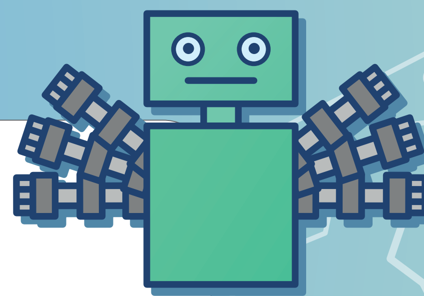
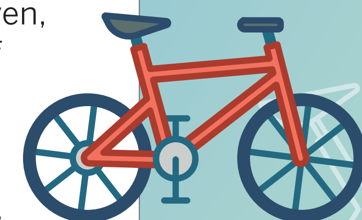


Unit 3

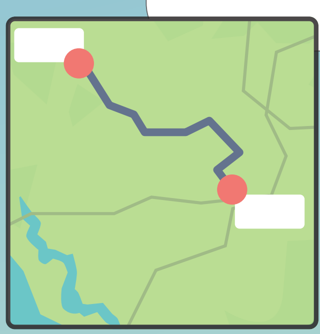
Unit Rates and Percentages

In Unit 2, you discovered how to use ratios to compare quantities and determine unknown amounts. In this unit, you'll explore unit rates and percentages. They can help you answer questions like: *How much soft serve can I buy? How fast can a model train travel? How does someone make money selling t-shirts?* And even, *Which country has the greatest percentage of young people?*



Essential Questions

- How are the terms *same rate*, *constant rate*, and *unit rate* alike and different?
- What is the relationship between unit rates and percentages?
- How are percentages used to estimate and compare quantities?



Summary | Lesson 1

Units of measurement can be used to describe things like length, volume, and weight or mass. Certain units of measurement might be more appropriate to use than others, depending on what you're measuring. Here are some examples of units of measurement, arranged from the *smallest* unit to the *largest* unit.

Length	Volume	Weight
millimeter	milliliter	gram
centimeter	fluid ounce	ounce
inch	cup	pound
foot	quart	kilogram
yard	liter	ton
meter	gallon	
kilometer		
mile		

Try This

What unit would you use for each measurement? Explain your thinking.

a. The height of a school building.

b. The amount of water in a swimming pool.

Summary | Lesson 2

When you're measuring the same quantity with different units, you need more of the smaller unit and fewer of the larger unit to describe the measurement. So a room that's 3 meters long will measure approximately 10 feet in length because a meter is longer than a foot.

The size of the object can help to determine the best unit of measurement.

For example, the length of the bottom edge of this notebook is 22 centimeters or 220 millimeters.

- It takes more millimeters to describe the length because millimeters are smaller than centimeters.
- You may choose to describe the length in centimeters instead of millimeters because of the size of the notebook.

Try This

Aditi has a bookshelf with 11-inch-tall shelves. She has a book that is 29 centimeters tall.

1 inch = 2.54 centimeters

a Would it take more inches or more centimeters to measure the heights of the book and the shelf? Explain your thinking.

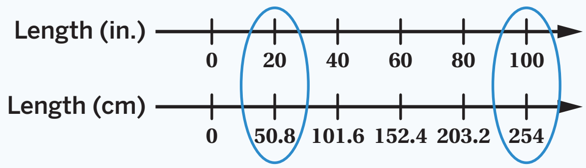
b Is there enough space for the book to stand up on the shelf? Explain your thinking.

Summary | Lesson 3

You can use equivalent ratios to convert measurements from one unit to another.

For example, if you know that 100 inches $= 254$ centimeters, you can use a double number line or a table to convert 20 inches to centimeters, too.

Double Number Line



Ratio Table

	Inches	Centimeters	
	100	254	
$\div 100$	1	2.54	$\div 100$
$\times 20$	20	50.8	$\times 20$

Try This

A macaw eats 6 kilograms of food per month.

About how many pounds of food is that?

10 kilograms $\approx$ 22 pounds

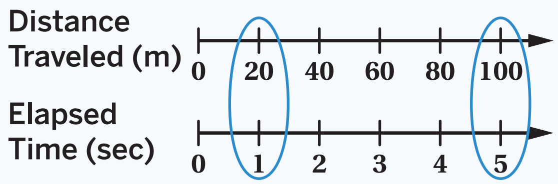
Summary | Lesson 4

A **rate** is a number that compares two quantities with different units. For example, 16 meters per 2 seconds is a rate that compares distance and time.

