AmplifyFractions

Teaching fractions through adaptive storytelling

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Abstract

Amplify Fractions is a new digital supplemental program that specifically addresses fractions, a topic that is important for all students—and challenging for most. The program includes lessons that blend narratives with interactive moments offering personalized feedback, as well as low-stakes adaptive practice. To date, two pilot classrooms have used a beta version of Amplify Fractions. Both classrooms performed significantly better on fixed-form post-tests than on corresponding pre-tests ($p < 10^{-4}$), with shifts between 1 and 2 σ that correspond to improvement indices in excess of 40 points. Such large improvements are consistent with previous studies in which students were exposed to mastery-based learning and/or 1:1 personal tutoring. Amplify Fractions also offers author-facing analytics that are being used to improve the content over time, as more students use the program.

Fractions are essential, but problematic

Fractions represent a critical juncture within the mathematics curriculum. More than any other elementary school-level mathematics or student demographics, fractions predict student achievement in high school-level mathematics.[1] Rational numbers (i.e., numbers that can be written as the quotient of two integers, and the formalization of fractions) are ubiquitous in all math and science beyond elementary school. One can't help but work with fractions when solving linear equations with integer coefficients. In other words, in math the answer is often not a whole number — it's something *between* whole numbers. Fractions are the first numbers students encounter that can quantify what's in between the wholes.

Yet fractions are challenging for most students because they appear to contradict the instincts they have developed around whole numbers. With whole numbers, larger numbers mean larger values. The operations of addition and subtraction are well-established. Multiplication can be defined as repeated addition, and division is the inverse operation. But with fractions, addition and subtraction are among the most complex operations to perform, while the connection between multiplication and addition seemingly breaks down when the number of repeated additions is a fractional amount. Division by fractions presents complexities as well, as students find it simpler to memorize an algorithm ("flip the second fraction") that can be misapplied, rather than understand the meaning of the operation.

Fractions are essential, but problematic. Amplify Fractions was specifically created to address this important challenge in mathematics education.

[1] Siegler RS, Duncan GJ, Davis-Kean PE, Duckworth K, Claessens A, Engel M, Susperreguy MI, and Chen M. "Early Predictors of High School Mathematics Achievement." *Psychological Science*, 23: 691-697 (2012).

Program overview

Amplify Fractions is a supplemental digital program under development by Amplify Education. When completed, it will consist of 49 episodes, each consisting of an interactive lesson and an adaptive practice. These episodes are bundled into three "sets," covering all fractions-related standards (grades 3 through 6): *Intro to Fractions* (15 episodes), *Adding, Subtracting, Equivalence, and Comparison* (17 episodes), and *Multiplying and Dividing* (17 episodes). The first set reviews division, introduces fractions as numbers, defines notation and proper versus improper fractions and mixed numbers, and covers locating fractions on the number line. The remaining two sets cover operations and equivalence and comparison, as described by their respective titles.

