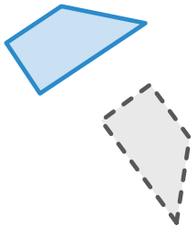
Rotate $\triangle ABC$ around point A until point C goes to point C'.



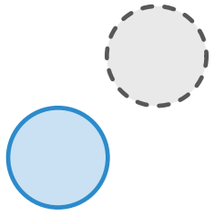
Lesson Practice
Geo.1.11

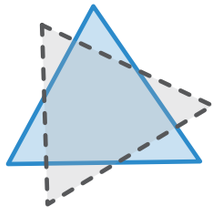
Name: ..... Date: ..... Period: .....

1. Which of these transformations could *not* be a single rotation?

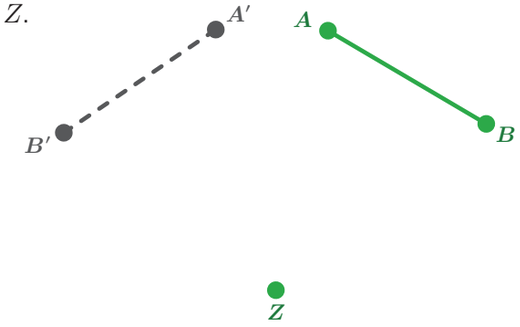
A. 

B. 

C. 

D. 

Problems 2–3: Rudra tried to rotate $\overline{AB}$ around point Z.

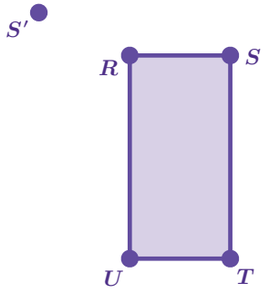


2. What is one thing Rudra did well?

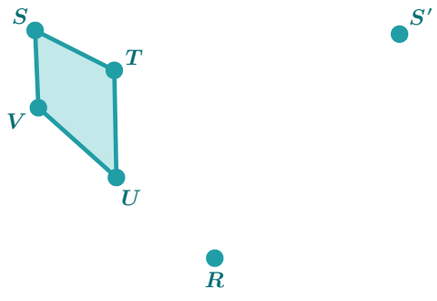
3. How could Rudra improve his rotation?

Problems 4–5: Rotate each quadrilateral around point R so that point S goes to S'.

4.



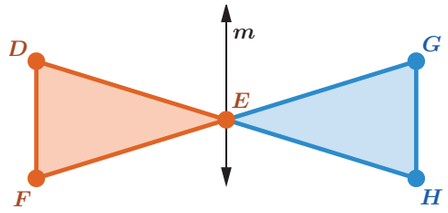
5.



Lesson Practice
Geo.1.11

Name: _____ Date: _____ Period: _____

6. Describe two different single transformations that move one of these triangles onto the other.



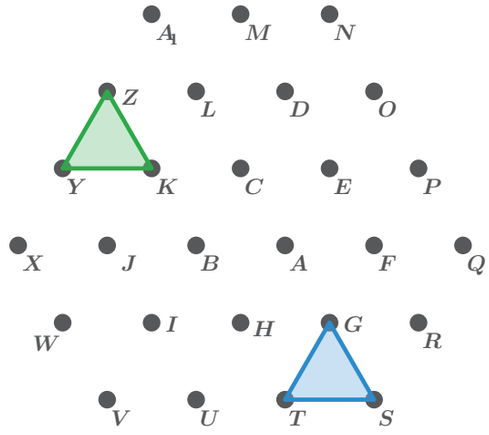
Problems 7–8: Use the grid.

7. Rotate each triangle *exactly once* to create the outline of a parallelogram.

8. Fill in the blanks to describe your rotations.

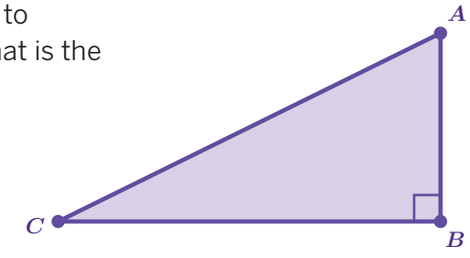
Rotate $\triangle YZK$ around point _____ until point Z goes to point _____.

Rotate $\triangle GST$ around point _____ until point T goes to point _____.



Spiral Review

Problems 9–10: Here is right triangle ABC. Describe how to use only rotations or only reflections to create an image that is the outline of a rectangle when combined with triangle ABC.



9. Description using only rotations:

10. Description using only reflections:

Reflection

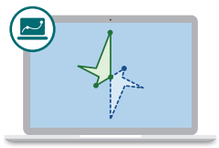
1. Put a star next to a problem you want to understand better.

2. Use this space to ask a question or share something you're proud of.

Unit 1
Lesson
12

Name: ..... Date: ..... Period: .....

Going Off the Grid, Part 2



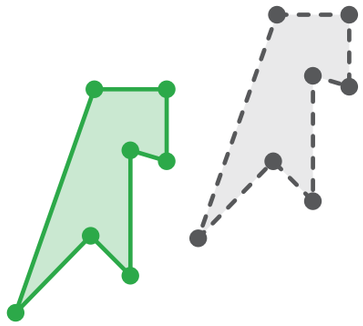
Entire Lesson

Things to Remember:

Select Screens

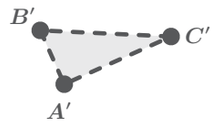
5 Transform the solid polygon onto the dashed polygon.

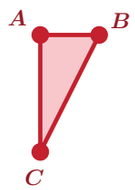
Notes from class discussion:



10 Transform triangle ABC onto triangle $A'B'C'$.

What details are helpful to include when describing transformations?

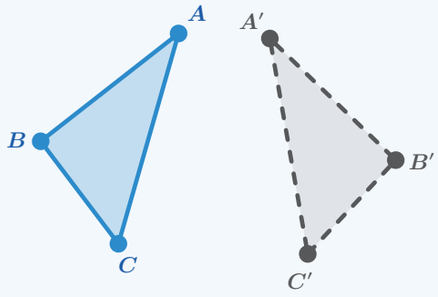




Lesson Summary

There are many strategies to transform one figure onto another.

Here are three ways you could transform triangle ABC onto triangle $A'B'C'$.

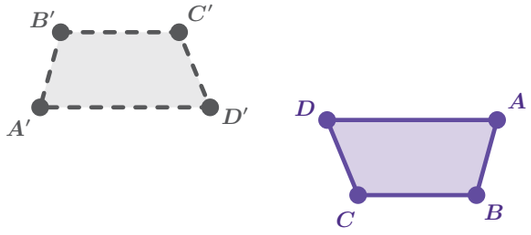


1. Translate triangle ABC so that point B goes to B'. Rotate the image around B' until the point that was originally C moves onto C', and then reflect over segment $B'C'$.
2. Reflect triangle ABC over the perpendicular bisector of segment AA'.
3. Rotate triangle ABC around the midpoint between points A and A' until segment AB aligns with segment $A'B'$. Then reflect the image over segment $A'B'$.

Try This

Describe how to transform figure $ABCD$ onto figure $A'B'C'D'$.

Use tracing paper if it helps with your thinking.

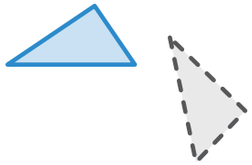


Lesson Practice
Geo.1.12

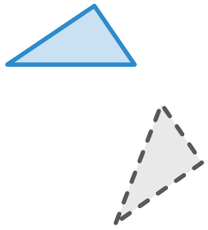
Name: _____ Date: _____ Period: _____

1. Which set of figures does *not* show a rigid transformation?

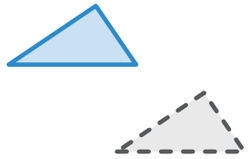
A.



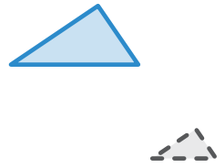
B.



C.



D.



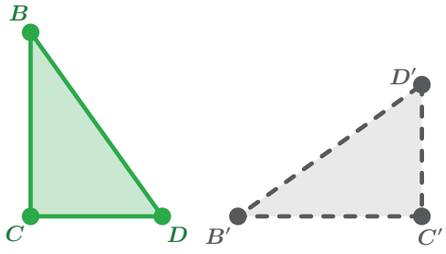
2. Select *all* the transformations that could move point A onto point B.



- ☐ **A.** Reflect point A over $\overline{AB}$.
- ☐ **B.** Reflect point A over the perpendicular bisector of $\overline{AB}$.
- ☐ **C.** Translate point A so that it moves onto point B.
- ☐ **D.** Rotate point A around point B.
- ☐ **E.** Rotate point A around the midpoint of $\overline{AB}$.

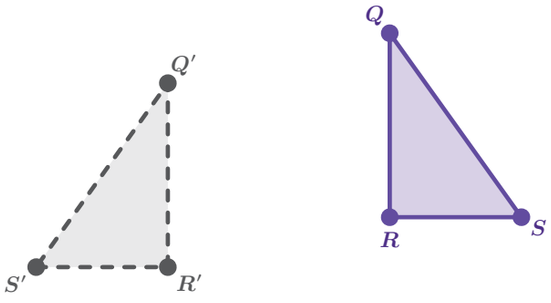
Problems 3–4: Transform the solid triangle onto the dashed triangle. Then describe the transformations you performed.

3.



Description:

4.

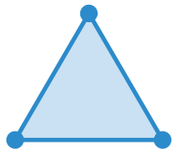


Description:

Lesson Practice
Geo.1.12

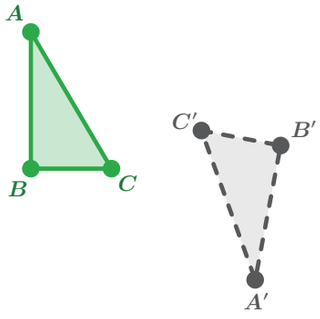
Name: _____ Date: _____ Period: _____

5. Here is an equilateral triangle. Create a larger triangle using only reflections. Show or explain your thinking.



Spiral Review

6. Describe a strategy for determining the center of rotation for the transformation that would move triangle ABC onto triangle $A'B'C'$.



7. A triangle has one angle that measures 55°. What are possible values for the other two angles?

Reflection

1. Put a star next to a problem you're still wondering about.

2. Use this space to ask a question or share something you're proud of.

Unit 1
Lesson
13

Name: ..... Date: ..... Period: .....

Tiles and Patterns



Entire Lesson

Things to Remember:

Select Screens

2 Spend some time researching and learning more about how artists like Samira Mian recreate geometric patterns found in works by Muslim architects.

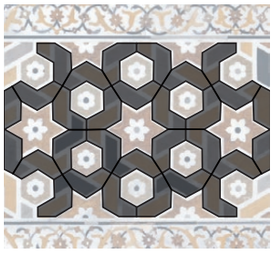
Record something from your research that you found interesting:



Samira Mian

7 This design is related to the geometric pattern found in the Tomb of I'timād-ud-Daulah.

Notes from class discussion:



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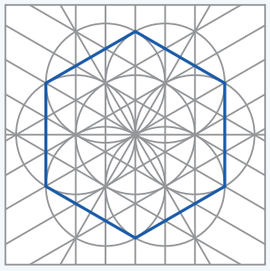
Lesson Summary

Artists and mathematicians throughout history have developed strategies for making geometric patterns. Here is an example of a geometric pattern from the Tomb of I'timād-ud-Daulah in India.

There are many strategies to construct and recreate designs. You can use circles, intersection points, and line segments to precisely construct figures. You can transform those figures using translations, rotations, and reflections. By combining constructions and transformations, you can create complex designs made up of beautiful shapes and patterns.

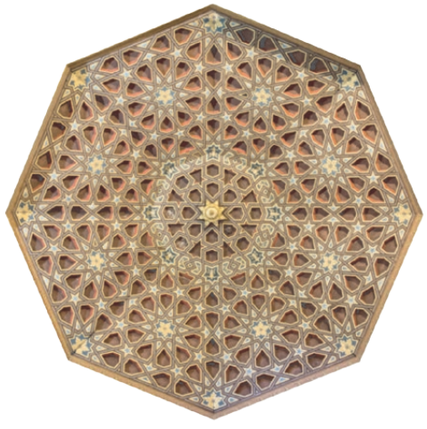


© 2014 Royroydeb / CC-BY-SA 4.0



Try This

Describe transformations that could be used to recreate this pattern from the Royal Alcazar in Spain. Draw on the image if it helps to show your thinking.



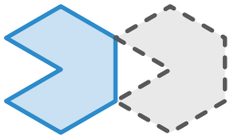
"Royal Alcazar, Seville, Spain" by GaryCampbell-Hall via Flickr

Lesson Practice
Geo.1.13

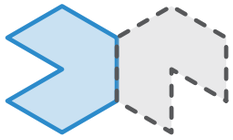
Name: _____ Date: _____ Period: _____

1. Which transformation cannot be achieved with a translation?

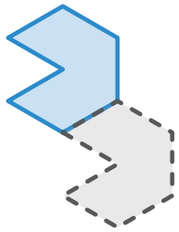
A.



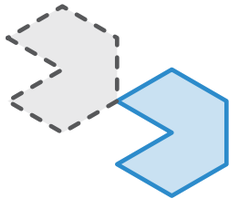
B.



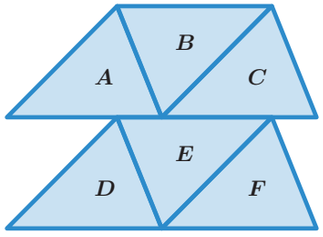
C.



D.



Problems 2–3: These triangles are rigid transformations of triangle A.



2. Select *all* the triangles that could be a translation of triangle A.

☐ B ☐ C ☐ D ☐ E ☐ F

3. Select *all* the triangles that could be a rotation of triangle A.

☐ B ☐ C ☐ D ☐ E ☐ F

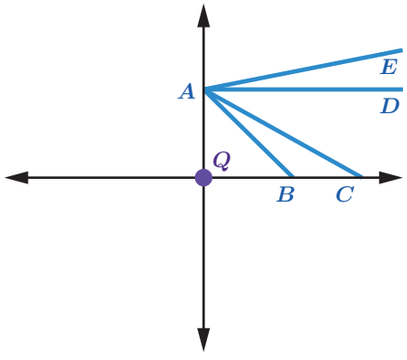
4. Which segment, when rotated around point Q into each quadrant, will form a square?

A. Segment AB

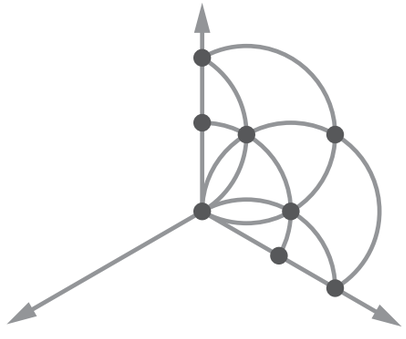
B. Segment AC

C. Segment AD

D. Segment AE



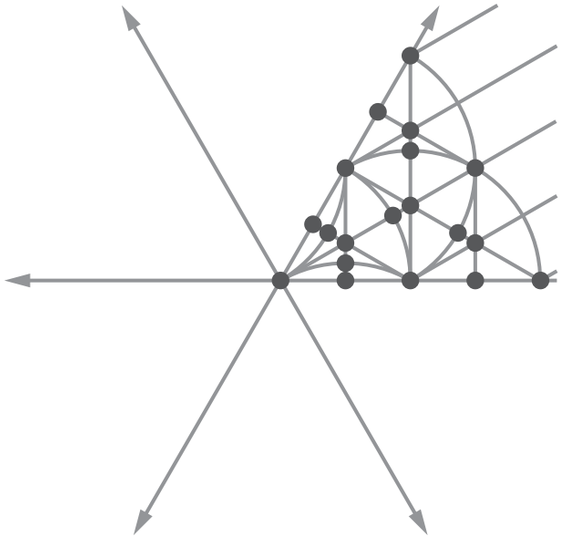
5. Use the intersection points to construct a line segment that will form an equilateral triangle when rotated.



Lesson Practice
Geo.1.13

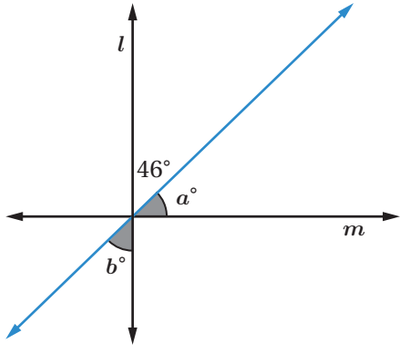
Name: _____ Date: _____ Period: _____

6. Connect points to create a polygon that, when rotated, will make a design that also includes at least one translation and one reflection. Use tracing paper to replicate your polygon to complete the design.



Spiral Review

Problems 7–8: Line l is perpendicular to line m. Determine the values of a and b.



7. $a =$ _____

8. $b =$ _____

Reflection

1. Put a heart next to the problem you found most interesting.

2. Use this space to ask a question or share something you're proud of.

Name: ..... Date: ..... Period: .....

Practice Day 2



Let's practice what you've learned so far in this unit!

You will use task cards and the digital companion. Record your responses here.

Task A: My Favorite Tools

1. Choose one to construct below and one to construct using the digital companion.

a	Right triangle	☐ Constructed below	☐ Digital companion
b	Regular hexagon	☐ Constructed below	☐ Digital companion

2. For geometric constructions, what is one advantage of using:

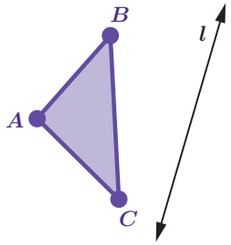
a Digital tools?

b Paper-based tools?

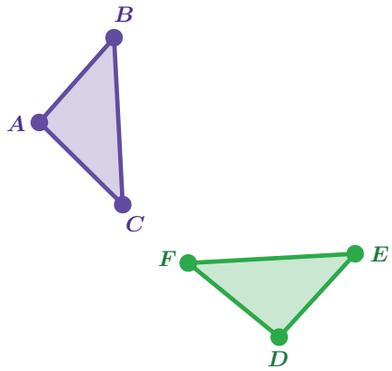
Task B: Reflected Triangles

1. Explanation:

2.



