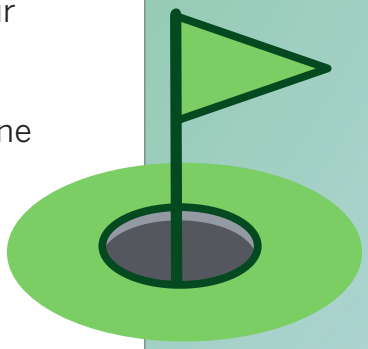
Unit 2

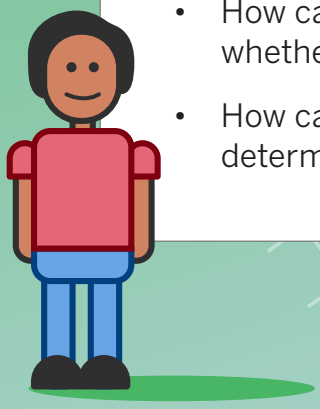
Dilations, Similarity, and Slope

We use dilations in everyday life to make objects smaller or bigger, like printing pictures in different sizes or zooming in and out on our phone screens. Unlike rigid transformations, dilations change the dimensions of a shape. We'll learn how dilations can help us determine similarity and how similarity can help us understand slope.



Essential Questions

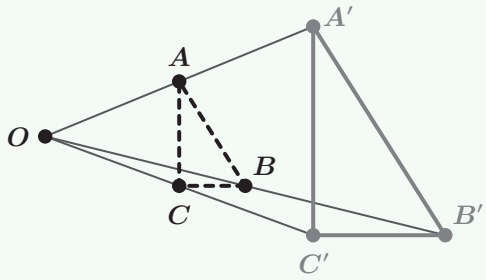
- What does it mean to dilate a figure?
- How can transformations be used to decide whether two figures are similar?
- How can similar triangles be used to determine the slope of a line?



Summary | Lesson 1

A **dilation** is a type of transformation that creates *scaled copies*. Dilating a figure means moving each of its vertices along a line that's extended from a given point. The original distance from the given point to each vertex on the *pre-image* is multiplied by the same number to create the dilated image.

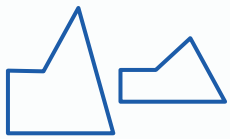
For example, Triangle ABC was dilated from the point O to create triangle $A'B'C'$.



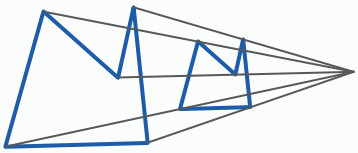
Try This

Which of these is an example of a dilation? Explain your thinking.

A.



B.



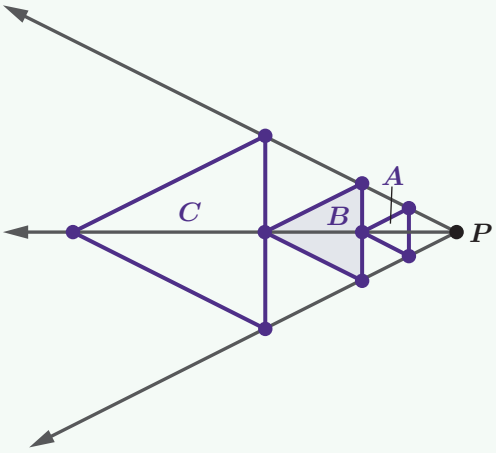
Summary | Lesson 2

A *dilation* is a transformation that involves a **center of dilation** and a **scale factor**.

One strategy for dilating a figure is to measure the distance between the center of dilation and one of the pre-image points, multiply that distance by the scale factor, then place the image point that distance away from the center of dilation along the same line. Repeat this strategy with all the other points in the pre-image.

In this example, triangle B is the pre-image.

- Triangle A is a dilation of triangle B using point P as the center of dilation and a scale factor of $\frac{1}{2}$.
- Triangle C is a dilation of triangle B using point P as the center of dilation and a scale factor of 2.

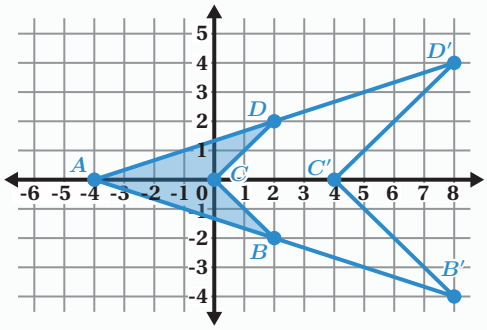


Try This

Figure $ABCD$ is dilated to create figure $A'B'C'D'$.

a What is the center of dilation?

b What is the scale factor?