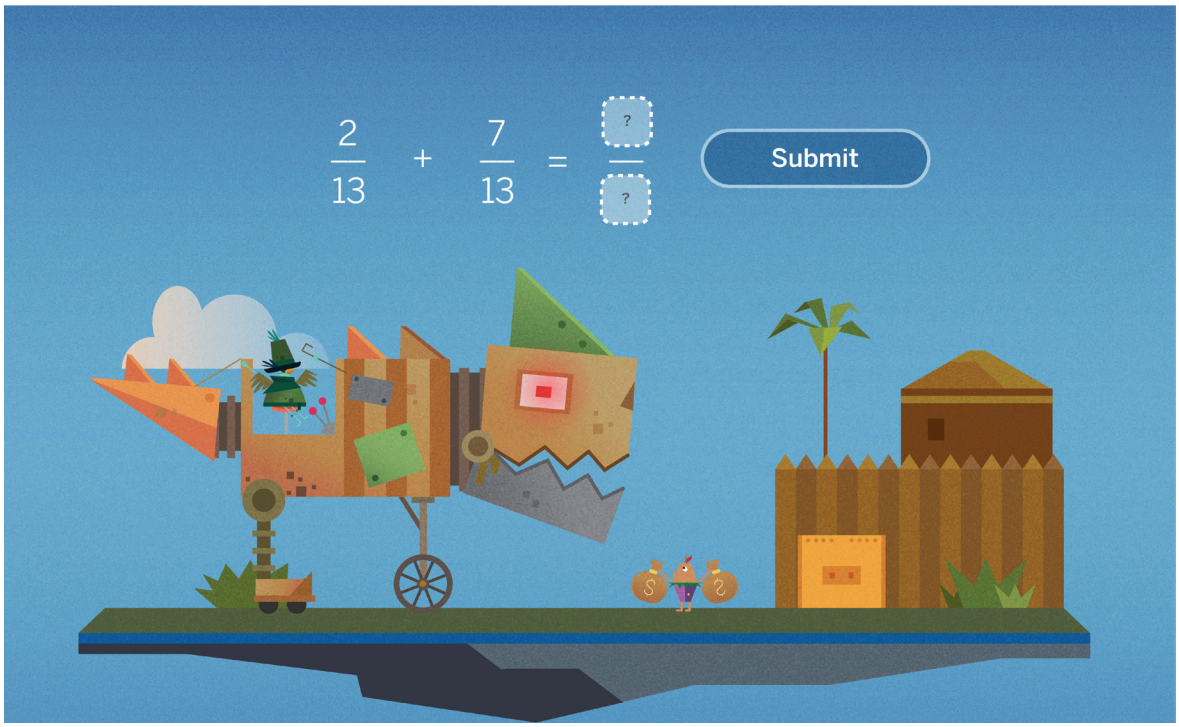

Fig. 1: Lessons in Amplify Fractions contain narratives that are broken up by interactive moments, where students must answer contextual fractions-related questions in order to advance the story.

Lessons in Amplify Fractions blend animated narratives with highly personalized feedback. In each lesson, students encounter one of several storylines (e.g., the Carnivorous Jungle, Bob Da Vinci's Workshop, etc.), and advance the story by answering key fractions-related questions along the way (Fig. 1). The lessons are adaptive in that they provide students with different questions and explanations in an automated way, depending on on the answers they give and the choices they make within the lesson. When students answer incorrectly, they often activate a "digital tutor" who offers audio and visual feedback that addresses a specific misconception. The digital tutor is pre-recorded, and can write on the screen and interact with the same manipulatives the students use (Fig. 2). Over time, as more students use Amplify Fractions, more feedback from the digital tutor is authored to address the answers students provide.

Fig. 2: The digital tutor illustrating that the student counted tick marks, rather than the spaces *between* tick marks, in trying to plot 7/4. The student instead plotted 6/4 using the number line manipulative.

The practice that accompanies each lesson in Amplify Fractions is a low-stakes, adaptive experience (Fig. 3). Students are initially presented with more scaffolded questions that address less sophisticated skills (e.g., recognizing equivalent fractions with area models). As they demonstrate proficiency, the system presents questions that gradually become more conceptual and address more complex skills (e.g., generating equivalent fractions numerically). Throughout each practice, students are presented with a Mastery Bar that reflects their performance (i.e., their Mastery Score). The Mastery Score weights recent items more heavily than previously seen items, and similarly weights skills deemed critical by our content experts. Unlike simpler metrics, such as the percentage of items answered correct, Mastery Score represents how well a student is performing at a given moment, taking into account learning over time and retention. Thus, it is typical for a student to struggle with the first few questions in a practice, "figure it out," and then proceed to attain a high or even perfect Mastery Score -- something not possible with many simpler metrics.

The lesson and practice within each episode cover the same concepts and skills, organized according to Amplify's learning progressions for fractions. The learning progressions provide a rich framework for dynamic student learning, and describe not only learning targets (e.g., standards), but also the multiple pathways students take to reach those targets. These learning progressions also correspond to state standards, and Amplify Fractions as a whole covers all fractions-related standards.