3.

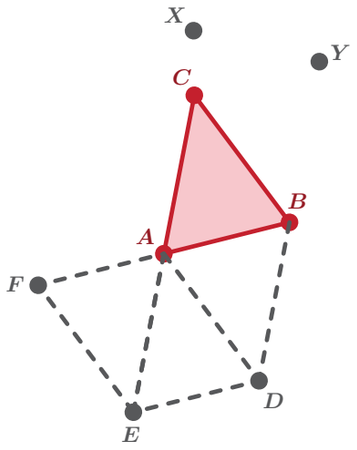


Name: Date: Period:

Practice Day 2 (continued)

Task C: Translated Triangles

1.



2. Description:

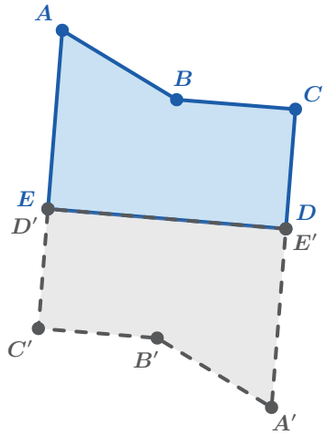
3. Circle one: Yes No

Explanation:

Task D: Two Transformations

1. Sequence of rigid transformations #1:

Sequence of rigid transformations #2:



2. Circle one: Yes No

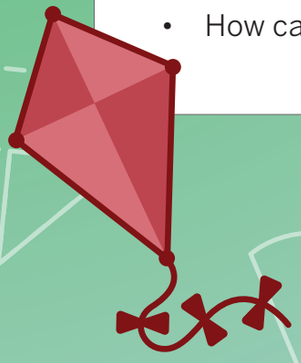
Unit 2

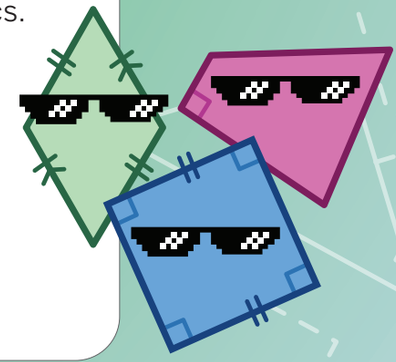
Congruence

In geometry, we often try to prove when two figures are exactly the same. In this unit, you'll use rigid transformations to prove congruent relationships in angles, triangles, and parallelograms. You'll also grow your skills for making a convincing argument in mathematics.

Essential Questions

- How can I use rigid transformations to prove relationships in angles, triangles, and quadrilaterals?
- How can I prove a conjecture?





Sub-Unit

1

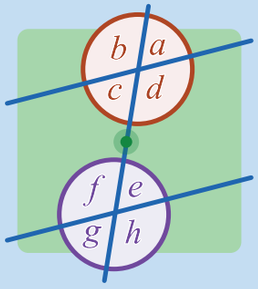
All About Angles



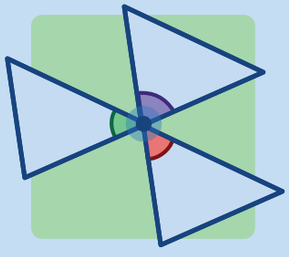
Lesson 1
Is It? Or Is It Not?



Lesson 2
Always True?



Lesson 3
Angles and Transversals



Lesson 4
Doing a 180

Unit 2
Lesson
1

Name: ..... Date: ..... Period: .....

Is It? Or Is It Not?

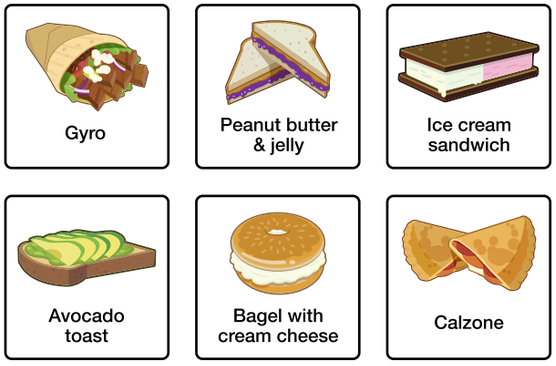


Entire Lesson

Things to Remember:

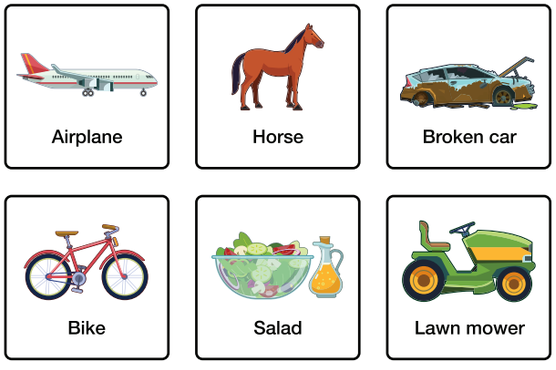
Select Screens

3 Think about which of these foods you'd consider to be a sandwich.



Write a definition of *sandwich* that is true for you.

6 Think about which of these things you'd consider to be a vehicle.



Write a definition of *vehicle* that is true for you.

Lesson Summary

Definitions help us communicate with one another. They are often created to help us understand one another as we live in community. We can use examples and non-examples to help us think about what to include in a definition.

Definitions are often as unique as the people that create them. For example, one person's definition of a vehicle can be very different from another person's, and that is okay. When members of the same community use different definitions, we can begin by asking how each person defines a term so that we can better understand each other.

There are also times when a shared definition can help a community speak clearly and precisely with one another. Creating a shared definition can have consequences, and a community that has a shared definition can make sense of those consequences together.

Try This

a Select *all* the activities that you think are sports.

- ☐ Poker
- ☐ Chess
- ☐ Fishing
- ☐ Dominoes
- ☐ Bowling

b Write a definition of *sport* that is true for you and the choices you made above.

Lesson Practice
Geo.2.01

Name: _____ Date: _____ Period: _____

1. Select *all* the items that you think are cake.

☐ A.



☐ B.



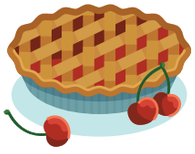
☐ C.



☐ D.



☐ E.



☐ F.

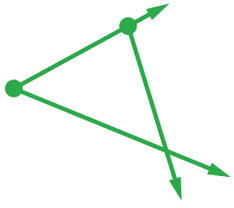


2. Write a definition of *cake* that is true for you and the choices you made in the previous problem.

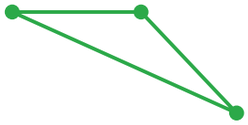
Problems 3–5: Raine's definition of a triangle is a figure that has three angles.

3. Select *all* the figures that fit Raine's definition.

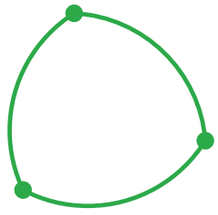
☐ A.



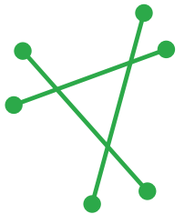
☐ B.



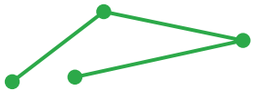
☐ C.



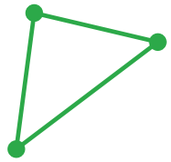
☐ D.



☐ E.



☐ F.



Lesson Practice
Geo.2.01

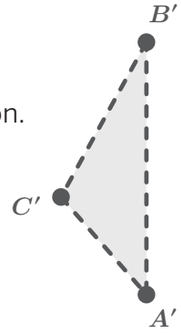
Name: ..... Date: ..... Period: .....

4. Arnav has a different definition of a triangle, which includes only choices *B*, *D*, *E*, and *F*. What is a possible definition that Arnav is using?

5. What is a possible definition of a triangle that includes only choices *B* and *F*?

Spiral Review

Problems 6–8: Here is a diagram.



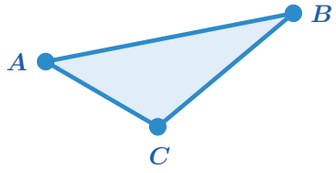
6. Select the rigid transformation that moves $\triangle ABC$ onto $\triangle A'B'C'$ in a single transformation.

A. Rotation

B. Translation

C. Reflection

7. Describe a strategy for completing the transformation that moves $\triangle ABC$ onto $\triangle A'B'C'$.



8. Sketch a new triangle that could be the result of a rotation of $\triangle ABC$, followed by a reflection.

Reflection

1. Circle the problem you feel least confident about.

2. Use this space to ask a question or share something you're proud of.

Unit 2
Lesson
2

Name: ..... Date: ..... Period: .....

Always True?

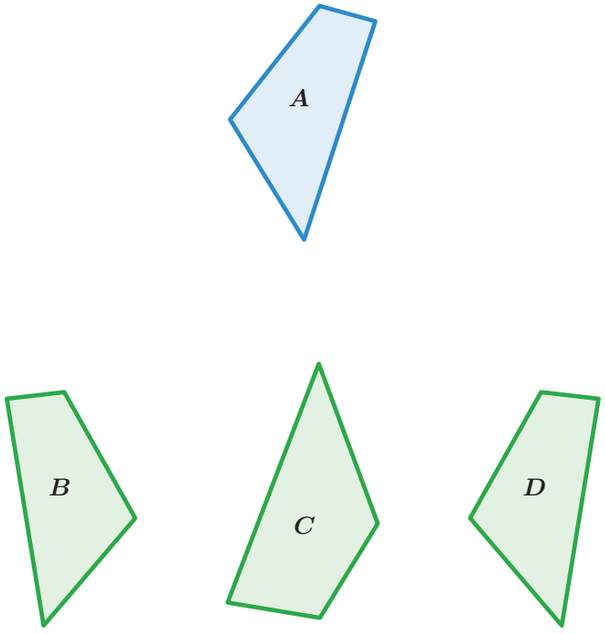
Let's make convincing arguments about transformations and angles.



Warm-Up

1. Circle *all* the figures that can be made by performing a sequence of rigid transformations on figure A.

Describe the sequence of transformations for each figure you circle. Use the geometry tools if they help with your thinking



Activity 1

Name: _____ Date: _____ Period: _____

Applying Rigid Transformations

What happens when we apply rigid transformations to rays, lines, and angles?

Ray



Line



Angle



2. Your teacher will show a series of challenges. Work with a new partner for each challenge and describe a rigid transformation that completes the challenge.

	Partner's Name	Start With	Make	Rigid Transformation
Challenge 1				
Challenge 2				
Challenge 3				

3. Write your own challenge. Make sure it can be solved using rigid transformations.

a. **My challenge:** Starting with _____, make _____.

b. Complete someone else's challenge. Classmate's name: _____

Rigid transformation solution:

Activity 1

Name: Date: Period:

Applying Rigid Transformations (continued)

A student wrote this challenge: *Starting with a line, make a parallel line.*

Here are the rigid transformations that Rio and Ariel used to solve it.

Rio:	**Ariel:**
Rotate the line $180°$.	*Translate the line.*

4. Circle one student's name. Explain why their rigid transformation can solve the challenge.

Rio and Ariel each made a claim that their transformation would always create a parallel line. Circle whether you agree or disagree with each claim and explain your reasoning.

5. Rio's claim: *Rotating a line by* $180°$ *always creates a parallel line.*

Agree Disagree

Explanation:

6. Ariel's claim: *Translating a line always creates a parallel line.*

Agree Disagree

Explanation:

Activity 2

Name: _____ Date: _____ Period: _____

Conjecture Venture

A **conjecture** is a reasonable claim that we want to prove. Let's use rigid transformations to prove a conjecture about angles

7. Use construction tools to draw two intersecting lines. Label the angles you created.

8. Fill in the blanks to complete a conjecture. Use tracing paper if it helps with your thinking.

In my diagram, ∠ _____ is *congruent* to ∠ _____.

9. **a** Describe a rigid transformation that proves your conjecture.

b Describe a different rigid transformation you could use or explain why that's not possible.

10. Compare papers with a partner.

Discuss:

- How are your diagrams for Problem 7 different? How are they alike?
- Will *your partner's* transformation in Problem 9 also prove *your* conjecture?

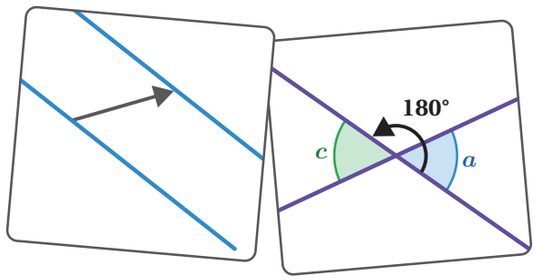
When two lines intersect, the angles that are opposite each other are called vertical angles.

11. Here's another conjecture: *Vertical angles are congruent*. How might you convince someone that this conjecture is always true?

Synthesis

12. How can rigid transformations help you explore conjectures and show if they are always true?

Use the examples if they help with your thinking.



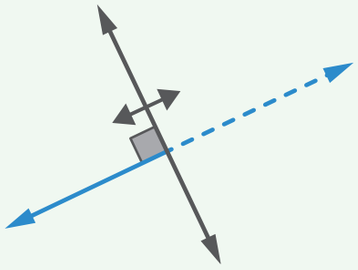
Lesson Summary

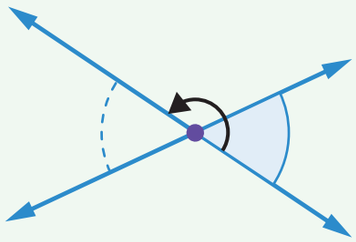
You can use rigid transformations and their definitions to explore geometric relationships.

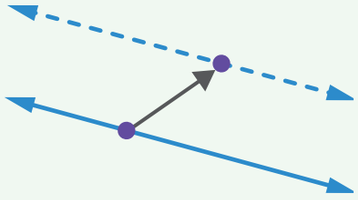
- A translation moves any point a specific distance and direction.
- A reflection moves any point to a point on the opposite side of a line of reflection.
- A rotation moves any point around a center of rotation by a specific angle and direction.

Because rigid transformations keep all angles and distances the same, they can be helpful tools for proving **conjectures** about geometric relationships.

Reflection	Rotation	Translation
You can reflect a ray across a perpendicular line to make another line.	You can rotate an angle $180°$ around its vertex to prove that *vertical angles* are *congruent*.	You can translate a line to a point off of the line in order to create a parallel new line.





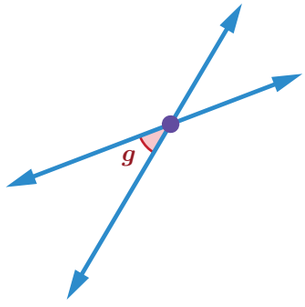


Try This

a If you applied a $180°$ rotation to a line, what could be the result? Select *all* that apply.

- ☐ **A.** A line that's parallel to the original line
- ☐ **B.** A line that's perpendicular to the original line
- ☐ **C.** Just the original line

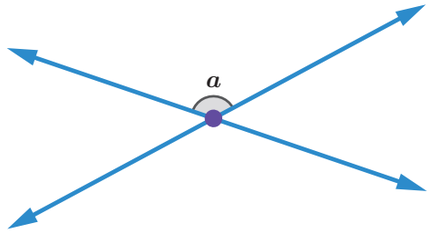
b The diagram shows two lines and the angle g. Describe the result of rotating everything by $180°$ around the intersection point.



Lesson Practice
Geo.2.02

Name: ..... Date: ..... Period: .....

1. Construct one ray to make an angle that is congruent to angle a.



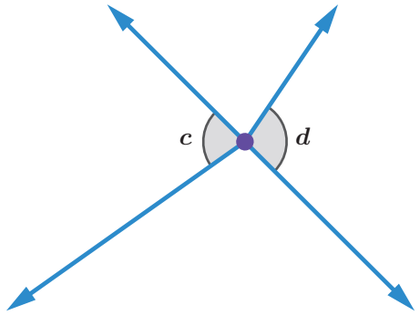
2. Which type of transformation would move $\angle c$ exactly onto $\angle d$?

A. Reflection

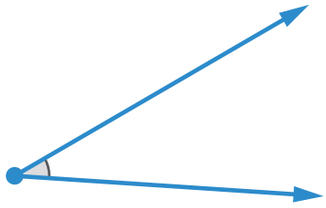
B. Rotation

C. Translation

D. None of these



Problems 3–5: Here is an angle.



3. Show or describe a rigid transformation (or sequence of transformations) that can create a vertical angle.

4. How does your transformation prove that the angle and its vertical angle are congruent?

5. Is there an angle between $0°$ and $180°$ that *cannot* be transformed to make a vertical angle? Explain your thinking.

Lesson Practice
Geo.2.02

Name: _____ Date: _____ Period: _____

6. Which statement is false?

A. A ray can be rotated around its endpoint to form an angle.

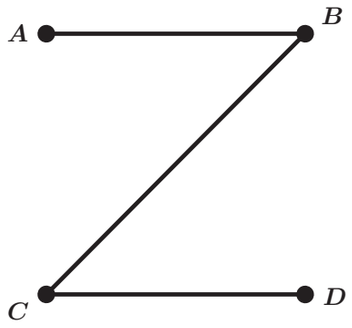
B. A ray can be rotated $180°$ around its endpoint to form a line.

C. A ray can be reflected to form an angle.

D. A ray can be translated to form a line.

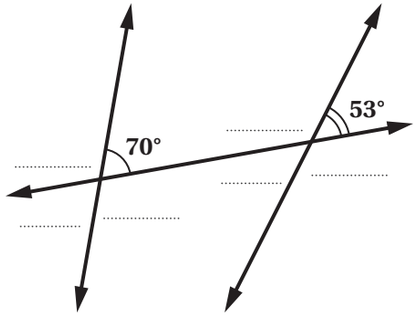
7. Three line segments form the letter Z. Rotate the figure counterclockwise around the midpoint of $\overline{BC}$ by $180°$.