Summary | Lesson 3

Dilations can be combined with other *sequences of transformations*.

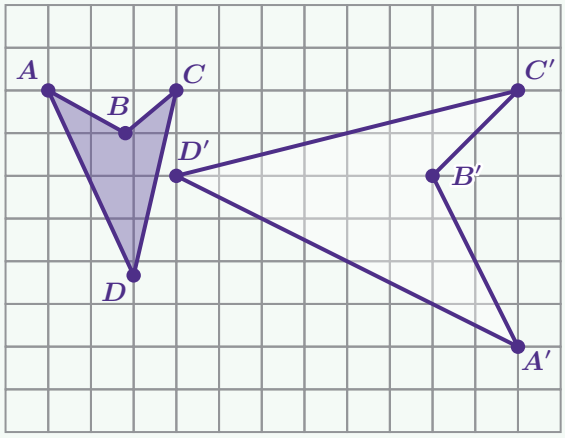
Here is a sequence of transformations that moves figure $ABCD$ onto figure $A'B'C'D'$

Step 1: Dilate figure $ABCD$ using point D as the center of dilation and a scale factor of 2.

Step 2: Translate the image after Step 1 so that point D moves onto point D'.

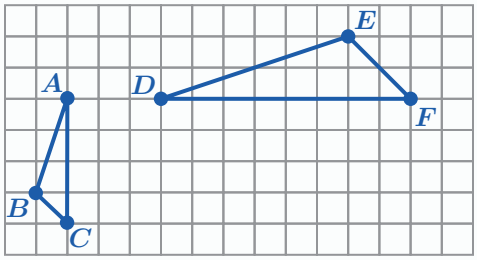
Step 3: Rotate the new image 90° clockwise around point D'.

Step 4: Reflect the new image across a horizontal line that contains points D' and B'.



Try This

Describe a sequence of transformations that moves triangle ABC onto triangle DEF.



Summary | Lesson 4

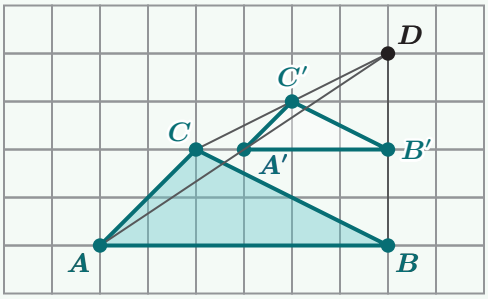
Another strategy for dilating a figure is to use a grid. Count the vertical and horizontal grid squares between the center of dilation and the pre-image, multiply each value by the scale factor, then count that number of grid squares away from the center to get the image.

If the scale factor of the dilation is:

- Greater than 1, the image will be *larger* than the pre-image and further from the center of dilation.
- Equal to 1, the image will be *the same size* as the pre-image and just as far from the center.
- Between 0 than 1, the image will be *smaller* than the pre-image and closer to the center.

For example, triangle ABC is dilated using point D as the center of dilation and a scale factor of $\frac{1}{2}$.

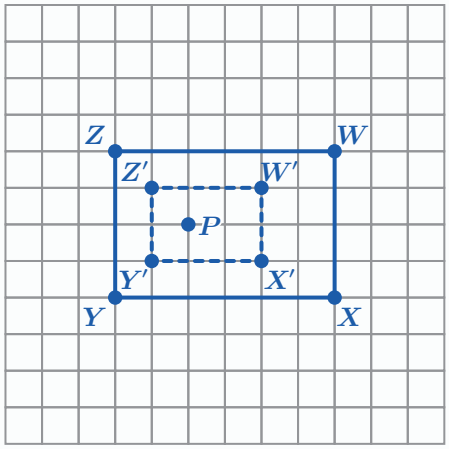
Since the scale factor is less than 1, the image is smaller than the pre-image and closer to the center of dilation.



Try This

Rectangle $WXYZ$ is dilated using point P as the center of dilation to create rectangle $W'X'Y'Z'$.

Is the scale factor greater than 1, equal to 1, or between 0 and 1? Explain your thinking.