A **unit rate** is a rate where one of the quantities is 1.

Let's say there's a train traveling 100 meters in 5 seconds. You can use a table of equivalent ratios or a double number line to calculate the unit rate, which is 20 meters per 1 second.

Distance (m)	Time (sec)
100	5
20	1



Now you can use the unit rate to answer other questions about the train. For example, to determine how far the train travels in 30 seconds, you can just multiply the unit rate of 20 meters per second by 30 to get 600. That means the train travels 600 meters in 30 seconds.

Try This

At a track meet, Natalia ran the 200-meter dash in 25 seconds.

a. What was Natalia's speed as a unit rate?

b. How far could Natalia run in 30 seconds if she ran at the same speed?

Summary | Lesson 5

When you're comparing different rates, like speeds, it's helpful to convert the rates to the same units of measurement. Then you can use equivalent ratios or unit rates to more accurately compare the rates.

Let's compare the speeds of two runners competing in different races.

- Runner A runs the 400-meter dash in 50 seconds.
- Runner B runs a 5-kilometer race in 20 minutes.

We can convert both of these speeds to meters per second.

Runner A

Seconds	Meters
50	400
1	8

8 meters per second

Runner B

5 kilometers = 5000 meters

20 minutes = 1200 seconds

Seconds	Meters
1,200	5,000
1	$4\frac{1}{6}$

$4\frac{1}{6}$ meters per second

Runner A runs at a faster rate because they ran a greater distance (8 meters) than Runner B ($4\frac{1}{6}$ meters) in the same amount of time (1 second).

Try This

1 pound = 16 ounces

Kimaya needs a pound of lentils for a recipe.

The store sells a 1-pound bag of lentils for $2.49. They also sell lentils in the bulk section at a price of $0.26 for 2 ounces.

Which is a better deal? Explain your thinking.

Summary | Lesson 6

When two quantities are related in a ratio, you can describe the relationship using two different unit rates.

For example, the ratio $A : B$ can be represented as:

- The amount of Quantity A per 1 of Quantity B.
- The amount of Quantity B per 1 of Quantity A.

In situations that involve money, one of the two possible unit rates is the **unit price** (the price per unit of an item).

Let's say a store advertises 4 pounds of granola for $\$5$.

You can use a table to determine the two different unit rates.

- Price per 1 pound: $\$1.25$ per pound of granola. This is the unit price.
- Number of pounds per $\$1$: 0.8 pounds of granola per dollar.

Granola (lb)	Price ($)
4	5.00
1	1.25
0.8	1.00

Try This

At a certain store, it costs $\$4$ to buy 5 pounds of potatoes.

a) How much would it cost to buy 1 pound of potatoes at this store?

b) How many pounds of potatoes could a customer buy with $\$1$?

Summary | Lesson 7

Unit rates can help you determine missing values in a table.

For example, let's say 4 pounds of apples cost $\$10$.

That means the cost of 1 pound of apples is $\$2.50$. So you can calculate the cost of any amount of apples by multiplying the weight by 2.5.

That also means that for $\$1$, you can buy 0.4 pounds of apples. So you can calculate the number of pounds of apples you can buy for any amount of money by multiplying the amount of money by 0.4.

$\div 0.4$
$\cdot 2.50$

Pounds	Dollars
4	10
2	5
1	2.5
0.4	1

$\div 2.5$
$\cdot 0.4$

Try This

A factory can make 12 cereal boxes in 3 minutes.

a Complete the table for different numbers of cereal boxes.

b Choose *one* row from the table and explain how you determined the unknown value.

Cereal Boxes	Time (min)
12	3
1	
40	
	8

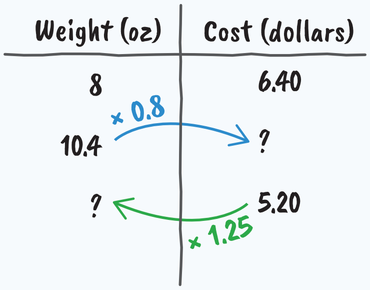
Summary | Lesson 8

You can describe the relationship between the same two quantities using two different unit rates.