Explain why the result looks like the same letter Z. Use tracing paper if it helps with your thinking.



Spiral Review

8. This diagram shows three lines with some missing angle measures. Complete the diagram by writing in all of the missing angle measures.



Reflection

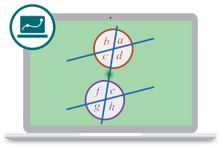
1. Put a star next to a problem you want to understand better.

2. Use this space to ask a question or share something you're proud of.

Unit 2
Lesson
3

Name: ..... Date: ..... Period: .....

Angles and Transversals



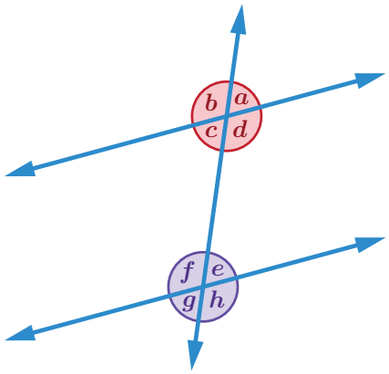
Entire Lesson

Things to Remember:

Select Screens

2 Here are some terms that come up when two lines are crossed by a third line, called a *transversal*.

- Alternate exterior angles
- Alternate interior angles
- Same side exterior angles
- Same side interior angles
- Corresponding angles

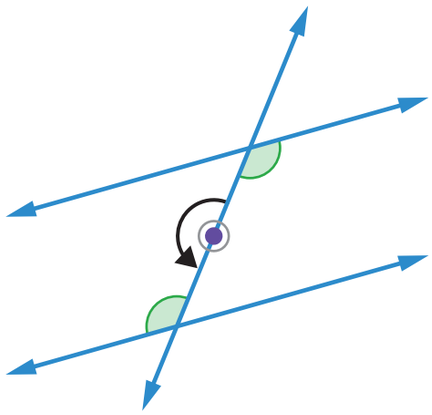


Notes from discussion:

8 Nyanna and Avi made this transformation. They each wrote a conjecture:

Nyanna: *When a transversal crosses parallel lines, alternate interior angles are congruent.*

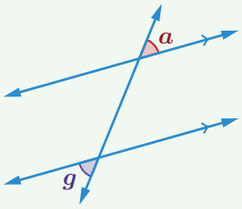
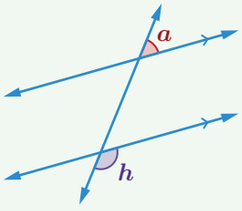
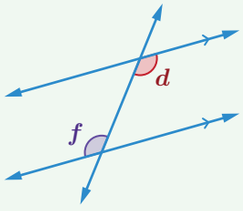
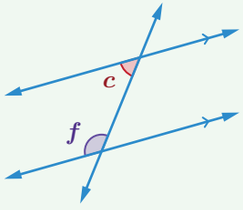
Avi: *When alternate interior angles are congruent, the two lines are parallel.*



Notes from class discussion:

Lesson Summary

There are often several different ways to explain why statements are true. Comparing the different ways can lead to new insights or more flexible understanding. Consider the different angles that form when two parallel lines are cut by a transversal.

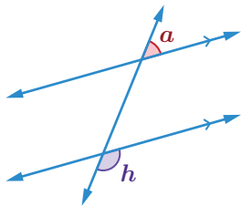
Alternate Exterior Angles	Same Side Exterior Angles	Alternate Interior Angles	Same Side Interior Angles
			

You can **prove** that there are relationships between these angles using rigid transformations, such as translations, reflections, and rotations. As an example for alternate exterior angles, by translating angle a to the vertex of angle g, you can prove that angle a is congruent to angle g because they become vertical angles.

Try This

Here is a conjecture.

When two parallel lines are crossed by a transversal, same side exterior angles are supplementary.



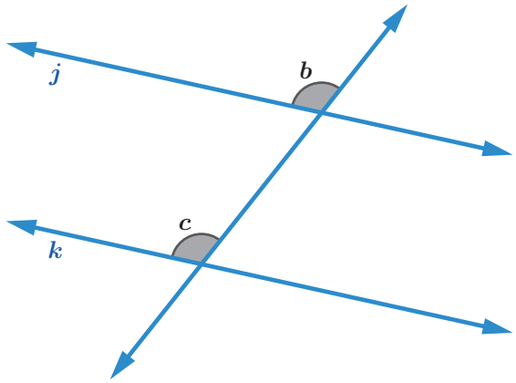
a Describe a transformation that would show this relationship.

b Explain how this proves the conjecture.

Lesson Practice
Geo.2.03

Name: ..... Date: ..... Period: .....

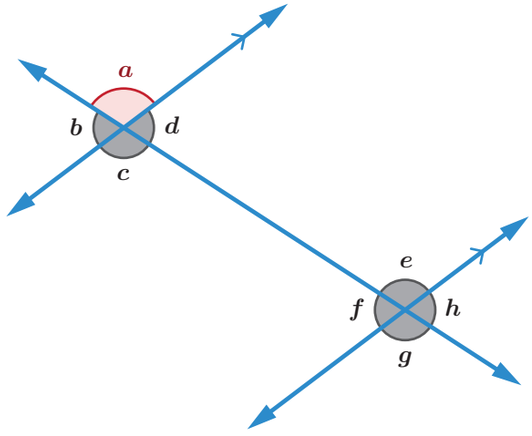
1. Describe one more piece of information you would need to show that angle c and angle b are congruent.



Problems 2–5: Two parallel lines are crossed by a transversal.

2. Select *all* the angles that are congruent to angle a.

- ☐ b
- ☐ c
- ☐ d
- ☐ e
- ☐ f
- ☐ g
- ☐ h



3. Identify a pair of vertical angles.

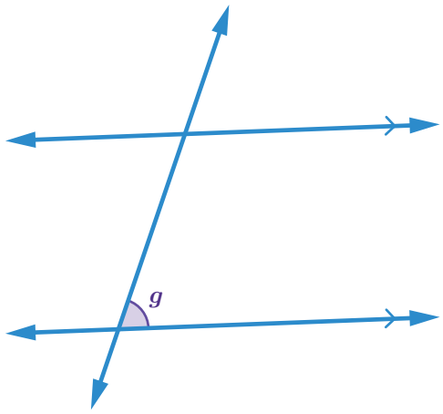
4. Identify a pair of alternate interior angles.

5. Identify a pair of corresponding angles.

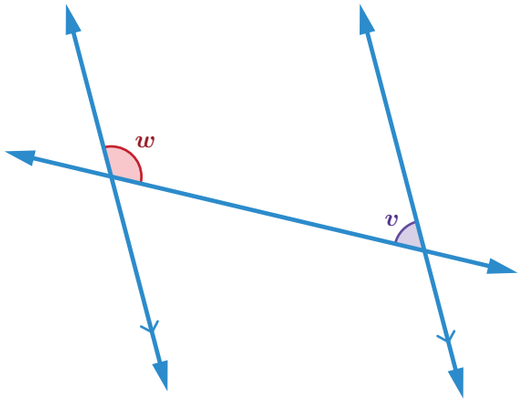
Lesson Practice
Geo.2.03

Name: ..... Date: ..... Period: .....

6. Show or describe a sequence of rigid transformations that carries angle g onto angle f.



Problems 7–8: Two parallel lines are crossed by a transversal.

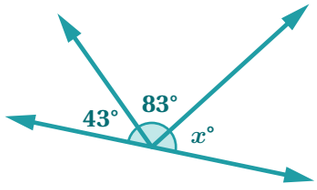


7. Write a conjecture about angles v and w.

8. Describe a transformation that proves your conjecture. Explain your thinking.

Spiral Review

Problems 9–10: Here are three angles that form a straight angle.



9. Write an equation that includes x.

10. Determine the value of x.

$x =$ .....

Reflection

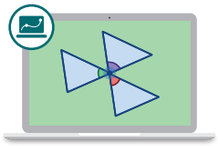
1. Put a star next to a problem where you revised your thinking.

2. Use this space to ask a question or share something you're proud of.

Unit 2
Lesson
4

Name: ..... Date: ..... Period: .....

Doing a 180



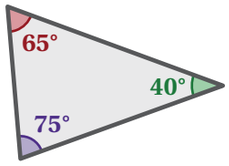
Entire Lesson

Things to Remember:

Select Screens

3 Here is a conjecture.

> The interior angles of a triangle sum to 180°.



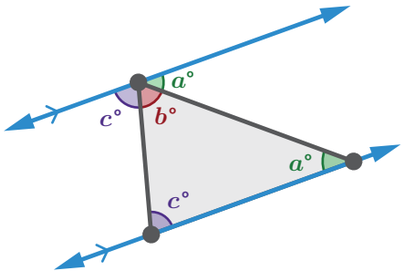
How can you prove this conjecture is always true?

Record the different relationships you explored for proving the conjecture.

5 Joel made two changes to his diagram:

- Replaced the numbers with variables.
- Made the points movable.

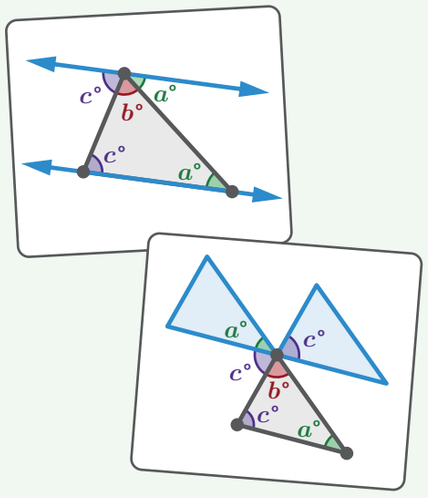
How do these changes strengthen his argument?



Notes from class discussion:

Lesson Summary

Conjectures can be proven in many different ways. As you make a plan to prove a conjecture, it can be helpful to begin with a relationship or property that has already been proven true. For example, to prove that the interior angles of a triangle always sum to $180°$, you might begin with what you know about vertical angles, alternate interior angles, *straight angles*, or other angle relationships.



Proving a conjecture is true for one case is not the same as proving that a conjecture is always true.

There are several strategies for proving a conjecture is always true.

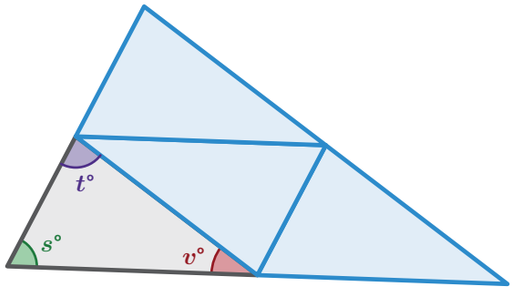
- You can explain why the reasoning that applies to one case can be generalized to all other possible cases.
- You can use draggable points to show that a relationship stays the same for all of the possible cases.
- You can also replace numbers with variables to show that a relationship stays the same for *all* cases.

Try This

A student is trying to prove this conjecture: *The interior angles of a triangle sum to* $180°$.

The student starts by making a triangle, and then uses rigid transformations to make three copies of the triangle.

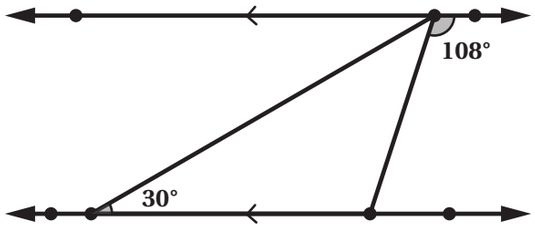
Label each angle in the copies with the appropriate variable.



Lesson Practice
Geo.2.04

Name: ..... Date: ..... Period: .....

1. Label all the angle measurements that are possible to know from the given information.



Problems 2–5: Here is a conjecture: *The interior angles of a triangle sum to* 180°. A student started to prove the conjecture by making a triangle and then a copy of the triangle.

2. Which rigid transformation was used to make the copy of the triangle?

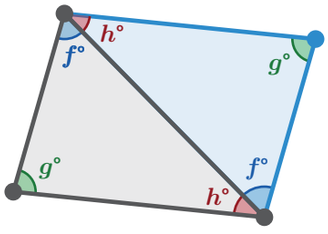
A. Translation

B. Reflection

C. Rotation

3. Show or describe what the student could do next to prove the conjecture.

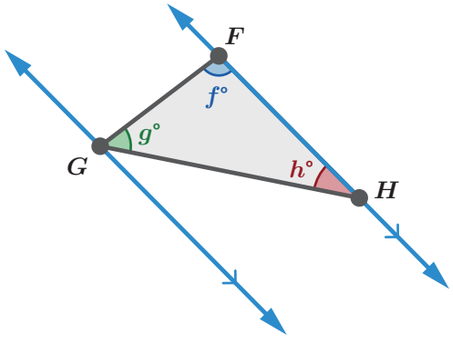
Diagram A



4. Here is a new diagram that proves the same conjecture. Label the angles at point *G* with the appropriate variables.

5. Explain how to prove the conjecture is always true using one or both of the diagrams.

Diagram B

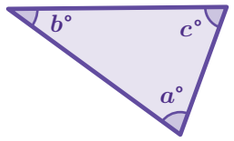


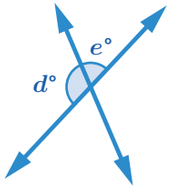
Lesson Practice
Geo.2.04

Name: _____ Date: _____ Period: _____

6. Determine the average of a, b, c, d, and e.

Explain your thinking.

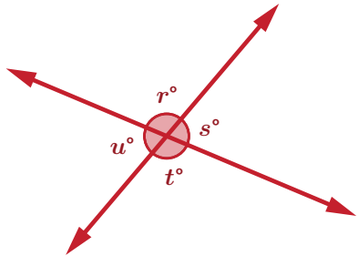




Spiral Review

7. Select *all* the true statements.

- ☐ **A.** $r + s = 180$
- ☐ **B.** $u + s = r + t$
- ☐ **C.** $t + r = 180$
- ☐ **D.** $s = u$
- ☐ **E.** $u + t = r + s$

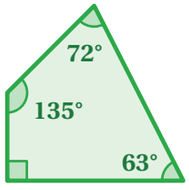


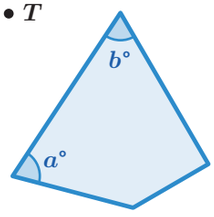
Problems 8–9: A polygon is rotated around point T. Its image is shown.

Determine the value of each variable.

8. $a =$ _____

9. $b =$ _____





Reflection

1. Put a heart next to the problem you're most proud of.

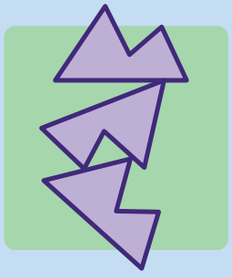
2. Use this space to ask a question or share something you're proud of.

Student Edition pages

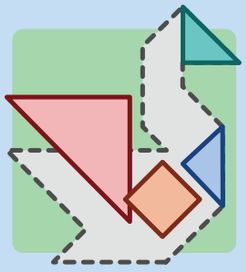
Sub-Unit

2

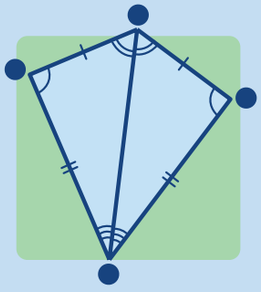
Triangles: Congruence, Criteria, and Conjectures



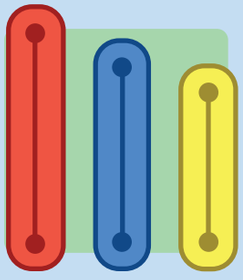
Lesson 5
Stack 'Em Up



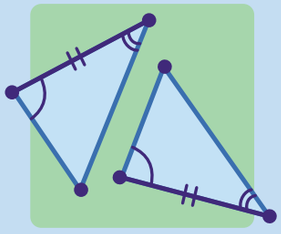
Lesson 6
Parts and Places



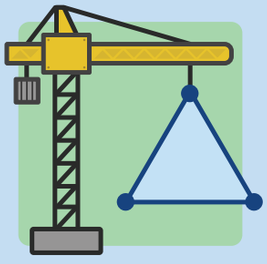
Lesson 7
Corresponding Components



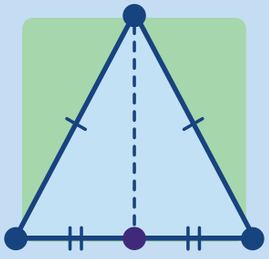
Lesson 8
Less Is More



Lesson 9
Congruence Conditions



Lesson 10
Resuming Construction

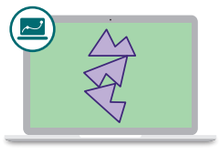


Lesson 11
Proving Points

Unit 2
Lesson
5

Name: ..... Date: ..... Period: .....

Stack 'Em Up



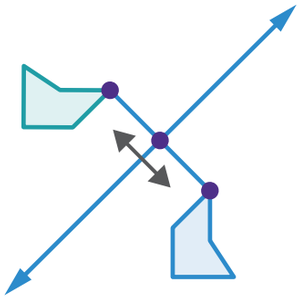
Entire Lesson

Things to Remember:

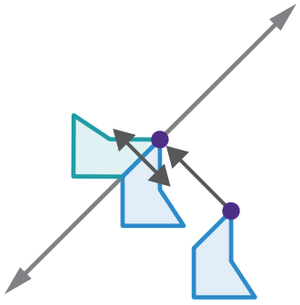
Select Screens

6 Dakota and Sakari used different methods to solve the previous challenge.

Dakota



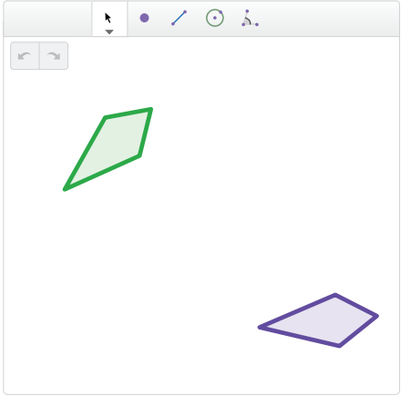
Sakari



Notes from class discussion:

9 Here are two new figures.

Record your favorite sequence of transformations that solved this challenge.