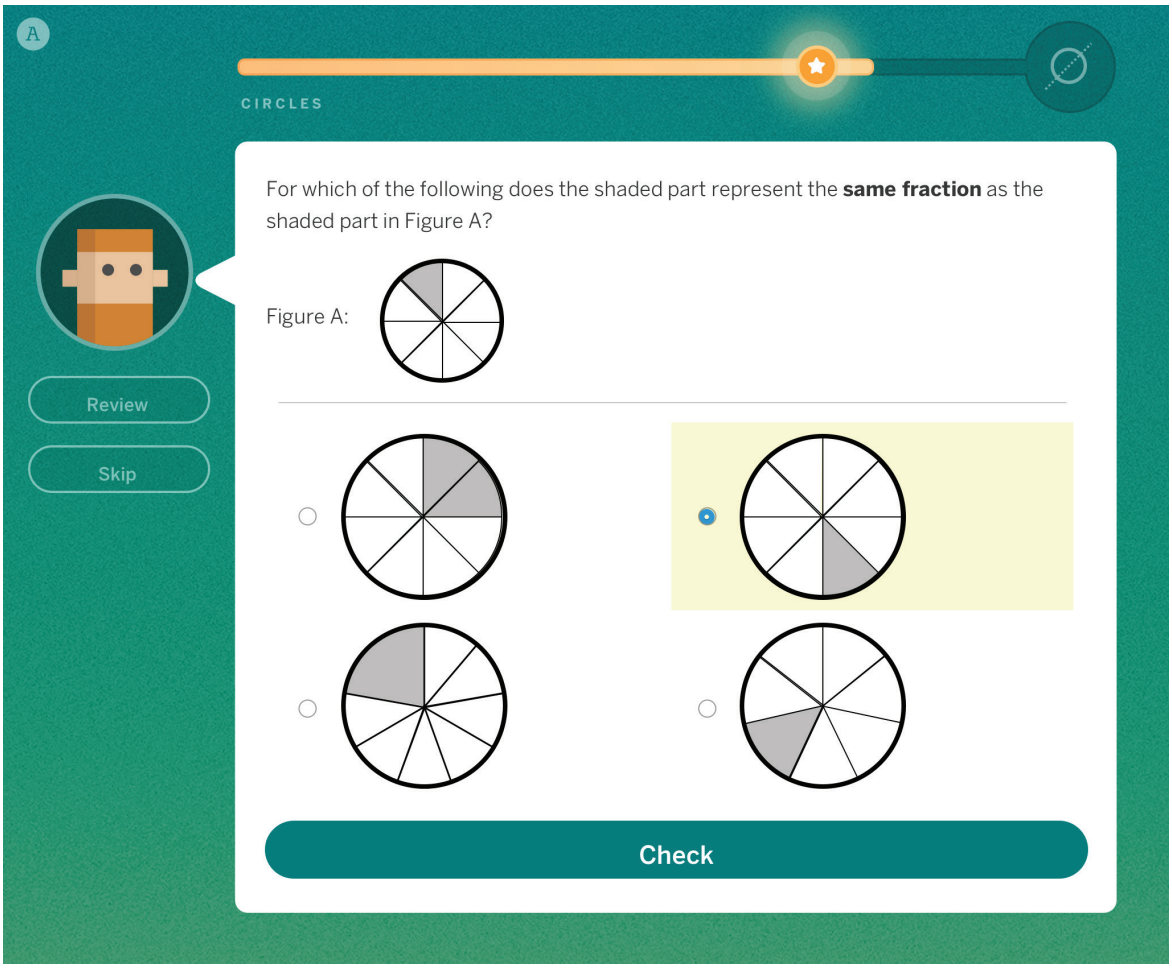

Fig. 3: A practice in Amplify Fractions, with an adaptively selected item and a mastery bar (top).

Pedagogical principles

Amplify Fractions has been designed using foundational and research-based pedagogical strategies. Specifically, Amplify Fractions includes: (1) feedback that is personalized to each student and how they're thinking about the mathematics, (2) story-driven instruction designed to keep students engaged and motivated, and (3) a scope and sequence that completely covers fractions standards, tied to learning progressions of conceptual understanding as well as standards.

Personalized feedback

Bloom and colleagues demonstrated decades ago that (i) 1:1 tutoring, and (ii) mastery-based instruction can dramatically improve student outcomes (by 1 σ each, and 2 σ together).[2] Delivering an authentic tutor-like experience in a digital setting, however, is technologically challenging and has been the focus of much research.[3] Amplify Fractions tackles this problem by offering students the type of feedback that a tutor might provide, directly on the screen. As more students use Amplify Fractions, this feedback will become more comprehensive.

Story-driven instruction

Research has shown that the inclusion of narrative improves learning outcomes, offering students context for the concepts they encounter.[4] In Amplify Fractions, the "big idea" of each lesson is grounded in a story, from fairly dividing a treasure among a jealous group of sharers (equipartitioning), to communicating a distance between mile markers on a map (the number line), to outwitting a thief by choosing the larger fractional quantity (equivalence and comparison). The seamless blend of story and math is intended to make fractions more approachable, and illustrate the necessity of rational numbers.

A focus on fractions

Rather than offering a "mile wide, inch deep" approach to mathematics, Amplify Fractions is laser-focused on fractions, a problem area for many students. Amplify Fractions covers every standard related to fractions, across multiple grade levels. The scope and sequence is constructed around Amplify's learning progressions, which are based on years of pedagogical research and insight into the conceptual basis of how students think about fractions. Both the lesson and practice experiences are explicitly tied to these learning progressions.

[2] Bloom BS. "The 2 Sigma Problem: The Search for Methods of Group Instruction as Effective as One-to-One Tutoring." *Educational Research*, 13: 4-16 (1984).
[3] Ma W, Adesope OO, Nesbit JC, and Liu Q. "Intelligent Tutoring Systems and Learning Outcomes: A Meta-Analysis." *Journal of Educational Psychology*, 106: 901-918 (2014).
[4] McQuiggan SW, Rowe JP, Lee S, and Lester JC. "Story-Based Learning: The Impact of Narrative on Learning Experiences and Outcomes." *International Conference on Intelligent Tutoring Systems*: 530-539 (2008).

Pilots

As the first set of content was completed in the fall of 2017, several pilot studies were initiated at various elementary and middle schools throughout the U.S. A complete pilot consisted of a fixed-form pre-test, followed by in-class or out-of-class usage of the first set of Amplify Fractions (*Intro to Fractions*), and then completed with a different fixed-form post-test and exit survey. Both the pre-test and post-test included 27 questions, with little overlap and distinct from questions encountered within the application. Students and teachers were not informed of student performance on these tests until after the pilots were complete. To date, two such pilots have been completed: one in a 5th grade class in Golden, CO, and the other in a 4th grade in Columbus, GA.