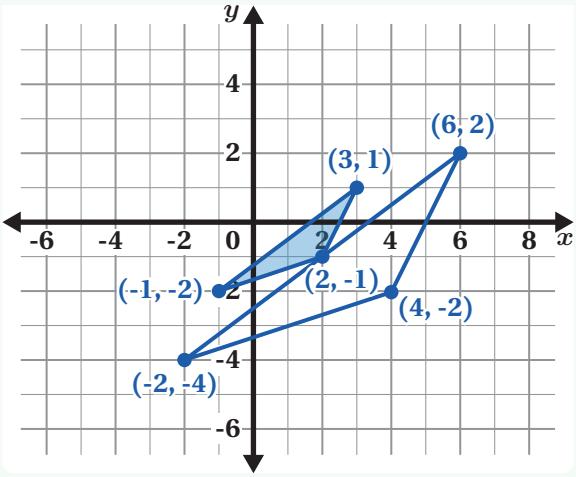


Summary | Lesson 5

Coordinates can help us communicate geometric information, such as dilations.

Let's say you're dilating this shaded pre-image using the center of dilation $(0, 0)$ and a scale factor of 2.

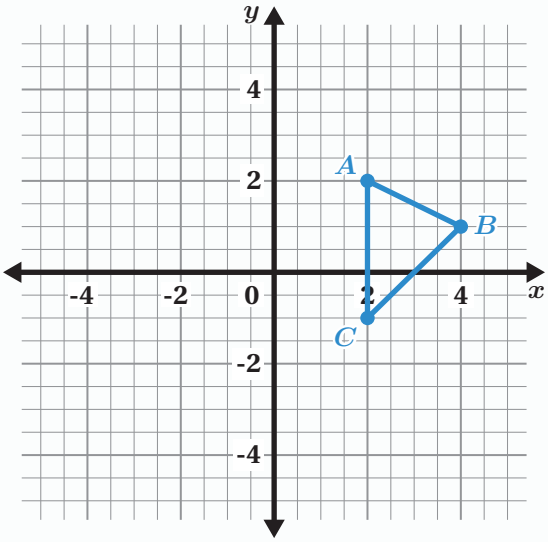
You can use the coordinate plane to measure the horizontal and vertical distances of each point from the center of dilation. Then you can multiply those distances by the scale factor to get the distances between the center of dilation and the points on the image.



In this example, the center of dilation is $(0, 0)$, so you can just multiply the pre-image coordinates by the scale factor to get the image coordinates.

Try This

Dilate triangle ABC using point $(-4, 0)$ as the center of dilation and a scale factor of $\frac{1}{2}$.



Summary | Lesson 6

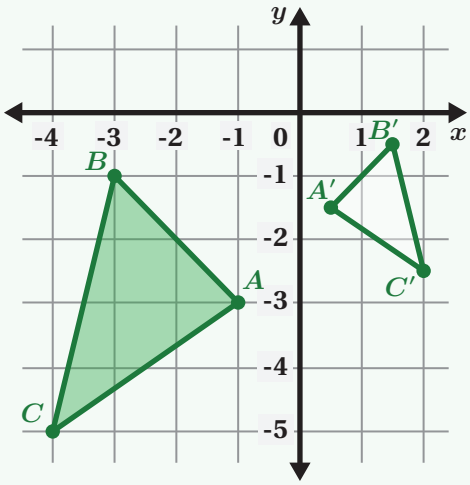
Two figures are *congruent* if one figure can be moved exactly onto the other using a sequence of rigid transformations. Two figures are **similar** if one figure can be moved exactly onto the other using a sequence of dilations and rigid transformations.

We use the symbol ~ to say that two figures are similar. For example, triangle ABC ~ triangle $A'B'C'$.

In congruent figures, the *corresponding* angles and corresponding sides are congruent.

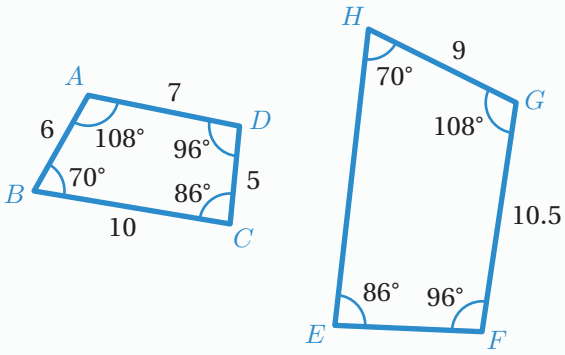
In similar figures, the corresponding angles are congruent and the corresponding side lengths are proportional but *not* always congruent.