Lesson Summary

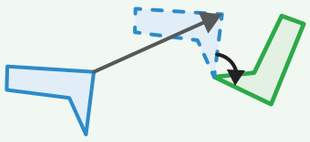
You can use sequences of rigid transformations to decide if two figures are **congruent**. *Translations*, *rotations*, and *reflections* are all *rigid transformations*, which means they keep the size or shape of a figure the same. If two figures stack perfectly using rigid transformations, then they are congruent.

Let's look at two examples.

These figures are congruent because there is a sequence of transformations that will make them stack perfectly.

One way to stack the figures perfectly is to translate and then rotate the figure.

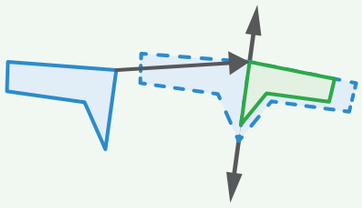
Congruent



These two figures are *not* congruent.

There is no sequence of rigid transformations that will stack the figures perfectly.

Not Congruent

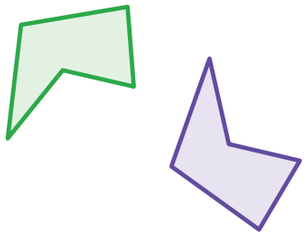


Try This

Are these two figures congruent? Circle one.

Yes No I'm not sure

Explain your thinking.



Lesson Practice
Geo.2.05

Name: _____ Date: _____ Period: _____

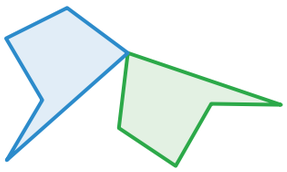
1. Select *all* transformations that carry point C onto point D.



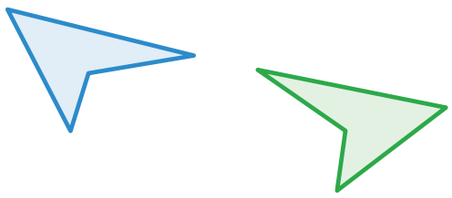
- ☐ **A.** Rotate $180°$ with point C as the center
- ☐ **B.** Rotate $180°$ with point D as the center
- ☐ **C.** Rotate $180°$ around the midpoint of segment CD
- ☐ **D.** Reflect over the perpendicular bisector of segment CD
- ☐ **E.** Reflect over segment CD

Problems 2–3: Show or explain *one* rigid transformation that can determine if the two figures are congruent.

2.

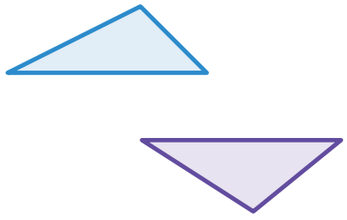


3.

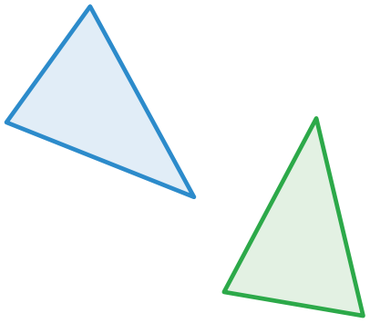


Problems 4–5: Are the two figures congruent? Circle your response and show or explain your sequence of rigid transformations. Use tracing paper if it helps with your thinking.

4. Congruent Not congruent



5. Congruent Not congruent

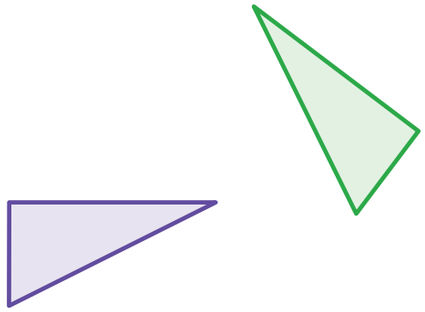


Lesson Practice
Geo.2.05

Name: _____ Date: _____ Period: _____

6. Use at least one reflection, rotation, and translation to determine if the figures are congruent. Circle your response and show or explain your thinking.

Congruent Not congruent



Spiral Review

Problems 7–8: Imagine figure B is an image of figure A.

7. Select *all* transformations that could have been used.

- ☐ **A.** Translation
- ☐ **B.** Rotation
- ☐ **C.** Reflection
- ☐ **D.** Dilation

8. Select *all* transformations that could not have been used.

- ☐ **A.** Translation
- ☐ **B.** Rotation
- ☐ **C.** Reflection
- ☐ **D.** Dilation

Reflection

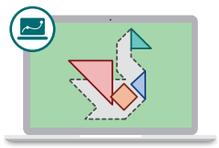
1. Circle the problem that was the most challenging for you.

2. Use this space to ask a question or share something you're proud of.

Unit 2
Lesson
6

Name: ..... Date: ..... Period: .....

Parts and Places

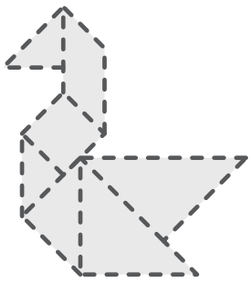


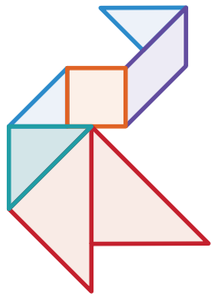
Entire Lesson

Things to Remember:

Select Screens

4 Here are the two tangram swans from the previous screen.



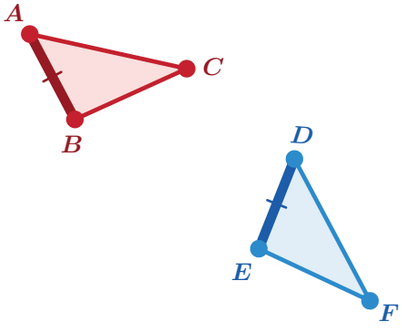


Discuss:
How are Aki's and Miriam's strategies alike? How are they different?

Notes from class discussion:

10 $\triangle ABC$ is congruent to $\triangle DEF$.

How did you decide which sides and angles correspond in the figures?

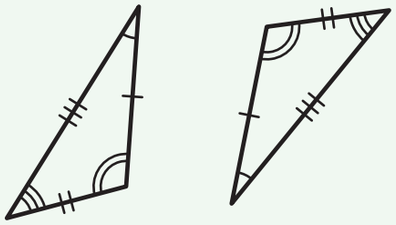


Lesson Summary

You can determine if two figures are congruent in two different ways:

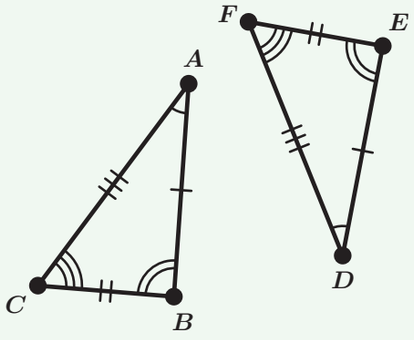
- Determining a sequence of rigid transformations that stacks the figures perfectly.
- Verifying that all the **corresponding** parts in the figures are congruent.

Corresponding parts are in the same position and orientation in each figure. If *all* the pairs of congruent sides and angles are corresponding in each figure, then the figures are congruent.



To show that congruent segments are corresponding, you can mark them with tick marks. To show that congruent angles are corresponding, you can mark them with arcs.

If two triangles are congruent, then they have 6 sets of corresponding congruent parts: 3 pairs of sides and 3 pairs of angles.

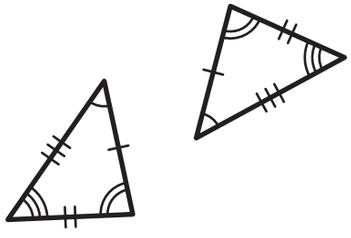


$\overline{AB} \cong \overline{DE}$ $\quad$ $\overline{BC} \cong \overline{EF}$ $\quad$ $\overline{AC} \cong \overline{DF}$

$\angle A \cong \angle D$ $\quad$ $\angle B \cong \angle E$ $\quad$ $\angle C \cong \angle F$

Try This

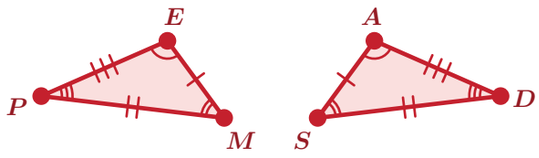
Explain how you know that these two triangles must be congruent.



Lesson Practice
Geo.2.06

Name: _____ Date: _____ Period: _____

Problems 1–2: Here is a pair of triangles.



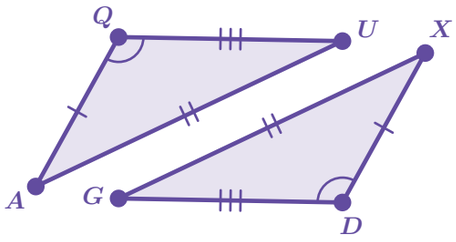
1. Explain how you know that $\triangle EMP \cong \triangle ASD$.

2. Write the congruence statements for the corresponding parts in the triangles.

$\angle P \cong \angle$_____ $\angle E \cong \angle$_____ $\angle M \cong \angle$_____

$\overline{EM} \cong$ _____ $\overline{PE} \cong$ _____ $\overline{PM} \cong$ _____

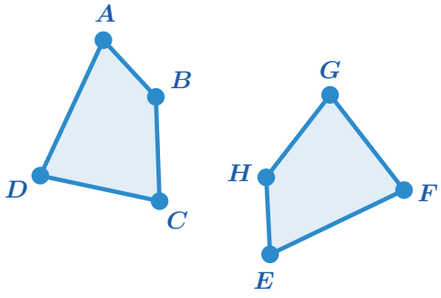
3. What two additional congruence statements are needed to make $\triangle QUA \cong \triangle DGX$ true?



_____ $\cong$ _____

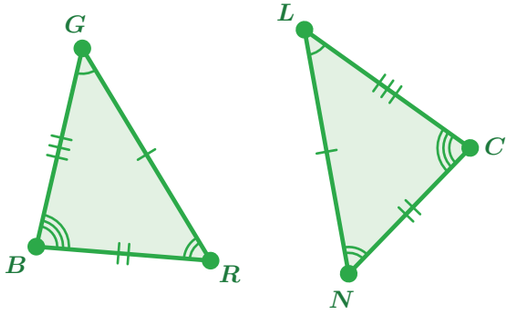
_____ $\cong$ _____

4. Quadrilateral $ABCD$ is congruent to quadrilateral $EHGF$. Select *all* the statements that must be true.



- ☐ **A.** $\angle C \cong \angle F$
- ☐ **B.** $\overline{BC} \cong \overline{GF}$
- ☐ **C.** $\angle B \cong \angle H$
- ☐ **D.** $\overline{EH} \cong \overline{AB}$
- ☐ **E.** $\angle A \cong \angle E$
- ☐ **F.** $\overline{DA} \cong \overline{EF}$

5. Here are two congruent triangles, $\triangle GRB \cong \triangle LNC$.



Complete the following congruence statements:

$\angle R \cong$ _____ $\overline{BG} \cong$ _____ $\angle B \cong$ _____

$\overline{GR} \cong$ _____ $\angle G \cong$ _____ $\overline{RB} \cong$ _____

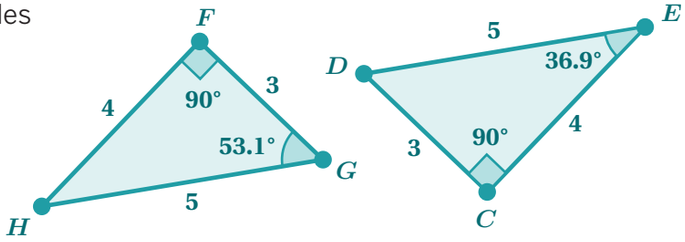
Lesson Practice
Geo.2.06

Name: _____ Date: _____ Period: _____

6. DeShawn thinks that these two triangles are congruent. Lucy does not think these triangles are congruent. Whose thinking is correct? Circle one.

DeShawn's Lucy's Neither

Explain your reasoning.



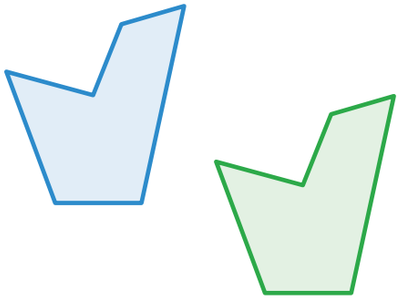
7. Draw two triangles, $\triangle LMN$ and $\triangle PQR$, that have 3 pairs of congruent angles.

8. Do your triangles from Problem 7 have to be congruent?

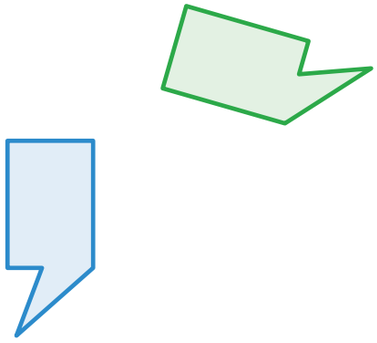
Spiral Review

Problems 9–10: Sketch one rigid transformation that shows if the two figures are congruent.

9.



10.



Reflection

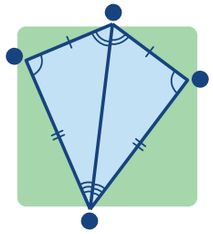
1. Circle the problem you feel least confident about.

2. Use this space to ask a question or share something you're proud of.

Unit 2
Lesson
7

Name: ..... Date: ..... Period: .....

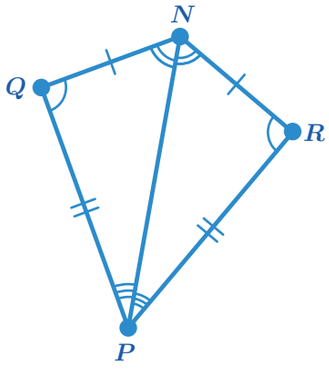
Corresponding Components



Let's explore how congruence statements and corresponding parts are related.

Warm-Up

1. Here is a pair of congruent triangles that share $\overline{NP}$. Ashley claims that $\angle N \cong \angle N$.



Discuss:

- Why might the name $\angle N$ be confusing?
- How would you rename the angles that include vertex N to help Ashley be more specific with her claim?

Activity 1

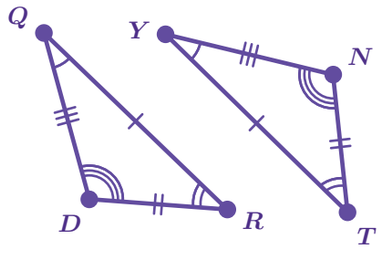
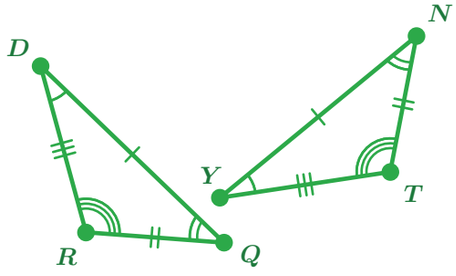
Name: Date: Period:

Two Truths and a Lie

2. Here are six statements and two different pairs of triangles with congruence markings.

Statement 1	Statement 2	Statement 3
$\triangle DRQ \cong \triangle NTY$	$\triangle RQD \cong \triangle TYN$	$\angle N \cong \angle Q$
Statement 4	**Statement 5**	**Statement 6**
$\overline{DR} \cong \overline{TN}$	$\overline{RQ} \cong \overline{TN}$	$\angle Q \cong \angle Y$

Use each statement once so that each triangle pair has two true statements and one false statement. Use tracing paper if it helps with your thinking.

Pair A	Pair B
	
True:	True:
True:	True:
False:	False:

3. The order of the vertices in a triangle congruence statement tells a story about the sequence of rigid transformations that will move one figure onto the other.

a Let's look at the false triangle congruence statement for Pair B.

Discuss: Why is this statement false?

b Describe a sequence of rigid transformations that will stack the triangles in Pair B.

c Which of the triangles' vertices stack in your sequence of rigid transformations?

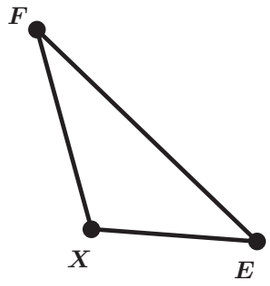
d Write a *true* triangle congruence statement for Pair B.

Activity 1

Name: _____ Date: _____ Period: _____

Two Truths and a Lie (continued)

4. Here is $\triangle FXE$.



a Rotate the triangle a half turn with a center at E to create a new image.

- Label the image RQE so that $\triangle RQE \cong \triangle FXE$.
- Add congruence markings that make the triangle congruence statement true.

b Write two true statements and one false statement (in any order) for this pair of triangles.

Statement 1: _____

Statement 2: _____

Statement 3: _____

c Swap papers with a classmate and circle the false statement in their problem.

Name: _____

Activity 2

Name: ..... Date: ..... Period: .....

Build a Pair

5. Here are three true statements about two congruent triangles.

- $\angle JBH$ and $\angle DBG$ are vertical angles.
- $\overline{JB} \cong \overline{DB}$
- $\angle J \cong \angle D$

Draw a pair of congruent triangles that make all three statements true. Remember to include vertex labels and congruence markings.