Let's say a shop charges $\$6.40$ for 8 ounces of soft serve. The unit rates in this situation are:

- Dollars per ounce: $\$0.80$ per ounce $(6.40 \div 8 = 0.80)$
- Ounces per dollar: 1.25 ounces per dollar $(8 \div 6.40 = 1.25)$

The unit rate you need for your calculations depends on what information you're given and what you want to determine.



So if you're given the number of ounces and want to determine the cost, you'll need to multiply by the dollars per ounce.

But if you're given the cost and want to determine the number of ounces you can get, you'll need to multiply by the ounces per dollar instead.

It's often helpful to determine *both* unit rates, so you can answer as many kinds of questions about the situation as possible!

Try This

A parking meter says that the price is $\$3$ for 60 minutes of parking.

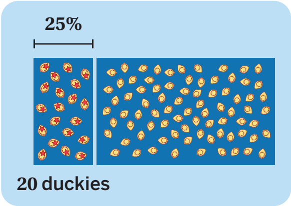
a What are the two unit rates in this situation?

b Nathan needs to park for 45 minutes. How much will this cost?

c Amir has $\$1.35$ for parking. How many minutes of parking can he pay for?

Percent means for every 100. It's represented by the percent symbol, $\%$.

Each of the different Ducky Games in this lesson had a certain percentage of ducks with stars: 10%, 25%, 50%, or 75%. Fractions and tape diagrams can help us interpret these percentage problems.

<table>
<tr><th>Example Problem</th><th>Using Fractions</th><th>Using Tape Diagrams</th></tr>
<tr>
<td>25% of the 80 duckies have stars.

</td>
<td>25% of something means $\frac{25}{100}$ or $\frac{1}{4}$.
$\frac{1}{4}$ of 80 duckies is 20 duckies.
80 total duckies

</td>
<td>There are four 25s in 100, so the tape diagram can be split into 4 pieces. The total number of duckies can also be split into 4 parts, so there are 20 duckies in each section.
80 total duckies
| 20 | 20 | 20 | 20 |</td>
</tr>
</table>

Try This

Rudra bought a box with 40 bags of chips. The chips come in different flavors.

a The box says that 25% of the bags of chips have cheese. How many bags of cheesy chips are there?

b The box says that 10% of the chips are spicy. How many bags of spicy chips are there?

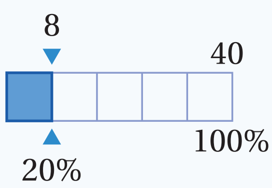
Summary | Lesson 10

You can represent percentages using tape diagrams, double number lines, and tables. The strategies you've already used to solve ratio problems can help you think about and solve percentage problems, too!

Let's say a biker traveled 8 kilometers, which is 20% of their goal distance. What's their goal distance?

Here are three ways to represent and solve this percentage problem.

Tape Diagram

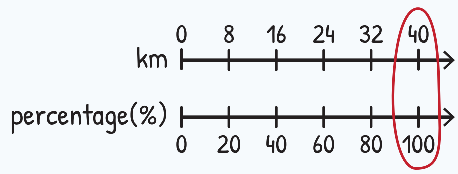


Table

km	%
8	20
40	100

×5 ×5

Double Number Line

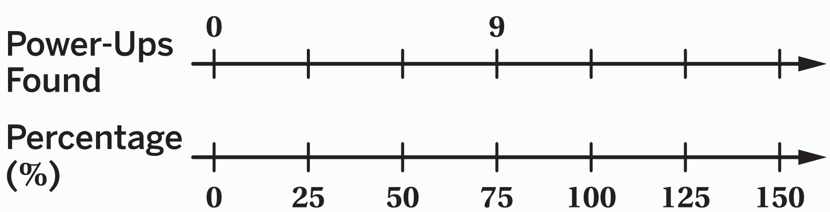


So the biker's goal distance is 40 kilometers.

Try This

Sneha is playing a video game. She's found 9 hidden power-ups, which is 75% of all the power-ups in the game. How many total power-ups are in the game?

Use the double number line if it helps with your thinking.



Summary | Lesson 11

You can use tables, tape diagrams, and double number lines to solve percentage problems.

There are three main types of percentage problems.

- Determine the whole when you're given the part and the percentage.
- Determine the percentage when you're given the part and the whole.
- Determine the part when you're given the percentage and the whole.