Metz Westminster Public Schools

A total of 29 students participated in the Metz Westminster pilot, with 25 of the students completing both the pre-test and the post-test. The teacher informed us that her class operated via a station (i.e., rotation) model, and she tasked her students with individually working on Amplify Fractions during one of these stations. Students were incentivized to work through the program with small rewards (stickers for their notebooks).

Of the 25 students who completed both the pre-test and the post-test, the mean score (+/- standard deviation) was 13.6 +/- 4.0 on the pre-test, and 19.8 +/- 4.1 on the post-test. The average improvement was 6.2 questions, for an improvement of 46%, or 23 percentage points on the 27-question test. The scores of three students decreased from pre-test to post-test, while the remaining 22 students had score increases (Fig. 4).

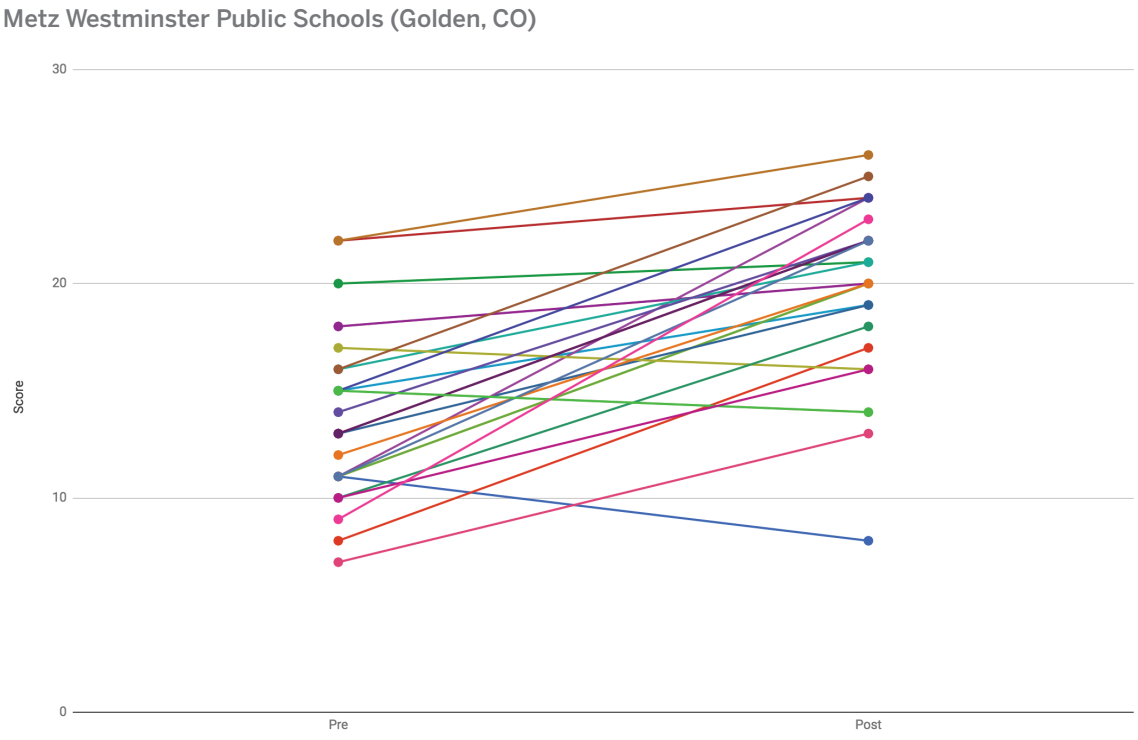

Fig. 4: Individual performance of 25 students on the pre-test and post-test in Metz Westminster Public Schools in Golden, CO. Both assessments had 27 total questions.

A histogram of overall class performance on the pre-test and post-test reveals an approximately normal distribution that shifted to the right (Fig. 5). A one-tailed, paired t-test found that this class' improvement was significant ($p < 10^{-6}$). Moreover, the size of the shift was **1.54 σ**, corresponding to an **improvement index of 44 percentage points** (i.e., a student in the 50th percentile would be expected to perform in the 94th percentile after using Amplify Fractions, relative to a control population). Note that for larger improvements, such as the one observed here, the multiple of σ becomes a more useful measurement, as the improvement index converges to a maximum value of 50.