For example, $\angle A$ is congruent to $\angle A'$, but segment AB is not the same length as segment $A'B'$. Segment $A'B'$ is twice as long as segment AB because the scale factor from triangle ABC to triangle $A'B'C'$ is 2.



Try This

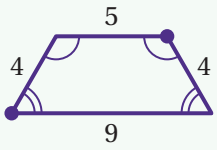
Is figure $ABCD$ similar to figure $GHEF$? Explain your thinking.

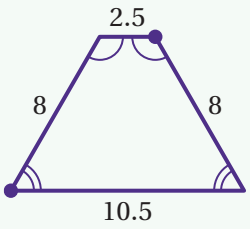


Summary | Lesson 7

Similar figures have congruent corresponding angles. Is knowing that the corresponding angles in two figures are congruent enough to show that the figures are also similar? It depends!

Here are two figures that have congruent corresponding angles but are not similar figures.

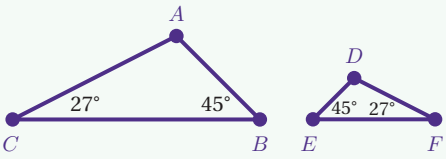




For triangles, knowing that the corresponding angles are congruent is enough to know that the triangles are similar.

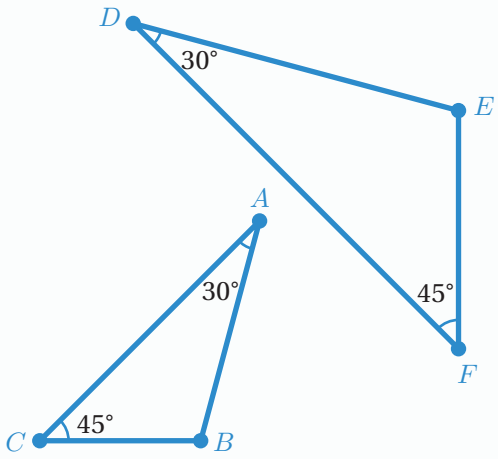
This is even true if you only know two angle measures, because we can use the fact that the sum of the interior angles of any triangle is $180°$ to figure out the third angle measure.

For example, the unknown angles in triangles CAB and FDE are each $108°$. All the corresponding angles are congruent, which means triangle CAB is similar to triangle FDE.



Try This

Is triangle ABC similar to triangle DEF? Explain your thinking.



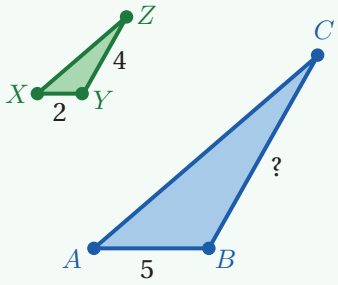
Summary | Lesson 8

There are a variety of strategies for determining unknown side lengths of similar triangles.

You can use ratios that compare two triangles using the scale factor between them.

You can also determine the ratio of two side lengths in the same triangle, then apply that ratio to the corresponding sides in a similar triangle.

Here are two similar triangles, triangle ABC and triangle XYZ. You can use each of these strategies to determine the length of side BC.



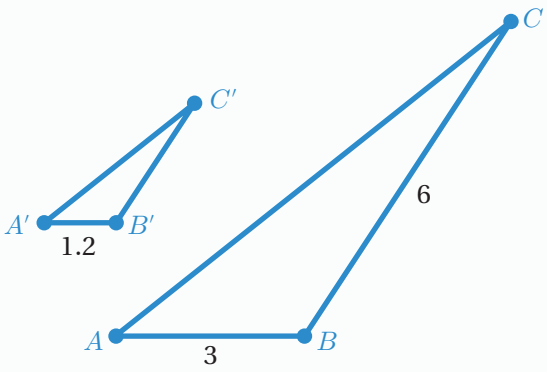
Using the scale factor between the triangles.
Side XY and side AB are corresponding sides. The ratio of their side lengths is $\frac{5}{2}$, which means the scale factor is 2.5. To calculate the length of side BC, you can multiply the length of side YZ by the scale factor. $4 \cdot 2.5 = 10$, so side BC is 10 units long.

Using ratios of side lengths within one triangle.
The ratio of side YZ to side XY is $4 : 2$, or 2. That means the length of side BC is twice the length of side AB. $5 \cdot 2 = 10$, so side BC is 10 units long.

Try This

Triangles ABC and $A'B'C'$ are similar.

Determine the length of side $B'C'$.