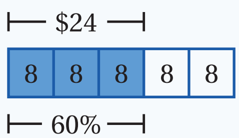
Let's say the sale price of a sweater is $24. The sweater is on sale for 60% of the original price. So how much did the sweater cost before the sale?

Here are three representations that you can use to find the whole (the original price of the sweater) given the part and the percentage.

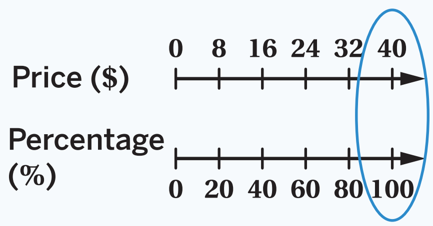
Table

	Price ($)	Percentage	
	24	60%	
÷3			÷3
	8	20%	
×5			×5
	40	100%	

Tape Diagram



Double Number Line



Try This

For each question, use a tape diagram, double number line, or table to determine the unknown value.

a Pilar has read 144 out of the 200 pages in a book. What percent of the book has she read?

b Tay is reading a book with 300 pages. Tay's e-reader says Tay is 80% finished. How many pages has Tay read?

c Axel has read 60 pages of his book. His e-reader says he is 40% finished. How many pages are in the book?

Summary | Lesson 12

When solving percentage problems related to money, you can:

- Determine the value of 1% and multiply that by the percentage you're looking for.
- Determine how many cents per dollar a given percentage represents.

Let's say a pair of pants costs $\$42$. If the factory makes a *profit* of 14% on the price of a pair of pants, how many dollars of profit does the factory make from each sale?

Strategy 1

Cost (dollars)	Percentage (%)
42	100
$\frac{42}{100}$	1
$\frac{42}{100} \cdot 14$	14

$$\frac{42}{100} \cdot 14 = 5.88$$

The factory makes $\$5.88$ of profit from each sale.

Strategy 2

- 14% profit means 14 cents of each dollar is profit.

  $\frac{14}{100} = 0.14$
- The price of the pants is $\$42$.
- $\frac{14}{100} \cdot 42 = 5.88$

The factory makes $\$5.88$ of profit from each sale.

Try This

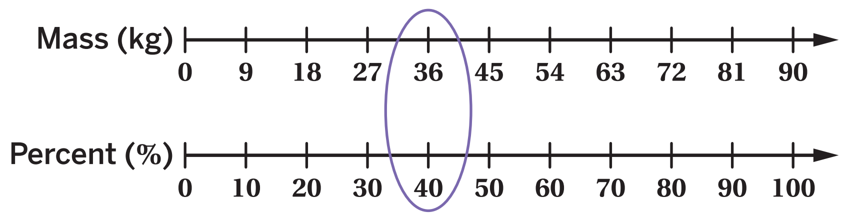
A pair of shoes costs $\$60$. The store's profit is 22% of the cost.

How much profit does the store make on each pair of shoes?

Show or explain your thinking.

You can use ratios to determine what percent one amount is compared to another amount.

Let's say an adult giant panda weighs 90 kilograms and a giant panda cub weighs 36 kilograms. You can determine the cub's weight as a percent of the adult's weight using several strategies.

<table>
<tr><td>Double Number Line</td><td colspan="2">
</td></tr>
<tr><td>Ratio Tables</td><td>
<table>
<tr><th></th><th>Mass (kg)</th><th>Percent (%)</th><th></th></tr>
<tr><td></td><td>90</td><td>100</td><td></td></tr>
<tr><td>$\times \frac{1}{90}$</td><td>1</td><td>$\frac{1}{90} \times 100$</td><td>$\times \frac{1}{90}$</td></tr>
<tr><td>$\times 36$</td><td>36</td><td>$\frac{36}{90} \times 100$</td><td>$\times 36$</td></tr>
</table>
</td><td>

- Determine the unit rate (what percent matches 1 kilogram).
- Use the unit rate to determine what percent 36 kilograms is of 90 kilograms.

</td></tr>
<tr><td>Expressions</td><td>$36 \div 90 \cdot 100 = \frac{36}{90} \cdot 100 = 40$</td><td>Evaluate $\frac{p}{w} \cdot 100$ to determine what percent one value, p, is of the other value, w.</td></tr>
</table>

Try This

Taylor set a goal of running 8 laps around a track. Taylor actually ran 12 laps.

