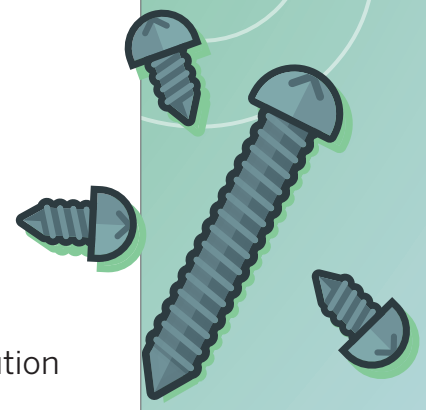
Unit 2

Linear Equations and Inequalities

In this unit, you will use what you know about solving one-variable equations and inequalities to make sense of multi-variable equations and two-variable inequalities.

Essential Questions

- What does it mean to be a solution to an equation or inequality?
- How are the different representations of equations or inequalities connected?
- How do you determine and graph the solution of an inequality?



Summary | Lesson 1

Solving an equation means taking steps to determine a **solution**. A solution to an equation is a value that makes the equation true. There are many ways to solve an equation, including working backwards, inverse operations, and moves that keep the equation balanced.

Here are two strategies for solving the equation $-4(x + 2) = 20$:

-2 ÷-4

x	$x + 2$	$-4(x + 2)$
-7	-5	20

$$\frac{-4(x + 2)}{-4} = \frac{20}{-4}$$

$$x + 2 = -5$$

$$-2 \quad -2$$

$$x = -7$$

You can check that the value you determined is a solution to an equation by substituting the value back into the equation to see if it makes the equation true. The solution to the equation $-4(x + 2) = 20$ is $x = -7$ because $-4(-7 + 2) = 20$ is a true statement.

Try This

Here is an equation: $-30 = 5(x - 2)$.

a Solve the equation. Use the table if it helps with your thinking.

x	$x - 2$	$5(x - 2)$

b Check your solution.

Summary | Lesson 2

You can solve one-variable equations by creating **equivalent equations**. To create equivalent equations, use solving moves that keep the equation balanced, such as combining like terms or using inverse operations to move a variable from one side of the equation to the other.

Here is an example of a set of solving moves that keep an equation balanced:

$-3m + 5 + m = 2(6m + 3)$	This is the original equation.
$-2m + 5 = 12m + 6$	We combined like terms on the left and distributed on the right.
$5 = 14m + 6$	We added $2m$ to each side of the equation.
$-1 = 14m$	We subtracted 6 from each side of the equation.
$\frac{-1}{14} = m$	We divided each side of the equation by 14.

All of the equations created at each step of this solution process are *equivalent equations* because they have the same solution: $m = \frac{-1}{14}$.

Try This

Emma made an error while solving the equation $\frac{1}{2}(x + 4) = -10 + 3x$.

a Describe one thing that Emma did well.

b Circle the error and explain what the error was.

Emma

$\frac{1}{2}(x + 4) = -10 + 3x$

$x + 4 = -20 + 3x$

$x + 24 = 3x$

$24 = 2x$

$12 = x$

c Solve $\frac{1}{2}(x + 4) = -10 + 3x$.

Summary | Lesson 3

Not all one-variable linear equations have a single solution. Some linear equations have **infinitely many solutions**, and some have **no solution**.

In the process of solving, you will be able to see a difference between equations with one solution, no solution, or infinitely many solutions:

- In an equation with one solution, *a single value* of x will make the equation true.
- In an equation with no solution, *no value* of x will make the equation true.
- In an equation with infinitely many solutions, *any value* of x will make the equation true.

Here are examples of equations with one solution, no solution, and infinitely many solutions.

One Solution	No Solution	Infinitely Many Solutions
$3x + 4 = 2x + 10$	$2x + 4 = 2x + 10$	$2(x + 5) = 2x + 10$
$3x = 2x + 6$	$4 = 10$	$2x + 10 = 2x + 10$
$x = 6$	This is *never* true!	$10 = 10$
		This is *always* true!

If the variable in an equation is eliminated during the solving process, that tells you that the equation has either no solution or infinitely many solutions. If the statement remaining is false, the equation has no solution. If the statement remaining is true, the equation has infinitely many solutions.

Try This

Group the equations based on their number of solutions.

$4t + 7 = 2(2t + 2)$	$3x = 10 - 3x$	$10 - 2(t + 5) = -2t$
$4t = 6t$	$5t + 7 = -3 + 5t$	

One Solution	No Solution	Infinitely Many Solutions

Summary | Lesson 4

Two-variable linear equations can be represented in different forms. Sometimes the different forms of an equation can reveal information that is useful for solving problems. Depending on what information you are looking for, you might choose to use one form or the other.

Here is an example of two equivalent equations. They are each represented in different forms, and they each reveal different information about seating and standing capacity. In each equation, t is the seating capacity and d is the standing capacity.

$$4t + 2d = 300$$

- Each seated passenger requires 4 square feet.
- Each standing passenger requires 2 square feet.
- The total area of the subway car is 300 square feet.

$$d = 150 - 2t$$

- When there are no seats ($t = 0$), 150 passengers can stand in the car.
- For every seat that is added, the standing capacity decreases by 2 square feet.

Try This

Tiara is trying to save $240 for a new gaming console. To earn the money she needs, she works at the pool for $8 an hour and tutors Spanish for $12 an hour.

Tiara wrote the equation $8p + 12t = 240$ to represent her situation.

a What do the variables p and t represent in Tiara's situation?

b Which equation is equivalent to Tiara's equation?

A. $t = 240 - 8p$ **B.** $t = 20 - \frac{2}{3}p$ **C.** $t = 30 - \frac{3}{2}p$ **D.** $t = -\frac{2}{3}p + 30$

Summary | Lesson 5

You can solve equations that contain multiple variables using some of the same strategies for solving equations with one variable. These strategies include working backwards, using inverse operations, and keeping the equation balanced.

When solving problems that model real-world situations, it can be helpful to rearrange an equation to highlight a variable of interest. Rearranging equations can also reveal different relationships between the variables and the quantities that they represent.

Here is how you could rearrange the equation $y = mx + b$ to solve for m or b.

Solving for m

$$\begin{aligned} y &= mx + b \\ y - b &= mx + b - b \\ \frac{y - b}{x} &= \frac{mx}{x} \\ \frac{y - b}{x} &= m \end{aligned}$$

Solving for b

$$\begin{aligned} y &= mx + b \\ y - mx &= mx + b - mx \\ y - mx &= b \end{aligned}$$

Rearranging equations for a specific variable can help make some calculations easier. For example, if you know the values of r and s for $6r + 4s = 240$, solving the equation for s can make it simpler to test different values of r and s to see how they affect the value of s.