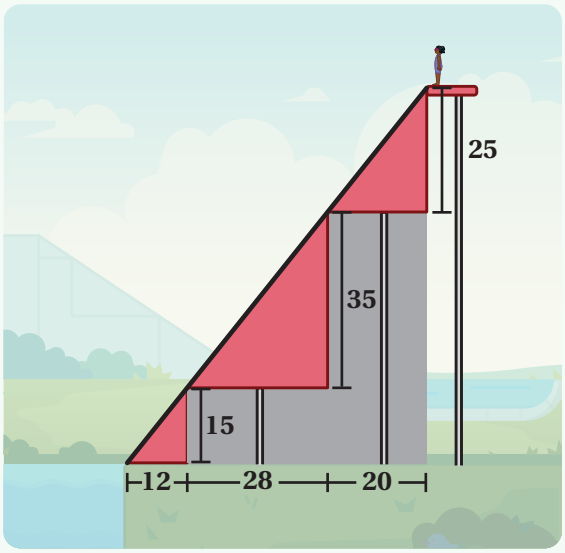
Summary | Lesson 9

Here are three ramps made up of similar triangles with proportional corresponding side lengths. Because of this, they line up perfectly to create a smooth slide.

To line up the longest side of each triangle, the **slope** of each triangle must be the same. Slope is the height-to-base *ratio* of a triangle, which describes the steepness of its longest side.

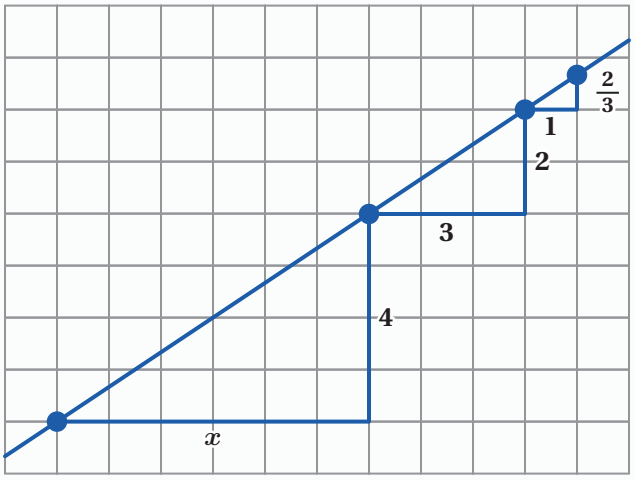
For example, the slope of this slide is $\frac{5}{4}$ because $\frac{15}{12} = \frac{35}{28} = \frac{25}{20} = \frac{5}{4}$.

You could sketch infinite triangles on the same line that all have the same height-to-base ratio. Any of those triangles can be used to determine the slope.



Try This

What is the value of x? Explain your thinking.

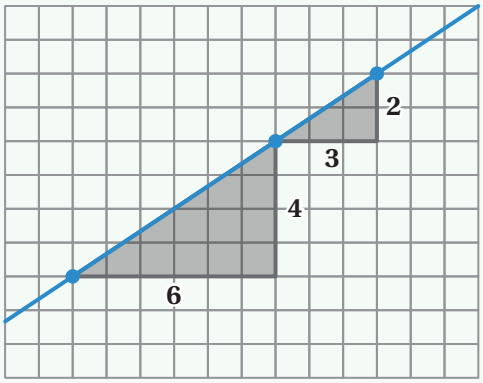


You can determine the slope of a line by drawing similar right triangles, called **slope triangles**, between any two points on the line. The height of the slope triangle represents the vertical distance between the points, and the base of the triangle represents the horizontal distance between the points.

Slope is the ratio of the height of a slope triangle to its base.

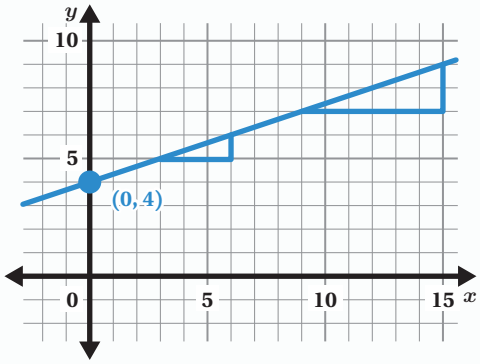
Here's an example of two possible slope triangles that you could use to calculate the slope of this line.

The slope of this line is $\frac{4}{6}$, or $\frac{2}{3}$, or any equivalent value.



Try This

Determine the slope of this line.