What percent of the goal did Taylor run?

Create a table or double number line if it helps with your thinking.

Summary | Lesson 14

Working with real-world data and information can be interesting, but it presents challenges, like working with very large numbers or information presented in different forms.

Ratios, rates, and percentages can help you make sense of real-world situations and compare very large numbers.

Some of the benefits of ratios, rates, and percentages are:

- They allow you to compare quantities that are on different scales because they describe things in terms of multiplying and dividing instead of adding and subtracting.
- They bring everything to the same scale, most commonly with a reference point of either 1 or 100, which makes comparing numbers more straightforward.

For example, percentages can help us compare different-sized groups of people around the world to see what the distribution of people really looks like.

Try This

Here are some facts about the Philippines in 2024.

- The population is about 118 million people.
- 53% of people have access to the internet.
- 30 out of 100 people are under 15 years old.

a How many people in the Philippines have access to the internet?

b How many people in the Philippines are under 15 years old?

Lesson 1

a *Responses vary*. You could use any unit of length, but it might make sense to use feet, yards, or meters so you get a number that's easy to work with. You would need a very large number of millimeters or centimeters and a very small number of kilometers or miles to measure a building's height.

b *Responses vary*. You could use any unit of volume, but it makes sense to choose a larger unit like gallons or liters because swimming pools hold a lot of water.

Lesson 2

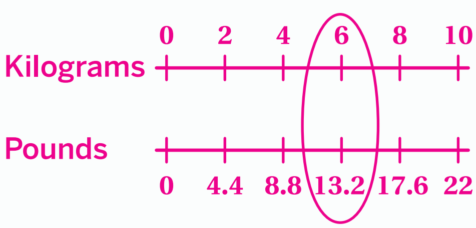
a Centimeters. *Explanations vary*. Centimeters are smaller than inches, so you need more of them to measure the same length.

b No. *Explanations vary*. Since 1 inch is equal to 2.54 centimeters, 11 inches is equal to $11 \cdot 2.54 = 27.94$ centimeters. The book is 29 centimeters tall, so it won't fit.

Lesson 3

About 13.2 pounds. [Here are two possible strategies for calculating this answer:

Strategy 1



Strategy 2

Kilograms	Pounds
10	22
1	2.2
6	13.2

6 kilograms is about 13.2 pounds]

Lesson 4

a 8 meters per second. [This is a unit rate because it tells how many meters Natalia can run in 1 second.]

b 240 meters. [One strategy is to multiply Natalia's speed (8 meters per second) by the amount of time she runs (30 seconds).]

Lesson 5

$0.26 for 2 ounces is a better deal. [One strategy is to multiply $0.26 by 8 to determine the price of buying 1 pound of bulk lentils. $0.26 \cdot 8 = \$2.08$, so the price of buying the bulk lentils is lower than the price of buying a 1-pound bag ($2.49).]

Lesson 6

a. $0.80

b. 1.25 pounds

Lesson 7

a.

Cereal Boxes	Time (min)
12	3
1	$\frac{1}{4}$ (or equivalent)
40	10
32	8

b. *Responses vary.* I figured out that it would take 10 minutes to make 40 boxes by multiplying the time for 1 box by 40. $40 \cdot \frac{1}{4} = 10$.

Lesson 8

a. 20 minutes per dollar and $0.05 per minute

b. $2.25. [One strategy is to multiply the unit rate $0.05 per minute by 45 minutes. $0.5 \cdot 45 = 2.25$.]

c. 27 minutes. [One strategy is to multiply the unit rate 20 minutes per dollar by $1.35. $20 \cdot 1.35 = 27$.]

Lesson 9

a. 10 bags have cheese.

b. There are 4 bags of spicy chips.

Lesson 10

12 power-ups. Here is an example of a completed double number line.