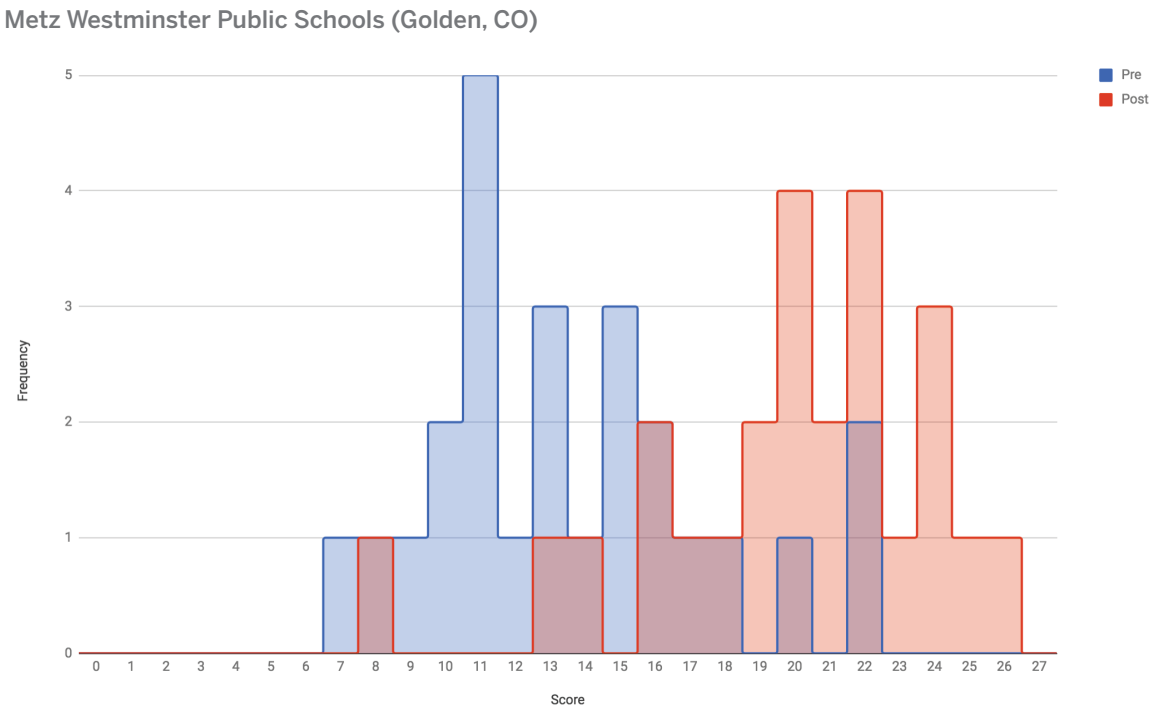

Fig. 5: Histogram showing class performance on the pre-test (blue) and post-test (red) in Metz Westminster Public Schools. Both assessments had 27 total questions.

Double Churches Elementary School

Similar gains were observed in Double Churches Elementary School, where 21 students completed the pre-test and post-test. The mean score (+/- standard deviation) was 10.1 +/- 4.4 on the pre-test, and 16.2 +/- 5.0 on the post-test. The average improvement was 6.1 questions, for an improvement of 60%, or 23 percentage points on the 27-question test. One student's score decreased, while all other students improved from pre-test to post-test (Fig. 6).

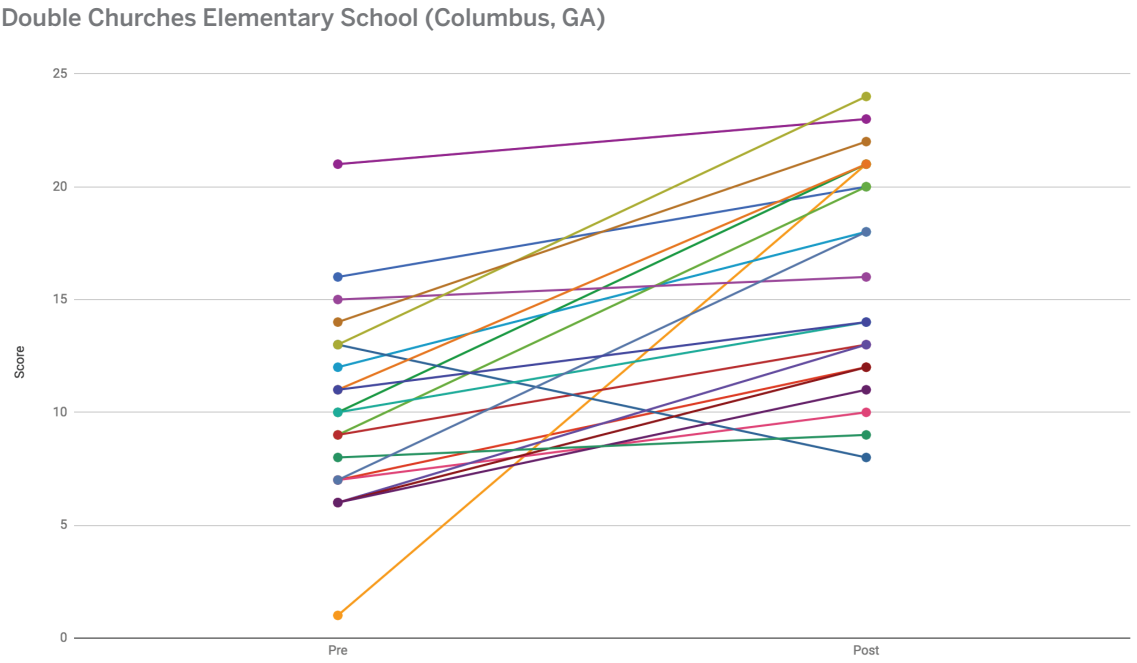

Fig. 6: Individual performance of 21 students on the pre-test and post-test in Double Churches Elementary School in Columbus, GA. Both assessments had 27 total questions.