Lesson 1

B. *Explanations vary.* Choice B is a dilation because the vertices are extended from a point, which results in scaled copies. The figures in choice A are not scaled copies because the side lengths have different ratios.

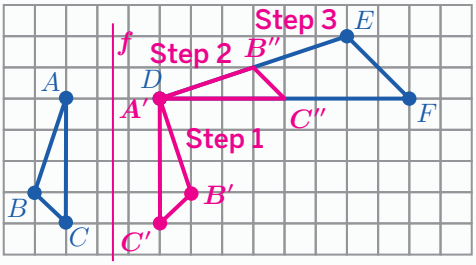
Lesson 2

a. The center of dilation is point A because the points of figure $A'B'C'D'$ can be determined by drawing a straight line from point A.

b. The scale factor is 2 because the points in figure $A'B'C'D'$ are twice as far from A as the points in figure $ABCD$. For example, point C is 4 units from point A, while point C' is 8 units from point A.

Lesson 3

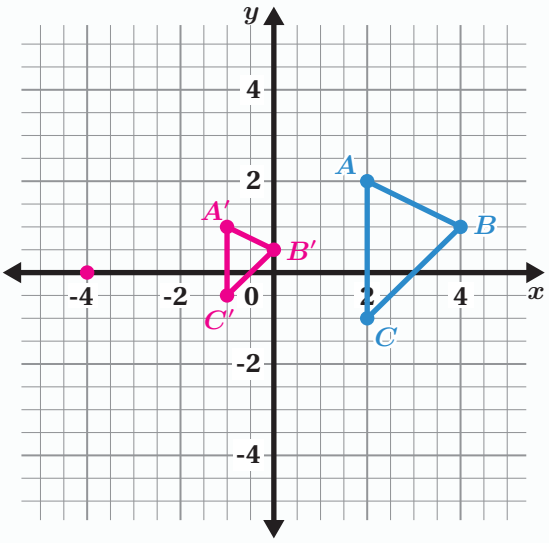
Responses vary.
Step 1: Reflect triangle ABC over a vertical line directly in between triangles ABC and DEF.
Step 2: Rotate triangle $A'B'C'$ 90° counterclockwise around point A'.
Step 3: Dilate triangle $A'B''C''$ using point A' as the center of dilation and a scale factor of 2.



Lesson 4

The scale factor is between 0 and 1 because the image is smaller than the pre-image and closer to the center of dilation.

Lesson 5



Lesson 6

Yes, these figures are similar. *Explanations vary*. The corresponding angle measures are congruent and the corresponding side lengths are proportional. For example, $\angle ADC$ and $\angle GFE$ are corresponding angles and both have measures of $96°$. Sides AB and GH are corresponding sides with a ratio of $6:9$ or $\frac{2}{3}$. Sides AD and GF are also corresponding sides with a ratio of $\frac{7}{10.5} = \frac{2}{3}$.

Lesson 7

Yes, triangles ABC and DEF are similar. *Explanations vary*. Since two angles of each triangle measure $45°$ and $30°$, the third angle must be $105°$, because $180 - 45 - 30 = 105$. When two triangles have three congruent angles, they are similar triangles.

Lesson 8

$B'C' = 2.4$ units. The ratio of side AB to side BC is $1:2$ or $\frac{1}{2}$. That means side BC is two times the length of side AB, so side $B'C'$ is two times the length of side $A'B'$. Another way to determine the measure of side $B'C'$ is to notice that the ratio of side AB to side $A'B'$ is $3:1.2$. $\frac{3}{1.2} = 2.5$, so side BC is 2.5 times the length of side $B'C'$. That means $B'C'$ measures $6 \div 2.5 = 2.4$ units.

Lesson 9

$x = 6$. *Explanations vary*. The triangles all have the same slope, so their height-to-base ratios are equivalent. For example, $\frac{2}{3} = \frac{4}{6}$.

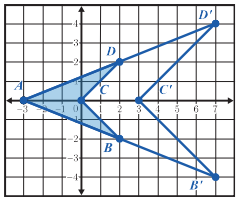
Lesson 10

The slope of the line is $\frac{1}{3}$. There is a slope triangle on the graph with a height of 1 unit and a base of 3 units, which is a ratio of $\frac{1}{3}$.

Grade 8 Unit 2 Glossary/8.º grado Unidad 2 Glosario

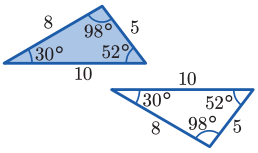
C

center of a dilation The point from which we measure distances in a dilation.