Activity 2

Name: Date: Period:

Build a Pair (continued)

6. Here is a list of statements.

- ☐ $m\angle C + m\angle F = 180°$
- ☐ $\angle A \cong \angle D$
- ☐ $\overline{BC} \cong \overline{DF}$
- ☐ $\overline{BA} \cong \overline{ED}$
- ☐ $\angle ABC \ncong \angle DEF$
- ☐ $\angle A \ncong \angle E$
- ☐ $\overline{AC} \cong \overline{DF}$
- ☐ $\angle BAC \cong \angle DEF$

Draw and label a pair of congruent triangles with congruence markings using as many statements as you can. Hint: Not all of the statements can be used at the same time for your triangle pair.

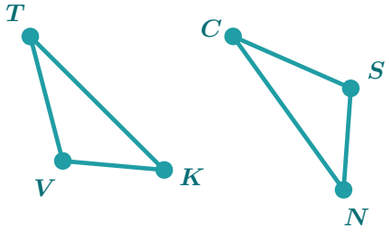
Check off the statements as you use them.

Synthesis

7. How can a triangle congruence statement communicate which pairs of segments and angles correspond?

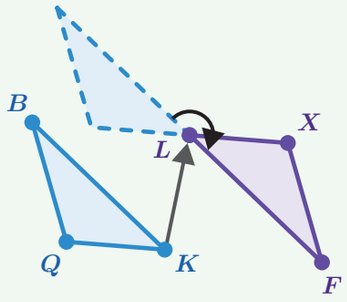
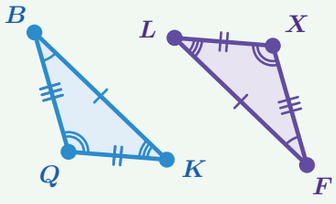
Use the example if it helps with your thinking.

$\triangle TVK \cong \triangle CSN$



Lesson Summary

The congruent parts that are in the same position and orientation within congruent figures are called *corresponding parts*. You can identify them in multiple ways:

Transformations	Diagrams	Congruence Statements
		
The sequence of transformations shows a translation that stacks K on L and a rotation that stacks Q on X and B on F. Since $\overline{BQ}$ stacks perfectly on $\overline{FX}$, $\overline{BQ} \cong \overline{FX}$.	The markings on the pair of triangles show which parts are congruent and corresponding. Since $\angle Q$ and $\angle X$ are both marked with a double arc, $\angle Q \cong \angle X$.	The order in which vertices appear in triangle congruence statements shows which vertices correspond in two figures. Since K and L are both the third letter in the triangle congruence statement, $\angle K \cong \angle L$.

Try This

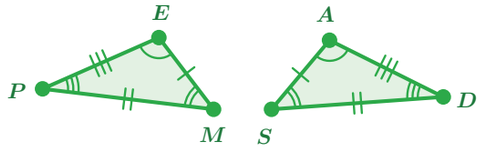
Draw and label two triangles so that $\triangle YMK \cong \triangle JFK$. Remember to include congruence markings.

Lesson Practice
Geo.2.07

Name: _____ Date: _____ Period: _____

Problems 1–2: Here are two triangles.

1. Which rigid transformation stacks one triangle on the other perfectly?

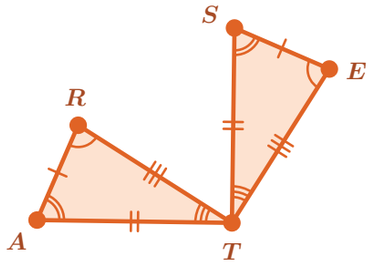


2. What are two possible triangle congruence statements for this pair?

△_____ ≅ △_____

△_____ ≅ △_____

Problems 3–5: Use the triangle pair to determine whether the statements are true or false. Circle one.



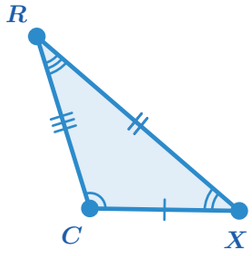
3. $\overline{RT} \cong \overline{ST}$ True False

4. $\triangle ART \cong \triangle SET$ True False

5. $\angle ATR \cong \angle ETS$ True False

Problems 6–7: Here is $\triangle CXR$.

6. Translate the triangle until point C goes to point X and label the image so $\triangle CXR \cong \triangle XZT$.

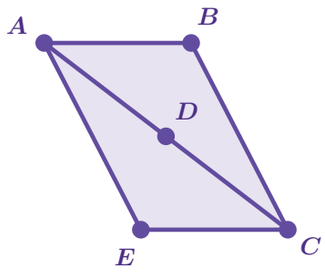


7. Add congruence markings that make the triangle congruence statement true.

Lesson Practice
Geo.2.07

Name: ..... Date: ..... Period: .....

8. Triangle CEA is the image of triangle ABC after a 180° rotation around point D. Select *all* statements that must be true.



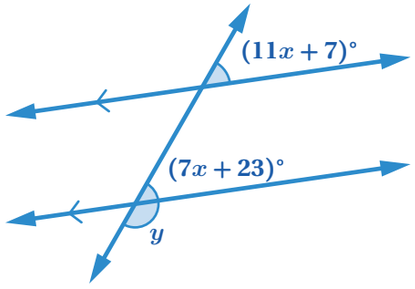
- ☐ **A.** $\triangle ABC \cong \triangle AEC$
- ☐ **B.** $\angle BAC \cong \angle EAC$
- ☐ **C.** $\triangle BAC \cong \triangle ECA$
- ☐ **D.** $\overline{AE} \cong \overline{AB}$
- ☐ **E.** $\angle EAC \cong \angle BCA$
- ☐ **F.** $\overline{EC} \cong \overline{BA}$

9. Draw a pair of congruent triangles that make all three statements true. Remember to include vertex labels and congruence markings.

- $\angle X \cong \angle W$
- $\angle XVY \cong \angle WVY$
- $\overline{XY} \cong \overline{WY}$

Spiral Review

Problems 10–11: Determine the value of each variable.



10. x = ...

11. y = ...

Reflection

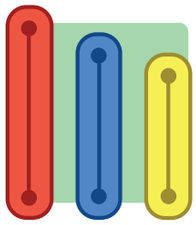
1. Put a star next to the problem you spent the most time on.

2. Use this space to ask a question or share something you're proud of.

Unit 2
Lesson
8

Name: ..... Date: ..... Period: .....

Less Is More



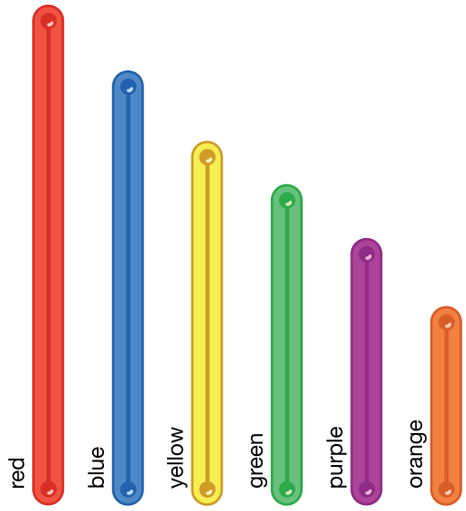
Let's investigate properties of triangle congruence.

Warm-Up

1. You will use a set of AngLegs® for this activity.

a Form a triangle by snapping the ends of the AngLegs® together.

b Compare your triangle to your partner's triangle. What are some similarities between the two? What are some differences?



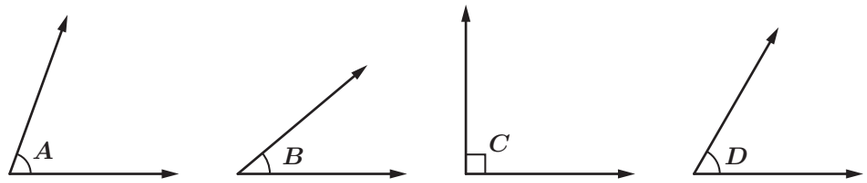
Activity 1

Name: Date: Period:

Congruent or Not?

Work alongside a partner to build triangles with a set of AngLegs®. Hide your triangles until it's time to reveal! Then, share and compare your triangles with your partner's triangles.

2. Use these angles to help you create the triangles in the table.



Partner's name:

	Create a triangle with . . .	Congruent?	Can you make triangles that fit the constraints and are *not* congruent? Explain your thinking.
a	• An orange side • A green side • A yellow side	Yes No	
b	• A purple side • Angle A • Angle B	Yes No	
c	• Angle C • Angle D	Yes No	

3. Haruto says: *You don't need to check all three corresponding sides and all three corresponding angles to know if two triangles are congruent. You just need to know that three corresponding pairs of parts are congruent. Then the triangles will always be congruent.*

a What is correct about Haruto's claim?

b What would you change to make Haruto's claim more precise?

Activity 2

Name: Date: Period:

Investigating Triangle Congruence

4. Use the AngLegs® and the information in the table to form two triangles. Determine if the triangles must always be congruent. Record your thinking in the table; the first row is done for you.

Create two triangles with these pairs of congruent parts:	Congruent?	How do you know?
• Three sides • Three angles	Yes No	If all the corresponding parts are congruent, then the triangles must be congruent. This means that a rigid transformation will stack one triangle onto the other.
• Three sides • One angle	Yes No	
• Three sides	Yes No	
• Two sides • One angle	Yes No	
You choose:	Yes No	
You choose:	Yes No	

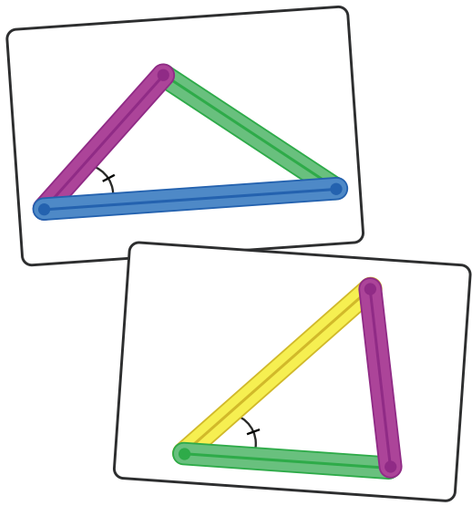
5. Haruto says: *When two sides and one angle are congruent in a pair of triangles, the triangles will be congruent.*

Is Haruto's statement *always*, *sometimes*, or *never* true?

Explain your thinking.

Synthesis

6. **a** Describe some combinations of congruent parts that will ensure a pair of triangles are congruent.

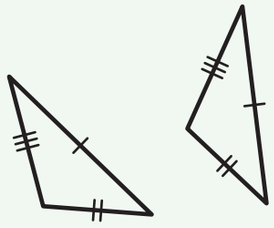


b Describe some combinations of congruent parts that will *not* ensure a pair of triangles are congruent.

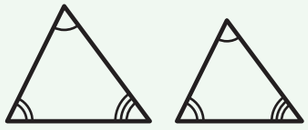
Lesson Summary

If you compare two triangles and every corresponding pair of sides and angles is congruent, then you know the two triangles are congruent. Sometimes, comparing just a few corresponding parts is enough to determine if two triangles are congruent.

Having three pairs of corresponding congruent sides means the triangles will always be congruent.



But there are exceptions to keep in mind! For example, having three pairs of corresponding congruent angles does not mean that the triangles will always be congruent.



Try This

Azul made two triangles with three pairs of corresponding congruent angles.

Are these triangles always congruent? Explain your thinking.

Lesson Practice
Geo.2.08

Name: _____ Date: _____ Period: _____

Problems 1–2: Determine the number of sides and angles that . . .

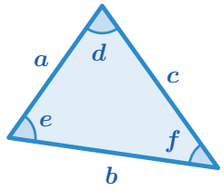
1. *Always* ensures a pair of congruent triangles.

_____ side(s) and _____ angle(s)

2. *Sometimes* ensures a pair of congruent triangles.

_____ side(s) and _____ angle(s)

3. Two congruent triangles have the same congruent corresponding parts. Circle a combination of sides and angles that would *not* ensure that the triangles are congruent.



4. What is the minimum number of pairs of congruent sides that ensures two triangles are congruent, without considering any angles?

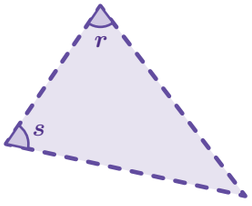
A. 1 **B.** 2 **C.** 3 **D.** Not possible

5. What is the minimum number of pairs of congruent angles that ensures two triangles are congruent, without considering any sides?

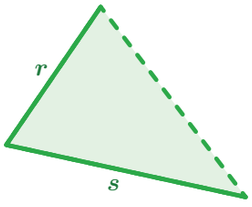
A. 1 **B.** 2 **C.** 3 **D.** Not possible

Problems 6–7: The congruent parts *r* and *s* are given for a pair of triangles. Ensure the triangles are congruent by marking . . .

6. One congruent side.



7. One congruent angle.



Lesson Practice
Geo.2.08

Name: _____ Date: _____ Period: _____

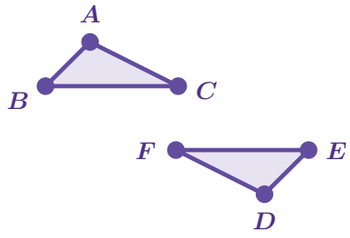
8. Congruent parts can also be used to determine if other types of polygons are congruent.

Can four congruent sides ensure that two quadrilaterals are congruent? Show or explain your thinking.

Spiral Review

Problems 9–10: Triangle ABC is congruent to triangle DEF.

9. Determine a sequence of rigid transformations that carries triangle ABC onto triangle DEF.



10. Determine a *different* sequence of rigid transformations that carries triangle ABC onto triangle DEF.

Reflection

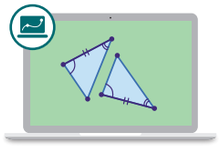
1. Put a star next to a problem you're still feeling confused about.

2. Use this space to ask a question or share something you're proud of.

Unit 2
Lesson
9

Name: ..... Date: ..... Period: .....

Congruence Conditions



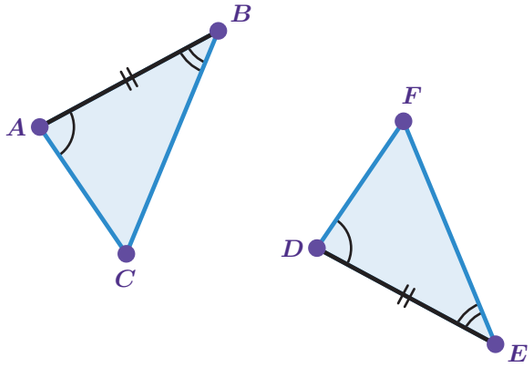
Entire Lesson

Things to Remember:

Select Screens

3 Kwasi says these triangles will always be congruent because:

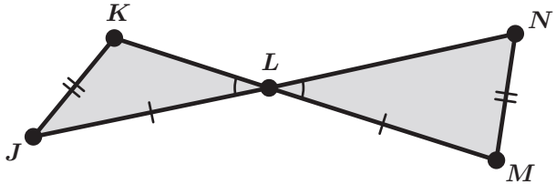
- Point A is translated to point D.
- Point B will move to point E because $\overline{AB} \cong \overline{DE}$.



Draft your thinking before you discuss:

9 **Discuss:**

- Are the triangles congruent?
- What can you conclude about side-side-angle (SSA) congruence from these triangles?

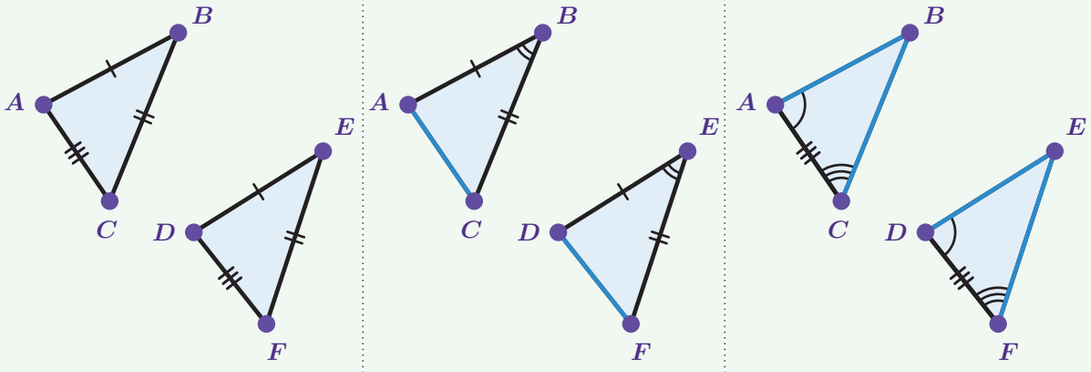


Notes from class discussion:

Lesson Summary

You can use specific triangle congruence criteria to create triangles that are always congruent. Two triangles are congruent when they have these pairs of corresponding congruent parts:

SSS (side-side-side)	SAS (side-angle-side)	ASA (angle-side-angle)
Three sides	Two sides and the angle between them	Two angles and the side between them



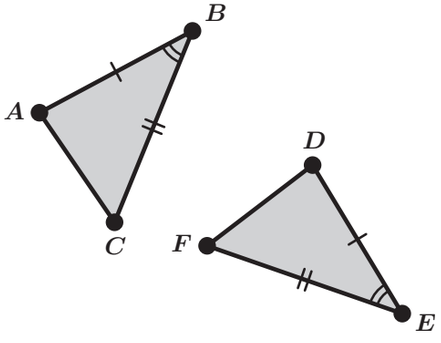
However, two corresponding congruent sides and an adjacent pair of corresponding angles (known as SSA or side-side-angle) do not always create congruent triangles.

Try This

Which criteria helps you know that these triangles are congruent? Circle one.

SSS SAS ASA

Explain your thinking.