As with the Metz Westminster pilot, overall class performance here revealed an approximately normal distribution that shifted to the right (Fig. 7). Once again, a one-tailed, paired *t*-test found significant improvement ($p < 10^{-4}$). This time, the size of the shift was **1.39 σ**, corresponding to an **improvement index of 42 percentage points.**

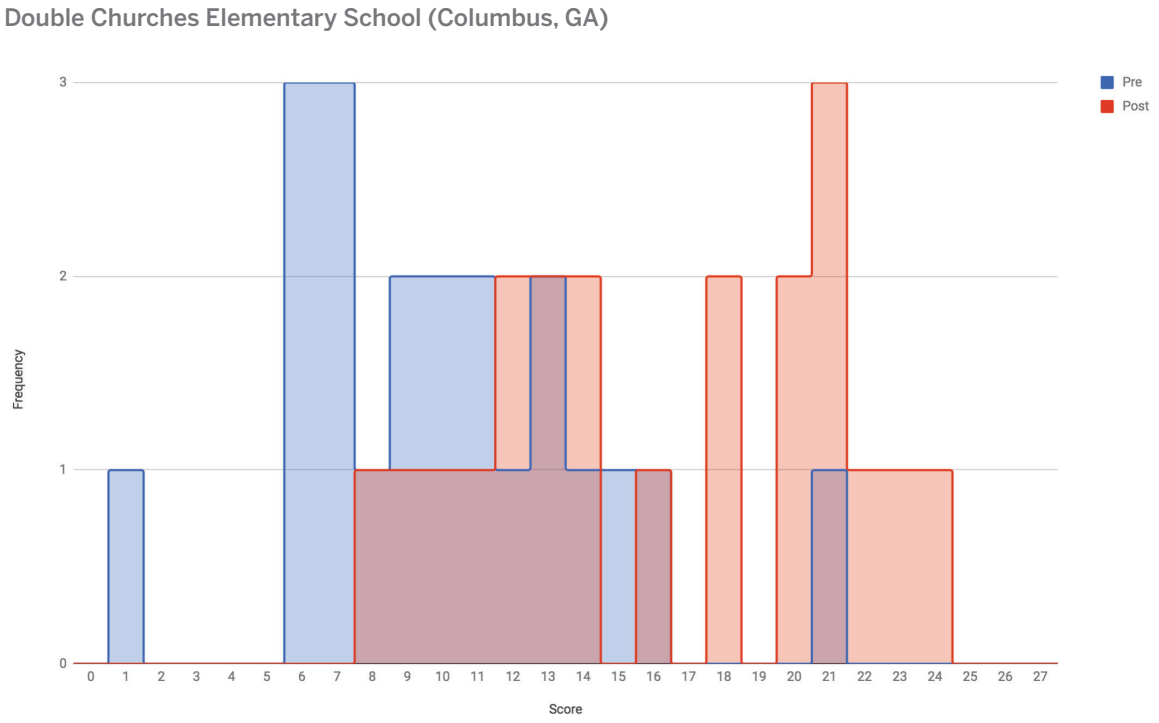

Fig. 7: Histogram showing class performance on the pre-test (blue) and post-test (red) in Double Churches Elementary School. Both assessments had 27 total questions.

Conclusions from pilots

These pilot studies exhibit several limitations: content was limited to the first set of Amplify Fractions (the only fully authored set at the time of testing), implementation was not tightly controlled and left to the discretion of the teacher, and there was no control group for comparative purposes (i.e., testing was longitudinal with single groups of students).

Despite these limitations, the sizable effects seen in both of these pilots suggest that Amplify Fractions has the potential to drastically improve student outcomes in fractions-related mathematics. The ~1.5 σ improvement seen in both of our pilots is consistent with the findings of Bloom and colleagues (1 to 2 σ), suggesting that Amplify Fractions may sufficiently emulates a 1:1 tutor in a digital environment. In the future we hope to test Amplify Fractions against other products in a controlled A/B format or a randomized control trial, to better gauge the true effect.

We will continue to update this white paper as more pilot studies of Amplify Fractions are completed, and as content beyond the first set is made available.