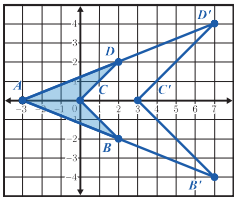
The center of dilation in this example is point A.

congruent One figure is congruent to another if it can be moved with translations, rotations, and reflections to fit exactly over the other.



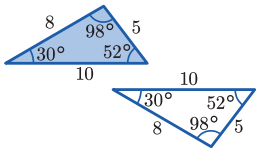
correspond (corresponding parts) To correspond is to match. When part of an original figure matches up with part of a copy, we call them corresponding parts. These could be points, segments, angles, or distances.

centro de una dilatación El punto desde el cual medimos las distancias en una dilatación.



El centro de dilatación en este ejemplo es el punto A.

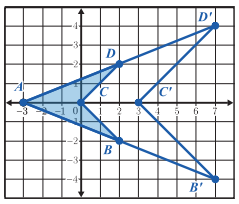
congruente Una figura es congruente con otra si se puede mover por medio de traslaciones, rotaciones y reflexiones de forma tal que coincida exactamente con la otra.



corresponder (partes correspondientes) Corresponder es coincidir. Cuando una parte de una figura original corresponde con una parte de una copia, se llaman partes correspondientes. Pueden ser puntos, segmentos, ángulos o distancias.

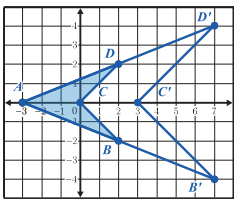
D

dilation A type of transformation that creates scaled copies. A dilation moves every point in a figure away from a center of dilation by the distance of a scale factor.



In this example, polygon $ABCD$ is dilated using point A as the center of dilation and a scale factor of 2. Point D' is twice as far away from point A as point D along the same ray.

dilatación Un tipo de transformación que produce copias a escala. Una dilatación aleja cada punto de una figura del centro de dilatación de acuerdo con un factor de escala.



Aquí, el polígono $ABCD$ se dilata usando punto A como centro de dilatación y un factor de escala de 2. El punto D' está al doble de la distancia del punto A que el punto D en la misma semirrecta.

I

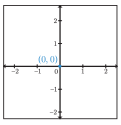
image A new figure that is created after a transformation of an original figure (called the pre-image). Every part of the pre-image moves in the same way to match up with a part of the image.

imagen Una nueva figura que se produce después de la transformación de una figura original (denominada preimagen). Todas las partes de la preimagen se mueven de la misma forma para coincidir con cada parte de la imagen.

English

O

origin The point $(0, 0)$ on the coordinate plane. This is where the x-axis and the y-axis intersect.

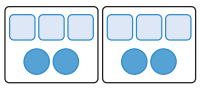


P

pre-image The name of a figure before any transformations are performed.

R

ratio A ratio $a{:}b$ is a relationship between two quantities. For every a of the first, there are b of the second.

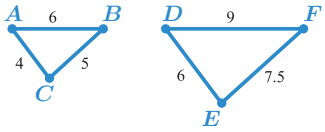


If the ratio of apples to oranges in a fruit bowl is $2{:}3$, then for every 2 apples, there are 3 oranges. There are several ways to describe ratios.

- For every 3 squares, there are 2 circles.
- The ratio of squares to circles is 3 to 2.
- The ratio of squares to circles is $3{:}2$.

S

scaled copy A copy of an image that may change in size, but always maintains the shape and angle measurements of the original. If a shape looks squished or stretched when compared to its original, it is not a scaled copy. To create a scaled copy, we multiply every length in the original figure by the same number.

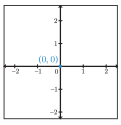


For example, triangle DEF is a scaled copy of triangle ABC.

Español

O

origen El punto $(0, 0)$ en el plano de coordenadas. El punto en el que se intersecan el eje x y el eje y.

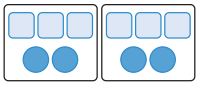


P

preimagen El nombre de una figura antes de realizar una transformación.

R

razón Una razón $a{:}b$ es una relación entre dos cantidades. Por cada a del primero, hay b del segundo.

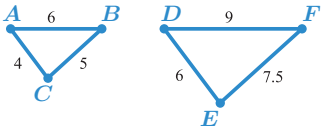


Si la razón de manzanas a naranjas en un frutero es $2{:}3$, entonces por cada 2 manzanas hay 3 naranjas. Hay varias formas de describir razones.