Lesson Practice
Geo.2.09

Name: _____ Date: _____ Period: _____

1. Which combination of congruent parts is *not* sufficient to know whether two triangles are congruent?

A. Angle-angle-angle

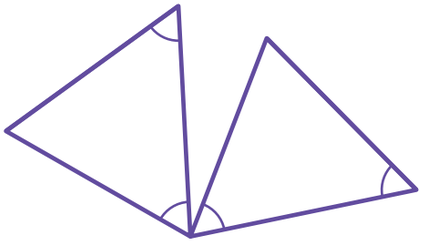
B. Side-side-side

C. Angle-side-angle

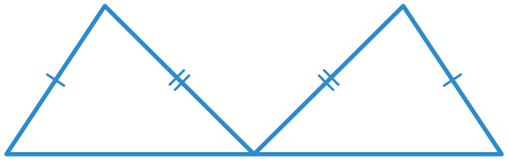
D. Side-angle-side

Problems 2–3: Mark one additional pair of congruent corresponding parts to match the given combination of congruent parts.

2. AAS



3. SAS



Problems 4–6: Which criteria proves the triangles are congruent?

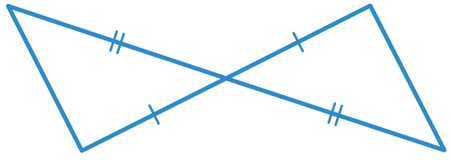
4. Circle one.

A. SSS

B. ASA

C. SAS

D. SAA



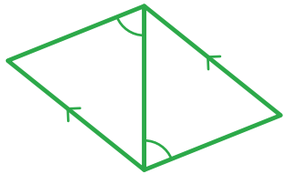
5. Circle one.

A. SSS

B. ASA

C. SAS

D. SAA



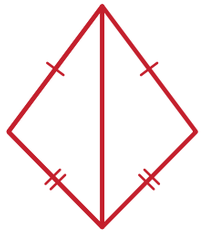
6. Circle one.

A. SSS

B. ASA

C. SAS

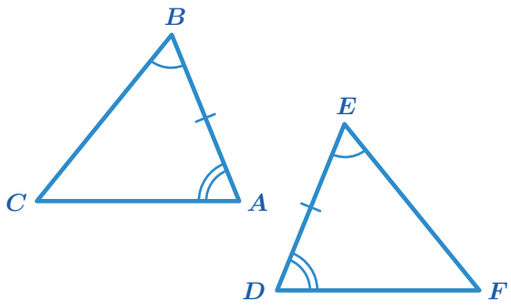
D. SAA



Lesson Practice
Geo.2.09

Name: _____ Date: _____ Period: _____

7. Fill in the blanks to prove $\triangle ABC \cong \triangle DEF$.



- I can translate $\triangle ABC$ so that point A goes to point _____.
- If I rotate this image around point D, I know that point B will align with point E because _____ $\cong \overline{DE}$.
- I can reflect this image over $\overline{DE}$.
- I know that $\angle A$ will align with $\angle$ _____ and $\angle B$ will align with $\angle$ _____ because angle measure is preserved with rigid transformations.
- After the reflection, I know that point C will stack onto point _____.
- After proving all of the corresponding vertices align, I know that _____ $\cong$ _____.

8. Here is a conjecture.

> Two triangles must be congruent if they have these pairs of corresponding congruent parts:
> - Two sides
> - One angle not between the two sides

When is this conjecture true? Circle one. Always Sometimes Never

Explain your thinking.

Spiral Review

9. Construct an equilateral triangle using $\overline{RS}$ as a side.



Reflection

1. Put a star next to a problem where you revised your thinking.

2. Use this space to ask a question or share something you're proud of.

Unit 2
Lesson
10

Name: ..... Date: ..... Period: .....

Resuming Construction

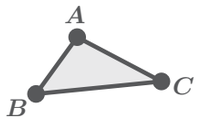


Entire Lesson

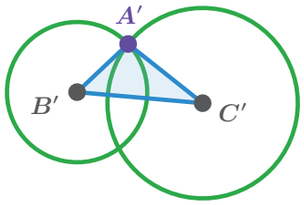
Things to Remember:

Select Screens

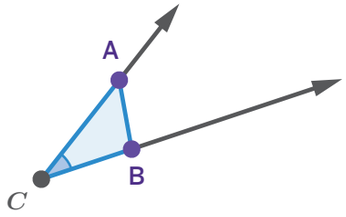
5 Anong says: *I can prove that I copied the triangle successfully using SSS congruence.*



Notes from class discussion:

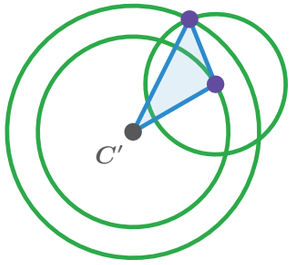


8 Here is Caleb's construction for copying $\angle C$ onto $\angle C'$.



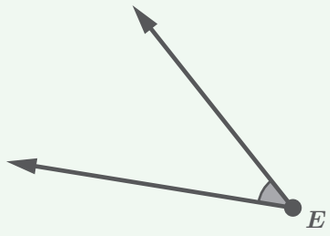
How can you be sure that $\angle C'$ is congruent to $\angle C$?

Make a connection to the Warm-Up in your response.



Lesson Summary

When constructing a copy of an angle, it can be helpful first to construct a copy of a triangle. For example, you can add a point on each ray of $\angle E$. If you connect these points, then you have constructed a triangle.

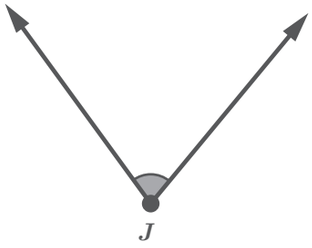


The "circle by length" tool can be used to copy the side lengths of this triangle. The intersection of the new circles represents the vertices of the copied triangle. The constructed triangle is congruent to the original because of the SSS congruence criteria.

By constructing a congruent triangle, you have also constructed a copy of $\angle E$ because corresponding parts of congruent triangles are always congruent.

Try This

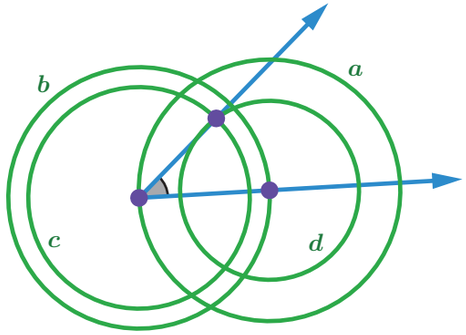
Describe how to construct a copy of $\angle J$ using a straightedge and a compass.



Lesson Practice
Geo.2.10

Name: _____ Date: _____ Period: _____

Problems 1–2: Here is an angle being copied.



1. Which circle is not necessary to copy the angle?

A. Circle a

B. Circle b

C. Circle c

D. Circle d

2. Which circle copies the segment opposite the angle?

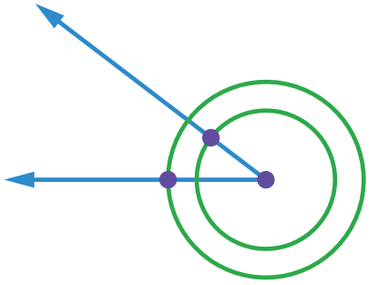
A. Circle a

B. Circle b

C. Circle c

D. Circle d

Problems 3–4: Here is part of the construction to copy an angle.



3. Construct a circle to complete the first half of the construction.

4. Which congruence criteria can be used to prove that the angle is copied perfectly?

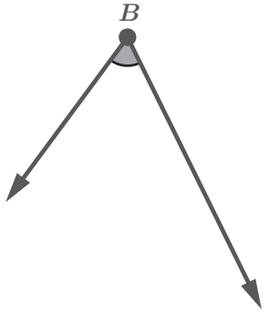
A. SAS

B. AAS

C. ASA

D. SSS

5. Copy $\angle B$ with the vertex at point B'.

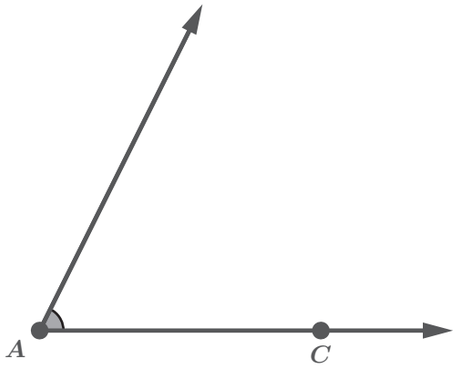


B'

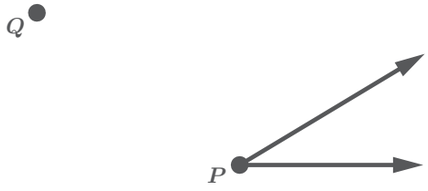
Lesson Practice
Geo.2.10

Name: _____ Date: _____ Period: _____

6. Copy $\angle A$ with the vertex at point C to construct an isosceles triangle ABC.

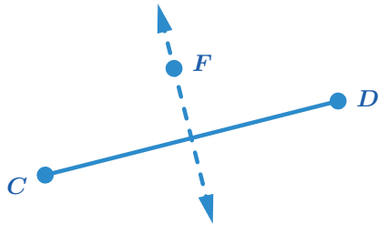


7. Construct an angle at point Q that is double the measure of $\angle P$.



Spiral Review

8. Point F is on the perpendicular bisector of $\overline{CD}$. Which statement must be true about point F?



A. It is closer to point C than to point D.

B. It is closer to point D than to point C.

C. It is equal distance to points C and D.

D. There is not enough information to know if it is closer to point C or point D.

Reflection

1. Put a star next to a problem you could explain to a classmate.

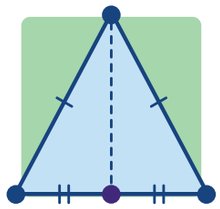
2. Use this space to ask a question or share something you're proud of.

Unit 2
Lesson
11

Name: ..... Date: ..... Period: .....

Proving Points

Let's use congruent triangles to prove the properties of perpendicular bisectors.



Warm-Up

Here is a conjecture. Let's make sense of it together

> If point C is the same distance from point A as it is from point B, then point C must be on the perpendicular bisector of $\overline{AB}$.

1. What is this conjecture about?

2. Draw a diagram to represent this relationship.

3. Do you believe this conjecture is true? Why or why not?

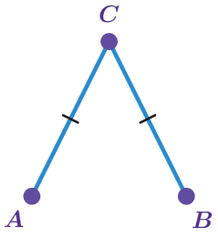
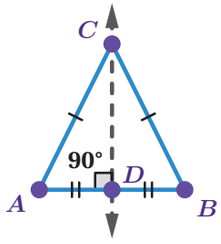
Activity 1

Name: ..... Date: ..... Period: .....

Finish It

If point C is the same distance from point A as it is from point B, then point C must be on the perpendicular bisector of $\overline{AB}$.

Here is Wayra's plan to prove the conjecture.

What do I know?	What do I want to prove?	What strategies can I use?
		I could construct $\overline{AB}$ and its midpoint D and then prove that $\overline{CD}$ must be perpendicular to $\overline{AB}$.

4. Discuss:

- What's in Wayra's second diagram that's not in the first diagram? Why?
- How can constructing $\overline{CD}$ help Wayra use congruent triangles to prove the conjecture?

5. Why would proving $\triangle ACD \cong \triangle BCD$ help show that point C must be on the perpendicular bisector of $\overline{AB}$?

6. Which triangle congruence criteria does Wayra have enough information to use? Circle one.

SSS SAS ASA AAS

List the congruent pairs for each part of the congruence criteria and the reason you know they are congruent.

Congruent Pair	Reason they are congruent
..... $\cong$	
..... $\cong$	
..... $\cong$	

Activity 1

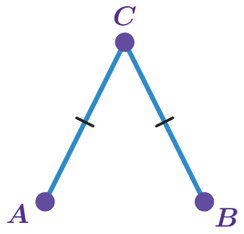
Name: Date: Period:

Finish It (continued)

You can make a proof more convincing by including a reason each statement must be true. A table can help organize reasons and statements.

7. Use a set of cards to finish the table and complete Wayra's proof.

If point C is the same distance from point A as it is from point B, then point C must be on the perpendicular bisector of $\overline{AB}$.



Statement	Reason that this must be true
$\overline{AC} \cong \overline{BC}$	This information is given in the conjecture.
$\overline{AD} \cong \overline{BD}$	Constructing point D at the midpoint of $\overline{AB}$ means that these segments must be congruent.
$\overline{CD} \cong \overline{CD}$	
$\triangle ACD \cong \triangle BCD$	
$\angle ADC \cong \angle BDC$	
$m\angle ADC + m\angle BDC = 180°$	
$m\angle ADC = 90°$ and $m\angle BDC = 90°$	
$\overline{CD}$ is a perpendicular bisector of $\overline{AB}$.	

8. What do you find convincing about Wayra's proof? What questions do you have?

Activity 2

Name: Date: Period:

Start It

Here is a similar (but different) conjecture:

> If point C is on the perpendicular bisector of $\overline{AB}$, then point C is the same distance from point A as it is from point B.

9. **Discuss:** How is this conjecture like the one from the previous activity? How is it different?

10. Make a plan like Wayra's to show how you might prove this conjecture.

What do I know?	What do I want to prove?	What strategies can I use?

11. Show your proof plan to a partner. Share feedback to strengthen your proofs.

Notes from Partner 1	Notes from Partner 2

Activity 2

Name: Date: Period:

Start It (continued)

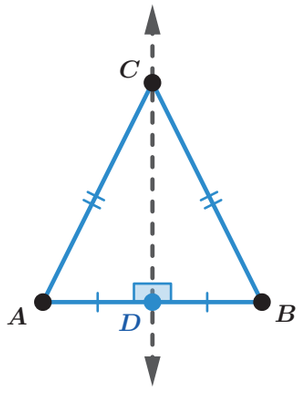
12. Prove this conjecture. Be sure to include a reason for each statement explaining why it must be true.

> If point C is on the perpendicular bisector of $\overline{AB}$, then point C is the same distance from point A as it is from point B.

Synthesis

By proving the two conjectures in this lesson, you proved this theorem:

Points on a perpendicular bisector of a line segment are exactly those *equidistant* from the segment's endpoints.

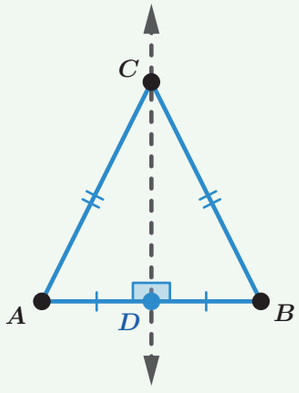


a How did proving the two conjectures work together to prove this theorem?

b What are some strategies for proving a conjecture?

Lesson Summary

The *perpendicular bisector* of a line segment is *exactly* those points that are the same distance from both endpoints of the line segment.



This **theorem** can be split into two conjectures:

1. If a point is on the perpendicular bisector of a segment, then it must be the same distance from both endpoints of the segment.
2. If a point is the same distance from both endpoints of a segment, then it must be on the perpendicular bisector of the segment.

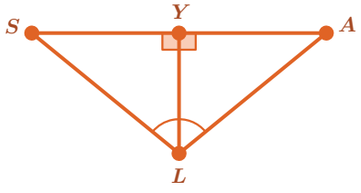
Convincing arguments include a reason explaining how you know that each statement must be true. A table can help you stay organized and make sure there is a reason for each statement.

One first step when writing a convincing argument is to decide if you believe the conjecture is true. This can help you focus on the information that must be true, and you can make a plan that only uses facts to prove the conjecture. Writing a first draft and getting feedback from a partner can help make your proof stronger and clearer.

Try This

Fill in the missing reasons to prove this conjecture:

If $\overline{YL}$ is the perpendicular bisector of $\overline{SA}$, then $\overline{LS} \cong \overline{LA}$.



	Statement	Reason
	$\overline{YL}$ is the perpendicular bisector of $\overline{SA}$.	Given in the conjecture.
	$\overline{SY} \cong \overline{AY}$, $m\angle SYL = 90°$, and $m\angle AYL = 90°$.	Definition of a perpendicular bisector.
a	$\angle SYL \cong \angle AYL$	
	$\overline{YL} \cong \overline{YL}$	The same segment in both triangles.
b	$\triangle YSL \cong \triangle YAL$	
c	$\overline{LS} \cong \overline{LA}$	

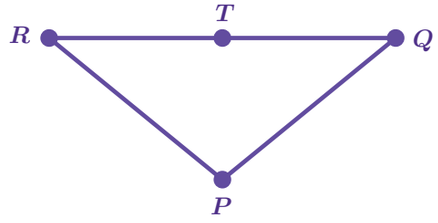
Lesson Practice
Geo.2.11

Name: _____ Date: _____ Period: _____

Problems 1–2: Mark the diagram with congruency markings to match the statement.

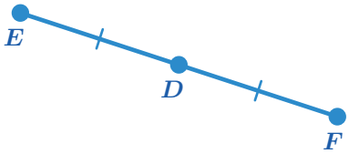
1. Point P is the same distance from points Q and R.

2. Point T is the midpoint of $\overline{QR}$.

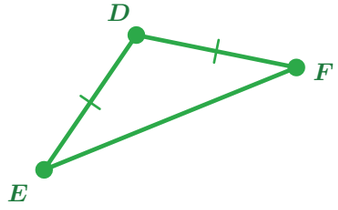


Problems 3–4: Does point D have to be on the perpendicular bisector of $\overline{EF}$? Circle one.

3. Yes No

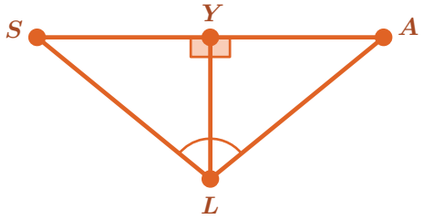


4. Yes No



Problems 5–7: Fill in the missing reasons to complete this proof.

If $\angle SYL$ and $\angle AYL$ are right angles and $\angle SLY \cong \angle ALY$, then $\overline{YL}$ must be the perpendicular bisector of $\overline{SA}$.