Improving the program

Iterating content

Amplify Fractions is authored in such a way that feedback from the digital tutor can be readily added at any time. Previous work by one of the authors suggests there is a strong correlation, and perhaps causation, between receiving personalized feedback and continuing to use optional digital instruction.[5] Indeed, this is consistent with the work of Bloom and colleagues regarding the value of 1:1 tutoring, an idealized form of personalized instruction.

We therefore will be updating Amplify Fractions at regular intervals so that frequent wrong answers by students are met with meaningful feedback, appearing in real time directly on their screens with audio, handwriting, animation, and manipulative control. While students can give just about any answer to open-ended questions within the lessons, most wrong answers typically represent only a handful of misconceptions. For example, when dividing circles into thirds, common wrong answers include (1) a circle divided into one half and two fourths, (2) two symmetric 3/8 parts with a 1/4 part at the bottom, or (3) three parts that are almost the same size (e.g., 3/10, 3/10, and 2/5). It is unnecessary to provide specific feedback for rare answers, such as 1/10, 1/10 and 4/5 — in such instances, generic feedback is suitable (e.g., "Let's compare the three parts you made. Do they look equal in size to you?").

Based on Amplify's learning progressions, we are able to author lessons with an initial set of feedback from the digital tutors. We have found that these provide feedback to approximately 50% of all student answers. In early September 2017, we reviewed common wrong answers throughout the lessons that had been released, and authored new feedback to address them. In the month of September, the feedback rate then jumped to almost 65% (Fig. 8). Since then, as more content has been released and as more students are using Amplify Fractions, this figure has slowly declined back toward 60%, still well above the initial value of 50%.

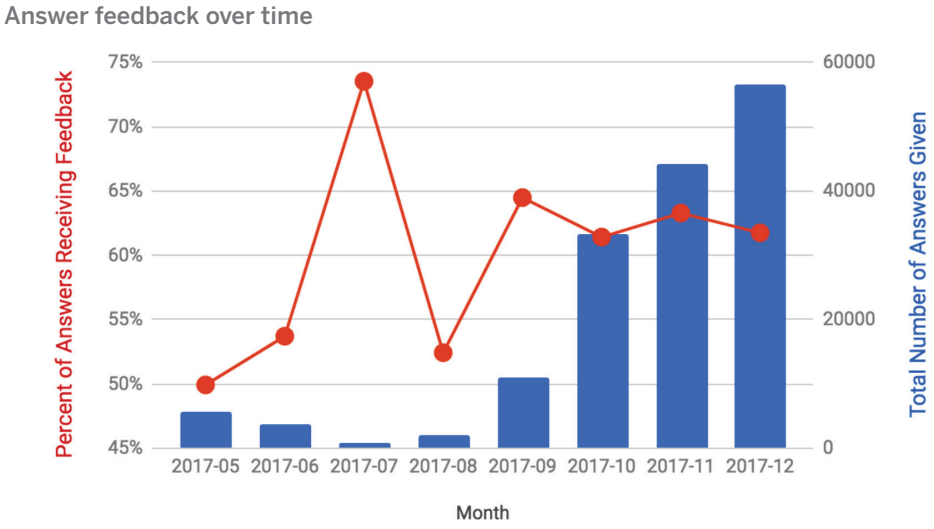

Fig. 8: Monthly traffic (i.e., total number of answers given by students, in blue) and the percentage of those answers that were met with specific feedback (red) in Amplify Fractions, from May 2017 to December 2017.

[5] Wissner-Gross, Z. "How to Make a Course Adaptive Without Really Trying." School Yourself blog post. Available at: http://blog.schoolyourself.org/2015/08/how-to-make-course-adaptive-without.html

We believe that this trend can continue: that over time, the feedback rate will decay as new content is added, and can be "boosted" up to a higher value by reviewing student analytics, authoring more feedback from the digital tutor, and otherwise tweaking lesson content. Once all of the content has been released, we expect the feedback rate to then increase nearly monotonically.

Analyzing mastery trajectories

In addition to pilot studies, student performance within Amplify Fractions can be measured via the practices. Within each practice, questions become increasingly challenging and more conceptual as students advance through the learning progressions. Thus, the trajectory of Mastery Score versus number of items encountered can reveal whether a student is "Proficient," "On Track" toward being Proficient, or "Struggling" within the practice. A student is deemed Proficient upon achieving a mastery score of at least 70. The boundary between On Track and Struggling is determined by recent trends in student performance.

As of January 2018, of the 7361 students who started the first practice within Amplify Fractions (a review of fair sharing and division), 3195 students (43%) were Proficient, 3892 students (53%) were On Track toward proficiency, and 274 students (4%) were Struggling. These three regimes, and the trajectories students followed, are shown in Fig. 9.