Try This

Solve each equation on the left. Then use the same strategy to solve the equation on the right.

a Solve for t.

$10 = 4 - 3t$

$10 = v - at$

b Solve for m.

$\frac{m}{3} + 7 = -4$

$\frac{m}{a} + t = h$

Summary | Lesson 6

Equations, tables, and graphs are all different ways to model a situation. The graph of a linear equation represents all the pairs of values that are solutions to the equation and make the equation true.

Linear equations can be written in different but equivalent forms. Rearranging equations into different but equivalent forms helps reveal new information, such as the **x-intercept** and **y-intercept**, which we can see in a graph, table, or description of a situation.

Let's say a lemonade stand sold lemonade for \$3 per cup and cookies for \$2 each. The stand made \$12. ℓ represents the number of cups of lemonade sold and c represents the number of cookies sold. This situation can be represented in many different ways:

Equation

$$3\ell + 2c = 12$$

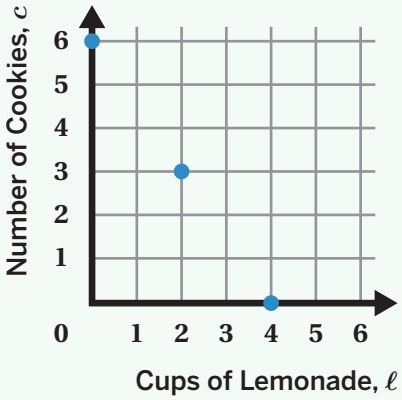
Equation Solved for c

$$c = 6 - \frac{3}{2}\ell$$

Table

ℓ	0	2	4
c	6	3	0

Graph



Try This

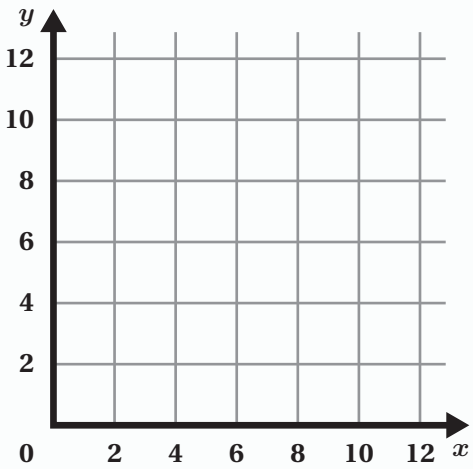
Here is an equation: $6x + 2y = 24$.

a Solve the equation for y.

b Graph the equation.

Use the table if it helps with your thinking.

x	y

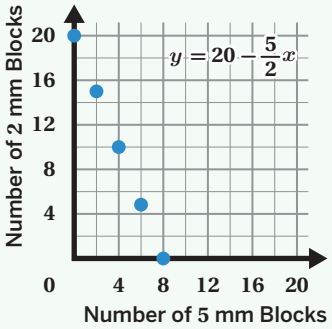


Summary | Lesson 7

There are lots of ways to represent a situation that involves two variables. Each of these representations is connected and reveals information about how the quantities in the situation are related.

A description explains how the quantities are related verbally or in writing.
For example: A snail needs to cross a gap that is 40 mm tall using 5 mm and 2 mm blocks.

A graph reveals the *slope* and y-*intercept*. This graph shows a slope of $-\frac{5}{2}$ and a y-intercept of 20.



A table reveals several combinations of 5 mm and 2 mm blocks that let the snail cross the gap.

x	0	2	4	6
y	20	15	10	5

The ordered pairs can be used to make a graph or find the rate of change (slope).

Different forms of an equation reveal unique information. In this situation, standard form reveals the relationship between combinations; slope-intercept form reveals how the blocks are changing in relationship with one another.

Standard form

$5x + 2y = 40$

Slope-intercept form (solved for one variable)

$y = 20 - \frac{5}{2}x$

Try This

Neel won 20 tickets at the fair to trade in for prizes. He can get a sticker for 2 tickets and a pencil for 3 tickets. Let s represent the number of stickers Neel gets and p represent the number of pencils.

a Write an equation to represent how many of each kind of prize Neel can get.

b Solve your equation for s.

c What can you determine about the situation by solving the equation for s?

Summary | Lesson 8

We can use *inequalities* to model situations with constraints. A **constraint** is a limitation on what values are possible in a model or situation.

Here is an example of a situation, the constraint, and the inequality that models them.

Situation	Constraint	Inequality
Tasia is ordering pizza for a party. Each plain pizza costs $\$12$ and there is a delivery fee of $\$8$.	Tasia can spend *up to* $\$140$.	$12p + 8 \leq 140$ Where p represents the number of pizzas.

When writing inequalities to model situations, you can use the symbols $>$, $\geq$, $<$, and $\leq$ to represent the nature of the constraint. Terms like *greater than, less than, at most, at least*, or *up to* can help you determine which inequality symbol to use.

Try This

Valeria wants to donate at least $\$120$ to her local food bank. She has already saved $\$64$ and is planning to save $\$8$ each week.

a Why is Valeria's situation an example of a constraint?

b Write an inequality to match Valeria's situation.

Use w to represent the number of weeks that Valeria will save $\$8$.

Summary | Lesson 9

The *solution* set of an inequality contains all the values that make the inequality true. You can represent a solution set on a number line by marking the *boundary point* and then shading the region of values that make the inequality true. To identify the boundary point, you can solve the equation that corresponds to the inequality. Then you can test one or more values to determine whether the *solution region* is greater than or less than the boundary point.

Here is an example of how you can determine and represent the solution set for $2x - 4 \geq 8$:

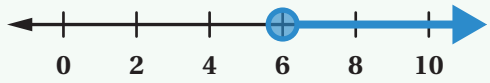
Determine the boundary point:

$$2x - 4 = 8$$
$$2x = 12$$
$$x = 6$$

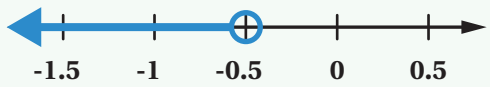
Determine the region:

$$2x - 4 \geq 8$$
$$2(0) - 4 \geq 8$$
$$4 \geq 8$$ ***False!***

- 6 is the boundary point and the $\geq$ symbol means it is included in the solution set.
- Since this statement is false when we substitute 0, we know 0 is not in the solution set.



When a boundary point is not included in the solution set, this is represented with an open circle on the number line. Here is an example of the solution $-0.5 > x$ graphed on a number line.

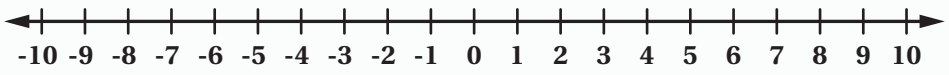


Try This

Here is an inequality: $5 + 3x < 32$.

a Determine the boundary point.

b Graph all the solutions to the inequality. Show or explain your thinking.