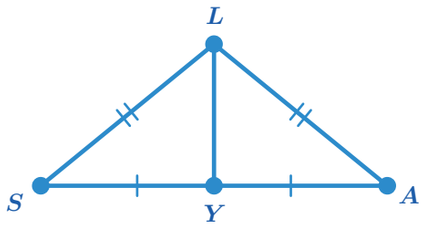
	Statement	Reason
	$\angle SYL$ and $\angle AYL$ are right angles and $\angle SLY \cong \angle ALY$.	Given in the conjecture.
5.	$\angle SYL \cong \angle AYL$	
	$\overline{YL} \cong \overline{YL}$	The same segment in both triangles.
6.	$\triangle YSL \cong \triangle YAL$	
7.	$\overline{SY} \cong \overline{AY}$	
	$\overline{YL}$ must be the perpendicular bisector of $\overline{SA}$.	$\overline{YL}$ is perpendicular to $\overline{SA}$, and since $\overline{SY} \cong \overline{AY}$ we know $\overline{YL}$ bisects $\overline{SA}$.

Lesson Practice
Geo.2.11

Name: _____ Date: _____ Period: _____

8. Prove this conjecture:

> If $\overline{SY} \cong \overline{AY}$ and $\overline{SL} \cong \overline{AL}$, then $\overline{YL}$ must be the perpendicular bisector of $\overline{SA}$.



Spiral Review

9. Which property must a square always have that a rhombus might not have?

A. Parallel sides
B. Four congruent sides
C. Two pairs of supplementary angles
D. All congruent angles

10. Which property must a rectangle always have that a parallelogram might not have?

A. Parallel sides
B. Four congruent sides
C. Two pairs of supplementary angles
D. All congruent angles

Reflection

1. Circle the problem you feel least confident about.

2. Use this space to ask a question or share something you're proud of.

Name: _____ Date: _____ Period: _____

Practice Day 1

Let's practice what you've learned so far in this unit!



You will use task cards for this Practice Day. Record all of your responses here.

Task A: Angle Properties

1. **a** _____

b _____

c _____

2. Explanation:

3. $m\angle a = 66°$ $m\angle b =$ _____ $m\angle c =$ _____ $m\angle d =$ _____

$m\angle e =$ _____ $m\angle f =$ _____ $m\angle g =$ _____ $m\angle h = 122°$

Task B: Triangle Congruence Criteria

1. SSS

ASA

SAS

Not congruent

2. SSS

ASA

SAS

Not congruent

3. SSS

ASA

SAS

Not congruent

4. SSS

ASA

SAS

Not congruent

5. Pair: _____

6. Pair: _____

7. Pair: _____

Name: ..... Date: ..... Period: .....

Practice Day 1 (continued)

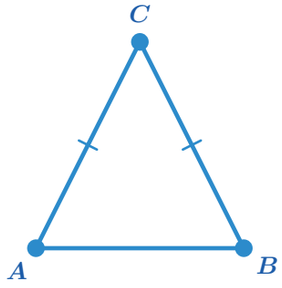
Task C: Build a Pair

- ☐ A. $\angle N \cong \angle Q$
- ☐ B. $\overline{WN} \cong \overline{WQ}$
- ☐ C. $\overline{NX} \cong \overline{WQ}$
- ☐ D. $\angle NWX \cong \angle QXW$
- ☐ E. $\overline{WX} \ncong \overline{QX}$
- ☐ F. $m\angle NXQ + m\angle QXW = 180°$
- ☐ G. $\overline{XN} \cong \overline{XQ}$
- ☐ H. $\overline{NW} \cong \overline{WX}$

Task D: Prove It

In an isosceles triangle, the base angles are congruent.

Response:

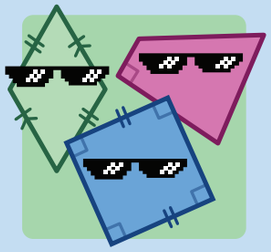


Student Edition pages

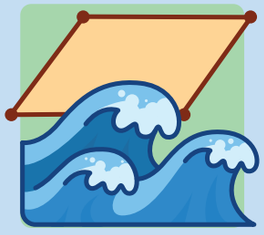
Sub-Unit

3

Proving Theorems About Quadrilaterals



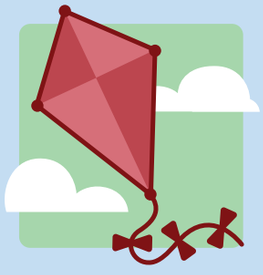
Lesson 12
The Quad Squad



Lesson 13
Go With the Flow



Lesson 14
A-Door-Able Parallelograms

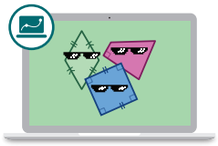


Lesson 15
Let's Go Build a Kite

Unit 2
Lesson
12

Name: ..... Date: ..... Period: .....

The Quad Squad



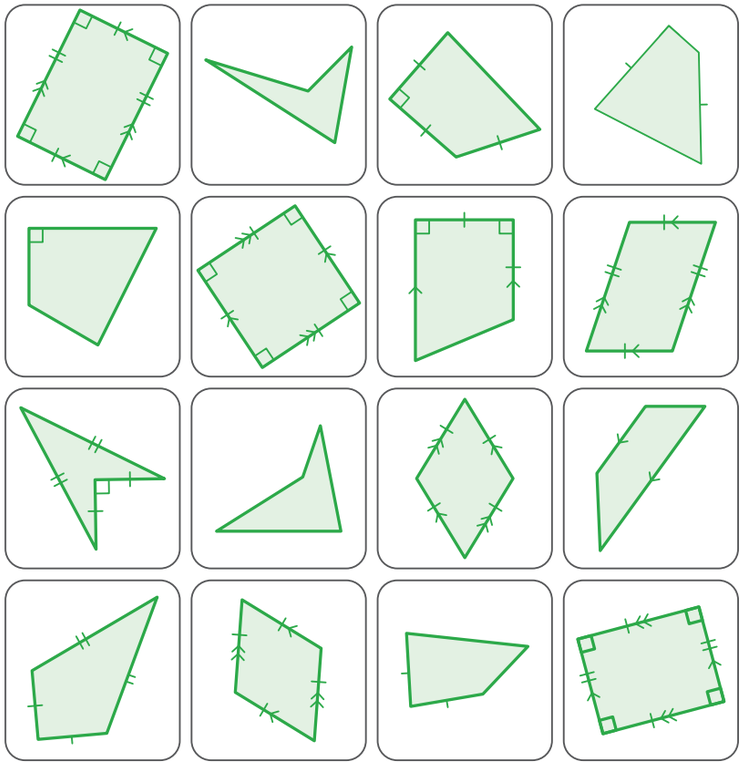
Entire Lesson

Things to Remember:

Select Screens

3 Here are five names of quadrilaterals.

- Kite
- Rectangle
- Parallelogram
- Rhombus
- Square



Discuss:

- What do you notice? What do you wonder?
- Are there any figures with more than one name? Why?

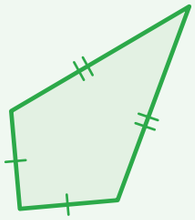
Notes from discussion:

Lesson Summary

Quadrilaterals can look very different from each other, and some quadrilaterals are defined by specific properties. Here are the names and definitions of five quadrilaterals.

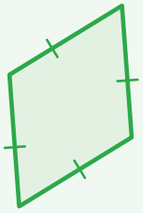
Kite

A quadrilateral with two pairs of adjacent congruent sides.



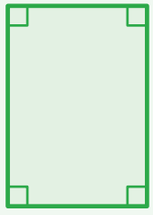
Rhombus

A quadrilateral with four congruent sides.



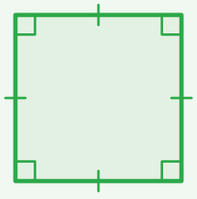
Rectangle

A quadrilateral with four congruent angles.



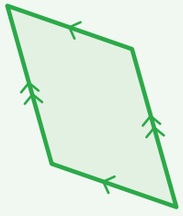
Square

A quadrilateral with four congruent angles and sides.



Parallelogram

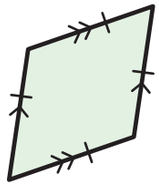
A quadrilateral with opposite sides that are parallel.



These defining properties can sometimes overlap. For example, a rhombus is also a kite because it has two pairs of adjacent sides. A square is both a rhombus and a rectangle because it has four congruent sides and four congruent angles. The definitions of quadrilaterals often imply that there are additional properties. For example, a rectangle has four congruent angles, which means that each of its angles must be a right angle.

Try This

Select *all* the names that apply to this quadrilateral.



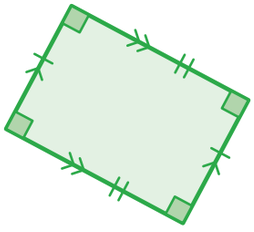
- ☐ **A.** Rectangle
- ☐ **B.** Square
- ☐ **C.** Parallelogram
- ☐ **D.** Kite
- ☐ **E.** Rhombus

Lesson Practice
Geo.2.12

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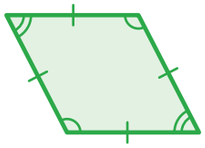
Problems 1–2: Select *all* the names that apply to the quadrilateral.

1.



- ☐ **A.** Rectangle
- ☐ **B.** Square
- ☐ **C.** Parallelogram
- ☐ **D.** Kite
- ☐ **E.** Rhombus

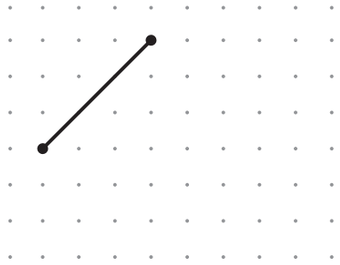
2.



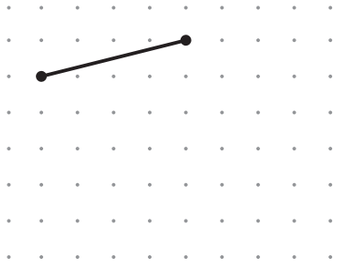
- ☐ **A.** Rectangle
- ☐ **B.** Square
- ☐ **C.** Parallelogram
- ☐ **D.** Kite
- ☐ **E.** Rhombus

Problems 3–5: Draw the remaining three sides to complete each quadrilateral.

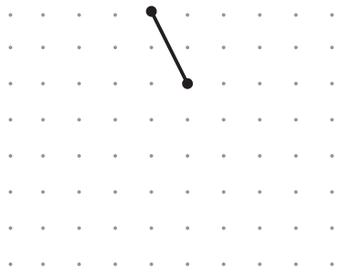
3. Rectangle



4. Parallelogram



5. Kite



6. Which of these quadrilaterals are possible to make? Select *all* that apply.

- ☐ **A.** A parallelogram that is also a rhombus
- ☐ **B.** A rectangle that is a kite
- ☐ **C.** A rhombus that is *not* a rectangle
- ☐ **D.** A square that is *not* a rhombus
- ☐ **E.** All of the above

7. Is it possible to construct a figure that can be described by *exactly* four of these quadrilateral names? Quadrilateral names: rectangle, square, parallelogram, kite, rhombus
Explain your thinking.

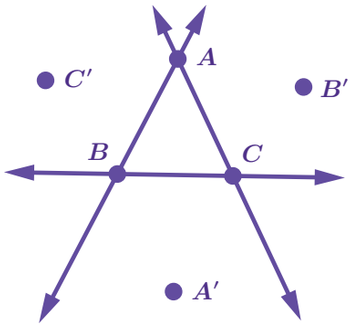
Lesson Practice
Geo.2.12

Name: _____ Date: _____ Period: _____

8. In this diagram:

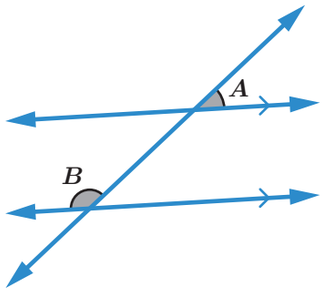
- $\overline{AB} \cong \overline{AC}$.
- Points A', B', and C', are all reflections over the opposite side of triangle ABC.

How many kites can you make using any of these points as vertices?



Spiral Review

Problems 9–10: Two parallel lines are crossed by a transversal.



9. Determine a possible pair of values for angle A and angle B.

Angle A: _____

Angle B: _____

10. Could angle A and angle B both be obtuse angles? Explain your thinking.

Reflection

1. Circle the problem you feel least confident about.

2. Use this space to ask a question or share something you're proud of.

Unit 2
Lesson
13

Name: ..... Date: ..... Period: .....

Go With the Flow



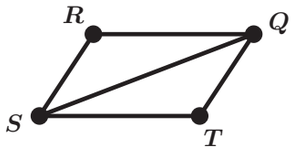
Entire Lesson

Things to Remember:

Select Screens

6 Here is the first draft of a proof.

If $QRST$ is a parallelogram,
then its opposite sides are congruent.



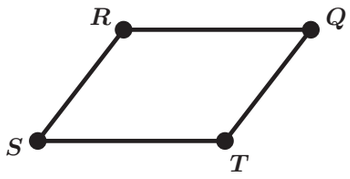
$QRST$ is a parallelogram. → $\triangle SRQ \cong \triangle QTS$ ASA → $\overline{RQ} \cong \overline{TS}$ and $\overline{RS} \cong \overline{TQ}$ Corresponding parts of congruent triangles.

Discuss: What do you think is clear about the proof? What could you add to make the proof stronger?

Notes from class discussion:

9 Opposite angles conjecture:

If $QRST$ is a parallelogram, then its opposite angles are congruent.



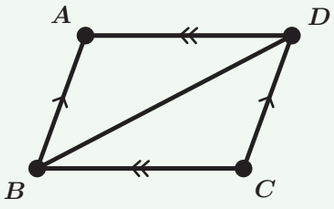
Use this space to plan your proof:

Lesson Summary

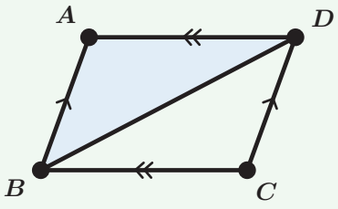
Constructing diagonals can be a helpful strategy for proving the properties of parallelograms.

When you construct a diagonal in a parallelogram, you create congruent triangles. You can use the triangles' corresponding pairs of congruent parts to prove the properties of parallelograms.

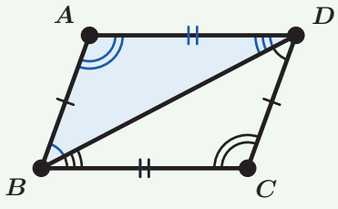
Here's one way you can use diagonals to prove that parallelograms have congruent opposite sides and congruent opposite angles.



Construct the diagonal $\overline{BD}$.



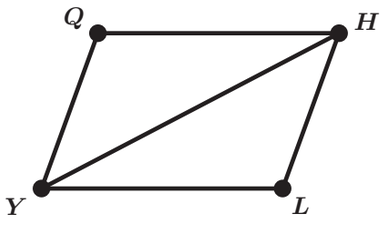
Prove $\triangle ABD \cong \triangle CDB$.



Look for corresponding congruent parts.

Try This

Here is parallelogram $YQHL$ with diagonal $\overline{YH}$.



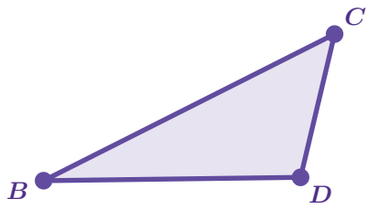
Select *all* the true statements.

- ☐ **A.** $\overline{QY} \cong \overline{LY}$
- ☐ **B.** $\angle QYH \cong \angle LHY$
- ☐ **C.** $\overline{QH} \cong \overline{LY}$
- ☐ **D.** $\angle YHL \cong \angle YHQ$
- ☐ **E.** $\overline{YQ} \cong \overline{HL}$
- ☐ **F.** $\angle YQH \cong \angle HLY$

Lesson Practice
Geo.2.13

Name: ..... Date: ..... Period: .....

Problems 1–2: Here is $\triangle BCD$.



1. If $\triangle BCD$ was rotated around the midpoint of $\overline{BC}$, it would create a:

A. Kite **B.** Rectangle

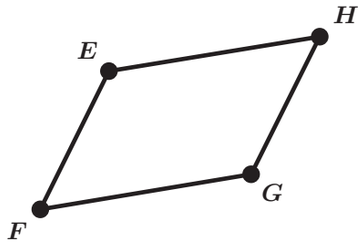
C. Parallelogram **D.** Rhombus

2. If $\triangle BCD$ was reflected over $\overline{BC}$, it would create a:

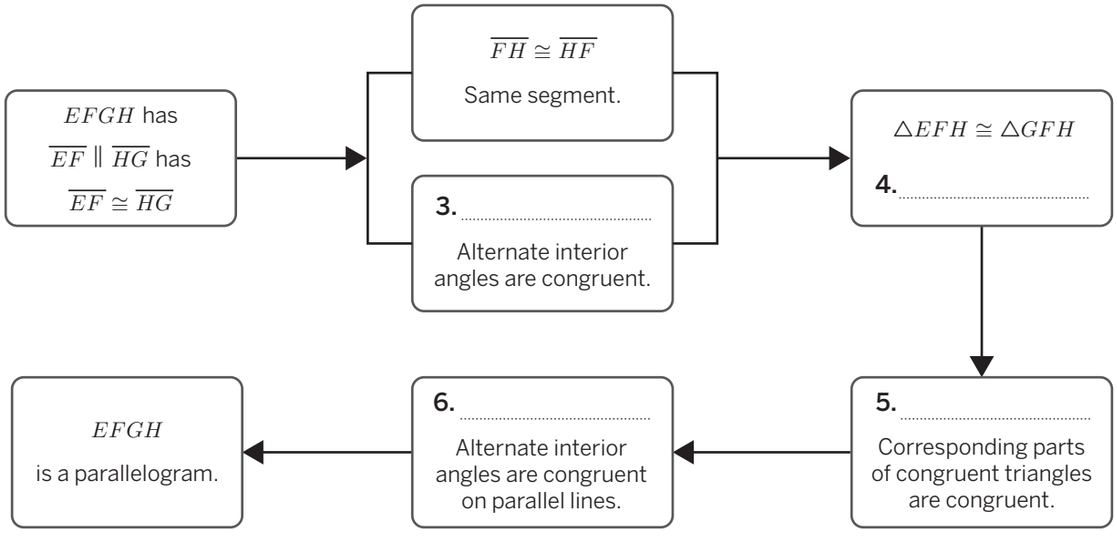
A. Kite **B.** Rectangle

C. Parallelogram **D.** Rhombus

Problems 3–6: Use quadrilateral $EFGH$ to fill in the missing information and prove the conjecture:



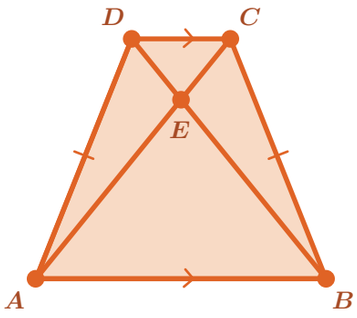
If $EFGH$ is a quadrilateral with $\overline{EF} \parallel \overline{GH}$ and $\overline{EF} \cong \overline{GH}$, then it is a parallelogram.