- Por cada 3 cuadrados hay 2 círculos.
- La razón de cuadrados a círculos es de 3 a 2.
- La razón de cuadrados a círculos es $3{:}2$.

S

copia a escala Una copia de una imagen que puede cambiar de tamaño, pero que siempre conserva la forma y las medidas de los ángulos de la imagen original. Si una figura luce aplastada o estirada en comparación con la original, no es una copia a escala. Para crear una copia a escala, multiplicamos todas las longitudes de la figura original por el mismo número.

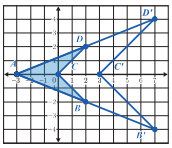


Por ejemplo, el triángulo DEF es una copia a escala del triángulo ABC.

Grade 8 Unit 2 Glossary/8.º grado Unidad 2 Glosario

English

scale factor The number used to create a dilation. All distances from the pre-image to the center of dilation are multiplied by the same number, called the scale factor.

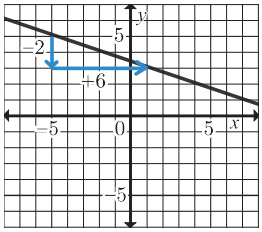


For example, the scale factor from polygon $ABCD$ to polygon $A'B'C'D'$ is 2.

sequence of transformations A set of translations, rotations, reflections, and dilations on a figure. The transformations are performed in a given order.

similar One figure is similar to another if it can be moved with translations, rotations, reflections, and dilations to fit exactly over the other. Two figures are similar if one is a scaled copy of the other.

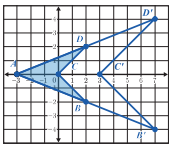
slope A number that describes the direction and steepness of a line. Slope represents the amount that y changes when x increases by 1. That's why the slope of a line is sometimes called a rate of change. To calculate the slope, divide the vertical distance between any two points on the line by the horizontal distance between those points.



The slope of this line is $-\frac{2}{6} = -\frac{1}{3}$.

Español

factor de escala El número que se usa para hacer una dilatación. Todas las distancias de la preimagen al centro de dilatación se multiplican por el mismo número, que se denomina factor de escala.

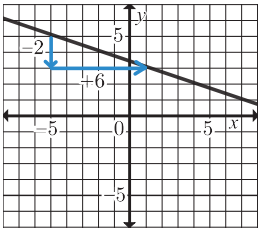


Por ejemplo, el factor de escala del polígono $ABCD$ al polígono $A'B'C'D'$ es 2.

secuencia de transformaciones Un conjunto de traslaciones, rotaciones, reflexiones y dilataciones aplicadas a una figura. Las transformaciones se ejecutan en un determinado orden.

semejante Una figura es semejante a otra si se puede mover por medio de traslaciones, rotaciones, reflexiones y dilataciones de forma tal que coincida exactamente con la otra. Dos figuras son semejantes si una es una copia a escala de la otra.

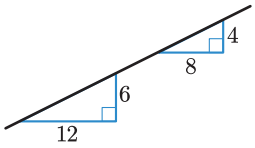
pendiente Un número que describe la dirección e inclinación de una línea. La pendiente representa la cantidad en la que cambia y cuando x se incrementa en 1. Es por eso que la pendiente de una recta a veces se denomina tasa de cambio. Para calcular la pendiente, la distancia vertical entre dos puntos cualesquiera en la recta se divide entre la distancia horizontal entre dichos puntos.



La pendiente de esta recta es $-\frac{2}{6} = -\frac{1}{3}$.

English

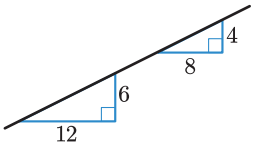
slope triangle A triangle drawn to include two points on a line in order to determine the slope of that line. The longest side lies on the line and the other two sides are vertical and horizontal. The height of the slope triangle represents the vertical distance between the points, and the base of the triangle represents the horizontal distance between the points.



Both of these triangles are slope triangles.

Español

triángulo de pendiente Un triángulo que se traza para incorporar dos puntos de una recta y así determinar la pendiente de dicha recta. El lado más largo se sitúa sobre la recta y los dos lados restantes son uno vertical y otro horizontal. La altura del triángulo de pendiente representa la distancia vertical entre los puntos, mientras que la base del triángulo representa la distancia horizontal entre los puntos.



Estos dos triángulos son triángulos de pendiente.