Summary | Lesson 10

One strategy that you can use to solve any one-variable inequality is to:

- Solve the equation that corresponds with the inequality to determine the boundary point.
- Then test value(s) greater than or less than the boundary point to determine where the solution region is located.

Here is an example of how you could solve the inequality $10 > -3x - 2$.

Solve the corresponding equation:

$$10 = -3x - 2$$
$$12 = -3x$$
$$-4 = x$$

The boundary point is $x = -4$.

Test $x = -5$

$10 > -3(-5) - 2$
$10 > 13$
False!

Test $x = 0$

$10 > -3(0) - 2$
$10 > -2$
True!

When $x = -5$, the inequality is false, so the solutions are greater than -4.

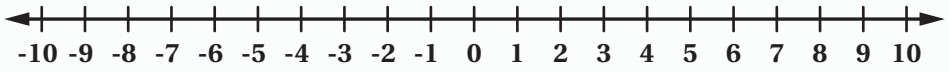
The solution to an inequality does not always have the same inequality symbol as the original inequality. Since the value that is *greater than* -4 is true, the solution set to the inequality can be written as $-4 < x$ or $x > -4$.

Try This

Here is an inequality: $11 - 2x \geq 5$.

a Solve this inequality.

b Graph all the solutions to this inequality.



Summary | Lesson 11

The **absolute value** of a number is its distance from 0 on a number line. We can use the definition of absolute value to help us understand how to solve absolute value equations and inequalities.

For an absolute value equation, there are usually two input values with the same output. This means there are often two solutions that make an absolute value equation true.

When solving and graphing absolute value inequalities, we can use the solve-and-test strategy we used when solving one-variable inequalities.

- First, solve the two corresponding absolute value equations to determine the boundary points.
- Then decide whether the boundary points are included in the solution set. 1 and -7 are not included because of the inequality symbol $>$. Draw open circles on the number line to represent these boundary points.
- Next, test a value between the two boundary points to decide which region of values makes the inequality true.
- Finally, graph the solution region.

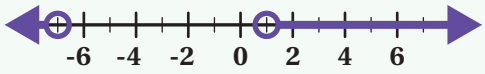
$$|x + 3| > 4$$

$$x + 3 = 4 \qquad -(x + 3) = 4$$

$$x = 1 \qquad x = -7$$

$$|(0) + 3| > 4$$

False!

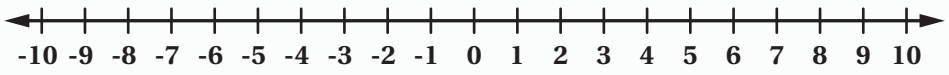


Try This

Here is an inequality: $|x - 3| \leq 2$.

a List four solutions to this inequality.

b Graph all the solutions to this inequality. Show or explain your thinking.



Summary | Lesson 12

We can write the solution to an absolute value inequality using a **compound inequality** that joins two or more inequalities with the words *and* or *or*. Using the word "and" communicates that the solutions are located between the two boundary points on the number line where the graph of the first inequality and the graph of the second inequality overlap. Using the word "or" communicates that the solutions are located to the left of one boundary point and to the right of the other.

Here is an example of absolute value inequalities with their solutions written as compound inequalities.

And

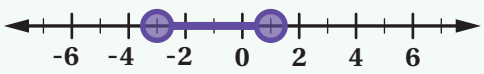
$|x + 1| \geq -2$

$x + 1 = -2 \qquad -(x + 1) = -2$

$x = -3 \qquad x = 1$

$|0 + 1| \geq -2$ ***True!***

$x \geq -3$ **and** $x \leq 1$



Or

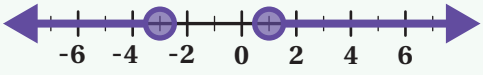
$|x + 1| \geq 2$

$x + 1 = 2 \qquad -(x + 1) = 2$

$x = 1 \qquad x = -3$

$|0 + 1| \geq 2$ ***False!***

$x \leq -3$ **or** $x \geq 1$



Try This

Match each absolute value inequality with its solutions.
One set of solutions will have no match.

$x < 4$ and $x > -5$ | $x > 4$ or $x < -5$ | $x < -4$ or $x > 5$ | $x > -4$ and $x < 5$

$\|2x + 1\| > 9$	$\|2x + 1\| < 9$	$\|2x - 1\| < 9$

Summary | Lesson 13

The solutions to a two-variable inequality are all of the ordered pairs that make the inequality true. Graphs can help us visualize all of these solutions. To check if an ordered pair is a solution symbolically, you can substitute the x- and y-values into the inequality to see if it makes the inequality true.

Here is an example of how you can determine if an ordered pair is a solution to an inequality:

Marco is making bracelets. Each bracelet needs to cost no more than $\$10$. Planet beads cost $\$1$ and oval beads cost $\$2$. Marco wants to know if he can make a bracelet with 3 planet beads and 4 oval beads.

To check, Marco looked at the graph of this situation to see if the point $(4, 3)$ was in the solution region, but he wasn't sure. Marco substituted 4 and 3 into the inequality $2x + y \leq 10$:

$$2(4) + (3) \leq 10$$
$$8 + (3) \leq 10$$
$$11 \leq 10 \quad \textbf{\textit{False}}!$$

That means Marco *cannot* make a bracelet with 3 planet beads and 4 oval beads while staying within his budget.

Try This

The Theater Club makes $\$5$ for every student ticket that they sell, x, and $\$7$ for every adult ticket, y. They want to make at least $\$180$ to buy costumes for their next show.

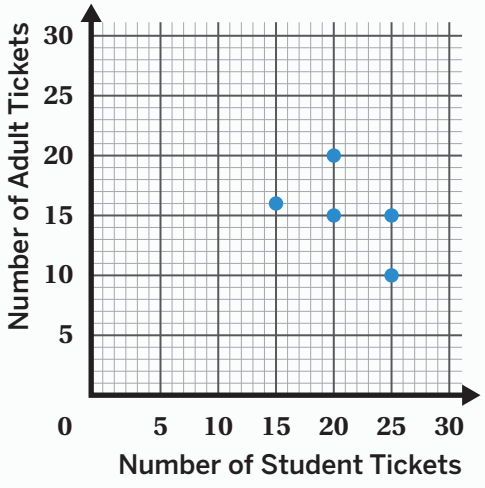
a Which inequality or equation represents this situation?

A. $5x + 7y \leq 180$ **B.** $5x + 7y = 180$ **C.** $5x + 7y \geq 180$ **D.** $7y = 5x + 180$

b This graph shows some solutions to the Theater Club's situation.