Lesson Practice
Geo.2.13

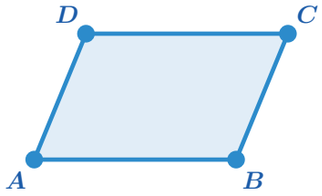
Name: _____ Date: _____ Period: _____

7. Select *all* the true statements based on quadrilateral $ABCD$.



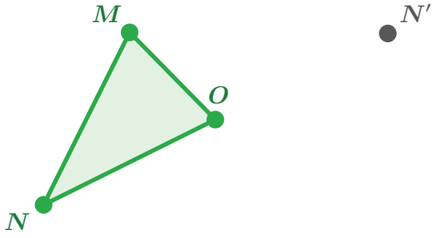
- ☐ **A.** Segment EB is congruent to segment AD.
- ☐ **B.** Segment DB is congruent to segment CA.
- ☐ **C.** Angle CBE is congruent to angle ABE.
- ☐ **D.** Angle CEB is congruent to angle DEA.
- ☐ **E.** Triangle AED is congruent to triangle BEC.

8. Prove that the adjacent angles in a parallelogram are supplementary. Use parallelogram $ABCD$ if it helps with your thinking.



Spiral Review

Problems 9–10: Here is triangle MNO and point N'.



9. Rotate triangle MNO around point O until point N aligns with point N'.

10. How are segments MN and $M'N'$ related to each other?

Reflection

1. Circle the problem you feel most confident about.

2. Use this space to ask a question or share something you're proud of.

Unit 2
Lesson
14

Name: ..... Date: ..... Period: .....

A-Door-Able Parallelograms



Let's prove some properties of parallelograms.

Warm-Up

1. Let's meet The Frame Fitter 5000!

This machine is fairly fast at finding out if a door is a rectangle.

Discuss:

- Which doors look like rectangles?
- Which doors do not look like rectangles?

Activity 1

Name: ..... Date: ..... Period: .....

Door to Door

You will use the activity sheet and tracing paper for this activity.

2. Identify if the angles and sides of the doors are congruent parts. Complete the table with the relationships that you notice. Use tracing paper and scissors to confirm your thinking.

	Door A	Door B	Door C	Door D
Angles		Two pairs of congruent angles		
Opposite Sides (First Pair)				
Opposite Sides (Second Pair)				
Diagonals			Not congruent	

3. Which of the doors are parallelograms? Explain your thinking.

4. Which of the doors are rectangles? Explain your thinking.

5. What are some properties you think all rectangles have in common?

Activity 2

Name: Date: Period:

Rectangle Diagonals

Here is a conjecture about rectangles:

> Rectangles have congruent diagonals.

6. Make a plan for how you might prove this conjecture.

What do I know?	What do I want to prove?	What strategies can I use?

7. Prove this conjecture using any structure you'd like. Be sure to include a reason for each statement explaining why it must be true.

8. Show your proof to at *least* one partner. Share feedback to strengthen your proofs.

Notes from Partner 1	Notes from Partner 2

Activity 3

Name: Date: Period:

Parallelogram Diagonals

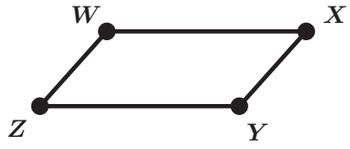
Here is a conjecture about parallelograms:

> Parallelograms have diagonals that bisect each other.

9. Take another look at the doors. Is this conjecture true for *all* four doors? Circle one.

Yes No I'm not sure

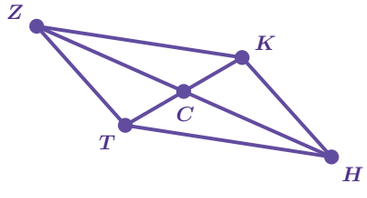
10. Make a plan for how you might prove this conjecture is always true.

What do I know?	What do I want to prove?	What strategies can I use?
		

11. Prove this conjecture using any structure you'd like. Be sure to include a reason for each statement explaining why it must be true.

Synthesis

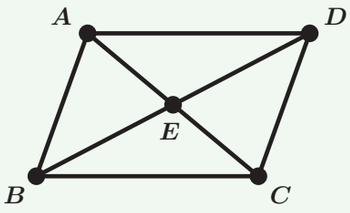
12. How can you use diagonals to help prove the properties of parallelograms?



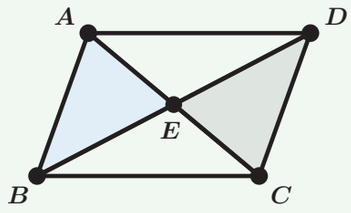
Lesson Summary

When you construct one or two diagonals in a parallelogram, they make congruent triangles. Since congruent triangles have corresponding pairs of congruent parts, this step helps prove the properties of parallelograms, such as opposite sides and angles being congruent.

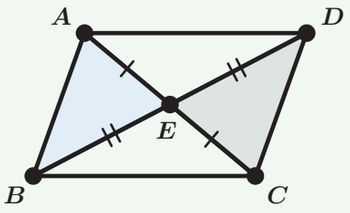
Here is one example of how constructing diagonals can be a helpful strategy for proving this property of parallelograms: *The diagonals of a parallelogram bisect each other.*



Construct the diagonals $\overline{AC}$ and $\overline{BD}$. Label the intersection point E.



Prove $\triangle AEB \cong \triangle CED$.



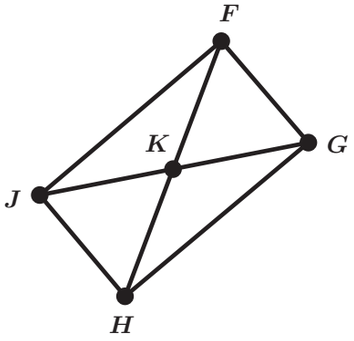
Look for corresponding congruent parts. $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$, so point E bisects both $\overline{AC}$ and $\overline{BD}$.

Try This

Figure $FGHJ$ is a rectangle with diagonals $\overline{JG}$ and $\overline{FH}$.

Select *all* the true statements.

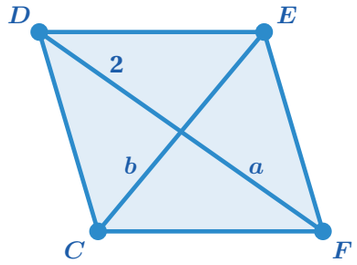
- ☐ **A.** $\overline{JK} \cong \overline{FG}$
- ☐ **B.** $\overline{JK} \cong \overline{GK}$
- ☐ **C.** $\overline{JG} \cong \overline{FH}$
- ☐ **D.** $\overline{FH} \cong \overline{JF}$
- ☐ **E.** $\overline{JH} \cong \overline{FG}$
- ☐ **F.** $\overline{GH} \cong \overline{JH}$



Lesson Practice
Geo.2.14

Name: _____ Date: _____ Period: _____

Problems 1–2: Here is quadrilateral $CDEF$.



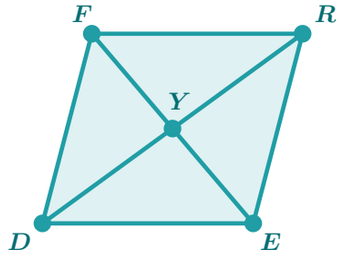
1. What is the value of a if figure $CDEF$ is a parallelogram?

$a =$ _____

2. What is the value of b if figure $CDEF$ is a rectangle?

$b =$ _____

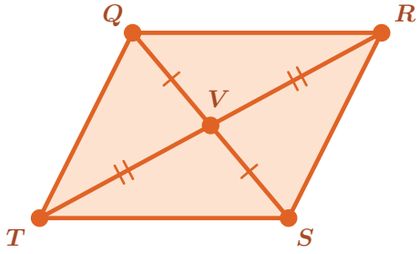
Problems 3–4: Name two different pairs of congruent triangles in parallelogram $FRED$.



3. _____ $\cong$ _____

4. _____ $\cong$ _____

Problems 5–7: Here is quadrilateral $QRST$.



5. Which triangle congruence criteria can be used to prove $\triangle SVT \cong \triangle QVR$?

A. SSS

B. SAS

C. ASA

D. SAA

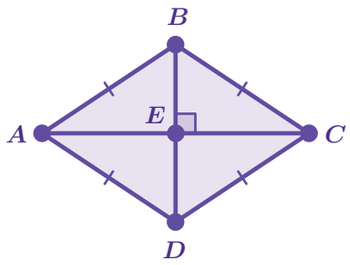
6. If $\triangle SVT \cong \triangle QVR$, mark a pair of angles that show $\overline{QR} \parallel \overline{ST}$.

7. Prove that $\overline{QT} \parallel \overline{SR}$.

Lesson Practice
Geo.2.14

Name: _____ Date: _____ Period: _____

8. Prove that the diagonals of a rhombus bisect each other. Use the rhombus $ABCD$ if it helps with your thinking.

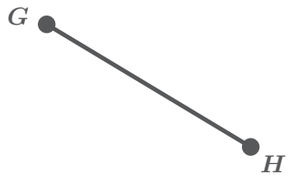


Spiral Review

Problems 9–10: Here is $\overline{GH}$.

9. Plot a point that is equidistant to points G and H.

10. Construct an isosceles triangle that is *not* equilateral. Use $\overline{GH}$ as the base of your triangle.



Reflection

1. Circle the problem you feel most confident about.

2. Use this space to ask a question or share something you're proud of.

Unit 2
Lesson
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Name: ..... Date: ..... Period: .....

Let's Go Build a Kite



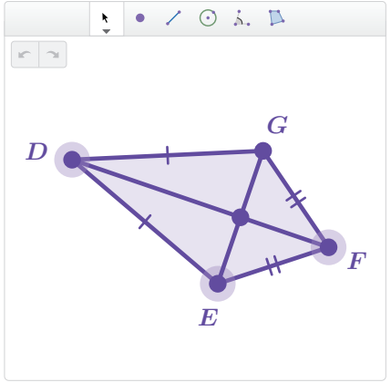
Entire Lesson

Things to Remember:

Select Screens

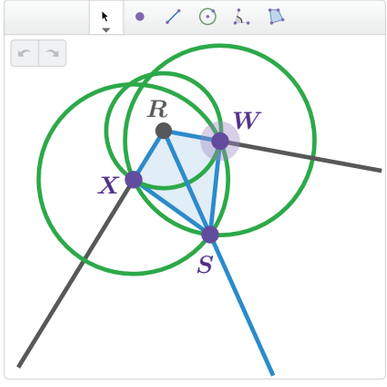
5 An **angle bisector** is a line, segment, or ray that divides an angle into two congruent angles.

What strategies can you use to prove something is an angle bisector?



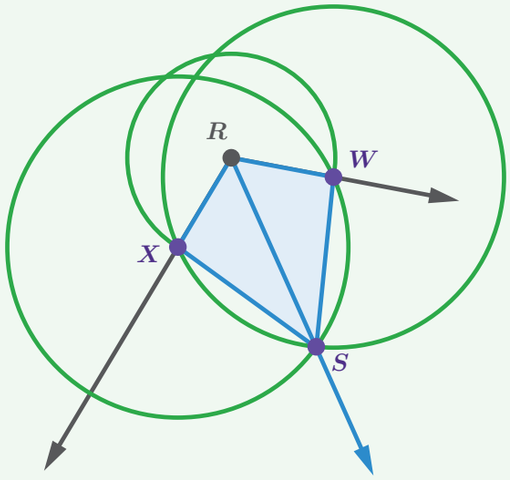
9 Let's take a closer look at Malik's strategy for constructing an angle bisector.

Explain how you know that $\angle WRS$ is congruent to $\angle XRS$.



Lesson Summary

An **angle bisector** is a line, segment, or ray that divides an angle into two adjacent congruent angles.

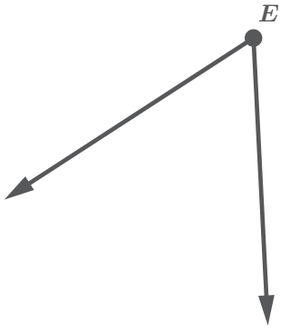


You can make sense of the construction of an angle bisector by using figures that form angle bisectors. For example, the diagonal that connects the vertices between two congruent sides of a kite will also bisect two of the kite's angles. If you can construct a kite, then you can use your understanding of a kite's properties to construct an angle bisector.

Other quadrilaterals, such as squares and rhombuses, also have diagonals that are angle bisectors. You can use figures that you already know how to construct to guide your thinking about figures that you haven't constructed yet.

Try This

Construct the angle bisector of $\angle E$.

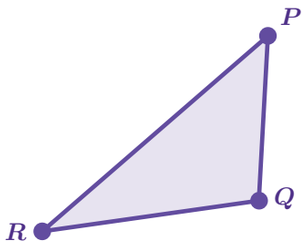


Lesson Practice
Geo.2.15

Name: ..... Date: ..... Period: .....

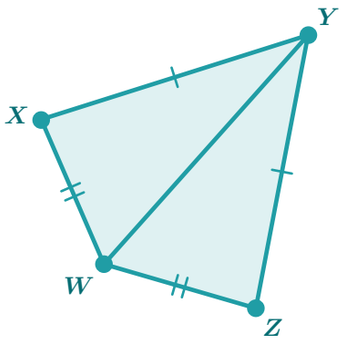
Problems 1–2: Here is $\triangle PQR$.

1. Reflect $\triangle PQR$ over $\overline{QR}$. Explain why the resulting quadrilateral is a kite.

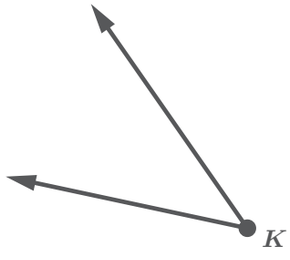


2. If you reflected over a different side of $\triangle PQR$, would you still form a kite? Explain your thinking.

3. Kiandra claims that the diagonal $\overline{WY}$ bisects angles ZWX and ZYX. Is her claim correct? Explain your thinking.



4. Construct the angle bisector of $\angle K$.



5. Is it possible to construct an angle bisector of an angle that is larger than $180°$? Explain your thinking.

Lesson Practice
Geo.2.15

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6. Construct a triangle where the angle bisectors are the same lines as the perpendicular bisectors. Use $\overline{FG}$ as one side of the triangle.

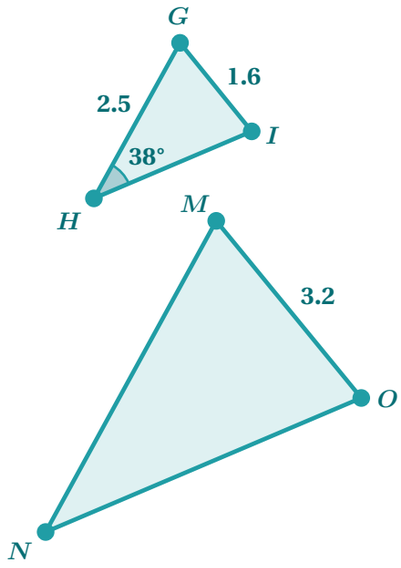


Spiral Review

Problems 7–8: Triangles GHI and MNO are similar triangles.

7. What is the length of $\overline{MN}$?

8. What is the measure of angle N?



Reflection

1. Circle a problem you want to talk to a classmate about.

2. Use this space to ask a question or share something you're proud of.

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Practice Day 2



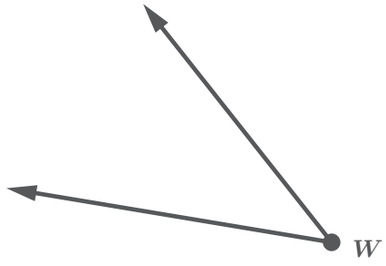
Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Construct It!

1.

2.



3. Explanation:

4. Explanation:

Task B: Match It!

1.

Side-Angle-Side (SAS)	Angle-Side-Angle (ASA)	Side-Side-Side (SSS)

2. **a** Figure _____ **b** Figure _____

c Figure _____ **d** Figure _____

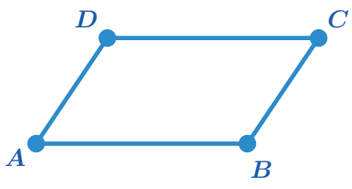
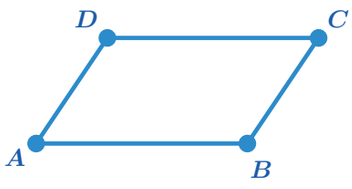
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Practice Day 2 (continued)

Task C: Plan It!

What do I know?	What do I want to prove?	What strategies could I use?

Task D: Prove It!

	What do I know?	What do I want to prove?	What strategies could I use?
1.			

2.

Notes



Notes



Notes



Amplify Desmos Math is a curiosity-driven program that builds lifelong math proficiency.

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