Power-Ups Found	0	3	6	9	12	15	18
Percentage (%)	0	25	50	75	100	125	150

Lesson 11

a 72%. Here is one strategy:

	Pages	Percentage (%)	
	200	100	
÷ 100	2	1	÷ 100
• 72	144	72	• 72

b 240 pages. Here is one strategy:

	Pages	Percentage (%)	
	300	100	
÷ 10	30	10	÷ 10
• 8	240	80	• 8

c 150 pages. Here is one strategy:

Pages	0	30	60	90	120	150
Percentage (%)	0	20	40	60	80	100

Lesson 12

$13.20. Here are two possible strategies:

Strategy 1

	Cost (dollars)	Percentage (%)	
	60	100	
÷ 100	$\frac{60}{100}$	1	÷ 100
• 22	13.20	22	• 22

The store makes $13.20 of profit on each pair of shoes.

Strategy 2

- 22% means that $\frac{22}{100}$ of each dollar is profit.
- The price of the shoes is $60.
- $\frac{22}{100} \cdot 60 = 13.20$
- The store makes $13.20 of profit on each pair of shoes.

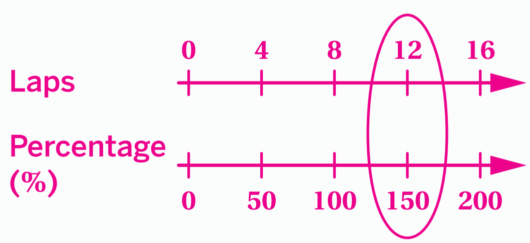
Lesson 13

150%. Here are two strategies:

Strategy 1

	Laps	Percentage (%)	
	8	100	
÷ 2	4	50	÷ 2
• 3	12	150	• 3

Strategy 2



Lesson 14

a. 62.54 million people

b. 35.4 million people

Grade 6 Unit 3 Glossary/6.º grado Unidad 3 Glosario

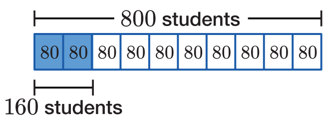
English

P

per The word *per* means "for each."

For example, if the price is $5 per ticket, that means that each ticket costs $5. Buying 4 tickets would cost $5 \cdot 4 = \$20$.

percent (percentage) Percent means "for every 100." It is represented by the percent symbol, %. We use percentages to represent ratios and fractions.



For example, 20% means 20 for every 100. 20% of a number means $\frac{20}{100}$ or $\frac{1}{5}$ of that number. Let's say there are 800 students in a school. If 20% of them are on a field trip, that means 160 students because 20 students are on the trip for every 100 students total.

profit The money left after paying for business expenses.

R

rate A comparison, or ratio, that describes how two quantities change together.

U

unit price The price per unit of an item. You can count how many units by counting the items or the weight.

For example, if 4 avocados cost $12, then the unit price is $\frac{12}{4} = \$3$ per avocado.

unit rate A rate that describes how one quantity changes when the other quantity changes by exactly 1 unit.

For example, if 12 people share 3 pizzas equally, then one unit rate is 4 people per pizza. Another unit rate in this situation is $\frac{1}{4}$ pizza per person.

Español

P

por, por cada La palabra *por* puede significar "por cada".

Por ejemplo, si el precio es $5 por boleto, esto significa que cada boleto cuesta $5. Comprar 4 boletos costaría $5 \cdot 4 = \$20$.

por ciento (porcentaje) Por ciento significa "por cada 100". Se representa con el símbolo de porcentaje, %. Usamos porcentajes para representar razones y fracciones.

Por ejemplo, 20% significa 20 por cada 100. 20% de un número significa $\frac{20}{100}$ o $\frac{1}{5}$ de dicho número. Supongamos que hay 800 estudiantes en una escuela. Si el 20% de ellos está en una excursión, eso significa 160 estudiantes porque 20 están de viaje por cada 100 estudiantes.

ganancia El dinero que sobra después de pagar los gastos del negocio.

R

tasa Una comparación, o razón, que describe cómo cambian juntas dos cantidades.

U

precio unitario El precio por unidad de un artículo. Puedes contar la cantidad de unidades contando los artículos o el peso.

Por ejemplo, si 4 aguacates cuestan $12, el precio unitario es $\frac{12}{4} = \$3$ por aguacate.

tasa unitaria Una tasa que describe cómo cambia una cantidad cuando la otra cantidad cambia en exactamente 1 unidad.

Por ejemplo, si 12 personas se reparten 3 pizzas en partes iguales, entonces una tasa unitaria es 4 personas por pizza. Otra tasa unitaria en esta situación es $\frac{1}{4}$ de pizza por persona.