Choose *one* solution.

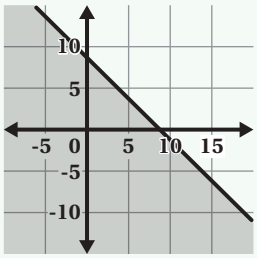
Show that this point is a solution to the inequality or equation that you chose in the previous problem.



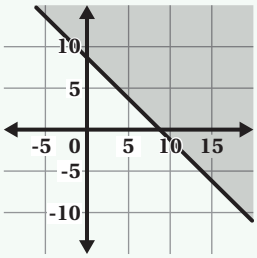
Summary | Lesson 14

The solutions to a two-variable linear inequality can be represented on a graph as a *half-plane*. A **boundary line** separates the plane into the region that contains solutions and the region that does not. The shaded area represents all of the solutions, which are the values of (x, y) that make the inequality true.

$x + y \leq 9$

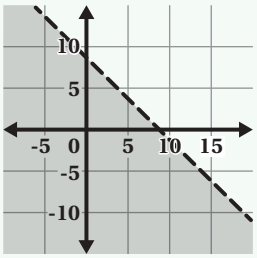


$x + y \geq 9$

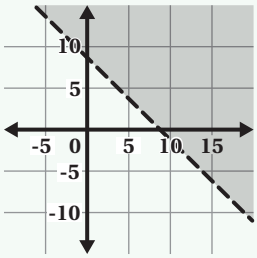


A solid line means that the points on the boundary line *are* included in the solutions. This is represented by the $\leq$ and $\geq$ symbols.

$x + y < 9$



$x + y > 9$

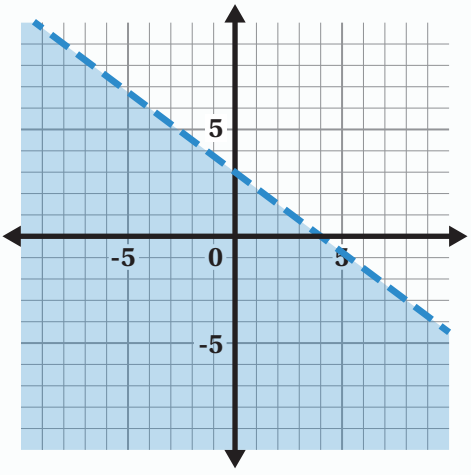


A dashed line means the points on the boundary line are *not* included in the solutions. This is represented by the $<$ and $>$ symbols.

To determine which of the half-planes is the *solution region*, you can test points on either side of the *boundary line* to see whether they make the inequality true or false.

Try This

Here is the graph of an inequality.



a Which inequality does this graph represent?

A. $3x + 4y < 12$

B. $3x + 4y \leq 12$

C. $3x + 4y > 12$

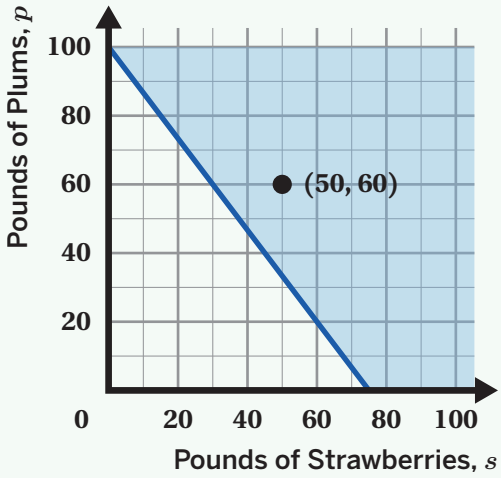
D. $3x + 4y \geq 12$

b How would the inequality change if the boundary line were solid instead of dashed? Explain your thinking.

Looking at solutions to two-variable inequalities on a graph can help us make sense of different situations.

Here is an example: Angel makes \$4 per pound of strawberries and \$3 per pound of plums that they sell. The inequality $4s + 3p \geq 300$ represents the pounds of strawberries, s, and pounds of plums, p, that Angel needs to sell to meet their goal of making at least \$300.

- To determine the solutions to the inequality, graph the corresponding equation
- $4s + 3p = 300$. Decide whether the points on the line will reach the goal by looking at the original inequality symbol.
- Then test a value, such as $(50, 60)$, to identify the solution region. $4(50) + 3(60) \geq 300$ ***True!***



Because the point $(50, 60)$ makes the inequality true, the *half-plane* that includes $(50, 60)$ is the solution region. So any combination of strawberries and plums in the shaded region, including those on the line, would meet Angel's goal.

But not all solutions to the inequality will make sense for the situation. For example, the point $(90, -20)$ makes the inequality true, but it doesn't make sense for Angel to sell -20 pounds of plums.

Try This

A group of students is installing a garden at their school. A vegetable bed will cost \$15 per square foot and a flower bed will cost \$12 per square foot. Their budget for the project is \$300.

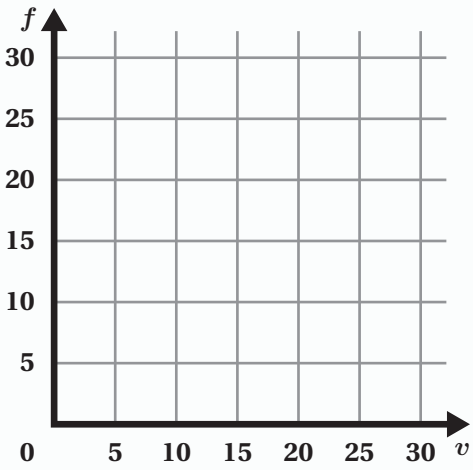
a If $15v + 12f \leq 300$ represents this situation, define v and f.

v represents . . .

f represents . . .

b Graph the corresponding equation: $15v + 12f = 300$.

c Shade in the region that represents the solutions to the inequality $15v + 12f \leq 300$.



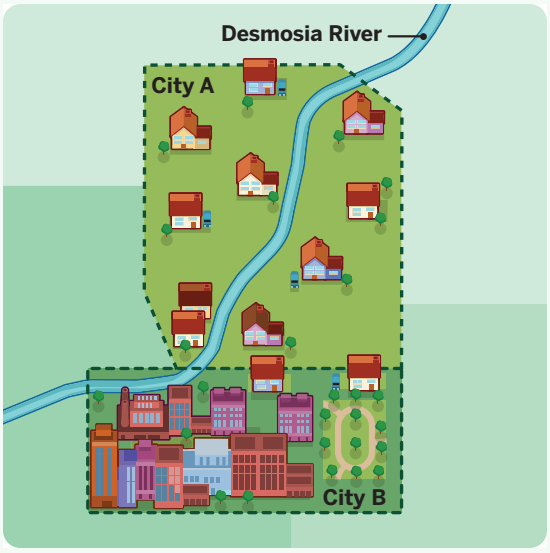
Summary | Lesson 16

We can use two-variable linear inequalities to model real-world constraints and make sense of issues in society, such as fair water distribution.