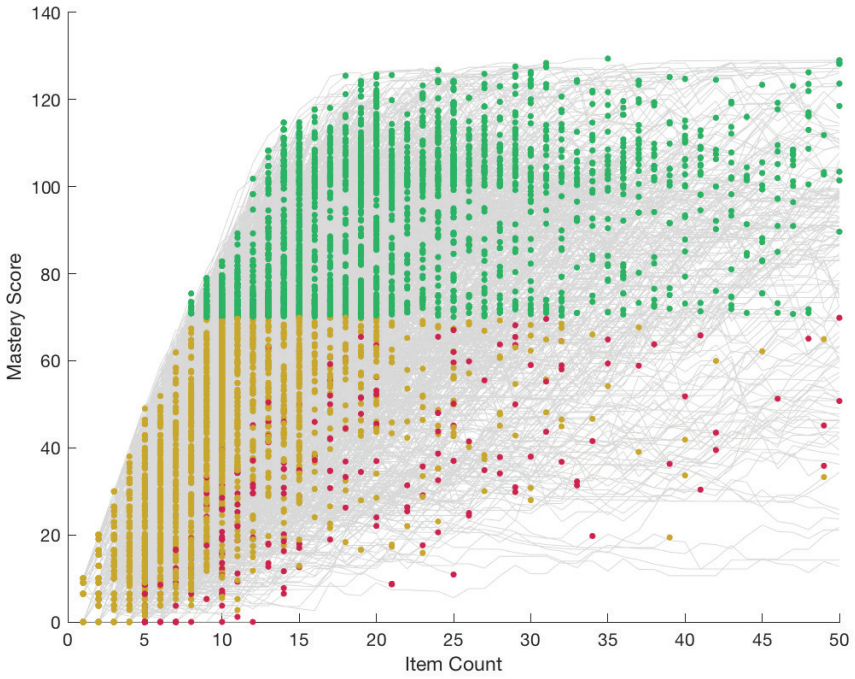

Fig. 9: Practice Mastery Score vs. number of items encountered for 7361 students in Practice 1 of Amplify Fractions (covering division and fair sharing). Proficiency is defined as having a Mastery Score of at least 70%.

[5] Wissner-Gross, Z. "How to Make a Course Adaptive Without Really Trying." School Yourself blog post. Available at: http://blog.schoolyourself.org/2015/08/how-to-make-course-adaptive-without.html

Later on, in the third practice, students encounter equipartitioning of rectangles and notation and naming conventions of fractional parts. Here, students do very well, as shown in Fig. 10. Of the 2649 students who started this practice as of January 2018, 1775 students (67%) were Proficient, 811 students (31%) were On Track toward proficiency, and 63 students (2%) were Struggling.

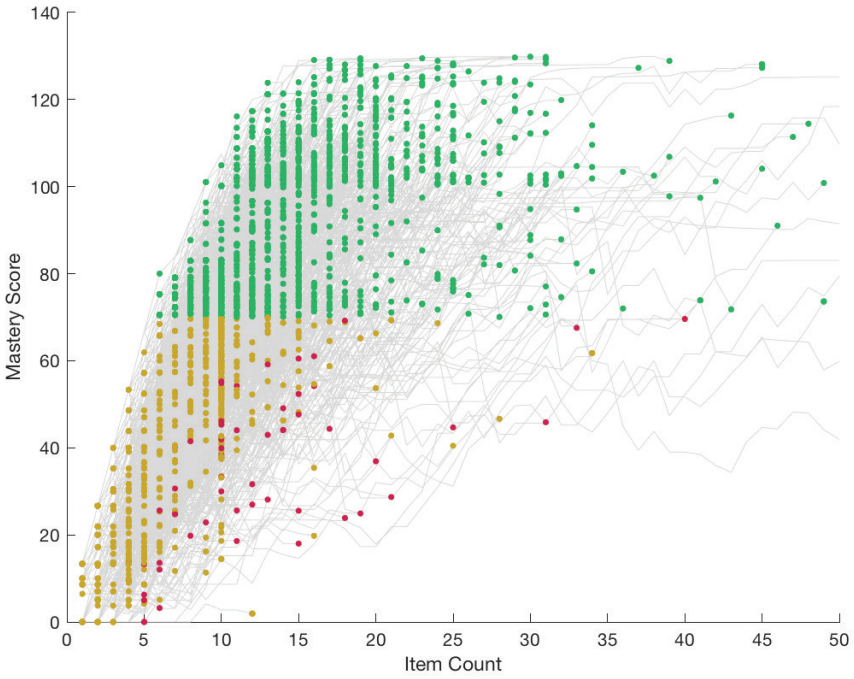

Fig. 10: Practice Mastery Score versus number of items encountered for 2649 students in Practice 3 of Amplify Fractions (covering equipartitioning of rectangles and notation and naming conventions of fractional parts). Proficiency is defined as having a Mastery Score of at least 70%.

Thus far, we have found similar trajectories across all practices within Amplify Fractions, with less than 5% of students in the Struggling regime. As we iterate upon the content and add more personalized feedback, as well as more support within practice itself, we will assess whether there is any effect on the frequency at which students struggle. We will also attempt to correlate performance in practice with performance on external assessments, such as post-tests and standardardized tests.