When modeling, we often make an initial decision based on the information and variables we're given. But we might find additional variables to consider when we look deeper.

In the example of fair water distribution, some additional variables to consider are:

- The population of each city.
- The amount of land in each city.
- The amount of water each city uses.
- Any future predictions for change in the river's flow.



When we consider additional variables, we have the opportunity to revise our initial model to make it better and more precise. Additional variables, along with empathy, personal experiences, and listening to the experiences of others can help us make better models that meet the needs of the diverse groups of individuals who make up a community.

Try This

A theater wants to pay its actors and other workers more. To do this, it needs to make $\$1{,}800$ in ticket sales for each performance. Each adult ticket costs $\$12$ and each child ticket costs $\$8$.

a Why might someone want to create a mathematical model for this situation?

b Write an inequality to represent this situation.

- Use x to represent the number of adult tickets sold.
- Use y to represent the number of child tickets sold.

Lesson 1

a $x = -4$. One strategy for solving the equation is to determine the order of operations to evaluate the expression and then do the inverse operations backwards. For example, since the last step in evaluating $5(x - 2)$ is to multiply by 5, the first step in solving is to divide both sides by 5.

b
$$-30 = 5([-4] - 2)$$
$$-30 = 5(-6)$$
$$-30 = -30$$

Lesson 2

a *Responses vary.*

- Emma tried to multiply by 2 on both sides because $2 \cdot \frac{1}{2} = 1$.
- She added 20 to both sides because $-20 + 20 = 0$.
- She subtracted x from both sides because $x - x = 0$.

b Emma made an error in the first step when she multiplied -10 by 2 but not $3x$.

c $x = 4.8$ (or equivalent)

Here is one way to solve the equation that builds on Emma's strategy.

$$\frac{1}{2}(x + 4) = -10 + 3x$$
$$x + 4 = -20 + 6x$$
$$4 = -20 + 5x$$
$$24 = 5x$$
$$4.8 = x$$

Lesson 3

One Solution	No Solution	Infinitely Many Solutions
$4t = 6t$ $3x = 10 - 3x$	$5t + 7 = -3 + 5t$ $4t + 7 = 2(2t + 2)$	$10 - 2(t + 5) = -2t$

Lesson 4

a The variable p represents the number of hours Tiara works at the pool. The variable t represents the number of hours she works tutoring Spanish.

b B. $t = 20 - \frac{2}{3}p$

One strategy for determining which equation is equivalent is to rewrite the equation in the form $t =$.

$$8p + 12t = 240$$
$$12t = 240 - 8p$$
$$\frac{12t}{12} = \frac{240 - 8p}{12}$$
$$t = 20 - \frac{2}{3}p$$

Lesson 5

a Left: $t = -2$

Right: $t = \frac{10 - v}{-a}$ (or equivalent)

Here is one strategy for solving the equation on the right.

$$10 = v - at$$
$$10 - v = -at$$
$$\frac{10 - v}{-a} = \frac{-at}{-a}$$
$$\frac{10 - v}{-a} = t$$

b Left: $m = -33$

Right: $m = a(h - t)$ (or equivalent)

Here is one strategy for solving the equation on the right.

$$\frac{m}{a} + t = h$$
$$\frac{m}{a} = h - t$$
$$a\left(\frac{m}{a}\right) = a(h - t)$$
$$m = a(h - t)$$

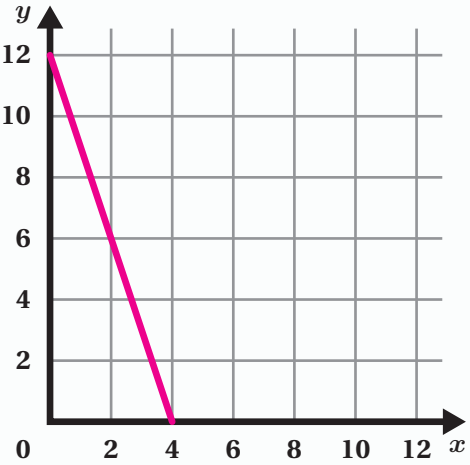
Lesson 6

a $y = 12 - 3x$ (or equivalent)

Here is one way to solve the equation for y.

$$6x + 2y = 24$$
$$2y = 24 - 6x$$
$$\frac{2y}{2} = \frac{24 - 6x}{2}$$
$$y = 12 - 3x$$

b



Here are two strategies for graphing this relationship.

- Use the equation $6x + 2y = 24$ to identify the x-intercept and y-intercept. The x-intercept is the value of x when $y = 0$. The x-intercept is $(4, 0)$ because $6x + 2(0) = 24$ and $6x = 24$. The y-intercept is the value of y when $x = 0$. Using similar reasoning, the y-intercept is $(0, 12)$.
- Use the equation $y = 12 - 3x$ to identify the y-intercept and slope. The y-intercept is $(0, 12)$ and the slope is -3.

Lesson 7

a $20 = 2s + 3p$ (or equivalent)

b $10 - \frac{3}{2}p = s$ (or equivalent)

c *Responses vary.* Solving the equation for s tells you how many stickers Neel can get if he gets no pencils. It also shows the slope of the graph because the equation is in slope-intercept form.

Lesson 8

a *Responses vary.* Valeria wants to donate at least $120. This is a constraint because she can save more than that amount but not less.

b $120 \leq 64 + 8w$ (or equivalent)

Lesson 9

a $x = 9$. One strategy for calculating the boundary point is to solve the corresponding equation, $5 + 3x = 32$.

b

Explanations vary. There is an open circle at $x = 9$ because that is the boundary point but the inequality has a $<$ symbol, which means that the inequality is not true when $x = 9$. If you test $x = 0$, $5 + 3(0) < 32$ is true, so the solutions include $x = 0$ (and all of the numbers in the same direction).

Lesson 10

a $x \leq 3$. One strategy for solving this inequality is to determine the boundary point, then test values. To determine the boundary point, solve the corresponding equation $11 - 2x = 5$. If you test $x = 0$, $11 - 2(0) \geq 5$ is true, so the solutions include $x = 0$ (and all of the numbers in the same direction).

b

Lesson 11

a Responses between 1 and 5 are considered correct.

b

Explanations vary. The equation $|x - 3| \leq 2$ is saying that the distance between x and 3 has to be less than or equal to 2. The numbers that are exactly 2 units away from 3 are 1 and 5. Since the distance has to be less than or equal to 2, any number between 1 and 5 is a solution.

Lesson 12

$\lvert 2x + 1 \rvert > 9$	$\lvert 2x + 1 \rvert < 9$	$\lvert 2x - 1 \rvert < 9$
$x > 4$ or $x < -5$	$x < 4$ and $x > -5$	$x > -4$ and $x < 5$

Lesson 13

a C. $5x + 7y \geq 180$

b *Responses vary*. One solution on the graph is (25, 10).

$5(25) + 7(10) \geq 180$

$125 + 70 \geq 180$

$195 \geq 180$

True!

Lesson 14

a A. $3x + 4y < 12$

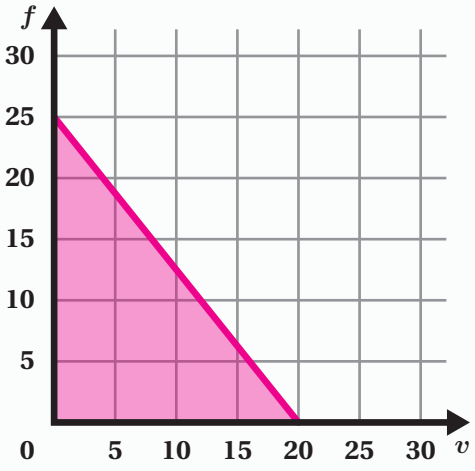
b The equation would become $3x + 4y \leq 12$. *Explanations vary*. The $\leq$ symbol means less than or equal to, which means that the boundary line is included in the solutions to the inequality. The way to show that on a graph is by using a solid line.

Lesson 15

a v represents the number of square feet of vegetable bed the students want to install. f represents the number of square feet of flower bed they want to install.

b Response shown on graph.

c Response shown on graph.



Lesson 16

a *Responses vary*. The theater needs to make at least a certain amount of money (or more), so it might be helpful to have an inequality or graph to see all the combinations of tickets they could sell and still earn enough.

b $12x + 8y \geq 1800$ (or equivalent)

Algebra 1 Unit 2 Glossary/Álgebra 1 Unidad 2 Glosario

English

A

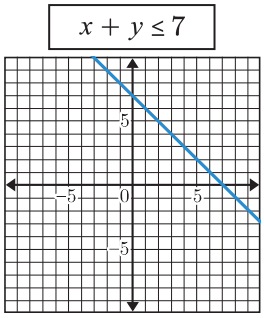
absolute value The absolute value of a number is its distance from 0 on the number line.

For example, the absolute value of -3 is 3 because -3 is 3 units away from 0. This is written as $|-3| = 3$. This means $|-3| = 3$ and $|3| = 3$.

$|x - 2| = 3$ means the value of the expression $x - 2$ is 3 units away from 0, so $x - 2 = 3$ or $x - 2 = -3$.

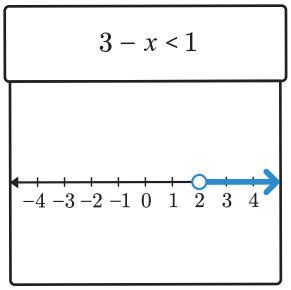
B

boundary line The line that separates the solution region of a linear inequality from non-solutions. A linear inequality (e.g., $y < 2x + 5$) has a boundary line that is represented symbolically by the corresponding equation (e.g., $y = 2x + 5$). A solid boundary line indicates that these points are included in the solution set (e.g., $y \leq x$). A dashed boundary line indicates that they are not (e.g., $y < x$).



The solution region to $x + y \leq 7$ has a boundary line at $x + y = 7$. The line is solid because the points on the line $x + y = 7$ are included in the solution region.

boundary point The value that separates the solution set of an inequality from non-solutions. A solid boundary point indicates that the point is included in the solution set. An empty boundary point indicates the point is not included in the solution set.



The solution set to $3 - x < 1$ has a boundary point at $x = 2$. The point at 2 is empty because 2 is not included in the solution set.

Español

A

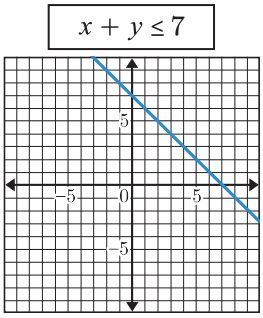
valor absoluto El valor absoluto de un número es su distancia al 0 en la recta numérica.

Por ejemplo, el valor absoluto de -3 es 3 porque -3 está a 3 unidades del 0. Esto se escribe $|-3| = 3$. Significa que $|-3| = 3$ y $|3| = 3$.

$|x - 2| = 3$ quiere decir que el valor de la expresión $x - 2$ está a 3 unidades del 0, por lo tanto, $x - 2 = 3$ o $x - 2 = -3$.

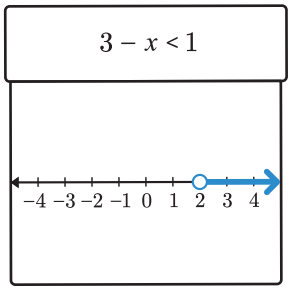
B

recta límite La línea que separa la región solución de una desigualdad lineal de todos los valores que no son soluciones. Una desigualdad lineal (p. ej., $y < 2x + 5$) tiene una recta límite representada simbólicamente por la ecuación correspondiente (p. ej., $y = 2x + 5$). Una recta límite continua indica que esos puntos están incluidos en el conjunto de soluciones (p. ej., $y \leq x$). Una recta límite discontinua indica que no lo están (p. ej., $y < x$).



La región solución de $x + y \leq 7$ tiene una recta límite en $x + y = 7$. La recta es continua porque los puntos en la recta $x + y = 7$ se encuentran dentro de la región solución.

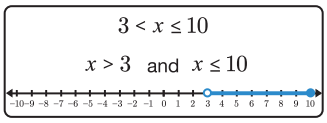
punto límite El valor que separa el conjunto de soluciones de una desigualdad de todos los valores que no son soluciones. Un punto límite sólido (rellenado) indica que el punto está incluido en el conjunto de soluciones. Un punto límite vacío indica que el punto no está incluido en el conjunto de soluciones.



El conjunto de soluciones de $3 - x < 1$ tiene un punto límite en $x = 2$. El punto en 2 no está relleno porque 2 no está incluido en el conjunto de soluciones.

C

compound inequality Two or more inequalities joined together. A compound inequality can be written using symbols or the words "and" or "or."



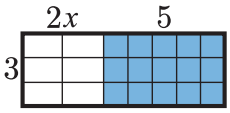
The numbers greater than 3 and less than or equal to 10 can be written as:
$x > 3$ and $x \leq 10$ or $3 < x \leq 10$

constraint A limitation on the possible values of variables in a model. Equations and inequalities are often used to represent constraints.

The constraint that "you must be 36 inches or taller to ride the Ferris wheel" can be represented by the inequality $h \geq 36$.

D

distributive property Multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding them together.



Area as a Product

$3(2x + 5)$

For example, $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

E

equation Two expressions with an equal sign between them.

$-3(2x + 5) = 5 + x$ is an example of an equation.

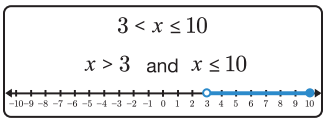
equivalent equations Equations that have the exact same solution(s).

$3x + 4 = 10$ and $9x + 12 = 30$ are equivalent equations because if you multiply the first equation by 3, you create the second. The solution to each equation is $x = 2$.

Español

C

desigualdad compuesta Dos o más desigualdades juntas. Una desigualdad compuesta puede escribirse con símbolos o las palabras "y" u "o".



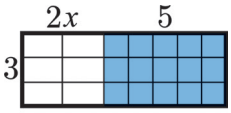
Los números mayores que 3 y menores o iguales que 10 pueden escribirse de la siguiente forma:
$x > 3$ y $x \leq 10$ o $3 < x \leq 10$

restricción Una limitación de los posibles valores de las variables en un modelo. Suelen usarse ecuaciones o desigualdades para representar restricciones.

La restricción "debes medir 36 pulgadas o más para subirte a la rueda de la fortuna" puede representarse con la desigualdad $h \geq 36$.

D

propiedad distributiva Multiplicar un número por la suma de dos o más términos equivale a multiplicar el número por cada término individualmente antes de sumarlos.



El área como un producto

$3(2x + 5)$

Ejemplo: $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

E

ecuación Dos expresiones con un signo igual entre ambas.

$-3(2x + 5) = 5 + x$ es un ejemplo de una ecuación.

ecuaciones equivalentes Ecuaciones que tienen exactamente la misma o las mismas soluciones.

$3x + 4 = 10$ y $9x + 12 = 30$ son ecuaciones equivalentes porque si se multiplica la primera por 3, se forma la segunda. La solución de cada ecuación es $x = 2$.

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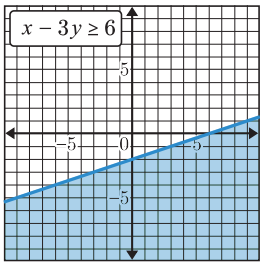
English

H

half-plane A region that defines the solution set of a single two-variable linear inequality. The boundary line of the inequality splits the coordinate plane into two equal half-planes.



The graph shows the solution region to $x - 3y \geq 6$. The boundary line $x - 3y = 6$ divides the coordinate plane into two equal half-planes.

I

infinitely many solutions An equation has infinitely many solutions if it is true for any value of the variable. A system of equations has infinitely many solutions if the equations in the system are equivalent. In a system of equations with infinitely many solutions, every point on the graph is a solution to the system.

For example, the equation $3x + 6 = 3(x + 2)$ has infinitely many solutions because the equation is true for any value of x.

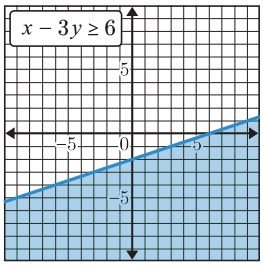
inverse operations Two operations that undo each other are called inverse operations.

Adding and subtracting are inverse operations. Multiplying and dividing are inverse operations.

Español

H

semiplano Una región que define el conjunto de soluciones de una desigualdad lineal con dos variables. La recta límite de la desigualdad divide el plano de coordenadas en dos semiplanos iguales.



La gráfica muestra la región solución de $x - 3y \geq 6$. La recta límite $x - 3y = 6$ divide el plano de coordenadas en dos semiplanos iguales.

I

infinitas soluciones Una ecuación tiene infinitas soluciones si es verdadera para cualquier valor de la variable. Un sistema de ecuaciones tiene infinitas soluciones si las ecuaciones del sistema son equivalentes. En un sistema de ecuaciones con infinitas soluciones, cada punto de la gráfica es una solución del sistema.

Por ejemplo, la ecuación $3x + 6 = 3(x + 2)$ tiene infinitas soluciones porque la ecuación es verdadera con cualquier valor de x.

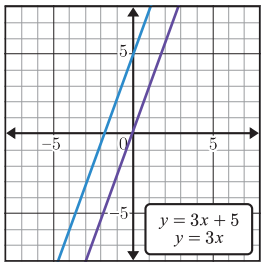
operaciones inversas Dos operaciones que se anulan entre sí se llaman operaciones inversas.

La suma y la resta son operaciones inversas. La multiplicación y la división son operaciones inversas.

N

English

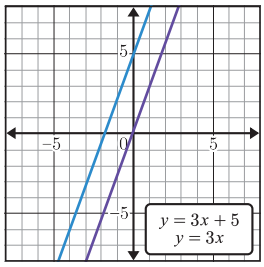
no solution An equation has no solution if there is no value of the variable that will make the equation true. A system of equations has no solution if there is no set of values that makes all the equations in the system true. In a system of equations with no solution, there is no point that is on the graph of every equation in the system.



For example, the system of equations containing $y = 3x + 5$ and $y = 3x$ has no solution because the graphs are parallel and never intersect.

Español

sin solución Una ecuación no tiene solución si no hay ningún valor de la variable que haga que la ecuación sea verdadera. Un sistema de ecuaciones no tiene soluciones si no hay ningún conjunto de valores que haga que todas las ecuaciones de ese sistema sean verdaderas. En un sistema de ecuaciones sin solución, no hay ningún punto que esté en la gráfica de cada una de las ecuaciones del sistema.

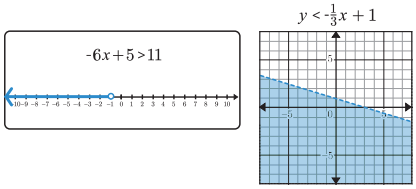


Por ejemplo, el sistema de ecuaciones que contiene $y = 3x + 5$ y $y = 3x$ no tiene soluciones porque las gráficas son paralelas y nunca se intersecan.

R

English

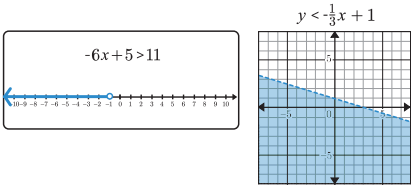
region The set of points on a number line or in a coordinate plane that are solutions to one or more inequalities. Shading is often used to indicate the location of a solution region.



The region to the left of -1 on the number line is the solution region to the inequality $-6x + 5 > 11$. The half-plane below the boundary line $y = -\frac{1}{3}x + 1$ is the solution region for the inequality $y < -\frac{1}{3}x + 1$.

Español

región El conjunto de puntos en una recta numérica o en un plano de coordenadas que son soluciones de una o más desigualdades. Se suele indicar la ubicación de una región solución con colores o sombreado.



La región a la izquierda del -1 en la recta numérica es la región solución de la desigualdad $-6x + 5 > 11$. El semiplano debajo de la recta límite $y = -\frac{1}{3}x + 1$ es la región solución de la desigualdad $y < -\frac{1}{3}x + 1$.

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S

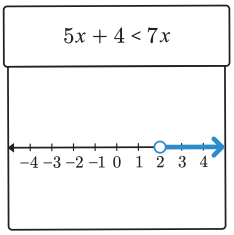
slope-intercept form A way to write a linear equation that highlights the slope and the y-intercept of the line it represents. Slope-intercept form equations are written as $y = mx + b$, where m represents the slope, b represents the y-intercept of the line, and x and y are variables.

The equations $y = 2x + 4$ and $y = -5x - 10$ are in slope-intercept form. The equation $2x + 5y = 20$ is not in slope-intercept form.

solution A value or set of values that makes an equation or inequality true.

For example, $x = 2$ is a solution to the equation $3x + 4 = 10$.
$x > 2$ is the solution to the inequality $3x + 4 > 10$.
The ordered pair $(1, 2)$ is a solution to the equation $3x + 4y = 11$.

solution set (to an inequality) The set of all values that makes an inequality true. To describe a solution set symbolically, we often use inequalities. To describe a solution set graphically, we often shade a portion of a number line or a region of the coordinate plane.



The solution set for the inequality $5x + 4 < 7x$ includes all of the values that are larger than 2. 2 is not included in the solution set.

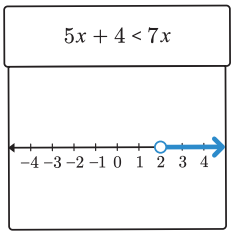
forma pendiente-intersección Una forma de escribir una ecuación lineal que destaca la pendiente y la intersección con el eje y de la recta que representa. Las ecuaciones en forma pendiente-intersección se escriben como $y = mx + b$, donde m representa la pendiente, b representa la intersección con el eje y de la recta, y tanto x como y son variables.

Las ecuaciones $y = 2x + 4$ y $y = -5x - 10$ están en forma pendiente-intersección. La ecuación $2x + 5y = 20$ no está en forma pendiente-intersección.

solución Un valor o conjunto de valores que hacen que una ecuación o una desigualdad sean verdaderas.

Por ejemplo, $x = 2$ es una solución de la ecuación $3x + 4 = 10$.
$x > 2$ es la solución de la desigualdad $3x + 4 > 10$. El par ordenado $(1, 2)$ es una solución de la ecuación $3x + 4y = 11$.

conjunto de soluciones (de una desigualdad) El conjunto de todos los valores que hacen verdadera una desigualdad. Para describir con símbolos un conjunto de soluciones suelen emplearse desigualdades. Para describir con gráficas un conjunto de soluciones suele colorearse o sombrearse una parte de una recta numérica o una región del plano de coordenadas.



El conjunto de soluciones de la desigualdad $5x + 4 \geq 7x$ incluye todos los valores mayores que 2. El 2 no está incluido en el conjunto de soluciones.

English

standard form (of a linear equation) Linear equations that are written in the form $ax + by = c$, where a, b, and c are constants and x and y are variables, are written in standard form.

The equations $2x + 5y = 20$ and $3x - 4y = -10$ are in standard form. The equation $y = 2x + 4$ is not in standard form.

substitute Replace a variable or expression with a value or another expression. $4x = 4(5)$

In this example, 5 is substituted for x in the expression $4x$.

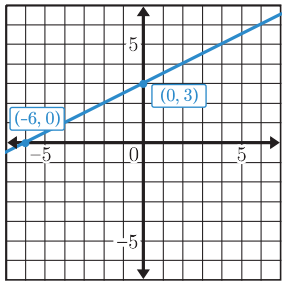
V

variable A letter or symbol that represents a value or set of values.

In the expression $10 - x$, the variable is x.

X

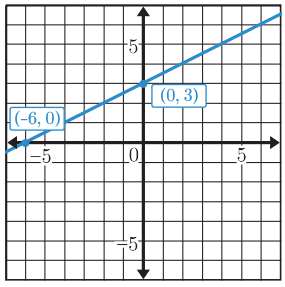
x-intercept A point where the graph of an equation or function crosses the x-axis or when $y = 0$.



The x-intercept of the graph of $-2x + 4y = 12$ is $(-6, 0)$, or just -6.

Y

y-intercept A point where the graph of an equation or function crosses the y-axis or when $x = 0$.



The y-intercept of the graph $-2x + 4y = 12$ is $(0, 3)$, or just 3.

Español

forma estándar (de una ecuación lineal) Las ecuaciones lineales que se escriben en la forma $ax + by = c$, donde a, b y c son constantes, y tanto x como y son variables, se conocen como ecuaciones en forma estándar.

Las ecuaciones $2x + 5y = 20$ y $3x - 4y = -10$ están en forma estándar. La ecuación $y = 2x + 4$ no está en forma estándar.

sustituir Reemplazar una variable o expresión por un valor u otra expresión. $4x = 4(5)$

En este ejemplo, el 5 sustituye a la x en la expresión $4x$.

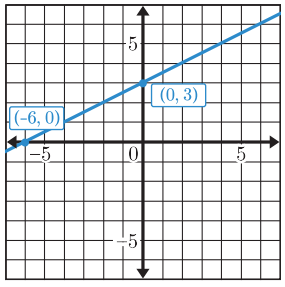
V

variable Una letra o un símbolo que representa un valor o un conjunto de valores.

En la expresión $10 - x$, la variable es x.

X

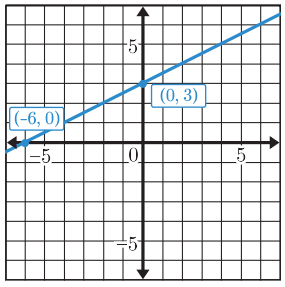
intersección con el eje x Un punto donde la gráfica de una ecuación o función cruza el eje x, o cuando $y = 0$.



La intersección con el eje x de la gráfica de $-2x + 4y = 12$ es $(-6, 0)$, o simplemente -6.

Y

intersección con el eje y Un punto donde la gráfica de una ecuación o función cruza el eje y, o cuando $x = 0$.



La intersección con el eje y de la gráfica de $-2x + 4y = 12$ es $(0, 3)$, o simplemente 3.